COMMODITY PRICES AND INFLATION: EVIDENCE FROM SEVEN LARGE INDUSTRIAL COUNTRIES

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ABSTRACT

This paper examines the relationships between movements in primary
commodity prices and changes in inflation in the large industrial countries.
It begins by developing a two-country model in order to examine the theoretical
effects of monetary, fiscal, and supply-side disturbances on commodity and
manufactures prices and on exchange rates. It is shown that if monetary shocks
dominate, then commodity prices should lead general price movements, and the
level of commodity prices should be correlated with the general inflation rate.
Non-monetary shocks generally weaken these relationships, but such disturbances
may cancel out for broad indexes covering a wide range of commodities.
Country-specific commodity price indexes are developed for the major
industrial countries. The weights assigned to different commodities vary
substantially across countries. Nonetheless, when the indexes are expressed in
a common currency, they tend to be highly correlated over time, except when
sharp movements occur in certain commodity prices. The major source of
contrast across countries in the behavior of the indexes derives from exchange
rate movements.
Several empirical tests broadly support the conclusions of the theoretical
model, with relatively few differences across countries. Three main tendencies
may be cited. First, low inflation in industrial countries has tended to be
associated with low levels of commodity prices, and conversely; commodity-price
levels are cointegrated with consumer-price inflation rates. Second, there has
been some tendency for movements in commodity prices to precede changes in
general inflation rates by a few months, although it is not clear whether this
tendency is strong enough to be a reliable aid in forecasting the rate of
inflation. Third, there is a strong and fairly reliable tendency for turning
points in general inflation rates. Commodity prices thus appear to contribute
to predictions of turning points in inflation, predictions of inflation rates
but more strongly to predictions of turning points in inflation.

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I.   Introduction and Summary

Considerable interest has arisen regarding the relationship between commodity prices and broader measures of inflationary pressures.   An earlier paper (Boughton and Branson, 1988) argued that a conventional trade-weighted index of world commodity prices contains useful information about future changes in the average inflation rate in large industrial countries.   This paper extends that research in two directions: (1) by developing a theoretical open-economy model in which countries differ in the role that is played by commodity prices; and (2) by deriving commodity price indexes that are specific to each country and testing the relationships between those indexes and inflation in the countries concerned.

Section II describes the theoretical model and suggests a number of hypotheses to be tested. A conclusion of this section is that commodity price movements should lead general price movements as long as disturbances are predominantly monetary. If commodity prices jump in response to such disturbances, while other prices respond more gradually, then the level of commodity prices should be positively correlated (and cointegrated) with the rate of inflation in manufactures prices. These conclusions should apply more to broad commodity price indexes, on which the effects of supply shocks to specific commodities are relatively weak, than to narrow indexes. Even so, unaccommodated fiscal policies and other general real disturbances could create substantial noise in the relationships between general price inflation and movements in broad commodity price indexes.

The new commodity price indexes are presented and discussed in Section III. The distribution of weights among commodities varies

markedly across countries, but as long as the indexes are denominated in the same currency the variation in their intertemporal movements is limited. The major source of contrast in the behavior of the individual-country indexes derives from exchange rate movements.

Empirical tests of the relationships between the commodity price indexes and general inflation rates are the subject of Sections IV and V. Section IV presents some cointegration tests that suggest the possibility that the level of commodity prices may be cointegrated with the rate of inflation in consumer prices; that is, low inflation in industrial countries appears to be associated with low levels of commodity prices, and conversely. Various single-equation tests of predictive power indicate that there is some tendency for movements in commodity prices to precede consumer price changes and to contribute to inflation predictions; this tendency, however, is not observed in all countries, nor in all time periods. Commodity price indexes have a stronger predictive relationship with turning points in consumer price inflation than with the level of the inflation rate.

Section V discusses evidence regarding the relationship between commodity prices and output deflators. No evidence is found to support the hypothesis that the linkages with output deflators are stronger in countries that have a relatively large share of output attributed to primary commodities. In general, commodity prices contribute to predictions of output deflators with slightly less power than in the case of consumer prices. The paper concludes with a brief discussion of some econometric model simulations that broadly confirm the major predictions of the theoretical model and the single-equation tests.

II. A Two-Country Model of Commodity Prices, Exchange Rates, and Inflation

This section presents a dynamic model of commodity and industrial prices in a framework of two goods and two countries. The goods are commodities and manufactures, with their prices determined in a world market in a common currency, the Group Currency Unit (GCU). 1/ The two countries have a floating exchange rate and different structures of production. The exchange rate of each against the GCU translates the prices of the two goods into the home currencies. The weights of the two goods in production and consumption in each country then determine the movements of the GNP deflator and the CPI, respectively. The model has two levels. The world market (subsection 1) determines the relative goods prices, while a relative monetary model (subsection 2) determines exchange rates. The two together give home-currency prices.

1. World model of commodities and manufactures prices

This subsection discusses a dynamic model of the interaction of commodity and manufactures prices at the level of the world market. Commodity prices are determined in flexible world markets with forward-looking expectations. Manufactures prices are set by sellers and adjust gradually. The model is essentially the same as the one presented in Boughton and Branson (1988), interpreted now as a world model in GCU.

Equilibrium in the world money market is described in the standard form of equation (1):

1/ Conceptually, the GCU is an implicit weighted average of the two currencies. It is defined for the general multi-country case in Boughton and Branson (1988).

(1) $m - \alpha p_m - (1 - \alpha)p_c = \phi y - \lambda i$.

Here m, p_m, p_c, and y are the logarithms of nominal world money, the price of manufactures, the price of commodities, and world real output, all expressed in GCU; i is the average world nominal short-term interest rate; and α is the share of manufactures in world output, and in the world CPI. Commodity price inflation and the interest rate are related by the arbitrage condition:

(2) $i = \dot{p}_c + b$,

where b is the real return to holding commodities for final use, net of storage costs, and $\dot{p}_c$ is the <u>expected</u> rate of change of the commodity price. Substitution of equation (2) into (1) yields the first dynamic equation:

(3) $m - \alpha p_m - (1 - \alpha)p_c = \phi y - \lambda(\dot{p}_c + b)$.

The downward-sloping locus of points where $\dot{p}_c = 0$ is shown in Figure 1; its slope is $-(1 - \alpha)/\alpha$.

For a point above the $\dot{p}_c = 0$ line to be consistent with money market equilibrium, P_c must be expected to rise. This is because above the line, P_c is higher, and real balances are lower, than on it, so the interest rate must exceed b, and P_c must therefore be expected to rise. If expectations exhibit perfect foresight, P_c must actually be rising above the line. These dynamics of the GCU commodity price are summarized by the horizontal arrows in Figure 1.

Figure 1

COMMODITY AND MANUFACTURES PRICES:
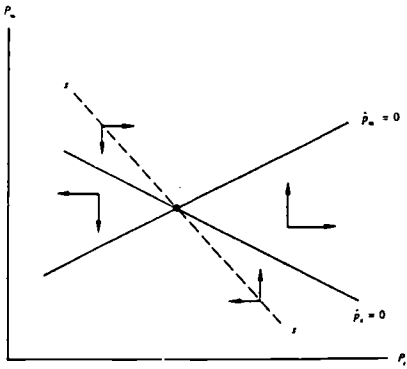WORLD EQUILIBRIUM AND DYNAMICS

Figure 2

WORLD MARKET RESPONSE TO UNANTICIPATED
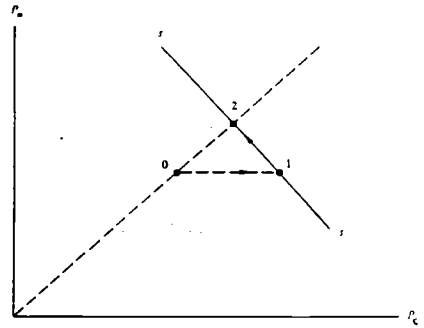MONETARY EXPANSION

Figure 3

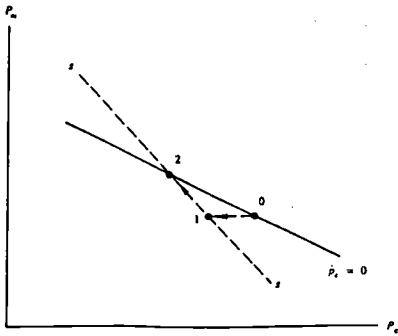WORLD MARKET RESPONSE TO REAL
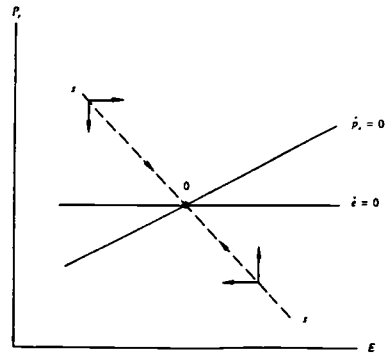DISTURBANCE ALTERING RELATIVE PRICES

Figure 4

EQUILIBRIUM BETWEEN COUNTRIES

The supply of real industrial output y_m is assumed to be constant. Demand is assumed to be an increasing function of the price of commodities relative to manufactures, P_c/P_m, and a decreasing function of the real interest rate in terms of manufactures. Thus demand is given by

(4) $d = \delta(p_c - p_m) - \sigma(i - \dot{p}_m) + g,$

where g is the logarithm of exogenous expenditure, which can represent aggregate world fiscal policy. We assume that a fiscal expansion increases the relative demand for industrial goods.

The price of manufactures is assumed to adjust gradually to eliminate excess demand:

(5) $\dot{p}_m = \pi[\delta(p_c - p_m) - \sigma(i - \dot{p}_m) + g - y_m].$

The terms in p_m can be consolidated to yield the second dynamic equation:

(6) $\dot{p}_m = \eta[\delta(p_c - p_m) - \sigma i + g - y_m],$

where $\eta = \pi/(1 - \pi\sigma)$. This term must be positive if a positive shock to excess demand is to raise the price of manufactures.

The positively sloped $\dot{p}_m = 0$ line in Figure 1 is the locus of points along which excess demand for manufactured goods is zero, given the value of the world money stock, which influences the world nominal interest rate i. The slope is less than unity because an increase in the price level raises the interest rate and reduces the demand for manufactured output. At points above the $\dot{p}_m = 0$ line, there is an excess world supply of manufactured goods, and their CGU price is falling, assuming $\eta > 0$. Below the line, there is excess demand and the price is rising. The

dynamics of adjustment of the GCU manufactured price are summarized by the vertical arrows in Figure 1.

The intersection of the two equilibrium lines in Figure 1 gives the equilibrium pair of GCU world prices for given world money stock, fiscal policy, and real supply conditions. 1/ Dynamic adjustment to the equilibrium proceeds along the stable saddle path ss. This path has two essential properties: it leads to the equilibrium (technically, satisfies the transversality conditions implicit in the model), and along it the expected rate of change of the commodity price is realized. All other paths with expectations realized from period to period explode away from the equilibrium; they are speculative bubbles. The assumption that the market seeks out the stable ss path following a disturbance is the same as assuming that speculative bubbles are unsustainable.

The model of Figure 1 can be used to show the potential usefulness of movements of commodity prices as an indicator of future inflation. Consider an unanticipated increase in the world money supply, originating in either country. In the long run, both prices would increase proportionately in GCU. Thus in Figure 1 both the $\dot{p}_c - 0$ and the $\dot{p}_m - 0$ lines shift out, so that the long-run equilibrium moves out proportionately. This is shown as the movement from point 0 to point 2 in Figure 2, which also shows the new ss path into the new equilibrium. The commodity price can adjust instantaneously to reflect the expected inflation, while the manu-

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1/ There is a difficulty in interpreting the model's equilibrium as a steady-state position, because interest rate data tend not to be stationary. Specifically, while money stocks, GNP, and GNP delators are generally integrated of degree zero, interest rates may be integrated of degree one.

factures price can adjust only over time. Thus in Figure 2, the flexible commodity price jumps onto the new ss path at point 1, and then the two prices adjust gradually along the ss path to point 2. In response to an unanticipated monetary shock, commodity prices jump and overshoot, and lead the subsequent inflation in industrial prices.

In response to repeated monetary shocks, jumps in the level of commodity prices would lead changes in the rate of inflation of industrial prices. This should produce a positive correlation between the level of world GCU commodity prices and the subsequent rate of inflation of manufactures prices if repeated monetary disturbances are empirically important. Cointegration of the level of commodity prices and the rate of inflation in consumer prices is not rejected in the statistical tests in Section IV (Table 4), below.

The effects of an unanticipated real disturbance that changes the equilibrium relative price P_m/P_c are shown in Figure 3. There we assume that the disturbance increases the equilibrium relative price. This could be due to a supply shock that increases commodity supply relative to the supply of manufactures; for example, an innovation in commodity production. It could also result from a shift in demand toward manufactures; for example, a fiscal expansion. This disturbance shifts the $\dot{p}_m = 0$ line up along the $\dot{p}_c = 0$ line to a new long-run equilibrium at point 2, which lies on a steeper ray from the origin that characterizes a higher p_m/p_c ratio. If there is no monetary accommodation to the disturbance, the $\dot{p}_c = 0$ line does not move. The result is that the commodity price jumps down onto the new ss path at point 1, and then continues to fall, while the industrial price rises to the new equilibrium at point 2. The com-

modity price <u>under</u>shoots in response to a real disturbance. It moves

before the industrial price, but in the opposite direction. With no

monetary accommodation, for the P_m/P_c ratio to rise, P_m must increase and

P_c must fall.

It could appear from this analysis that commodity prices would not

be a useful indicator of future inflation in the presence of unaccommo-

dated real disturbances. In fact, in this model, as long as the real

disturbance is not accommodated, there is no general inflation. As one

price rises, the other falls to keep the weighted CPI in GCU constant.

Thus a stochastic series of aggregate real shocks would produce stochastic

behavior of the commodity price with no general inflation. If in empiri-

cal implementation we use an index of a variety of commodity prices, real

supply shocks would come from differing sources, depending on the commod-

ity. To the extent that these are independent, offsetting negative and

positive disturbances at the individual commodity level would minimize

the contribution of supply shocks to the variance of the index. Thus the

path of the index would be dominated by monetary disturbances, with real

disturbances producing noise around that path. The contribution of real

supply shocks to the variance of the index would be minimized to the

extent that they are uncorrelated. This would improve the usefulness of

the index as an inflation indicator.

At this point we can summarize the results from the world model of

commodity and manufactures prices in GCU. Monetary disturbances lead to

jumps in the commodity price index, followed by gradual movement in the

manufactures price in the same direction. The long run movement of both

prices is proportional to the monetary disturbance. Thus a series of

positive monetary shocks would produce a series of jumps in the level of commodity prices and smoother inflation in manufactures prices, with a common trend to both. Real disturbances yield jumps in the commodity price, followed by gradual movement of the manufactures price in the opposite direction. If the real disturbances are not accommodated by monetary policy, they have no long-run effect on the CPI, but they produce noise in the commodity price series. If the real disturbances have in them a trend that raises the relative price of manufactures, they will generate noise around that trend.

2. Relative model of prices and exchange rates

. The world model gives us the movements of commodity and manufactures prices in GCU, as functions of world monetary and real disturbances. To translate these into domestic price indexes in individual countries, we can use a model of the exchange rate and relative price levels between two countries. This will give us the relative effects of disturbances, depending on the country of their source. The relative effects can then be combined with the world effects in GCU to obtain the results for CPIs and GDP deflators in home currencies. The relative model is an application of Aoki's (1981) average and difference method to the Dornbusch (1976) model of prices and the exchange rate.

We begin by defining relative variables between two "representative" countries as the differences between the logarithms of the variables for the domestic GNP deflators, money stocks, GNPs, and fiscal variables; and the arithmetic differences for interest rates, with each relative variable being the standard mnenonic with a subscript "r". Thus for relative

prices, we have

$$p_r = \ln P - \ln P^*,$$

where P and P^* are the home and foreign GNP deflators, respectively. Relative money stocks (m_r), real GNP (y_r), and fiscal variables (g_r) are similarly defined. Each country's GNP will be a combination of commodity, manufactures, and services output, and its GNP deflator will be the corresponding weighted average. The interest differential, equal to the expected rate of change of the nominal exchange rate, E, is

$$i_r = i - i^* = \dot{e}.$$

In equilibrium with $\dot{e} = 0$, i_r would also be 0. 1/

The model of relative prices and the **exchange rate is** specified similarly to the model of manufactures and commodity prices described above in subsection 1. Here the exchange rate is the forward-looking variable that jumps in response to unanticipated disturbances, and the relative GNP deflator adjusts slowly to relative excess demand. The relative equilibrium condition for the money market is

$$(7) \quad m_r - p_r = \phi y_r - \lambda i_r = \phi y_r - \lambda \dot{e},$$

using the uncovered interest parity condition to substitute $\dot{e}$ for i_r.

1/ A risk premium related to relative stocks of outstanding assets could be added to the interest differential. While that extension might provide additional insights, especially regarding the longer-run effects of fiscal policies, it would complicate the analysis without modifying the qualitative conclusions regarding the model's dynamics.

We can set $\dot{e} = 0$ (or $i_r = 0$) in equation (7) to obtain the $\dot{e} = 0$ line in Figure 4, which shows the relative price p_r that would clear the money market for given values of m_r and y_r. In contrast to the $\dot{p}_c = 0$ line in Figure 1, the $\dot{e} = 0$ line here is horizontal. For a point above the $\dot{e} = 0$ line to be consistent with money-market equilibrium, E must be rising. This is because p_r is higher than on the line, so i_r must be positive above the line. This would mean that E would be expected to rise (so $\dot{e}$ is positive), and on the assumption of perfect foresight, E would actually rise. This reasoning is exactly the same as that behind the p_c dynamics in Figure 1.

Relative demand, d_r, for the two real GNPs is specified as

(8) $\quad d_r = \delta(e - p_r) - \sigma(i_r - \dot{p}_r) + g_r.$

As in the earlier model, the relative GNP deflator adjusts gradually to excess demand, so that

(9) $\quad \dot{p}_r = \pi(d_r - y_r).$

Substituting (8) into (9) and solving for $\dot{p}_r$ yields the second dynamic equation,

(10) $\quad \dot{p}_r = \eta[\delta(e - p_r) - \sigma i_r + g_r - y_r],$

where $\eta = \pi/(1 - \pi\sigma)$, which is required to be positive. Setting $\dot{p}_r = 0$ in equation (10) yields the positively sloped $\dot{p}_r = 0$ line in Figure 4, for relative GNP deflator equilibrium. As long as η is positive, p_r adjusts toward the $\dot{p}_r = 0$ line.

The two equilibrium lines in Figure 4 show the equilibrium relative price, measured in the ratio of the two currencies, and the equilibrium exchange rate at point 0 for given relative money stocks, fiscal positions, and supply conditions. Dynamic adjustment to equilibrium proceeds along the saddle path ss, as in the earlier model of Figure 1. The only analytical difference between Figure 1 and Figure 4 is the slope of the curve representing monetary equilibrium. In both models, the saddle path is negatively sloped, and the flexible price jumps horizontally onto the saddle path following an unanticipated disturbance, while the sticky price adjusts gradually along the saddle path.

The effects of a strictly relative monetary disturbance (an increase in the money supply in the home country and a decrease of the same percent in the foreign country) are shown in Figure 5. There is no change at the world level of subsection 1, since there is no effect on the world money stock. But in the relative model, the long run equilibrium moves out proportionately to point 2 in Figure 5, with the exchange rate and relative price both increasing in proportion to the change in the relative money supply. The home price level rises and the foreign price level declines. The home currency depreciates against the GCU and the foreign currency appreciates. In the short run, the exchange rate between the two countries jumps to point 1 on the new ss path, and then the exchange rate falls and the relative price rises to point 2. This is the basic result from Dornbusch (1976) for two countries.

The effects of a strictly relative fiscal disturbance, an increase in g_r in equation (10), are shown in Figure 6. We assume that the home government increases, and the other government decreases, expenditure on

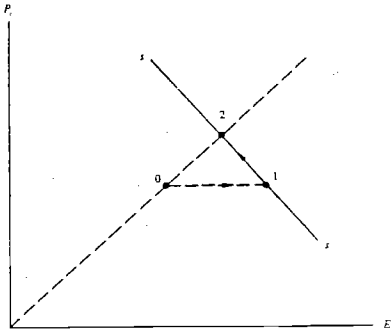RELATIVE EFFECTS OF MONETARY EXPANSION

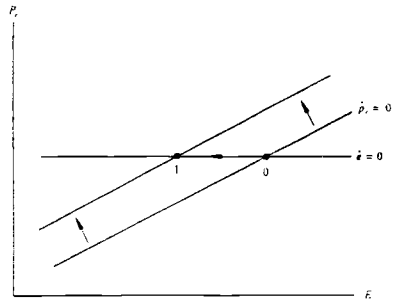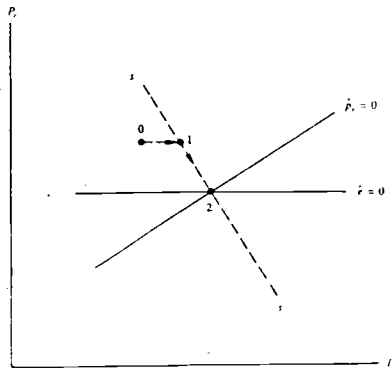Figure 6
RELATIVE EFFECTS OF FISCAL DISTURBANCE

Figure 7
RELATIVE EFFECTS OF SUPPLY DISTURBANCE

manufactures by the same amount. There is no change in the world market in manufactures or commodities, but in the relative model, the $\dot{p}_r = 0$ line in Figure 6 shifts up. The equilibrium relative price level of the home country, where the fiscal expansion took place, increases for a given level of the exchange rate. There is no shift of the $\dot{e} = 0$ line, since the fiscal variable does not enter the money-market equilibrium condition, equation (7). As a result, the exchange rate falls in a jump from point 0 to point 1 in Figure 6. The home currency appreciates by enough to offset the effect of the fiscal shift on relative demand, with no effect on either price level in home currency. The nominal appreciation of the home currency produces a real appreciation of the same amount. This result is implicit in Dornbusch (1976), and explicit in Mundell (1963).

3.   The two models combined

The model of the world markets in commodities and manufactures in GCU and the relative model of exchange rates and relative prices in home currencies can now be combined to analyze the effects of the various types of unanticipated disturbances originating in one country. We will consider, in turn, monetary, fiscal, and supply shocks.

Monetary Expansion in One Country

First we illustrate the effects of an unanticipated monetary expan-
sion in one country. At the level of the world model of commodity and
manufactures prices, the results are illustrated in Figure 2. Measured in
GCU, the commodity price jumps, and then the manufactures price rises
gradually, as the commodity price falls. The jump in the level of the
commodity price leads the gradual increase in the manufactures price.

Eventually both prices rise proportionately to the increase in the money stock, all measured in GCU.

The relative effect is shown in Figure 5. The exchange rate of the expanding country against the second country jumps from point 0 to point 1; that is, the expanding country's currency depreciates. Subsequently, that country's price level in home currency rises and its exchange rate falls, converging to point 2. In the new long-run equilibrium, the exchange rate and the price level in the expanding country rise proportionately to the monetary expansion.

Fiscal Expansion in One Country

The interesting case of an unanticipated fiscal expansion is one that alters world relative demand for manufactures and commodities, as well as the real exchange rate as shown in Figure 6. To use a specific example, consider a fiscal expansion in the manufacturing country that increases the world relative demand for manufactures. As a convenient simplification, think of this country as Japan, and the other country, specializing in commodities production, as Italy. 1/ The effects at the world level in GCU are shown in Figure 3. The commodity price jumps down, and then declines gradually further as the manufactures price rises. At the new equilibrium point 2, P_m has risen and P_c has fallen, in GCU.

The relative effects are shown in Figure 6. With no direct representation of the fiscal variable in the financial equilibrium equation (7), the $\dot{e} = 0$ line does not move. The fiscal expansion in Japan shifts the

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1/ Table 9 below shows that among the large industrial countries, Japan is the least intensive in production of primary commodities, and Italy the most.

$\overset{\bullet}{p}_r = 0$ line left, moving the equilibrium point from 0 to 1 in a jump. The home real and nominal exchange rates appreciate, with no change in relative GDP deflators, in home currency. The appreciation of the yen just offsets the increase in P_m in GCU, and the depreciation of the lira just offsets the fall in P_c in GCU.

The effects of the fiscal expansion in the manufacturing country on prices in all three currencies are summarized in panel (a) of Table 1. P_m is up and P_c is down, in GCU. P_m in lira rises both because of the increase in GCU and because of the depreciation of the lira against the GCU, while in yen P_m is unchanged. P_c is unchanged in lira, while it falls in yen both from the fall in GCU and from the yen appreciation. Thus if both countries consume both goods, the CPI will fall in Japan and rise in Italy, reflecting the terms-of-trade effect, with both GDP deflators unchanged.

Supply Expansion in One Country

A supply shock also alters the world relative prices, as in Figure 3, and the equilibrium real exchange rate. The analysis is slightly more complicated than that of a fiscal expansion, though, since both equilibrium lines in Figure 6 shift with a supply shock. Consider an increase in productivity in the commodity-producing country (Italy) that increases the world relative supply of commodities. The effects at the level of the world market in GCU are again shown in Figure 3. The commodity price jumps down from point 0 to point 1 when the supply increase becomes known to the market. Then P_c falls gradually as P_m rises to point 2. In GCU the price of commodities falls and the price of manufactures rises.

Table 1. Theoretical Effects on Prices from Various Disturbances 1/

(a)				(b)			
Responses to a Fiscal Expansion in the Manufactures-Producing Country (Japan)				Responses to a Supply Increase in the Commodities-Producing Country (Italy)			

	Measured in:				Measured in:		
Effect on	GCU	Lira	Yen	Effect on	GCU	Lira	Yen
P_m	+	+ +	0	P_m	+	+ +	+
P_c	-	0	- -	P_c	-	-	- -

(c)

Summary of Effects of Disturbances on Home-Currency Prices

	Effects on Prices in:					
	Originating Country			Other Countries		
Shock	P_c	P_m	CPI	P_c	P_m	CPI
Increase in Money Stock in Either Country	+	+	+	0	0	0
Fiscal Expansion in Manufactures Producer	-	0	-	0	+	+
Supply Increase in Commodities Producer	-	+	?	-	+	?

1/ Price increases are denoted by + and decreases by -; 0 indicates no change. The symbols + + and - - indicate relatively large changes.

The relative effects are shown in Figure 7, with Italy as the home country. The productivity shock increases y_r in both equations (7) and (10), shifting both equilibrium lines down. The $\dot{p}_r = 0$ line shifts down by a factor of $-1/\delta$ from equation (10), and the $\dot{e} = 0$ line shifts down by $-\phi$ from equation (7). If the shift in the $\dot{p}_r = 0$ line is sufficiently larger than the shift in the $\dot{e} = 0$ line, the new saddle path will pass to the right of the original equilibrium point 0, as shown in Figure 7. This would be the case if the supply shock's effects were felt mainly in the goods markets, compared with the effects in the money market via an increase in income. In this case, the exchange rate of Italy would jump up to point 1, and then rise further as P_r falls to point 2. That is, the lira would depreciate in both nominal and real terms against the GCU.

One cannot rule out the possibility that the downward shift in the $\dot{e} = 0$ line would be large enough that the new saddle path would pass to the left of point 0. In that case the exchange rate could jump down, and then rise as P_r fell to the new equilibrium. Equation (10) shows that in the new equilibrium the real exchange rate $(e - p_r)$ must rise with an increase in y_r by $d(e - p_r)/dy_r = 1/\delta$. We know, therefore, that in Figure 7, along the new equilibrium line $\dot{p}_r = 0$ the lira has depreciated in real terms, regardless of the direction of its original jump. So the result from the relative model is that the real exchange rate of the country with the positive supply shock rises (home currency depreciates) relative to the GCU, and the other country's real exchange rate falls.

The results for the positive supply shock in the commodity-producing country are summarized in panel (b) of Table 1. The price of commodities falls and the price of manufactures rises in GCU. With the lira

depreciating in real terms, the lira price of manufactures rises even more, and the lira price of commodities falls less than in GCU. As the yen appreciates in real terms, the yen price of commodities falls even more, and the yen price of manufactures rises less, than in GCU. With the two prices moving in opposite directions in both currencies, the effects on the two CPIs are ambiguous. However, since the movement in the real exchange rate adds to the downward movement in commodity prices in Japan, and to the upward movement of manufactures prices in Italy, it seems likely the the CPI would fall in Japan and rise in Italy.

Summary

The effects of the unanticipated disturbances in single countries are summarized in panel (c) of Table 1. The effects of an unanticipated expansion in the money supply are familiar. In the home country, prices rise proportionately to the expansion. The currency of the foreign country appreciates, and local currency prices in the foreign country are unchanged. In this case the commodity price jumps and leads a general inflation in the home country.

The effects of a fiscal expansion or a supply shock are less familiar. A fiscal expansion in the manufacturing country that raises world relative demand for manufactures raises the GCU price of manufactures and reduces the GCU price of commodities. It also appreciates the currency of the expanding country and depreciates the currencies of the others, in nominal and real terms. The result is a negative effect on the CPI in the expanding country and a positive effect on the others, owing to the movement of the exchange rate. This is similar to the effect of the fiscal expansion in the United States on the U.S. inflation rate via the

dollar exchange rate in the early 1980s. In this case the downward jump in the commodity price following the disturbance contains no general signal for inflation unless it includes a general shift in monetary policy. If unaccommodated fiscal disturbances are randomly distributed across countries and over time, they will create noise in the relation between commodity price movements and subsequent inflation.

Finally, a positive supply shock lowers the price of the good whose supply has increased and depreciates the currency of the suppliers in real terms. The net result is to reduce the price of the good in question, and increase the prices of the others, in all currencies. The effects on home-currency CPIs depend on the weights of the goods in the CPI. In this case the initial downward jump in the GCU commodity price also contains no signal for subsequent inflation. By using an index over a large number of commodities, individual supply shocks such as these are averaged out to some degree in the empirical work that follows.

III. Country-Specific Commodity Price Indexes

The empirical tests developed in Sections IV and V of this paper evaluate the relationships between general movements in world prices of primary commodities and inflation in the large industrial countries. In order to isolate the effects of global developments on individual countries, it is necessary to develop separate commodity-price indexes for each country. This task has two dimensions: determining the weights that each commodity should have in a country's index, and denominating the index in that country's currency. The first dimension is at least potentially important in order to take account of variation between countries in the shares of key commodities in consumption patterns; the second

ensures that the commodity-price and general-price indexes are denominated in the same currency units.

The commodity-price indexes have been constructed using spot prices for major commodities and weights that reflect the pattern of consumption of these commodities in each country. A large number of price quotations are included (thirty-one primary commodities represented by thirty-six international price quotations), in order to minimize the effects of price fluctuations caused by supply shocks or other real disturbances in individual commodity markets. Weights are based on consumption, rather than production or imports, since these relate most directly to movements in the consumer price index.

Table 2 provides a comparison of patterns of consumption of primary commodities in the seven major industrial countries. 1/ These data indicate that the portion of total private consumption attributed to the consumption of primary commodities 2/ is quite uniform among countries, at about 8 percent (second row of Table 2). Consumption of individual commodities, however, varies substantially from country to country. For example, while the consumption of grains is relatively constant as a share of primary consumption for most countries (about 14 percent except for Germany), the composition of grain consumption is more diversified. Weights representing the consumption of wheat in France, Italy, and the United Kingdom (each 10 percent); maize in the United States (11 percent);

1/ Data sources and methodology are described in an appendix, available on request.
2/ This statement refers to the ratio of the value of consumption of the 31 selected primary commodities to the value of total private consumption.

Table 2. Consumption of Primary Commodities in Major
Industrial Countries, 1983-85

Commodity	Canada	France	Germany	Italy	Japan	United Kingdom	United States	G-7
Total, All Primary Commodities								
In millions of U.S. dollars (yearly average)	17,515	27,749	36,225	24,545	55,566	22,284	183,172	367,055
In percent of total consumption	9.0	8.9	9.7	9.4	7.5	8.1	7.5	8.0
Weights (in percent)								
Cereals	10.5	14.0	6.5	16.3	15.5	11.3	15.4	14.0
Wheat	4.2	9.8	5.3	9.6	2.2	10.1	3.3	4.7
Maize	6.2	4.0	1.1	5.1	4.1	1.1	11.1	7.3
Rice	0.1	0.2	0.1	1.7	9.2	0.0	1.0	2.0
Vegetable oils	3.1	4.3	4.4	4.1	3.6	3.0	5.7	4.8
Soybeans	0.8	0.3	0.8	0.7	1.3	0.3	1.9	1.4
Soybean meal	1.5	3.0	2.1	2.2	1.2	1.5	2.0	1.9
Soybean oil	0.6	0.2	0.8	0.7	0.8	0.6	1.5	1.1
Palm oil	0.1	0.1	0.2	0.2	0.2	0.5	0.1	0.1
Coconut oil	0.1	0.2	0.3	0.1	0.1	0.1	0.2	0.2
Groundnut oil	0.0	0.5	0.1	0.1	0.0	0.0	0.0	0.1
Meat	14.5	18.6	9.7	16.2	2.6	15.1	14.9	12.9
Beef	14.3	17.0	9.4	15.5	2.4	11.7	14.8	12.4
Lamb	0.2	1.6	0.3	0.6	0.3	3.4	0.2	0.5
Sugar	0.7	2.8	2.3	2.5	0.7	4.0	2.0	2.0
Bananas	0.5	0.5	0.5	0.5	0.4	0.5	0.5	0.5
Beverages	2.2	3.9	6.2	3.8	1.9	4.8	2.4	3.0
Coffee	1.4	2.8	3.8	2.9	1.1	1.3	1.6	1.9
Cocoa	0.6	1.1	2.2	0.8	0.3	1.6	0.7	0.9
Tea	0.2	0.1	0.1	0.0	0.5	1.9	0.1	0.3
Agricultural Raw Materials	19.1	15.2	24.8	14.8	21.1	18.4	11.5	15.5
Timber	16.1	9.9	20.3	5.9	15.3	12.7	7.7	10.8
Fibers	0.6	2.2	1.5	3.1	3.0	2.7	1.3	1.8
Cotton	0.5	1.0	1.0	1.8	2.1	0.4	1.1	1.2
Wool	0.1	1.3	0.6	1.3	0.9	2.4	0.2	0.6
Natural rubber	0.5	0.5	0.5	0.4	0.9	0.5	0.4	0.5
Tobacco	1.4	1.5	0.9	1.9	1.2	1.2	1.0	1.2
Hides	0.5	1.0	1.5	3.4	0.6	1.2	1.1	1.2
Metals and Phosphate Rock	8.4	9.7	13.3	8.2	16.4	8.7	7.4	9.7
Copper	1.8	2.1	3.1	2.1	3.3	2.3	1.6	2.1
Aluminum	2.2	2.7	3.9	2.3	3.8	1.9	2.9	3.0
Iron ore	2.2	2.2	3.3	1.9	5.8	1.8	1.0	2.2
Tin	0.3	0.4	0.4	0.2	0.7	0.5	0.3	0.4
Nickel	0.2	0.6	1.0	0.4	1.1	0.6	0.4	0.6
Zinc	0.7	0.8	0.9	0.7	1.1	0.7	0.4	0.7
Lead	0.2	0.3	0.4	0.4	0.3	0.5	0.3	0.3
Phosphate Rock	0.7	0.7	0.2	0.3	0.2	0.3	0.5	0.4
Petroleum 1/	41.0	30.9	32.5	33.8	37.9	34.2	40.2	37.6

Sources: see text and Appendix I (available on request).
1/ 1986 data.

and rice in Japan (9 percent) are significantly higher than average consumption of these commodities by the group of seven countries.

Similarly, beef consumption is relatively low in Japan (2.4 percent) and Germany (9.4 percent), compared with the group (12.4 percent); these two countries consume relatively more seafood and pork products, respectively, which are not included in the indexes. 1/ Tea consumption, as might be expected, is relatively high in the United Kingdom (1.9 percent), as is coffee consumption in Germany (3.8 percent). The relatively low level of consumption of timber products in Italy (5.9 percent) reflects a preference for stone and cement in housing construction, compared with a greater preference for timber in construction in Germany (20.3 percent).

There is remarkable consistency in the share of consumption of primary commodities attributed to petroleum, which--at roughly 38 percent--is the largest of any commodity by far. Weights for petroleum were based on consumption and import unit values in 1986, rather than the 1983-85 average that was used for all other commodities. This ad hoc procedure is intended to avoid overstating the importance of petroleum, in light of the very sharp decline in oil prices that occurred in 1986 and again in 1988. 2/

A caveat on the estimates of the pattern of consumption in the large industrial countries relates to the difficulty in separating end-use consumption of primary commodities from intermediate use in the production of other primary commodities or manufactured products for export. Some

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1/ Broadly representative and timely price series are not available for seafood or pork products.

2/ The use of average 1983-85 data would give a weight to petroleum of around 50 percent in each country.

adjustments were made to consumption values to avoid double counting of commodities at various stages of production, such as in the production of soybean and timber products. 1/ It was not generally possible, however, to adjust for the consumption of one commodity in the production of another. Hence there is some double counting for all countries, on account of maize and soybean meal used as cattle feed, and phosphate rock used in the production of grains, soybeans, and fibers.

Consumption weights also may be overestimated for a few commodities in countries where there are significant exports of finished manufactures. Relatively high weights for metals in Japan (16.4 percent) and Germany (13.3 percent) compared with the group average (9.7 percent) reflect a greater rate of production of automobiles, more than half of which are exported. Relatively high weights for hides in Italy (3.3 percent) and sugar and cocoa in Germany (about 2 percent each) reflect the production of shoes and confectionery for export in these countries. The bias in the indexes caused by overestimated weights may not extend to the esti-mation of weights for the group of countries, however, since manufactures exported by one country are often imported by another country in the group. In addition, some underestimation of weights occurs, when commod-

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1/ The consumption weights in Table 1 were modified to prevent double counting of soybeans and soybean meal production; logs, timber, plywood, and paperboard production; and tobacco and cigarette production. The last two adjustments were needed to account for the export of commodities at a higher value-added level (principally plywood and paper exports from Canada and cigarette exports from the United States).

ities are imported at an advanced stage of production from outside the
group of seven countries. 1/

A comparison of commodity price indexes, using weights from Table 2
and denominating commodity prices in a common currency, is presented in
Chart 1. A casual inspection reveals the indexes to be quite similar,
despite the differences in weights used to construct them. In particular,
indexes for all countries that include large, similar weights for petro-
leum are found to be roughly identical, with only minor variations evident
in the indexes that exclude petroleum. These variations, in turn, are
caused by the presence or absence of large weights for commodities whose
prices moved differently than did prices of the group.

As examples of differences in movements in prices, indexes for most
countries rose in 1973-74 to 80 percent or more of their respective 1980
levels, largely owing to a more than doubling of grain prices from 1972.
The exception to this was the price index for Germany, which remained
below 80, on account of a weight for grains equal to roughly one half of
that in other countries. Other distinctive movements in Germany's index
include a sharp, extended rise in the index during 1978-80, owing in part
to a doubling of the international price of logs, and a subsequent rise in
1988, to well over 100 percent of the 1980 level, again related to a rise

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1/ An example of underestimation of weights is given by iron ore, which
is often imported and consumed as steel products. An adjustment to the
iron ore weight to account for the import of steel products is difficult
because of the variety of these products and their high value-added
component; the value of steel at an advanced stage of production (cold
rolled steel) is ten times that of iron ore. The United States is a
significant importer of semi-finished and finished steel products, with an
apparent consumption nearly equal to that of Japan, in tonnage terms; this
compares with an estimated weight for iron ore consumption in the United
States that is about one fifth that in Japan.

Chart 1. Seven Large Industrial Countries: Commodity Prices,
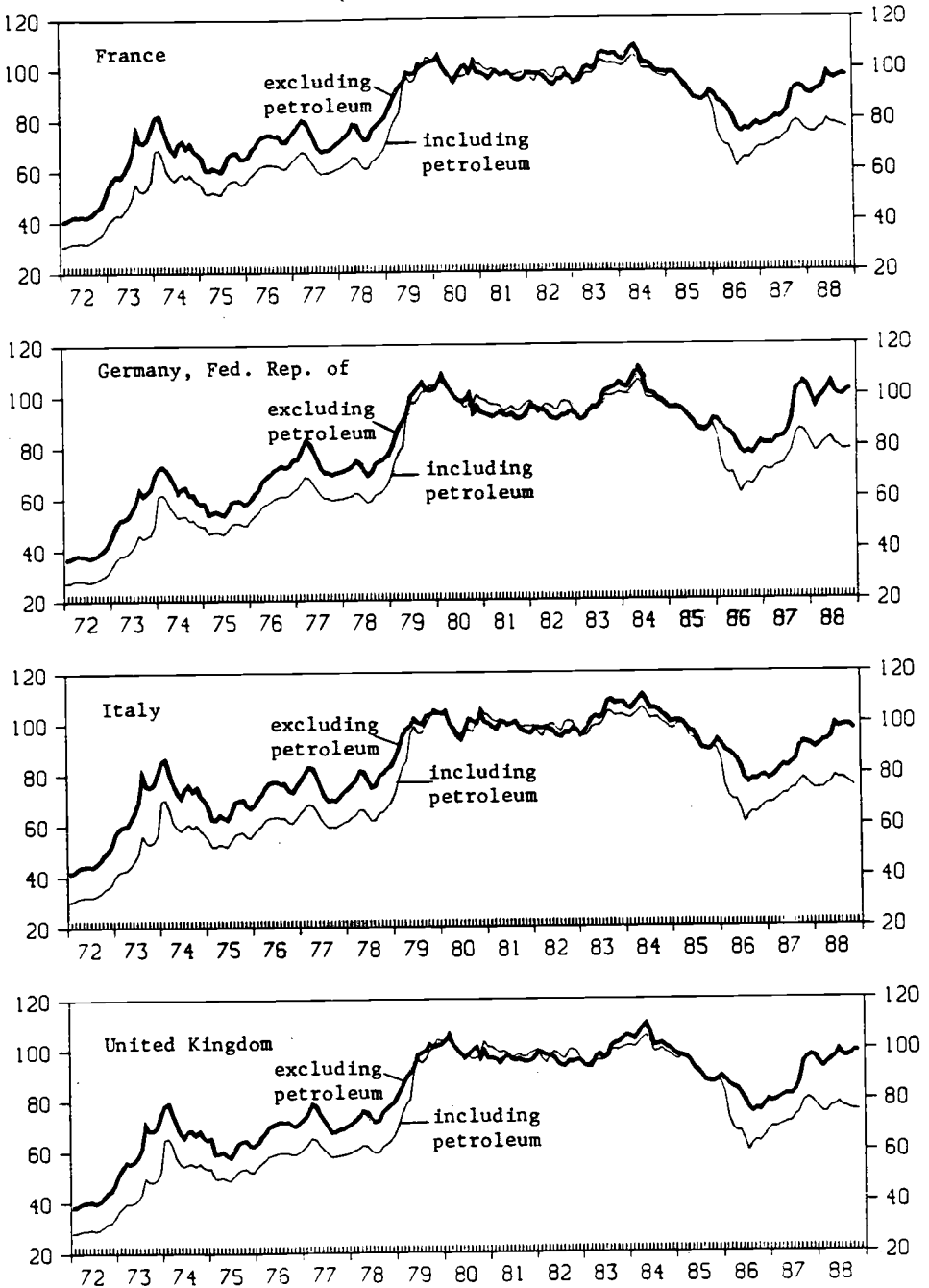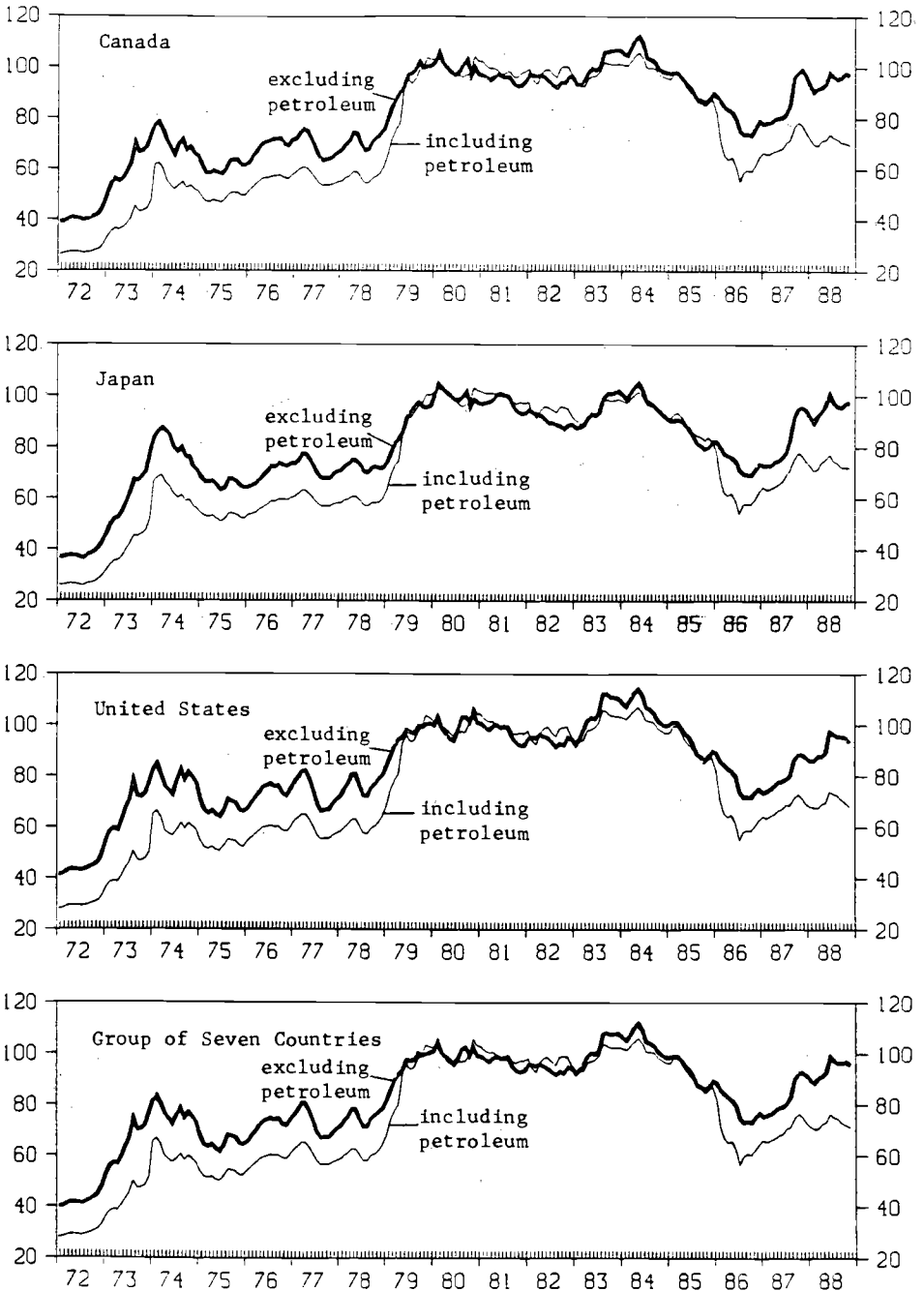In a Common Currency, 1957-88.

(Index: 1980=100)

Chart 1 (continued).   Seven Large Industrial Countries: Commodity Prices,
In a Common Currency, 1957-88.

(Index: 1980=100)

VI. CONCLUSION

This paper has illustrated the tradeoffs policy makers would face when choosing the width of a band, or when converging to an extremely narrow band in order to shift to a unique currency. These tradeoffs concern the degree of exchange- rate, interest-rate and fundamentals variability. They depend on the assumed process (constant drift or mean reversion) for the disturbances which affect money demand and/or supply.

They also depend on the type of coordination which would take place in case of foreign exchange crisis. In this last respect, we have been careful to make Assumption 2 so as to avoid altogether the problem of speculative attacks and needed reserves. We have assumed that central banks credibly and unconditionally intervene so as prevent speculative attacks. Indefinite intervention is possible if the central bank of the currently strong currency supports the other currency by printing money.

We have generalized the result of Svensson (1989) that the variability of the exchange rate is translated into variability in the interest rate differential.

We have also found that the degree of fundamentals variability which would be tolerated by the market when a target zone is extremely narrow is very large (a difference of two orders cf magnitude) in the case of a constant drift disturbance and in the case of mean reversion with a band exactly centered on the reversion point. The result is lost if the band is not precisely centered there.

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APPENDIX A

MODEL INTERPRETATION

It is conceivable to interpret v either as a supply or a demand shock.
Consider the following two-country log-linear monetary exchange rate model
which has been extensively used in recent work (see for example Delgado and
Dumas (1990) and all the references given there).

$$\bar{m} = \ell n(D + R) + z_1 = m + z_1 \tag{A1}$$

$$\bar{m}^* = \ell n(D^* + R^*) + z_1^* = m^* + z_1^* \tag{A2}$$

$$\bar{m} - p = \psi y - \gamma i + z_2 \tag{A3}$$

$$\bar{m}^* - p^* = \psi y^* - \gamma i^* + z_2^* \tag{A4}$$

$$i = i^* + E\{dS \mid \Phi(t)\}/dt + z_3 \tag{A5}$$

All starred variables are the foreign variables corresponding to the non-
starred domestic variables. Z_1 being a multiplicative shock which affects $(D + R)$ and is assumed to follow a geometric Brownian motion process, z_1 is the log
of Z_1.[11] $\bar{m}$ is the total money supply that can be broken down into two
components, m the controllable money supply, and z_1 the uncontrollable
component. The right-hand sides of equations (A3) and (A4) are money demands;
z_2 and z_2^* are money demand shocks. p is the price level. y is domestic output.
i is the domestic interest rate.

E is the expectations operator. $\Phi(t)$ is the information set used by

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[11]In the classic speculative attack literature [Krugman (1979) and Flood
and Garber (1983); see also Dornbusch (1984) and Claessens (198(A3)], shocks
came from growth in domestic credit. In our formulation, the terms Z_1 and Z_1^*
are shocks applied to the sum of R and D, not to D.

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(1989a, b) put it, all information regarding not only the evolution of the variables in the system (A1)-(A5), but the implicit as well as explicit government policies regarding exchange rate regimes in particular and monetary policy in general. Of importance for our purposes is the market perception that once the central bank whose currency is weak runs out of reseves, the other central bank will continue intervention to support the weak currency.

Subtracting (A3) from (A4) and replacing (A1), (A2) and (A3) we can obtain equation (1), where $v = (z_1 - z_1^*) + \psi(y - y^*) + (z_2 - z_2^*) + z_3$ is a cumulative shock. Given that v includes terms in z_1 and z_1^* it can be interpreted as either a demand or a supply shock to money.

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MONOTONIC RELATIONSHIP BETWEEN

THE BAND ON FUNDAMENTALS AND THE BAND ON THE EXCHANGE RATE

For simplicity let's define the following functions:

$$NF(\tilde{X},\underline{X}) = -\beta e^{\alpha\tilde{X} - \beta\underline{X}} - \alpha e^{\alpha\underline{X} - \beta\tilde{X}} + (\alpha+\beta)e^{(\alpha-\beta)\tilde{X}} \tag{B1}$$

$$NG(\tilde{X},\underline{X}) = \alpha e^{\alpha\tilde{X} - \beta\underline{X}} + \beta e^{\alpha\underline{X} - \beta\tilde{X}} - (\alpha+\beta)e^{(\alpha-\beta)\underline{X}} \tag{B2}$$

$$DF(\tilde{X},\underline{X}) = -\alpha\beta(e^{\alpha\tilde{X} - \beta\underline{X}} - e^{\alpha\underline{X} - \beta\tilde{X}}) \tag{B3}$$

$$F(\tilde{X},\underline{X};\tilde{S},\underline{S}) = - \tilde{X} + \tilde{S} - \gamma\mu + NF(\tilde{X},\underline{X})/DF(\tilde{X},\underline{X}) = 0 \tag{B4}$$

$$G(\tilde{X},\underline{X};\tilde{S},\underline{S}) = - \underline{X} + \underline{S} - \gamma\mu + NG(\tilde{X},\underline{X})/DF(\tilde{X},\underline{X}) = 0 \tag{B5}$$

Using the implicit function theorem for $F(\cdot) = 0$ and $G(\cdot) = 0$ we can obtain the two derivatives $\partial\tilde{X}/\partial\tilde{S}$ and $\partial\underline{X}/\partial\underline{S}$ which we are looking for.

Defining the following functions

$$F_{x1}(\cdot) = \partial F(\cdot)/\partial\tilde{X} \tag{B6}$$

$$F_{x2}(\cdot) = \partial F(\cdot)/\partial\underline{X} \tag{B7}$$

$$G_{x1}(\cdot) = \partial G(\cdot)/\partial\tilde{X} \tag{B8}$$

$$G_{x2}(\cdot) = \partial G(\cdot)/\partial\underline{X} \tag{B9}$$

$$F_{s1}(\cdot) = \partial F(\cdot)/\partial\tilde{S} \tag{B10}$$

$$F_{s2}(\cdot) = \partial F(\cdot)/\partial\underline{S} , \tag{B11}$$

we can determine the signs of the following expressions

$$\partial \bar{X}/\partial \bar{S} - \begin{bmatrix} F_{s1} & F_{x2} \\ & \\ G_{s1} & G_{x2} \end{bmatrix} / J \geq 0 \qquad (B12)$$

$$\partial \underline{X}/\partial \underline{S} - \begin{bmatrix} F_{x1} & F_{s2} \\ & \\ G_{x1} & G_{s2} \end{bmatrix} / J \geq 0 \qquad (B13)$$

where:

$$J - \begin{bmatrix} F_{x1} & F_{x2} \\ & \\ G_{x1} & G_{x2} \end{bmatrix} > 0 \qquad (B14)$$

Replacing all definitions in (B12) and (B13), and remembering that $1/\alpha$ - $1/\beta - \gamma\mu$, we obtain the signs of the partial derivatives. Q.E.D.

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APPENDIX C

ASYMPTOTIC BEHAVIOR FOR WIDE BANDS

UNDER CONSTANT DRIFT

Let us call δ the joint asymptotic value of the distance of $\bar{X}$ from the diagonal line and of the distance of $\underline{X}$ from the free-float line; let us also call 2ϵ the width of the exchange rate band, $\bar{S} = S_0 + \epsilon$ and $\underline{S} = S_0 - \epsilon$. We have:

$$\bar{X} = S_0 + \epsilon + \delta$$

$$\underline{X} = S_0 - \epsilon - \gamma\mu - \delta \qquad \text{and}$$

$$\underline{X} = \bar{X} - 2(\epsilon + \delta) - \gamma\mu$$

To study the behavior of wide bands we can substitute these relationships into (14), simplify, and neglect terms that approach zero as we make δ, $\epsilon \to \infty$. We obtain a simple relationship:[12]

$$\delta = \epsilon - \gamma\mu + 1/\alpha = \epsilon + 1/\beta \tag{C1}$$

Q.E.D.

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[12]Svensson obtained a similar relationship for the special case of zero drift ($\mu = 0$) in which the two roots are of equal magnitude: $\alpha = -\beta$. The distance from the free-float (confounded in his case with the $45°$ line) was $1/\alpha = -1/\beta$.

CUBIC RELATIONSHIP FOR NARROW BANDS BETWEEN

THE BAND ON FUNDAMENTALS AND THE BAND ON THE EXCHANGE RATE

UNDER CONSTANT DRIFT

To obtain the relationship between the widths of the bands for small widths we are going to use a linear approximation to the exponential function:

$$e^{\lambda x} \approx 1 + \lambda x + (\lambda x)^2/2! + (\lambda x)^3/3! + \ldots \tag{D1}$$

The expansion is carried out as far as is needed. Additional terms are added if the prior order terms vanish or lead to an identity.

We proceed with the linearization around S_0. The four variables with respect to which the linearization is to be performed can be written as:

$$\bar{S} - S_0 + \epsilon \tag{D2}$$

$$\underline{S} - S_0 - \epsilon \tag{D3}$$

$$\hat{X} - S_0 + \bar{\delta} \tag{D4}$$

$$\underline{X} - S_0 - \underline{\delta} \tag{D5}$$

Replacing (D2)-(D5) in (A14)-(A15) the system is written as:

$$\bar{\delta} = \bar{\epsilon} - \gamma\mu + \frac{-\beta e^{\alpha\bar{\delta} + \beta\underline{\delta}} - \alpha e^{-(\alpha\underline{\delta} + \beta\bar{\delta})} + (\alpha+\beta)e^{(\alpha-\beta)\bar{\delta}}}{-\alpha\beta(e^{\alpha\bar{\delta} + \beta\underline{\delta}} - e^{-(\alpha\underline{\delta} + \beta\bar{\delta})})} \tag{D6}$$

$$\underline{X} - \underline{S} - \gamma\mu + \frac{\alpha e^{\alpha\bar{\delta} + \beta\underline{\delta}} + \beta e^{-(\alpha\underline{\delta} + \beta\bar{\delta})} - (\alpha+\beta)e^{-(\alpha-\beta)\underline{\delta}}}{-\alpha\beta(e^{\alpha\bar{\delta} + \beta\underline{\delta}} - e^{-(\alpha\underline{\delta} + \beta\bar{\delta})})} \tag{D7}$$

The last two equations can also be written in matrix form as

$$\begin{bmatrix} \epsilon \\ \\ -\epsilon \end{bmatrix} - \begin{bmatrix} \gamma\mu + \bar{\delta} \\ \\ \gamma\mu - \underline{\delta} \end{bmatrix} + \begin{bmatrix} e^{\alpha\bar{\delta}} & e^{-\beta\bar{\delta}} \\ \\ e^{-\alpha\underline{\delta}} & e^{\beta\underline{\delta}} \end{bmatrix} \begin{bmatrix} 1/\alpha & 0 \\ \\ 0 & -1/\beta \end{bmatrix} \begin{bmatrix} e^{\alpha\bar{\delta}} & e^{-\beta\bar{\delta}} \\ \\ e^{-\alpha\underline{\delta}} & e^{\beta\underline{\delta}} \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ \\ -1 \end{bmatrix} \tag{D8}$$

Using $\gamma\mu - 1/\alpha - 1/\beta$ and expanding (D8), the following expression is obtained:

$$\epsilon \approx (\alpha + \beta)^2 (\bar{\delta} + \underline{\delta})^3/12 \tag{D9}$$

Q.E.D.

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Legends for figures

Figure 1: Target zones of different widths

The figure is constructed by changing the width of the band around $S_0 = 4.5$. This means solving the system (14)-(15) for different values of $\bar{S}$ and $\underline{S}$ *positioned symmetrically around* S_0. The two straight lines in the middle are the 45° diagonal line which contains the fixed exchange points, and the free float. The thick line is the locus of tangencies implied by the smooth pasting conditions. Points above S_0 are pairs $(\bar{X}, \bar{\ })$; points below S_0 are pairs $(\underline{X}, \underline{S})$. Numerical values of parameters are: $\sigma^2 = 0.25$, $\mu = 0.5$ and $\gamma = 0.5$. Units on the two axes are not the same.

Figure 2: Extreme values of interest rate differential

The figure is constructed by changing the width of the exchange rate band symmetrically around $S_0 = 4.5$. Points above S_0 have abscissae equal to $\bar{X}$ and ordinates equal to the interest rate differential (assumed equal to the conditionally expected exchange rate change) reached when $X = \bar{X}$. Points below S_0 have abscissae equal to $\underline{X}$ and ordinates equal to the interest rate differential (assumed equal to the conditionally expected exchange rate change) reached when $X = \underline{X}$. Numerical values of parameters are: $\sigma^2 = 0.25$, $\mu = 0.5$ and $\gamma = 0.5$.

Figure 3: Exchange rate and interest rate differential

When the band is wide as in the case of this figure ($\underline{S} = 1.999$, $\bar{S} = 7.001$), the exchange-rate curve follows the free-float line and the interest-rate curve is flat over most of the range of allowed variations. Numerical values of parameters are: $\sigma^2 = 0.25$, $\mu = 0.5$ and $\gamma = 0.5$.

Figure 4: Exchange rate and interest rate differential

When the band is narrow as in the case of this figure ($\underline{S}$ - 4.4999, $\dot{S}$ - 4.5001), the exchange-rate curve is S-shaped and is situated far from the free-float line. The interest-rate curve is practically a straigtht line reflecting the imminent intervention. Numerical values of parameters are: σ^2 - 0.25, μ - 0.5 and γ - 0.5.

Figure 5: Target zones of different widths: mean reverting case; symmetric solution with reversion point A_0 - 7.8.

The figure is constructed by changing the width of the band around S_0 - A_0 - 7.8. This means solving the system (26) for different values of $\dot{S}$ and $\underline{S}$ positioned symmetrically around S_0 - A_0. The two straight lines in the middle are the 45° diagonal line which contains the fixed exchange points, and the free float which is not at a 45° incline in this case. The thin curve is an example of an exchange rate curve. The thick line is the locus of tangencies implied by the smooth pasting conditions. Points above S_0 are pairs $(\bar{X}, \dot{S})$; points below S_0 are pairs $(\underline{X}, \underline{S})$. Observe the flatness of the locus around 7.8. Numerical values of parameters are: ρ - 0.5, σ^2 - 0.2, A_0 - 7.8 and γ - 0.5. Units on the two axes are not the same.

Figure 6: Target zones of different widths: mean reverting, non-symmetric case.

The figure is similar to Figure 5. While the reversion point A_0 is still equal to 4.8, the center of the exchange rate band is now at S_0 - 9. The free-float line and the 45° line would intersect at 7.8. The thin curve is an example of an exchange rate curve. The thick line is the locus of tangencies implied by the smooth pasting conditions. Observe that the locus is not flat around the central point S_0 - 9.
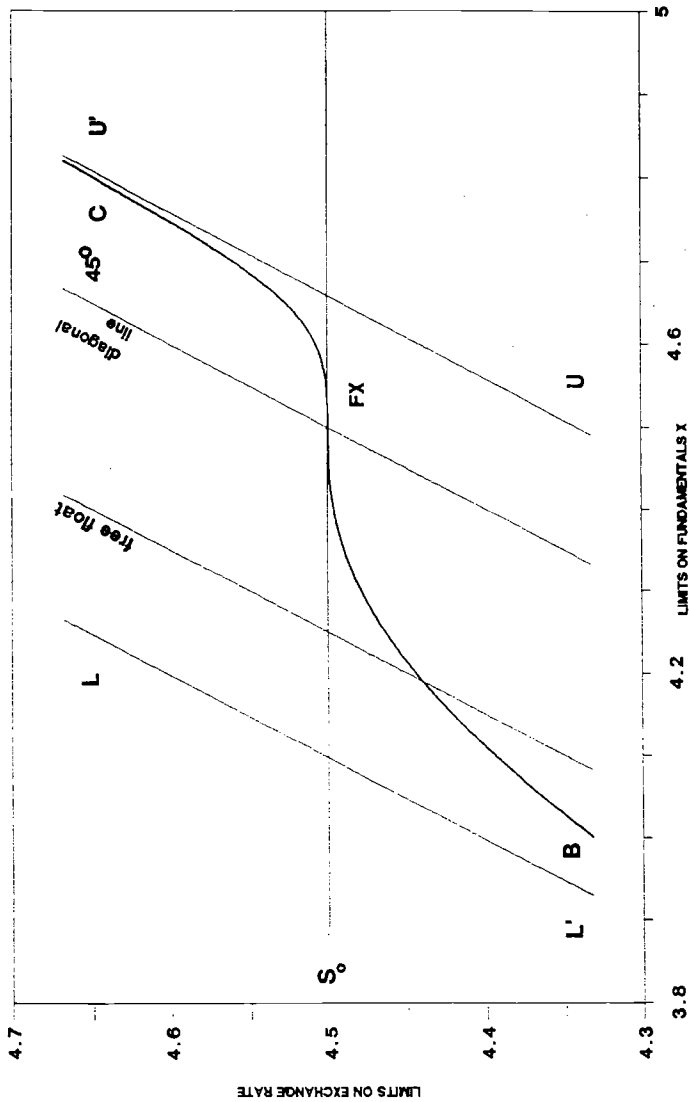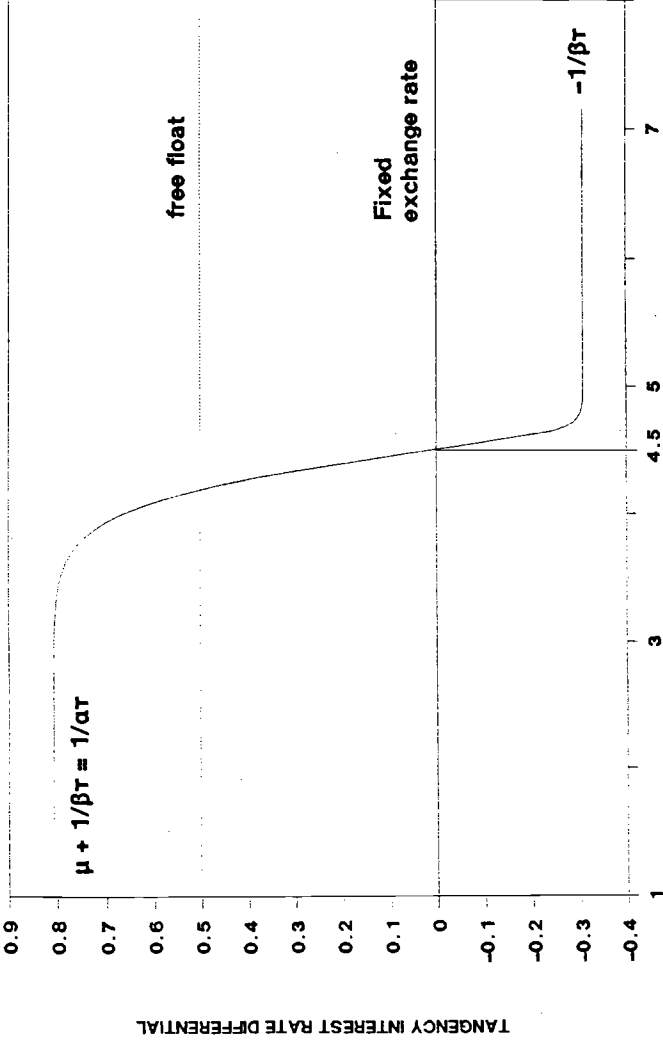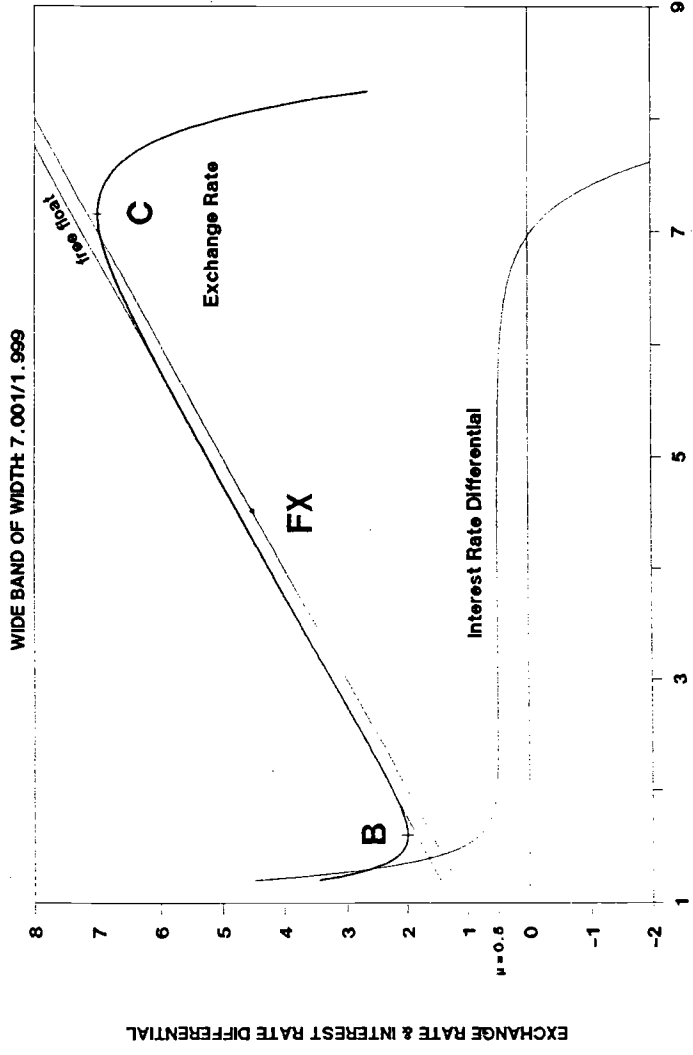
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TARGET ZONES OF DIFFERENT WIDTHS

LIMITS ON FUNDAMENTALS X

FIGURE 1

EXTREME VALUES OF INTEREST RATE DIFFERENTIAL

FIGURE 2

EXCHANGE RATE AND INTEREST RATE DIFFERENTIAL

WIDE BAND OF WIDTH± 7.001/1.999

FUNDAMENTALS X

FIGURE 3

EXCHANGE RATE AND INTEREST RATE DIFFERENTIAL

NARROW BAND OF WIDTH: 4.5001/4.4999

INTEREST RATE DIFFERENTIAL

EXCHANGE RATE

FX

INTEREST RATE DIFFERENTIAL
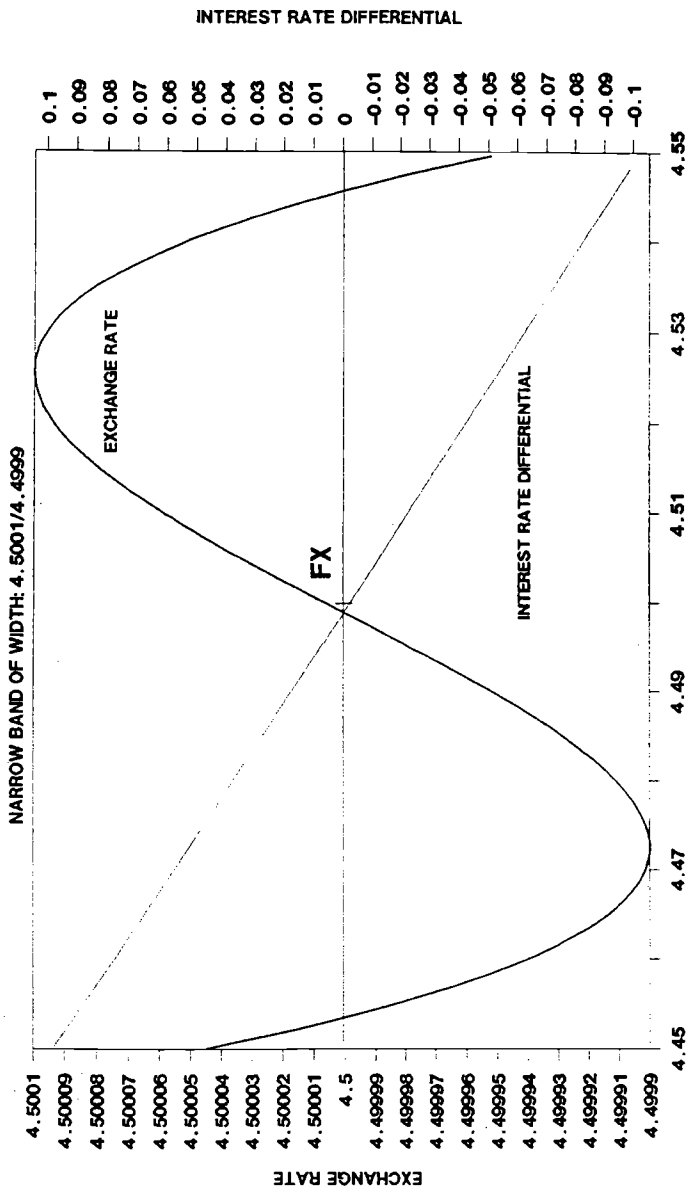
FUNDAMENTALS X

FIGURE 4

TARGET ZONES OF DIFFERENT WIDTHS: MEAN REVERTING CASE
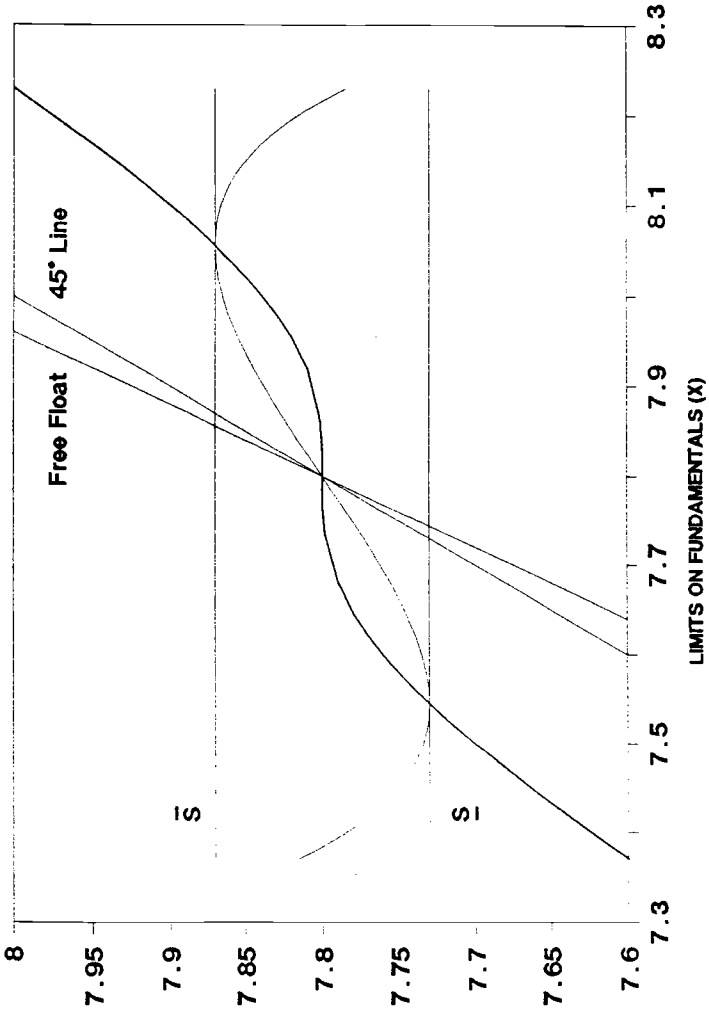
SYMMETRIC SOLUTION Ao = 7.8

FIGURE 5

LIMITS ON FUNDAMENTALS (X)

TARGET ZONES OF DIFFERENT WIDTHS

MEAN REVERTING NON-SYMETRIC CASE

45° Line

EXCHANGE RATE

Free Float

LIMITS ON FUNDAMENTALS (X)

FIGURE 6