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ABSTRACT

Global and local methods used to study open-economy incomplete-markets models yield different cyclical moments, impulse responses, spectral densities and precautionary savings. Endowment and RBC model solutions obtained with first-order, higher-order, and risky-steady-state local methods are compared with fixed-point-iteration global solutions. Analytic and numerical results show that the differences are due to the near-unit-root nature of net foreign assets under incomplete markets and inaccuracies of local methods in computing their autocorrelation. In a Sudden Stops model, quasi-linear methods that handle occasionally binding constraints understate the size of credit constraint multipliers, financial premia and macroeconomic responses.

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A data appendix is available at http://www.nber.org/data-appendix/w31544

1. Introduction

Incomplete asset markets play a key role in major strands of the international macroeconomics literature (e.g., business cycles, sovereign default, sudden stops, global imbalances, macroprudential regulation, currency carry trade, etc.). Since the dynamics of external wealth (or net foreign assets, NFA) generally lack analytic solutions, researchers rely on numerical methods. However, choosing the appropriate method is difficult for several reasons. First, deterministic models yield stationary equilibria dependent on initial conditions. Second, in stochastic models, the evolution of wealth is state-contingent and driven by precautionary savings (i.e., certainty equivalence fails). Third, with standard preferences, if the interest rate equals the rate of time preference, precautionary savings make NFA diverge to infinity.

The literature follows two approaches to address these issues. The first, based on the seminal work of Schmitt-Grohé and Uribe (2003), modifies the models by inducing stationarity with one of three assumptions: a debt-elastic interest-rate (DEIR) function, preferences with endogenous discounting (ED), or asset holding costs (AHC).[1] These assumptions support a well-defined deterministic steady state of NFA independent of initial conditions. The models are then solved with a first-order approximation (1OA) around that steady state, recovering certainty equivalence. Innovations to local methods have occurred since then, including higher-order methods (e.g., Schmitt-Grohé and Uribe, 2004; Devereux and Sutherland, 2010; Fernández-Villaverde et al., 2011), the risky steady state (RSS) method (Coeurdacier et al. (2011)), and quasi-linear methods for handling occasionally binding constraints (QLOBC), including OccBin by Guerrieri and Iacoviello (2015) and DynareOBC by Holden (2016, 2021).[2]

Table 1 summarizes the numerical methods used in a set of research papers and policy applications. Among local methods, 1OA is the most common in research papers and ubiquitous in policy applications. Among stationarity inducing assumptions, DEIR is the most common. Of these, the majority set the value of the debt elasticity parameter, ψ, to an arbitrary small number (ranging from 0.00001 to 0.01, with the value of 0.001 used by Schmitt-Grohé and

[1] They show business cycle moments and impulse response functions of an RBC small open-economy model obtained with any of these assumptions are very similar.

[2] Boehl and Strobel (2022) and Kulish et al. (2017) have also produced similar algorithms.

Uribe (2003) the most common), with the aim of preventing the DEIR function from playing a role other than inducing stationarity.[3] In other cases, ψ is calibrated or estimated.

The second approach, introduced to solve an RBC small open-economy model by Mendoza (1991), uses global approximation (GA) methods to solve for the nonlinear decision rules and long-run distribution of external wealth of the models in their original form. These methods are similar to those used in closed-economy models of heterogeneous agents with incomplete markets. The existence of a well-defined stochastic steady state follows from the same condition as in those models (see Ch. 18 of Ljungqvist and Sargent, 2018): the interest rate must be lower than the rate of time preference.

This condition is a general equilibrium result in multicountry models, because if the interest rate equals the rate of time preference, all countries desire infinitely large NFA for self-insurance, which is inconsistent with market clearing (see Mendoza et al., 2009). Hence, assuming the interest rate is lower than the rate of time preference in small open-economy models is an *implication* of the assumption that the interest rate is a world-determined price. With local methods, the stationarity inducing assumption is constructed so that, at a chosen deterministic steady state, the interest rate equals the rate of time preference.

While global methods solve the models in their original form and capture NFA dynamics more accurately, they suffer from the curse of dimensionality—becoming exponentially inefficient with the number of endogenous state variables. In contrast, local methods can solve larger-scale models efficiently but require a stationarity-inducing assumption that is not part of the original model. This tradeoff poses four key questions: Are local solutions accurate? If not, why not? Are the inaccuracies economically meaningful? Can they be reduced?

This paper answers these questions by analytically and numerically comparing global and local solutions for two small open-economy models: 1) An endowment model and 2) a model of sudden stops (SS), which is an RBC model with an occasionally-binding collateral constraint.

[3]Garcia-Cicco et al. (2010) explain that, following Schmitt-Grohé and Uribe (2003), it is standard to set ψ to a small value because the DEIR function aims to obtain independence of the deterministic steady state from initial conditions without affecting cyclical dynamics. Garcia-Cicco et al. (2010) also study a model in which ψ represents a financial friction and is estimated. More broadly in the literature, DEIR functional forms vary and some papers use quarterly frequency while others use annual frequency, and hence ψ values are not directly comparable.

In Appendix C of de Groot et al. (2019) we also compared solutions for a standard RBC model with similar findings as those reported here.

For the global solution, we use an accurate fixed-point iteration approach and calibrate the model to Mexican data.[4] For the local methods, we consider 1OA, second-order approximation (2OA), RSS, and QLOBC.[5] RSS and QLOBC can be used with or without stationarity-inducing assumptions, and we study both cases.

Given the prevalence of the DEIR function in the literature, we focus on this stationarity inducing assumption in most of our analysis. We solve "baseline calibrations" with $\psi = 0.001$ and "targeted calibrations" with ψ calibrated to match the ratio of the standard deviations of consumption and output in the GA solution, which is the same as in the data.[6] In both instances, the reference value of NFA in the DEIR function is calibrated to match the mean NFA position in the GA solution (also the same as in the data). In addition, we study the implications of using AHC and ED instead of DEIR to induce stationarity, and, for RSS and QLOBC, compare variants without DEIR in which the interest rate is lower than the rate of time preference. We compare across solutions statistical moments, impulse response functions (IRFs), spectral densities (in Appendix B.3.4), Euler-equation errors, and solution run times.

The results show that global and local solutions differ significantly due to the near-unit root nature of the NFA equilibrium process, one of the main endogenous state variables in open-economy analysis. This is a typical property of incomplete-markets models caused by the persistence of precautionary savings behavior. NFA is a near-unit-root process in our global and local calibrated solutions (the NFA autocorrelation always exceeds 0.96). In the local solutions, we show formally how this autocorrelation is determined by ψ and the center of approximation, whereas in the global solution it is a moment of the endogenous ergodic distribution of

[4]We use the *FiPIt* algorithm developed by Mendoza and Villalvazo (2020). This algorithm modifies the standard iteration-on-Euler-equation approach to avoid both solving simultaneous non-linear equations (as with standard time iteration methods) and irregular interpolation (as with endogenous grid methods). For comparison, Appendix B.1.2 solves the model with value function iteration.

[5]In Appendix B.3.7 of de Groot et al. (2019), we presented third-order-approximation (3OA) results, and found that 3OA is unnecessary unless stochastic volatility is introduced (see de Groot, 2016). For QLOBC, we use the DynareOBC algorithm. DynareOBC and OccBin give the same solution when the equilibrium is unique. DynareOBC, however, has the advantage that it converges in finite time and can test for equilibium multiplicity.

[6]In de Groot et al. (2019), we study targeted calibrations set to match the first-order autocorrelation of NFA instead. The qualitative features of our findings are unchanged.

NFA. Because they are near-unit-roots, slight differences in these NFA autocorrelations cause large differences in unconditional moments, IRFs and spectral densities.

The effect on two key moments is particularly striking. First, small differences in the NFA autocorrelations of 2OA and RSS relative to GA solutions yield large differences in precautionary savings (i.e., the unconditional mean of NFA). This problem is also particularly acute for QLOBC when the collateral constraint binds at the deterministic steady state but not at the ergodic mean of the global solution.

Second, small differences in NFA autocorrelations yield large differences in net exports (nx) autocorrelations, because nx is a quasi first-difference of the near-unit-root NFA process. For example, in the endowment model with the baseline calibration, the global solution predicts that raising the persistence of income from 0.5 to 0.95 increases the autocorrelation of NFA from 0.957 to 0.997 and that of net exports from 0.444 to 0.983. In contrast, 2OA and RSS predict that the autocorrelation of NFA always exceeds 0.99 over the same interval of income persistence, while that of nx increases from 0.874 to above 0.999. The local solutions always overstate the autocorrelations of NFA and nx and, as a result, overstate also consumption and nx volatility and understate their income correlations.

Comparing across local methods, 2OA and RSS yield similar second- and higher-order moments, IRFs and spectral densities for all endogenous variables. To explain these results, we analytically solve the endowment model and show that i) the coefficient on lagged NFA in the NFA decision rule is nearly the same when ψ is small (less than 0.1), unless the deterministic and risky steady state of NFA differ by a large margin (at least 40 percentage points of GDP); ii) the coefficients in the square and interaction terms of 2OA decision rules are small.

The local solutions with the "targeted calibrations" better match the global solution, which in turn yields a reasonable approximation to the data moments. However, this approach has two drawbacks. One, it requires obtaining the mean NFA and consumption standard deviation from the global solution to calibrate ψ and the center of approximation, and re-doing this for any parametric change that alters those two moments. Two, targeted calibrations require increasing ψ from 0.001 to 0.042. This sharp increase in the elasticity of the DEIR function makes NFA "sticky," because it is analogous to making deviations of NFA from steady state too costly. As a result, all the results, even the first moments, of the 2OA and RSS methods become similar to the 1OA solution (i.e., precautionary savings vanish and certainty equivalence approximately holds).

The QLOBC method that we used to solve the SS model with its occasionally binding collateral constraint, DynareOBC, retains some of the benefits of local methods while handling such discontinuities. DynareOBC works by introducing news shocks that hit every time the constraint is violated to push the relevant variables back to the constraint. For consistency with rational expectations, these news shocks are constructed as if they were expected along a perfect-foresight path and so are akin to being endogenous. This method, however, ignores precautionary savings; the possibility of alternative future paths in which the constraint may or may not bind; and the equity risk premium.

Findings from the endowment model extend to the SS model. In addition, QLOBC yields large differences relative to the global solution in the amount of precautionary savings induced by the collateral constraint, the tightness of the constraint, the probability of hitting it, and its effect on financial premia. Lower equity returns imply higher equity prices and investment when the constraint binds, and hence higher borrowing capacity. As a result, QLOBC both with the constraint binding or not-binding at steady state does not match the macroeconomic effects of sudden stops found in the GA solution.

In terms of computational performance, the global algorithm is slower than 2OA methods for solving the endowment model. The trade-off is that the local methods yield less accurate results in terms of Euler equation errors and differences in decision rules. However, once an occasionally binding constraint is introduced, QLOBC methods lose much of their speed advantage relative to the *FiPIt* global method. This is because, with the near-unit-root nature of NFA, QLOBC methods require multiple, long perfect-foresight paths and long time-series simulations to attain convergence of long-run moments.

In summary, we find that the choice and parameterization of stationarity-inducing assumptions when using local methods are not innocuous, as is often assumed in the literature. Our results serve as a cautionary guide as to why and when these stationarity inducing assumptions have meaningful economic implications. And, while advances in local methods such as

RSS and QLOBC methods provide ways to dispense with stationarity-inducing assumptions, their considerable additional computational cost relative to standard local methods means that the benefits of not using a global method becomes less clear.

Related literature This paper is related to several recent studies comparing global and local solutions. Rabitsch et al. (2015) compares the local method proposed by Devereux and Sutherland (2010) for solving portfolio allocations in a two-country incomplete-markets model, with a global solution. Devereux and Sutherland use ED preferences to induce stationarity. The paper finds that this method is accurate when the countries are symmetric with zero long-run NFA, but not when the countries are asymmetric and the center of approximation differs from the ergodic mean of the global solution.

Global and local solutions with occasionally binding constraints have been compared in the closed-economy New-Keynesian literature on the zero-lower-bound (ZLB) on interest rates. These models typically formulate a Taylor rule with the ZLB constraint (rather than studying constraints on the agents' optimization problems); assume complete markets; private bonds in zero net supply; and a rate of time preference equal to the steady-state interest rate. Hence, the effects of precautionary savings on the dynamics of bond positions and the center of approximation of local solutions, which are essential to our findings, are not at issue in this literature. Fernández-Villaverde et al. (2015) solve a ZLB model using a global (projection) method with one endogenous state (price dispersion).[7] They found that the ZLB yields important nonlinearities that local methods miss. Gust et al. (2017) also solved a ZLB model with projection methods and compared the results with a QLOBC method (using OccBin). They found the latter poorly approximates the global solution and that the differences have implications for the propagation of shocks and estimation results.[8] Atkinson et al. (2020) examined model estimation in a ZLB model but, in contrast, conclude there are more accuracy gains from es-

[7]In their model, the ergodic mean and deterministic steady state are nearly identical, whereas a key finding of our analysis is that precautionary savings causes large differences in the ergodic mean and steady state of NFA.

[8]Solving our SS model using projection methods is difficult because the global basis functions are not defined in points of the state space where it is infeasible to satisfy the collateral constraint with positive consumption. The boundary varies as capital, NFA and the capital pricing function vary. This problem can be avoided using uneven grids but this is also difficult because the debt limit imposed by the collateral constraint is not a pre-determined value. These hurdles do not arise in ZLB models and models with constant, uni-dimensional debt limits.

timating a richer (less misspecified) model using QLOBC methods than estimating a stylized model using global methods.

In the literature on financial frictions, Dou et al. (2019) compared global, 1OA, 2OA and QLOBC (using OccBin) methods for closed-economy models and found that the local solutions poorly approximated the nonlinear dynamics and yield biased IRFs. Holden (2016) shows that DynareOBC yields similar results as a global solution for a small open-economy endowment model with quadratic utility (which rules out precautionary savings) and NFA adjustment costs to ensure stationarity. In contrast, we find the global and DynareOBC solutions of our endowment model with an ad-hoc debt limit and CRRA utility (which allows for precautionary savings) differ sharply. We also used DynareOBC to solve the SS model, which has two endogenous states (capital and NFA) and a collateral constraint that depends on both states and endogenous asset prices, and find the results again differ markedly from the global solution. Benigno et al. (2020) propose an alternative perturbation method for solving models with an occasionally binding constraint and applied it to a similar SS model. Their method uses the DEIR function to induce stationarity and models constraint regime-switching as driven by draws of regime realizations and regime-transition probabilities determined by parameterized logistic functions that depend on the slack in the credit constraint and the constraint multiplier.[9]

The rest of the paper is organized as follows. Section 2 presents the endowment model and compares solution methods, providing both analytic and numerical results. Section 3 presents the SS model and compares solution methods. Section 4 concludes. The Appendix provides further details on the solution methods, analytic derivations and additional results.

2. Endowment model

2.1. Model structure and equilibrium

We first consider a small open-economy model with stochastic endowment income and use it to derive analytical results and characterize NFA dynamics under incomplete markets. The

[9]Other promising approaches that attempt to adopt the benefits of the global approach while maintaining computational feasibility include Ajevskis (2017) and Mennuni and Stepanchuk (2022).

economy is inhabited by a representative agent with preferences given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}, \quad u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \tag{1}$$

where β is the subjective discount factor, c_t is consumption and σ is the CRRA coefficient. The economy's resource constraint is given by

$$c_t = y_t - A + b_t - \frac{b_{t+1}}{R}, \tag{2}$$

where $y_t = e^{z_t}\bar{y}$ denotes income, $z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$, and $\varepsilon_{z,t}$ is i.i.d. from $N(0, \sigma_\varepsilon^2)$.[10] Hence, the variance and autocorrelation of (loged, demeaned) income are $\sigma_z^2 = \sigma_\varepsilon^2/(1-\rho_z^2)$ and ρ_z, respectively; b_t denotes NFA in one-period, non-state-contingent discount bonds traded in a global market where R is the gross world interest rate; and A represents constant investment and government spending, necessary for model calibration.[11]

The agent chooses the sequences of bonds and consumption to maximize (1) subject to (2). This optimization problem is analogous to the one solved by a single agent in heterogeneous-agents models (e.g., Aiyagari, 1994). Since the marginal utility of consumption, $u_c(c_t)$, tends to infinity as c_t tends to zero from above, the economy faces a Natural Debt Limit (NDL), by which net foreign debt never exceeds the annuity value of the worst realization of net income $b_{t+1} \geq b^{NDL} \equiv -\frac{R}{R-1}\min(e^{z_t}\bar{y} - A)$, otherwise agents are exposed to the possibility of non-positive consumption with positive probability. Following Aiyagari (1994), we also impose a tighter ad-hoc debt limit, φ, such that $b_{t+1} \geq \varphi \geq b^{NDL}$, which is useful for model calibration.

Using the resource constraint, we can express the Euler equation for bonds as

$$u_c\left(e^{z_t}\bar{y} - A + b_t - \frac{b_{t+1}}{R}\right) = \beta R E_t\left[u_c\left(e^{z_{t+1}}\bar{y} - A + b_{t+1} - \frac{b_{t+2}}{R}\right)\right] + \mu_t, \tag{3}$$

[10] $\bar{y}$ is normalized to 1. Technically, we require z_t to be truncated below. However, even if we assume z_t is truncated such that $y_{min} = 0.5$, the probability of this bound binding is essentially zero and thus it has no affect on the solution to local approximations. It also ensures the ad hoc debt limit is always tighter than the natural limit for the global approximation.

[11] Later, we allow R to be stochastic.

where μ_t is the Lagrange multiplier of the debt limit.

Under complete markets of contingent claims, and assuming income shocks are idiosyncratic to the small open economy, the economy diversifies away all of its income risk. Consumption is constant and the economy's wealth is time- and state-invariant. The solution is akin to that of a perfect-foresight model with $\beta R = 1$ and wealth (the present value of income plus initial NFA) scaled to represent the same wealth as in the complete-markets economy.

With incomplete markets, the equilibrium differs because wealth becomes state-contingent and consumption is not perfectly smoothed. Equation (3) implies that $M_t \equiv \beta^t R^t u_c(c_t)$ forms a supermartingale, which converges almost surely to a non-negative random variable because of the Supermartingale Convergence Theorem (see Ljungqvist and Sargent, 2018, Chap. 18). If $\beta R \geq 1$, consumption and NFA diverge to infinity because marginal utility converges to zero almost surely, causing the non-stationarity problem that necessitated the DEIR function for local methods. The economy builds an infinitely large stock of precautionary savings and self-insurance sustains a consumption process for which M_t converges and $u_c(c_t) \geq \beta R E_t u_c(c_{t+1})$ holds. In contrast, if $\beta R < 1$, the economy has a well-defined stochastic steady state with finite unconditional means of assets and consumption. Intuitively, the opposing forces of the pro-saving incentive for self-insurance and the pro-borrowing incentive due to $\beta R < 1$ keep NFA moving within an ergodic set. If NFA falls (rises) too much the first (second) force prevails.

2.2. *Global methods*

For the global solution, we solve the model in recursive form over a discrete state space of (b, z) pairs using the *FiPIt* method of Mendoza and Villalvazo (2020).[12] The AR(1) income process is approximated with a discrete Markov chain with transition probability matrix $\pi(z', z)$. We solve for the NFA decision rule, $b'(b, z)$, which together with the shock Markov process produces a joint ergodic distribution of NFA and income $\lambda(b, z)$. The method solves for $b'(b, z)$ by iterating on a recursive representation of the Euler equation.

[12]This method is in the class of global methods that iterate on Euler equations, including endogenous grids, time iteration and projection methods (see Rendahl, 2015, for an overview). *FiPIt* performs better than time iteration and endogenous grids for models with two endogenous states and occasionally binding constraints because time iteration requires solving nonlinear Euler equation systems and endogenous grids require interpolation techniques for irregular grids. *FiPIt* solves Euler equations directly using linear interpolation.

The global method solves the model without imposing assumptions to induce stationarity. If $\beta R = 1$, NFA diverges to infinity, which is undesirable but is the equilibrium solution. However, $\beta R < 1$ is the relevant case because, as discussed above, it is implied by world general equilibrium. Note that with $\beta R < 1$ the *deterministic* stationary state converges to the debt limit, φ, with consumption falling at gross rate $(\beta R)^{1/\sigma}$. Hence, theory predicts that the unconditional mean of NFA in the stochastic, incomplete-markets model can differ significantly from the deterministic steady state and that the difference is due to precautionary savings.

2.3. Local methods

The local methods solve a local approximation of the optimality conditions (2)–(3) around the deterministic steady state, b^{dss}, for 1OA and 2OA or the risky steady state, b^{rss}, for RSS. Since assuming $\beta R = 1$ implies that b^{dss} depends on initial conditions and under uncertainty NFA diverges to infinity, 1OA and 2OA require a stationarity-inducing assumption. As documented earlier, the most common assumption is to introduce the DEIR function

$$R_t = R + \psi \left[e^{b^* - B_{t+1}} - 1 \right], \tag{4}$$

where b^* and ψ are parameters, with ψ determining the elasticity of R_t with respect to NFA, and B_{t+1} is the *aggregate* NFA position (i.e., treated as exogenous by agents). At equilibrium, $b_{t+1} = B_{t+1}$. Since DEIR applications assume $\beta R = 1$, (3) implies $b^{dss} = b^*$.

We implement 1OA, 2OA, and 3OA using Dynare 5.3 and RSS following Coeurdacier et al. (2011).[13] 1OA/2OA yield local approximations around b^{dss} by solving a first- or second-order approximation to the decision rules with same-order approximations to the model's optimality conditions. In contrast, RSS solves a linear approximation around b^{rss} and assumes $\beta R < 1$.

RSS takes into account future risk, so the center of approximation may better capture precautionary savings. The value b^{rss} is obtained from a second-order approximation to the conditional expectation of the steady-state Euler equation, solved jointly with the coefficients of a first-order approximation to the decision rules. This requires a conditional second-order ap-

[13]A detailed description of all the global and local methods we used is given in Appendix B.

proximation of the full equilibrium conditions' Jacobian, which implies third derivatives of those conditions. de Groot (2014) explains why the third derivatives are necessary to obtain stationary NFA dynamics. We also consider a variant of RSS in which b^{rss} is computed as above but is combined with the DEIR function and standard first-order approximations to the decision rules and equilibrium conditions to obtain stationarity. We denote the original as *full* and the DEIR alternative as *partial* RSS.

2.4. Calibration

Table 2 lists the parameter values of the baseline calibration, which is based on the calibration constructed by Mendoza (2010) using quarterly Mexican data at annualized rates for the 1993-2005 period. We use the same values of $\sigma = 2$ and $R = 1.086$. For the income process, we set $\sigma_z = 0.0272$ and $\rho_z = 0.749$ to match Mendoza's estimates of the standard deviation and first-order autocorrelation of the cyclical component of Mexico's GDP. The implied standard deviation of income innovations is $\sigma_\varepsilon = 0.01802$. The local methods use this income process directly. In the GA solution, we approximate it as a five-point Markov chain using the improved Tauchen and Hussey (1991) quadrature method developed by Flodén (2008).[14]

The mean NFA-GDP and consumption-GDP ratios in the Mexican data are also taken from Mendoza's calibration ($E(b/y) = -0.363$, $E(c/y) = 0.65$). Given the value of R and the resource constraint, the value of the autonomous spending share that captures investment and government absorption is $A = 0.32$.

In the global calibration, $\varphi = -0.435$ and $\beta = 0.917$ are set so as to match Mexico's mean NFA-GDP ratio of -0.363 and standard deviation of private consumption of 3.397%. Two parameters are required to identify this calibration, because while the mean NFA-GDP ratio can be matched by adjusting φ, this can result in a stochastic steady state in which the distribution of NFA is clustered near the debt limit and consumption fluctuates too much, or NFA has a high variance and consumption fluctuates too little.

[14]The Markov process is discrete with bounded support whereas the AR(1) is normally-distributed with unbounded support. However, Flodén (2008) showed that for other than highly-persistent shocks, Markov processes using quadrature methods match closely the unconditional moments of the AR(1) even with few nodes. In Appendix B.3.9 of de Groot et al. (2019), we showed that increasing the nodes to 11 gives near-identical results.

In the baseline calibration for the 2OA and partial RSS local solution (which use the DEIR function), we follow the standard practice of setting $\beta = 1/R$ so $b^{dss} = b^*$ and we use the inessential interest-rate elasticity $\psi = 0.001$. We set b^* so that the local solutions yield the same $E(b/y) = -0.363$ as in the GA solution and the Mexican data, this yields $b^{dss} = -0.724$ (-0.552) for the 2OA (partial RSS) solution. The full RSS solution uses the same β value as the GA solution and does not need a b^* value because it does not use the DEIR function. In the *targeted* calibrations, we set ψ so that the solutions match the standard deviation of consumption in the Mexican data. This yields $\psi = 0.042$ for both 2OA and partial RSS. As we show below, these targeted calibrations yield moments closer to those of the GA solution, which also approximate well the untargeted data moments. We also find, however, that once parametric changes are introduced (e.g., policy or counterfactual experiments), the results start to differ from the comparable GA solution.

2.5. Results

NFA decision rule and net exports Two key moments of open-economy models are the autocorrelations of NFA and net exports. The former because it is a key driver of the dynamics of capital flows and their cyclical co-movements with other macro variables, and the latter because of its relevance in the international RBC literature (e.g., Garcia-Cicco et al. (2010)). Hence, we start our comparison of solution methods by exploring their implications for these two key moments.

Assume for now that b_{t+1} follows an AR(1) process with autocorrelation coefficient ρ_b (as will turn out to be the case in the GA and local solutions). Since nx is a quasi first-difference of NFA ($nx_t = \frac{b_{t+1}}{R} - b_t$), the autocorrelation of net exports, ρ_{nx}, can be expressed as

$$\rho_{nx}(\rho_b) = \frac{\rho_b(1+R^2) - R(1+\rho_b^2)}{R^2 - 2R\rho_b + 1}. \tag{5}$$

In Appendix B.3.2, we prove that ρ_{nx} is increasing and convex in ρ_b. To get a sense of what the convexity implies, note that $\rho_{nx} \approx -0.5$ when $\rho_b = 0$ (since R is close to 1), turns positive when $\rho_b = 1/R$, and reaches $+1$ when $\rho_b = 1$. For $R = 1.06$, increasing ρ_b from 0.94 to 0.995 causes

ρ_{nx} to rise from 0 to 0.65. If ρ_b is close to 1, as is typical in incomplete-markets models, small differences in ρ_b induce large differences in ρ_{nx}. Thus, small errors in the local solutions for ρ_b can yield large errors in ρ_{nx} (and in the moments of other variables that depend on b).

The GA solution determines ρ_b as a moment of the joint stationary distribution $\lambda(b, z)$. The local solutions determine ρ_b by solving for the coefficients of the NFA local decision rule, and ψ and b^* are key determinants of these coefficients, as we show next.

The 2OA decision rule is given by

$$\tilde{b}_{t+1} = h_b \tilde{b}_t + h_z z_t + \frac{1}{2}\left(h_{bb}\tilde{b}_t^2 + h_{zz} z_t^2\right) + h_{bz}\tilde{b}_t z_t + \frac{1}{2} h_{\sigma_z \sigma_z}, \tag{6}$$

where $\tilde{b}_t \equiv b_t - b^{dss}$. The 1OA decision rule contains only the first two right-hand-side terms, with the exact same values of h_b and h_z. The RSS decision rules are of the same form as 1OA but with b^{rss} replacing b^{dss} and RSS-specific values of h_b and h_z. The coefficient of interest is h_b because it is the main determinant of ρ_b. This is the case even for 2OA solutions because in all our experiments the nonlinear terms—h_{bb}, h_{zz} and h_{bz}—are small.[15] The term $h_{\sigma_z \sigma_z}$ matters because it isolates the effect of income risk on mean NFA and thus captures precautionary savings in the 2OA solution. Since $h_{\sigma_z \sigma_z}$ is the only quantitatively relevant term that distinguishes 2OA from 1OA, their second- and higher-order moments are very similar.

For the RSS method, de Groot (2014) showed that income risk matters for determining b^{rss} because the coefficient of variation of consumption (relative to its risky steady state) is constant and depends on β, r and σ.[16] Intuitively, this captures precautionary savings because, if income risk rises and the shares of income allocated to savings remains unchanged, the volatility of consumption would rise. But, by increasing NFA relative to endowment income, more disposable income comes from interest income, so that the coefficient of variation of consumption can remain constant. However, since the RSS decision rule has a linear form, ρ_b differs from the 1OA solution only to the extent that b^{dss} and b^{rss} differ. As we show below, this re-

[15] Appendix B.3.3 shows the robustness of this result. In particular, h_{bb}, h_{bz}, and h_{zz} are irrelevant for the variance and autocorrelation of NFA for a range of ψ, σ and ρ_z values. For mean NFA, these terms are only important if ρ_z is high or ψ is very small.

[16] Corollary 5 in de Groot (2014) gives $\frac{var(c)}{(c^{rss})^2} = \frac{2}{\sigma(1+\sigma)} \frac{1-\beta R}{\beta R}$.

quires larger differences than those implied by our calibrations. Hence, 1OA, 2OA and partial RSS moments are likely to be very similar, except for their first moments.

Next, we show how ψ and b^* determine h_b. Assuming log-utility, an i.i.d income process, and $R_t = R\psi e^{b^*-B_{t+1}}$ for tractability, we obtain the following solution for h_b:

$$h_b(\psi, b^*) = \frac{R + e^{b^*\psi}(1 - b^*\psi + \psi) - \sqrt{R^2 + 2e^{b^*\psi}(b^*\psi + \psi - 1)R + e^{2b^*\psi}\left(1 - b^*\psi + \psi\right)^2}}{2e^{b^*\psi}}, \quad (7)$$

where $b^* = b^{dss}$ for 1OA/2OA and $b^* = b^{rss}$ for RSS. Since we find that h_{bb}, h_{zz} and h_{bz} are quantitatively irrelevant, it follows that $\rho_b(\psi, b^*) \approx h_b(\psi, b^*)$ for 1OA, 2OA and RSS. Hence, (7) describes how ψ and b^* determine the autocorrelation of NFA in local solutions. It also shows that the h_b obtained with 1OA/2OA differs from RSS only to the extent that b^{dss} and b^{rss} differ. Moreover, (7) shows that calibrating ψ implicitly imposes the value of ρ_b. In particular, given b^* and R, choosing a low ψ implies a ρ_b close to 1.[17] Finally, (7) illustrates the non-stationarity of the local solutions without a stationarity-inducing assumption. If $\psi = 0$, the solution of $h_b(\psi, b^*)$ has two roots, 1 and $R\,(> 1)$. In contrast (and assuming $b^* = 0$ for tractability), if $\psi > 0$ the smaller of the two roots is less than unity, and thus yields a stationary solution.[18]

To numerically study how variations in ψ and b^* alter ρ_b, we solve the calibrated endowment economy model with the 2OA and RSS methods to determine the value of ρ_b for $\psi \in [0, 0.5]$ and three values of b^*: 0, -0.552 (b^{rss}) and -0.724 (b^{dss}). The results, plotted in Figure 1, show that ρ_b is nearly identical across 2OA and RSS for any $0 \leq \psi \leq 0.15$, which includes both the baseline and targeted calibrations and the values used in all but one of the 76 articles using local solutions included in Table 1.[19] This is a key result, because it means that, for ψ values used in the literature, approximating around b^{dss} or b^{rss} or solving with 1OA, 2OA or partial RSS makes little difference. For $\psi \leq 0.03$, even solving with $b^* = 0$ makes little difference. Non-negligible differences between RSS and 2OA require $\psi > 0.15$ and/or large differences between b^{dss} and

[17]For RSS, the mapping is non-trivial since b^{rss} is solved jointly with the coefficients of the decision rule for b_{t+1}, which also depend on ψ.

[18]Schmitt-Grohé and Uribe (2003) show the same for an endowment model with ED preferences.

[19]The highest ψ was 2.8 from Garcia-Cicco et al. (2010), which they estimated for a model with financial frictions. We study later the implications of local solutions with higher ψ values.

b^{rss}. Moreover, since under the baseline and targeted calibrations the nonlinear terms of the NFA 2OA decision rule are small, we can expect the 2OA and RSS solutions to produce similar variances and correlations for all endogenous variables (as we show below).[20]

The above findings indicate that the implications of ρ_b for ρ_{nx} conjectured in (5) by *assuming* NFA follows an AR(1) apply to the *equilibrium* processes produced by the local methods. As we document next, the DEIR function with small ψ imposes values of ρ_b near 1, and small differences between them and the global solution result in large differences in ρ_{nx}. In contrast, in the global solution, ρ_b and ρ_{nx} are moments implied by the endogenous limiting distribution of NFA ($\lambda(b, z)$), the NFA decision rule ($b'(b, z)$), and the definition of nx.

Table 3 compares the global and local solutions of ρ_b and ρ_{nx} as ρ_z increases from 0 to $0.0.95$.[21] In general, 2OA and (partial) RSS always yield similar results for ρ_b and ρ_{nx}, because the gap between b^{dss} and b^{rss}, and the nonlinear terms in the 2OA decision rules, are too small to yield large differences.[22] Panel i) shows that for the GA solution, as ρ_z rises from 0.5 to 0.95, ρ_b rises from 0.957 to 0.997 while ρ_{nx} rises from 0.444 to 0.983. Thus, as (5) predicts, small variations in ρ_b near 1 cause large changes in ρ_{nx}. Panel ii), in contrast, shows that for the baseline local solutions ($\psi = 0.001$), ρ_b exceeds 0.999 and ρ_{nx} rises from about 0.87 to near 1. The GA results are always smaller and the differences are sizable. Panel iii) shows that the local solutions perform better with the targeted calibrations ($\psi = 0.042$, $b^* = -0.374$), particularly for ρ_z larger than the calibrated value of 0.75. For $\rho_z \leq 0.7$, they still yield higher values of ρ_b and ρ_{nx} than the GA solution. Panel iv) shows that the local solutions yield values of ρ_b and ρ_{nx} closer to those of the GA solutions if we re-calibrate ψ and b^* to match the mean NFA and standard deviation of consumption generated by the GA solution for each value of ρ_z. The required ψ values are, however, in the $0.047 - 0.126$ interval, far exceeding the 0.001 value that keeps the DEIR inessential and making deviations of NFA from steady state very costly, as we explain later. As a result, b^* needs to rise with ρ_z so as to track the increase in $E(b)$ produced by the GA

[20]While the analytic solution for $h_b(\psi, b^*)$ is for log-utility and i.i.d. shocks, the implications of the analysis hold quantitatively in solutions with AR(1) shocks.

[21]For the analysis in this table, we use Rouwenhorst (1995) method to create the Markov chain that approximates the income process, which is more accurate than the quadrature methods when persistence is high.

[22]The 1OA and 2OA solutions are near-identical, hence we omit 1OA from the table.

solution (see panel i)). Moreover, the problem arises that re-calibrating the model for each ρ_z value requires solving the model globally first.

Long-run moments. Table 4 compares unconditional cyclical moments, including also the moments from the Mexican data.[23] The GA solution yields moments that approximate well the data cylical moments, except for the countercyclical trade balance. This is a well-known limitation of endowment models that is corrected in models with investment and/or credit constraints (see Mendoza (1991, 2010)). With the baseline calibration, the local solutions poorly match both the GA and data moments.

The GA, 2OA and partial RSS solutions have the same mean NFA-GDP ratio of the Mexican data (-36.3%) by construction, since they were calibrated to match it. In contrast, the full RSS solution (which has the same $\beta R < 1$ condition as GA and does not use DEIR) has a much lower mean NFA-GDP ratio of -704.1%. This is because it lacks the debt-limit, φ, of the global solution.[24] As a result, it also overstates the variability of consumption.

As predicted by our analysis of the relationship between ρ_b and ρ_{nx}, and since NFA is a near-unit root process, the baseline local solutions have a slightly higher NFA autocorrelation than the GA solution but they yield a sharply higher autocorrelation of net exports. They also overstate the volatility of net exports and the volatility and persistence of consumption, and understate their GDP correlations.[25] Notably, given the literature's emphasis on explaining consumption volatility in emerging markets, all the baseline local solutions significantly overstate consumption volatility relative to GDP.

The local methods perform better at approximating most of the GA and data moments with the targeted calibration. They yield the same variability of consumption and mean NFA-GDP ratio by construction (ψ and b^* are set to match them), but most of the standard deviations, GDP-correlations, and autocorrelations are also much closer to their GA counterparts. The

[23]Since a key aspect of the model are its low frequency properties (e.g., precautionary savings), we report unfiltered moments. We also study spectral densities and show that the solution methods yield different volatility at both mid and low frequencies (see Appendix B.3.4).

[24]The GA solution without an ad-hoc debt limit (i.e., $\varphi = NDL$) has a mean NFA-GDP ratio of -650%, similar order of magnitude as in the full RSS solution, but yields much larger consumption variability than in the data.

[25]Since $\rho_b \approx h_b$ and the nonlinear terms are small, the volatility of b_t rises with ρ_b because $\sigma_b = h_z\sigma(z)/\sqrt{1-h_b^2}$ and the correlation with GDP falls because $\rho_{b,z} = (\rho_z/(1-h_b\rho_z))\sqrt{1-h_b^2}$.

autocorrelation of consumption, however, still overstates the GA and data moments.

2OA and partial RSS yield near-identical results for all moments under baseline or targeted calibrations. This is in line with our previous finding showing that the difference between b^{dss} and b^{rss} and the higher-order terms in the 2OA decision rules are quantitatively irrelevant.

Now we compare the solution methods in terms of their implications for precautionary savings. We examine their predictions for the value of $E(b/y)$ as the variance of the income process σ_z changes, keeping all other parameter values unchanged. Figure 2 shows that the local methods yield significantly different results compared with the GA solution. For 1OA (not shown), certainty equivalence implies no precautionary savings with the $E(b/y)$ remaining at b^{dss} for all values of σ_z (and of ψ). The solid-blue curve for the GA solution shows that increasing σ_z from 1% to 8% increases the mean NFA-GDP ratio from -42% to near 30%. In contrast, under the baseline calibration in Panel a., both 2OA (green-dot-dash curve) and partial RSS (red-dash curve) overstate the increase in precautionary savings, with a gap that widens as σ_z rises.[26] Note that, even though second- and higher-order moments of 2OA and RSS solutions are similar, $E(b/y)$ differs because b^{rss} is significantly smaller than b^{dss}.

The local methods with targeted calibrations in Panel b. yield almost no precautionary savings, with the mean NFA-GDP ratio barely rising above b^{dss}or b^{rss} as σ_z rises. Thus, while the targeted calibrations bring most second- and higher-order moments of the local solutions closer to the GA solution, they also remove precautionary savings, which then renders 2OA and RSS solutions approximately consistent with certainty equivalence and the 1OA solution.[27]

Table 3 shows that increases in ρ_z also alter mean NFA, indicating again that precautionary savings differ sharply across global and local solutions, and that this is due again to differences in the values of ρ_b they produce. As ρ_z rises from 0 to 0.95, the standard deviation of income rises from 1.8% to 5.8%. The GA solution predicts an increase in $E(b)$ from -0.41 to -0.26 compared with an increase from -0.69 (-0.53) to 1.36 (0.87) with the 2OA (RSS) baseline solution (see panels i) and ii)). Panel iv) shows that again the $E(b)$ stays close to b^* as ψ and

[26]The curves intersect at the calibrated value of $\sigma_z = 0.0272$ because the baseline and targeted calibrations are designed so that 2OA, RSS and GA solutions have the same mean NFA.

[27]Appendix B.3.3 shows this analytically for log-utility and i.i.d. shocks.

b^* are re-calibrated ($E(b)$ is the same as in panel i) because of the re-calibration of b^*).

The intuition for why mean NFA stays close to b^* in targeted calibrations follows from Schmitt-Grohé and Uribe (2003) who show that the DEIR setup is similar to a setup that uses instead quadratic costs, $\tilde{\psi}(b_{t+1} - b^{dss})^2/2$, for deviating from b^{dss}. The log-linear Euler equation of the setups are equivalent if $\tilde{\psi} = \psi/R$.[28] Moreover, by rewriting b_{t+1} as $E(b) + (b_{t+1} - E(b))$ and hence the cost function as $\tilde{\psi}((b_{t+1} - E(b)) + \big(E(b) - b^{dss}\big))^2/2$, it is clear the cost has variable and fixed components. If the fixed cost is larger than the benefit derived from precautionary savings, it is suboptimal to let mean NFA deviate from b^{dss}. Thus, local solutions using targeted calibrations have the shortcoming that a modest increase in ψ makes precautionary savings vanish and render 1OA, 2OA and RSS solutions near-identical.

Table 4 also shows execution times and Euler equation errors. The full RSS solves in 0.3 seconds because, given the simplicity of the endowment model, we can split the algorithm into a step that derives the non-linear system of equations in Mathematica and a step that solves it using Matlab. Partial RSS takes longer (3.1 seconds) because it does both steps within Matlab, building on a toolkit developed by Schmitt-Grohé and Uribe (2004). The Dynare 2OA solutions run in 0.6 seconds and the *FiPIt* GA solution in 1.46 seconds. Hence, the local methods are faster. However, the GA solution is more accurate, inasmuch as it yields much smaller maximum and mean Euler equation errors. Relaxing the *FiPIt* convergence criterion to yield Euler equation errors of similar magnitude as the local solutions lowers its execution time to 0.75 seconds, closer to the 2OA execution time.

Impulse response functions Figure 3 compares IRFs for a negative, one-standard-deviation income shock starting at the unconditional means. Consumption and output are shown in percent deviations from those means, while b/y and nx/y are in absolute deviations. The IRFs for 1OA (not shown), 2OA and RSS are near-identical, in line with the results that the h_b coefficients of NFA decision rules are similar and nonlinear terms of 2OA solutions are small.

The local solutions with the baseline calibration and the GA solution yield very different IRFs. GA predicts a smaller initial decline in the NFA-GDP ratio (i.e., less borrowing) and

[28] With DEIR, for $b_{t+1} < b^{dss}$ ($b_{t+1} > b^{dss}$) agents pay more (get less) for borrowing (saving) more.

much faster mean reversion (in about 50 periods instead of about 650). As a result, consumption falls nearly twice as much initially in the GA solution and also displays faster mean reversion. This also implies smaller trade deficits earlier on and faster convergence with smaller surpluses. Local solutions with targeted calibrations yield IRFs that are closer to the GA IRFs, but overstate the fall in consumption on impact and without a J-shaped response.

In Appendix B.3.4 we compared the global and local solutions in the frequency domain using nonparametric periodograms of simulated data. The local methods under the baseline calibration overstate the contribution of low frequency movements of b, c and nx relative to the GA solution, in line with the earlier findings of slow mean-reversion and higher ρ_b. For targeted calibrations, GA and local periodograms of b are more similar, but the local solutions still understate the relative contribution of consumption fluctuations at the business cycle frequencies to overall consumption variance.

Interest-rate shocks Next, we add interest-rate shocks to facilitate comparison with the Sudden Stops model of the next Section. It is also important because Coeurdacier et al. (2011) and de Groot (2014) show the RSS method yields higher precautionary savings with these shocks.

The gross interest rate is $R_t = e^{\nu_t} R$, where ν_t is an exogenous shock and R is the mean interest rate. The endowment and interest-rate shocks have a diagonal VAR representation

$$\begin{bmatrix} z_t \\ \nu_t \end{bmatrix} = \begin{bmatrix} \rho_z & 0 \\ 0 & \rho_r \end{bmatrix} \cdot \begin{bmatrix} z_{t-1} \\ \nu_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{z,t} \\ \varepsilon_{r,t} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma^2_{\varepsilon_z} & \sigma_{\varepsilon_z,\varepsilon_r} \\ \sigma_{\varepsilon_z,\varepsilon_r} & \sigma^2_{\varepsilon_r} \end{bmatrix}, \tag{8}$$

where Σ is the innovation variance-covariance matrix. The DEIR function takes the form

$$R_t = e^{\nu_t} R + \psi \left[e^{b^{dss} - B_{t+1}} - 1 \right]. \tag{9}$$

As in the original calibration, $\rho_z = 0.749$. To minimize the size of the state space in the GA solution, we use a bi-variate, two-point Markov chain defined by the *simple persistence rule*, which imposes the same autocorrelation on both shocks (see Appendix B.3.5 in de Groot et al. (2019)). Hence, $\rho_r = 0.749$. We also keep $\sigma_z = 0.0272$ ($\sigma_{\varepsilon_z} = 0.018$), as in the baseline calibration. For the interest-rate process, we solve the model with values of σ_{ε_r} and $\sigma_{\varepsilon_z,\varepsilon_r}$ such that σ_ν

takes values ranging from 0 to 2.5% and the correlation between income and the interest rate is $\rho_{z,R} = -0.669$, which matches the interest rate-TFP correlation in Mendoza (2010).

A well-defined limiting distribution of NFA requires $\beta R < 1$, otherwise $\beta^t \Pi_{j=1}^t R_j$ diverges to infinity (see Chamberlain and Wilson, 2000). In addition, there are long histories of realizations with R_t lower (higher) than R, which imply much weaker (stronger) precautionary savings incentives than with a constant interest rate. For example, histories with $\beta R_t > 1$ produce sequences where b_{t+1} can grow very large, since there is no pro-borrowing effect offsetting the precautionary savings incentive.[29] At some point, each of these histories shifts to histories with sufficiently low R_t to induce NFA mean-reversion. Note the NDL is computed with the highest realization of R_t, and so is tighter than when computed with R. Importantly, these effects are at work only in the GA solution, because they result from expectations of histories of future shocks that take the economy far from $E(b/y)$ and b^{dss}.

Table 5 shows key moments given by the various solution methods for $\sigma_\nu \in \{0, 0.5, \ldots, 2.5\}$. The baseline and targeted calibrations are as in Table 2. For the GA solution, we show results with both the calibrated ad-hoc debt limit ($\varphi = -0.435$) and the NDL, with the aim of comparing the roles of debt limits and interest-rate shocks in inducing higher mean NFA.

For the partial RSS and 2OA baseline calibration, we find the local solutions overstate sharply the increase in mean NFA in response to increased interest-rate risk relative to the GA solution. $E(b/y)$ increases by 58 (189) percentage points (pp.) for the partial RSS (2OA) solution and turns from negative to positive, while in GA it increases by 10pp. High interest-rate risk also alters the result that the RSS and 2OA solutions have similar second- and higher-order moments.

These findings suggest that interest-rate shocks in the local solutions with baseline ψ could be helpful for matching mean NFA, playing the role of φ in the GA model. This strategy, however, results in too much consumption volatility. The baseline local solutions (2OA and full and partial RSS) overstate consumption volatility relative to the baseline GA solutions for each value of σ_ν by wide margins.

For the targeted calibrations, the adjustment-cost-like effect of higher ψ keeping NFA close

[29]Reducing R while keeping σ_ν constant accentuates these effects, because histories with larger gaps between β and R_t are possible and with higher probability.

to b^{dss} dominates. The local solutions yield only small increases in $E(b/y)$ and second- and higher-order moments for RSS and 2OA are very similar. Hence, the result that higher ψ removes precautionary savings and yields very similar 1OA, 2OA and RSS local solutions is robust to adding interest-rate shocks.

Table 5 also shows that, with interest-rate shocks, full and partial (baseline) RSS do not yield similar second- and higher-order moments. Full RSS generates higher (lower) consumption (NFA) volatility, generally higher autocorrelation of nx, and much lower mean NFA-GDP ratio. In fact, full RSS is closer to the GA solution with the NDL than to the baseline or targeted partial RSS solutions. However, both full RSS and the GA solution with the NDL have the shortcoming of producing mean NFA-GDP ratios of -2 to -7. Moreover, if R is higher than the calibrated value of 1.086 such that βR is almost 1, full RSS yields a much lower mean NFA-GDP ratio than the GA solution with either the ad-hoc debt limit or NDL. Conversely, for low R, the RSS solution frequently violates the NDL. Hence, although at the calibrated R full RSS gets close to the mean NFA-GDP ratio of the GA solution with NDL, in general full RSS poorly approximates this model moment.

Endogenous discounting As shown in Table 1, ED preferences and AHC specifications of asset holding costs are alternatives to DEIR used to induce stationarity in local solutions. We showed earlier that AHC and DEIR are similar, because a higher ψ is akin to making NFA deviations from b^* costlier. Hence, we examine next the robustness of our findings to using the ED approach (Appendix B.3.5 provides full details). The ED approach assumes that the discount factor depends on aggregate consumption (i.e., private agents ignore this dependency).

First, we compare analytically DEIR and ED local decision rules assuming log-utility and i.i.d. shocks. In line with Schmitt-Grohé and Uribe (2003), DEIR and ED are equivalent to first-order: A nonlinear mapping determines the elasticity of the discount factor with respect to consumption (ψ^{ED}) for a given ψ such that the decision rules are the same. 2OA solutions, however, are not equivalent.[30] Varying ψ while adjusting ψ^{ED} so that the h_b coefficients using DEIR and ED are equal, yields a h_{bb} coefficient for DEIR that is increasing and concave in ψ

[30]Seoane (2015) compares approaches to induce stationarity using 3OA methods and, in line with our results, finds that different approaches generate large differences when calibrated to Argentina.

while that for ED is slightly decreasing and near-linear. The $h_{\sigma\sigma}$ coefficient is nearly invariant to r and ψ using DEIR, yet is decreasing and convex in ψ and sensitive to r using ED. These differences are due to a key difference between the two approaches: When consumption rises as the economy borrows, r_t rises using DEIR but β_t falls using ED. Hence, the marginal benefit of savings, $\beta_t(1 + r_t)u'(c_{t+1})$, rises in the DEIR solution but falls in the ED solution. The latter weakens the precautionary savings incentive in the ED solution relative to the GA solution with standard preferences and $\beta R < 1$, as Durdu et al. (2009) show.

Second, we compare quantitative results for the ED model using 2OA and GA methods (see Table 3 of Appendix B.3.5). We consider two GA cases. Case I is the baseline from Table 4, in which the "true model" is one with standard time-separable preferences and $\beta R < 1$. Case II corresponds to the global solution for a "true model" with ED preferences (GA-ED). In contrast with local ED solutions, in this global solution agents internalize the dependency of the discount factor on consumption. This introduces an "impatience effect," by which all future utility flows are discounted more heavily as today's consumption rises. These two cases are compared with two 2OA solutions: the baseline DEIR solution from Table 4 and an ED solution in which ψ^{ED} is calibrated to match the same $E(b/y)$ data target as the DEIR solution.

2OA-ED yields moments similar to those of the GA-ED solution because in this case the "true model" has preferences that support a well-defined deterministic steady state independent of initial conditions. It does not need a stationary-inducing transformation because it is stationary in its original form. A second-order approximation is enough to get close to the GA-ED solution because the impatience effect that is present in GA-ED but not in 2OA-ED is quantitatively small, in line with results obtained by Schmitt-Grohé and Uribe (2003).

In contrast, comparing local solutions vis-a-vis the GA solution for the model with standard preferences (i.e., using ED to induce stationarity), we find that 2OA-ED still yields different results although it is closer to the GA moments than the 2OA-DEIR solution. The variability of consumption, net exports and NFA still exceeds those of the GA solution but by smaller margins, and their autocorrelations and GDP-correlations are also closer to the GA outcomes.[31]

[31] The means are the same because the calibration of the three solutions targets the same data moments.

Examining how precautionary savings respond to higher income risk, however, the 2OA-ED solution yields much smaller increases in $E(b/y)$ than the GA solution (because of the self-correcting effect of the discount factor explained earlier). Thus, 2OA-DEIR overstates sharply precautionary savings while 2OA-DE does the opposite.

Exact-solution model Both local and global solutions are approximations, although we have showed that the latter is more accurate inasmuch as Euler equation errors are much smaller. Still, it is worth exploring how both solutions compare relative to an exact solution. For this purpose, consider two assumptions that allow us to solve the model in closed form: i) income is a multiplicative return on a risky asset with a log-normal i.i.d process; ii) consumption is chosen before the return is observed (see Appendix B.3.7 for details).[32] The analytic solutions are $c_t = \lambda(\sigma_\varepsilon)b_t$ and $b_{t+1} = (1-\lambda(\sigma_\varepsilon))R_t b_t$, where the savings rate is given by

$$1-\lambda(\sigma_\varepsilon) = \beta^{1/\sigma} E(R)^{\frac{1-\sigma}{\sigma}} \exp\left(-(1-\sigma)\frac{\sigma_\varepsilon^2}{2}\right). \tag{10}$$

If $\sigma > 1$, the precautionary savings effect is evident since a mean-preserving increase in the volatility of R_t (i.e., higher σ_ε keeping $E(R)$ constant) increases the savings rate. NFA (in logs) follow a random walk with drift, $\ln(b_{t+1}) = \ln(1-\lambda(\sigma_\varepsilon)) + \ln(b_t) + \ln(R_t)$, and hence so does consumption. However, consumption growth is a log-i.i.d. process: $c_{t+1}/c_t = (1-\lambda(\sigma_\varepsilon))R_t$.

Appendix B.3.7 implements GA, the local solutions up to fourth-order (4OA), and RSS by expressing the model in ratios of b_t. We set $\beta = 0.94$, $E(R) = 1.7$ (in line with the assumption that b is a risky asset that provides all the economy's income), and vary σ_ε from 0 to 0.45 while keeping $E(R)$ constant.[33] The exact, GA, and 4OA solutions are virtually identical for all values of σ_ε. The accuracy of RSS and 2OA deteriorates for $\sigma_\varepsilon > 0.3$, understimating the savings rate by up to 15% and 5%, respectively. Moreover, for $\sigma_\varepsilon > 0.45$, 2OA and RSS predict feasible saving rates when the true solution is unfeasible.

Since this model is non-stationary, ρ_b and the center of approximation of NFA do not con-

[32] The resource constraint becomes $b_{t+1} = R_{t+1}(b_t - c_t)$, where $\log(R_t) = \mu + \sigma_\varepsilon \varepsilon_{t+1}$, $\varepsilon_{t+1} \sim N(0,1)$. Note that b is now a risky asset.

[33] For $\sigma_\varepsilon > 0.45$ the equilibrium is infeasible with $\lambda(\sigma_\varepsilon) < 0$.

tribute to the inaccuraccies of the local solutions. In this model, the inaccuracies of 2OA and RSS are only a result of expanding the Euler equation. Hence, a higher-order approximation such as 4OA improves accuracy in this case. However, for the canonical endowment model we studied earlier this is not the case because the problems result from pinning down ρ_b and the center of approximation, which continue to be a problem even at higher-order approximations.

3. Sudden Stops model

This section compares global and local solutions of the model of Sudden Stops in a small open economy proposed by Mendoza (2010). This is an RBC model augmented with an occasionally binding collateral constraint. We compared solutions of the RBC model itself in an earlier version of this paper (see de Groot et al. (2019)) and found that local solutions have similar problems with regard to NFA, net exports, and consumption as in the endowment model. Supply-side variables are similar in local and global solutions because there is no wealth effect on labor supply and the equity premium is small.[34]

3.1. Model structure

The model's competitive equilibrium is represented as the solution to a representative firm-household problem. Gross output is produced with a Cobb-Douglas technology using capital, k_t, labor, L_t, and imported inputs, υ_t.

$$e^{z_t} F(k_t, L_t, \upsilon_t) = e^{z_t} k_t^{\gamma} L_t^{\alpha} \upsilon_t^{\eta}, \quad 0 \leq \alpha, \gamma \leq 1, \quad \eta = 1 - \alpha - \gamma. \tag{11}$$

Gross output is a tradable good sold at a world-determined price which is the numeraire and set to 1. The relative price of imported inputs is also world-determined and given by $p_t = e^{u_t}\bar{p}$, where $\bar{p}$ is the mean price and u_t is a terms-of-trade shock. The model also includes TFP (z_t) and interest-rate (ν_t) shocks. A standard working capital constraint requires a fraction ϕ of the cost of L_t and υ_t to be paid in advance of sales. Working capital loans are obtained from

[34] As Mendoza (1991) noted, these features render the capital decision rule similar to that implied by the risk-neutral arbitrage of returns on capital and NFA, which implies the Fisherian separation of investment from consumption and savings decisions nearly holds. Hence, in the capital decision rule, the coefficient on lagged NFA in the local solutions and the elasticities of k' with respect to b in the GA solution are negligible.

foreign lenders at the beginning of each period and repaid at the end, so the financing cost of inputs is the net interest rate, $R_t - 1$. Capital is costly to adjust, with adjustment costs per unit of net investment, $k_{t+1} - k_t$, given by $\Psi(\frac{k_{t+1}-k_t}{k_t}) = \frac{a}{2}\left(\frac{k_{t+1}-k_t}{k_t}\right)$, with $a \geq 0$. This functional form satisfies Hayashi's conditions so average and marginal Tobin's Q are equal in equilibrium.

The representative firm-household chooses $[c_t, L_t, i_t, \upsilon_t, b_{t+1}, k_{t+1}]_{t=0}^{\infty}$ to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\sigma}}{1-\sigma} \right\}, \tag{12}$$

subject to

$$c_t(1+\tau) + i_t = e^{z_t} F(k_t, L_t, \upsilon_t) - p_t \upsilon_t - \phi(R_t - 1)(w_t L_t + p_t \upsilon_t) - \frac{b_{t+1}}{R_t} + b_t, \tag{13}$$

$$\frac{b_{t+1}}{R_t} - \phi R_t (w_t L_t + p_t \upsilon_t) \geq -\kappa q_t k_{t+1}. \tag{14}$$

The utility function is of Greenwood-Hercowitz-Huffman (GHH) form, which removes the wealth effect on labor supply. The market prices of labor and capital, denoted w_t and q_t, are taken as given by the agent. The left-hand-side of the resource constraint (13) is the sum of consumption, inclusive of an ad-valorem tax τ used to calibrate the ratio of government expenditures to GDP, plus gross investment, i_t, where $i_t = \delta k_t + (k_{t+1} - k_t)\left[1 + \Psi\left(\frac{k_{t+1}-k_t}{k_t}\right)\right]$ and δ is the depreciation rate. The right-hand-side equals total supply, which consists of GDP, $y_t \equiv e^{z_t} F(k_t, L_t, \upsilon_t) - p_t \upsilon_t$, net of foreign interest payments on working capital loans, $\phi(R_t - 1)(w_t L_t + p_t \upsilon_t)$, minus net resources lent abroad, $\frac{b_{t+1}}{R_t} - b_t$. Net exports is given by $nx_t = \frac{b_{t+1}}{R_t} - b_t + \phi(R_t - 1)(w_t L_t + p_t \upsilon_t) = y_t - c_t(1+\tau) - i_t$. The Fisherian collateral constraint (14) prevents debt and working capital credit exceeding a fraction κ of the market value of capital.

The competitive equilibrium is defined by sequences of allocations $[c_t, L_t, k_{t+1}, b_{t+1}, \upsilon_t, i_t]_0^{\infty}$ and prices $[w_t, q_t]_0^{\infty}$ such that (a) the representative firm-household solves its optimization problem given $[w_t, q_t]_0^{\infty}$ and initial conditions (k_0, b_0), and (b) $[w_t, q_t]_0^{\infty}$ satisfy the corresponding market equilibrium conditions.

3.2. Solution methods

Solutions of this model involve an occasionally binding constraint and an additional endogenous state variable, k_t. For the global solution, we use *FiPIt* defining grids of k and b with 30 and 80 nodes, respectively.[35] For the quasi-local (QLOBC) method, we use the DynareOBC toolkit. This toolkit treats the occasionally binding constraint as a source of endogenous news about the future along perfect-foresight paths (see Appendix B.3.6 for details). If the constraint is (is not) binding at the deterministic steady state, the algorithm uses news shocks to solve for unconstrained (constrained) periods along those paths by solving a mixed-integer linear programming problem. Suppose the constraint does not bind at steady state. If agents anticipate the constraint will bind at $t + j$ conditional on the date-t state variables, this provides "news" that b_{t+1} will follow a path higher than otherwise. This approach is akin to assuming that there is no constraint, but whenever agents are on a path that would lead them to borrow more than the constraint allows, a series of news shocks hit that makes them borrow only what is allowed and moderates their borrowing before that happens.[36]

The main output of DynareOBC is a time-series simulation constructed by stitching together the date-t values of perfect-foresight paths conditional on $(k_t, b_t, z_t, u_t, \nu_t)$. Each path is obtained using an extended path algorithm that traces equilibrium dynamics up to period $t + T$. The extended path can be obtained using first- or higher-order approximations, but we report only results based on the former.[37] The path computed for a given starting date t determines the values of (k_{t+1}, b_{t+1}). The rest of the path is discarded and the process is repeated at $t + 1$ to generate the values of the time-series simulation for that period.

The efficiency of this method depends on three factors: (a) T: This parameter needs to be large enough so that after T no further news shocks are needed (if the constraint does (does not) bind in steady state, after T the constraint must always (never) bind). A model with

[35] See Mendoza and Villalvazo (2020) for details, including User Guide and Matlab code.

[36] The model is similar to the model without the constraint but with sequences of news shocks chosen to yield the same equilibrium as the model with the constraint. This equivalence holds exactly if the model is linear and shock variances are zero, such that the news shocks are unanticipated.

[37] Holden (2016) shows that using 2OA and integrating over future uncertainty can approximate precautionary savings in an endowment model with a simple constraint. However, this method is slower than the global method and for our SS model produced results that deviate sharply from the GA and first-order DynareOBC solutions. In particular, investment and net exports had negative serial autocorrelation and NFA had near-zero autocorrelation.

persistent dynamics requires a larger T and a larger T increases the search time for the sequence of shocks that supports the equilibrium; (b) Frequency of binding constraint: In each period for which the perfect foresight path requires news shocks, the search for the equilibrium sequence of news shocks needs to be repeated. A model in which the constraint binds frequently requires more time-costly searches; and (c) Time-series simulation length, N: This parameter needs to be large enough for long-run moments of the endogenous variables to converge. The algorithm is therefore less efficient in models with persistent dynamics (requiring a large T and N), and models in which the news shocks are needed frequently.

Figure 4 illustrates the DynareOBC method using the endowment model, with $b_{t+1} \geq \varphi$ as an occasionally binding constraint.[38] Panels (a)-(b) show a stochastic simulation for c_t and b_{t+1} for $t = 90$ to 250 (black-solid lines) and eleven of the perfect-foresight paths (red-dash lines) with the corresponding date-t solution (red circle). In Panel (b), the constraint binds in four of the perfect-foresight paths (the shaded area corresponds to $b_{t+1} < \varphi$). Panels (c)-(d) isolate the path that defines the equilibrium in $t = 141$. The comparable path of b_{t+1} without the collateral constraint is the black-dot line in (d). The constraint first binds along the perfect-foresight path at $t = 144$. Relative to the model without the constraint, agents choose higher b_{t+1} (less debt) earlier, in anticipation of the constraint becoming binding with perfect foresight (i.e., the red-dashed curve is above the black-dotted curve at $t = 142, 143$). Since income rises gradually back to steady state, the constraint continues to bind for several periods, until income is high enough for b_{t+1} to also rise back towards steady state (after $t = 170$).

Quasi-linear methods ignore the *risk* of moving between regions of the state space where the constraints binds or not. In particular, at each t, DynareOBC only considers the perfect-foresight path conditional on the date-t state and ignores the histories of future shocks and associated allocations and prices that can occur. Hence, wealth and precautionary-saving effects of the constraint are ignored, and forward-looking objects like asset prices and excess returns also abstract from them. These effects are central to SS models, because when the collateral constraint binds, a sudden stop with a deep recession and collapsing prices can occur.

[38]DEIR is used since the constraint does not bind in the steady state (see Appendix B.3.6 for details).

The risk of a sudden stop strengthens precautionary savings and is priced in asset markets. (see Mendoza, 2010; Durdu et al., 2009).

3.3. Calibration

Table 6 shows the calibration parameters, most of which were taken from Mendoza (2010). The main difference is that φ and β in the GA solution are set following a strategy similar to that used in the endowment model, by targeting them so the RBC version of the model approximates the mean NFA-GDP ratio and the volatility of consumption in Mexican data.[39]

The three shocks have a diagonal VAR representation given by

$$\begin{bmatrix} z_t \\ \nu_t \\ u_t \end{bmatrix} = \begin{bmatrix} \rho_z & 0 & 0 \\ 0 & \rho_r & 0 \\ 0 & 0 & \rho_p \end{bmatrix} \cdot \begin{bmatrix} z_{t-1} \\ \nu_{t-1} \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{z,t} \\ \varepsilon_{r,t} \\ \varepsilon_{p,t} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_z^2 & \sigma_{z,r} & 0 \\ \sigma_{z,r} & \sigma_r^2 & 0 \\ 0 & 0 & \sigma_p^2 \end{bmatrix}. \tag{15}$$

Following empirical evidence in Mendoza (2010), the co-movement between TFP and interest-rate shocks is driven only by the covariance of their innovations and the price shock is independent of the other two. The calibration of the autocorrelation and variance-covariance matrices also follows Mendoza (2010). The discrete approximation to the VAR in the GA solution is constructed using the Simple Persistence Rule, requiring $\rho_z = \rho_r$ (see Appendix C.2 in de Groot et al. (2019) for details).

For the quasi-linear method, the steady-state equilibrium is well-defined without the DEIR function when the constraint binds at the deterministic steady state. The bonds Euler equation becomes $1 = \beta R + \mu^{dss}/u'(c^{dss})$, where μ is the multiplier on the constraint. Since $\beta R < 1 \iff \mu^{dss} > 0$, having the constraint bind at steady state requires $\beta R < 1$. When the constraint does not bind at the deterministic steady state, the DEIR function is used to induce stationarity.

We study DynareOBC solutions with $\mu^{dss} > 0$ (labeled "DynareOBC-$\beta R < 1$") and $\mu^{dss} = 0$ (labeled "DynareOBC-DEIR"). For the former, β is the same as in the GA solution, and hence the DynareOBC-$\beta R < 1$ and GA calibrations are identical. For the latter, $\beta = 1/R$ by construc-

[39]This was necessary because we use standard preferences with $\beta R < 1$ while Mendoza (2010) used ED preferences allowing agents to consider the dependency of the discount factor on the history of consumption.

tion and we calibrate ψ and $(b/y)^*$ in two steps. First, we set them so as to match the data moments for the mean NFA-GDP ratio and the standard deviation of consumption (relative to that of GDP) in a 1OA solution of an RBC variant of the model without the credit constraint.[40] These same moments are also matched by the calibration of the GA-RBC solution used in the GA solution of the Sudden Stops model. Second, in line with the "targeted" approach to calibrate DEIR solutions, we adjust b^* so that the DynareOBC-DEIR solution yields the same $E(b/y)$ as the GA solution of the Sudden Stops model. This procedure yields $(b/y)^* = -0.008$ and $\psi = 0.0044$. The rationale for looking at DynareOBC-DEIR is that in the GA solution the constraint rarely binds and $E(b/y) > b^{dss}/y^{dss}$. Hence, a local approximation around an unconstrained steady state is in line with the unconstrained long-run equilibrium of the GA solution.

3.4. Results

Long-run moments Table 7 shows that several moments of the DynareOBC solutions differ sharply from their GA counterparts, with smaller differences for supply-side variables.[41] The latter occurs because, around the stochastic steady state, the model is still close to Fisherian separation of savings and investment, as is the case for the RBC version of the model.

The collateral constraint causes a large increase in precautionary savings that DynareOBC-$\beta R < 1$ understates significantly. Relative to a mean NFA-GDP ratio of -37% in the RBC model, the GA solution of the Sudden Stops model yields 1.5% and DynareOBC-$\beta R < 1$ yields -13.5% (DynareOBC-DEIR has the same mean NFA as GA because it was calibrated to match it). This has implications for both research and policy. For example, quantifying optimal macroprudential regulation or foreign reserves to manage Sudden Stop risk first requires determining how NFA responds to this risk without policy intervention (see Durdu et al., 2009; Bianchi and Mendoza, 2018). By underestimating precautionary savings without policy intervention, the DynareOBC solution would result in excessive accumulation of reserves and macroprudential regulation that is too tight.

Certainty equivalence does not hold in the quasi-linear solutions even though the perfect-

[40] We used 1OA to be consistent with the first-order DynareOBC algortihm we used.

[41] GA and DynareOBC-DEIR yield the same means by construction, because the latter was calibrated to yield the same $E(b/y)$ as the former.

foresight paths are first-order approximations. In the DynareOBC-$\beta R < 1$ (DynareOBC-DEIR) solution, $b^{dss}/y^{dss} = -0.192$ (-0.008) while $E(b/y) = -0.135$ (0.015). This is due to asymmetric responses to shocks induced by the constraint, not precautionary savings. This asymmetry is illustrated in Figure 4 (see also Appendix B.3.6). A negative shock that causes the constraint to bind along the perfect-foresight path determining the date-t value of the solution reduces b_{t+1} by less than the increase in b_{t+1} in response to the same size positive shock. Hence, the quasi-linear time-series is "biased" above b^{dss}, implying a mean above b^{dss}/y^{dss}.[42] The global solution has a similar asymmetry but it also has precautionary savings effects due to the risk of future shocks causing the constraint to bind.

Performance metrics Table 7 also reports performance metrics of the different solution methods. In the model with the occasionally binding collateral constraint, there is no longer a clear speed advantage to using local methods. Relative to the *FiPIt* global solution, DynareOBC-$\beta R < 1$ is about 9% faster but DynareOBC-DEIR is 24% slower. This is due to the three determinants of the efficiency of DynareOBC noted earlier and the near-unit-root nature of the NFA process. Each extended path required at least 60 periods and the full simulation needed 10^6 periods to converge to invariant moments.[43]

Speed comparisons need to be made carefully. Global methods suffer from the curse of dimensionality and solutions are slower in models that require a root-finder when the constraint binds.[44] But, once the decision rules are solved, generating time-series simulations is fast. In contrast, the number of state variables is not an issue for QLOBC methods, but execution time rises with the length of perfect-foresight paths; the iterations needed to compute news-shocks sequences that implement the constraint; and the length of the time-series simulation needed for convergence of unconditional moments. In Appendix C.2.2, we show DynareOBC-$\beta R < 1$ becomes slower than *FiPIt* with a simulation length of 1.5×10^6 periods $(350$ v. 268 seconds$)$, TFP shocks only $(230$ v. $42)$, or $\kappa = 0.3$ $(228$ v. $137)$. Using DynareOBC with a second-order

[42]The constraint in this example is a fixed debt limit while in the SS model it depends on $q_t k_{t+1}$.

[43]The estimators of the mean and autocorrelation of an AR(1) process are consistent but biased in finite samples. The bias is higher the closer the true autocorrelation is to 1 but falls as the sample size rises. A near-unit-root process needs a long sample to ensure negligible estimation bias.

[44]In the SS model without working capital in the constraint, this is not needed, reducing the *FiPIt* run time by 57% (see Mendoza and Villalvazo, 2020).

approximation and/or integrating over future uncertainty would further increase run times.

In terms of accuracy, the global solution produces small maximum errors in the bonds and capital Euler equations. Since QLOBC solutions only produce time-series simulations, we follow Holden (2016) to evaluate their accuracy by constructing consumption simulations of the GA solution for the same initial conditions and sequence of shocks as in the DynareOBC solutions, and compute the maximum absolute values of the differences across them. The maximum differences are about 0.19% for both DynareOBC-$\beta R < 1$ and DynareOBC-DEIR, much larger than Holden's 0.0038% estimate from the solution of an endowment model with debt limit without integrating over future uncertainty (see Fig. 2 in Holden (2016).

Impulse responses & periodograms Figure 5 shows IRFs for a one-standard deviation, negative TFP shock conditional on starting at the unconditional means. The DynareOBC IRFs differ significantly from the GA ones. With DynareOBC-$\beta R < 1$, the NFA-GDP ratio hardly moves and the NX-GDP ratio moves into a surplus on impact, reflecting reduced demand for imported inputs. This occurs because the constraint binds at date-0 and the TFP shock tightens the constraint more. For DynareOBC-DEIR, the NFA-GDP ratio falls, offsetting the fall in imported inputs, leaving the NX-GDP ratio almost unchanged. In contrast, in the GA solution, the NX-GDP ratio jumps on impact nearly twice as much as under DynareOBC-$\beta R < 1$ and the NFA-GDP ratio rises gradually to peak roughly 1.5 pp. above its mean. For capital, in both quasi-linear solutions it falls on impact and falls slightly further before recovering. In contrast, capital in the GA solution is nearly unchanged on impact before falling around three times as much, reaching 1.5% below its mean before recovering. Qualitatively, the responses of c, i, L, v and y are similar in all solutions, but the declines on impact are larger in the GA solution.[45]

Appendix C.2.1 compares periodograms for the DynareOBC and GA solutions and shows that they differ sharply. DynareOBC assigns significantly less consumption variability to business cycle and lower frequencies than the GA solution. Net exports show higher persistence

[45]For the GA solution, the IRFs of the RBC and SS models are very similar, because the constraint binds only in the left tail of the ergodic distribution (see Appendix C.2.3). Hence, IRFs, which are triggered by shocks of standard magnitudes and start from long-run means, are nearly unaffected by the credit friction. In contrast, the IRFs for the SS model obtained with DynareOBC-$\beta R < 1$ are different from the IRFs that the local methods produce for the RBC model.

in the DynareOBC-DEIR solution while DynareOBC-$\beta R < 1$ and GA have similar persistence. The GA solution has less variability at all frequencies. Investment has higher persistence in the GA than in the local solutions, and it has uniformly higher variability at all frequencies.

Collateral constraint multipliers, Sudden Stops, and risk effects The DynareOBC-$\beta R < 1$ and GA solutions also differ sharply in that the collateral constraint binds much more frequently in the former (20% instead of 2.6% of the time).[46] This is partly because QLOBC methods disregard precautionary savings. Moreover, these methods yield smaller credit-constraint multipliers and financial premia than the GA solution, and the sudden-stop responses of macro variables differ sharply. To demonstrate these results, we compare the multipliers and the shadow interest-rate premium (SIP), the equity premium (EP), its components due to unpledgeable capital ($(1-\kappa)SIP$) and risk premium (RP), and the Sharpe ratio (S). For macro responses in sudden-stop episodes, we compare deviations from unconditional means in c, nx/y, i, y, L and v.

SIP_t is the amount by which the intertemporal marginal rate of substitution, $u_{c,t}/\beta E_t u_{c,t+1}$, exceeds R_t. The bonds Euler equation gives

$$SIP_t = \frac{R_t \mu_t (1+\tau)}{u_{c,t} - \mu_t(1+\tau)}. \tag{16}$$

SIP_t is only relevant when $\mu_t > 0$ and rises as the constraint becomes more binding, because μ_t rises and $E_t u_{c,t+1}$ falls, since the constraint forces agents to defer consumption.

The equity premium is $EP_t \equiv E_t[R^q_{t+1}] - R_t$, where $R^q_{t+1} \equiv (d_{t+1} + q_{t+1})/q_t$ is the return on equity and d_{t+1} is the dividend payment, where $d_t \equiv \exp(\epsilon^A_t)F_{k,t} - \delta + \frac{a}{2}\frac{(k_{t+1}-k_t)^2}{k_t^2}$. Using the Euler equations for bonds and capital it follows that

$$EP_t = (1-\kappa)SIP_t + RP_t, \qquad RP_t \equiv -\frac{COV_t[u_{c,t+1}, R^q_{t+1}]}{E_t u_{c,t+1}}. \tag{17}$$

EP_t has two components: the standard risk premium (RP_t) driven by $COV_t[u_{c,t+1}, R^q_{t+1}]$ and

[46]DynareOBC-DEIR has a similar frequency of binding constraint as the GA solution because it was calibrated to the seam mean NFA and with $\mu^{dss} = 0$.

the fraction of SIP_t pertaining to the share of k_{t+1} that cannot be pledged as collateral $(1-\kappa)$. EP_t rises when $\mu_t > 0$ for two reasons: First, SIP_t rises, as explained above. Second, RP_t rises, because $COV_t[u_{c,t+1}, R^q_{t+1}]$ becomes more negative as consumption is harder to smooth and $E_t u_{c,t+1}$ falls as the collateral constraint forces consumption into the future. Thus, EP_t reflects both the tightness of the constraint via SIP_t and the larger *risk* premium that the constraint induces. The Sharpe ratio measures the compensation for risk-taking, defined as $S_t = E[EP]/\sigma(R^q)$. Following standard practice, we compute S_t using unconditional moments.

For the GA solution, the financial premia are computed for each triple (b, k, ε) in the state space (see Appendix C.2.3). Means are then computed using the conditional and unconditional distributions of (b, k, ε). For QLOBC, SIP_t is computed using the time-series simulations that DynareOBC produces. Since $RP_t = 0$ by construction, because each date-t solution is determined by a perfect-foresight path, the equity premium is $EP_t = (1-\kappa)SIP_t$ (the DynareOBC simulations also produce very small values for $COV[u_c(\cdot), R^q]$).

Table 8 reports quintile distributions of μ conditional on $\mu > 0$, the associated within-quintile averages of financial and macro variables, their overall means and medians, and the Sharpe ratios.[47] Consider first the multipliers and financial premia. Results are similar across DynareOBC-DEIR and DynareOBC-$\beta R < 1$. Relative to the GA solution, however, the multipliers and financial premia are markedly smaller in the quasi-local solutions, and the differences grow larger for higher μ (i.e., in the fourth and fifth quintiles). Overall, the mean (median) of μ across all quintiles is about 35.6% (24.6%) larger in the GA solution. For GA, the overall means of SIP, EP, $(1-\kappa)SIP$ and RP are $2.59, 2.17, 2.07$, and 0.1%, respectively, while DynareOBC-$\beta R < 1$ (DynareOBC-DEIR) yields $1.54, 1.23, 1.23, 0$ $(1.42, 1.14, 1.14, 0)$. In the GA solution, RP is about 0.1% on average in each of the five quintiles of μ, but EP increases sharply with μ because $(1-\kappa)SIP$ rises sharply. In the fifth quintile, GA yields means for SIP, EP, and $(1-\kappa)SIP$ of $6.59, 5.38$, and 5.27%, respectively, while DynareOBC-$\beta R < 1$ (DynareOBC-

[47]Variables are assigned into quintiles according to the quintile distribution of μ. If a given μ_i belongs to a particular quintile of μ, then the corresponding values of the other variables are assigned to the same quintile. μ is small in general because it is in units of marginal utility with $CRRA$ preferences and $\sigma = 2$. For instance, at the unconditional means of c and L, marginal utility is -4.688 in log base 10. But small μ values do not imply that the constraint is irrelevant for financial and macro outcomes, as Table 8 shows.

DEIR) yields $3.52, 2.82, 2.82$ $(2.94, 2.36, 2.36)$. Thus, the quasi-local solutions understate SIP and EP. They also miss the risk premium, but this accounts for a small fraction of the gap in EP. Since RP is small in the GA solution and zero in the local solutions, the differences in S are due to SIP. The compensation for risk-taking is also much higher in the GA solution, which yields a Sharpe ratio of 1.16, compared with 0.08 and 0.72 for DynareOBC-$\beta R < 1$ and DynareOBC-DEIR, respectively.

The sizable differences in SIP and EP result in different sudden-stop responses. To explain why, we follow Mendoza and Smith (2006) in expressing the price of capital as

$$q_t = E_t \left(\sum_{i=1}^{\infty} \left[\prod_{j=0}^{i} \frac{1}{E_t R^q_{t+1+j}} \right] d_{t+1+i} \right). \tag{18}$$

Since (17) implies $E_t R^q_{t+1} = (1-\kappa) SIP_t + RP_t + R_t$, lower financial premia with QLOBC implies higher q_t when $\mu_t > 0$, which in turn implies weaker Fisherian deflation effects of the binding collateral constraint. Moreover, since q_t is a monotonic function of investment due to the Tobin-Q investment setup, k_{t+1} is higher and so is borrowing capacity $(\kappa q_t k_{t+1})$, which is key for determining allocations when $\mu_t > 0$. This also affects future dividends, creating feedback effects into q_t and borrowing capacity.

The differences in sudden-stop responses reported in Table 8 reflect the above arguments. In the GA solution, the responses are in line with standard Sudden Stop features (i.e., large recessions and sharp reversals in the external accounts). The mean percent declines in c, i, y, L, and v (relative to their unconditional means) are -3.6, -4.1, -1.0, -0.7, and -1.8, respectively while nx/y rises 2.6 pp. on average. The responses are generally larger when the constraint binds more, reaching means of -4.9 for c and -13.5 for i and a trade balance reversal of 5.1 pp. in the fifth quintile of μ. DynareOBC-$\beta R < 1$ yields smaller mean declines in consumption (-1.03), investment (-0.53), GDP (-0.48), labor (-0.29) and inputs (-1.00) and a smaller mean increase in net exports (0.40). It also fails to match the property that the responses should be larger when the constraint binds more, displaying instead the largest responses in the first quintile of μ. DynareOBC-DEIR performs worse, producing *positive* mean responses for i, y, L and v and smaller mean decline in c. Moreover, these counterfactual responses grow larger when the constraint binds more, in the fourth and fifth quintiles of μ.

There are tradeoffs in choosing between DynareOBC-$\beta R < 1$ and DynareOBC-DEIR. Both yield solutions that differ sharply from the GA solution, but Dynare-$\beta R < 1$ does better at approximating the effects of the collateral constraint, uses the same calibration as the GA solution and does not require extra assumptions to impose stationarity. On the other hand, DynareOBC-DEIR yields unconditional moments and a frequency of hitting the collateral constraint that are closer to the GA solution. The inability to produce Sudden Stops when the constraint binds, however, is an important shortcoming of DynareOBC-DEIR.

4. Conclusions

In this paper, we compared global and local solutions of open-economy models with incomplete markets using an endowment model and a model of Sudden Stops with an occasionally binding collateral constraint (de Groot et al. (2019) examined also an RBC model with similar qualitative findings as those reported here). Local solutions were produced using 1OA, 2OA, 3OA, RSS and DynareOBC algorithms and global solutions were generated using the *FiPIt* algorithm. Most local methods need a stationarity-inducing assumption, for which we chose the widely-used DEIR function that makes the interest rate a decreasing function of NFA.

We found large differences between global and local solutions relative to untargeted data moments and when we examined parametric changes. In particular, local methods approximate poorly the effects of precautionary savings on NFA, net exports and consumption, even when using higher-order methods such as 2OA and RSS. For the Sudden Stops model, quasi-linear methods have two additional disadvantages: they understate the magnitude of the multipliers of the collateral constraint and its effects on financial premia and macro variables, and they do not capture *risk* effects of the collateral constraint and their implications for precautionary savings and forward-looking variables like asset prices.

Standard local methods (such as 2OA) are faster than *FiPIt* for the endowment and RBC models, but the variation of the RSS method with the DEIR function is slower. *FiPIt* yields significantly smaller Euler equation errors, but the curse of dimensionality remains a limitation. For the Sudden Stops model, *FiPIt* and first-order DynareOBC are of comparable speed.

Hence, some of the recent advances in local methods that do not require stationarity-inducing assumptions, such as RSS and QLOBC, come with a large increase in solution times relative to both standard local methods and global methods.

The differences across local and global solutions originate in the near-unit-root nature of the equilibrium stochastic process of NFA, a typical feature of incomplete-markets models with non-state-contingent assets because of the persistence induced by precautionary savings. We provide analytic and quantitative results showing that small errors in approximating the NFA autocorrelation cause sizable differences in the unconditional means of NFA, consumption and net exports and in business cycle moments, impulse responses and spectral densities. Interestingly, 1OA, 2OA, and RSS produce very similar second- and higher-order moments, impulse responses and periodograms, because they yield decision rules with similar first-order terms and small higher-order terms. Local solutions that target mean NFA and the variability of consumption perform better but require knowing the global solution. Moreover, these targeted calibrations require higher DEIR elasticities that make moving NFA from its steady state so costly as to neutralize the precautionary savings motive. In this case, even the first moments of 1OA, 2OA, and RSS solutions are similar.

These findings suggest caution in assessing results obtained with local solutions, especially when dynamics of non-state-contingent assets are the key variable of interest (e.g., studies examining global imbalances, sovereign default, optimal foreign reserves, or macroprudential policy) or when assessing the frequency and magnitude of Sudden Stops. Good practice in studies inducing stationarity with, for example, a DEIR function, is to examine the robustness of the results to the value of the debt-elasticity parameter. Alternatively, quasi-linear or RSS methods can be used without inducing stationarity.

Our results are robust to several modifications, including setting the DEIR elasticity to its inessential low value versus targeting it to the global solution; replacing DEIR with an interest rate lower than the rate of time preference, with an endogenous discount factor or with costs of holding foreign assets; introducing different shocks and changing their variability; and examining a model with an exact solution. We did find, however, that inducing stationarity using endogenous discounting is relatively more accurate than using DEIR or asset holding costs.

For the DynareOBC solutions of the Sudden Stops model, the results are robust to whether the credit constraint is binding or non-binding at the steady state.

We see our findings as suggesting that local and global methods are best seen as complements. For parsimonious models, a global solution is feasible and desirable, and innovations in hardware and algorithm design are making global solutions of larger models more feasible. But for larger models that cannot be solved globally, it is best to use local methods while being mindful of their limitations. Complementing them with global solutions for simplified versions of large models would shed light on the size and direction of those limitations. Our results also show that calibrating the DEIR elasticity and the center of approximation to match the observed mean of NFA and variability of consumption yields results closer to a global solution calibrated to the same targets, but the local solutions would still approximate poorly the effects of parametric changes (e.g., counterfactual and policy experiments).

Our findings also have useful policy implications. For instance, in the case of the Sudden Stops model, we found that local solutions underestimate the effects of credit constraints on precautionary savings and would thus lead to excessive accumulation of foreign reserves or macroprudential regulation that is too tight.

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Table 1: Summary of Numerical Methods used in Open-Economy Models

	Global	Local						Total
Research papers	33	68						101
		Stationarity Assumption				Approximation		
		AHC	DEIR	ED	Other	1OA	Higher	
		16	32	8	12	62	6	
Policy models	0	8						8
		Stationarity Assumption				Approximation		
		AHC	DEIR	ED	Other	1OA	Higher	
		0	5	1	2	8	0	

Note: This table presents a survey of 101 research papers and 8 policy models. The stationarity inducing assumptions are asset holding costs (AHC), debt-elastic interest-rate (DEIR), endogenous discounting (ED), and Other. The local approximation are first-order approximation (1OA) and Higher, which includes higher-order perturbation methods and RSS. Appendix A explains the survey methodology and includes comprehensive details of all the papers and models surveyed.

Table 2: Calibration of the Endowment Model

Notation	Description	Value
Common parameters		
σ	Coefficient of relative risk aversion	2
y	Mean endowment income	1
A	Absorption constant	0.321
R	Gross world interest rate	1.086
σ_z	St. dev. of income	0.0272
ρ_z	Autocorrelation of income	0.749
Global solution parameters		
β	Discount factor	0.917
φ	Ad-hoc debt limit	-0.435
Local solution parameters		
β	Discount factor $(1/R)$	0.921
ψ	DEIR elasticity coefficient	0.001
b^*	DEIR steady-state NFA (2OA)	-0.724
b^*	DEIR steady-state NFA (RSS)	-0.552

Table 3: Autocorrelations of NFA, Net Exports, and Income in the Endowment Model

	ρ_z	0	0.1	0.3	0.5	0.7	0.9	0.95
i)	**Global**							
	ρ_{nx}	-0.102	-0.007	0.195	0.444	0.688	0.945	0.983
	ρ_b	0.801	0.845	0.912	0.957	0.985	0.997	0.997
	$E(b)$	-0.409	-0.407	-0.402	-0.392	-0.370	-0.288	-0.259
ii)	**Local: Baseline calibration**							
	2OA							
	ρ_{nx}	0.497	0.593	0.753	0.874	0.955	0.995	0.999
	ρ_b	0.997	0.997	0.998	0.999	0.999	1.000	1.000
	$E(b)$	-0.688	-0.681	-0.656	-0.602	-0.445	0.433	1.362
	RSS							
	ρ_{nx}	0.493	0.590	0.752	0.875	0.957	0.997	1.000
	ρ_b	0.997	0.997	0.998	0.999	0.999	1.000	1.000
	$E(b)$	-0.534	-0.530	-0.518	-0.490	-0.409	0.116	0.870
iii)	**Local: Targeted calibration** $(\psi = 0.042, b^* = -0.374)$							
	2OA							
	ρ_{nx}	0.022	0.123	0.327	0.529	0.729	0.920	0.963
	ρ_b	0.927	0.940	0.960	0.975	0.987	0.996	0.998
	$E(b)$	-0.372	-0.372	-0.371	-0.369	-0.364	-0.348	-0.340
	RSS							
	ρ_{nx}	0.022	0.123	0.327	0.529	0.728	0.919	0.962
	ρ_b	0.927	0.940	0.960	0.975	0.987	0.996	0.998
	$E(b)$	-0.373	-0.372	-0.371	-0.370	-0.365	-0.348	-0.336
iv)	**Local: Targeted calibration for each** ρ_z							
	2OA							
	ρ_{nx}	-0.019	0.081	0.283	0.493	0.729	0.916	0.968
	ρ_b	0.835	0.869	0.923	0.960	0.987	0.996	0.998
	b^*	-0.410	-0.408	-0.404	-0.395	-0.380	-0.309	-0.298
	ψ	0.191	0.172	0.133	0.094	0.042	0.043	0.030
	RSS							
	ρ_{nx}	-0.019	0.080	0.282	0.491	0.721	0.929	0.968
	ρ_b	0.834	0.869	0.922	0.959	0.985	0.997	0.998
	b^*	-0.410	-0.409	-0.404	-0.396	-0.379	-0.320	-0.308
	ψ	0.192	0.173	0.135	0.096	0.048	0.030	0.030

Note: 2OA and RSS denote the second-order and partial risky-steady state solutions, respectively. Targeted calibrations for each ρ_z in panel iv) set (ψ, b^*) so as to match the value of $E(b)$ in the corresponding GA solution, shown in Panel i), and the standard deviation of consumption (not shown).

Table 4: Long-run Moments: Endowment Model

	Data	Global	Local				
			Baseline calibration			Targeted calibration	
			2OA	RSS	RSS	2OA	RSS
			DEIR	$\beta R < 1$	DEIR	DEIR	DEIR
DEIR parameters							
ψ	·	·	0.001	·	0.001	0.042	0.042
b^*	·	·	-0.724	·	-0.552	-0.374	-0.374
Calibration targets from Mexican data in Mendoza (2010)							
$sd(c)/sd(y)$	1.247	1.352	·	·	·	1.363	1.358
$E[b/y]$	-0.363	-0.363	-0.363	·	-0.363	-0.363	-0.363
Cyclical moments							
Standard deviation relative to GDP							
c	1.247	1.352	3.326	15.080	3.469	1.363	1.358
nx/y	0.775	0.540	2.168	1.799	2.268	0.649	0.650
b/y	·	9.766	71.159	3.274	74.774	11.457	11.499
Correlation with GDP							
c	0.895	0.834	0.213	0.250	0.202	0.759	0.758
nx/y	-0.688	0.431	0.223	-0.072	0.227	0.458	0.464
b/y	·	0.531	0.150	0.364	0.130	0.585	0.582
First-order autocorrelation							
c	0.701	0.849	0.997	0.995	0.997	0.947	0.947
nx/y	0.797	0.768	0.972	0.999	0.973	0.788	0.787
b/y	·	0.971	0.999	0.984	0.999	0.984	0.984
Performance metrics							
Run time (sec)	·	1.46	0.6	0.3	3.1	0.6	3.1
rel. to GA	·	·	0.41	0.21	2.12	0.21	2.12
EE errors	·	9.46E-11	6.05e-05	1.64e-02	1.81e-04	4.11e-05	8.50e-05
	·	(1.28E-04)	(2.56e-04)	(3.11e-02)	(1.03e-03)	(1.74e-04)	(3.80e-04)
Decision rule diff b	·	·	0.036	1.066	0.036	0.029	0.029
	·	·	(0.110)	(2.302)	(0.111)	(0.071)	(0.075)
Decision rule diff c	·	·	0.015	0.716	0.015	0.009	0.009
	·	·	(0.074)	(1.347)	(0.073)	(0.047)	(0.047)

Note: 2OA and RSS refer to second-order and risky steady state, respectively. Results were obtained using Matlab2022a in a Linux cluster with 128GB of RAM, 2×16-core Intel Xeon(R) Gold 6142 CPU @ 2.6GHz processors, and a Samsung SSD 840 512GB hard drive. The number of CPUs called by the parallel computing toolbox was set to minimize run time. Run times include elapsed time up to the solution of decision rules. Mean and maximum (in brackets) Euler equation (EE) errors and decision rule differences are computed for all (b, z) pairs in the state space of the Global solution. Decision rule differences in the last two rows are differences between the local and GA solutions in percent of the latter conditional on bond values with positive probability in the ergodic distribution of the GA solution.

Table 5: Endowment Model with Income and Interest-Rate Shocks

	Interest Rate Standard Deviation (%)					
	0.0	0.5	1.0	1.5	2.0	2.5
Global calibrated						
$E(b/y)$	-0.358	-0.355	-0.345	-0.326	-0.296	-0.256
$\sigma(c)/\sigma(y)$	1.321	1.342	1.407	1.527	1.706	1.922
$\sigma(b)/\sigma(y)$	8.565	9.156	11.082	14.946	21.703	31.889
$\rho(y, nx/y)$	0.469	0.452	0.411	0.362	0.317	0.283
$\rho_{nx/y}$	0.738	0.740	0.748	0.763	0.780	0.791
$\rho_{b/y}$	0.961	0.964	0.972	0.981	0.987	0.989
Global with NDL						
$E(b/y)$	-6.523	-5.428	-4.227	-3.314	-2.602	-2.027
$\sigma(c)/\sigma(y)$	9.710	10.797	8.968	7.498	6.256	5.175
$\sigma(b)/\sigma(y)$	2.487	5.719	8.157	10.121	11.634	12.690
$\rho(y, nx/y)$	-0.022	0.040	0.065	0.086	0.109	0.136
$\rho_{nx/y}$	0.999	0.999	0.999	0.992	0.980	0.959
$\rho_{b/y}$	0.999	0.999	0.999	0.999	0.999	0.999
Full RSS ($\beta R < 1$)						
$E(b/y)$	-7.041	-6.232	-4.996	-3.887	-2.937	-2.124
$\sigma(c)/\sigma(y)$	15.080	13.454	11.171	8.982	6.935	5.150
$\sigma(b/y)/\sigma(y)$	3.274	4.838	7.441	9.856	11.743	13.071
$\rho(y, nx/y)$	-0.072	-0.033	-0.003	0.017	0.037	0.062
$\rho_{nx/y}$	0.999	0.995	0.991	0.986	0.975	0.952
$\rho_{b/y}$	0.984	0.992	0.996	0.997	0.998	0.998
2OA DEIR Baseline calibration						
$E(b/y)$	-0.363	-0.287	-0.060	0.318	0.847	1.527
$\sigma(c)/\sigma(y)$	3.326	3.360	3.454	3.593	3.758	3.932
$\sigma(b/y)/\sigma(y)$	71.263	91.542	457.581	93.541	38.613	23.901
$\rho(y, nx/y)$	0.223	0.218	0.207	0.190	0.172	0.153
$\rho_{nx/y}$	0.972	0.971	0.969	0.965	0.962	0.961
$\rho_{b/y}$	0.999	0.999	0.999	0.999	0.999	0.999
Partial RSS (DEIR) Baseline calibration						
$E(b/y)$	-0.363	-0.340	-0.270	-0.154	0.007	0.213
$\sigma(c)/\sigma(y)$	3.469	3.490	3.555	3.672	3.863	4.179
$\sigma(b/y)/\sigma(y)$	74.774	80.658	104.452	192.309	4369.169	167.436
$\rho(y, nx/y)$	0.227	0.226	0.223	0.217	0.208	0.194
$\rho_{nx/y}$	0.973	0.972	0.972	0.971	0.970	0.971
$\rho_{b/y}$	0.999	0.999	1.000	1.000	1.000	1.000
2OA DEIR Targeted calibration						
$E(b/y)$	-0.363	-0.362	-0.358	-0.351	-0.342	-0.331
$\sigma(c)/\sigma(y)$	1.363	1.384	1.445	1.541	1.667	1.814
$\sigma(b/y)/\sigma(y)$	11.458	11.659	12.257	13.249	14.636	16.439
$\rho(y, nx/y)$	0.458	0.445	0.412	0.371	0.329	0.291
$\rho_{nx/y}$	0.788	0.784	0.772	0.759	0.747	0.738
$\rho_{b/y}$	0.984	0.984	0.984	0.985	0.985	0.986
Partial RSS (DEIR) Targeted calibration						
$E(b/y)$	-0.363	-0.363	-0.361	-0.359	-0.355	-0.351
$\sigma(c)/\sigma(y)$	1.358	1.361	1.369	1.383	1.402	1.427
$\sigma(b/y)/\sigma(y)$	11.499	11.516	11.566	11.650	11.770	11.928
$\rho(y, nx/y)$	0.464	0.463	0.458	0.450	0.439	0.427
$\rho_{nx/y}$	0.787	0.780	0.761	0.731	0.692	0.648
$\rho_{b/y}$	0.984	0.984	0.984	0.984	0.984	0.985

Note: The volatility and persistence of endowment shocks are kept as in Table 2. GA, 2OA and RSS refer to the global, second-order and risky-steady state solutions, respectively.

Table 6: Calibration of the Sudden Stops Model

Notation	Description	Value
Common parameters		
σ	Coefficient of relative risk aversion	2
R	Gross world interest rate	1.0857
α	Labor share in gross output	0.592
γ	Capital share in gross output	0.306
η	Imported inputs share in gross output	0.102
δ	Depreciation rate of capital	0.088
ω	Labor exponent in the utility function	1.846
ϕ	Working capital constraint coefficient	0.258
a	Investment adjustment cost parameter	2.750
τ	Consumption tax	0.168
κ	Collateral constraint coefficient	0.2
ρ^A	TFP autocorrelation	0.555
ρ^R	Interest rate autocorrelation	0.555
ρ^p	Input price autocorrelation	0.737
$\sigma^2_{u^A}$	Variance of TFP innovations	1.0273e-04
$\sigma^2_{u^R}$	Variance of interest rate innovations	2.4387e-04
$\sigma^2_{u^p}$	Variance of input price innovations	5.1097e-04
σ_{u^A,u^R}	Covariance of TFP and interest rate innovations	-0.0047
Global solution parameters		
β	Discount factor	0.920
φ	Ad-hoc debt limit as a share of y^{dss}	-0.505
Local solutions parameters		
DynareOBC with DEIR		
β	Discount factor $(1/R)$	0.9211
ψ	DEIR elasticity coefficient	0.0044
$(b/y)^*$	DEIR steady-state NFA	-0.008

Note: For the Sudden Stops model, the GA solution has two credit constraints, namely φ and the collateral constraint. Credit is constrained at the deterministic steady state, since $\beta R < 1$, but φ is set low enough so that the collateral constraint binds first.

Table 7: Long-run Moments: Sudden Stops Model

	Global	DynareOBC	
	GA	$\beta R < 1$	DEIR
Mean relative to GDP			
c	0.696	0.686	0.695
i	0.171	0.171	0.172
nx/y	0.015	0.027	0.015
b/y	0.015	-0.135	0.015
$lev.ratio$	-0.102	-0.173	-0.101
v	0.108	0.108	0.108
Standard deviation relative to GDP			
$\sigma(c)/\sigma(y)$	1.023	0.971	0.938
$\sigma(i)/\sigma(y)$	3.383	3.224	3.419
$\sigma(nx/y)/\sigma(y)$	0.746	0.582	0.687
$\sigma(b/y)/\sigma(y)$	4.980	2.100	3.384
$\sigma(lev.ratio)/\sigma(y)$	2.340	0.979	1.570
$\sigma(v)/\sigma(y)$	1.495	1.513	1.510
$\sigma(L)/\sigma(y)$	0.599	0.598	0.599
Correlations with GDP			
$\rho(y,c)$	0.842	0.951	0.901
$\rho(y,i)$	0.641	0.685	0.641
$\rho(y,nx/y)$	-0.117	-0.257	-0.118
$\rho(y,b/y)$	-0.120	-0.044	-0.200
$\rho(y,lev.rat.)$	-0.111	0.0085	-0.168
$\rho(y,v)$	0.832	0.832	0.830
$\rho(y,L)$	0.994	0.995	0.995
First-order autocorrelations			
$\rho(y)$	0.825	0.816	0.817
$\rho(c)$	0.829	0.797	0.795
$\rho(i)$	0.500	0.474	0.500
$\rho(nx/y)$	0.601	0.407	0.520
$\rho(b/y)$	0.990	0.978	0.985
$\rho(lev.rat.)$	0.992	0.986	0.990
$\rho(v)$	0.777	0.769	0.772
$\rho(L)$	0.801	0.785	0.795
Credit constraint			
Prob.(μ>0)	2.58	20.05	2.80
Performance metrics			
Run time (sec)	268	244	332

Note: See note to Table 4. For the DEIR solution, we set $\psi = 0.004$ (which ensures the model matches the $\sigma(c)/\sigma(y)$ ratio in the data when the constraint is not binding) and $b^* = -0.008$ (which ensures we match $E(b/y)$ value that is consistent with the global approximation).

Table 8: Collateral Constraint Multiplier, Macro & Financial Variables Conditional on $\mu > 0$

	$\log(\mu)$		Financial Premia				Macro variables					
			means				*means of deviations from long-run averages*					
	upper limit	mean	SIP	EP	$(1-\kappa)SIP$	RP	c	nx/y	i	y	L	v
Panel a. GLB												
Quintiles of μ												
First	-6.563	-6.941	0.32	0.37	0.26	0.10	-2.76	1.98	-1.76	-0.60	-0.26	0.35
Second	-6.320	-6.428	1.07	0.96	0.85	0.11	-2.17	1.37	2.70	0.08	0.15	-1.25
Third	-6.088	-6.187	1.82	1.56	1.46	0.10	-3.80	2.30	-3.00	-1.35	-0.82	-1.29
Fourth	-5.843	-5.968	2.98	2.48	2.38	0.09	-4.72	2.58	-5.46	-2.26	-1.42	-3.35
Fifth	-3.374	-5.636	6.59	5.38	5.27	0.10	-4.86	5.10	-13.45	-1.21	-1.37	-2.98
Overall mean		-6.038	2.59	2.17	2.07	0.10	-3.64	2.64	-4.05	-1.04	-0.73	-1.78
Overall median		-6.198	1.79	1.52	1.43	0.11	-3.22	1.60	-1.64	-1.02	-0.57	-2.15
			Ex-post Sharpe ratio = 1.16									
Panel b. DynareOBC-$BetaR < 1$												
Quintiles of μ												
First	-6.930	-7.352	0.15	0.12	0.12	0.00	-3.39	1.00	-6.90	-3.00	-1.75	-3.68
Second	-6.538	-6.691	0.69	0.55	0.55	0.00	-0.22	0.20	1.85	0.32	0.25	0.20
Third	-6.318	-6.415	1.30	1.04	1.04	0.00	-0.44	0.20	1.33	0.09	0.08	-0.28
Fourth	-6.128	-6.219	2.05	1.64	1.64	0.00	-0.45	0.30	0.96	0.15	0.06	-0.33
Fifth	-5.603	-5.987	3.52	2.82	2.82	0.00	-0.65	0.50	0.13	0.04	-0.11	-0.89
Overall mean		-6.343	1.54	1.23	1.23	0.00	-1.03	0.40	-0.53	-0.48	-0.29	-1.00
Overall median		-6.418	1.29	1.03	1.03	0.00	-0.74	0.40	0.28	-0.19	-0.12	-0.83
			Ex-post Sharpe ratio = 0.08									
Panel c. DynareOBC DEIR												
Quintiles of μ												
First	-6.818	-7.105	0.27	0.21	0.21	0.00	-1.39	1.30	2.77	0.67	0.48	0.42
Second	-6.531	-6.650	0.76	0.61	0.61	0.00	-1.13	1.27	3.71	1.01	0.66	0.65
Third	-6.350	-6.434	1.26	1.01	1.01	0.00	-0.92	1.23	4.51	1.28	0.80	0.87
Fourth	-6.190	-6.268	1.86	1.49	1.49	0.00	-0.82	1.18	5.15	1.42	0.87	0.82
Fifth	-5.785	-6.073	2.94	2.36	2.36	0.00	-0.24	1.12	6.97	2.16	1.27	2.03
Overall mean		-6.386	1.42	1.14	1.14	0.00	-0.90	1.22	4.62	1.31	0.81	0.96
Overall median		-6.437	1.26	1.01	1.01	0.00	-1.04	1.26	4.52	1.20	0.75	0.92
			Ex-post Sharpe ratio = 0.72									

Note: SIP is the Shadow interest premium, EP is the equity premium, and RP is the risk premium component of EP. The quintile distribution of μ is conditional on $\mu > 0$. Means for other variables are computed using the distribution of μ within each quantile and the overall distribution of μ conditional on $\mu > 0$. $\log(\mu)$ is the base-10 logarithm of the multiplier on the collateral constraint. The moments for GA are computed using the recursive equilibrium decision rules and ergodic distribution of the model. For EP, we compute the equity premium conditional on all date-t states (b, k, s) and then calculate the mean from the ergodic distribution. The Sharpe ratio is computed using the conditional ergodic distribution for $\mu > 0$. The moments for DynareOBC are ex-post moments, computed using time-series simulation output of DynareOBC. RP is set to zero because the covariance between future equity returns and marginal utility is zero along the perfect-foresight paths that determine each date-t solution in DynareOBC.

Figure 1: First-order coefficient of 2OA NFA decision rule

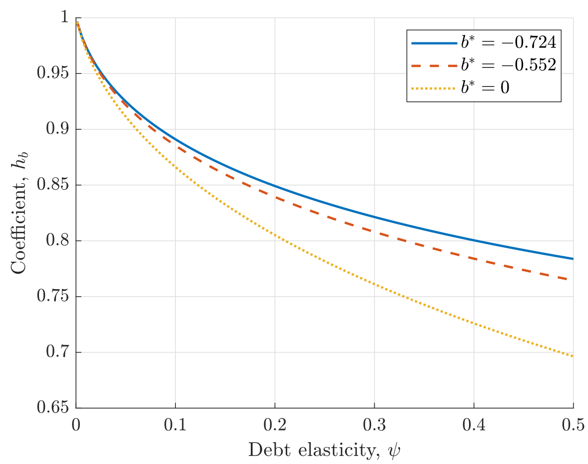
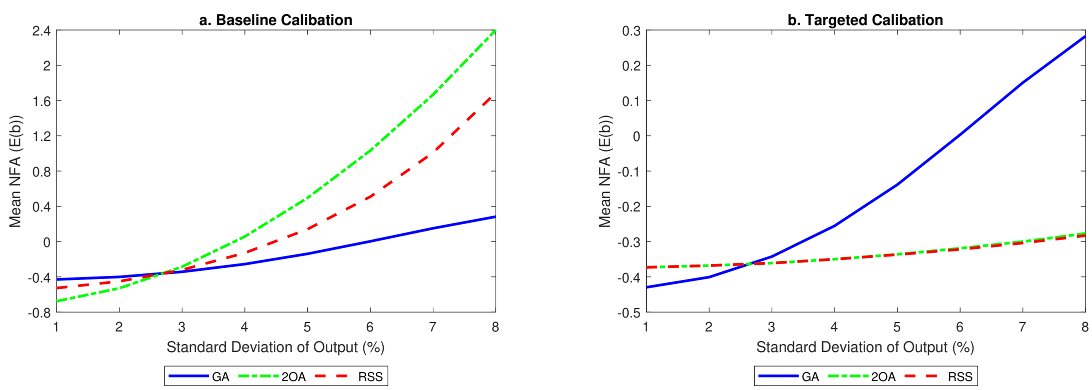
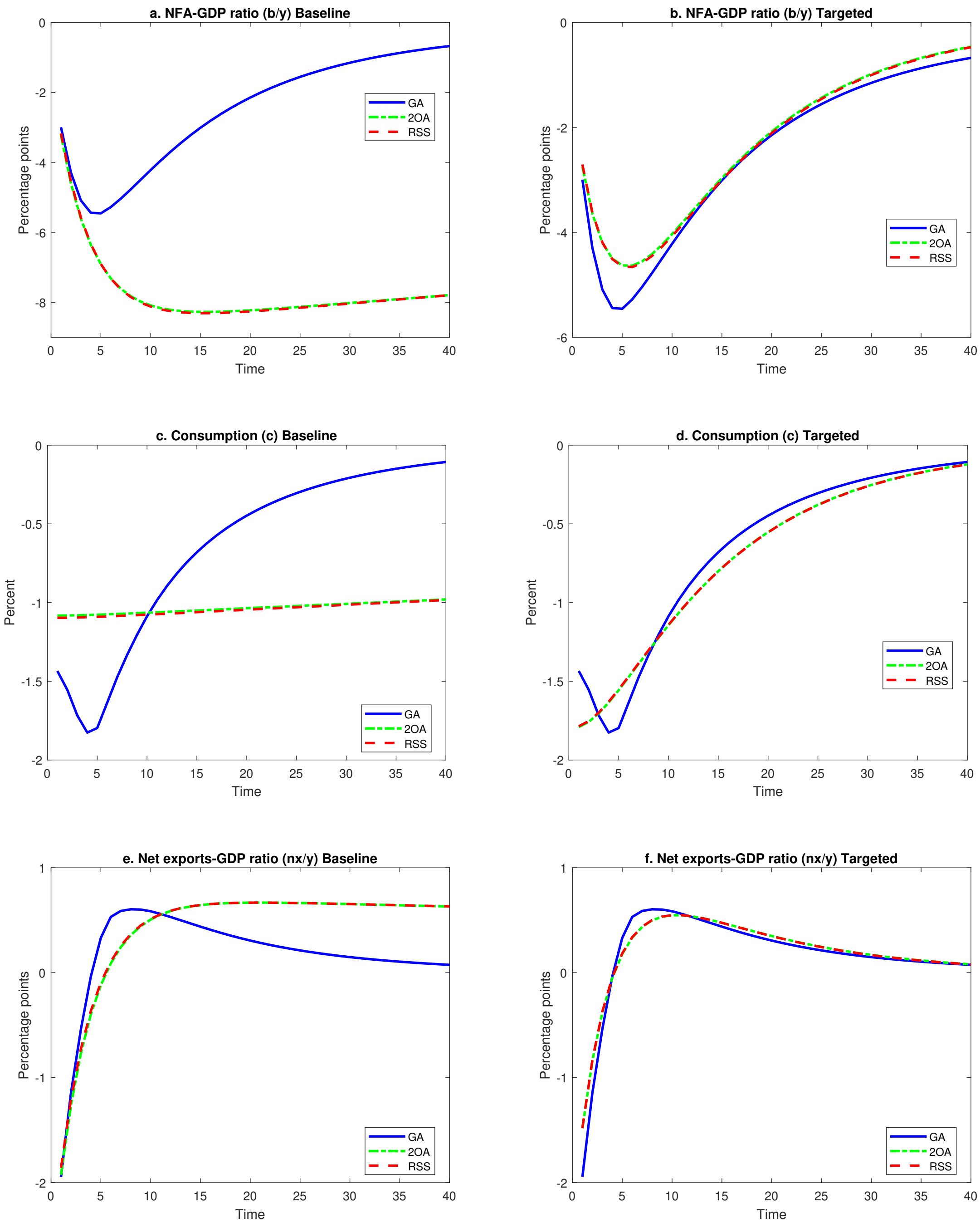


Figure 2: Income Risk and Mean NFA in the Endowment Model



Note: GA refers to global solution, 2OA refers to second-order solution, RSS refers to risky-steady state solution.

Figure 3: Endowment Model Impulse Responses to a Negative Income Shock



Note: GA, 2OA and RSS denote global, second-order and risky-steady state solutions, respectively. GA impulse responses are forecast functions of the equilibrium Markov processes of the endogenous variables with initial conditions set to $E[b]$ and a value of z equal to a one-standard-deviation shock.