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SPONTANEOUS VOLATILITY OF OUTPUT AND INVESTMENT

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ABSTRACT

Spontaneous shifts in output originating within the business sector are an important factor in aggregate fluctuations. This paper develops a simple two-component decomposition of the movement of real GNP. One component is the path that GNP would have followed in order to deliver the volume of goods and services actually taken by consumers, government, and the rest of the world. The second component, noise, is the residual between actual GNP and the theoretical calculation. The two components are of roughly the same size, but noise has more of its power at higher frequencies.

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The output of goods and services of the business sector of the U.S. economy displays considerable volatility. Understanding this volatility is one of the central concerns of macroeconomics. This lecture considers the following basic question: Are the fluctuations in output primarily a response to changes in the volume of goods and services delivered to users outside the business sector (consumers, government, or foreigners), or do the fluctuations arise spontaneously within the sector? The research demonstrates a substantial spontaneous element of output. In particular, much of the variability of output from one year to the next is unrelated to changes in deliveries of goods and services to users. On the other hand, much of the lower-frequency movement of output is associated with changes in the volume of goods delivered to outside users.

By definition, goods produced by the business sector but not delivered to outside users constitute investment. Because this research considers output conditional on deliveries, it is as much about the volatility of investment as it is about the volatility of production. And it is no surprise to any student of the aggregate U.S. economy that investment, especially inventory investment, is volatile. The literature on inventory investment has noted many times that contractions in that component of investment alone are often about as large as the contractions in total GNP at the outsets of recessions. What is novel here is a full consideration of the extent to which the movements of investment are induced by actual or expected changes in deliveries. The model gives a full rational-expectations treatment to the accelerator. Interestingly, it finds that a large part of the cyclical movements of inventory investment are induced by changes in deliveries. The spontaneous element of investment comes mainly, but not exclusively, from fixed investment.

1. The basic approach to measuring the spontaneous element of output

Consider an economy whose business sector delivers goods and services in volume z_t. The variable z_t is not exogenous in any sense; rather, I simply consider the problem of scheduling output conditional on z_t. The business sector produces output y_t. Any output not shipped is accumulated as capital, k_t, with survival rate δ, so the capital stock evolves according to

$$k_t = y_t - z_t + \delta k_{t-1} \tag{1.1}$$

The technology is most efficient when there are ψ units of capital for each unit of output produced; the cost function is

$$\frac{1}{2} \sum (k_t - \psi y_{t+1})^2 \tag{1.2}$$

Note that capital is measured at the end of the period and the capital must be in place in advance in order for production to take place.

Let

$$\omega = \frac{\psi}{1 + \psi\delta} < 1 \tag{1.3}$$

The solution to the problem of minimizing expected cost is

$$k_t^e = \omega k_{t-1} + \omega E_t \sum_{i=0}^{\infty} \omega^i (z_{t+i+1} - \omega z_{t+i}) \tag{1.4}$$

That is, capital is a weighted average of future expected sales. The ^{e} indicates that this is the theoretical value for output without noise. Actual capital is its theoretical values plus noise:

$$k_t = k_t^e + s_t \tag{1.5}$$

Following an insight of LeRoy-Porter [1981] and Shiller [1981], I will find it useful to introduce the "perfect foresight" variable,

$$k_t^* = \omega k_{t-1} + \omega \sum_{i=0}^{\infty} \omega^i (z_{t+i+1} - \omega z_{t+i}) \tag{1.6}$$

The * variables are observable by the econometrician long after the fact. Let $\epsilon_{t,i}$ be the difference between actual sales at time $t+i$ and sales expected for time $t+i$ as of time t. Then the observable discrepancy between the actual and the * versions of the variables can be expressed as

$$k_t - k_t^* = s_t - \omega \sum_{i=0}^{\infty} \omega^i (\epsilon_{t,i+1} - \omega \epsilon_{t,i})$$

$$= s_t - \nu_t \tag{1.7}$$

That is, the discrepancy between the actual capital stock and the perfect foresight value is noise less a composite expectation error, ν_t. The expectation error obeys the standard rational expectations orthogonality condition:

$$E(\nu_t | x_t) = 0 \tag{1.8}$$

where x_t is any vector of data known to the firm when the investment decision is taken. Note that the composite expectation error, ν_t, is serially correlated, but the correlation does not involve a failure of the orthogonality condition because lagged values of ν are not in the information set; ν_t is not observed until long after time t.

To obtain information about the noise, I make use of the technique developed in Durlauf and Hall [1988]. Let M_X be the projection operator onto current and lagged value of x_t; for a time series u_t, $M_X u_t$ is the fitted values of the regression of u_t on x_t, x_{t-1}, ... Then the projection of the discrepancy in x yields information about noise:

$$M_X(k_t - k_t^*) = M_X \nu_t + M_X s_t \tag{1.9}$$

The first term, $M_X \nu_t$, is zero by rational expectations. Regressing the discrepancy on the x-variables eliminates the expectation error. The remainder,

$$\hat{s}_t = M_X s_t \tag{1.10}$$

is a conservative estimate of the noise in the following sense: The variance of $\hat{s}_t$ is less than the variance of noise, s_t. The reason is simple. The noise variable can be decomposed into the fitted value, $\hat{s}_t$, plus an orthogonal residual:

$$s_t = \hat{s}_t + u_t \quad , \tag{1.11}$$

so

$$V(s_t) = V(\hat{s}_t) + V(u_t) \tag{1.12}$$

and

$$V(s_t) \geq V(\hat{s}_t) \quad . \tag{1.13}$$

The procedure I will use to make inferences about the spontaneous element of investment is the following. First, I will form the discrepancy between actual capital and the amount of capital mandated by the model under perfect foresight. The discrepancy arises because of expectation errors and because of the spontaneous element. Then I will regress the discrepancy on well-chosen variables known to firms when investment decisions are made. The test for the existence of noise is simply the test if any of the variables have non-zero coefficients in the regression. Finally, the fitted value from the regression is a conservative estimate of the noise time series.

2. *Fixed capital and inventories*

It is important to extend the model by distinguishing two major components of the capital stock–fixed capital and inventories. Investment in fixed capital involves a lag of a year or more for planning and installation. Inventories respond almost immediately to changes in the economy. I denote the stock of inventories as v_t and extend the cost function as

$$\frac{1}{2} \sum \gamma^t[(k_t - \psi y_{t+1})^2 + \alpha(v_t - \phi y_{t+1})^2] \ . \qquad (2.14)$$

Here $\gamma<1$ is the discount ratio and α is the relative weight of inventory discrepancies. Let τ be the time to build fixed capital. At time $t-\tau$ the firm makes its plan for capital to be in place in period t. This decision is conditional on the amount of fixed capital already committed to be in place in period $t-1$ and on the level of inventories expected at the end of period $t-1$:

$$k_t^* = \omega^k k_{t-1} + E_{t-\tau}[\omega^v v_{t-1} + \sum_{i=0}^{\infty} \omega_i z_{t+i}] \ . \qquad (2.15)$$

Then at time t, firm makes its plan for inventories to be in place at the end of period t. The capital stock for periods t, $t+1$, ..., $t+\tau-1$ is given, so the plan is conditional on the known values of $k_{t-1},\ldots,k_{t+\tau-1}$ and v_{t-1}. Thus

$$v_t^* = \sum_{j=0}^{\tau} \beta_j^k k_{t-j-1} + \beta^v v_{t-1} + E_{t-\tau} \sum_{i=0}^{\infty} \beta_i z_{t+i} \ . \qquad (2.16)$$

Calculation of the coefficients in these formulas follows standard principles for quadratic optimization.

The information available at time $t-\tau$ relevant to the determination of k_t includes all variables known at the beginning of period $t-\tau$ plus the committed values of the capital stock through period t. If candidate x-variables are time averages through the period, then the latest admissable variables are those observed during period $t-\tau-1$. In addition, the capital stock through period $t-1$ is admissable. If the assumption implicit in the cost function is literally true—that cost depends only on the end-of-period stocks—then variables dated t would be admissable as well. However, if the discrete-time problem is really an approximation to a continuous-time problem, then the variables dated t are not admissable. In this work, I do not use the contemporaneous variables.

For the inventory equation, I use variables dated $t-1$ and earlier.

Estimation

I will use the value of 0.9 for the discount ratio γ and 0.9 for the capital survival rate, δ. For the parameters ψ and ϕ, I use the sample averages of the fixed capital/output ratio and the inventory/output ratio; these values are 1.32 and 0.245. The only econometric slope parameter I estimate is the relative cost of inventory discrepancies, α. To estimate this parameter, I note that the difference between the first-order condition for fixed capital, k_t and the one for inventories, v_t, is

$$k_t-\psi y_t = \alpha\frac{1-\psi(1-\delta)}{1+\phi(1-\delta)}(v_t-\phi y_t) \;. \qquad (2.17)$$

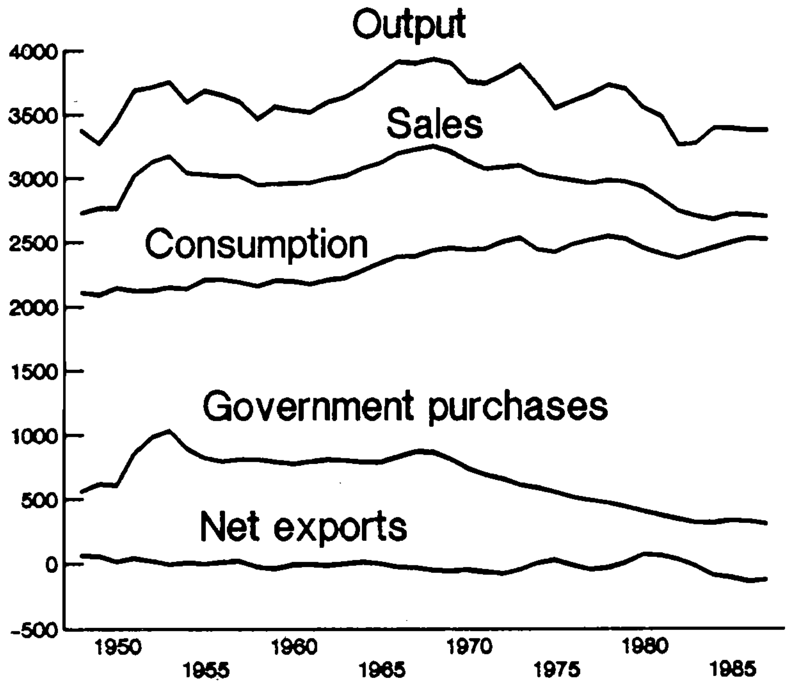
According to the model, this relation is deterministic. Of course, in practice, there is a random disturbance in the relation. However, it is not very large in comparison to the movements in the variables. Under an errors-in-variables interpretation, with errors uncorrelated with the true values of the variables, the value of α lies between the regression estimate of equation 2.17 and the estimate obtained by reversing left and right-hand variables and taking the reciprocal of the regression coefficient. Because the fit is quite good, the bound is fairly tight. I used the average of the two estimates.

3. The data

The study uses annual data from the U.S. National Income and Product Accounts. The sales of the business sector, z_t, are consumption, government purchases of goods and services less government production, and net exports. Production of the business sector, y_t, consists of sales plus investment (inventory investment and fixed investment). Note that the provision of housing services is considered part of the business sector—one of the sources of spontaneous volatility is fluctuations in construction of new housing. All other consumer durables are treated as deliveries to consumers and do not contribute to business volatility.

Figure 3-1 shows the basic data in detrended form (econometric results are obtained from the original data, not the detrended data). Output always exceeds sales because of the deterioration of capital. Plainly, output is more volatile than sales.

Figure 3-1. Output, sales, and components



The recessions of 1973-75 and 1981-82 stand out as times when production fell dramatically but sales remained roughly constant. The relative volatility of the two series does not answer the question posed in this lecture because it fails to account for the accelerator. However, the accelerator is unable to explain the jagged movements of output, as the results of the next section show.

In the first half of the sample period, changes in military purchases were a major source of volatility in sales. The sharp peak of purchases for the Korean War in 1953 and the smaller peak in 1968 for Vietnam show clearly in total sales. Sales reached a trough around 1960 between the two wars and at a time when consumption was low. Then sales grew rapidly through the 1960s to a peak around 1970. Consumption was the key component in this growth. Starting in the mid-1970s, volatility in net exports became important. Sales have been relatively constant since 1970, but net exports have tended to move in the opposite direction from consumption and government purchases.

It would not be plausible to suggest that sales or its components, save government purchases, were exogenous with respect to production. Sales occur in a market that allocates the productive capacity of the business sector. The idea in this lecture is to examine production conditional on sales; there is no hypothesis that sales cause production.

4. *Results*

The parameters of the model are the capital/output ratio,

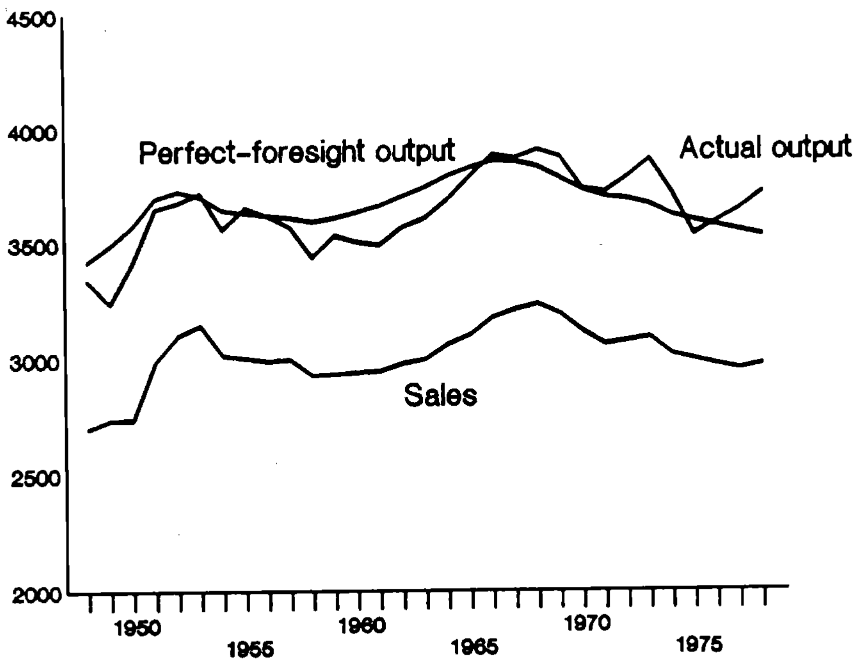
ψ, the survival rate of capital, δ, the inventory/output ratio, ϕ, the discount ratio, γ, and the relative cost of inventory discrepancies, α. There are robust estimators for ψ and ϕ that do not require econometric estimation of slope coefficients. Take an estimate of the survival rate of capital—I used 90 percent per year. Compute the capital stock from a reasonable initial benchmark and the recursion,

$$k_t = \delta k_{t-1} + i_t \tag{4.1}$$

where i_t is gross investment in fixed capital from the national income and product accounts. Then estimate the capital/output ratio, ψ, as the average of the ratio of k_t to y_{t+1}. For ϕ, simply take the average of the ratio of the stock of inventories as reported in the NIPA to production. As I noted earlier, the only slope parameter is α The estimate of α obtained by treating equation 2.17 as a regression is 3.27 with a standard error of 0.40. The estimate obtained by reversing the variables and taking the reciprocal of the coefficient is 5.58. My estimate of α is the average of the two, 4.72. The results would hardly be affected by different choices of α within this interval; α affects only the short-run adjustment to surprises and not the relation between future sales and current investment..

Figure 4-1 shows the level of output that would have occurred if firms had known the actual path of sales many years in advance. This output series is different from the one implied by the * variables because they are computed year-by-year with actual capital and inventory stocks, whereas the output series in Figure 4-1 does not depend on initial stocks.. This output series cannot

Figure 4-1. Perfect-foresight output



form the basis of a rigorous noise measurement, but it does give an indication of the overall departures from the path that would have been optimal according to the model with perfect foresight. The departures of actual from predicted output in Figure 4-1 are a mixture of expectation errors and model noise.

In Figure 4-1, perfect-foresight output, y_t^*, has the same basic low-frequency movements as actual output, and those movements are dictated by similar movements in deliveries, z_t. However, there are some important differences between perfect-foresight and actual output. In particular, in the early 1960s, the business sector would have produced substantially more output than it did had it known that sales would grow so much later in the 1960s. A similar shortfall of output occurred before the Korean War. After 1970, with sales relatively smooth, output would have been fairly smooth as well, if the business sector had obeyed the model and had perfect foresight about sales. Instead, actual output was quite jagged. The peak of output in 1973 was not related to any increase in sales and the sharp contraction in 1975 came a year after a modest decline in sales. The question to be answered in this research is whether the ups and downs in actual output not warranted by the subsequent movements in deliveries can be attributed to imperfect information or whether they are spontaneous noise.

Noise in fixed investment

The level of gross investment mandated by the model is

$$i_t^e = k_t^e - \delta k_{t-1} \quad . \tag{4.2}$$

When gross investment is determined in period $t-\tau$, the amount is sufficient to bring the capital stock for period t up to the optimal level, k_t^e, from its level already committed for the previous period, k_{t-1}. Actual gross investment is

$$i_t = k_t - \delta k_{t-1} \quad . \tag{4.3}$$

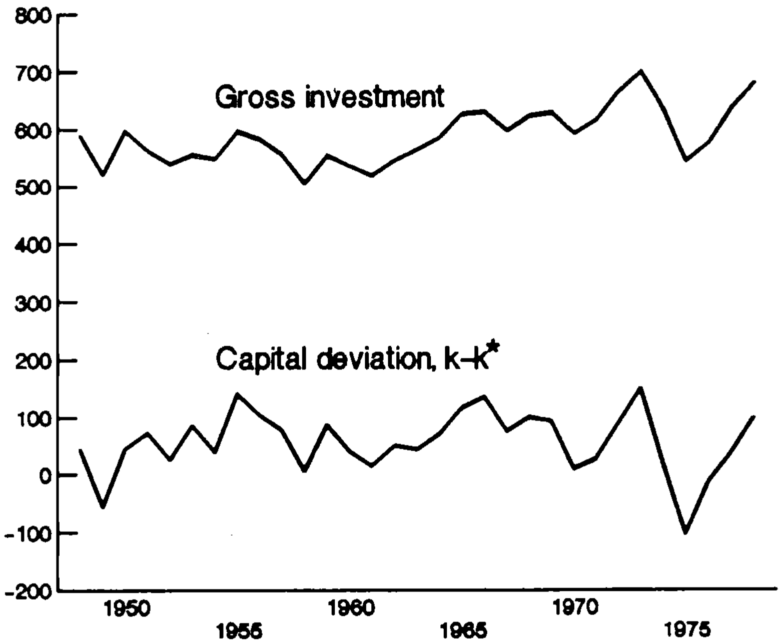
Hence noise in investment is

$$i_t - i_t^e = k_t - k_t^e = s_t \quad . \tag{4.4}$$

Even though investment is approximately the first difference of capital, the level of noise in investment is equal to noise in capital. The fitted values for the regression of $k_t - k_t^*$ on the x-variables are estimates of noise in investment.

Figure 4-2 shows $k_t - k_t^*$ for the period 1951 through 1978 (later years are lost in the calculation of k_t^*). The figure also shows the level of gross investment. Both variables are detrended by the trend of deliveries. Except for the higher level of investment, the two series are very similar. The similarity is paradoxical for the rational expectations accelerator model—according to that model, the difference between k_t and k_t^* should be an expectation error, uncorrelated with any information available when k_t was chosen.

Figure 4-2. Gross investment and capital deviation



Under the assumption that the time to build is two years or less, variables dated $t-3$ or earlier are eligible regressors for detecting noise. In addition, future capital is known over the horizon of the time to build, so values of k dated $t-1$ and earlier are eligible regressors. The regression using just the lagged values of the actual capital stock is

$$k_t - k_t^* = 33.9 + .87k_{t-1} - 1.69k_{t-2} + .82k_{t-3} \qquad (4.5)$$
$$R^2 = .402 \qquad F = 5.37$$

The large coefficients on lagged actual capital show the presence of a large noise component. Because k_{t-1} contains the noise variable, it is the logical candidate to be the most powerful x-variable in the noise-detection regression. I have not been able to find other variables that have unambiguous predictive value for $k_t - k_t^*$ three years in advance, once lagged capital is included in the noise-detection regression.

Figure 4-3 shows the estimated conservative noise series (the fitted values from equation 4.5) in comparison to actual fixed investment. Noise is shown to be a major factor in the overall movements of fixed investment. The rational expectations accelerator model leaves much of the volatility of investment unexplained; the hypothesis that the volatility can be attributed to information limitations is refuted by the regression.

Figure 4–3. Fixed investment and investment noise

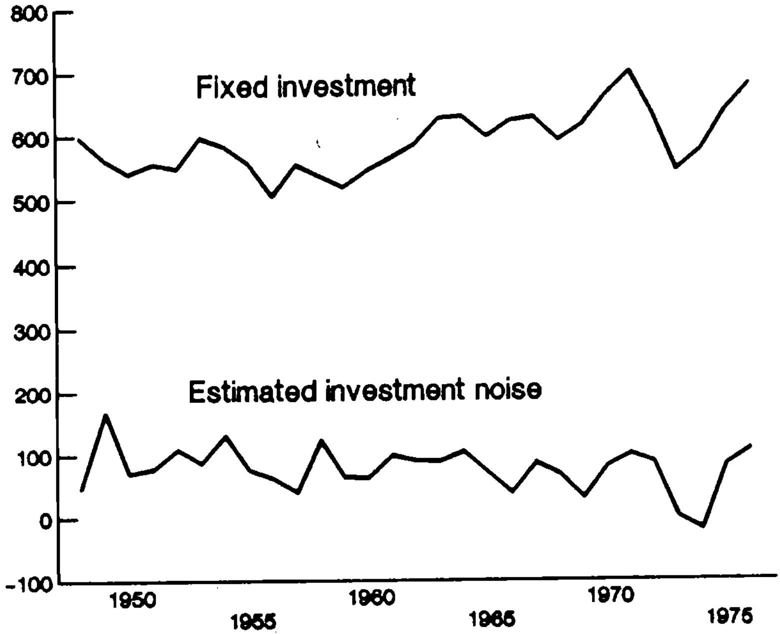
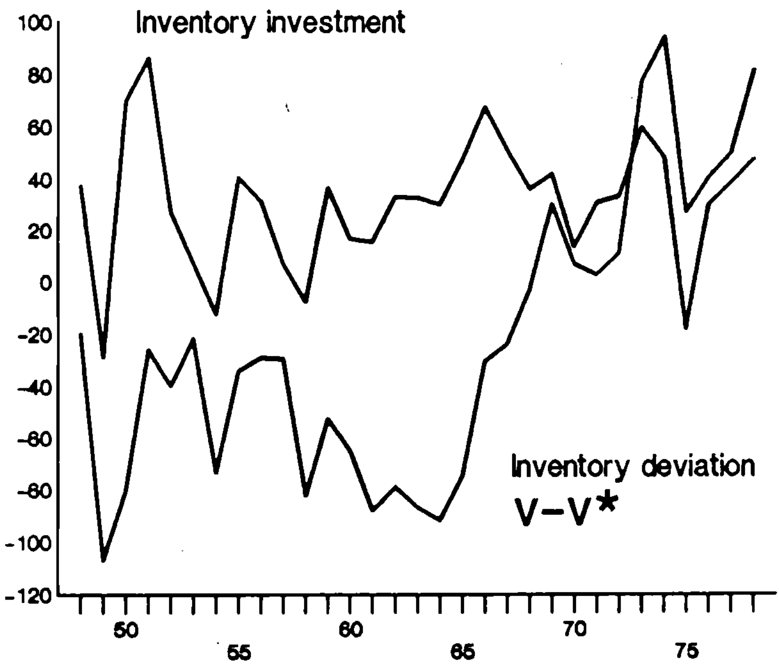


Figure 4-4 shows $v_t - v_t^*$ in relation to actual inventory investment. The calculation of perfect-foresight inventories, v_t^*, takes the actual level of capital as given for years t and $t+1$ because of a two-year time to build for fixed capital–this restriction has a large effect on the coefficients relating inventory investment to its determinants but little effect on the actual values of $v_t - v_t^*$. Again, movements in the difference, $v_t - v_t^*$ are strongly correlated at business-cycle frequencies. This correlation is paradoxical within a model where the difference arises purely from an expectation error. In addition, there is a rise in $v_t - v_t^*$ in the late 1960s. During the remaining years of the sample, the actual level of inventories is chronically well above the level mandated by the model The model does not account for the permanent rise in the inventory/production ratio that occurred in the late 1960s. Instead, it asks that there be inventory disinvestment each year because it finds that the level of inventories coming into the year is excessive by the standards of the model. As a result, there is much more noise in calculations that take the previous year's inventory stock as given than there is in the calculations underlying Figure 4-1, where the previous inventory level is taken to be the level mandated by the model instead of the actual level.

I assume that there is no important time to build inventories, at least in an annual model. Therefore all variables dated $t-1$ and earlier are eligible as noise-detecting regressors. The variables I use are a time trend (to deal with the trend in the inventory/production ratio), lagged inventories, and a financial

Figure 4-4. Inventory investment and inventory deviation



variable. The latter is the spread between the 6-month commercial paper rate and the 3-month Treasury bill rate, an indicator of financial stress that has been shown to be a good predictor of recessions. The noise-detecting regression is

$$v_t - v_t^* = -89 - 8.0t + .65v_{t\text{-}1} - 16.1R_{t\text{-}1} \tag{4.6}$$

$$R^2 = .860 \qquad F = 49.25$$

Figure 4-5 shows the conservative noise series from this regression, in comparison to inventory investment. Again, the bulk of the movements in inventory investment come from sources other than those considered in the rational-expectations accelerator model.

Noise in output

Figure 4-6 summarizes the findings about the spontaneous element of investment and output. It decomposes total output into sales, the part of fixed and inventory investment explained by the rational expectations accelerator, and noise. The explained part is simply total fixed and inventory investment less the two noise series. At lower frequencies, sales and output move together with amplitudes greater than either part of investment. At higher frequencies, noise is somewhat larger than the accelerator part of investment. In a number of fluctuations, noise was a dominant part of the story.

The noise series in Figure 4-6 is conservative. Only the part of the noise that can be associated with variables observed at least a year earlier is included. Any other elements of noise are included in the rational expectations accelerator term. Unless movements

Figure 4-5. Inventory investment and inventory noise

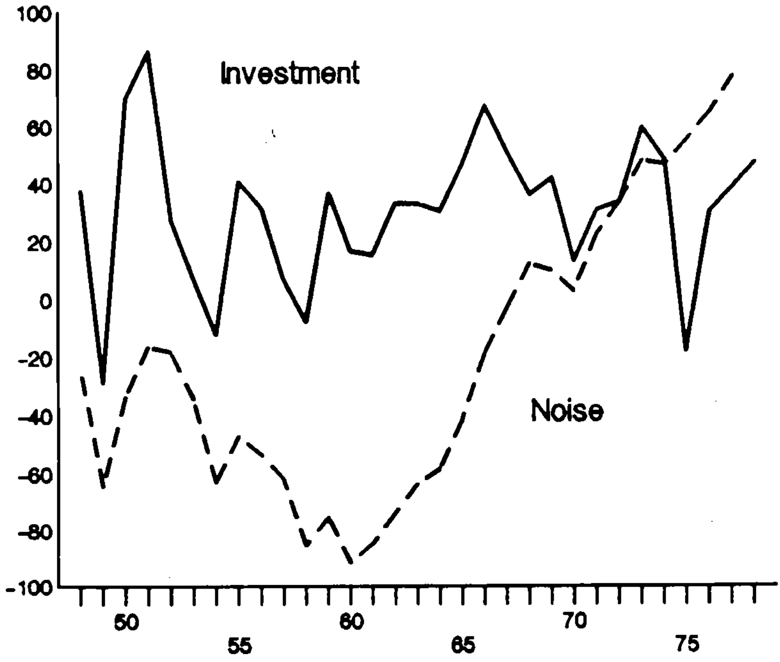
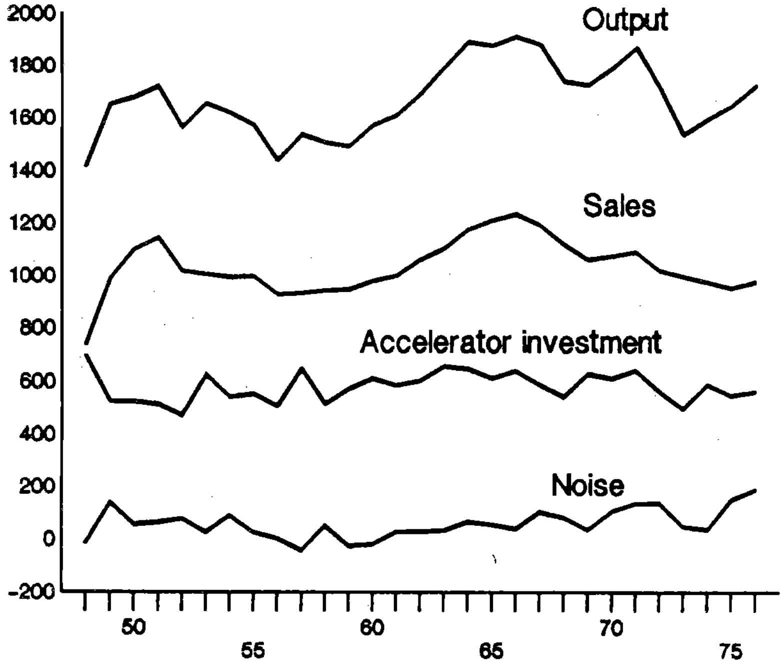


Figure 4-6. Output and its components



of noise are found to be correlated with some observed variable, there is no way to refute the hypothesis that they are expectation errors—there is no content to the concept of an expectation error apart from lack of correlation with variables known at the time the expectation is formed.

5. *Conclusions*

Output is substantially more volatile than it would be if the timing of sales were the same but the business sector scheduled production to minimize deviations from a prescribed capital-output ratio. Noise detection regressions show clearly that there is a large element of investment and hence output that cannot be explained by the rational-expectations accelerator model.

Factor substitution and financial responses are potentially important influences omitted from the accelerator model which are therefore included in measured noise. The finding that inventory investment noise is associated with the spread between commercial paper and Treasury bills is one indication that a period of financial stress causes an inventory selloff that is not contemplated by the accelerator model.

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