NBER WORKING PAPER SERIES

A THEORY OF PRICE CAPS ON NON-RENEWABLE RESOURCES

Simon Johnson
Lukasz Rachel
Catherine Wolfram

Working Paper 31347
http://www.nber.org/papers/w31347

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2023

We thank Ben Moll, Jose-Victor Rios-Rull, and seminar participants at UCL and Ohio State for comments. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

A Theory of Price Caps on Non-Renewable Resources
Simon Johnson, Lukasz Rachel, and Catherine Wolfram
NBER Working Paper No. 31347
June 2023
JEL No. F51,L13,L71,Q41

ABSTRACT

In December 2022, following Russia's invasion of Ukraine, a G7-led coalition of countries imposed a $60 per barrel price cap on the sales of Russian oil that use western services. This paper provides a theoretical and quantitative analysis of this new tool. We build a tractable equilibrium model in which the financially constrained exporter of a non-renewable resource optimally exerts market power, and the price of the resource varies stochastically. An important insight from this framework is that the supply curve is inelastic and can even be downward sloping, rationalizing the patterns we observe in the data. Contrary to the fears that an introduction of the price cap will cause a damaging oil supply shock, the exporter may have strong incentives to increase extraction following the introduction of a binding price cap. In fact, when the producer is large and has market power, a price cap that applies to all or most sales significantly limits the degree to which market power is used in equilibrium and stabilizes world oil prices. But if the cap is poorly enforced, or if the sanctioned state has access to a non-compliant "shadow" fleet, the cap is less effective at stabilizing world prices.

Simon Johnson
MIT Sloan School of Management
100 Main Street, E52-562
Cambridge, MA 02142
and NBER
sjohnson@mit.edu

Lukasz Rachel
University College London
30 Gordon Street
London, WC1H 0AX
United Kingdom
and Centre for Macroeconomics
l.rachel@ucl.ac.uk

Catherine Wolfram
Haas School of Business
University of California, Berkeley
Berkeley, CA 94720-1900
and NBER
wolfram@haas.berkeley.edu

1 Introduction

On December 5, 2022, the Price Cap Coalition, consisting of the G7, the European Union (EU), and Australia, responded to the continuing Russian invasion of Ukraine by imposing a cap on the price of seaborne Russian oil sold into global markets. Companies based in coalition countries are currently allowed to provide services that support Russian oil sales, including shipping, insurance and trade finance, but only if the price paid to Russia does not exceed $60 per barrel. The Coalition's goal is to reduce Russian revenue from oil sales, while also ensuring the uninterrupted flow of Russian oil to global markets, hence preventing a negative supply shock that could have adverse short-term consequences for the rest of the world.[1]

The implementation of this price cap on Russian oil is a significant development in the realm of international economic policy. It represents a novel approach to sanctions in an era of globalization when some markets are dominated by few large autocratic producers. Russia is one of the top three oil producers, every day exporting about 12% of the daily global supply of crude oil and oil products combined. Given its importance to global markets, conventional approaches to sanctioning oil producers, such as outright bans or embargoes, would have had major impacts on world prices and, by extension, the global economy. If the price cap proves effective, no country is too large to escape the consequences of sanctions imposed by a sufficiently determined alliance of financial and shipping-related service providers. Depending on its success, the price cap on Russian oil could become a blueprint for future sanctions, as well as international economic and trade policy more generally.

However, to date there has been little formal economic analysis of how the policy might impact both the sanctioned country and the global oil market. Some of the public discourse on the price cap applies outdated logic based on previous outright embargoes and suggests that any oil shipments out of Russia are a violation of sanctions. Most of the policy discussion continues to focus on static analysis, such as whether the price cap should be set close to the short- or long-run marginal cost of production, raising the question of whether the

---

[1] The price policy was developed in the context of the EU's "6th Sanctions Package", which was adopted in early June 2022. These measures included an embargo on the purchase of Russian oil from December 5, 2022, along with a ban on EU countries providing services in support of Russian oil exports. A similar ban on Russian oil products was slated for February 5, 2023. Since western services were used for a large share of Russian exports – over 70% by most accounts in the case of Russian seaborne crude trade (see Craig Kennedy, forthcoming) – there were concerns that this EU policy package could keep large volumes of Russian crude and product out of the market, effectively squeezing global supply and sharply raising oil prices everywhere. The price cap mechanism was designed to maintain the supply of Russian oil to world markets while squeezing Russian government revenue and sustaining the EU embargo.

same intuition applies in a dynamic setting. Furthermore, policymakers and analysts lack a quantitative framework that would help inform choices, such as whether to maintain the original $60 per barrel limit on crude or lower the price cap. Discussions of implementation and enforcement priorities can also benefit from the simple yet robust framework proposed in this paper. In particular, our framework emphasizes the importance of preventing – or delaying as much as possible – the development of a shadow fleet that does not operate out of the G7-led coalition countries.

Our dynamic model features a state exporter of a non-renewable resource who decides on an extraction path – i.e. a canonical cake-eating problem, in the spirit of Hotelling (1931).[2] The producer lives, to some degree, hand-to-mouth. Our main motivation for this is the potential presence of financial frictions, such as borrowing constraints, as well as frictions and risks (to the exporting country) associated with accumulating financial assets.[3] The on-shore storage capacity of Russia is also quite limited.[4]

Section 3 builds intuition using the illustrative case of perfect foresight, but our main model features a price of oil that is stochastic. Fluctuations of the world oil price reflect a complex pattern of demand, supply, and sentiment (expectation) shocks. In our model, the oil price follows a diffusion process which we estimate, that captures the empirically relevant moments of the oil price series.[5]

The final key element of our framework is that the producer has market power. This is realistic in the current context, given that Russia is one of the the world's leading oil producers, oil prices spiked immediately after the 2022 invasion began, and a principal rationale for implementing the price cap policy was that a complete EU embargo – refusing to buy Russian oil and effectively blocking sales to third countries – could lead to a contraction in world oil supply and a spike in world prices of oil. We develop a tractable approach

---

[2] Throughout this paper, we use the terms "exporter", "producer" and "Russia" interchangeably, given the context. We also refer to the commodity in question as oil. But our analysis is general and applies to any entity that extracts a commodity over time.

[3] These frictions are driven, in part, by both the (ex-ante) anticipated possibility of future sanctions and by the imposition of sanctions. Russia has substantial official foreign reserves, but these were "frozen" by the G7 immediately after the February 2022 Russian invasion of Ukraine. Since that initial freeze, Russia has been allowed to sell oil, and some other commodities, accumulating foreign assets in Gazprombank and other "private" entities. Russian authorities may be concerned about potential future freezes of those assets.

[4] Oil can also be stored "on the sea". But this involves loading oil that has not yet been sold, and shipping costs are incurred until sale and delivery.

[5] While OPEC Plus, the original Organization of Petroleum Exporting Countries (OPEC) plus Russia and several smaller countries, influences world prices by setting production quotas, in practice the oil price varies a great deal. At the end of 2021, the price of the Brent benchmark was around $75 per barrel, rising close to $90 in January 2021, and jumping sharply after the invasion. After peaking above $120 per barrel in early June 2022, the oil price fell during the rest of that year.

that integrates the market power of the producer with a stochastic process for the price of oil. The producer's market power is endogenous since it depends on the market share of the producer, which evolves over time, depending on extraction decisions. While we do not model the strategic interaction between Russia and other global producers, our model features parameters that reflect the responsiveness of other producers, such as OPEC, to shocks originating from Russia or elsewhere.

This approach offers four main findings. First, the producer's supply curve is inelastic and, for plausible parameters, may be downward sloping – i.e., it may be optimal to increase production when faced with a lower price of oil. We view this finding as consistent with the evidence of a negative correlation between the price of oil and Russian extraction that we present in Section 2, and with the observation that Russian production has changed little in the face of large fluctuations in the oil price over the past few years. This downward slope comes from two distinct effects: a "want cash now" effect and a "smoothing" effect. The "want cash now" effect captures the degree to which the oil producer uses oil revenues to finance current consumption. It arises even in a setting with no uncertainty, when the price of oil is known and fixed forever. With realistic (non-homothetic) preferences that reflect e.g. the fact that the producer has other source of funds beyond oil sales, when the price of oil is low, the producer is more willing to substitute oil revenue flows across periods – becomes more inter-temporally elastic – which makes it behave as if it were more impatient. Thus more of the resources are extracted today, increasing supply. The "smoothing effect" operates when the price of oil fluctuates. A high price today means that the marginal utility of revenues today is low and is expected to increase; thus, the producer cuts production today, effectively saving for tougher times ahead when the price is expected to be lower. When prices are low and the marginal utility is high, the producer smooths revenues by extracting more, effectively using up the implicit savings that it had accumulated in the good times.[6]

Second, a "perfect" price cap – a policy that applies to all or nearly all of the producer's export sales – tends to increase the producer's oil exports.[7] This is an important and perhaps

---

[6] When prices are stochastic there is also a "option value" effect: when prices are low, the producer cuts production, saving the oil in the ground to be sold only when prices rise in the future. This effect causes the supply curve to be upward sloping, but, as we discuss in detail below, is dominant only at prices that are just above the marginal cost of production.

[7] Throughout the paper we treat the price cap as a once-and-for-all policy. That is, we abstract from any effects that come from the anticipation that the level of the cap will be adjusted in the future, or any probability that the cap might be lifted altogether. Such analysis is beyond the scope of this paper but should be pursued in future research.

surprising finding that runs counter to the popular notion that the producer's incentives to extract are diminished when prices are capped. The impact on production is a net effect of two forces. First, a binding price cap brings down the sale price, and thus induces a movement *along* a downward sloping supply curve. The "want cash now" and the "smoothing" effects create economic incentives to increase extraction. Second, a binding price cap removes the upside shocks to future revenues that the producer would otherwise benefit from in expectation. Thus the cap makes the producer poorer, diminishing its ability to smooth revenues (smoothing is effectively a normal good). Less smoothing *shifts* the supply curve inwards and to the left. We show that, if the price cap is sufficiently binding – i.e. if it is set significantly below the ongoing world price – the movement down the supply curve dominates the inward shift, and the net effect is that the pace of extraction goes up.

Third, in the presence of market power, implementing a binding, "perfect" price cap significantly diminishes the incentives to exercise market power in equilibrium. The logic behind this is simple: when the price cap is binding, curbing supply leads to lower volumes but unaltered prices, thereby rendering the use of market power ineffective. This finding has important implications for the impact of the price cap. Most notably, a binding price cap can actually drive down world oil prices and act as a stabilizer of the global oil market. Such positive effects of a price cap are stronger the greater is the degree of market power of the producer. This is because the cap eradicates the use of market power in equilibrium, so the greater the influence of market power at the outset, the more significant the favorable impact of a cap. Overall, our study demonstrates that a price cap can be a potent tool and suggests that its benefits might actually be greater if it is applied to the exports of a producer with significant power in a market for a given commodity.

Our fourth finding concerns the effects of an imperfect price cap, i.e., a cap that applies to only a share of a country's sales of a commodity. This analysis is important because monitoring and enforcement of any cap is likely to be imperfect and the sanctioning coalition (e.g., of buyers) is likely to be able to impact only a certain part of exporter's sales.[8] Our findings show that if world prices are high, so that sufficient revenues can be generated through sales outside of the price cap regime, the producer may have strong incentives to "shut-in" production and instead sell the reduced quantities only outside of the price cap regime. The incentives to shut-in production are greater when the price cap is lower, so that the threshold for the world price above which the producer shuts in production

---

[8] For example, expert estimates and market intelligence suggest that, as of March 2023, western insurance and transport services are used in around 70% of Russian oil sales.

falls with the cap. Thus, with an imperfect cap, the incentives to exercise market power remain. This eliminates the powerful stabilizing effects that exist when the cap is perfect. However, our simulations also suggest that shutting in production can be costly in terms of contemporaneous revenue, and the impact on world prices is manageable. In the current context the exercise emphasizes the paramount economic importance of preventing Russia from building up a large shadow fleet, as well as the need for effective enforcement of the price cap.

Literature and contribution. The price cap is a new and live policy and there is little direct literature on this topic, which motivates this project. Early analysis of the economics of the price cap appears in Wolfram et al. (2022) and in Johnson et al. (2023) (forthcoming). In a recent paper that complements ours with the empirical analysis of the cap, Babina et al. (2023) use customs data to provide evidence on the effectiveness of the cap imposed by the G7 on Russia. They find that sanctions have led to a fragmentation of the oil market, with the oil that was destined to Europe trading at steep discounts and below the cap, while the oil sold elsewhere trading at close to global prices. They conclude that a lower level of the price cap coupled with effective enforcement is warranted, and our analysis lends further support to this recommendation. In a related theoretical contribution, Salant (2023) studies the effects of pre-announcing the price cap.

The framework we employ in the analysis builds on the classic work by Hotelling (1931), but appends it with the stochastic price of oil as studied in the finance literature (see Cox et al. (1985), Longstaff and Schwartz (1992), Chen and Scott (1993), Duffie and Kan (1996) for models of interest rates, and Schwartz and Smith (2000) and Pindyck (1999) for models of commodity prices) and develops a tractable way to think about market power. The Hotelling framework has been studied and extended in numerous studies.[9] A notable contribution is that of Anderson et al. (2018) who study the role of capacity constraints and drilling decisions – both margins which we abstract from – and an earlier work by Salant (1976) who studies the extraction problem in a framework with realistic industrial organization structure of the world market. Our paper analyzes the impact of the price cap on the extraction decisions and world oil prices, stopping short of analyzing the general equilibrium impact on the global economy. A complementary paper by Bornstein et al. (2023) develops a quantitative general

---

[9] Classic references include Solow (1974), Stiglitz (1976), Dasgupta and Heal (1974), Pindyck (1980), Arrow and Chang (1982). For an overview of work in the 50 years after the publication of Hotelling's article, see Devarajan and Fisher (1981). Recent work includes van der Ploeg and Withagen (2012), Newell and Prest (2017), Salant (2012) and Gaudet (2013).

equilibrium macroeconomic model with oil production sector, and uses it to study the advent of fracking.

This paper makes two main contributions. First, it provides a dynamic framework for analysis of the economics of the price cap policy with prices that vary stochastically, and demonstrates how to incorporate market power considerations into such framework. Second, it applies the model to study the economic incentives of Russia, and the policy options of the Price Cap Coalition, in the current context, thus explicitly addressing one of the timely and pressing policy questions.

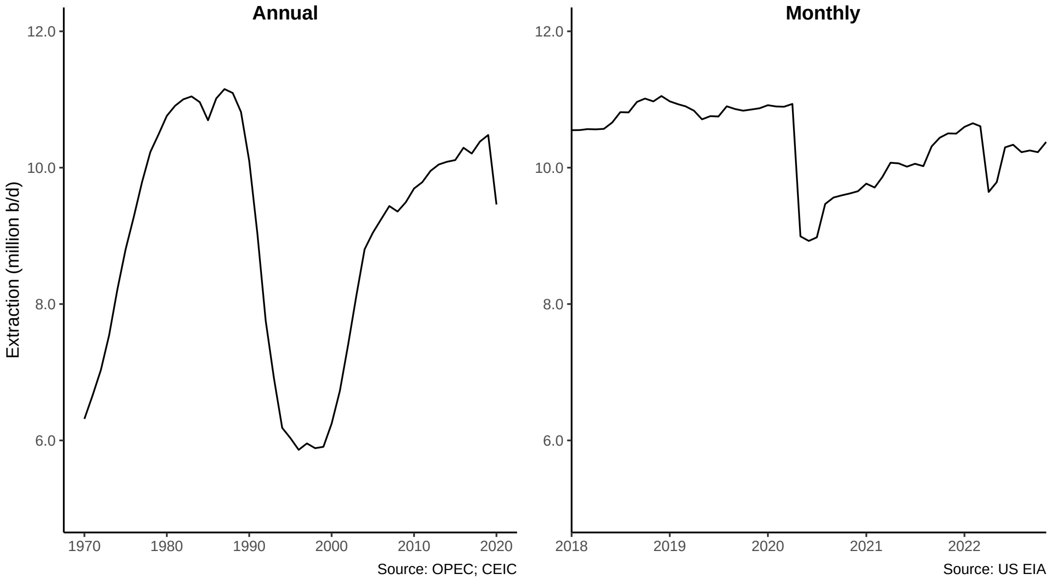
Finally, it is useful to highlight what we are not doing in this paper. The decision to go to war is economically costly and naturally increases the pressure on the state budget, raising the marginal value of funds today, relative to the future. This presents an additional incentive for Russia to increase extraction at given prices – a straightforward way of capturing this in a model would be to raise the discount rate of the producer, making it more impatient. We abstract from this important effect in our analysis in order to focus on how the price cap policy itself affects the producer's incentives.

Structure. The rest of the paper is structured as follows. Section 2 describes Russia's oil sector, including its costs, the prices it faces and typical export volumes and routes, and provides some institutional context on the price cap that is relevant to our model. Section 3 introduces our main assumptions in a setting with no uncertainty. Section 4 adds the stochastic process for the reference price of oil. Section 5 studies the effects of the price cap in a partial equilibrium setting. In Section 6 we construct our equilibrium model with market power, and study the effects of the price cap on the degree of market power exercised in equilibrium. Section 7 considers the case where the producer can partially bypass the price cap. Finally, Section 8 concludes with a discussion of policy implications.

2 The facts

2.1 Russian extraction historically

Russia has a long-established oil sector and in the 1970s was the world's largest oil producer. With the fall of the Soviet regime, oil production dropped to as low as 6 million barrels of oil per day. However, investment in the mid-1990s, along with access to western oil field services, helped to restore production to more than 10 millions barrels per day in 2019, making Russia the third largest oil producer in the the world (after the US and Saudi



Source: OPEC; CEIC

Source: US EIA

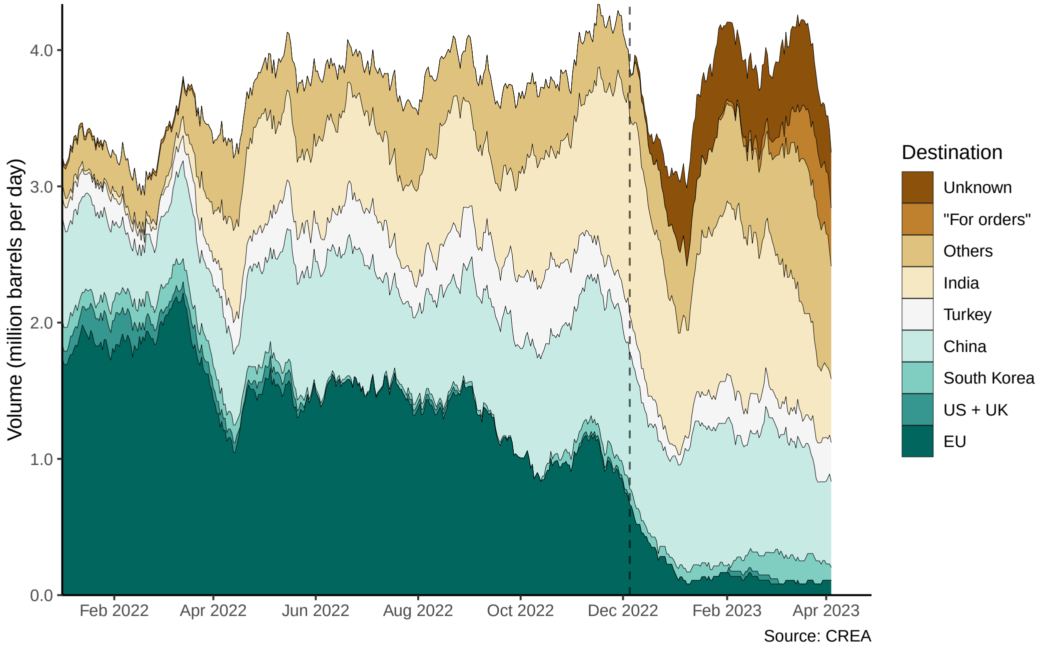
Figure 1: Russia's oil extraction historically: annual, 1970-2020 (left panel) and monthly, January 2018-December 2022 (right panel)

Arabia). In recent years, most Russian production has been exported (7.5-8 million barrels per day, out of production of 10-10.5 million barrels per day), making Russia the world's top exporter of combined crude oil and product. The left panel of Figure 1 plots annual Russian oil production since 1970 and the right panel plots monthly production in the last 5 years, highlighting the major disruptions around the pandemic and the war.

2.2 Russian oil production and export markets prior to the war

In 2021, Russia exported 7.5 million barrels of oil per day, of which crude was 4.7 million barrels and refined products were 2.8 million barrels. A single barrel of crude oil can be processed to produce multiple refined products such as gasoline, diesel, jet fuel, and other derivatives of oil. Refineries can be designed to produce different mixes of refined products, although the scope to change this is limited, especially in the short run. As of 2021, Russia's refining industry had the capacity to serve domestic gasoline demand and the country exported the remaining products. Substituting between exporting crude and exporting refined products is possible to some degree, but the infrastructure differs and there are pipeline and port constraints on each.

Most Russian oil is produced in Western Siberia and transported by pipeline to refineries and shipping facilities in Russia's Western ports. Before the war, Russia's largest oil customer



Source: CREA

Figure 2: Russia's seaborne crude oil exports by destination, January 2022 - April 2023. Dashed line indicates the start of the price cap policy for crude oil on December 5, 2022.

was the European Union, which received 0.7 million barrels of crude oil per day by pipeline and 1.5 million barrels by sea in 2021. The EU also bought 1.2 million barrels of oil product, almost all of which arrived by sea. Overall, the EU imported about half of Russia's total exports. Most of the tankers carrying these fossil fuels to the EU departed from three sets of ports: in the Black Sea, the Baltic Sea, and Murmansk in the far north.

China was also an important customer, primarily supplied from Russian ports in the far East. In 2021, China received 1.6 million barrels of crude per day, half by pipeline and half by sea. China did not previously buy a significant quantity of Russia's refined product.

Figure 2 plots Russia's seaborne crude oil exports by destination since January 2022. It does not reflect the approximately 1.5 million barrels of crude oil per day exported via pipeline, roughly half of which used to go to the EU and half to China. Figure 3 plots Russia's oil product exports by destination since January 2022, almost all of which travels by ship.

Russia's ability to shift supply across customers faces several constraints. First, crude oil and refined products travel in different kinds of ships. Crude oil is generally moved in very large tankers, capable of carrying 1 million barrels or more. Refined product travels in smaller tankers, not ideally suited to long-distance voyages. In addition, while Russia

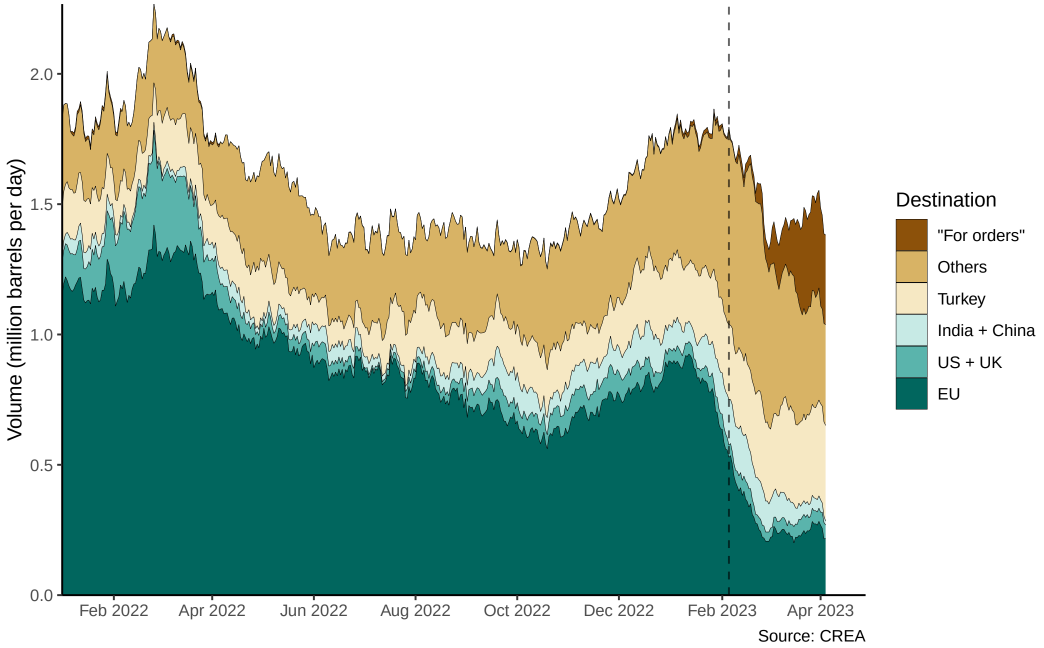


Figure 3: Russia's oil product exports by destination, January 2022 - April 2023. Dashed line indicates the onset of the price cap policy for oil products on February 5, 2023.

has some ability to route oil to different transshipment points within the country, there are pipeline constraints – including out to the far East. In 2021, Russia's pipeline to the far East was already roughly at capacity (1.6 million barrels per day).

Russia has very limited on-shore storage available, and most of this was already full when the 2022 invasion of Ukraine started. Storage "on the sea" is available, but this requires chartering and insuring ships for the duration – an expensive proposition.

Oil producers can also "shut-in" production, meaning they can close down wells. However, this process is costly and creates a risk that it will be expensive to restart production subsequently. This is a particular concern for Russia, as some of its oil fields are old and access to advanced western technologies – which would likely be required to re-open closed wells or open up new ones – are curtailed due to sanctions. However, some observers assess that Russia could shut in several million barrels per day and face low costs to restarting production (temporarily reducing production to approximately 8 million barrels per day and exports to approximately 6 million barrels per day).

2.3 Russian extraction costs and sensitivity to oil price fluctuations

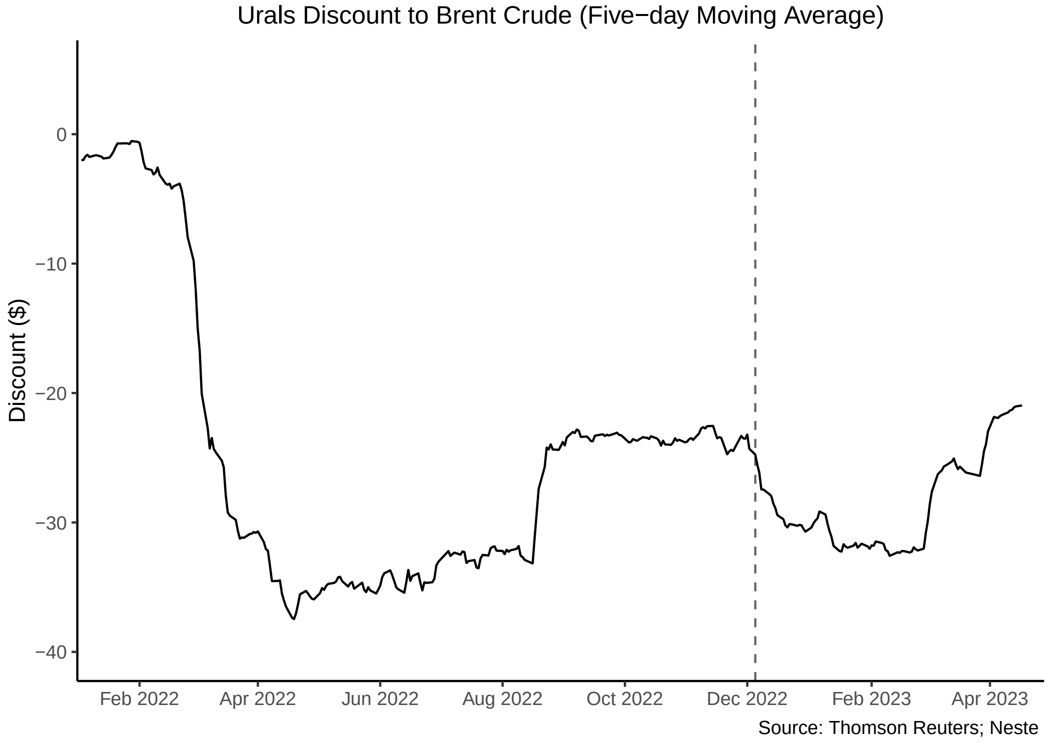
This paper aims to answer a central question regarding the responsiveness of Russia's production to the price it faces. One element of the production decision will be marginal costs. Various estimates peg marginal costs at most Russian fields at \$10 to \$40 per barrel, with the high end generally reflecting longer run marginal costs of developing new fields. At its low point at the beginning of the COVID pandemic, the price of oil was around \$20-\$25 per barrel. A presentation to investors by Rosneft, Russia's largest state-owned oil company, indicated that this price still covered short-run marginal cost, which appears to have been around \$15. According to careful analysis by CREA (a Finnish research institute) of fiscal regime operation in 2022, it appears that the Russian fiscal authorities adjust tax rates so that producers received (post-tax) around \$25 per barrel. Given the power of the Russian state over its oil companies and its ability to require payment of ex-post profit taxes, it seems reasonable to model responses to the price of oil (and attempts to exert market power) as a national level decision.

The price Russia receives for its oil is heavily influenced by the world price of oil. OPEC Plus, which periodically sets production quotas, has considerable influence on world prices. Observers suggest that OPEC Plus quotas announced in early 2023 targeted world oil prices of \$80 to \$90 per barrel. In addition to participating in OPEC Plus actions, Russia, as one of the largest oil producers, can exert short-run influence over oil prices through its announcements and actions. The 2022 invasion of Ukraine, for example, pushed world oil prices up by nearly 40 percent, presumably because participants in the oil market were concerned about potential disruptions to Russian supply. In response to the announcement of a potential price cap regime, some western analysts predicted that Russia would reduce supply and drive up oil prices – one team (at JPMorgan Chase) predicted that oil could reach \$380 per barrel as a result.[10]

Immediately after the invasion, some shipping companies and customers declined to do business with Russia, resulting in a stigma that lowered the price paid for Russian oil. Consequently, Russian oil sold for a discount from the world benchmark price. Exchanges quote prices for "Urals" oil, which describes the mix typically sold by Russia, including oil from fields in the Urals, Volga and Western Siberian regions. Figure 4 plots the Urals

---

[10] In Section 6 we provide estimates of elasticities of oil demand and deduce the likely price impact of a production shut-in. For prices to go above \$300 per barrel on a sustained basis, one needs to assume that Russia essentially shuts in all of its supply and that the world demand is extremely inelastic.

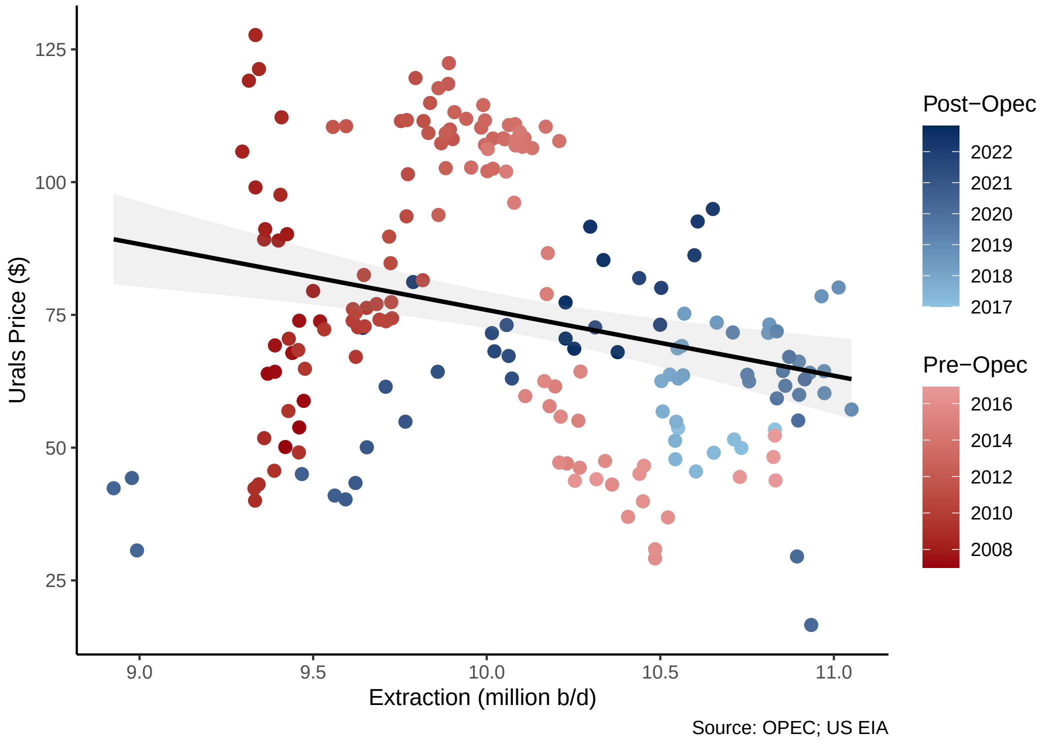


Source: Thomson Reuters; Neste

Figure 4: Russian Urals price minus Brent price January 2022 - April 2023

discount (Urals price minus Brent price) since just before the invasion through the end of April 2023. Urals prices are not based on publicly posted transactions but are collected by reporting services, like Argus Media and S&P Global, who request quotes from traders. There is some question about the accuracy and representativeness of the prices that the reporting services are able to collect, particularly after the price cap was enacted. Before the war, the Urals discount was usually small, reflecting the market value of Russia's blend of primarily heavy sour oil. The discount was largest at nearly $40 in mid-April 2022, and then declined before increasing again in early December, as the price cap was imposed. Oil sold out of Russia's Eastern ports, primarily to China, is priced relative to the benchmark "ESPO" price, referencing the Eastern Siberia-Pacific Ocean oil pipeline. ESPO prices are even less transparent than Urals prices, but historically traded close to Urals and have been discounted less than Urals since the war began.

To get another perspective on Russia's production decisions, Figure 5 plots monthly production against the Urals price from January 2007 to November 2022. Months before Russia joined OPEC Plus are denoted with red circles and months after it joined OPEC Plus are denoted with blue circles. The dark fitted line suggests that, if anything, production has



Source: OPEC; US EIA

Figure 5: Russia's oil extraction versus Urals price

decreased at higher prices. Consistent with this, early 2023 reports suggest that Russia's crude oil production increased in the first four months after the price cap was imposed. For example, the International Energy Agency in April 2023 concluded that,"Russian oil exports in March soared to the highest [level] since April 2020 ... Total oil shipments rose by 0.6 [million barrels per day] to 8.1 [million barrels per day], with products climbing 450 [thousand barrels per day, month-on-month] to 3.1 [million barrels per day]. Estimated oil export revenues rebounded by $1 billion to $12.7 billion but were 43% lower than a year ago."[11]

2.4 Importance of oil revenues for Russia's current account and government budget

In 2021, oil (crude and product) was Russia's largest export by category, followed by natural gas and coal. In total, energy accounted for over 50% of all export revenues, with oil accounting for 75% of energy exports. Reflecting the dependency on oil revenues, Russia's

---

[11] https://www.iea.org/reports/oil-market-report-april-2023

fiscal planning processes benchmark an oil price.

In the first nine months after the February 2022 invasion, Russia received significant revenue from the export of natural gas to Europe as it curtailed supply and prices rose to more than offset the lost revenue. Global oil prices also rose in the beginning of 2022, offsetting the growing discount so that the price for Russian oil was higher in 2022 than 2021. But European gas prices declined precipitously, by about 75%, in the six months beginning December 2022. Coal exports are also subject to sanctions, although Russia appears to have diverted some coal exports to China. Looking forward, if Russia is to earn a significant amount of foreign exchange, it is under pressure to keep oil production and exports as high as possible.

Reinforcing this pressure is the fact that Russia cannot freely access official foreign exchange reserves that were accumulated before February 2022. Russia ran a current account surplus for many years, and had a pre-invasion stock of foreign assets owned or controlled by the central bank of around $500bn. Immediately after the invasion, the G7 forbade western counterparties from transacting with the central bank for these amounts, effectively freezing the funds. However, Russia was allowed to receive payment for ongoing oil (and other) exports, and claims on foreign entities built up during 2022 in Gazprombank and other nominally private entities. Given the continuing conflict, it is possible that those balances would be subject to an additional freeze, amounting to a form of ex post price cap.

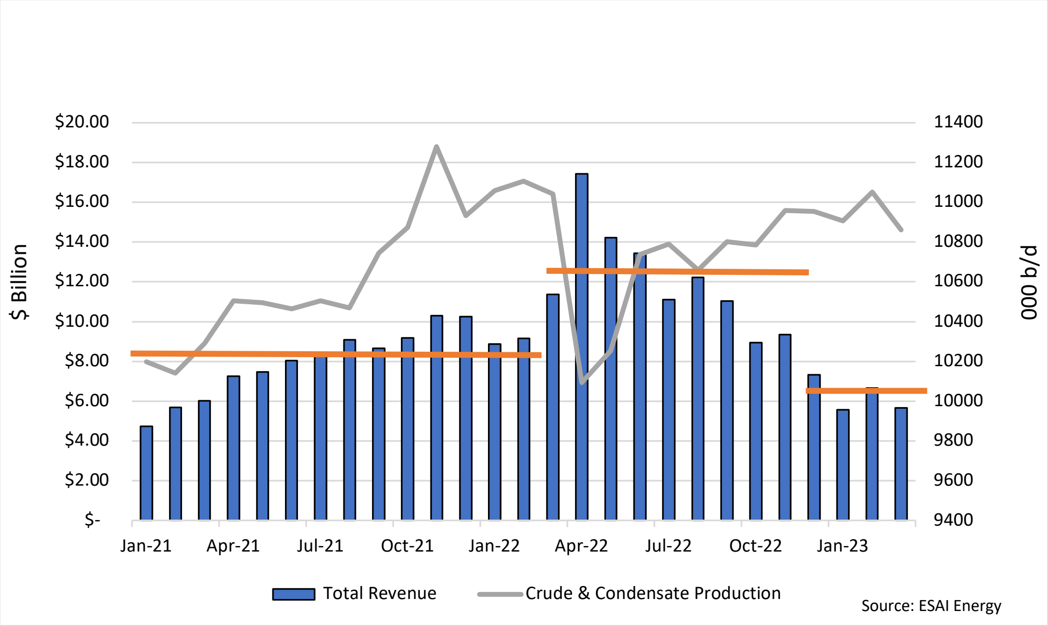
2.5 Price cap implementation details

The price cap operates by setting terms and conditions on the provision of western financial and shipping services. Specifically, services can only be provided for shipping Russian oil by companies located in price cap coalition countries if the price paid to Russia is at or below the cap.[12] The caps were initially set at $60 per barrel for crude, $100 per barrel for high-value refined products (including diesel, gasoline and kerosene) and $45 per barrel for low-value refined products (including fuel oil and naptha).

As discussed above, the price cap was implemented in response to the EU's 6th sanctions package, which would have banned the provision of services for shipments of Russian oil altogether and could have reduced the supply of Russian oil to world markets considerably. The price cap effectively allows for an exception to that outright ban. Before the price cap

---

[12] In addition to the G7, EU and Australia, Albania, Bosnia and Herzegovina, Iceland, Liechtenstein, Montenegro, North Macedonia, Norway, Switzerland and Ukraine have all pledged to follow EU sanctions against Russia.



Source: ESAI Energy

Figure 6: Russia's oil production and government tax revenue

was put in place, some market observers expressed concern that, if compliance with it was too complicated, western service providers would de-risk and pull back from the Russia trade completely – driving up world oil prices in much the same way as was feared under the EU's 6th sanctions package.

However, in contrast to any dire predictions, setting a price cap on Russian oil at $60 per barrel seems to have had four broad effects. First, the Kremlin's oil-related revenues have fallen by 49% compared to the March to November 2022 period and 23% compared to the January 2021 to January 2022 period. Specifically, the blue vertical bars in Figure 6 reflect Russian government revenue from the mineral extraction and export taxes by month. The orange horizontal bars reflect averages during the pre-war, post-war and pre-price cap and post-price cap period (see also Hilgenstock et al. (2023)). Second, Russia's oil production has if anything increased (see the grey line in Figure 6). Third, the advent of the EU embargo (for crude in December and refined products in February) did not result in a spike in world oil prices. Fourth, most western service providers have remained engaged in the Russia trade. Data from CREA suggest that about 60% of crude oil shipments and 75% of product shipments from Russia's ports in April 2023 were covered by insurers from the EU, G7 or Norway.

As a result, Russia's reported current account surplus has declined commensurately. And this has happened without a large supply shock in the world market for oil. These outcomes have been achieved through the combination of the embargo and the price cap which, relative to the 6th sanctions package originally announced, has allowed Russian oil to flow to world markets.

There are, however, serious concerns about what comes next. Russia is amassing access to a shadow fleet, which does not operate out of western countries. A major concern has already appeared with regard to trade out of Kozmino (the Russian "far east") – some of this oil is moving to China and, reportedly, to Australia despite the fact that the oil is bought at a price above the capped level and shipped using western services.

There are also questions about who exactly in the value chain is making higher than normal profits from this arrangement. If an entity, e.g., in India, buys crude at or below the cap, it is allowed to sell the refined product at world prices. This arrangement is expected to encourage the flow of Russian oil and helps explain why Russian deliveries to the world market are largely unchanged. But who exactly is profiting from the difference (world price minus capped price) remains shrouded in some mystery. As one example, a *Wall Street Journal* article in April 2023 cited evidence that Saudi Arabia and the United Arab Emirates were importing Russian oil products at low prices and earning high profits (Faucon and Said (2023)), but no systematic accounting of where the rents have gone exists.

In the remainder of this paper, we provide an analytical framework intended to help organize thinking on the price cap.

3 Warm-up: no uncertainty

3.1 Producer's problem

Consider a problem of a country endowed with x_0 amount of natural exhaustible resource. Given the motivation and context for writing this paper, we refer to the country as Russia and to the commodity as oil. We normalize $x_0 = 1$ without loss of generality. Let y_t denote the amount of oil extracted at time t and $p > 0$ denote the price of oil in the global market, in dollars per barrel, which is a fixed parameter (in this warm-up section only), and which Russia takes as given (we also relax this later). The producer discounts the future at rate ρ and its instantaneous utility $\tilde{u}$ takes as an argument the profits, $\pi := (p - c)y_t$, where c is marginal cost of extraction which we assume to be constant. The idea is that the state does not consume the vast majority of the oil it extracts, but instead sells oil globally and uses the proceeds to finance purchases of items it deems useful. We define the function

$u(y) := \tilde{u}(\pi(y))$. The producer solves the following problem:[13]

$$\max_{y_t} \left[\int_0^\infty e^{-\rho t} u(y_t) dt\right] \text{ subject to } dx_t = -y_t dt,\ x_t \geq 0,\ y_t \geq 0. \tag{1}$$

The constraints say that the stock of reserves x_t diminishes by the amount extracted y_t, and that reserves and extraction must be non-negative.[14] Note that there is no expectations operator in (1), as everything is known at time 0 – this is a perfect foresight environment.

We assume that $u : \mathbb{R}^{\geq 0} \to \mathbb{R}$ satisfies $u_y > 0$, $u_{yy} \leq 0$, and that $u(0)$ and $u_y(0)$ are bounded. We now discuss the interpretation of these assumptions in more detail.

3.2 Properties of preferences

We make an important assumption that profits from oil sales are an argument in the utility function. This implies that the curvature of the utility function determines the degree to which the exporter is hand-to-mouth. To see why, note that one can think of an underlying utility maximization problem where the consumption of real goods and services appear as an argument in the utility function of the exporter, and revenues from oil sales determine the budget set of the problem. If the producer has frictionless access to borrowing and lending opportunities, the time path of revenues becomes irrelevant – the only thing that matters is the net present value of a path of revenues. Instead, problem (1) features frictions in the ability of the exporter to finance consumption out of the stock of accumulated financial savings and out of the claims to oil that will be extracted in the future. The degree of these frictions is captured in a reduced form way by the degree of concavity of the utility function. In the limit, a linear u in (1) means that the exporter can perfectly smooth consumption intertemporally, with no preference for smoothing oil revenues, so that only the net present value of the revenue stream matters. But if the producer is constrained in how it can substitute intertemporally, it must finance its consumption out of current revenues, meaning that the time path of revenues matters – and oil revenue today can be worth more than the present discounted value of even completely certain sales tomorrow.

Beyond the financial frictions interpretation, the formulation of the problem in (1) can also be motivated by the presence of dividend smoothing motives, as in the corporate finance

---

[13] This problem is also known as a canonical cake-eating problem, here set in continuous time. An agent has a cake of size $x_0 = 1$ and decides on the optimal way of eating the cake, given time-separable preferences and the instantaneous utility u over consumption of cake y_t and a discount rate ρ. x_t is the size of the cake at t; $dx_t = -y_t dt$ simply says that the size of the cake gets smaller with each bite.

[14] Note also that this formulation effectively assumes that the marginal cost of production is zero; we could easily incorporate a positive constant marginal cost $c > 0$ in the analysis. We discuss this more later.

literature (Lintner (1956), Fama and Babiak (1968), Cui (2022)), but applied to public finance. Smoother revenues from oil sales may aid fiscal (and war) planning and might help in achieving a smoother path for taxes that finance the budget (Barro (1979)).

We also assume that the level of utility at zero extraction $u(0)$, and the marginal utility at that point $u_y(0)$, are both bounded.[15] We make this assumption for realism: there are plenty of countries in the world which do just fine without extracting any oil. An alternative way to think about this is that the producer finances its consumption from several sources, one of which is oil sales. Thus, if no oil is extracted, welfare-relevant measure of consumption of the exporter is positive, with bounded utility and bounded marginal utility.

It is important to point out that this assumption implies that complete shut-in of production – limiting extraction all the way to zero – is a potential option for the producer facing sanctions. Thus, in addition to adding realism to our model, this assumption serves the role of not precluding the outcome that policymakers in the current context are most concerned about: that Russia will very significantly, or perhaps even completely, limit the supply of oil to the global market.[16]

3.3 The Hamilton-Jacobi-Bellman and Euler Equations

The Hamilton-Jacobi-Bellman equation associated with problem (1) is

$$\rho v(x) = \max_{y} u(y) - y v_x(x). \tag{2}$$

The solution satisfies the Euler equation

$$\frac{\dot{y}}{y} = -\mathcal{E}(y)\rho \tag{3}$$

---

[15] Note that our restrictions are violated if utility is of a simple Constant Relative Risk Aversion (CRRA) form, $u(\pi) = \frac{\pi^{1-\gamma}}{1-\gamma}$. While this utility function is particularly tractable in the current setting, it has the unrealistic implication that the marginal value of the last barrel of oil in the ground tends to infinity, and that utility goes to minus infinity as reserves are depleted. We view these implications as unreasonable in this context. We illustrate below how our results would differ had we assumed CRRA preferences.

[16] Conversely, if utility diverged at the point of zero extraction, a complete shut-down of production would never be optimal.

where $\mathcal{E}(y) := \frac{-u_y}{u_{yy}y}$ is the intertemporal elasticity of substitution (IES).[17] Since IES>0, equation (2) says that extraction is declining over time, and that it is declining faster, the more willing is the producer to substitute revenue intertemporally. This is intuitive: since the producer discounts the future at a positive rate, absent any smoothing considerations it would extract all the resources at once. Desire for smooth extraction profile – a finite IES – puts a limit on this process. The greater is the IES, the greater is the tolerance for uneven pattern of extraction over time, which raises the level of extraction today and increases the pace at which extraction declines going forward.[18]

3.4 Parametrizing preferences

We assume that utility function u belongs to a HARA (hyperbolic absolute risk aversion) class:[19]

$$u(y) = \frac{\sigma}{1-\sigma}\left(\frac{\gamma\pi}{\sigma} + b\right)^{1-\sigma} \tag{4}$$

with $\sigma > 0$, $\gamma > 0$ and $b > 0$. This broad class of utility functions includes notable special cases such as linear, quadratic, exponential, and isoelastic utility functions. The restriction that parameter b be strictly positive ensures that the utility function meets the boundedness requirements at $y = 0$ that we discussed above.

With preferences in (4), the intertemporal elasticity of substitution is

$$\mathcal{E}(y) = \frac{1}{\sigma} + \frac{b}{\gamma(p-c)y}.$$

Thus, the IES is inversely related to p: when p is low, the producer is more willing to substitute intertemporally. Put differently, it has a greater tolerance for a more uneven path of revenues. A useful intuition comes also from noting that with a p that is a fixed parameter, varying p is equivalent to varying the quantity of reserves: both affect the consumption-equivalent value of the pool of reserves in the ground symmetrically. With non-homothetic

---

[17] Along the optimal extraction path the marginal value of the exhaustible resource grows at the rate of time preference – the celebrated Hotelling rule. To see this, note that the envelope condition is $\rho v_x(x) = -y v_{xx}(x)$, so that letting $P := v_x(x)$, we have $\frac{\dot{P}}{P} = \frac{v_{xx}(x)}{v_x(x)}\dot{x} = \rho$, where the final equality follows from the resource accumulation equation.

[18] Equation (3) also illustrates why the CRRA case is so tractable in this setting: with constant $\mathcal{E}(y)$, the Euler equation implies that extraction declines exponentially. It is then straightforward to show that extraction is a constant fraction of reserves, and that reserves decline towards zero only asymptotically. Still, this case is not realistic for the reasons outlined above.

[19] For future reference, note that marginal utility is $u_y(y) = (p-c)\cdot\gamma\left(\frac{\gamma}{\sigma}\pi + b\right)^{-\sigma}$ so that $u_y(0) = (p-c)\cdot\gamma b^{-\sigma}$.

preferences, such changes in the value of reserves affect the optimal extraction rate.

Two useful special cases of the utility function are Constant Absolute Risk Aversion (CARA) utility, obtained by setting $\sigma \to \infty$ and $b = 1$:[20]

$$u(y) = -e^{-\gamma\pi} \tag{5}$$

and power utility, obtained by setting $\sigma = \gamma$:

$$u(y) = \gamma\frac{(\pi+b)^{1-\gamma}}{1-\gamma}. \tag{6}$$

3.5 Downward sloping supply of oil

In the main text of this paper we focus on the case with CARA utility, although all our results are robust to the alternative assumption within the broad HARA class. The following proposition characterizes the optimal dynamic path of extraction, and focuses in particular on the contemporaneous supply function y_0 (all proofs are in the Appendix).

Proposition 1. *When utility is CARA as in (5), $x_0 = 1$ and the price of oil is known and equal to $p > c$, the optimal interior paths for extraction and reserves satisfy*

$$y_t = \sqrt{\frac{2\rho}{\gamma(p-c)}} - \frac{\rho}{\gamma(p-c)}t \qquad\qquad x_t = \frac{1}{2}\frac{\rho}{\gamma(p-c)}t^2 - \sqrt{\frac{2\rho}{\gamma(p-c)}}t + 1. \tag{7}$$

that is, the amount extracted declines linearly over time. Reserves are used up after $T = \sqrt{\frac{2\gamma(p-c)}{\rho}}$ years. Contemporaneous level of supply is $y_0 = \min\left\{1, \sqrt{\frac{2\rho}{\gamma(p-c)}}\right\}$, and hence it is inversely related to the price of oil.

How does the price of oil impact producer's decisions? As long as $p > c$, we have

$$\begin{aligned}\frac{\partial y_0}{\partial p} &= -\frac{1}{2}\sqrt{\frac{2\rho}{\gamma(p-c)}}\frac{1}{(p-c)} < 0\\ \frac{\partial T}{\partial p} &= \frac{1}{2}\sqrt{\frac{2\gamma}{\rho(p-c)}} > 0\end{aligned}$$

That is, for a *lower* price, the producer extracts *more* oil today and depletes the stock of reserves more rapidly. Thus, the model suggests that the (contemporaneous) supply curve

---

[20] We have $\lim_{\sigma\to\infty}\frac{\sigma}{1-\sigma}\left(\frac{\gamma}{\sigma}\pi+1\right)^{1-\sigma} = -\lim_{\sigma\to\infty}\left(\left(1+\frac{\gamma\pi}{\sigma}\right)^{\sigma}\right)^{\frac{1-\sigma}{\sigma}}$, since $\frac{\sigma}{1-\sigma}$ goes to -1 as $\sigma\to\infty$. Using the limit definition of the exponential, this limit equals $-\lim_{\sigma\to\infty}\left(e^{\gamma\pi}\right)^{\frac{1-\sigma}{\sigma}} = -e^{-\gamma\pi}$.

is *downward sloping*. This result follows straightforwardly from the intuition developed earlier, that with non-homothetic preferences, a lower p acts to make the producer more intertemporally elastic, as if the producer was more impatient. Thus, faced with a lower price, the producer has an incentive to ramp up contemporanous production levels. We term this mechanism a *"want cash now"* effect.

3.6 Numerical solution

To further illustrate the results in the Proposition above, we parameterize our simple perfect foresight model (at annual frequency) and compute the numerical solution. We set: $\rho = 0.05,\ \gamma = 2,\ p = \$80$ and $c = \$15$. With these values, we obtain:

$$\begin{aligned} y_0 &= 2.8\% \\ T &= 72 \text{ years} \end{aligned}$$

That is, current rate of extraction is 2.8% of the stock, implying that at the current extraction levels, the stock will last just short of 40 years, broadly in line with the available estimates.[21] However, the model also predicts that extraction will decline in the future; hence it will take 72 years before the reserves are fully depleted.

Figure 7 shows the optimal time profile of extraction and reserves depletion. Both fall monotonically over time. The ratio of the two $\frac{y_t}{x_t}$ is the extraction rate, which is flat initially but increases sharply towards 100% as reserves are depleted in finite time.

The figure also plots the solution under a lower price of oil (dashed lines). This is the *want cash now* effect in action: the extraction path becomes steeper, with a higher starting level.

Figure 8 plots the contemporaneous supply as a function of the price. The supply curve is downward sloping, but over much of the price range is it inelastic, essentially close to vertical. This accords well with the patterns observed in the data which we discussed in Section 2.

3.7 Discussion of the assumptions

There are three assumptions worth discussing briefly.

The first is that the producer takes the world price of oil as given. In practice, Russia is a

---

[21] Source: `https://warsawinstitute.org/russian-oil-gas-resources-much-long/`.

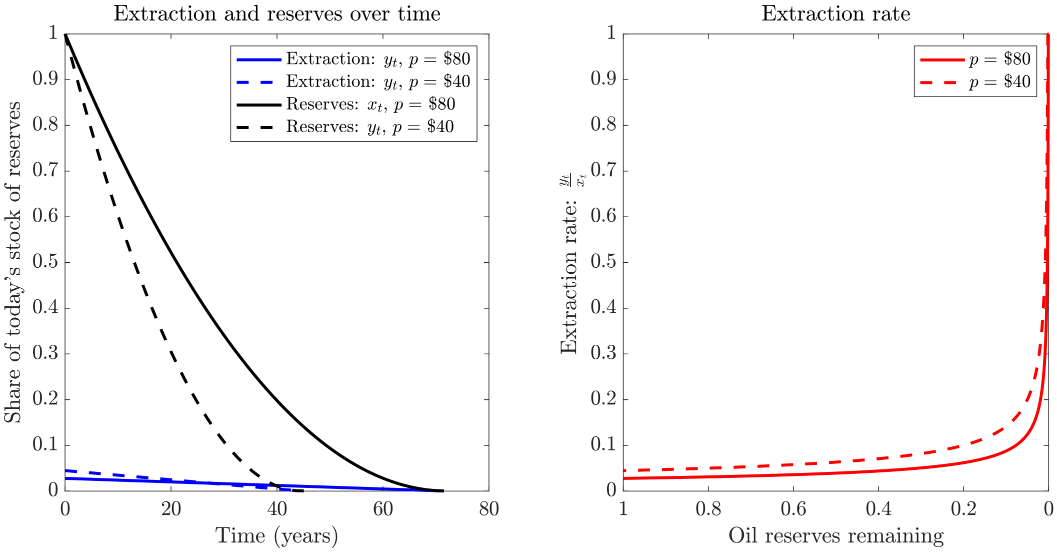


Figure 7: Optimal extraction path when there is no uncertainty and the oil price is fixed at \$80 (or \$40) per barrel

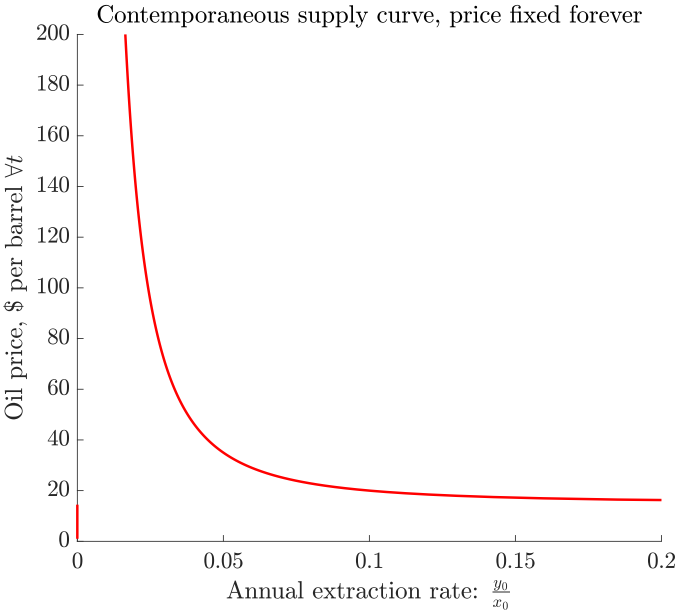


Figure 8: Supply curve is downward sloping

large producer with pricing power in the global market – in other words, it could internalize the effect of its own actions on the global price of oil. We will relax this assumption shortly.

The second assumption is that over time as the producer's oil reserves are depleted, the price does not move deterministically and instead stays constant. Instead, one might expect the price to be trending as oil is depleted globally. We have made this assumption for two reasons. First, the stock of Russia's oil reserves is small relative to global stocks: at current production levels Russia's reserves are likely to be exhausted a long time before stocks globally are gone (if that ever happens). Second, even as fossil fuel resources have been pumped out of the ground and burnt in past decades, there has been no discernible trend in the (real) price of oil since WW2, notwithstanding large spikes during the energy crises. Additional oil reserves have been discovered, while technological progress has driven increasingly efficient energy use and generated alternative sources of energy, which has meant that oil is not becoming economically more scarce (Jones and Volrath (2013)).[22]

The third assumption is that the producer faces a known price forever and that there is no uncertainty. This is of course unrealistic as the oil price fluctuates every day. In the remainder of this paper, we incorporate fluctuations of world oil prices into the analysis.

4 Optimal extraction with price fluctuations

4.1 Stochastic process for the price of oil

A rich and complex combination of demand, supply, and market shocks result in daily fluctuations in oil prices. To capture the empirically relevant volatility of the price, we model it using a stochastic process called Cox–Ingersoll–Ross model (also known as a Feller square root process):

$$dp_t = D(\tilde{p} - p)dt + \sigma\sqrt{p}dW_t \tag{8}$$

where W_t is the standard Wiener process and $\tilde{p}$, D, and σ are (strictly positive) parameters that satisfy $2D\tilde{p} > \sigma^2$. Parameter D determines how quickly the gap between the current price and the average price $\tilde{p}$ closes (i.e. it determines the speed of mean reversion). Parameter σ determines the volatility of the price, driven by standard Brownian motion.

The process in (8) ensures that the price always stays positive: as $p \to 0$, the importance of Brownian noise diminishes, and mean reversion drives the price away from zero. There is

---

[22] Papers that study long-run determinants of real oil prices include Barsky and Kilian (2004), Kaufmann et al. (2004), Fattouh (2005), Smith (2005), Hamilton (2009), Alquist and Kilian (2010), among others.

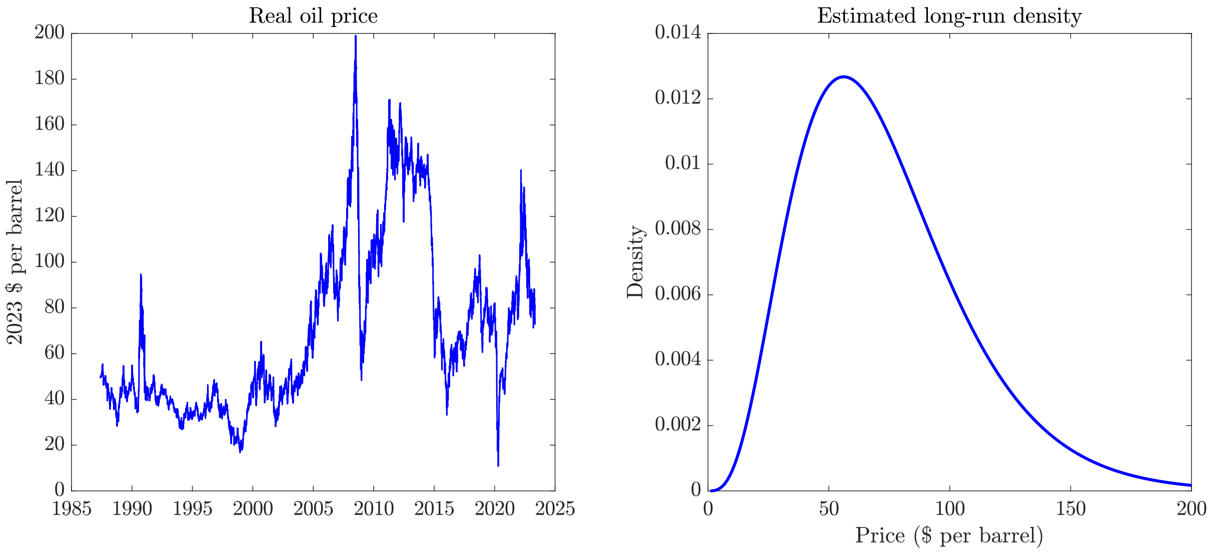


Figure 9: Data on real Brent oil prices used in estimation, and the long-run distribution of the estimated process.

no upper bound to the price: we have $p_t \in (0, \infty) \forall t$. Due to mean reversion, as time becomes large, the distribution of p_∞ will approach a Gamma distribution with the probability density function

$$f(p_\infty; D, \tilde{p}, \sigma) = \frac{\beta^\alpha}{\Gamma(\alpha)} p_\infty^{\alpha-1} e^{-\beta p_\infty},$$

where $\beta := \frac{2D}{\sigma^2}$, $\alpha := \frac{2D\tilde{p}}{\sigma^2}$ and $\Gamma(\alpha)$ is the Gamma function.[23]

We estimate the process in (8) using daily data on real oil prices from 1987.[24] We obtain $\tilde{p} = \$72$ (in today's prices), $\sigma = 2.97$ and $D = 0.26$. Under these esimated values, the limiting distribution of the oil price is skewed to the right (Figure 9). Estimated standard deviation is \$35.

4.2 Optimal extraction when the price is stochastic

With stochastic prices, the problem of the oil producer becomes:

$$\max_{y_t} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} u(y) dt \right] \text{ subject to } dx_t = -y_t dt, \; x_t \geq 0, \; y_t \geq 0$$

---

[23] The variance of the limiting distribution is $\frac{2D\tilde{p}}{\sigma^2}$.

[24] We obtain our data series from the FRED database. We deflate the daily nominal Brent oil price (code DCOILBRENTEU) by US CPI index (code CPIAUCSL_NBD20230401) set to 1 in April 2023 (we extrapolate from monthly to daily data using monthly averages). We use maxiumum likelihood estimation, making use of the numerical implementation by Kladivko (2013).

and (8), where we now have $\pi_t = (p_t - c)y_t$ (p_t is stochastic and is a state variable of the problem). The HJB equation now reflects the drift and uncertainty of the price:

$$\rho v(x,p) = \max_{y_t} u(y) - v_x(x,p)y + v_p(x,p)D(\tilde{p} - p) + \frac{1}{2}v_{pp}(x,p)\sigma^2 p. \tag{9}$$

The first order condition is

$$u_y(y) = v_x,$$

implying the optimal rate of extraction[25]

$$y = u_y^{-1}(v_x).$$

4.3 The three effects shaping the supply curve

We solve the model assuming that the oil price follows the estimated CIR process (the solution method is outlined in the Appendix). We assume the same parameter values as in the previous section. Figure 10 depicts contemporaneous supply curve with price fluctuations, and compares it against the deterministic case.[26]

The *need-cash-now* effect still operates in this setting, but there are also two additional forces that operate in presence of price fluctuations.

The first of these forces is the *option-value-of-waiting* effect. This effect dominates extraction decisions when prices are low, close to the short-run marginal cost. The intuition is straightforward: if the price is low today, and it is expected to recover in the future, then the producer reduces extraction today and sells only when the prices are more favorable.[27]

The second effect is the *smoothing* effect. At high prices revenues are high and the marginal utility of extraction is low. In that region the producer saves oil reserves for more challenging times, that is, leaves them in the ground. These reserves (in the ground) are then used to smooth revenues when prices are in the low-to-mid range.

---

[25] The constraints $x \geq 0$ and $y \geq 0$ give rise to a state boundary condition $u_y(0) = v_x(0,p)$. This is because at $x = 0$, extraction must be zero.

[26] Figure 26 in the Appendix shows the complete policy function. Here we focus on how uncertainty about the price affects optimal extraction contemporaneously.

[27] Interestingly, the range of (low) prices at which producer shuts in production expands as reserves are used up: at today's reserves, the producer shuts in only at extremely low prices like \$2 per barrel. However, once left with only 10% of the reserves, the shut-in region expands to about \$30 a barrel. In other words, the option value of waiting increases as reserves decline. The intuition for this pattern is that at high reserves, the producer expects to sell plenty of oil for high prices sometime in the future, thus giving it a chance to build a buffer against future low price realizations. At low reserves the future does not hold that promise, so the value of oil in the ground as a cushion is greater then.

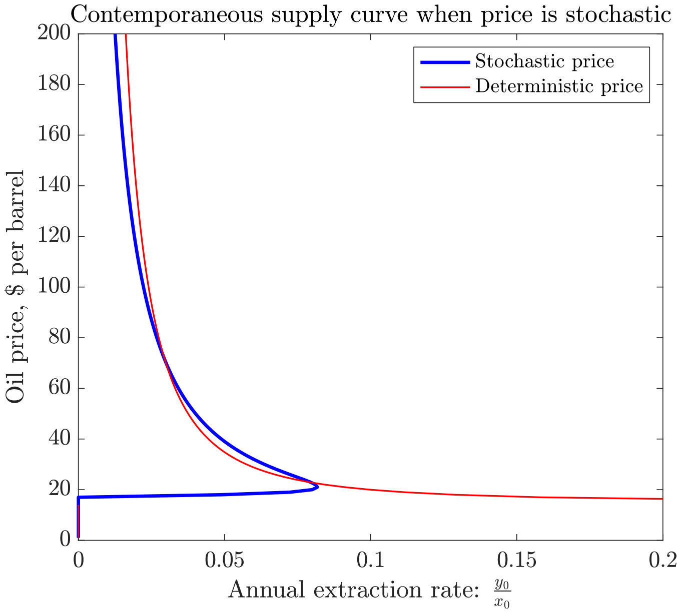
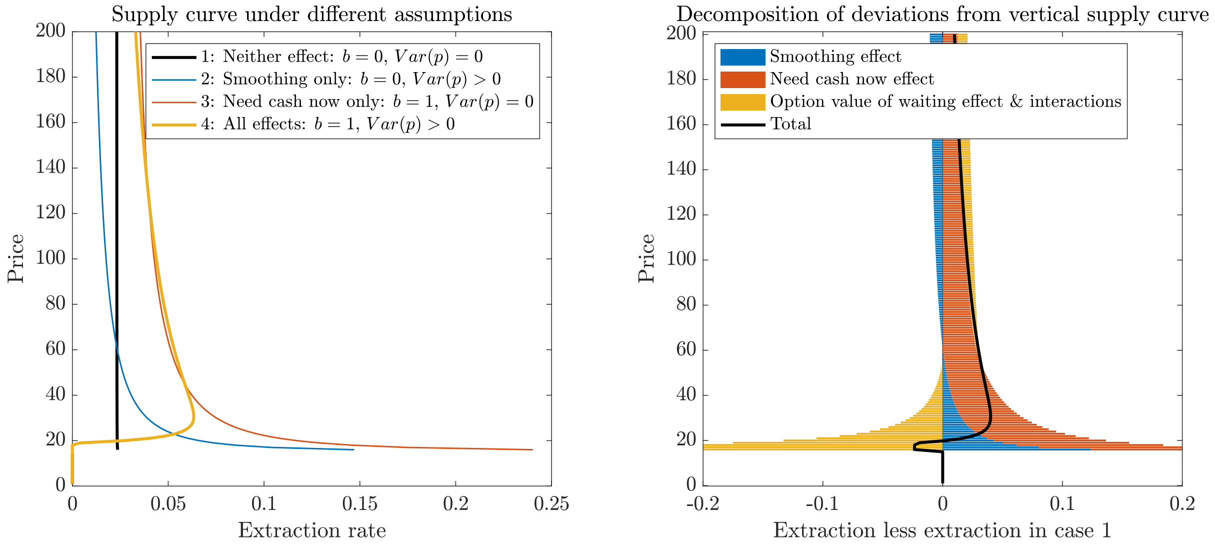


Figure 10: Supply curve when price is stochastic

To further illustrate how these effects operate, we consider for a moment a power utility function in (6). While this is not the specification we pursue in the rest of the paper, this detour is useful because with this utility function we can switch different effects on and off individually one-by-one (whereas with exponential utility we have $b = 1$ so it is no longer a free parameter that we can vary). Figure 11 illustrates the results of our experiment, with the left panel showing the supply curves under various assumptions and the right panel showing the importance of the various effects.

Our benchmark case (case 1, solid black line in the figure) features no non-homotheticity ($b = 0$) and no volatility in the price of oil. In this case the supply curve is vertical, and in fact we can solve for the extraction rate analytically: $\frac{y_0}{x_0} = \frac{\rho}{\sigma}$ (this holds as long as price is above marginal cost; of course, supply is zero otherwise). Changing the assumption on the volatility of oil prices (assuming that volatility is positive, as estimated above) shows the impact of the smoothing effect alone: since $b = 0$, the need-cash-now effect does not operate; moreover, the option value effect is mostly absent since, with $b = 0$, the utility cost of reducing profits when prices are low is very high (recall that in this case the marginal utility tends to infinity as profits approach zero). Conversely, assuming prices are fixed but setting $b > 0$ isolates the need-cash-now effect. Finally, letting both the b parameter and the volatility be positive captures all three effects and their interactions. The right panel of the figure plots the marginal effects on extraction due to our three effects, making the point



Note: this Figure assumes power utility as in (6).

Figure 11: Smoothing, need-cash-now and option-value-of-waiting effects, decomposed

that the downward slope is a result of the need-cash-now and the smoothing effects. These effects are overwhelmed by the option-value-of-waiting effect at prices that are just above the marginal cost of extraction. This experiment aids our understanding of how the three effects operate. For the remainder of the paper we assume that the utility function is CARA (as in (5)), which features all of these effects.

4.4 Supply effects of varying severity of financial constraints

The balance of the three effects depends on parameter values. We now study how the supply curve changes as we vary the degree of financial frictions. Figure 12 shows the supply curve in our baseline calibration and the supply curves under different parameterizations of γ. To interpret the findings in this Figure, recall that in the case of perfect certainty and in the limit when the utility function is linear, the producer wants to extract all resources immediately: the future is discounted at the positive discount rate ρ, so the Net Present Value, NPV, is maximized when all resources are extracted immediately.

Against this benchmark, the first exercise is to compute the supply curve when the price is stochastic and $\gamma \to 0$. In that case extraction varies discontinuously with the price: at prices below some price, the producer ceases to produce and waits for the price to increase. At prices above this threshold, the producer extracts all resources at once. The intuition parallels that in the case of no uncertainty. Because the producer cares only about the NPV

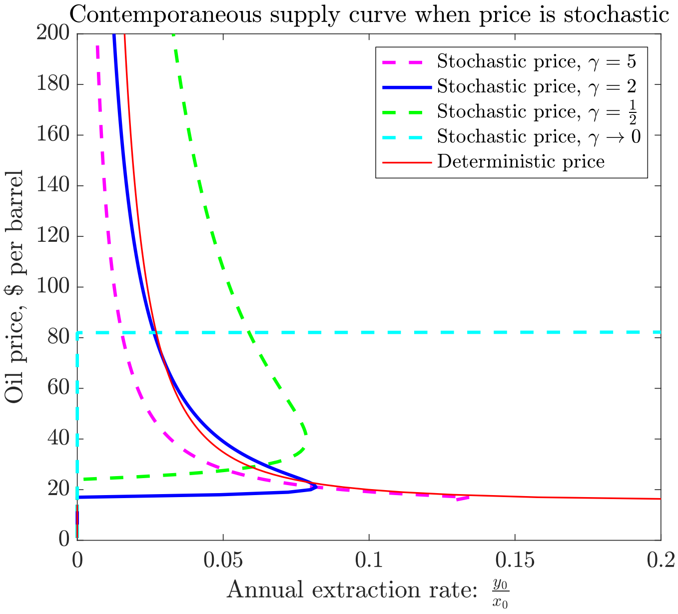


Figure 12: Supply curve when price is stochastic for different parameterizations of γ

and not about the smoothness of the extraction profile, the option value effect is stark.

However, even for small but positive values of γ (e.g. $\gamma = 1/2$) the supply curve becomes downward sloping over a broad range of prices. The lesson here is that a small degree of financial friction and the associated inability to smooth consumption out of oil revenues translates into strong economic incentives to supply oil even at low prices.

5 Price cap

This Section incorporates a price cap policy into the framework outlined in the paper so far. The price cap we consider in this section is “perfect”, in the sense that it applies to all of the exporter’s sales, and is permanent. We continue to assume that the producer has no market power in the global market for oil (i.e., that it faces a perfectly elastic demand). We relax both assumptions in subsequent analysis.

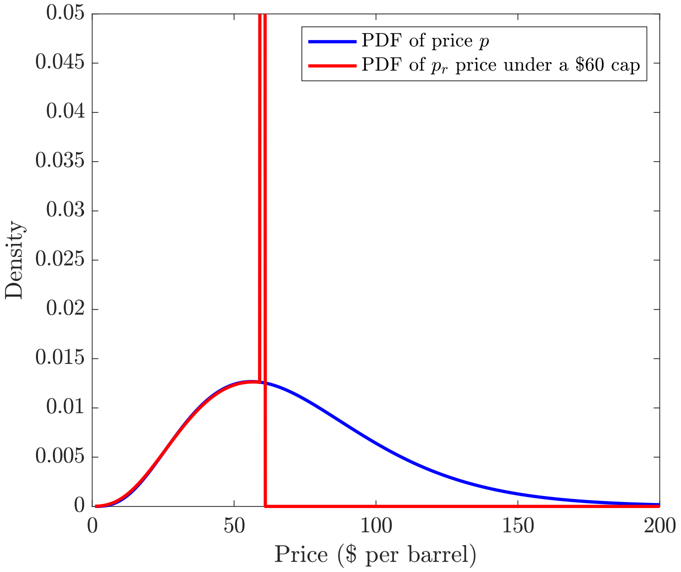


Figure 13: Distribution of the oil prices faced by Russia under the cap

5.1 Price that the producer receives under a price cap

A price cap limits upside exposure to the volatility in oil prices. Denoting with p_r the price received by the producer, we have

$$p_r = \min \{p, \bar{p}\} .$$

The price that Russia receives for its oil is simply the cap $\bar{p}$ whenever the price cap is binding, and the ongoing price when it is not. The resulting distribution of prices faced by Russia is depicted in Figure 13.[28]

5.2 How does a price cap affect Russia's supply?

The previous section demonstrated that the producer's supply schedule could be downward sloping. In the current context, this means that Russia may face economic incentives to increase supply if the global price of oil declines. But this does not necessarily imply that Russian production increases when a binding price cap is introduced. This is because the price cap fundamentally changes the stochastic behavior of the price at which the producer can sell its oil. This results in an endogenous response: a shift of the entire supply curve. The effect on extraction rates needs to take into account both the shift and the move down

---

[28] Formally, there is a Dirac point mass at $\bar{p}$.

and along the supply curve.

There are two effects that a price cap has on optimal extraction behavior and thus on the supply schedule.

First, when the price cap is binding, the fluctuations in the price do not affect the exporter's revenues. As a result, the supply curve becomes insensitive to global prices at and above the price cap.[29]

Second, for prices below the cap, there is less smoothing and so the supply curve shifts in and to the left. The economic intuition is as follows. The price cap eliminates exposure to upward swings in the oil price (above the cap), thus making the producer poorer and limiting its ability to smooth by saving (i.e., extracting less) during times of high prices. Consequently, the producer has fewer resources to cushion the impact of low prices. There is less extraction during the challenging times of low prices, compared to the scenario without the cap.

Figure 14 shows how this intuition translates into a supply curve under a price cap. It shows the supply schedules under no cap and under three alternative caps, the \$60 cap that has been implemented, the lower \$45 cap, and the \$30 cap proposed e.g., by The International Working Group on Russian Sanctions (2023). As anticipated above, in each case the supply curve features a close to vertical segment above the price cap, as the producer's decisions become insensitive to fluctuations in p. For prices below the cap, the supply curve shifts in and to the left, as the producer can afford less smoothing. It is clear from the figure that, if the price cap is sufficiently below the world price, the movement along and down the supply curve, driven by the *need-cash-now* and *smoothing* effects, dominates. In particular, the locus of the points that are right at the price cap (the black squares) form a downward sloping schedule. We summarize these findings in a proposition:

Proposition 2. *In a model with no market power, if the contemporaneous supply schedule is downward sloping, a sufficiently binding perfect price cap can lead to an increase in sanctioned exporter's supply to the global market.*

---

[29] The supply schedule is not *exactly* vertical above $\bar{p}$, because the expected duration of the price being above the cap is different at different levels of the reference price: if the price today is at \$200 per barrel, it will take some time to cross the $\bar{p} = \$60$ threshold, while if it is \$65, there is a good chance it will be below the cap soon.

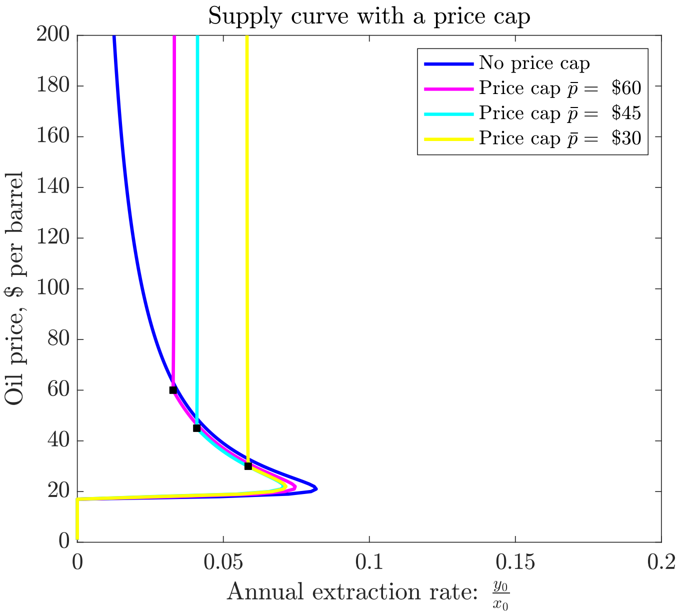


Figure 14: Russia's supply curve under three price cap regimes

5.3 How much does a price cap hurt the producer?

Price caps are the new weapons of economic warfare. But how powerful are they? Back-of-the-envelope calculations can give us some sense of the revenue losses, but tell us little about the dynamic *welfare* losses. Fortunately, we can use our model to compute the loss of welfare that Russia suffers as a result of the price caps set at different levels. Figure 15 plots the model-based measure of welfare (the value function $v(x, p)$). Of course the welfare of the producer depends somewhat on the *current* level of prices (although this dependence is not very strong unless the price process is very persistent). The three panels in the Figure depict welfare assuming current price is equal to \$30, \$60 or \$90 per barrel. Within each panel, the x-axis denotes the amount of reserves still in the ground, so that the right-most point corresponds to welfare from having today's level of oil reserves.

The three panels are nearly identical and suggest three implications. First, given the relatively low persistence of the price of oil, current prices do not affect the economic value of oil in the ground much (the exception is the red-dashed line, which plots welfare under the assumption that the price is fixed forever at a given value).

Second, comparing the magenta and blue lines in any of the panels, the model suggests that the impact of the \$60 cap imposed so far – recall that we assume that the cap is permanent – is to reduce the welfare from oil by about 10%, equivalent to reduction in

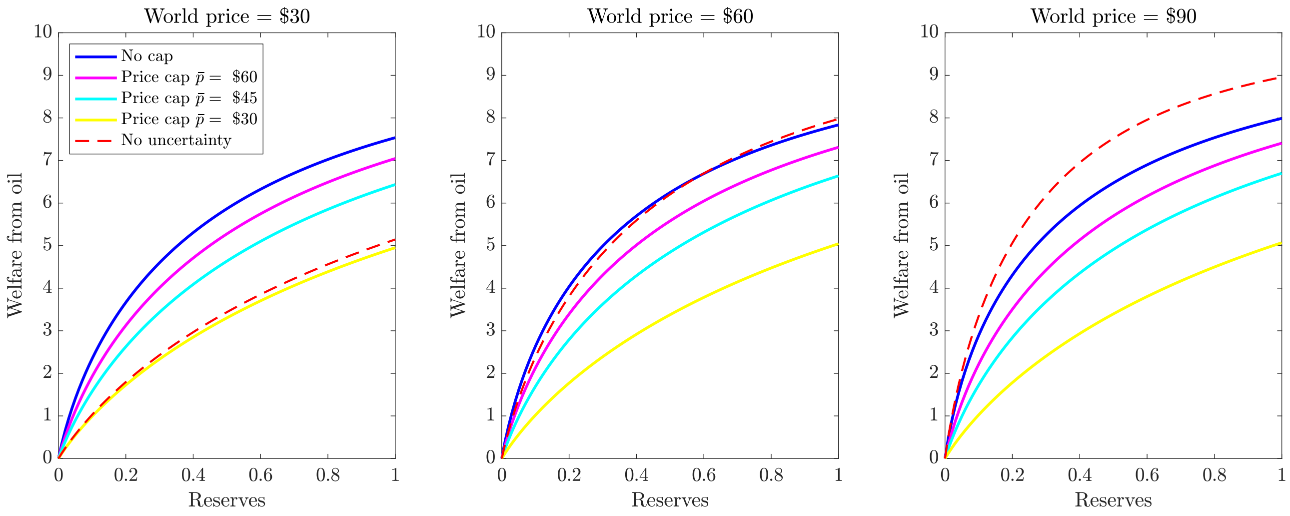


Figure 15: Value functions

reserves of about 20%.

Third, lowering the price cap further would deal an even more significant blow to Russia. With a $30 cap, the hit to oil welfare would be in the region of 35%, equivalent to wiping out 70% of Russia's reserves.

The welfare results must be interpreted with caution, since they miss important feedbacks that are due to Russia's potential to exercise market power and miss the fact that the cap does not apply to all exports and is in any case not perfectly enforced. We return to the welfare calculations below, once we introduce these important elements into our framework.

6 Market power and equilibrium

We now enrich our model by considering a producer with market power. In the current context, we want to study the implications of the fact that by limiting supply, Russia can increase global equilibrium prices, potentially causing a damaging energy supply shock.

6.1 World demand for oil and producer's market power

Denote the world price of oil with p_w. We assume that the global demand for oil is an isoelastic function

$$p_{w,t} = \delta_t Y_t^{-\epsilon} \tag{10}$$

where parameter $\epsilon \geq 0$ determines the elasticity of demand, with $\varepsilon_D := \frac{1}{\epsilon}$.[30] Y_t is global oil production at time t and δ_t represents shocks to global oil demand and supply. These shocks result in a stochastic process for the *reference oil price,* p, as specified in equation (8).

The *reference price* p is a hypothetical price of oil that would prevail in equilibrium *if the exporter did not exercise its market power.* We introduce the following notation: we denote the level of production for the exporter under the counterfactual hypothesis of no use of market power with $y_{N,t}$, we denote the actual production (with whatever is the optimal use of market power) with y_t, and we denote the "normal" level of world production with $\bar{y}$.

At any point in time $y_{N,t}(x,p)$ is a function of the state variables of the producer's problem, namely the level of reserves x and the reference price p. And we have already characterized this function: it is simply the optimal level of extraction absent market power, computed in Section 4 and depicted in Figure 26 in the Appendix.

The definitions of the variables introduced so far imply

$$p_{w,t} = \delta_t \left(\bar{y} + y_t(x,p)\right)^{-\epsilon} \tag{11}$$

$$p_t = \delta_t \left(\bar{y} + y_{N,t}(x,p)\right)^{-\epsilon}. \tag{12}$$

where $p_{w,t}$ is the equilibrium price and p_t is the reference price that would prevail in the market if Russia exercised no market power. Equation (11) re-writes global output Y_t in equation (10) as the sum of the normal level of global output and the optimal output of the producer. Equation (12) states that, in a counterfactual scenario if the producer did not use market power and instead extracted y_N, the price would be equal to the shadow price.

Taking the ratio, and dropping the time subscripts where it does not create confusion, we get:

$$\frac{p_w}{p} = \left(\frac{\bar{y} + y(x,p)}{\bar{y} + y_N(x,p)}\right)^{-\epsilon},$$

therefore we can express the equilibrium world price as follows:

$$p_w = p\left(\frac{\bar{y}}{\bar{y} + y_N(x,p)} + \frac{y_N(x,p)}{\bar{y} + y_N(x,p)}\frac{y(x,p)}{y_N(x,p)}\right)^{-\epsilon}. \tag{13}$$

By changing extraction rate y, the producer can influence the world price p_w. The degree of producer's market power is governed by the slope of the demand curve ϵ and the shadow market share $\psi(x,p) := \frac{y_N(x,p)}{\bar{y}+y_N(x,p)}$. The greater is this share, the greater is the ability of the

---

[30] The model collapses to the no market power case considered so far as we set $\epsilon = 0$, so that $\varepsilon_D = \infty$.

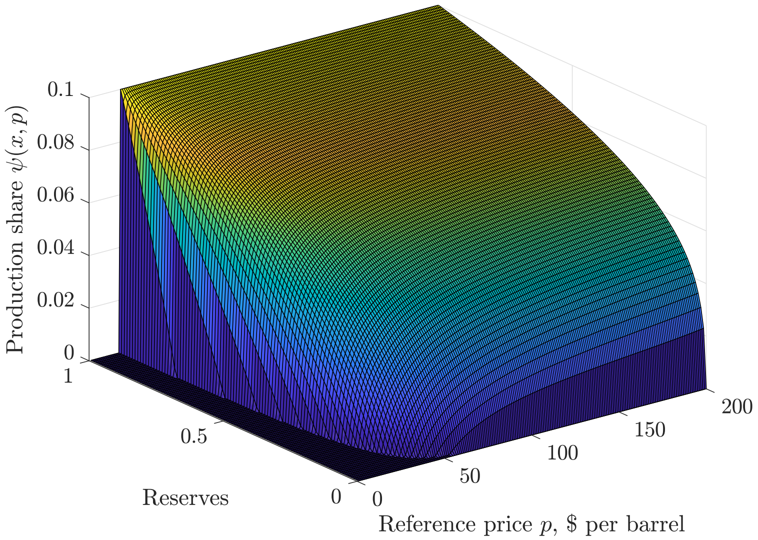


Figure 16: Russia's shadow market share – the $\psi(x,p)$ function

Note: this and other figures in the paper calibrate α directly, setting it to 0.1.

producer to affect global prices. In the limit, if the producer's share in global production is close to zero, such producer cannot affect world prices.[31]

We denote the contemporaneous value of ψ with α:

$$\alpha := \frac{y_N(1,p)}{\bar{y} + y_N(1,p)}$$

The reference (no-market-power) production share $\psi(x,p)$ is then

$$\psi(x,p) = \frac{y_N(x,p)}{\frac{1-\alpha}{\alpha} y_N(1,p) + y_N(x,p)}. \tag{14}$$

Figure 16 shows the $\psi(x,p)$ function.[32] As Russia's reserves deplete, it becomes a less significant global player, and its influence in the global market wanes.

Combining equations (14) and (13) we obtain the expression for the equilibrium world price:

$$p_w = p\left((1-\psi(x,p)) + \psi(x,p)\cdot\frac{y_R(x,p)}{y_N(x,p)}\right)^{-\epsilon}. \tag{15}$$

---

[31] This, coupled with the assumption that $\bar{y}$ remains constant, means that as the producer depletes its oil reserves, its market power will wane over time. However, our results are robust to keeping the market power of the producer constant over time.

[32] We calibrate α directly at 0.1, reflecting its size as the global producer but downsizing it slightly to reflects its obligations towards the OPEC+ cartel.

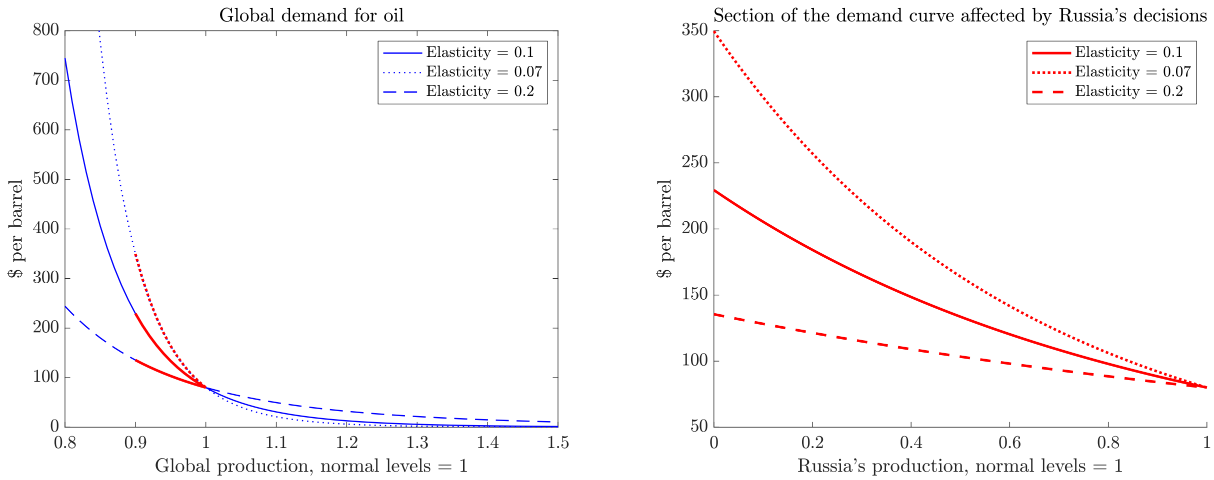


Figure 17: World demand for oil

The equilibrium price of oil is driven by two forces in this model. First, demand and supply shocks buffet the global economy and the oil market, and drive the stochastic evolution of the reference price p, according to (8). Second, the producer exercises its market power, reducing the amount produced from y_N and therefore moving the world price above from the reference price p.

To illustrate this further, Figure 17 plots the global demand curve for three different values of the demand elasticity $\varepsilon_D := 1/\epsilon$, and highlights the section of the demand curve that can be influenced by Russia's supply decisions contemporaneously. Over time the section of the global demand that is under Russia's influence will shrink, as in Figure 16.

6.2 OPEC

While in our framework we do not attempt to explicitly model the strategic behavior of the OPEC cartel, there is a close mapping from the parameters of our framework into OPEC's behavior.

First, to the extent that the cartel targets the specific level of the price of oil in global markets, the average price parameter $\tilde{p}$ in equation (8) can be thought as reflecting that target level.

Second, the responsiveness of the OPEC cartel to shocks emanating from outside of Russia is subsumed in the parameter D in the process for the reference price of oil in equation (8). This parameter determines the speed of mean-reversion of the oil price. If OPEC stands ready to adjust the production levels of oil to stabilize oil prices, this mean reversion parameter is high, and consequently oil prices will move within narrow bands around $\tilde{p}$. Conversely, if the control OPEC has over prices is weak, large and persistent deviations from the target price would occur, and parameter D would be estimated to have a lower value.

Third, the responsiveness of the OPEC cartel to shocks emanating from inside of Russia is one of the forces that ultimately shapes the value of the elasticity parameter ϵ. This parameter determines by how much the oil price responds to production cuts by Russia. A strong OPEC response would make these efforts futile, which can be captured in a low ϵ and a high demand elasticity. Conversely, if OPEC responds to Russia's cuts in supply by holding production steady, one might expect the world price to be significantly more responsive. This can be captured by setting ϵ to a high value, making world demand that enters Russia's decision problem inelastic.

We now turn to the producer's problem to determine whether and how Russia will exert its market power in this environment. We start with an environment with no price cap in place.

6.3 Producer's problem: no price cap

The optimization problem of the producer becomes:

$$\max_{y_t} E_0 \left[\int_0^\infty e^{-\rho t} \tilde{u}(\pi_t) dt \right] \text{ subject to } dx_t = -y_t dt, \; x_t \geq 0, \; y_t \geq 0 \tag{16}$$

and (8), where now

$$\pi_t = (p_{w,t} - c) y_t = \left(p_t \left((1 - \psi_t) + \psi_t \frac{y_t}{y_{N,t}} \right)^{-\epsilon} - c \right) y_t \tag{17}$$

and $y_{N,t}$ is the policy function that solves (9). Note that when the producer extracts the no-market-power reference level of output y_N, profits are simply equal to $(p - c)y$.

6.4 Equilibrium

An *equilibrium* is a policy function $y(x, p)$ that solves producer's problem (16) and the price function $p_w(y(x, p), p)$ that clears the market for oil (and thus satisfies (15)).

6.5 Parametrizing the elasticity of world demand for oil

Estimating oil demand elasticity is a subject of an extensive empirical literature. Meta-analysis in Uria-martinez et al. (2018) suggests the range for this elasticity in the short-run (around one year) is in the $[0.07, 0.14]$ range, while the long-run elasticity (after over a decade) is within the $[0.26, 0.82]$ range.[33] Intuitively, the price responds more in the short-run than in the long-run. In our analysis we focus on the short-run, since we suspect that Russia's decision horizon is relatively short at present, and we want to err on endowing Russia with significant market power and investiagte the consequences. We thus set world demand elasticity to $\varepsilon_D = 0.1$ (i.e. we set $\epsilon = 10$).

6.6 Characterization

We begin by considering how market power alters Russia's contemporaneous supply curve. The left panel of Figure 18 shows that when Russia has market power, so that its level of production can affect the global market price, it indeed exercises this power, producing less and selling at higher prices.

The contemporaneous supply curve is plotted in Figure 18. The supply curve with market power is shifted in relative to the case where the producer faces a perfectly elastic demand (and hence takes prices as given). Still, the shape of the supply curve is little changed: while the incentives to manipulate the price upward are strongest in the middle to low price ranges, this does not change the big picture: the supply curve remains downward sloping in the range where the price is well above the marginal cost.

The right panel of the Figure shows the impact of the restricted supply on oil prices. In relative terms the biggest impact occurs at low prices – the blue curve essentially tracks the difference between the supply curves in the left panel. The impact in absolute terms – expressed in dollars per barrel – is increasing in the shadow price in close to linear fashion.

Figure 19 shows the policy functions over the entire state space for the case with and without market power. The left panel shows that quantity is always restricted, but by how much depends on the ongoing price and on the remaining reserve levels. The right panel illustrates the price impact across the state space, making the point that as time goes by and Russia 's reserves are depleted, its ability to affect world prices diminishes.

---

[33] We report the absolute value of the elasticity; of course the demand curve is downward sloping.

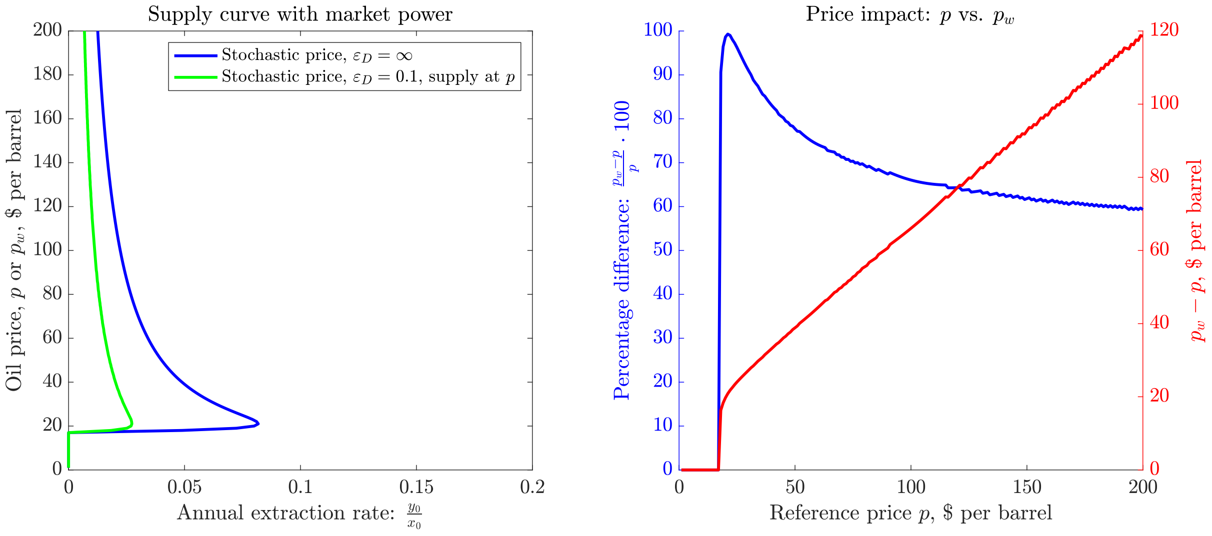
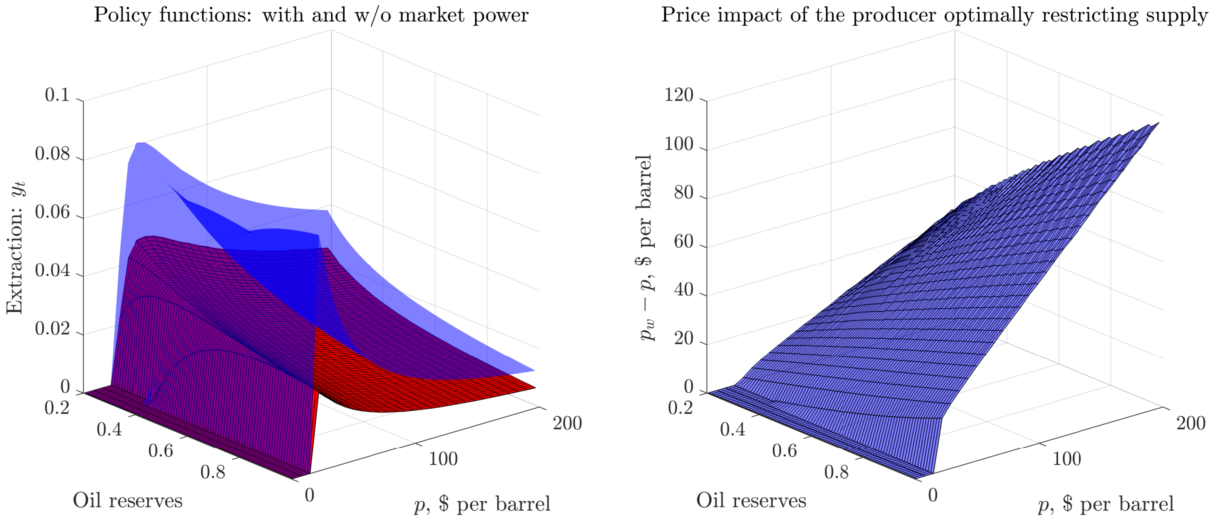
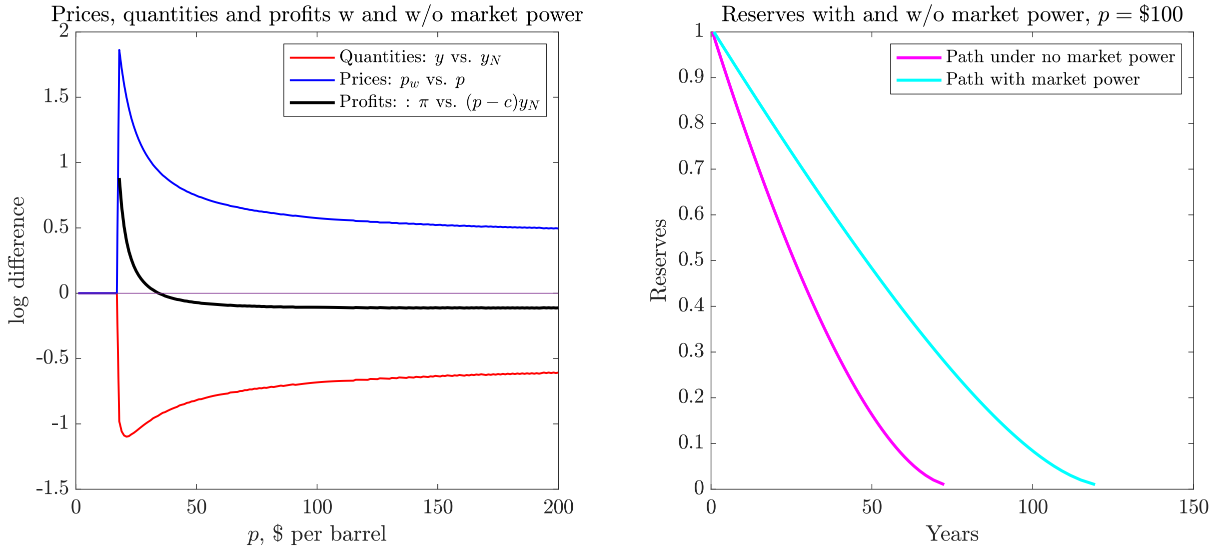


Figure 18: Contemporaneous supply curve with market power



Note: in the left panel, the blue surface shows the extraction rate with optimal use of market power, and the red surface shows the extraction when the use of market power is not allowed. The right panel shows the price impact.

Figure 19: Policy functions with and without market power, and the price impact



Note: the right-hand panel assumes that the reference price of oil happens to be equal to \$80 per barrel throughout.

Figure 20: Policy functions with and without market power, and the price impact

6.7 How does Russia gain from restricting supply?

The answer to this question is somewhat surprising – it turns out that it is possible that producer's contemporaneous revenues are *lower* when it can optimally exercise its market power, relative to the case when it cannot. That is, the effect of lower quantities can dominate over the effect of higher prices. This turns out to be true when world prices are significantly above marginal cost (the left panel in Figure 20). This is surprising because, with market power, it is still feasible for the producer to recreate the no-market-power revenues, simply by setting $y(x,p) = y_N(x,p)$ for all x and p.

The reason behind this finding is that the lower extraction rate means that oil reserves last longer: unlike in the static setting, in a dynamic setting optimal behavior can result in lower per period revenues and profits but a longer period over which extraction is carried out.

This intuition is illustrated in Figure 20. The left panel shows the revenue impact of exercising market power. The revenue impact is positive when prices are low, but is mildly negative for higher p, as lower quantity dominates the price effect. The benefit of these lower extraction rates can be seen in the right panel – at price of \$80 a barrel, for example, the benefits of producing oil accrue for a longer period of time.

This more drawn out extraction time path raises producer's welfare even as contempo-

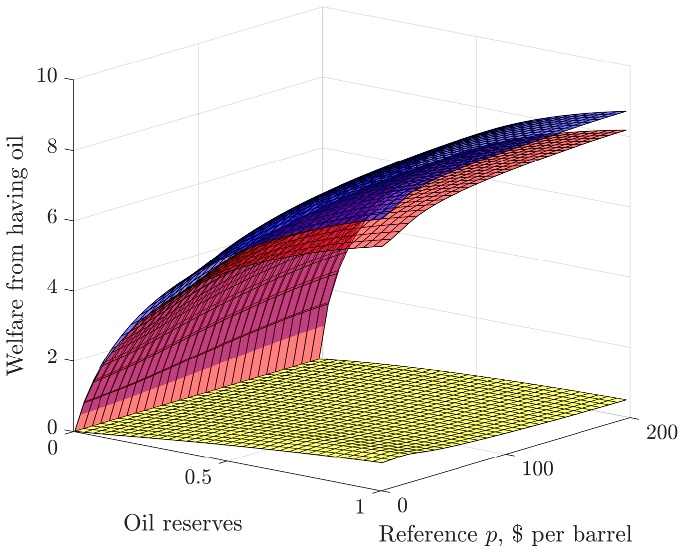


Figure 21: Value functions with and without market power

Note: The blue surface is the value function with $\varepsilon_D = 0.1$. The red surface is the value function with $\varepsilon_D = \infty$. The yellow surface is the difference between the two. This difference is always positive, meaning that having market power always increases producer's welfare.

raneous profits are lower. The value of having oil in the ground is always higher when the producer has market power, as illustrated in Figure 21.

6.8 Price cap when Russia has market power

When the producer has market power and a price cap that applies to all of its sales is introduced, the per barrel price that the producer receives is given by

$$p_r = \min \{\bar{p}, p_w\} \tag{18}$$

where $\bar{p}$ is the level of the price cap and p_w is the equilibrium price of oil in the world market, given by equation (15).

The price cap and market power of the producer interact in two noteworthy ways.

First, the price cap limits the extent to which the producer exercises its market power *when the cap is binding*, i.e. when it is below the ongoing equilibrium world price. To understand this, observe that restricting supply does not result in higher sale prices if $p_w > \bar{p}$: prices remain at the cap, so reducing supply leads to a proportional decrease in revenues. Consequently, the producer has no economic incentive to limit supply.

Second, the price cap constrains supply *when it is not binding*, for the same reason as in

Section 5: a non-binding cap limits the potential upside from future oil price spikes, thereby weakening the motivation for smoothing revenues.

These effects lead to a perhaps surprising conclusion that *implementing a price cap can decrease global prices when the cap is binding and increase them when it is not.* Figure 22 demonstrates these mechanisms in the calibrated model by examining the impact of the \$60 price cap on equilibrium prices. The left panel of the Figure displays the supply curve; the middle panel presents the prices received by the producer as a function of the reference price; and the right panel illustrates the equilibrium world price. The dark blue lines in the figure represent the price cap equilibrium, with the other three lines (cyan, green, and red) denoting benchmarks which we have discussed in previous sections.

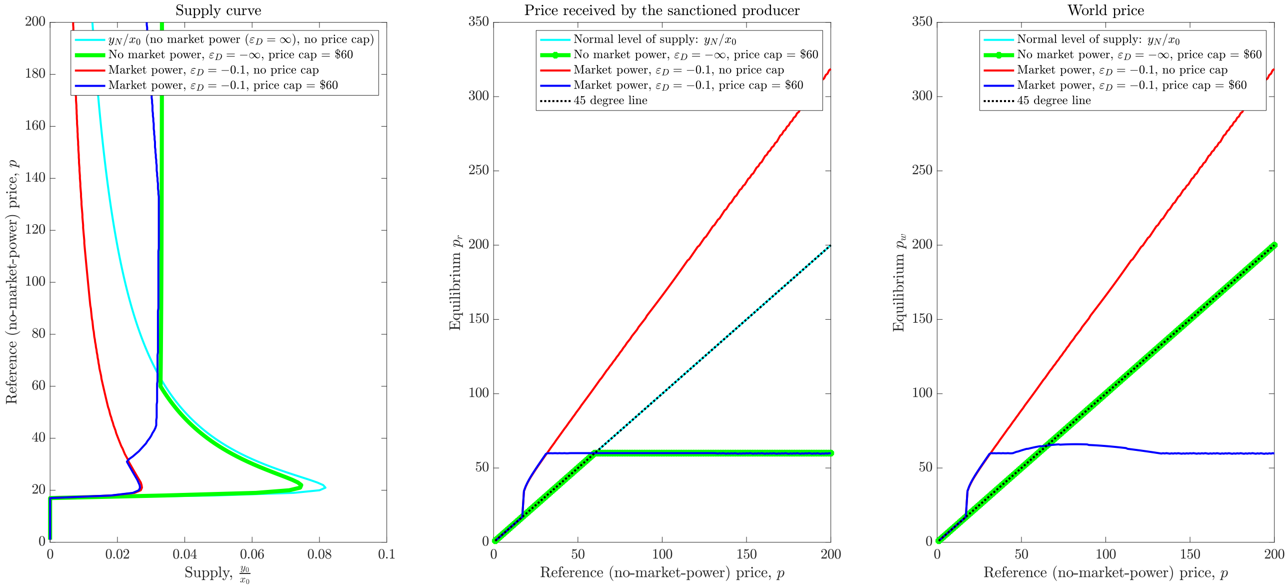


Figure 22: Equilibrium prices with and without market power, with and without a price cap

Note: The lines in cyan represent the standard supply levels, denoted by y_N, and their corresponding prices. Since Russia does not have market power in this scenario, it is considered a price taker, meaning that it receives the shadow price, which is equivalent to the world price: $p_r = p_w = p$. The thick green lines represent what happens when a price cap of \$60 is imposed, assuming Russia has no market power. The supply curve remains the same as in Figure 14. The red lines indicate the equilibrium with market power ($\varepsilon_D = 0.1$), but no price cap. The producer optimizes by limiting supply, resulting in equilibrium world prices that exceed the reference price p: $p_r = p_w > p$. The blue lines illustrate the equilibrium with market power and a price cap.

Start with the supply curve in the left panel. With a price cap in place, the supply curve overlaps the no-market-power supply schedule when prices are high, and is shifted-in when prices are low. Since the producer receives the cap for each barrel it sells, it is counterproductive to exercise market power. Doing so would only result in reduced revenues proportional to lower sales. This effect begins to kick in at a shadow price that is below the cap, because the producer already exercises market power in equilibrium, creating a wedge between p and p_w.

The most striking outcome of this is that production is higher with a cap than without one when the cap is binding. As a result, implementing a binding price cap on a country such as Russia can *lower* global oil prices. Observe that in the figure the blue schedule in the right panel is below the red schedule when the shadow price is high. In fact, due to the downward-sloping supply schedule, this effect is especially powerful at high shadow prices, causing the equilibrium world price to also fall below the reference price p: in the right panel, the blue line is not only below the red line but also below the 45-degree line when p is high.

Furthermore, with the price cap in place, the equilibrium world price barely changes with the reference price – a stark contrast to the equilibrium without a cap (the red line), where the world price moves more than the reference price. These findings suggest that the price cap can be an effective stabilization tool in global oil markets.

It is crucial to note that these effects are more pronounced when Russia has substantial market power. This is because the gap between production levels with and without market power naturally increases with the degree of market power, and it is this gap that the price cap eliminates.

We summarize these results in the following Proposition:

Proposition 3. *When the sanctioned producer has market power, introducing a price cap that applies to all sales has the following effects:*

When the price cap is binding, so that $p_w > \bar{p}$:

(1) the cap limits the extent to which Russia uses its market power in equilibrium;

(2) this reduces equilibrium world prices: p_w *declines;*

(3) the decline in p_w *is larger the higher is the reference price* p*;*

(4) the cap makes equilibrium world oil prices less responsive to shocks – hence more stable;

(5) for high reference prices p*, the equilibrium* p_w *can decline below the reference price* p*;*

(6) these effects are particularly powerful when Russia commands significant market power.

When the price cap is not binding, so that $p_w < \bar{p}$*:*

(1) the cap limits the potential upside from higher future oil prices, making Russia poorer and limiting the degree to which it intertemporally smooths revenues in equilibrium;

(2) at low prices this translates to lower extraction and higher equilibrium world prices.

7 Shadow fleet

So far, this paper has examined cases where a price cap applies to all sales made by the exporter. In reality, however, the price cap might only affects a specific portion of the exporter's oil sales. In the case of Russia, the G7 price cap currently restricts only the transaction price for sales of seaborne oil and products that utilize Western services, such as transportation and insurance. There is an intense debate and speculation among experts, policymakers and the media about the ability of Russia to do without these western services, and on its ability to build up such capacity – termed "shadow fleet" – over time. While there is a significant uncertainty about this, experts estimate that about 30-40% of the flow of oil exports can be legally sold outside of the cap at the moment. Furthermore, the price cap might not be perfectly enforced. How do these factors alter the analysis and conclusions?

Let us represent the percentage of the producer's current oil reserves that can be exported outside of the cap with $\kappa \in [0, 1]$. For instance, with $\kappa = 0.01$, the producer can export 1% of its reserves each year without being subject to the price cap. $\kappa = 0$ represents the case of a perfect price cap that applies to all of exports (meaning that the producer cannot sell outside the price cap regime), as described in previous sections. $\kappa = 1$ corresponds to a situation where the price cap is irrelevant, since the producer can export all of its oil without using any Western services. In practice, since none of the scenarios considered here involve Russia extracting more than 10% of today's reserves, a κ value greater than 0.1 is sufficient to render the price cap inconsequential in the present context. We assume that κ remains constant over time.[34]

7.1 Optimality conditions with shadow fleet

With a shadow fleet, the revenues from oil sales are:

$$\pi = \int_0^{\kappa} (p_w - c) ds + \int_{\kappa}^{y} (\min\{p_w, \bar{p}\} - c) ds \tag{19}$$

---

[34] Future work might fruitfully revisit this and explore cases with variable κ, reflecting, for example, Russia's potential expansion of its capacity to sell oil outside of the price cap regime.

where p_w is the equilibrium oil price in equation (15). The first term on the right hand side of (19) denotes the revenues from oil that is transported out of Russia using the shadow fleet. Up to extraction level κ the profits are simply the amount of oil extracted times the world price (our model abstracts from the fact that the price cap might increase the bargaining power of non-coalition purchasers of Russian oil). The second term says that any oil sold above the κ threshold brings in the lower of the world price and the cap in revenues. With CARA utility, equation (19) implies that

$$\tilde{u}_\pi = e^{-\gamma\pi} = e^{-\gamma[\min\{\kappa,y\}\cdot(p_w-c)+\max\{0,y-\kappa\}\cdot(\min\{p_w,\bar{p}\}-c)]}$$

so that the first order condition of the producer's problem is

$$v_x = \begin{cases} \tilde{u}_\pi \cdot (p_w\,(1-\varepsilon_D)-c) & \text{if } y<\kappa \\ \tilde{u}_\pi \cdot (p_w\,(1-\varepsilon_D)-c) & \text{if } y>\kappa \text{ and } p_w<\bar{p} \\ \tilde{u}_\pi \cdot \left(\bar{p}+\kappa\frac{\partial p_w}{\partial y}-c\right) & \text{if } y>\kappa \text{ and } p_w>\bar{p} \end{cases} \tag{20}$$

where ε_D is the elasticity of demand given in equation (29). Equation (20) illustrates how the price cap and the shadow fleet interact to result in endogenous degree of market power, depending on the level of production. When production is low, so that all oil can be transported outside of the cap regime (the first row in (20)), the marginal utility of extracting an additional barrel is given by the marginal utility of oil profits times the world price adjusted downwards for the impact that this extraction has on the prevailing oil price. This is also true if the marginal barrel is sold using the coalition services and so under the price cap regime, but if the price cap is not binding (the second row). Finally, when the marginal barrel is sold at a cap, the marginal benefit is just the price cap adjusted for the price impact that the sales of a marginal barrel have on the revenues from the sales of the infra-marginal κ barrels (the final row).[35]

---

[35] Note that the last line can be re-written as

$$\tilde{u}_\pi\left(p_w\left(\frac{\bar{p}}{p_w}-\frac{\kappa}{y}\varepsilon_D\right)-c\right)=v_x.$$

7.2 The effects of the price cap in presence of a shadow fleet

Figure 23 illustrates equilibrium production and equilibrium world prices with a price cap on the producer who has access to a shadow fleet capable of carrying 1% of its reserves (i.e. approximately a third of its current oil production levels ($\kappa = 0.01$)). The left panel displays the supply schedule, while the right panel presents the equilibrium world price of oil. The dotted lines represent the equilibrium with the price cap that is applied to all production, as discussed in the previous section and summarized in Proposition 3.

The solid red and blue lines demonstrate the equilibrium under \$60 and \$45 price caps, respectively. In addition, the thick broken lines illustrate how the results change when the producer's capabilities to carry oil outside of the price cap regime are lower ($\kappa = 0.5\%$).

The key result that emerges is that the presence of the shadow fleet means that a binding price cap no longer eliminates the incentive to exercise market power. Consider first the \$60 price cap – the red lines in the figure. When reference prices are low and the cap is not binding, all of the oil is sold for the world equilibrium price p_w. The incentives to limit production are thus similar to the case with market power and no cap (the black line). Moving up the price spectrum, as the cap becomes binding, the incentives to hold back production initially decrease, for the same reasons that we discussed before: the marginal barrel of oil is sold at the cap, so the producer has less incentives to utilize market power. The supply curve slopes upwards, as binding cap acts to lessen the use of market power.

However, when prices are higher still, the producer generates substantial revenues, and the temptation to "shut in" production – so that sales can rely solely on exports carried through the shadow fleet – grows more significant. Indeed, when prices are sufficiently high – at about $p_w = \$65$ per barrel in our calibration – the producer cuts exports all the way to κ and sells only outside of the price cap regime.

A more aggressive cap of \$45 a barrel induces a supply response that is similar, albeit the shut-in occurs at slightly lower equilibrium prices of about $p_w = \$60$ dollars. So, on important lesson from this analysis is that for a given size of the shadow fleet, a lower price cap affects the incentives to shut-in production only marginally.

The margin that is more important is the capacity of the shadow fleet – the κ itself. The thick dashed line in the left panel shows the supply curve when κ is lower, set to 0.5% instead of 1%, assuming that the price cap is set at the current level of $\bar{p} = \$60$. This leads to the deeper shut-in which occurs at much higher prices. This is intuitive: the producer will shut in and rely only on exports outside of the price cap regime if these exports are sufficiently profitable. Thus, as long as κ is not high, the stabilization benefits of the price cap can be

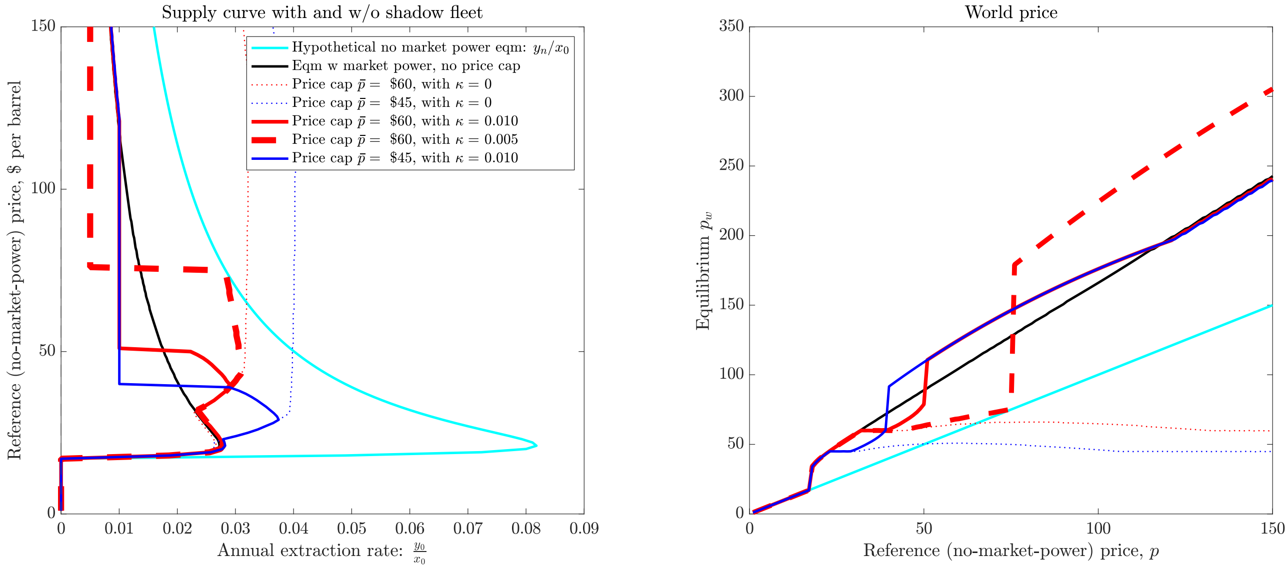


Figure 23: Equilibrium supply and prices under a price cap when Russia has access to a shadow fleet

very substantial: notice that the dashed lines in the right panel show that equilibrium prices remain around \$70 per barrel even as the reference prices reach over \$120.

The bottom line from this analysis is that the capacity of the shadow fleet matters a lot. In particular, the presence of a fleet that can transport the flow of oil equivalent to about a third of pre-war exports ($\kappa = 1\%$) negates much of the stabilization benefits of introducing the cap. Thus, the analysis underscores the importance of the existence of the shadow fleet for the implications of the price cap on Russia's behavior and the resulting dynamics in the world oil markets.

7.3 The impact on Russia

We have now endowed the producer with market power and we have made it possible to partially circumvent the price cap regime by exporting oil using a shadow fleet of tankers and services. We are ready to revisit the question about the effectiveness of the price cap as a tool of economic warfare. What impact does the price cap have on Russia, in the presence of market power and a possibility of partial go-around the cap regime with a shadow fleet?

Figure 24 offers an answer, both in terms of contemporaneous profits from oil sales in the left panel, as well as welfare from having oil in the ground (i.e., the value function).

The dotted lines in the left panel show that a perfect price cap essentially caps the profits

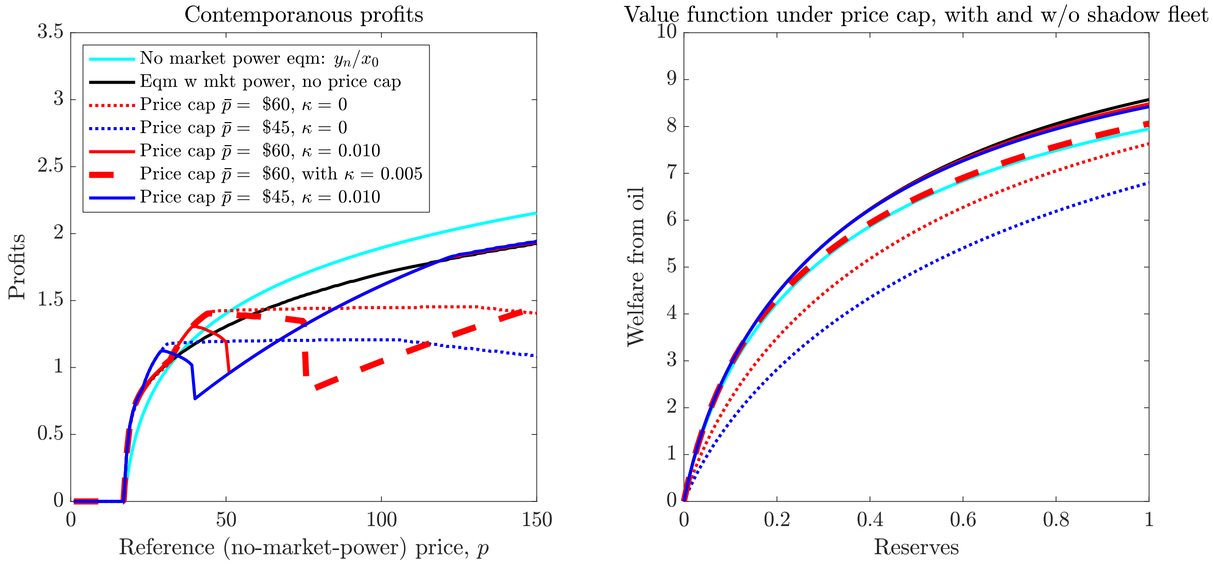


Figure 24: Effects of price caps on Russia's contemporaneous revenues and welfare when Russia has access to a shadow fleet or the cap is imperfectly enforced

Note: the right-panel assumes that the (current) reference no-market-power price of oil p is \$80.

of the producer. This is intuitive given the findings documented above: we have found that the quantity exported by the producer stays constant even as market conditions (and hence the reference price) vary, as long as the cap is binding. Since the extraction rate is constant and the producer receives a fixed price (equal to the cap), the revenues and hence the profits are essentially constant even as market conditions fluctuate.[36] This capping of profits – and the associated inability to enjoy any of the upside of future energy shocks – significantly reduce the welfare from having oil. The reductions in welfare from a \$60 and a \$45 price cap is equivalent in welfare terms to the loss of 40% and 60% reduction in reserves, respectively. These are large effects.

The welfare effects of the price cap are much more modest, however, once one considers the availability of the shadow fleet. With $\kappa = 1\%$, the caps of \$60 or \$45 are equivalent to the loss of 5-10 years of reserves. while this sounds large, this reduction occurs at an already flat part of the value function. A larger welfare response occurs when the producer has limited ability to export outside of the price cap regime.

---

[36] An eagle-eyed reader will notice that profits actually decline slightly when the reference prices are high, above \$100 per barrel. This is because of the smoothing effect in the hypothetical equilibrium without market power: recall that for high prices the smoothing effect reduces extraction, thus limiting the reference output y_N. The increasing positive gap between the actual and reference extraction would ultimately push prices below the cap; to prevent this, the producer limits production to keep world prices at the cap.

Still, an interesting result is that the contemporaneous profits are significantly reduced by the shutting in of production. That is, our model suggests that even with a highly inelastic demand, the sharp reduction in exports does not generate a price response that is sufficiently strong to make the shut-in a profitable strategy in the short term. In other words, higher prices in the shut-in scenario do not compensate for the lost revenues due to lower volumes. According to the model, shutting in production to κ is optimal not because it raises contemporaneous profits, but because it allows for a more spread out production profile over time (see also the relevant discussion in Section 6).

8 Conclusions

This paper tackled the problem of extraction decisions of a state that has access to reserves of a non-renewable resource. The question we posed was how the optimal supply decisions are determined, in particular when the producer faces frictions in its financial planning, when the price of the resource fluctuates daily in the market, and when the producer's actions affect the equilibrium prices through endogenously determined market power. Our particular focus was on the effects of the new instrument of international policy – a price cap.

To answer these questions we have developed a tractable dynamic model of extraction decisions. Having estimated a flexible and realistic process for the oil price, we developed a novel and elegant way of embedding this stochastic process in an equilibrium structure where the producer has market power which changes dynamically and endogenously. We have developed robust solution algorithms that can handle these problems highly efficiently.

We have uncovered and analyzed the different effects driving extraction choices: the *need-cash-now*, the *smoothing*, and the *option value of waiting* effects, as well as the impact of market power, showing the important role that the dynamics play in the economics of the price cap.

Beyond these contributions, our analysis has important policy implications in the current context of the war in Ukraine and sanctions against the Russian Federation.

First, our economic framework supports the idea that Russia's supply curve is inelastic and may even be downward sloping, helping to explain why Russian oil production levels have remained relatively stable despite considerable fluctuations in prices in recent years.

Second, a binding oil price cap may increase Russia's supply to the market, stabilizing the price of oil globally. The cap may not be effective in the long run, however, if Russia can sell enough of its exports outside the price cap regime (i.e., without using western transportation and financial services). This highlights the importance of lowering the cap before Russia finds alternative ways to export its oil and the need for strict enforcement of the cap.

Third, even when a commodity producer has significant market power, this need not deter western policymakers from imposing – and lowering – the price cap. In fact, the oil price cap can effectively neutralize Russia's market power, which it already uses in equilibrium.

Finally, our simulations suggest that a lower price cap, perhaps around $45 per barrel, could significantly impact Russia's revenue flows, and depending on the capacity of the shadow fleet, potentially also its welfare.

Since the present paper is one of the first attempts to study the price cap in a rigorous way, it must be seen as the useful starting point to a longer research journey. Several issues are worth pursuing in the next stage. For example, it would be useful to incorporate supply constraints or convex costs of production into our setting, and think more carefully about the incentives to build up shadow fleet, by making its size – κ – endogenous and determined optimally, perhaps subject to adjustment costs. Another useful avenue for future research to explore would be to explicitly embed the setting developed here within a general equilibrium model of a world economy, with strategic interactions across groups of producers. A contribution that might be very relevant to the current policy debate is to use the present setting to study the effects of forward guidance-type of announcements with regards to the price cap.

References

Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll, "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach," Technical Report, National Bureau of Economic Research, Cambridge, MA aug 2017.

Alquist, Ron and Lutz Kilian, "What Do We Learn from the Price of Crude Oil Futures?," *Journal of Applied Econometrics*, 2010, *25* (4), 539–573.

Anderson, Soren T., Ryan Kellogg, and Stephen W. Salant, "Hotelling under pressure," *Journal of Political Economy*, 2018, *126* (3), 984–1026.

Arrow, Kenneth J and Sheng-Cheng Chang, "Optimal pricing, use, and exploration of uncertain natural resource stocks," *Journal of Environmental Economics and Management*, 1982, *9* (1), 1–10.

Babina, Tania, Benjamin Hilgenstock, Oleg Itskhoki, Maxim Mironov, and Elina Ribakova, "Assessing the Impact of International Sanctions on Russian Oil Exports," 2023.

Barro, Robert J, "On the Determination of the Public Debt," 1979, *87* (5), 940–971.

Barsky, Robert B and Lutz Kilian, "Oil and the macroeconomy since the 1970s," *Journal of Economic Perspectives*, 2004, *18* (4), 115–134.

Bornstein, Gideon, Per Krusell, and Sergio Rebelo, "A World Equilibrium Model of the Oil Market," *The Review of Economic Studies*, 2023, *90* (1), 132–164.

Chen, Ren-Raw and Louis Scott, "Maximum likelihood estimation for a multifactor equilibrium model of the term structure of interest rates," *The Journal of Fixed Income*, 1993, *3* (3), 14–31.

Cox, John C., Jonathan E. Ingersoll, and Stephen A. Ross, "A Theory of the Term Structure of Interest Rates," *Econometrica*, mar 1985, *53* (2), 385.

Cui, Wei, "Macroeconomic Effects of Delayed Capital Liquidation," *Journal of the European Economic Association*, 2022, *20* (4), 1683–1742.

Dasgupta, Partha S and Geoffrey M Heal, "The optimal depletion of exhaustible resources," *The Review of Economic Studies*, 1974, *41* (Supplement), 3–28.

Devarajan, S. and A. C. Fisher, "Hotelling's "Economics of Exhaustible Resources" Fifty Years Later," 1981.

Duffie, Darrell and Rui Kan, "A Yield-Factor Model of Interest Rates," *Mathematical Finance*, 1996, *6* (4), 379–406.

Fama, Eugene F and Harvey Babiak, "Dividend policy: An empirical analysis," *Journal of the American Statistical Association*, 1968, *63* (324), 1132–1161.

Fattouh, Bassam, "The Causes of Crude Oil Price Volatility," *The Middle East Economic Survey*, 2005, *48* (11).

Faucon, Benoit and Summer Said, "Saudi Arabia, U.A.E. Scoop Up Russian Oil Products at Steep Discounts," *The Wall Street Journal*, 2023, *April.*

Gaudet, Gérard, "The Economics of Exhaustible Resources," in Geoffrey M Heal and Theodore Panayotou, eds., *Handbook of Environmental Economics*, Vol. 4, Elsevier, 2013, pp. 121–176.

Hamilton, James D, "Causes and consequences of the oil shock of 2007-08," *Brookings Papers on Economic Activity*, 2009, *2009* (1), 215–261.

Hilgenstock, Benjamin, Elina Ribakova, Nataliia Shapoval, Tania Babina, Oleg Itskhoki, and Maxim Mironov, "Russian Oil Exports Under International Sanctions," 2023, (April), 1–27.

Hotelling, Harold, "The Economics of Exhaustible Resources," *Journal of Political Economy*, 1931, *39* (2), 137–175.

Johnson, Simon, Lukasz Rachel, and Catherine Wolfram, "Design and Implementation of the Price Cap on Russian Oil Exports," *Journal of Comparative Economics*, 2023.

Jones, Charles I and Dietrich Volrath, *Introduction to economic growth*, third edit ed., New York: W.W. Norton & Company, 2013.

Kaufmann, Robert K, Stephane Dees, Pavlos Karadeloglou, and Marcelo Sánchez, "Does OPEC matter? An econometric analysis of oil prices," *The Energy Journal*, 2004, *25* (4), 67–90.

Kladivko, Kamil, "CIR Maximum Likelihood Implementation in MATLAB," *Department of Statistics and Probability Calculus, University of Economics*, 2013, *1* (3).

Lintner, John, "Distribution of Incomes of Corporations Among Dividens , Retained Earnings , and Taxes John Lintner The American Economic Review , Vol . 46 , No . 2 , Papers and Proceedings of the Sixty-eighth Annual Meeting of the American Economic Association . (May ,," *Taxes*, 1956, *46* (2), 97–113.

Longstaff, Francis A and Eduardo S Schwartz, "Interest rate volatility and the term structure: A two-factor general equilibrium model," *The Journal of Finance*, 1992, *47* (4), 1259–1282.

Newell, Richard G and Brian C Prest, "The Unconventional Oil Supply Boom: Aggregate Price Response from Microdata," *The Energy Journal*, 2017, *38* (2).

Pindyck, Robert S, "Uncertainty and Exhaustible Resource Markets," *Journal of Political Economy*, 1980, *88* (6), 1203–1225.

_ , "The long-run evolution of energy prices," *The Energy Journal*, 1999, pp. 1–27.

Salant, Stephen W., "Exhaustible Resources and Industrial Structure: A Nash-Cournot Approach to the World Oil Market," *Journal of Political Economy*, 1976, *84* (5), 1079–1093.

Salant, Stephen W, "The Hotelling Principle, Backwardation of Futures Prices and the Values of Developed and Undeveloped Oil Fields," *The Energy Journal*, 2012, *33* (3), 1–28.

_ , "Targeted Price Ceilings in Competitive Models," 2023.

Schwartz, Eduardo S and James E Smith, "Short-term variations and long-term dynamics in commodity prices," *Management Science*, 2000, *46* (7), 893–911.

Smith, James L, "Inscrutable OPEC? Behavioral Tests of the Cartel Hypothesis," *The Energy Journal*, 2005, *26* (1), 51–82.

Solow, Robert M, "The Economics of Resources or the Resources of Economics," *The American Economic Review*, 1974, *64* (2), 1–14.

Stiglitz, Joseph E, "Monopoly and the Rate of Extraction of Exhaustible Resources," *The American Economic Review*, 1976, *66* (4), 655–661.

The International Working Group on Russian Sanctions, "Strengthening Financial Sanctions against the Russian Federation - Action Plan 2.0," 2023.

Uria-martinez, Rocio, Paul Leiby, Gbadebo A Oladosu, David C Bowman, and Megan M. Johnson, *Using Meta-Analysis to Estimate World Oil Demand Elasticity* number February 2018.

van der Ploeg, Frederick and Cees Withagen, "Too Much Coal, Too Little Oil," *Journal of Public Economics*, 2012, *96* (1-2), 62–77.

Wolfram, Catherine, Simon Johnson, and Lukasz Rachel, "The Price Cap on Russian Oil Exports , Explained," *Belfer Center Policy Brief*, 2022, (December).

Appendix

A Proofs

A.1 Proof of Proposition 1

The HJB equation is

$$\rho v(x) = \max_{y} u(y) - yv'(x)$$

The two necessary conditions are

$$\begin{aligned} u_y(y) &= v_x(x) \\ \rho v_x(x) &= -yv_{xx}(x) \end{aligned}$$

Taking logs and differentiating the first of those with respect to time gives

$$\frac{u_{yy}y}{u_y}\frac{\dot{y}}{y} = \frac{v_{xx}\dot{x}}{v_x}.$$

Using the definition of the intertemporal elasticity of substitution, the envelope condition and the fact that $\dot{x} = -y$ we obtain

$$\frac{\dot{y}}{y} = -\mathcal{E}(y)\rho.$$

With CARA utility as in (5) we get:

$$\dot{y} = -\frac{\rho}{\gamma(p-c)}. \tag{21}$$

Integrating gives the optimal extraction path

$$y_t = y_0 - \frac{\rho}{\gamma(p-c)}t, \tag{22}$$

where y_0 is the (optimally chosen) extraction level today – more on this in moment. Equation (22) says that the amount of resource extracted over time decreases linearly. From the constraint we obtain:

$$\dot{x} = \frac{\rho}{\gamma(p-c)}t - y_0,$$

and so:

$$x_t = \frac{1}{2}\frac{\rho}{\gamma(p-c)}t^2 - y_0 t + 1 \tag{23}$$

Along the optimal path a transversality condition must hold. Intuitively, as t approaches infinity, all oil must be extracted. Clearly, equation (23) then implies that oil is used up in finite time. Let us denote with T the time when the last barrel of Russian oil is pumped out of the ground. Then equations (22) and (23) imply that

$$\frac{1}{2}\frac{\rho}{\gamma(p-c)}T^2 - y_0 T + 1 = 0$$

and

$$y_0 = \frac{\rho}{\gamma(p-c)}T.$$

Combining these two conditions we obtain an analytical characterization for the time when Russia runs out of oil, as well as optimal current level of production (we focus on interior solutions):

$$y_0 = \sqrt{\frac{2\rho}{\gamma(p-c)}} \tag{24}$$

$$T = \sqrt{\frac{2\gamma(p-c)}{\rho}}. \tag{25}$$

Note the dependence on the curvature parameter γ: as $\gamma \to 0$, it becomes optimal to extract all the oil immediately. This is intuitive: if there is no curvature, the producer aims to maximize the net present value of oil sales. Since the price of oil is fixed, and since there is discounting of future payoffs, this NPV is maximized by extracting all the resources today.[37]

A.2 Optimal extraction when producer has market power

We can still define $u(y) := \tilde{u}(\pi(y))$. The HJB equation is

$$\rho v(x,p) = \max_y u(y) - v_x(x,p)y + v_p(x,p)D(\bar{p}-p) + \frac{1}{2}v_{pp}(x,p)\sigma^2 p, \tag{26}$$

---

[37] This suggests yet another interpretation of $\gamma > 0$, which is that it represents a convex cost of increasing production (so that the net-of-cost utility from extracting a given amount of resource fails to keep pace with the revenues that such extraction generates).

and the first order condition is

$$u_y(y) = v_x. \tag{27}$$

By the chain rule, the left hand side is

$$u_y(y) = \tilde{u}_\pi(\pi) \cdot \frac{\partial \pi}{\partial y} = \tilde{u}_\pi(\pi) \cdot p_w \cdot \left(1 - \frac{c}{p_w} + \frac{\partial p_w}{\partial y}\frac{y}{p_w}\right). \tag{28}$$

It is easy to show that

$$\varepsilon_D := -\frac{\partial p_w}{\partial y}\frac{y}{p_w} = \frac{\epsilon y}{\frac{1-\psi}{\psi} y_N + y}. \tag{29}$$

Using (15), (28) and (29), the FOC (27) thus becomes

$$\tilde{u}_\pi(\pi) \cdot \left(p \cdot \left((1-\psi) + \psi \frac{y}{y_N(x)}\right)^{-\epsilon} \cdot \left(1 - \frac{\epsilon y}{\frac{1-\psi}{\psi} y_N + y}\right) - c\right) = v_x. \tag{30}$$

The boundary condition is

$$\tilde{u}_\pi(0) \cdot (p - c) = u_y(0) = v_x(0, p), \tag{31}$$

since with no resources the producer does not have market power: $\psi(0, p) = 0 \forall p$.

We solve the problem using the finite difference method, as before. The main challenge now is that the first order condition takes a more complicated form: while for a given value function we can express y using the FOC in closed form when the demand curve is perfectly elastic, with market power the FOC in equation (27) is more complicated and cannot be solved in closed form. Specifically, with CARA utility equation (27) becomes

$$e^{-\gamma\left(p\left((1-\psi)+\psi\frac{y}{y_N}\right)^{-\epsilon} - c\right)y} \left(p\left((1-\psi) + \psi\frac{y}{y_N}\right)^{-\epsilon}\left(1 - \frac{\epsilon y}{\frac{1-\psi}{\psi} y_N + \psi}\right) - c\right) = v_x,$$

where the function $\psi(x, p)$ is given by equation (14). We must solve this non-linear equation for y for every point in the $x - p$ grid when searching for the solution. With CARA utility, the boundary condition in (31) is simply

$$p - c = v_x(0, p).$$

B Policy functions

Figures 25 and 26 show the complete policy functions for the calibrated model under no uncertainty and with uncertain price, respectively.

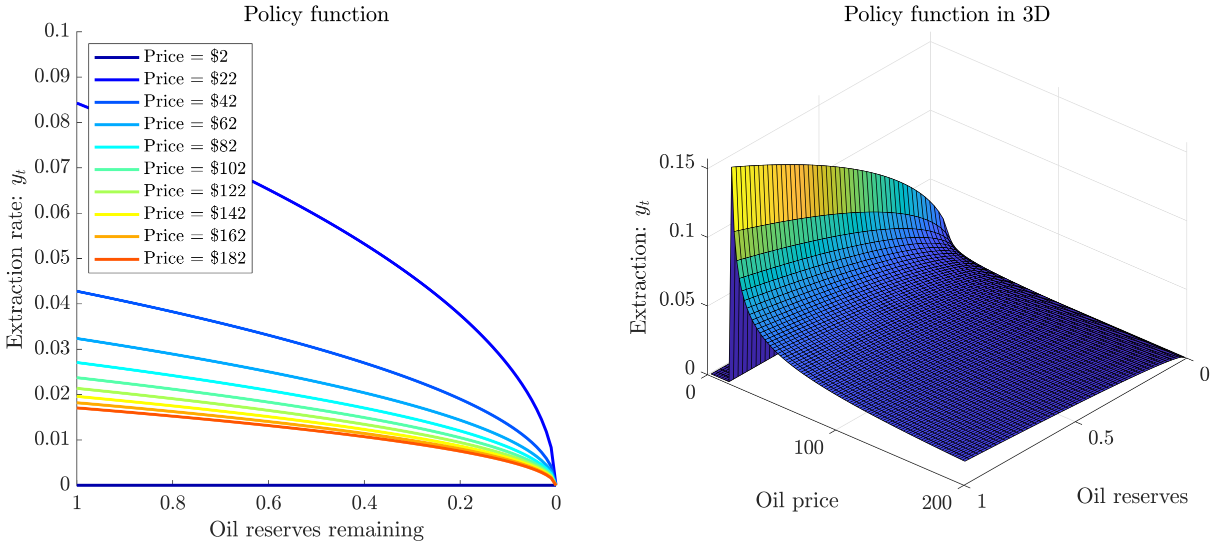


Figure 25: Policy function when p is a known parameter

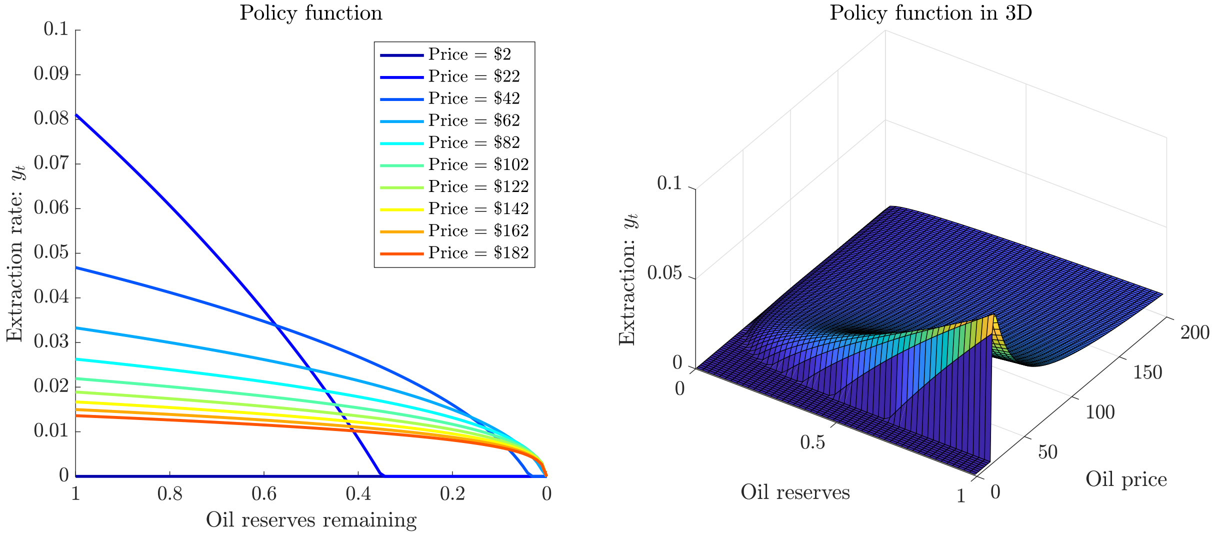


Figure 26: Optimal extraction when prices are stochastic

C Numerical appendix

This appendix provides a summary of the numerical procedure used to solve the model.

C.1 Baseline case

We start with a model with no market power. We seek to solve the HJB equation (9), reproduced here:

$$\rho v(x,p) = \max_{y_t} u(\pi) - v_x(x,p)y + v_p(x,p)D(\tilde{p}-p) + \frac{1}{2}v_{pp}(x,p)\sigma^2 p.$$

We use a finite difference method; a useful reference in the macroeconomics literature is Achdou et al. (2017). We approximate the function v at I discrete points in the reserves grid, $x_i,\ i \in 1,...,I$ with $x_1 = 0$ and J discrete points in the price dimension, $p_j,\ j \in 1,...,J$. We use equispaced grids with Δx and Δp the distance between gridpoints. Since oil is a non-renewable resource, reserves can only stay constant or fall. Thus, the drift is always (weakly) negative. When reserves are zero, the derivative of the value function is pinned down by the boundary condition. Therefore, we approximate the derivative of the value function in the x-dimension with

$$\partial_x v_{i,j} = \begin{cases} u_y(0) & \text{if } i = 1 \\ \frac{v_{i,j}-v_{i-1,j}}{\Delta x} & \text{if } i \geq 2. \end{cases}$$

The approximations of the derivatives in the p dimensions are:

$$\begin{aligned} \partial_{p,B} v_{i,j} &= \frac{v_{i,j} - v_{i,j-1}}{\Delta p} \\ \partial_{p,F} v_{i,j} &= \frac{v_{i,j+1} - v_{i,j}}{\Delta p} \\ \partial_{pp} v_{i,j} &= \frac{v_{i,j+1} - 2v_{i,j} + v_{i,j-1}}{(\Delta p)^2}. \end{aligned}$$

We use the appropriate approximation depending on whether the price is falling or increasing. For any variable z, we use the notation $z^+ := \max\{z, 0\}$ and $z^- := \min\{z, 0\}$. The finite difference approximation to the HJB equation is then:

$$\begin{aligned}\rho v_{i,j} &= u(y_{i,j}) - \partial_x v_{i,j} y + \partial_{p,F} v_{i,j} \left[D(\tilde{p}-p)\right]^+ + \partial_{p,B} v_{i,j} \left[D(\tilde{p}-p)\right]^- + \frac{1}{2}\partial_{pp} v_{i,j}\sigma^2 p \\ y_{i,j} &= (u_y)^{-1}(\partial_x v_{i,j}).\end{aligned}$$

Algorithm The algorithm for finding the solution to the HJB eqiuation is as follows. Guess $v^0_{i,j},\ i = 1, ..., I,\ j = 1, ..., J$ and for $n = 0, 1, 2, ...$ follow

1. Compute $\partial_x v^n_{i,j}$.
2. Compute optimal extraction y^n assuming that the marginal barrel is priced at the cap if the cap is binding. In the model without market power, compute $y^n_{i,j} = (u_y)^{-1}(\partial_x v^n_{i,j})$ where u_y^{-1} is evaluated at $\min\{p_j, \bar{p}\}$.
3. Compute extraction as if there was no price cap, $\tilde{y}^n$: $\tilde{y}^n_{i,j} = (u_y)^{-1}(\partial_x v^n_{i,j})$, where u_y^{-1} is evaluated at p_j.
4. For i, j where $y^n_{i,j} < \kappa$, set $y^n_{i,j} = \tilde{y}^n_{i,j}$.
5. Compute $\tilde{u}^n_{i,j}(\pi)$ with $\pi^n_{i,j} = \min\left\{\kappa, y^n_{i,j}\right\} \cdot p_j + \max\left\{0, y^n_{i,j} - \kappa\right\} \cdot \min\left\{p_j, \bar{p}\right\}$
6. Find v^{n+1} using the implicit method described below.
7. If v^{n+1} is close enough to v^n, stop. Otherwise go to step 1.

The implicit method for finding v^{n+1}. With the implicit method, we update the value function as follows. With a given step size Δ, v^{n+1} is defined by the following equation:

$$\frac{v^{n+1}_{i,j} - v^n_{i,j}}{\Delta} + \rho v^{n+1}_{i,j} = u(y^n_{i,j}) - \partial_x v^{n+1}_{i,j} y^n_{i,j} + \partial_{p,F} v^{n+1}_{i,j} \left[D(\tilde{p}-p_j)\right]^+ + \partial_{p,B} v^{n+1}_{i,j} \left[D(\tilde{p}-p_j)\right]^- + \frac{1}{2}\partial_{pp} v^{n+1}_{i,j}\sigma^2 p_j$$

Substituting in for the deivatives and collecting together the terms that are multiplied by the same gridpoint of v, this equation can be written as follows:

$$\frac{v^{n+1}_{i,j} - v^n_{i,j}}{\Delta} + \rho v^{n+1}_{i,j} = u(y^n_{i,j}) + v^{n+1}_{i,j} z_{i,j} + v^{n+1}_{i,j}\nu_{i,j} + v^{n+1}_{i,j-1}\chi_j + v^{n+1}_{i,j+1}\zeta_j$$

where $z_{i,j} = -\frac{y^n_{i,j}}{\Delta x}$, $\nu_{i,j} = \left[\frac{D(\tilde{p}-p_j)}{\Delta p}\right]^- - \left[\frac{D(\tilde{p}-p_j)}{\Delta p}\right]^+ - \frac{\sigma^2}{(\Delta p)^2}$, $\chi_j = -\left[\frac{D(\tilde{p}-p_j)}{\Delta p}\right]^- + \frac{\sigma^2}{2(\Delta p)^2}$ and $\zeta_j = \left[\frac{D(\tilde{p}-p_j)}{\Delta p}\right]^+ + \frac{\sigma^2}{2(\Delta p)^2}$. We can now write this in a matrix form:

$$\frac{1}{\Delta}(v^{n+1}-v^n)+\rho v^{n+1}=u^n+\mathbf{A}^n v^{n+1}$$

where v^n is a vector of length $I\cdot J$ with entries $(v_{1,1},...,v_{I,1},v_{1,2},...v_{I,2},...,v_{I,J})$ and $\mathbf{A}^n$ is a $(I\times J)\times(I\times J)$ matrix that has $z,\ \nu,\ \chi,\ \zeta$ as entries. Collecting terms, we get

$$\left(\frac{1}{\Delta}+\rho-\mathbf{A}^n\right)v^{n+1}=u^n+\frac{1}{\Delta}v^n.$$

This is a system $Bx=b$ which we can solve efficiently in MATLAB (note that the A matrix is sparse) using the B/b command.

C.2 Market power

The model with market power is solved in an analogous fashion. The only complication is that the first order condition cannot be solved in closed form, that is, to obtain $y^n_{i,j}$ in step 2 above it is necessary to solve the FOC (30) numerically for very point i,j in the grid.

We solve the FOC equation (30) by setting up the following functions:

1. The world price of oil function:

$$p_w(y_{ij})=\begin{cases}p_j\cdot\left((1-\psi_{ij})+\psi_{ij}\cdot\frac{y_{ij}}{y_{Nij}}\right)^{-\epsilon} & \text{if } y_{Nij}>0\\ p_j & \text{if } y_{Nij}=0\end{cases}$$

2. The profit function:

$$\pi(y_{ij})=\min\{\kappa,y_{ij}\}\cdot(p_w(y_{ij})-c)+\max\{0,y_{ij}-\kappa\}\cdot(\min\{p_w(y_{ij}),\bar{p}\}-c)$$

3. The utility function:

$$u(y_{ij})=e^{-\gamma\pi(y_{ij})}$$

4. The elasticity function:

$$\varepsilon_D(y_{ij})=\frac{\epsilon y_{ij}}{\frac{1-\psi_{ij}}{\psi_{ij}}y_{Nij}+y_{ij}}$$

5. The LHS (of the FOC) function:

$$LHS(y_{ij}) = \begin{cases} u(y_{ij}) \cdot \Big(p_w(y_{ij}) \cdot \big(\frac{\bar{p}}{p_w(y_{ij})} - \frac{\kappa}{y_{ij}} \cdot \varepsilon_D(y_{ij})\big) - c\Big) & \text{if } y_{ij} > \kappa \text{ and } p_w(y_{ij}) > \bar{p} \\ u(y_{ij}) \cdot (p_w(y_{ij}) \cdot (1 - \varepsilon_D(y_{ij})) - c) & \text{otherwise} \end{cases}$$

We then solve the non-linear equation

$$LHS(y_{ij}) - v^n_{x,ij} = 0 \tag{32}$$

for each point in the grid. The solution is an $I \times J$ matrix with entries corresponding to $y^n_{i,j}$. Note that this involves solving $I \times J$ non-linear equations at each iteration n. A robust routine that achieves this is as follows:

1. Construct a grid for candidate solutions, $z_s \in [0, 1]$, $s = 1, ..., S$. We use a power spaced grid in the $[0, 0.5]$ interval and a linear grid in the $[0.5, 1]$ interval, since any solution that is ultimately found to be optimal is well below 0.5. Then, for each point i, j in the $x - p$ grid:

2. Evaluate $LHS(y_{ij}) - v^n_{x,ij}$ at each z_s.

3. If the resulting vector is negative for all s, meaning that the value of the resource is greater than the marginal utility of extraction at that point in the grid, set $y_{ij} = 0$.

4. If the resulting vector is positive for all s, meaning that the value of the resource is lower than the marginal utility of extraction at that point in the grid, set $y_{ij} = x_i$.

5. If the resulting vector has entries of either sign, find the lowest index s at which the vector crosses zero, then set $y_{ij} = z_s$.