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ABSTRACT

The classic models of staggered adjustment of Taylor and Blanchard takes the frequency of price or wage adjustment as exogenous. This paper develops a model in which the frequency of price changes is endogenous. It then uses the model to analyze the effects of changes in the parameters of the economy on the frequency of adjustment and the real effects of monetary shocks.

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## Introduction

In an important set of papers, Taylor (1980) and Blanchard (1983, 1986) show that the fact that not all prices and wages are changed simultaneously can cause the real effects of a nominal disturbance to persist long after all prices and wages have been changed. Thus staggered adjustment may play an important role in the microeconomic foundations of nominal rigidity. These models of staggering have a serious limitation, however: they assume that the interval between each firm's price or wage changes is fixed and equal to two periods. As a result, they do not investigate the determinants of the frequency of adjustment, or the possibility that the frequency might change in response to changes in the economic environment.

This note's purpose is to remedy this limitation of previous staggering models. Section I develops a model that can be solved for the behavior of the economy for an arbitrary frequency of price changes. Section II then assumes that more frequent adjustment is more costly and solves for the Nash equilibrium frequency. The model is used to find the effect of changes in the parameters of the economy on the real consequences of nominal shocks. But the main usefulness of the model is likely to be as a tool in other analyses (see for example Jinushi and Romer, 1988, and Ball and Romer, 1989).<sup>1</sup>

The key simplification that permits the model to be solved analytically is an assumption that a firm's price changes, rather than occurring at regular intervals, occur stochastically. Specifically, following Calvo (1983), I assume that a firm's price changes are a Poisson process. I generalize Calvo's model by solving explicitly for aggregate dynamics for a given arrival rate of price changes and by allowing firms to choose the arrival rates of their price changes. Since the assumption of Poisson adjustment times is almost surely less realistic than the assumption of deterministic times, the

model involves some sacrifice in realism in exchange for considerable gain in tractability.<sup>2</sup>

### I. Price Dynamics for a Given Frequency of Price Changes

Let  $p(t)$  be the log price level and  $y(t)$  log aggregate output at  $t$ . The number of firms is large, so each firm takes the behavior of  $p(t)$  and  $y(t)$  as given. Firm  $i$  faces a downward-sloping demand curve for its good,

$$(1) \quad y_i(t) = y(t) - \epsilon(p_i(t) - p(t)), \quad \epsilon > 1,$$

and has marginal cost curve

$$(2) \quad c_i(t) = \ln \left( \frac{\epsilon - 1}{\epsilon} \right) + \beta y(t) + \gamma y_i(t), \quad \beta \geq 0, \gamma \geq 0, \beta + \gamma > 0,$$

where  $c_i(t)$  is the log of real marginal cost and where the constant term is chosen for convenience. Aggregate output  $y$  affects costs by changing the costs of inputs (for example, the real wage may be higher when aggregate output is higher);  $y_i$  affects costs through diminishing returns.

(1) and (2) imply that the profit-maximizing levels of output and price are:

$$(3) \quad y_i^*(t) = \frac{1 - \epsilon\beta}{1 + \gamma\epsilon} y(t),$$

$$(4) \quad p_i^*(t) = p(t) + \frac{\gamma + \beta}{1 + \gamma\epsilon} y(t).$$

For  $y \approx 0$ , the loss in real profits from failing to set  $p_i = p_i^*$  is approximately

$$(5) \quad \frac{1}{2} (\epsilon-1)(\gamma\epsilon+1) \left[ p_i(t) - p_i^*(t) \right]^2 = \frac{1}{2} K \left[ p_i(t) - p_i^*(t) \right]^2.$$

Let  $\alpha$  be the arrival rate of price changes. Thus if a price change for firm  $i$  occurs at time  $t$ , the probability that the price set at  $t$  is still in effect at  $t+s$  is  $e^{-\alpha s}$ . The average interval between price changes is  $1/\alpha$ . In this section I take  $\alpha$  as given.

Using (5) to approximate the costs of departures from  $p_i^*$ , the firm chooses  $p_i(t)$  to minimize

$$(6) \quad E_t \int_{s=0}^{\infty} e^{-\alpha s} e^{-rs} \frac{1}{2} K \left[ p_i(t) - p_i^*(t+s) \right]^2 ds,$$

where  $r$  is the real interest rate. Thus,

$$(7) \quad p_i(t) = (\alpha+r) \int_{s=0}^{\infty} e^{-(\alpha+r)s} E_t p_i^*(t+s) ds.$$

I now consider the behavior of the economy as a whole. I model aggregate demand by just positing a simple money demand equation,

$$(8) \quad y(t) = \phi(m(t) - p(t)), \quad \phi > 0,$$

where  $m(t)$  is the log money stock and where  $p(t)$ , the log price level, is the average of the  $p_i(t)$ 's. I assume that  $m(t)$  follows a random walk with drift:

$$(9) \quad m(t) = gt + Z(t),$$

where  $Z(t)/\sigma_m$  is Wiener.<sup>3</sup> (8) and (4) imply

$$(10) \quad p_i^*(t) = vm(t) + (1-v)p(t), \quad v = \frac{\gamma+\beta}{1+\gamma\epsilon} \phi.$$

Arrivals of price changes are assumed to be independent across firms. In addition, I assume that the number of firms is large, so that the fraction of firms changing prices at any moment is constant and so that fraction  $e^{-\alpha s}$  of prices set at  $t$  are in effect at  $t+s$ .

Let  $\pi(t)$  denote the value of prices that are set at  $t$ . Substituting (10) into (7) and using the fact that  $m$  is a random walk with drift yields

$$(11) \quad \pi(t) = \frac{vg}{\alpha+r} + vm(t) + (1-v)(\alpha+r) \int_{s=0}^{\infty} e^{-(\alpha+r)s} E_t p(t+s) ds.$$

Since  $p(t)$  is the average of prices in effect at  $t$ ,

$$(12) \quad p(t) = \alpha \int_{s=0}^{\infty} e^{-\alpha s} \pi(t-s) ds.$$

Substituting this expression into (11) yields

$$(13) \quad \pi(t) = \frac{vg}{\alpha+r} + vm(t) + (1-v) \frac{(\alpha+r)\alpha}{2\alpha+r} \left[ \int_{s=0}^{\infty} e^{-\alpha s} \pi(t-s) ds + \int_{s=0}^{\infty} e^{-(\alpha+r)s} E_t \pi(t+s) ds \right].$$

To solve for the behavior of  $\pi(t)$  in terms of the shocks to aggregate demand and the parameters of the economy, I use the method of undetermined coefficients. Specifically, I guess a solution of form

$$(14) \quad \pi(t) = A + Bt + \int_{s=0}^{\infty} (a+be^{-cs}) dZ(t-s),$$

and then find values of  $A$ ,  $B$ ,  $a$ ,  $b$ , and  $c$  such that (13) holds. This yields:<sup>4</sup>

$$A = \left[ \frac{v(\alpha+r)-r}{(\alpha+r)v\alpha} \right] g,$$

$$B = g,$$

$$(15) \quad a = 1,$$

$$b = -\frac{\alpha-c}{\alpha},$$

$$c = \frac{-r + \sqrt{r^2 + 4v\alpha(\alpha+r)}}{2}.$$

This result, together with (8) and (12), implies that the path of aggregate output is given by

$$(16) \quad y(t) = \frac{\phi r}{(\alpha+r)\alpha v} g + \phi \int_{s=0}^{\infty} e^{-cs} dZ(t-s).$$

For the case of  $r=0$ , (16) simplifies to:

$$(17) \quad y(t) = \phi \int_{s=0}^{\infty} e^{-\alpha\sqrt{v}s} dZ(t-s).$$

Expression (16) (or (17)) shows the response of output to aggregate demand disturbances. Because prices are not continually adjusted, a nominal shock raises output in the short run. As prices are changed, the real effect of the shock diminishes; in the long run output returns to the natural rate. The speed of adjustment is given by  $c$ , which is increasing in  $v$  (given  $\alpha$ ) and in  $\alpha$ . A higher  $\alpha$  means that prices are adjusted more often. From (8) and (10), a higher  $v$  corresponds either to a larger responsiveness of  $p_i^* - p$  to  $y$  -- a larger responsiveness of the profit-maximizing real price to real output -- or to a smaller income elasticity of money demand. The effect of

the real interest rate  $r$  (given  $\alpha$ ) on the speed of adjustment is more complex. A higher  $r$  causes firms to be more concerned with charging the optimal price in the short run. The elasticity of the optimal price with respect to  $m$  in the very short run (that is, with  $p$  fixed) is  $v$ ; in the very long run (that is, with  $p=m$ ) it is 1. As a result, increases in  $r$  reduce the speed of adjustment if  $v < 1$  but raise the speed of adjustment if  $v > 1$ .<sup>5</sup>

## II. The Equilibrium Frequency of Price Changes

I now assume that firms choose the arrival rates of their price changes. As emphasized at the outset, I do not allow firms to choose the exact times of their changes; firm  $i$  can only choose  $\alpha_i$ , the arrival rate of its price changes. Price changes are assumed to be costly.

Assume without loss of generality that a price change for firm  $i$  occurs at  $t=0$ . If firm  $i$ 's arrival rate of price changes is  $\alpha_i$  and all other firms' is  $\alpha$ , the reduction in the expected present value of firm  $i$ 's profits relative to the case in which it can continually and costlessly adjust its nominal price is

$$(18) \quad L(\alpha_i, \alpha) = F + \min_{p_i} \left[ \int_{t=0}^{\infty} e^{-(\alpha_i + r)t} \frac{1}{2} KE \left[ (p_i - p_i^*(t))^2 \right] dt \right] \\ + \int_{t=0}^{\infty} \alpha_i e^{-(\alpha_i + r)t} L(\alpha_i, \alpha) dt,$$

where  $F$  denotes the cost that must be incurred when a price change occurs. The first term of (18) reflects the cost of setting the new price; the second term represents the lost profits from  $p_i \neq p_i^*$  during the time that the price set at  $t=0$  is in effect; and the final term reflects lost profits



after the first price change. (18) implies:

$$(19) \quad L(\alpha_i, \alpha) = \frac{\alpha_i + r}{r} \left[ F + \min_{p_i} \left[ \int_{t=0}^{\infty} e^{-(\alpha_i + r)t} \frac{1}{2} KE \left[ (p_i - p_i^*(t))^2 \right] dt \right] \right].$$

The first order condition for firm  $i$ 's choice of  $\alpha_i$  is  $\partial L(\alpha_i, \alpha) / \partial \alpha_i = 0$ . The condition for  $\alpha$  to be a Nash equilibrium is that this first order condition hold at  $\alpha_i = \alpha$ . Thus, the equilibrium arrival rate of price changes is defined by:

$$(20) \quad \frac{\alpha + r}{r} \int_{t=0}^{\infty} (-t) e^{-(\alpha + r)t} \frac{1}{2} KE \left[ (p_i - p_i^*(t))^2 \right] dt \\ + \frac{1}{r} \int_{t=0}^{\infty} e^{-(\alpha + r)t} \frac{1}{2} KE \left[ (p_i - p_i^*(t))^2 \right] dt + \frac{F}{r} = 0,$$

where  $p_i$  is the price the firm sets at time 0 and where the Envelope Theorem has been used to exclude terms reflecting the fact that  $p_i$  changes when  $\alpha_i$  changes.

If one uses (14)-(15) to substitute for  $p_i$  (which is simply  $\pi(0)$ ) and (10) and (16) to substitute for  $p_i^*(t)$ , tedious manipulation shows that the equilibrium condition (20) can be written as

$$(21) \quad \frac{2g^2}{(\alpha + r)^3} + \sigma_m^2 \left[ \frac{1}{(\alpha + r)^2} - \frac{(1-v)(1+\frac{c}{\alpha})}{(\alpha + r + c)^2} \right] = \frac{2F}{K}.$$

To interpret (21), it is useful to begin with the case  $r=0$ . Then the equilibrium condition is

$$(22) \quad \frac{2g^2}{\alpha^3} + \sigma_m^2 \frac{\sqrt{v}}{\alpha^2} = \frac{2F}{K}.$$

Increases in the variance of nominal shocks, increases in the absolute value of trend inflation, decreases in the cost of price changes, and increases in the cost of not charging the optimal price all raise the equilibrium frequency of price changes. Increases in  $v$  -- the extent to which profit-maximizing prices depend on  $m$  -- also raise equilibrium  $\alpha$ . All of these changes other than the change in  $v$  affect the speed of adjustment to shocks only by affecting  $\alpha$ ; an increase in  $v$  increases the speed of adjustment both directly and through  $\alpha$ .<sup>6</sup> Finally, expression (21) shows that an increase in  $r$  reduces the equilibrium  $\alpha$ . The reason is simply that the costs of a price change are borne immediately while the benefits are spread out over the future.

Notes

1. Parkin (1986) develops a model of staggered price setting with endogenous frequency of adjustment in which there are no shocks; the frequency of adjustment thus depends only on trend inflation, and he does not solve for the dynamic response of the economic to shocks. Ball (1987) presents a model of endogenous frequency of wage changes in which wages are "predetermined" but not "fixed," and in which the real effects of nominal shocks therefore do not persist after all contracts have expired. Finally, Caplin and Spuller (1987) develop a model with endogenous frequency of adjustment in which firms follow "state dependent" rather than "time dependent" rules for changing prices. As Blanchard (1987) stresses, these two types of rules have dramatically different consequences for the real effects of nominal shocks, and there are theoretical and empirical arguments to be made in favor of each type of rule. This paper does not seek to resolve this debate but simply to extend previous work on time dependent rules.

2. But see Benabou (1987) for a model in which randomization of adjustment times arises endogenously.

3. Thus I follow the nearly universal practice in this literature of focusing on the "aggregate supply" side of the economy -- the determination of the price level -- while simply assuming the existence of an "aggregate demand" side (that is, an inverse relationship between aggregate output and the price level). Calvo builds up the aggregate demand side of his model from utility-maximization (with real money balances assumed to enter utility).

4. There is also a negative value of  $c$  that solves (14). This solution is unstable.

5. The first term of (16) shows that if the real interest rate is positive, higher trend inflation increases mean output (given  $\alpha$ ). The source of this effect is that trend inflation causes the expected profit-maximizing price to be rising over time and that a positive real interest rate causes firms to put relatively greater weight on current rather than future optimal prices; thus they charge less than the weighted average expected profit-maximizing price.

6. It is possible to find an expression for the equilibrium frequency of price changes in the case of deterministic intervals between changes if there are no aggregate shocks and if  $r=0$ . The expression analogous to (22) is  $g^2 \lambda^3 / 6 = 2F/K$ , where  $\lambda$  is the interval between price changes. Comparison with (22) shows that in this simple case, the average interval between price changes is shorter by a factor of  $12^{1/3}$  (that is, about 2.3) when firms have Poisson adjustment times than when they can adjust at deterministic intervals.

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