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SUPPLY, DEMAND, INSTITUTIONS, AND FIRMS:
A THEORY OF LABOR MARKET SORTING AND THE WAGE DISTRIBUTION

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ABSTRACT

This paper builds a general equilibrium framework with firm and worker heterogeneity, monopsony power, and task-based production to quantify the long-run effects of education, biased demand shocks, and minimum wage. I take it to Brazilian data for 1998 and 2012 and find that (i) supply and demand shocks increase sorting of high-wage workers to high-wage firms, (ii) increased entry of high-wage firms boosts the effect of rising schooling attainment on mean log wages by 25%, and (iii) the minimum wage reduces formal wage inequality but also causes wage loss for mid-productivity workers and disemployment for those at the very bottom.

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A data appendix is available at http://www.nber.org/data-appendix/w31318

1 Introduction

Brazil experienced a dramatic reduction in wage inequality between the mid-1990s and the early 2010s. In a literature review, Firpo and Portella (2019) point to three shocks as plausible causes of that phenomenon: an increased supply of skilled labor due to rising educational attainment, labor demand shocks that favored unskilled workers (mostly due to the 2000s commodities boom), and large real increases in the federal minimum wage. Understanding the labor market effects of these shocks is important for not only those interested in the Brazilian case but also those seeking to remedy rising wage inequality in other contexts.

To that end, this paper develops a tractable framework that describes how supply, demand, and minimum wage jointly determine the long-run wage distribution in imperfectly competitive labor markets. I employ matched employer-employee data to test its theoretical predictions and to structurally estimate a local labor markets model of the Brazilian economy. Finally, I simulate counterfactual scenarios based on the estimated model to quantify the individual impacts of each shock, as well as their interactions.

Current academic literature employs two separate frameworks to study the labor market effects of those shocks. Supply and demand factors are typically examined under the assumption of perfect competition, using models with representative firms (e.g., Bound and Johnson, 1992; Card and Lemieux, 2001) or assignment models based on comparative advantage (e.g., Teulings, 1995; Acemoglu and Autor, 2011). In such models, inequality trends reflect changes in productivity gaps between workers. By contrast, leading quantitative models of the minimum wage, such as those developed by Flinn (2006) and Engbom and Moser (2022), are imperfectly competitive. Those models emphasize the contribution of cross-firm wage differentials between equally productive workers (henceforth, firm wage premiums) to overall wage inequality.

Although the use of different frameworks for different shocks facilitates tractability, it also imposes restrictions on causal pathways. In competitive models, supply and demand factors cannot affect wages through firm wage premiums or *sorting*, defined in this paper as the assortativeness between worker skill and the firm wage premium they earn at their current employer. But those channels may be quantitatively important. For example, Card, Heining and Kline (2013) and Song et al. (2018) show that long-run changes in sorting account for significant shares of the overall increase in wage inequality in West Germany and the US, respectively. If those changes in sorting are driven by supply and demand factors, competitive models may provide an incomplete account of their labor market effects. On the minimum wage side, the leading models impose strong restrictions on how productivity gaps between workers may change by assuming perfect substitutability between worker types, ruling out changes in technologies firms may use, or disallowing cost pass-throughs.

A descriptive analysis of the Brazilian case shows that these restrictions may be consequential. I use matched employer-employee data to calculate labor market statistics for 151 *microregions* comparable to US commuting zones. Those statistics include several measures of wage inequality, minimum wage bindingness, and formal employment rates for 1998 and 2012. I also use the methodology detailed by Kline, Saggio and Sølvsten (2018) to obtain reduced-form estimates of the importance of firm wage premiums and sorting, based on two-way fixed effects regressions in the tradition of Abowd, Kramarz and Margolis (1999).

Many of my descriptive findings align with previous work on Brazil: the fall in inequality is large, widespread, and associated with the reduced dispersion of firm wage premiums (Alvarez et al., 2018). However, I also document a new fact not readily explained by existing theoretical approaches: assortative matching rises in most regions. Although papers such as Engbom and Moser (2022) allow for the minimum wage to impact sorting, it acts in the opposite direction.

Motivated by these findings, I develop a new framework to investigate whether the transformations observed in Brazilian labor markets can be parsimoniously explained by supply, demand, and minimum wage shocks and, if so, to determine what role each of them plays. It features rich worker and firm heterogeneity, a task-based model of production, monopsony power based on idiosyncratic worker preferences, general equilibrium in the market for goods, and free entry of firms. The distinguishing feature of my framework is that it combines the two theoretical perspectives mentioned above by allowing all shocks to affect wages via changes in labor productivity, the dispersion of firm wage premiums, and sorting.

This unified approach provides novel insights into how these shocks affect wage inequality. The first insight is a new explanation for why increases in the supply of skilled labor may have limited effects on the aggregate skill wage premium, or may even widen it (Blundell, Green and Jin, 2021; Carneiro, Liu and Salvanes, 2022). This phenomenon is typically explained using models of endogenous innovation, which creates non-convexities in the aggregate production function (Acemoglu, 1998, 2007). My framework features no such non-convexities. Instead, the aggregate skill premium can rise when the supply shock leads to the creation of skill-intensive, high-wage firms, and the gains in firm premiums for skilled workers reallocated to those new firms outweigh decreases in productivity differentials by skill.[1]

I also show that the combination of monopsony power, firm heterogeneity, and task-based production can lead to qualitative changes in minimum wage effects. Workers of different skill levels may be complements at high-wage firms that use a broad set of tasks in production, but substitutes in low-wage firms specialized in simple tasks. When the minimum wage reallocates unskilled labor from low- to high-wage firms, productivity gaps between skilled and unskilled workers may widen within the destination firms. But there may be no change in productivity differentials at the origin firms. As a result, the impact of minimum wages may be negative in the middle of the wage distribution and positive at the top, contrasting with the smooth inequality-reducing effects predicted by competitive task-based models (Teulings, 2000).

The theoretical results have broader relevance to minimum wage literature, as they call attention to identification threats in reduced-form designs. Some studies measure minimum wage effects using panel data at the firm level, defining treatment and control firms based on the initial fraction of their employees with wages below the new minimum. As discussed, high-wage firms may be affected by the minimum wage due to reallocation inflows, even in the absence of general equilibrium responses. However, because such firms are likely to have a low "fraction affected" due to their wage premium, the regression uses them as control units. Similar concerns may apply to designs comparing workers with initial wages below the new minimum and others in the same region with higher wages.

With the objective of performing policy counterfactuals, I estimate a parsimonious parameterization of the framework using a simultaneous equation nonlinear least squares procedure. Conceptually, the exercise resembles Katz and Murphy (1992) or Krusell et al. (1999), who use supply/demand models to explain rising wage inequality in the US. I target an array of endogenous outcomes at the region-time level: wage inequality between and within three educational groups, the variance of firm effects, the covariance of firm and worker effects, minimum wage bindingness metrics (including the size of the minimum wage *spike*), and formal employment rates by education type. Although over-identified, the model fits the data well. I interpret the quality of fit as demonstrating that, at least in the Brazilian context,

[1]This mechanism is comparable to that of Acemoglu (1999) but differs in that it is not based on search frictions. In addition, firms in my model are large and simultaneously employ many worker types, with within-firm imperfect substitution between skill levels. This generates smooth labor market responses to supply shocks instead of the discrete regime changes predicted by Acemoglu (1999).

secular trends in wage inequality, the dispersion of firm wage premiums, and sorting can be largely explained by supply, demand, and minimum wage.

Armed with the estimated model, I measure the labor market impacts of each shock and their interactions. Consistent with previous work, I find that demand shocks and the minimum wage are the main causes of the decline in wage inequality in Brazil's formal sector. I also find significant interactions that would not be detectable without a unified framework. The inequality effects of the minimum wage are twice as large when that shock acts in isolation, compared to a scenario where it is accompanied by supply and demand transformations. Supply and demand shocks increase measured sorting, with their effect magnified when they act together. The minimum wage reduces sorting, but its effect is weaker when supply and demand are also changing.

I also conduct two decomposition exercises that demonstrate the quantitative relevance of the new theoretical pathways. In the first exercise, I show that increased entry of high-wage firms amplifies the effects of rising schooling achievement on mean log wages by 25%. The second exercise finds that the minimum wage has negative wage effects on workers in the middle of the productivity distribution due to endogenous changes in wages posted by high-wage firms. These effects differ markedly from simulated "minimum wage spillovers" because the minimum wage causes disemployment for very low-skilled workers (such that spillovers arise from truncation of the latent productivity distribution). I include a discussion of the reasons that my results differ from the recent work of Engbom and Moser (2022), who observe small employment effects in Brazil using both reduced-form methods and a structural model.

The paper proceeds as follows. The next section details how this work builds upon and contributes to different strands of literature. The third section contains a descriptive analysis of the Brazilian data. The fourth section presents the task-based model of production and some of its implications in partial equilibrium. The fifth section describes the complete general equilibrium framework and discusses its predictions concerning the effects of supply, demand, and minimum wage. The sixth section contains the quantitative exercises. The final section concludes with directions for further research.

2 Literature and contribution

This paper's framework can rationalize a large set of empirical facts documented in recent years. It can explain why the contribution of firm wage premiums and sorting to wage inequality may change in the long run (Card, Heining and Kline, 2013; Song et al., 2018; Alvarez et al., 2018). Sorting originates from differences in demand for skills between firms, as documented by Deming and Kahn (2018). Because firms use production functions featuring complementarity between worker types, the framework rationalizes changes in within-firm wages in response to shifts in its internal workforce composition, such as those documented by Jäger and Heining (2022). Minimum wage can cause positive employment effects, reallocation of workers from low- to high-wage firms (Dustmann et al., 2021), spillovers (Fortin, Lemieux and Lloyd, 2021), and changes in how selective firms are when hiring (Butschek, 2022). Minimum wages may also precipitate changes in the types of firms operating in the economy (Rohlin, 2011; Aaronson et al., 2018) and relative consumer prices (Harasztosi and Lindner, 2019). Including all those potential channels lends credibility to the model's quantitative predictions.

On the theoretical side, my task-based model of production builds upon the work of Sattinger (1975) and Teulings (1995), among many others. I derive new formulas for elasticities of complementarity between worker types and provide computationally efficient parameterization. But the core contribution to this literature is characterizing task-based production in an environment with monopsony power and heterogeneous firms. I show that the optimal assignment of workers to tasks may differ between firms and find support for that prediction in the data. I also discuss how substitution patterns differ between firms and why that matters for comparative statics.

The second strand of literature I build upon concerns monopsony models of labor markets based on idiosyncratic worker preferences for firms. I embed the model developed by Card et al. (2018) into a general equilibrium framework with task-based production, firm entry, endogenous participation decisions, and minimum wages. I show how firm heterogeneity in skill intensity and wage premiums emerge from differences in production technologies available to entrepreneurs when they create firms. I also show that the elasticity of labor supply to individual firms—a key component of monopsony models—can be identified from the size of the minimum wage "spike" in log wage distributions.[2,3]

[2] I thank an anonymous referee for this suggestion.

[3] Within the monopsony literature, my paper resembles the work of Lamadon, Mogstad and Setzler (2022),

More broadly, this paper relates to models that quantify the effects of changing supply of and demand for skills. Within that literature, it is closest to those where supply/demand shocks alter the composition of jobs in the economy. Some work in that tradition, such as Kremer and Maskin (1996) and Lindenlaub (2017), abstract from the role of firm wage premiums. Others, such as Helpman et al. (2017), Shephard and Sidibe (2019), and Lise and Postel-Vinay (2020), feature imperfect competition and firm wage premiums but assume workers are perfect substitutes within firms (or that each firm hires only one worker). In such models, labor market imperfections are the only reason for observing skills dispersion within a firm type. By contrast, firms in my model hire multiple types of workers to benefit from the division of labor, even when labor markets are competitive. Accurate firm-worker sorting patterns are important for capturing the part of the effects of supply/demand shocks that derive from endogenous firm entry and changing prices.[4]

Finally, I describe how my framework differs from quantitative models of minimum wages developed in recent years. Engbom and Moser (2022) build a model with on-the-job search in the style of Burdett and Mortensen (1998). Similar to my study, they estimate their model using Brazilian data and match moments from two-way fixed effects decompositions. Because their model features search frictions, it is better suited to studying job ladders and transitions into and out of unemployment. However, it abstracts from non-wage amenities and assumes perfect substitutability between worker types.

Berger, Herkenhoff and Mongey (2022*b*) and Hurst et al. (2022) build monopsonistic minimum wage models with imperfect substitution across labor types. Berger, Herkenhoff and Mongey (2022*b*) include cross-firm differences in productivity and allow for variation in markdowns depending on the firm size relative to the market. Hurst et al. (2022) abstract from firm heterogeneity but include search frictions and a putty-clay technology that allows them to distinguish between short- and long-run minimum wgae effects. They also study how minimum wage can be paired with transfers to achieve redistribution goals.

whose model also generates realistic firm wage premiums and sorting patterns. They allow for worker reallocation across regions and richer forms of firm heterogeneity but do not model within-firm complementarities between worker types, endogenous participation decisions, firm entry, or minimum wages.

[4] Eeckhout and Pinheiro (2014) and Trottner (2019) also model large firms with multiple jobs, but with common elasticities of substitution across all pairs of worker types. Herkenhoff et al. (2018) allows for search frictions and within-firm complementarities, but firms may only employ up to two workers. Models of hierarchical firms in the tradition of Garicano (2000), Garicano and Rossi-Hansberg (2006), and Antràs, Garicano and Rossi-Hansberg (2006) imply within-firm division of labor, but the modeling of costly information transmission within firms reduces their tractability. My production structure can be viewed as a hierarchical firms model without that cost and without the restriction that hierarchies need to be pyramidal.

As a tool for evaluating minimum wages, my framework is unique in four ways. First, substitution patterns between worker types depend on whether they are close or distant in terms of skill and also on the task demands of the firm employing them. Second, it allows for cost pass-throughs and endogenous changes in the composition of firms operating in the economy. Third, it measures how minimum wages interact with educational trends and many types of labor demand shocks. Fourth, it includes an estimation procedure based on regional and time variation. That procedure showcases the model's tractability (because each iteration of the estimation procedure requires solving for equilibria more than 15 thousand times) and its ability to explain cross-sectional variation in features such as the minimum wage spike. It also allows for measuring how minimum wage effects differ between local labor markets, which may be important in contexts with significant regional heterogeneity.

3 Wage inequality and sorting in Brazil

In this section, I present descriptive statistics that motivate the theoretical framework. I use two data sources. The first is the RAIS (*Relação Anual de Informações Sociais*), a confidential linked employer-employee dataset maintained by the Brazilian Ministry of Labor. Firms are mandated by law to report to the RAIS at the establishment level. The dataset contains information about both the establishment (including legal status, economic sector, and the municipality in which it is registered) and each worker it formally employs (including education, age, earnings in December, contract hours, and hiring and separation dates).

The other data come from the Brazilian censuses of 1991, 2000, and 2010. From them, I obtain statistics for the overall population, such as the number of adults in each educational group and the proportion of those who hold formal jobs. I also extract from the Census the share of workers in agriculture, manufacturing, or other sectors.[5]

I focus on individuals between 18 and 54 years of age. In the RAIS, I select individuals in that age range who are working in December, having been hired in November or earlier. If a worker has more than one job in the same year, I only keep the highest-paying one.

All the statistics are calculated at the local level. I use the concept of "microregion" as defined by the Brazilian Statistical Bureau (IBGE). Microregions group nearby, economi-

[5]The 1998 outcomes are interpolated using the 1991 and 2000 Censuses. The 2012 outcomes are extrapolated using 2000 and 2010. The interpolations and extrapolations are linear for formal employment rates and sectoral shares, and linear in logs for population counts.

cally connected municipalities ("IBGE", 2003). They are commonly used to define local labor market models in Brazil (e.g., Costa, Garred and Pessoa, 2016; Ponczek and Ulyssea, 2021).[6]

I use a local labor markets approach for two reasons. First, regional variation helps identify key parameters of the structural model. Second, local labor markets more closely map theory to empirics. If firm-worker sorting is measured nationally, it will largely reflect geographical barriers in addition to the supply-demand-minimum wage dynamics emphasized by the framework. I return to this point at the end of the paper when I compare my results to previous work studying the Brazilian case.

The final sample is restricted to microregions with at least 15,000 workers in the RAIS data in 1998 and 2012 and at least 1,000 formal workers in each of the three educational groups defined below.[7] That leaves a set of 151 microregions encompassing 73% of the adult population. Appendix Table D1 presents the consequent sample sizes.

Differing from the pattern in many high-income countries, wage inequality has been downward trending in Brazil since the 1990s. The first two panels in Table 1 report the evolution of several inequality metrics calculated at the microregion level and averaged nationally using total formal employment in both base years as weights (this means that region weights are constant over time). Almost all metrics are declining, some of them dramatically. The one exception is the college premium, which widened in 47 out of 151 regions. Because those regions tend to be larger, the average college premium increased.

I gauge the contribution of firm wage premiums and sorting using region-specific variance decompositions based on two-way fixed effects regressions of log wages (henceforth AKM regressions after Abowd, Kramarz and Margolis, 1999). The log wage of worker i in region r at time τ is written as:

$$\log y_{i,r,\tau} = \nu_{i,r} + \psi_{J(i,r,\tau)} + \delta_{r,\tau} + u_{i,r,\tau}$$

where $\nu_{i,r}$ is the worker fixed effect, ψ_j is establishment j's fixed effect, $J(i,r,\tau)$ denotes the

[6]Using data for 2000 and 2010, Dix-Carneiro and Kovak (2017) calculate that less than 5% of workers lived in one region and worked in another. That number, combined with their average size, makes Brazilian microregions analogous to commuting zones in the US. After combining some microregions to ensure that their boundaries remain constant throughout the study period, my base sample features 486 microregions.

[7]My structural estimation procedure requires a low level of measurement error in formal employment rates by educational group and minimum wage bindingness. Those restrictions also yield better estimates of the contribution of firm wage premiums and sorting to local wage inequality.

Table 1: Evolution of wage inequality measures and sorting

	1998	2012
Panel A: Variances of log wages in base years		
All workers	0.715	0.544
Less than secondary	0.410	0.241
Secondary	0.684	0.355
Tertiary	0.702	0.624
Panel B: Mean log wage gaps in base years		
Secondary / less than secondary	0.498	0.168
Tertiary / secondary	0.965	1.038
Panel C: Variance decomposition using three-year panels		
Total variance	0.688	0.577
Variance of worker effects	0.419	0.384
Variance of establishment effects	0.116	0.056
2×Covariance worker, estab. effects	0.098	0.097
Correlation worker, establishment effects	0.224	0.315

Notes: Panels A and B display average wage inequality measures for the base years of 1998 and 2012. Panel C shows the average outcomes of region-specific log wage decompositions based on Equation (1), using the estimator provided by Kline, Saggio and Sølvsten (2018). All numbers are averaged over regions using the total number of formal workers in both base years as weights.

establishment employing worker i in region r at time τ, $\delta_{r,\tau}$ is a region-time effect, and $u_{i,r,\tau}$ is a residual. Then, the within-region variance of log wages can be written as follows:

$$\begin{aligned}\text{Var}\left(\log y_{i,r,\tau}|r\right) = \text{Var}\left(\nu_{i,r}|r\right) + \text{Var}\left(\psi_{J(i,r,\tau)}|r\right) + 2\text{Cov}\left(\nu_{i,r}, \psi_{J(i,r,\tau)}|r\right) \\ + \text{Var}\left(\delta_{r,\tau}|r\right) + 2\text{Cov}\left(\nu_{i,r} + \psi_{J(i,r,\tau)}, \delta_{r,\tau}|r\right) + \text{Var}\left(u_{i,r,\tau}|r\right) \quad (1)\end{aligned}$$

If wages differ substantially across establishments for similar workers, the variance of establishment effects may be large, adding to overall wage dispersion. If high-wage workers are more likely to work at high-wage establishments, then the first covariance term will be positive, further boosting inequality. Based on this logic, the correlation between establishment and worker fixed effects is often used as a simple measure of labor market sorting.

Estimating the variance decomposition (1) is not trivial. I use the method developed by Kline, Saggio and Sølvsten (2018) (henceforth KSS), which is not subject to the limited mobility bias discussed by Andrews et al. (2008). I run the KSS model separately for each microregion and period, using three-year panels centered on either 1998 or 2012. Because that procedure requires a leave-one-out connected set, small establishments are under-represented in that sample. Appendix D.2 provides details about the procedure.

Average results for the decompositions appear in Panel C of Table 1. Both worker effects and establishment effects contributed to the fall of inequality in Brazilian microregions. However, the covariance term remains virtually unchanged. Thus, it accounts for a larger share of the variance of log wages in 2012. The measured correlations between worker and establishment effects increase in most microregions (104 out of 151).[8,9]

The interpretability of AKM decompositions relies on categorizing establishments as high- or low-wage. However, in many economic models of sorting, including this paper's, wages are not log-additive in worker and establishment components: Some establishments may pay some worker types more and other worker types less. Still, indirect inference can be used to extract identifying information from the AKM decomposition. I employ this strategy in this paper.

Now I consider the potential explanations for the falling inequality in Brazil. The most conspicuous are increased educational achievement and rising minimum wages. Table 2 shows the magnitude of those shocks. Panel A displays the average share of adults in each of three educational groups: less than secondary (that is, a level of achievement lower than completing high school, or between zero and ten years of schooling), secondary (combining complete high school and college dropouts, or between 11 and 14 years or schooling); and tertiary (complete college or more). The pattern is striking: In the span of 14 years, the share of adults completing high school or further education increases by 20 percentage points (a 68% increase). This represents the outcome of educational reforms and policies traceable to the 1980s, including minimum government expenditure requirements on education, construction of schools, cash transfers conditional on school enrollment, and vouchers for tertiary education.

Panel B shows that the minimum wage became more binding over the study period. The Brazilian national minimum wage increased by 66 log points in real terms (93.7%) between December 1998 and December 2012, which increased the "bite" of the minimum wage into the wage distribution regardless of the bindingness metric used. The apparent compression

[8] The KSS estimate of the *correlation* between worker and establishment effects is not guaranteed to be unbiased. In the structural estimation exercise, I target the unbiased covariance estimates rather than the correlations.

[9] Alvarez et al. (2018) and Engbom and Moser (2022) also find that establishment effects explain a significant fraction of the decline in wage inequality in Brazil. However, they find that the covariance term also falls, such that there is no increase in measured sorting. The key difference between my approach and theirs is that whereas my decompositions are performed at the local labor market level, they use national models. National-level sorting can fall if, for example, gains in educational achievement are stronger in areas with low-wage firms.

Table 2: Trends in schooling achievement and minimum wage bindingness

	1998	2012
Panel A: Share of adults by education group		
Less than secondary	0.699	0.493
Secondary	0.229	0.383
Tertiary	0.072	0.124
Panel B: Minimum wage bindingness		
Log minimum wage minus mean log wage	-1.418	-0.922
Log minimum wage minus log median wage	-1.220	-0.719
Share up to log minimum wage + 0.3	0.086	0.212

Notes: All numbers are averaged over regions using the total number of formal workers in both base years as weights.

of wage distribution is shown in Appendix Figure D2.

A third factor emphasized in the Brazilian case is labor demand shocks associated with international trade. During the study period, Brazilian regions were still adapting to the trade liberalization of the early 1990s, which, according to Dix-Carneiro and Kovak (2017), had long-lasting impacts. During the 2000s, the "rise of China" led to significant changes in terms of trade. Costa, Garred and Pessoa (2016) study that shock and also find evidence of differential labor market impacts at the microregion level. Trade liberalization seemingly benefitted skilled workers, while the commodities boom benefited unskilled workers.

These transformations are not easily explained using existing quantitative frameworks. One could be tempted to conclude that rising education and demand for commodities increase the relative productivity of unskilled workers, while the minimum wage further reduces markdowns for unskilled workers and reallocates some of them to high-wage firms (Engbom and Moser, 2022). But that simple story does account for the fact that sorting is rising. Indeed, the minimum wage effects just described would imply *decreases* in sorting. That is the motivation for building a framework where supply and demand factors affect wages through not only worker productivity but also firm wage premiums and assortative matching.

4 The task-based production function

Task-based models of comparative advantage are increasingly used to model wage inequality. Acemoglu and Autor (2011) show that these models are better suited than the "canonical" constant elasticity of substitution (CES) model of labor demand to study inequality trends in the US. Teulings (2000, 2003) shows that substitution patterns implied by assignment models make them particularly suitable for studying minimum wages. Costinot and Vogel (2010) develop a task-based model to study the consequences of trade integration and offshoring.

In this section, I demonstrate an additional advantage of the task-based approach: It allows for intuitive, tractable, and parsimonious modeling of firm heterogeneity in both competitive and imperfectly competitive labor markets. All proofs appear in Appendix A.

4.1 Setup, definitions, and the assignment problem

Workers are characterized by their labor type $h \in \{1,\dots,H\}$ and the amount of labor efficiency units they can supply, $\varepsilon \in \mathbb{R}_{>0}$. They use their labor to produce tasks that are indexed by complexity $x \in \mathbb{R}_{>0}$.[10] Although all labor types are perfect substitutes in the production of any particular task, their productivities are not the same:

Definition 1. *The* ***comparative advantage function*** $e_h : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ *denotes the rate of conversion of worker efficiency units of type h into tasks of complexity x. It is continuously differentiable and log-supermodular:* $h' > h \Leftrightarrow \frac{d}{dx}\left(\frac{e_{h'}(x)}{e_h(x)}\right) > 0 \; \forall x$.

To fix ideas, consider an example with two workers. Alice, characterized by h, ε, can use a fraction $r \in [0,1]$ of her time to produce $r\varepsilon e_h(x)$ tasks of complexity x. Bob (h', ε'), who belongs to a lower type $(h' < h)$, can still produce more of those tasks than Alice, provided his quantity of efficiency units is sufficiently high $(\varepsilon' > \varepsilon e_h(x)/e_{h'}(x))$. But Alice has a comparative advantage: Moving toward more complex tasks increases her productivity relative to Bob's.

It is easy to see that the sum of efficiency units of each type is a sufficient statistic for production. Thus, this section provides definitions and results in terms of total efficiency units of each type available to the firm, denoted by $\boldsymbol{l} = \{l_1,\dots,l_H\}$. The distinction between efficiency units and workers will be relevant later in the paper.

Each good, indexed by $g = 1,\dots,G$, is produced by combining tasks in fixed proportions:

Definition 2. *The* ***blueprint*** $b_g : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ *is a continuously differentiable function that denotes the density of tasks of each complexity level x required for the production of a unit of*

[10] In the quantitative exercises, worker skill is mapped to educational achievement, meaning more complex tasks should be interpreted as those better performed by formally educated workers. The assumption of a single complexity dimension is maintained throughout. Quantitative models using multi-dimensional skills and tasks include Lindenlaub (2017) and Lise and Postel-Vinay (2020).

consumption good g. Blueprints satisfy $\int_0^\infty b_g(x)/e_H(x)dx < \infty$ (production is feasible given a positive quantity of the highest labor type).

Consider a firm trying to produce good g after hiring $\boldsymbol{l}$ efficiency units of labor in the labor market. Tasks cannot be traded. The firm assigns workers to tasks with the goal of maximizing output q, subject to two constraints: producing the required amount of tasks and using no more labor than what it has hired. I assume firms can split workers' time across tasks according to assignment functions $m_h : \mathbb{R}_{>0} \to \mathbb{R}_{\geq 0}$, assumed to be right-continuous.

Definition 3. *The **task-based production function** is given by*

$$
\begin{aligned}
f(\boldsymbol{l}; b_g) = \max_{q \in \mathbb{R}_{\geq 0}, \{m_h(\cdot)\}_{h=1}^{H}} \quad & q \\
s.t. \qquad q b_g(x) &= \sum_h m_h(x) e_h(x) \quad \forall x \in \mathbb{R}_{>0} \\
l_h &\geq \int_0^\infty m_h(x) dx \quad \forall \in \{1, \dots, H\}
\end{aligned}
$$

and is defined for all $\boldsymbol{l} \in \mathbb{R}_{\geq 0}^{H-1} \times \mathbb{R}_{>0}$ and valid blueprints b_g.

This definition assumes a positive amount of labor of type H, which is not restrictive for my applications. See Appendix B.1 for a brief discussion.

The next subsections characterize the properties of this production function under different labor market structures. Before arriving there, I present a general result for optimal assignment:

Lemma 1 (Optimal allocation is assortative)**.** *For every combination of inputs $\boldsymbol{l}, b_g$, there exists a unique set of $H-1$ complexity thresholds $\bar{x}_1 < \cdots < \bar{x}_{H-1}$ that defines the range of tasks performed by each worker type in an optimal allocation: $m_h(x) > 0 \Longleftrightarrow x \in [\bar{x}_{h-1}, \bar{x}_h)$, with $\bar{x}_0 = 0$ and $\bar{x}_H = \infty$. Thresholds satisfy:*

$$
\frac{e_{h+1}(\bar{x}_h)}{e_h(\bar{x}_h)} = \frac{f_{h+1}}{f_h} \quad h \in \{1, \dots, H-1\} \tag{2}
$$

where $f_h = \frac{d}{dl_h} f(\boldsymbol{l}, b_g(\cdot))$ denotes the marginal product of labor h, which is strictly positive.

Lower types specialize in low-complexity tasks and vice-versa. Equation (2) means that the shadow cost of using neighboring worker types is equalized at the task that separates them. This result is the starting point for obtaining compensated labor demands, as I describe in the following subsection.[11]

4.2 Compensated labor demand in competitive labor markets

To study the implications of task-based production for labor demand, I start with a partial equilibrium analysis. Consider an individual firm, which produces good g, attempting to minimize labor costs given a production target q. The labor market is competitive, such that unit costs per efficiency unit of each labor type are constants $\boldsymbol{w} = \{w_1, \ldots, w_H\}$.

Optimality requires that marginal product ratios equal wage ratios. Then, from Equation (2):

$$\frac{e_{h+1}(\bar{x}_h)}{e_h(\bar{x}_h)} = \frac{w_{h+1}}{w_h}$$

Because the left-hand side is strictly increasing in $\bar{x}_h$, this expression pins all task thresholds as functions of wage ratios and comparative advantage functions. That is, thresholds are strictly increasing functions $\bar{x}_h(w_{h+1}/w_h)$. This renders the compensated labor demand as follows:

$$l_h(q, b_g, \boldsymbol{w}) = q \int_{\bar{x}_{h-1}(w_h/w_{h-1})}^{\bar{x}_h(w_{h+1}/w_h)} \frac{b_g(x)}{e_h(x)} dx \tag{3}$$

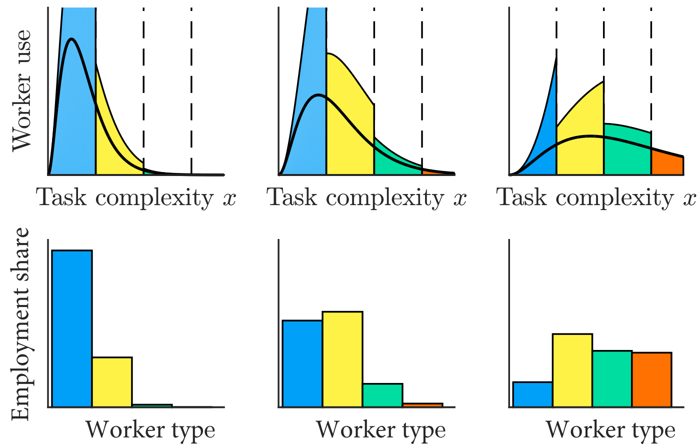
Now suppose that different firms produce different goods in this partial equilibrium, competitive environment. Because neither efficiency functions nor labor costs are firm-specific, all firms choose the same task thresholds.

Figure 1 illustrates how blueprints determine demand for skills. The graphs at the top show the discussed compensated labor demand integral. The heavy, continuous line is the blueprint. The vertical dashed lines are the thresholds defining the ranges of tasks assigned to each worker type. The colored areas represent the labor demand integrals from Equation 3. The bottom panels show corresponding factor intensities as histograms.

Due to the infinite-dimensional blueprints and efficiency functions, the task-based structure might appear exceedingly flexible at first glance. Proposition 1 extends the results of Teulings (2005) and shows that, on the contrary, there are strong constraints on substitution

[11] In general, the task-based production function and its derivatives do not have simple closed-form representations. To evaluate output and marginal productivities as a function of labor inputs, one must first solve the system of H compensated labor demand equations (3) on q and the $H-1$ thresholds. Next, use equation (2) to calculate marginal productivity gaps. Finally, use the constant returns relationship $q = \sum_h l_h f_h$ to normalize marginal productivities.

Figure 1: Compensated labor demand in competitive labor markets



patterns.[12]

Proposition 1 (Curvature)**.** *The task-based production function is concave, features constant returns to scale, and is twice continuously differentiable with strictly positive first derivatives. Appendix A provides formulas for elasticities of complementarity and substitution.*

Corollary 1 (Distance-dependent complementarity)**.** *For a fixed h, the partial elasticity of complementarity between that type and another type h' is strictly increasing in h' for $h' \geq h$ and strictly decreasing in h' for $h' \leq h$.*

The curvature of the task-based production function reflects the division of labor within the firm. Suppose that, initially, a firm only employs Alice, who belongs to the highest type H. In this case, output is linear in the quantity of labor bought from Alice. Adding a lower-type worker, Bob, increases Alice's productivity by enabling her to specialize in complex tasks while Bob takes care of simpler tasks. At that point, decreasing returns to Alice's hours reflect a reduction in gains from specialization.

The impact of adding a third worker on the marginal productivities of Alice and Bob depends on the third worker's labor type. Close types perform similar tasks and are net substitutes; distant types perform different tasks and are complements.

[12] Teulings (2005) derives elasticities of complementarity for a similar model but using parametric efficiency functions and taking a limit where the number of worker types grows to infinity. In an application of assignment models to optimal taxation, Ales, Kurnaz and Sleet (2015) derive elasticities of substitution in a model of production where output is CES in tasks, instead of Leontief .

Figure 2: Distance-dependent complementarity

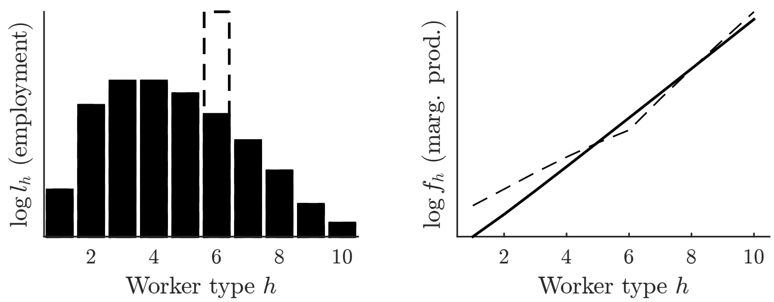


Figure 2 illustrates distance-dependent complementarity. The left panel shows baseline log employment by worker type (black bars) and a shock to the employment of workers of type 6 (dashed contour). The right panel shows baseline log marginal productivities (solid line) and marginal productivities after the employment shock (dashed). Workers of type 6 suffer the largest relative decline in marginal productivity, followed by types 7 and 5. Marginal productivities increase for low- and high-skilled types further away.

Teulings (2000) shows that distance-dependent complementarity can explain minimum wage spillovers, that is, changes in the distribution of wages at quantiles where the minimum wage does not bind. If a minimum wage causes disemployment of low-skilled workers, then the logic of Figure 2 implies that marginal products—and hence wages—should increase for workers close to the minimum. The core contribution of Teulings (2000) is to show that, differing from a "canonical" CES approach, a task-based model with many worker types can explain realistic levels of spillovers even when the disemployment effects are small.[13] My framework differs from Teulings (2000) in that I allow for firm heterogeneity and imperfect competition, which I start discussing in the next subsection.

4.3 Labor demand in a monopsonistic labor market

Suppose that firms have wage-setting power. Each firm j posts a prices per efficiency unit w_{hj} for each type h. At that posted wage, it is able to attract a quantity of labor equal to $l_{hj} = l_h(w_{hj}) = L_h \cdot \left(\frac{w_{hj}}{\omega_h}\right)^{\beta}$.[14] The core implication of upward-sloping supply curves to the

[13] Teulings and van Rens (2008) derives a sufficient statistic that can be used to compare the degree of substitution across worker types in different models. For some combinations of shocks and outcomes of interest, task-based models and the canonical model can produce very similar predictions. But this is typically not true for minimum wage shocks.

[14] This expression is consistent with the general equilibrium model described in Section 5, in a special case with no minimum wage. β is the firm-level elasticity of labor supply, L_h is the aggregate supply of labor of

firm is that the more intensely a factor is used, the higher its marginal cost. Thus, if firms differ in their skill intensity because they use different blueprints, their marginal product ratios differ. Equation (2) implies that their optimal assignments will also differ:

Lemma 2 (Differences in skill intensity, monopsony, and task assignment)**.** *Consider a partial equilibrium environment where firms have wage-setting power as described above. Suppose that the optimal labor choices of two firms indexed by $j \in \{1,2\}$ satisfy $\frac{l_{h+1,2}}{l_{h,2}} > \frac{l_{h+1,1}}{l_{h,1}}$ for some h. Then, $\bar{x}_{h,2} > \bar{x}_{h,1}$ (where $\bar{x}_{h,j}$ denotes the task threshold $\bar{x}_h$ at firm j).*

When a worker moves from one firm to another that is more skill-intensive, they will be assigned to more complex tasks. I test that prediction in Subsection 6.1. Lemma 2 also shows that wage-setting power may generate productive mismatch, similarly to how search frictions introduce mismatch in Teulings and Gautier (2004).[15]

Another implication of wage-setting power and task-based production is that an aggregate shock may produce different responses at different firms:

Proposition 2 (Complementarity patterns may differ between firms)**.** *Consider a partial equilibrium model with three worker types ($H = 3$), two goods with positive prices, and wage-setting power as described above. Good $g = 1$ has a degenerate blueprint requiring a unit measure of low-complexity tasks, $x = 0$. Good $g = 2$ has a regular blueprint. Then:*

1. *Firms producing either good employ workers of all types h.*
2. *If there is an increase in L_1 but all other supply parameters remain unchanged, posted wages do not change for firms producing good $g = 1$. But for firms producing good $g = 2$, all posted wage gaps $w_{h+1,j}/w_{h,j}$ become larger.*

The first part of this proposition exemplifies the production mismatch mentioned above. In a competitive market, firms that only need tasks $x = 0$ would not hire workers of high types. However, given isoelastic firm-level supply curves, it is sensible to hire at least a few such workers because, at sufficiently low employment levels, they become very cheap. More generally, there is less employment specialization under monopsony, although we should still expect firms demanding more complex tasks to be more skill-intensive.

The second part of Proposition 2 highlights a key feature of my framework: Firms differ in terms of not only demand for skill but also substitution patterns. For firms using the

type h, and ω_h is a sufficient statistic for labor demand by other firms in the market.

[15]Specifically, a planner that maximizes aggregate output given any vector of prices for goods will choose a different assignment of workers to tasks, compared to the monopsonistic allocation.

regular blueprint, $g = 2$, an increase in the aggregate supply of labor type $h = 1$ widens all within-firm skill wage differentials. This reflects distance-dependent complementarity. For firms producing the low-complexity good $g = 1$, posted wages do not change: The shock increases employment of workers of type $h = 1$ but has no other impact .

The degenerate blueprint used in the Proposition is very stylized, but it serves to illustrate a more general pattern. Suppose that the blueprints are those shown in Figure 1. Then, the intuition from Proposition 2 still applies: We should expect wage responses to be more muted for firms using the blueprint in the left panel because workers are closer substitutes in those firms. In Subsection 5.6, I show that this property has implications for the equilibrium effects of minimum wages.

4.4 Exponential-Gamma parameterization

In the quantitative exercises, I employ a parameterization with exponential efficiency functions and blueprints shaped like the density of a Gamma distribution:

$$e_h(x) = \exp(\alpha_h x) \qquad b_g(x) = \frac{x^{k-1}}{\Gamma(k)\theta_g{}^k} \exp\left(-\frac{x}{\theta_g}\right)$$

The coefficients α_h are increasing and determine the degree of comparative advantage of a labor type. The parameter θ_g relates to average task complexity. All else being equal, goods with higher θ_g require more complex tasks and thus have a higher demand for skills. Given that the shape parameter k is assumed to be common across firms, goods with higher θ_g also have more diffuse task requirements, meaning that workers are more likely to be complements at those firms.

Appendix C presents the mapping between marginal productivity gaps and task thresholds for a generalized version of this parametrization, as well as formulas for compensated labor demand integrals in terms of incomplete gamma functions or power series. These formulas do not require numerical integration, making them computationally efficient.

5 Markets and wages

This section builds a general equilibrium model with monopsonistic firms and free entry. The first subsection lays out the structure of the economy. The second subsection describes the functioning of labor markets, solves the problem of the firm, and presents an important property of the model: Goods encapsulate firm heterogeneity in skill intensity and wages. The third subsection describes firm wage differentials. The remaining subsections discuss comparative statics with respect to supply, demand, and minimum wage shocks.

Although the model is static, Appendix C.3 discusses a simple dynamic extension that can be used to simulate moments that require a panel dimension. Unless otherwise noted, all parameters are assumed to be strictly positive.

5.1 Factors, goods, technology, and preferences

There are two factors of production. The first is labor. The total number of workers of type h is denoted by N_h, and the distribution of efficiency units ε within group h is continuous with density $r_h(\cdot)$ and support over the positive real line. The second factor is an entry input used to create firms. The total stock of the entry input is normalized to one, and it is fully owned by a representative entrepreneur.

The economy features G firm-produced goods. Firms can only produce one of the goods, and the decision of which good the firm produces is made when the firm is created. The entry cost per firm, F_g, depends on the chosen good. The entrepreneur's action is to choose the number of firms J_g, conditional on the entry input constraint $\sum_g F_g J_g \leq 1$.

Firm-produced goods are sold in competitive markets at prices p_g. Consumers (workers or the representative entrepreneur) combine them into the final consumption good using a constant elasticity of substitution (CES) aggregator:

$$c = z \left[\sum_{g=1}^{G} \gamma_g Q_g^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}}$$

where z is a productivity parameter and γ_g is a taste shifter. The elasticity of substitution σ may depend on the interpretation of goods in the model: lower for different sectors, higher for different varieties within sectors, or close to infinity for different production technologies used to produce the same good. A large σ can also be an approximation for a small open economy where all goods are tradable.[16] I use the corresponding price index as the numeraire

[16] In the empirical exercise, I do not map goods to industries because the within-industry dimension is important. In many contexts, changes in inequality happen within industries (see Card, Heining and Kline, 2013; Song et al., 2018). The validation exercise in Subsection 6.1 suggests substantial task heterogeneity within finely defined sectors.

in this economy: $P \equiv \left[\sum_{g=1}^{G} \gamma_g^\sigma p_g^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = 1$.

Alternatively, workers that choose not to work for any firm can produce the final good via home production. A worker of type (h, ε) can produce $c = \varepsilon z_{0,h}$ units for its own consumption. The productivity parameters $z_{0,h}$ are intended to capture the value of outside options such as informal employment, self-employment, and government transfers to unemployed adults. The quantitative section allows those parameters to vary flexibly at the region, time, and education levels.

The entrepreneur's preferences are monotonic in the final good. Worker preferences depend on not only consumption but also where they are employed:

$$U_i(c, j) = c \cdot \left[\exp\left(\eta_{ij}\right)\right]^{\frac{1}{\lambda}}$$

where i denotes worker identity, c is its final good consumption, and j denotes the employment choice. Home production is denoted by $j = 0$. Employment in any of the firms is denoted by $j = 1, \ldots, J$ where $J = \sum_g J_g$. The η_{ij} parameters denote idiosyncratic preferences of workers towards their employment options. The importance of those components relative to consumption is regulated by λ.

The idiosyncratic preference components capture match-specific features, such as distance to the workplace, personal relationships with the manager or other coworkers, and how much they like staying at home for $j = 0$. The full vector of idiosyncratic preferences for a worker is drawn from the following cumulative distribution function:

$$CDF\left(\{\eta_{ij}\}_{j=0}^{J}\right) = \exp\left\{-\exp\left(-\eta_{i0}\right) - \left[\sum_{j=1}^{J} \exp\left(-\eta_{ij} \cdot \frac{\beta}{\lambda}\right)\right]^{\frac{\lambda}{\beta}}\right\}$$

This is a nested logit, with all firms included in one nest and home production in another. The parameter $\beta \geq \lambda$ denotes the correlation in preferences between firms. In the following section, I demonstrate that λ pins down the macro elasticity of labor supply to all firms, while β determines the firm-level elasticity of labor supply.

5.2 Labor markets, the problem of the firm, and equilibrium

Throughout this section, it is important to distinguish between quantities of workers, denoted by n, and quantities of labor, denoted by l. Worker earnings are denoted by y, while prices for efficiency units of labor are denoted by w.

Labor regulations prevent firms from paying a total compensation of less than $\underline{y}$ to any worker. I refer to $\underline{y}$ as the minimum wage. Because the model has no variation in hours worked, earnings and hourly wages are interchangeable. And because workers with low ε might have a marginal product of labor below $\underline{y}$ at some firms, I allow firms to reject workers with productivity below some minimum value $\underline{\varepsilon}_{hj}$.

5.2.1 Firm-level labor supply and labor costs

The timing of the labor market is as follows. First, all firms post rejection cutoffs $\underline{\varepsilon}_{hj}$ and earnings schedules $y_{hj}(\varepsilon) : [\underline{\varepsilon}_{hj}, \infty) \to [\underline{y}, \infty)$. Second, workers observe all $\underline{\varepsilon}_{hj}$ and $y_{hj}(\varepsilon)$ and choose their employment option j. Third, firms observe (h, ε) of workers who applied to them (but not idiosyncratic preference shifters η_{ij}) and hire those with $\varepsilon \geq \underline{\varepsilon}_{hj}$. Finally, production occurs and hired workers are paid. Rejected workers, if any, earn zero income.

To study worker choices in step 2, consider the indirect utility of a worker i characterized by (h, ε), if this worker chooses option j. It can be written as:

$$
\begin{aligned}
V_{ih}(\varepsilon, j) =& \exp\left(\lambda \log\left(\varepsilon z_{0,h}\right) + \eta_{ij}\right)^{\frac{1}{\lambda}} && \text{if } j = 0 \\
V_{ih}(\varepsilon, j) =& \mathbf{1}\left\{\varepsilon \geq \underline{\varepsilon}_{hj}\right\} \exp\left(\lambda \log y_{hj}(\varepsilon) + \eta_{ij}\right)^{\frac{1}{\lambda}} && \text{if } j \geq 1
\end{aligned}
$$

Given the distribution of η_{ij}, the probability of a worker (h, ε) choosing a particular option j is given by:

$$
\begin{aligned}
\Pr\left(0 = \underset{j' \in \{0,1,\ldots,J\}}{\arg\max}\ V_{ih}(\varepsilon, j')\right) &= \frac{(\varepsilon z_{0,h})^{\lambda}}{(\varepsilon z_{0,h})^{\lambda} + \omega_{\varepsilon,h}^{\lambda}} \\
\Pr\left(j = \underset{j' \in \{0,1,\ldots,J\}}{\arg\max}\ V_{ih}(\varepsilon, j')\right) &= \frac{\omega_{\varepsilon,h}^{\lambda}}{(\varepsilon z_{0,h})^{\lambda} + \omega_{\varepsilon,h}^{\lambda}} \times \left(\frac{\mathbf{1}\{\varepsilon \geq \underline{\varepsilon}_{hj}\} y_{hj}(\varepsilon)}{\omega_{\varepsilon,h}}\right)^{\beta} \qquad \text{for } j \geq 1 \\
\text{where } \omega_{\varepsilon,h} &= \left(\sum_{j=1}^{J} \mathbf{1}\{\varepsilon \geq \underline{\varepsilon}_{hj}\} y_{hj}(\varepsilon)^{\beta}\right)^{\frac{1}{\beta}}
\end{aligned}
$$

The "inclusive value" $\omega_h(\varepsilon)$ is a measure of demand for skills coming from firms. The employment rate for workers with productivity (h, ε) is given by a logit formula comparing that value against those workers' efficacy at home production. The macro elasticity of labor supply with respect to $\omega_h(\varepsilon)$ is given by λ multiplied by the share of those workers in home production.

As in Card et al. (2018), I assume that firms ignore their own contribution to $\omega_h(\varepsilon)$, an approximation that is adequate when firms are small relative to the size of the labor market. Under that assumption, each firm's labor supply for workers of a particular type (h, ε) is given by β.

The number of workers choosing a particular firm, the resulting supply of efficiency units of labor, and total labor costs are increasing in posted earnings and decreasing in rejection cutoffs:

$$n_h(y_{hj}, \underline{\varepsilon}_{hj}) = N_h \int_{\underline{\varepsilon}_{hj}}^{\infty} \frac{\omega_{\varepsilon,h}^{\lambda}}{(\varepsilon z_{0,h})^{\lambda} + \omega_{\varepsilon,h}^{\lambda}} \left(\frac{y_{hj}(\varepsilon)}{\omega_h(\varepsilon)} \right)^{\beta} r_h(\varepsilon) d\varepsilon \tag{4}$$

$$l_h(y_{hj}, \underline{\varepsilon}_{hj}) = N_h \int_{\underline{\varepsilon}_{hj}}^{\infty} \frac{\omega_{\varepsilon,h}^{\lambda}}{(\varepsilon z_{0,h})^{\lambda} + \omega_{\varepsilon,h}^{\lambda}} \left(\frac{y_{hj}(\varepsilon)}{\omega_h(\varepsilon)} \right)^{\beta} \varepsilon r_h(\varepsilon) d\varepsilon \tag{5}$$

$$C_h(y_{hj}, \underline{\varepsilon}_{hj}) = N_h \int_{\underline{\varepsilon}_{hj}}^{\infty} \frac{\omega_{\varepsilon,h}^{\lambda}}{(\varepsilon z_{0,h})^{\lambda} + \omega_{\varepsilon,h}^{\lambda}} \frac{y_{hj}(\varepsilon)^{\beta+1}}{\omega_h(\varepsilon)^{\beta}} r_h(\varepsilon) d\varepsilon \tag{6}$$

5.2.2 Problem of the firm

Firms maximize profit by choosing posted earnings schedules and rejection cutoffs:

$$\pi_j = \max_{\boldsymbol{y}_j, \underline{\boldsymbol{\epsilon}}_j} p_g f\left(\boldsymbol{l}(\boldsymbol{y}_j, \underline{\boldsymbol{\epsilon}}_j), b_g \right) - \sum_{h=1}^{H} C_h(y_{hj}, \underline{\varepsilon}_{hj})$$

The following Lemma shows that this problem has intuitive solutions and that the model admits a representative firm for each good:

Lemma 3. *Firms producing the same good g choose the same earnings schedules and rejection criteria, denoted by y_{hg} and $\underline{\varepsilon}_{hg}$. Optimal earnings schedules have the form $y_{hg}(\varepsilon) = \max\{w_{hg}\varepsilon, \underline{y}\}$. The following first-order conditions define prices per efficiency unit w_{hg} and hiring thresholds:*

$$p_g f_h\left(\boldsymbol{l}(\boldsymbol{w}_g, \underline{\boldsymbol{\epsilon}}_g), b_g \right) \frac{\beta}{\beta+1} = w_{hg} \qquad h = 1, \ldots, H \tag{7}$$

$$p_g f_h\left(\boldsymbol{l}(\boldsymbol{w}_g, \underline{\boldsymbol{\epsilon}}_g), b_g \right) \underline{\varepsilon}_{hg} = \underline{y} \qquad h = 1, \ldots, H \tag{8}$$

Equation 7 defines optimal prices per efficiency unit $w_{h,g}$ as constant markdowns of their marginal revenue products, a common result in monopsony models with a constant elasticity of labor supply to the firm. Equation 8 is the first-order condition on the rejection cutoffs. A lower cutoff brings in additional workers with $\varepsilon = \underline{\varepsilon}_{hj}$, each of which increases revenues by $p_g f_h \underline{\varepsilon}_{hj}$. When firms choose thresholds optimally, that additional revenue equals the minimum wage $\underline{y}$, which is the cost of labor at that margin.

5.2.3 Firm creation and equilibrium

A finite σ engenders positive firm creation for all goods for two reasons. First, with the CES functional form for the consumption aggregator, marginal utilities for each good are unbounded as consumption moves to zero, enabling arbitrarily high equilibrium prices even if entry and marginal costs are large. Second, firms are guaranteed to record positive profits due to the constant markdowns of log wages.[17]

An equilibrium of this model is defined by vectors of aggregate consumption $\{Q_g\}_{g=1}^G$, firm entry $\{J_g\}_{g=1}^G$, choices by representative firms $\{\boldsymbol{w}_g, \underline{\boldsymbol{\epsilon}}_g\}_{g=1}^G$, and prices $\{p_g\}_{g=1}^G$ such that:

1. Markets for firm-produced goods clear:

$$Q_g = \gamma_g^\sigma p_g^{-\sigma} I = J_g f\left(\boldsymbol{l}(\boldsymbol{y}_g, \underline{\boldsymbol{\epsilon}}_g), b_g\right) \quad \forall g \tag{9}$$
$$\text{where } I = \sum_{g=1}^G J_g \left[\pi_g + \sum_{h=1}^H C_h(w_{hg}, \underline{\varepsilon}_{hg})\right] = \sum_{g=1}^G J_g p_g f\left(\boldsymbol{l}(\boldsymbol{y}_g, \underline{\boldsymbol{\epsilon}}_g), b_g\right)$$

2. For all g, firm choices solve the first-order conditions (7) and (8).

3. Firm creation is optimal and feasible:

$$\frac{\pi_g}{F_g} = \frac{\pi_{g'}}{F_{g'}} \; \forall (g, g') \text{ and } \sum_g J_g F_g = 1 \tag{10}$$

Labor market clearing is embedded in the firm-level labor supply curve. Appendix C presents an efficient numerical algorithm to solve for equilibrium given a set of parameters.

[17] I assume that the number of firms in every market is sufficiently large that we can ignore the integer constraint in optimal firm creation. Accordingly, I treat J_g as a continuous variable.

5.3 Firm wage premiums

The following proposition describes how wages vary between firms:

Proposition 3. *1. If $b_g(x) = b(x)/z_g$ for scalars $z_1, \ldots, z_G$ and F_g is the same for all firm-produced goods, then there are no firm wage premiums:*

$$\log y_{hg}(\varepsilon) = \max\left\{\upsilon_h + \log\varepsilon, \log \underline{y}\right\}$$

where $\upsilon_1, \ldots, \upsilon_H$ are scalar functions of parameters.

2. If there is no minimum wage and $b_g(x) = b(x)/z_g$, wages are log additive:

$$\log y_{hg}(\varepsilon) = \upsilon_h + \log\varepsilon + \frac{1}{1+\beta}\log\left(F_g\right)$$

3. If there is no minimum wage and there are firm types g, g' and worker types h' h such that $\ell_{h'g'}/\ell_{hg'} > \ell_{h'g}/\ell_{hg}$ (that is, good g' is relatively more intensive in h'), then:

$$\frac{y_{h'g'}(\varepsilon)}{y_{hg'}(\varepsilon)} > \frac{y_{h'g}(\varepsilon)}{y_{hg}(\varepsilon)}$$

The first part of Proposition 3 shows that wage dispersion for similar workers exists only if there are differences in the shapes of blueprints (such that firms differ in skill intensity) or entry costs. Notably, differences in physical productivity across goods (z_g) or in taste shifters (γ_g) are insufficient to generate wage differentials between firms. This is because if entry costs are the same, differences in physical productivity or tastes lead to additional entry and reduced marginal utility of consumption of the good with greater productivity, up to the point where the marginal revenue product of labor is equalized across firms.

The second part highlights the role of entry costs in generating wage differences across firms. Optimal firm creation implies that all else being equal, firms producing goods with higher entry costs need to operate at a larger scale. To hire more workers, these firms must post higher wages. At equilibrium, prices for those goods will also be higher, such that worker earnings are proportional to the marginal revenue product of labor.

The third part of Proposition 3 shows how skill intensity heterogeneity generates differential wage gaps across firms. Firms using some factors more intensively than others must pay a relative premium for that factor. This model's inability to simultaneously generate log-additive wages and assortative matching echoes some results in the literature on labor market sorting, such as those in Eeckhout and Kircher (2011). However, it is possible for skill-intensive firms to pay all workers a positive wage premium if those firms have high entry costs, such that the model can still include "high-wage firms" as a meaningful concept.

Appendix B.2 adds vertical differentiation of non-wage amenities to the model. Those extra parameters can be used to match firm sizes without affecting the rest of the theory.

5.4 Supply shocks

It is possible that labor supply, labor demand, and the minimum wage evolve in concert, making the economy more productive while leaving wage distribution unchanged (see Proposition 7 in Appendix B.3). However, if there are imbalances in this race, relative prices for goods and labor may change.

I start with supply shocks. To focus on what general equilibrium and firm entry add to the model, the following proposition abstracts from within-firm complementarities by assuming that each good only requires one task (i.e., workers are perfect substitutes within firms):

Proposition 4 (Supply shock and reallocation)**.** *Consider an economy with two comparative advantage types, two goods, full employment ($z_{0,h} = 0$), and no minimum wage. Assume both goods $g = 1,2$ have degenerate blueprints such that each unit of output requires a unit measure of tasks of complexity x_g, with $x_2 > x_1$. Then:*

$$\frac{d\left(\frac{s_{2,1}\log w_{2,1}+s_{2,2}\log w_{2,2}}{s_{1,1}\log w_{1,1}+s_{1,2}\log w_{1,2}}\right)}{d\log\left(L_2/L_1\right)} = \frac{d\log\left(\frac{p_2}{p_1}\right)}{d\log\left(\frac{L_2}{L_1}\right)}\left[(s_{2,2}-s_{1,2})+(\beta+1-\sigma)\left(s_{2,1}s_{2,2}\log\frac{w_{2,2}}{w_{2,1}}-s_{1,1}s_{1,2}\log\frac{w_{1,2}}{w_{1,1}}\right)\right]$$

where $s_{h,g}$ denotes the share of efficiency units of labor of type h employed by firms producing good g, and $\frac{d\log(p_2/p_1)}{d\log(L_2/L_1)} < 0$.

Corollary 2. *For any set of parameters satisfying the conditions of Proposition 4, there exists a number $\bar{\beta}$, such that by changing β to $\beta' > \bar{\beta}$ and F_g to $F_g' = F_g\frac{\beta+1}{\beta'+1}$, the effect of rising supply on the mean log wage gap is negative.*

The effect of increased supply of skills on the aggregate skill wage premium has two components. The first is the direct effect of the supply shock on marginal products of labor via prices. That component is always negative because positive supply shocks reduce p_2/p_1 and $s_{2,2} > s_{1,2}$. The second component is the reallocation of labor across firms paying different wage premiums. If the reallocation effect is positive and sufficiently large, the aggregate skill premium can widen in response to the supply shock.

The strength of the reallocation effect depends on the magnitude of firm wage premiums, initial sorting patterns, and the elasticities β and σ. Those elasticities also determine the direction of net reallocation flows. As mentioned, the supply shock reduces p_2/p_1. Because that price change passes on to wages, individual firms producing $g = 1$ can attract more workers, with elasticity β. However, the reduction in p_2/p_1 also shifts consumption toward the second good, increasing relative firm entry J_2/J_1. If $\sigma > \beta + 1$, the second effect wins, and there is net reallocation to firms producing $g = 2$.

Corollary 2 emphasizes how imperfect competition is essential to the result that positive supply shocks may widen the aggregate skill premium. By moving the parameters close to the competitive limit ($\beta \to \infty$, $F_g \to 0$), supply shocks are guaranteed to compress the skill wage premium. This result exemplifies how Proposition 4 differs fundamentally from the directed technical change channel emphasized by Acemoglu (1998, 2007).

In a more general environment with non-degenerate blueprints, the expression for the change in the aggregate skill wage premium would include additional terms deriving from imperfect substitution within firms. The total impact of supply shocks on the aggregate skill premium may be positive even in these cases, as the quantitative analysis demonstrates.

5.5 Demand shocks

There are three ways to model skill-biased demand shocks in this economy. The first is by changing blueprints in a way that increases the demand for complex tasks. Analogously to the monotone comparative statics used by Costinot and Vogel (2010), this should increase all wage gaps w_{h+1}/w_h in a competitive economy with a single good.

The second form of skill-biased shock is an increase in demand for skill-intensive goods, which may represent improvements in the quality of those goods or trade shocks affecting demand for goods that are more skill intensive.[18]

[18] In the latter interpretation, Proposition 5 is in the same spirit as the classic result of Stolper and Samuelson (1941). At the limit $\sigma \to \infty$, the model is equivalent to a small open economy with prices $p_2/p_1 = \gamma_2/\gamma_1$.

Proposition 5 (Demand for goods and returns to skill)**.** *Consider a competitive version of this economy ($\beta \to \infty$, $F_g = 0$) with full employment ($z_{0,h} = 0$), two goods, and no minimum wage. Assume $b_2(x)/b_1(x)$ is increasing in x (good $g = 2$ is more intensive in high-complexity tasks). Then, an increase in γ_2/γ_1 increases all wage gaps w_{h+1}/w_h.*

Proposition 5 has a more general implication: If other shocks change aggregate consumption patterns in the direction of more or less complex tasks, there may be secondary effects on skill wage premiums. I return to this point in the discussion of general equilibrium effects of minimum wage policies.

The third type of skill-biased demand shock is a reduction in relative entry costs F_2/F_1 when good 2 is more skill intensive. It reallocates labor towards more complex tasks by reducing relative prices p_2/p_1 and increasing relative entry J_2/J_1. As Proposition 3 describes, that shock also reduces the magnitude of firm wage premiums when skill-intensive firms are also high-wage. The net effect on inequality measures is ambiguous.

In the empirical exercise, I allow for regional and time differences in these three dimensions of labor demand.

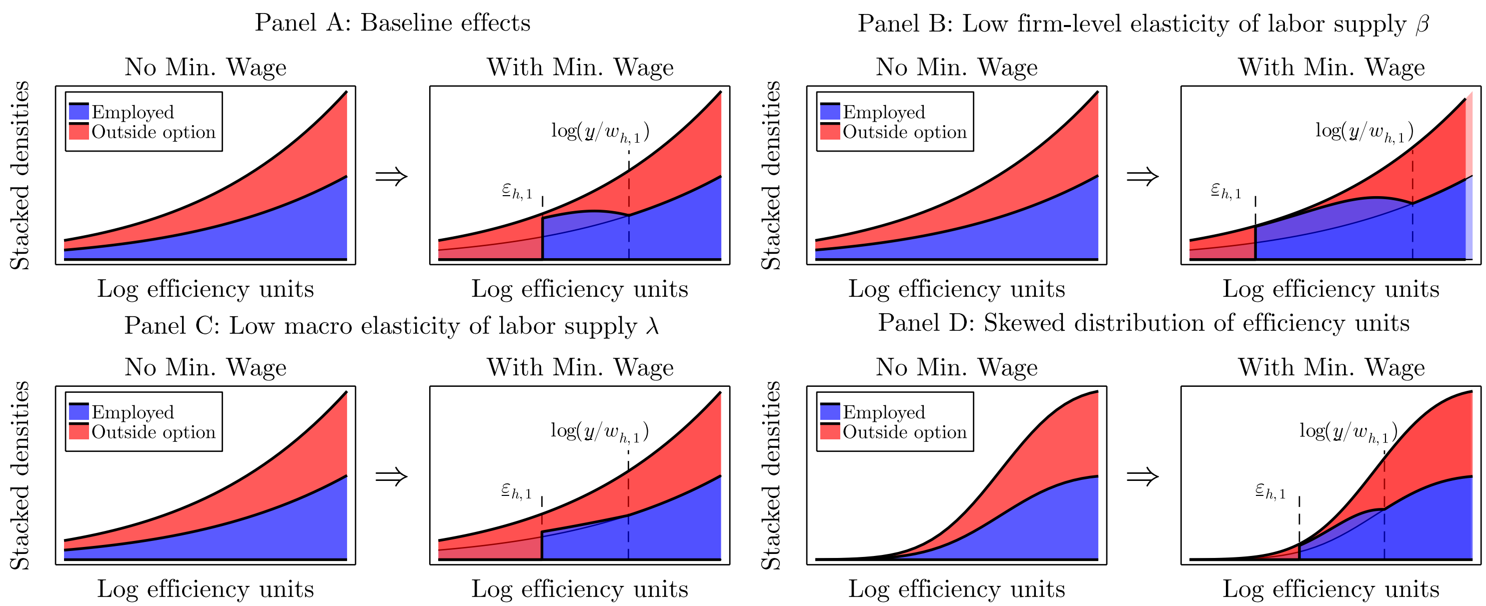
5.6 Minimum wage

In this section, I explain how minimum wage affects the model economy. This discussion serves two purposes. First, it includes some novel insights that may be of value to economists who study minimum wages, including potential pitfalls to avoid in reduced-form empirical studies. Second, it clarifies what channels are accounted for in the simulations presented later in the paper. Appendix B.4 discusses causal pathways not included in this framework and explains why their omission may not be consequential in the Brazilian context.

5.6.1 Channel 1: "monopsony" (mechanical wage increases, disemployment, positive employment effects, and reallocation)

Suppose that, starting from an initial equilibrium, the minimum wage increases to $\underline{y}' > \underline{y}$. I update earnings schedules from $y_{h,g}(\varepsilon)$ to $y'_{h,g}(\varepsilon) = \max\left\{y_{h,g}(\varepsilon), \underline{y}'\right\}$. I also update the minimum hiring thresholds to account for the fact that, keeping marginal products of labor constant, some low-skilled workers become unprofitable under the new minimum. Then, I allow workers to change their employment options based on the new earnings schedules and hiring thresholds. All other equilibrium variables remain unchanged.

Figure 3: Minimum wage effects with a single firm type



Notes: This figure shows the "monopsony" effects of minimum wages on worker employment options in a context with a single firm type. The graph shows the lower tail of the distribution of efficiency units ε for a particular worker group h. Each vertical slice of the graphs shows the stacked densities of employed and non-employed workers for each ε, such that the total height corresponds to the total density $N_h r_h(\varepsilon)$. Workers with $\varepsilon < \underline{\varepsilon}_{h,1}$ lose their formal jobs. Workers with $\varepsilon \in [\underline{\varepsilon}_{h,1}, \log(\underline{y}/w_{h,1})]$ earn exactly the minimum wage. These are the workers for whom formal wages are increasing mechanically and for whom we should see positive employment effects.

Panel B shows that a lower β increases the range of productivity levels where the minimum wage may cause positive employment effects. **Panel C** shows that if λ is small, positive employment effects are also likely to be small. **Panel D** shows that estimates of minimum wage effects depend crucially on assumptions about the shape of the underlying productivity distribution.

Figure 3 illustrates the counterfactual employment choices in a model with a single good. The graphs show the mass of workers by employment choice and worker productivity, providing a close-up view of the left tail of the productivity distribution.

Consider the baseline scenario in Panel A. For workers with $\varepsilon > \log(\underline{y}/w_{h,1})$, employment options remain unchanged, as do their optimal choices. Because workers with $\varepsilon < \underline{\varepsilon}_{hj}$ are no longer employable at formal firms, all of them move to their outside options. Finally, workers with $\varepsilon \in \left[\underline{\varepsilon}_{h,1}, \log(\underline{y}/w_{h,1})\right]$ are the ones receiving a mechanical "wage boost" at formal firms. If they choose to work there, they earn exactly the minimum wage. Thus, the blue mass of workers in that interval corresponds to the minimum wage spike.

Positive employment effects of minimum wage arise from workers in that middle interval. One important takeaway is that, even if the *total* change in employment is non-negative, the minimum wage may still cause disemployment for very low-productivity workers.[19]

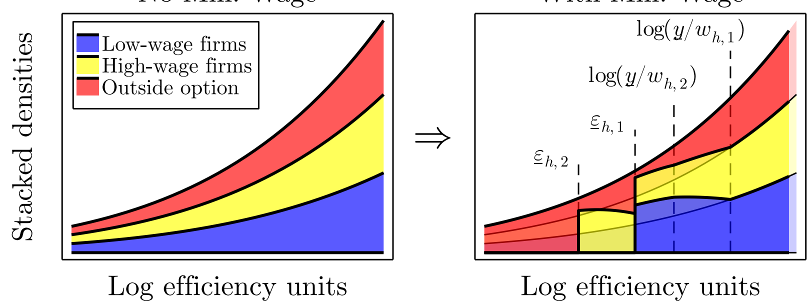
Panels B, C, and D in Figure 3 illustrate how minimum wage effects depend on the firm-level elasticity of labor supply, the aggregate elasticity of labor supply, and the shape of the underlying productivity distribution. In the quantitative section, I estimate the two elasticities and allow for flexible distributions of worker ability within educational groups.

Figure 4 resembles Figure 3 except that it shows a scenario with two goods. The initial equilibrium has workers evenly split between low-wage firms ($g = 1$), high-wage firms ($g = 2$), and home production. The high-wage firms have higher revenue productivity and can afford to hire workers with lower ε after the introduction of the minimum wage. This generates reallocation from low- to high-wage firms for workers with $\varepsilon \in \left[\underline{\varepsilon}_{h,2}, \underline{\varepsilon}_{h,1}\right]$, a pattern that is the model analog of the empirical results in Dustmann et al. (2021).

The model also predicts some reallocation from high- to low-wage firms, especially for workers with $\varepsilon \approx \log(\underline{y}/w_{h,2})$. This is because the minimum wage does not affect their wage at high-wage firms but makes low-wage firms more attractive. This result has implications for empirical studies of minimum wages that compare workers based on their initial wage. Even if there are no strategic wage-posting responses and no general equilibrium effects, workers earning more than the new minimum may still be affected by the minimum wage, precluding them from being a valid control group.

[19] The inability of minimum wages to correct monopsony-induced underemployment for all worker types simultaneously was first noted by Stigler (1946).

Figure 4: Minimum wage effects with two firm types



Notes: This figure shows the impact of minimum wage on worker employment options when there are two firm-produced goods (equivalently, two firm types). The "high-wage firms", $g=2$, have higher revenue productivity and can afford to hire workers with lower ε after the introduction of the minimum wage. This generates reallocation from low- to high-wage firms for workers with $\varepsilon \in \left[\underline{\varepsilon}_{h,2}, \underline{\varepsilon}_{h,1}\right]$. The neighborhood around $\log(\underline{y}/w_{h,2})$ may feature the opposite type of reallocation (from high-wage to low-wage firms).

5.6.2 Channel 2: Wage-posting responses and within-firm returns to skill

To quantify the role of this channel, I calculate a partial equilibrium where prices p_g and firm creation J_g are kept constant following the increase in the minimum wage. Firms can reoptimize earnings schedules $y_{h,g}(\varepsilon)$ and hiring thresholds $\underline{\varepsilon}_{hj}$. Then, I compare the simulated outcomes of this partial equilibrium to the baseline equilibrium and subtract the contribution of the "Monopsony" channel described in the previous subsection.

Why would firms choose different posted earnings following the introduction of a minimum wage? Holding earnings schedules constant, disemployment and reallocation effects imply changes in factor shares within firms. Because the production function is concave, marginal products of labor also change. Then, firms need to adjust w_{hg} to ensure that they are proportional to the marginal revenue products of labor.

The combination of monopsony power, firm heterogeneity, and task-based production generates novel predictions regarding minimum wage effects compared to both competitive task-based models (Teulings, 2000) and monopsonistic models of minimum wage (Engbom and Moser, 2022). Suppose that there are two firms with blueprints that are equally skill-intensive but with $F_2 \gg F_1$. A newly introduced minimum wage may bind for low-h workers at firms producing good $g=1$ but not good $g=2$. This may generate reallocation of low-h labor. Within-firm skill premiums could fall at firms producing $g=1$ and widen at firms producing $g=2$.

Perhaps a more typical scenario is one where low-wage firms are also low-skill. Suppose that good $g = 1$ has a blueprint fully concentrated in tasks of complexity $x = 0$, as described in Proposition 2. Then, internal skill premiums at firms producing that good will not respond to the minimum wage. Reallocation will still widen skill premiums at firms producing $g = 2$. Combining those effects, it is possible that wage changes induced by the minimum wage are ultimately less progressive, especially for middle-skill workers. The quantitative section shows that this channel is responsible for the negative wage effects of minimum wages for workers in the middle of Brazil's productivity distribution.[20]

This theoretical prediction also has implications for empirical minimum wage designs. Some papers compare firms in the same region based on the proportion of their workers that earns below the new value of the minimum wage. The preceding discussion demonstrates that the minimum wage may also affect high-wage firms, albeit in a fundamentally different way. This means that those high wage firms may not constitute an appropriate control group. Note that because this mechanism does not depend on changes in prices and firm entry, arguing that there are no general equilibrium effects is insufficient to validate the "fraction affected" design.

5.6.3 Channel 3: General equilibrium

Finally, I account for minimum wage-induced changes in prices p_g and firm creation J_g. The strength of those equilibrium effects depends crucially on the elasticity of substitution in consumption σ. To make the analysis concrete, consider a scenario with two goods in which skill-intensive firms are also high-wage.

Start with the Leontief case, $\sigma = 0$. Minimum wages reduce profits at low-wage firms by compressing their markdowns. In general equilibrium, falling profits at low-wage firms induce an increase in J_2/J_1. In the Leontief world, $Q_2/Q_1 = (J_2/J_1) \cdot (q_2/q_1)$ is constant, so q_2/q_1 must fall. That change in relative scale can only be achieved by compressing firm wage premiums because minimum-wage-induced reallocation tends to increase q_2/q_1. Consequently, the cost ratio falls, as does the price ratio p_2/p_1.

Now consider the other extreme with perfect substitution: $\sigma \to \infty$. Relative prices are now invariant, $p_2/p_1 = \gamma_2/\gamma_1$, and changes in relative profits induce changes in firm entry. There

[20]This channel may not always cause reductions in real wages for low-wage workers at high-wage firms. As an example, in a scenario where minimum wage causes strong mechanical increases in wages at low-wage firms but not much disemployment, the resulting increase in $\omega_{h,\varepsilon}$ can lead to positive wage effects at high-wage firms.

is more reallocation of labor from low- to high-wage firms because there is no need for offsetting entry with scale responses to keep quantities constant.

Comparing both scenarios, we should expect minimum wages to be less progressive if σ is large. With a low σ, low- and medium-skilled workers benefit from the increase in the relative price for low-skill goods even if the minimum wage does not mechanically increases their wages. An increase in p_1 also attenuates disemployment effects. With a large σ, firm-creation responses increase aggregate demand for complex tasks, benefiting skilled workers.

6 Quantitative exercises

I now apply the framework to the data. The first subsection uses reduced-form regressions to test basic implications of the theory. The second subsection structurally estimates a parametric model of the Brazilian economy. The third subsection contains counterfactual exercises.

6.1 Firm heterogeneity, task assignment, and wage premiums

In this subsection, I test four implications of the model: (i) skill-intensive firms have more demand for complex tasks (Figure 1); (ii) within firms, more skilled workers are assigned to more complex tasks (Lemma 1); (iii) with monopsony power, workers moving to more skill-intensive firms are reallocated to more complex tasks (Lemma 2); and (iv) wage gaps between high- and low-skill firms should be larger for skilled workers (Proposition 3).

To test these predictions, I need proxies for worker skill and task complexity. Skill is measured by years of schooling. Appendix Table D2 reports results for an alternative measure. For task complexity, I use the non-routine analytical task content of Brazilian occupations created by de Sousa (2020). That measure reflects whether O*NET survey respondents believe that their occupation requires mathematical reasoning and was created following the methodology in Deming (2017).[21]

The first two columns in Table 3 test the first two predictions using data for 1997. Column (1) reports a firm-level regression of the establishment's average task complexity on the

[21] The O*NET survey asks workers in the US about their jobs, including skill requirements and the degree of automation in the occupation. Deming (2017) describes how that survey is collected and processed to produce data that describe each occupation as a combination of tasks of varying intensities. de Sousa (2020) links SOC occupation codes with occupation codes in the RAIS data before calculating the task content of occupations using O*NET data and the procedures in Deming (2017).

Table 3: Validation of the task-based production function.

	Non-routine cognitive task content				Log wage
	(1)	(2)	(3)	(4)	(5)
Mean schooling in establishment	0.07921 (0.00049)				
Own schooling		0.06304 (0.00159)			
Mean schooling of coworkers in establishment			0.00663 (0.00077)	0.00343 (0.00086)	
Own × mean schooling of coworkers in estab.					0.00162 (0.00045)
Microregion-time fixed effects			✓	✓	✓
Establishment fixed effects		✓			✓
Sector fixed effects				✓	
Worker fixed effects			✓	✓	✓
r2	0.26216	0.40172	0.84463	0.85033	0.95789
N	93,606	11,551,108	2,673,660	2,673,659	14,996,848

Notes: RAIS data, largest connected set in each of the 151 selected microregions. Columns (1) and (2) use data from 1997. Columns (3)–(5) use two years, 1997 and 1999. In Column 1, the unit of observation is the establishment, and the dependent variable is the establishment average. Columns (2)– (5) are at the worker level. The samples for Columns (3) and (4) only include workers who move between establishments. The dependent variable in columns (1)–(4) is the analytical non-routine task content of the occupation (averaged across workers employed by the establishment in Column (1)). Standard errors (in parenthesis) are robust in Column (1), clustered at the establishment level in Column (2), and two-way clustered at the worker and establishment levels in the others. The standard deviation of the task content variable is approximately one.

average years of schooling of that establishment's employees. Consistent with the theory, I find a positive relationship. Column (2) is a worker-level regression of the task content of the worker's occupation on that worker's schooling, controlling for firm fixed effects. The positive coefficient confirms the prediction for within-firm assignment.

Next, I use worker transitions between establishments to test the third prediction. Specifically, I regress the analytical task content of the worker's occupation on mean schooling of other workers in the same establishment, controlling for worker fixed effects. That regression uses data from 1997 and 1999 and only includes movers. Column (3) demonstrates that the estimate is positive and significant, although the correlation is weaker than in Column (2). Workers moving to firms with more educated colleagues tend to be assigned to more analytical occupations, consistent with differences in optimal assignment across firms in imperfectly competitive environments.

I also investigate whether changes in assignment are driven by workers moving between sectors. Column (4) shows results for a specification similar to Column (3) but with sector fixed effects.[22] I find that the coefficient falls by about half but remains highly significant. This suggests sizable within-sector variation in skill intensity and task content of occupations, consistent with the interpretation that goods in the model might represent differentiated varieties or technologies within industries.

Finally, Column (5) tests the fourth prediction, again using panel data. It reports a regression of log wage on worker fixed effects, firm fixed effects, and the interaction between a worker's years of schooling and the average schooling of coworkers in their workplace. I find a positive, statistically significant estimate, consistent with the theory. In Appendix Table D2, I demonstrate that this result is not a mechanical consequence of the minimum wage.

6.2 Structural estimation

6.2.1 Parameterization

Each microregion-time combination is treated as an isolated economy, indexed by (r,t). For each, the general equilibrium model specifies a mapping from the estimated parameters to simulated endogenous outcomes. The estimation procedure minimizes deviations between the observed endogenous outcomes and their simulated values. In this subsection, I describe the parameterization of the model. In the following subsection, I formalize the data-generating process and discuss identification, estimation, and inference.

Worker types: I set $H = 10$. The comparative advantage functions for these ten groups are fixed.[23] The exogenous number of workers N_h is determined by the observed shares of the adult population in each educational group $\hat{h} \in \{1,2,3\}$ (less than high school, high school, and college or more) according to the following probabilities:

$$\Pr(h=1|\hat{h}) = \Phi\left(\frac{1.5-\mu_{\hat{h}}}{\rho_{\hat{h}}}\right)$$
$$\Pr(h|\hat{h}) = \Phi\left(\frac{h+0.5-\mu_{\hat{h}}}{\rho_{\hat{h}}}\right) - \Phi\left(\frac{h-0.5-\mu_{\hat{h}}}{\rho_{\hat{h}}}\right) \qquad h \in \{2,\dots,9\}$$

[22] There are 560 "CNAE10" sectors in the regression sample. 507 include at least 100 movers.

[23] $e_h(x) = \exp(\alpha_h x)$, with $\alpha_h = -1 + \left(\sum_{h'=1}^{h-1}\frac{1}{h'}\right) / \left(\sum_{h'=1}^{H-1}\frac{1}{h'}\right)$. That formulation implies that the highest type the same productivity in all tasks, while the lowest type has $e_1(x) = \exp(-x)$. The values for intermediate types are such that if task thresholds are equally spaced for a firm g, then ratios of marginal products of labor between neighboring worker types are identical for all types. Although not essential, this property helps make skill premiums between groups reasonably uniform.

$$\Pr(h=10|\hat{h}) = 1-\Phi\left(\frac{9.5-\mu_{\hat{h}}}{\rho_{\hat{h}}}\right)$$

where Φ is the cumulative distribution function of a standard Normal. Those probabilities resemble an "ordered Probit" model with thresholds $1.5, 2.5, \ldots, 9.5$. I normalize $\mu_{\hat{h}=1} = 3$ and $\mu_{\hat{h}=3} = 8$. That is, the median worker with less than high school corresponds to $h = 3$, and the median college worker has $h = 8$. The comparative advantage of the median high-school worker is given by the estimated parameter $\mu_{\hat{h}=2}$. The model allows for dispersion in comparative advantage within an educational group, depending on the magnitude of $\rho_{\hat{h}}$.

The distribution of efficiency units ε within latent group h is a mean-zero Skew Normal:

$$\begin{aligned} r_h(\varepsilon) &= \frac{2}{S_h}\phi\left(\tilde{\varepsilon}_h\right)\Phi\left(\chi\tilde{\varepsilon}_h\right) \\ \tilde{\varepsilon}_h &= \frac{\varepsilon}{S_h} - \chi\sqrt{\frac{2}{\pi(1+\chi^2)}} \\ S_h &= \sum_{\hat{h}=1}^{3} \Pr(\hat{h}|h)\hat{S}_{\hat{h}} \end{aligned}$$

where ϕ is the density of a standard Normal. The skewness is determined by χ. This degree of freedom helps the model fit the left tail of the wage distribution, which is essential for the effects of minimum wages. The parameters $\hat{S}_{\hat{h}}$ determine the dispersion of the efficiency units associated with each educational group $\hat{h}$.

Worker preferences and outside options

The preference parameters β and λ, which determine the firm-level and the macro elasticities of labor supply, are common across regions and periods. The value of outside options, which helps determine formal employment rates, is determined by:

$$\begin{aligned} z_{0,h,r,t} &= \sum_{\hat{h}=1}^{3} \Pr(\hat{h}|h)\hat{z}_{0,\hat{h},r,t} \\ \text{where } \hat{z}_{0,\hat{h},r,t} &= \hat{z}^{HT}_{0,\hat{h},t} \cdot \hat{z}^{RH}_{0,r,\hat{h}} \cdot \hat{z}^{RT}_{0,r,t}{}^{\left(1+\Lambda\mathbf{1}\{\hat{h}=3\}\right)} \\ \text{and normalizing: } \hat{z}^{HT}_{0,\hat{h},t} &= 1 \quad \text{if } t = 1998 \text{ or } \hat{h} = 2 \\ \text{and } \hat{z}^{RH}_{0,r,\hat{h}} &= 1 \quad \text{if } \hat{h} = 2 \end{aligned}$$

The easiest way to understand that formulation is to focus on $\hat{z}_{0,\hat{h},r,t}$, the average value for educational group $\hat{h}$. It is determined by flexible education-time (HT), region-education (RH), and region-time (RT) components, which absorb confounders determining formal employment such as regional differences in the enforcement of labor regulation. The region-time shocks are allowed to have stronger or weaker effects on college workers ($\hat{h}=3$) depending on the Λ parameter.

Once the outside options for the three educational groups are known, they can be transformed into outside options for latent worker groups, $z_{0,h,r,t}$, using the conditional probabilities $\Pr(\hat{h}|h)$ (similarly to the approach for the dispersion of efficiency units).

Labor demand: There are $G=2$ goods in each region. Blueprints follow the Exponential-gamma parameterization discussed in Subsection 4.4. Good $g=1$ is assumed to have blueprint complexity $\theta_{g=1,r,t}=0$, meaning that one unit of that good requires a unit mass of tasks of complexity $x=0$. Along with exponential efficiency functions, this means workers of all types are perfect substitutes and equally productive in the production of that good.

There are four demand-side parameters that vary at the region-time level. The first is the productivity parameter $z_{r,t}$, which is unrestricted. The others are blueprint complexities $\theta_{g=2,r,t}$, relative entry costs $F_{2,r,t}/F_{1,r,t}$, and relative consumer preference $\gamma_{2,r,t}/\gamma_{1,r,t}$.[24] They are determined by region-time-specific covariates as follows:

$$\begin{aligned} D^d_{r,t} =& \delta^{d,t}_0 + \delta^{d,t}_1 ShareHighSchool_{r,1998} + \delta^{d,t}_2 ShareCollege_{r,1998} \\ &+ \delta^{d,t}_3 ShareAgriculture_{r,1998} + \delta^{d,t}_4 ShareManufacturing_{r,1998} \\ &+ \delta^{d,t}_5 \left(log(min.wage) - meanLogWage\right)_{r,t} \end{aligned}$$

for $d \in \{\theta, F, \gamma\}$, where:

$$D^\theta_{r,t} = \log\theta_{2,r,t} \qquad D^F_{r,t} = \log\left(\frac{F_{2,r,t}}{F_{1,r,t}}\right) \qquad D^\gamma_{r,t} = \log\left(\frac{\gamma_{2,r,t}}{1-\gamma_{2,r,t}}\right)$$

There are a total of 36 $\delta^{d,t}_i$ parameters, six for each demand shock in each period.

The demand parameters are partly determined by initial educational shares. This formulation allows for labor demand patterns to be systematically correlated with *initial* educational levels. Furthermore, it allows *changes* in labor demand to correlate with initial education.

[24] Entry costs only matter in relative terms because I do not target average firm sizes. For computational purposes, I set $F_{2,r,t}=1$. Consumer preferences also only matter in relative terms, given that outside option parameters are fully flexible. As such, I normalize $\gamma_{1,r,t}+\gamma_{2,r,t}=1$.

Thus, educational shares are not assumed to be orthogonal to labor demand. The variation that is used to identify the effects of supply is the *change* in educational shares relative to regions that began with about the same educational level.

Initial shares of the workforce engaged in agriculture and manufacturing are used as additional predictors of labor demand shocks. This approach is analogous to the "shift-share designs" used to evaluate the consequences of labor demand shocks on employment and wages, where the "shift" component is effectively a dummy for $t = 2012$.

Finally, biased labor demand parameters are also allowed to correlate with how binding the minimum wage is in each period. It may be unclear why the gap between the minimum wage and the mean log wage is used as a covariate in the structural model, given that the mean log wage is an endogenous outcome. To understand the usefulness of this formulation, note that conditioning on local supply and demand factors, the mean log wage is a function of the region-time-specific productivity parameter $z_{r,t}$. Thus, the bindingness metric should be interpreted as a proxy for local productivity. Including it in those equations allows for correlations between regional productivity shifters and other demand-side parameters.[25]

Summing up, there are 51 estimated parameters common across regions: eight defining latent worker types and their supply; two outside option shifters at the education-time level, along with one determining the relative relevance of regional outside options to college workers; 36 determinants of local demand; blueprint shape k; and the elasticities σ, β, and λ. In addition, there are six region-specific parameters: four formal employment rate shifters and two time-specific TFPs.

6.2.2 The data-generating process and identification

The data-generating process is:

$$\boldsymbol{Y}_r = a(\boldsymbol{Z}_r, \boldsymbol{\theta}_0^G, \boldsymbol{\theta}_r^R) + \boldsymbol{u}_r \qquad r \in \{1, \ldots, R\}$$

where $\boldsymbol{Y}_r$ is a vector of 26 endogenous outcomes for both periods (1998 and 2012). It includes inequality measures within and between groups, variance components from the

[25]One may wonder why the expressions for these demand parameters are not specified in terms of $z_{r,t}$ instead of the effective minimum wage. As the next section demonstrates, the estimation procedure requires "inverting" region-specific parameters, including $z_{r,t}$, from the observed moments. If the expressions for the demand parameters were written in terms of $z_{r,t}$, that inversion procedure would be impossible. Writing the expression in terms of minimum wage bindingness enables the model to capture the key endogeneity concern—correlation between TFP and other demand parameters—while keeping the model tractable.

AKM decomposition, formal employment rates, and minimum wage bindingness measures. The full list corresponds to the non-italicized moments in Table 5. The vector $\boldsymbol{Z}_r$ includes all region-specific covariates. The 51 general parameters are represented by the $\boldsymbol{\theta}_0^G$ vector (where the subscript denotes the true value). Finally, $\boldsymbol{\theta}_r^R$ represents the six region-specific parameters. The function $a(\cdot)$ simulates the endogenous outcomes using the model parameters implied by $(\boldsymbol{Z}_r, \boldsymbol{\theta}_0^G, \boldsymbol{\theta}_r^R)$. The residuals $\boldsymbol{u}_r$ combine model misspecification and sampling error in the endogenous variables.[26]

Let $PB(\boldsymbol{Y})$ be a function that selects the following six moments from $\boldsymbol{Y}$: formal employment rates for each of the educational groups in $t = 1998$, the formal employment rate for high school workers in $t = 2012$, and minimum wage bindingness in both years (defined as log minimum wage minus mean log wage). These endogenous outcomes are used to "invert" the region-specific parameters given a guess of the other parameters, as formalized in the following identification assumptions:

Assumption 1 (Exogeneity)**.** $E[\boldsymbol{u}_r | Z_r, \boldsymbol{\theta}_r^R] = \mathbf{0}_{26\times1}$.

Assumption 2 (Independence between microregions)**.** *If $r \neq r'$, then $E[\boldsymbol{u}_r \boldsymbol{u}_{r'}'] = \mathbf{0}_{26\times26}$.*

Assumption 3 (Correct specification of employment and bindingness)**.** $PB(\boldsymbol{u}_r) = \mathbf{0}_{6\times1}\ \forall r$.

Assumption 4 (Invertibility of outside options and TFP)**.** *For all r and all allowable $\boldsymbol{\theta}^G$, there is a function $\hat{\boldsymbol{\theta}}^R(\cdot | \boldsymbol{Z}_r, \boldsymbol{\theta}^G)$ such that: $\boldsymbol{Y} = a(\boldsymbol{Z}_r, \boldsymbol{\theta}^G, \boldsymbol{\theta}^R) \Leftrightarrow \boldsymbol{\theta}^R = \hat{\boldsymbol{\theta}}^R(PB(\boldsymbol{Y}) | \boldsymbol{Z}_r, \boldsymbol{\theta}^G)$.*

Assumption 5 (Rank condition)**.** *Define:*

$$\tilde{a}\left([\boldsymbol{Z}_r', PB(\mathbf{Y}_r)']', \boldsymbol{\theta}^G\right) = a\left(\boldsymbol{Z}_r, \boldsymbol{\theta}^G, \hat{\boldsymbol{\theta}}_r^R\left(PB(\mathbf{Y}_r) | \boldsymbol{Z}_r, \boldsymbol{\theta}^G\right)\right)$$

Denote the 51×1 gradient of the o-eth endogenous outcome of the $\tilde{a}(\cdot)$ function, with respect to $\boldsymbol{\theta}^G$, in region r, by $J_{r,o}(\boldsymbol{\theta}^G)$. Then, the following matrix exists and is nonsingular:

$$\boldsymbol{A}_0 = \underset{R\to\infty}{plim}\ \frac{1}{R}\sum_{r=1}^{R}\sum_{o=1}^{26} J_{r,o}(\boldsymbol{\theta}_0^G) J_{r,o}(\boldsymbol{\theta}_0^G)'$$

[26] One source of misspecification is that the sample used to calculate the AKM decomposition moments differs from that used for the other moments. It includes more years and restricts attention to the leave-one-out connected set, meaning that it selects for larger firms. Table 1 shows that the total variances of log wages are similar—but not exactly the same—between samples. Engbom and Moser (2022) addresses this concern by using only the connected set to calculate all statistics, with the cost that all of the model's outcomes become subject to potential bias associated with endogenous selection into the sample. The best solution would be to formally model selection into the leave-one-out set, but that would add significant complexity to the paper and is thus left to future work.

Assumption 6 (Limited dispersion of structural residuals)**.** *The following matrix exists and is positive definite:*

$$\boldsymbol{B}_0 = \underset{R\to\infty}{plim}\,\frac{1}{R}\sum_{r=1}^{R}\sum_{o=1}^{26}\sum_{o'=1}^{26} J_{r,o}(\boldsymbol{\theta}_0^G)J_{r,o'}(\boldsymbol{\theta}_0^G)' u_{r,o}u_{r,o'}$$

These assumptions allow for the identification of model parameters:

Proposition 6 (Identification, estimation, and inference)**.** *Under Assumptions 1 through 6, the following nonlinear least squares estimator*

$$\hat{\boldsymbol{\theta}}^G = \arg\min_{\boldsymbol{\theta}^G}\sum_{r=1}^{R}\left[\boldsymbol{Y}_r - \tilde{a}\left([\boldsymbol{Z}_r', PB(\boldsymbol{y}_r)']', \boldsymbol{\theta}^G\right)\right]'\left[\boldsymbol{Y}_r - \tilde{a}\left([\boldsymbol{Z}_r', PB(\boldsymbol{y}_r)']', \boldsymbol{\theta}^G\right)\right]$$

has the following asymptotic distribution:

$$\sqrt{R}(\hat{\boldsymbol{\theta}}^G - \boldsymbol{\theta}_0^G) \xrightarrow{d} \mathcal{N}\left(\boldsymbol{0}, \boldsymbol{A}_0^{-1}\boldsymbol{B}_0\boldsymbol{A}_0^{-1}\right)$$

Appendix D.4 contains a thorough discussion of identification. First, it demonstrates how the invertibility assumption allows for addressing unobserved heterogeneity at the regional level without causing incidental parameter bias. Next, it provides an intuitive description of how each parameter is identified. It also proposes a parallel between my estimator and a nonlinear instrumental variables design, with standard errors clustered at the region level. Finally, it discusses the identification assumptions in the Brazilian context and considers threats such as regional differences in schooling quality.

The empirical model is over-identified. For an example of how the theory may constrain the quality of fit, consider the minimum wage spike (measured as the share of workers earning up to log minimum wage plus five log points). As Figure 3 illustrates, the spike depends on the elasticities β and λ, along with the shape of the latent productivity distribution (in the empirical model, the skewness parameter χ). But these parameters also matter for several other moments. Both elasticities are crucial for formal employment rates, with β also determining how firm premiums vary by skill (generating implications for wage inequality moments). And the skewness parameter is essential for fitting the share of workers within 30 log points of the minimum wage. Thus, there is no free parameter that can be used to nail the spike. If the model can match its size reasonably well—not only on average but also with respect to differences over time and between regions—then one could argue that the corresponding economic mechanisms are reasonable.

This is not the only important constraint imposed by the parameterization. It is possible, for example, that the formulation of labor demand with only two firm types is too simple to explain regional and time differences in the variance of establishment effects and sorting. In the following, I show that, despite those concerns, the model fits the data rather well.[27]

6.2.3 Estimated parameters

I estimate the model using the Levenberg-Marquardt method with region and equation weights. Region weights are identical to those used in Section 3: total formal employment in the region (adding up both years). Equations were weighted by the inverse mean squared error from the "Simple" regressions described in Appendix D.5.5. In essence, the procedure downweights moments that have more residual variation after eliminating the linear contributions of time effects, educational composition, and minimum wage bindingness.

Estimation is computationally costly because, for each region, one must invert the regional parameters based on the subset of endogenous variables, find the equilibrium, and then simulate all moments. Each optimization step requires performing that procedure 15,704 times: 151 regions $\times$ 2 time periods $\times$ (1 base value + 51 Jacobian columns). Furthermore, because the loss function is not globally concave, several starting points must be used. Appendix D.5 details the implementation, describing, for example, how the inversion and equilibrium finding procedures can be performed simultaneously.

Table 4 shows a subset of the estimated parameters. The others—labor demand determinants $\delta_i^{d,t}$—appear in Appendix Table D3. Before interpreting the results, I note that two parameters were estimated at the boundary of the parametric space. The first is the dispersion in comparative advantage for workers with less than secondary education, $\hat{S}_{\hat{h}=1} = 0$. The second is the elasticity of substitution between goods, $\sigma \to \infty$. The asymptotic formulas are not valid for parameters at the boundary, so Table 4 does not report standard errors for them.

In Appendix D.5.3, I discuss the estimation of σ in detail because the parameter is important for comparative statics. I show that the large estimated value of σ is not driven by numerical issues nor sensitive to the choice of starting points. I also show that a lower σ decreases quality of fit in ways that are consistent with the discussion of identification in Appendix D.4.

[27] I used two goods to keep the model as simple as possible. There is no technical impediment to using a larger number of goods. The estimator proposed by Bonhomme, Lamadon and Manresa (2019) may be helpful in higher-dimensional applications.

Table 4: Selected parameter estimates

Parameter	Estimate	Std. Error
Panel A: Worker types		
$\mu_{\hat{h}=2}$ (modal comp. adv. type, secondary)	2.93	(0.09)
$\hat{S}_{\hat{h}=1}$ (dispersion in comp. adv., less than secondary)	0	-
$\rho_{\hat{h}=2}$ (, secondary)	2.91	(0.06)
$\rho_{\hat{h}=3}$ (, tertiary)	4.74	(0.25)
$\hat{S}_{\hat{h}=1}$ (dispersion in abs. adv., less than secondary)	0.88	(0.01)
$\hat{S}_{\hat{h}=2}$ (, secondary)	0.39	(0.01)
$\hat{S}_{\hat{h}=3}$ (, tertiary)	0.52	(0.08)
χ (skewness of abs. adv. distribution)	-1.39	(0.12)
Panel B: Worker preferences		
β (firm-level elast. labor supply)	10.20	(0.67)
λ (aggregate labor supply parameter)	1.78	(0.14)
$\log \hat{z}^{HT}_{0,\hat{h}=1,t=2}$ (outside option shock, less than secondary)	0.01	(0.01)
$\log \hat{z}^{HT}_{0,\hat{h}=3,t=2}$ (outside option shock, tertiary)	-1.26	(0.63)
Λ (rel. effect of regional part. shocks on tertiary)	0.58	(0.15)
Panel B: Labor demand		
σ (elast. of substition between goods)	∞	-
k (blueprint shape)	1.83	(0.24)

Notes: Standard errors are cluster-robust at the region level. They are calculated using the asymptotic formula in Proposition 6, using sample analogs for the populational matrices A_0 and B_0.

All simulations based on the estimated model are calculated with $\sigma = 100$.

The high level of σ opens space for significant reallocation effects in the long run. To assess whether the magnitudes are plausible, I calculate shares of employed workers at firms producing good $g = 2$ in each region and period, based on the estimated model parameters. The mean change in that share is -0.076, with a standard deviation of 0.075. The largest positive change is from 0.274 to 0.346, while the largest reduction is from 0.752 to 0.384. That means that the production possibilities frontier implied by the model is "concave enough" to prevent unrealistic reallocation responses and corner solutions, despite the large σ and the fact that the shocks affecting the Brazilian economy are substantial.

Moving to the elasticities of labor supply, I find a large value for β, which implies that wages are set to 91% of marginal products of labor for workers earning more than the minimum wage spike. That value is higher than recent estimates for the US, but not dramatically so.[28] The estimated λ implies aggregate labor supply elasticities to the formal sector of

[28] Lamadon, Mogstad and Setzler (2022) estimate firm-level elasticities of labor supply between 6.02 and 6.52, corresponding to markdowns around 86%. Berger, Herkenhoff and Mongey (2022*a*) find average firm-

Table 5: Quality of fit and comparison to benchmark predictive models

	Data		Model		R2	Benckmark R2	
	1998	2012	1998	2012	Model	Simple	Large
Moments	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Wage inequality measures							
Secondary / less than secondary	0.498	0.168	0.486	0.15	0.77	0.78	0.812
Tertiary / secondary	0.965	1.038	0.995	0.932	0.131	0.167	0.406
Within less than secondary	0.41	0.241	0.387	0.225	0.575	0.706	0.791
Within secondary	0.684	0.355	0.647	0.335	0.831	0.761	0.86
Within tertiary	0.702	0.624	0.69	0.644	0.051	0.254	0.378
Total variance of log wages	*0.715*	*0.544*	*0.722*	*0.504*	*0.749*		
Two-way fixed effects decomposition							
Variance establishment effects	0.116	0.056	0.117	0.057	0.652	0.619	0.66
Covariance worker, estab. effects	0.049	0.048	0.058	0.048	0.421	0.408	0.55
Variance worker effects	*0.419*	*0.384*	*0.417*	*0.301*	*0.293*		
Correlation worker, estab. effects	*0.224*	*0.315*	*0.256*	*0.361*	*0.196*		
Formal employment rates							
Less than secondary	0.266	0.337	0.266	0.336	0.951	0.956	0.979
Secondary	0.435	0.508	0.435	0.508	1.0	1.0	1.0
Tertiary	0.539	0.629	0.539	0.631	0.878	0.93	0.95
Minimum wage bindingness							
Log min. wage - mean log wage	-1.418	-0.922	-1.418	-0.922	1.0	1.0	1.0
Share < log min. wage + 0.05	0.031	0.053	0.03	0.074	0.696	0.575	0.784
Share < log min. wage + 0.30	0.086	0.212	0.099	0.218	0.892	0.738	0.904

Notes: Moments targeted by the estimation procedure appear as plain text. Untargeted moments are *italicized*. Columns (1) through (4) report national averages of the corresponding moments for each year, calculated using region weights based on total formal employment. Column (5) reports the usual R2 metric $r_e^2 = 1 - \left[\sum_{t=1}^{2}\sum_{r=1}^{151} s_r \left(Y_{e,r,t} - \hat{Y}_{e,r,t}\right)^2\right] / \left[\sum_{t=1}^{2}\sum_{r=1}^{151} s_r \left(Y_{e,r,t} - \bar{Y}_e\right)^2\right]$, where e indexes the specific target moment, $\hat{Y}_{e,r,t}$ is the model prediction, and $\bar{Y}_e$ is the sample average using the region weights s_r. Columns (6) and (7) report analogous R2 metrics for benchmark OLS models for comparison purposes (see Appendix D.5.5).

around 0.7 for college workers. These values are in the upper range of steady-state elasticities inferred from microdata in the US but are significantly below the values between 1 and 2 that are typically used in macroeconomic models (Keane and Rogerson, 2012). Elasticities are larger for less skilled workers, reaching 1.3 for those with less than high school in 1998. This difference aligns well with informality being an important outside option for those workers.[29]

6.2.4 Quality of fit

Columns (1)—(4) in Table 5 show that the model closely tracks averages in the data, successfully capturing the overall decline in inequality (especially within groups) and the increase in sorting. The most significant deviation is in the mean return to college (tertiary/secondary), which increases in the data but falls in the estimated model. That moment has the lowest estimation weight. Although the model fails to capture the average increase in the college premium, there are 19 regions in the estimated model where the college premium rises, compared to 47 such regions in the data.

A more comprehensive measure of fit is the R2 statistic for each individual moment, reported in Column (5). The statistics are all positive, even for the college premium. But it is difficult to make sense of that metric without context. A low R2 may come from either a failure of the model to fit the data or a lack of sufficient explanatory power in the covariates used by the model. To distinguish between these two possibilities, I estimate benchmark predictive models based on Ordinary Least Squares (OLS) regressions. The "Simple" model is constructed to have the same number of parameters as the structural model. It includes the minimum wage bindingness measure, educational shares for secondary and tertiary, and time dummies as regressors. The "Large" model includes several other variables, such as initial sectoral shares and a quadratic component for minimum wage bindingness. It features a total of 112 parameters, more than twice as many as in the structural model. Those models are guaranteed to match time-specific averages for all moments. See Appendix D.5.5 for details.

My model fits the data approximately as well as the Simple OLS benchmark. It is worse for inequality measures and participation rates among college workers but better for AKM moments and bindingness measures. Although the Large OLS model has a better R2 for all moments, for many of them, the difference is not substantial.

To further validate the model, I verify the quality of fit for outcomes not directly targeted by the estimation procedure. Table 5 shows that the model has predictive power for the overall variance of log wages, the variance of worker effects, and the correlation between worker and establishment effects. Appendix D.5.6 shows a series of additional measures of fit, including histograms of log wages and measures of minimum wage bindingness by educational group.

level markdowns of 78% or 89%, depending on whether the average is weighted by payroll or not.

[29] Dix-Carneiro and Kovak (2017) has found evidence of significant formal-informal transitions in Brazilian microregions more affected by trade liberalization.

Table 6: Effects of supply, demand, minimum wage, and their interactions

	Base	All	Individual effects			Interactions			
	value	changes	S	D	M	S+D	S+M	D+M	Triple
Outcome	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Mean log real wage	1.42	0.15	0.25	-0.06	0.17	-0.13	-0.04	-0.05	0.02
Variance of log wages	0.72	-0.22	0.04	-0.20	-0.14	0.02	0.01	0.05	0.00
Corr. worker, estab. effects	0.26	0.10	0.08	0.08	-0.16	0.04	0.03	0.04	-0.01

Notes: Each row shows within-region effects averages across all 151 regions using total formal employment summed over 1998 and 2012 as weights. "S" is for supply (the rise in educational achievement of the adult population), "D" is for the combined changes in demand-side parameters, and "M" is for the real minimum wage increase of 65 log points. See the text for an explanation of each column.

It also shows that good quality of fit is not an artifact of using region weights. These exercises reinforce the conclusion that the model provides a good approximation for Brazilian labor markets.

6.3 Counterfactual exercises

This subsection presents the counterfactual analyses that I use to understand how supply, demand, and minimum wage shocks affected Brazilian labor markets between 1998 and 2012. The supply shock is the change in the educational composition of the adult population, and the minimum wage shock is an increase of 65 log points in the minimum wage relative to the price index in all regions. The demand shock combines all other time-varying factors in the model: TFP, task requirements, relative entry costs, relative consumer taste, and outside option parameters. In Appendix D.6.2, I discuss why outside options are included in the demand shock and separately show comparative statics for different demand parameters.

6.3.1 Supply, demand, minimum wage, and their interactions

Table 6 shows the impact of those shocks on mean log wages, the variance of log wages, and sorting measured using the AKM decomposition. Columns (1) and (2) show base levels and total changes for each outcome, averaged over regions. Columns (3), (4), and (5) explore counterfactuals where only one factor changes. Columns (6), (7), and (8) show pairwise interactions. Specifically, I simulate the combined effect of two shocks and then subtract the corresponding individual effects. Finally, Column (9) shows the triple interaction, that is, the difference between Column (2) and the sum of Columns (3)–(8).

The combination of demand shock and the minimum wage had strong inequality-reducing

Table 7: Decomposition of the impact of supply shocks

Outcome	Total supply effect (1)	Compositional effect (2)	Firm choices (3)	Entry and prices (4)
Mean log real wage	0.25	0.17	0.03	0.05
Variance of log wage	0.04	0.10	-0.05	-0.01
Corr. worker, estab. effects	0.08	0.14	-0.06	0.00

Notes: Column (1) repeats Column (3) from Table 6. Column (2) measures changes induced by a re-weighting of worker types, keeping log wages and employment shares unchanged. Column (3) measures changes from that scenario to a "partial equilibrium" where firms and workers allowed to reoptimize while but firm entry and log prices are kept constant. Column (4) corresponds to the change from the partial equilibrium to the new general equilibrium, accounting for entry and price responses.

effects in Brazil, while the supply shock had a weak—but positive—effect. The model also reveals significant interactions that would not be detectable without a unified approach. For example, if minimum wages were the only change happening between 1998 to 2012, the variance of log wages would have fallen by 0.14. However, another meaningful counterfactual involves considering what would have happened if supply and demand changed, but the minimum wage stayed at the 1998 level. In that case, inequality in 2012 would be higher, but only by 0.08.

Interactions are even more important for explaining changes in sorting. Both supply and demand shocks help explain why assortative matching increases (on average) within Brazilian labor markets. The interaction boosts their combined individual effects by 25%. Minimum wages decrease the correlation between worker and firm effects by 0.16 if acting in isolation but only by 0.10 when also including the effects of supply and demand shocks.

Appendix D.6.1 shows similar decompositions for a broader set of outcomes.

6.3.2 Supply effects: composition, returns to skill, or reallocation?

Supply shocks may affect wage distribution via a purely compositional effect. With more skilled workers, average wages should increase. The variance in log wages should also increase because there is more within-group productivity dispersion among more educated adults. The compositional change may also have a statistical effect on measured sorting.

The model also specifies two types of endogenous responses to the supply shock. The first derives from firms reoptimizing their wage-posting decisions. To isolate this effect, I calculate a partial equilibrium in which firm creation and prices for goods remain at their initial levels. This channel is powered by the concavity of the task-based production function and represents the central component of competitive models that focus on between-group inequality.

The other endogenous response derives from the changes in firm entry and prices emphasized in Proposition 4. Given that the estimated σ is large, we should expect net reallocation of labor toward high-wage, skill-intensive firms.

Table 7 reports the magnitudes of each of those channels. Although I find that the compositional effect is the most important, equilibrium effects cannot be ignored. Wage posting responses cut the inequality and sorting effects associated with compositonal changes by half. Meanwhile, entry responses boost the effect of supply on the mean log real wage by 25%.

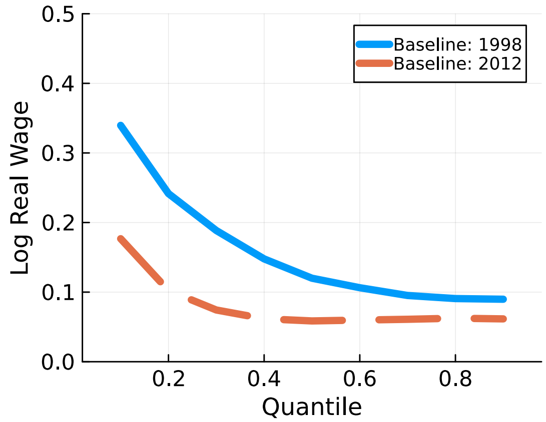
In the discussion of Proposition 4, I argued that positive supply shocks may widen the aggregate skill wage premium. To verify that possibility in the Brazilian context, I simulate the effects of small increases in the share of workers with complete college, with a corresponding reduction in the share with less than high school. If the baseline models are the 1998 equilibria, the mean log wage gap between those educational groups falls in all regions. However, if the baseline models are the 2012 equilibria, mean log wage gaps increase in 133 out of 151 regions. That exercise reinforces the importance of accounting for firm wage premiums, sorting, and endogenous firm entry when calculating the long-run effects of educational shocks. It also illustrates how reallocation effects depend on not only structural elasticities but also the characteristics of the initial equilibrium, such as the level of segregation by skill.

6.3.3 The impacts of the rising minimum wage

The effects of minimum wages on wage distribution are often measured by its spillovers—that is, causal effects on wage distribution quantiles. Figure 5 shows average within-region spillovers implied by the estimated model. Real log wages increase for all quantiles, especially the lowest. The difference between the two curves illustrates the significant interactions I have described.

Unfortunately, spillover graphs are not informative about causal effects for any particular worker because the same quantile of the wage distribution might correspond to different workers before and after the introduction of the minimum wage. If the minimum wage

Figure 5: Minimum wage spillovers



Notes: This figure shows minimum wage impacts on quantiles of within-region log wage distributions, averaged over all regions. The blue line corresponds to a 65 log point increase in the minimum wage starting from the 1998 equilibria. The orange dashed line corresponds to a similar-sized reduction starting from the 2012 equilibria.

causes disemployment of low-skilled workers, spillovers may reflect sample selection, as explained by Lee (1999). And Figure 3 shows that, even when net employment effects are zero, there may still be compositional changes that would be reflected in spillover graphs.

To address these concerns, Table 8 reports minimum wage impacts for stable groups of adults, starting at the 1998 equilibria. The workers are grouped at the national level based on their productivity (relative to the minimum wage) if they were employed at skill-intensive firms based on their region. Column (3) shows the mean wage for the subset of adults employed at the initial equilibrium. Columns (4), (5), and (6) show how the mean wage for employed workers changes within that group of adults. Each column isolates one of the channels described in Subsection 5.6: "monopsony" (disemployment, positive employment effects, mechanical wage increases, and cross-firm reallocation), "returns to skill" (the partial equilibrium analysis with firm creation and prices fixed at their initial levels), and "general equilibrium" (corresponding firm creation and price responses).

The numbers in Table 8 paint a very different picture compared to the spillovers implied by the blue line in Figure 5. Although workers in the bottom three groups see increases in the mean real wages (conditional on being employed), the effects are negative for groups four through eight. Those negative effects stem from the returns to skill channel. As low-skilled workers reallocate from low- to high-wage firms, the marginal returns of middle-skill workers decrease. Highly skilled workers in the top group benefit from the complementarities

Table 8: Wage and employment effects of the minimum wage

Prod. decile	Pop. share	Base wage	Mean wage changes: Monops.	Ret. sk.	Gen. eq.	Base emp.	Emp. elasticities w.r.t.: Min.	Mean	·, monops.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	0.15	1.33	0.75	-0.04	-0.01	0.21	-0.33	-1.21	-0.96
2	0.11	1.76	0.92	-0.09	-0.02	0.28	-0.12	-0.48	-0.34
3	0.11	2.34	0.24	-0.06	0.01	0.30	-0.02	-0.55	-0.20
4	0.10	2.96	0.01	-0.07	0.03	0.31	-0.01		
5	0.10	3.68	-0.00	-0.08	0.03	0.32	-0.01		
6	0.10	4.61	-0.00	-0.08	0.03	0.33	-0.01		
7	0.09	5.91	-0.00	-0.08	0.03	0.35	-0.00		
8	0.09	7.94	0.00	-0.07	0.04	0.37	-0.00		
9	0.08	11.71	-0.00	-0.03	0.05	0.41	-0.00		
10	0.07	25.07	-0.00	0.12	0.07	0.49	0.00		

Notes: Each row shows causal effects of an increase of 65 log points in the minimum wage in all regions for a subset of adults. Adults are grouped based on productivity at the skill-intensive firms, such that each row corresponds to 10% of the employed population (i.e., the product of Columns (2) and (7) is constant across rows). Wage effects are decomposed as described in Subsection 5.6: monopsony, returns to skill, and general equilibrium. Columns (8) and (9) report elasticities of employment with respect to the log real minimum wage and the mean wage for the group. Column (10) resembles Column (9) but only considers the monopsony channel.

with low-skilled labor and also from general equilibrium effects (which make high-wage firms more common in the economy).

The causal effects differ from simulated spillovers because the model predicts disemployment for workers with very low productivity. Columns (8) and (9) show that the implied employment elasticities for the lowest group, measured either relative to the minimum wage or the mean wage increase in the group, are in the lower range of estimates for the US (Harasztosi and Lindner, 2019; Neumark and Shirley, 2021). Employment elasticities are closer to zero for the second and third groups.

The strong heterogeneity in the effects of minimum wages may be hard to detect in reduced-form approaches. Papers such as Dustmann et al. (2021) group workers based on their initial wages to define the extent to which they are "treated" by the minimum wage shock. In an imperfectly competitive labor market, such grouping confounds worker productivity and firm wage premiums. The lowest bins include low-productivity workers at high-wage firms, who are at the greatest disemployment risk, and higher-productivity workers at low-wage firms, who may find employment elsewhere after the minimum wage shock. In Appendix D.6.3, I show that if workers are grouped by wages instead of productivity, the heterogeneity patterns are much less salient.

Appendix D.6.4 presents a long discussion of why the employment results I observe differ from recent empirical work studying the rising minimum wage in Brazil. I argue that, in the Brazilian context, it is difficult to account for the confounding effects of supply and demand shocks using reduced-form methods. In addition, those methods may capture short-run rather than long-run effects (Sorkin, 2015).

I end this section by examining why my estimates of employment effects differ from the simulations based on the model of Engbom and Moser (2022). This is partly because my model includes channels that are not present in theirs. Column (10) in Table 8 shows that ignoring the returns to skill and general equilibrium effects would significantly lower employment elasticities with respect to the mean wage. The simulated shock is also smaller in their paper (0.577 versus 0.65).

Nonetheless, the main reason for the different predictions concerning employment is likely to be my use of a local labor markets approach. In Engbom and Moser (2022), disemployment effects for very low-skilled workers are dampened by reallocation to firms in the top 5% of the productivity distribution (see their Appendix Figure E.3). Many of these firms may be in the richest parts of the country, while the displaced workers may be in the poorest. My model does not allow for geographical mobility, limiting the extent of minimum wage-induced reallocation. This approach is consistent with Dix-Carneiro and Kovak (2017), who document that the Brazilian microregions most affected by tariff reductions in the 1990s saw declines in formal employment but no systematic out-migration responses.

7 Conclusion

The unified framework proposed in this paper combines two labor economics perspectives: supply/demand models focusing on endogenous productivity gaps between workers and imperfectly competitive labor market models focusing on firm wage differentials and sorting. I have demonstrated that there are important interactions between these two traditions. Including firm wage premiums in a supply/demand framework may lead to qualitative changes in the effects of education on between-group wage differentials. Meanwhile, the combination of task-based production, firm heterogeneity, and monopsony power generates new channels through which the minimum wage affects employment and wages. In the paper's empirical component, I have shown that both results are quantitatively relevant in Brazil.

According to my simulations, although minimum wages effectively reduce inequality in the Brazilian formal sector, part of that reduction derives from disemployment effects concentrated on low-skilled workers in the country's poorer regions. If the national minimum wage is raised, policymakers should consider parallel efforts to provide support to the most vulnerable workers. The framework developed in this paper can be used to identify the regions most in need of such support.

An important technical contribution of the paper is the task-based production function, a convenient tool for studying labor markets with rich worker and firm heterogeneity. It offers a tractable, intuitive, and parsimonious means of modeling cross-firm differences in labor demand patterns. It also enables the modeling of different forms of technical change. One avenue for further research is understanding the effects of *routine-biased technical change* (Autor, Levy and Murnane, 2003; Acemoglu and Autor, 2011) in a context with firm heterogeneity and imperfect competition.

References

Aaronson, Daniel, Eric French, Isaac Sorkin, and Ted To. 2018. "INDUSTRY DYNAMICS AND THE MINIMUM WAGE: A PUTTY-CLAY APPROACH." *International Economic Review*, 59(1): 51–84.

Abowd, John M, Francis Kramarz, and David N Margolis. 1999. "High Wage Workers and High Wage Firms." *Econometrica*, 67(2): 251–333.

Acemoglu, Daron. 1998. "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality." *The Quarterly Journal of Economics*, 113(4): 1055–1089.

Acemoglu, Daron. 1999. "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence." *The American Economic Review*, 89(5): 1259–1278.

Acemoglu, Daron. 2007. "Equilibrium Bias of Technology." *Econometrica*, 75(5): 1371–1409.

Acemoglu, Daron, and David Autor. 2011. "Skills, Tasks and Technologies: Implications for Employment and Earnings." In *Handbook of Labor Economics*. Vol. 4, Part B, , ed. David Card and Orley Ashenfelter, Chapter 12, 1043–1171. Elsevier.

Ales, Laurence, Musab Kurnaz, and Christopher Sleet. 2015. "Technical Change, Wage Inequality, and Taxes." *American Economic Review*, 105(10): 3061–3101.

Almeida, Rita, and Pedro Carneiro. 2012. "Enforcement of Labor Regulation and Informality." *American Economic Journal: Applied Economics*, 4(3): 64–89.

Alvarez, Jorge, Felipe Benguria, Niklas Engbom, and Christian Moser. 2018. "Firms and the Decline in Earnings Inequality in Brazil." *American Economic Journal: Macroeconomics*, 10(1): 149–89.

Andrews, M. J., L. Gill, T. Schank, and R. Upward. 2008. "High wage workers and low wage firms: negative assortative matching or limited mobility bias?" *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 171(3): 673–697.

Antràs, Pol, Luis Garicano, and Esteban Rossi-Hansberg. 2006. "Offshoring in a Knowledge Economy." *The Quarterly Journal of Economics*, 121(1): 31–77.

Autor, David H., Alan Manning, and Christopher L. Smith. 2016. "The Contribution of the Minimum Wage to US Wage Inequality over Three Decades: A Reassessment." *American Economic Journal: Applied Economics*, 8(1): 58–99.

Autor, David H, Frank Levy, and Richard J Murnane. 2003. "The Skill Content of Recent Technological Change: An Empirical Exploration." *The Quarterly Journal of Economics*, 118(4): 1279–1333.

Azar, José, Emiliano Huet-Vaughn, Ioana Marinescu, Bledi Taska, and Till von Wachter. 2019. "Minimum Wage Employment Effects and Labor Market Concentration." Working Paper 26101.

Beaudry, Paul, and David A. Green. 2003. "Wages and Employment in the United States and Germany: What Explains the Differences?" *American Economic Review*, 93(3): 573–602.

Berger, David, Kyle Herkenhoff, and Simon Mongey. 2022*a*. "Labor Market Power." *American Economic Review*, 112(4): 1147–93.

Berger, David, Kyle Herkenhoff, and Simon Mongey. 2022*b*. "Minimum Wages, Efficiency and Welfare." *Unpublished manuscript.*

Bezanson, Jeff, Alan Edelman, Stefan Karpinski, and Viral B Shah. 2017. "Julia: A fresh approach to numerical computing." *SIAM review*, 59(1): 65–98.

Blundell, Richard, David A Green, and Wenchao Jin. 2021. "The U.K. as a Technological Follower: Higher Education Expansion and the College Wage Premium." *The Review of Economic Studies*, 89(1): 142–180.

Bonhomme, Stéphane, Thibaut Lamadon, and Elena Manresa. 2019. "A Distributional Framework for Matched Employer Employee Data." *Econometrica*, 87(3): 699–739.

Bound, John, and George Johnson. 1992. "Changes in the Structure of Wages in the 1980's: An Evaluation of Alternative Explanations." *The American Economic Review*, 82(3): 371–392.

Burdett, Kenneth, and Dale T. Mortensen. 1998. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review*, 39(2): 257–273.

Butschek, Sebastian. 2022. "Raising the Bar: Minimum Wages and Employers' Hiring Standards." *American Economic Journal: Economic Policy*, 14(2): 91–124.

Card, David, Ana Rute Cardoso, Joerg Heining, and Patrick Kline. 2018. "Firms and Labor Market Inequality: Evidence and Some Theory." *Journal of Labor Economics*, 36(S1): S13–S70.

Card, David, and Thomas Lemieux. 2001. "Can Falling Supply Explain the Rising Return to College for Younger Men? A Cohort-Based Analysis." *The Quarterly Journal of Economics*, 116(2): 705–746.

Card, David, Jörg Heining, and Patrick Kline. 2013. "Workplace Heterogeneity and the Rise of West German Wage Inequality." *The Quarterly Journal of Economics*, 128(3): 967.

Carneiro, Pedro, Kai Liu, and Kjell G Salvanes. 2022. "The Supply of Skill and Endogenous Technical Change: Evidence from a College Expansion Reform." *Journal of the European Economic Association*, 21(1): 48–92.

Costa, Francisco, Jason Garred, and João Paulo Pessoa. 2016. "Winners and losers from a commodities-for-manufactures trade boom." *Journal of International Economics*, 102: 50–69.

Costinot, Arnaud, and Jonathan Vogel. 2010. "Matching and Inequality in the World Economy." *Journal of Political Economy*, 118(4): 747–786.

Deming, David, and Lisa B. Kahn. 2018. "Skill Requirements across Firms and Labor Markets: Evidence from Job Postings for Professionals." *Journal of Labor Economics*, 36(S1): S337–S369.

Deming, David J. 2017. "The Growing Importance of Social Skills in the Labor Market." *Quarterly Journal of Economics*, 132(4): 1593–1649.

de Sousa, Gustavo. 2020. "The Labor Market Consequences of (In)Appropriate Technology." Unpublished manuscript.

Dix-Carneiro, Rafael, and Brian K. Kovak. 2017. "Trade Liberalization and Regional Dynamics." *American Economic Review*, 107(10): 2908–46.

Dustmann, Christian, Attila Lindner, Uta Schönberg, Matthias Umkehrer, and Philipp vom Berge. 2021. "Reallocation Effects of the Minimum Wage*." *The Quarterly Journal of Economics*, 137(1): 267–328.

Eeckhout, Jan, and Philipp Kircher. 2011. "Identifying Sorting—In Theory." *The Review of Economic Studies*, 78(3): 872–906.

Eeckhout, Jan, and Roberto Pinheiro. 2014. "Diverse Organizations and the Competition for Talent." *International Economic Review*, 55(3): 625–664.

Engbom, Niklas, and Christian Moser. 2022. "Earnings Inequality and the Minimum Wage: Evidence from Brazil." *American Economic Review*, 112(12): 3803–47.

Engerman, S.L., and K.L. Sokoloff. 2012. *Economic Development in the Americas Since 1500: Endowments and Institutions. NBER Series on Long-Term Facto*, Cambridge University Press.

Firpo, Sergio, and Alysson Portella. 2019. "Decline in Wage Inequality in Brazil: A Survey." World Bank Policy Research Working Paper 9096.

Flinn, Christopher J. 2006. "Minimum Wage Effects on Labor Market Outcomes under Search, Matching, and Endogenous Contact Rates." *Econometrica*, 74(4): 1013–1062.

Fortin, Nicole M., Thomas Lemieux, and Neil Lloyd. 2021. "Labor Market Institutions and the Distribution of Wages: The Role of Spillover Effects." *Journal of Labor Economics*, 39(S2): S369–S412.

Garicano, Luis. 2000. "Hierarchies and the Organization of Knowledge in Production." *Journal of Political Economy*, 108(5): 874–904.

Garicano, Luis, and Esteban Rossi-Hansberg. 2006. "Organization and Inequality in a Knowledge Economy." *The Quarterly Journal of Economics*, 121(4): 1383–1435.

Hahn, Jinyong, and Guido Kuersteiner. 2002. "Asymptotically Unbiased Inference for a Dynamic Panel Model with Fixed Effects When Both n and T Are Large." *Econometrica*, 70(4): 1639–1657.

Hahn, Jinyong, and Whitney Newey. 2004. "Jackknife and Analytical Bias Reduction for Nonlinear Panel Models." *Econometrica*, 72(4): 1295–1319.

Harasztosi, Peter, and Attila Lindner. 2019. "Who Pays for the Minimum Wage?" *American Economic Review*, 109(8): 2693–2727.

Helpman, Elhanan, Oleg Itskhoki, Marc-Andreas Muendler, and Stephen J Redding. 2017. "Trade and Inequality: From Theory to Estimation." *The Review of Economic Studies*, 84(1): 357.

Herkenhoff, Kyle, Jeremy Lise, Guido Menzio, and Gordon M Phillips. 2018. "Production and Learning in Teams." National Bureau of Economic Research Working Paper 25179.

Hurst, Erik, Patrick Kehoe, Elena Pastorino, and Thomas Winberry. 2022. "The Distributional Impact of the Minimum Wage in the Short and Long Run." *Unpublished manuscript.*

"IBGE", Instituto Brasileiro de Geografia e Estatística. 2003. *Metodologia do Censo Demográfico 2000.* IBGE.

Jäger, Simon, and Jörg Heining. 2022. "How Substitutable Are Workers? Evidence from Worker Deaths." *Unpublished manuscript*.

Katz, Lawrence F, and Kevin M Murphy. 1992. "Changes in Relative Wages, 1963-1987: Supply and Demand Factors." *The Quarterly Journal of Economics*, 107(1): 35–78.

Keane, Michael, and Richard Rogerson. 2012. "Micro and Macro Labor Supply Elasticities: A Reassessment of Conventional Wisdom." *Journal of Economic Literature*, 50(2): 464–476.

Kline, P., R. Saggio, and M. Sølvsten. 2018. "Leave-out estimation of variance components." *ArXiv e-prints*.

Kremer, Michael, and Eric Maskin. 1996. "Wage Inequality and Segregation by Skill." National Bureau of Economic Research Working Paper 5718.

Krusell, Per, Lee E Ohanian, José-Víctor Ríos-Rull, and Giovanni L Violante. 1999. "Capital-skill Complementarity and Inequality: A Macroeconomic Analysis." *Econometrica*, 68(5): 1029–1053.

Lamadon, Thibaut, Magne Mogstad, and Bradley Setzler. 2022. "Imperfect Competition, Compensating Differentials, and Rent Sharing in the US Labor Market." *American Economic Review*, 112(1): 169–212.

Lee, David S. 1999. "Wage Inequality in the United States during the 1980s: Rising Dispersion or Falling Minimum Wage?" *The Quarterly Journal of Economics*, 114(3): 977–1023.

Lindenlaub, Ilse. 2017. "Sorting Multidimensional Types: Theory and Application." *The Review of Economic Studies*, 84(2): 718–789.

Lise, Jeremy, and Fabien Postel-Vinay. 2020. "Multidimensional Skills, Sorting, and Human Capital Accumulation." *American Economic Review*, 110(8): 2328–76.

Neumark, David, and Peter Shirley. 2021. "Myth or Measurement: What Does the New Minimum Wage Research Say about Minimum Wages and Job Loss in the United States?" Working Paper 28388.

Neumark, David, and William Wascher. 1992. "Employment Effects of Minimum and Subminimum Wages: Panel Data on State Minimum Wage Laws." *ILR Review*, 46(1): 55–81.

Ponczek, Vladimir, and Gabriel Ulyssea. 2021. "Enforcement of Labour Regulation and the Labour Market Effects of Trade: Evidence from Brazil." *The Economic Journal*, 132(641): 361–390.

Rohlin, Shawn M. 2011. "State minimum wages and business location: Evidence from a refined border approach." *Journal of Urban Economics*, 69(1): 103–117.

Sattinger, Michael. 1975. "Comparative Advantage and the Distributions of Earnings and Abilities." *Econometrica*, 43(3): 455–468.

SEDLAC. 2022. "Socio-Economic Database for Latin America and the Caribbean." CEDLAS and The World Bank.

Shephard, Andrew, and Modibo Sidibe. 2019. "Schooling Investment, Mismatch, and Wage Inequality." PIER Working Paper 19-013.

Song, Jae, David J Price, Fatih Guvenen, Nicholas Bloom, and Till von Wachter. 2018. "Firming Up Inequality*." *The Quarterly Journal of Economics*, 134(1): 1–50.

Sorkin, Isaac. 2015. "Are there long-run effects of the minimum wage?" *Review of Economic Dynamics*, 18(2): 306–333.

Stigler, George J. 1946. "The Economics of Minimum Wage Legislation." *The American Economic Review*, 36(3): 358–365.

Stolper, Wolfgang F., and Paul A. Samuelson. 1941. "Protection and Real Wages." *The Review of Economic Studies*, 9(1): 58–73.

Teulings, Coen, and Thijs van Rens. 2008. "Education, Growth, and Income Inequality." *The Review of Economics and Statistics*, 90(1): 89–104.

Teulings, Coen N. 1995. "The Wage Distribution in a Model of the Assignment of Skills to Jobs." *Journal of Political Economy*, 103(2): 280–315.

Teulings, Coen N. 2000. "Aggregation Bias in Elasticities of Substitution and the Minimum Wage Paradox." *International Economic Review*, 41(2): 359–398.

Teulings, Coen N. 2003. "The Contribution of Minimum Wages to Increasing Wage Inequality." *The Economic Journal*, 113(490): 801–833.

Teulings, Coen N. 2005. "Comparative Advantage, Relative Wages, and the Accumulation of Human Capital." *Journal of Political Economy*, 113(2): 425–461.

Teulings, Coen N, and Pieter A Gautier. 2004. "The Right Man for the Job." *The Review of Economic Studies*, 71(2): 553–580.

Trottner, Fabian. 2019. "Who Gains From Scale? Trade and Wage Inequality Within and Between Firms." Unpublished manuscript.

Verhoogen, Eric A. 2008. "Trade, Quality Upgrading, and Wage Inequality in the Mexican Manufacturing Sector*." *The Quarterly Journal of Economics*, 123(2): 489–530.