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ABSTRACT

"Debt overhang" models have motivated the possibility of Pareto–improving "market–based debt–reduction schemes" under an assumption of creditor seizure in bad states. These models usually reach the conclusion that while pure debt forgiveness is in the interest of debtor nations, debt repurchase programs are not.

This paper introduces a "safe sector" into the debtor nation which is unexposed to seizure during default states. Two important results which emerge are that debt forgiveness is not necessarily in the interest of all debtors, and the potential for Pareto–improving debt–equity swaps is magnified.

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1. Introduction

The recent shift in United States policy concerning highly indebted countries (HIC's) has attracted greater attention to the growing work on investment disincentives due to "debt overhang." The discussion has centered on disincentives for economic reform [Sachs and Huizinga (1987)] as well as investment disincentives [Krugman (1989), Froot (1989)]. Arguments for mutually beneficial "market-based" debt reduction schemes have centered on the possibility of a "debt-relief Laffer curve" [Sachs (1988a,b), Krugman (1989), Froot (1988)] in which reductions in nominal debt, by lowering investment distortions, can actually increase the market value of remaining outstanding debt. Other discussions of mutually-beneficial responses to debt overhang have centered on risk-sharing issues [Helpman (1987)].

Although our results generally concur with the Bulow and Rogoff (1989) conclusions regarding the dominance of pure debt-reduction over market-based schemes, two important distinctions emerge. First, the use of resources to buy back old debt has an impact on the value of old debt not retired. By breaking the separability between equity sales and debt buybacks that arises in the one sector framework, we generate the theoretical potential for a welfare improving debt for equity swap. Secondly, while Bulow and Rogoff show that the country chooses the take-it-or-leave-it offer in the case of debt buybacks and exit bonds, in the two-sector model the decision is complicated by the distributional consequences of these plans. Entrenched domestic interests may offer resistance to strategies that may otherwise seem appealing.

The reliance upon the debt-relief-Laffer-curve for Pareto-improving market-based debt reduction schemes leads Bulow and Rogoff to skepticism concerning the desirability of these programs for debtors. Bulow and Rogoff clearly show that pure buyback schemes would only be undertaken when profitable investment opportunities exist within the debtor nation. However, if these opportunities exist, debtors should undertake these investments directly, rather than conducting a debt conversion. Similar skepticism on market–based schemes is voiced against debt–equity swaps. These swaps can be described as a two–step procedure: first equity is sold on the open market; second, the proceeds from that sale are used to repurchase debt. Bulow and Rogoff argue that the two steps are "separable," i.e. that the second step, the debt repurchase, is not necessary for the first, the sale of domestic equity. It follows that debt–equity swaps are no more desirable than buyback schemes.

In our analysis we question the separability of the equity sale and debt repurchase. The analysis focusses on the role of intersectoral capital mobility in a two–sector model characterized by distinct threats of creditor penalties in the event of default. Previous investigations of debt overhang [Froot (1988), Krugman (1989)] concentrated on assumed "penalty functions" in the form of "gunboat technologies" in which creditors seize output in the event of default. Investment disincentives arose in states in which the probability of output seizure was high, leading rational agents to substitute first–period consumption for investment.

When considering the empirical validity of the gunboat technology, however, one would question whether all sectors of an economy would be equally exposed to this expropriation threat. In particular, we examine a two–sector model in which default only results in seizure in the "international sector." This "seizable sector" may be interpreted as exportables and raw materials, whereas the nonseizable sector would comprise import–competing goods production and cottage industry. In this economy investment in the "non–seizable" sector is a viable alternative to additional first–period consumption as a response to debt overhang.

The introduction of a non–seizable sector yields two distinctions from the previous debt overhang literature. First, distributional considerations arise in analyses of debt–reduction schemes. The role of the non–seizable sector as a "safe haven" for investment has implications for relative returns to factors across sectors. Assuming sector-specificity, factors complementary to new investment, such as labor, would benefit from their sector's "safe haven" role. Removal of debt overhang would remove this advantage. The statement that "pure debt forgiveness is in the interest of debtors" [Froot (1988)] no longer applies. Debt forgiveness may be in the interest of some agents, and not others. These distributional considerations may play a role in explaining lack of popularity of market-based debt reduction schemes.

Secondly, a debt repurchase serves to alter the allocation of investment across the two sectors and yield capital gains to the owners of old equity in the seizable sector. This result provides a channel by which the Bulow and Rogoff (1988) results may be altered. If the anticipation of a debt repurchase affects the terms of equity sales of some domestic assets favorably, debt-equity swaps may be in the interest of debtors. This result may hold even in instances where pure debt repurchases are not in the debtor's interest, as was shown to be likely by Bulow and Rogoff (1989).

Below, the decisions facing the creditors, the debtor government and consumer/investors are modelled within a five stage complete information process. In the first stage the country "inherits" a stock of debt, domestic consumer/investors inherit an endowment and given sectoral work assignments.[1] In the second stage the country chooses an amount of reserves to devote to a debt buyback conditioned on levels of debt forgiveness by creditors. In the third stage consumer/investors decide the allocation of their financial capital across periods and sectors. Uncertainty is resolved in the fourth stage when investment output is realized in the seizable and nonseizable sectors. Output realizations determine tax revenues received by the government, and in turn determine

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[1]As in Bulow and Rogoff (1989), any negotiated restructuring of the country's debt occurs prior to the start of the game and is already reflected in the size of inherited debt.

whether debt payments are made in full or not at all. If the debtor defaults, creditors, employing a "gunboat technology," seize available output. It is assumed that the output of one sector is seizable whereas the output in the other sector is immune from this threat.[2] After the creditors extract their rents, in the fifth stage residual output in both sector is distributed to workers in the form of wages and to investors in the form of dividends.

Since the government and creditor behavior are conditioned on the anticipated response of the private consumer/ investors, for subgame perfection, we first analyze the private sectors' behavior in Section 2. In Section 3 we explore the debt forgiveness, buy-back, and debt-equity swaps proposals offered to debtors and creditors. In Section 4 the distributional consequences of these proposals are discussed. Section 5 concludes.

2. The Consumption/ Investment Choice

We utilize a two-period model to explore the consumption/ investment allocation decision faced by domestic agents. The economy is comprised of two sectors: seizables (traded goods and raw materials) and nonseizables (import substitutes and cottage industry), denoted by subscripts s and n respectively. Each consumer, designated L_n and L_s depending upon whether his labor is specific to the nonseizable or seizable sector, allocates his initial endowment W across first period consumption, C^1 and investment in each sector, I_n and I_s. Investment is assumed to be the only means by which agents can transfer first period purchasing power into second consumption.[3] The new investment is in

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[2]Clearly goods would fall somewhere between complete "seizability" or "isolation." The analysis below would be robust to two sectors which differed only in severity of exposure to foreign intervention.

[3]Seizable and non-seizable goods are assumed to be distinct only on the production side, leaving the relative price of the two goods fixed throughout the analysis. Weakening this assumption would require further restrictions concerning cross-elasticity of demand to motivate the distributional conclusions below. However, the conclusions concerning the viability of market-based schemes would be robust to this complication.

"putty" form and is sold on competitive markets to "firms" which base their investment decisions on maximizing the returns to old equity capital, $\tilde{K}_i$ and new capital I_i.

Formally, the consumer/ investor maximizes

$$(2.1) \qquad U = U(C^1) + \beta E(C^2),$$

where

$$(2.2) \qquad C^1 = W - I_n - I_s$$

and

$$E(C^2) = E(\ r_s \cdot K_s^{\,f} + r_n \cdot K_n^{\,f} + w_n L_n + w_s L_s + T\ ).$$

where $K_s^{\,f} = \tilde{K}_s + I_s$ and $K_n^{\,f} = \tilde{K}_n + I_n$. As in Froot (1988), the linearity of the two period consumption function simplifies the analysis, while the assumption that $U(\cdot)$ satisfies Inada conditions preserves finite elasticity of intertemporal substitution. β is the time discount factor, and E(T) are expected second period transfers from the government to the private sector.

Wages and dividends on investments are derived by analyzing the hiring and production activities in the economy. The sectoral production technologies are:

$$(2.3) \qquad y_i = f_i(L_i,\ K_i + I_i) + e_i \qquad\qquad (i = n,s)$$

where $f(\cdot)$ satisfies the Inada conditions and e_i has support $[\bar{e},\ \underline{e}]$. For each sector $y'(L)>0$, $y''(L)<0$, $y'(I)>0$, $y''(I)<0$, and new investment is assumed to be complementary to labor, $\delta^2 y/\delta L \delta I > 0$. In equilibrium, the allocation of capital across sectors takes into account the effects of taxes on output and the risk that output will be seized by foreign creditors. Only output in sector s bears the risk of seizure by creditors when government tax revenues are too low to meet full debt repayment.

The government is assumed to tax both sectors at the upper boundary, τ.[4] Notice that the specification of τ determines the presence or absence of "repudiation" in

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[4]Alternatively, taxes could be levied on producer profits and on the dividend and wage income of consumer/investors. So long as the tax rate is the same across sectors and domestic labor and investment are the only inputs into production, government tax revenues will be identical to the case we assume.

the model. The government's short–run tax capabilities are assumed to be exogenous, leaving the possibility of default susnequent to bad realizations of ϵ. Furthermore, when foreign creditors utilize a gunboat technology in a low output state of the world, they are effectively circumventing the tax rate ceiling on seizable sector output which binds the domestic government.[5]

Maximizing profits with respect to the factor market yields the familiar first–order conditions

$$(2.4) \qquad r_i = (1-\tau)f_i^k(I_i) \qquad\qquad (i = n,s)$$

Notice that investment decision of firms is distorted only by the expected tax on output. In particular, the threat of output seizure does not affect the hiring decision in the seizable sector because these firms pay their factors only in non–seizure states. In bad states, the s–sector firms end up in bankrupcy and pay neither dividends or wages to the consumer/investors.

With probability (1–G) the investors fail to get paid dividends in the seizable sector. Consequently the rental rate on capital exceeds the expected return on "putty" investment capital in the seizable sector. To equalize ex–ante expected rates of return across sectors, contractual rates of return must satisfy

$$(2.5) \qquad G \cdot r_s = r_n \ .$$

Combined with (4) the marginal products of capital are related by

$$(2.6) \qquad G \cdot f_s^k = f_n^k \ .$$

As in Helpman (1988a,b) and Froot (1988), G represents the probability that investment will yield output and tax revenues sufficient to cover debt payments in the second period, so that

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[5]It is implicitly assumed that total government revenues never exceed total output of the seizable sector in default states. Under this realization, creditors would choose to take what the government could raise, rather than conducting seizure activities. This possibility would be unlikely unless the seizable sector of the debtor nation became extremely small relative to its entire economy.

$$(2.7)\qquad G = G(I_s^*, I_n^*, x) = 1 - \int_{\epsilon^*}^{\infty} g(\epsilon)d\epsilon, \quad \text{with } \delta G/\delta x > 0$$

where,

$$(2.8)\qquad \epsilon^* = \tau[f_n(I_n) + f_s(I_s)] - (D-x)$$

and at $\epsilon=\epsilon^*$ transfer payments are equal to debt obligations. Implicitly, the probability is a function of first period investment in each sector and output realizations. For $\epsilon > \epsilon^*$ output levels generate tax revenues $\tau[f_n(I_n) + f_s(I_s)]$ sufficient to cover debt obligations. Revenues in excess of those required to service debt are distributed to private agents in the form of transfers. For $\epsilon < \epsilon^*$, the government defaults on debt obligations. Since creditors seize all sector s output in bad states, the government receives tax payments on n sector output only. These tax revenues are distributed to the domestic consumer/investors who would otherwise realize wage and dividend income only on their investments of financial capital or labor in the nonseizable sector. The boundary on the probability of a good state, ϵ^* shifts with debt forgiveness according to

$$(2.9)\qquad \frac{\partial \epsilon^*}{\partial x} = 1 + \tau\left\{ f_n^k \frac{\partial I_n}{\partial x} + f_s^k \frac{\partial I_s}{\partial x} \right\}$$

A sufficient condition for debt forgiveness to lower the probability of default is

$$(2.10)\qquad G\frac{\partial I_n}{\partial x} + \frac{\partial I_s}{\partial x} > 0 \quad .$$

Expected transfers to the domestic consumer/investors are also a function od investment and output realizations:

$$(2.11)\qquad \begin{array}{ll} \tau[f_n(I_n) + f_s(I_s)] - (D - x) & \text{if } \epsilon > \epsilon^* \\ \tau[f_n(I_n)] & \text{if } \epsilon \leq \epsilon^* \end{array}$$

The consumer/investor operates in a perfectly competitive market, so that he perceives his impact on wages, returns on equity, transfers and default probabilities to be zero and therefore exogenous to his consumption decision. The first order conditions on investment and consumption are given by (1.12).

$$\text{(2.12a)} \quad U'(C^1) = \beta(1-\tau)Gf_s^{\,k}(I_s)$$

$$\text{(2.12b)} \quad U'(C^1) = \beta(1-\tau)f_n^{\,k}(I_n)$$

The marginal utility of first period consumption is equal to the marginal utility of future consumption. In order for a debt disincentive for investment to exist, the expected returns to investment must be decreasing in the level of outstanding debt. The condition for this result is presented as Proposition 1.

<u>Proposition 1:</u> Seizable sector investment is increasing in x, the level of debt write–down, while nonseizable sector investment is decreasing in G, the probability that that output will not be seized.

Differentiating equation 12 and making appropriate substitutions yields,

$$\text{(2.13)} \quad \frac{\partial I_s^*}{\partial x} = \frac{-[U''+\beta\tau(1-\tau)G\cdot G'(f_s^{\,k})^2](\partial I_n/\partial x) - \beta(1-\tau)G'f_s^{\,k}}{U''+\beta G(1-\tau)f_s^{\,kk} + \beta\tau(1-\tau)G'(f_s^{\,k})^2}$$

which depends both on a second order condition for the utility function and the form of capital productivity in the seizable and nonseizable sectors. The differential of (12b) with respect to debt forgiveness is

$$\text{(2.14)} \quad \frac{\partial I_n^*}{\partial x} = \frac{-U''\ \partial I_s/\partial x}{U'' + \beta(1-\tau)f_n^{\,kk}} < 0$$

which is negative so long as the investment in the seizable sector increases with forgiveness. Combining (1.13) and (1.14), the condition for Proposition 1 to hold is given by

$$\text{(2.15)} \quad \frac{\partial I_s^*}{\partial x} = \frac{-f_s^{\,k}G'[U''+\beta(1-\tau)f_n^{\,kk}]}{[\tau G'(f_s^{\,k})^2+Gf_s^{\,kk}][U''+\beta(1-\tau)f_n^{\,kk}] + U''[f_n^{\,kk} - G\cdot G'\tau(f_s^{\,k})^2]} > 0$$

The responsiveness of investment in the seizable to debt forgiveness is a function of the concavity of production technologies in investment, the concavity of utility in consumption, and the level of taxation on second period production output. Sufficient conditions for $\partial I_s^*/\partial x > 0$ are: U" sufficiently negative and $[\tau G'(f_s^k)^2 + G f_s^{kk}] < 0$. Helpman's (1988) conclusions regarding the consequences of debt write-downs under different levels of risk aversion are confirmed in this model. When U" approaches zero, agents tend toward risk neutrality. Without concavity of the utility function (U"=0), debt relief has no effect on investment in the nonseizable sector and serves to stimulate investment in the seizable sector under much more restrictive conditions. It is more likely that the income effect caused by debt forgiveness increases demand for first period consumption by levels which dominate the substitution toward investment in the threatened sector.

In summary, private agents adjust their "putty" investments in response in anticipation of the effects that the debt write-down will have on competitive returns to investment in each sector. In particular, under reasonable conditions the write down will raise the expected return to capital in the seizable sector. Lowering the "debt overhang" has the effect of lowering the degree of "excess" investment in the nonseizable sector, since agents place a lower probability on the seizure of other capital. Under reasonable conditions, debt forgiveness reduces investment in nonseizables, such as import competing goods and cottages industries, and increases investment in seizable goods such as those produced in export promoting industries.

3. Distributional Effects of Debt Write-Downs

To analyze possible distributional effects from debt relief, assume that in addition to sector specific capital, the production function includes sector specific labor. Although at some point in the past this allocation was consistent with equal returns to labor across sectors, under labor specificity debt forgiveness can have a perverse effect on the wages of labor specific to the non-seizable sector. If the debt forgiveness policy succeeds in moving investment out of the non–seizable sector, as well as lowering first–period consumption, some agents of the debtor nation can be made worse off.

Proposition 2: Wages of seizable sector labor are unambiguously increasing in the level of debt forgiveness. However, the wages of labor in the non–seizable sector are unambiguously decreasing in debt relief.

Proof:

Assume that laborers are price takers in their respective sectors. Profit maximization by firms assures that labor earns the value of its marginal product after taxes:

$$E(w_s) = (1-\tau)Gf_s^{L} \qquad i=s,n \tag{3.1a}$$

$$E(w_n) = (1-\tau)f_n^{L} \qquad i=s,n \tag{3.1b}$$

Debt write–downs, in the form of increasing x, will have the impact of increasing seizable sector investment and decreasing non–seizable sector investment. It follows that the impact wages will be:

$$\frac{\partial w_s}{\partial x} = (1-\tau)\left\{ G\ f_s^{Lk} \frac{\partial I_s}{\partial x} + \frac{\partial G}{\partial x}\ f_s^{L} \right\} > 0 \tag{3.2a}$$

$$\frac{\partial w_n}{\partial x} = (1-\tau)\ f_n^{Lk} \frac{\partial I_n}{\partial x} < 0 \tag{3.2b}$$

Although laborers in the non–seizable sector benefit from an increase in the level of government transfers, their marginal products are lowered by the outflow of capital from the non–seizable sector. It follows that the standard conclusion that " ... pure debt relief always makes debtors better off" [Froot (1988)] requires a further restriction. In particular, if the output of a sector is non–seizable, debt relief will lead to an outflow of capital from that sector, unambiguously lowering the wage–bill of laborers in the non–seizable sector. Clearly, it need no longer be the case that all laborers will be better off as a result of a debt write-down.

As a group, moreover, it need not even be the case that the returns to labor as a whole increase in as a result of debt write-downs. The reason for this surprising result is that all of the benefits of the debt write-down may accrue to owners of capital. The total wage bill satisfies:

$$(3.3) \qquad w_T = (1-\tau)[L_s G f_s^L + L_n f_n^L]$$

Differentiating with respect to x and appropriate substitutions yields:

$$(3.4) \quad \frac{\partial w_T}{\partial x} = (1-\tau)L_s G' f_x^L$$

$$+(1-\tau)\frac{\partial I_s}{\partial x}\left\{\frac{L_s[U''f_s^{LK}+U''G'f_s^k f_s^L(1-G)+\beta(1-\tau)f_n^{kk}(f_s^{Lk}+G'f_s^L f_s^k)] - L_n U'' f_n^{Lk}}{[U'' + \beta(1-\tau)f_n^{kk}]}\right\}$$

It is clear that the total wage bill is increasing in debt forgiveness provided that the direct effect of capital mobility between sectors has an impact on wage payments to seizable sector labor. A weak sufficient condition for the total wage bill to increase is

$$(3.5) \quad L_s[U''f_s^{LK}+U''G'f_s^k f_s^L(1-G) + \beta(1-\tau)f_n^{kk}(f_s^{Lk}+G'f_s^L f_s^k)] - L_n U'' f_n^{Lk} < 0$$

A stronger sufficient condition compares only the direct effect of forgiveness on total sectoral rents without adding on the reinforcing effect of altered probabilities of default,

$$(3.6) \quad L_s f_s^{LK} - L_n U'' f_n^{Lk} \geq 0$$

Although the results of the model are in agreement with the standard literature on the effect of debt write-downs on aggregate consumption, the effects on distribution can be perverse. In particular, we have shown that debt forgiveness is not necessarily in the interest of <u>all debtors</u>. The simple reason behind this result is that laborers in some industries were benefitting ex-ante from the incentive effects of the debt overhang. Removal of the debt overhang redressed the initial wage "distortions".

4. Buybacks, Write–Downs and Debt–Equity Swaps

4.1 Pure Debt Relief: Before assessing the combination of equity sales and write–downs, in this section we establish the conditions for a debt write–down alone to be welfare enhancing for the debtor nation. The analysis implies:

<u>Proposition 3:</u> Debtor welfare generally increases with levels of debt forgiveness.

Proof:

The welfare function of the debtor nation satisfies

$$(4.1)\ U = U(C^1) + \beta \cdot C^2 , \quad \text{where}$$

$$C^1 = W - I_n - I_s$$

$$E(C^2) = E\left\{ K_n r_n + I_n r_n + K_s r_s + I_s r_s + w_n L_n + w_s L_s + T \right\}$$

and expected transfers are

$$E(T) = \begin{cases} \tau[f_n(I_n) + f_s(I_s)] - (D - x) & \text{prob} = G \\ \tau[f_n(I_n)] & \text{prob} = 1-G \end{cases}$$

Using the values of wages and rental rates from profit maximizing behavior of firms, the welfare function of the debtor nation reduces to:

$$(4.2)\quad U = U(W - I_n - I_s) + \beta[f_n + G(f_s - (D-x))]$$

Differentiating the objective function with respect to levels of debt write–down, the following condition is derived:

$$(4.3)\quad \frac{\delta U}{\delta x} = G + G'[f_s - (D-x)] + \frac{\delta I_s}{\delta x}\{ A \} ,$$

where

$$A = \frac{G'[f_s-(D-x)][U''(1-G)+\beta(1-\tau)\tau f_s^{\,k} f_n^{\,kk}] + \beta(1-\tau)f_n^{\,kk}[-U'+\beta(1-\tau)Gf_s^{\,k}]}{[U'' + \beta(1-\tau)f_n^{\,kk}]}$$

Three forces attributed to pure debt forgiveness determine the utility gains of the debtor nation. First, there is a direct income effect [G] from increased expected second period transfers. Second, debt forgiveness reduces debt overhang and increases the probability of retaining transfers in the second period $[G'\{f_s-(D-x)\}]$. Third, the increase in seizable sector investment in response to debt overhang increase debtor utility provided that $A > 0$. A sufficient condition for debtor utility to unambiguously increase with debt forgiveness is:

(4.4) $-U' + \beta(1-\tau)Gf_s^{\,k} \geq 0$

Recall that this is precisely equal to zero in the individual consumers maximization problem. If the private investment decision ignores positive externalities associated with investment in the seizable sector, the marginal utility of first period consumption will be uniformly less than the social utility of second period consumption from the national viewpoint. With the existence of such positive externalities condition (4.4) will be satisfied unambiguously. If negative externalities are associated with investment in the seizable sector, debt forgiveness schemes could potentially worsen debtor welfare.

The sufficient conditions for debt forgiveness alternatively can be expressed in terms of the deep parameters of $\delta I_s/\delta x$. Sufficient conditions for forgiveness to increase debtor utility are

(4.5a) $-U' + \beta(1-\tau)Gf_s^{\,k} \geq 0$

(4.5b) $\tau G'(f_s^{\,k})^2 + Gf_x^{\,kk} \leq 0$

Condition (3.5b) may be interpretted as a restriction on the shape of the seizable sector transformation curve.

4.2 Debt Buybacks and Debt–Equity Swaps:

In our model old sector–specific capital is a substitute for new investment. Our argument suggested that a debt repurchase would alter the allocation of investment across the seizable and non–seizable sectors. In this section we show that this reallocation of new investment can have a capital gains effect on the value of old capital equity.

The capital gains effect enhances the possibility of a mutually beneficial debt-equity swap. If an expected debt repurchase improves the terms of equity sales of domestic assets, the two components of a debt-equity swap cannot be separated as is suggested by the Bulow and Rogoff framework. This implies that debt-equity swaps may be in the interest of debtors, even in situations in which pure debt repurchases are not.

Consider the loss experienced by debtors in a buyback program.The value of creditor claims on outstanding debt after a debt repurchase of x are:

$$(4.6)\quad V(x) = E\Big[\min[D - x, f_s]\Big]$$

alternatively:

$$(4.7)\quad V(x) = G\cdot(D - x) + (1 - G)f_s(I_s, x)$$

Letting c(x) be the cost to debtors of a write-down of x dollars of nominal debt, Bulow and Rogoff show that:

$$(4.8)\qquad \frac{x}{c(x)} = \frac{v(\theta,x)}{D - c(x)}$$

where the right-hand side term represents the market price, or average value, of outstanding debt subsequent to a buyback. Bulow and Rogoff show that the average value of outstanding debt is always at least as large as the marginal value of debt: $\partial v(\theta,x)/\partial(D - x)$. Based on this result, Bulow and Rogoff conclude that buybacks are rarely in the interest of debtors relative to investment in the debtor country. The reason is that some level of profitable domestic investment is necessary for a debt overhang to exist. Given this profitable investment opportunity, it is unlikely that a buyback scheme would dominate direct investment from the debtor's point of view.

This result has led to the further conclusion [Bulow and Rogoff (1988) and Froot (1988)] that debt-equity swap schemes are also not in the interest of debtors. The reasoning behind this conclusion is that the first portion of the swap, the equity sale, could take place without the buyback. However, as we have shown above, the proceeds from the equity sale are directly affected by an anticipated upcoming debt repurchase. This leads to the final proposition:

Proposition 4: In the case where debt–equity swaps lead to capital gains on the value of equity sales, the swaps may be in the interest of debtors, even if the "buyback" phase of the transaction is not.

Proof:

Utility in the debtor nation depends on three sources: returns to labor, returns to capital, and transfers:

$$(4.9) \quad U = U_1 + \beta\left[\, w_T + r_T + T \,\right]$$

where:[6]

$$w_T = (1-\tau)[L_s G f_s^{\,L} + L_n f_n^{\,L}]$$

$$r_T = (1-\tau)[K_s^{\,f} G f_s^{\,k} + [(1-\theta)K_n^{\,f} + L_n^{\,f}] f_n^{\,k}$$

$$T = G[\tau f_s - D + x] + \tau f_n$$

and θ is the share of nonseizable sector output sold to foreign investors. To provide an intuitive result on the utitlity effects of debt–equity swaps, we continue by differentiating these components individually. We have shown in proposition 2 that w_T is increasing in x under the sufficient but not necessary condition:

$$(4.10) \qquad L_s f_s^{\,lk} G > L_n f_n^{\,lk}.$$

This condition is quite strong since it would result in an increase to labor as a whole solely as a result of the capital redistribution, even if G did not change. However, since we are interested in the capital gains effect here, we will not assume the sign of $\partial w_T/\partial x$.

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[6]We prove Proposition 4 in terms of a swap of non–seizable sector equity for outstanding debt. The analysis would go through equivalently with seizable sector equity, since equilibrium in capital markets requires that investors are indifferent between holding assets in either sector. To allow for intuition, the analysis is conducted in a disaggregated framework. A more aggregative, albeit less intuitive, derivation is presented in the appendix.

To determine the response of expected transfers to x, differentiation yields:

$$(4.11)\quad \frac{\partial T}{\partial x} = \frac{\partial G}{\partial x}[\tau f_s - D + x] + \left[\, G\tau f_s{}^k \frac{\partial I_s}{\partial x} + \tau f_n{}^k \frac{\partial I_n}{\partial x} \,\right] + G > 0$$

where we can sign $\left[\,\cdot\,\right]$ because capital inflows into the seizable sector exceed outflows from the non-seizable sector. An unambiguous increase in transfers is attributed to the decreased probability of default and increased expected revenues in nondefault states.

Finally, differentiating r_T with respect to θ yields:

$$(4.12)\quad \frac{\partial r_T}{\partial x} = (1-\tau)\Big[\, K_s{}^f f_s{}^k \frac{\partial G}{\partial x} + G f_s{}^{kk} \frac{\partial I_s}{\partial x} K_s{}^f + G f_s{}^k \frac{\partial I_s}{\partial x} + f_n{}^k \frac{\partial I_n}{\partial x}$$

$$+ \left[\, (1-\theta)K_n{}^f + I_n{}^f \,\right] f_n{}^{kk} \frac{\partial I_n}{\partial x}$$

The rental bill, at constant G, is unambiguously increasing in debt forgiveness if the gains to domestic owners of old equity exceeds the losses to old equity owners in the seizable sector. Ex–post, at the new probability of default, equity returns are expected to be equalized.

The overall welfare response is summarized by

$$(4.13)\quad \frac{\partial U}{\partial \theta} = \left\{\, U_1{}' \,\frac{\partial C_1}{\partial x} + \beta\left[\, \frac{\partial w_T}{\partial x} + \frac{\partial r_T}{\partial x} + \frac{\partial T}{\partial x} \,\right]\right\} \frac{\partial x}{\partial \theta} \;-\; \beta(1-\tau)f_n{}^k K_n$$

where the additional term in equation (4.13), $-\ \beta(1-\tau)f_n{}^k K_n$, represents the cost of swapping a θ share of old equity. If debt–write–downs are increasing in equity sales, debtors benefit from debt–equity swaps if the loss of second period earnings and decline in first period consumption are dominated by increased second period wages, rents and transfers.

In the appendix, we show the conditions under which an increase in equity value in the non–seizable sector increases the amount of debt which can be written down per unit of equity swapped. We are of course ruling out the irrelevant range in which $\partial x/\partial \theta$

is negative since in this range there is clearly no hope for a debt–equity swap which would be in the interest of the debtor. $\partial x/\partial \theta$ satisfies:

$$(4.14) \quad \frac{\partial x}{\partial \theta} = \frac{\upsilon(x)}{c'(x)\cdot x + c(x) - \theta\cdot\upsilon'(x)} > 0$$

where c(x) is the cost of repurchasing one unit of debt and $\upsilon(x)$ is the value of one unit of equity. $\upsilon'(x)$ is equal to $(1-\tau)f_n^{kk}\partial I_n/\partial x$.

It can be clearly seen that the "capital gains effect" of the debt–equity swap enters positively in two separate ways: first, it increases $\partial U/\partial x$ by increasing the returns on retained equity in the nonseizable sector; secondly, it increases $\partial x/\partial \theta$ by allowing the debtor nation to write down more debt per unit of equity sold. Ignoring this second effect would underestimate the potential for a successful debt–equity swap.

We can alternatively term $x\cdot c'(x)$ the "buyback effect" as discussed in Bulow and Rogoff. By repurchasing marginal debt at average prices, debtors are pushing up the value of remaining outstanding debt. Clearly, the larger is this effect, the lower is $\partial x/\partial \theta$, and hence the less likely is a debt–equity exchange beneficial to debtors. However, even if the "buyback effect" alone leaves $\partial U/\partial \theta$ negative, the positive capital gains effect might make a debt–equity swap viable.

Proposition 4 is derived in an aggregative framework, ignoring distribution considerations, in the appendix. Again, the capital gains effect is shown to increase the likelihood that debt–equity–swaps will be in the interest of debtors.

4.3 Barriers to Welfare–improving Debt–Equity Swaps

Although equation 4.13 shows that the debt–equity swap may be welfare–improving from the point of view of the debtor nation as a whole, a swap may still not occur. The debtor government may be precluded from engaging in a beneficial debt–equity swap by the revenue raising constraint. The constraint arises within the two-step procedure described above: In the first step, the capital gains accrue to private holders of credit in the sale of domestic equity. In the second step the unprofitable buyback is undertaken by the debtor nation government. Although the net effect on the nation as a whole may be positive, the government cannot undertake the swap unless it can raise the necessary financing.

The swap can be financed domestically only if the expected change in government revenues, conditioned on τ, is sufficient to make up for the loss in the government buyback. To cast the possibility of a swap in its worst light, assume further that domestic debt is unambiguously junior to foreign debt. Then, the debtor government will lose on the debt-equity swap unless:

$$(4.15) \quad \tau \left[\frac{\partial y_n}{\partial \theta} + \frac{\partial y_s}{\partial \theta} \right] \geq \theta f_n^* - \left[\frac{\partial G}{\partial \theta} \cdot x \right]$$

where the left-hand side of equation (4.15) represents the increase in government revenues due to increased output while the right-hand side represents the expense associated with buybacks described by Bulow and Rogoff. Since satisfaction of equation (4.15) clearly requires a large output response within the debtor nation as a whole, the ability of debtors to undertake even potentially-profitable debt-equity swaps is questionable.

5. Conclusions

In this paper, we have weakened the implicit assumption in the literature that all sectors within a debtor nation bear the same exposure to sovereign risk. This minor change has yielded two important distinctions from the "debt-forgiveness" literature which relied on "gunboat technologies" applied within one sector models [Helpman (1988), Bulow and Rogoff (1988)]. First, the misalignment of capital in the presence of a sectorally-distorted overhang has implications for the potential of so-called "market-based debt-reduction schemes." Our model showed that the value of equity in the debtor nation will rise in anticipation of an upcoming debt write–down from any source. In anticipation of a future write–down, capital will move towards its default–risk free allocation, resulting in a capital gain in sectors whose returns on capital are lowered by the debt overhang. This removes the "separability" argument concerning the equity sale and debt repurchase portions of a debt–equity exchange [Froot (1988)]. Since the terms of the equity sale are now positively dependent upon the upcoming anticipated debt write–down, the potential for a debt–equity swap in the debtor's interest are enhanced.

Second, by introducing two sectors, we called into question the notion of debtor nation "welfare". It was clearly demonstrated in the text that some agents may benefit while others are harmed by a debt overhang. In the model above, factors specific to "safe haven" sectors for investment saw their marginal products rise in the presence of a debt overhang. Although it may be argued from a policy point of view that removal of the overhang merely removes an initial distorting relative wage difference, the decline in wages in the safe sector is no less real. These agents have an incentive to maintain the debt overhang, and will resist efforts by the debtor government to retire debt.

Two logistical problems remain concerning the actual swap transaction. First, the government will need funds to acquire domestic equity from its private sector.[7] We have shown above that this may prove an insurmountable problem if the government is unable to raise its domestic tax base. It follows that even potentially "welfare–increasing" (in an additive sense) debt–equity swaps may be precluded by domestic revenue–raising difficulties. Secondly, there is the issue of credibility in the debt–equity swap exchange. We have assumed here that the debtor conducts both portions of the exchange simultaneously. If this were not the case, there would be a credibility problem since, as

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[7]As in Bulow and Rogoff (1988), any completely free revenues would be better spent directly upon investment than on debt repurchase schemes.

shown in Bulow and Rogoff, the debtor loses on the repurchase portion of the debt–equity swap. Given the ability to not conduct the repurchase ex–post, if the debtor would choose to do so, and therefore the capital flows, and the resulting equity capital gains, would not take place.

Official agency intervention can play a crucial role in aiding a successful swap outcome. A bridge loan would allow the debtor to first repurchase debt, and then sell domestic equity. Changing the order of the transactions in this manner would remove the credibility problem and allow for a mutually–beneficial debt–equity swap to take place. However, unless the government's receipts were expected to grow sufficiently for it to service the bridge loan, its desirability from an official agency viewpoint would be unclear.

While the possibility of welfare enhancing debt–equity swaps arises in the two–sector model, this is likely to occur only when there is sufficient resource reallocation among sectors. Since the feasibility conditions are difficult to satisfy in our model, which assumes the extreme conditions that output in one sector is fully seizable by gunboat technology while output in the other sector is immune from seizure threat, it is even less likely that feasible debt–for–equity swaps will occur in a world where all sectors are differentially threatened by less extreme degrees.

Bibliography

Bulow, Jeremy, and Kenneth Rogoff (1988) "The Buyback Boondoggle", Brookings Papers on Economic Activity, 2.

——————, (1989) "Sovereign Debt Repurchases: No Cure for Overhang", N.B.E.R. Working Paper No. 2850, February.

Dooley, Michael P. (1988) "Self–Financed Buy–Backs and Asset Exchanges," I.M.F. Staff Papers, vol. 35, no. 4., 714–723.

Froot, Kenneth A. (1988) "Buybacks, Exit Bonds, and the Optimality of Debt and Liquidity Relief," N.B.E.R. Working Paper No. 2675, August.

Helpman, E. (1988a) "Voluntary Debt Reduction: Incentives and Welfare", N.B.E.R. Working Paper no. 2692, August.

—————— (1988b) "The Simple Analytics of Debt–Equity Swaps and Debt Forgiveness", American Economic Review, forthcoming 79.

Krugman, Paul (1989) "Market–Based Debt Reduction Schemes", International Monetary Fund Staff Papers, 36, forthcoming.

Sachs, Jeffrey (1988) "Conditionality, Debt Relief, and the Developing Country Debt Crisis", Harvard University.

—————— (1988) "The Debt Overhang of Developing Countries", in De Macedo and Findlay eds., Diaz Memorial Volume, Wider Institute, Helsinki.

Sachs, Jeffrey and Harry Huizinga (1987) "U.S. Commercial Banks and the Developing Country Debt Crisis", Brookings Papers on Economic Activity, 2, 555–601.

Appendix

In this appendix, we derive the conditions for a debt-equity swap to be welfare-improving from the point of view of the nation as a whole. Ignoring wealth redistributions, the utility of the nation as a whole may be represented:

$$\text{(A.1)}\quad U = U_1(C_1) + \beta[G(f_s-D+x) + f_n - (1-\tau)\theta f_n^k]$$

Differentiating with respect to θ:

$$\text{(A.2)}\quad \frac{\partial U}{\partial \theta} = \frac{\partial U_1 \partial C_1}{\partial C_1 \partial \theta} + \beta\Big[\frac{\partial G \partial x}{\partial x \partial \theta}(f_s-D+x)+Gf_s^{\,k}\frac{\partial I_s \partial x}{\partial x\ \partial \theta}+G\frac{\partial x}{\partial \theta}+f_n^{\,k}\frac{\partial I_n \partial x}{\partial x\ \partial \theta}$$

$$- (1-\tau)[f_n^k + \theta f_n^{kk}\frac{\partial I_n}{\partial x}\frac{\partial x}{\partial \theta}]\Big]$$

Since $I_s+I_n+C_1 = W$,

$$\text{(A.3)}\quad \frac{\partial I_s \partial x}{\partial x \partial \theta} = -\Big[\frac{\partial I_n}{\partial x} - \frac{\partial C_1}{\partial x}\Big]\frac{\partial x}{\partial \theta}$$

Substituting into (A.2):

$$\text{(A.4)}\quad \frac{\partial U}{\partial \theta} = \Big[\frac{\partial U_1}{\partial C_1} - \beta G f_s^{\,k}\Big]\frac{\partial C_1 \partial x}{\partial x\ \partial \theta} + \beta\Big[(f_n^{\,k}-Gf_s^{\,k})\Big]\frac{I_n \partial x}{\partial x\ \partial \theta} + \beta\Big[\frac{\partial G \partial x}{\partial x \partial \theta}(f_s-D+x)+G\frac{\partial x}{\partial \theta}\Big]$$

$$- (1-\tau)\beta[f_n^k + \theta f_n^{kk}\frac{\partial I_n}{\partial x}\frac{\partial x}{\partial \theta}]\Big]$$

Equilibrium in the capital market requires that $f_n^{\,k}=Gf_s^{\,k}$. The expression therefore simplifies to:

$$\text{(A.5)}\quad \frac{\partial U}{\partial \theta} = \beta\Big[\frac{\partial G \partial x}{\partial x \partial \theta}(f_s-D+x)+G\frac{\partial x}{\partial \theta} + (\tau-1)\beta[f_n^k + \theta f_n^{kk}\frac{\partial I_n}{\partial x}\frac{\partial x}{\partial \theta}]\Big]$$

From above:

$$\text{(A.6)}\quad G'(\theta) = (G')\Big[\tau f_s^{\,k}\frac{\partial I_s}{\partial x} + \tau f_n^{\,k}\frac{\partial I_n}{\partial x} + 1\Big]\frac{\partial x}{\partial \theta}$$

where $[\cdot] > 0$.

By equation (1.14), we can specify $I_n^{\,k}(x)=QI_s^{\,k}(x)$ where:

$$\text{(A.7)}\quad Q = \frac{-U''}{U'' + \beta(1-\tau)f_n^{\,kk}}$$

where $0>Q>-1$. Note that Q is increasing in absolute value in U'' and decreasing in β and f_n^{kk}. Substituting into (A.7):

$$\text{(A.8)} \quad \frac{\partial U}{\partial \theta} = \beta\Big[(G')\Big[\tau(f_s^k+Qf_n^k)\frac{\partial I_s}{\partial x} + 1 \Big] (f_s-D+x)\frac{\partial x}{\partial \theta} + \beta G\frac{\partial x}{\partial \theta}$$

$$- (1-\tau)\beta[f_n^k + \theta f_n^{kk} \frac{\partial I_n}{\partial x}\frac{\partial x}{\partial \theta}]\Big]$$

Rearranging terms and letting $Z=f_s-D+x$:

$$\text{(A.10)} \quad \frac{\partial U}{\partial \theta} = \beta\Big[\frac{\partial I_s}{\partial x} [G'\tau(f_s^k+Qf_n^k)Z] +(\tau-1)\theta f_n^{kk}Q]+G'Z+G \Big] \frac{\partial x}{\partial \theta} + \beta(\tau-1)f_n^k$$

By equation (1.15) we can specify $H=\partial I_s/\partial x$ where:

$$\text{(A.11)} \quad \frac{\partial I_s}{\partial x} = \frac{-f_s^kG'(U'' + \beta(1-\tau)f_n^{kk})}{U''[\tau G'f_s^k(1-G)-(Gf_s^{kk}+f_n^{kk})]-\beta(1-\tau)f_n^{kk}[Gf_s^{kk}-\tau G'(f_s^k)^2]} = H>0$$

Recall that $H>0$.

Examining (A.11) more closely, it is clear that the impact of "deep parameters" such as U'', f_s^k, and f_n^{kk} is ambiguous. However, $\partial I_s/\partial x$ is unambiguously decreasing in f_n^k.

Substituting into (A.8):

$$\text{(A.12)} \quad \frac{\partial U}{\partial \theta} = \beta\Big[(G')Z[H\tau(f_s^k+ Qf_n^k)+1]-(1-\tau)\theta f_n^{kk}QH+G \Big] \frac{\partial x}{\partial \theta} + \beta(\tau-1)f_n^k$$

A special case of (A.12) is $x>0$ and $\theta=0$, i.e. pure debt forgiveness. Debtor utility is increasing in pure debt forgiveness when:

$$\text{(A.13)} \quad \frac{\partial U}{\partial x} = \beta\Big[(G')Z[H\tau(f_s^k+ Qf_n^k)+1]+G \Big] > 0$$

It is clear that in order for a debt-equity swap to be in debtor interest, a pure debt write-down must be in the interest of the debtor. Moreover, since $-(1-\tau)\theta f_n^{kk}QH > 0$ in the case of a debt-equity swap, we can conclude that $[\cdot] > 0$ in equation (A.12). It follows that $\partial U/\partial \theta$ is increasing in $\partial x/\partial \theta$.

The budget constraint faced by the debtor nation in conducting a debt for equity swap is:

$$\text{(A.14)} \quad (x)\cdot\theta = c(x)\cdot x$$

Totally differentiating with respect to θ and x we obtain:

$$\text{(A.15)} \quad \frac{\partial x}{\partial \theta} = \frac{\upsilon(x)}{c'(x)\cdot x + c(x) - \theta\cdot\upsilon'(x)}$$

The value of a nominal unit of debt satisfies:

$$\text{(A.16)} \quad c(x) = G(D-x) + (1-G)f_s$$

Differentiating with respect to x:

$$\text{(A.17)} \quad c'(x) = (1-G)f_s^{*}\frac{\partial I_s}{\partial x} - G - G'(x)(D-x)$$

The value of a unit of equity satisfies:

$$\text{(A.18)} \quad \upsilon(x) = (1-\tau)Gf_n'$$

Differentiating with respect to x:

$$\text{(A.19)} \quad \upsilon'(x) = (1-\tau)\left[G'(x)f_n^{*} + Gf_n^{**}Q\frac{\partial I_s}{\partial x} \right]$$

Substituting into (A.15):

$$\frac{\partial x}{\partial \theta} = \frac{(1-\tau)Gf_s^{*}}{[(1-G)f_s^{*}-\theta(1-\tau)f_s^{**}Q]H\cdot x+G(D-x)+(1-G)f_s-G'[\tau Hf_s^{*}(1+Q)+1][D-x+\theta(1-\tau)f_n^{*}]- G}$$

(A.20)

It can be seen that $\partial x/\partial\theta$ may be either increasing or decreasing in H, or $\partial I_s/\partial\theta$. This depends upon whether $c'(x)$ is greater than $\upsilon'(x)$. Both are positive since a debt write-down will increase the value of outstanding debt, increasing $c(x)$, and the outflow of capital from the non-seizable sector will increase the value of equity in that sector, increasing $\upsilon(x)$. We can say that $\partial x/\partial\theta$ is increasing in $\partial I_s/\partial x$ when:

$$\text{(A.21)} \quad (1-G)f_s^{*} < \theta(1-\tau)f_s^{**}Q+G'[\tau f_s^{*}(1+Q)+1][D-x+\theta(1-\tau)f_n^{*}]$$

However, satisfaction of (A.20) would also imply that $\partial x/\partial\theta<0$. Since debt-equity swaps would clearly not be in the interest of the debtor nation under these conditions, we can conclude that in the relevant range $\partial x/\partial\theta$ is decreasing in $\partial I_s/\partial\theta$.
Substituting into (A.12):

$$\text{(A.22)} \quad \frac{\partial U}{\partial \theta} = \frac{\beta\left[(G')Z[H\tau f_s^{*}(1+Q)+1]-(1-\tau)\theta f_n^{**}QH+G \right](1-\tau)Gf_s^{*}}{[(1-G)f_s^{*}-\theta(1-\tau)f_s^{**}Q]H-G'[\tau Hf_s^{*}(1+Q)+1][D-x+\theta(1-\tau)f_n^{*}]-G} + \beta(\tau-1)f_n^{*}$$

From the discussion above, it can be seen that the effect of the responsiveness of investment, as measured by $\partial I_s/\partial\theta$, on $\partial U/\partial\theta$ is ambiguous. This is due to the fact that while $\partial U/\partial x$ is increasing in $\partial I_s/\partial\theta$, $\partial x/\partial\theta$ is decreasing in $\partial I_s/\partial\theta$.