#### NBER WORKING PAPER SERIES

ASSET PRICES AND INTEREST RATES IN CASH-IN-ADVANCE MODELS

Alberto Giovannini

Pamela Labadie

Working Paper No. 3109

NATIONAL BUREAU OF ECONOMIC RESEARCH 1050 Massachusetts Avenue Cambridge, MA 02138 September 1989

We are grateful to John Donaldson and to participants of the 1989 NBER Summer Institute for comments. Alberto Giovannini was partially supported by a CEPR grant from the Ford Foundation and the Alfred P. Sloan Foundation. This paper is part of NBER's research program in Financial Markets and Monetary Economics. Any opinions expressed are those of the authors not those of the National Bureau of Economic Research.

NBER Working Paper #3109 September 1989

#### ASSET PRICES AND INTEREST RATES IN CASH-IN-ADVANCE MODELS

#### ABSTRACT

This paper develops a method to solve and simulate cash-in-advance models of money and asset prices. The models are calibrated to US data spanning the period from 1890 to 1987 and are used to study some empirical regularities observed in the US data over this period. The phenomena which are the focus of the paper include the average level of stock returns and returns on nominal bonds, the covariation of realized real interest rates and real asset returns with inflation, and the ability of nominal interest rates to predict inflation and nominal stock returns.

Alberto Giovannini CEPR, NBER, and Columbia University 622 Uris Hall Graduate School of Business New York, NY 10027

Pamela Labadie Columbia University 622 Uris Hall Graduate School of Business New York, NY 10027

#### Introduction 1

Empirical regularities involving nominal interest rates, asset prices and inflation are determined by the role and effects of money in the economy. In this paper we ask whether or not popular models of money and asset prices can help to interpret those regularities. The models we employ are general equilibrium, stochastic models. They are ideally suited to describe explicitly and to evaluate the distortions produced by monetary policy. They cannot, however, be solved analytically, given any realistic assumptions about the stochastic behavior of exogenous shocks. The general Markov structure that we assume for the forcing variables does not allow us to perform comparative dynamics analysis without resorting to numerical simulations. Our strategy is to develop an algorithm for computing equilibria produced by the models, to simulate them, to ask whether or not the simulated data resemble the actual data, and to intepret differences between reality and simulations.

The main virtues of general equilibrium, stochastic models of money and asset prices are their internal consistency and the simplicity of their structure. Their potentially serious drawback, of course, is a lack of "realism". The simple structure of markets and transactions underlying the equations can at best be considered approximations to reality. Our objective in this paper is to determine whether these "approximations" are acceptable for the purpose of interpreting the comovements of inflation, interest rates, and stock returns. We think it is useful to evaluate the empirical predictions of these models for two reasons. First, their simplicity allows us to interpret the results of numerical simulations more easily. Second. as Prescott (1986) stresses, once internal consistency is regarded as a necessary condition, it is better to start from simple models: their ability to explain empirical regularities may help to sort out what are the important effects that macroeconomic models should include, and what omissions or approximations are acceptable.

The models we simulate are the representative-agent, cash-inadvance models developed by Lucas (1982) and by Svensson (1985). Our choice is mainly motivated by our intention to provide the closest formal analog to real models of asset prices commonly used to interpret US data, which employ the infinite-horizon, representative-

 $\mathbf{1}$ 

agent specification. Furthermore, alternatives like the overlappinggenerations models rely on assumptions very similar to those we employ to make agents willingly hold an asset, like money, that is dominated by other real assets.<sup>1</sup> Finally, it is known that cash-inadvance models and money-in-the-utility-function models can be reconciled by appropriately parametrizing tastes and technology.<sup>2</sup>

This paper is complementary to that of Hodrick, Kocherlakota and Lucas (1989), who ask whether models from the same family as those we study can explain the observed behavior of the velocity of money in the United States, and to those of Kydland (1987) and Cooley and Hansen (1988), who study the effects of introducing money in real business cycle models. Like the papers of Leroy (1984a,b), Danthine and Donaldson (1986) and Marshall (1988), our paper discusses the ability of representative-agent monetary models to explain correlations between inflation, nominal interest rates, and stock returns. Leroy (1984a,b) first formally applied general-equilirbium models of money in the utility function to analyze the relations between inflation and asset returns. He considers only two-state models where the source of randomness is either the endowment process or the money supply process. Danthine and Donaldson also study a similar model of money in the utility function with a more general stochastic structure, where inflation uncertainty is a function of endowment uncertainty, but money growth is nonstochastic: hence, in their model, all sources of fluctuation of the price level are money demand shocks. Marshall employs a model where money demand arises from a transactions costs function in the budget constraint. By contrast, we consider an economy where money supply is stochastic, and money demand arises from a cash-in-advance constraints. While Marshall studies his model under simplifying assumptions on the forcing variables' processes, we assess the predictive power of our models by simulating them with processes for the forcing variables that are estimated

<sup>&</sup>lt;sup>1</sup>See, for example, Sargent and Wallace (1982).

<sup>&</sup>lt;sup>2</sup>This result is due to Feenstra (1986). As Ostroy and Starr (1988) point out, however, that equivalence relies on the assumption that for every dollar received for sales of labor there is a dollar with which to buy commodities-i.e. that money buys goods and services, but goods and services do not buy other goods and services. It thus appears more adequate to make the constraint explicit through cash-in-advance equations.

from the US data.<sup>3</sup>

Section 2 presents the data set and the empirical regularities we choose to discuss. Section 3 describes the models and establishes the notation. Section 4 constructs an algorithm that solves both models. Section 5 discusses the results of the simulations. Section 6 contains a few concluding remarks.

#### $\mathbf{2}$ Data and Empirical Regularities

Our objective is to point to empirical facts that are sufficiently "general" and, broadly speaking, largely independent of changes in monetary institutions, since the model we use to assess these regularities are clearly ill-equipped to deal with the institutional changes underlying many important episodes of US monetary history. For this reason, we chose the long sample of Grossman and Shiller (1981) and Mehra and Prescott (1985), and augment it with the estimates of the US money stock provided by Friedman and Schwartz (1982).

The data set includes:

- · Annual Average S&P Composite Stock Price Index. 1889-1975 from the Mehra-Prescott data set.<sup>4</sup> 1976-1987 from Citibank Database
- Annual Dividends from S&P Series. 1889-1975 from the Mehra-Prescott data set. 1976-1987 from Citibank Database.
- Nominal Yield on Short Term Securities. 60- and 90-day prime commercial paper prior to 1920, Treasury certificates for the 1920-1930 period, and 90-day Treasury Bills for the 1931-1975 period (Mehra-Prescott data). 1976-1987: 90-day Treasury Bills from Citibank Database (annual averages).
- · Per Capita Consumption of Nondurables. Kuznets-Kendrik USNIA.1889-1975 from the Mehra-Prescott data set. 1976-1987 from Citibank Database.

<sup>&</sup>lt;sup>3</sup>The differences between our solution algorithm and those employed by Marshall and Hodrick, Kocherlakota and Lucas will be highlighted below. The basic logic of our algorithm is similar to the one by Danthine and Donaldson.

<sup>&</sup>lt;sup>4</sup>We thank Rajnish Mehra for kindly providing us with the data set.

- · Consumption Deflator, measured in 1972 dollars, from Kuznets-Kendrik USNIA, 1889-1975 from the Mehra-Prescott data set. 1976-1987 from Citibank Database.
- Money Stock. Sums of currency held by the public plus adjusted deposits at all commercial banks less large negotiable CDs since 1961, divided by population. 1889-1958 from Friedman and Schwartz (1982): annual averages of monthly data. The data for 1958-1987 are constructed following the methods outlined in Friedman and Schwartz (1970). We use the definition of M2 from the Statistical Releases of the Board of Governors of the Federal Reserve System which is available in the Citibank Database (series MS2), and subtract from it "small denomination time deposits at thrift institutions" (series MSTT) and "savings deposits at thrift institutions" (series MSVT). The monthly data are averaged to obtain annual data. The rates of growth of this series match very closely those of the corresponding series from Friedman and Schwartz in the years of overlap: 1959 to 1975. As Friedman (1988) points out, however, in 1983 the money supply series calculated this way displays exceptional rates of growth, largely because of shifts out of savings accounts and into money-market accounts. These shifts net out in the Federal Reserve definition of M2. Following Friedman (1988) we correct for this problem by updating the net-of-savings-accounts series constructed above with the growth rate of the Fed M2 series from 1983.

The choice of the broader monetary aggregate is justified by two considerations. First, as Friedman and Schwartz (1963, 1982) convincingly argue, M2 is preferred because it has undergone fewer redefinitions over the sample period. Second, as Hodrick, Kocherlakota and Lucas (1989), among others, have pointed out, at least in the second postwar period, M1 velocity clearly displays nonstationarity, a feature inconsistent with the basic assumptions of the models we are exploring. The nonstationarity of M1 velocity might be due to the redefinitions of that aggregate, but also to technological progress in the transactions technology, a feature that our model does not capture.

There is, of course, no firm criterion to determine—in a sample that includes annual data from 1889 to 1987-what represents an empirical regularity. Indeed, it could be argued that, over such a long historical period, very few phenomena of interest involving money and asset prices have maintained the same characteristics from start to end. Our choices do not conform to a single criterion. We regard some of the phenomena we look at as important because of the statistical significance of certain correlations in the data, while other phenomena are-in our view-equally interesting for the opposite reason: certain correlations are statistically insignificant. Furthermore, several of the phenomena we study are included because of the great interest they have aroused in finance and monetary theory, even though they might have previously been studied on much shorter sample intervals. The reason why we include them is that many of the theories offered to explain them are largely independent of institutional factors.<sup>5</sup> Finally, we let the data suggest the appropriateness of studying subsets of our sample period, by simple visual examination of the plots of all series.

These plots are contained in Figures 1 to 4. Figure 1 shows realized nominal returns on stocks and the nominal interest rate. Except for the large swings in stock returns in the 1930s, these data do not display appreciable patterns over the sample.

Figures 2 and 3 report growth rates in real per-capita consumption and per-capita money balances. The variance of the consumption data decreases remarkably after World War II. To some extent, the variance of money supply growth also appears to decrease in the second postwar. While the models we study can, in principle, produce marked changes in conditional variances of money growth and consumption growth, we interpret these figures as suggesting the presence of potentially important differences in the data before and after the end of World War II. Hence we evaluate the predictions of the monetary models both over the whole sample and, separately, for the second postwar period.<sup>6</sup>

<sup>&</sup>lt;sup>5</sup>This, however, is not true in many important cases, including all models of nominal nonneutralities based on features of the US tax system.

<sup>&</sup>lt;sup>6</sup>This strategy is further justified by the work of Romer (1986), suggesting the presence higher sampling errors in the pre-war consumption data.

Figure 4 reports the rates of growth of velocity (in terms of nondurables consumption). Once again we observe some differences before and after World War II: while the average growth rate of velocity is close to zero over the whole sample, it is lower in the first half than in the second half.

Table 1 reports summary statistics from the data in our sample. One remarkable fact documented in the Table is the similarity of the summary statistics in the whole sample with those in the second part of the sample, despite the significant changes in economic and monetary institutions in the recent years. The more recent period, however, is characterized by higher and more volatile inflation, higher and less volatile stock returns, and higher and more volatile nominal interest rates. Consumption velocity has on average hardly changed, although the variability of velocity (as measured by the standard error) in the second part of the sample is half of that in the whole sample. This decrease in the variability of velocity is consistent with the decrease in volatility of consumption growth and money growth documented in Figures 2 and 3. One important feature of the data in Table 1 is the difference between the average realized real return on stocks (7.16 percent in the whole sample, and 7.83 percent in the second postwar) and the average realized real interest rate (1.03 in the whole sample, and 0.36 percent in the second postwar years). This difference, also called "equity premium", is 6.13 percent in the whole sample, and 7.47 percent in the second postwar period. Mehra and Prescott (1985) showed that traditional asset pricing models can generate equity premia that are only a small fraction (of the order of 1-tenth) of those observed in the data. Mehra and Prescott used a "real" (that is without money) version of the models we study here. We wish to determine whether the introduction of money modifies the conclusions of Mehra and Prescott.

We turn next to the comovements of interest rates, stock returns, and inflation. Table 2 describes the covariation of realized real interest rates and stocks returns with inflation. These are contemporaneous correlations: inflation, real stock returns and real interest rates are all measured from time  $t$  to  $t + 1$ . The Table shows that, when inflation is high, realized real stock returns and interest

rates are low and viceversa. The relationship between realized real interest rates and inflation is documented, for the postwar period, by Fama and Schwert (1977) and Mishkin (1981) (among others). Ibbotson and Sinquefield (1976) and Summers (1983) document it over the period 1926-74 and 1860-1979, respectively. The coefficients in Table 1 largely confirm the results of these authors. In particular, the larger negative correlation between real interest rates and inflation in the years before World War II is noted by Summers (1983). The relationship between realized stock returns and inflation is documented by Fama and Schwert (1977), Jaffe and Mandelker (1976), Nelson (1976), Schwert (1981), and Summers (1983) for the second postwar period. Summers (1983) also looks at the period 1870-1979, Kaul (1987) studies the 1930's. These findings are again consistent with ours. Summers (1983) claims that, over low frequencies, the negative relation between stock returns and inflation is much stronger in the 1970s, a decade characterized by historically high inflation, and a depressed stock market.<sup>7</sup> This fact is reflected in the difference between the estimated coefficients in the two samples: the (negative) response of realized stock returns to inflation in the second postwar period is more than three times larger than in the whole sample.

In Table 3 we document the relation between nominal interest rates and subsequent inflation and nominal stock returns. We regress inflation and nominal stock returns from year  $t$  to  $t + 1$ on the nominal interest rate in year t. Without theoretical priors, these regressions can be thought of measuring the forecasting ability of nominal interest rates, a question of interest in its own right. Under the assumption that economic agents correctly use the available information to form expectations, so that inflation and stock-returns innovations are orthogonal to agents' expectations (the rational-expectations assumption), these regressions provide information about the comovements of of ex-ante real interest rates and stock returns and expected inflation. The relation between interest rates and subsequent inflation is discussed by Fama (1975), Fama and Gibbons (1982), Mishkin (1981, 1989) and Sum-

<sup>&</sup>lt;sup>7</sup>Summers uses band-spectral regressions which filter out frequencies shorter than 5 years. Indeed, he estimates positive coefficients in a regression (estimated over 1870-1940) of inflation on realized stock yields.

mers (1983). The pre-war evidence is studied by Barski (1987) (who emphasizes, like Mishkin (1989) the shifts in the stochastic properties of inflation) and Summers (1983). Our evidence is again consistent with the findings in the literature: the coefficient of the nominal interest rate is significantly less than 1. Under the rational expectations hypothesis, this evidence suggests that real interest rates are not constant over time, and might be negatively correlated with expected inflation.<sup>8</sup>. Indeed, nominal interest rates appear to be an unbiased predictor of subsequent inflation only in certain periods (in particular during the years from 1951 to 1979) but the relation is not stable over longer samples: the low values of the Durbin-Watson statistics in Table 3 suggest the presence of in-sample instability of the equation. The bottom panel of the Table reports regressions of nominal stock returns (from time t to time  $t + 1$  on the nominal interest rate at time t. The estimated coefficients are negative but insignificant. The R-square statistics indicate that nominal interest rates are very poor predictors of stock returns. Similar results have been reported by Fama and Schwert (1977), who use monthly and quarterly data from 1953 to 1977, and Giovannini and Jorion (1987) who use weekly data from 1973 to 1984. These authors estimate negative, but significant coefficients; their R-square statistics never exceed 3 percent.

#### 3 The Models

The two monetary models we simulate are Lucas's (1982) and Svensson's (1985). The models are characterized by representative agents maximizing a time-separable isoelastic utility function, subject to a liquidity constraint, which compels them to buy goods with money. and a wealth constraint. Endowment is stochastic and exogenous and growing over time. There is no storage or investment technology available. Money supply is also exogenous and stochastically growing, and is distributed lump-sum to the agent. The crucial difference between the two models is in the timing of transactions in goods and asset markets.

<sup>&</sup>lt;sup>8</sup>In section 5 we identify and discuss all the components in the slope coefficients of these regressions.

In Lucas's model individuals can acquire money after observing the state of the economy but before purchasing the consumption good; hence given their risk and return characteristicsmoney and assets are equally suitable to intertemporal consumption smoothing. In Svensson's model, by contrast, individuals begin the period with predetermined money balances, which they need to purchase the consumption good. This feature introduces a wedge between the marginal utility of consumption and the marginal utility of wealth. Money is not perfectly substitutable with other assets for the purpose of intertemporal consumption smoothing, it is more "liquid" than other assets.<sup>9</sup>

Formally, the consumer's problem in the Lucas model is:

$$
\max_{\{c_t, x_t, M_t^d\}} E_0 \left[ \sum_{t=0}^{\infty} \delta^t \frac{1}{1 - \gamma} c_t^{1 - \gamma} \right] \tag{1}
$$

subject to:

$$
c_{t} \leq M_{t}^{d} \pi_{t}
$$
\n
$$
(2)
$$
\n
$$
M_{t}^{d} \pi_{t} + z_{t} q_{t} \leq \left( \frac{\pi_{t}}{\pi_{t-1}} y_{t-1} + q_{t} \right) z_{t-1} + \left( \omega_{t} - 1 \right) M_{t-1} \pi_{t} + \left( M_{t-1}^{d} \pi_{t} - c_{t-1} \frac{\pi_{t}}{\pi_{t-1}} \right)
$$
\n
$$
(3)
$$

The notation is the standard one: notice, in particular, that we use  $\pi$  to indicate the inverse of the price level, i.e. the purchasing power of money; z stands for the shares of the productive asset— $\ddot{a}$ claim to future dividends-held and demanded by the consumer. Equation  $(2)$  is the liquidity constraint while equation  $(3)$  is the wealth constraint. The evolution of exogenous variables is:

$$
y_t = \eta_t y_{t-1} \tag{4}
$$

$$
M_t = \omega_t M_{t-1} \tag{5}
$$

The timing of transactions is as follows. At the beginning of a period, individuals learn the realizations of the monetary and endowment shocks, respectively  $\omega$  and  $\eta$ , and receive their monetary

<sup>&</sup>lt;sup>9</sup>See Giovannini (1989). For this reason the model gives rise to a form of "precautionary" money demand.

transfer, which in real terms is  $(\omega_t - 1)M_{t-1}\pi_t$ . In the assets market, they obtain dividend payments from "firms",  $z_{t-1} \frac{\pi_t}{\pi_{t-1}} y_{t-1}$ , and the value of their stock holdings,  $q_t z_{t-1}$ .<sup>10</sup>

They use these resources to purchase money balances and stocks. In the goods market they use currency to purchase the consumption good. Notice that the money balances turned over by consumers to the "firm" are held by it until the asset market opens at the beginning of the next period. Hence the inflation tax is levied directly on the firm.

The market clearing conditions are:

$$
c_t = y_t \tag{6}
$$

$$
M_t^a = M_t \tag{7}
$$

$$
z_t = 1 \tag{8}
$$

In the Svensson model, the monetary transfer, which is observed at the beginning of each period, is received in the assets market. However, goods trade occurs before asset trade. Consumers use their money balances at the beginning of the period to purchase the consumption good, and then enter the assets market with any remaining cash balances. The money stock evolves as:

$$
M_t = \omega_{t-1} M_{t-1} \tag{9}
$$

which implies that at time t agents know the nominal stock of money available to purchase goods at time  $t + 1$ . The consumer's problem is:

$$
\max_{\{c_t, z_t, M_t^d\}} E_0 \left[ \sum_{t=0}^{\infty} \delta^t \frac{1}{1 - \gamma} c_t^{1 - \gamma} \right] \tag{10}
$$

subject to:

$$
c_t \leq M_t^d \pi_t \tag{11}
$$

$$
M_{t+1}^d \pi_t + z_t q_t \leq [(y_t + q_t)z_{t-1}] + (\omega_t - 1)M_t \pi_t + (M_t^d \pi_t - c_t)
$$
\n(12)

The sequence of market equilibrium conditions is, of course, identical to equations  $(6)$  to  $(8)$ .

<sup>10</sup> This interpretation differs from Lucas's, where equities' dividend payments occur in the same period, although they can be spent only in the next period.

In this section we derive the first-order necessary conditions and sufficient conditions for existence and uniqueness of the equilibrium. These conditions are the basis for the algorithm used to compute the equilibrium realizations of the model-determined variables. Both versions of the model can be formulated as a dynamic program with unbounded returns.

Since the endowment is growing over time and the isoelastic utility function is unbounded, the maximization problem posed in equations (1) and (10) may not be well defined because total expected utility may be infinite. To ensure finite expected utility, we assume that:

$$
\lim_{t \to \infty} \delta^t E_0 y_t^{1-\gamma} = 0 \tag{13}
$$

An equilibrium for either version of the model is a set of functions:  $q, \pi$ , a value function, and associated multiplier functions  $\mu$ (the liquidity constraint multiplier) and  $\lambda$  (the wealth constraint multiplier). Given the  $\pi$  and  $q$  functions, a representative agent can solve his maximization problem. For each set of price functions, standard arguments can be used to show that there is a unique, continuous and bounded value function, which can then be used to construct the multiplier functions associated with the problem. The main task is then to compute the pair of equilibrium price functions.

For notational consistence, we denote the state of the economy at time  $t$  as  $s_t$ . For the Lucas version of the model, any pair of equilibrium price functions must satisfy the first order conditions of the individual optimization problem together with the market equilibrium conditions. Substituting market equilibrium conditions into the first order conditions with respect to  $c_t$ ,  $M_t^d$  and  $z_t$  we have:

$$
y_t^{-\gamma} = \lambda(s_t) \tag{14}
$$

$$
\lambda(s_t)\pi(s_t) = \delta E_t [\lambda(s_{t+1})\pi(s_{t+1})] + \mu(s_t)\pi(s_t) \qquad (15)
$$

$$
\lambda(s_t)q(s_t) = \delta E_t \left[ \lambda(s_{t+1})(q(s_{t+1}) + y_t \frac{\pi_{t+1}}{\pi_t}) \right]
$$
(16)

Since  $\lambda$  is already determined by (14), we use (15) and the cashin-advance constraint to compute  $\pi$ . This information is then used to determine the equilibrium equity price function from (16).

We start by defining implicitly a function  $K$  as the inverse of velocity:

$$
\pi(s_t) = \frac{y_t K(s_t)}{M_t}
$$

By the implicit function theorem, we can study the properties of the function K as well as the function  $\pi$  since y and M are strictly positive and are determined exogenously. The cash-inadvance constraint (2) implies that the value of the function  $K$ for any state cannot fall below unity (since this violates the lower bound on the inverse of the price level imposed by the constraint). If  $K(s_t)$  exceeds 1, the cash-in-advance constraint is nonbinding at  $s_t$  while, if  $K(s_t)$  equals 1, the constraint is binding.

Substituting for  $\pi$  in (15) and using (14) we obtain after some simplification,

$$
K(s_t)\mu(s_t) = y_t^{-\gamma} \left( K(s_t) - \delta E_t \left[ \frac{\eta_{t+1}^{1-\gamma} K(s_{t+1})}{\omega_{t+1}} \right] \right) \tag{17}
$$

Our iterative procedure is based on equation (17). If the cashin-advance constraint is not binding,  $\mu_t = 0$ ,

$$
K(s_t) = \delta E_t \left[ \frac{\eta_{t+1}^{1-\gamma} K(s_{t+1})}{\omega_{t+1}} \right]
$$
 (18)

On the other hand, if the cash-in-advance constraint is binding at  $s_t$ ,  $\mu_t > 0$ ,  $K(s_t) = 1$  and

$$
\mu(s_t) = y_t^{-\gamma} \left( 1 - \delta E_t \left[ \frac{\eta_{t+1}^{1-\gamma} K(s_{t+1})}{\omega_{t+1}} \right] \right) \tag{19}
$$

This reasoning suggests that the function  $K$  at  $s_t$  is:

$$
K(s_t) = \max\left(1, \delta E_t\left[\frac{\eta_{t+1}^{1-\gamma}K(s_{t+1})}{\omega_{t+1}}\right]\right) \tag{20}
$$

To study the properties of the function  $K$ , we start by defining the operator  $S_1$  by:

$$
S_1 K(s_t) = \delta E_t \left[ \frac{\eta_{t+1}^{1-\gamma} K(s_{t+1})}{\omega_{t+1}} \right]
$$
 (21)

If K is a continuous, bounded, and nonnegative function, then the operator in (21) is well defined. We define a second operator  $T$  by

$$
TK(s_t) = \max(1, K(s_t)),\tag{22}
$$

so that the composite operator is:

$$
T \cdot S_1 K(s_t) = \max(1, S_1 K(s_t)) = \max\left(1, \delta E_t \left[ \frac{\eta_{t+1}^{1-\gamma} K(s_{t+1})}{\omega_{t+1}} \right] \right) \tag{23}
$$

Let  $H_1$  denote the composite operator  $T \cdot S$ . The following theorem shows that, under certain conditions, the composite operator  $H_{1}^{n}$  (the operator applied *n* times) is a contraction mapping. Let  $\zeta$ denote the space of continuous, bounded functions that are defined over the state space, which has a supremum norm associated with it. The properties of the solution to the functional equation (23) are described by the following

Theorem 1 If  $E_0\left[\frac{n_{i+1}^{1-\gamma}}{\omega_{i+1}}\right]<\frac{1}{\delta}$ , there exists exactly one continuous bounded function  $K$  that solves equation (23).

Proof: See the Appendix.

In the proof we show that  $H_1$  is a contraction, hence we can use the method of successive approximations to find the fixed point  $K$ . Once the fixed point is found, the equilibrium purchasing power of money is determined as

$$
\pi(s_t) = \frac{y_t K(s_t)}{M_t},
$$

where K is the fixed point of  $H_1$ . The function  $\pi$  is used in (15) to determine the equilibrium multiplier function  $\mu$ .

An important property of the solution is that the fixed point  $K$ is a function of a subset of the state space, namely  $(\eta, \omega)$ , and is not a function of the levels of the endowment or of the money stock (see the proof in the Appendix). This property greatly simplifies the computation of the equilibrium.

Furthermore, the condition used to establish the existence and uniqueness of the fixed point provides some valuable insight into the sufficient conditions for the existence of an equilibrium where money is valued. If  $E_t[\eta_{t+1}^{1-\gamma}/\omega_{t+1}] < 1/\delta$  for all states, the cashin-advance constraint is always binding. If the unconditional expectation,  $E_0[\eta_{t+1}^{1-\gamma}/\omega_{t+1}] > 1/\delta$ , a monetary equilibrium does not exist because the cash-in-advance constraint is never binding. For an equilibrium to display variable velocity (alternating between a binding and a nonbinding cash-in-advance constraint), the argument above suggests that  $E_0[\eta_{t+1}^1/\omega_{t+1}] < 1/\delta$  must be true, but  $E_i[n_{i+1}^{1-\gamma}/\omega_{i+1}] > 1/\delta$  should hold in some states.

The equilibrium  $K$  function is then used to construct the equilibrium equity price. The equity price function q must satisfy the functional equation:

$$
y_t^{-\gamma}q(s_t) = \delta E_t \left[ y_{t+1}^{-\gamma} (q(s_{t+1}) + y_t \frac{\pi_{t+1}}{\pi_t}) \right].
$$
 (24)

Using the definition of  $K$ , we have

$$
y_t^{-\gamma}q(s_t) = \delta E_t \left( y_{t+1}^{-\gamma}q(s_{t+1}) + \delta E_t \left[ \left( \frac{y_{t+1}^{1-\gamma}}{\omega_{t+1}} \right) \frac{K(s_{t+1})}{K(s_t)} \right].
$$
 (25)

Factoring out  $y_t^{1-\gamma}$  from the two sides of the equation, and letting  $\xi_t = q_t/y_t$  (the price/earnings ratio), we have:

$$
\xi(s_t) = \delta E_t \left( \eta_{t+1}^{1-\gamma} \xi(s_{t+1}) \right) + \delta E_t \left[ \left( \frac{\eta_{t+1}^{1-\gamma}}{\omega_{t+1}} \right) \frac{K(s_{t+1})}{K(s_t)} \right]. \tag{26}
$$

We iterate on the function  $\xi$ . Define the operator  $T_1$  as:

$$
T_1\xi(s_t) = \delta E_t \left( \eta_{t+1}^{1-\gamma} \xi(s_{t+1}) \right) + \delta E_t \left[ \left( \frac{\eta_{t+1}^{1-\gamma}}{\omega_{t+1}} \right) \frac{K(s_{t+1})}{K(s_t)} \right]. \tag{27}
$$

The mapping  $T_1$  takes continuous bounded functions into continuous bounded functions. If we start with an initial guess that is in the space of continuous bounded functions, the application of the operator  $T_1$  results in a function that is an element of the same space. The operator  $T_1$  is monotone but does not display the discounting property when applied to any arbitrary function in  $\zeta$ . When  $T_1$  is applied repeatedly there is some positive integer *n* such that  $T_1^n$  is a contraction mapping. The stock price function can now be solved in two steps. In the first step we use equation (26) to solve for the price/earnings ratio, the function  $\xi$ . Notice that, once again, we are able to limit the domain of  $\xi$  to a subset of the state space. Finally we obtain the stock price simply by multiplying  $\xi$  times  $y_t$ .

For the Svensson version of the model, the first-order conditions and the market equilibrium conditions imply:

$$
y_t^{-\gamma} = \mu(s_t) + \lambda(s_t) \tag{28}
$$

$$
\lambda(s_t)\pi(s_t) = \delta E_t [(\lambda(s_{t+1}) + \mu(s_{t+1}))\pi(s_{t+1})]
$$
 (29)

$$
\lambda(s_t)q(s_t) = \delta E_t \left[ \lambda(s_{t+1})(q(s_{t+1}) + y_{t+1}) \right] \tag{30}
$$

Substituting our definition of  $K$  into equation (29), we have

$$
\lambda(s_t)K(s_t) = \frac{\delta y_t^{-\gamma}}{\omega_t} E_t[\eta_{t+1}^{1-\gamma} K(s_{t+1})]. \tag{31}
$$

Multiplying both sides of (28) by  $K(s_t)$ , and substituting from (31), we have the basic recursive equation in  $K$ :

$$
K(s_t)\mu(s_t) = y_t^{-\gamma}\left(K(s_t) - \delta \frac{E_t\left[\eta_{t+1}^{1-\gamma}K(s_{t+1})\right]}{\omega_t}\right) \tag{32}
$$

If  $\mu(s_t) = 0$ ,

$$
K(s_t) = \frac{\delta}{\omega_t} E_t \left[ \eta_{t+1}^{1-\gamma} K(s_{t+1}) \right]
$$
 (33)

If  $\mu(s_t) > 0$ ,  $K(s_t) = 1$ , and:

$$
\lambda(s_t) = \frac{\delta y_t^{-\gamma}}{\omega_t} E_t[\eta_{t+1}^{1-\gamma} K(s_{t+1})]
$$
(34)

Hence, in equilibrium, the function  $K$  satisfies the following:

$$
K(s_t) = \max\left(1, \frac{\delta}{\omega_t} E_t[\eta_{t+1}^{1-\gamma} K(s_{t+1})]\right) \tag{35}
$$

A comparison of equations (35) and (20) demonstrates that we apply an essentially identical solution procedure to both models. The only difference between the two equations is in the timing of the nominal shock  $\omega$ : this difference reflects the basic assumption that, in the Svensson model, individuals start every period with a predetermined money stock and have to acquire money balances for purchases in the future periods.

As above, we define the composite operator operator  $H_2$  as

$$
H_2K(s_t) = \max(1, S_2K(s_t)) = \max\left(1, \frac{\delta}{\omega_t} E_t[\eta_{t+1}^{1-\gamma} K(s_{t+1})]\right)
$$
(36)

The proof of Theorem 1 (in the Appendix) can be used to verify. that  $H_2^n$  is a contraction mapping for some positive integer n, and hence  $H_2$  is a contraction. Once the function K is computed,  $\lambda$ can be obtained by solving equation (31).

To compute stock prices, we start by rewriting the first-order condition (30) in terms of the price/dividend ratio  $\xi(s_t)$ :

$$
\xi(s_t) = \delta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \eta_{t+1} \xi(s_{t+1}) \right) + \delta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \eta_{t+1} \right) \tag{37}
$$

The function  $\lambda$  was determined earlier. We iterate on the function  $\xi$ , and the associated mapping can be shown—with arguments similar to those used above—to be a contraction.

Our procedure to solve the two models significantly differs from those applied by Hodrick, Kocherlakota and Lucas (1989), and Marshall (1988), who also simulate general equilibrium monetary models. Hodrick, Kocherlakota and Lucas examine only equilibrium functions that are separable in the endowment. They show that the mapping converges to a stationary equilibrium if one exists, but do not prove that it is unique.<sup>11</sup> Marshall approximates the conditional expectations in the first-order conditions with an exponential function of a polynomial. The approximation error becomes arbitrarily small as the order of the polynomial tends to infinity. The mapping he defines is not necessarily a contraction.

#### **Simulation Results** 5

We use the consumption series and money stock series described in section 2 to compute the growth rates of the endowment  $(\eta)$  and of the money supply  $(\omega)$ . Since the realization of the monetary shock is observed at different times in the two versions of the model, the bivariate autoregressive systems that we estimate are carefully constructed to reflect this difference. We estimate first-order and second-order bivariate autoregressive processes for the two versions of the model over both the full sample and second postwar sample. We then apply a likelihood-ratio test to determine the order of the system; in all cases we cannot reject the hypothesis that the bivariate autoregressions are first-order.

We then fit a Markov process by discretizing the state space (the space of  $\eta$  and  $\omega$ ) and using Tauchen's quadrature method.<sup>12</sup> The properties of this procedure are described by Tauchen (1986). In particular, the law of motion of the simulated discrete process

$$
m(\eta_t, \omega_t) = \frac{M_t \pi_t}{y_{t-1}}
$$

If the cash-in-advance constraint is binding,  $m(\eta_t, \omega_t) = \eta_t$ , otherwise  $m(\eta_t, \omega_t) > \eta_t$ . The equilibrium first-order condition (15) becomes:

$$
\mu(\eta_t, \omega_t) = \eta_t^{-\gamma} - \frac{4\eta_t^{1-\gamma} E_t \eta_{t+1}^{-\gamma} m(\eta_{t+1}, \omega_{t+1})}{\omega_t m(\eta_t, \omega_t)}
$$

An initial guess  $m^0 = \eta^{-\gamma}$  is used in the right-hand-side of this equation to determine  $\mu^0$ . Depending on the sign of  $\mu^0$  (evaluated for each  $(\eta, \omega)$ ) the algorithm is terminated if  $\mu^0 < 0$ for all states or  $\mu^0 > 0$  for all states. Or else a new function  $\mu^1$  is computed such that for all those  $(\eta_t, \omega_t)$  for which  $\mu^0 < 0$ , m in the denominator insures that the whole expression equal to zero. For all other states,  $m^1 = m^0$ . This procedure is repeated until there is convergence to a fixed point.

<sup>12</sup>The algorithm was kindly supplied to us by George Tauchen.

 $11$ Using our notation their algorithm for the Lucas version of the model is described as follows. Define a function  $m(\eta,\omega)$  as follows:

is constructed to approximate closely the estimated law of motion. Tauchen's quadrature method results in conditional transition probabilites and a state space grid for  $\eta$  and  $\omega$ . In our simulations there are 64 possible realizations of these two variables.

Simulated series of  $\eta$  and  $\omega$  can then be used, together with initial values of  $M$  and  $y$  to obtain realization of the money-stock and endowment processes. The endowment and money-stocks series are used to compute the equity price, the price level and other endogenous variables. The price level is determined by evaluating the function  $K$  as described in section 4.

In the simulations we vary the utility discount rate from 1 % per annum to 3 % per annum. The elasticity parameter in the utility function ranges from 0.5 to 10. The size of the simulated samples is 100.

#### $5.1$ Average Returns and Inflation

Tables 4a and 4b, 5a and 5b report summary statistics for inflation, velocity, real stock returns and real interest rates for the two models and the two samples, respectively. The first column in these tables reports the coefficient of variation of velocity. We find that velocity is much less volatile, relative to its average, than in the sample data. This just confirms the findings of Hodrick. Kocherlakota, and Lucas (1989), who studied the Svensson model (and others) over the second postwar period. Tables 4a and 5a show that, when the model is calibrated with data from the longer sample, velocity displays greater variability, that is, the liquidity constraint is binding more frequently in the simulations based on the second postwar period. Furthermore, the liquidity constraint appears to be binding more frequently in the Lucas model than in the Svensson model. This difference is consistent with the models' predictions about nominal interest rates. The Lucas model implies that only when the liquidity constraint is binding are nominal interest rates positive. Since the model is calibrated using consumption and money data exclusively, and since nominal interest rates are positive in the data, the model's predictions are, in this sense, correct.

The second column of these tables reports simulated means and

standard errors of the inflation rate. Notice that these statistics are largely independent of the values of the preference parameters. While there are differences in the means (ranging from 1 percent to 2 percent), the standard errors of inflations are very similar to the sample values in the case of the Lucas model, and only slightly higher in the case of the Svensson model.

The last three columns in the tables report real returns on stocks and one-period bonds, and the equity premium. To interpret the results, recall that, in these models, both nominal bonds and stocks are risky assets (in real terms). As we showed in Section 4, the equilibrium expected returns on these assets have to satisfy the following conditions:

 $\label{eq:2.1} \mathcal{L}_{\text{max}} = \frac{1}{2} \left( \frac{1}{\sqrt{2}} \right)^{1/2} \left( \frac{1}{2} \right)^{1/2} \left( \frac{1}{2} \right)^{1/2}$ 

$$
1 = \delta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} R_{t+1} \right] \tag{38}
$$

$$
1 = \delta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} R_{t+1}^n \right] \tag{39}
$$

where  $R$  is the real return on stocks, and  $R<sup>n</sup>$  the real return on nominal bonds. Similarly, we can derive the rate of return on an indexed bond (the risk free rate), which has to be equal to the reciprocal of the marginal rate of substitution of present and future wealth. Substituting for the definition of  $R^n$  in (39), and rearranging, we obtain the familiar asset-pricing equations:

$$
E(R_{t+1}) - R_t^f = -\frac{COV(\frac{\lambda_{t+1}}{\lambda_t}, R_{t+1})}{E(\frac{\lambda_{t+1}}{\lambda_t})}
$$
(40)

$$
E(R_{t+1}^{n}) - R_{t}^{f} = -(1+i_{t}) \frac{COV(\frac{\lambda_{t+1}}{\lambda_{t}}, \frac{\pi_{t+1}}{\pi_{t}})}{E(\frac{\lambda_{t+1}}{\lambda_{t}})}
$$
(41)

where *i* represents the nominal interest rate. While in the Lucas model the risk free rate only depends on the distribution of real shocks, in the case of the Svensson model, the marginal utility of wealth, and hence the risk free rate, are affected by monetary shocks. Stochastic inflation affects risk premia in the Lucas model by affecting payoffs to stocks and bonds and through the covariance of nominal and real disturbances. By contrast, in the Svensson

model nominal shocks change the marginal rate of substitution between present and future wealth even if they are independent of real disturbances.<sup>13</sup>

The estimated equity premia in Table 4a range from .42 percent to 1.91 percent. Simulating over the second postwar sample (Table 4b), we find that equity premia are virtually unchanged, ranging from .62 percent to 1.54 percent. These values are much larger than those reported by Mehra and Prescott (1985) but fall well below the data (the tables report the actual values of the statistics in the first row). Tables 5a and 5b show that, in the Svensson model, equity premia can be negative.<sup>14</sup> They range from -1.23 percent to 1.23 percent.

Equations (40) and (41) indicate that the equity premium is "small" when the risk premia on stocks and bonds are of comparable size, either large or small. To interpret this evidence we compute both premia choosing  $\gamma = 2$ , a utility discount factor equal to 3 percent, and the processes for the forcing variables corresponding to the full sample.<sup>15</sup> Figures 5 and 7 report the ex-ante real returns on stocks, nominal bonds, and indexed bonds, in the

$$
q_t = \frac{1}{y_t^{-\gamma}} E_t \sum_{j=0}^{\infty} \delta^{j+1} y_{t+j+1}^{-\gamma} y_{t+j} \frac{\pi_{t+j+1}}{\pi_{t+j}}
$$

In the Svensson model, we solve recursively equation (30) and substituting for  $\lambda_{t+i}$  from equation (29), we obtain:

$$
q_t = \frac{1}{\delta E_t y_{t+1}^{-\gamma} \frac{\pi_{t+1}}{\pi_t}} E_t \sum_{i=1}^{\infty} \delta^{i+1} \left( y_{t+i+1}^{-\gamma} \frac{\pi_{t+i+1}}{\pi_{t+i}} \right) y_{t+i}
$$

In both models, real stock prices are affected by the discounted future path of the rate of deflation. The difference between the two models is, as we pointed out above, in the timing of transactions in the money market. This difference is highlighted by the effects of next-period expected inflation on current stock prices. In the Lucas model, inflation levies a direct tax on dividend payments. In particular, expected inflation, other things equal, lowers the expected return on stocks. In the Svensson model, inflation does not affect dividend payments directly. but affects the marginal utility of wealth. In particular, other things equal, expected inflation increases the marginal rate of substitution between present and future wealth (since current wealth is, in part, accounted for by money balances that will be given at the beginning of next period) hence the expected return on stocks is lowered, through an increase in the current stock prices. By contrast, the marginal rate of substitution is unchanged in the Lucas model.

<sup>14</sup>This does not occur, as equations (40) and (41) demonstrate, when the equity premium is computed using indexed bonds. See Labadie (1989).

<sup>15</sup>Other parameter combinations do not change the evidence in any appreciable way.

<sup>13</sup> Stochastic inflation also affects the equilibrium stock prices. In the Lucas model, solving equation (16) recursively, and using (14) we obtain:

Lucas and Svensson models, respectively. The figures show that the expected return on the three assets is nearly identical. To highlight the behavior of risk premia, we plot them directly in Figures 6 and 8. The striking implication of the four figures is that the only important factor in the fluctuation of ex-ante asset returns appears to be the marginal rate of substitution in wealth. Fluctuations in expected returns generated by these models are generally not due to fluctuations in risk premia. Shiller (1982), among others, has emphasized that asset pricing models like those we study need to explain the wide fluctuations in ex-ante returns of different assets.<sup>16</sup> Our results confirm the observation of Weil (1988), who simulated a model without money, and concluded that the "equity premium puzzle" is really a puzzle about the differences between the behavior of real interest rates and the marginal rates of substitution generated by asset pricing models (the "risk free rate puzzle").

#### Inflation, Stock Returns and Real Interest Rates  $5.2$

Tables 6a and 6b and 7a and 7b contain the regression coefficients of realized real returns on stocks and 1-period bonds on contemporaneous inflation. The value of the slope coefficient of stock returns on inflation is equal to, by definition, the covariance between ex-ante real stock returns and expected inflation, plus the covariance between innovations in stock returns and inflation innovations, divided by the variance of inflation. Stocks are good "inflation hedges" whenever the covariance between innovations in stock returns and inflation innovations is positive. This occurs in the Lucas model, when simulated using postwar data, and in the Svensson model, for high values of gamma. The results of the simulations in the Lucas model contrast with the data, where the negative relation between realized real stock returns and inflation is more apparent especially in the second postwar period. The positive coefficients observed in the simulations of the Svensson model in Tables 7a and 7b correspond to the cases where the equity premium turns to negative values, and, at least in part, explain that

<sup>&</sup>lt;sup>16</sup>See also Barsky (1986).

phenomenon. Since stocks are good hedges against inflation, they command a very small or negative inflation premium, and, as a consequence, their ex-ante returns are driven towards the ex-ante real returns on nominal bonds.

The two columns on the right of Tables 6 and 7 contain the estimated covariations between ex-post real interest rates and inflation. By definition these are equal to the covariance between the ex-ante real interest rate and expected inflation, minus the variance of the inflation innovation, divided by the variance of inflation. Hence the numbers would be negative whenever the covariance between ex-ante real rates and expected inflation, if positive, does not exceed the variance of inflation innovations. This condition is met by most models, in the majority of cases.

#### Interest Rates as Predictors of Inflation and Nominal Stock  $5.3$ Returns

Finally Tables 8a and 8b, 9a and 9b contain regressions of inflation on nominal interest rates, and of realized nominal stock returns on nominal interest rates. The coefficient of the interest rates in the inflation equation equals the covariance between the ex-ante real interest rate and expected inflation, plus the variance of expected inflation, divided by the variance of the nominal interest rate. It tends to 1 whenever the variance of expected inflation accounts for the largest fraction of the total variation of the nominal interest rate, i.e. the variance of the real rate is small (hence its covariance with the expected rate of inflation is also small). In the data, it appears that, if the rational expectations hypothesis is true, either the variance of the real interest rate is significant, or the covariance between the ex-ante real interest rate and expected inflation is negative, or both propositions are true. In the simulations, most regressions coefficients tend to cluster around the values estimated by Fama (1975) and others using data from the early fifites until the late seventies. As Figures 5 and 7 indicate, however, ex-ante real interest rates vary significantly. The estimated coefficients are then due to very low correlations between ex-ante real interest rates and expected inflation.

Finally, probably the most striking results of our simulations

appear in the regressions of ex-post nominal stock returns on the nominal interest rate. We find the nominal interest rate to be an extremely good predictor of subsequent nominal stock returns. All estimated coefficients are very close to unity, and, surprisingly, the R-square statistics in the regressions can be as high as 97 percent. These results just confirm our observations above: most of the covariation between interest rates and stock returns is driven by their common factor-the reciprocal of the marginal rate of substitution of wealth, that is the risk free real interest rate. Any time varying risk premia between the two financial assets are far too small to explain the data.

#### $6$ **Concluding Remarks**

We have calibrated two standard cash-in-advance models to US data to determine their ability to reproduce important empirical regularities affecting asset returns and inflation. The most important result of our analysis is that the models predict a very high covariation between ex-ante returns on stocks and nominal bonds: accounting for money-supply uncertainty does not add significantly to the variation of conditional risk premia. The high covariation of ex-ante returns explains our findings that, in the data generated by the model (assuming that the money growth and consumption growth processes resemble those observed in the US economy), the equity premium is only a small fraction of that observed in the US data. It also explains why nominal interest rates implied by the models turn out to predict nominal stock returns extremely well. Finally, we find that the real returns on stocks are not in many cases negatively related to inflation-as they are in the data (especially in the more recent years), and that ex-ante real interest rates are uncorrelated with expected inflation.

# Appendix A Proof of Theorem 1

The steps are to show first that  $H_1$  takes bounded continuous functions into bounded continuous functions and, second, that  $H_1$  is a contraction.

Both T and  $S_1$  are linear operators. If  $K^0 \in \zeta$  then  $S_1 K^0$  is an element of  $\zeta$  since  $E$  is a continuous linear operator. The operator T is bounded and, because a linear operator is bounded if and only if it is continuous [Luenberger (1969, page 144)], it is also continuous. This establishes that,  $H_1$  takes bounded continuous functions into bounded continuous functions.

The next step is to verify Blackwell's sufficient conditions (monotonicity and discounting property) for a contraction mapping. For notational convenience, let  $x_t$  denote  $\frac{\eta_{t+1}^{1-\gamma}}{\omega_{t+1}}$ . To determine if the composite operator is monotone, notice that, for any  $f > g$ ,  $S_1 f(s)$  $S_1g(s)$  for all s. When T is applied.

$$
(T\cdot S_1)f(s_t)=\max[1,S_1f(s_{t+1})]\geq (T\cdot S_1)g(s_t)=\max[1,S_1g(s_{t+1})],
$$

hence the composite operator is monotone.

To determine whether or not the composite mapping has the discounting property, notice that, because  $\delta E_i x_{i+1}$  may exceed unity, application of the composite operator to an arbitrary function in C generally will not have the discounting property because

$$
H_1(f+a)(s_t) = \max[1, S_1(f+a)(s_t)] = \max[1, \delta E_t x_{t+1}(f(s_{t+1}) + a)]
$$
  
 
$$
\leq \max[1, \delta E_t x_{t+1}f(s_{t+1})] + \max[1, a\delta E_t x_{t+1}]
$$

where the coefficient for a in the last term may exceed unity. Since x is by assumption a stationary process, there exists an  $N_t$  such that

# $\delta E_t x_{t+N}$ . < 1.

Let  $N$  denote the maximum over the  $N_t$ . Start with an initial guess  $K^0 \in \mathcal{C}$  and define

$$
K^{1}(s_{t}) = H_{1} K^{0}(s_{t}) = (T \cdot S_{1}) K^{0}(s_{t})
$$
  
= max[1,  $\delta E_{t} x_{t+1} (K^{0}(s_{t+1}))$ ].

Applying  $H_1$  again,

$$
H_1K^1(s_t) = \max[1, \delta E_t x_{t+1}(K^1(s_{t+1}))]
$$
  
=  $\max[1, \delta E_t x_{t+1} T(\delta E_{t+1} x_{t+2} K^0(s_{t+2}))].$ 

When  $H_1$  is applied M times,

$$
H_1^M K^0(s_t) = (T \cdot S_1)^M K^0(s_t) = \max[1, \delta E_t x_{t+1}(K^{M-1}(s_{t+1}))]
$$
  
=  $(t \cdot S_1)^{M-1} \max[1, \delta E_t x_{t+1}(K^0(s_{t+1}))].$ 

To determine if  $H_1^M$  has the discounting property, define

$$
H_1^M(K^0 + a)(s_t) = (T \cdot S_1)^M (K^0 + a)(s_t)
$$
  
=  $(T \cdot S_1)^{M-1} \max[1, \delta E_t x_{t+1}(K^0(s_{t+1}) + a)]$   
 $\leq (T \cdot S_1)^{M-1} \max[1, \delta E_t x_{t+1}(K^0(s_{t+1})] + (\delta E_t x_{t+M})a$ 

If  $M > N$  then  $\delta E_t x_{t+M} a = \beta a$  where  $0 < \beta < 1$ , and hence  $H_1^M$ has the discounting property. By theorem 2 of Luenberger (1969, page 275), if  $H_1^M$  is a continuous mapping from a closed subset of a Banach space into itself and if  $H_1^M$  is a contraction for some positive integer  $M$ , then  $H_1$  is a contraction. Q.E.D.

Barsky, R.B. (1987): "The Fisher Hypothesis and the Forecastability and Persistence of Inflation," Journal of Monetary Economics, 19, 3-23.

Barsky, R.B. (1986): "Why don't the Prices of Stocks and Bonds Move Together?" National Bureau of Economic Research Working Paper No. 2047.

Cooley, T.F. and G.D. Hansen (1988): "The Inflation Tax in a Real Business Cycle Model", mimeo University of Rochester, August.

Danthine, J.P. and J. B. Donaldson (1986): "Inflation and Asset Prices in an Exchange Economy," Econometrica 54, 585-606.

Fama, E.F. (1975): "Short Term Interest Rates as Predictors of Inflation." American Economic Review, 65, 269-282.

Fama, E.F. and M.R. Gibbons (1982): "Inflation, Real Returns and Capital Investment," Journal of Monetary Economics, 9, 297-323.

Fama, E.F. and G.W. Schwert (1977): "Asset Returns and Inflation," Journal of Financial Economics, 5, 115-146.

Feenstra, R.C. (1986): "Functional Equivalence between Liquidity Costs and the Utility of Money," Journal of Monetary Economics, 17, 271-291.

Friedman, M. (1988): "Money and the Stock Market," Journal of Political Economy, 96, n. 2, 221-245.

Friedman, M. and A.J. Schwartz (1963): A Monetary History of the United States: 1867-1960. Princeton: Princeton University Press (for NBER).

Friedman, M. and A.J. Schwartz (1970): Monetary Statistics of the United States: Estimates, Sources, Methods. New York: Columbia University Press (for NBER).

Friedman, M. and A.J. Schwartz (1982): Monetary Trends in the United States and the United Kingdom: Their Relation to Income. Prices and Interest Rates, 1867-1975. Chicago: University of Chicago Press (for NBER).

Giovannini, A. (1989): "Uncertainty and Liquidity," Journal of Monetary Economics, 23, 239-258.

Giovannini A. and P. Jorion (1987): "Interest Rates and Risk Premia in the Stock Market and in the Foreign Exchange Market." Journal of International Money and Finance, 6, 107-123.

Grossman S.J. and R.J. Shiller (1981): "The Determinants of the Variability of Stock Market Prices" American Economic Review, 71, 222-227.

Hodrick, R.J., N. Kocherlakota and D. Lucas (1989): "The Variability of Velocity in Cash-in-Advance Models." January.

Ibbotson, R.G. and R.A. Sinquefield (1976): "Stocks, Bonds, Bills, and Inflation: Year-by-Year Historical Returns (1926-1974)", Journal of Business, 49, 11-47.

Jaffe J.F. and G. Mandelker (1976): "The 'Fisher Effect' for Risky Assets: An Empirical Investigation", Journal of Finance, XXXI, 447-458.

Kaul, G. (1987): "Stock Returns and Inflation: The Role of the Monetary Sector", Journal of Financial Economics, 18.

Kydland, F.E. (1987): "The Role of Money in a Business Cycle Model", mimeo, Carnegie-Mellon University, December.

Labadie, P. (1989a): "Stochastic Inflation and the Equity Premium," forthcoming, Journal of Monetary Economics.

Labadie, P. (1989b): "Stock Prices and the Inflation Tax," mimeo. Columbia University.

LeRoy S.F. (1984a): "Nominal Prices and Interest Rates in General Equilibrium: Money Shocks", Journal of Business, 57,  $177 - 196.$ 

LeRov S.F. (1984b): "Nominal Prices and Interest Rates in General Equilibrium: Endowment Shocks", Journal of Business, 57, 199-213.

Lucas, R.E., Jr. (1982): "Interest Rates and Currency Prices in a Two-Country World", Journal of Monetary Economics, 10, 335-360.

Luenberger, D.G. (1969): Optimization by Vector Space Methods. New York: John Wiley & Sons.

Marshall, D.A. (1987): "Inflation and Asset Returns in a Monetary Economy with Transactions Costs," mimeo, Carnegie Mellon University.

Mehra, R. and E.C. Prescott (1985): "The Equity Premium: A Puzzle," Journal of Monetary Economics15, no. 2, 145-162.

Mishkin, F.S. (1981): "The Real Interest Rate: An Empirical Investigation", in Carnegie-Rochester Conference Series in Public Policy 15, 151-200.

Mishkin, F.S. (1989): "Is the Fisher Effect for Real?", mimeo, Columbia Business School, June.

Nelson, C.R. (1976): "Inflation and Rates of Return on Common Stocks", Journal of Finance, XXXI, 471-483.

Prescott, E.C. (1986): "Theory Ahead of Business Cycle Measurement", Carnegie-Rochester Conference Series in Public Policy.

Romer, C.D. (1986): "Is the Stabilization of the Postwar Economy a Figment of the Data?" American Economic Review, 76, 3, June, 314-334.

Sargent, T.J. and N. Wallace (1982): "The Real Bills Doctrine vs. the Quantity Theory: A Reconsideration," Journal of Political Economy, 90, December, 1212-36.

Schwert G.W. (1981): "The Adjustment of Stock Prices to Information About Inflation" Journal of Finance, XXXVI, 15-29.

Shiller, R. (1982): "Consumption, Asset Markets, and Macroe-

conomic Fluctuations." Carnegie-Rochester Conference Series on Public Policy, 17.

Summers, L.H. (1983): "The Nonadjustment of Nominal Interest Rates: A Study of the Fisher Effect," in J. Tobin (ed.) Macroeconomics, Prices, and Quantities, Washington, D.C.: The **Brookings Institution.** 

Svensson, L.E.O. (1985): "Money and Asset Prices in a Cashin-Advance Economy," Journal of Political Economy, 93, no. 5,  $919 - 944$ .

Tauchen, G. (1986): "Quadrature-Based Methods for Obtaining Approximate Solutions to the Integral Equations of Nonlinear Rational Expectations Models", mimeo, Duke University, March.

Weil P. (1988): "The Equity Premium Puzzle and the Riskfree Rate Puzzle," mimeo, Harvard University, May.

#### Table 1:

#### Summary Statistics



 $\bar{z}$ 

 $\overline{a}$ 

See text for data sources.

 $30<sub>o</sub>$ 

#### Table 2:

#### Comovements of Inflation and

#### Ex-Post Real Stock Returns and Real Interest Rates



See text for data sources. Standard Errors in parentheses.

 $\bar{z}$ 

 $31$ 

#### Table 3:

#### Nominal Interest Rates as Predictors of

#### Inflation and Nominal Stock Returns



See text for data sources. Standard Errors in parentheses.

 $32$ 

#### Table 4a:

# Sample means and standard deviations

#### Obtained from the Lucas Version of the Cash-in-Advance Model

#### Simulations Over the Whole Sample

#### (in percent)

Returns:



 $\rho = 1/6 \cdot 1$ . Standard Errors in Parentheses.

 $\sim$   $^{\prime}$ 

 $33<sub>1</sub>$ 

#### Table 4b:

#### Sample means and standard deviations

#### Obtained from the Lucas Version of the Cash-in-Advance Model

#### Simulations Over the Second-Postwar Sample

(in percent)

Returns: Velocity Inflation Stock Returns Interest Rate Equity  $(Coef,Var.)$ rate Real Real Premium 11.49 Data:  $4.43(3.02)$  $7.83(14.64)$  $0.23(3.20)$  $7.60(13.87)$ Simulations:  $(\rho, \gamma)$  $(.01..5)$  $0.21$  $5.36(3.53)$  $2.04(1.59)$  $1.43(2.64)$  $0.62(3.25)$  $0.05 (.01, 1)$  $5.35(3.49)$  $3.00(1.42)$  $2.36(2.58)$  $0.64(2.94)$  $(.01, 2)$  $0.00$  $5.35(3.48)$  $4.93(1.22)$  $4.24(2.62)$  $0.69(2.50)$  $0.00$  $(.01, 5)$  $5.35(3.48)$  $10.80(1.83)$  $9.92(2.91)$  $0.88(1.51)$  $(.01, 10)$  $0.00$  $5,35(3.48)$  $20.77(4.02)$  $19.59(3.84)$  $1.18(1.22)$  $(.03,.5)$  $0.00$  $5.35(3.48)$  $4.28(1.56)$  $3.55(2.55)$  $0.72(3.05)$  $(.03, 1)$  $0.00$  $5.35(3.48)$  $5.26(1.42)$  $4.50(2.57)$  $0.75(2.84)$  $(.03, 2)$  $0.00$  $5.35(3.48)$  $7.23(1.27)$  $6.41(2.63)$  $0.82(2.44)$  $(.03, 5)$  $0.00$  $5.35(3.48)$  $13.23(1.93)$  $12.20(2.94)$  $1.04(1.52)$  $(.03, 10)$  $0.00$  $5.35(3.48)$  $23.43(4.10)$  $22.06(3.90)$  $1.36(1.25)$  $(.05,.5)$  $5.35(3.48)$  $0.00$  $6.51(1.55)$  $5.67(2.56)$  $0.84(2.97)$  $(.05, 1)$  $0.00$  $5.35(3.48)$  $.7.52(1.43)$  $6.64(2.58)$  $0.88(2.77)$  $(.05, 2)$  $0.00$  $5.35(3.48)$  $9.54(1.33)$  $8.58(2.64)$  $0.96(2.40)$  $(.05, 5)$  $0.00$  $5.35(3.48)$  $15.67(2.03)$  $14.48(2.98)$  $1.19(1.54)$  $(.05, 10)$  $0.00$  $5.35(3.48)$  $26.08(4.20)$   $24.54(3.97)$  $1.54(1.30)$ 

 $\rho = 1/6$ -1. Standard Errors in Parentheses.

#### Table 5a:

#### Sample means and standard deviations

# Obtained from the Svensson Version of the Cash-in-Advance Hodel

#### Simulations Over the Whole Sample

(in percent)



 $\rho = 1/6$ -1. Standard Errors in Parentheses.

 $35$ 

#### Table 5b:

#### Sample means and standard deviations

Obtained from the Svensson Version of the Cash-in-Advance Model

# Simulations Over the Second-Postwar Sample

(in percent)



 $\rho = 1/\delta$ -1. Standard Errors in Parentheses.

#### Table 6a:

#### Comovements of Inflation and

#### Ex-Post Real Stock Returns and Real Interest Rates

#### Obtained from the Lucas Version of the Cash-in-Advance Model

#### Simulations Over the Whole Sample



 $\rho = 1/6-1$ .

#### Table 6b:

#### Comovements of Inflation and

#### Ex-Post Real Stock Returns and Real Interest Rates

# Obtained from the Lucas Version of the Cash-in-Advance Model

#### Simulations Over the Second-Postwar Sample



 $\rho = 1/6-1$ .

#### Table 74:

#### Comovements of Inflation and

#### Ex-Post Real Stock Returns and Real Interest Rates

#### Obtained from the Svensson Version of the Cash-in-Advance Model

#### Simulations Over the Whole Sample



 $\rho = 1/6-1$ .

39

#### Table 7b:

#### Comovements of Inflation and

#### Ex-Post Real Stock Returns and Real Interest Rates

# Obtained from the Svensson Version of the Cash-in-Advance Model

# Simulations Over the Second-Postwar Sample



 $\rho = 1/6 - 1$ .

#### Table 8a:

# Nominal Interest Rates as Predictors of

# Inflation and Nominal Stock Returns

# In the Lucas Version of the Cash-in-Advance Model

# Simulations Over the Whole Sample



 $\rho = 1/6-1$ .

# Table 8b:

# Nominal Interest Rates as Predictors of

# Inflation and Nominal Stock Returns

# In the Lucas Version of the Cash-in-Advance Model

# Simulations Over the Second-Postwar Sample



 $\rho = 1/\delta - 1$ .

#### Table 9a:

#### Nominal Interest Rates as Predictors of

#### Inflation and Nominal Stock Returns

#### In the Svensson Version of the Cash-in-Advance Model

#### Simulations Over the Whole Sample



 $\rho = 1/6 \cdot 1$ .

#### Table 9b:

#### Nominal Interest Rates as Predictors of

#### Inflation and Nominal Stock Returns

# In the Svensson Version of the Cash-in-Advance Model

# Simulations Over the Second-Postwar Sample



 $p = 1/6-1$ .



Percent per Annum







Changes in Velocity Figure 4

# Lucas Model

 $\begin{array}{ll} \mathsf{Figcup} \mathsf{C} \\ \mathsf{Eigcup} \mathsf{Retums} \mathsf{on} \ \mathsf{Stocks}, \ \mathsf{Nominal} \ \mathsf{Bonds} \ \mathsf{and} \ \mathsf{Indeed} \ \mathsf{Bonds} \end{array}$ 





Risk Premia in the Lucas Model

# Svensson Model

Expected Real Returns on Stocks, Nominal Bonds and Indexed Bonds Figure 7



