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CAPITAL CONTROLS AND TRADE POLICY

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ABSTRACT

How does the conduct of optimal cross-border financial policy change with prevailing trade agreements? We study the joint optimal determination of trade policy and capital-flow management in a two-country, two-good model with trade in goods and assets. While the cooperative optimal allocation is efficient, a country-planner can achieve higher domestic welfare by departing from free trade in addition to levying capital controls, absent retaliation from abroad. However, time variation in the optimal tariff induces households to over- or under-borrow through its effects on the path of the real exchange rate. As a result, optimal capital controls can be larger when used in conjunction with optimal tariffs in specific cases; and in others, the optimal trade tariff partly substitutes for the use of capital controls. Accounting for strategic retaliation, we show that committing to a free-trade agreement can reduce incentives to engage in costly capital-control wars for both countries.

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1 Introduction

Trade and capital-flow management have long been key topics of macroeconomic policy and have, once more, come into sharp focus following the Covid-19 pandemic. Following at least two decades of integration (Baier and Bergstrand, 2007), the process of trade liberalization appears to have stalled. The decline in average tariff rates observed worldwide in the years prior to the global financial crisis has abated (green line, Figure 1). Alongside this, there has been a decline in the number of new regional-trade agreements and a deceleration of global value chain integration.[1] More recent events like the US-China trade war and supply-chain pressures have since contributed to substantially heightened uncertainty around world trade (Ahir, Bloom, and Furceri, 2022).[2] After moving in line with trade openness in the years prior to the global financial crisis, financial liberalization too has slowed in recent years and the International Monetary Fund has partially revised their 'institutional view' to emphasize a role for managing capital flows in specific circumstances (Qureshi, Ostry, Ghosh, and Chamon, 2011).[3] Partly reflecting this, the trend decline in capital-flow restrictions (Fernández, Klein, Rebucci, Schindler, and Uribe, 2016) seen prior to 2007 has since reversed (blue line, Figure 1).

However, despite trade policy and capital-flow management growing in prominence since the global financial crisis as this evidence suggests, academics and policymakers have typically discussed these measures separately. Trade policy discussions often balance economic forces (e.g., comparative advantage) with political factors (e.g., consequences of de-industrialization, trade sanctions), while recent debates about capital controls have centred on their role in insulating countries from large and volatile cross-border flows.

In this paper, we show that adjustments to trade policy influence optimal capital-flow management through their effects on the path for real exchange rates and, in turn, private incentives to borrow on international financial markets. We provide a unifying framework to study the joint optimal determination of trade policy and capital-flow management, in a model where both instruments are driven by a common motive: to exploit a country's monopoly power in markets. Within this setup, we assess how prevailing trade arrangements influence the incentives for, and the size of, optimal capital flows. We then extend this framework to account for strategic interactions between countries and use it to assess the implications of international trade and financial arrangements for global welfare and the likelihood that capital-control wars can emerge.

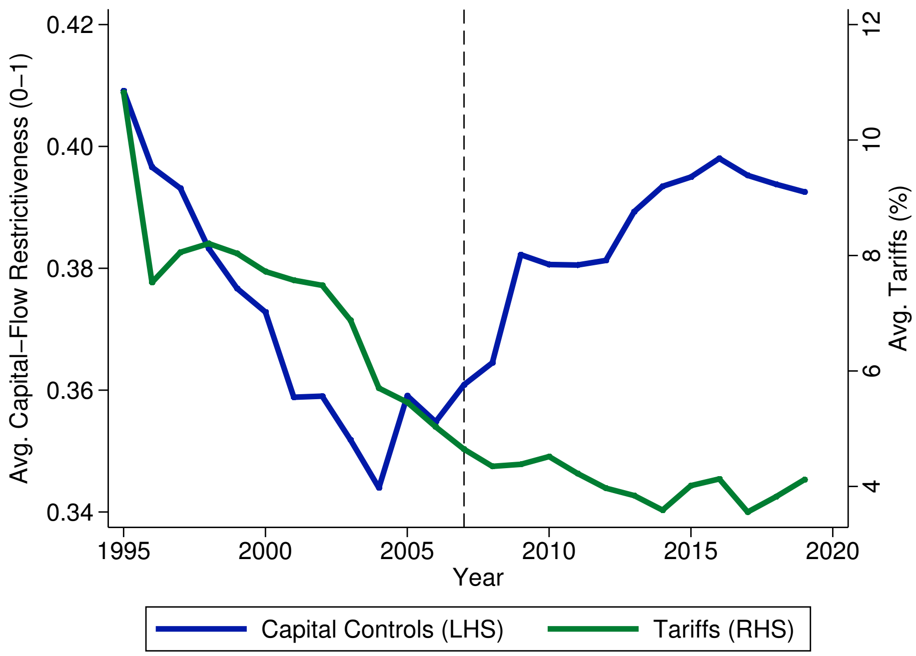
The starting point for our analysis is a canonical two-country, two-good endowment economy model, absent nominal or financial frictions. Households make an inter-temporal consumption-savings decision and choose their optimal consumption bundle intra-temporally. In the laissez-faire or decentralized allocation, relative consumption growth across countries is proportional to

[1] See D'Aguanno, Davies, Dogan, Freeman, Lloyd, Reinhardt, Sajedi, and Zymek (2021) for further discussion.

[2] See Amiti, Redding, and Weinstein (2019) and Itskhoki and Mukhin (2022) for studies into the US-China trade war and recent sanctions on Russian goods, respectively.

[3] See, for example, the prevalence of 'macroprudential' foreign-exchange interventions targeting cross-border flows (Ahnert, Forbes, Friedrich, and Reinhardt, 2020).

Figure 1: Capital Controls and Trade Policy Over Time



Notes: Average capital-control restrictions (0-1 index) and tariff rates (%) across 99 countries from 1995-2019 (annual frequency). Capital-control restrictions data from Fernández, Klein, Rebucci, Schindler, and Uribe (2016). Measure used here captures all restrictions (inflow and outflow). Tariff rates from World Development Indicators (World Bank). Metric used here uses weighted mean applied tariff for all products, which measures the average of effectively applied tariffs weighted by the product import shares corresponding to each partner country. Vertical dashed line at 2007.

the relative decrease in price levels—i.e., the rate of real exchange rate depreciation.[4] However, households do not internalize the effect of their actions on relative prices. These pecuniary externalities, described in Geanakoplos and Polemarchakis (1986), imply that a country planner maximizing domestic welfare has an incentive to manipulate the inter- and intra-temporal terms of trade—i.e., world interest rates and relative goods prices, respectively—even though the laissez-faire allocation is optimal from a global perspective. Within this setup, Costinot, Lorenzoni, and Werning (2014) show that, when domestic households borrow between two periods, the planner tends to levy capital-inflow taxes to delay consumption relative to the decentralized allocation, but must trade off the incentive to drive down the world interest rate with second-best effects on relative goods prices.[5] In this paper, we ask: what more can a country planner achieve by deviating from a free-trade agreement (FTA), and what might this imply for the conduct of optimal capital controls and world welfare?

[4]This condition, highlighted in Backus and Smith (1993) and Kollmann (1995), reflects the perfect risk-sharing underlying open-economy macroeconomic models with complete international asset markets.

[5]Capital controls introduce a wedge to the risk-sharing condition, so consumption growth can be slower than the rate of exchange-rate depreciation. This wedge can also be understood as a measure of exchange-rate misalignment induced by the planner at the optimal allocation (e.g., Corsetti, Dedola, and Leduc, 2020).

Our key contribution is to relax the constraint imposed on the planner by a FTA and assess the interactions between optimal capital-flow management and trade policy within a tractable environment, using the primal approach of Lucas and Stokey (1983). To illustrate the mechanisms at play, we initially focus on an implementation with capital controls and import tariffs. We begin by assessing the incentives of a country-planner acting unilaterally to maximize domestic welfare, without retaliation from abroad. Consider a scenario in which domestic households borrow between two periods, driven by a temporarily lower endowment of the good consumed with home bias (the 'domestic good'). The planner will seek to delay aggregate consumption inter-temporally by taxing capital inflows, but also has an intra-temporal incentive to reduce consumption of the relatively expensive domestic good. Absent a FTA, the planner achieves higher domestic welfare by levying a temporary subsidy on imports. However, this puts pressure on the real exchange rate to depreciate. All else equal, such trade policy will result in an adjustment in capital controls due to its effect on the path for real exchange rates. Intuitively, the exchange-rate depreciation encourages 'over-borrowing' by households—insofar as a larger capital-flow tax is required to induce a constrained-efficient path for consumption—because their domestic consumption bundle becomes cheaper today relative to the future.

Whether capital controls are larger or smaller when the FTA is relaxed depends on the alignment of the planner's incentives to manipulate the terms of trade inter- and intra-temporally. In the above example, the capital-inflow tax is larger (and the domestic welfare gains from taxing capital flows are higher) in the presence of tariffs, and incentives are aligned. Inter-temporally, the planner leans against the private desire to over-borrow today while, intra-temporally, the planner offsets the private desire to consume the relatively expensive domestic good. In contrast, when the domestic endowment of the 'foreign good' is temporarily low, inter- and intra-incentives are misaligned. In this case, the optimal unilateral tariff puts pressure on the real exchange rate to appreciate, which, absent further action, incentivizes under-borrowing. So, at the optimal allocation, trade policy acts as a partial substitute for capital controls, requiring smaller capital-flow taxes than the FTA case. Calibrating the stylized framework to standard values used in the literature, we show these interactions can be large, with the capital-inflow tax being almost one-third larger absent a FTA when motives are aligned.

Our analysis applies to more general environments, including where capital-flow management is driven by alternative incentives. To show this, we extend the model to include production of non-tradable goods subject to nominal-wage rigidities. In this setting, policy is driven by a demand-management motive (via an aggregate-demand externality), as in Farhi and Werning (2016). We show that the planner faces an additional incentive to bring forward (delay) consumption when the economy is demand-constrained (overheating). They achieve this either through an optimal policy mix of a capital-inflow subsidy and an import tariff which depreciates the exchange rate stimulating aggregate demand.

Our results also generalize to settings in which a wider range of policy instruments are used, and are not specific to capital-flow taxes and tariffs. Within our baseline framework, the planner targets an optimal risk-sharing wedge, and an optimal relative demand wedge which are interrelated. In an extension that allows for segmented financial markets, as in Gabaix and Maggiori (2015), we show that foreign-exchange interventions (FXI) can be used to target the same risk-sharing wedge as capital controls and interact similarly with trade policy.[6] In turn, the relative-demand wedge need not originate from optimally set tariffs, but can reflect disruptions to trade more generally—such as multilateral sanctions or global supply-chain issues. Faced with a temporarily high cost of imports, the planner taxes inflows and, when unconstrained by a FTA, subsidizes imports. Because inter- and intra-temporal incentives are aligned in this case, our theory suggests that capital-flow taxes will be *larger* absent a FTA in response to trade disruptions.

Moreover, we show that policy interactions can persist in small-open economies where an individual country's ability to manipulate the world interest rate disappears. We highlight the special case of unitary trade and inter-temporal elasticities of substitution, as in Cole and Obstfeld (1991). If inter- and intra- temporal incentives are aligned, the optimal capital-inflow tax is invariant to the size of the economy since the tax needed to address the inter-temporal margin exactly coincides with that required to address intra-temporal incentives. The optimal tariff is still non-zero since countries are always large in their domestic goods market.

Finally, we return to our baseline model to analyse a strategic setting where both countries' planners retaliate to the other by setting policy as a best response to each other. In the strategic setting, we find that capital-flow wedges tend to be larger absent a FTA, *both* when the domestic or foreign goods are away from their long-run level, due to the effects of tariff competition on the path for real exchange rate. Policy wars follow an 'inverse elasticity rule'. Capital controls are larger when the elasticity of inter-temporal substitution is low and tariffs are more prevalent when the intra-temporal elasticity of substitution between goods is low.[7] While the joint application of capital controls and tariffs may be unilaterally optimal for an individual country when there is no retaliation, the costs to global welfare, as well as in both countries individually, are disproportionately large. Using our strategic framework, we show that when countries are not committed to a FTA, the incentives to depart from a 'free-financial-flows agreement' (FFFA) and engage in costly capital-control wars are heightened, providing a novel argument in favor of free trade. FTAs reduce incentives for an individual country to levy capital controls, which could prompt retaliation, since, consistent with our results, tariffs distort the path of real exchange rates over time. In short: retaining openness in trade can help to sustain financial openness.

Related Literature. Our work builds on Costinot et al. (2014) who study the role of capital controls as a means of dynamic terms-of-trade manipulation in large-open endowment

[6]Our insights on the interaction of capital-flow management and policy interventions will also apply to any policies that do not directly induce a wedge in the risk-sharing condition (e.g., monetary policy, fiscal policy).

[7]The findings mirror those in the optimal taxation literature (e.g., Atkinson and Stiglitz, 1980; Chari and Kehoe, 1999), where the planner taxes inelastic commodities more.

economies.[8] We depart from their assumption of free trade to study the interaction of trade and financial policy. In doing so, our work combines analyses of inter-temporal incentives to manipulate the terms of trade, a key part of the broader literature on capital controls surveyed in Rebucci and Ma (2019) and Bianchi and Lorenzoni (2022), with intra-temporal incentives, for which tariffs are regularly applied in practice (Broda, Limao, and Weinstein, 2008).[9] We show that trade policy itself can give rise to incentives to levy capital controls, through their impact on real exchange rates and, in turn, incentives to over-/under-borrow.[10]

Our analysis also contributes to the broader literature on capital controls. Most notably, we show how the mechanisms that underpin the interaction between optimal trade and financial policy persist in settings where policy intervention is driven by aggregate-demand externalities in models with nominal rigidities (see, e.g., Farhi and Werning, 2014, 2016; Schmitt-Grohé and Uribe, 2016; Marin, 2022). In addition, within our deterministic setting, we show that anticipated shocks can engender preemptive policy interventions, akin to the precautionary motives underpinning prudential capital controls in small-open economy models with borrowing constraints (see, e.g., Mendoza, 2002; Bianchi, 2011).

The literature on trade tariffs has predominantly focused on environments with no trade in assets, albeit with a richer supply-side setup with monopolistic (and often heterogeneous) firms (see, e.g., Demidova and Rodriguez-Clare, 2009; Caliendo, Feenstra, Romalis, and Taylor, 2021). We contribute to this literature by evaluating the scope for tariffs to be used as second-best instruments to manipulate the cost of borrowing in a dynamic setting.

Our analysis also relates to the literature on FXI. In particular, Fanelli and Straub (2021) show that FXI and capital controls are isomorphic when there is partial segmentation in international markets, up to implementation costs. They share our focus on an endowment economy and pecuniary externalities, but, unlike us, abstract from trade policy.[11]

Finally, our paper contributes to a growing literature assessing the joint role of trade and macroeconomic stabilization policies. Bergin and Corsetti (2020) study the optimal response of monetary policy to tariff shocks. Auray, Devereux, and Eyquem (2020) study the scope for trade wars and currency wars in a New-Keynesian small-open economy model but their model features balanced trade, so there is no scope for capital controls. Jeanne (2021) studies monetary policy and the accumulation of foreign reserves, emphasizing the distinction between a 'Keynesian regime' where instruments are used to achieve full employment and a 'classical regime' where tariffs are used to manipulate the terms of trade.

[8] Costinot et al. (2014) note that optimal capital controls are not guided by the absolute desire to alter the inter-temporal price of goods produced in a given period, but rather by the relative strength of this desire between two consecutive periods, generalising the results from a two-period environment in Obstfeld and Rogoff (1996). Heathcote and Perri (2016) study capital controls in a two-country, two-good model with incomplete markets and capital, but do not derive the optimal policy.

[9] Like us, Ju, Shi, and Wei (2013) analyze the relationship between inter- and intra-temporal trade. Their analysis is conducted within a Heckscher-Ohlin model.

[10] Jeanne (2012) and Farhi, Gopinath, and Itskhoki (2014) analyse the effects of tariffs on real exchange rates.

[11] In an extension with price rigidities, Fanelli and Straub (2021) consider the interaction between FXI and monetary policy in which FXI can recover the flexible-price allocation when monetary policy is constrained.

Outline. The remainder of the paper is structured as follows. Section 2 describes the model environment. Section 3 characterizes the optimal unilateral planning allocation. Section 4 discusses policy implementation and macroeconomic outcomes. Section 5 considers a number of model generalisations and extensions. Section 6 studies strategic cross-country interactions. Section 7 considers global welfare and the likelihood of capital-control wars emerging with and without a FTA. Section 8 concludes.

2 Basic Environment

There are two countries, Home H and Foreign F, each populated by a continuum of identical households. Time is discrete and infinite, $t = 0, 1, ...,$ and there is no uncertainty. The preferences of the representative Home consumer are denoted by the time-separable utility function:

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(C_t)$$

where C_t is aggregate Home consumption and $u(C)$ is a twice continuously differentiable, strictly increasing and strictly concave function with $\lim_{C\to 0} u'(C) = \infty$. $\beta \in (0, 1)$ is the discount factor. The preferences of the representative Foreign consumer are analogous, with asterisks denoting Foreign variables.

Consumers in both countries consume two goods, good 1 and good 2. We denote the representative Home consumer's consumption of good 1 and good 2 by $c_{1,t}$ and $c_{2,t}$, respectively, and group them into the vector $\mathbf{c}_t = [c_{1,t}\ c_{2,t}]'$. Home aggregate consumption is defined by the aggregator $C_t \equiv g(\mathbf{c}_t)$, where $g(\cdot)$ is a function that is twice continuously differentiable, strictly increasing, concave and homogeneous of degree one. We define the Jacobian of $g(\mathbf{c}_t)$ by $\nabla g(\mathbf{c}_t) = [g_{1,t}\ g_{2,t}]'$, where $g_{i,t} = \frac{\partial g(\mathbf{c}_t)}{\partial c_{i,t}}$ for $i = 1, 2$, while second derivatives are written as $g_{ij,t} = \frac{\partial^2 g(\mathbf{c}_t)}{\partial c_{i,t} \partial c_{j,t}}$ for $i, j = 1, 2$. The aggregator for the representative Foreign consumer is written as $C_t^* \equiv g^*(\mathbf{c}_t^*)$, with analogously defined derivatives.

The Home (Foreign) consumer's period-t endowments of goods 1 and 2 are denoted by $y_{1,t}$ ($y_{1,t}^*$) and $y_{2,t}$ ($y_{2,t}^*$), respectively, and are weakly positive in all periods.[12] Throughout, without loss of generality, we assume that Home consumers have a 'home bias' for good 1, and we therefore describe this as the 'domestic good'. Defining the Home expenditure share on domestic goods as α, then 'home bias' implies $\alpha > 0.5$. Likewise, Foreign consumers prefer good 2 (the 'foreign good') and we assume $\alpha^* = \alpha$. The total world endowment of goods 1 and 2 are $Y_{1,t} \equiv y_{1,t} + y_{1,t}^*$ and $Y_{2,t} \equiv y_{2,t} + y_{2,t}^*$, respectively.

The inter-temporal budget constraint for the Home household expressed as:

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t - \mathbf{y}_t) \leq 0 \tag{1}$$

[12] We focus on an endowment setup for mathematical tractability. Section 5 discusses how our findings carry over to production economies.

where $\mathbf{p}_t = [p_{1,t}\ p_{2,t}]'$ denotes the vector of period-t world goods prices and $\mathbf{y}_t = [y_{1,t}\ y_{2,t}]'$ is the vector of Home endowments.

We define two additional quantities. First, the terms of trade is given by $S_t = p_{2,t}/p_{1,t}$ and, since good 1 is the 'domestic good' and good 2 the 'foreign good', we refer to an increase in S_t as a deterioration of the Home terms of trade. Second, the real exchange rate is given by the ratio of consumer price indices $Q_t = P_t^*/P_t$, where $P_t^{(*)} \equiv \min_{\mathbf{c}_t^{(*)}} \{\mathbf{p}_t \cdot \mathbf{c}_t^{(*)} : g^{(*)}(\mathbf{c}_t^{(*)}) \geq 1\}$. An increase in Q_t corresponds to a depreciation of the Home real exchange rate.

Free-Trade Agreements and the Pareto Frontier. In the presence of a FTA, households' consumption allocations are Pareto efficient (from an individual-household perspective) and can be summarized by:[13]

$$C^*(C_t) = \max_{\mathbf{c}_t, \mathbf{c}_t^*} \{g^*(\mathbf{c}_t^*) \quad \text{s.t.} \quad \mathbf{c}_t + \mathbf{c}_t^* = \mathbf{Y}_t \text{ and } g(\mathbf{c}_t) \geq C_t\} \tag{2}$$

for some C_t, where $\mathbf{Y}_t = [Y_{1,t}\ Y_{2,t}]'$. The Pareto frontier summarizes efficient combinations of consumption $\{c_{1,t}, c_{2,t}\}$ for a given level of aggregate consumption C_t. The Home and Foreign Pareto frontiers are defined by $\mathbf{c}(C)$ and $\mathbf{c}^*(C^*)$ and the full expressions are presented in Appendix A.2, which reflect individual households' optimization of consumption bundles given an aggregate consumption C.

Specific Functional Forms. For our numerical exercises, we use a constant relative risk aversion (CRRA) specification for per-period utility $u(C) \equiv \frac{C^{1-\sigma}-1}{1-\sigma}$, where $\sigma > 0$ denotes the coefficient of relative risk aversion. The aggregate consumption of the representative agent is given by the Armington (1969) aggregator:

$$C_t \equiv g(\mathbf{c}_t) = \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{3}$$

where $\phi > 0$ is the elasticity of substitution between good 1 and 2.

An interesting analytical special case arises when $\sigma = \phi = 1$. This corresponds to the parametrization studied in Cole and Obstfeld (1991), henceforth described as the 'CO case'. In subsequent sections, we use this special case to build intuition around the interaction between capital-flow taxes and trade policy at the optimal allocation.

3 Unilateral Planning Allocation

We begin by considering an equilibrium in which the Home planner maximizes domestic welfare, while the Foreign planner is passive—i.e., does not levy taxes in response to Home policy. We compare the equilibrium with a FTA in place (corresponding to the two-good environment studied in Costinot et al., 2014) to an equilibrium where the Home planner is unconstrained by

[13] This coincides with the contract curve for the representative Home and Foreign consumers when there are no goods-specific taxes.

a FTA. In both cases, the equilibrium conditions of the representative Foreign household act as a constraint for the unilateral Home planner. With λ^* denoting the Lagrange multiplier on the Foreign inter-temporal budget constraint, the first-order conditions for the Home planner are:[14]

$$\beta^t u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*) = \lambda^* \mathbf{p}_t \tag{4}$$

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) = 0 \tag{5}$$

3.1 With Free Trade

In the presence of a FTA, the Home government chooses the sequence of Home aggregate consumption $\{C_t\}$ to maximize the discounted lifetime utility of the Home representative consumer subject to: (i) the representative Foreign consumer's utility maximization at world prices; (ii) market clearing in each period; and (iii) the Pareto frontier arising from the FTA. Conditions (i) and (ii) can be summarized in a single implementability condition (Lucas and Stokey, 1983), described in the following lemma:

Lemma 1 (Implementability for Unilateral Planner) *When the Foreign country is passive, an allocation $\{\mathbf{c}_t, \mathbf{c}_t^*\}$, together with world prices $\mathbf{p}_t$, form part of an equilibrium if they satisfy:*

$$\sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

where $\boldsymbol{\rho}(C_t) \equiv u^{*\prime}(C^*(C_t))\nabla g^*(\mathbf{c}_t^*(\mathbf{c}_t))$ denotes the price of consumption at each t.

Proof: See Appendix B.1. □

The Home planning problem can then be written as:

$$\max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{P-Unil-FTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

$$\mathbf{c}_t = \mathbf{c}(C_t), \quad \mathbf{c}_t^* = \mathbf{c}^*(C_t) \tag{FTA}$$

where the third line (FTA) summarizes the Pareto frontier constraint imposed by the presence of a FTA. After substituting (FTA) into (IC), we assume that $\boldsymbol{\rho}(C_t) \cdot [\mathbf{c}(C_t) - \mathbf{y}_t]$ is a strictly convex function of C_t to guarantee a unique solution to (P-Unil-FTA).

Optimal Allocation. Since utility is time-separable, the first-order condition is given by:

$$u'(C_t) = \mu \mathcal{MC}_t^{FTA} \tag{6}$$

[14] See Appendix B.1 for a full statement of the representative Foreign household's optimization problem.

where μ is the multiplier on the implementability constraint and:

$$\begin{aligned}\mathcal{MC}_t^{FTA} \equiv & u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*(C_t)) \cdot \mathbf{c}'(C_t) + u^{*\prime\prime}(C_t^*)C^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*(C_t)) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\ & + u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t(C_t))}{\partial C_t} \cdot [\mathbf{c}_t - \mathbf{y}_t]\end{aligned}$$

The left-hand side of equation (6) is the marginal utility from one additional unit of aggregate consumption for the representative Home consumer. The right-hand side represents the marginal cost of that unit of consumption, captured by $\mathcal{MC}_t^{FTA}$. The first term in $\mathcal{MC}_t^{FTA}$ is the price of one unit of consumption, which can be shown to be equal to $u^{*\prime}(C_t^*)Q_t^{-1}$. The second term reflects how the inter-temporal price of consumption changes when importing one additional unit of consumption, for given relative goods prices. The final term reflects how relative goods prices change with aggregate consumption. If endowments and consumption outcomes coincide, $\mathbf{c}_t = \mathbf{y}_t$, (6) collapses to $u'(C_t) = \mu u^{*\prime}(C_t^*)Q_t^{-1}$, which corresponds to the decentralized allocation.

An interesting special case arises in the CO setting, where $\sigma = \phi = 1$. The following lemma, which matches Proposition 3 in Costinot et al. (2014), clarifies how aggregate consumption is procyclical for the planner at this parametrization when a FTA is in place.

Lemma 2 (Procylical Consumption with a FTA in the CO Case) *In the limit as $\sigma \to 1$ and $\phi \to 1$, aggregate consumption is procyclical with respect to domestic endowments in the unilateral planning allocation with a FTA: $\frac{\mathrm{d}C_t}{\mathrm{d}y_{i,t}} > 0$ for $i = 1, 2$.*

Proof: See Appendix B.2. □

This result implies that, with a FTA, the unilateral Ramsey planner will optimally delay consumption, leaning against capital inflows, when $y_{i,t} < y_{i,t+1}$. This contrasts with the decentralized allocation, in which aggregate consumption $C_t^{(*)}$ is invariant to domestic endowments $y_{i,t}^{(*)}$ when there is no aggregate risk—i.e., when $Y_{i,t} = \overline{Y}_i$ for all t.

3.2 Without Free Trade

Without free trade, the Home planner—unconstrained by the Pareto frontier—chooses the allocation of both goods 1 and 2. The Home planner's problem is:

$$\max_{\{c_{1,t}, c_{2,t}\}} \sum_{t=0}^{\infty} \beta^t u(C_t) \qquad \text{(P-Unil-nFTA)}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \qquad \text{(IC)}$$

$$C_t = g(\mathbf{c}_t) \qquad \text{(nFTA)}$$

where the third line (nFTA) reflects that aggregate consumption C_t can then be backed out of the consumption aggregator $g(\mathbf{c}_t)$. The implementability condition is unchanged and, as in the

FTA-case, we assume that $\boldsymbol{\rho}(g(\mathbf{c}_t)) \cdot [\mathbf{c}_t - \mathbf{y}_t]$ is strictly convex.

Optimal Allocation. The first-order conditions—with respect to $c_{1,t}$ and $c_{2,t}$, respectively—are given by:

$$u'(C_t)g_{1,t} = \mu \mathcal{MC}_{1,t}^{nFTA} \tag{7}$$

$$u'(C_t)g_{2,t} = \mu \mathcal{MC}_{2,t}^{nFTA} \tag{8}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\begin{aligned}
\mathcal{MC}_{1,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)g_1^*(\mathbf{c}_t) + u^{*\prime\prime} g_1^*(\mathbf{c}_t^*)\nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
& + u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
\mathcal{MC}_{2,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)g_2^*(\mathbf{c}_t) + u^{*\prime\prime} g_2^*(\mathbf{c}_t^*)\nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
& + u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t]
\end{aligned}$$

Like equation (6), equations (7) and (8) equate the marginal benefit from a unit of good-specific consumption to its marginal cost—for goods 1 and 2, respectively. Without free trade, the planner optimizes over the consumption allocation good by good. Considering equation (7), the first term on the right-hand, as before, reflects the price of one unit of good 1. The next term reflects how the cost of borrowing a unit of aggregate consumption changes. The final term, captures the intra-temporal margin—specifically how each good-specific price changes with respect to c_1.

Without a FTA, at the CO parametrization, a separation arises between the optimal consumption of goods 1 and 2. This result is summarized in the following lemma, which extends Lemma 2 to the no-FTA case.

Lemma 3 (Procyclical Consumption without a FTA in the CO Case) *In the limit as $\sigma \to 1$ and $\phi \to 1$, consumption of each good is procyclical with respect to each endowment, but independent from other good endowments, in the unilateral planning allocation without a FTA: $\frac{\mathrm{d}c_{i,t}}{\mathrm{d}y_{i,t}} > 0$ and $\frac{\mathrm{d}c_{i,t}}{\mathrm{d}y_{j,t}} = 0$ for $i, j = 1, 2$ and $i \neq j$.*

Proof: See Appendix B.3. □

As in the FTA case, the unilateral planner will optimally delay consumption when endowments are relatively low today. However, in this no-FTA setting in the CO case, the optimal consumption path is linked solely to the endowment path for each specific good.

3.3 Comparing Optimal Allocations

For the Home planner the first-order condition under a FTA (6), represents a constrained first-best allocation. However, the no-FTA optimality conditions (7) and (8), represent the first-best outcome for the Home country, as the following proposition explains.

Proposition 1 (Optimal Unilateral Allocations without a FTA) *In the absence of a FTA, the unilateral optimal allocation* $\mathbf{c}_t$ *satisfies* (7) *and* (8)*. Moreover:*

(i) *the level of welfare* U_0 *achieved in* (P-Unil-nFTA) *is always weakly higher than that achieved in* (P-Unil-FTA)*;*

(ii) *if the optimal allocation* $\mathbf{c}$ *in* (P-Unil-nFTA) *violates the Pareto frontier* (2) *given by a FTA, then* (i) *holds strictly; and*

(iii) *the welfare achieved, and corresponding allocation* $\mathbf{c}$*, in* (P-Unil-FTA) *and* (P-Unil-nFTA) *coincide only when endowments are proportional to consumer preferences,* $y_1 \propto \alpha$*,* $y_2 \propto 1-\alpha$*,* $y_1^* \propto 1-\alpha$ *and* $y_2^* \propto \alpha$*.*

Proof: See Appendix B.4. □

Proposition 1 implies that away from the no-trade point detailed in (iii), active trade policy is desirable in addition to capital-flow management. To illustrate this, Figure 2 plots the optimal allocations with (blue) and without (green) a FTA, alongside the loci of $\{c_1, c_2\}$ which attain different levels of aggregate consumption (grey, and black for $C = 1$), in the long run where endowments are constant.

In Figure 2, the blue line maps the Pareto frontier: the efficient combinations of $\{c_1, c_2\}$ for different levels of long-run aggregate consumption C, which are consistent with a FTA. When not constrained by a FTA, the planner achieves a higher level of consumption by changing the Home allocation $\{c_1, c_2\}$, as in parts (i) and (ii) of Proposition 1. For $y_1 > \alpha$—the area above the black line, where good 1 is abundant—the long-run allocation absent FTA is more biased towards c_1. Whereas for $y_1 < \alpha$—the area below the black line, where good 1 is scarce—the allocation is more biased towards c_2. The FTA and no-FTA allocations only coincide in the case $y_1 = y_2^* = \alpha$—part (iii) of Proposition 1.

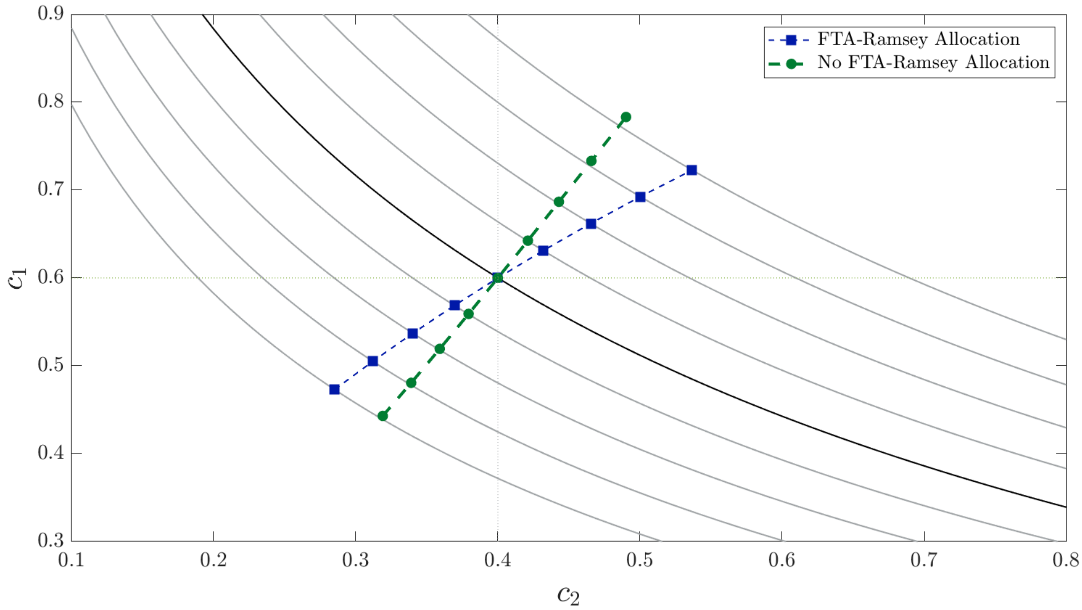
4 Policy and Macro Outcomes at the Optimal Allocation

In this section, we describe the implementation of the unilateral planner's optimal allocation and highlight how policy instruments interact. We then contrast the macroeconomic dynamics at the planning allocation with and without a FTA, comparing to the decentralized case.

4.1 Implementation

We consider an implementation where policy instruments map directly to wedges in the Euler and relative goods demand equations. We assume households can trade in non-contingent bonds, denominated in each good variety. The Home planner can impose the same proportional tax

Figure 2: Optimal Allocations and the Pareto Frontier



Notes: Plot of optimal consumption allocations for Home consumer from Ramsey capital flow taxation (i) with a FTA in place (blue circles, i.e., the Pareto frontier) and (ii) absent a FTA, with goods-specific taxation (green crosses) at different Home endowments. Specifically using nine equally-spaced allocations for $y_1 \in [\alpha - 0.25, \alpha + 0.25]$, with $y_1^* = 1 - y_1$, $y_2 = 1 - \alpha$ and $y_2^* = \alpha$. Other model parameters are: $\beta = 0.96$, $\sigma = 2$, $\phi = 1.5$, and $\alpha = 0.6$. Grey/black lines denote loci of $\{c_1, c_2\}$ which attain different levels of aggregate consumption (black for $C = 1$, grey otherwise). Horizontal (vertical) dotted lines denote α $(1 - \alpha)$, and intersect at the 'no-trade' point—part (iii) of Proposition 1.

θ_t on the gross returns to net lending in all bond markets. So the per-period budget constraint for the Home consumer can be written as:

$$\mathbf{p}_{t+1} \cdot \mathbf{a}_{t+1} + \tilde{\mathbf{p}}_t \cdot \mathbf{c}_t = \mathbf{p}_t \cdot \mathbf{y}_t + (1 - \theta_{t-1}) (\mathbf{p}_t \cdot \mathbf{a}_t) - T_t$$

where $\tilde{\mathbf{p}}_t = \mathbf{p}_t$ when a FTA is in place, $\mathbf{a}_t$ denotes the vector of asset positions and T_t is a lump-sum rebate. Given a no-Ponzi condition, $\lim_{t \to \infty} \tilde{\mathbf{p}}_t \cdot \mathbf{a}_t \geq 0$, the first-order conditions associated with Home households' utility maximization are given by:

$$u'(C_t) g_i(\mathbf{c}_t) = \beta(1 - \theta_t)(1 + r_{i,t}) u'(C_{t+1}) g_i(\mathbf{c}_{t+1}) \tag{9}$$

for $i = 1, 2$, where $r_{i,t} \equiv \frac{p_{i,t}}{p_{i,t+1}} - 1$ is a good-specific interest rate. Combining this with the analogous Foreign Euler equation, and using $g_{i,t}/p_{i,t} = 1/P_t$, yields the Backus and Smith (1993) condition with a wedge reflecting capital-flow taxation:

$$(1 - \theta_t) = \frac{u'(C_t)}{u'(C_{t+1})} \frac{u^{*\prime}(C_{t+1}^*)}{u^{*\prime}(C_t)} \frac{Q_t}{Q_{t+1}} \tag{10}$$

A tax on capital inflows (or a subsidy for outflows) is then captured by values of $\theta_t < 0$, which can also be interpreted as a tax on current consumption relative to future consumption.

Without a FTA, the Home planner can additionally levy a proportional import tax τ_t, and $\tilde{\mathbf{p}}_t = \boldsymbol{\tau}_t \cdot \mathbf{p}_t$ where $\boldsymbol{\tau}_t = [1 \;\; \tau_t]'$, and an import tariff is captured by $\tau_t > 0$. The representative Home household faces an import price $p_{2,t}(1+\tau_t)$, so their relative demand is given by:

$$\frac{c_{1,t}}{c_{2,t}} = \frac{\alpha}{1-\alpha}\left(\frac{1}{S_t(1+\tau_t)}\right)^{-\phi} \tag{11}$$

Alternative Instruments. While we focus on an implementation using capital controls and tariffs, the policy problem solves for the optimal wedges in the risk-sharing and relative demand equations. So, consistent with the fact that the implementation of the Ramsey optimal allocation via taxation is generally non-unique (Chari and Kehoe, 1999), any instruments which map to these wedges could instead be used. In Section 5 and Appendix C.2, we detail an extension of the model with segmented markets and show that FXI (see, e.g., Bianchi and Lorenzoni, 2022; Fanelli and Straub, 2021) can achieve similar outcomes to capital controls, with interactions between trade and financial policy persisting. Similarly, time-variation in the optimal relative-demand wedge may reflect manipulation of non-tariff barriers or regulation, evidenced in Broda et al. (2008).[15]

4.2 Interactions Between Optimal Trade and Financial Policy

To investigate the interactions between the capital-flow tax and tariffs, we decompose the (log) risk-sharing condition (10) into the following two wedges:

$$\ln(1-\theta_t) \approx -\theta_t = \underbrace{-\sigma\left(\hat{C}_t - \hat{C}_{t+1} + \hat{C}^*_{t+1} - \hat{C}^*_t\right)}_{\text{Consumption Wedge}} + \underbrace{\left(\hat{Q}_t - \hat{Q}_{t+1}\right)}_{\text{RER Wedge}} \tag{12}$$

where $\hat{x}$ denotes the natural logarithm of x. The 'consumption wedge' component captures the target consumption growth for the planner. The 'RER wedge' reflects capital-flow taxation incentives are connected to the evolution of the real exchange rate Q.

For a given target consumption growth, a higher RER wedge (corresponding to a depreciated exchange rate) implies that a larger capital-inflow tax is required to implement the optimal consumption allocation—so there is over-borrowing in the decentralized equilibrium. When goods are perfectly substitutable, i.e., $\phi \to \infty$, the consumption wedge will be the same with and without a FTA, but tariffs can still effect the RER wedge. A lower import tariff leads to a higher RER wedge, in turn requiring a larger capital-inflow tax.

Optimal Instruments in CO Case. We present the general expressions for optimal capital-flow taxes and tariffs in Appendix B.5. However, to build intuition, we discuss the CO case

[15] In addition, De Loecker, Goldberg, Khandelwal, and Pavcnik (2016) show that exporters charge variable markups over marginal costs in foreign markets, suggesting that trade agreements do not necessarily constrain relative prices.

(i.e., the limit as $\sigma \to 1$ and $\phi \to 1$) which yields stark predictions for the interaction of policy instruments at the optimal allocation.

Proposition 2 (Optimal Instruments in the CO Case) *Evaluating the allocation defined by equations (7) and (8), the optimal instruments for the Home unilateral Ramsey planner absent a FTA in the limit as $\sigma \to 1$ and $\phi \to 1$ are given by:*

$$\theta_t^{nFTA} = 1 - \frac{1 + \frac{c_{1,t} - y_{1,t}}{\overline{Y}_1 - c_{1,t}}}{1 + \frac{c_{1,t+1} - y_{1,t+1}}{\overline{Y}_1 - c_{1,t+1}}} \quad \text{and} \quad \tau_t^{nFTA} = \frac{1 + \frac{c_{2,t} - y_{2,t}}{\overline{Y}_2 - c_{2,t}}}{1 + \frac{c_{1,t} - y_{1,t}}{\overline{Y}_1 - c_{1,t}}} - 1$$

In contrast, when an FTA is in place, evaluating the allocation defined by equation (6) yields the following expression for the optimal capital-inflow tax for the Home unilateral Ramsey planner with a FTA in the limit as $\sigma \to 1$ and $\phi \to 1$:

$$\theta_t^{FTA} = 1 - \frac{1 + \omega_{1,t} \frac{c_{1,t} - y_{1,t}}{\overline{Y}_1 - c_{1,t}} + \omega_{2,t} \frac{c_{2,t} - y_{2,t}}{\overline{Y}_2 - c_{2,t}}}{1 + \omega_{1,t+1} \frac{c_{1,t+1} - y_{1,t+1}}{\overline{Y}_1 - c_{1,t+1}} + \omega_{2,t+1} \frac{c_{2,t+1} - y_{2,t+1}}{\overline{Y}_2 - c_{2,t+1}}}$$

where $\omega_{1,t} \equiv (1-\alpha) \frac{c_1^{*\prime}(C_t^*)}{c_{1,t}^*}$ and $\omega_{2,t} \equiv \alpha \frac{c_2^{*\prime}(C_t^*)}{c_{2,t}^*}$.

Proof: See Appendix B.5. □

In the CO setting, there is a separation between the objectives of trade and financial policy which follows from the independence of goods allocations detailed in Lemma 3. Without a FTA, the optimal tariff τ_t^{nFTA} depends on the relative excess demand across goods (i.e., $c_{2,t} - y_{2,t}$ relative to $c_{1,t} - y_{1,t}$) weighted by the respective valuation of each good (i.e., $(\overline{Y}_i - c_{i,t})^{-1}$ for $i = 1, 2$). If there is higher excess demand for good 2, all else equal, a positive import tariff is levied so that households consume less of the relatively expensive good. In turn, when the optimal tariff is set, the optimal capital-inflow tax θ_t^{nFTA} depends on the growth rate of excess demand for good 1 only (i.e., $c_{1,t} - y_{1,t}$ relative to $c_{1,t+1} - y_{1,t+1}$) weighted by valuations.[16] If there is higher excess demand for good 1 at time t, a capital-inflow tax is levied $\theta_t^{nFTA} < 0$. In contrast, under a FTA, the optimal capital-inflow tax at the CO parametrization θ_t^{FTA} depends on the growth of a weighted average of excess demand for both goods.

Consider a setting in which the Home endowment of good 1 is growing such that $y_{1,t} < y_{1,t+1}$. Proposition 2 indicates that, under a FTA, the optimal capital-inflow tax at the CO parametrization will depend on the excess demand for both goods 1 and 2, which, in this case, will move in opposite directions. When taxing capital inflows, the Home planner balances the incentives to tax inflows of good 1 with the desire to subsidize inflows of good 2. In contrast, in the no-FTA setting, the optimal capital-inflow tax depends only on the growth in excess demand for good 1, since any inefficiency in relative goods demand is dealt with by the optimal tariff. Framed in terms of equation (12), since consumption is procyclical, the RER wedge will

[16] This result depends on the fact that tariffs are levied on good 2 only. If tariffs were levied on good 1, then capital-inflow taxes would depend only on good-2 excess demand.

reinforce movements in the consumption wedge. So, in the CO case, the capital-inflow tax will be larger absent a FTA when $y_{1,t} < y_{1,t+1}$.

In contrast, in response to variation in the good-2 endowment, the optimal capital-flow tax will always be zero absent a FTA in the CO case, implying that optimal trade policy can substitute for capital-flow management. The following corollary clarifies this.

Corollary 1 (Trade Policy as a Substitute for Capital Controls) *In the limit as $\sigma \to 1$ and $\phi \to 1$, when $y_{2,t} < y_{2,t+1}$, the Home unilateral planner will tax capital inflows under a FTA $\theta_t^{FTA} < 0$. However, the optimal capital-inflow tax is zero in the no-FTA case: $\theta_t^{nFTA} = 0$.*

Proof: This follows from Lemmas 2 and 3, and Proposition 2. □

In this knife-edge case, the RER wedge moves to perfectly offset changes in the consumption wedge. Away from the CO case, in our numerical simulations, we verify that, in response to fluctuations in the good-2 endowment, variation in optimal capital-inflow taxes is still smaller absent a FTA—implying that trade policy can act as a substitute for capital controls more generally.

4.3 Model Simulation

To illustrate the macroeconomic dynamics and implementation of the optimal allocations away from the CO case, we describe two simulation scenarios that generalize the intuition discussed in the previous sub-section—focusing on fluctuations in the good-1 and good-2 endowments in turn. Our simulations are deterministic.

We specify initial and terminal values for the country-good endowments, and construct the full sequence of endowments for all periods by assuming that endowments follow a first-order autoregressive process:

$$\begin{aligned} y_{i,t+1}^{(*)} &= \left(1 - \rho_i^{(*)}\right) \overline{y}_i^{(*)} + \rho_i^{(*)} y_{i,t}^{(*)}, \quad \forall t > 0 \text{ and } i = 1, 2, \\ \mathbf{y}_0 &= [y_{1,0}\ y_{2,0}]' \\ \mathbf{y}_0^* &= \left[y_{1,0}^*\ y_{2,0}^*\right]' \end{aligned}$$

where for simplicity we assume $\rho_1 = \rho_2 = \rho_1^* = \rho_2^*$. In both scenarios, we assume there is no change in the aggregate endowment ($Y_{1,t} = \overline{Y}_1$ and $Y_{2,t} = \overline{Y}_2$ for all t). This is a useful benchmark, because households are able to fully insure their consumption against known changes in their endowment in the decentralized allocation, and perfect consumption smoothing is achieved.[17]

Based on the CRRA per-period utility function and the Armington (1969) specification for aggregate consumption, the model calibration is detailed in Table 1. In each scenario, we compare the decentralized allocation, the unilateral Ramsey planning allocation with a FTA in

[17]This assumption merely serves to sharpen comparison with the decentralized allocation and clarify the mechanisms driving our results. The same factors are at play when the aggregate endowment is allowed to fluctuate.

place, and one without a FTA. To focus on the dynamic implications of the three variants in a consistent manner, we equalize the long-run equilibrium (i.e., 'steady state') of each model by using a steady-state import tariff for the Home country.[18]

Table 1: Benchmark Model Calibration

Parameter	Value	Description
β	0.96	Discount factor, annual frequency
σ	2	Coefficient of relative of risk aversion
ϕ	1.5	Elasticity of substitution between goods 1 and 2
α	0.6	Share of good 1 (good 2) in Home (Foreign) consumption basket
ρ	0.8	Persistence of endowments

4.4 Scenario 1: Temporarily Low Endowment of Domestic Good

Consider a scenario in which the Home economy is recovering from a domestic downturn, or is growing more quickly than its Foreign counterpart. Specifically, the Home country's endowment in good 1 is low in the near term, and grows towards its long-run level. Denoting initial endowment values by $y_{i,0}^{(*)}$ and long-run levels by $\overline{y}_i^{(*)}$ for $i = 1, 2$, we assume that $y_{1,0} = 0.9\overline{y}_1$ and $y_{2,0} = \overline{y}_2$. To ensure there is no aggregate uncertainty: $y_{1,0}^* = 1 - y_{1,0}$ and $y_{2,0}^* = 1 - y_{2,0}$. Figure 3 shows the resulting allocations.

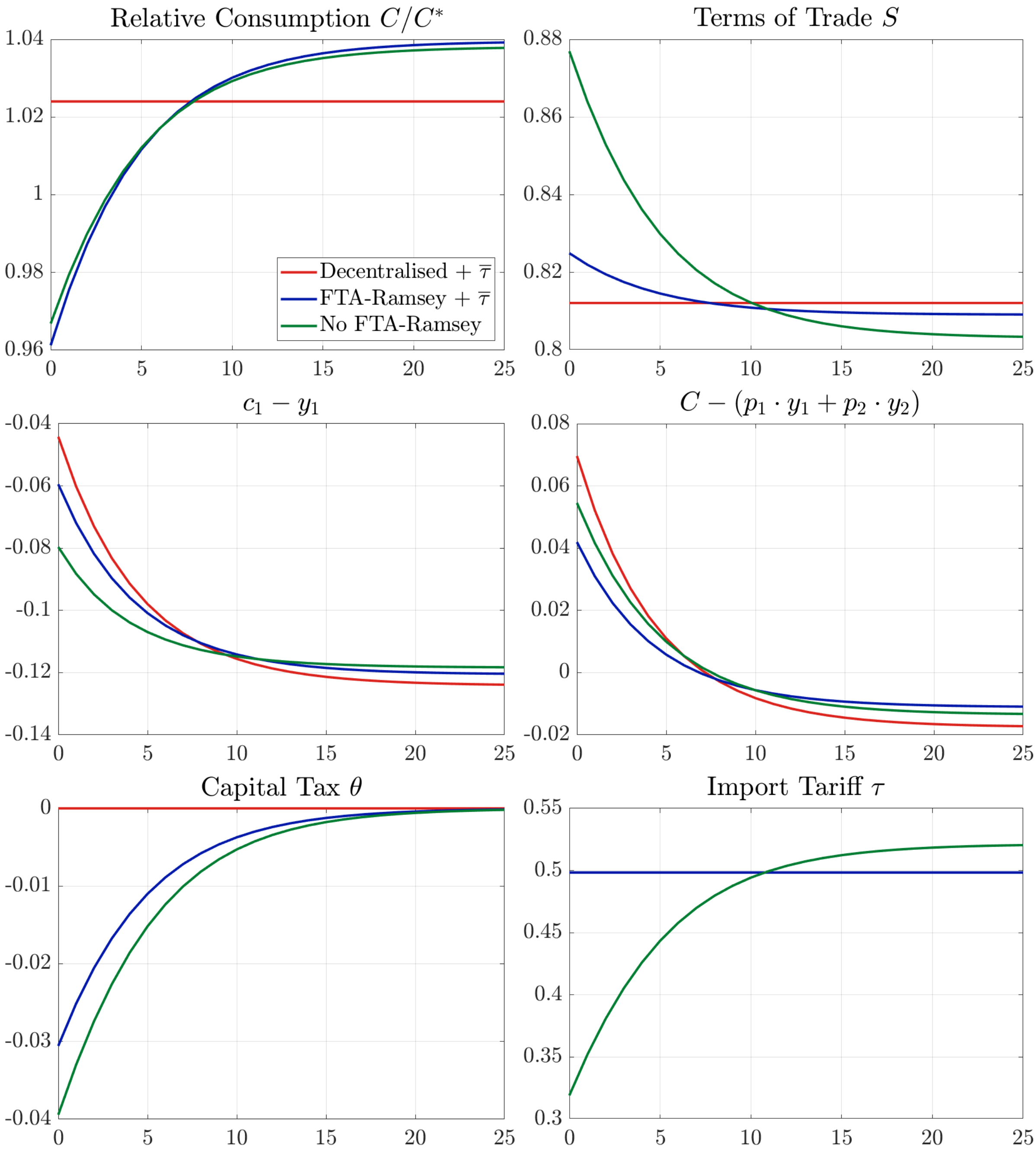
Faced with a higher stream of endowments in the future, Home households will borrow to smooth consumption—as demonstrated by the flat line in the top-left panel of Figure 3. However, each additional unit of consumption brought forward raises the cost of borrowing (i.e., inter-temporal margin). Additionally, the Home household will buy relatively more units of the domestic good (good 1) from abroad, at a time when it is relatively more expensive to do so. As a consequence, the path for $c_1 - y_1$ in the middle-left panel of Figure 3 is most steep for the decentralized allocation (i.e., intra-temporal margin).

The optimal policy, both with and without a FTA, involves leaning against capital flows to delay consumption.[19] This is shown in the middle-right panel, plotting the evolution of the balance of payments, which varies by less under the two planning solutions relative to the decentralised outcome. Additionally, because the Home endowment of good 1—the good consumed with home bias domestically—is initially lower, the planner has an incentive to restrict the global excess demand for good 1 over and above their endowment y_1. As a result, when the good-1 endowment deviates from its long-run level, the planner's inter- (pertaining to the cost of borrowing) and intra- temporal (pertaining to relative goods prices) incentives to manipulate the terms of trade are aligned. The planner chooses to both delay aggregate consumption *and* consumption of good 1, in expectation that the future price of C and c_1 will fall. Therefore,

[18] This approach is similar to that used in the normative New-Keynesian literature that studies allocations where the steady state is first best (or constrained first best).

[19] This echoes the general result in Fanelli and Straub (2021) that the planner has a greater incentive to smooth flows than households.

Figure 3: Time Profile of Optimal Allocations as the Home Endowment of Good 1 Rises in Scenario 1



Notes: Time profile for macroeconomic outcomes in Scenario 1, simulated for 100 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The "Decentralized" and FTA-Ramsey models include a steady-state tariff to ensure that their steady-state allocations replicate the No FTA-Ramsey case.

the planner taxes aggregate consumption C via a capital-inflow tax $\theta_t < 0$ and, in the absence of a FTA, levies an increasing path for import tariffs $\tau_{t+1} > \tau_t$.

In the presence of a FTA, the planner achieves the desired allocation by choosing a lower level of near-term aggregate consumption C. Due to home bias for good 1 this implies lower Home consumption of good 1 (c_1). However, in the long run, the planner delivers higher consumption, both in aggregate and of good 1, in comparison to the decentralized case, by allocating consumption to periods when it is relatively cheaper. The required capital-inflow tax is around 3% in the near term and approaches zero as the endowment returns to its long-run level.

When unconstrained by a FTA, the planner can also restrict the net global supply of good 1 via an import tariff on good 2, which incentivizes Home consumers to consume more of good-1. However, tariffs have second-best effects on the terms of trade which, in turn, influence optimal capital-flow taxes. In this scenario, the planner wants to reduce consumption of good 1 in the near term. The planner implements this using a rising path for tariffs over time—starting at 30% (a subsidy relative to the steady-state tariff) but increasing to over 50% in the long term. However, the effective subsidy leads to a near-term depreciation of the terms of trade, resulting in a more variable terms of trade than in the FTA case. Since this implies that consumption is relatively cheap for Home households, all else equal, this would lead to further over-borrowing. As a consequence, and consistent with the logic of Proposition 2, the capital-inflow tax is roughly one-third larger without a FTA, at 4% in the near term, than in the FTA case.[20]

Figure 4 plots the decomposition of optimal capital-flow taxes from equation (12). The decomposition implies that the capital-flow tax is larger when the terms of trade is depreciated in the near term, only if target consumption growth is relatively unchanged. The left-hand plot indicates that the consumption wedge explains a substantial portion of the overall variation in the capital-flow tax θ but is very similar across the FTA and no-FTA cases. In contrast, the RER wedge is significantly more negative in the no-FTA case, shown in the right-hand panel, and this drives the increase in the capital inflow tax. Nevertheless, both the consumption and RER wedge have the same sign, reflecting the alignment of inter- and intra-temporal incentives in this scenario.

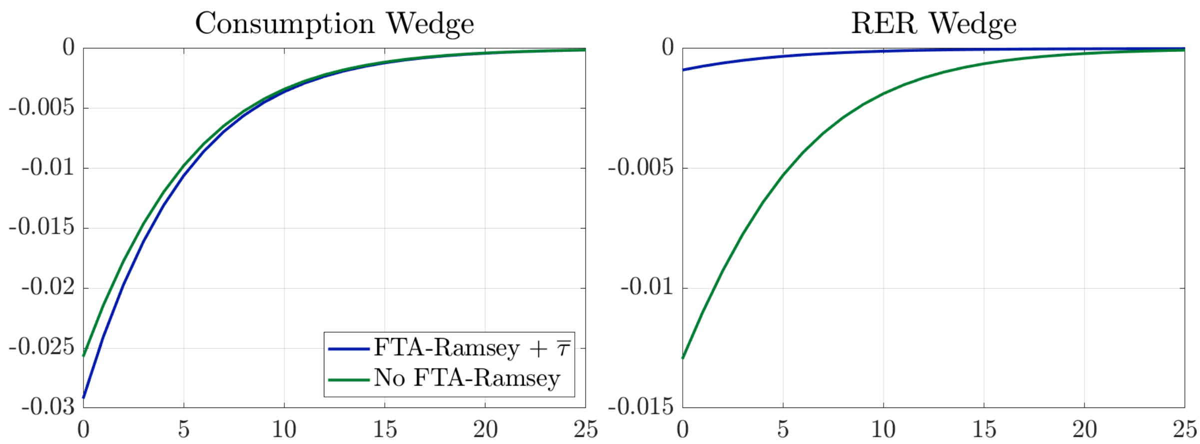
4.5 Scenario 2: Temporarily Low Endowment of Foreign Good

Next, consider the case in which the Home endowment of the foreign good (good 2) starts at a low value relative to its long-run level. We assume that $y^*_{2,0} = 1.1\overline{y}^*_2$ and $y^*_{1,0} = \overline{y}^*_1$. To ensure there is no aggregate uncertainty: $y_{1,0} = 1 - y^*_{1,0}$ and $y_{2,0} = 1 - y^*_{2,0}$. The resulting time profiles for the allocations are plotted in Figure 5.

As in scenario 1, households borrow in the near term in the decentralized allocation, knowing that their endowment will increase in the future. However, the net supply of good 1 that Home sells abroad rises, because c_1 falls while y_1 is unchanged. The Home planner wants to delay aggregate consumption C inter-temporally, but has an incentive to act monopolisticaly and

[20]The difference between taxes in the FTA and no-FTA cases is even larger absent the steady-state tariff.

Figure 4: Decomposition of Optimal Capital-Flow Taxes for Scenario 1



Notes: Time profile for Home capital-flow tax components in Scenario 1, simulated for 100 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place.

drive up the price of good 1 (intra-temporally).

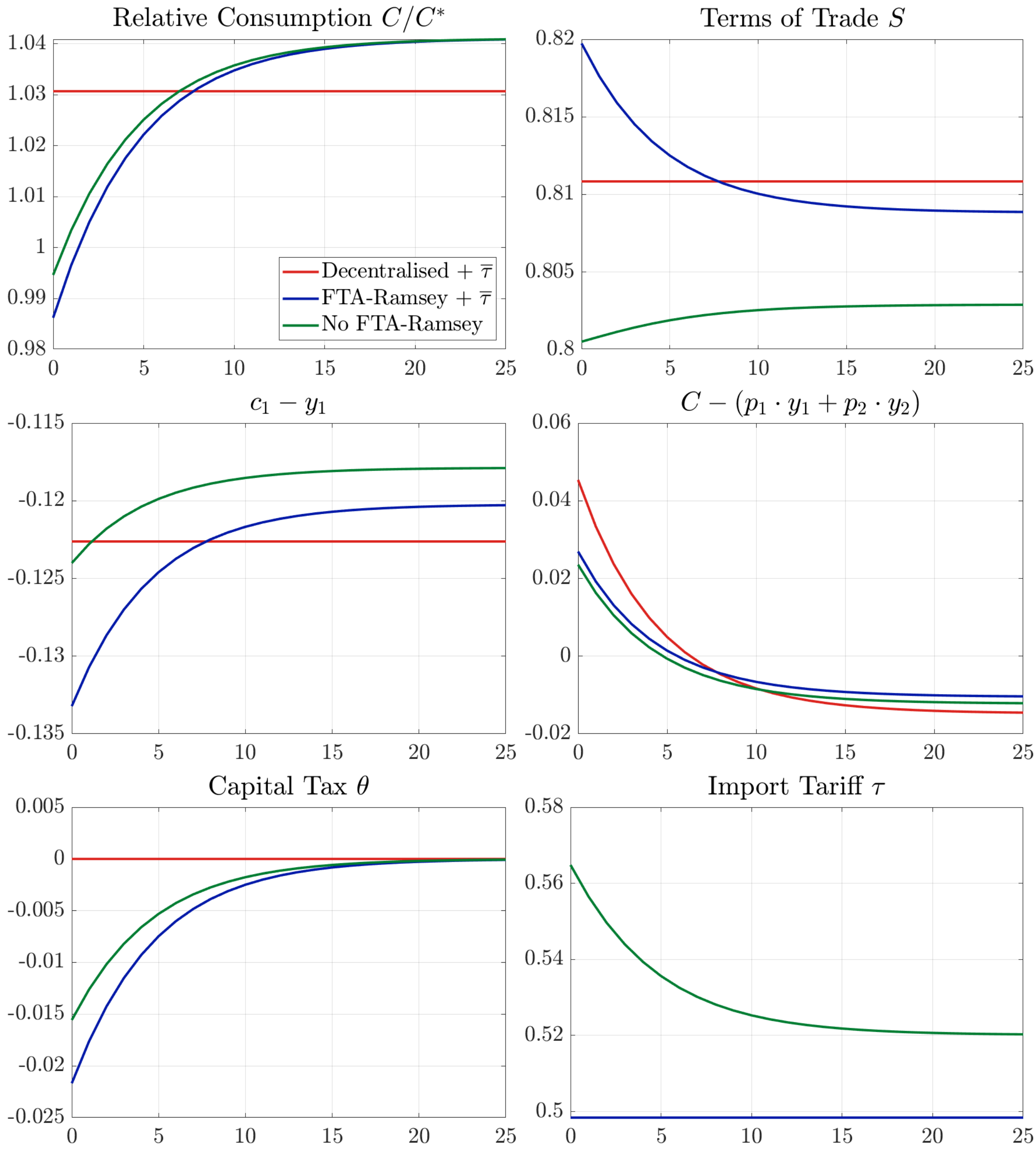
With a FTA, the planner levies a capital-inflow tax in the near term, which diminishes as endowments approach their long-run level. This implies a disproportionately lower consumption of good 1, trading off inter- and intra-temporal incentives to manipulate the terms of trade.

Absent a FTA, the planner levies a relatively high import tariff in the near term to increase Home demand for good 1, c_1, and drive up its relative price. The declining path for tariffs, all else equal, implies that the terms of trade will depreciate over time—making Home consumption relatively expensive in the near-term and discouraging borrowing. As a result, time-varying tariffs act as a substitute for the capital-inflow tax, which is lower in the no-FTA case—but only partially so away from the CO parametrization.

Figure 6 plots the corresponding risk-sharing wedges. As in scenario 1, the consumption wedge explains the majority of overall variation in the capital-flow tax but the differences between the FTA and no-FTA cases are small. However, in contrast to scenario 1, the right-hand panel demonstrates that the RER wedge has the opposite sign for the planner when there is no FTA. This reflects the misalignment of inter- and intra-temporal incentives in this scenario. As a consequence of this, when the planner levies tariffs to monopolistically drive the price of good 1 up, at the same time, this appreciates the terms of trade in the near term, which discourages households from borrowing and reduces the need for a capital-inflow tax.

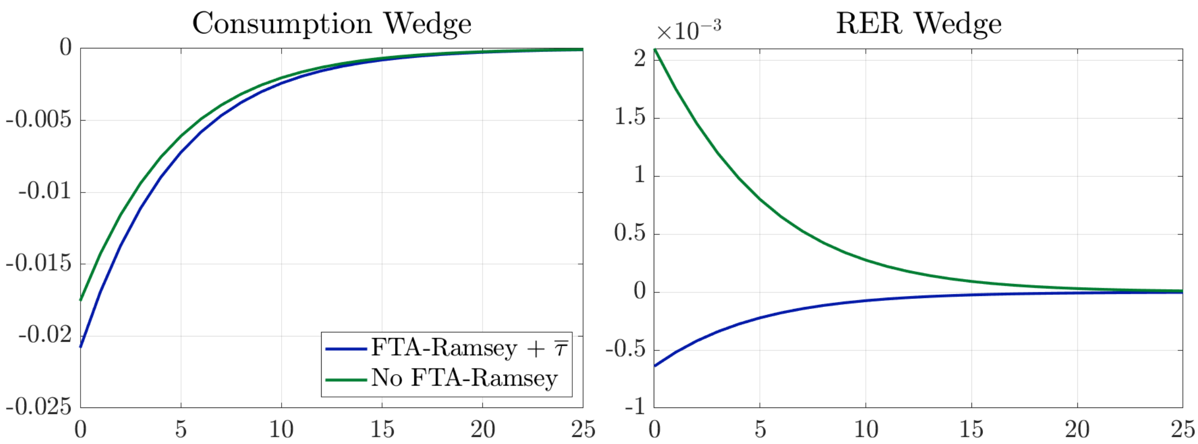
A comparison of Figures 4 and 6 clarifies the role of inter- and intra-temporal incentives in driving the interaction between trade and financial policy. In scenario 1, the alignment of incentives results in reinforcing consumption and RER wedges and, in turn, larger capital-inflow taxes in the absence of an FTA. In contrast, in scenario 2, or more generally when inter- and intra-temporal incentives are misaligned, high import tariffs in early periods appreciate the real exchange rate disincentivizing consumption and, in this case, optimal trade policy partly

Figure 5: Time Profile of Optimal Allocations as the Foreign Endowment of Good 2 Falls in Scenario 2



Notes: Time profile for macroeconomic outcomes in Experiment 2, simulated for 100 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The "Decentralized" and FTA-Ramsey models include a steady-state tariff to ensure that their steady-state allocations replicate the No FTA-Ramsey case.

Figure 6: Decomposition of Optimal Capital Flow Taxes for Scenario 2



Notes: Time profile for Home capital-flow tax components in Scenario 2, simulated for 100 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place.

substitutes for capital-flow management.

Comparative Statics. For both scenarios 1 and 2, two parameters are key influences on the size of inter- and intra-temporal motives: the respective elasticities of substitution. When the elasticity of inter-temporal substitution $1/\sigma$ is low—i.e., σ is high—countries levy larger capital controls in an attempt to reallocate consumption inter-temporally. When the intra-temporal (trade) elasticity ϕ is low, countries set larger tariffs. This aligns with the well-understood *inverse-elasticity* result in public finance that a planner optimally chooses to tax commodities for which demand is price-inelastic.[21]

Anticipated Changes in Endowments. The mechanisms that underpin optimal allocations in scenarios 1 and 2 also carry over to instances in which changes in endowments are anticipated. In these cases, optimal trade and financial policy involves preemptive action in advance of the shock itself—similar to Mendoza (2002); Bianchi (2011). For instance, in events where a temporary fall in the good-1 endowment is anticipated in the future, the Home planner will subsidize capital-inflows prior to the shock to facilitate borrowing to help smooth consumption. Without a FTA in place, the planner only employs tariffs to subsidise consumption of the relatively abundant good once the endowment fall has been realized. We discuss anticipated scenarios further in Appendix B.7, where we operationalize the shocks within our deterministic setting by supposing that, at $t = 0$, a change in endowment at some time period $\bar{t}$ is fully and accurately anticipated.

[21] We explore these comparative statics further in Appendix B.6.

4.6 Optimal Tariffs Absent Capital Controls: Free Financial Flows

To further clarify the interactions between optimal trade policy and capital controls, we also consider a setting where the planner optimally chooses tariffs while capital controls are ruled out by a FFFA. This case serves both as a useful benchmark to evaluate the welfare consequences of policy interventions, but also illustrates how tariffs can be implemented as a second-best instrument to manipulate the cost of borrowing over time.

In scenario 1, because the optimal tariff in the no-FTA case is low in early periods and, all else equal, exacerbates over-borrowing, the optimal tariff in the FFFA case deviates less from its long-run value. In scenario 2, where inter- and intra-temporal planning incentives work in opposite directions, the optimal tariff in the no-FTA case is relatively high in early periods and induces under-borrowing—in part, substituting for financial policy. As a result, absent capital controls, the optimal tariff is even higher in the early periods. The full details of the FFFA case are shown in Appendix B.8.

5 Generality of Results

In this section, we discuss how our results apply to more general and realistic environments.

Production, Nominal Rigidities and Aggregate-Demand Externalities. We first consider how our results generalize when policy is driven by alternative *incentives*, specifically demand management. We extend the model to feature production of traded and non-traded goods (denoted with subscripts T and NT, respectively), endogenous labor supply and nominal-wage rigidities. Non-traded goods are produced with a linear production technology $y_{NT,t} = A_t L_t$ under perfect competition (where A_t denotes productivity and L_t labor). The associated firm maximization yields $p_{NT,t} = A_t w_t$, where we assume the nominal wage is perfectly rigid $w_t = \overline{w}$, and $c_{NT,t} = y_{NT,t}$ in equilibrium. A full model exposition is presented in Appendix C.1.

In this setting, the marginal benefit to the unilateral Home planner from a unit of tradable consumption $c_{T,t}$ can be written as: $u'(C_t) g_{T,t} \left(1 + \frac{\omega}{1-\omega} \tau_t^L\right)$, where $g_{T,t}$ is the derivative of the aggregate consumption aggregator with respect to the tradable good, ω represents the expenditure share on non-tradable goods, and τ_t^L is the labor wedge, given by:

$$\tau_t^L = 1 + \frac{1}{A_t} \frac{\upsilon_{L,t}}{u'(C_t) g_{T,t}}$$

where $\nu_{L,t}$ represents the marginal disutility of labor supply for the household at time t. It is positive when the economy is demand constrained and households are involuntarily unemployed. The marginal benefit of a unit of tradable consumption is higher when the economy is demand constrained, generating an additional incentive for a planner to bring forward consumption with policy interventions.[22]

[22]The labor wedge is acyclical and constant under the CO parametrization.

Returning to the planner's problem, the implementability constraint (Lemma 1) is unchanged. Absent a FTA, the first-order conditions with respect to goods 1 and 2 are given by:

$$\begin{aligned} u'(C_t)g_{1,t}\left(1+\frac{\omega}{1-\omega}\tau_t^L\right) &= \mu\mathcal{MC}_{1,t} \\ u'(C_t)g_{2,t}\left(1+\frac{\omega}{1-\omega}\tau_t^L\right) &= \mu\mathcal{MC}_{2,t} \end{aligned}$$

Moreover, since the risk-sharing condition of this model is unchanged relative to the baseline setup, tariffs affect the path of the exchange rate for tradables in the same way. And so, faced with constrained demand and unemployment ($\tau_t^L > 0$), the planner brings consumption forward with an optimal mix of capital-inflow subsidy or an import subsidy, which puts pressure on the exchange rate to depreciate, as in the baseline model. Proposition C1 in Appendix C.1 details the optimal instruments in the model with nominal rigidities.

Segmented Markets and Quantity Interventions. We also consider how our results generalize when alternative *instruments* are used to deliver the optimal allocation. Importantly, we show that a similar outcome can be achieved if the planner uses quantity interventions (e.g., open-market operations or FXI) in place of capital controls.[23] To show this, we consider a model extension with non-traded goods and segmented financial markets, which we detail in Appendix C.2. There is a single asset in each economy, denominated in units of the domestic non-traded good, which households trade with financial intermediaries—where positions are denoted by a_{t+1} and a^I_{t+1}, respectively.[24] The intermediation problem implies one additional equilibrium condition:

$$\left[R^{*\,-1}_{NT,t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R^{-1}_{NT,t}\right] = \Gamma a^I_{t+1} \tag{13}$$

where $R^{(*)}_{NT,t}$ is the cost of borrowing in the Home (Foreign) countries and $\mathcal{E}_t = p^*_{NT,t}/p_{NT,t}$. The parameter Γ captures how binding limits to arbitrage are, with $\Gamma \to 0$ being the limiting case with frictionless financial markets.

We additionally allow the planner to take a position a^G_{t+1} in domestic assets, financed by selling foreign assets, such that market clearing requires: $a_{t+1} + a^G_{t+1} + a^I_{t+1} = 0$. Because of this, the planner can affect risk sharing via the balance sheet of financial intermediaries a^I_{t+1}. If $a^G_{t+1} = 0$, (13) indicates that when households are borrowing ($a_{t+1} < 0$) they face higher borrowing costs ($R_{NT,t+1}$ rises) because financiers with limits to arbitrage must be compensated to take the opposite position ($a^I_{t+1} > 0$). Planner intervention (e.g., in the form of FXI) can reduce the size of imbalances that need to be intermediated ($a_{t+1} < a^G_{t+1} < 0$) and so the spread narrows. Proposition C2 in Appendix C.2 formalizes that FXI can target a similar allocaiton

[23] It is now well understood that for quantity interventions to be effective, we must allow for imperfect mobility of capital across countries, as in Gabaix and Maggiori (2015), to break the result of Backus and Kehoe (1989).

[24] Since the model is deterministic, the exact denomination does not affect the spanning properties of the asset. Without a non-traded good, trade in real bonds would imply interest-rate equalization by the law of one price.

to capital controls when markets are segmented and that trade and financial policy remain interconnected. Moreover, under specific restrictions on preferences detailed in Appendix C.2, the direction of the interaction is unchanged as well.

Trade Disruptions and Sanctions. We can also show that the mechanisms we describe apply to a wide range of *sources* of economic fluctuations. For example, we can consider the case of global trade disruptions and multilateral sanctions within our setting by modelling increases in the (iceberg) cost of importing for the planning country.[25] Faced with trade disruptions that increase the cost of imports (and therefore of aggregate consumption), the optimal policy mix prescribes capital inflow taxes, which we find are larger when allowing for tariffs.

Concretely, suppose there are sanctions in place which will be relaxed in the near future. The Home planner will seek to tax capital inflows and delay consumption. Absent a FTA, the planner also wants to subsidize good 2 in the near future, partly offsetting the wedge introduced in the relative demand by the sanctions. As a result, inter- and intra-temporal incentives are aligned and, consistent with our theory, the optimal capital inflow tax rises.[26]

Country Size. Finally, we describe how our results generalize with respect to *country size.* Within our two-country model, countries are large in goods *and* financial markets. So planners internalize the effects of domestic allocations on both goods prices and the world real interest rate. Appendix C.3 details a small-open economy setting, with $N \to \infty$ foreign countries. In this case, as Costinot et al. (2014) show, countries remain large in goods markets for their *domestic* variety, although their ability to influence the world interest rate along the inter-temporal margin disappears.[27] While there are a range of outcomes in the small-open economy setting, there is an interesting knife-edge case when $\sigma = \phi = 1$ (Cole and Obstfeld, 1991). Here, the required size of capital controls for inter- and intra-temporal incentives is the same: in scenario 1, as $N \to \infty$, the optimal size of capital controls in both the FTA and no-FTA case is unchanged. Moreover, even though the optimal import tariff falls, it is always non-zero since Home goods become more scarce. Moving away from this limiting case, when $\sigma > \phi$, the size of capital controls will fall as N rises since the inter-temporal motive dominates. In scenario 2, consistent with Proposition 1, the optimal capital-inflow tax is 0 absent an FTA.

[25] Unlike a tariff, the iceberg costs are not rebated to households, nor do they reallocate consumption across countries. We can interpret multilateral sanctions as a case in which the sanctions are set by an international organisation or coalition, or a third-party country.

[26] Varying σ and ϕ indicates that, under the FTA, inter- and intra-temporal incentives are opposed. This is true only when we are restricted to capital flow taxes, because capital controls cannot offset the wedge induced by sanctions.

[27] Egorov and Mukhin (2020) show that in the presence of nominal rigidities and dollar currency pricing, i.e., when world exports are priced in dollars, US prices affect the world stochastic discount factor and the US is able to manipulate the inter-temporal terms of trade even if it is small.

6 Strategic Planning Allocation

We next consider a Nash equilibrium, where each planner chooses allocations taking the other's tax sequence $\{\theta_t^{(*)}, \tau_t^{(*)}\}$ as given, where τ_t^* denotes Foreign import tariffs levied on good 1. The Nash equilibrium with a FTA is discussed in Costinot et al. (2014), so we defer a full exposition of this to Appendix D.1. The derivations result in a strategic counterpart to equation (6), which indicate that the ratio of marginal costs from bringing forward a unit of consumption in each country should be proportional to the bargaining power of each country. In this section, we first present the Nash equilibrium without a FTA, before discussing the interaction of trade and financial policy.

6.1 Without Free Trade

Defining the vector of Foreign goods-specific tariffs by $\boldsymbol{\tau}_t^* \equiv [(1+\tau_t^*)^{-1}\ 1]'$, the following lemma details the implementability constraint for the Home planner.

Lemma 4 (Implementability for Nash Planner without FTA) *The Home allocation forms part of an equilibrium without an FTA if it satisfies:*

$$\sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\leq 0 \tag{IC-Nash-nFTA}$$

Proof: See Appendix D.3. □

With this, the Home planning problem, accounting for the optimal response by the Foreign planner, is given by:

$$\max_{\{\mathbf{c}_t\}} \quad \sum_{t=0}^{\infty}\beta^t u(C_t) \tag{P-Nash-nFTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\leq 0 \tag{IC-Nash-nFTA}$$

$$C_t \equiv g(\mathbf{c}_t) \tag{nFTA}$$

which is comparable to the unilateral problem (P-Unil-nFTA), albeit with additional terms in the implementability constraint reflecting the Foreign capital-flow tax θ_t^* and tariff τ_t^*.

Optimal Allocation. Problem (P-Nash-nFTA) yields the optimality conditions:

$$u'(C_t)g_1(\mathbf{c}_t) = \mu\hat{\mathcal{MC}}_{1,t}^{nFTA} \tag{14}$$

$$u'(C_t)g_2(\mathbf{c}_t) = \mu\hat{\mathcal{MC}}_{2,t}^{nFTA} \tag{15}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\begin{aligned}\hat{\mathcal{MC}}_{1,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)(1+\tau_t^*)^{-1}g_1^*(\mathbf{c}_t^*) + u^{*\prime\prime}g_1^*(\mathbf{c}_t^*)\boldsymbol{\tau}_t^{*-1} \cdot \nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\ & + u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1} \cdot \frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t] \\ \hat{\mathcal{MC}}_{2,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)g_2^*(\mathbf{c}_t^*) + u^{*\prime\prime}g_2^*(\mathbf{c}_t^*)\boldsymbol{\tau}_t^{*-1} \cdot \nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\ & + u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1} \cdot \frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t]\end{aligned}$$

The Foreign planner undertakes an analogous maximization. Combining the optimality conditions of the Home and Foreign planners yields the equilibrium allocation, summarized in the following proposition.

Proposition 3 (Capital Controls and Tariff Wars) *In a Nash equilibrium where each country chooses optimal capital controls $\{\theta_t, \theta_t^*\}$ and tariffs $\{\tau_t, \tau_t^*\}$ for all $t \geq 0$, the allocations $\{\mathbf{c}_t, \mathbf{c}_t^*\}$ satisfy:*

$$\frac{\hat{\mathcal{MC}}_{1,t}^{nFTA}}{\hat{\mathcal{MC}}_{1,t}^{*nFTA}} = \alpha_{1,0}^{nFTA} \qquad \frac{\hat{\mathcal{MC}}_{2,t}^{nFTA}}{\hat{\mathcal{MC}}_{2,t}^{*nFTA}} = \alpha_{2,0}^{nFTA} \tag{16}$$

where

$$\alpha_{i,0}^{nFTA} \equiv \frac{\hat{\mathcal{MC}}_{i,0}^{nFTA}}{\hat{\mathcal{MC}}_{i,0}^{*nFTA}} \quad \textit{for } i = 1, 2$$

Proof: See Appendix D.3. □

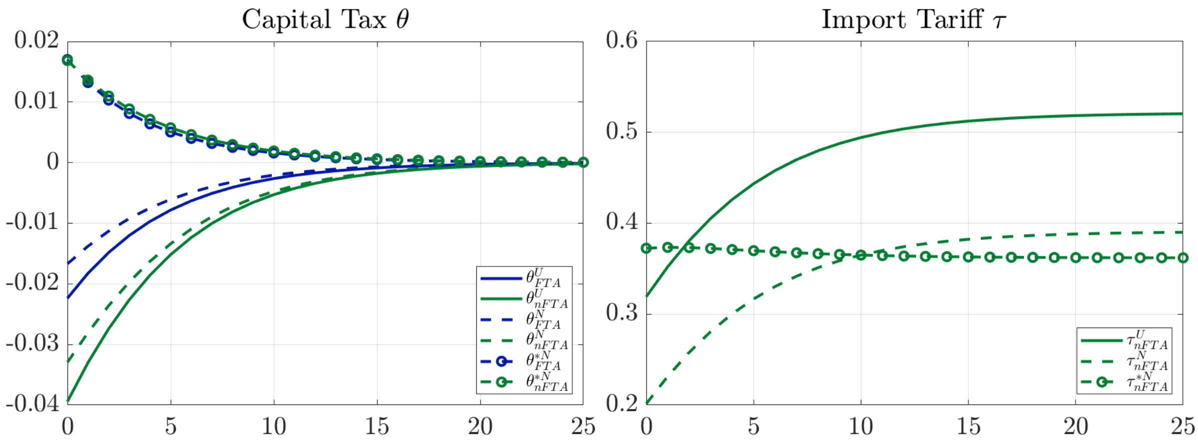
The equilibrium conditions state that the ratio of the marginal cost of a unit of consumption for the planner across the Home and Foreign country, for each good variety, is equal to a constant. The constants $\{\alpha_{i,0}^{nFTA}\}$ reflect the bargaining power of the Foreign country relative to the Home with respect to each good, and they depend on initial conditions. The interpretation of the marginal cost terms is consistent with that in Section 3.

6.2 Numerical Simulations

We now revisit the two scenarios from Section 4 to assess how strategic interactions affect policy outcomes and the macroeconomic allocations.

Scenario 1. Figure 7 presents the optimal capital-inflow taxes and import tariffs for scenario 1, comparing the FTA (blue) and no-FTA (green) cases in the strategic (dashed) and unilateral (solid) settings. Because the Home endowment of good 1 is temporarily low relative to its long-run value, Home households will over-borrow in the decentralized setting. In the strategic setting, the Home planner will delay consumption using a capital-inflow tax, while the Foreign planner brings forward consumption with a capital-inflow subsidy, both with and without a

Figure 7: Optimal Capital-Inflow Taxes and Import Tariffs for Home and Foreign in the Nash Equilibrium for Scenario 1



Notes: Optimal capital controls and taxes. 'U' subscript denotes unilateral optimal policy result (for Home, solid lines). 'N' denotes Nash outcome for Home (dashed lines) and Foreign (dashed lines with circle markers).

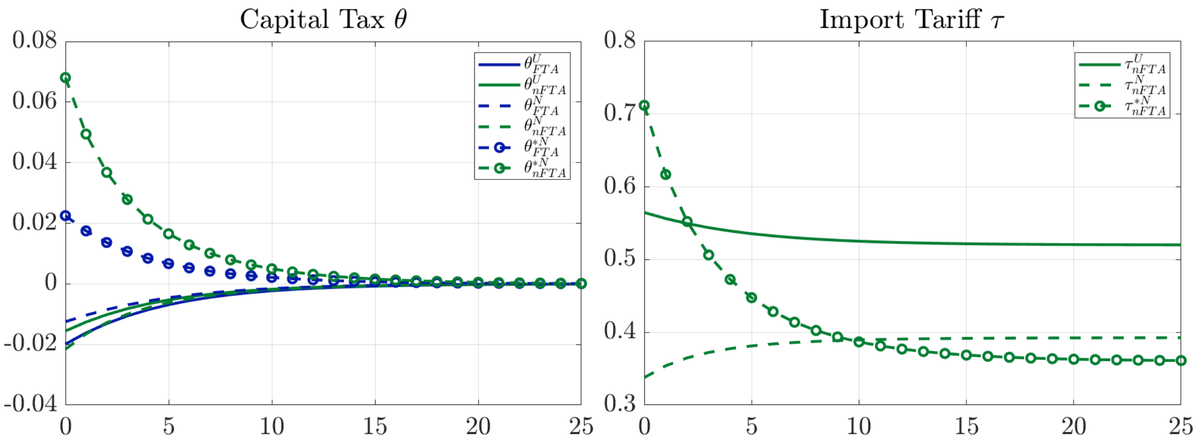
FTA. The required capital-inflow tax set by the Home planner in the strategic setting is smaller than that in the unilateral case, since Foreign policy is reinforcing and helps to tilt near-term consumption from Home to Foreign households, relative to the decentralized case.

As in the unilateral case, the Home planner has an incentive to delay consumption of good 1. To achieve this, the Home planner sets an increasing path for tariffs today. However, while the Home planner's inter- and intra-temporal incentives are aligned, they are opposed for the Foreign planner, and so Foreign import tariffs decline somewhat over time. Overall, since the Home tariff varies more, trade policy implies a relative depreciation of the terms of trade. So, consistent with the unilateral case, the capital-inflow tax is larger absent a FTA.

Scenario 2. Figure 8 presents the corresponding figures for scenario 2. As in scenario 1, the Home planner delays consumption by levying a capital-inflow tax in the near term, while the Foreign planner brings forward consumption with a subsidy. Strategic interactions make a difference in the implementation of tariffs. In the Foreign country, where good 2 is relatively abundant in the near term, the planner seeks to increase the price of good 2 and does so by setting a declining path for tariffs on good 1. Because the Foreign country is large in the market for good 2, this effect dominates the Home planner's incentive to manipulate relative prices for this model parametrization. As a consequence, the Home planner faces a real exchange rate depreciation (relative to the FTA case) which encourages Home households to borrow further—requiring a larger capital-inflow tax absent an FTA in scenario 2 *as well as* in scenario 1.

Comparative Statics. As in the unilateral case, the size of capital controls and import tariffs depend strongly on values of the inter- and intra-temporal elasticities of substitution. Lower values for the elasticity of inter-temporal substitution $1/\sigma$ are associated with more capital-

Figure 8: Optimal Capital-Inflow Taxes and Import Tariffs for Home and Foreign in the Nash Equilibrium for Scenario 2



Notes: Optimal capital controls and taxes. 'U' subscript denotes unilateral optimal policy result (for Home, solid lines). 'N' denotes Nash outcome for Home (dashed lines) and Foreign (dashed lines with circle markers).

control wars, while lower trade elasticities ϕ are associated with more tariff wars.[28]

7 Welfare and Policy Games

In this section, we analyze the welfare costs of capital controls and trade tariffs. We show that capital-control wars are less likely to emerge when countries are committed to a FTA.

7.1 Welfare Losses Relative to the Global-Cooperation Benchmark

As a starting point, consider the problem faced by a world planner maximizing joint (world) welfare:

$$\max_{\{\mathbf{c}_t, \mathbf{c}_t^*\}} \quad \sum_{t=0}^{\infty} \beta^t \left[u(g(\mathbf{c}_t)) + \kappa u(g^*(\mathbf{c}_t^*)) \right] \tag{P-Coop}$$

$$\text{s.t.} \quad \mathbf{c}_t + \mathbf{c}_t^* = \mathbf{Y}_t \tag{RC}$$

$$\mathbf{c} = \mathbf{c}(C), \quad \mathbf{c}^* = \mathbf{c}^*(C) \tag{FTA}$$

where κ is the relative weight attributed to Foreign welfare. Intuitively, since there are no frictions in the global economy, the world planner's problem yields the globally first-best allocation, as the following proposition clarifies.

Proposition 4 (Global Cooperation Allocation) *In the cooperative allocation resulting from* (P-Coop)*, no intervention is optimal such that, if* $\kappa = 1$*,* $\theta_t = \theta_t^* = \tau_t = \tau_t^* = 0$.

Proof: See Appendix D.6. □

[28] We discuss this further in Appendix D.5.

Table 2: Welfare and Spillovers: % Consumption-Equivalent Welfare Losses from Alternative Planning Allocations

	H	F	Global $\sum_{H,F}$
Scenario 1			
Unilateral-Home Allocation:			
with FTA	-0.020	0.032	0.006
without FTA	-1.992	3.442	0.815
(from dynamics)	*(-0.052)*	*(0.007)*	-
without FTA, with FFFA	-1.944	3.350	0.777
(from dynamics)	*(-0.003)*	*(-0.081)*	-
Nash Allocation:			
with FTA	0.009	0.017	0.014
without FTA	1.757	1.534	1.668
without FTA, with FFFA	1.751	1.709	1.725
Scenario 2			
Unilateral-Home Allocation:			
with FTA	-0.015	0.025	0.004
without FTA	-2.277	3.956	0.964
(from dynamics)	*(-0.018)*	*(0.169)*	-
without FTA, with FFFA	-2.271	3.943	0.959
(from dynamics)	*(-0.012)*	*(0.157)*	-
Nash Allocation:			
with FTA	0.025	0.003	0.015
without FTA	2.269	1.653	2.007
without FTA, with FFFA	2.137	2.153	2.133

Notes: Table presents the % of extra consumption that a country (or the world) would require in the planning allocation to deliver the same welfare as in the decentralized allocation in scenarios 1 and 2. A positive (negative) number represents a welfare loss (gain) in the planning allocation relative to the decentralised allocation. Results come from 100-period simulation of scenarios 1 and 2. Home (Foreign) consumption-equivalent expressed in units of Home (Foreign) aggregate consumption. Global consumption-equivalent expressed in units of PPP-weighted world aggregate consumption.

Moreover, since the cooperative outcome is first best, this corollary follows:

Corollary 2 (Negative Spillovers) *Any policy intervention which improves welfare in one country necessarily reduces global welfare by disproportionately worsening welfare in the other.*

Proof: Follows directly by combining Propositions 1 and 4. □

To analyze the welfare implications of policies, we revisit scenarios 1 and 2. We consider the allocations arising from both the unilateral and strategic planning allocations, with and without a FTA and a FFFA. We compare welfare by assessing consumption-equivalent variation relative to the global-cooperative allocation (i.e., one of no intervention).

Table 2 presents the consumption-equivalent welfare losses for each country, as well as the globally.[29] Three results stand out from the unilateral outcomes, in which the Home planner sets policy optimally while the Foreign planner is passive. First, the capital-flow taxes and tariffs levied by the Home planner are distortionary and change consumption paths in a manner that is inefficient for the Foreign country. Consistent with the corollary above, unilateral policy does not simply reallocate consumption across borders: the Home welfare gain is small in comparison to the welfare costs to the Foreign country for both scenarios 1 and 2. Therefore, world welfare is lower relative to the globally-cooperative allocation in all cases. Second, departing from a FTA can generate larger welfare gains for the Home country relative to the FTA case, both in levels (i.e., without equalizing steady states with a constant tax) and dynamically (i.e., with a steady-state tax). However, the corresponding losses for the Foreign economy are also larger such that, in level terms, the loss in world welfare is substantially larger without a FTA (increasing from 0.006% with a FTA in scenario 1 to 0.815%). Third, comparing the FTA with the difference between the no FTA and FFFA, we see that welfare gains from capital controls are larger in the absence of a FTA in scenario 1 but smaller in scenario 2, consistent with our analysis in Section 4.

The key result from the Nash outcomes is that the country-level and global welfare costs from policy wars are disproportionately larger when countries depart from a FTA. Introducing a distortion along the intra-temporal margin will exacerbate over-/under-borrowing through the impact of tariffs on the real exchange rate.

7.2 FTAs and Prospects for Capital-Control Wars

Can commitment to a FTA discourage costly capital-control wars? To answer this question, we consider a dynamic setting with an open-loop Nash equilibrium in which country planners begin in an equilibrium without capital controls (i.e., a FFFA), either with a FTA in place or with optimal tariffs in place (i.e., no FTA).[30] We then assess the incentive for the Home planner to deviate from the FFFA and levy capital controls. To do this we assume that the Foreign planner initially sets no tariffs in the FTA case and the strategically optimal tariff with no FTA. They assume that the Home planner will adopt the same strategy for tariffs (i.e., either none with a FTA, strategically optimal for no FTA) and will never levy capital controls (i.e., they assume Home will also stick to the FFFA).

We then consider a case in which the Home planner deviates from the FFFA, and begin our simulation from here. Initially, the Home planner sets capital-flow taxes (and tariffs in the no-FTA case) as if they are acting unilaterally—i.e., they assume the Foreign planner is passive. However, we allow the Foreign planner to retaliate after $\bar{t}$ periods by re-optimizing and choosing

[29] Home (Foreign) consumption-equivalents are expressed in units of Home (Foreign) aggregate consumption. Global consumption-equivalent expressed in units of PPP-weighted world aggregate consumption.

[30] In an open-loop Nash equilibrium, players cannot observe the actions of their opponents and therefore do not respond optimally to each others' change in strategy. This assumption is made for tractability (see, e.g., Fudenberg and Levine, 1988).

Table 3: Welfare Losses (% Consumption Equivalent) when Home Deviates from FFFA (with and without FTA) and Foreign Retaliates $\bar{t} = 5$ Periods Later

	Scenario 1		**Scenario 2**	
	H	F	H	F
with FTA	−0.134	0.119	−0.178	0.159
without FTA	−0.188	0.188	−0.535	0.483

Notes: Table presents the % of extra consumption that a country would require to deliver the same welfare as in the strategic allocation in scenarios 1 and 2. A positive (negative) number represents a welfare loss (gain) in the planning allocation relative to the strategic allocation. Results come from 100-period simulation of scenarios 1 and 2. Home (Foreign) consumption-equivalent expressed in units of Home (Foreign) aggregate consumption.

both capital controls (and tariffs in the no-FTA case).[31] At this stage, the assumptions of the game are the same as in the strategic allocation: each planner sets policy assuming the other will also do the same.

We present the path for instruments and consumption, in both the FTA and no-FTA cases for scenarios 1 and 2 in Appendix D.7. In both cases, the Home planner attains a higher consumption level in the first $\bar{t}$ periods, which comes at the cost of Foreign consumption. After $\bar{t}$ periods, allocations coincide with the strategic outcome.

Using these simulations, we assess the consumption-equivalent welfare losses and gains for the Home and Foreign country to assess the magnitude of incentives to depart from a FFFA with and without a FTA in place. The results, shown in Table 3, demonstrate that a (credible) commitment to a FTA reduces the incentive for a country to deviate from a FFFA and unilaterally levy capital controls. In scenario 1, deviation from the FFFA increases Home welfare by 0.134% with a FTA in place compared to 0.188% without one; in scenario 2, the welfare gains are over three-times larger without a FTA. Moreover, the costs of one country's deviation for countries that do not deviate is significantly larger when there is no FTA in place. In scenario 1, Foreign losses are over 50% larger without a FTA; in scenario 2, the losses are over three-times larger. Intuitively, if the initial equilibrium is one where there is competition over tariffs, either country faces a distorted path for aggregate consumption over time due to the effects of time-variation in tariffs on the path for real exchange rates and, therefore, the welfare gains from departing from a FFFA and levying capital controls are larger.

8 Conclusion

In this paper, we provide a unified framework for the analysis of capital-flow management and trade policy. We show that introducing tariffs, or trade disruptions more generally, distorts

[31] Such a strategy is often referred to as a 'Grim Trigger' strategy (see, e.g., Friedman, 1971). For our experiments, we use $\bar{t} = 5$, but this parameter does not have important implications for the main message of these experiments.

the cost of borrowing over time. In turn, this reduces efficiency and gives rise to a novel motive for managing capital flows. When tariffs are optimally chosen, we show that whether optimal capital controls are larger or smaller depends on whether the inter- and intra-temporal incentives to manipulate the terms of trade are aligned. While, in our baseline setting, we focus on pecuniary (terms-of-trade) externalities, we generalize our results to settings in which policy is driven by aggregate-demand management. In addition to this, our results also apply to a range of alternative policy instruments (e.g., FXI when markets are segmented), different sources of shocks (e.g., trade disruptions and sanctions), and persist within small-open economies.

While employing tariffs in addition to capital controls can improve welfare domestically when a planner faces no retaliation, this comes at a disproportionate cost to foreign welfare. In a Nash equilibrium with retaliation, capital controls tend to be larger absent a FTA in all states of the economy because of the effect of tariff wars on the real exchange rate. As a consequence, the welfare costs from capital-control wars are disproportionately larger at both a country-level and globally when there is no FTA in place—giving rise to tariff wars too.

Finally, we conduct a policy experiment and show that commitment to a FTA can reduce incentives to depart from a FFFA and use capital controls. Since capital-control wars are costly for global welfare, our analysis highlights a novel argument in favor of FTAs: namely that retaining openness in trade can help to sustain financial openness.

Our framework leaves open numerous avenues for future research. There is scope to consider the role of incomplete markets. While policy cannot improve upon the cooperative allocation absent additional frictions when financial markets are complete, this is not the case with financial-market incompleteness. Moreover, the interaction between capital-flow taxes and tariffs will depend on the currency denomination of debt. In a nominal model, for example, foreign-currency debt alters the balance between inter- and intra-temporal incentives facing the planner due to the desire to the inflate away debt obligations. Moreover, while our results generalize to settings that include production, in models with capital or intermediate inputs (to mimic real-world global value chains), there may be scope for asymmetries in factor intensities to drive policy incentives.

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Appendix

A Mathematical Preliminaries

A.1 Derivatives of the Consumption Aggregator

In this appendix, we define the derivatives of the Armington (1969) aggregator which arise in the Ramsey-planning first-order conditions. We present the expressions for the representative Home consumer only, but they are analogous for the representative Foreign consumer. The first derivatives of the Home aggregator are given by:

$$g_1(\mathbf{c}_t) \equiv \frac{\partial g(\mathbf{c}_t)}{\partial c_{1,t}} = \alpha^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} = \alpha^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} C_t^{\frac{1}{\phi}}$$

$$g_2(\mathbf{c}_t) = \frac{\partial g(\mathbf{c}_t)}{\partial c_{2,t}} = (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} = (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} C_t^{\frac{1}{\phi}}$$

The second derivatives are:

$$\begin{aligned}
g_{11}(\mathbf{c}_t) =& -\frac{1}{\phi} \alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{-1-\phi}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} \\
&+ \frac{1}{\phi} \alpha^{\frac{2}{\phi}} c_{1,t}^{-\frac{2}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{2-\phi}{\phi-1}} \\
g_{12}(\mathbf{c}_t) =& \frac{1}{\phi} \alpha^{\frac{1}{\phi}} (1-\alpha)^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{2-\phi}{\phi-1}} \\
g_{21}(\mathbf{c}_t) =& g_{12}(\mathbf{c}_t) \\
g_{22}(\mathbf{c}_t) =& -\frac{1}{\phi} (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{-1-\phi}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} \\
&+ \frac{1}{\phi} (1-\alpha)^{\frac{2}{\phi}} c_{2,t}^{-\frac{2}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{2-\phi}{\phi-1}}
\end{aligned}$$

A.2 Pareto Frontier

This appendix provides derivations for the Pareto frontier defined in Section 2. The Pareto frontier summarizes combinations of consumption allocations $\{c_{1,t}, c_{2,t}\}$ which are Pareto efficient, given a level of aggregate consumption C_t.

The Home representative household chooses their consumption by minimizing expenditure, for a given level of aggregate consumption $\overline{C}$:

$$\min_{c_{1,t}, c_{2,t}} \quad p_{1,t} c_{1,t} + p_{2,t} c_{2,t} \quad \text{s.t.} \quad \overline{C} = g(\mathbf{c}_t)$$

The first-order conditions for this problem yield the Home relative demand equation:

$$\frac{g_{1,t}}{g_{2,t}} = \frac{p_{1,t}}{p_{2,t}} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\phi}} \left(\frac{c_{2,t}}{c_{1,t}}\right)^{\frac{1}{\phi}} \tag{A1}$$

where $p_{1,t}/p_{2,t} \equiv 1/S_t$ and S_t refers to the terms of trade.

To derive the Pareto frontier, note that the analogous Foreign relative demand curve is:

$$\frac{g^*_{1,t}}{g^*_{2,t}} = \frac{p_{1,t}}{p_{2,t}} = \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\phi}} \left(\frac{c^*_{2,t}}{c^*_{1,t}}\right)^{\frac{1}{\phi}} \tag{A2}$$

Equating relative prices across countries, equations (A1) and (A2) yield:

$$\frac{c^*_{2,t}}{c^*_{1,t}} = \left(\frac{\alpha}{1-\alpha}\right)^2 \frac{c_{2,t}}{c_{1,t}} \tag{A3}$$

This expression for optimal relative consumption must be consistent with goods market clearing ($Y_{i,t} = c_{i,t} + c^*_{i,t}$ for $i = 1, 2$). Combining (A3) with goods market clearing, we attain the following expressions for consumption:

$$c_{1,t} = \frac{b c_{2,t} Y_{1,t}}{Y_{2,t} - (1-b)c_{2,t}} \tag{A4}$$

$$c_{2,t} = \frac{c_{1,t} Y_{2,t}}{b Y_{1,t} + (1-b)c_{1,t}} \tag{A5}$$

where $b \equiv \left(\frac{\alpha}{1-\alpha}\right)^2$.

Solving for $\mathrm{d}c_i(C)/\mathrm{d}C$. Rearranging the Armington aggregator, we can show that:

$$c_{1,t}(C_t) = \left[\frac{C_t^{\frac{\phi-1}{\phi}} - (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}}{\alpha^{\frac{1}{\phi}}}\right]^{\frac{\phi}{\phi-1}} \tag{A6}$$

$$c_{2,t}(C_t) = \left[\frac{C_t^{\frac{\phi-1}{\phi}} - \alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}}}{(1-\alpha)^{\frac{1}{\phi}}}\right]^{\frac{\phi}{\phi-1}} \tag{A7}$$

Equating (A5) with (A7) yields:

$$\left[C_t^{\frac{\phi-1}{\phi}} - \alpha^{\frac{1}{\phi}} c_{1,t}(C_t)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} (bY_{1,t} + (1-b)c_{1,t}(C_t)) = c_{1,t}(C_t) Y_{2,t} (1-\alpha)^{\frac{1}{\phi-1}}$$

Totally differentiating this expression and rearranging yields:

$$\frac{\mathrm{d}c_{1,t}(C_t)}{\mathrm{d}C_t} = \frac{C_t^{-\frac{1}{\phi}}(1-\alpha)^{-\frac{1}{\phi}}c_{2,t}^{\frac{1}{\phi}}(bY_{1,t}+(1-b)c_{1,t}(C_t)}{Y_{2,t}-c_{2,t}(C_t)(1-b)+\alpha^{\frac{1}{\phi}}c_{1,t}(C_t)^{-\frac{1}{\phi}}(1-\alpha)^{-\frac{1}{\phi}}c_{2,t}^{\frac{1}{\phi}}(bY_{1,t}+(1-b)c_{1,t}C_t)}$$

The expression for $\mathrm{d}c_{2,t}(C_t)/\mathrm{d}C_t$ can be derived analogously.

A.3 Deriving Price Indices

Repeating the expenditure minimization exercise in Appendix A.2 while allowing for import tariffs:

$$\min_{c_{1,t},c_{2,t}} p_{1,t}c_{1,t} + p_{2,t}c_{2,t}(1+\tau_t) \quad \text{s.t. } \overline{C} = g(\mathbf{c}_t)$$

yields the relative demand condition (11).

Substituting this into total expenditure yields:

$$\begin{aligned} c_{1,t} &= \frac{\hat{y}_t p_1^{-\phi}\alpha^{-1}}{\alpha p_{1,t}^{1-\phi} + (1-\alpha)p_{2,t}^{1-\phi}(1+\tau_t^{1-\phi})} \\ c_{2,t} &= \frac{\hat{y}_t p_2^{-\phi}(1-\alpha)^{-1}}{\alpha p_{1,t}^{1-\phi} + (1-\alpha)p_{2,t}^{1-\phi}(1+\tau_t^{1-\phi})} \end{aligned}$$

where $\hat{y}_t$ denotes the sum of endowment income and lump-sum transfers to the household.

Finally, substituting this into the constraint of the minimization above and setting $\overline{C} = 1$ and replace $\overline{y}_t$ by P_t:

$$P_t = \left[\alpha p_{1,t}^{1-\phi} + (1-\alpha)p_{2,t}^{1-\phi}(1+\tau_t)^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

Solving an analogous problem for the foreign country yields:

$$P_t^* = \left[(1-\alpha)p_{1,t}^{1-\phi}(1+\tau_t^*)^{1-\phi} + \alpha p_{2,t}^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

The real exchange rate is defined as the ratio P^*/P. This coincides with the ratio of CPI, rather than the PPI, and thus includes sales taxes.

B Unilateral Planning Allocation

B.1 Foreign Household Optimization and Proof to Lemma 1

This appendix details the representative Foreign consumer's optimization problem, which acts as a constraint for the unilateral Home planner. Foreign households maximize their discounted lifetime utility subject to their inter-temporal budget constraint, given world prices $\mathbf{p}_t$:

$$\begin{aligned}\max_{\{\mathbf{c}_t\}} \quad & U_0^* = \sum_{t=0}^{\infty} \beta^t u^*(g^*(\mathbf{c}_t)) \\ \text{s.t.} \quad & \sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) \leq 0\end{aligned}$$

The first-order conditions for this problem are given by (4) and (5), where λ^* is the Lagrange multiplier on the Foreign inter-temporal budget constraint.

B.2 Proof to Lemma 2

First, evaluate (6) at $\sigma \to 1$ and $\phi \to 1$:

$$\begin{aligned} C_t^{-1} =& \mu \left[\frac{(1-\alpha)C_t^*}{c_{1,t}^*} c_1'(C_t) + \frac{\alpha C_t^*}{c_{2,t}^*} c_2'(C_t) \right] \times \\ & \left\{ C_t^{*-1} - C_t^{*-2} \left[\frac{(1-\alpha)C_t^*}{c_{1,t}^*}(c_{1,t} - y_{1,t}) + \frac{\alpha C_t^*}{c_{2,t}^*}(c_{2,t} - y_{2,t}) \right] \right. \\ & \left. + C_t^{*-1} \left[\left(\frac{1-\alpha}{c_{1,t}^*} - \frac{(1-\alpha)C_t^*}{{c_{1,t}^*}^2} c_1^{*\prime}(C_t^*) \right)(c_{1,t} - y_{1,t}) + \left(\frac{\alpha}{c_{2,t}^*} - \frac{\alpha C_t^*}{{c_{2,t}^*}^2} c_2^{*\prime}(C_t^*) \right)(c_{2,t} - y_{2,t}) \right] \right\} \end{aligned}$$

which simplifies to:

$$\begin{aligned} C_t^{-1} =& \mu \left[\frac{(1-\alpha)C_t^*}{c_{1,t}^*} c_1'(C_t) + \frac{\alpha C_t^*}{c_{2,t}^*} c_2'(C_t) \right] \left\{ C_t^{*-1} - \right. \\ & \left. C_t^{*-1} \left[\left(\frac{(1-\alpha)C_t^*}{{c_{1,t}^*}^2} c_1^{*\prime}(C_t^*) \right)(c_{1,t} - y_{1,t}) + \left(\frac{\alpha C_t^*}{{c_{2,t}^*}^2} c_2^{*\prime}(C_t^*) \right)(c_{2,t} - y_{2,t}) \right] \right\} \end{aligned}$$

and further:

$$\begin{aligned} C_t^{-1} =& \mu \left[\frac{(1-\alpha)C_t^*}{c_{1,t}^*} c_1'(C_t) + \frac{\alpha C_t^*}{c_{2,t}^*} c_2'(C_t) \right] C_t^{*-1} \left\{ 1 - \right. \\ & \left. \left[\left(\frac{(1-\alpha)C_t^*}{c_{1,t}^*} c_1^{*\prime}(C_t^*) \right) \frac{c_{1,t} - y_{1,t}}{c_{1,t}^*} + \left(\frac{\alpha C_t^*}{c_{2,t}^*} c_2^{*\prime}(C_t^*) \right) \frac{c_{2,t} - y_{2,t}}{c_{2,t}^*} \right] \right\} \end{aligned}$$

As in Costinot et al. (2014), taking the total derivative of this with respect to $\mathbf{y}_t$, yields

$\mathrm{d}C_t > 0$ if $\sum_i \frac{\mathrm{d}u'(C_t^*)\nabla g_{i,t}^*}{\mathrm{d}C_t}\mathrm{d}y_{i,t} > 0$. Evaluating this at the CO point yields:

$$1 + \frac{P_t C_t}{P_t^* C_t^*} > \frac{2\alpha - 1}{\alpha} \tag{B1}$$

The right-hand side attains a maximum at $\alpha = 1$, so the inequality is trivially satisfied. □

B.3 Proof to Lemma 3

Evaluate equations (7) and (8) as $\sigma \to 1$ and $\phi \to 1$. In this case, equation (7) can be written as:

$$\begin{aligned}\frac{\alpha}{c_{1,t}} =& \mu\Bigg\{C_t^{*-1}(1-\alpha)\frac{C_t^*}{c_{1,t}^*} + C_t^{*-2}(1-\alpha)\frac{C_t^*}{c_{1,t}^*}\left[(1-\alpha)\frac{C_t^*}{c_{1,t}^*}(c_{1,t}-y_{1,t}) + \alpha\frac{C_t^*}{c_{2,t}^*}(c_{2,t}-y_{2,t})\right] \\ & - C_t^{*-1}\left(-(1-\alpha)\frac{C_t^*}{{c_{1,t}^*}^2} + (1-\alpha)(1-\alpha)\frac{C_t^*}{{c_{1,t}^*}^2}\right)(c_{1,t}-y_{1,t}) + \alpha(1-\alpha)\frac{1}{c_{1,t}^*}\frac{C_t^*}{c_{2,t}^*}(c_{2,t}-y_{2,t})\Bigg\}\end{aligned}$$

which can be rewritten as:

$$\begin{aligned}\frac{\alpha}{c_{1,t}} =& \mu(1-\alpha)\frac{1}{c_{1,t}^*}\Bigg\{1 + \left[(1-\alpha)\frac{1}{c_{1,t}^*}(c_{1,t}-y_{1,t}) + \alpha\frac{1}{c_{2,t}^*}(c_{2,t}-y_{2,t})\right] \\ & + \left[\left(\frac{1}{c_{1,t}^*} - (1-\alpha)\frac{1}{c_{1,t}^*}\right)(c_{1,t}-y_{1,t}) - \alpha\frac{1}{c_{2,t}^*}(c_{2,t}-y_{2,t})\right]\Bigg\}\end{aligned}$$

This then simplifies to:

$$\frac{\alpha}{c_{1,t}} = \mu(1-\alpha)\frac{1}{c_{1,t}^*}\left\{1 + \left[\left((1-\alpha)\frac{1}{c_{1,t}^*} + \alpha\frac{1}{c_{1,t}^*}\right)(c_{1,t}-y_{1,t}) + \left(\alpha\frac{1}{c_{2,t}^*} - \alpha\frac{1}{c_{2,t}^*}\right)(c_{2,t}-y_{2,t})\right]\right\}$$

and further:

$$\frac{\alpha}{c_{1,t}}\left(\mu\frac{1-\alpha}{\overline{Y}_1 - c_{1,t}}\right)^{-1} = 1 + \frac{c_{1,t}-y_{1,t}}{\overline{Y}_1 - c_{1,t}}$$

Analogously, (8) simplifies to :

$$\frac{1-\alpha}{c_{2,t}}\left(\mu\frac{\alpha}{\overline{Y}_2 - c_{2,t}}\right)^{-1} = 1 + \frac{c_{2,t}-y_{2,t}}{\overline{Y}_2 - c_{2,t}}$$

The left-hand side of these last two expressions are decreasing in $c_{1,t}$ and $c_{2,t}$, respectively. Similarly, the right-hand side is increasing in $c_{i,t}$ and decreasing in $y_{i,t}$ (for $i = 1, 2$, respectively), but $y_{j,t}$ does not feature in the first-order condition for $c_{i,t}$ for $j \neq i$. From this it follows that $\frac{\mathrm{d}c_{i,t}}{\mathrm{d}y_{i,t}} > 0$ and $\frac{\mathrm{d}c_{i,t}}{\mathrm{d}y_{j,t}} = 0$, verifying the lemma. □

B.4 Proof to Proposition 1

First, note that any outcome achievable in (P-Unil-FTA) is achievable in (P-Unil-nFTA). Part (i) follows immediately since (P-Unil-nFTA) is a relaxed version of (P-Unil-FTA). Therefore the planner achieves weakly better outcomes when the FTA is relaxed. However, we analyze this further. Equations (6), (7), and (8) satisfy the following total derivative rule:

$$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}C} = \frac{\partial \mathcal{L}}{\partial c_1} c_1'(C) + \frac{\partial \mathcal{L}}{\partial c_2} c_2'(C)$$

The solution to (P-Unil-FTA) (when an FTA is in force) satisfies $\frac{d\mathcal{L}}{dC} = 0$ at the (constrained) optimal allocation. Since $c_1'(C)$ and $c_2'(C)$ are positive and increasing functions in Appendix A.2, generally $sign(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}) = -sign(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2})$ indicating an incentive to adjust consumption across varieties remains at the constrained-optimal allocation.

In contrast, the solution to (P-Unil-nFTA) given by (7) and (8) implies $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1} = \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2} = 0$ which necessarily implies aggregate consumption is (unconstrained) optimal as well. Formally, denote:

$$\overline{C} = \{C : \max \quad \mathcal{L}(C) \mid c_1(C), c_2(C) \text{ on Pareto frontier}\}, \tag{B2}$$

where $\overline{C}$ is a scalar because $\mathcal{L}$ is strictly concave in the region of interest. Then note that $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}_{|c_H(\overline{C}),c_2(\overline{C})}, \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2}_{|c_1(\overline{C}),c_2(\overline{C})} \neq 0$. If, for example, $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}_{|c_1(\overline{C}),c_2(\overline{C})} > 0$, then $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2}_{|c_1(\overline{C}),c_2(\overline{C})} < 0$ and there exists an ϵ perturbation such that a $c_1(\overline{C}) \pm \epsilon, c_2(\overline{C}) \pm \epsilon$ are preferred.

Furthermore, (ii) follows since it must be then that $c_1'(C)$ and $c_2'(C)$ implied by (7) and (8) violate Lemma 1 if $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}_{|c_1(\overline{C}),c_2(\overline{C})}, \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2}_{|c_1(\overline{C}),c_2(\overline{C})} \neq 0$. Conversely, if $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}C} = 0$ then $\frac{\partial\mathcal{L}}{\partial c_1} = 0$ and $\frac{\partial\mathcal{L}}{\partial c_2} = 0$ if $c_1'(C)$ and $c_2'(C)$ are not binding, i.e., the constraints are identical to the correspondence implied by (7) and (8).

(iii) follows since the allocations coincide when there is no trade in goods in equilibrium as the households' choice is optimal for the planner. □

B.5 Interaction Between Optimal Instruments and Proof to Proposition 2

In this appendix, we first express the optimal capital controls and tariffs from the unilateral planner's allocation in the general form. We then proceed to prove Proposition 2 by considering the CO limit as $\sigma \to 1$ and $\phi \to 1$.

With Free Trade. Rearranging equation (6) using the following to do so, $Q_t^{-1} = -\frac{\mathrm{d}C_t^*}{\mathrm{d}C_t} = \nabla g^*(\mathbf{c}_t^*)\mathbf{c}_t'(C_t) = -\nabla g^*(\mathbf{c}_t^*)\mathbf{c}_t^{*\prime}(C_t)$, yields:

$$\frac{u'(C_t)}{\mu u^{*\prime}(C_t^*)} Q_t = 1 - \left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)} \nabla g^*(\mathbf{c}_t^*(C_t)) + \frac{\mathrm{d}\nabla g^*(\mathbf{c}_t^*(C_t))}{\mathrm{d}C_t^*} \right) \cdot [\mathbf{c}_t - \mathbf{y}_t]$$

Combining this with equation (10) yields:

$$\theta_t^{FTA} = 1 - \frac{1 - \left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*(C_t)) + \frac{\mathrm{d}\nabla g^*(\mathbf{c}_t^*(C_t))}{\mathrm{d}C_t^*}\right) \cdot [\mathbf{c}_t - \mathbf{y}_t]}{1 - \left(\frac{u^{*\prime\prime}(C_{t+1}^*)}{u^{*\prime}(C_{t+1}^*)}\nabla g^*(\mathbf{c}_{t+1}^*(C_{t+1})) + \frac{\mathrm{d}\nabla g^*(\mathbf{c}_{t+1}^*(C_{t+1}))}{\mathrm{d}C_{t+1}^*}\right) \cdot [\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]}$$

Without Free Trade. Rearranging (7) yields:

$$\frac{u'(C_t)g_{1,t}}{u^{*\prime}(C_t^*)g_{1,t}^*} = 1 - \left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*) + \frac{1}{g_{1,t}^*}\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}^*}\right) \cdot [\mathbf{c}_t - \mathbf{y}_t]$$

Combining this with equation (10) yields, and using $Q_t = \frac{g_{1,t}}{g_{1,t}^*}$, allows us to derive the optimal capital-inflow tax:

$$\theta_t^{nFTA} = 1 - \frac{1 - \left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*) + \frac{1}{g_{1,t}^*}\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}^*}\right) \cdot [\mathbf{c}_t - \mathbf{y}_t]}{1 - \left(\frac{u^{*\prime\prime}(C_{t+1}^*)}{u^{*\prime}(C_{t+1}^*)}\nabla g^*(\mathbf{c}_{t+1}^*) + \frac{1}{g_{1,t+1}^*}\frac{\partial \nabla g^*(\mathbf{c}_{t+1}^*)}{\partial c_{1,t+1}^*}\right) \cdot [\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]}$$

Using expressions (A1) and (A2), the optimal tariff can then be expressed as:

$$1 + \tau_t^{nFTA} = \frac{g_{2,t}/g_{1,t}}{g_{2.t}^*/g_{1,t}^*}$$

and using equations (7) and (8):

$$\tau_t^{nFTA} = \frac{1 - \left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*) + \frac{1}{g_{2,t}^*}\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}^*}\right) \cdot [\mathbf{c}_t - \mathbf{y}_t]}{1 - \left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*) + \frac{1}{g_{1,t}^*}\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}^*}\right) \cdot [\mathbf{c}_t - \mathbf{y}_t]} - 1$$

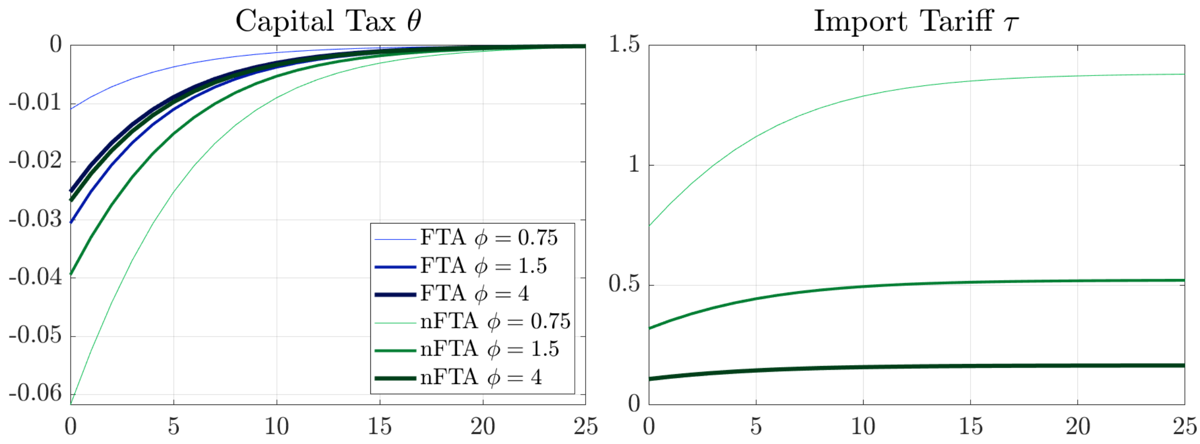
Proof to Proposition 2. This follows from the above after substituting for CRRA per-period utility and the Armington aggregator as $\sigma \to 1$ and $\phi \to 1$. □

B.6 Comparative Statics

Within the model, two parameters are particularly important for governing the size of the planner's intra- and inter-temporal incentives to manipulate the terms of trade: the intra-temporal elasticity of substitution between goods ϕ (i.e., the trade elasticity) and the coefficient of relative risk aversion σ (i.e., the inverse inter-temporal elasticity of substitution). In doing so, these parameters influence the size of both the optimal capital inflow taxes and optimal import tariffs. They do so in a manner that is inversely related to the elasticity: the lower the elasticity, the higher the taxes, and vice versa.

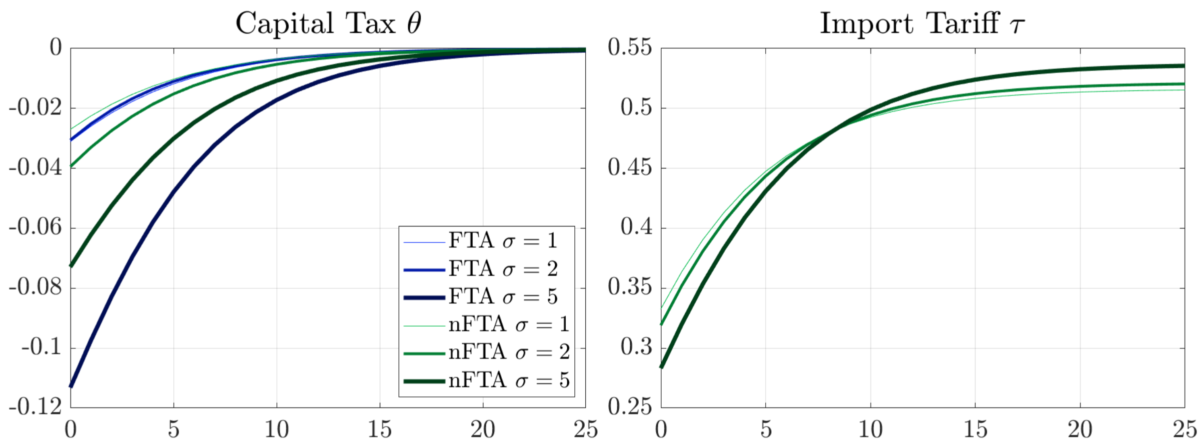
Figure B1 demonstrates this for the intra-temporal trade elasticity in the context of scenario 1 for the unilateral Home planner—although the 'inverse elasticity rule' holds in both scenarios. As the right-hand figure shows, optimal import tariffs are both larger and vary more over time when the trade elasticity is lower. These intra-temporal incentives interact with the optimal

Figure B1: Comparative Statics of Optimal Capital-Flow Taxes and Tariffs with Respect to the Intra-temporal Trade Elasticity ϕ in Scenario 1



Notes: Time profile for Home capital-flow tax and import tariff in scenario 1, simulated for 100 periods, with three different values of intra-temporal elasticity of substitution between goods 1 and 2 ϕ. See Table 1 for calibration details. "(n)FTA" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The FTA-Ramsey model includes a steady-state tariff to ensure that the steady-state allocation replicates the nFTA-Ramsey case.

Figure B2: Comparative Statics of Optimal Capital-Flow Taxes and Tariffs with Respect to the Coefficient of Relative Risk Aversion σ (Inverse Inter-temporal Elasticity of Substitution) in Scenario 1



Notes: Time profile for Home capital-flow tax and import tariff in scenario 1, simulated for 100 periods, with three different values of the coefficient of relative risk aversion σ (i.e., inverse inter-temporal elasticity of substitution). See Table 1 for calibration details. "(n)FTA" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The FTA-Ramsey model includes a steady-state tariff to ensure that the steady-state allocation replicates the nFTA-Ramsey case.

capital-flow taxes too, which are higher for lower trade elasticities, regardless of the prevailing trade agreement.

Similarly, Figure B2 shows that optimal capital-flow taxes are larger when the inter-temporal elasticity of substitution is lower (i.e., higher coefficient of relative risk aversion σ). In turn, variation in import tariffs is larger when σ is high.

B.7 Anticipated Changes in Endowments

In this appendix, we discuss how optimal capital-flow taxes and tariffs are levied in the face of anticipated changes in endowments. To operationalize this within our deterministic simulations, we assume that at $t = 0$, a change in the endowment at some time period $\bar{t}$ is fully and accurately anticipated by all agents in the economy.

As an example, Figure B3 plots the optimal policy instruments from the unilateral setting when the dynamics from scenario 1 are anticipated to occur at $\bar{t} = 5$ (rather than on impact). Concretely, initial endowments are defined as: $y_{i,t}^{(*)} = \bar{y}_i^{(*)}$ for $i = 1, 2$ and $t = 0, 1, 2, 3, 4$. Period-5 endowments at Home are given by $y_{1,5} = 0.9\bar{y}_1$ and $y_{2,5} = \bar{y}_2$, and to ensure no aggregate uncertainty $y_{1,5}^* = 1 - y_{1,5}$ and $y_{2,5}^* = 1 - y_{2,5}$. From period 5 onwards, endowments are assumed to return to their long-run values gradually. So, in essence, this example represents an anticipated negative, but temporary, endowment shock for the Home country.

We choose $\bar{t} = 5$ as an example. Since capital-flow taxes depend on income in adjacent periods (i.e., t and $t + 1$), a key insight from Costinot et al. (2014), this is sufficient to capture how anticipated shocks can generate preemptive policy action.

Consistent with the logic in the main body of the paper, optimal trade and financial policy involves action in advance of the shock—akin to the 'precautionary' motives for intervention in small-open economy models with borrowing constraints (see, e.g., Mendoza, 2002; Bianchi, 2011). For instance, in Figure B3, the Home unilateral planner subsidizes capital-inflows in period 4, prior to the shock, both with and without a FTA, since good 1 is relatively abundant at that time. This facilitates borrowing to help smooth Home consumption. Thereafter, since good 1 becomes relatively scare, the Home planner taxes capital-inflows—as in scenario 1. Without an FTA in place, however, the policymaker is able to smooth Home consumption by more, relative to the FTA case, by employing tariffs alongside preemtive capital-flow taxes.

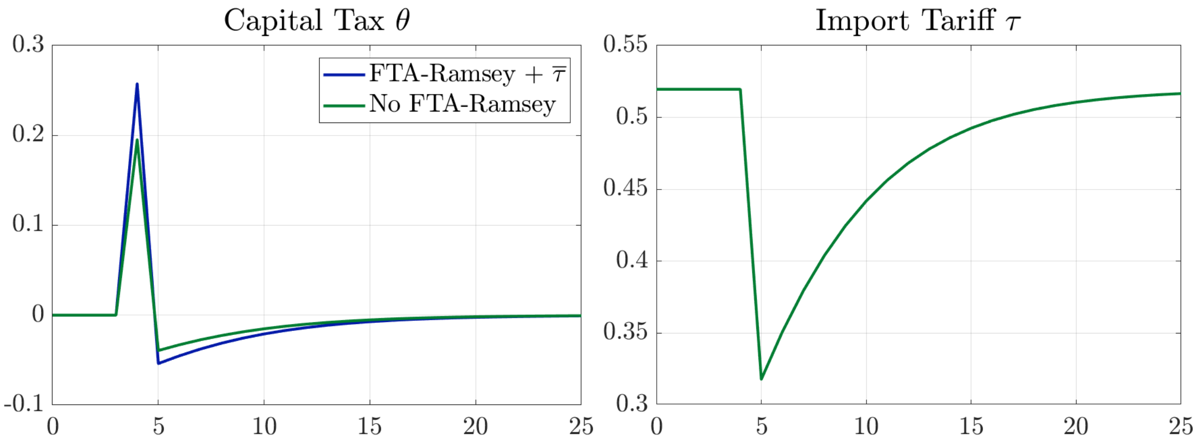
B.8 Ruling Out Capital Controls

We also consider the case where the planner optimally chooses tariffs, but capital controls are contractually ruled out—i.e., by a 'free-financial-flows agreement' (FFFA). This case serves both as a useful benchmark to evaluate welfare gains from capital controls and illustrates how tariffs can act as a second-best instrument to manipulate the cost of borrowing over time.

To rule out capital controls, the allocation must satisfy:

$$\frac{u'(C_{t+1})}{u'(C_t)} \frac{u^{*\prime}(C_t)}{u^{*\prime}(C_{t+1}^*)} = \frac{Q_t}{Q_{t+1}}$$

Figure B3: Time Profile of Optimal Instruments for Anticipated and Temporary Fall in Home Endowment of Good 1 (Scenario 1, Anticipated at $\bar{t} = 5$)



Notes: Time profile for Home capital-flow tax and import tariff in anticipated variant of scenario 1, simulated for 100 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to optimal instruments for Home planner acting unilaterally with (without) a FTA in place.

which corresponds to the Backus and Smith (1993) condition. While this condition rules out capital-flow taxation, it can allow for tariffs, which can be seen by rewriting it as follows:

$$\frac{u'(C_{t+1})}{u'(C_t)}\frac{u^{*\prime}(C_t)}{u^{*\prime}(C^*_{t+1})}\frac{1+\tau^*_t}{1+\tau^*_{t+1}} = \frac{g_{1,t}}{g^*_{1,t}}\left(\frac{g_{1,t+1}}{g^*_{1,t+1}}\right)^{-1}$$

We further impose that this holds period-by-period:

$$u'(C_t)g_{i,t} = \kappa u'(C^*_t)g^*_{i,t}\frac{1}{1+\tau^*_t} \quad \forall t \tag{B3}$$

where κ is a risk-sharing constant, calculated in an equilibrium with the optimal tariffs in place to ensure no transfers are needed.

Considering a setting with no FTA, but a FFFA. The first-order conditions for a unilateral Home planner with respect to $c_{1,t}$ and $c_{2,t}$ when capital controls are ruled out become:

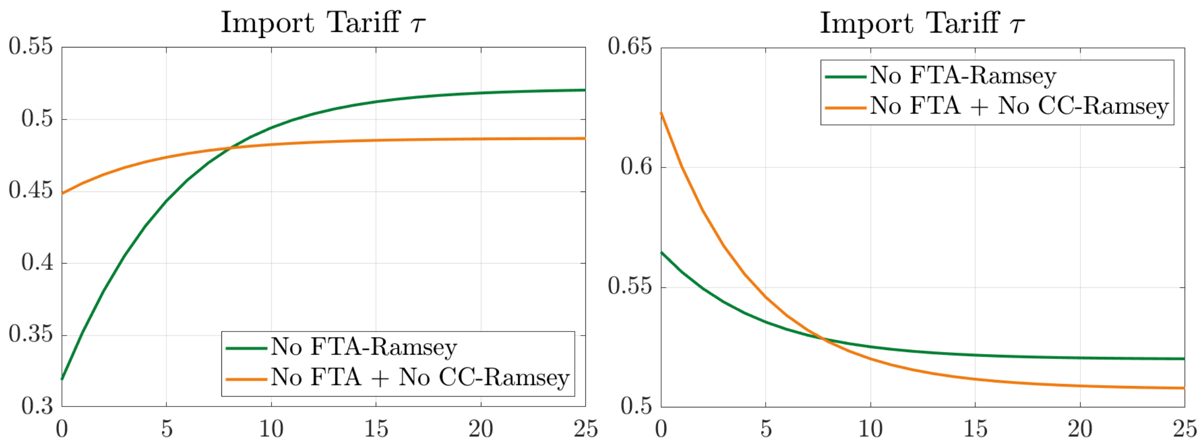
$$u'(C_t)g_{1,t} = \mu\mathcal{MC}^{nFTA}_{1,t} + \zeta_t\mathcal{RS}_{1,t}$$
$$u'(C_t)g_{2,t} = \mu\mathcal{MC}^{nFTA}_{2,t} + \zeta_t\mathcal{RS}_{2,t}$$

where ζ_t is the multiplier on the risk-sharing condition (B3) and, for $i = 1, 2$:

$$\mathcal{RS}_{i,t} = u''(C_t)g_{i,t} - \kappa u''(C^*_t)g_{i,t}\frac{g^*_{1,t}}{g_{1,t}}\frac{1}{1+\tau^*_t} - \kappa u'(C^*_t)\frac{-g^*_{1i,t}g_{1,t} - g^*_{1,t}g_{1i,t}}{g^2_{1,t}}\frac{1}{1+\tau^*_t}$$

Intuitively, the planner now internalizes the effect of an additional unit of consumption of good 1 and 2 respectively on the risk-sharing condition. An increase in C_t is only permitted if the

Figure B4: Optimal Tariffs when Capital Controls are Ruled Out by a FFFA: Tariffs Acting as Second-Best Instrument in Scenarios 1 (Left) and 2 (Right)



Notes: Time profile for optimal tariffs in Scenario 1 (left) and 2 (right), simulated for 100 periods. See Table 1 for calibration details. "No FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally without a FTA in place. This is compared to the "No FTA + No CC" allocation, in which capital controls are ruled out by a FFFA.

allocation of c_1 and c_2 is such that there is a sufficient depreciation in the real exchange rate.

The paths for the optimal tariffs when capital controls are ruled out are displayed in Figure B4. In scenario 1, the path for tariffs is less variable with a FFFA, compared to the no-FTA case. This occurs because good 1 is relatively scarce in the near term, so the optimal path for import tariffs (on good 2) is increasing. All else equal, this would incentivise over-borrowing in the near-term, with knock-on effects for optimal capital controls in the no-FTA case. Consequently, in the absence of capital controls, variation in the optimal tariff is smaller.

In contrast, the optimal path for tariffs in the no-FTA case for scenario 2 will, all else equal, disincentivise over-borrowing because inter- and intra-temporal incentives oppose. As a consequence, the path for tariffs is more variable in this case, with a larger optimal import tariff in the near term than in the no-FTA case. In this instance, tariffs in effect act as a second-best instrument to stabilise borrowing.

C Model Extensions and Generalizations

C.1 Production and Nominal Rigidities

In this appendix, we explain how the incentives to manipulate the terms of trade remain in a model with non-traded goods, endogenous labour supply and nominal wage rigidities. Specifically, a planner will have an additional motive to bring forward consumption with policy interventions when output is demand constrained due to the presence of an aggregate-demand externality.

Setup. We consider a minimal model of production and nominal rigidities to illustrate that the results in the main body generalize to more complex environments. Households consume non-traded goods NT in addition to traded T goods 1 and goods 2 as in the baseline model. Their instantaneous period-by-period utility function is given by:

$$\mathcal{U} = u(c_1, c_2, c_{NT}) + v(L)$$

where u is CRRA with risk aversion σ and v represents captures disutility from labor supply L. Aggregate consumption C takes a nested CES form:

$$C_t = \left[(1-\omega)^{\frac{1}{\phi}} c_{T,t}^{\frac{\phi-1}{\phi}} + \omega^{\frac{1}{\phi}} c_{NT,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}$$

$$c_{T,t} = \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}$$

Non-traded goods are produced with a linear production function $y_{NT,t} = A_t L_t$ under perfect competition. Firm maximization yields $p_{NT,t} = A_t w_t$ and we assume wages are perfectly rigid, $w_t = \overline{w}$.

The budget constraint for the Home representative household is given by:

$$\tilde{\mathbf{p}}_t \cdot \mathbf{c}_t - \mathbf{p}_t \mathbf{y}_t + \mathbf{p}_{T,t+1} \cdot \mathbf{a}_{T,t+1} \leq \overline{w} L_t + \mathbf{p}_{T,t} \cdot \mathbf{a}_{T,t} + T_t \tag{C1}$$

where $\tilde{\mathbf{p}}_t$ captures prices of goods after taxes. We assume households trade in good 1 and good 2 denominated bonds and earn wages. The consolidated present-value budget constraint, assuming no initial assets, a no-Ponzi condition, substituting in profits, and market clearing $y_{NT,t} = c_{NT,t}$ can be written as:

$$\sum_{t=0}^{\infty} \mathbf{p}_T \cdot [\mathbf{c}_T - \mathbf{y}_T] \leq 0$$

The indirect utility function is given by:

$$V\left(c_{T,t}, \frac{p_{T,t}}{p_{NT}}\right) = u\left(c_{T,t}, \frac{\omega}{1-\omega}\left(\frac{p_{T,t}}{p_{NT}}\right)^{\phi} c_{T,t}\right) + v\left(\frac{1}{A_t}\frac{\omega}{1-\omega}\left(\frac{p_{T,t}}{p_{NT}}\right)^{\phi} c_{T,t}\right)$$

The marginal benefit to the planner for a unit of c_T is written as:

$$\frac{\partial V_t}{\partial c_{T,t}} = u'(C_t) g_{T,t} \left(1 + \frac{\omega}{1-\omega} \tau_t^L\right)$$

where τ_t^L is the labor wedge, given by:

$$\tau_t^L = 1 + \frac{1}{A_t} \frac{v_{L,t}}{u'(C_t) g_{T,t}}$$

The labor wedge is positive when the economy is demand constrained and households are involuntarily unemployed. The marginal benefit of a unit of tradable consumption is higher when the economy is demand constrained. The labour wedge is acyclical and constant under the CO parametrization.

Returning to the planner's problem, the implementability constraint is unchanged albeit with a different maximand. Absent a FTA, the first-order conditions with respect to goods 1 and 2 are given by:

$$u'(C_t) g_{T,t} \left(1 + \frac{\omega}{1-\omega} \tau_t^L\right) \frac{g_{1,t}}{g_{T,t}} = \mu \mathcal{MC}_{1,t} \tag{C2}$$

$$u'(C_t) g_{T,t} \left(1 + \frac{\omega}{1-\omega} \tau_t^L\right) \frac{g_{2,t}}{g_{T,t}} = \mu \mathcal{MC}_{2,t} \tag{C3}$$

Suppose $\tau_t^L > 0$ because the economy is demand constrained. The planner now has an additional inter-temporal incentive to bring forward consumption to stimulate employment, as reflected by a higher marginal benefit form a unit of tradable consumption.[32]

Implementation and Policy Interactions. Revisiting the implementation in this setting, the risk-sharing equation when capital controls are in place is given by:

$$(1 - \theta_t) = \frac{u'(C_t) g_{T,t}}{u'(C_{t+1}) g_{T,t+1}} \frac{u'(C_{t+1}^*) g_{T,t+1}}{u'(C_t^*) g_{T,t}} \frac{Q_{T,t}}{Q_{T,t+1}}$$

where $Q_{T,t} = P_{T,t}^* / p_{T,t}$ and $p_{T,t}$ has the same form as the aggregate price level in the baseline model with only goods 1 and 2.

Proposition C1 *The capital-flow tax, absent a FTA, is given by:*

$$\theta_t^{noFTA} = 1 - \frac{\left(1 + \frac{\omega}{1-\omega}\tau_t^L\right)\left(1 - \left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)} \nabla g^*(\mathbf{c}_t^*) + \frac{1}{g_{1,t}^*} \frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}^*}\right) \cdot [\mathbf{c}_t - \mathbf{y}_t]\right)}{\left(1 + \frac{\omega}{1-\omega}\tau_{t+1}^L\right)\left(1 - \left(\frac{u^{*\prime\prime}(C_{t+1}^*)}{u^{*\prime}(C_{t+1}^*)} \nabla g^*(\mathbf{c}_{t+1}^*) + \frac{1}{g_{1,t+1}^*} \frac{\partial \nabla g^*(\mathbf{c}_{t+1}^*)}{\partial c_{1,t+1}^*}\right) \cdot [\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]\right)}$$

[32] Jeanne (2021) considers an environment with tradables production and shows that when the economy is demand constrained there is an incentive to use trade policy to stimulate demand for the domestic good through a substitution argument. Here, we emphasise trade policy can be used to stimulate aggregate demand as a substitute for a capital-inflow subsidy.

and the optimal tariff is unchanged. When an FTA is in place, the optimal capital-flow tax is:

$$\theta_t^{FTA} = 1 - \frac{\left(1+\frac{\omega}{1-\omega}\tau_t^L\right)\left(1-\left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*(C_t)) + \frac{\mathrm{d}\nabla g^*(\mathbf{c}_t^*(C_t))}{\mathrm{d}C_t^*}\right)\cdot[\mathbf{c}_t - \mathbf{y}_t]\right)}{\left(1+\frac{\omega}{1-\omega}\tau_{t+1}^L\right)\left(1-\left(\frac{u^{*\prime\prime}(C_{t+1}^*)}{u^{*\prime}(C_{t+1}^*)}\nabla g^*(\mathbf{c}_{t+1}^*(C_{t+1})) + \frac{\mathrm{d}\nabla g^*(\mathbf{c}_{t+1}^*(C_{t+1}))}{\mathrm{d}C_{t+1}^*}\right)\cdot[\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]\right)}$$

Proof: Follows from the Proof to Proposition 2, replacing the first-order conditions with equation (C2). □

Since the risk-sharing condition is unchanged, tariffs affect the path of the exchange rate for tradables in the same way as in the baseline model. Consistent with this, tradables consumption can be brought forward either with a capital-inflow tax or an import subsidy which puts pressure on Q_T to depreciate, as in the baseline model.

C.2 Segmented Markets and Quantity Interventions

In this appendix, we explain how a similar outcome to our baseline model (with capital controls and tariffs) can be achieved if the planner uses quantity interventions (e.g., open-market operations or FXI) in place of capital controls.

Setup. To show this, we present a model with non-trade goods and financial intermediation with international segmentation. Within it, we allow for tariffs but not capital controls. The budget constraint for the Home representative household is given by:

$$\mathbf{p}_t \cdot [\mathbf{c}_t - \mathbf{y}_t] + p_{NT,t+1}a_{t+1} \leq p_{NT,t}a_t + \Pi_t^f + T_t$$

where Π_t^f are rebated profits from financial intermediaries and T_t is the lump-sum rebate from the government. Normalising $p_{NT,t} = 1$ yields:

$$\tilde{\mathbf{p}}_t \cdot \mathbf{c}_t - \mathbf{p}_t\mathbf{y}_t + R_{NT,t}^{-1}a_{t+1} + \Pi_t^f + T_t \leq a_t$$

where $R_{NT,t}^{-1}$ is the price of an asset highlighting that the NT good is the numèraire in the economy. We define $\mathcal{E}_t = \frac{p^*_{NT,t}}{p_{NT,t}}$ as the exchange rate, as in Gabaix and Maggiori (2015).

The households' maximization yields the following Euler equation for non-traded goods:

$$\beta\frac{u'(C_{t+1})g_{NT,t+1}}{u'(C_t)g_{NT,t}} = R_{NT,t}^{-1}$$

Moreover, the relative demand equation is given by,

$$\frac{g_{NT,t}}{g_{i,NT}} = \frac{p_{NT,t}}{p_{i,t}(1+\tau_{i,t})} \tag{C4}$$

where $g_{NT,t} = \frac{\partial C_t}{\partial c_{NT,t}}$. Foreign households undertake an analogous maximization.

Monetary Authority. The planner, in this case a monetary authority, can take a position $p_{NT,t+1}a^G_{t+1}$ in domestic assets. We assume this is financed by an exactly opposition position $p^*_{NT,t+1}a^{G\,*}_{t+1}$ in foreign assets. If the monetary authority cannot borrow in foreign assets, there must be sufficient reserves to sell and carry out the operation. The monetary authority also provides a lump-sum transfer T_t to households.

Financial Intermediaries. A continuum of financial intermediaries indexed by $k \in [0, \overline{k}]$ trade in one-period assets with households in both countries. Each financier starts with no initial capital, faces a participation cost k and position limits $\{\overline{\alpha}, \underline{\alpha}\}$. The variable k corresponds to both the financiers' cost of participating and their index. Financiers choose a position in the asset $\alpha^I_{t+1}(k)$, financed by a position $-\alpha^I_{t+1}(k)\mathcal{E}_t$ in the foreign asset to maximize profits earned at t, subject to breaking even at $t+1$. The $t+1$ break-even condition is $\alpha^I_{t+1}(k)+\mathcal{E}^*_{t+1}\alpha^{*\,I}_{t+1}(k) = 0$. The problem of an individual financier, indexed by k, at time t can be summarized as:

$$\max_{\{\alpha^I_{t+1}(k))\in[\overline{\alpha},\underline{\alpha}]\}} \left[R^{*-1}_{NT,t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R^{-1}_{NT,t} \right] \alpha^I_{t+1}(k) - k$$

In equilibrium, a measure $\boldsymbol{k} = |R^{*-1}_{NT,t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R^{-1}_{NT,t}|\overline{\alpha}$ participates. The total position taken up by is given by $\alpha^I_{t+1} = \boldsymbol{k}\overline{\alpha}$. Defining $\Gamma = \frac{1}{\overline{\alpha}^2}$ yields:

$$\left[R^{*-1}_{NT,t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R^{-1}_{NT,t} \right] = \Gamma\alpha^I_{t+1}$$

Market clearing requires:

$$a_{NT,t+1} + a^G_{NT,t+1} + \alpha^I_{t+1} = 0 \tag{C5}$$

Due to limited participation by financiers and limits to arbitrage, the cost of borrowing is not equalised across countries. The more constrained the position that financiers can take, the higher the Γ and the larger the gap in the cost of borrowing when there are imbalances. Specifically, if the Home country is a net borrower, $\alpha^I_{t+1} > 0$, and the cost of borrowing for Home households $R_{NT,t}$ will be relatively high.

Substituting in the Euler equations for $R_{NT,t}$ and $R^*_{NT,t}$, yields a modified risk-sharing condition:

$$\left[\beta\frac{u^{*\prime}(C^*_{t+1})g^*_{NT,t+1}}{u^{*\prime}(C^*_t)g^*_{NT,t}}\frac{p^*_{NT,t}}{p_{NT,t}}\frac{p_{NT,t+1}}{p^*_{NT,t+1}} - \beta\frac{u'(C_{t+1})g_{NT,t+1}}{u'(C_t)g_{NT,t}} \right] = \Gamma\alpha^I_{t+1} \tag{C6}$$

Using the relative demand (C4), home and abroad, market clearing for assets (C5), and simplifying:

$$\left[\frac{\frac{p_{NT,t+1}}{P_{t+1}}}{\frac{p_{NT,t}}{P_t}} \right] \left[\beta\frac{u'(C_{t+1})}{u'(C_t)} - \beta\frac{u^{*\prime}(C^*_{t+1})}{u^{*\prime}(C^*_t)}\frac{Q_{t+1}}{Q_t} \right] = \Gamma(a_{NT,t+1} + a^G_{NT,t+1}) \tag{C7}$$

Relationship Between Instruments. Suppose Home households are borrowing $a_{NT,t+1} < 0$. By taking an opposing position and purchasing these assets $a^G_{NT,t+1} > 0$, funded by selling Foreign reserves ($a^{G*}_{NT,t+1} < 0$), the planner reduces the size of the imbalance that needs to be intermediated. As a result, this lowers the cost of borrowing for Home households. Below, we illustrate that such an intervention in a model with $\Gamma_t > 0$ can target the same wedge in risk-sharing as a capital-inflow tax in the baseline model.

Proposition C2 (Capital Controls and Quantity Intervention Equivalence) *Any path for risk-sharing wedges* $\frac{u'(C_t)}{\mu u'(C_t^*)}Q_t - 1$ *implemented with capital controls in the model with perfect financial markets can be implemented by FXI in the model with international financial frictions.*

Proof: To see the relationship between capital controls θ_t and open-market interventions $a^G_{NT,t+1}$, we first define a risk-sharing wedge as in Costinot et al. (2014):

$$\psi_t = \frac{u'(C_t)}{\mu u'(C_t^*)}Q_t$$

In the baseline model, capital controls (on assets denominated in traded varieties) can implement a desired risk-sharing wedge through the following mapping:

$$\theta_t = 1 - \frac{1+\psi_{t+1}}{1+\psi_t}$$

The risk-sharing condition, allowing for capital-flow taxes, can be written as:

$$\left[\beta\frac{u'(C_{t+1})}{u'(C_t)} - \beta\frac{u^{*\prime}(C_t)}{u^{*\prime}(C^*_{t+1})}\frac{Q_{t+1}}{Q_t}\right] = \theta_t\beta\frac{u'(C_{t+1})}{u'(C_t)}, \quad \text{for } i = \{1,2\} \tag{C8}$$

Combining the definition of the risk-sharing wedge and (C7) suggests that, in the model with non-traded goods and financial frictions, FXI can implement a desired risk-sharing wedge through the following mapping:

$$a^G_{NT,t} = \frac{1}{\Gamma}\left[\left(1 - \frac{1+\psi_{t+1}}{1+\psi_t}\right)\beta\frac{u'(C_{t+1})}{u'(C_t)}\right]\left[\frac{\frac{p_{NT,t+1}}{P_{t+1}}}{\frac{p_{NT,t}}{P_t}}\right] - a_{NT,t+1} \tag{C9}$$

To ensure the two models yield equivalent allocations is to determine lump-sum transfers and allocate profits when $\Gamma > 0$. Financiers earns $\Gamma(\alpha^I_{t+1})^2$ total profits. which we assume are fully rebated to Home households.[33] The monetary authority earns $-\Gamma\alpha^I_{t+1}a^G_{NT,t+1}$ on its FXI, which is potentially a loss. We assume these losses are imposed on households through lump-sum transfers.

Finally, we can rewrite the consolidated budget constraint, summing up the position of Home households, the monetary authority and financial intermediaries. Substituting $\Pi^f_t = \Gamma(\alpha^I_t)^2$, $T_t = \tau_{2,t}p_{2,t}c_{2,t} + \Gamma\alpha^I_{t+1}a^G_{NT,t+1}$, imposing a no-Ponzi condition ($p_{NT,\infty}a_\infty \to 0$) and assuming

[33]Relaxing this condition would create a quadratic-cost term as in Fanelli and Straub (2021). This would provide an additional motive for the monetary authority to narrow the spread.

zero initial assets ($p_{NT,0}a_0 + \Pi_t^f = 0$) yields the budget constraint:

$$\mathbf{p}_t \cdot [\mathbf{c}_t - \mathbf{y}_t] + R_t^* \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} a_{NT,t+1} \leq a_{NT,t+1}$$

where $R_t^* \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} = \left(\frac{p^*_{NT,t+1}}{p^*_{NT,t}}\right)\left(\frac{p^*_{NT,t+1}}{p_{NT,t+1}}\right)^{-1} p^*_{NT,t} = p_{NT,t+1}$. Iterating this forward yields:

$$\sum_{t=1}^{\infty} \mathbf{p}_t \cdot [\mathbf{c}_t - \mathbf{y}_t] \leq 0$$

Since $c_{NT,t} = y_{NT,t}$ $\forall t$, the present-value budget constraint is unchanged relative to the baseline two-good model with trade in bonds denominated in units of goods 1 and 2. As a result, the planning problem's implementability condition is also unchanged. □

Between Financial and Trade Policy. Inspecting (C9) yields two key insights. First, the interaction between trade and financial policy persists, since the path for real exchange rates is contained in $\left(1 - \frac{1+\psi_{t+1}}{1+\psi_t}\right)$. Second, the interaction now also depends on the evolution for the ratio of the price of non-traded goods to the aggregate price level.

Consider the special case where aggregators are Cobb-Douglas and utility has a logarithmic form, then:

$$a^G_{NT,t} = \frac{1}{\Gamma}\left[\left(1 - \frac{1+\psi_{t+1}}{1+\psi_t}\right)\right] \frac{\frac{\chi_{t+1}}{y_{NT,t+1}}}{\frac{\chi_t}{y_{NT,t}}} - a_{NT,t+1}$$

where χ_t is the share of expenditure spent on non-tradables. If $\chi_t = y_{NT,t}$ in every period such that variations in the marginal utility of tradables is neutralised, as assumed in Gabaix and Maggiori (2015), then our results on the direction of interactions between capital controls and trade policy go through for the case of open-market operations.

C.3 Country Size

In this appendix, we explain how incentives to manipulate relative prices remain for a small-open economy, as they remain large in goods markets. We then focus on an interesting knife-edge case in which the required size of capital controls for inter- and intra-temporal motives is the same in both the FTA and no-FTA case.

Setup. We adopt the small-open economy limit of Costinot et al. (2014). To do so, we define aggregate consumption for the rest of the world as:

$$C^* = \frac{c_1^{*\,\frac{1}{N}} c_2^{*\,1-\frac{1}{N}}}{N-1}$$

where N is the number of countries. In the small economy, aggregate consumption is given by:

$$C = c_1^{\frac{1}{2}} c_2^{\frac{1}{2}}$$

$g_{ii}^{(*)}, g_{ij}^{(*)}$, $i, j = 1, 2$, denote the partial derivatives of the aggregators, similar to those defined in Section A.1. The expenditure minimization problem used to derive the price index is analogous to Section A.3. Taking the ratio of the two price indices yields the real exchange rate:

$$Q_t = (N-1)\,\frac{p_{2,t}}{p_{1,t}}^{\frac{1}{2}-\frac{1}{N}}\left[2\left(\frac{1}{N}\right)^{\frac{1}{N}}\left(1-\frac{1}{N}\right)^{1-\frac{1}{N}}\right] \tag{C10}$$

The market-clearing conditions are given by: $c_1 + c_1^* = y_1$ and $c_2 + c_2^* = y_2 + (N-1)y_2^*$.

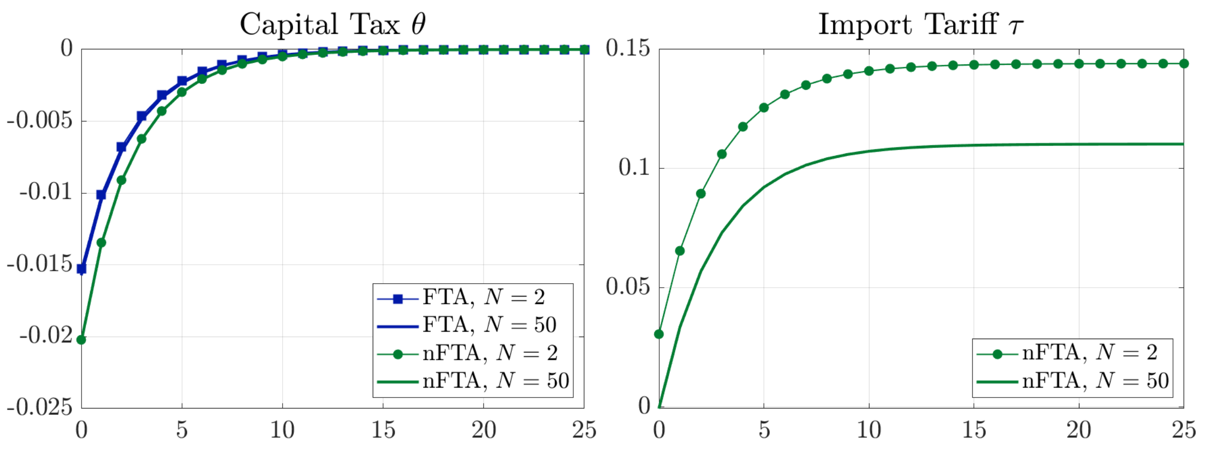
In the limit $N \to \infty$, the Home country becomes a small-open economy. It follows that $C_t^* \to c_{2_t}^* = Y_{2,t}$ resulting in $\frac{\mathrm{d}C^*}{\mathrm{d}C} \to 0$.[34] The Home small-open economy planner maximizes utility subject to:

$$\sum_t (N-1)\ u^{*\prime}(C_t^*)\nabla \mathbf{g}_t^* \cdot [\mathbf{c}_t - \mathbf{y}_t] \tag{C11}$$

with the $(N-1)$ appearing because C^* is defined as per-country aggregate consumption. The first-order conditions are derived analogously as in Section 3.

Optimal Policy and Country Size. While there are a range of outcomes in the small-open economy setting, there is an interesting knife-edge case when $\sigma = \phi = 1$ (Cole and Obstfeld, 1991) that we discuss here. At this parametrization, the required size of capital controls for inter- and intra-temporal incentives is the same. We demonstrate this in Figure C1 for scenario 1. Here we plot the optimal size of capital controls in both the FTA and no-FTA cases as $N \to \infty$, as well as tariffs in the no-FTA case.

Figure C1: Time Profile for Optimal Taxes and Tariffs in a Small-Open Economy as the Home Endowment of Good 1 Rises in Scenario 1



Notes: Time profile for optimal capital-flow taxes and tariffs in Scenario 1, simulated for 100 periods, for two-country case ($N = 2$) and small-open economy case ($N = 50$). "(n)FTA" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. See Table 1 for additional calibration details.

[34] Moreover, as before, $\frac{\mathrm{d}C_t^*}{\mathrm{d}C_t} = -\frac{1}{Q_t} \to 0$ as $Q_t \to \infty$.

D Strategic Planning Allocation

D.1 With Free Trade

In this appendix, we present the details of the strategic planning allocation with a FTA. This corresponds to the two-good Nash allocation discussed in Costinot et al. (2014).

Focusing on the Home planning problem, we can characterize the optimal allocation with a FTA in place, taking the sequence of Foreign capital flow taxes $\{\theta_t^*\}$ as given. Faced with these taxes, the Foreign Euler equations, for $i = 1, 2$ can be written:

$$u^{*\prime}(C_t^*)g_i^*(\mathbf{c}_t^*) = \beta(1-\theta_t^*)(1+r_{i,t})u^{*\prime}(C_{t+1}^*)g_i^*(\mathbf{c}_t^*) \tag{D1}$$

These Foreign optimality conditions, the Home inter-temporal budget constraint and the market-clearing conditions yield an implementability condition for the Home planner, which is described in the following lemma.

Lemma (Implementability for Nash Planner with FTA) *Since* $1 + r_{i,t} \equiv p_{i,t}/p_{i,t+1}$*, when the Foreign country seeks to set* $\{\mathbf{c}_t^*\}$ *in order maximize domestic welfare, then the Home allocation* $\{\mathbf{c}_t\}$ *forms part of an equilibrium if it satisfies:*

$$\sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t - \mathbf{y}_t] \leq 0 \tag{IC-Nash-FTA}$$

The Home planning problem, accounting for the optimal response by the Foreign planner, is given by:

$$\max_{\{C_t\}} \quad \sum_{t=0}^{\infty}\beta^t u(C_t) \tag{P-Nash-FTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t - \mathbf{y}_t] \leq 0 \tag{IC-Nash-FTA}$$

$$\mathbf{c}_t = \mathbf{c}(C_t), \quad \mathbf{c}_t^* = \mathbf{c}^*(C_t) \tag{FTA}$$

which is comparable to the unilateral problem (P-Unil-FTA), albeit with an additional term in the implementability constraint reflecting the Foreign capital flow tax θ_t^*.

Optimal Allocation. Problem (P-Nash-FTA) yields the optimality condition:

$$u'(C_t) = \mu\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\hat{\mathcal{MC}}_t^{FTA} \tag{D2}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\hat{\mathcal{MC}}_t^{FTA} \equiv u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*)\cdot\mathbf{c}'(C_t) + u^{*\prime\prime}(C_t^*)C^{*\prime}(C_t)\nabla g^*(\mathbf{c}^*)\cdot[\mathbf{c}_t - \mathbf{y}_t]$$

$$+ u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial C_t} \cdot [\mathbf{c}_t - \mathbf{y}_t]$$

Taking the ratio of t and $t+1$ optimality conditions further implies that:

$$\frac{u'(C_t)}{u'(C_{t+1})} = \frac{1}{1-\theta_t^*}\frac{\hat{\mathcal{MC}}_t^{FTA}}{\hat{\mathcal{MC}}_{t+1}^{FTA}} \tag{D3}$$

Combining equation (D3) with the Foreign Euler equations (D1) and the analogous Home Euler equations, yields an expression for $1-\theta_t$. The planning problem of the Foreign government is symmetric, so an analogous expression for $1-\theta_t^*$ can be derived. After some simplification, the combination of these expressions yields a mutual best response function, given by:

$$\frac{\hat{\mathcal{MC}}_t^{FTA}}{\hat{\mathcal{MC}}_t^{*FTA}} = \alpha_0^{FTA} \tag{D4}$$

where

$$\alpha_0^{FTA} \equiv \frac{\hat{\mathcal{MC}}_0^{FTA}}{\hat{\mathcal{MC}}_0^{*FTA}}$$

This is the strategic counterpart of equation (6). In the Nash setup, α_0^{FTA} can be interpreted as the bargaining power of the Foreign country relative to the Home.

D.2 Derivation of Strategic Planning Allocation Without Free Trade

Consider the problem faced by the Foreign planner:

$$\max_{\{\mathbf{c}_t^*\}} \quad \sum_{t=0}^{\infty} \beta^t \; u\left(g(\mathbf{c}_t^*)\right) \tag{$P1^{*\ Nash}$}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \beta^t u'(g(\mathbf{c}_t))\boldsymbol{\tau}_t^{-1}\boldsymbol{\nabla} g(\mathbf{c}_t) \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) \leq 0 \tag{$IC^{*\ Nash}$}$$

where:

$$\boldsymbol{\tau}_t = \begin{bmatrix} 1 & 0 \\ 0 & (1-\tau_t) \end{bmatrix}$$

The first-order conditions for the Foreign country with respect to $c_{H,t}^*$ and $c_{F,t}^*$ are given by:

$$C_t^{*\ -\sigma} g_{1,t}^* = \mu \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \Bigg\{ C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t} \begin{bmatrix} g_{1,t}(c_{1,t}^* - y_{1,t}^*)+ \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} - C_t^{-\sigma} \begin{bmatrix} g_{11,t}(c_{1,t}^* - y_{1,t}^*)+ \\ g_{21,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} \Bigg\} \tag{D5}$$

such that:

$$C_t^{*\ -\sigma} g_{1,t}^* = \mu \ \hat{\mathcal{MC}}_{1,t}^*$$

and:

$$C_t^{*\ -\sigma} g_{2,t}^* = \mu \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \left\{ C_t^{-\sigma} g_{2,t}(1-\tau_t)^{-1} + \sigma C_t^{-\sigma-1} g_{2,t} \left[\begin{array}{c} g_{1,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array} \right] - C_t^{-\sigma} \left[\begin{array}{c} g_{12,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{22,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array} \right] \right\}$$

such that

$$C_t^{-\sigma} g_{2,t}^* = \mu\ \hat{\mathcal{MC}}_{2,t}^*$$

D.3 Proof to Proposition 3

We derive mutual best responses, for goods 1 and 2 respectively. Dividing (14) by its $t+1$ analogue yields:

$$\frac{C_t^{-\sigma} g_{1,t}}{C_{t+1}^{-\sigma} g_{1,t+1}} = \frac{1}{1-\theta_t^*} \frac{\hat{MC}_{1,t}}{\hat{MC}_{1,t+1}}$$

Introduce $1-\theta_t$ using the Home Euler (9) and substitute out $\frac{1}{1-\theta_t^*}$ using the Foreign Euler equation. This yields the expression for the optimal tax on capital flows levied by the Home country:

$$1-\theta_t = \frac{1 + \sigma C_t^{*-1} \left[\begin{array}{c} g_{1,t}^*(c_{1,t} - y_{1,t}) + \\ g_{2,t}^*(1-\tau_t^*)^{-1}(c_{2,t} - y_{2,t}) \end{array} \right] - \frac{1}{g_{1,t}^*} \left[\begin{array}{c} g_{11,t}^*(c_{1,t} - y_{1,t}) + \\ g_{21,t}^*(1-\tau_t^*)^{-1}(c_{2,t} - y_{2,t}) \end{array} \right]}{1 + \sigma C_{t+1}^{*-1} \left[\begin{array}{c} g_{1,t+1}^*(c_{1,t+1} - y_{1,t+1}) + \\ g_{2,t+1}^*(1-\tau_{t+1}^*)^{-1}(c_{2,t+1} - y_{2,t+1}) \end{array} \right] - \frac{1}{g_{1,t+1}^*} \left[\begin{array}{c} g_{11,t+1}^*(c_{1,t+1} - y_{1,t+1}) + \\ g_{21,t+1}^*(1-\tau_{t+1}^*)^{-1}(c_{2,t+1} - y_{2,t+1}) \end{array} \right]} \tag{D6}$$

Abroad, dividing (D5) by its $t+1$ analog yields:

$$\frac{C_t^{*\ -\sigma} g_{1,t}^*}{C_{t+1}^{*\ -\sigma} g_{1,t+1}^*} = \frac{1}{1-\theta_t} \frac{C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t} \left[\begin{array}{c} g_{1,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array} \right] - C_t^{-\sigma} \left[\begin{array}{c} g_{11,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{21,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array} \right]}{C_{t+1}^{-\sigma} g_{1,t+1} + \sigma C_{t+1}^{-\sigma-1} g_{1,t+1} \left[\begin{array}{c} g_{1,t+1}(c_{1,t+1}^* - y_{1,t+1}^*) + \\ g_{2,t+1}(1-\tau_{t+1})^{-1}(c_{2,t+1}^* - y_{2,t+1}^*) \end{array} \right] - C_{t+1}^{-\sigma} \left[\begin{array}{c} g_{11,t+1}(c_{1,t+1}^* - y_{1,t+1}^*) + \\ g_{21,t+1}(1-\tau_{t+1})^{-1}(c_{2,t+1}^* - y_{2,t+1}^*) \end{array} \right]}$$

$$= \frac{1}{1-\theta_t} \frac{\hat{\mathcal{MC}}^*_{1,t}}{\hat{\mathcal{MC}}^*_{1,t+1}}$$

and following the analogous steps as for (D6) yields the expression for the optimal tax on capital flows levied by the Foreign country:

$$1-\theta^*_t = \frac{1+\sigma C_t^{-1}\begin{bmatrix} g_{1,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{2,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{bmatrix} - \frac{1}{g_{1,t}}\begin{bmatrix} g_{11,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{21,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{bmatrix}}{1+\sigma C_{t+1}^{-1}\begin{bmatrix} g_{1,t+1}(c^*_{1,t+1}-y^*_{1,t+1})+ \\ g_{2,t+1}(1-\tau_{t+1})^{-1}(c^*_{2,t+1}-y^*_{2,t+1}) \end{bmatrix} - \frac{1}{g_{1,t+1}}\begin{bmatrix} g_{11,t+1}(c^*_{1,t+1}-y^*_{1,t+1})+ \\ g_{21,t+1}(1-\tau_{t+1})^{-1}(c^*_{2,t+1}-y^*_{2,t+1}) \end{bmatrix}} \tag{D7}$$

To reach the conditions characterizing allocations in a Nash equilibrium, combine (D6) and (D7) yields:

$$\frac{C_t^{*-\sigma}g^*_{1,t}+\sigma C_t^{*-\sigma-1}g^*_{1,t}\begin{bmatrix} g^*_{1,t}(c_{1,t}-y_{1,t})+ \\ g^*_{2,t}(1-\tau^*_t)^{-1}(c_{2,t}-y_{2,t}) \end{bmatrix} - C_t^{*-\sigma}\begin{bmatrix} g^*_{11,t}(c_{1,t}-y_{1,t})+ \\ g^*_{21,t}(1-\tau^*_t)^{-1}(c_{2,t}-y_{2,t}) \end{bmatrix}}{C_t^{-\sigma}g_{1,t}+\sigma C_t^{-\sigma-1}g_{1,t}\begin{bmatrix} g_{1,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{2,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{bmatrix} - C_t^{-\sigma}\begin{bmatrix} g_{11,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{21,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{bmatrix}} = \alpha_{1,0},$$

The constant $\alpha_{1,0}$ is given by,

$$\alpha_{1,0} = \frac{C_0^{*-\sigma}g^*_{1,0}+\sigma C_0^{*-\sigma-1}g^*_{1,0}\begin{bmatrix} g^*_{1,0}(c_{1,0}-y_{1,0})+ \\ g^*_{2,0}(1-\tau^*_0)^{-1}(c_{2,0}-y_{2,0}) \end{bmatrix} - C_0^{*-\sigma}\begin{bmatrix} g^*_{11,0}(c_{1,0}-y_{1,0})+ \\ g^*_{21,0}(1-\tau^*_0)^{-1}(c_{2,0}-y_{2,0}) \end{bmatrix}}{C_0^{-\sigma}g_{1,0}+\sigma C_0^{-\sigma-1}g_{1,0}\begin{bmatrix} g_{1,0}(c^*_{1,0}-y^*_{1,0})+ \\ g_{2,0}(1-\tau_0)^{-1}(c^*_{2,0}-y^*_{2,0}) \end{bmatrix} - C_0^{-\sigma}\begin{bmatrix} g_{11,0}(c^*_{1,0}-y^*_{1,0})+ \\ g_{21,0}(1-\tau_0)^{-1}(c^*_{2,0}-y^*_{2,0}) \end{bmatrix}}$$

Undertaking the same operations for the good-2 first-order conditions yields:

$$\frac{\begin{array}{c} C_t^{*-\sigma} g_{2,t}^* (1-\tau_t^*)^{-1} + \sigma C_t^{*-\sigma-1} g_{2,t}^* \left[\begin{array}{c} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{2,t}^*(1-\tau_t^*)^{-1}(c_{2,t}-y_{2,t}) \end{array} \right] - \\ C_t^{*-\sigma} \left[\begin{array}{c} g_{21,t}^*(c_{1,t}-y_{1,t})+ \\ g_{22,t}^*(1-\tau_t^*)^{-1}(c_{2,t}-y_{2,t}) \end{array} \right] (1-\tau_t)^{-1} \end{array}}{\begin{array}{c} C_t^{-\sigma} g_{2,t} (1-\tau_t)^{-1} + \sigma C_t^{-\sigma-1} g_{2,t} \left[\begin{array}{c} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^*-y_{2,t}^*) \end{array} \right] - \\ C_t^{-\sigma} \left[\begin{array}{c} g_{12,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{22,t}(1-\tau_t)(c_{2,t}^*-y_{2,t}^*) \end{array} \right] (1-\tau_t^*)^{-1} \end{array}} \frac{1-\tau_t^*}{1-\tau_t} = \alpha_{2,0},$$

and $\alpha_{2,0}$ is given by:

$$\alpha_{2,0} = \frac{1-\tau_0^*}{1-\tau_0} \frac{\begin{array}{c} C_0^{*-\sigma} g_{2,0}^* (1-\tau_0^*)^{-1} + \sigma C_0^{*-\sigma-1} g_{2,0}^* \left[\begin{array}{c} g_{1,0}^*(c_{1,0}-y_{1,0})+ \\ g_{2,0}^*(1-\tau_0^*)^{-1}(c_{2,0}-y_{2,0}) \end{array} \right] - \\ C_0^{*-\sigma} \left[\begin{array}{c} g_{12,0}^*(c_{1,0}-y_{1,0})+ \\ g_{22,0}^*(1-\tau_0^*)^{-1}(c_{2,0}-y_{2,0}) \end{array} \right] (1-\tau_0)^{-1} \end{array}}{\begin{array}{c} C_0^{-\sigma} g_{2,0} (1-\tau_t)^{-1} + \sigma C_0^{-\sigma-1} g_{2,0} \left[\begin{array}{c} g_{1,0}(c_{1,0}^*-y_{1,0}^*)+ \\ g_{2,0}(1-\tau_0)^{-1}(c_{2,0}^*-y_{2,0}^*) \end{array} \right] - \\ C_0^{-\sigma} \left[\begin{array}{c} g_{12,0}(c_{1,0}^*-y_{1,0}^*)+ \\ g_{22,0}(1-\tau_0)(c_{2,0}^*-y_{2,0}^*) \end{array} \right] (1-\tau_0^*)^{-1} \end{array}}$$

Finally, substituting out τ_t and τ_t^* yields:

$$\frac{\begin{array}{c} C_t^{*-\sigma} g_{1,t}^* + \sigma C_t^{*-\sigma-1} g_{1,t}^* \left[\begin{array}{c} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{1,t}^* S_t(c_{2,t}-y_{2,t}) \end{array} \right] - \\ C_t^{*-\sigma} \left[\begin{array}{c} g_{11,t}^*(c_{1,t}-y_{1,t})+ \\ g_{21,t}^* \dfrac{g_{1,t}^*}{g_{2,t}^*} S_t(c_{2,t}-y_{2,t}) \end{array} \right] \end{array}}{\begin{array}{c} C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t} \left[\begin{array}{c} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{1,t} S_t(c_{2,t}^*-y_{2,t}^*) \end{array} \right] - \\ C_t^{-\sigma} \left[\begin{array}{c} g_{11,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{21,t} \dfrac{g_{1,t}}{g_{2,t}} S_t(c_{2,t}^*-y_{2,t}^*) \end{array} \right] \end{array}} = \alpha_{1,0}$$

and:

$$\frac{C_t^{*-\sigma}g_{2,t}^{*}+\sigma C_t^{*-\sigma-1}g_{2,t}^{*}\left[\begin{array}{c}g_1^{*}(c_{1,t}-y_{1,t})+\\ g_{1,t}^{*}S_t(c_{2,t}-y_{2,t})\end{array}\right]-C_t^{*-\sigma}\left[\begin{array}{c}g_{12,t}^{*}(c_{1,t}-y_{1,t})+\\ g_{22,t}^{*}\frac{g_{1,t}^{*}}{g_{2,t}^{*}}S_t(c_{2,t}-y_{2,t})\end{array}\right]}{C_t^{-\sigma}g_{2,t}+\sigma C_t^{-\sigma-1}g_{1,t}\left[\begin{array}{c}g_{1,t}(c_{1,t}^{*}-y_{1,t}^{*})+\\ g_{1,t}S_t(c_{2,t}^{*}-y_{2,t}^{*})\end{array}\right]-C_t^{-\sigma}\left[\begin{array}{c}g_{12,t}(c_{1,t}^{*}-y_{1,t}^{*})+\\ g_{22,t}\frac{g_{1,t}}{g_{2,t}}S_t(c_{2,t}^{*}-y_{2,t}^{*})\end{array}\right]}\frac{g_{1,t}}{g_{2,t}}\frac{g_{2,t}^{*}}{g_{1,t}^{*}}=\alpha_{2,0}$$

which completes the proof. □

To derive the optimal tariffs, divide the Foreign by the Home optimality condition for good 1 and use the Euler to substitute in the Home optimal tariff on the left-hand side. Use the Foreign Euler to substitute out the Foreign optimal tariff:

$$1-\tau_t=\frac{1}{S_t}\frac{C_t^{*-\sigma}g_{1,t}^{*}S_t+\sigma C_t^{*-\sigma-1}g_{2,t}^{*}\left[\begin{array}{c}g_{1,t}^{*}(c_{1,t}-y_{1,t})+\\ g_{1,t}^{*}S_t(c_{2,t}-y_{2,t})\end{array}\right]-C_t^{*-\sigma}\left[\begin{array}{c}g_{12,t}^{*}(c_{1,t}-y_{1,t})+\\ g_{22,t}^{*}\frac{g_{1,t}^{*}}{g_{2,t}^{*}}S_t(c_{2,t}-y_{2,t})\end{array}\right]}{C_t^{*-\sigma}g_{1,t}^{*}+\sigma C_t^{*-\sigma-1}g_{1,t}^{*}\left[\begin{array}{c}g_{1,t}^{*}(c_{1,t}-y_{1,t})+\\ g_{1,t}^{*}S_t(c_{2,t}-y_{2,t})\end{array}\right]-C_t^{*-\sigma}\left[\begin{array}{c}g_{11,t}^{*}(c_{1,t}-y_{1,t})+\\ g_{21,t}^{*}\frac{g_{1,t}^{*}}{g_{2,t}^{*}}S_t(c_{2,t}-y_{2,t})\end{array}\right]}$$

and then:

$$1-\tau_t^{*}=\frac{1}{S_t}\frac{C_t^{-\sigma}g_{1,t}S_t+\sigma C_t^{-\sigma-1}g_{2,t}\left[\begin{array}{c}g_{1,t}(c_{1,t}^{*}-y_{1,t}^{*})+\\ g_{1,t}S_t(c_{2,t}^{*}-y_{2,t}^{*})\end{array}\right]-C_t^{-\sigma}\left[\begin{array}{c}g_{12,t}(c_{1,t}^{*}-y_{1,t}^{*})+\\ g_{22,t}\frac{g_{1,t}}{g_{2,t}}S_t(c_{2,t}^{*}-y_{2,t}^{*})\end{array}\right]}{C_t^{-\sigma}g_{1,t}+\sigma C_t^{-\sigma-1}g_{1,t}\left[\begin{array}{c}g_{1,t}(c_{1,t}^{*}-y_{1,t}^{*})+\\ g_{1,t}S_t(c_{2,t}^{*}-y_{2,t}^{*})\end{array}\right]-C_t^{-\sigma}\left[\begin{array}{c}g_{11,t}(c_{1,t}^{*}-y_{1,t}^{*})+\\ g_{21,t}\frac{g_{1,t}}{g_{2,t}}S_t(c_{2,t}^{*}-y_{2,t}^{*})\end{array}\right]}$$

D.4 Nash Equilibrium With a FTA

Consider the Nash problem when a FTA is in place for both Home and Foreign planners. If a FTA is in place $\boldsymbol{\tau}_t, \boldsymbol{\tau}_t^* = 1$, the Home planner chooses C_t and the Foreign C_t^* and $\mathbf{c}(C_t), \mathbf{c}^*(C_t^*)$ are given by Lemma 1. Then the allocations C_t, C_t^* in a Nash equilibrium must satisfy:

$$\frac{\begin{gathered} C_t^{*-\sigma}(g_{1,t}^* c_{1,t}'(C_t) + g_{2,t}^* c_{2,t}'(C_t)) + \\ \sigma C_t^{*\ -\sigma-1} C_t^{*\prime}(C_t)\left[g_{1,t}^*(c_{1,t} - y_{1,t}) + g_{2,t}^*(c_{2,t} - y_{2,t})\right] + \\ C_t^{*-\sigma}\left[(g_{11,t}^* + g_{21,t}^*) c_{1,t}'(C_t)(c_{1,t} - y_{1,t}) + (g_{12,t}^* + g_{22,t}^*) c_{2,t}'(C_t)(c_{2,t} - y_{2,t})\right] \end{gathered}}{\begin{gathered} C_t^{-\sigma}(g_{1,t} c_{1,t}^{*\prime}(C_t) + g_{2,t} c_{2,t}^{*\prime}(C_t)) + \\ \sigma C_t^{-\sigma-1} C_t'(C_t^*)\left[g_{1,t}(c_{1,t}^* - y_{1,t}^*) + g_{2,t}(c_{2,t}^* - y_{2,t}^*)\right] + \\ C_t^{-\sigma}\left[(g_{11,t} + g_{21,t}) c_{1,t}^{*\prime}(C_t^*)(c_{1,t}^* - y_{1,t}^*) + (g_{12,t} + g_{22,t}) c_{2,t}^{*\prime}(C_t^*)(c_{2,t}^* - y_{2,t}^*)\right] \end{gathered}} = \alpha_0^{FTA}$$

Optimal capital controls levied by the Home country are given by:

$$1 - \theta_t = \frac{\begin{gathered} (g_{1,t}^* c_{1,t}'(C_t) + g_{2,t}^* c_{2,t}'(C_t)) + \\ \sigma C_t^{*\ -1} C_t^{*\prime}(C_t)\left[g_{1,t}^*(c_{1,t} - y_{1,t}) + g_{2,t}^*(c_{2,} - y_{2,t})\right] + \\ \left[\begin{gathered} (g_{11,t}^* + g_{21,t}^*) c_{1,t}'(C_t)(c_{1,t} - y_{1,t}) + (g_{12,t}^* + \\ g_{22,t}^*) c_{2,t}'(C_t)(c_{2,t} - y_{2,t}) \end{gathered}\right] \end{gathered}}{\begin{gathered} (g_{1,t+1}^* c_{1,t+1}'(C_{t+1}) + g_{2,t+1}^* c_{2,t+1}'(C_{t+1})) + \\ \sigma C_{t+1}^{*\ -1} C_{t+1}^{*\prime}(C_{t+1})\left[g_{1,t+1}^*(c_{1,t+1} - y_{1,t+1}) + g_{2,t+1}^*(c_{2,t+1} - y_{2,t+1})\right] + \\ \left[\begin{gathered} (g_{11,t+1}^* + g_{21,t+1}^*) c_{1,t+1}'(C_{t+1})(c_{1,t+1} - y_{1,t+1}) + \\ (g_{12,t+1}^* + g_{22,t+1}^*) c_{2,t+1}'(C_{t+1})(c_{2,t+1} - y_{2,t+1}) \end{gathered}\right] \end{gathered}}$$

with an analogous condition for the Foreign.

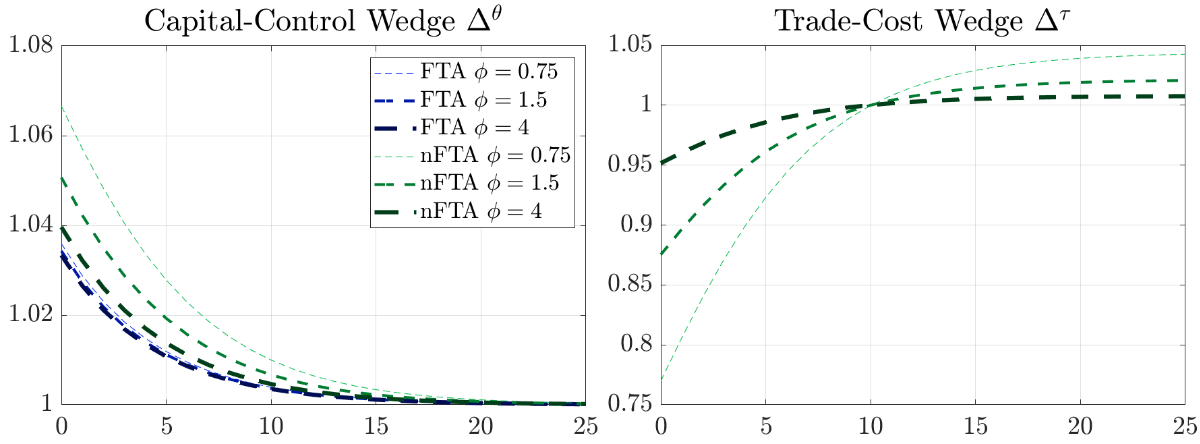
D.5 Comparative Statics in Nash Setting

To analyze the comparative statics of distortions in the Nash equilibria, it is useful to define two quantities to capture the difference in the cost of borrowing in the Home *vis-à-vis* the Foreign country, and the relative ratio of tariffs at Home *vis-à-vis* Foreign:

$$\Delta^\theta = \frac{1 - \theta_t}{1 - \theta_t^*} \quad \text{and} \quad \Delta^\tau = \frac{1 + \tau_t}{1 + \tau_t^*}$$

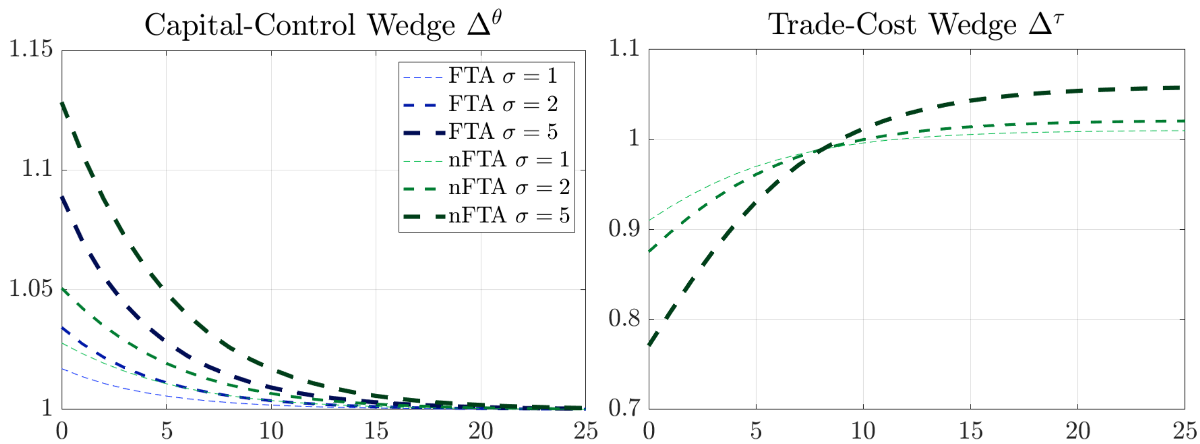
The distance of these quantities from unity captures the total distortion to the inter- and intratemporal margins, respectively. Figures D1 and D2 demonstrate the 'inverse elasticity' relationship between the inter- and intra-temporal wedges and the corresponding inter- and intra-temporal elasticities of substitution.

Figure D1: Comparative Statics of Wedges in Strategic Allocation with Respect to the Intra-Temporal Elasticity of Substitution ϕ in Scenario 1



Notes: Time profile of capital-flow and import-tariff wedges in scenario 1, simulated for 100 periods, with three different values of intra-temporal elasticity of substitution between goods 1 and 2 ϕ. See Table 1 for calibration details. "(n)FTA" refers to allocation arising from strategic allocation with (without) a FTA in place.

Figure D2: Comparative Statics of Wedges in Strategic Allocation with Respect to the Coefficient of Relative Risk Aversion σ (Inverse Inter-temporal Elasticity of Substitution) in Scenario 1



Notes: Time profile of capital-flow and import-tariff wedges in scenario 1, simulated for 100 periods, with three different values of the coefficient of relative risk aversion σ (i.e., inverse inter-temporal elasticity of substitution). See Table 1 for calibration details. "(n)FTA" refers to allocation arising from strategic allocation with (without) a FTA in place.

D.6 Proof to Proposition 4

When a FTA is in place, the optimal cooperative allocation satisfies:

$$u'(g(\mathbf{c}_t)) + \kappa u'(g(\mathbf{c}_t^*)) \frac{\mathrm{d}C^*}{\mathrm{d}C} = 0 \tag{D8}$$

where $\frac{dC_t^*}{dC_t} = -\frac{P_t}{P_t^*}$, yielding the decentralized risk sharing condition (10) with $\kappa = \frac{u'(g(\mathbf{c}_{t-1}))}{u'(g(\mathbf{c}_{t-1}^*))}\frac{P_{t-1}^*}{P_{t-1}}$ implying $\theta_t = 0$. Relaxing the FTA does not change the optimal allocation (since goods taxes are zero at the optimal). With a FTA, the first-order condition follows straightforwardly by substituting $\frac{dC_t^*}{dC_t} = -\frac{P_t}{P_t^*}$.

Relaxing the FTA, we get two first-order conditions,

$$u'(g(\mathbf{c}_t))g_1 + \kappa u'(g(\mathbf{c}_t^*))g_1^* \frac{dc_1^*}{dc_1} = 0 \tag{D9}$$

$$u'(g(\mathbf{c}_t))g_2 + \kappa u'(g(\mathbf{c}_t^*))g_2^* \frac{dc_2^*}{dc_2} = 0 \tag{D10}$$

Note that $\frac{g_1}{g_1^*} = \frac{dC}{dc_1}\frac{dc_1^*}{dC^*} = \frac{dC}{dC^*}\frac{dc_1^*}{dc_1} = -\frac{dC}{dC^*}$, therefore both of the above conditions imply (D8), as in the FTA case. □

D.7 Allocations from Dynamic Policy Game

Figures D3 to D6 plot the allocations from the policy games discussed in Section 7.2.

Figure D3: Scenario 1: Allocations and Policy Instruments when Home Deviates from FFFA with FTA and Foreign Retaliates $\bar{t} = 5$ Periods Later

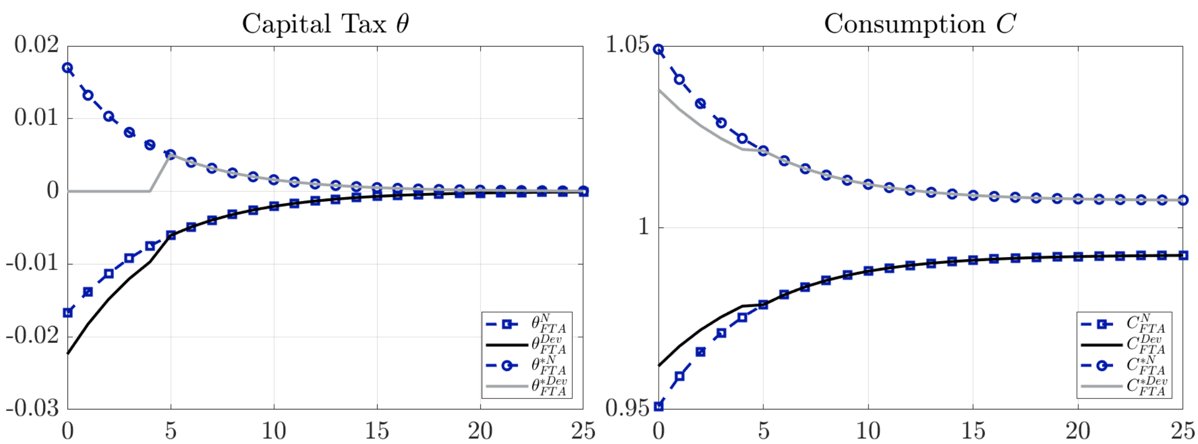


Figure D4: Scenario 2: Allocations and Policy Instruments when Home Deviates from FFFA with FTA and Foreign Retaliates $\bar{t} = 5$ Periods Later

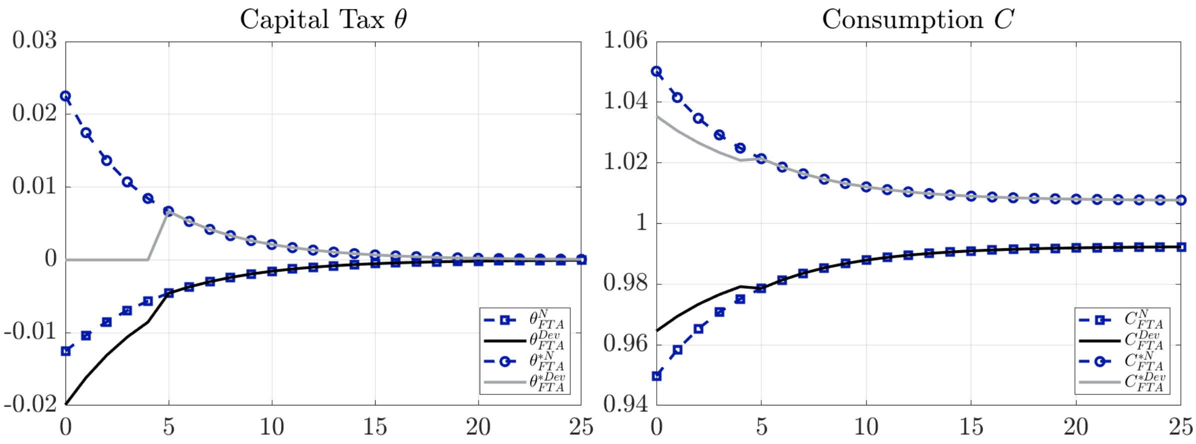


Figure D5: Scenario 1: Allocations and Policy Instruments when Home Deviates from FFFA without FTA and Foreign Retaliates $\bar{t} = 5$ Periods Later

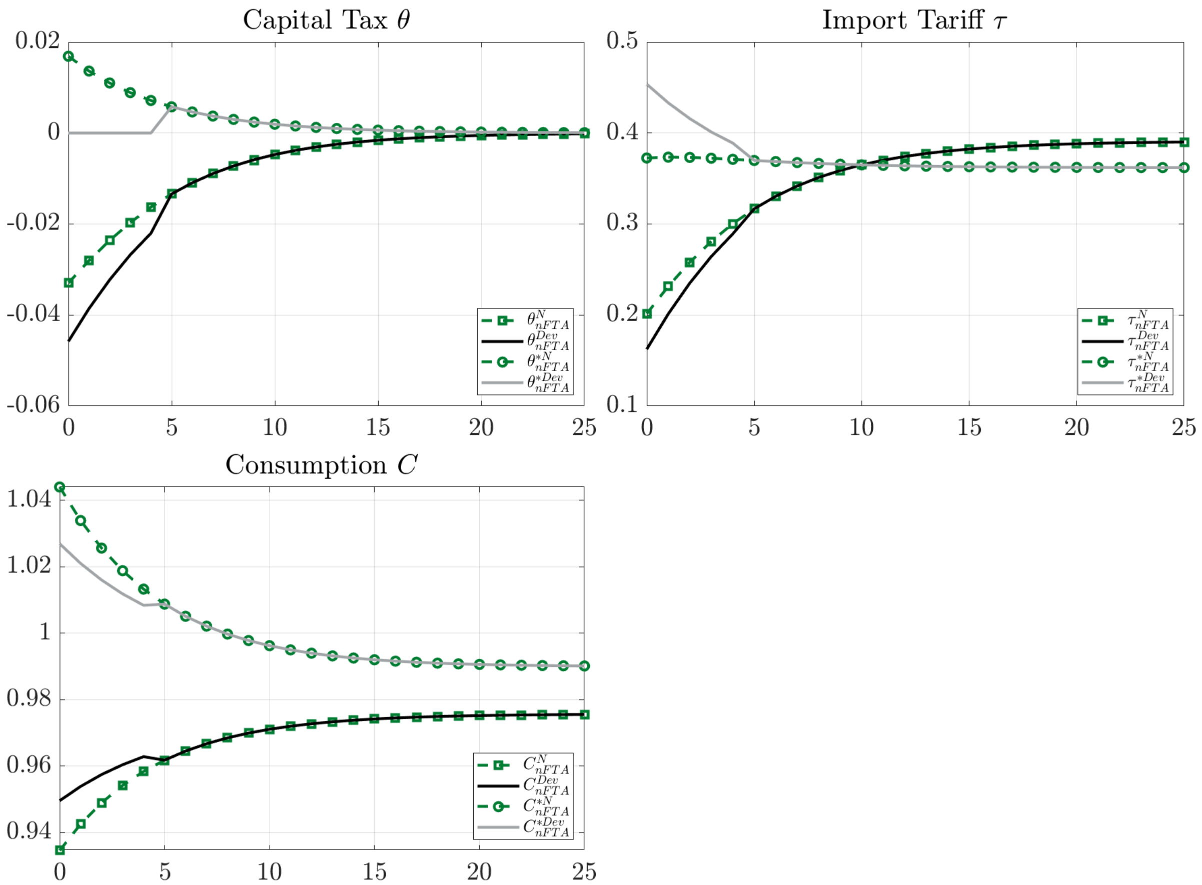


Figure D6: Scenario 2: Allocations and Policy Instruments when Home Deviates from FFFA without FTA and Foreign Retaliates $\bar{t} = 5$ Periods Later