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Zhengyang Jiang

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**ABSTRACT**

I develop a general characterization of the effect that market incompleteness has on exchange rate dynamics. On the one hand, it weakens the pass-through from a country's marginal utility shocks to its own exchange rate movements; on the other hand, it gives rise to additional variations in exchange rates and propagates one country's marginal utility shocks to other countries' exchange rate movements. This novel international spill-over effect gives rise to both exchange rate disconnect from local fundamentals and exchange rate comovements in the cross-section of currencies, offering a novel channel for understanding these salient features of exchange rate behaviors.

Zhengyang Jiang
Kellogg School of Management
Northwestern University
2211 Campus Drive
Evanston, IL 60208
and NBER
zhengyang.jiang@kellogg.northwestern.edu

A data appendix is available at http://www.nber.org/data-appendix/w30856

# 1 Introduction

Two salient features of exchange rates stand out: first, each country's exchange rate movements appear to be disconnected from its own economic fundamentals, exhibiting a lack of correlation with macro variables that should drive marginal utility growth (Meese and Rogoff, 1983; Backus and Smith, 1993; Engel and West, 2005); second, exchange rate movements are highly correlated across countries, exhibiting a strong factor structure (Lustig, Roussanov, and Verdelhan, 2011; Verdelhan, 2018). Using intuition from complete-market models, one explanation is that agents in different countries are exposed to correlated shocks. This literature searches for specifications of frictions or preferences such that the equilibrium marginal utilities are driven by latent factors that (i) have different dynamics than the observed consumption process and (ii) are correlated in the cross-section. For example, if the marginal utilities are driven by long-run consumption risks that are correlated across countries, then, the equilibrium exchange rate movements do not closely follow short-run consumption growth patterns while they share a common factor from the correlated long-run risks.

In this paper, I show market incompleteness offers a new and complementary explanation for the patterns of exchange rate disconnect and comovements. I develop a general framework to show that market incompleteness has two effects on exchange rate dynamics. First, it weakens the pass-through from marginal utility shocks to exchange rate movements by generating a wedge that acts as a "shock absorber" in bilateral exchange rate dynamics. This result has been noted in prior works (Backus, Foresi, and Telmer, 2001; Lustig and Verdelhan, 2019). In this paper, it is characterized in a more convenient way to analyze the cross-section of currencies. Second, market incompleteness propagates a country's marginal utility shocks to the exchange rate movements between other countries. This international spill-over effect has not been recognized in the prior literature, which largely focuses on the two-country case. I show that this effect generates exchange rate disconnect and comovements even when marginal utilities are uncorrelated across countries.

To show these results, I develop a general framework to characterize exchange rate dynamics in incomplete markets. This framework builds on the no-arbitrage approach of Backus, Foresi, and Telmer (2001), who show that market incompleteness introduces a stochastic wedge between two countries' stochastic discount factors (SDFs) and their bilateral exchange rate movements. I take one step further by decomposing this wedge into an SDF component and an IM (incomplete-market) component. The SDF component describes how this wedge affects how the SDF shocks pass through to the exchange rate dynamics, a key parameter that I will refer to as the *SDF-FX pass-through*. When markets are complete, the exchange rate movement is equal to the SDF differential, implying an SDF-FX pass-through coefficient of one. When markets are incomplete, this pass-through coefficient can be less than one, reflecting a partial reaction of the exchange rate to SDF shocks. The IM component captures additional variations in the exchange rate that arise due to market incompleteness. By construction, this component is orthogonal to the home country's SDF shocks, but it may be correlated with other shocks in the economy.

This characterization is general and does not rely on specific assumptions of preferences, frictions, and shocks. As a result, it can be applied to a large set of equilibrium international macroeconomic models. In this paper, the economic restriction is imposed by assuming investors in each country can freely trade domestic and foreign risk-free bonds. Opening these basic bond markets implies symmetric Euler equations that price a country's risk-free bond from the perspective of another country's investor. For other contingent claims such as equity and long-term bonds, the financial markets *may or may not* be open, which allows me to model a flexible degree of market incompleteness.

While allowing agents to trade risk-free bonds is a general assumption, it is not devoid of economic content. In prior work, Lustig and Verdelhan (2019) use this assumption to characterize the relation between the cyclicality, volatility and risk premium of the bilateral exchange rate between two countries. In this paper, I am going to show that this assumption imposes additional restrictions when I consider more than two countries, and that as a result the *symmetric* Euler equations implied from cross-country bond holdings will give rise to *asymmetric* pass-through from SDF shocks to exchange rates.

Specifically, asymmetric SDF-FX pass-through means that the home currency's exchange rates against different foreign currencies respond to the home SDF shock to different degrees. Then, by Triangular arbitrage, the home country's SDF shock affects not only its own exchange rates against other currencies, but also the bilateral exchange rate between any two foreign countries. More concretely, suppose a 1% increase in the U.K. households' marginal utility appreciates the British pound by 1% against the U.S. dollar but only by 0.5% against the Japanese yen. Then, yen will appreciate by 0.5% against the dollar in response to this local shock in the U.K. This is the key result in this paper that generates exchange rate disconnect and comovements in the presence of market incompleteness.

To derive this result, I first pick an arbitrary country as the home country and study the Euler equations for the cross-country holdings of risk-free bonds between this country and each foreign country. As the markets may be incomplete, for given SDF processes, there may be multiple exchange rate dynamics that are consistent with these Euler equations. Still, these Euler equations impose restrictions on the equilibrium exchange rate behavior: they pin down the drift of the bilateral exchange rate movement and the magnitude of the IM shock for a given level of the SDF-FX pass-through and a given correlation between SDF and IM shocks.

Next, I consider the bilateral dynamics between any two foreign countries. On the one hand, Triangular arbitrage rules out any discrepancies in spot exchange rates: once I know the bilateral exchange rate between the home and each foreign country, the bilateral exchange rate between any two foreign countries is directly implied. On the other hand, when markets are incomplete, the Euler equations between two foreign countries for their cross-holdings of risk-free bonds are *not* implied from the Euler equations between the home and each foreign country. As a result, when I extend the analysis from two to three countries, the number of independent exchange rates increases from 1 to 2 (i.e., home-foreign 1 and home-foreign 2), while the number of sets of unique restrictions from the Euler equations increases from 1 to 3 (i.e., home-foreign 1, home-foreign 2, and foreign 1-foreign 2). The Euler equations between foreign countries, which are not possible to study in the two-country models in prior works, impose additional constraints and reduce the set of admissible exchange rate dynamics.

Specifically, I derive what these constraints imply under *symmetric* SDF-FX pass-through, in which case the home country's SDF shock affects different foreign countries' exchange rates by the same degree. In other words, when the home country's marginal utility increases while foreign marginal utilities remain constant, the home country's exchange rate appreciates against all foreign countries equally. This is a natural benchmark, as it holds in complete markets from which the majority of our intuition about exchange rate dynamics is derived. Importantly, symmetric SDF-FX pass-through does not rule out heterogeneous exposures that different countries' SDFs may have with respect to common and idiosyncratic shocks—it only describes how a given SDF shock affects the exchange rate movement, not how the SDF shocks are correlated.

The main result in this paper is that, under symmetric SDF-FX pass-through, the Euler equations imply that the covariance between bilateral exchange rate movements has to be identical to the covariance between SDF differentials of the same country pairs. In other words, under symmetric SDF-FX pass-through, market incompleteness is incapable of relaxing the tight link between the exchange rate covariance and the SDF covariance that also characterizes the complete-market case.

This implication is incompatible with known features of SDF and exchange rate dynamics in the data. I make three empirical observations. First, the correlation between exchange rate movements is much higher than the correlation between consumption growth differentials. If the agents have identical CRRA preferences, the correlation between consumption growth differentials proxies for the correlation between SDF differentials. Similarly, motivated by the long-run risk literature, I also use the change in each country's stock price-to-dividend ratio as a proxy for the long-run growth component in its SDF, and show that the correlation between this SDF proxy is also much lower than the exchange rate correlation. I also obtain similar results using covariance instead of correlation. These patterns are representative of a robust pattern in international macroeconomics, that consumption correlation is much lower than what full risk-sharing implies.

Second, a salient feature of the exchange rate data is that a small number of common factors explain large fractions of variations in exchange rate movements. If the equilibrium constraints under symmetric SDF-FX pass-through hold, the same factor structure must also manifest itself in fundamental quantities that drive the SDFs. Empirically, evidence for such a factor structure in fundamental quantities has been elusive.

Third, cross-country financial holdings data reveal a severe home bias in international portfolio positions. Under certain standard assumptions about household preferences and income shocks, this portfolio home bias implies poor international risk-sharing and hence a low correlation in marginal utility growth across countries.

These three observations suggest that the correlation in SDF differentials should be lower than the observed correlation in exchange rate movements. Therefore, this necessary condition for symmetric SDF-FX pass-through is unlikely to hold in the data, which implies that the asymmetry in SDF-FX pass-through is a fundamental feature of the exchange rate dynamics. To the extent that our intuition about exchange rate dynamics is largely derived from complete-market models which always have symmetric SDF-FX pass-through, this result calls for a reassessment of our intuition.

As discussed above, the key effect of asymmetric SDF-FX pass-through is to propagate one country's SDF shocks to other countries' bilateral exchange rate movements. This international spill-over effect gives rise to a novel mechanism through which common variations in exchange rates endogenously arise, even when the SDF shocks happen to be uncorrelated across countries. To test this spill-over effect in the data, I consider a null in which markets are complete and SDF shocks have a factor structure (Lustig, Roussanov, and Verdelhan, 2011; Verdelhan, 2018). Each country's SDF is driven by common factors and an idiosyncratic shock. Under this null, the home currency's average exchange rate movement, which captures its own SDF shock, does not explain bilateral exchange rate movements between two foreign currencies once common exchange rate factors are controlled for.

However, I find the opposite in the data. Regardless of the choice of the home country, the home currency's average exchange rate movement explains a large number of bilateral exchange rate movements between two foreign countries, even after common exchange rate factors are controlled for. To the extent that these common factors capture the comovements in the SDFs, this result rejects the null and suggests that the spill-over effect from asymmetric SDF-FX pass-through is a general phenomenon.

Moreover, in a numerical example, I show a possibility result that asymmetric SDF-FX pass-through can generate a quantitatively much greater exchange rate correlation relative to the SDF correlation, which is the lower bound of the exchange rate correlation attained when the SDF-FX pass-through is symmetric. In this numerical example, market incompleteness indeed gives rise to endogenous comovements and factor structure in the exchange rate dynamics, above and beyond the comovements and factor structure embedded in the SDFs.

In conclusion, these results help us tie together the exchange rate comovements and disconnect under one mechanism based on market incompleteness. This novel mechanism complements the prior works that are based on complete markets and correlated latent SDF shocks. In doing so, it sheds light on the origins of exchange rate movements and comovements. Moreover, as the exchange rates play an important role in the transmission of global shocks to domestic conditions, these results have additional implications for the real economy. While this paper focuses on the theoretical possibility, quantifying the relative contributions of SDF shocks and the incomplete-market effects for exchange rate patterns and real outcomes is an exciting next step.

*Literature Review.*

Market incompleteness is a salient feature of the international financial markets. Prior works have proposed various settings with specific preferences and frictions and shown how market incompleteness impacts the dynamics of business cycles and exchange rates (see Alvarez, Atkeson, and Kehoe (2002, 2009); Kehoe and Perri (2002); Chari, Kehoe, and McGrattan (2002); Corsetti, Dedola, and Leduc (2008); Pavlova and Rigobon (2010, 2012); Hassan (2013); Bruno and Shin (2015); Favilukis, Garlappi, and Neamati (2015); Gabaix and Maggiori (2015); Maggiori (2017); Itskhoki and Mukhin (2021a,b)). While these results are important and speak to the relevance of modeling incomplete markets, they all focus on specific settings. In this paper, I explore whether we can make any general statements about the role of market incompleteness in the foreign exchange markets.

This paper builds on the preference-free approach to modeling incomplete-market exchange rate dynamics developed by Backus, Foresi, and Telmer (2001); Brandt, Cochrane, and Santa-Clara (2006), which has been applied to understand welfare implications (Lewis and Liu, 2022) and the role of non-Gaussian shocks (Maurer and Tran, 2021). The closest paper in this literature is Lustig and Verdelhan (2019), who characterize the exchange rate volatility, cyclicality and risk premium in a two-country setting. I further develop this preference-free approach in two ways. First, much of the focus has been given to the analysis of the bilateral exchange rate dynamics between two countries, while the generalization to a multi-country setting is presumed to be a trivial extension. In this paper, I show that this is not true: incomplete markets give rise to non-trivial constraints and exchange rate dynamics in a general multi-country setting, and they allow me to study the patterns in currency comovements that two-country models cannot address. Second, I decompose exchange rate dynamics in a new way, which is economically meaningful and convenient for interpreting the effect of market incompleteness and characterizing the asymmetry in the SDF-FX pass-through. We also adopt this approach in concurrent work Jiang, Krishnamurthy, Lustig, and Sun (2021), with a different objective of characterizing the bilateral exchange rate dynamics in the presence of Euler equation wedges.

To my best knowledge, this paper is the first to show how market incompleteness can generate comovements and common factors in multi-currency dynamics. The prior literature explains these patterns by various mechanisms that generate common factors in SDFs (Farhi and Gabaix, 2015; Corte, Riddiough, and Sarno, 2016; Colacito, Croce, Gavazzoni, and Ready, 2018; Mueller, Stathopoulos, and Vedolin, 2017; Jiang and Richmond, 2019). In contrast, this paper shows that asymmetric SDF-FX pass-through arises in incomplete markets and is able to generate comovements in exchange rates even when SDFs are uncorrelated. This paper also offers some empirical evidence that helps distinguish this effect of asymmetric pass-through in incomplete markets from that of heterogeneous SDF exposures in complete markets.

In the broader literature, this paper contributes to our understanding of the asset pricing dynamics in incomplete markets. Complementary to the prior works that study specific instances of incomplete-market models, my paper derives a general characterization of the exchange rate dynamics for arbitrary SDFs. My approach is also complementary to Bakshi, Cerrato, and Crosby (2018) and Korsaye, Trojani, and Vedolin (2020); Sandulescu, Trojani, and Vedolin (2021), which use asset return data to discipline the SDF dynamics and characterize the roles of incompleteness and segmentation in international asset markets, respectively.

The rest of this paper is organized as follows. Section 2 presents my main theoretical result in a simple setting. Section 3 discusses the implication of the main result and its empirical relevance. Section 4 discusses generalizations of the model. Section 5 concludes. Appendix contains the proof and data sources.

# 2 Exchange Rate Characterization in a Simple Economy

In this section, I use a very stylized setting to illustrate the main results. There is only one time period, and all shocks are drawn from a multivariate normal distribution. In Section 4.2, I show the results hold much more generally in a dynamic, infinite-horizon, continuous-time setting.

## 2.1 Accounting for Bilateral Exchange Rate Movements

I begin with analyzing the dynamics between two countries, indexed by $i \in \{0, 1\}$. I refer to country $0$ as the home country. I assume there is no arbitrage and there exist stochastic discount factors (SDFs) that price all tradable assets. Let $\Delta m^{(i)}$ denote country $i$'s log SDF and let $\tilde{r}$ denote the return of an arbitrary tradable asset in its numeraire. Then, the Euler equation $\mathbb{E}[\exp(\Delta m^{(i)} + \tilde{r})] = 1$ holds. The SDF captures the marginal utility growth and the intertemporal marginal rate of substitution (IMRS) of the country's representative agent, which can depend on the agent's preference, the nature of economic shocks, and space of tradable financial assets in the underlying economy.

Let $\Delta s^{1/0}$ denote the log bilateral exchange rate movement[1], which takes a higher value if the currency in country $0$ is stronger. Without imposing any structure except joint normality, I can describe the system of SDFs and exchange rate as

$$\begin{pmatrix} \Delta m^{(0)} \\ \Delta m^{(1)} \\ \Delta s^{1/0} \end{pmatrix} = \begin{pmatrix} -\mu_0 \\ -\mu_1 \\ \alpha_1 \end{pmatrix} + \begin{pmatrix} \sigma_{0,0} & 0 & 0 \\ \sigma_{1,0} & \sigma_{1,1} & 0 \\ \eta_{1,0} & \eta_{1,1} & \zeta_{1,1} \end{pmatrix} \begin{pmatrix} \varepsilon_m^{(0)} \\ \varepsilon_m^{(1)} \\ \varepsilon_s^{(1)} \end{pmatrix}, \tag{1}$$

where $\varepsilon_m^{(0)}$, $\varepsilon_m^{(1)}$, and $\varepsilon_s^{(1)}$ are three i.i.d. normal random variables with a mean of 0 and a volatility of 1. The last row in Eq. (1) describes the exchange rate dynamics, which can be rewritten as

$$\Delta s^{1/0} = x^{(1)} + z^{(1)}(\Delta m^{(0)} - \Delta m^{(1)}) + \left[(\eta_{1,1} + z^{(1)}\sigma_{1,1})\varepsilon_m^{(1)} + \zeta_{1,1}\varepsilon_s^{(1)}\right],$$

where $z^{(1)} = \frac{\eta_{1,0}}{\sigma_{0,0}-\sigma_{1,0}}$ and $x^{(1)} = \alpha_1 + z^{(1)}(\mu_0 - \mu_1)$. Define $y^{(1)} = \sqrt{(\eta_{1,1} + z^{(1)}\sigma_{1,1})^2 + \zeta_{1,1}^2}$. Then, I can express this equation as

$$\Delta s^{1/0} = x^{(1)} + z^{(1)}(\Delta m^{(0)} - \Delta m^{(1)}) + y^{(1)}\varepsilon_y^{(1)}, \tag{2}$$

where $\varepsilon_y^{(1)} = \left[(\eta_{1,1} + z^{(1)}\sigma_{1,1})\varepsilon_m^{(1)} + \zeta_{1,1}\varepsilon_s^{(1)}\right]/y^{(1)}$ is also a standard normal variable with a mean of 0 and a volatility of 1.

This equation decomposes the exchange rate movement into two components. The first component $z^{(1)}(\Delta m^{(0)} - \Delta m^{(1)})$ describes how the two countries' SDFs affect their bilateral exchange rates, and the parameter $z^{(1)}$ can be interpreted as the pass-through from SDF shocks to the exchange rate movement, or *SDF-FX pass-through* for simplicity. When markets are complete, the SDF shocks impact the exchange rate movement one-for-one (i.e., $\Delta s^{1/0} = \Delta m^{(0)} - \Delta m^{(1)}$), implying a pass-through of $z^{(1)} = 1$. When markets are incomplete, $z^{(1)}$ can take values lower than

[1] I define the SDFs, exchange rates, and bonds in real terms, but this analysis also applies if I relabel them to their nominal counterparts. Among developed countries, inflation has been low and stable in recent decades until recently. As a result, for the issues I study in this paper, evaluating the model using either nominal and real variables leads to similar results.

1, reflecting a partial pass-through from the SDF shocks to the exchange rate: a 1% shock to the SDF differential only generates an exchange rate movement smaller than 1%.

The second component $y^{(1)}\varepsilon_y^{(1)}$ captures additional variations in the exchange rate movement that are not fully captured by the SDF shocks. This component allows the exchange rate movement to be disconnected from the fundamental variables that drive the SDFs, which is a common feature in the incomplete-market models of exchange rates. I refer to the variable $\varepsilon_y^{(1)}$ as the IM (incomplete-market) shock. By construction, this IM shock is uncorrelated with the home SDF $\Delta m^{(0)}$. As such, Eq. (2) can be thought of as a projection of the exchange rate movement on the home SDF shock, with the IM shock $\varepsilon_y^{(1)}$ representing the home currency's exchange rate movement unexplained by its SDF. It is also worth noting that this shock is not necessarily "non-fundamental". For example, in the incomplete-market model of Pavlova and Rigobon (2012), this additional shock reflects investors' time-varying preferences for different goods across time.

I take the preference-free approach and start with exogenously given SDFs and SDF parameters $\mu_i$ and $\sigma_{i,j}$. When markets are incomplete, there may exist multiple exchange rate solutions that are consistent with the asset market equilibrium that I next describe. These solutions correspond to different values in the 4 exchange rate parameters $(\alpha_1, \eta_{1,0}, \eta_{1,1}, \zeta_{1,1})$. Equivalently, these 4 degrees of freedom can be described by $(x^{(1)}, z^{(1)}, y^{(1)}, corr(\varepsilon_m^{(1)}, \varepsilon_y^{(1)}))$ in Eq. (2). However, this model does not necessarily imply multiple equilibria. A full specification of the model with additional conditions on goods and asset market clearing usually pins down a unique equilibrium among the family of equilibria described by this approach.

Everything so far is just accounting. Eq. (2) remains a general description of the exchange rate dynamics in a log-normal world. Next, I impose economic restrictions by assuming that investors can freely trade both home and foreign risk-free bonds. Let $r^{(i)}$ denote the risk-free rate in country $i$. Opening these basic markets implies the following four Euler equations, which impose restrictions on the exchange rate dynamics:

$$
\begin{aligned}
1 &= \mathbb{E}[\exp(\Delta m^{(0)} + r^{(0)})] = \mathbb{E}[\exp(\Delta m^{(1)} + r^{(1)})], \\
1 &= \mathbb{E}[\exp(\Delta m^{(0)} - \Delta s^{1/0} + r^{(1)})] = \mathbb{E}[\exp(\Delta m^{(1)} + \Delta s^{1/0} + r^{(0)})].
\end{aligned}
$$

The financial markets for other contingent claims may or may not open for either country's investors, which allows me to model a flexible degree of market incompleteness. If other contingent claims are tradable, they could impose additional restrictions on the equilibrium exchange rate dynamics.

The first two Euler equations describe the valuation of risk-free bonds from domestic perspectives, which determine the equilibrium risk-free rates as functions of the SDF parameters in Eq. (1). For example, $r^{(0)} = \mu_0 - \frac{1}{2}\sigma_{0,0}^2$.

The last two Euler equations describe the valuation of risk-free bonds from foreign perspectives, which impose two restrictions on the relationships between the four exchange rate parameters $(x^{(1)}, z^{(1)}, y^{(1)}, corr(\varepsilon_m^{(1)}, \varepsilon_y^{(1)}))$. One of these restrictions govern the drift $x^{(1)}$ of the exchange rate movement, and the other can be expressed as

$$
-cov(\Delta m^{(0)}, -\Delta s^{1/0}) - \frac{1}{2}var(\Delta s^{1/0}) = cov(\Delta m^{(1)}, \Delta s^{1/0}) + \frac{1}{2}var(\Delta s^{1/0}),
$$

which has a simple interpretation. Note that the log currency risk premium from the perspective of country $0$ is captured by $cov(\Delta m^{(0)}, -\Delta s^{1/0})$, the covariance between country $0$'s SDF and its log foreign exchange rate. Similarly, the log currency risk premium from the perspective of country $1$ is captured by $cov(\Delta m^{(1)}, \Delta s^{1/0})$, the covariance between country $1$'s SDF and its log foreign exchange rate. Then, this expression states that the log currency risk premium from the perspective of country $0$ should match the inverse of the log currency risk premium from the perspective of country $1$, after adjusting for second-order Jensen's terms.

These two restrictions take out 2 degrees of freedom and we have $4-2=2$ degrees of freedom left. Specifically, I use these two restrictions to determine $x^{(1)}$ and $y^{(1)}$.

**Lemma 1.** *The Euler equations between countries* $0$ *and* $1$ *imply*

$$
\begin{aligned}
y^{(1)} &= (\frac{1}{2}-z^{(1)})(-\sigma_{1,1}corr(\varepsilon_m^{(1)},\varepsilon_y^{(1)})) \\
&\pm \sqrt{\left(\frac{1}{2}-z^{(1)}\right)^2(\sigma_{1,1}corr(\varepsilon_m^{(1)},\varepsilon_y^{(1)}))^2+z^{(1)}(1-z^{(1)})((\sigma_{0,0}-\sigma_{1,2})^2+\sigma_{1,1}^2)} \\
x^{(1)} &= (1-z^{(1)})(\mu_1-\mu_0)-\frac{1}{2}(\sigma_{1,0}^2+\sigma_{1,1}^2-\sigma_{0,0}^2)+\frac{1}{2}(z^{(1)})^2(\sigma_{0,0}-\sigma_{1,0})^2+\frac{1}{2}(z^{(1)})^2\sigma_{1,1}^2+\frac{1}{2}(y^{(1)})^2 \\
&- z^{(1)}y^{(1)}\sigma_{1,1}cov(\varepsilon_m^{(1)},\varepsilon_y^{(1)})-z^{(1)}\sigma_{0,0}(\sigma_{0,0}-\sigma_{1,0})
\end{aligned}
$$

This lemma states that if investors can freely trade home and foreign risk-free bonds, the bilateral exchange rate dynamics can be described by two parameters $(z^{(1)}, corr(\varepsilon_m^{(1)}, \varepsilon_y^{(1)}))$ and a dummy indicating which root of $y^{(1)}$ is picked. Of these two parameters, $z^{(1)}$ reflects the SDF-FX pass-through, while $corr(\varepsilon_m^{(1)}, \varepsilon_y^{(1)})$ governs the correlation between country $1$'s SDF shock and the IM shock. These two parameters along with the SDF parameters ($\sigma_{i,j}$ and $\mu_i$) determine $x^{(1)}$ and $y^{(1)}$. To guarantee a real solution for $y^{(1)}$, the parameters have to satisfy

$$
\left(\frac{1}{2}-z^{(1)}\right)^2(\sigma_{1,1}corr(\varepsilon_m^{(1)},\varepsilon_y^{(1)}))^2+z^{(1)}(1-z^{(1)})((\sigma_{0,0}-\sigma_{1,2})^2+\sigma_{1,1}^2)\geq 0,
$$

which implies a bound on the SDF-FX pass-through coefficient $z^{(1)}$. In particular, if the IM shock is uncorrelated with the SDF shock, i.e., $corr(\varepsilon_m^{(1)}, \varepsilon_y^{(1)}) = 0$, then, $z^{(1)}$ has to be bounded between 0 and 1, which guarantees a partial pass-through. Section 2.3 discusses this special case in detail.

Lastly, it is worth noting that my approach is closely related to the representation of incomplete-market exchange rate dynamics in Backus et al. (2001), which shows that the exchange rate movement is equal to the SDF differential plus an incomplete-market wedge $\eta$:

$$
\Delta s^{1/0} = (\Delta m^{(0)} - \Delta m^{(1)}) + \eta.
$$

My approach decomposes this incomplete-market wedge into an SDF component $(z^{(1)} -$

$1)(\Delta m^{(0)} - \Delta m^{(1)})$ and an IM component $y^{(1)}\varepsilon_y^{(1)}$:

$$\eta = x^{(1)} + (z^{(1)} - 1)(\Delta m^{(0)} - \Delta m^{(1)}) + y^{(1)}\varepsilon_y^{(1)},$$

which allows me to offer an economic interpretation for the effect of market incompleteness on the pass-through from SDF shocks to exchange rates, and to characterize its properties in my subsequent analysis.

## 2.2 Three Countries and Triangular Arbitrage

Next, I extend my analysis to the case of three countries, indexed by $i \in \{0, 1, 2\}$. Without loss of generality, I again set country $0$ as the home country. Then, assuming joint normality, the following equation system describes the SDF and exchange rate dynamics in a general way:

$$\begin{pmatrix} \Delta m^{(0)} \\ \Delta m^{(1)} \\ \Delta m^{(2)} \\ \Delta s^{1/0} \\ \Delta s^{2/0} \end{pmatrix} = \begin{pmatrix} -\mu_0 \\ -\mu_1 \\ -\mu_2 \\ \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{pmatrix} \sigma_{0,0} & 0 & 0 & 0 & 0 \\ \sigma_{1,0} & \sigma_{1,1} & 0 & 0 & 0 \\ \sigma_{2,0} & \sigma_{2,1} & \sigma_{2,2} & 0 & 0 \\ \eta_{1,0} & \eta_{1,1} & \eta_{1,2} & \zeta_{1,1} & 0 \\ \eta_{2,0} & \eta_{2,1} & \eta_{2,2} & \zeta_{2,1} & \zeta_{2,2} \end{pmatrix} \begin{pmatrix} \varepsilon_m^{(0)} \\ \varepsilon_m^{(1)} \\ \varepsilon_m^{(2)} \\ \varepsilon_s^{(1)} \\ \varepsilon_s^{(2)} \end{pmatrix}. \tag{3}$$

Similar to the two-country case, I take the SDF parameters as exogenously given, and regard the $11$ exchange rate parameters $(\alpha_1, \alpha_2, \eta_{1,0}, \eta_{1,1}, \eta_{1,2}, \zeta_{1,1}, \eta_{2,0}, \eta_{2,1}, \eta_{2,2}, \zeta_{2,1}, \zeta_{2,2})$ as free parameters. If I rewrite this equation system following Eq. (2), I obtain

$$\Delta s^{1/0} = x^{(1)} + z^{(1)}(\Delta m^{(0)} - \Delta m^{(1)}) + y^{(1)}\varepsilon_y^{(1)}, \tag{4}$$

$$\Delta s^{2/0} = x^{(2)} + z^{(2)}(\Delta m^{(0)} - \Delta m^{(2)}) + y^{(2)}\varepsilon_y^{(2)}, \tag{5}$$

with

$$
\begin{aligned}
z^{(1)} &= \frac{\eta_{1,0}}{\sigma_{0,0}-\sigma_{1,0}}, \\
z^{(2)} &= \frac{\eta_{2,0}}{\sigma_{0,0}-\sigma_{2,0}}, \\
y^{(1)} &= \sqrt{(\eta_{1,1}+z^{(1)}\sigma_{1,1})^2+\eta_{1,2}^2+\zeta_{1,1}^2}, \\
y^{(2)} &= \sqrt{(\eta_{2,1}+z^{(2)}\sigma_{2,1})^2+(\eta_{2,2}+z^{(2)}\sigma_{2,2})^2+\zeta_{2,1}^2+\zeta_{2,2}^2}, \\
\varepsilon_y^{(1)} &= \left[(\eta_{1,1}+z^{(1)}\sigma_{1,1})\varepsilon_m^{(1)}+(\eta_{1,2})\varepsilon_m^{(2)}+\zeta_{1,1}\varepsilon_s^{(1)}\right]/y^{(1)}, \\
\varepsilon_y^{(2)} &= \left[(\eta_{2,1}+z^{(2)}\sigma_{2,1})\varepsilon_m^{(1)}+(\eta_{2,2}+z^{(2)}\sigma_{2,2})\varepsilon_m^{(2)}+\zeta_{2,1}\varepsilon_s^{(1)}+\zeta_{2,2}\varepsilon_s^{(2)}\right]/y^{(2)}.
\end{aligned}
$$

The 11 degrees of freedom can be described by $(x^{(1)}, x^{(2)}, z^{(1)}, z^{(2)}, y^{(1)}, y^{(2)}, corr(\varepsilon_m^{(1)}, \varepsilon_y^{(1)}), corr(\varepsilon_m^{(1)}, \varepsilon_y^{(2)}), corr(\varepsilon_m^{(2)}, \varepsilon_y^{(1)}), corr(\varepsilon_m^{(2)}, \varepsilon_y^{(2)}), corr(\varepsilon_y^{(1)}, \varepsilon_y^{(2)}))$. Similar to the two-country case, the home SDF $\Delta m^{(0)}$ is orthogonal to the IM (incomplete-market) shocks $\varepsilon_y^{(1)}$ and $\varepsilon_y^{(2)}$ by construction.

Once we know these bilateral exchange rate dynamics, Triangular arbitrage implies the bilateral exchange rate movement between countries 1 and 2: $\Delta s^{1/2} = \Delta s^{1/0} - \Delta s^{2/0}$, which can be expressed as

$$
\begin{aligned}
\Delta s^{1/2} &= (x^{(1)}-x^{(2)}) + (z^{(1)}-z^{(2)})\Delta m^{(0)} - z^{(1)}\Delta m^{(1)} + z^{(2)}\Delta m^{(2)} \\
&+ y^{(1)}\varepsilon_y^{(1)} - y^{(2)}\varepsilon_y^{(2)},
\end{aligned}
\tag{6}
$$

which increases when country 2's currency appreciates.

An interesting question is whether the SDF-FX pass-through coefficients $z^{(1)}$ and $z^{(2)}$ are identical. If they are identical, the SDF-FX pass-through is said to be *symmetric*; if they are not, the SDF-FX pass-through is said to be *asymmetric*. The case of asymmetric pass-through is interesting because, according to Eq. (6), $(z^{(1)} - z^{(2)}) \neq 0$ implies that the bilateral exchange rate movement between foreign countries 1 and 2 has to load on the home country 0's SDF, giving rise to a new mechanism for international spill-over.

To answer this question, I divide the general case into three special cases which exhaust all possibilities. In each case, I characterize the conditions under which the SDF-FX pass-through is symmetric.

## 2.3 Case I: Uncorrelated SDF and IM Shocks within and across Countries

First, I consider the case in which the IM shocks in Eq. (4) and (5) are not correlated with the SDF shocks:

**Assumption 1.** *(a)* $corr(\Delta m^{(2)}, \varepsilon_y^{(1)}) = corr(\Delta m^{(1)}, \varepsilon_y^{(2)}) = 0$.

*(b)* $corr(\Delta m^{(1)}, \varepsilon_y^{(1)}) = corr(\Delta m^{(2)}, \varepsilon_y^{(2)}) = 0$.

This assumption describes a useful benchmark. If Assumption 1(a) does not hold, i.e., $corr(\Delta m^{(2)}, \varepsilon_y^{(1)}) \neq 0$ for example, country 2's SDF $\Delta m^{(2)}$ affects the bilateral exchange rate between countries $0$ and $1$ via the IM term $\varepsilon_y^{(1)}$, which already gives rise to some international spill-over effects. To characterize conditions under which the SDF-FX pass-through is symmetric, I start with the case without such spill-over.

Moreover, Assumption 1(b) implies that two countries' SDFs have symmetric effects on their bilateral exchange rate: a $1\%$ increase in $\Delta m^{(0)}$ corresponds to $z^{(1)}\%$ appreciation in $\Delta s^{1/0}$, whereas a $1\%$ increase in $\Delta m^{(1)}$ corresponds to $z^{(1)}\%$ depreciation in $\Delta s^{1/0}$. This symmetry is always preserved with $z^{(1)} = z^{(2)} \equiv 1$ in complete markets, from which the majority of our intuition about FX dynamics is derived. In contrast, if $corr(\Delta m^{(1)}, \varepsilon_y^{(1)}) \neq 0$, a shock to country $1$'s SDF also affects the IM shock $\varepsilon_y^{(1)}$. Then, the response of the bilateral exchange rate $\Delta s^{1/0}$ to country $1$'s SDF shock is different from its response to country $0$'s SDF shock. I consider this more general case in the next subsection.

This assumption pins down 4 degrees of freedom $(corr(\varepsilon_m^{(1)}, \varepsilon_y^{(1)}), corr(\varepsilon_m^{(1)}, \varepsilon_y^{(2)}), corr(\varepsilon_m^{(2)}, \varepsilon_y^{(1)}),$ $corr(\varepsilon_m^{(2)}, \varepsilon_y^{(2)}), corr(\varepsilon_y^{(1)}, \varepsilon_y^{(2)}))$. So we have $11 - 4 = 7$ degrees of freedom left, which can be described by $(x^{(1)}, x^{(2)}, z^{(1)}, z^{(2)}, y^{(1)}, y^{(2)}, corr(\varepsilon_y^{(1)}, \varepsilon_y^{(2)}))$. The $x$, $y$, and $z$ parameters describe the bilateral exchange rate dynamics, and the last parameter $corr(\varepsilon_y^{(1)}, \varepsilon_y^{(2)})$ describes the cross-country comovements in the IM shocks, which affect the cross-country correlation between the exchange rate movements $\Delta s^{1/0}$ and $\Delta s^{2/0}$.

Next, I consider the Euler equations for the cross-country risk-free bond holdings between the home country and each foreign country:

$$\begin{aligned}1 &= \mathbb{E}[\exp(\Delta m^{(0)} - \Delta s^{1/0} + r^{(1)})] = \mathbb{E}[\exp(\Delta m^{(1)} + \Delta s^{1/0} + r^{(0)})], \\ 1 &= \mathbb{E}[\exp(\Delta m^{(0)} - \Delta s^{2/0} + r^{(2)})] = \mathbb{E}[\exp(\Delta m^{(2)} + \Delta s^{2/0} + r^{(0)})].\end{aligned}$$

As discussed in the two-country case, the two Euler equations for the country pair $(0, 1)$ pin down the parameters $x^{(1)}$ and $y^{(1)}$. Similarly, the two Euler equations for the country pair $(0, 2)$ pin down the parameters $x^{(2)}$ and $y^{(2)}$. These restrictions remove 4 more degrees of freedom, leaving $7 - 4 = 3$ left, which can be described by $(z^{(1)}, z^{(2)}, corr(\varepsilon_y^{(1)}, \varepsilon_y^{(2)}))$.

Finally, I consider the Euler equations for the country pair $(1, 2)$:

$$1 = \mathbb{E}[\exp(\Delta m^{(2)} - \Delta s^{1/2} + r^{(1)})] = \mathbb{E}[\exp(\Delta m^{(1)} + \Delta s^{1/2} + r^{(2)})].$$

While the bilateral exchange rate movements for country pairs $(0, 1)$ and $(0, 2)$ directly imply the bilateral exchange rate movements for the country pair $(1, 2)$, the Euler equations for country pairs $(0, 1)$ and $(0, 2)$ *do not* imply these Euler equations for the country pair $(1, 2)$ in general.[2] As a result, the Euler equations for the country pair $(1, 2)$ imposes an additional independent restriction on the exchange rate parameters, leaving only $3 - 1 = 2$ degrees of freedom described by $(z^{(1)}, z^{(2)})$ for the three-country dynamics under Assumption 1. In other words, these two pass-through coefficients uniquely determine not only the bilateral exchange rate dynamics between country pairs $(0, 1)$ and $(0, 2)$, but also the cross-country correlations between the IM shocks and between the exchange rate movements.

To interpret this result, one intuition from the complete-market models is that, once we model

[2] The only exception is the complete-market case, in which the Euler equations for country pairs $(0, 1)$ and $(0, 2)$ directly imply the Euler equations for the country pair $(1, 2)$.

the bilateral dynamics between the home country $0$ and each foreign country, we will know the bilateral dynamics between any two foreign countries. In incomplete markets, this is not the case, as the Euler equations between two foreign countries impose additional restrictions. These additional restrictions further restrict the exchange rate correlation between the two country pairs. In particular, if we impose symmetric SDF-FX pass-through, we obtain a specific restriction on the equilibrium exchange rate comovements:

**Proposition 1** (Conditions for Symmetric SDF-FX Pass-Through, Case I)**.** *Under Assumption 1, symmetric SDF-FX pass-through, i.e. $z^{(1)} = z^{(2)} = z$, implies the following restriction on the covariances of exchange rate movements and SDF differentials:*

$$cov(\Delta s^{1/0}, \Delta s^{2/0}) = z \cdot cov(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(0)} - \Delta m^{(2)}).$$

*In correlation form,*

$$corr(\Delta s^{1/0}, \Delta s^{2/0}) = corr(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(0)} - \Delta m^{(2)}). \tag{7}$$

This result imposes a tight constraint on the equilibrium exchange rate dynamics: the correlation of exchange rate movements has to be exactly the same as that of SDF differentials. The complete-market case is a special case of symmetric SDF-FX pass-through, with $z = 1$, $\Delta s^{1/0} = \Delta m^{(0)} - \Delta m^{(1)}$, and $\Delta s^{2/0} = \Delta m^{(0)} - \Delta m^{(2)}$. Under symmetric SDF-FX pass-through, market incompleteness cannot alter the correlation structure of bilateral exchange rate movements to deviate from that of the SDF differentials. Asymmetric SDF-FX pass-through (i.e., $z^{(1)} \neq z^{(2)}$), on the other hand, allows more flexibility.

## 2.4 Discussion

Proposition 1 identifies a necessary condition for symmetric SDF-FX pass-through. Before I generalize this result, I discuss whether it is consistent with the data. In this discussion, I make three empirical observations that all suggest this condition is unlikely to hold, which makes a case for asymmetric SDF-FX pass-through.

First, a major puzzle of international economics is the lack of correlation in the consumption growth data across countries, which suggests that risks are poorly shared internationally (Backus, Kehoe, and Kydland, 1992; Backus and Smith, 1993). In comparison, the correlation between bilateral exchange rates is much higher.

The first panel in Figure (1) plots the distribution of the correlations between bilateral exchange rate movements. Here I take the USA as the base country. For example, the correlation between the AUS/USA bilateral exchange rate movement and the GBR/USA bilateral exchange rate movement is 0.64, which contributes to one data point in this histogram. On average, the correlation is 0.57, which is marked by the vertical dashed line.

For comparison, the second panel in Figure (1) plots the distribution of the correlations between consumption growth differential. For example, the correlation between the AUS/USA consumption growth differential and the GBR/USA consumption growth differential is $-0.03$. On average, the correlation is 0.29, which is about half of the average correlation of bilateral exchange rate movements. Therefore, consistent with the findings in the earlier literature, the international correlation in consumption growth is much lower than that in exchange rate movements. To the extent that the consumption growth shocks proxy for SDF shocks, this result suggests that the right-hand side of Eq. (7) should be much lower than its left-hand side.

Alternatively, SDFs can be driven by forces beyond the concurrent consumption growth. For example, the long-run risk literature develops SDFs that are driven by persistent long-run consumption news, and uses the stock market dividend-price ratio as its empirical proxy (Bansal et al., 2007; Colacito and Croce, 2011; Colacito et al., 2018). Without explicitly estimating a specification of the long-run risk model, I calculate the correlation matrix for the change in log stock dividend-price ratio differential. The last panel in Figure (1) plots the distribution. For example, the correlation between the change in the AUS/USA log dividend-price ratio differential and the change in the GBR/USA log dividend-price ratio differential is 0.52. On average, the correlation

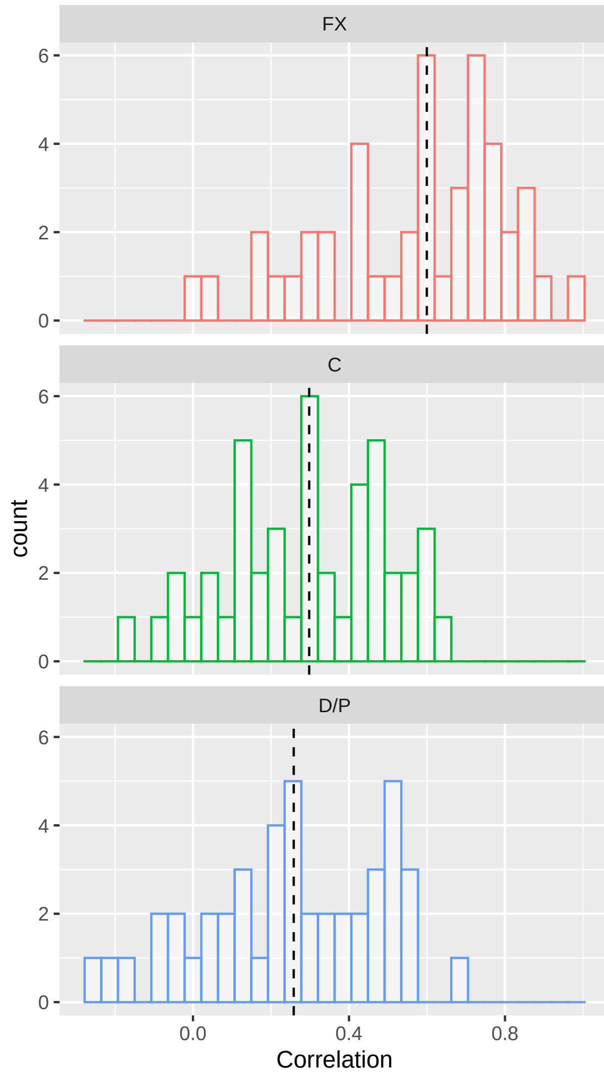



**Figure 1:** Correlations of Exchange Rates and Economic Quantities

This histogram reports the distribution of the pairwise correlations of bilateral exchange rate, bilateral log consumption growth differential, and bilateral stock log price-dividend ratio growth differential. The underlying data are reported in Appendix Table (A.1).

is 0.25, which is again about half of the average correlation of bilateral exchange rate movements.

Second, a salient feature of the exchange rate data is the existence of common factors that explain large fractions of variations in exchange rate movements (Lustig et al., 2011). Verdelhan (2018) shows that two common factors based on the returns of long and short positions on currencies account for 20% to 90% of the exchange rate movements in developed countries. If the condition (7) holds, the same factor structure must also manifest itself in the fundamental quantities that drive the SDFs. For example, if consumption growth drives the SDFs, then, under symmetric SDF-FX pass-through, two similarly constructed consumption factors should be able to explain the majority of variations in consumption growth across countries. However, common factors in economic fundamentals have been elusive in the literature. The possible exceptions are international capital flows (Verdelhan, 2018) and country-level stock price-dividend ratios (Colacito et al., 2018), which are themselves quantities related to asset price and exchange rate movements.

My third observation is that international portfolio holdings exhibit large home bias, another major puzzle in international macroeconomics (Lewis, 1995). For example, in 2019 among developed countries, the share of domestic assets is 70% in equity portfolio and 71% in bond portfolio (Jiang et al., 2022). In models with standard preferences and heterogeneous country-level income processes, the high degree of home bias implies poor international risk-sharing through holdings of external financial assets and therefore a low correlation in the marginal utility growth across countries. This different approach of assessing international risk-sharing also implies that the SDF correlations should be lower than the observed exchange rate correlations.

Admittedly, it is possible to modify investor preferences or other model ingredients and generate correlated marginal utility shocks across countries even with a high degree of portfolio home bias, just as it is possible to generate high correlations in SDFs without high correlations in consumption growth. This paper does not go over an exhaustive list of theories to rule out such possibility. Instead, this paper is about considering the possibility of asymmetric SDF-FX pass-through and understanding what it implies for exchange rate dynamics.

## 2.5 Case II: Uncorrelated SDF and IM Shocks across Countries

Is the result in Proposition 1 a general feature of the exchange rate dynamics in incomplete markets? I next relax Assumption 1 in two steps. First, I allow the bilateral exchange rate between the home country $0$ and each foreign country $i$ to load differently on their respective SDFs, by removing part (b) from the assumption:

**Assumption 2.** *(a)* $corr(\Delta m^{(2)}, \varepsilon_y^{(1)}) = corr(\Delta m^{(1)}, \varepsilon_y^{(2)}) = 0.$

By removing part (b) in Assumption 1, Assumption 2 allows a foreign country's SDF and its IM shock to be correlated. For example, if $corr(\Delta m^{(1)}, \varepsilon_y^{(1)}) \neq 0$, the bilateral exchange rate movement between countries $0$ and $1$ has different responses to the two countries' SDF shocks:

$$\begin{aligned} cov(\Delta s^{1/0}, \Delta m^{(0)}) &= z^{(1)} cov(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(0)}), \\ cov(\Delta s^{1/0}, \Delta m^{(1)}) &= -z^{(1)} cov(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(1)}) + y^{(1)} cov(\Delta m^{(1)}, \varepsilon_y^{(1)}), \end{aligned}$$

which already imposes a certain degree of asymmetry to the bilateral exchange rate dynamics that is absent in the complete-market case. On the other hand, Assumption 2(a) is maintained: the IM shock for the country pair $(0, 1)$ cannot load on country $2$'s SDF shock.

The following proposition shows that in this more general case, the main result is maintained.

**Proposition 2** (Conditions for Symmetric SDF-FX Pass-Through, Case II)**.** *Under the more general Assumption 2, symmetric SDF-FX pass-through, i.e.* $z_t^{(1)} = z_t^{(2)} = z_t$*, implies the same relationship between the covariance of exchange rate movements and the covariance of SDF differentials:*

$$cov(\Delta s^{1/0}, \Delta s^{2/0}) = z \cdot cov(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(0)} - \Delta m^{(2)}). \tag{8}$$

*In correlation form, the correlation of SDF differentials is augmented with additional terms:*

$$\begin{aligned} & corr(\Delta s^{1/0}, \Delta s^{2/0}) \\ = & \frac{cov(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(0)} - \Delta m^{(2)})}{\sqrt{var(\Delta m^{(0)} - \Delta m^{(1)}) + y^{(1)} cov(\Delta m^{(1)}, \varepsilon_y^{(1)})/z} \sqrt{var(\Delta m^{(0)} - \Delta m^{(2)}) + y^{(2)} cov(\Delta m^{(2)}, \varepsilon_y^{(2)})/z}}. \end{aligned}$$

The covariance form (8) holds in both Proposition 1 and 2. So, in this more general case, symmetric SDF-FX pass-through still implies a tight relationship between exchange rate covariance and SDF covariance.

To directly evaluate this covariance result, I conduct a similar test as in Figure (1). I fix the U.S. as the base country $0$, and calculate the ratio between the covariance between bilateral exchange rate movements and the covariance between bilateral consumption growth differentials: $\frac{cov_t(\Delta s_{t+1}^{i/0},\Delta s_{t+1}^{j/0})}{cov_t(\Delta c_{t+1}^{(0)}-\Delta c_{t+1}^{(i)},\Delta c_{t+1}^{(0)}-\Delta c_{t+1}^{(j)})}$, for two foreign countries $i$ and $j$.

If households have identical CRRA preference with risk aversion $\gamma$, then, the SDF can be expressed as $\Delta m^{(i)} = -\alpha - \gamma \Delta c^{(i)}$. By Proposition 2, under symmetric SDF-FX pass-through, we expect this covariance ratio to be identical across country pairs $(i, j)$:

$$\frac{cov_t(\Delta s_{t+1}^{i/0}, \Delta s_{t+1}^{j/0})}{cov_t(\Delta c_{t+1}^{(0)} - \Delta c_{t+1}^{(i)}, \Delta c_{t+1}^{(0)} - \Delta c_{t+1}^{(j)})} = \gamma^2 \frac{cov_t(\Delta s_{t+1}^{i/0}, \Delta s_{t+1}^{j/0})}{cov_t(\Delta m_{t+1}^{(0)} - \Delta m_{t+1}^{(i)}, \Delta m_{t+1}^{(0)} - \Delta m_{t+1}^{(j)})} = \gamma^2 z.$$

Figure (2), panel (a) reports these ratios in the data. Contrary to being identical, these ratios vary across country pairs. Certainly, there are other possibilities if we insist on having symmetric SDF-FX pass-through. One possibility is that the investor preferences are very different across countries. If so, these ratio estimates suggest a high degree of heterogeneity in preferences.

Another possibility is that the SDFs are driven by other economic quantities. In the same spirit as in the last exercise in Figure (1), I use the change in the stock market dividend-price ratio as a proxy for the SDF shock in an economy with long-run risk. If the households have the same preference, I expect the ratio between the covariance between bilateral exchange rate movements

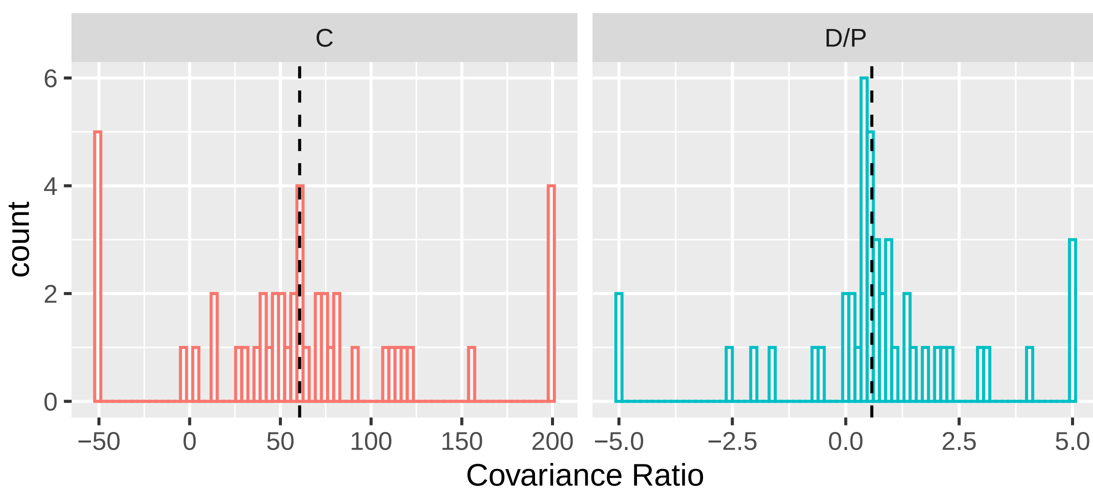



**Figure 2:** Covariance Ratio Test

This histogram reports the ratio between the covariance between bilateral exchange rate movements and the covariance between bilateral growth differentials in economic quantities. The underlying data are reported in Appendix Table (A.2). The covariance ratios are winsorized at $(-50, 200)$ and $(-5, 5)$ to improve visibility.

and the covariance between the bilateral differential in the change of the stock market dividend-price ratio to be identical:

$$\frac{cov_t(\Delta s_{t+1}^{i/0}, \Delta s_{t+1}^{j/0})}{cov_t(\Delta dp_{t+1}^{(0)} - \Delta dp_{t+1}^{(i)}, \Delta dp_{t+1}^{(0)} - \Delta dp_{t+1}^{(j)})} = k.$$

Figure (2), panel (b) reports these covariance ratios in the data, which also exhibit a large degree of heterogeneity across country pairs. Therefore, to the extent each country's stock market dividend-price ratio proxies for its SDF shock, this result does not support the implication of symmetric SDF-FX pass-through.

### 2.6 Case III: Correlated SDF and IM Shocks across Countries

Lastly, I consider the remaining case in which the SDF shocks and the IM shocks are correlated across country pairs.

**Assumption 3.** *(a') $corr(\Delta m^{(2)}, \varepsilon_y^{(1)}) \neq 0$ or $corr(\Delta m^{(1)}, \varepsilon_y^{(2)}) \neq 0$.*

Without loss of generality, I focus on the case in which $corr(\Delta m^{(1)}, \varepsilon_y^{(2)}) \neq 0$. In this case, the SDF shock in country $1$ affects the bilateral exchange rate dynamics between countries $0$ and $2$. As $\Delta m^{(1)} = -\mu_1 + \sigma_{1,0}\varepsilon_m^{(0)} + \sigma_{1,1}\varepsilon_m^{(1)}$ and by construction $corr(\varepsilon_m^{(0)}, \varepsilon_y^{(2)}) = 0$, then, $corr(\Delta m^{(1)}, \varepsilon_y^{(2)}) \neq 0$ implies $\kappa = corr(\varepsilon_m^{(1)}, \varepsilon_y^{(2)}) \neq 0$. I can thus decompose the IM shock $\varepsilon_y^{(2)}$ into country $1$'s SDF shock $\varepsilon_m^{(1)}$ and an orthogonal component:

$$\tilde{\varepsilon}_y^{(2)} = \frac{1}{\sqrt{1-\kappa^2}}(\varepsilon_y^{(2)} - \kappa\varepsilon_m^{(1)})$$

that satisfies $corr(\varepsilon_m^{(1)}, \tilde{\varepsilon}_y^{(2)}) = 0$.

Then, I can rewrite the bilateral exchange rate movement between countries $0$ and $2$ as

$$\begin{aligned}\Delta s^{2/0} &= x^{(2)} + z^{(2)}(\Delta m^{(0)} - \Delta m^{(2)}) + y^{(2)}(\kappa\varepsilon_m^{(1)} + \sqrt{1-\kappa^2}\tilde{\varepsilon}_y^{(2)}) \\ &= \tilde{x}^{(2)} + \frac{y^{(2)}\kappa}{\sigma_{1,1}}\Delta m^{(1)} + \tilde{z}^{(0)}\Delta m^{(0)} - z^{(2)}\Delta m^{(2)} + \tilde{y}^{(2)}\tilde{\varepsilon}_y^{(2)},\end{aligned}$$

with $\tilde{y}^{(2)} = y^{(2)}\sqrt{1-\kappa^2}$, $\tilde{x}^{(2)} = x^{(2)} + \frac{y^{(2)}\kappa}{\sigma_{1,1}}\mu^{(1)} - \frac{y^{(2)}\kappa}{\sigma_{1,1}}\frac{\sigma_{1,0}}{\sigma_{0,0}}\mu^{(0)}$, and $\tilde{z}^{(0)} = z^{(2)} - \frac{y^{(2)}\kappa}{\sigma_{1,1}}\frac{\sigma_{1,0}}{\sigma_{0,0}}$.

To interpret this equation, let us take a different perspective and regard country 1 as the base country. Then, the term $\frac{y^{(2)}\kappa}{\sigma_{1,1}}\Delta m^{(1)}$ indicates that the bilateral exchange rate dynamics between countries 0 and 2 is exposed to country 1's SDF. Moreover, since $\tilde{z}^{(0)} \neq z^{(2)}$ when $\sigma_{1,0} \neq 0$, we can also interpret the pass-through from country 0's SDF and country 2's SDF to their bilateral exchange rate as being asymmetric. Therefore, this expression is very similar to Eq. (6) in the earlier Cases I and II, reproduced below,

$$\Delta s^{1/2} \;=\; (x^{(1)} - x^{(2)}) + (z^{(1)} - z^{(2)})\Delta m^{(0)} - z^{(1)}\Delta m^{(1)} + z^{(2)}\Delta m^{(2)} + y^{(1)}\varepsilon_y^{(1)} - y^{(2)}\varepsilon_y^{(2)},$$

which regards country 0 as the base country. In this baseline case, when the SDF-FX pass-through is asymmetric (i.e., $z^{(1)} - z^{(2)} \neq 0$), the bilateral exchange rate dynamics between countries 1 and 2 is exposed to country 0's SDF.

In other words, Assumption 3 generates asymmetric pass-through and the cross-country spillover from SDF shocks to exchange rate movements from the perspective of a different base country. Together with Cases I and II, these results exhaust all possibilities and suggest that the asymmetric SDF-FX pass-through is a general feature in the exchange rate dynamics. In the next section, I explore the implications of this feature in detail.

# 3 Implications of Asymmetric SDF-FX Pass-through

The results above characterize the exchange rate dynamics between the base country 0 and two foreign countries 1 and 2. In particular, Eq. (6) provides a general expression for the bilateral exchange rate between the two foreign countries:

$$\Delta s^{1/2} \;=\; (x^{(1)} - x^{(2)}) + (z^{(1)} - z^{(2)})\Delta m^{(0)} - z^{(1)}\Delta m^{(1)} + z^{(2)}\Delta m^{(2)} + y^{(1)}\varepsilon_y^{(1)} - y^{(2)}\varepsilon_y^{(2)}.$$

To derive some intuition, let us first assume that the SDF shocks are uncorrelated across countries, i.e., $corr(\Delta m^{(i)}, \Delta m^{(j)}) = 0$ for $i \neq j$, and consider the following three scenarios.

First, when the markets are complete, the bilateral exchange rate movement is entirely determined by the two countries' SDF shocks. That is, Eq. (6) can be simplified to

$$\Delta s^{1/2} = \Delta m^{(2)} - \Delta m^{(1)}.$$

In this scenario, since country $0$'s SDF $\Delta m^{(0)}$ is uncorrelated with either foreign country's SDF, it does not comove with the exchange rate movement $\Delta s^{1/2}$.

Second, when the markets are incomplete but the SDF-FX pass-through is symmetric, i.e., $z^{(1)} = z^{(2)} = z$, then, Eq. (6) can be simplified to

$$\Delta s^{1/2} = (x^{(1)} - x^{(2)}) + z(\Delta m^{(2)} - \Delta m^{(1)}) + y^{(1)}\varepsilon_y^{(1)} - y^{(2)}\varepsilon_y^{(2)}.$$

By construction, the IM shocks $\varepsilon_y^{(1)}$ and $\varepsilon_y^{(2)}$ are uncorrelated with the base country $0$'s SDF $\Delta m^{(0)}$. So, symmetric SDF-FX pass-through also implies zero correlation between country $0$'s SDF $\Delta m^{(0)}$ and the exchange rate movement $\Delta s^{1/2}$.

Finally, when the markets are incomplete and the SDF-FX pass-through is asymmetric, then, the term $(z^{(1)} - z^{(2)})\Delta m^{(0)}$ in Eq. (6) is nonzero. It allows country $0$'s SDF, which is fully idiosyncratic by assumption, to affect the bilateral exchange rate movement between countries $1$ and $2$. This term gives rise to a spill-over effect that propagates one country's SDF shock to other countries' bilateral exchange rate movements, which is absent in the previous two scenarios.

Given that the theoretical results in the previous section support the case with asymmetric SDF-FX pass-through, this international spill-over effect should be a general phenomenon. I next test whether this spill-over effect is consistent with exchange rate patterns in the data.

### 3.1 Empirical Implication: Is This Spill-Over Effect Consistent with Data?

If the SDFs are observable, I can test this spill-over effect by directly regressing the bilateral exchange rate movments between two countries $j$ and $k$ on their SDFs and the SDF of a third country $i$:

$$\Delta s_t^{j/k} \;=\; \alpha^{j/k} + \beta \Delta m_t^{(i)} + \gamma^j \Delta m_t^{(j)} + \gamma^k \Delta m_t^{(k)} + u_t^{j/k}.$$

I am interested in knowing whether the coefficient $\beta$ is zero. I control for the SDFs $\Delta m_t^{(j)}$ and $\Delta m_t^{(k)}$ in order to absorb the potential comovements in the SDFs. When the SDF-FX pass-through is symmetric (which includes the case of complete markets), I expect $\beta = 0$; when the SDF-FX pass-through is asymmetric, I expect $\beta \neq 0$ for a general choice of countries $i$, $j$, and $k$.

Unfortunately, SDFs are not directly observable. I have to set up my test with an auxiliary assumption. Specifically, I consider a null hypothesis in which markets are complete and the SDFs have the following factor structure:

$$\Delta m_t^{(\ell)} \;=\; a_{t-1}^{(\ell)} + b^{(\ell)} f_t + v_t^{(\ell)}$$

for $\ell \in \{i, j, k\}$, where $f_t$ captures one or more common SDF factors, and $v_t^{(\ell)}$ is a country-specific shock. This factor structure has been proposed to understand exchange rate comovements (Lustig, Roussanov, and Verdelhan, 2011; Verdelhan, 2018).

Under this null, country $i$'s average exchange rate contains information about its SDF. More precisely, the average exchange rate movement for country $i$ is

$$\Delta \overline{s}_t^i \;=\; \frac{1}{N} \sum_{\ell} \Delta s_t^{i/\ell},$$

which describes the average performance of country $i$'s currency against other currencies (Lustig and Richmond, 2020; Aloosh and Bekaert, 2022). The (equal-weighted) dollar index is a good example, which describes the performance of the U.S. dollar relative to an average foreign currency.

The null hypothesis implies that $\Delta \overline{s}_t^i = \Delta \overline{m}_t - \Delta m_t^{(i)}$, where $\Delta \overline{m}_t$ is the global average of the SDFs. Even when the markets are incomplete, as long as the SDF-FX pass-through is above zero, this average exchange rate movement still captures the home country's SDF shock while averages out the foreign countries' idiosyncratic shocks.

Next, I regress the bilateral exchange rate movments between two other countries $j$ and $k$ on country $i$'s average exchange rate movement, and control for the two common factors in the exchange rate literature, namely dollar and carry (Verdelhan, 2018):

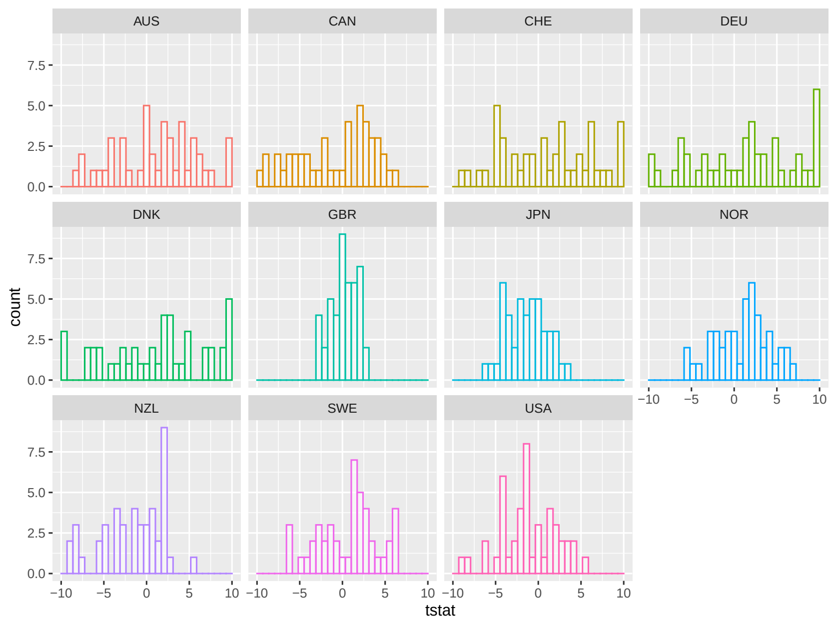
$$\Delta s_t^{j/k} \quad = \quad \alpha^{j/k} + \beta^i \Delta \overline{s}_t^i + \beta^d dollar_t + \beta^c carry_t + u_t^{j/k}, \text{ for } i \notin \{j,k\}. \tag{9}$$

Under the null, as country $i$'s average exchange rate movement is only exposed to these common factors and its own country-specific shock, the foreign exchange rate's loading $\beta^i$ on its SDF will be zero. In contrast, when markets are incomplete, asymmetric pass-through propagates a country's SDF shocks to other countries' bilateral exchange rates, which leads to nonzero $\beta^i$. In this way, this test helps us distinguish exchange rate comovements that arise from common SDF factors from comovements that arise from asymmetric SDF-FX pass-through.

To carry out this test, I first consider a version of the regression that does not contain the common factors:

$$\Delta s_t^{j/k} \quad = \quad \alpha^{j/k} + \beta^i \Delta \overline{s}_t^i + u_t^{j/k}, \text{ for } i \notin \{j,k\}. \tag{10}$$

Figure (3)(a) reports the distribution of the $t$-statistics for the $\beta^i$ coefficient. In each panel, I use a different base country $i$ and run this regression for all foreign country pairs $(j,k)$. For example, the USA panel reports the distribution of $t$-statistics taking the U.S. as the base country $i$. Many of the reported $t$-statistics are above 2 or below $-2$, implying that the $\beta^i$ coefficient tends to be nonzero and statistically significant for many country pairs. In other words, the dollar's average exchange rate movement tends to comove with other countries' bilateral exchange rate movements. While this result is hardly surprising given the central role that the U.S. plays in the international trade




(a) Without Controlling Common Currency Factors


(b) With Controlling Common Currency Factors

**Figure 3:** Distribution of $t$-Statistics for the $\beta^i$ Coefficient.

The regression equations in the two panels are (10) and (9). Each panel reports the distribution of $t$-statistics using a different base country $i$. The $t$-statistics are winsorized at $\pm 10$ to improve visibility. The exchange rate data are quarterly, 1988—2020.

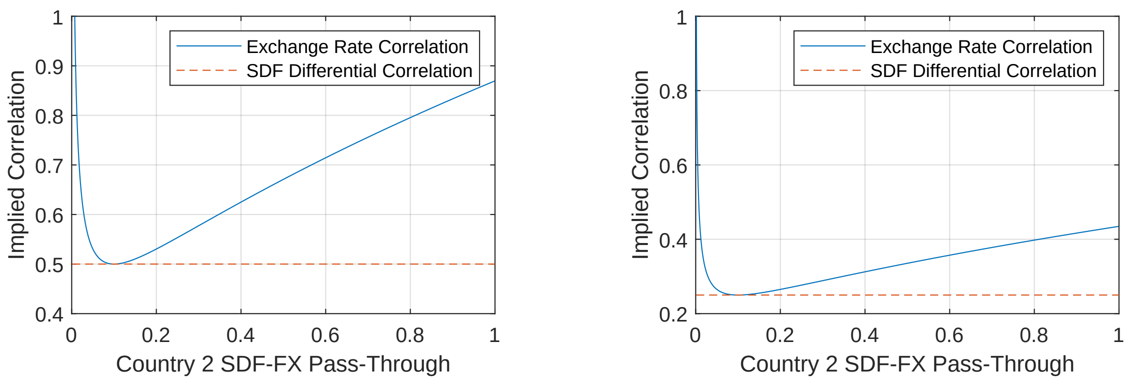
and financial networks, this pattern also holds when I pick a different base country. I report these results in different panels: many of the $t$-statistics have absolute values greater than 2 and some are greater than 5, which is suggestive of the spill-over effect.

Then, to rule out the possibility that this result is simply driven by common variations in SDFs, I run regression (9) that controls for the common factors. Figure (3)(b) reports the distribution of the $t$-statistics for the $\beta^i$ coefficient. The panel USA is omitted because the U.S. dollar's average exchange rate movement is co-linear with the dollar factor. The remaining panels show that the $\beta^i$ coefficient remains statistically significant for many base countries and bilateral currency pairs. If we take the factor model of exchange rates in Lustig et al. (2011); Verdelhan (2018) literally, the bilateral exchange rate movement $\Delta s_t^{j/k}$ after controlling for the common factors only reflects the idiosyncratic SDF shocks specific to countries $j$ and $k$. In contrast, this empirical result suggests that this idiosyncratic exchange rate movement is strongly correlated with currency $i$'s average exchange rate movement $\Delta \overline{s}_t^i$.

This result rejects the null and supports the view that bilateral exchange rates between any two countries tend to comove with other countries' SDFs, which is consistent with the implication of the asymmetry in the SDF-FX pass-through. This mechanism gives rise to a novel channel through which exchange rate comovements arise endogenously. This channel complements traditional ingredients that generate exchange rate comovements by generating correlated SDFs. Moreover, the spill-over into exchange rate movements may give rise to further spill-over effects on other countries' real outcomes, which could be an interesting path to pursue in future research.

## 3.2 Numerical Magnitude: How Large is This Spill-Over Effect?

Quantitatively, how large is the effect of asymmetric SDF-FX pass-through on the FX correlation structure? In this section, I present a numerical result that quantifies how much deviation of the exchange rate correlation from the SDF correlation can be expected under asymmetric SDF-FX pass-through. There are three countries in this example. Suppose the pairwise correlation between SDF shocks is $\rho_{0,1} = \rho_{0,2} = \rho_{1,2} = 0.5$, and the SDF volatility is $1$ in each country. Then, the




*Panel (a)* $\rho_{1,2} = 0.5$ *Panel (b)* $\rho_{1,2} = 0.25$

**Figure 4:** FX Correlation as SDF-FX Pass-through Varies

We plot the correlation between the SDF differentials $\Delta m^{(0)} - \Delta m^{(1)}$ and $\Delta m^{(0)} - \Delta m^{(2)}$ and the correlation between the bilateral exchange rate movements $\Delta s^{1/0}$ and $\Delta s^{2/0}$ as we vary the SDF-FX pass-through in two numerical examples.

implied correlation between the SDF differentials is $corr(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(0)} - \Delta m^{(2)}) = 0.5$. For simplicity, I maintain Assumption 1 to focus on the IM shocks $\varepsilon_y^{(i)}$ that are not correlated with the SDF shocks.

Then, I fix country 1's SDF-FX pass-through $z^{(1)}$ at 0.1 and vary country 2's SDF-FX pass-through $z^{(2)}$, and trace out the implied correlation between the bilateral exchange rate movements $\Delta s^{1/0}$ and $\Delta s^{2/0}$ in Figure (4), Panel (a). When $z^{(2)} = z^{(1)} \equiv 0.1$, I recover the result in Proposition 1: under symmetric SDF-FX pass-through, the exchange rate correlation equals the SDF differential correlation.

When $z^{(2)}$ is either higher or lower than $z^{(1)}$, the implied exchange rate correlation becomes higher relative to the SDF correlation. This pattern is consistent with Figure (1), which shows that the correlation between bilateral exchange rate movements tends to be higher than the correlation between proxies of SDF differentials. Numerically, the exchange rate correlation can approach 1, which allows us to quantitatively escape the tight constraint between the exchange rate correlation and the SDF differential correlation that characterizes the case of symmetric SDF-FX pass-through.

Figure (4), Panel (b), reports a different case in which the SDF correlation between countries

1 and 2 is $\rho_{1,2} = 0.25$. In this case, the implied correlation between SDF differentials is lower: $corr(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(0)} - \Delta m^{(2)}) = 0.25$, which is closer to the average correlation between economic fundamentals in Figure (1). Even so, the implied exchange rate correlation can still become much higher as the SDF-FX pass-through of country 2 deviates from that of country 1.

Lastly, there is an interesting connection to Brandt et al. (2006) who also study the exchange rate dynamics in incomplete markets. Incomplete markets allow *multiple SDFs* to be consistent with a given exchange rate process. Brandt et al. (2006) show that, among these SDFs, there exists a unique choice such that the exchange rate movements are equal to the SDF differentials, just as in the complete-market case. These SDFs are the *minimum-variance SDFs*, because other admissible SDFs are equal to these SDFs plus additional shocks that are unspanned by the asset payoffs, and thus have higher variances.

My paper takes a different but related approach. I fix the SDFs, which could come from some underlying consumption processes. Then, incomplete markets allow *multiple exchange rate processes* to be consistent with these SDFs. Among these exchange rate processes, symmetric SDF-FX pass-through generates the *minimum-correlation exchange rate movement*, which attains the lowest exchange rate correlation that is equal to the SDF correlation. As I introduce asymmetric SDF-FX pass-through by either raising or lowering $z^{(2)}$ relative to $z^{(1)}$, I obtain an exchange rate correlation that is higher and above the SDF correlation.

# 4 Generalizations

## 4.1 More Than Three Countries

The main result in this paper holds in a general multi-country setting. To recap, my setting so far contains only three countries indexed by $0, 1, 2$. I designate country $0$ as the base country, and the Euler equations between foreign countries $1$ and $2$ imply one set of equilibrium constraints via Triangular arbitrage. For given values of $z^{(1)}$ and $z^{(2)}$, this set of equilibrium constraints pin down the correlation between IM shocks $corr(\varepsilon_y^{(1)}, \varepsilon_y^{(2)})$, which further pins down the correlation between bilateral exchange rate movements $corr(\Delta s^{1/0}, \Delta s^{2/0})$.

With $N+1$ countries indexed by $0, 1, \ldots, N$, where $N \geq 2$, the results I derive in the paper must hold for any three countries in this set. Moreover, I can similarly designate country $0$ as the base country, and derive $N(N-1)/2$ unique sets of equilibrium constraints from the pairs of foreign countries $(i, j)$ such that $1 \leq i < j \leq N$. For given values of $z^{(i)}$ and $z^{(j)}$, these sets of equilibrium constraints pin down the $N(N-1)/2$ correlations between foreign incomplete-market shocks $corr(\varepsilon_y^{(i)}, \varepsilon_y^{(j)})$, which further pin down the $N(N-1)/2$ correlations between bilateral exchange rate movements $corr(\Delta s^{i/0}, \Delta s^{j/0})$. Therefore, the $(N+1)$-country case is a straightforward generalization of the three-country case.[3]

## 4.2 General Exchange Rate Dynamics in Continuous Time

The results hold more generally in a dynamic, infinite-horizon, and continuous-time economy. I fix a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and a given filtration $\{\mathcal{F}_t : t \geq 0\}$ satisfying the usual conditions. I assume that all stochastic processes are adapted to this filtration.

Let $\mathcal{A}[X_t]$ denote the drift of the process $X_t$; the operator $\mathcal{A}$ is also known as the generator. With slight abuse of notation, let $[dX_t, dY_t]$ denote the instantaneous conditional covariance between two diffusion processes $X_t$ and $Y_t$. Formally, it is defined as $[dX_t, dY_t] = d[X, Y]_t/dt$ where $[X, Y]_t$ is the standard quadratic covariation process between $X_t$ and $Y_t$.

Countries are indexed by $i \in \{0, 1, 2\}$. Let $M_t^{(i)}$ and $m_t^{(i)}$ denote country $i$'s cumulative SDF and its log. The log SDF follows a diffusion process:

$$dm_t^{(i)} = -\mu_t^{(i)} dt - \sigma_t^{(i)} dZ_t^{(i)},$$

and let $\rho_{i,j,t} = [dZ_t^{(i)}, dZ_t^{(j)}]$ denote the correlation between two countries' SDFs.

The mean, volatility and correlation parameters $\mu_t^{(i)}$, $\sigma_t^{(i)}$, and $\rho_{i,j,t}$ can be time-varying. I

[3] That said, with more than 2 foreign countries, the correlation matrix for $corr(\varepsilon_y^{(i)}, \varepsilon_y^{(j)})$ has to be positive semidefinite, which could impose additional constraints on parameter values.

assume they follow

$$
\begin{aligned}
d\sigma_t^{(i)} &= a_{\sigma,t}^{(i)}dt + b_{\sigma,t}^{(i)}dW_{\sigma,t}^{(i)}, \\
d\mu_t^{(i)} &= a_{\mu,t}^{(i)}dt + b_{\mu,t}^{(i)}dW_{\mu,t}^{(i)}, \\
d\rho_{i,j,t} &= a_{\rho,t}^{(i)(j)}dt + b_{\rho,t}^{(i)(j)}dW_{\rho,t}^{(i)(j)},
\end{aligned}
$$

where all stochastic processes are adapted to the filtration $\mathcal{F}_t$ and the Brownian motions $W_{\sigma,t}^{(i)}, W_{\mu,t}^{(i)}, W_{\rho,t}^{(i)(j)}$ are allowed to be correlated with each other.

Without loss of generality, I fix country $0$ as the base country. Let $S_t^{i/0}$ denote the exchange rate between country $0$ and country $i$, which increases when the currency in country $0$ appreciates. Let $s_t^{i/0}$ denote the log exchange rate.

If markets are complete, there is a unique exchange rate process determined by the SDF differential:

$$
ds_t^{cm,i/0} = dm_t^{(0)} - dm_t^{(i)} = -(\mu_0 - \mu_i)dt - (\sigma_0 dZ_t^{(0)} - \sigma_i dZ_t^{(i)}).
$$

Under incomplete markets, I assume that investors can still trade both home and foreign risk-free bonds. Their risk-free bond prices follow

$$
dP_t^{(i)} = r_t^{(i)} P_t^{(i)} dt.
$$

As investors in each country can hold domestic bonds, we have the following set of first-order conditions:

$$
0 = \mathcal{A}[M_t^{(i)} P_t^{(i)}];
$$

moreover, as investors can also hold foreign bonds, we have the following set of first-order conditions between country $0$ and country $i$:

$$0 = \mathcal{A}[M_t^{(0)}(S_t^{i/0})^{-1}P_t^{(i)}] = \mathcal{A}[M_t^{(i)}S_t^{i/0}P_t^{(0)}].$$

Following my derivation in the simple case, I can without loss of generality express the bilateral exchange rate movement as

$$ds_t^{i/0} = x_t^{(i)}dt + z_t^{(i)}(dm_t^{(0)} - dm_t^{(i)}) + y_t^{(i)}dY_t^{(i)},$$

where $(dm_t^{(0)} - dm_t^{(i)})$ is the SDF differential between countries $0$ and $i$, $z_t^{(i)}$ describes the SDF-FX pass-through, and $dY_t^{(i)}$ is an additional shock that arises due to market incompleteness. Let $\pi_{i,j,t} = [dZ_t^{(i)}, dY_t^{(j)}]$ denote the correlation between the SDF shock and the IM shock. By construction, I pick $z_t^{(i)}$ to ensure zero correlation between the IM shock $dY_t^{(i)}$ and country $0$'s SDF shock $dZ_t^{(0)}$: $[dZ_t^{(0)}, dY_t^{(i)}] = 0$.

Then, the first-order conditions for bond holdings between countries $0$ and $i$ imply equilibrium restrictions on the relationship between $x_t^{(i)}$, $y_t^{(i)}$, and $z_t^{(i)}$:

$$\begin{aligned} 0 &= (y_t^{(i)})^2 + (2z_t^{(i)} - 1)\sigma_t^{(i)}\pi_{i,i,t}y_t^{(i)} + ((z_t^{(i)})^2 - z_t^{(i)})((\sigma_t^{(0)})^2 + (\sigma_t^{(i)})^2 - 2\rho_{0,i,t}\sigma_t^{(0)}\sigma_t^{(i)}) \\ x_t^{(i)} &= (1 - z_t^{(i)})(\mu_t^{(i)} - \mu_t^{(0)}) - \frac{1}{2}((\sigma_t^{(i)})^2 - (\sigma_t^{(0)})^2) \\ &+ \frac{1}{2}(z_t^{(i)})^2((\sigma_t^{(0)})^2 + (\sigma_t^{(i)})^2 - 2\rho_{0,i,t}\sigma_t^{(0)}\sigma_t^{(i)}) \\ &+ \frac{1}{2}(y_t^{(i)})^2 - z_t^{(i)}((\sigma_t^{(0)})^2 - \rho_{0,i,t}\sigma_t^{(0)}\sigma_t^{(i)}) + z_t^{(i)}\sigma_t^{(i)}y_t^{(i)}\pi_{i,i,t}. \end{aligned}$$

Now, I consider the equilibrium conditions for multiple foreign currencies, which are restricted by Triangular arbitrage. Specifically, since

$$\begin{aligned} ds_t^{1/0} &= x_t^{(1)}dt + z_t^{(1)}(dm_t^{(0)} - dm_t^{(1)}) + y_t^{(1)}dY_t^{(1)}, \\ ds_t^{2/0} &= x_t^{(2)}dt + z_t^{(2)}(dm_t^{(0)} - dm_t^{(2)}) + y_t^{(2)}dY_t^{(2)}, \end{aligned}$$

then, the bilateral exchange rate movement between the two foreign currencies 1 and 2 has to follow

$$\begin{aligned} ds_t^{1/2} &= (x_t^{(1)} - x_t^{(2)})dt + (z_t^{(1)} - z_t^{(2)})dm_t^{(0)} - z_t^{(1)}dm_t^{(1)} + z_t^{(2)}dm_t^{(2)} \\ &+ y_t^{(1)}dY_t^{(1)} - y_t^{(2)}dY_t^{(2)}, \end{aligned} \tag{11}$$

which increases when country 2's currency appreciates.

The following proposition states the main results in this continuous-time setting.

**Proposition 3** (Conditions for Symmetric SDF-FX Pass-Through, Continuous Time)**.** *(a) If the IM shock $dY_t^{(i)}$ in a country $i$ is not correlated with the SDF shock in another country $j$, i.e., $[dZ_t^{(1)}, dY_t^{(2)}] = [dZ_t^{(2)}, dY_t^{(1)}] = 0$, then, symmetric SDF-FX pass-through, i.e. $z_t^{(1)} = z_t^{(2)} = z_t$, implies a tight relationship between the conditional covariance of exchange rate movements and the conditional covariance of SDF differentials:*

$$[ds_t^{1/0}, ds_t^{2/0}] = z_t[dm_t^{(0)} - dm_t^{(1)}, dm_t^{(0)} - dm_t^{(2)}].$$

*(a1) If we further assume $[dZ_t^{(1)}, dY_t^{(1)}] = [dZ_t^{(2)}, dY_t^{(2)}] = 0$, there is a starker restriction in correlation form:*

$$\frac{[ds_t^{1/0}, ds_t^{2/0}]}{\sqrt{[ds_t^{1/0}, ds_t^{1/0}]}\sqrt{[ds_t^{2/0}, ds_t^{2/0}]}} = \frac{[dm_t^{(0)} - dm_t^{(1)}, dm_t^{(0)} - dm_t^{(2)}]}{\sqrt{[dm_t^{(0)} - dm_t^{(1)}, dm_t^{(0)} - dm_t^{(1)}]}\sqrt{[dm_t^{(0)} - dm_t^{(2)}, dm_t^{(0)} - dm_t^{(2)}]}}.$$

*(b) If the IM shock $dY_t^{(i)}$ in a country is correlated with the SDF shock in another country, i.e., $[dZ_t^{(1)}, dY_t^{(2)}] \neq 0$ without loss of generality, then, we can similarly derive*

$$ds^{2/0} = \tilde{x}_t^{(2)}dt - \frac{y_t^{(2)}\kappa_t}{\sigma_t^{(1)}\sqrt{1-\rho_{0,1,t}^2}}dm_t^{(1)} + \tilde{z}_t^{(0)}dm_t^{(0)} - z_t^{(2)}dm_t^{(2)} + \tilde{y}_t^{(2)}d\tilde{Y}_t^{(2)},$$

*which indicates that country $1$'s SDF shock affects the bilateral exchange rate movement between countries $0$ and $2$.*

The proof and the definition of the relevant parameters is provided in a separate Online Appendix. This proposition replicates the results in the one-period, discrete-time model: part (a) corresponds to Case II above, part (a1) corresponds to Case I, and part (b) corresponds to Case III. In fact, the continuous-time model bears strong similarity to the discrete-time model, so that the derivation is almost identical. So, the results about asymmetric SDF-FX pass-through also hold in this more general set-up with time-varying drifts, volatilities, and correlations.

# 5 Conclusion

In this paper, I develop a framework that attributes exchange rate movements in incomplete markets to SDF and non-SDF shocks. This framework requires freely tradable domestic and foreign risk-free bonds, but is otherwise flexible to allow general characterizations of exchange rate dynamics in incomplete markets. Under this framework, I show that symmetric SDF-FX pass-through imposes a tight relation between exchange rate correlation and SDF differential correlation, which is unlikely to hold in the data. Therefore, asymmetry in SDF-FX pass-through is a general feature of incomplete markets, which propagates a country's SDF shock to bilateral exchange rate movements between foreign countries. These results offer a new way of understanding the disconnect and comovements in exchange rate dynamics.

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# Appendix

This appendix contains proof for the main results in the paper and some empirical tables that demonstrate the underlying data. There is an online appendix that contains proof for the more general continuous-time setting, which is available on the author's personal website.

# A Proof

## A.1 Lemma 1

The Euler equations for domestic risk-free bonds imply

$$r^{(0)} = \mu_0 - \frac{1}{2}(\sigma_{0,0}^2), \quad r^{(1)} = \mu_1 - \frac{1}{2}(\sigma_{1,0}^2 + \sigma_{1,1}^2).$$

The Euler equations for foreign risk-free bonds imply

$$\begin{aligned}
1 &= \mathbb{E}[\exp(\Delta m^{(0)} - \Delta s^{1/0} + r^{(1)})] \\
&= \exp(\mathbb{E}[\Delta m^{(0)} - \Delta s^{1/0} + r^{(1)}] + \frac{1}{2}(\sigma_{0,0}^2 + var(\Delta s^{1/0}) - 2cov[\Delta m^{(0)}, \Delta s^{1/0}]) \\
0 &= -\mu_0 - \mathbb{E}[\Delta s^{1/0}] + r^{(1)} + \frac{1}{2}\sigma_{0,0}^2 + \frac{1}{2}var(\Delta s^{1/0}) - cov[\Delta m^{(0)}, \Delta s^{1/0}] \\
&= -\mathbb{E}[\Delta s^{1/0}] + r^{(1)} - r^{(0)} + \frac{1}{2}var(\Delta s^{1/0}) - cov[\Delta m^{(0)}, \Delta s^{1/0}]
\end{aligned}$$

and similarly

$$0 = \mathbb{E}[\Delta s^{1/0}] + r^{(0)} - r^{(1)} + \frac{1}{2}var(\Delta s^{1/0}) + cov[\Delta m^{(1)}, \Delta s^{1/0}]$$

Take the sum and plug in the first set of first-order conditions,

$$0 = var(\Delta s^{1/0}) - cov[\Delta m^{(0)} - \Delta m^{(1)}, \Delta s^{1/0}] \tag{A.1}$$

Plug in

$$\Delta s^{1/0} = x^{(1)} + z^{(1)}(\Delta m^{(0)} - \Delta m^{(1)}) + y^{(1)}\varepsilon_y^{(1)}; \tag{A.2}$$

then, Eq. (A.1) implies an equilibrium restriction on the relationship between $z^{(i)}$ and $y^{(i)}$,

$$\begin{aligned}
0 &= (z^{(1)})^2 var(\Delta m^{(0)} - \Delta m^{(1)}) + 2z^{(1)} cov[\Delta m^{(0)} - \Delta m^{(1)}, y^{(1)}\varepsilon_y^{(1)}] + (y^{(1)})^2 \\
&- cov[\Delta m^{(0)} - \Delta m^{(1)}, z^{(1)}(\Delta m^{(0)} - \Delta m^{(1)}) + y^{(1)}\varepsilon_y^{(1)}] \\
0 &= (y^{(1)})^2 + ((z^{(1)})^2 - z^{(1)}) var(\Delta m^{(0)} - \Delta m^{(1)}) \qquad \text{(A.3)} \\
&+ (2z^{(1)} - 1)y^{(1)} cov[\Delta m^{(0)} - \Delta m^{(1)}, \varepsilon_y^{(1)}] \\
&= (y^{(1)})^2 + ((z^{(1)})^2 - z^{(1)})((\sigma_{0,0} - \sigma_{1,0})^2 + \sigma_{1,1}^2) + (2z^{(1)} - 1)y^{(1)} cov[\Delta m^{(0)} - \Delta m^{(1)}, \varepsilon_y^{(1)}]
\end{aligned}$$

which implies the solution for $y^{(1)}$, noting that $cov[\Delta m^{(0)} - \Delta m^{(1)}, \varepsilon_y^{(1)}] = -cov[\Delta m^{(1)}, \varepsilon_y^{(1)}] = -\sigma_{1,1} corr(\varepsilon_m^{(1)}, \varepsilon_y^{(1)})$.

Next, based on the FX dynamics (2),

$$\begin{aligned}
\mathbb{E}[\Delta s^{1/0}] &= x^{(1)} + z^{(1)}(\mu_1 - \mu_0) \\
var(\Delta s^{1/0}) &= var(z^{(1)}(\sigma_{0,0}\varepsilon_m^{(0)} - \sigma_{1,0}\varepsilon_m^{(0)} - \sigma_{1,1}\varepsilon_m^{(1)}) + \eta_{1,1}\varepsilon_m^{(1)} + z^{(1)}\sigma_{1,1}\varepsilon_m^{(1)} + \zeta_{1,1}\varepsilon_s^{(1)}) \\
&= var(z^{(1)}(\sigma_{0,0} - \sigma_{1,0})\varepsilon_m^{(0)} + \eta_{1,1}\varepsilon_m^{(1)} + \zeta_{1,1}\varepsilon_s^{(1)}) \\
&= (z^{(1)})^2(\sigma_{0,0} - \sigma_{1,0})^2 + \eta_{1,1}^2 + \zeta_{1,1}^2
\end{aligned}$$

and

$$\begin{aligned}
cov[\Delta m^{(0)}, \Delta s^{1/0}] &= cov[\sigma_{0,0}\varepsilon_m^{(0)}, z^{(1)}(\sigma_{0,0}\varepsilon_m^{(0)} - \sigma_{1,0}\varepsilon_m^{(0)} - \sigma_{1,1}\varepsilon_m^{(1)}) + (\eta_{1,1} + z^{(1)}\sigma_{1,1})\varepsilon_m^{(1)} + \zeta_{1,1}\varepsilon_s^{(1)}] \\
&= z^{(1)}\sigma_{0,0}(\sigma_{0,0} - \sigma_{1,0})
\end{aligned}$$

Then, we can recover $x^{(1)}$ from the cross-country Euler equations:

$$\begin{aligned}
x^{(1)} &= -z^{(1)}(\mu_1 - \mu_0) + r^{(1)} - r^{(0)} + \frac{1}{2}var(\Delta s^{1/0}) - cov[\Delta m^{(0)}, \Delta s^{1/0}] \\
&= (1 - z^{(1)})(\mu_1 - \mu_0) - \frac{1}{2}(\sigma_{1,0}^2 + \sigma_{1,1}^2 - \sigma_{0,0}^2) + \frac{1}{2}((z^{(1)})^2(\sigma_{0,0} - \sigma_{1,0})^2 + \eta_{1,1}^2 + \zeta_{1,1}^2) \\
&- z^{(1)}\sigma_{0,0}(\sigma_{0,0} - \sigma_{1,0})
\end{aligned}$$

## A.2 Proposition 1 and 2

Under symmetric pass-through, i.e., $z^{(1)} = z^{(2)} = z$ and following Eq. (A.3) in the proof for Lemma 1, the cross-country Euler equations for country pairs $(0, 1)$ and $(0, 2)$ imply

$$0 = (z^2 - z)[(\sigma_{0,0} - \sigma_{1,0})^2 + \sigma_{1,1}^2] + (y^{(1)})^2 + (1 - 2z)y^{(1)}cov(\Delta m^{(1)}, \varepsilon_y^{(1)}) \tag{A.4}$$

$$0 = (z^2 - z)[(\sigma_{0,0} - \sigma_{2,0})^2 + \sigma_{2,1}^2 + \sigma_{2,2}^2] + (y^{(2)})^2 + (1 - 2z)y^{(2)}cov(\Delta m^{(2)}, \varepsilon_y^{(2)}) \tag{A.5}$$

Similar Euler equations apply to the country pair $(1, 2)$, which imply

$$0 = var(\Delta s^{1/2}) - cov[\Delta m^{(2)} - \Delta m^{(1)}, \Delta s^{1/2}] \tag{A.6}$$

That is,

$$\begin{aligned}
0 &= (z^2 - z)(\sigma_{2,0} - \sigma_{1,0})^2 + (z^2 - z)(\sigma_{1,1} - \sigma_{2,1})^2 + (z^2 - z)\sigma_{2,2}^2 \\
&+ (1 - 2z)[y^{(1)}\sigma_{1,1}cov(\varepsilon_m^{(1)}, \varepsilon_y^{(1)}) + y^{(2)}\sigma_{2,1}cov(\varepsilon_m^{(1)}, \varepsilon_y^{(2)}) + y^{(2)}\sigma_{2,2}cov(\varepsilon_m^{(2)}, \varepsilon_y^{(2)})] \\
&+ (y^{(1)})^2 + (y^{(2)})^2 - 2y^{(1)}y^{(2)}cov(\varepsilon_y^{(1)}, \varepsilon_y^{(2)})
\end{aligned}$$

Plug in Assumption 2 and Eq. (A.4) and (A.5), this implies

$$y^{(1)}y^{(2)}cov(\varepsilon_y^{(1)}, \varepsilon_y^{(2)}) = -(z^2 - z)\left((\sigma_{0,0} - \sigma_{1,0})(\sigma_{0,0} - \sigma_{2,0}) + \sigma_{1,1}\sigma_{2,1}\right)$$

Then, by Assumption 2 again,

$$\begin{aligned} cov(\Delta s^{1/0}, \Delta s^{2/0}) &= cov(z(\Delta m^{(0)} - \Delta m^{(1)}) + y^{(1)}\varepsilon_y^{(1)}, z(\Delta m^{(0)} - \Delta m^{(2)}) + y^{(2)}\varepsilon_y^{(2)}) \\ &= z^2 cov(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(0)} - \Delta m^{(2)}) + y^{(1)}y^{(2)}cov(\varepsilon_y^{(1)}, \varepsilon_y^{(2)}) \\ &= z\left((\sigma_{0,0} - \sigma_{0,1})(\sigma_{0,0} - \sigma_{0,2}) + \sigma_{1,1}\sigma_{2,1}\right) \\ &= z \cdot cov(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(0)} - \Delta m^{(2)}) \end{aligned}$$

Lastly, I also use Eq. (A.4) and (A.5) to solve for the FX variance:

$$\begin{aligned} var(\Delta s^{1/0}) &= z^2\left((\sigma_{0,0} - \sigma_{1,0})^2 + \sigma_{1,1}^2\right) + (y^{(1)})^2 + (-2z)y^{(1)}\sigma_{1,1}cov(\varepsilon_m^{(1)}, \varepsilon_y^{(1)}) \\ &= z\left((\sigma_{0,0} - \sigma_{1,0})^2 + \sigma_{1,1}^2\right) - y^{(1)}cov(\Delta m^{(1)}, \varepsilon_y^{(1)}) \\ var(\Delta s^{2/0}) &= z\left((\sigma_{0,0} - \sigma_{2,0})^2 + \sigma_{2,1}^2 + \sigma_{2,2}^2\right) - y^{(2)}cov(\Delta m^{(2)}, \varepsilon_y^{(2)}); \end{aligned}$$

then,

$$\begin{aligned} & corr(\Delta s^{1/0}, \Delta s^{2/0}) \\ = & \frac{cov(\Delta s^{1/0}, \Delta s^{2/0})}{\sqrt{var(\Delta s^{1/0})}\sqrt{var(\Delta s^{2/0})}} \\ = & \frac{z \cdot cov(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(0)} - \Delta m^{(2)})}{\sqrt{z \cdot var(\Delta m^{(0)} - \Delta m^{(1)}) + y^{(1)}cov(\Delta m^{(1)}, \varepsilon_y^{(1)})}\sqrt{z \cdot var(\Delta m^{(0)} - \Delta m^{(2)}) + y^{(2)}cov(\Delta m^{(2)}, \varepsilon_y^{(2)})}} \\ = & \frac{cov(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(0)} - \Delta m^{(2)})}{\sqrt{var(\Delta m^{(0)} - \Delta m^{(1)}) + y^{(1)}cov(\Delta m^{(1)}, \varepsilon_y^{(1)})/z}\sqrt{var(\Delta m^{(0)} - \Delta m^{(2)}) + y^{(2)}cov(\Delta m^{(2)}, \varepsilon_y^{(2)})/z}} \end{aligned}$$

With Assumption 1, the correlation formula can be simplified as

$$corr(\Delta s^{1/0}, \Delta s^{2/0}) = corr(\Delta m^{(0)} - \Delta m^{(1)}, \Delta m^{(0)} - \Delta m^{(2)})$$

# B Data Sources and Empirical Tables

The exchange rate data are from Datastream. The consumption data are from World Bank Development indicators. The equity price-dividend data are from MSCI.

## **Table A.1:** Correlation Matrix of Exchange Rates and Economic Quantities

This table reports the correlation matrix of different bilateral variables, between the pair of country $i$ and USA and the pair of country $j$ and USA. Exchange rate data are annual, 1988—2020. Consumption data are constant price in local currency terms, and annual, 1970—2020. Price-dividend ratio data are constructed from the cum- and ex-dividend returns of the MSCI country-level aggregate stock portfolios in local currency units, annual, 1990—2016.

*Panel (a) Correlation of bilateral log exchange rate movements* $\Delta s_t^{i/USA}$

| | AUS | CAN | CHE | DEU | DNK | GBR | JPN | NOR | NZL | SWE |
|---|---|---|---|---|---|---|---|---|---|---|
| AUS | | | | | | | | | | |
| CAN | 0.78 | | | | | | | | | |
| CHE | 0.35 | 0.19 | | | | | | | | |
| DEU | 0.53 | 0.44 | 0.82 | | | | | | | |
| DNK | 0.53 | 0.44 | 0.82 | 1.00 | | | | | | |
| GBR | 0.64 | 0.54 | 0.42 | 0.58 | 0.58 | | | | | |
| JPN | 0.34 | 0.03 | 0.47 | 0.30 | 0.29 | -0.02 | | | | |
| NOR | 0.73 | 0.68 | 0.61 | 0.75 | 0.75 | 0.72 | 0.18 | | | |
| NZL | 0.89 | 0.71 | 0.41 | 0.58 | 0.58 | 0.67 | 0.17 | 0.79 | | |
| SWE | 0.73 | 0.60 | 0.72 | 0.84 | 0.84 | 0.77 | 0.24 | 0.84 | 0.70 | |

*Panel (b) Correlation of log consumption growth differential* $\Delta c_t^{(i)} - \Delta c_t^{(USA)}$

| | AUS | CAN | CHE | DEU | DNK | GBR | JPN | NOR | NZL | SWE |
|---|---|---|---|---|---|---|---|---|---|---|
| AUS | | | | | | | | | | |
| CAN | 0.55 | | | | | | | | | |
| CHE | 0.44 | 0.46 | | | | | | | | |
| DEU | 0.42 | 0.48 | 0.62 | | | | | | | |
| DNK | 0.27 | 0.14 | 0.14 | 0.30 | | | | | | |
| GBR | -0.03 | 0.29 | 0.13 | 0.03 | -0.18 | | | | | |
| JPN | 0.23 | 0.30 | 0.37 | 0.60 | 0.12 | 0.20 | | | | |
| NOR | 0.47 | 0.50 | 0.28 | 0.44 | 0.56 | -0.05 | 0.20 | | | |
| NZL | 0.43 | 0.36 | 0.28 | 0.03 | 0.15 | 0.12 | -0.08 | -0.00 | | |
| SWE | 0.63 | 0.61 | 0.50 | 0.49 | 0.32 | 0.10 | 0.18 | 0.45 | 0.33 | |

*Panel (c) Correlation of log stock dividend-price ratio change* $\Delta dp_t^{(i)} - \Delta dp_t^{(USA)}$

| | AUS | CAN | CHE | DEU | DNK | GBR | JPN | NOR | NZL | SWE |
|---|---|---|---|---|---|---|---|---|---|---|
| AUS | | | | | | | | | | |
| CAN | -0.05 | | | | | | | | | |
| CHE | 0.37 | -0.16 | | | | | | | | |
| DEU | 0.03 | 0.20 | 0.56 | | | | | | | |
| DNK | -0.10 | 0.26 | 0.08 | 0.44 | | | | | | |
| GBR | 0.52 | 0.07 | 0.47 | 0.33 | 0.25 | | | | | |
| JPN | 0.06 | 0.26 | 0.18 | 0.22 | 0.19 | 0.29 | | | | |
| NOR | 0.15 | -0.03 | 0.49 | 0.45 | 0.49 | 0.25 | 0.52 | | | |
| NZL | 0.48 | -0.24 | 0.26 | -0.11 | -0.23 | 0.36 | 0.12 | 0.28 | | |
| SWE | 0.13 | 0.35 | 0.22 | 0.52 | 0.67 | 0.56 | 0.56 | 0.50 | 0.01 | |

## **Table A.2:** Covariance Ratio Test

This table reports the ratio between the covariance between bilateral exchange rate movements and the covariance between bilateral growth differentials in economic quantities. Exchange rate data are annual, 1988—2020. Consumption data are constant price in local currency terms, and annual, 1970—2020. Price-dividend ratio data are constructed from the cum- and ex-dividend returns of the MSCI country-level aggregate stock portfolios in local currency units, annual, 1990—2016.

*Panel (a) Ratio between FX covariance and consumption growth differential covariance*

| | AUS | CAN | CHE | DEU | DNK | GBR | JPN | NOR | NZL | SWE |
|---|---|---|---|---|---|---|---|---|---|---|
| AUS | | | | | | | | | | |
| CAN | 61.35 | | | | | | | | | |
| CHE | 36.23 | 14.76 | | | | | | | | |
| DEU | 54.24 | 30.37 | 46.51 | | | | | | | |
| DNK | 91.49 | 111.74 | 222.18 | 120.86 | | | | | | |
| GBR | -1032.36 | 60.62 | 109.39 | 563.28 | -111.44 | | | | | |
| JPN | 56.36 | 3.09 | 39.46 | 14.79 | 79.37 | -2.78 | | | | |
| NOR | 71.32 | 48.80 | 80.87 | 60.78 | 52.19 | -456.93 | 27.85 | | | |
| NZL | 75.33 | 55.79 | 43.48 | 490.15 | 115.07 | 157.05 | -52.07 | -9243.45 | | |
| SWE | 61.59 | 40.51 | 62.61 | 71.07 | 119.16 | 319.20 | 49.74 | 82.65 | 74.28 | |

*Panel (b) Ratio between FX covariance and stock dividend-price ratio differential covariance*

| | AUS | CAN | CHE | DEU | DNK | GBR | JPN | NOR | NZL | SWE |
|---|---|---|---|---|---|---|---|---|---|---|
| AUS | | | | | | | | | | |
| CAN | -14.87 | | | | | | | | | |
| CHE | 0.57 | -0.57 | | | | | | | | |
| DEU | 10.08 | 1.07 | 0.44 | | | | | | | |
| DNK | -2.61 | 0.63 | 2.32 | 0.54 | | | | | | |
| GBR | 1.97 | 10.28 | 0.70 | 1.41 | 1.44 | | | | | |
| JPN | 4.04 | 0.07 | 0.94 | 0.49 | 0.42 | -0.06 | | | | |
| NOR | 2.94 | -12.23 | 0.36 | 0.49 | 0.35 | 2.22 | 0.12 | | | |
| NZL | 1.34 | -1.67 | 0.55 | -1.98 | -0.70 | 1.75 | 0.62 | 0.98 | | |
| SWE | 3.07 | 0.75 | 0.86 | 0.44 | 0.27 | 1.00 | 0.14 | 0.44 | 26.17 | |