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Laurent Bouton
Aniol Llorente-Saguer
Antonin Macé
Adam Meirowitz
Shaoting Pi
Dimitrios Xefteris

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ABSTRACT

We study how investors' beliefs about firm value, and hence their willingness to trade, respond to the release of public information. We consider a standard rational expectations model with homogeneous investors (common preferences, priors, and opinions) with the novelty that information, both public and private, pertains to the decisions the firm will make in the future and whether it is value-enhancing (what we refer to as the path-forward), instead of being directly about the value of the firm. Our analysis shows that, counter to the received wisdom, standard models can explain the well-documented pattern of increased in disagreement and trade volume after public announcements. Two economic insights emerge. First, investors holding different information about the path-forward of the firm may nonetheless have the same assessment of the firm's value. The release of public information may then reinforce or contradict interim beliefs about the path-forward, and hence lead to divergence in investors' assessments of the firm's value and then an increase in trade volume. Second, investors who participate in shareholders' meetings may have an informational advantage relative to investors that observe only public information about the meeting. The former group know both how they voted and their private information before voting, while others only know the total vote tally. The exploitation of that advantage leads to a surge in trade after public disclosure of meeting outcomes.

Laurent Bouton
Georgetown University
Department of Economics
37th & O Streets, NW
Washington,DC 20057
and CEPR
and also NBER
boutonllj@gmail.com

Aniol Llorente-Saguer
School of Economics and Finance
Queen Mary, University of London
Mile End Road, London E1 4NS
United Kingdom
a.llorente-saguer@qmul.ac.uk

Antonin Macé
Paris School of Economics
48 boulevard Jourdan
75014 Paris
antonin.mace@gmail.com

Adam Meirowitz
Yale University
Department of Political Science
115 Prospect St.
Rosenkranz Hall
New Haven, CT 06520
adam.meirowitz@yale.edu

Shaoting Pi
Iowa State University
Department of Finance
3127 Gerdin Business Bldg.
1200, 2167 Union Dr.
Ames, IA 50011
shaoting@iastate.edu

Dimitrios Xefteris
University of Cyprus
Department of Economics
xefteris.dimitrios@ucy.ac.cy

1 Introduction

It is a well-established fact that public announcements, such as those related to a firm's earnings or decisions made at shareholders' meetings, have the potential to significantly increase trading volumes on the stock markets.[1] This empirical regularity is perceived as a challenge to the standard rational expectations models with homogeneous agents (who share common preferences and priors): A consensus has emerged in the literature that standard models such as representative agent models (e.g., Merton 1973 and Lucas 1978) and differential information models (e.g., Grossman and Stiglitz 1976, 1980, Diamond and Verrecchia 1981, Kyle 1985, and Verrecchia 1991a, 1991b), are unable to explain the observed patterns of trade after public announcements (see, e.g., the discussions in Wang 1994, Kandel and Pearson 1995, Banerjee and Kremer 2010, and Kondor 2012). As a testimony of that consensus, the literature offers alternative, non-standard, models that could explain the empirical regularity. These models introduce heterogeneous preferences (e.g., due to different trading horizons as in Kondor 2012 and Cespa et al. 2015, or to different risk tolerance as in Kim and Verrecchia 1991), heterogeneous information quality (Pasquariello and Vega 2007, and Banerjee and Green 2015), heterogeneous opinions on how to interpret information (Varian 1989, Harris and Raviv 1993, Kandel and Pearson 1995, Banerjee and Kremer 2010, Bollerslev et al. 2018, and Li et al. 2022), or behavioral frictions (e.g. overconfidence as in Odean 1998 and Scheinkman and Xiong 2003), in order to generate increases in disagreement among agents, and hence trade, after public announcements.[2]

In this paper, we challenge that consensus and show that, in a standard rational expectations model with homogeneous agents, public announcements can increase disagreement among investors about the firm's valuation, and then their willingness to trade. The key novelty is that, in our model, public and private information is not directly informative about the value of the firm. Instead, information pertains to what we refer to as the *path-forward* of the firm (i.e., which decision the firm will make in the future and whether it is value-enhancing), and beliefs about that path-forward shape investors' beliefs about the value of the firm.[3] It then follows that, while public announcements necessarily reduce disagreement among investors about the path-forward, they can nonetheless increase disagreement about

[1]Beaver (1967) and Bamber (1985), Kim and Verrechhia 1991, 1994, Kandel and Pearson (1995), Krinsky and Lee (1996), Fleming and Remolona (1999), Green (2004), Chae (2005), Evans and Lyons (2008), Love and Payne (2008) Bollerslev et al. (2018), Li et al. (2022), Ben-Rephael et al. (2022)

[2]An alternative explanation for these trade patterns that relies on standard models with homogeneous agents is that public announcements actually comprise a combination of public and private signals (as in Kim and Verrecchia 1997 and Evans and Lyons 2001).

[3]Bond and Eraslan (2010) also consider a model with asymmetric information in which firm's choice affects its value and show that trading can then generate information to be used by the eventual owner. They thus also have the key ingredient in our conception of the path-forward.

the value of the firm, and then lead to an increase in trade volume.

We develop a standard rational expectations model with homogeneous agents who have common preferences, priors, and opinions, and hence interpret information in the same way. Each investor is endowed with a private signal, following which they all receive a public signal. Then, based on that public signal, the firm makes a decision which can be value-increasing or decreasing. The key novelty is that, given the existence of that payoff-relevant decision, the private and the public signals provide information both about which decision the firm will make and about the quality of that decision (i.e., the path-forward of the firm). We consider two different types for the public signal. In the first, the public signal is a public disclosure of information made either by the firm itself (e.g., earnings announcements) or by another entity (e.g., FOMC decisions). In the second, the public signal is the outcome of a vote at the shareholders' meeting (e.g., about a merger or an board member election). In both cases, investors understand that the public signal is informative about the future decision of the firm. It can be either because it directly determines it, as in the case of a shareholders' vote, or because it affects the decision of the firm's management (e.g., Initial Public Offerings).

Two key economic intuitions drive our findings. First, even when investors possess different interim beliefs[4] about the optimal course of action for the firm, they may still hold similar interim beliefs about the firm's value if they expect the firm's decision to be guided by the public signal.[5] This is because investors with different information about the optimal course of action for the firm will also have different beliefs about which course of action will be chosen. As an example of this, consider investors that anticipate management will use future information to make a value maximizing decision. If they have different ex-ante beliefs about which decision is best they will also have different ex-ante beliefs about the likelihood that future informative signals will favor each course of action and they will thus have different ex-ante beliefs about which decision management will make. They may not however disagree very much about the likelihood that management will ultimately make the right decision. The revelation of a public signal will alter both their beliefs about the optimal course of action and which course of action will be taken. A public signal favoring one course of action will lead an investor that already thought that course of action was best (and likely to be chosen) to be even more optimistic. By contrast, it will lead an investor who thought that course of action was sub-optimal to now believe both that it is better than they thought, and it is more likely to be chosen. For that latter investor, this creates a con-

[4]That is, their beliefs after receiving their private signals but not the public one.

[5]In the terms of Mongin (2016), we may say that the investors then exhibit spurious unanimity about firm's value.

flict between their earlier information and the new information, leading to reduced optimism about the future course of action of the firm relative to an investors for whom the public signal is added confirmation that their initial belief was correct. Accordingly, while public information makes investors' beliefs about what course of action is best more congruent, it may not lead to a convergence of their beliefs about the likelihood that the correct course of action will be chosen. This divergence in beliefs about the path-forward is what causes the divergence in valuations of the firm, and hence increase the willingness to trade.

Second, another relevant source of public information is shareholders meetings. In that case, the public signal stems from collective choices made by the investors. We identify an additional force that may drive apart the beliefs of the shareholders that were involved in the vote (for others, the effect on beliefs is only driven by the force described in the previous paragraph). Here, we draw on the existing literature showing that we should not expect shareholders to fully reveal the information that they possess at those meetings. (Austen-Smith and Banks 1996; Feddersen and Pesendorfer 1996; Myerson 1998; Maug and Rydqvist 2009; Meirowitz and Pi 2022). Hence, even the vote tally does not reveal fully the actual support in favor and against the decision made at the meeting. As a consequence, the shareholders themselves will have heterogeneous beliefs about the likelihood that the alternative that won the vote is in fact optimal. We find that the public disclosure of the outcome of the vote may actually increase the level of disagreement among shareholders about firm's value, and thus trading volume might increase as well. This happens when the vote tally is not too lopsided, and there is then substantial uncertainty about whether the decision is in fact optimal. Then, private information has a large impact on shareholders' beliefs, and hence on how they value the firm.

After developing these intuitions through the study of canonical and purposely sparse models, we extract predictions about the dynamics of market action. We then connect our findings to existing research on trading behavior following public announcements. Our analysis justifies a more optimistic interpretation of standard models with homogeneous investors and recently articulated empirical patterns. Although there may be compelling reasons to move beyond these models in the future, we conclude that previous work may have been too hasty in dismissing this classic approach.

In addition to challenging the consensus about the inability of standard models to explain empirical patterns of trade following public announcements, our findings bring to light two critical nuances that the previous literature has overlooked. First, it is essential to distinguish between disagreement about the path-forward of the firm and disagreement about its value. Investors who disagree about the former may still agree about the latter, and this can shape the impact of public announcements on trade volumes. Second, it is crucial to differentiate between public announcements on forward-looking decisions and those on backward-looking decisions. Our findings only apply to the former type of announcements. In our model, public announcements relevant to the valuation of the firm only through previously enacted management decisions can only reduce disagreement among investors, leading to lower trading volumes.

2 Model

We have a finite set of shareholders, N, with cardinality n, assumed to be odd. At any point in time, shareholder $i \in N$ estimates the value of the firm, x, using all the information available to her. The real value of the firm x depends on the state of the world $\omega \in \Omega = \{\alpha, \beta\}$ and on a decision $k \in K = \{A, B\}$. We assume that:

$$\begin{aligned} x(A, \alpha) &= x(B, \beta) = 1, \\ x(B, \alpha) &= x(A, \beta) = 0. \end{aligned} \tag{1}$$

The model has four discrete stages:

First stage. Nature draws the state of the world from Ω with equal probability. The shareholders do not observe the state, but they are aware of the state generating process.

Second stage. Each shareholder $i \in N$ receives an independent signal $s \in S = \{s_a, s_b\}$, drawn in the following manner: if the state is α then s_a is drawn with probability q_α, and s_b is drawn with probability $1 - q_\alpha$; while, if the state is β then s_a is drawn with probability q_β, and s_b is drawn with probability $1 - q_\beta$, with $q_\alpha > \frac{1}{2} > q_\beta$. The values of q_α and q_β are common knowledge.

A decision $k \in K = \{A, B\}$ is made either by command or by majority voting. If the decision is made by command,[6] the decision is affected by a public signal. We assume the public signal is realized in a different period (the third stage) from the decision-making period (the fourth stage). If the decision is made by majority voting, there is no third stage and the decision simply depends on the voting result (the fourth stage).

Third stage. A public signal $\hat{s} \in \hat{S} = \{\hat{s}_a, \hat{s}_b\}$ is drawn and revealed to the shareholders. If the state is α (β), then $\hat{s}_a$ ($\hat{s}_b$) is drawn with the commonly known probability $Q > \frac{1}{2}$, and $\hat{s}_b$ ($\hat{s}_a$) is drawn with probability $1 - Q$. In section 3.1 we relax the symmetry assumption and allow for state dependent conditional probabilities.

Fourth stage. If the decision is made by command, then it is A with probability $p > \frac{1}{2}$ and B with probability $1 - p$ if the public signal is $\hat{s}_a$, and B with probability p and A with

[6] This is meant to capture the case in which another actor, e.g., the manager, is in charge of the decision.

probability $1-p$ if the public signal is $\hat{s}_b$. If the decision is made by majority voting, then shareholders vote simultaneously either for A or B, and the alternative with the most votes wins. Abstention is not allowed. At the end, uncertainty about the state of the world is resolved, and the value of the firm is revealed.

We measure disagreement at a given period by the difference between the most optimistic and the most pessimistic belief about x. Disagreement at time t may depend on shareholders' public history H (public signal, decision, voting outcome) and private history $h = (h_i)_{i\in N}$ (private signals, individual voting decisions). Formally, we write

$$d^t(H,h) = \max_{i\in N} E[x \mid H, h_i] - \min_{i\in N} E[x \mid H, h_i].$$

When disagreement does not vary with private histories h, nor with the public history H, we simply write d^t for $d^t(H,h)$.

3 Decision Made by Command

If the decision is made by command and $p = 1$, then expectations about the value of the firm do not change between stages three and four, for every possible realization of the public and the private signals. This is true because the shareholders understand that the decision is fully determined by the public signal (A if $\hat{s}_a$ and B if $\hat{s}_b$).[7] Hence, when $p = 1$ the only interesting comparison for the evolution of disagreement is between the second and the third period. When $p \in (\frac{1}{2}, 1)$, the level of disagreement can change both between stage two and stage three, and between stage three and stage four.

We assume throughout that at least two shareholders have received different private signals—otherwise, $d^4 = d^3 = d^2 = 0$—and focus first on the second stage of the model, before the public signal is revealed.

Proposition 1. *After receiving their private signal, but before the public signal is revealed ($t = 2$), shareholders have the same expectation of the value of the firm even if they receive different private signals.*

The intuition behind Proposition 1 is the following. Before the public signal is revealed, shareholders with different private information disagree on the likelihood of each state of the world (because $\Pr(\omega = \alpha|s_a) \neq \Pr(\omega = \alpha|s_b)$). However, they do not disagree on the expected value of the firm x, because this is determined by the probability that the

[7]Of course, even in this case, shareholders who received different signals in stage two can hold very different beliefs.

commander makes the correct decision. And this probability is equal to $pQ+(1-p)(1-Q)$ in both states. Hence, the private signals play no role in determining expectations regarding the value of the firm, x, before the public signal is realized.

Example 1. *Assume that* $q_\alpha = 0.6$*,* $q_\beta = 0.1$*,* $Q = 0.75$ *and* $p = 0.9$*. Note that we use these values of the parameters for all the examples in the paper. Given these parameters, the shareholders' beliefs at the end of stage 2 about the state and the policy to be implemented are given by the following table:*

	$s = s_a$	$s = s_b$
$\Pr(\omega = \alpha, k = A \vert s)$	0.600	0.215
$\Pr(\omega = \alpha, k = B \vert s)$	0.257	0.092
$\Pr(\omega = \beta, k = A \vert s)$	0.043	0.208
$\Pr(\omega = \beta, k = B \vert s)$	0.100	0.485
$E[x \vert s]$	0.700	0.700

Shareholders who receive different signals have profound disagreements about the likelihood of different states and policies. But the expected value of the firm, which is equal to $\Pr(\omega = \alpha, k = A \vert s) + \Pr(\omega = \beta, k = B \vert s)$ *is the same for both types of shareholders.*

The following proposition focuses on disagreement after the public signal is revealed ($t = 3$) and after the decision is implemented ($t = 4$):

Proposition 2. *The public signal results in investors with different private signals disagreeing about the expected value of* x *at* $t = 3$ *(i.e.,* $d^3 > d^2$*). If* $p < 1$*, this decision exacerbates the disagreement further about the expected value of* x *at* $t = 4$ *(i.e.,* $d^4 > d^3$*).*

To explain the result, let us focus first on the evolution of disagreement between stages 2 and 3. Observe that if two shareholders agree on the expected value of the firm after they observe the public signal, they must have agreed on its expected value also in the previous stage. But the converse is not true. If shareholders receive different private signals, they have different expectations both about the state of the world and about the policy to be implemented: each shareholder believes that the state of the world likely matches with her own signal, and that the decision is likely to match the true state of the world (with probability $pQ + (1-p)(1-Q)$). However, since the relevant feature to form expectations about x is to match the state with the right policy, both type of shareholders attach the same value to x at the second stage.

Why does the public signal increase disagreement? Suppose, that shareholders observe the public signal $\hat{s}_a$. Shareholders then learn that: (i) state α is more likely than expected and (ii) policy A is more likely to be implemented than policy B. While shareholders still disagree on the state of the world ($Pr(\alpha|s_a, \hat{s}_a) > Pr(\alpha|s_b, \hat{s}_a)$), they agree that the likelihood of A being implemented is equal to p. This implies that a shareholder who received signal s_a (resp. s_b) is now more optimistic (resp. pessimistic) about the adequacy of the likely decision. This generates the divergence in beliefs about the value of the firm between the two types of shareholders.

Let us now turn our attention to the evolution of disagreement between the third and the fourth stage of the game, i.e., between the revelation of the public signal and the decision. If $p < 1$, there is still uncertainty about the policy implemented at the end of stage 3. This uncertainty brings the expectations of two shareholders who received opposite signals closer because two shareholders who received different private signals realize that both the policy they believe is best and the one they believe is worst have a chance to be implemented. The uncertainty about which policy will be implemented then decreases the expected valuation of a shareholder whose signal coincides with the public signal and increase the one of a shareholder whose signal does not coincide. When the policy is finally implemented, the policy-related uncertainty is resolved and hence expectations diverge even more.

We illustrate these intuitions by continuing our previous example.

Example 1 (continued)**.** *The next table shows how shareholders' beliefs about the state and the policy to be implemented change after the public signal $\hat{s}_a$ is revealed:*

	$s = s_a$	$s = s_b$
$\Pr(\omega = \alpha, k = A \| s, \hat{s}_a)$	0.853	0.514
$\Pr(\omega = \alpha, k = B \| s, \hat{s}_a)$	0.095	0.057
$\Pr(\omega = \beta, k = A \| s, \hat{s}_a)$	0.047	0.386
$\Pr(\omega = \beta, k = B \| s, \hat{s}_a)$	0.005	0.043
$E[x \| s, \hat{s}_a]$	0.858	0.557

After receiving the public signal $\hat{s}_a$, while shareholders still disagree about the state of the world ($Pr(\alpha|s_a, \hat{s}_a) = 0.947$ and $Pr(\alpha|s_b, \hat{s}_a) = 0.571$), they now agree that the likelihood of A being implemented is 0.9 (before the public signal, the beliefs that A would be implemented were 0.643 and 0.423, respectively). This generates the divergence in beliefs about the value of the firm: $d_3 = 0.301$.

The next table indicates the beliefs when a policy is implemented:

	$k=A$		$k=B$	
	$s=s_a$	$s=s_b$	$s=s_a$	$s=s_b$
$\Pr(\omega=\alpha, k=A \mid s, \hat{s}_a, k)$	0.947	0.571	–	–
$\Pr(\omega=\alpha, k=B \mid s, \hat{s}_a, k)$	–	–	0.947	0.571
$\Pr(\omega=\beta, k=A \mid s, \hat{s}_a, k)$	0.053	0.429	–	–
$\Pr(\omega=\beta, k=B \mid s, \hat{s}_a, k)$	–	–	0.053	0.429
$E[x \mid s, \hat{s}_a, k]$	0.947	0.571	0.053	0.429

When a policy is finally implemented, it reduces the uncertainty about the policy implemented further, and this generates even higher disagreement. See that independently of the commander's decision, the disagreement about the expected value of the firm is equal to $d_4 = 0.376 > d_3$.

3.1 Extension: Asymmetric Precision of the Public Signal

One might wonder whether our result that disagreement increases after a public signal is revealed (i.e., $d^3 > d^2$) is an artifact of the assumption that the public signal is "correct" with the same probability in either state of the world. Indeed, the finding that $d^2 = 0$ requires this assumption. But the result that $d^3 > d^2$ does not. To see this, assume now that if the state is α (resp. β), then $\hat{s}_a$ is drawn with probability $Q > \frac{1}{2}$ (resp. $1 - Q + \gamma$), and $\hat{s}_b$ is drawn with the complimentary probability, with $\gamma \in (Q-1, Q-\frac{1}{2})$. By continuity of $E[x|s]$ and $E[x|s, \hat{s}]$ in γ it follows that $\lim_{\gamma \to 0} d^2 = 0$ and $\lim_{\gamma \to 0} d^3 > \delta$, for some $\delta > 0$, and hence our main observation remains valid even when the public signal is asymmetric, provided that the asymmetry (i.e., the value of $|\gamma|$) is not too large. In fact, even if the asymmetry is large, there is at least one realization of the public signal such that $d^3 > d^2$. In other words, a weaker version of our result (i.e., there is a positive probability that disagreement will increase after the revelation of the public signal) is true independently of the degree of asymmetry in the private and the public signals.

4 Decision Made by Voting

Before discussing the details, it is important to draw a distinction between the findings on shareholder meetings and the findings from a study of other public events and announcements. Our first model (in Section 3) applies to both classes of analysis because a large fraction of potential traders are not involved in shareholder meetings and so to them the shareholder meeting is just a public source of information. But our second model (in Section

4) captures additional incentives and behavior that is germane to agents that are involved in both voting and trading.

In this section, we consider the case of a decision made through shareholder voting and examine whether expectations of shareholders who participated in the meeting might diverge after the vote tally is made public. If elections fully aggregated information, all shareholders would end up with the same beliefs regardless of their information at stage 2. Therefore, we focus on the relevant case in which voting is not fully revealing (sometimes called non-sincere voting).

In our setup, whether fully revealing voting is an equilibrium phenomenon depends on whether the signals differ in their informativeness (this is because the states of nature are equiprobable). Hence, we assume that $q_\alpha < 1 - q_\beta$, so that, at the voting stage, there is a mixed strategy equilibrium in which shareholders who receive signal s_a vote for A and shareholders who receive signal s_b mix between voting for A and voting B.[8]

To ease notation, we denote a vote in favor of A by $v_i = 1$ and a vote in favor of B by $v_i = 0$. The quantity $v = \sum_{i=0}^{n} v_i$ denotes the number of shareholders who voted for A. The quantity $v' = \sum_{j \in N-\{i\}} v_j$ denotes the number of shareholders except shareholder i herself who voted for A. We also denote by $m = Pr(v_i = 0 \mid s_b)$ the share of shareholders who vote for B among those who receive a private signal s_b in the mixed equilibrium.

The following proposition focuses on disagreement among shareholders before the vote:

Proposition 3. *Before the vote ($t = 2$), shareholders receiving different private signals have different expectations of the value of the firm (i.e., $d_2 > 0$).*

When the decision is made by voting, the probability of making the correct decision in state α is different than in state β. The reason is that voting involves the aggregation of private information, which has asymmetric informativeness in different states. In particular, since we have assumed $q_\alpha < 1 - q_\beta$, the probability of making the correct decision in state α is lower than that in state β. This is simply because shareholders' private signal is more informative in state β, which helps information aggregation. Thus, before voting, a shareholder with signal s_b has a higher expectation of firm value than a shareholder with signal s_a.

Note that this result is closely related to our discussion about decision by command that $d^2 > 0$ when the precision of the public signal is asymmetric. In that case, we also have that

[8] See that $q_\alpha < 1 - q_\beta$ is a necessary but not sufficient condition for having a mixed strategy equilibrium. If $1 - q_\beta$ is close to q_α, and n is not too large, sincere voting is an equilibrium and therefore full information aggregation is achieved. However, for any $q_\alpha \neq 1 - q_\beta$ there is an $\overline{n} < \infty$ such that for any $n > \overline{n}$ there is a mixed strategy equilibrium. The exact condition for a mixed strategy equilibrium to exist is $\log\left(\frac{q_\alpha}{q_\beta}\right) \geq \frac{n+1}{n-1} \log\left(\frac{1-q_\alpha}{1-q_\beta}\right)$. This is a standard result in the voting literature. See, e.g., Bouton et al. (2018) and references therein.

the decision is more likely to be correct in one state than the other, which drives a wedge in the valuations of shareholders who receive different signals.

We now illustrate this with a numerical example.

Example 2. *Consider the parameters* $n = 9$*,* $q_\alpha = 0.6$ *and* $q_\beta = 0.1$*. With these parameters there is a mixed strategy equilibrium in which shareholders who received signal* s_a *vote for A and shareholders who received signal* s_b *vote for A (B) with probability 0.156 (0.844). Taking into account their signal and equilibrium play, the shareholders' beliefs at the end of stage 2 about the state of the world and the policy to be implemented are given by the following table:*

	$s = s_a$	$s = s_b$
$\Pr(\omega = \alpha, k = A \mid s)$	0.778	0.279
$\Pr(\omega = \alpha, k = B \mid s)$	0.079	0.028
$\Pr(\omega = \beta, k = A \mid s)$	0.014	0.070
$\Pr(\omega = \beta, k = B \mid s)$	0.128	0.622
$E[x \mid s]$	0.906	0.902

Note that, while the valuations differ, the difference is small (0.004*, which compares to a maximum disagreement of* 1*). As we will see below, this is a feature common to all values of the parameters, with the difference vanishing when* n *grows large.*

The next proposition focuses on disagreement among shareholders after the vote:

Proposition 4. *After the vote (*$t = 4$*), shareholders who received different private signals but voted similarly disagree on the expected firm value (i.e.,* $d_4 > 0$*).*

To understand the intuition of this result, let us consider two shareholders who receive different signals but nonetheless vote for the same alternative, A. Given a vote tally v, these two shareholders draw exactly the same inference about the number of votes for A other shareholders cast. This means that, without taking into account their private signals, these two shareholders have the same post-voting beliefs about the state of the world. Yet, because they received different private signals, they end up with different post-voting beliefs.

Note that shareholders who receive the same signal but vote differently also disagree about the expected firm value after the vote. For instance, shareholders who received s_b and voted for B put a higher weight on $\omega = \alpha$ than those who received s_b but voted for A. This is because, for a given vote tally v, shareholders who voted for B observe a higher number of votes in favor of A among others' votes. Such shareholders have intermediate disagreement with respect to those described in the theorem, which is why we ignore them in the result's statement (remember that we focus on the maximal disagreement).

It remains to determine whether shareholders' disagreement increases or decreases after the vote. The answer actually depends on the realized vote tally v. We illustrate this point building on the same numerical example as above.

Example 2 (continued). *Recall that, before knowing the tally, the difference in beliefs about x between players with different private information was* 0.004. *How does knowing the tally affect this difference? If the total number of votes for A is between 1 and 8, the difference in beliefs between a shareholder who received the private signal s_a and another one who received signal s_b and voted for A* ***increases*** *(see column d_4 in the table below). See that the difference is particularly striking when the election is (close to) a tie ($v = 4$ or $v = 5$).*

v	k	$\Pr(\alpha \vert v)$	$E[x \vert s_a, v' = v-1]$	$E[x \vert s_b, v' = v-1]$	d_4
0	B	0.001	-	-	-
1	B	0.004	0.991	0.999	0.008
2	B	0.025	0.946	0.996	0.049
3	B	0.139	0.740	0.975	0.235
4	B	0.500	0.315	0.861	0.546
5	A	0.861	0.931	0.500	0.431
6	A	0.975	0.988	0.861	0.127
7	A	0.996	0.998	0.975	0.023
8	A	0.999	1.000	0.996	0.004
9	A	1.000	1.000	0.999	0.001

This result is the confluence of two effects. First, knowing the tally eliminates the uncertainty on which policy will be implemented: A is implemented if $v \geq 5$ and B is implemented otherwise. Second, because shareholders receiving signal s_b use a mixed strategy, there is asymmetric information between s_a and s_b shareholders. In particular, an s_b shareholder who has voted for A knows that there has been $v - 1$ votes for A plus one vote for A after a signal of s_b, while an s_a shareholder knows that there has been $v - 1$ votes for A plus one vote for A after a signal of s_a. Thus, the two types of shareholders infer a different overall number of signals s_a and s_b in the population, with the s_b shareholders inferring about one more signal s_b. And, as suggested by the first column of the table, for relatively close elections, one extra signal in favor of B has a very substantial impact on the belief about the state of nature.

By contrast, when the vote tally is $v = 9$, i.e., all shareholders vote for A, then the disagreement among two shareholders with different signals decreases after the vote. This is because, in that case, the vote tally becomes overwhelmingly informative that the state of nature is α. Even if the two types of shareholders still infer a different overall number of signals s_a and s_b in the population from the vote tally, they are both almost sure that the state is α and hence that the decision is correct.

The result in the example proves to have some generality. In particular, we can prove the following result when n grows large:

Proposition 5. *If the election outcome is close to a tie (i.e., $v = \frac{n+1}{2} + r$ with $r \in \mathbb{Z}$) then if n is sufficiently large, the disagreement after voting is larger than the disagreement before voting (i.e., $\lim_{n\to\infty} d^4 > \lim_{n\to\infty} d^2$).*

Proposition 5 proves that when n grows large, if the election is close to a tie, the disagreement among shareholders is necessarily larger after than before voting. Note that how large n needs to be depends on r. The intuition of this result is in two parts. First, when n grows large, the disagreement before the vote vanishes. This is because shareholders expect the outcome of the election to be the correct one (A in state α and B in state β) with a probability that tends to 1. Second, when n grows large, shareholders disagree after voting if the election is close to a tie. In that case, the outcome of the election does not provide much information to the shareholders about the state of nature. The difference in valuation for two shareholders who voted for A but received different signal is essentially the difference in beliefs about the state of nature given that there are $\frac{n-1}{2} + r$ signals s_a in the population or $\frac{n+1}{2} + r$ signals s_a in the population. This difference is positive even at the limit.

One might wonder whether the very substantial differences in beliefs in the event of a tie in the example above are an artifact of the small electorate, or if such a large difference can occur when n is large. As the next example shows, substantial disagreement can occur even when n is large. This is due to the s-shaped form of the beliefs about the state of nature given a number of signals s_a.

Example 2 (continued)**.** *Suppose now that $n \to \infty$. If the election outcome is close to a tie ($v = \frac{n+1}{2}$), the disagreement after voting is substantial:*[9]

$$\lim_{n\to\infty} d_4 = \left| \frac{1}{1 + \frac{q_\beta}{q_\alpha}\left(\frac{1-q_\alpha}{1-q_\beta}\right)} - \frac{1}{1 + \left(\frac{1-q_\alpha}{1-q_\beta}\right)^0} \right| = 0.43,$$

which compares to a maximum disagreement of 1.

Note however that the probability of an almost tied election (i..e, $v = \frac{n+1}{2}$) tends to zero when n grows large. Thus, Proposition 5 does not say anything about the expected disagreement after the vote.

[9] The formula to calculate $\lim_{n\to\infty} d_4$ is derived in the proof of Proposition 5. Here we consider the case with $r = 0$.

We can also prove that, when n grows large, the disagreement after the vote vanishes when the vote tally is unanimously in favor of A, i.e., $v = n$ (the proof is similar to the proof of Proposition 5). This generalizes another part of the example above.

5 Pricing Rules, Transaction Fee, and Trading

The previous sections discuss the divergence in expectations of firm value but do not explicitly discuss the implications for share trading. In this section, we model a financial market in which shareholders can trade their shares in the frameworks studied in Sections 3 and 4. We assume that the market is semi-efficient. Hence, the stock price of the firm reflects all public information.[10]

For the sake of expositional clarity, we work under the assumption that, when the decision is made by command, it is in line with the realization of the public signal with probability 1, i.e., $p = 1$. As discussed above, in that case, the expectations about firm value do not change between stages 3 and 4. It is thus unnecessary to distinguish between these two stages. In what follows, we thus allow for trade at two different stages: (i) stage 2, when shareholders only have their private signals, and (ii) stage 4, when the decision has been made (and the public signal has been revealed for the decision by command).

We denote the price at stage 2 by P_2 and the price at stage 4 by P_4. Then, $P_2 = Pr(k = \omega)$, which is the probability that the decision that will be made at stage 4 is correct. In the case of voting, this probability takes into account the voting equilibrium. At stage 4, $P_4(k, H) = Pr(k = \omega|H, k)$, which is the probability that the chosen policy k is correct conditional on the public information available H. In the case of decision by command, the public information available is the public signal and the decision by the manager. In the case of voting, H is the vote tally v and (from which the decision k can also be determined).

Shareholders who trade, buying or selling, need to pay a transaction fee, $\epsilon > 0$. So, at either stage, a shareholder wants to buy (sell) one share only if her expectation of the firm value is greater (smaller) than the stock price plus (minus) the transaction fee.

We remain somehow agnostic vis-a-vis the way the market clears. Most of our results indeed hold if there are liquidity traders that can help the market to clear (Kyle 1985's model), or if such traders do not exist and hence trading occurs only among shareholders explicitly modelled (No Additional Liquidity model, or NAL for short). We explicitly state when a result is sensitive to the market clearing assumption.

[10] Although there is a long debate over market efficiency (see the review by Malkiel 2003), recent empirical evidence shows that the stock market is efficient in quickly reflecting public events (e.g., Chordia and Miao 2020; Gregoire and Martineau 2022; Martineau 2021).

Decision Made by Command

For the case of decision by command, we can show that trade occurs only after the public signal is revealed (and the decision has been made):

Proposition 6. *There are positive transaction fees $\epsilon > 0$ such that (i) no shareholder wants to trade in stage 2, and (ii) both types of shareholders want to trade in stage 4.*

This result is a direct consequence of Propositions 1 and 2. The former shows that shareholders do not disagree in stage 2. Costly trade is thus not possible. The latter shows that shareholders disagree in stage 4. They thus want to trade even if there is a small transaction fee.

Decision Made by Voting

Similarly, for the case of decision by voting, we can show that trade occurs only after the vote:

Proposition 7. *Given an arbitrarily small transaction fee $\epsilon > 0$, there is a group size $\overline{n}$ such that, for any $n > \overline{n}$, (i) no shareholder wants to trade in stage 2, and (ii) there is trade in stage 4 after some vote tallies. Moreover, voting and stage 4 trading strategies are as follows:*

		Trading Strategy if...	
Signal	Vote	$k = A$	$k = B$
s_a	Vote for A	Buy	Sell
s_b	Vote for A	Sell	Buy
	Vote for B	Hold	Hold

The intuition is as follows. Before the vote (stage 2), the price P_2 reflects the fact that the voting decision in stage 4 is expected to be accurate (the probability the decision is correct tends to 1 when the number of shareholders grows large). The private signals lead to a difference in expectations of the firm value between shareholders who received different signals, but this difference is vanishingly small and hence not sufficient to compensate for the transaction fee. Thus, no shareholder wants to trade before the vote. After the vote (stage 4), the situation is different. As we have seen in Proposition 5, for some vote tallies, beliefs about the accuracy of the decision can be substantially affected by one extra signal in favor of A or B. Consider for instance a shareholder who received a signal s_b but voted for A. Compared to the public information revealed by the vote tally, this shareholder holds private information: she knows she voted insincerely for A while the market only attaches a probability to the vote being insincere. Thus, she has more confidence that the state is β than the market. When the transaction fee is sufficiently small, if the vote outcome is such that A (resp. B) is adopted, then she wants to sell (resp. buy), as her expectation about the value of the firm is strictly below (resp. above) the price. Finally, note that shareholders who vote for B have no valuable private information after the vote since, base on the vote tally, the market knows that they received a signal s_b. These shareholders thus never want to trade with a positive transaction fee.

Example 2 (continued)**.** *How much trade is there in the previous example? For what tallies is trading volume highest? Does voting against the winner increase the likelihood of selling? Assume that the transaction fee is* $\epsilon = 0.0035$*. In that case, no type wants to trade in stage 2 despite the difference in beliefs about* x *between players with different private information. However, there is trade with positive probability in stage 4. For the Kyle's market clearing model (i.e., all buying and selling orders on the market clear), trade occurs with probability* 80.95% *in state* $w = \alpha$ *and with probability* 91.58% *in state* $w = \beta$ *(overall probability of* 86.26%*). For the NAL model of market clearing (i.e., the short side of the market determines the volume of trade), trade occurs with probability* 24.9% *in* $w = \alpha$ *and with probability* 44.0% *in state* $w = \beta$ *(overall probability of* 34.4%*).*

While some discussion above highlighted belief spread at close tallies and thus might lead one to expect volume to be a single-peaked function of the vote tally, this is note the case. The relationship between these two quantities is indeed non-monotonic. The key forces at work here are that (i) in the equilibrium described in Proposition 7, a shareholder trades only if she voted for A (hence the number of shareholders willing to trade is increasing in the vote tally in favor of A), but (ii) for large vote tallies, the difference between the price and a shareholder's expectation of firm value vanishes (and hence the willingness to trade is not large enough to compensate for the trading fee). Note that this intuition is for the case in which all buying and selling orders on the market clear (i.e. the Kyle's model). If not, then the relationship between vote tally and volume of trade is also non-monotonic, but the reason is that there are too few sellers when the tally is large. Table 1 illustrate these results (columns 6 and 7).

v	P_2	k	P_4	$\|P_4 - P_2\|$	$E(Volume)$ NAL	$E(Volume)$ Kyle	$Pr(sell)$ $v_i \neq k$	$Pr(sell)$ $v_i = k$
0			0.999	0.096	0	0	-	0
1			0.996	0.092	0	1.000	0.418	0
2		B	0.975	0.071	0.478	2.000	0.428	0
3			0.861	0.042	0.663	3.000	0.484	0
4	0.903		0.500	0.403	0.789	4.000	0.661	0
5			0.861	0.042	0.608	5.000	0	0.162
6			0.975	0.071	0.599	6.000	0	0.107
7		A	0.996	0.092	0	0.674	0	0.096
8			0.999	0.096	0	0.756	0	0.095
9			1.000	0.097	0	0.000	-	0
Weighted Mean:			0.903	0.107	0.532	3.065	0.250	0.052

Table 1: Prices Before and After Voting And Expected Trading Volume. Columns 2-5 show price information and the winning alternative for each tally v. Columns 6 and 7 display the $E(volume)$. NAL captures expected number of orders that clear when there is no additional liquidity, and Kyle's market captures the expected number of orders that are placed (and would clear in the model of Kyle). Columns 8 and 9 show the odds of putting in an order to sell conditional on casting a vote against or in favor of the winner at the relevant tally.

Table 1 reveals several other interesting patterns. First, the expected price after voting in the market is U-shaped. Second, in terms of magnitudes, price change is rather low at all vote tallies but near ties. Third, we see that at low tallies dissenting votes are strongly predictive of offering to sell and, overall, the probability of selling when dissenting is higher than when not dissenting.

6 Discussion of Empirical Implications

In this section, we first connect our theoretical findings with extant empirical work. We then discuss new testable predictions highlighted by our model.

6.1 Empirical Connections

We now examine how the predictions of the model(s) match up with a relatively small number of documented patterns.

A series of papers have looked at how markets respond to public announcements and firm decisions. Taken together, the models analyzed in Sections 3 and 4 are capable of explaining several patterns that others have argued are inconsistent with standard rational expectations models with homogeneous agents.

Before discussing the details, it is important to draw a distinction between the findings of studies focused on shareholder meetings and the findings from studies of other public events and announcements. Our first model (in Section 3) applies to both classes of analyses because a large fraction of potential traders are not involved in shareholder meetings and so to them the shareholder meeting is just a public source of information. But our second model (in Section 4) captures additional incentives and behavior that is germane to shareholders that are involved in both voting and trading.

Using event studies, several papers have documented increased trading volume following public announcements non-related to shareholders' meetings. Kandel and Pearson (1995) focus on earnings reports and Bollerslev et al. (2018) focus on Federal Open Market Committee meetings. In both papers, abnormal trading volume is found just after the public announcements and in general there is less price change than one would expect given the volume. Our results show that these findings should not be taken as evidence against standard rational expectations models with homogeneous agents. Indeed, Proposition 6 (drawing on Proposition 2) says that trade should spike after a public announcement (e.g., earning report), if the information revealed affects shareholders' beliefs about the *path-forward* of the firm, i.e., beliefs about both what the best course of action is for the firm, and about what course of action will actually be chosen. This is in stark contrast with the effect of a public announcement that is relevant for the valuation of the firm only through previously enacted decisions. In that later case, the public announcement can only reduce disagreement among shareholders (since there is no uncertainty about what course of action taken by the firm), and hence trading volumes (as discussed, e.g., in Bollerslev et al. 2018).

Another important finding in these papers is the lack of substantial price change following public announcements. This is taken to be a puzzle and several previous studies (e.g., Kandel and Pearson 1995; Bollerslev et al. 2018; Li et al. 2022) take this as evidence that standard rational expectations models with homogeneous agents cannot fit the data. Our model of decision by command provides an explanation for the lack of price change. In the baseline version of that model ($p = 1$), the equilibrium price in period 2 is $P_2 = Q$ and at period 4 the price is also Q and thus there is a prediction of no change in price. When we relax the symmetry assumptions about the informativeness of the public signal, there is potentially some price change but this change is continuous in the source of asymmetry, i.e., γ.

Another relevant source of public information is shareholder meetings. Li et al. (2022) find high trading volume and small price changes after shareholder meetings. They argue that standard rational expectations models with homogeneous agents cannot account for such patterns. Our decision by command model offers a direct challenge to this argument for traders not involved in the vote. Moreover, our decision by voting model, and in particular Proposition 7, offers an additional equilibrium force that challenges this conclusion for shareholders involved in the vote. The key in that latter case is that voting does not perfectly aggregate the private signals, which seems reasonable in practice (for example, Maug and Rydqvist 2009 show that voting will only partially reveal the private information held by voters). Accordingly, we find that standard rational expectations models with homogeneous agents also yield the prediction "[...] that shareholder's beliefs may diverge more after observing voting outcomes." (Li et al. 2022, page 1813).

Empirical work generally finds small price changes after shareholders meetings. As mentioned already for traders that are not involved in voting, shareholder meetings are strategically equivalent to decision by command. And that model predicts no price change after the public event. In a hybrid model in which some traders are voters and some are not, we might see some forces for price change coming out of shareholders involved in the vote. Yet, as Table 1 (above) illustrates, the price changes are generally low (especially for the tallies likely to be realized in equilibrium).

Another natural question that has been asked is whether lopsided or close votes have larger effects on market behavior. Li et al. (2022) find that the vote tally does not matter and that abnormal trading volumes occur both after close and non-close votes. As we discussed at the end of Section 5, our model predicts a non-monotonic relationship between trade volume and the vote tally, which is in line with this empirical finding.[11]

We may also ask whether voting behavior is correlated with which side of the market a shareholder is on. Li et al. (2022) find that shareholders who voted for the losing alternative tend to sell. While we do not find that voting for the losing alternative always implies that one will want to sell, as mentioned above, it is the case that in our model this pattern is predicted for some ranges of parameters. From Proposition 7, and in particular the table characterizing trading strategies we can show that it requires that signal qualities are not too asymmetric. This is the case for Example 2 above.

6.2 Testable Predictions

Our model produces several new testable predictions. First, Proposition 1 says that shareholders may disagree about the path-forward of the firm, but nonetheless not disagree about the firm's value. This means that debate and competing arguments in traditional media and social media platforms about what course of action is best for the firm need not match up

[11]Note that testing empirically a monotonic relationship between volume of trade and the vote tally is challenging. Indeed, if a study pools over votes in which half the time a large tally is support for B (the policy that is optimal in the state that generates more informative private signals), and half the time a low tally is support for B, then a positive relationship between volume of trade and vote tally would show as flat in the data.

with trading volume prior to the time that either key information about the future decision is made public (see Proposition 6) or the firm actually makes the decision (see Proposition 7). Testing this result requires data about shareholders' beliefs that go beyond their valuation of the firm. For instance, it would be useful to explore shareholders' beliefs about key decisions that the firm has to make in the near future and check whether differences in such beliefs is compatible with shareholders having similar valuations of the firm. We are not aware of any such measure in the literature.[12]

Another testable prediction directly related to this result is that public announcements may have a very different effect on shareholders' valuations depending on whether it is relevant for the valuation of the firm only through previously enacted decisions by the firm, or through future decisions. In the former case, the public announcement can only reduce disagreement among shareholders, and hence reduce trade. In the latter case, it can increase disagreement, and then increase trade. One possibility to test that prediction would be to identify announcements that are directly related to previous decisions made by the firm (e.g., patent decision, authorization or ban on products, regional economic shocks for firm with clear regional patterns of foreign investments), and then examine if trading volume decreases after these announcements.

Finally, our model offers a more refined assessment of the association between voting behavior at a shareholders' meeting, and trading behavior after the meeting. In particular, our model predicts when selling should be positively associated with dissenting votes. To understand these predictions, recall that Proposition 7 and its table show that a shareholder wants to buy (sell) when her private information more strongly supports (challenges) the chosen policy than the inference that the market makes from her vote. Starting from there, it is helpful to treat separately the cases of each chosen policy.

Suppose first that B has been chosen. A vote against B can either come from a shareholder with private signal s_a playing a pure strategy or a shareholder with private signal s_b that happened to vote A. The former sells and the latter buys. A non-dissenting vote (for B) occurs only if a shareholder has type s_b and she voted for B. This shareholder does not trade (and more importantly she does not sell) and thus, if B was chosen, we unambiguously see that dissenting votes are predictive of both buying and selling as opposed to not trading.

Suppose now that A is chosen. A dissenting vote must have been cast by a shareholder with type s_b and this shareholder does not trade. On the other hand, a supporting vote could have been cast by a shareholder with either signal. In that case, the odds of buying

[12] Existing empirical literature typically measures disagreement among investors based on their sentiments about stock performance expressed on social media (e.g., Cookson and Niessner 2020, Giannini et al. 2019, and Antweiler and Frank 2004) or dispersion in analysts' earnings forecast's (e.g., Diether et al. 2002). Both these measures are closely related to the investors' valuation of the firm.

are higher as this occurs when the shareholder's signal supported the chosen policy (signal s_a), whereas selling requires getting the minority signal (s_b) and randomly opting to vote against this signal.

As discussed above, when the model is not too asymmetric, the average effect has the same direction as the finding in Li et al. (2022) (i.e., shareholders that voted for the losing alternative tend to sell). Table 1 shows that Example 2 above is one such specification. On the other hand, if the signal qualities are very asymmetric, then the opposite finding holds on average.

Pooling across chosen policies and signal qualities is likely to lead to too much measurement error, but building on this discussion we see that to test those predictions, one option would be to go back to the data in studies like Li et al. (2022) and determine when passage coincides with A (the correct choice in the state in which the signals are not very informative). We could then test whether selling is positively associated with dissenting votes when B is chosen but not when A is.[13]

7 Conclusions

We studied how investors' beliefs about firm value and their willingness to trade respond to the disclosure of public information. We consider two variations of a standard rational expectations models with homogeneous investors (who have common preferences, priors, and opinions). In the first variation, the focus is on beliefs of investors who are not involved in firm decision-making. The public information released can then be a public announcement by the firm itself (e.g., an earnings announcement or the release of decisions made at the shareholders' meeting), or by the another entity (e.g., a FOMC decision) as long as this announcement is payoff relevant for the firm. In the second variation, the focus is on the beliefs of investors who participate and vote in shareholders' meetings. The public information released is then an announcement about the result of a vote at the meeting.

A key novelty of our model is that both private and public information pertains to the path-forward of the firm (i.e., which decisions the firm will make in the future and whether it is value-enhancing), and is thus only indirectly informative about the value of the firm. This implies that investors holding different information (about the path-forward) may nonetheless have the same assessment of the firm value. We showed that shareholders may then react differently to the release of public information since, for some of them, this information reinforces their interim beliefs about the path-forward of the firm, while for

[13]Note that the data in Li et al. (2022) pool across realizations in which either outcome is chosen. It is then ambiguous which effect dominates.

others, this information contradicts their interim beliefs. We also showed that this divergence in beliefs can lead to an increase in trading volume after public announcements.

For investors who are involved in firm decision-making at the shareholders' meetings, we identified another force that may drive apart beliefs about firm value. This force materializes when (some) shareholders do not fully reveal their information through their vote at the meeting. Then, the vote tally does not reveal fully the actual support among shareholders in favor and against the decision made at the meeting, and it is rational for shareholders to use both their private information and the vote tally to assess whether the decision made is desirable. We found that, when the actual vote tally is not too lopsided, the public disclosure of that tally may actually increase the level of disagreement among shareholders about the firm value. The effect on trading volume is the same as for the first source of divergence in beliefs.

These findings are in line with empirical patterns of trade identified in the literature. Hence, these patterns should not be used as an argument to dismiss the standard rational expectations models with homogeneous investors, and its implications for corporate governance. This point is consequential as there are various reasons why it is important to determine whether corporate governance is generally a problem involving homogeneous shareholders, or if heterogeneity prevails.

A brief review of works assessing governance features and reforms illustrates that extant evaluations depend critically on whether shareholders are homogeneous. For instance, Hayden and Bodie (2008) discuss how the optimal allocation of voting rights among shareholders depends on whether shareholders share common goals. The desirability of communication among shareholders also depends on the approach taken: with common preferences, communication can aid information aggregation (Coughlan 2000; Gerardi and Yariv 2007), whereas with heterogeneous preferences it seems inconsequential. The desirability of regulations about vote trading and empty voting are different in homogeneous preference environments (Christoffersen et al. 2007; Esö et al. 2014; Brav and Mathews 2011) than in heterogeneous preferences ones (Hu and Black 2007, 2008; Casella et al. 2012). Finally, assessments of the consequences of regulation of proxy advisors also depend on the perspective (see Malenko and Malenko 2019; Malenko et al. 2021; Buechel et al. 2022; Ma and Xiong 2021 for the case of homogeneous preferences, and Matsusaka and Shu 2021 for the case of heterogeneous preferences).

Also, if shareholders' actions are mainly driven by differences in opinions, then several important corporate governance mechanisms would turn out to be inefficient or create more chaos. For example, the literature on agency problem and contract theory suggests that compensating CEOs based on stock performance can incentivize CEOs to work for the interests of shareholders (e.g., Holmström 1979). However, if stock prices mainly depend on marginal traders' opinions rather than information about firms' fundamental value, then CEOs are not necessarily induced to devote more effort into improving firms' fundamental value. Similarly, if selling is driven by different opinions rather than private information, then the exit of a blockholder should not be interpreted as a negative signal on firm value, and thus the "Wall Street Walk" (Admati and Pfleiderer 2009) would inefficiently discipline a hard-working manager who enhances firm value but is unfavored in the blockholder's idiosyncratic opinions. Finally, the desirability and organization of the communication between shareholders and managers also depends on whether shareholders have common or heterogeneous opinion (see Levit 2019, 2020 for a model with common opinion, and Kakhbod et al. 2022 for a difference-of-opinion model).

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A Proofs

Proof of Proposition 1. See that the expected value of x in period 2 is given by:

$$\begin{aligned} E[x|s] = \Pr(\omega=\alpha|s) \times Pr(k=A|\omega=\alpha) \\ + \Pr(\omega=\beta|s) \times Pr(k=B|\omega=\beta) \end{aligned} \tag{2}$$

Note that

$$\begin{aligned} &Pr(k=A|\omega=\alpha) \\ &= Pr(k=A|\hat{s}_a)Pr(\hat{s}_a|\omega=\alpha) + Pr(k=A|\hat{s}_b)Pr(\hat{s}_b|\omega=\alpha) \\ &= pQ + (1-p)(1-Q) \end{aligned} \tag{3}$$

and

$$\begin{aligned} &Pr(k=B|\omega=\beta) \\ &= Pr(k=B|\hat{s}_a)Pr(\hat{s}_a|\omega=\beta) + Pr(k=B|\hat{s}_b)Pr(\hat{s}_b|\omega=\beta) \\ &= (1-p)(1-Q) + pQ \end{aligned} \tag{4}$$

Thus,

$$E[x|s] = \Pr(\omega=\alpha|s) \times [pQ + (1-p)(1-Q)] \tag{5}$$

$$+ \Pr(\omega=\beta|s) \times [(1-p)(1-Q) + pQ]$$

$$= [pQ + (1-p)(1-Q)](Pr(\omega=\alpha|s) + Pr(\omega=\beta|s)) \tag{6}$$

$$= pQ + (1-p)(1-Q), \text{ for every } s \in S$$

The last step is because $Pr(\omega=\alpha|s) + Pr(\omega=\beta|s) = 1$ for every $s \in S$. Therefore, $d^2 = 0$. □

Proof of Proposition 2. Suppose, without loss of generality, that the realization of the public signal is $\hat{s}_a$.

$$\begin{aligned} E[x|s_a, \hat{s}_a] &= Pr(k=\omega|s_a, \hat{s}_a) \\ &= Pr(\omega=\alpha|s_a, \hat{s}_a)Pr(k=A \mid \hat{s}_a) + Pr(\omega=\beta|s_a, \hat{s}_a)Pr(k=B \mid \hat{s}_a) \\ &= pPr(\omega=\alpha|s_a, \hat{s}_a) + (1-p)(1-Pr(\omega=\alpha|s_a, \hat{s}_a)) \\ &= (1-p) + (2p-1)Pr(\omega=\alpha|s_a, \hat{s}_a) \end{aligned}$$

A similar formula applies to $E[x|s_b, \hat{s}_a]$. Hence, the disagreement between shareholders who received different signals is:

$$\begin{aligned} d_3 &= (2p-1)|Pr(\omega=\alpha|s_a,\hat{s}_a) - Pr(\omega=\alpha|s_b,\hat{s}_a)| \\ &= (2p-1)d_4 \end{aligned}$$

where d_4 measures the disagreement at time $t=4$ when the decision is $k=A$:

$$d_4 = |Pr(\omega=\alpha|s_a,\hat{s}_a) - Pr(\omega=\alpha|s_b,\hat{s}_a)|.$$

Note that d_4 also coincides with the disagreement at time $t=4$ when the decision is $k=B$ since $Pr(\omega=\alpha|\cdot) = 1 - Pr(\omega=\beta|\cdot)$. We may write:

$$d_4 = \left| \frac{q_\alpha Q}{q_\alpha Q + q_\beta(1-Q)} - \frac{(1-q_\alpha)Q}{(1-q_\alpha)Q + (1-q_\beta)(1-Q)} \right|.$$

Obviously, $d_4 \geq d_3 > d_2 = 0$ when $q_\alpha \neq q_\beta$. Moreover, when $p<1$, $d_4 > d_3$. □

Proof of Proposition 3. At stage 2, the expectation of firm value of a shareholder who received signal s_a is

$$\begin{aligned} &Pr(k=\omega|s_a, v_i=1) \\ &= Pr(k=A|\omega=\alpha, v_i=1)Pr(\omega=\alpha|s_a) + Pr(k=B|\omega=\beta, v_i=1)Pr(\omega=\beta|s_a) \\ &= Pr(v' \geq \tfrac{n-1}{2}|\omega=\alpha)\tfrac{q_\alpha}{q_\alpha+q_\beta} + Pr(v' < \tfrac{n-1}{2}|\omega=\beta)\tfrac{q_\beta}{q_\alpha+q_\beta} \end{aligned} \quad (7)$$

Since voters who receive s_b are indifferent between voting for A and B in equilibrium, we can simply consider those that vote for A. Therefore,

$$\begin{aligned} &Pr(k=\omega|s_b, v_i=1) \\ &= Pr(k=A|\omega=\alpha, v_i=1)Pr(\omega=\alpha|s_b) + Pr(k=B|\omega=\beta, v_i=1)Pr(\omega=\beta|s_b) \\ &= Pr(v' \geq \tfrac{n-1}{2}|\omega=\alpha)\tfrac{1-q_\alpha}{2-q_\alpha-q_\beta} + Pr(v' < \tfrac{n-1}{2}|\omega=\beta)\tfrac{1-q_\beta}{2-q_\beta-q_\alpha} \end{aligned} \quad (8)$$

Obviously, $Pr(k=\omega|s_a, v_i=1)$ is different from $Pr(k=\omega|s_b, v_i=1)$ as $q_\alpha \neq q_\beta$. □

Proof of Proposition 4. Throughout the proof, we assume that $k=A$ is the outcome of the voting process. An s_a-shareholder's expectation of firm value is

$$
\begin{aligned}
E[x|v', s_a, k = A] &= Pr(\omega = \alpha|v', s_a) \\
&= \frac{Pr(v', s_a|\omega = \alpha)}{Pr(v', s_a|\omega = \alpha) + Pr(v', s_a|\omega = \beta)} \\
&= \frac{\binom{n-1}{v'} z^{v'}(1-z)^{n-1-v'} q_\alpha}{\binom{n-1}{v'} z^{v'}(1-z)^{n-1-v'} q_\alpha + \binom{n-1}{v'} y^{v'}(1-y)^{n-1-v'} q_\beta} \\
&= \frac{1}{1 + \left(\frac{y}{z}\right)^{v'} \left(\frac{1-y}{1-z}\right)^{n-1-v'} \frac{q_\beta}{q_\alpha}},
\end{aligned} \tag{9}
$$

where

$$
\begin{aligned}
z &:= Pr(v_i = 1|\omega = \alpha) \\
&= Pr(v_i = 1|s_a)Pr(s_a|\omega = \alpha) + Pr(v_i = 1|s_b)Pr(s_b|\omega = \alpha) \\
&= q_\alpha + (1-m)(1-q_\alpha)
\end{aligned} \tag{10}
$$

and

$$
\begin{aligned}
y &:= Pr(v_i = 1|\omega = \beta) \\
&= Pr(v_i = 1|s_a)Pr(s_a|\omega = \beta) + Pr(v_i = 1|s_b)Pr(s_b|\omega = \beta) \\
&= q_\beta + (1-m)(1-q_\beta).
\end{aligned} \tag{11}
$$

An s_b-shareholder who votes for A expects x to be

$$
\begin{aligned}
E[x|v', s_b, k = A] &= Pr(\omega = \alpha|v', s_b) \\
&= \frac{Pr(v', s_b|\omega = \alpha)}{Pr(v', s_b|\omega = \alpha) + Pr(v', s_b|\omega = \beta)} \\
&= \frac{\binom{n-1}{v'} z^{v'}(1-z)^{n-1-v'}(1-q_\alpha)}{\binom{n-1}{v'} z^{v'}(1-z)^{n-1-v'}(1-q_\alpha) + \binom{n-1}{v'} y^{v'}(1-y)^{n-1-v'}(1-q_\beta)} \\
&= \frac{1}{1 + \left(\frac{y}{z}\right)^{v'} \left(\frac{1-y}{1-z}\right)^{n-1-v'} \frac{1-q_\beta}{1-q_\alpha}}.
\end{aligned} \tag{12}
$$

Since we assume that both shareholders vote for A, we have $v' = v - 1$ for both of them. Since $q_\alpha \neq q_\beta$, we have that $E[x|v', s_a, k = A]$ is different from $E[x|v', s_b, k = A]$. The proof for $k = B$ is similar, and thus omitted.

□

Proof of Proposition 5. We want to prove that for fixed r when $n \to \infty$, the disagreement among voters is larger after than before voting if the outcome of the election is a tie. We will first show that $\lim_{n\to\infty} d^2 = 0$ and then that $\lim_{n\to\infty} d^4 > 0$.

Note first that, the mixed strategy of s_b-shareholders, m, must be such that $\lim_{n\to\infty} z = \lim_{n\to\infty} 1-y$ (otherwise, conditional on being pivotal, one state becomes infinitely more likely than the other, see, e.g., Austen-Smith and Banks (1996)). One obtains from (10) and (11) that the following must hold at the limit:

$$q_\alpha + (1-m)(1-q_\alpha) = 1 - q_\beta - (1-m)(1-q_\beta) \quad \Leftrightarrow \quad (1-m)(2-q_\alpha-q_\beta) = 1-q_\alpha-q_\beta,$$

so that $m = \frac{1}{2-q_\alpha-q_\beta}$ at the limit. It follows that, at the limit, $z = \frac{1-q_\beta}{2-q_\alpha-q_\beta}$, which is strictly higher than $1/2$ since $q_\beta < q_\alpha$. Since $z = Pr(v_i = 1|\omega = \alpha)$, this directly implies that

$$\lim_{n\to\infty} \Pr\left(v' \geq \frac{n-1}{2}|\omega=\alpha\right) = \lim_{n\to\infty} \Pr\left(v' < \frac{n-1}{2}|\omega=\beta\right) = 1.$$

Plugging that in equations (3) and (4) , we obtain

$$\begin{aligned}\lim_{n\to\infty} \Pr(k=\omega|s_a) &= \lim_{n\to\infty}\left(\Pr\left(v' \geq \frac{n-1}{2}|\omega=\alpha\right)\frac{q_\alpha}{q_\alpha+q_\beta}\right.\\ &\left.+\Pr\left(v' < \frac{n-1}{2}|\omega=\beta\right)\frac{q_\beta}{q_\alpha+q_\beta}\right)\\ &= \frac{q_\alpha}{q_\alpha+q_\beta}+\frac{q_\beta}{q_\alpha+q_\beta} = 1,\end{aligned}$$

and

$$\begin{aligned}\lim_{n\to\infty} \Pr(k=\omega|s_b) &= \lim_{n\to\infty}\left(\Pr\left(v' \geq \frac{n-1}{2}|\omega=\alpha\right)\frac{1-q_\alpha}{1-q_\alpha+1-q_\beta}\right.\\ &\left.+\Pr\left(v' < \frac{n-1}{2}|\omega=\beta\right)\frac{1-q_\beta}{1-q_\alpha+1-q_\beta}\right)\\ &= \frac{1-q_\alpha}{1-q_\alpha+1-q_\beta}+\frac{1-q_\beta}{1-q_\alpha+1-q_\beta} = 1.\end{aligned}$$

Thus

$$\lim_{n\to\infty} d^2 = \lim_{n\to\infty} |\Pr(k=\omega|s_a) - \Pr(k=\omega|s_b)| = 0,$$

that is, for $n\to\infty$, the disagreement before voting vanishes.

We can now compute $\lim_{n\to\infty} d^4$, the limit disagreement after voting. In the mixed equilibrium, a shareholder who receives signal s_b is indifferent between voting for A and voting for B. This requires that

$$\Pr(\omega=\alpha|s_b)\Pr(v'=\tfrac{n-1}{2}|\omega=\alpha) = \Pr(\omega=\beta|s_b)\Pr(v'=\tfrac{n-1}{2}|\omega=\beta)$$

or:

$$\frac{\Pr(v'=\frac{n-1}{2}|\omega=\beta)}{\Pr(v'=\frac{n-1}{2}|\omega=\alpha)}=\frac{\Pr(\omega=\alpha|s_b)}{\Pr(\omega=\beta|s_b)}. \tag{13}$$

We also know that

$$\Pr(\omega=\alpha|s_b)=\frac{1-q_\alpha}{2-q_\alpha-q_\beta} \text{ and } \Pr(\omega=\beta|s_b)=\frac{1-q_\beta}{2-q_\alpha-q_\beta},$$

and that

$$\Pr(v'=\frac{n-1}{2}|\omega=\alpha)=\begin{pmatrix} n-1 \\ \frac{n-1}{2}\end{pmatrix}(z(1-z))^{\frac{n-1}{2}}, \text{ and}$$
$$\Pr(v'=\frac{n-1}{2}|\omega=\beta)=\begin{pmatrix} n-1 \\ \frac{n-1}{2}\end{pmatrix}(y(1-y))^{\frac{n-1}{2}}.$$

Plugging these objects in (13), we obtain

$$\left(\frac{y(1-y)}{z(1-z)}\right)^{\frac{n-1}{2}}=\frac{1-q_\alpha}{1-q_\beta}. \tag{14}$$

We now consider two shareholders who both voted for A but received different signals (when n is large, these two shareholders almost surely exist), so that $v'=v-1=\frac{n-1}{2}+r$ for both of them. Assuming that $k=A$ (i.e. $r\geq 0$), we may apply (5) and (6) to compute the expected value of the firm given that $v'=\frac{n-1}{2}+r$:

$$E[x|v'=\frac{n-1}{2}+r,s_a]=\frac{1}{1+\left(\frac{y}{z}\right)^{\frac{n-1}{2}+r}\left(\frac{1-y}{1-z}\right)^{\frac{n-1}{2}-r}\frac{q_\beta}{q_\alpha}},$$

and

$$E[x|v'=\frac{n-1}{2}+r,s_b]=\frac{1}{1+\left(\frac{y}{z}\right)^{\frac{n-1}{2}+r}\left(\frac{1-y}{1-z}\right)^{\frac{n-1}{2}-r}\frac{1-q_\beta}{1-q_\alpha}}.$$

Using (14), these expected valuations boil down to

$$E[x|v'=\frac{n-1}{2}+r,s_a]=\frac{1}{1+\frac{1-q_\alpha}{1-q_\beta}\frac{q_\beta}{q_\alpha}\left(\frac{y(1-z)}{z(1-y)}\right)^r}$$

and

$$E[x|v'=\frac{n-1}{2}+r,s_b]=\frac{1}{1+\left(\frac{y(1-z)}{z(1-y)}\right)^r}.$$

As $z = \frac{1-q_\beta}{2-q_\alpha-q_\beta} = 1-y$ at the limit, we have

$$\lim_{n\to\infty} \frac{y}{z} = \lim_{n\to\infty} \frac{1-z}{1-y} = \frac{1-q_\alpha}{1-q_\beta}. \tag{15}$$

We thus obtain:

$$E\left[x|v' = \frac{n-1}{2} + r, s_a\right] \quad \to_{n\to\infty} \quad \frac{1}{1+\frac{q_\beta}{q_\alpha}\left(\frac{1-q_\alpha}{1-q_\beta}\right)^{2r+1}} \tag{16}$$

and

$$E\left[x|v' = \frac{n-1}{2} + r, s_b\right] \quad \to_{n\to\infty} \quad \frac{1}{1+\left(\frac{1-q_\alpha}{1-q_\beta}\right)^{2r}}. \tag{17}$$

Note that shareholders voting for B have no valuable private information since their private signal can be inferred from the vote outcome, their expectation of the firm's value must thus be intermediate between the two expectations considered above. Therefore, we have

$$\begin{aligned}\lim_{n\to\infty} d_4 &= \lim_{n\to\infty}\left|E\left[x|v' = \frac{n-1}{2} + r, s_a\right] - E\left[x|v' = \frac{n-1}{2} + r, s_b\right]\right| \\ &= \left|\frac{1}{1+\frac{q_\beta}{q_\alpha}\left(\frac{1-q_\alpha}{1-q_\beta}\right)^{2r+1}} - \frac{1}{1+\left(\frac{1-q_\alpha}{1-q_\beta}\right)^{2r}}\right|,\end{aligned}$$

which is positive since $q_\alpha \neq q_\beta$. Thus, for $v = \frac{n+1}{2} + r$, with $r \geq 0$, we have that $\lim_{n\to\infty} d^4 > \lim_{n\to\infty} d^2$. The proof for $r \leq 0$ is similar and thus omitted. □

Proof of Proposition 6. The proof is straightforward. We know from Proposition 1 that, at stage 2, expectations about the value of the firm are independent of the signal received. This means that all shareholders have identical expectations about the value of the firm, and that these expectations are identical to the price $P_2 = Q$, as $p = 1$. Therefore, for any strictly positive transaction fee, there are no incentives to trade in stage 2.

At stage 4, given $p = 1$, the price is simply the probability that the public signal is correct: $P_4 = Q$. Proposition 2 shows that, at stage 4, there is disagreement among shareholders who receive different signals. From the proof of Proposition 2, it is straightforward to show that the price is necessarily strictly between the two expectations: $P_4 \in (\min_s \{E[x|s]\}, \max_s \{E[x|s]\})$. Therefore, for any ϵ smaller than both $P_4 - \min_s \{E[x|s]\}$ and $\max_s \{E[x|s]\} - P_4$, there is trade in stage 4. Note that the conclusion holds both in Kyle's trade model and in the NAL trade model. □

Proof of Proposition 7. **Stage 2.** We first prove that no one wants to trade before voting. When $n \to \infty$, P_2, the stock price before voting, converges to 1.

$$\begin{aligned}\lim_{n\to\infty} P_2 = & \lim_{n\to\infty} Pr^*(k=\omega) \\ & = \lim_{n\to\infty} Pr(t \geq \frac{n+1}{2}|\omega=\alpha)Pr(\omega=\alpha) \\ & + \lim_{n\to\infty} Pr(t \leq \frac{n-1}{2}|\omega=\beta)Pr(\omega=\beta) \\ & = 1,\end{aligned} \tag{18}$$

because $\lim_{n\to\infty} Pr(t \geq \frac{n+1}{2}|\omega=\alpha) = 1$ and $\lim_{n\to\infty} Pr(t \leq \frac{n-1}{2}|\omega=\beta) = 1$.

As for a shareholder's expectation of firm value, we know that $E[x|s_a] = Pr(k = \omega|s_a) \to_{n\to\infty} 1$ and $E[x|s_b] = Pr(k=\omega|s_b) \to_{n\to\infty} 1$ from the proof of Proposition 5.

Therefore, the difference between a shareholder's expectation of firm value and the price before voting goes to 0 as $n \to \infty$, no matter what signal the shareholder gets. That is:

$$\forall s \in S, \qquad \lim_{n\to\infty} |P_2 - E[x|s]| = 0.$$

Accordingly, given the arbitrarily small transaction fee, $\epsilon > 0$, a shareholder's trading profits are strictly negative at the limit, equal to $-\epsilon$. So, no shareholder wants to trade before voting.

Stage 4. We first show that for any n and any election outcome v, if the transaction fee is arbitrarily small, shareholders who voted for B never want to trade, while those who voted for A want to buy (resp. sell) if the chosen policy is aligned (resp. misaligned) with their private signal. In a second step, we show that there are (small) positive fees $\epsilon > 0$ such that trade occurs *for any* sufficiently large n, when the vote is sufficiently close.

Stage 4: Trading strategies. We first compute the the posterior probability of the state being α after any combination of public and private histories. Given the (public) vote outcome v, this probability is:

$$\begin{aligned}Pr(\omega=\alpha|v) &= \frac{Pr(v|\omega=\alpha)}{Pr(v|\omega=\alpha)+Pr(v|\omega=\beta)} \\ &= \frac{\binom{n}{v} z^v (1-z)^{n-v}}{\binom{n}{v} z^v (1-z)^{n-v} + \binom{n}{v} y^v (1-y)^{n-v}} \\ &= \frac{1}{1+\left(\frac{y}{z}\right)^v \left(\frac{1-y}{1-z}\right)^{n-v}}.\end{aligned} \tag{19}$$

Consider a shareholder who receives the signal of s_b. In the trading stage, the shareholder observes v and v_i, and privately knows $s_i = s_b$. Applying (12), the shareholder's posterior is

$$Pr(\omega = \alpha | v, v_i, s_b) = \frac{1}{1 + \left(\frac{y}{z}\right)^{v-v_i} \left(\frac{1-y}{1-z}\right)^{n-1-(v-v_i)} \frac{1-q_\beta}{1-q_\alpha}}$$

$$= \begin{cases} \dfrac{1}{1 + \left(\frac{y}{z}\right)^{v} \left(\frac{1-y}{1-z}\right)^{n-v} \frac{1-z}{1-y} \cdot \frac{1-q_\beta}{1-q_\alpha}} & \text{if } v_i = 0 \\ \dfrac{1}{1 + \left(\frac{y}{z}\right)^{v} \left(\frac{1-y}{1-z}\right)^{n-v} \frac{z}{y} \cdot \frac{1-q_\beta}{1-q_\alpha}} & \text{if } v_i = 1. \end{cases}$$

Since

$$\frac{1-z}{1-y} \cdot \frac{1-q_\beta}{1-q_\alpha} = \frac{m(1-q_\alpha)}{m(1-q_\beta)} \cdot \frac{1-q_\beta}{1-q_\alpha} = 1,$$

we have $Pr(\omega = \alpha | v, v_i = 0, s_b) = Pr(\omega = \alpha | v)$. This is because a shareholder voting for B has no valuable private information (at equilibrium, it is public knowledge that she received s_b). Since

$$\begin{aligned} \frac{z}{y} \cdot \frac{1-q_\beta}{1-q_\alpha} &= \frac{q_\alpha + (1-m)(1-q_\alpha)}{q_\beta + (1-m)(1-q_\beta)} \cdot \frac{1-q_\beta}{1-q_\alpha} \\ &= \frac{q_\alpha - q_\alpha q_\beta + (1-m)(1-q_\alpha)(1-q_\beta)}{q_\beta - q_\alpha q_\beta + (1-m)(1-q_\alpha)(1-q_\beta)} \\ &> 1, \end{aligned}$$

we know that $Pr(\omega = \alpha | v, v_i = 1, s_b) < Pr(\omega = \alpha | v)$. This is because a shareholder voting for A but receiving s_b has private information making her less optimistic about the state being α than if she only had public information.

Then, consider that a shareholder receiving the signal of s_a. Recall that $v_i(s_a) = 1$ in equilibrium. Applying (9), the shareholder's posterior is

$$Pr(\omega = \alpha | v, s_a) = \frac{1}{1 + \left(\frac{y}{z}\right)^{v-1} \left(\frac{1-y}{1-z}\right)^{n-1-(v-1)} \frac{q_\beta}{q_\alpha}}.$$

Because

$$\begin{aligned} \frac{z}{y} \cdot \frac{q_\beta}{q_\alpha} &= \frac{q_\alpha + (1-m)(1-q_\alpha)}{q_\beta + (1-m)(1-q_\beta)} \cdot \frac{q_\beta}{q_\alpha} \\ &= \frac{q_\alpha q_\beta + (1-m)q_\beta - (1-m)q_\alpha q_\beta}{q_\alpha q_\beta + (1-m)q_\alpha - (1-m)q_\alpha q_\beta} \\ &< 1, \end{aligned}$$

we have $Pr(\omega = \alpha | v, v_i = 1, s_a) > Pr(\omega = \alpha | v)$. This is because a shareholder receiving s_a has private information making her more optimistic about the state being α than if she only had public information.

We conclude that the following ordering of posteriors holds for any outcome v:

$$Pr(\omega=\alpha|v,v_i=1,s_b)<Pr(\omega=\alpha|v)=Pr(\omega=\alpha|v,v_i=0,s_b)<Pr(\omega=\alpha|v,s_a).$$

When $k=A$, the price after voting is $P_4(k=A,v)=Pr(\omega=\alpha|v)$ and we have for any information H, $E[x|H]=Pr(\omega=\alpha|H)$, so that:

$$E[x|v,v_i=1,s_b,k=A]<P_4(k=A,v)=E[x|v,v_i=0,s_b,k=A]<E[x|v,s_a,k=A].$$

Thus, when $k=A$, for an arbitrarily small transaction fee, shareholders voting for B want to hold, those voting for A want to buy if they received a signal s_a and they want to sell if they received a signal s_b.

Similarly, when $k=B$, we have $P_4(k=B,v)=1-Pr(\omega=\alpha|v)$ and for any information H, $E[x|H]=1-Pr(\omega=\alpha|H)$. We obtain:

$$E[x|v,s_a,k=B]<P_4(k=B,v)=E[x|v,v_i=0,s_b,k=B]<E[x|v,v_i=1,s_b,k=B].$$

Thus, when $k=B$, for an arbitrarily small transaction fee, shareholders voting for B want to hold, those voting for A want to buy if they received a signal s_b and they want to sell if they received a signal s_a.

Stage 4: Occurrence of trade.

We show that given any $\overline{r}\in\mathbb{N}$, there are small transaction fees ϵ such that trade occurs if $v=\frac{n+1}{2}+r$ with $r<\overline{r}$ whenever n is large enough.

First, assuming that $k=A$ (i.e. $r\geq 0$), we compute $P_4(k=A,v)$ using its definition (19), the mixed equilibrium condition (14) and then the limit condition (15):

$$\begin{aligned}P_4(k=A,v)=Pr(\omega=\alpha|v)&=\frac{1}{1+(\frac{y}{z})^{\frac{n+1}{2}+r}(\frac{1-y}{1-z})^{\frac{n-1}{2}-r}}\\&=\frac{1}{1+\frac{y}{z}(\frac{y(1-z)}{z(1-y)})^{r}\times\frac{1-q_\alpha}{1-q_\beta}}\\&\to_{n\to\infty}\frac{1}{1+(\frac{1-q_\alpha}{1-q_\beta})^{2r+2}}.\end{aligned}$$

We thus have that the price after voting converges to a limit of the form $\frac{1}{1+C}$ with $C:=$

$(\frac{1-q_\alpha}{1-q_\beta})^{2r+2}$. Now observe from (16) and (17) that

$$E_a := \lim_{n\to\infty} E[x|v = \frac{n+1}{2} + r, v_i = 1, s_a] = \frac{1}{1 + C\frac{q_\beta(1-q_\beta)}{q_\alpha(1-q_\alpha)}}$$

and

$$E_b := \lim_{n\to\infty} E[x|v = \frac{n+1}{2} + r, v_i = 1, s_b] = \frac{1}{1 + C(\frac{1-q_\beta}{1-q_\alpha})^2}.$$

As $q_\beta < q_\alpha < 1 - q_\beta$, we also have $q_\beta < 1 - q_\alpha < 1 - q_\beta$ and thus $(\frac{1-q_\beta}{1-q_\alpha})^2 > 1 > \frac{q_\beta(1-q_\beta)}{q_\alpha(1-q_\alpha)}$. We obtain that $E_b < \lim_{n\to\infty} P_4(k = A, v) < E_b$. Hence, for any given $r \geq 0$, there are small enough transaction fees such that trade occurs after a voting outcome $v = \frac{n+1}{2} + r$ whenever n is large enough. Note that the conclusion holds both in Kyle's and in the NAL trade models. The proof for $r < 0$ is similar and thus omitted. □