NBER WORKING PAPER SERIES

ENDOGENOUS EXCHANGE RATE REGIME SWITCHES

Gabriel de Kock

Vittorio Grilli

Working Paper No. 3066

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 1989

We would like to thank Antoine D'Autume, Barry Eichengreen, Paul Krugman, the participants to the second Franco-American Workshop and the colleagues at the International Research Department of the Federal Reserve Bank of New York for useful comments, and Lizabeth Miller for drawing the graphs. The opinions expressed in this paper are those of the authors alone and od not necessarily represent those of the Federal Reserve Bank of New York or of the Federal Reserve System. This research has been funded by NSF grant No. SES-8605866. This paper is part of NBER's research program in International Studies. Any opinions expressed are those of the authors not those of the National Bureau of Economic Research.

NBER Working Paper #3066
August 1989

ENDOGENOUS EXCHANGE RATE REGIME SWITCHES

ABSTRACT

In this paper we demonstrate that exchange rate regime switching is compatible with optimal government policies. Nominal exchange-rate regimes are formalized as equilibrium commitments on future seigniorage policies, and the collapse of an exchange-rate peg as an excusable default which allows the government to lump-sum tax private sector money holdings. We demonstrate that a regime in which the exchange-rate peg is allowed to collapse when government spending is unusually high is a trigger-strategy equilibrium. Such a regime can be superior to both fixed and flexible exchange rate because it combines some of the flexibility of the floating exchange rates with some of the benefits of precommitment afforded by fixed rates.

Gabriel de Kock
Federal Reserve Bank of New York
33 Liberty Street
New York, NY 10045

Vittorio Grilli
Yale University
Department of Economics
1972 Yale Station
New Haven, CT 06520

1. Introduction

Several recent studies have analyzed the properties of alternative exchange rate regimes. Work begun in papers by Helpman and Razin [1979] and Helpman [1981] compared the welfare levels achievable under a fullyflexible and a fixed exchange rate system. The authors showed that, in a highly simplified environment, the choice of the exchange system is irrelevant in the sense that the individual opportunity set is invariant across alternative exchange rate arrangements. This neutrality property tends to break down when richer analytical models are considered. For example, Helpman and Razin [1982] and Aschauer and Greenwood [1983] discuss cases in which a flexible exchange rate regime is superior to a fixed exchange rate regime. In general, however, it is difficult to provide an unambiguous ranking. Another line of research, initiated by Krugman [1987], Flood and Garber [1989], and Froot and Obstfeld [1989], has concentrated on the properties of a band-type system. But these studies have focused on the exchange rate stabilization effect of this type of system and not strictly on its welfare implications.

These analyses are conducted under the implicit assumption that a country makes an irrevocable decision to operate under one particular system. There is ample evidence, however, that policy makers "change their minds." It is not rare, in fact, to observe a country abandoning a fixed exchange rate or moving from free floating to some form of exchange rate control. The past hundred years of financial history have been characterized by switches from fixed exchange arrangements (e.g. Gold Standard, Gold-Exchange Standard, Bretton-Woods) to flexible exchange rate regimes (the interwar flexible periods and the post-1972 floating period). A more recent system like the EMS can be interpreted as a mixture of fixed and flexible regimes: usually the exchange rate is pegged around its official central parity, but sometimes it is left to drift away from the parity, and at other times the parity itself is changed.

The existence of switches between fixed and flexible exchange rates has long been recognized and has been the subject of a consistent body of research. Starting with Salant and Henderson [1978] and Krugman [1979], several papers have been written on the topic of speculative attacks and collapses of fixed exchange rates. In most of this literature, however, these switches are seen as the result of incompatible monetary and exchange rate policies , thus ruling out any rationale for this type of occurrence. Moreover, while much attention has been devoted to the forced abandonment of the fixed exchange rate, a similar effort has not been made to understand the welfare consequences of speculative attacks or the motives that would lead the authorities to fix the exchange rate. In fact, most of the speculative attack literature offers no justification for fixed exchange rate policies.

In this paper we show that exchange rate collapses may be consistent with optimal policies. We illustrate how superficially inconsistent policies--in this instance monetary growth that is incompatible with a previously announced fixed exchange rate--can be understood as part of a more complex regime in which the policy maker in well-defined circumstances is allowed an excusable default on the commitment to a fixed exchange rate. The analysis is based on a model in which policy makers use distortionary income taxation and seigniorage to finance an exogenously given expenditure stream. The policy maker has an incentive to generate surprise inflation to lump-sum tax money balances and, unless she can precommit future inflation, the equilibrium of the model is inefficient. However, as Barro and Gordon [1983a,b] have shown, an equilibrium superior to the time-consistent one can be implemented if the policy maker cares sufficiently about future outcomes and the private sector "punishes" her for surprise inflation by losing confidence in her resolve not to generate inflation surprises in the future. In this case, the inefficiency of future equilibria that would result if the private sector loses confidence deters the policy maker from generating surprise inflation. However, deterrence may not be sufficient to prevent surprise inflation in all states of nature. As Rotemberg and Saloner [1986] have shown in the context of an oligopoly example, there may exist other types of equilibria in which temporary breakdowns of cooperation (i.e. when deterrence is insufficient) do not preclude cooperation in other states[1]. The collapse of an exchange rate peg may be understood in exactly this way, as an excusable default on a commitment to a fixed rate that allows the policy maker to meet unusually large government spending by generating unexpected inflation, that is, by imposing a lump-sum tax on money balances. Support for this interpretation can be found in Figure 1, which reveals that historically the collapse of fixed exchange rate regimes coincided with periods of sudden rises in government expenditures while the return to pegged exchange rates has coincided with the return of expenditure to normal levels. In this figure, we plot both seigniorage and government expenditure (both as a fraction of domestic product) of the United Kingdom[2]. Unshaded areas correspond to periods in which the exchange rate was officially pegged. The association between expenditure levels and the exchange rate system is quite striking.

The plan of the paper is as follows. The next section discusses and motivates a simple model of optimal seigniorage. In sections 3 and 4 the

[1] Canzoneri and Henderson [1988] provide an application of this result in a policy coordination context.

[2] Seigniorage is measured as the rate of monetary growth multiplied by the real monetary base.

model is used to analyze the welfare properties of alternative exchange-rate regimes and to demonstrate the existence of an equilibrium regime that allows for the collapse and refixing of the exchange rate. Finally, the concluding section summarizes the results and discusses possible further research.

2. The Model

The model is designed to highlight the constraints on government financial policies imposed by alternative exchange rate regimes. Thus, we assume a small open economy that takes world prices and interest rates as given. World prices and interest rates are assumed to be constant and domestic and foreign goods to be perfect substitutes. Consequently, purchasing power parity holds and government seigniorage policy is the crucial determinant of nominal exchange-rate behavior, since the rate of exchange depreciation equals the domestic rate of inflation,

$$(2.1) \qquad \frac{S_t - S_{t-1}}{S_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} = 1 + \pi_{t-1} \quad ,$$

where S_t is the exchange rate (domestic currency price of foreign currency), P_t is the domestic price level, and π_{t-1} the inflation rate. The analysis focuses on the decision problem of a policy maker who seeks to minimize the cost of financing an exogenous stream of real government expenditure, $\{G_t\}_{t=0}^{\infty}$. We implicitly assume a private sector whose members have rational expectations and whose demand for real money balances depends on expected inflation.

The analysis is simplified considerably by taking G_t to be the government's revenue requirement after transactions in government bonds and foreign exchange. Thus, the policy maker chooses tax and money financing to satisfy the government budget constraint,

$$(2.2) \qquad G_t = T_t + \frac{M_t - M_{t-1}}{P_t} ,$$

where T_t and M_t are tax revenues and the nominal stock of high powered money outstanding. By pushing government borrowing and reserve use into the background, we are able to focus more clearly on the long-run factors underlying exchange-rate regime switches, illustrated in figure 1, although we thereby preclude the analysis of the short-run dynamics of the transition from one regime to another.[3]

We normalize government budget constraint by expressing variables as ratios to GNP,

$$(2.3) \qquad g_t = \tau_t + s_t ,$$

where g_t is the rate of government spending out of GNP, τ_t is the average income tax rate, and s_t is seigniorage revenues relative to GNP. To simplify the exposition we assume s_t to be a linear function of anticipated and unanticipated inflation,

$$(2.4) \qquad s_t = c_1(\pi_{t-1} - \pi^e_{t-1}) + c_2\pi^e_{t-1} , \qquad\qquad c_1, c_2 > 0$$

where $\pi^e_{t-1} = E_{t-1}\pi_{t-1}$ is the expected rate of inflation between period $t-1$ and period t, and π_{t-1} is the actual rate of inflation from period $t-1$ to period t

[3] Transitions as analyzed in the speculative attack literature are not inconsistent with our model. In fact, we consider our analysis to be complementary to that literature.

and E_t denotes an expectation conditional on information available in period t. Here c_2 is meant to capture the revenue from the distortionary taxation on money holdings associated with expected inflation, and c_1 the revenue from the lump-sum taxation of nominal government liabilities associated with surprise inflation. Note that by restricting c_2 to exceed zero, we are implicitly assuming the inflation-tax Laffer curve to be upward sloping at rates of inflation relevant to our analysis. We shall also assume that the parameters c_1 and c_2 remain constant across the different exchange rate regimes that we analyze.

The cost of financing government expenditures has four components: the excess burden of income taxes, the liquidity cost of (smaller money holdings arising from) anticipated inflation, the menu costs of actual inflation, and the cost of resource misallocation resulting from unanticipated inflation. A simple loss function capturing these costs is assumed for the policy maker in period t,

$$(2.5) \qquad L = E_t \sum_{i=0}^{\infty} \beta^i L_{t+i}$$

$$= E_t \sum_{i=0}^{\infty} \beta^i \left\{ a_1 \tau^2_{t-1+i} + a_2 (\pi^e_{t+i})^2 + a_3 (\pi_{t-1+i} - \pi^e_{t-1+i})^2 + a_4 \pi^2_{t+i} \right\},$$

where β , $0 < \beta \leq 1$, is the policy maker's discount factor, and L_{t+i} will be referred to as a one-period loss function. This loss function can be viewed as capturing, approximately, costs borne by a representative individual. Note also that the liquidity costs in period t depend on expected inflation from that period to the next, π^e_t , although it would not be possible for a policy maker who cannot precommit future policies to influence these expectations directly.

Finally, it is assumed that the government spending rate out of GNP is serially independent, with mean g and variance σ^2, and has bounded support $[\underline{g}, \bar{g}]$, with $0 < \underline{g} < \bar{g} < 1$.

In the absence of mechanisms that enable policy makers to precommit future policies credibly, the equilibrium of the model is inefficient. The source of the inefficiency is well known from the literature on time-consistent monetary policy (Kydland and Prescott [1977], Calvo [1978a, b]): from the policy maker's point of view, private sector real money holdings (or expectations of inflation) at the end of the preceding period are given. This creates an incentive to impose a capital levy on money balances through unanticipated inflation. Private agents recognize this incentive and choose real money holdings small enough to eliminate it. As a result, the equilibrium is characterized by an average inflation rate exceeding that which a policy maker who can precommit to a fixed rate of inflation would choose. In other words, the economy can be said to have an inflation bias stemming from a precommitment externality.

In this paper a nominal exchange-rate regime is viewed as a commitment to a set of future seigniorage policies. Such a commitment, if credible, enables the policy maker to fix private sector expectations. The exchange rate is a convenient indicator of government policies. It is responsive to policy changes and widely observed and, therefore, easy for private agents to monitor. Thus, it is reasonable to "write" an implicit social contract on seigniorage in terms of the exchange rate, even if such a contract need not be particularly simple.

3 Permanently Fixed vs. Permanently Floating Exchange Rates

A permanent exchange-rate peg can be viewed as a commitment on the part of government never to use the seigniorage tax. It enables the policy maker to fix private sector expectations of inflation at the cost of giving up the use of seigniorage altogether. A free float, on the other hand, allows the policy maker the flexibility to adjust income tax rates and inflation in response to government spending. As a simple way of capturing the potential costs of monetary independence, we assume that under a float the policy maker cannot precommit, that is, the equilibrium is the Nash equilibrium of the seigniorage policy game.

Under a permanent float, the policy maker in each period chooses the income tax rate and inflation rate so as to minimize the cost of taxation, taking as given government spending and private sector real money holdings. Because current policy actions do not affect future outcomes, the policy maker in period t solves the single-period optimization problem,

$$(3.1) \qquad \min_{\tau_t,\ \pi_{t-1}} \quad L_t = a_1\pi_{t-1}^2 + a_2(\pi_t^e)^2 + a_3(\pi_{t-1} - \pi_{t-1}^e)^2 + a_4\tau_t^2$$

subject to

$$(3.2) \qquad g_t = \tau_t + c_1(\pi_{t-1} - \pi_{t-1}^e) + c_2\,\pi_{t-1}^e ,$$

with π_{t-1}^e taken as given. The first-order conditions for tax and inflation rates,

$$(3.3) \qquad (a_1 + a_3)\pi_{t-1} = a_3\pi_{t-1}^e + a_4 c_1\tau_t \quad ,$$

along with the government budget constraint, eq. (3.2), yield the equilibrium tax and inflation rates under a float as

$$(3.4) \qquad \tau_t^F = E_{t-1}\tau_t^F + \frac{a_1 + a_3}{a_1 + a_3 + a_4 c_1^2}(g_t - E_{t-1}g_t)$$

$$= \frac{a_1}{a_1 + a_4 c_1 c_2} E_{t-1}g_t + \frac{a_1 + a_3}{a_1 + a_3 + a_4 c_1^2}(g_t - g)$$

$$= \frac{a_1}{a_1 + a_4 c_1 c_2} g + \frac{a_1 + a_3}{a_1 + a_3 + a_4 c_1^2}(g_t - g) \ ,$$

since g_t is i.i.d., and

$$(3.5) \quad \pi_{t-1}^F = E_{t-1}\pi_{t-1}^F + \frac{a_4 c_1}{a_1 + a_3 + a_4 c_1^2}(g_t - E_{t-1}g_t)$$

$$= \frac{a_4 c_1}{a_1 + a_4 c_1 c_2} g + \frac{a_4 c_1}{a_1 + a_3 + a_4 c_1^2}(g_t - g) \ ,$$

where the superscript "F" refers to equilibrium values of variables under floating exchange rates.

A government deciding which exchange rate system to choose, will evaluate the loss functions conditional on information available in the planning period $t-1$. Thus, using eqs. (3.4) and (3.5), we easily find that expected losses under a regime of freely floating exchange rates,

$$(3.6)\quad L^F = E_{t-1}\sum_{j=0}^{\infty}\beta^j\left\{a_1\left[\tau^F_{t-1+j}\right]^2 + a_2\left[\tau^{e,F}_{t+j}\right]^2 + a_3\left[\tau^F_{t-1+j} - \tau^{e,F}_{t-1+j}\right]^2 + a_4\left[\tau^F_{t+j}\right]^2\right\},$$

are

$$(3.7)\quad L^F = \frac{a_4}{1-\beta}\left\{\frac{a_1^2 + (a_1 + a_2)a_4c_1^2}{(a_1 + a_4c_1c_2)^2}\, g^2 + \frac{a_1 + a_3}{a_1 + a_3 + a_4c_1^2}\,\sigma^2\right\},$$

where $\tau^{e,F}_{t+j} = E_{t+j}\,\tau^F_{t+j}$.

If exchange rates are fixed permanently, both actual and expected inflation rates are equal to zero at all times. As a result, taxes and the minimized value of the policy maker's loss function are given by

$$(3.8)\quad \tau^P_t = g_t \quad ,$$

and

$$(3.9)\quad L^P = E_{t-1}\sum_{j=0}^{\infty}\beta^j\left\{a_4\left[\tau^P_{t+j}\right]^2\right\} = \frac{a_4}{1-\beta}\left\{g^2 + \sigma^2\right\}.$$

We can now compare welfare under the two alternative regimes. From

$$(3.10)\quad L^P - L^F = \frac{a_4}{1-\beta}\left[\left\{1 - \frac{a_1^2 + (a_1 + a_2)a_4c_1^2}{(a_1 + a_4c_1c_2)^2}\right\} g^2 + \frac{a_4c_1^2}{a_1 + a_3 + a_4c_1^2}\,\sigma^2\right],$$

we obtain the necessary and sufficient condition for floating exchange rates to be superior to a permanent peg:

$$(3.11) \qquad \sigma^2 > kg^2 ,$$

where

$$(3.12) \qquad k = \frac{(a_1 + a_3 + a_4c_1^2)\left[(a_1 + a_2)c_1 - 2a_1c_2 - a_4c_1c_2^2\right]}{c_1(a_1 + a_4c_1c_2)^2}.$$

The costs of floating are increasing in the average government spending rate (or revenue requirement) because high average spending implies a high expected inflation rate in a float. On the other hand, the benefits of a flexible exchange rate regimes are increasing in the variance of the government spending rate, as indicated by the expression $\dfrac{a_4c_1^2}{a_1 + a_3 + a_4c_1^2}\,\sigma^2$ in equation (3.10). This term captures the value of flexibility, that is, the value of the policy maker's being able to generate surprise in(de)flation under floating . Thus, our model predicts that a permanent float will be preferred in countries with low but highly volatile government spending[4]

A regime of floating exchange rates may be preferred even if government spending were nonstochastic. This would be the case when k is negative, that is:

$$(3.13) \qquad a_4c_1c_2 > (a_2 - a_1) + (a_1 + a_2)\left[\frac{c_1 - c_2}{c_2}\right].$$

[4] See Barro (1983) for a similar result.

Thus, a float is more desirable the greater the revenue from fully anticipated inflation (c_2), the smaller the liquidity cost of inflation relative to its direct cost (a_2 - a_1), and the greater the revenue from surprise inflation relative to that from fully anticipated inflation (c_1- c_2). Intuitively, (a_2- a_1) and (c_1 - c_2) are measures of the cost of the policy maker's inability to precommit inflation: the greater (a_2 - a_1), the smaller the cost entailed by the policy maker's inability to take into account the effect of his anticipated actions on real money holdings in the preceding period; and the greater (c_1 - c_2), the greater the incentive to generate surprise inflation and hence the higher the equilibrium inflation rate in the time-consistent equilibrium. Note also that if there is no excess burden of income taxation, $a_4 = 0$, the two regimes are equivalent. This can be seen from eqs. (2.3) and (3.5), which indicate that $\pi^F_{t-1} = 0$ and that $\tau^F_t = g_t$ if $a_4 = 0$.

It is clear, however, that a pegged exchange rate, even if preferable to a float on average, need not be preferred in all states of nature. Specifically, suppose that, if the policy maker resorts to unanticipated inflation (a devaluation of the domestic currency), private agents would lose confidence in the exchange rate peg forever and expect the policy maker to use discretionary seigniorage in all subsequent periods. Even if this "punishment mechanism" exists, the policy maker may have an incentive to generate an inflation surprise in periods with extraordinarily high government expenditures. The incentive to reduce the cost of income taxation at extraordinarily high rates may outweigh the cost of being in a floating rate regime afterwards.

4. Endogenous Exchange Rate Regime Switching

In this section we demonstrate the existence of an equilibrium exchange rate regime in which transitions between fixed and floating exchange rates are endogenous. In this regime the commitment to the exchange rate peg is understood to be a contingent one. The public knows that the government will, in well-defined circumstances, break its commitment to the simple exchange rate peg and loses confidence in the policy maker only if he resorts to the discretionary use of seigniorage outside these circumstances. Such a mixed or adjustable peg regime allows the government to trade off an inefficient average inflation rate for the ability to "lump sum"[5] tax money balances when government spending is unusually high; it dominates the polar extremes of permanently fixed and freely flexible exchange rates for a large subset of possible values of the parameters of the model.

A fixed exchange rate that is subject to collapse when government spending is unusually high can be formalized as a trigger-strategy equilibrium of the seigniorage policy game that we have outlined: Under fairly general conditions, there exists a trigger government spending rate out of GNP, g^* , such that the government will maintain the exchange rate peg when the actual spending rate, g_t , falls short of g^* and will collapse the exchange rate by resorting to unanticipated inflation when $g_t > g^*$. The private sector, recognizing the policy maker's incentives, will punish the government by losing confidence in the exchange-rate regime (playing Nash) forever only if the government uses discretionary seigniorage at spending rates below g^* .

[5] Properly speaking, unanticipated inflation is not a lump-sum tax in this model. In fact, we have assumed that unanticipated inflation is distortionary, because of its redistributive effects (that is, $a_3 > 0$). Still, since agents cannot preemptively reduce their money holdings when inflation is unexpected, unanticipated inflation does not have liquidity costs. It is in this sense that it has elements of a lump-sum tax.

Formally, the equilibrium trigger is defined as follows. Let $B^c(g_t, g^*)$ denote the benefit, and $C^c(g^*)$ the cost, of collapsing the exchange rate peg when it is not justified--an action that we will refer to as the policy maker "reneging" on the commitment to the exchange rate regime. Then the equilibrium trigger, g^*, satisfies one of the following three conditions:

$$\begin{aligned} &B^c(g^*, g^*) = C^c(g^*), \text{ and} \\ (4.1) \qquad &B^c(g_t, g^*) > C^c(g^*), \ g_t > g^*, \text{ and} \\ &B^c(g_t, g^*) < C^c(g^*), \ g_t < g^*, \end{aligned}$$

or

$$(4.2) \qquad g^* = \bar{g} \ \text{ if } B^c(g_t, \bar{g}) < C^c(\bar{g}), \text{ for all } g_t \in [\underline{g}, \bar{g}],$$

or

$$(4.3) \qquad g^* = \underline{g} \ \text{ if } B^c(g_t, \underline{g}) > C^c(\underline{g}), \text{ for all } g_t \in [\underline{g}, \bar{g}],$$

Eq. (4.1) defines the equilibrium for an adjustable peg regime, that is the case where $\underline{g} < g^* < \bar{g}$. It states that g^* is the highest government spending rate at which the private sector's loss of confidence can deter the policy maker from setting the exchange rate afloat and generating surprise inflation. A permanent peg corresponds to an equilibrium trigger equal to $\bar{g}$. According to (4.2) it is an equilibrium exchange-rate regime if the cost of collapsing exceeds the benefits at all possible values of g_t, when the private agents expect the government to maintain the peg forever. Similarly, eq. (4.3) states that a permanent float is an equilibrium when the policy maker never has an incentive to peg the rate if the private agents expect her not to do so.

In the case analyzed before, the policy maker made a once and for all choice of exchange-rate regime. Now we consider a situation where the policy maker in each period decides whether to maintain the exchange-rate regime or to renege. In this situation, the policy maker's present actions affect the exchange-rate regime and hence losses from next period onwards. For ease of exposition, we shall compare the benefit of reneging on the mixed exchange-rate regime (which are reaped in the current period), with the costs (which are borne in the future). This allows us to treat the policy maker's choice in parallel fashion with the analysis up to now.

4.1 Existence of an Equilibrium Trigger

To illustrate the conditions for existence of an equilibrium trigger, we consider the benefits and costs of collapsing the exchange rate peg when it is not justified, that is, when $g_t < g^*$.

Consider first the benefits from reneging on the mixed exchange rate regime. Benefits arise only in the current period, and are given by

$$(4.4) \qquad B^C(g_t\ , g^* \mid g_t < g^*\ ) = L_t^P(g_t, g^* \mid g_t < g^*\ ) - L_t^C(g_t, g^* \mid g_t < g^*\ )\ ,$$

where $L_t^P(g_t, g^* \mid g_t < g^*\ )$ denotes the policy maker's one-period loss function if the exchange rate is kept fixed, and $L_t^C(g_t, g^* \mid g_t < g^*\ )$ is the one-period loss function if the exchange rate is collapsed. Note that both $L_t^P(g_t, g^*)$ and $L_t^C(g_t, g^*)$ depend on the value of the trigger; the reason is that private agents' expectations of inflation in a mixed regime depend on g^*, as the notation $\pi(g^*)$ will emphasize.

The cost of collapsing the exchange-rate peg when it is not justified equals the discounted cost of being in a floating rate regime rather than a mixed regime from the next period onwards, and can be written as

(4.5) $C^C(g^*) = \beta(L^F - L^M)$,

where L^F and L^M are the policymaker's minimized loss functions under floating and in the mixed regime. The loss function under a mixed regime is given by

$$L^M(g^*) = E_t \sum_{j=1}^{\infty} \beta^j L^M_{t+j}(g_{t+j}, g^*),$$

where the one-period loss function under a mixed regime, $L^M_{t+j}(g_t, g^*)$, is given by

$$\left.L^M_{t+j}(g_{t+j}, g^*)\right\} \begin{aligned} &= L^P(g_{t+j}, g^* \mid g_{t+j} < g^*) \quad \text{if } g_t < g^* \\ &= L^C(g_{t+j}, g^* \mid g_{t+j} > g^*) \quad \text{if } g_t > g^*. \end{aligned}$$

Thus, expected one-period losses under a mixed regime are given by

$$\begin{aligned} (4.6) \quad E_t L^M_{t+j}(g_{t+j}, g^*) &= F(g^*) L^P_{t+j}(g_{t+j}, g^* \mid g_{t+j} < g^*) \\ &\quad + (1 - F(g^*)) L^C_{t+j}(g_{t+j}, g^* \mid g_{t+j} > g^* \\ &= L^P(g_{t+j}, g^*) - (1 - F(g^*)) B^C(g_{t+j}, g^* \mid g_{t+j} > g^*), \end{aligned}$$

where $F(g^*)$ is the probability that $g_t < g^*$. Using the definition of the policymaker's loss function under floating and eq. (4.5), the cost of reneging can be written as

$$(4.7) \qquad C^c(g^*) = E_t \sum_{j=1}^{\infty} \beta^j \left\{ L^F_{t+j}(g_{t+j}) - L^P_{t+j}(g_{t+j}, g^*) \right.$$

$$\left. + (1 - F(g^*))B^C(g_{t+j}, g^* \mid g_{t+j} > g^*) \right\} ,$$

The cost of reneging on the exchange rate regime equals the discounted cost of being in a floating regime forever, adjusted by the cost of having a non-zero expected rate of inflation under pegging, plus the cost of forgoing the ability to engineer excusable exchange-rate collapses under a mixed regime. Details of the derivation and the exact expressions for both $B^c(g_t,g^*)$, $C^c(g^*)$, and $L^M(g^*)$ are given in the Appendix.

To simplify the notation, we use $\psi(g_t,g^*)$ to denote the net benefit of reneging on the exchange-rate regime,

$$(4.8) \qquad \psi(g_t,g^*) \equiv B^c(g_t,g^*) - C^c(g^*) \ .$$

Then an equilibrium mixed exchange-rate regime that is, an equilibrium trigger satisfying eqs. (4.1) to (4.3), exists if there is a value of g^* , $\underline{g} \leq g^* \leq \bar{g}$ such that[6]

$$(4.9) \qquad \psi(g^*,g^*) = 0 \ , \text{ and}$$

$$(4.10) \qquad \frac{\delta}{\delta g_t} \left\{ \psi(g_t,g^*) \right\} \Bigg|_{g_t=g^*} > 0 \ .$$

---

[6] Recall that $g^* = \bar{g}$ correspond to a permanent peg and $g^* = \underline{g}$ to a free floating, respectively. Therefore, a true mixed regime requires $\underline{g} < g^* < \bar{g}$.

In the following we make the fairly unrestrictive assumption that:

$$(4.11)\quad (c_2 - c_1) < \frac{a_3}{a_4 c_1}$$

that is, expected inflation cannot be a much more effective source of revenue than unexpected inflation. In fact, we would in general expect the opposite to be true. We can now prove the following Lemma:

<u>Lemma 4.1:</u> $B^c(g_t, g^*)$, and thus $\psi(g_t, g^*)$, are monotonically increasing in g_t.

Proof: From the expression (A.11) in the Appendix, it is easy to show that

$$(4.12)\quad \frac{\delta}{\delta g_t}\left\{B^c(g_t, g^*)\right\} = 2\left\{\frac{a_4 c_1}{a_1 + a_3 + a_4 c_1^2}\left\{a_4 c_1 g_t + (a_3 + a_4 c_1(c_1 - c_2))\pi(g^*)\right\}\right\}.$$

Therefore, given (4.11), $\frac{\delta}{\delta g_t}\left\{B^c(g_t, g^*)\right\} > 0$, always. From the definition (4.9) of $\psi(\cdot)$ it is clear that $\frac{\delta}{\delta g_t}\left\{\psi(g_t, g^*)\right\} = \frac{\delta}{\delta g_t}\left\{B^c(g_t, g^*)\right\}$ ■.

Notice that Lemma 4.1 guarantees that condition (4.10) is always satisfied.

The discussion of existence is simplified by considering a trigger function, $g^T(g^*)$, defined by

$$(4.13)\qquad g^T(g^*) = \bar{g} \text{ if } \psi(g_t,g^*) < 0 \text{ for } \underline{g} \le g_t \le \bar{g}\ ,$$

$$= \underline{g} \text{ if } \psi(g_t,g^*) > 0 \text{ for } \underline{g} \le g_t \le \bar{g}\ , \text{ and}$$

$$= g^T \text{ such that } \psi(g^T,g^*) = 0 \text{ otherwise.}$$

Thus, the function $g^T(g^*)$ maps values of g^* into $[\underline{g}\ ,\ \bar{g}]$ as follows. If for a given g^* there is a value of g_t that equates the implied costs and benefits of reneging on the exchange rate regime, the trigger function picks out this value, denoted by g^T , as a candidate trigger. If for g^* the costs of reneging on the exchange rate regime exceed the benefits ($\psi(g_t,g^*) < 0$) for all g_t, the trigger function picks $\bar{g}$, that is a permanent peg. Finally, it logically picks a permanent float as a candidate equilibrium if the benefits of reneging exceed the costs at all possible values of g_t. An equilibrium trigger is a fixed point of the "trigger function" map, that is, a g^* satisfying

$$(4.14)\qquad g^T(g^*) = g^*\ ,$$

as illustrated in figure 2. Thus, to prove the existence of an equilibrium adjustable peg exchange rate regime, we have to show that $g^T(g^*)$ continuously maps the $[\underline{g}\ ,\ \bar{g}]$ interval into itself. For the regime to include the possibility of occasional collapses, however, the fixed point of the trigger function should occur in the interior of $[\underline{g},\bar{g}]$. Nonetheless, to establish the existence of an interior fixed point, is important to characterize what happens at the boundaries of the support of g_t. This is the purpose of the following two lemmas.

Lemma 4.2: $\underline{g}$ is an equilibrium trigger.

Proof: Recall that when $g^* = \underline{g}$ a mixed regime is equivalent to a permanent float. Therefore, from (4.5) we obtain that $C^c(\underline{g}) = 0$. From the expressions (A.11) for $B^c(g_t, g^*)$ it can be shown that:

$$(4.15)\quad \phi(g_t,\underline{g}) = B^c(g_t,\underline{g}) = (a_1 + a_2 + a_4c_1^2)^{-1}\left\{a_4c_1g_t + (a_2 + a_4c_1(c_1 - c_2))\pi_t^e\right\}^2 > 0,$$

that is $B^c(g_t,\underline{g}) > C^c(\underline{g}) = 0$ at all values of g_t, as shown in figure 3(a). Therefore, by (4.13), $g^T(g^*) = \bar{g}$. ∎

Intuitively, the policy maker will always prefer to choose π_{t-1} optimally, since reneging is costless when the exchange rate is expected to float forever from the next period onwards. As a corollary of lemma 4.2 notice that, since $\phi(g_t,g^*)$ is continuous in g_t and g^*, $\phi(g_t,g^*) > 0\ \forall\ g_t$ for g^* in the neighborhood of $\bar{g}$.

Similarly,

Lemma 4.3: If a permanent peg is time consistent, $\bar{g}$ is an equilibrium trigger.

Proof: By definition, a permanent peg is time consistent if $\phi(g_t,\bar{g}) < 0$, or $B^c(g_t,\bar{g}) < C^c(\bar{g})$, for all g_t, as shown in figure 3(b). Thus, by (4.13), $g^T(\bar{g}) = \bar{g}$. ∎

As a corollary of lemma 4.3 notice that $\phi(g_t,g^*) < 0\ \forall\ g_t$ for g^* in the neighborhood of $\bar{g}$, by continuity of $\psi(g_t, g^*)$.

Before proving the existence of an interior equilibrium trigger is convenient to prove the following two results:

Lemma 4.4: If a permanent peg is time consistent, the cost of an unjustified collapse is globally increasing in the trigger spending rate.

Proof: Note, first, that when $g^* = \bar{g}$, a mixed regime is equivalent to a permanent peg, and thus $L_t^M(g_t, \bar{g}) = L^P$. Consequently, (4.5) reduces to $C^c(\bar{g}) = \beta(L^F - L^P)$, which is positive because a permanent peg, to be time consistent, must be superior to a permanent float, that is $L^F > L^P$. Recalling from Lemma 4.2 that $C^c(\underline{g}) = 0$, we obtain $C^c(\bar{g}) > C^c(\underline{g}) = 0$. ■

Lemma 4.5: The benefit of an unjustified collapse is monotonically decreasing in the trigger spending rate, that is, $\partial B^c / \partial g^* < 0$.

Proof: From (A.11), we can derive

$$(4.16) \qquad \frac{\partial B^c(g_t, g^*)}{\partial g^*} = 2\pi'(g^*) \left\{ \frac{a_3 + a_4 c_1(c_1 - c_2)}{a_1 + a_3 + a_4 c_1^2} \left\{ a_4 c_1 g_t + \left[a_3 + a_4 c_1(c_1 - c_2)\right]\pi(g^*) \right\} + a_2 \pi(g^*) \right\},$$

where, from (A.8) in the appendix:

$$(4.17) \quad \pi'(g^*) = \frac{- a_4 c_1 f(g^*)}{F(g^*)(a_1 + a_3 + a_4 c_1^2) + (1 - F(g^*))(a_1 + a_4 c_1 c_2)} \text{ X}$$

$$\left\{g^* + \frac{a_3 + a_4c_1(c_1 - c_2)}{F(g^*)(a_1 + a_3 + a_4c_1^2) + \left[1 - F(g^*)\right](a_1 + a_4c_1c_2)}\int_{g^*}^{\bar{g}} g_t f(g_t)dg_t\right\} < 0.$$

Condition (4.11) guarantees that $\pi'(g^*)$, and thus $\partial B^c/\partial g^*$, are negative. ■

We are now in the position to prove the following proposition which establishes the existence of an equilibrium mixed exchange-rate regime.

<u>Proposition 4.1:</u> If a permanent peg is incentive compatible, there exists an equilibrium trigger, g^*, such that $\underline{g} < g^* < \bar{g}$.

Proof: We have to show that there always is a closed subset of $[\underline{g}, \bar{g}]$, $[g_1, g_2]$, such that $g^T(g^*)$ continuously maps $[g_1, g_2]$ into $[\underline{g}, \bar{g}]$. Recall that, by Lemma 4.5, $B^c(g_t, g^*)$ is monotonically decreasing in g^*. If $C(g^*)$ were monotonically increasing in g^*, the proof of the existence of a fixed point would be straightforward. In this case, in fact, we could define an interval $[g_1, g_2]$ $(\underline{g} < g_1 < g_2 < \bar{g})$ such that $\psi(\underline{g}, g_1) = 0$ and $\psi(\bar{g}, g_2) = 0$. Therefore, $g^T(g^*) = \underline{g}$ for $g^* \le g_1$ and $g^T(g^*) = \bar{g}$ for $g^* \ge g_2$. For all values of g^* such that $g_1 < g^* < g_2$, $g^T(g^*)$ is defined by $\psi(g^T, g^*) = 0$. By continuity of $\psi(g_t, g^*)$, $g^T(g^*)$ continuously maps $[g_1, g_2]$ into $[\underline{g}, \bar{g}]$. Moreover, because of the monotonicity of $B^c(g_t, g^*)$ and $C^c(g^*)$, also $g^T(g^*)$ is monotonic, which guarantees the uniqueness of the (interior) fixed point. The monotonicity of $C(g^*)$ is not guaranteed, however. In this case, $g^T(g^*)$ need not be monotonic, nor the (interior) equilibrium unique. Nevertheless, the existence of at least one interior fixed point can be proved, following the same type of argument used above. The reason why this is true can be seen by considering a transition between the definition of $g^T(g^*)$ as the solution to $\psi(g^T,g^*) = 0$ and $g^T = \underline{g}$ because $\psi(g_t,g^*) > 0 \; \forall \; g_t$. Because $B^c(g_t,g^*)$ is monotonically increasing in g_t and $C^c(g^*)$ is independent of g_t, such transitions between always occur at $g^T = \underline{g}$. Similarly, transition between the definition of $g^T(g^*)$ as the solution to $\psi(g^T,g^*) = 0$ and $g^T = \bar{g}$ because $\psi(g_t,g^*) < 0 \; \forall \; g_t$ always occur at $g^T = \bar{g}$. Thus, since $C(g^*)$ is globally increasing, see Lemma 4.4, it is always possible to define an interval $[g_1,g_2]$ such that $g^T(g_1) = \underline{g}$, $g^T(g_2) = \bar{g}$ and $\psi(g_t,g^*) = 0$ for $g_1 < g^* < g_2$. ■

Figure 4(a) illustrates an example of $g^T(g^*)$. Lemma 4.2 established that $\psi(g_t,\underline{g}) > 0 \; \forall \; g_t$. By continuity of $\psi(g_t,\underline{g})$, $g^T(g^*) = \underline{g}$ for some interval $[\underline{g}, g_a]$, as represented by the initial horizontal segment of $g^T(g^*)$. Analogously, Lemma 4.3 established that $\psi(g_t,\bar{g}) < 0 \; \forall \; g_t$, if a fixed exchange rate is time consistent. Hence, by continuity, $g^T(g^*) = \bar{g}$ for some interval $[g_d, \bar{g}]$, as represented by the final horizontal segment of $g^T(g^*)$.

In this example it is assumed that the non-monotonicity of $C^c(g^*)$ produces an interior interval of $g^T(g^*)$, $[g_b,g_c]$, such that for $g^* \in [g_b,g_c]$ $\psi(g_t,g^*) > 0 \; \forall \; g_t$. Jumps in the value of g^T, as shown at g_c in figure 4(b), cannot occur because transitions between the definition of $g^T(g^*)$ as the solution to $\psi(g^T,g^*) = 0$ and $g^T = \underline{g}$ because $\psi(g_t,g^*) > 0 \; \forall \; g_t$ always occur at $g^T = \underline{g}$.

Finally, it should be emphasized that the conditions assumed are sufficient but certainly not necessary for the existence of an equilibrium mixed exchange rate regime. In particular, although Proposition 4.1 is based on the assumption that a permanent peg is time consistent, the examples in section 5 will show that a mixed exchange rate regime may be an equilibrium even if a permanent peg is not incentive compatible. Figure 5 illustrates the equilibrium in this case. When a permanent peg is not time consistent, $\psi(\bar{g},\bar{g}) > 0$, therefore $g^T(\bar{g}) < \bar{g}$, as shown in the figure. Multiple equilibria arise naturally in this case: both g^*_1 and g^*_2 are equilibrium trigger spending rates.

4.2 Welfare under an Adjustable Peg Regime

An adjustable peg regime allows the policy maker the use of unanticipated inflation, generally presumed to be a very efficient source of revenue, when the government revenue requirement is unusually high. It does so at the cost of an inefficiently high expected rate of inflation; costly inflation expectations errors occur in periods when spending is low. In this section we show that the benefits may outweigh the cost: an adjustable peg may dominate both permanent peg and a permanent float. We take the trigger spending rate as a parameter and show that under intuitively plausible conditions an equilibrium regime with g^* sufficiently close to $\bar{g}$ will dominate a permanent peg. Since we still assume that the peg is time consistent, this also implies that the mixed regime also dominates a free float.

The minimized value of the policy maker's intertemporal loss function under an adjustable peg is given by

$$(4.18) \qquad L^M(g^*) = \frac{1}{1-\beta}\left\{EL_t^P(g_t,g^*) - (1 - F(g^*))E\left[\left[B^C(g_t\ ,\ g^*) \mid g_t > g^*\right]\right]\right\}$$

$$\approx \frac{1}{1-\beta}\ EL_t^P(g_t,\bar{g}) + \left.\frac{\delta L^M}{\delta g^*}\right|_{g^*=\bar{g}} (g^* - \bar{g})$$

$$= L^P + \frac{\delta L^M(\bar{g})}{\delta g^*} (g^* - \bar{g}) .$$

From eq. (A.16) in the appendix, it follows that

$$\text{(4.19)} \qquad \frac{\delta L^M}{\delta g^*} = - \frac{1}{\beta} \frac{dC^c}{dg^*} .$$

Substituting $\pi(\bar{g}) = 0$ and

$$\text{(4.20)} \qquad \pi'(\bar{g}) = \frac{a_4 c_1}{a_1 + a_3 + a_4 c_1^2} \bar{g} f(\bar{g}) ,$$

$$\text{(4.21)} \qquad L^M(g^*) \approx L^P - \frac{1}{1-\beta} \left\{ 2a_4(c_1 - c_2)\, g \frac{a_4 c_1}{a_1 + a_3 + a_4 c_1^2} \bar{g} f(\bar{g}) - \frac{(a_4 c_1)^2}{a_1 + a_3 + a_4 c_1^2} \bar{g}^2 f(\bar{g}) \right\} (g^* - \bar{g}) .$$

Hence, an adjustable peg is superior to a permanent peg for g^* sufficiently close to $\bar{g}$ $(L^M < L^P)$ if

$$\text{(4.22)} \qquad \frac{\bar{g}}{2g} > \frac{c_1 - c_2}{c_1} ,$$

that is, if the upper limit of g_t is high enough relative to its mean which is, again, a condition about the variability of g_t. A high variance of g_t will favor a mixed regime over a permanent peg.

5. An Example

We have shown that an equilibrium adjustable peg (or mixed) exchange rate regime exists if a permanent peg is incentive compatible and that such a regime may be superior to both a permanent float and a permanent peg if g^* happens to be close to $\bar{g}$. It is not clear, however, that the equilibrium adjustable peg regime corresponding to a particular set of parameter values would in fact dominate the polar extremes of pegged or freely floating exchange rates. To address this issue we now turn to some numerical examples that confirm the existence of equilibrium adjustable peg regimes that dominate free floating and permanent pegs, not only when a permanent peg is credible but also when it is not and even when free floating is superior to a permanent peg.

In the examples, we assume that government spending is uniformly distributed, with upper and lower bounds equal to 65 percent and 5 percent of GNP respectively. Thus the expected value of government spending equals 35 percent and it variance approximately 3 percent of GNP. The parameters of the policy maker's loss function are as follows: $a_1 = 0.004$, $a_2 = 0.005$ and $a_4 = 0.03$. In other words, the marginal cost of income taxation is six times the marginal liquidity cost and seven and a half times the marginal menu cost of inflation for $\tau = \tau^e$. Finally, c_1 is assumed to equal 0.08; unanticipated inflation at 1 percent per period yields government revenue equal to 0.08 percent of GNP. Variations in c_2, a_3 and β are used to generate the different examples.

Figure 6 illustrates the existence of a welfare improving adjustable peg when a permanent peg cannot be implemented $\left[B^c(\bar{g}, \bar{g}) > C^c(\bar{g})\right]$. It is based on the assumption that, $c_2 = 0.06$, and $\beta = 0.35$ and $a_3 = 0.002$. Thus, unanticipated inflation is assumed not to be very costly ($a_3 < a_1 < a_2$), as would be the case if the indexation is widespread. The top panel shows the equilibrium trigger spending rate as the intersection of the cost of reneging, $C^c(g^*)$, and the benefit of doing so when current government spending out of GNP equals the trigger spending rate, $B^c(g^*, g^*)$. There are two equilibria at trigger spending rates equal to 0.205 and 0.435. These equilibria correspond to the exchange rate being peg 26 percent and 64 percent of the time under an adjustable peg regime, respectively. The bottom panel plots the minimized values of the policy maker's loss function under alternative exchange rate regimes against the trigger. In this case a permanent peg is superior to a free float, while an adjustable peg dominates the permanent peg when the equilibrium trigger exceeds 0.265. Thus the adjustable peg regime corresponding to the equilibrium trigger equal to 0.435 is superior to both permanently fixed and freely floating exchange rates, while the one with g^* equal to 0.205 dominates a free float but not a permanent peg.

The case in which a permanent peg is incentive-compatible is shown in figure 7. The figure is drawn assuming that $c_2 = 0.07$, and $\beta = 0.5$ and $a_3 = 0.03$ -- unanticipated inflation is costly as the same rate as income taxation. In this example the equilibrium mixed regime is unique and corresponds to an equilibrium trigger, g^*, equal to about 0.15. As the lower panel shows, it dominates a permanent peg and a free float.

Finally, figure 8, which assumes $c_2 = 0.09$, and $\beta = 0.5$ and $a_3 = 0.002$ illustrates a case in which a free float is superior to a permanent peg. There are two equilibrium mixed regimes. Both dominate the polar extremes of free floating and permanent pegging. The equilibrium values of the trigger are 0.18 and 0.395. They correspond to regimes in which the exchange rate is allowed to float 78 and 42.5 percent of the time, respectively.

The results illustrated by these examples are surprisingly robust. In contrast with the theoretical analysis, the examples require few special assumptions to generate equilibrium adjustable peg regimes that dominate free floating and permanent pegs.

6. Conclusions

In this paper a simple model of of exchange rate determination was used to interpret alternative nominal exchange-rate regimes as commitments on future seigniorage policies. We interpreted the abandonment of a fixed exchange rate as an excusable default on the commitment to a particular set of seigniorage policies which, in well defined situations, allows the government to tax private sector money holdings by unanticipated inflation. Thus, what superficially appears to be inconsistent policies, can be viewed as part of a more complex regime of state contingent policies that partially internalize precommitment externalities by simple rules. This arrangement allows the policy maker some flexibility in states of nature when simple rules are not incentive compatible.

A simple model of optimal taxation was used to analyze three exchange rate regimes: a permanent peg, a free float, and a mixed peg-floating regime. A free float allows the policy maker revenue flexibility at the cost of lack of precommitment of future inflation. It is most desirable in the face of considerable uncertainty about future government financing needs. A permanent exchange rate peg, while avoiding precommitment externalities, does so at the expense of the government being unable to spread the excess burden of taxation over taxes and seigniorage. Whether a permanent peg would be preferred to a free float depends on both the stochastic properties of government expenditure and the parameters of the model. Even if a permanent peg is preferred, ex ante, to a free float, it may be impossible to implement it, because the government may not have the necessary precommitment technology. In this case, we show that a free float is not necessarily the only outcome. We demonstrate that it is possible to support a mixed regime, that is an exchange rate peg that sometime collapses in the face of large spending shocks, as a trigger-strategy equilibrium. We also show, with simulation exercises, that this mixed regime can be superior, from a welfare point of view, to either a permanent float and a permanent peg (even if the permanent peg were incentive compatible).

Our purpose is to provide a framework capable of reconciling exchange rate pegging and collapses with optimal government policies. The specifics of the analysis, based on budgetary problems, is most directly applicable to the experience of developing countries and of industrial countries in extremely adverse circumstances, like wars. However, the idea of optimal exchange rate regime switches is more general. For example, one could obtain similar results by assuming that the state variable triggering a regime switch is the real exchange rate, instead of government spending. This alternative approach may be more appropriate in modeling the recent experience of EMS countries.

Appendix

1. Derivation of the Benefit Function

We can compute $L_t^c(g_t, g^* \mid g_t < g^*)$ by solving the one-period optimization problem of a policy maker who reneges:

$$\text{(A.1)} \qquad \underset{\tau_t, \pi_{t-1}}{Min} \left\{ a_1 \pi_{t-1}^2 + a_2 (\pi_t^{e,F})^2 + a_3 (\pi_{t-1} - \pi(g^*))^2 + a_4 \tau_t^2 \right\},$$

subject to

$$\text{(A.2)} \qquad g_t = \tau_t + c_1 \left[\pi_{t-1} - \pi(g^*) \right] + c_2 \pi(g^*) \ .$$

Notice that two different expected rates of inflation: π_t^e and $\pi(g^*)$ appear in (A.1). The policy maker takes into account that if he reneges, expected inflation from the time of the collapse onward would be the one corresponding to a floating rate regime (that is $\pi_t^{e,F}$ based on equation (3.5)). However, in the previous period agents formed their expectations under the belief that the policy maker would not renege, so that unexpected inflation is based on $\pi(g^*)$.

From the first-order conditions of problem (A.1),

$$\text{(A.3)} \qquad (a_1 + a_3) \pi_{t-1} = a_3 \pi(g^*) + a_4 c_1 \tau_t \ ,$$

and the government budget constraint, eq. (A.2), we obtain the tax rate

$$(A.4)\qquad \pi_t = \frac{a_1 + a_3}{a_1 + a_3 + a_4c_1^2}\, g_t - \left(\frac{c_3(a_1 + a_3) - a_1c_1}{a_1 + a_3 + a_4c_1^2}\right) \pi(g^*) ,$$

the inflation rate,

$$(A.5)\qquad \pi_{t-1} = \frac{a_4c_1}{a_1 + a_3 + a_4c_1^2}\, g_t - \left(\frac{a_4c_1c_2 - (a_3 + a_4c_1^2)}{a_1 + a_3 + a_4c_1^2}\right) \pi(g^*) ,$$

and unanticipated inflation,

$$(A.6)\qquad \pi_{t-1} - \pi(g^*) = \frac{a_4c_1}{a_1 + a_3 + a_4c_1^2}\, g_t - \left(\frac{a_1 + a_4c_1c_2}{a_1 + a_3 + a_4c_1^2}\right) \pi(g^*) .$$

Private agents take the possibility of a justified collapse in the next period into account when forming their expectations of inflation in the mixed regime. Thus, the expected rate of inflation from period $t-1$ to period t is

$$(A.7)\qquad \pi(g^*) = F(g^*)\ E_{t-1}[\pi_{t-1} | peg\ in\ period\ t]$$

$$+ (1 - F(g^*))\ E_{t-1}[\pi_{t-1} | float\ in\ period\ t]$$

where $F(g^*)$ is the probability that $g_t < g^*$. Note that the expectation of inflation from period t to period $t+1$, formed in period $t-1$ conditional on the exchange rate regime being maintained, is $\pi(g^*)$. Thus substituting from eq. (A.5) into eq. (A.7) and simplifying, we get

$$(\text{A.8})\quad \pi(g^*) = \frac{a_4 c_1}{F(g^*)(a_1 + a_3 + a_4 c_1^2) + (1-F(g^*))(a_1 + a_4 c_1 c_2)} \int_{g^*}^{\bar{g}} g_t f(g_t) dg_t,$$

where $f(\cdot)$ is the density of g_t and $\bar{g}$ is its upper bound.

To calculate $L_t^P(g_t,\ g^* \mid g_t < g^*)$, we need to know the tax rate when the policy maker pegs the exchange rate, which is given by

$$(\text{A.9})\qquad \tau_t = g_t + (c_1 - c_2)\pi(g^*)\ ,$$

and unanticipated inflation,

$$(\text{A.10})\qquad \pi_{t-1} - \pi(g^*) = -\ \pi(g^*)\ .$$

Thus, if we substitute into the policy maker's one-period objective, the benefit of collapsing the exchange rate peg when $g_t < g^*$ is given by

$$(\text{A.11})\ B^c(g_t, g^* \mid g_t < g^*) = L_t^P(g_t,\ g^* \mid g_t < g^*) - L_t^c(g_t,\ g^* \mid g_t < g^*)$$

$$= (a_1 + a_3 + a_4 c_1^2)^{-1}\left\{a_4 c_1 g_t + (a_3 + a_4 c_1(c_1 - c_2))\pi(g^*)\right\}^2$$

$$+\ a_2 \left\{\pi(g^*)^2 - \pi_t^e(g^*)^2\right\}\ .$$

2. Derivation of the Cost Function

Recall that the cost of collapsing the exchange rate peg when it is not warranted equals

$$(A.12)\qquad C^c(g^*) = E_t \sum_{j=1}^{\infty} \beta^j \Big\{ L^F_{t+j}(g_{t+j}) - L^P_{t+j}(g_{t+j}, g^*)$$

$$+ (1 - F(g^*))B^C(g_{t+j}, g^* \mid g_{t+j} > g^*)\Big\} ,$$

Note that the benefit of a justified collapse, $B^C(g_t, g^* \mid g_t > g^*)$, differs from $B^C(g_t, g^*)$ given in eq. (A.11) in that only the first term appears, since a justified collapse is not followed by an increase of inflation expectations from $\pi(g^*)$ to π^e_t. Therefore:

$$(A.13)\quad B^C(g_{t+j}, g^* \mid g_{t+j} > g^*) = \frac{[a_4 c_1 g_t + (a_3 + a_4 c_1(c_1 - c_2))\pi(g^*)]^2}{[a_1 + a_3 + a_4 c_1^2]}$$

Thus, since g_t is i.i.d, the cost of an unjustified collapse is given by

$$(A.14)\quad C^c(g^*) = \frac{\beta}{1 - \beta} \Big\{ E\Big\{ L^F_t(g_t) - L^P_t(g_t, g^*) + (1 - F(g^*))B^C(g_{t+j}, g^* \mid g_{t+j} > g^*)\Big\}\Big\}$$

$$= \frac{\beta}{1 - \beta} \Bigg\{ a_4 \Bigg[\frac{a_1^2 + (a_1 + a_2)a_4 c_1^2}{(a_1 + a_4 c_1 c_2)^2} g^2 + \frac{a_1 + a_3}{a_1 + a_3 + a_4 c_1^2} \sigma^2 - g^2 - \sigma^2 \Bigg]$$

$$- \Big\{ \Big[a_2 + a_3 + a_4(c_1 - c_2)^2 \Big] \pi(g^*)^2 + 2a_4(c_1 - c_2)\pi(g^*) g \Big\}$$

$$+ \Big[a_1 + a_3 + a_4 c_1^2 \Big]^{-1} \int_{g^*}^{\bar{g}} \Big\{ a_4 c_1 g_t + (a_3 + a_4 c_1(c_1 - c_2))\pi(g^*) \Big\}^2 f(g_t)\, dg_t \Bigg\}.$$

3. Welfare under a Mixed Regime

Finally, we are in a position to derive the policymaker's loss function under an adjustable peg regime, $L^M(g^*)$. By definition:

$$(\text{A.15}) \qquad L^M(g^*) = E_t \sum_{j=1}^{\infty} \beta^j \; L^M_{t+j}(g_{t+j}, g^*)$$

and thus, using eq. (4.6) in the text along with eqs. (A.9), (A.10), and (A.13),

$$(\text{A.16}) \; L^M(g^*) = (1-\beta)^{-1}\{E_t L^P(g_{t+j}, g^*) - (1-F(g^*))E^C(g_{t+j}, g^* \mid g_{t+j} > g^*)\}$$

$$= (1-\beta)^{-1}\{a_4(g^2 + \sigma^2) + [(a_2 + a_3 + a_4(c_1 - c_2))\pi(g^*)^2 + 2a_4(c_1 - c_2)\pi(g^*)g]$$

$$- \left[a_1 + a_3 + a_4 c_1^2\right]^{-1} \int_{g^*}^{\bar{g}} [a_4 c_1 g_t + (a_3 + a_4 c_1(c_1 - c_2))\pi(g^*)]^2 f(g_t)\,dg_t\}.$$

References

Aschauer, D. and J. Greenwood (1983). "Inflation, Welfare and the Choice of Exchange-Rate Regime," Journal of Political Economy 91 (5):868-875.

Barro, R.J. (1983). "Inflationary Finance under Discretion and Rules," Canadian Journal of Economics 16(1):1-16.

__________ and D.B. Gordon (1983a). "Rules, Discretion and Reputation in a Model of Monetary Policy," Journal of Monetary Economics 12(1):102-121.

__________ (1983b). "A Positive Theory of Monetary Policy in a Natural Rate Model," Journal of Political Economics 91(4):589-620.

Calvo, G.A. (1978a). "Optimal Seigniorage from Money Creation," Journal of Monetary Economics 4:503-517.

__________ (1978b). "On the Time Consistency of Optimal Policy in Monetary Economy," Econometrica 46(6):1411-1428.

Canzoneri, M.D. and D.W. Henderson (1988). "Is Sovereign Policy Making Bad?," in K. Brunner and A.H. Meltzer (eds.), Stabilization Policies and Labor Markets, Carnegie-Rochester Conference on Public Policy 28, Spring.

Flood, R.P. and P.M. Garber (1989). "The Linkage Between Speculative Attack and Target Zone Models of Exchange Rates," NBER Working Paper, No 2918.

Froot, K.A. and M. Obstfeld (1989). "Exchange Rate Dynamics under Stochastic Regime Shifts: A Unified Approach," NBER Working Paper, No 2835.

Helpman, E. (1981). "An Exploration in the Theory of Exchange-Rate Regimes," Journal of Political Economy 89(5):865-890.

__________ and A. Razin (1979). "Toward a Consistent Comparison of Alternative Exchange-Rate Regimes," Canadian Journal of Economics 12:394-409.

__________ (1982). "A Comparison of Exchange-Rate Regimes in the Presence of Imperfect Capital Markets," International Economic Review 23(2):365-388.

Krugman, P.R. (1979). "A Model of Balance of Payments Crises," Journal of Money, Credit and Banking 11(3):311-325.

__________ (1987). "Trigger Strategies and and Price Dynamics in Equity and Foreign Exchange Markets," NBER Working Paper, No.2459.

__________ (1988). "Target Zones and Exchange-Rate Dynamics," NBER Working Paper No.2841.

Kydland F.E. and E.C. Prescott (1977). "Rules Rather than Discretion: The Inconsistency of Optimal Plans," Journal of Political Economy 85:473-491.

Rotemberg J.J. and G.Saloner (1986). "A Supergame-Theoretic Model of Price Wars During Booms," American Economic Review 76(3):390-407.

Salant S.W. and D.W. Henderson (1978). "Market Anticipations of Government POlicies and the Price of Gold," Journal of Political Economy 86:627-648.

Figure 1
UK Government Expenditure and Seigniorage
(five–year moving averages)

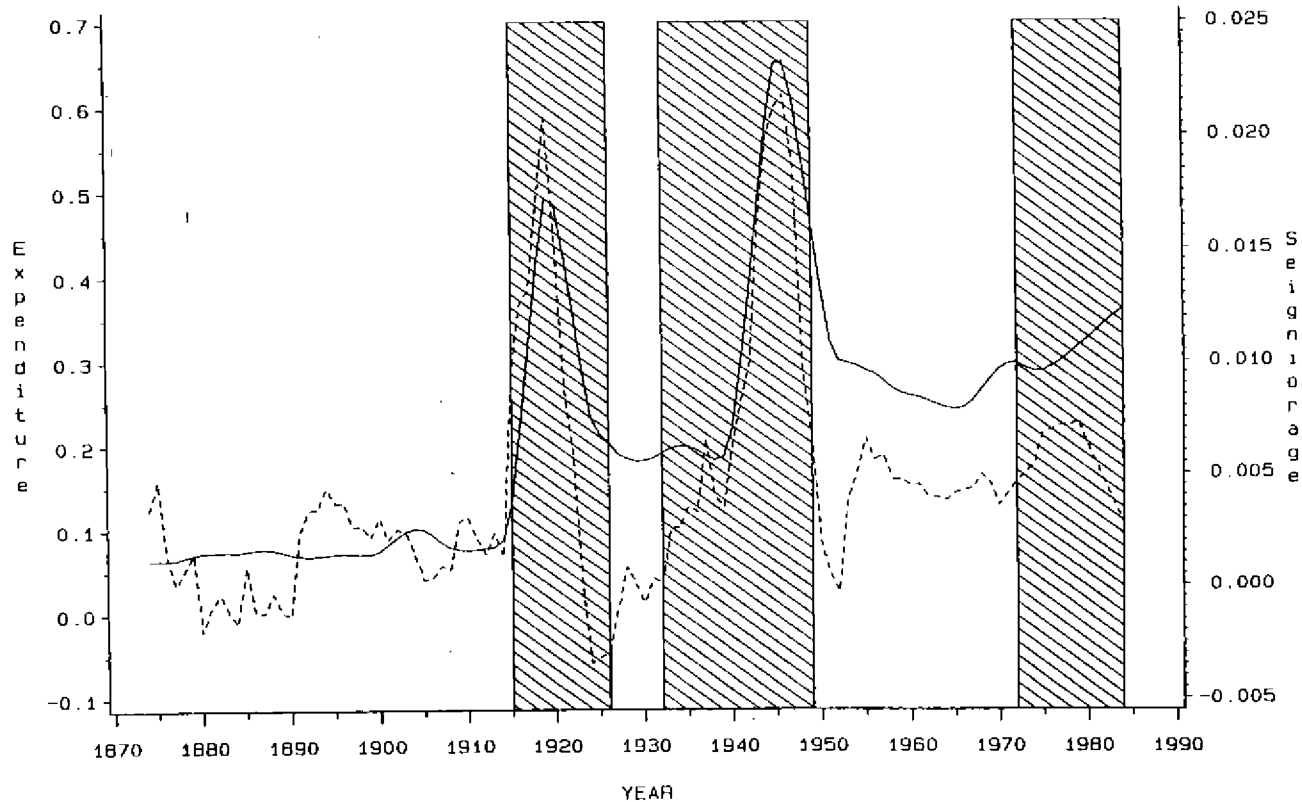
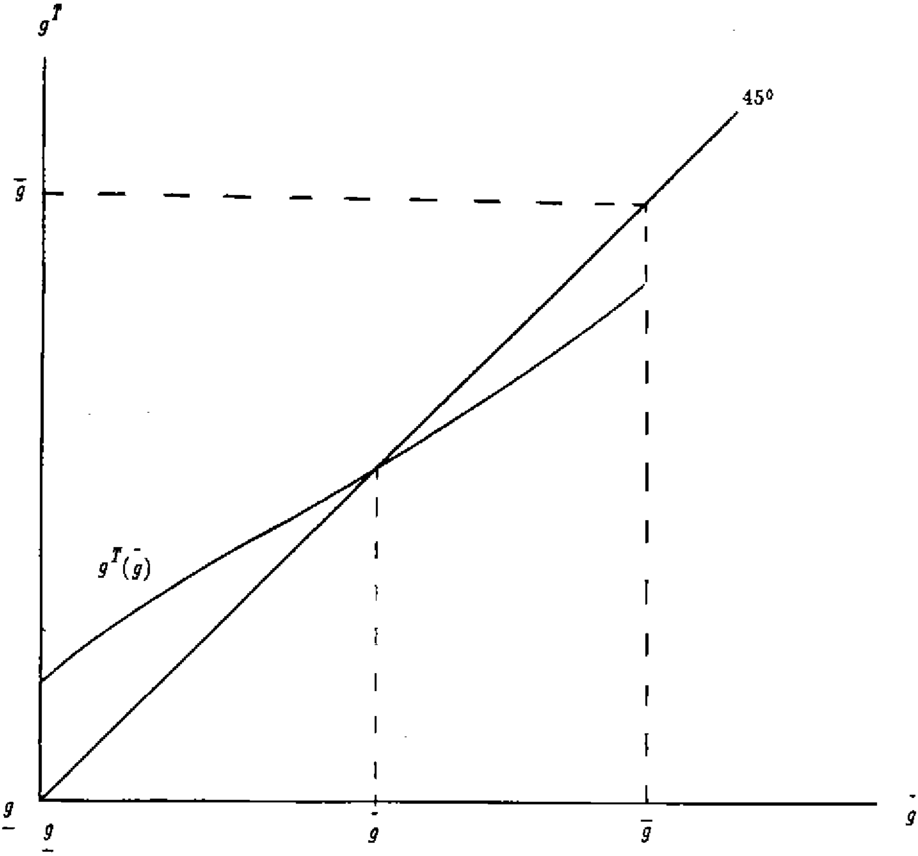


Figure 2



g^T
45°
$\bar{g}$
$g^T(\tilde{g})$
$\underline{g}$
$\underline{g}$
$\tilde{g}$
$\bar{g}$
$\tilde{g}$

Figure 3

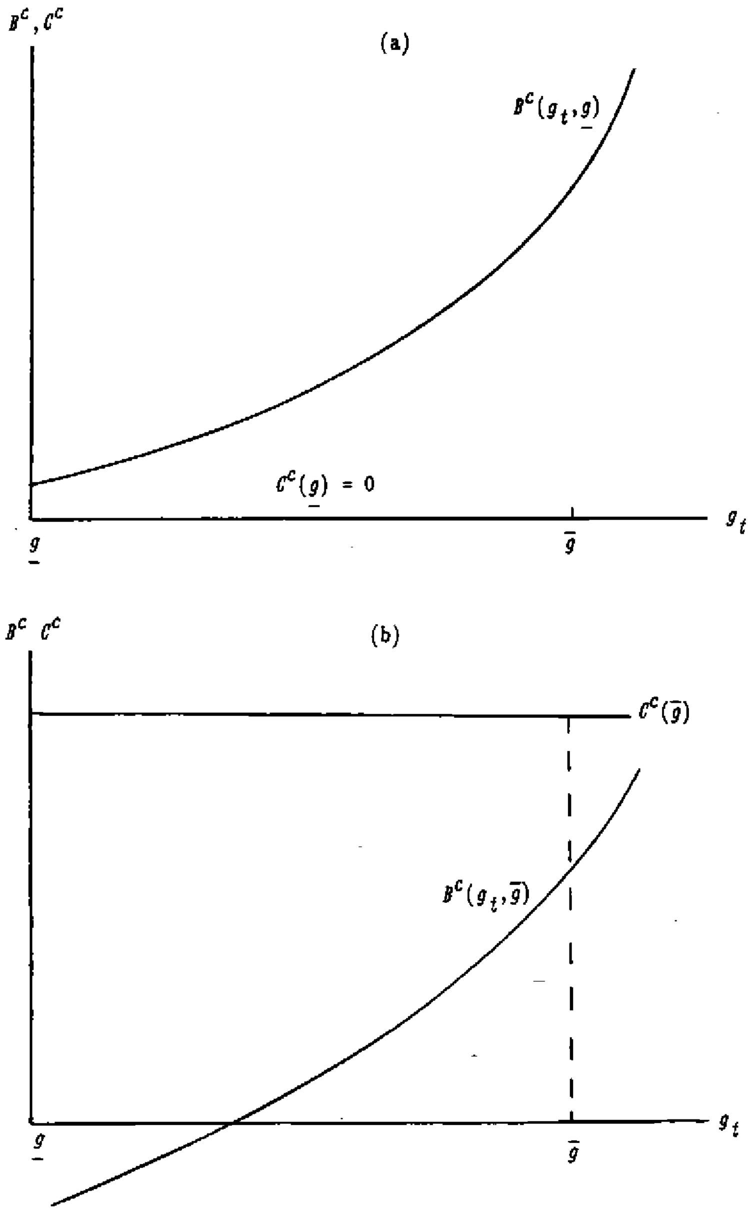
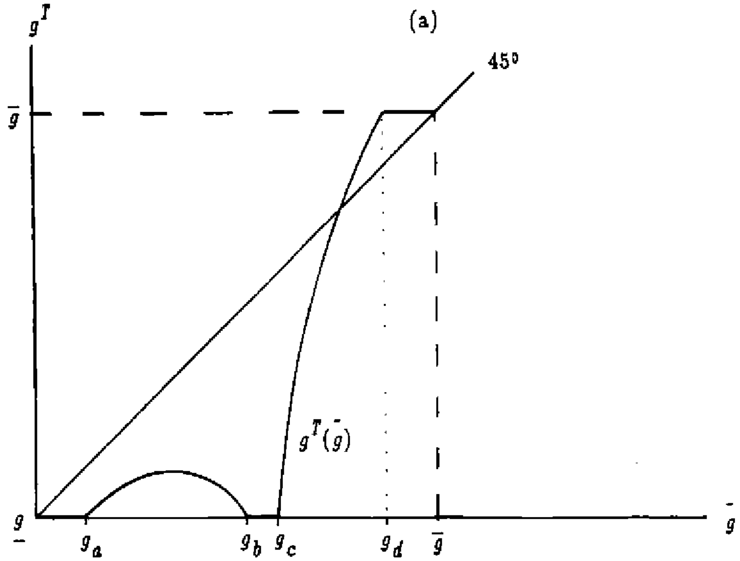

B^c, C^c
(a)
$B^c(g_t, \underline{g})$
$C^c(\underline{g}) = 0$
$\underline{g}$
$\bar{g}$
g_t
$B^c \; C^c$
(b)
$C^c(\bar{g})$
$B^c(g_t, \bar{g})$
$\underline{g}$
$\bar{g}$
g_t

Figure 4

(a)


g^T
45°
$\bar{g}$
$g^T(\bar{g})$
$\underline{g}$
g_a
g_b
g_c
g_d
$\bar{g}$
$\bar{g}$

(b)

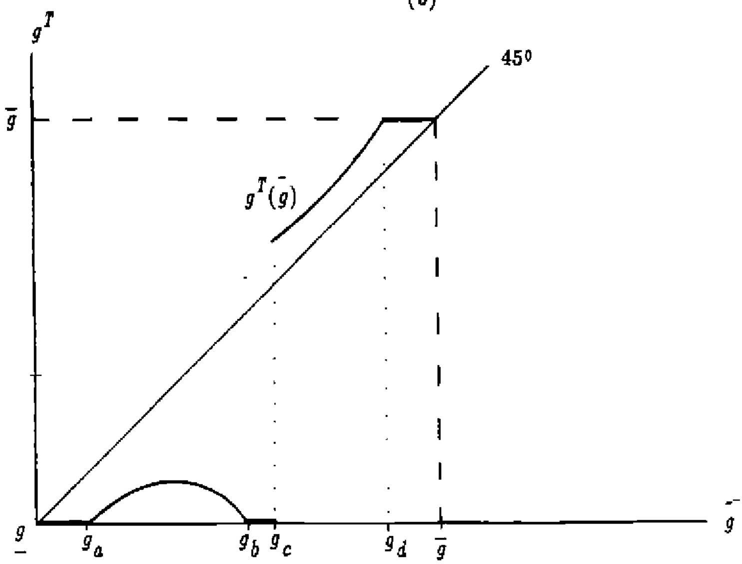
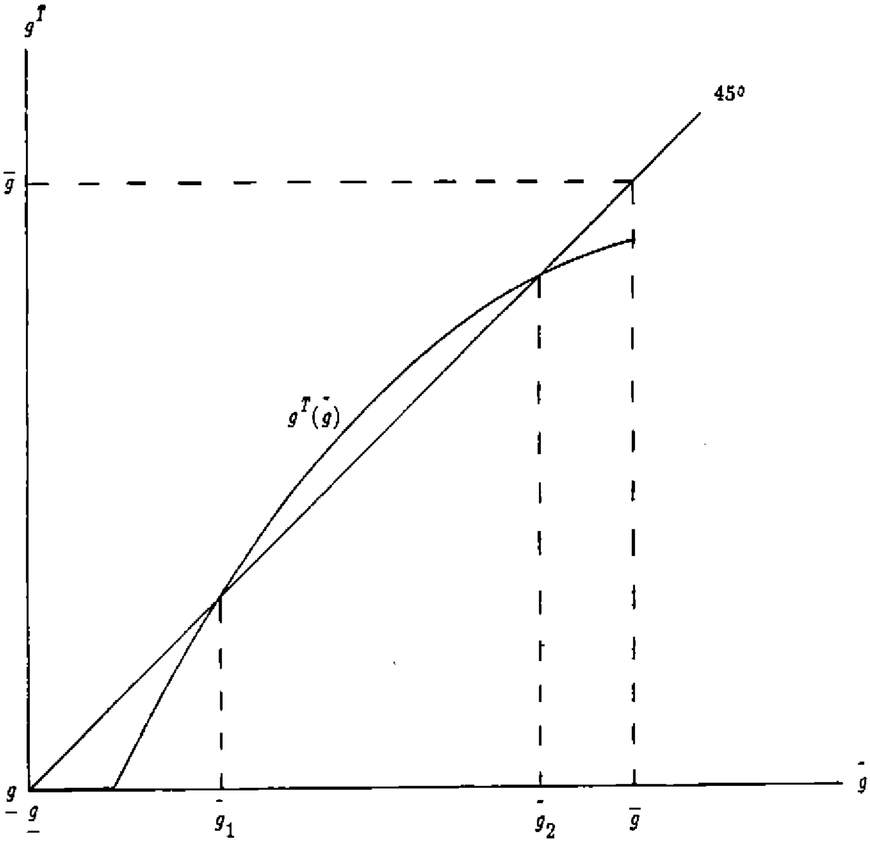
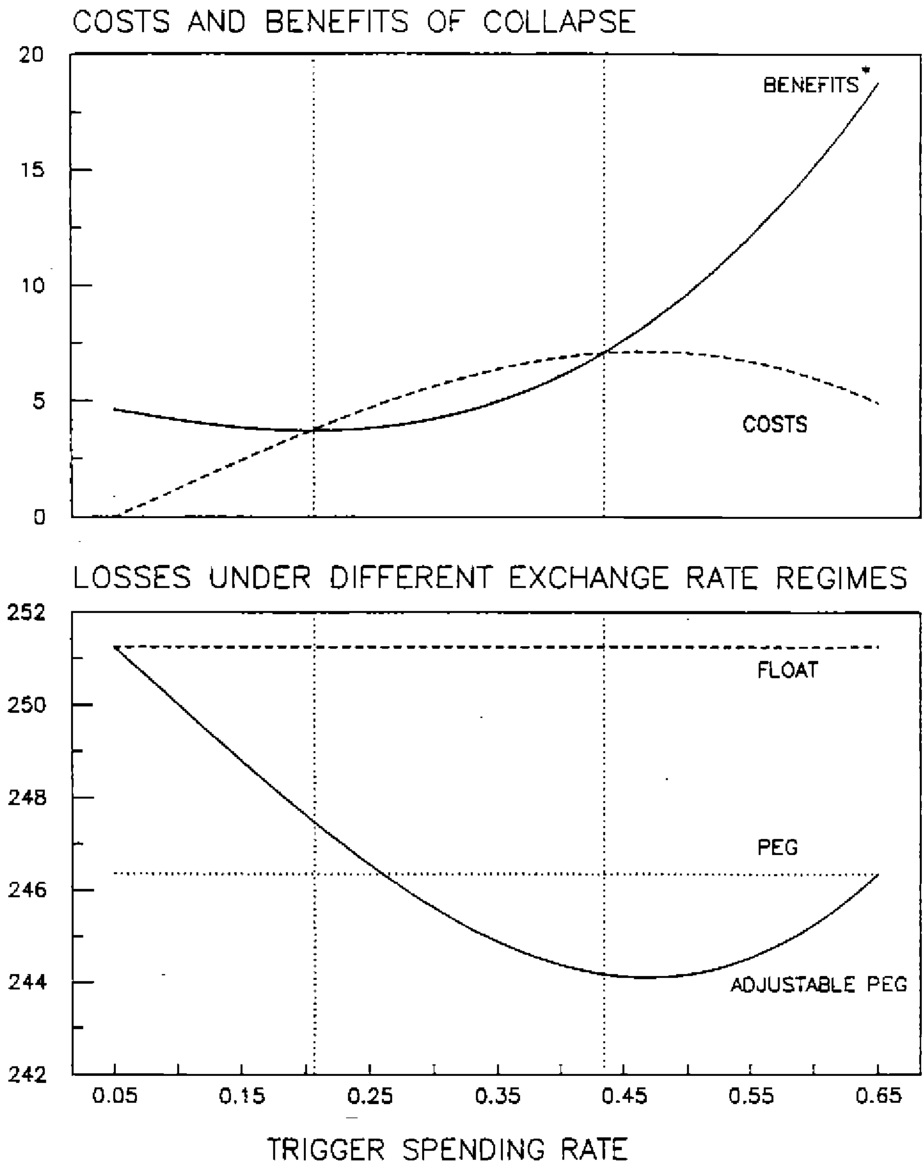

g^T
45°
$\bar{g}$
$g^T(\bar{g})$
$\underline{g}$
g_a
g_b
g_c
g_d
$\bar{g}$
$\bar{g}$

Figure 5



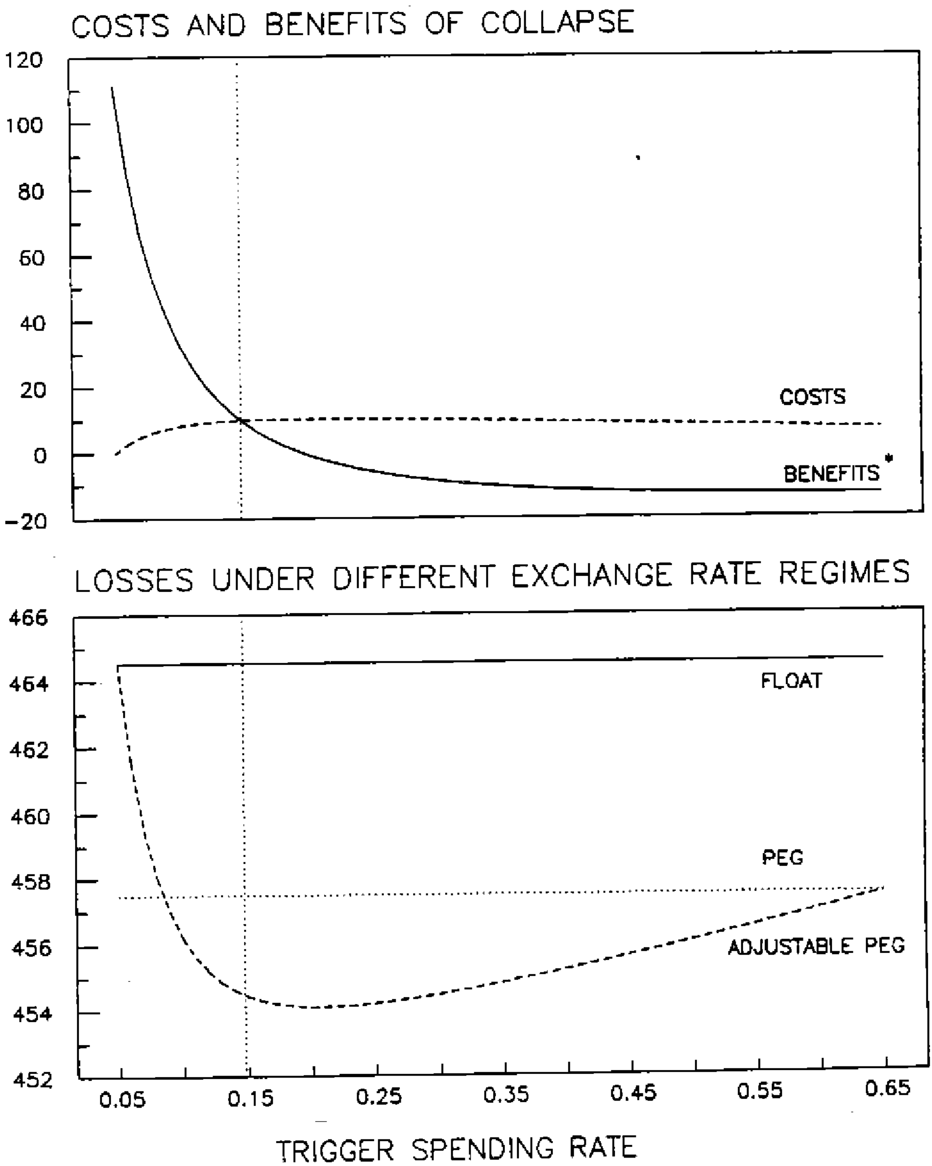
g^T
45°
$\bar{g}$
$g^T(\tilde{g})$
$\underline{g}$
$\tilde{g}$
$\underline{g}$
$\tilde{g}_1$
$\tilde{g}_2$
$\bar{g}$

Figure 6



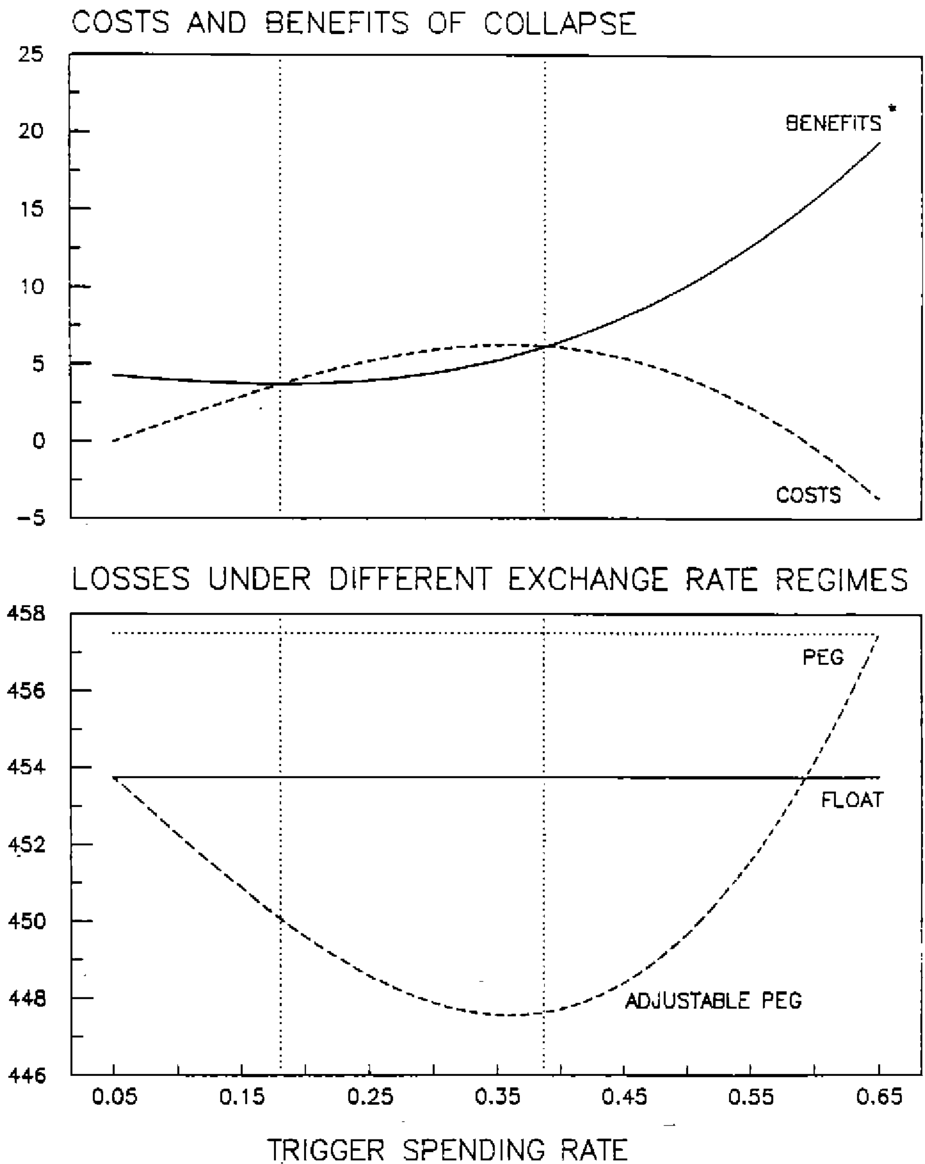
* Restrict current spending to equal the trigger level.
Note: All figures on the vertical axis are scaled up by 10^5.

Figure 7



* Restrict current spending to equal the trigger level.
Note: All figures on the vertical axis are scaled up by 10^5.

Figure 8



* Restrict current spending to equal the trigger level.
Note: All figures on the vertical axis are scaled up by 10^5.