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ABSTRACT

This paper applies some of the key insights of dynamic discrete choice models to continuous-time job search models. We propose a novel framework that incorporates preference shocks into search models, resulting in a tight connection between value functions and conditional choice probabilities. Including preference shocks allows us to establish constructive identification of all the model parameters. Our method also makes it possible to estimate rich nonstationary job search models in a simple and tractable way, without having to solve any differential equations. We apply our framework to rich longitudinal data from Hungarian administrative records, allowing for nonstationarities in offer arrival rates, wage offers, and in the flow payoff of unemployment. Longer unemployment durations are associated with substantially worse wage offers and lower offer arrival rates, which results in accepted wages falling over time.

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1 Introduction

This paper applies some of the key insights from the dynamic discrete choice literature to continuous-time job search models. The main idea of our approach is to adapt conditional choice probabilities (henceforth CCP) to a continuous-time job search environment. To do so, we incorporate preference shocks into the search framework, resulting in a tight connection between value functions and conditional choice probabilities. These preference shocks represent the relative attractiveness of a new job compared to the current state of the individual (employed or unemployed), and affect the instantaneous utility associated with that particular job. As a result, and consistent with recent empirical evidence that workers tend to accept particular job offers with probabilities that are significantly different from zero or one (Krueger and Mueller, 2016), future job offers associated with particular wages will only be accepted probabilistically from the perspective of the worker.

Our approach has two key advantages. The first one is related to identification. We consider a class of nonstationary job search models that incorporate on-the-job search, non-pecuniary job attributes, and involuntary wage transitions. We establish constructive identification of all of the model parameters, up to the discount rate. In particular, and in contrast with the well-known non-identification result of Flinn and Heckman (1982), we are able to separately identify the offered wage distribution both from employment and unemployment—the latter allowed to vary over the course of unemployment—without having to assume recoverability of the underlying distribution. Central to our identification strategy is the existence of preference shocks that allow us to trace out the full offered wage distribution from the observed job-to-job transitions, and express the employment and unemployment value functions as functions of the conditional probabilities of accepting particular job offers. Under this framework, we are able to derive closed-form expressions for most of the model parameters where the expressions depend on the hazard rates associated with the different types of labor market transitions.

The second advantage is computational. While the empirical labor search literature has been rapidly growing over the last few years, structural estimation of these models often remains challenging. This is particularly true for models in nonstationary environments, which tend to be the norm rather than the exception in the context of job search (van den Berg, 1990, 2001, Cahuc, Carcillo, and Zylberberg, 2014). We provide in this paper a novel empirical framework that makes it possible to estimate nonstationary job search models in a simple, tractable, and transparent way.

We apply our method using rich longitudinal administrative data from Hungary. The dataset consists of half of the population, i.e., 4.6 million individuals, who are linked across 900 thousand firms. An important feature of the Hungarian data is that individuals are observed on a monthly basis, making it possible to follow the labor force transitions at a high frequency.[1] In practice we consider a flexible parametric specification that allows for unobserved heterogeneity through worker types, and devise a novel sequential estimation procedure that adapts the insights of Arcidiacono and Miller (2011) to a continuous-time search environment.

The data reveal sharp decreases over time in accepted wages out of unemployment. Among those who find a job before benefit expiration, job seekers with the shortest 25% of unemployment durations were a little over half as likely to exit to a minimum wage job than those with the longest 25% of unemployment durations. Estimates of the model show that this, in part, is the result of the wage offer distribution shifting to the left as unemployment duration increases. With the offer arrival rate also declining over the course of unemployment, job seekers become increasingly less selective in which jobs they are willing to accept. The decline in accepted wages is then a result both of facing worse wage offer distributions but also changes in the job acceptance rate. An important takeaway from these results is that nonstationarities along multiple dimensions play a central role in accounting for the job search environment over the course of unemployment.

This paper fits into several literatures. First, it contributes to the literature on the identification and estimation of dynamic discrete choice models using conditional choice probabilities. Since the seminal articles of Hotz and Miller (1993) and Magnac and Thesmar (2002), CCP methods have been increasingly used as a way to identify, and estimate complex dynamic discrete choice models at a limited computational cost (see surveys by Aguirregabiria and Mira, 2010 and Arcidiacono and Ellickson, 2011).

[1] A substantial share of job-to-job transitions in our data involve a wage decrease, which our model rationalizes with preference shocks.

While CCP methods have been used a variety of settings, they have been mostly used in a discrete time environment. Exceptions are Arcidiacono, Bayer, Blevins, and Ellickson (2016), Agarwal, Ashlagi, Rees, Somaini, and Waldinger (2021) and work in progress by Llull and Miller (2018), who apply CCP methods to estimate continuous-time dynamic equilibrium models of market competition, an equilibrium model of kidney allocations, and a stationary dynamic model of job and location choices in the context of internal migration in Spain, respectively. In discrete-time setups, CCP methods are also generally used to estimate dynamic discrete choice models in the absence of search frictions, an exception being recent work by Ransom (2022). We contribute to this literature by exploring the use of CCP methods to constructively identify and estimate job search models in continuous time.

This paper also contributes to the empirical job search literature. Since the seminal work of Flinn and Heckman (1982), a large number of papers have structurally estimated various types of job search models (see Eckstein and van den Berg, 2007 for a survey, and French and Taber, 2011 for an overview of the identification of job search models). In this literature, structural parameters are generally estimated via maximum likelihood or indirect inference methods, where the full model needs to be solved within the estimation procedure, and are often based on a strict job acceptance cutoff based on whether the offer exceeds the reservation wage. Nonstationarity in job search, which arises in particular when the level of unemployment benefits varies over the unemployment spell, is an important case where the computational demands are especially high. Since the seminal work of van den Berg (1990) who structurally estimated a continuous-time nonstationary search model,[2] examples of structural estimates of nonstationary job search models remain scarce. Important exceptions include Cockx, Dejemeppe, Launov, and Van der Linden (2018), Launov and Walde (2013), Robin (2011), Lollivier and Rioux (2010), Paserman (2008), and Frijters and van der Klaauw (2006).

We contribute to this literature by providing a novel empirical framework, based on a constructive identification strategy, that makes it possible to estimate a rich class of nonstationary job search models in a simple and tractable way. Key to our

[2] See also Wolpin (1987), which is the first study to estimate a (discrete time) nonstationary search model.

identification strategy is the existence of preference shocks and, in that sense, our approach is similar in spirit to Sorkin (2018). Our paper also complements recent work by Sullivan and To (2014) and Taber and Vejlin (2020) who consider the identification of search models that allow for non-pecuniary job attributes. In contrast to these papers, we consider a nonstationary environment and establish constructive identification of the model parameters, most of them being obtained as closed-form expressions of the underlying hazard rates. However, an important difference with Taber and Vejlin (2020) is that they consider an equilibrium search framework, while our framework is set in partial equilibrium.

Finally, our application fits into the vast empirical literature that investigates the impact of unemployment benefit levels and duration on labor supply (see, e.g., Johnston and Mas, 2018, Nekoei and Weber, 2017, Le Barbanchon, Rathelot, and Roulet, 2017, Lollivier and Rioux, 2010, Card, Chetty, and Weber, 2007, van den Berg, 1990, and Schmieder and von Wachter, 2016 and Krueger and Meyer, 2002 for overviews of this literature). Consistent with many of these earlier studies, our estimation results provide evidence that nonstationarity plays an important role in describing the search environment over the course of the unemployment spell. A central and distinctive feature of our empirical strategy is that it leverages the direct links that exist between reduced form hazard rates from unemployment to employment, or from one job to another, and the structural parameters of the model. Beyond the specific application we consider in this paper, a similar approach can be readily used to identify and estimate a wide range of search models (see Gyetvai, 2021, for a recent application to occupational mobility).

The rest of the paper is structured as follows. In Section 2, we introduce and discuss the general setup of the nonstationary search model we consider throughout the paper. Section 3 establishes identification of the model parameters. In Section 4 we discuss the data used to estimate the model. Section 5 presents our estimation procedure, with Section 6 discussing the estimation results. Section 7 concludes.

2 Model

2.1 The environment

Consider an economy in continuous time with infinitely lived workers, who discount the future at a rate $\rho > 0$. Both employed and unemployed workers engage in job search. Job offers are characterized by a wage, w, and a job type, s. Job types capture non-wage characteristics such as firm, occupation, industry, or any particular non-monetary job attribute. The distribution of wages and job types are assumed to be discrete with a finite number of support points, denoted by W and S respectively. The support for wages and job types is given by $\Omega_w = \{\underline{w}, \ldots, \overline{w}\}$ and $\Omega_s = \{\underline{s}, \ldots, \overline{s}\}$. Conditional on receiving an offer from a particular job type s, the offered wage distributions depend on whether or not one is currently employed and, if not employed, the duration of unemployment, which we denote by t. The probability mass functions (pmf) of the wage offer distributions evaluated at wage w are given by f_w^s for the employed, and $g_w^s(t)$ for the unemployed at unemployment duration t.

We model job offer arrivals from the different job types as Poisson processes, and allow employed and unemployed workers to sample job offers at different frequencies. While working at a job of type s, the offer arrival rate for jobs of type s' is given by $\lambda^{ss'}$. The offer arrival rate for the unemployed for type-s jobs may vary with the duration of the unemployment spell, and is given by $\lambda^s(t)$. Unemployed workers also receive benefits that depend on the duration of the spell.[3] The wage offer distribution $(g_w^s(t))$, the unemployed offer arrival rates $(\lambda^s(t))$, and the flow payoff of unemployment $(b(t))$ are the three sources of nonstationarity in this setup.

While this model shares many of the features of the continuous-time job search models that have been estimated in the literature, a key distinction is that it incorporates preference shocks into the search framework. This feature is instrumental to our approach as it allows us to connect the value functions of unemployment and employment to the conditional choice probabilities of accepting particular job offers. Specifically, any given job offer is associated with a wage and a job type, but also with a preference shock. This preference shock, ε, is drawn independently whenever

[3]In practice, following much of the empirical search literature, we treat unemployment and non-participation as a single state.

a new job offer arrives. The preference shock represents the relative attractiveness of a new job compared to the current state of the individual (employed or unemployed), and affects the instantaneous utility. Our model also incorporates job switching costs, which in our application play an important role in fitting the observed job mobility flows.[4]

2.2 Value of employment

The flow payoff of employment is assumed to be the sum of two parts: the utility of the wage paid, u_w, and the non-pecuniary payoff of working in a job of type s, ϕ^s. Without loss of generality, we normalize $\phi^1 = 0$. Workers employed in a job (w, s) can experience three different types of transitions. First, they may be laid off and become unemployed, which happens at a rate δ_0^s.[5] Second, within the same firm, they may exogenously transition to a different wage w' and job type s'. These involuntary within-firm changes occur at a rate $\delta_{ww'}^{ss'}$, with the convention that $\delta_{ww}^{ss} = 0$. Third, workers may receive an offer from another firm for a job of type s' at a rate $\lambda^{ss'}$ and then decide whether to accept it or stay with their current job. These voluntary transitions are associated with an instantaneous cost of switching jobs, $c^{ss'}$, where we assume that the switching costs are symmetric (i.e. $c^{ss'} = c^{s's}$ for all s, s'). These cross-firm transitions occur both between ($s' \neq s$) and within ($s' = s$) job types.

We now turn to the value of employment. The Bellman equation for the value of employment V_w^s associated with a job (w, s) writes as follows:

$$\begin{aligned}\left(\rho + \delta_0^s + \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'} + \sum_{s'} \lambda^{ss'}\right) V_w^s \quad = \quad & u_w + \phi^s + \delta_0^s V_0(0) + \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'} V_{w'}^{s'} \\ & + \sum_{w'}\sum_{s'} \lambda^{ss'} f_{w'}^{s'} \mathbb{E}_\varepsilon \max\left\{V_{w'}^{s'} - c^{ss'} + \varepsilon, V_w^s\right\}\end{aligned} \tag{2.1}$$

where $V_0(0)$ is the value of unemployment immediately upon entering an unemployment spell ($t = 0$). Following McFadden (1978) and Arcidiacono and Miller (2011),

[4] Preference shocks have an alternative, isomorphic interpretation as the stochastic component of job switching costs.

[5] Our identification strategy would also readily apply to a more general setup where transitions to unemployment are allowed to be wage-specific. For simplicity, we focus on the case where these transition rates depend on job types only.

we can re-express Equation (2.1) such that some of the value functions on the right-hand side are eliminated. Namely, assuming that the shocks ε are drawn from a standard logistic distribution, we can rewrite the Bellman equation as:

$$\begin{aligned}\left(\rho + \delta_0^s + \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'}\right) V_w^s &= u_w + \phi^s + \delta_0^s V_0(0) + \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'} V_{w'}^{s'} \\ &\quad - \sum_{w'}\sum_{s'} \lambda^{ss'} f_{w'}^{s'} \ln\left(1 - p_{ww'}^{ss'}\right) \end{aligned} \tag{2.2}$$

where $p_{ww'}^{ss'}$ denotes the probability of accepting a new job offer of type s' at wage w' given the current job type s and wage rate w.

Prior to the realization of ε, the probability of a job of type s' paying w' being accepted given current job type s paying w is then:

$$p_{ww'}^{ss'} = \frac{\exp\left(V_{w'}^{s'} - c^{ss'}\right)}{\exp\left(V_w^s\right) + \exp\left(V_{w'}^{s'} - c^{ss'}\right)} \tag{2.3}$$

2.3 Value of unemployment

We now write the problem of the unemployed individuals. Indexing by t time spent unemployed, we first write the Bellman equation for the unemployment value function $V_0(t)$ in discrete time:[6]

$$\begin{aligned} V_0(t) &= b(t)\Delta t + \frac{\Delta t}{1+\rho\Delta t}\sum_w\sum_s \lambda^s(t) g_w^s(t+\Delta t)\mathbb{E}_\varepsilon \max\left\{V_w^s + \varepsilon, V_0(t+\Delta t)\right\} \\ &\quad + \frac{1 - \sum_s \lambda^s(t)\Delta t}{1+\rho\Delta t} V_0(t+\Delta t) \end{aligned}$$

where Δt denotes the discrete time unit and where the equation can be rewritten as:

$$\begin{aligned} \rho V_0(t) &= b(t)(1+\rho\Delta t) + \sum_w\sum_s \lambda^s(t) g_w^s(t+\Delta t)\mathbb{E}_\varepsilon \max\left\{V_w^s - V_0(t+\Delta t) + \varepsilon, 0\right\} \\ &\quad + \frac{V_0(t+\Delta t) - V_0(t)}{\Delta t} \end{aligned}$$

Next, letting $\Delta t \to 0$, and denoting by $\dot{V}_0(t)$ the derivative of $V_0(t)$ (with respect to

[6] Note that we implicitly normalize to zero the switching cost from unemployment to employment, which in our setup is not separately identified from the value of unemployment.

unemployment duration) and by $p_w^s(t)$ the probability of accepting a job offer of type s and wage w at time t, we obtain the following differential equation in $V_0(\cdot)$:

$$\rho V_0(t) = b(t) - \sum_w \sum_s \lambda^s(t) g_w^s(t) \ln\left(1 - p_w^s(t)\right) + \dot{V}_0(t) \tag{2.4}$$

A couple of remarks are in order. First, Equation (2.4) now involves the time derivative of the value of unemployment. This term represents the change in the option value of job search due to variation over time in the value of unemployment. In the particular case where nonstationarity arises as a result of over-time changes in the level of unemployment benefits, the option value of searching for a job will decrease as job seekers get closer to the unemployment benefit expiration date.

Second, Equation (2.4) is a simple linear first-order differential equation in $V_0(\cdot)$, which admits under standard regularity conditions an exact analytical solution as a function of the structural parameters and the conditional choice probabilities $p_w^s(t)$.[7] In the absence of preference shocks, $V_0(t)$ would satisfy instead the following nonlinear differential equation:

$$\rho V_0(t) = b(t) + \sum_s \sum_w \lambda^s(t) g_w^s(t) \max\left\{V_w^s - V_0(t), 0\right\} + \dot{V}_0(t)$$

This type of nonlinear differential equation would need to be solved numerically, similar to van den Berg (1990) in a simpler context without on-the-job search.

3 Identification

We have shown in the previous section that the unemployment and employment value functions can be expressed as a function of the structural parameters of the model, the wage offer distributions, and the conditional job acceptance probabilities. There are two fundamental differences compared to a Hotz-Miller type CCP-based approach for dynamic discrete choice models. First, in a search environment, choices (i.e., job offer acceptance or rejection) are partially unobserved by the analyst. Second, wage

[7]Sufficient regularity conditions are the continuity of the functions $t \mapsto \lambda^s(t)$, $t \mapsto g_w^s(t)$ and $t \mapsto p_w^s(t)$. As we discuss in the section below, $V_0(t)$ and $\dot{V}_0(t)$ can be directly identified (and estimated) from the log-odds ratios out of unemployment, without having to solve any differential equation.

offers are generally unobserved as well. Nonetheless, we provide in the following a simple and constructive identification strategy for the parameters of the job search model introduced in Section 2. These identification results hold in an empirical setting where one has access to longitudinal data on (i) across-firm job-to-job transitions, (ii) within-firm transitions, (iii) transitions from unemployment to employment, and (iv) transitions from employment to unemployment.

Recall that we assume that wages are drawn from a discrete distribution with finite support. This distribution can be thought of as a discrete approximation to an underlying continuous wage distribution. We maintain this assumption throughout our analysis for simplicity, but note that our identification strategy readily applies to the case of continuous wage distributions.[8]

3.1 Assumptions

We first introduce four assumptions that relate to the types of transitions that are observed in the data. Namely, we denote by A1, A2, A3 and A4, respectively, the assumptions that the following hazard rates are identified from the data:

- **A1** $h_{ww'}^{ss'}$, the hazard rate of moving from a job with wage w and type s to a job with wage w' and type s' (in a different firm);
- **A2** $h_w^s(t)$, the hazard rate out of unemployment at time t to a job that pays w and is of type s (assumed to be continuously differentiable);
- **A3** $\delta_{ww'}^{ss'}$, the hazard rate of within-firm wage (w to w') and type (s to s') changes;
- **A4** δ_0^s, the hazard rate from a type-s job to unemployment.

As is standard for this class of models, we also maintain the assumption that the discount rate ρ is known.

We next show that these hazard rates can be used to recover closed-form solutions for the employed and unemployed wage offer distributions (f_w^s and $g_w^s(t)$); the pecuniary

[8]Specifically, a key observation here is that, for any given pair of wages (w, w'), the hazard rates associated with the transitions to wage w' conditional on current wage w are directly identified from the data. Such hazard rates are also known in the statistical literature as the conditional mark-specific hazard function (see, e.g., Sun et al., 2009, Equation (1) p.395).

and non-pecuniary payoffs of the job (u_w and ϕ^s) each up to a constant; the cost of switching jobs ($c^{ss'}$); the job offer arrival rates for those who are employed and unemployed ($\lambda^{ss'}$ and $\lambda^s(t)$); and the flow payoff of unemployment ($b(t)$). All of our identification results are subject to the model specification given in Section 2.

3.2 Employed-side parameters

We begin by showing identification of the employed wage offer distributions for each job type, f_w^s, which we establish in the following lemma:

Lemma 1 *Assume that Assumption A1 holds. Then* f_w^s *is identified and can be written as follows:*

$$f_w^s = \frac{h_{ww}^{ss}}{\sum_{w'} h_{w'w'}^{ss}} \tag{3.1}$$

To prove this result, first note that the hazard $h_{ww'}^{ss'}$ can be expressed as the product of (i) the arrival rate of offers to job type s' given the current job type is s ($\lambda^{ss'}$), (ii) the pmf. of w' for offered wages in job type s' ($f_{w'}^{s'}$), and (iii) the probability of accepting a job of type s' paying wage w' given current job type s and wage w' ($p_{ww'}^{ss'}$):

$$h_{ww'}^{ss'} = \lambda^{ss'} f_{w'}^{s'} p_{ww'}^{ss'} \tag{3.2}$$

Now consider the hazard rate of a transition to a job that is of the same type and pays the same amount as the current job (h_{ww}^{ss}). From Equation (2.3), the probability of accepting a job in this case does not depend on w: $p_{ww}^{ss} = p_{w'w'}^{ss} = \frac{\exp(-c^{ss})}{1+\exp(-c^{ss})}$ for all $(w, w') \in \Omega_w^2$. That is, since for these transitions the wage and job type is held fixed, so too is the value function. Hence when the transitions are to same-type and same-pay jobs, the ratio of the hazards for two different initial wages is the ratio of the pmfs for the two wages:

$$\frac{f_w^s}{f_{w'}^s} = \frac{h_{ww}^{ss}}{h_{w'w'}^{ss}}$$

Summing over w' then gives the result:

$$f_w^s = \frac{h_{ww}^{ss}}{\sum_{w'} h_{w'w'}^{ss}}$$

Next, we consider in Lemma 2 below the identification of the on-the-job offer arrival rates ($\lambda^{ss'}$), which then immediately leads to identification of the conditional choice probabilities and switching costs.

Lemma 2 *(i) Assume that Assumption A1 holds and that there exists a triple* $(w, w', \tilde{w}) \in \Omega_w^3$ *such that* $f_{\tilde{w}}^s h_{ww'}^{ss} h_{w'w}^{ss} \neq f_{w'}^s h_{\tilde{w}w}^{ss} h_{w\tilde{w}}^{ss}$*. Then* λ^{ss}*,* $p_{ww'}^{ss}$ *and* c^{ss} *are identified.*

(ii) For $x \in \{w', \tilde{w}\}$ *and* $s \neq s'$*, let* $A_x = f_x^{s'} f_x^s h_{ww}^{ss'} h_{ww}^{s's} - f_w^{s'} f_w^s h_{xx}^{ss'} h_{xx}^{s's}$*,* $B_x = f_x^{s'} h_{xx}^{s's} h_{ww}^{ss'} h_{ww}^{s's} - f_w^{s'} h_{ww}^{s's} h_{xx}^{ss'} h_{xx}^{s's}$*, and* $C_x = f_w^s h_{xx}^{ss'} h_{ww}^{ss'} h_{ww}^{s's} - f_w^s h_{ww}^{ss'} h_{xx}^{ss'} h_{xx}^{s's}$*. Assume that Assumption A1 holds and that there exists a triple* $(w, w', \tilde{w}) \in \Omega_w^3$ *such that the following conditions hold:*

(a) $A_{w'} \neq 0$

(b) $B_{w'} A_{\tilde{w}} - B_{\tilde{w}} A_{w'} \neq 0$

(c) $A_{w'} C_{\tilde{w}} - A_{\tilde{w}} C_{w'} \neq 0$

then $\lambda^{ss'}$*,* $p_{ww'}^{ss'}$*,* $c^{ss'}$ *and* $V_{w'}^{s'} - V_w^s$ *are identified (the latter two under the symmetry assumption* $c^{ss'} = c^{s's}$*).*

Further, when the conditions stated in (i) and (ii) are met, there are closed-form expressions for $\lambda^{ss'}$*,* $c^{ss'}$*,* $p_{ww'}^{ss'}$ *and* $V_{w'}^{s'} - V_w^s$*, for all* $(w, w', s, s') \in \Omega_w^2 \times \Omega_s^2$*, as a function of the underlying hazard rates.*

We show identification and the closed-form expression for λ^{ss} in the text with the corresponding proof for $\lambda^{ss'}$ given in Appendix A.1.1. To begin, note that the distributional assumption on the preference shocks ε yields a simple relationship between probabilities of accepting a new job offer, the employment value functions, and the switching cost:[9]

$$\ln\left(\frac{p_{ww'}^{ss}}{1 - p_{ww'}^{ss}}\right) = V_{w'}^s - c^{ss} - V_w^s \tag{3.3}$$

implying:

$$\ln\left(\frac{p_{ww'}^{ss}}{1 - p_{ww'}^{ss}}\right) + \ln\left(\frac{p_{w'w}^{ss}}{1 - p_{w'w}^{ss}}\right) = -2c^{ss} \tag{3.4}$$

[9]Recall that the preference shocks are assumed to follow a standard logistic distribution. Note in particular that the scale parameter of the shocks distribution would not be separately identified from the flow utility of wages and unemployment.

Solving Equation (3.2) for $p^{ss}_{ww'}$,

$$p^{ss}_{ww'} = \frac{h^{ss}_{ww'}}{\lambda^{ss} f^s_{w'}} \tag{3.5}$$

it then follows that, for any given triple $(w, w', \tilde{w}) \in \Omega^3_w$:

$$\ln\left(\frac{h^{ss}_{ww'}}{\lambda^{ss} f^s_{w'} - h^{ss}_{ww'}}\right) + \ln\left(\frac{h^{ss}_{w'w}}{\lambda^{ss} f^s_{w} - h^{ss}_{w'w}}\right) = \ln\left(\frac{h^{ss}_{w\tilde{w}}}{\lambda^{ss} f^s_{\tilde{w}} - h^{ss}_{w\tilde{w}}}\right) + \ln\left(\frac{h^{ss}_{\tilde{w}w}}{\lambda^{ss} f^s_{w} - h^{ss}_{\tilde{w}w}}\right)$$

Solving for λ^{ss} under the assumption that $f^s_{\tilde{w}} h^{ss}_{ww'} h^{ss}_{w'w} \neq f^s_{w'} h^{ss}_{w\tilde{w}} h^{ss}_{\tilde{w}w}$—a condition that can be verified in the data—gives the result:

$$\lambda^{ss} = \frac{\left(f^s_w h^{ss}_{w\tilde{w}} + f^s_{\tilde{w}} h^{ss}_{\tilde{w}w}\right) h^{ss}_{ww'} h^{ss}_{w'w} + \left(f^s_w h^{ss}_{ww'} + f^s_{w'} h^{ss}_{w'w}\right) h^{ss}_{w\tilde{w}} h^{ss}_{\tilde{w}w}}{f^s_w f^s_{\tilde{w}} h^{ss}_{ww'} h^{ss}_{w'w} - f^s_w f^s_{w'} h^{ss}_{w\tilde{w}} h^{ss}_{\tilde{w}w}}$$

Given the expressions of $f^s_{w'}$ and λ^{ss}, closed-form expressions for $p^{ss}_{ww'}$ and c^{ss} then immediately result from Equations (3.5) and (3.4), as does the difference in value functions $V^s_{w'} - V^s_w$ from Equation (3.3).

Lemma 3 below states our main identification result for the remaining set of employed-side parameters, namely the utility of wages, u_w, and the non-pecuniary payoff of working in a job of type s, ϕ^s.

Lemma 3 *Given Assumptions A1, A3, and A4:*

(i) u_w *is identified up to a constant and has a closed-form expression.*

(ii) When workers have CRRA preferences so that $u_w = \frac{\alpha w^{1-\theta}}{1-\theta}$*, both* α *and the risk aversion parameter* θ *are identified.*

(iii) Given the normalization $\phi^1 = 0$*, the non-pecuniary payoffs* ϕ^s *are a known linear function of* $V_0(0)$*.*

We prove part (i) of Lemma 3 in the text with proofs of the remaining parts in Appendix A.1.2. We begin by eliminating the employment value functions on the right hand side of Equation (2.2). To do this, note that we can use the log-odds ratio to express $V_{w'}^{s'}$ as a linear function of V_w^s, the switching cost $c^{ss'}$, and the conditional choice probabilities $p_{ww'}^{ss'}$:

$$V_{w'}^{s'} = V_w^s + c^{ss'} + \ln\left(p_{ww'}^{ss'}\right) - \ln\left(1 - p_{ww'}^{ss'}\right) \tag{3.6}$$

Equation (2.2) can then be written as:

$$\begin{aligned} V_w^s &= \Big(u_w + \phi^s + \delta_0^s V_0(0) + \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'}\left[c^{ss'} + \ln\left(p_{ww'}^{ss'}\right) - \ln\left(1 - p_{ww'}^{ss'}\right)\right] \\ &\quad - \sum_{w'}\sum_{s'} \lambda^{ss'} f_{w'}^{s'} \ln\left(1 - p_{ww'}^{ss'}\right)\Big)\Big/(\rho + \delta_0^s) \end{aligned} \tag{3.7}$$

Normalizing the flow payoff of employment in the lowest-paying job, $u_{\underline{w}}$, to zero, it follows that we can express $\ln(p_{\underline{w}w}^{ss}/(1 - p_{\underline{w}w}^{ss}))$ as:

$$\begin{aligned} \ln\left(\frac{p_{\underline{w}w}^{ss}}{1 - p_{\underline{w}w}^{ss}}\right) &= V_w^s - V_{\underline{w}}^s - c^{ss} \\ &= \frac{u_w - \sum_{w'}\sum_{s'} \lambda^{ss'} f_{w'}^{s'}\left[\ln\left(1 - p_{\underline{w}w'}^{ss'}\right) - \ln\left(1 - p_{ww'}^{ss'}\right)\right]}{\rho + \delta_0^s} \\ &+ \frac{\sum_{\tilde{w}\in\{\underline{w},w\}}\sum_{w'}\sum_{s'} (-1)^{\tilde{w}=\underline{w}}\, \delta_{\tilde{w}w'}^{ss'}\left[c^{ss'} + \ln\left(p_{\tilde{w}w'}^{ss'}\right) - \ln\left(1 - p_{\tilde{w}w'}^{ss'}\right)\right]}{\rho + \delta_0^s} - c^{ss} \end{aligned} \tag{3.8}$$

As the only unknown in Equation (3.8) is u_w, solving for u_w gives the result.[10]

3.3 Unemployed-side parameters and main identification result

We now turn to the identification of the parameters governing the transitions out of unemployment. As with the employed-side parameters, we begin by recovering the wage offer distributions, $g_w^s(t)$, which is allowed to vary over the course of unemployment.

Lemma 4 *Given Assumptions A1 through A4 and $W \geq 3$, the unemployed wage offer distribution for job type s at time t, $g_w^s(t)$, satisfies a generally overdetermined*

[10]Note that the expression substantially simplifies when there are no within-job involuntary changes ($\delta_{ww'}^{ss'} = 0$), which is the case we will consider in our application.

linear system of $W-1$ unknowns and $\frac{W(W-1)}{2}-1$ equations. A unique solution exists when the system is of full rank.

To prove Lemma 4, we note that, for job type s, the difference in the log odds from accepting a job that pays w and accepting a job that pays w' can be written as the difference in the employment value functions:

$$\ln\left(\frac{p_w^s(t)}{1-p_w^s(t)}\right)-\ln\left(\frac{p_{w'}^s(t)}{1-p_{w'}^s(t)}\right)=V_w^s-V_{w'}^s \tag{3.9}$$

It follows that the difference in the log odds of accepting any two wage offers out of unemployment depends on the (identified) difference in the employment value functions associated with these two wages only. As such, it does not vary over the course of unemployment.

The conditional choice probabilities for accepting a job at time t ($p_w^s(t)$) on the left hand side of Equation (3.9) can then be expressed as a function of the hazard out of unemployment ($h_w^s(t)$), the arrival rate ($\lambda^s(t)$), and the probability that the offer pays w ($g_w^s(t)$):

$$p_w^s(t)=\frac{h_w^s(t)}{\lambda^s(t)g_w^s(t)} \tag{3.10}$$

Denote the differenced value function $V_w^s-V_{w'}^s$ as $\kappa_{ww'}^{ss}$. Recall that $\kappa_{ww'}^{ss}$ is known from Equation (3.3). Using Equations 3.9 and 3.10, we can then express $\lambda^s(t)$ as:

$$\lambda^s(t)=\frac{h_w^s(t)h_{w'}^s(t)(\exp(\kappa_{ww'}^{ss}-1))}{g_w^s(t)h_{w'}^s(t)\exp(\kappa_{ww'}^{s})-g_{w'}^s(t)h_w^s(t)} \tag{3.11}$$

Evaluating the right hand side of Equation (3.11) for an alternative pair of wages, $(\tilde{w},\tilde{w}')$, and differencing yields:

$$\frac{h_w^s(t)h_{w'}^s(t)(\exp(\kappa_{ww'}^{ss}-1))}{g_w^s(t)h_{w'}^s(t)\exp(\kappa_{ww'}^{ss})-g_{w'}^s(t)h_w^s(t)}-\frac{h_{\tilde{w}}^s(t)h_{\tilde{w}'}^s(t)(\exp(\kappa_{\tilde{w}\tilde{w}'}^{ss}-1))}{g_{\tilde{w}}^s(t)h_{\tilde{w}'}^s(t)\exp(\kappa_{\tilde{w}\tilde{w}'}^{ss})-g_{\tilde{w}'}^s(t)h_{\tilde{w}}^s(t)}=0 \tag{3.12}$$

Denote the numerators of the two terms as $A_{ww'}^s(t)$ and $A_{\tilde{w}\tilde{w}'}^s(t)$. These can be calculated from the unemployment hazards and the previously identified differences in employment value functions. Rearranging the terms yields:

$$\begin{aligned} 0 &= A^s_{\tilde{w}\tilde{w}'}(t)h^s_{w'}(t)\exp(\kappa^{ss}_{ww'})g^s_w(t) - A^s_{\tilde{w}\tilde{w}'}(t)h^s_w(t)g^s_{w'}(t) \\ &\quad -A^s_{ww'}(t)h^s_{\tilde{w}'}(t)\exp(\kappa^{ss}_{\tilde{w}\tilde{w}'})g^s_{\tilde{w}}(t) + A^s_{ww'}(t)h^s_{\tilde{w}}(t)g^s_{\tilde{w}'}(t) \end{aligned} \tag{3.13}$$

This is a simple linear equation in its unknowns, the wage offer distribution terms. Excluding, for any given job type s and unemployment duration t, redundant equations by evaluating Equation (3.13) at the following set of wage tuples:

$$\{(w, w', \tilde{w}, \tilde{w}') : w = 1, w' = 2, \tilde{w} < \tilde{w}', (\tilde{w}, \tilde{w}') \neq (1, 2)\}$$

and noting that $g^s_{\overline{w}}(t) = 1 - \sum_{w<\overline{w}} g^s_w(t)$ yields a linear system with $W - 1$ unknowns and $\frac{W(W-1)}{2} - 1$ equations. When this generally overdetermined system is of full rank, there exists a unique (closed-form) least squares solution for $(g^s_w(t))_{w\in\Omega_w}$.

Identification of the remaining unemployed-side parameters directly proceeds from the earlier steps:

Lemma 5 *Given Assumptions A1-A4, the offer arrival rates $\lambda^s(t)$, the conditional choice probabilities $p^s_w(t)$, the flow payoff of unemployment $b(t)$, the value function of unemployment and its derivative, $V_0(t)$ and $\dot{V}_0(t)$, are identified.*

An important implication of Lemma 5 is that the non-pecuniary payoffs ϕ^s, which from Lemma 3 were only known up to $V_0(0)$, are also identified (up to the normalization $\phi^1 = 0$).

Identification of $\lambda^s(t)$ follows directly from Equation (3.11) as all the terms on the right hand side are either directly identified from the data ($h^s_w(t)$) or identified from a previous step ($\kappa^{ss}_{ww'}$ and $g^s_w(t)$). Identification of $p^s_w(t)$ then follows immediately from Equation (3.10).

To recover the unemployment value function, we express the following log odds by normalizing the future value of working relative to staying at the same job:

$$\begin{aligned}
\ln\left(\frac{p_w^s(t)}{1-p_w^s(t)}\right) &= V_w^s - V_0(t) \\
&= \Big(u_w + \phi^s + \delta_0^s V_0(0) + \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'}\left[c^{ss'} + \ln\left(p_{ww'}^{ss'}\right) - \ln\left(1-p_{ww'}^{ss'}\right)\right] \\
&\quad - \sum_{w'}\sum_{s'} \lambda^{ss'} f_{w'}^{s'} \ln\left(1-p_{ww'}^{ss'}\right)\Big)\Big/(\rho+\delta_0^s) - V_0(t) \qquad (3.14)
\end{aligned}$$

where the second equality follows directly from Equation (3.7).

Evaluating the previous equation at the start of the unemployment spell ($t=0$) and solving for $V_0(0)$ yields:

$$\begin{aligned}
V_0(0) &= \frac{1}{\rho}\Big[u_w + \phi^s + \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'}\left[c^{ss'} + \ln\left(p_{ww'}^{ss'}\right) - \ln\left(1-p_{ww'}^{ss'}\right)\right] \\
&\quad - \sum_{w'}\sum_{s'} \lambda^{ss'} f_{w'}^{s'} \ln\left(1-p_{ww'}^{ss'}\right)\Big] - \frac{\rho+\delta_0^s}{\rho}\ln\left(\frac{p_w^s(0)}{1-p_w^s(0)}\right) \qquad (3.15)
\end{aligned}$$

Note that at this stage everything on the right hand side is known, so that this equality identifies $V_0(0)$. Plugging $V_0(0)$ into Equation (3.14) then identifies $V_0(t)$ (for all $t \geq 0$), and thus also $\dot{V}_0(t)$. It follows that one can directly identify the flow payoff of unemployment $b(t)$ from the Bellman equation (2.4):

$$b(t) = \rho V_0(t) + \sum_{w}\sum_{s} \lambda^s(t) g_w^s(t) \ln\left(1-p_w^s(t)\right) - \dot{V}_0(t) \qquad (3.16)$$

A remarkable implication of these results is that, by exploiting the tight connection between value functions and conditional choice probabilities, we are able to recover the structural parameters of this nonstationary job search model without solving any differential equation.

Finally, our main identification result follows from Lemmas 1 through 5:

Theorem 1 *Given Assumptions A1-A4, all of the employed and unemployed-side parameters are identified subject to a normalization of one* u_w *and one* ϕ^s*, and subject to the rank conditions given in Lemmas 2 and 4.*

Our identification strategy can be extended to more general search models. In particular, one can allow for aggregate shocks to the economy. Namely, we assume that the economy is in one of K states, $k \in \{1, \ldots, K\}$, with the transition rate from state k to k' denoted by $q_{kk'}$. Different states of the economy then affect the job destruction rates, δ^s_k, the within-employer type and wage transitions, $\delta^{ss'}_{ww'k}$, the offer arrival rates, $\lambda^{ss'}_k$, and the offer distributions, f^s_{wk}. Appendix A.2 shows that constructive identification holds in this case as well, under the assumption that the econometrician observes the market state, and therefore identifies $q_{kk'}$ and the hazards in A1 through A4, but now conditional on market state. The key insight is that, on the employed side, the introduction of market states has no effect on the identification proof for the offered wage distribution, offer arrival rates, conditional choice probabilities, and switching costs. Given that, identification of the remaining parameters follows trivially.

4 Application to job search in Hungary: background and data

4.1 Setup

We apply our method to a special case of the job search model described in Section 2, in which there is one job type only ($S = 1$) and no involuntary wage transitions ($\delta^{ss'}_{ww'} = 0$, for all (s, s', w, w')). While this model shares the key features of nonstationary job search models that have been estimated in the literature (see, in particular, van den Berg, 1990, Lollivier and Rioux, 2010), an important distinction is that it incorporates preference shocks into the search framework.

4.2 Institutional background

Our analysis focuses on the period from January 2003 to October 2005. During this period, Hungary had a two-tier unemployment insurance system. Only those were eligible for second-tier benefits who had a sufficiently long work history, and benefit payments in the second tier were lower than in the first. Those who exhausted benefits in both tiers were eligible for social assistance. Tier 1 benefits expired in 270 days and Tier 2 benefits expired in an additional 90 days. We focus on unemployed workers leaving unemployment in Tier 1, because Tier 2 benefits were low ($114 per month on average over our period of interest) and very similar to the amount of social assistance that anyone is eligible for, regardless of prior work history. As such, Tier 2 benefits likely did not provide significant further incentive to remain in unemployment.[11]

4.3 Data

We estimate the model using matched employer-employee data from Hungarian administrative records, provided by the Center for Economic and Regional Studies at the Hungarian Academy of Sciences (CERS-HAS). The dataset used in this analysis combines data from five administrative sources: (i) the National Health Insurance Fund of Hungary; (ii) the Central Administration of National Pension Insurance; (iii) the National Tax and Customs Administration of Hungary; (iv) the Public Employment Service National Labor Office; and (v) the Educational Authority. This dataset has been used in several recent papers, including DellaVigna, Lindner, Reizer, and Schmieder (2017), Harasztosi and Lindner (2019), and Verner and Gyöngyösi (2020).

The sample consists of half of the population, i.e., 4.6 million individuals, linked across 900 thousand firms. On the individual side, a de facto 50% random sample of the Hungarian population is observed; every Hungarian citizen born on January 1, 1927 and every second day thereafter are included. A key distinctive feature of the Hungarian data is their frequency: job spells are observed on a monthly basis, and unemployment spells are observed at a daily frequency. When working, one individual can be present in at most two work arrangements: labor market measures, such as wages and days worked, are observed separately for each one of them. We also have information on demographics, total earnings and days worked, as well as, for job seekers, unemployment benefit payments. On the firm side, all firms are included at which any sampled individuals are observed to have worked for at least one month. From these data, we can infer the length of the employment spells, as well as job-to-job transitions from changes in firm identifiers.

We estimate the model using a sample of employment spells over the period January

[11]In practice, we choose to censor durations at 269 days as a disproportionally large number of workers are recorded as claiming Tier 1 benefits up until exactly 270 days. This suggests that some of these workers might actually have started working before that point.

2003 to October 2005, and unemployment spells from January 2004 to October 2005.[12] We focus on males who were older than 18 in the beginning of our sample and younger than 40 at the end: we drop females from our sample to abstract from differential labor market flows resulting in part from childbearing decisions.[13] Furthermore, we drop older males to abstract from differential search behavior as retirement nears, with a retirement age of 43 for males in certain occupations.[14]

Because of some recoding of jobs around the first day of the year, for job-to-job transitions we treat employment spells that go past December 31st of a particular year as right-censored. Given that the employed data set tracks where the individuals are employed on the 15th of the month, there can be issues with distinguishing whether there was a job-to-job transition versus a short break between two jobs. As a result, we further right censor jobs at October 31st in each year to allow for a consistent coding of job-to-job transitions within a month. Appendix B describes our data cleaning process.

Table 1 shows summary statistics for the employment spells. In a given year, eleven percent of workers have two or more employment spells. Eighty percent of employment spells are right-censored. Among those that are not right-censored, 28% end in a transition to another job, with the remaining entailing transitions to unemployment.

For the purposes of estimation, we discretize wages into fifty bins. The first bin contains wages around the minimum wage (namely between 75 and 107% of the effective minimum wage in a given year), with the remaining bins set to be evenly distributed based on the distribution of current wages in each calendar year.[15] Whenever we use wage levels in a given bin (e.g., for the utility of wages), we take the mean wage in each bin of the distribution of current wages in 2004, except for the first bin where we use the 2004 minimum wage. For the purposes of describing the data below, we follow a similar procedure but discretize wages into ten bins.

Table 2 shows the number of employment-to-employment transitions to particular wage bins given the current wage bin. Excluding transitions to the first bin, the most

[12] Unemployment data is only available from January 2004 onwards.

[13] The female labor force participation rate in Hungary was 54.0 percent in 2004, 5.8 percentage points lower than the OECD average in the same time period.

[14] Our final sample consists of 1,314,384 employment and 15,454 unemployment spells.

[15] See Appendix B.3 for additional details on the wage discretization process.

Table 1: Summary statistics, employment spells

	Number of spells per year					
	1	*2*	*3*	*4*	*5*	*6*
In whole history (%)	16.8	15.6	48.9	14.1	3.6	0.9
In a given year (%)	88.9	10.4	0.7	0.0	0.0	0.0

	Destination		
	EE	*EU*	*RC*
Share (%)	5.7	14.4	79.9

	Mean	Percentiles				
		10	*25*	*50*	*75*	*90*
Duration (year)	0.621	0.148	0.380	0.833	0.833	0.833
Current wage (HUF)	3,681	1,726	1,874	2,536	4,238	7,025

Notes: The top panel shows the share of individuals with a given number of employment spells in their history, as well as the share of individual-years with a given number of employment spells. Durations are right-censored at October 31st each year. The middle panel shows the fraction of employment spells that end in an employment-to-employment, employment-to-unemployment transition, or are censored. The bottom panel shows summary statistics of the duration and current daily wage of employment spells. 200 HUF ≈ 1 USD in 2004.
Source: CERS-HAS, authors' own calculations.

Table 2: Employment-to-employment transition counts by wage bins

Current wage	Accepted wage: *1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	13,711	2,116	1,269	1,368	1,184	992	1,045	1,025	660	480
2	2,677	1,378	646	604	498	357	388	387	213	133
3	1,247	672	831	625	429	350	351	278	150	82
4	1,324	540	574	1,228	868	584	494	392	216	115
5	1,145	319	325	594	963	741	625	464	263	119
6	823	232	234	333	595	925	858	593	324	169
7	798	248	218	273	357	544	1,236	1,048	520	217
8	760	201	160	213	301	354	573	1,536	1,057	471
9	474	134	74	129	165	181	275	538	1,408	1,088
10	367	68	78	91	101	150	206	356	604	3,612

Notes: For exposition's sake, the table uses 10 wage bins instead of 50 as in our empirical illustration. The first bin contains wages between 75 and 107% of the effective minimum wage. Subsequent bins are equally sized percentiles of the distribution of current wages.
Source: CERS-HAS, authors' own calculations.

populous cells are those that involve within-bin transitions, the second most populous cells are ones involving a transition to one bin higher, and the third most populous cells are ones involving a transition to one bin lower.[16] There are also a number of transitions involving substantial wages changes in both directions.

Table 3: Summary statistics, employment-to-employment transitions

	Overall	**By wage change**		
		Less than −5%	*−5 to 5%*	*More than 5%*
Share (%)				
All E spells	5.7	30.6	28.3	41.1
Cur. wage is min.	3.2	–	54.1	45.9
Cur. wage above min.	6.3	34.8	24.8	40.4
Mean wage change (%)				
All E spells	18.8	−30.3	−0.3	68.6
Cur. wage is min.	38.6	–	0.5	83.4
Cur. wage above min.	16.1	−30.3	−0.5	66.3

Notes: The top panel shows the distribution of EE spells. The "Overall" column shows the share of EE spells within all E spells/within E spells with a current wage being equal to vs. higher than the minimum wage. Within each row, the columns titled "By wage change" show the conditional distribution of EE spells by wage change. The bottom panel shows the mean wage change within each category. Current and accepted wages are recoded as $w = \max(w, w_{\min})$.
Source: CERS-HAS, authors' own calculations.

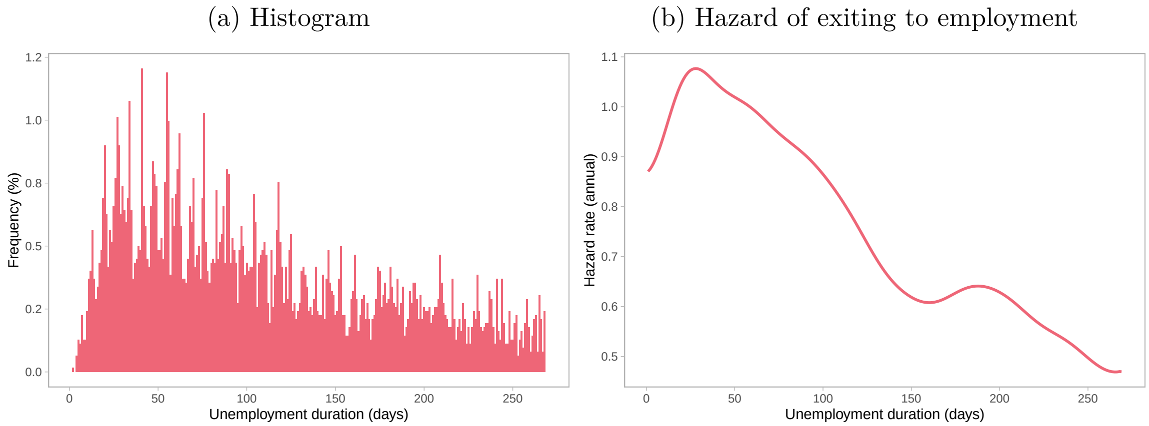
Table 3 takes this analysis one step further by looking at how often a job-to-job transition resulted in wage increases or decreases of particular levels. Over 30% of job-to-job changes involve a wage decrease of more than 5%; this number rises to 35% excluding jobs at the minimum wage level. Over 41% of job-to-job transitions entail a wage increase of more than 5%; 28% of job-to-job transitions result in a more incremental wage change, between negative five and plus five percent.

Taken together, the descriptives reported in Tables 2 and 3 provide support both for and against the model described in Section 2. On the one hand, there is clear evidence of individuals moving to jobs that involve significant wage cuts. This is consistent with a search model where individuals value more than just the wage. On the other hand, the large number of transitions along the diagonal in Table 2 strongly

[16]The sole exception is current wage bin 3, with 8% more transitions to one bin lower than higher.

suggests that the current wage may affect what wages are offered. This motivates the specification in our empirical application, where we allow for the possibility that the current wage affects the wage offer distribution.

Figure 1: Unemployment durations



Notes: Panel (a) shows the distribution of unemployment spells that end in exiting to employment. Spells are censored from the right at 269 days. Panel (b) shows the unconditional hazard rate of exiting unemployment. We calculate the hazard as the kernel-smoothed density of exiting unemployment to a job, divided by the kernel-smoothed survivor function. We use Gaussian kernels with optimal bandwidth selection and reflection for boundary correction.
Source: CERS-HAS, authors' own calculations.

Table 4: Summary statistics, unemployment-to-employment transitions

	Overall	**By unemployment duration (days)**				
		1–30	*31–60*	*61–90*	*91–180*	*181–269*
Mean U duration (days)	108.3	20.7	45.8	75.5	130.4	220.0
Mean acc. wage (HUF)	3,042	3,356	3,229	3,080	2,975	2,727
Share $\underline{w}$ (%)	32.8	23.8	24.7	32.2	35.4	43.2

Notes: The table shows summary statistics of spells that end in an unemployment-to-employment transition. Accepted wages are recoded as $w = \max(w, w_{\min})$. The last row shows the share of UE transitions to the lowest wage bin (75 to 107% of the minimum wage). Wage rates are daily; 200 HUF $\approx$ 1 USD in 2004.
Source: CERS-HAS, authors' own calculations.

Turning to the unemployment side, almost 43% of unemployment spells end in employment; most of the remaining spells are right-censored. Panel (a) of Figure 1 shows the distribution of unemployment durations for those who exited unemployment during our observation window; the mean duration is 108.3 days. Panel (b) of Figure

1 shows that the hazard rate of exiting unemployment to employment is downward-sloping, consistent with the existence of negative duration dependence. Next, we divide those who exited unemployment to a job into five categories based on their unemployment duration. Summary statistics for accepted wages for those who exited unemployment in each of these durations are presented in Table 4. Consistent with unemployed workers willing to accept lower wage offers over time, longer durations are associated with lower accepted wages and higher probabilities of accepting a job at the minimum wage. In particular, those whose unemployment durations were less than 30 days were a little over half as likely to exit to a job paying the minimum wage as those whose durations were in the top quartile.

5 Estimation procedure

For estimation, we specify $u_w = \alpha \ln(w)$ and set $\rho = 0.05$. Motivated by the job-to-job transition patterns discussed in Section 4.3, we further allow current wages to affect the on-the-job wage offer distribution. We use a flexible parametric specification for the employed and unemployed wage offer distributions as well as for the offer arrival rates out of unemployment, the latter two of which are time-dependent. We do this for two main reasons. First, the model is heavily over-identified. This parametric specification allows us to incorporate all of the relevant information in a disciplined fashion. Second, the model requires data on job-to-job transitions conditional on the current wage and unemployed-to-job transitions to specific wages at each moment in time. These conditional transition rates are inherently noisy, and our flexible parametric specification yields substantial precision gains.

Consider a workforce populated by N individuals, indexed by i. Workers may face different wage offer distributions, job offer arrival and destruction rates, as well as different flow payoffs of unemployment in ways that are unobserved to the econometrician. We allow for unobserved heterogeneity in the following manner. Each individual belongs to one of R unobserved types with probability q_{ir}; the population probability of type r is given by π_r. We set $R = 2$ in our application. Each individual experiences S_i employment spells indexed by s and $\tilde{S}_i$ unemployment spells indexed by $\tilde{s}$. The corresponding likelihoods for these spells for individual i of type r are given by $\mathcal{L}^E_{isr}(\theta^E_r)$ and $\mathcal{L}^U_{i\tilde{s}r}(\theta^E_r, \theta^U_r)$, respectively, where θ^E_r denote the employed-side parameters and θ^U_r the unemployed-side parameters. Note that the employed-side parameters enter the likelihood for the unemployment spells but the reverse is not true. We will exploit this sequential likelihood property in our estimation procedure.

To apply our identification arguments in the case with unobserved heterogeneity requires identifying the type-specific hazard functions, along with the distribution of heterogeneity types. One can apply in an initial step the identification results from Heckman and Singer (1984) for duration models with unobserved heterogeneity but without covariates to identify the distribution of unobserved types along with the type-specific hazards associated with the job-to-job transitions. One can identify in a second step the distribution of the type-specific hazards out of unemployment, taking as given the distribution of heterogeneity types. The identification arguments from Section 3 then still apply, yielding identification of the structural parameters that are now allowed to vary by unobserved heterogeneity type. In practice, we estimate a specification where the offer arrival rates, job destruction rates, wage offer distributions, and flow payoff of unemployment are allowed to be type-specific. Denoting by $f_{ww'r}$ the probability to receive a wage offer w' conditional on current wage w and type r, $\theta^E_r = (\delta_{0r}, \lambda_r, (f_{ww'r})_{w,w'}, (u_w)_w, c)'$ and $\theta^U_r = ((\lambda_r(t))_t, (g_{wr}(t))_{w,t}, (b_r(t))_t)'$.

For estimation, the unobserved type must be integrated out of the likelihood function. Note that there is an an initial conditions problem here as the initial wage may be affected by the type. These initial conditions, described in more detail in the next subsection, are indexed by a vector of parameters θ^I_r. We denote by $\mathcal{L}^I_{ir}(\theta^I_r)$ the probability of observing i's initial wage conditional on being of type r. Assuming spells are independent across individuals and, conditional on heterogeneity type, within individuals, the log-likelihood for the initial wage, employment and unemployment spells data is then:

$$\sum_i \ln \left(\sum_r \pi_r \mathcal{L}^I_{ir}(\theta^I_r) \prod_{s=1}^{S_i} \mathcal{L}^E_{isr}(\theta^E_r) \prod_{\tilde{s}=1}^{\tilde{S}_i} \mathcal{L}^U_{i\tilde{s}r}(\theta^E_r, \theta^U_r) \right) \tag{5.1}$$

We estimate the model parameters using a three-step estimation procedure. Following Arcidiacono and Jones (2003) and Arcidiacono and Miller (2011), we implement an adaptation of the Expectation-Maximization (EM) algorithm that restores the additive separability of the log-likelihood function (5.1). In particular, the EM algorithm treats the unobserved type as known at the maximization stage and weights the log-likelihoods of each individual i by the posterior probability of i being of unobserved type r, q_{ir}.

We build on Arcidiacono and Miller (2011) and use in a first step a reduced-form approximation of the employment duration models to estimate θ_r^I and the q_{ir}'s. The posterior probabilities obtained at this stage follow directly from Bayes' rule:

$$q_{ir} = \frac{\pi_r \mathcal{L}_{ir}^I(\theta_r^I) \prod_{s=1}^{S_i} \tilde{\mathcal{L}}_{isr}^E(\tilde{\theta}_r^E)}{\sum_r \pi_r \mathcal{L}_{ir}^I(\theta_r^I) \prod_{s=1}^{S_i} \tilde{\mathcal{L}}_{isr}^E(\tilde{\theta}_r^E)} \tag{5.2}$$

where $\tilde{\mathcal{L}}_{isr}^E(\tilde{\theta}_r^E)$ denotes the reduced-form likelihood associated with employment spell s. Given the q_{ir}'s, we then estimate θ_r^E and θ_r^U (holding θ_r^E fixed) in two sequential maximization steps.

5.1 Step 1: posterior type distributions

We use the initial wage and the job-to-job transitions to estimate the conditional probabilities of being each unobserved type, q_{ir}. We specify the job-to-job transitions between jobs that pay w to jobs that pay w' as the product between the hazard rate out of a job that pays w and the probability that the accepted wage is w' given that the current wage is w. The exact specification is given in Appendix C.1.

We specify the likelihood of the initial wage as following a tobit structure. Denote w_i^I as individual i's initial wage level, and X_i^I a set of observed characteristics that may affect this initial wage.[17] Denoting by $\Phi(.)$ and $\phi(.)$ the cdf and pdf of a standard normal distribution, the likelihood contribution of initial wages is then

$$\mathcal{L}_{ir}^I(\theta_r^I) = \left[\Phi\left(\frac{\ln(\underline{w}) - X_i^I\theta_{xr}^I}{\sigma_r^I}\right)\right]^{\mathbb{1}\left\{w_i^I = \underline{w}\right\}} \cdot \left[\frac{1}{\sigma_r^I}\phi\left(\frac{\ln(w_i^I) - X_i^I\theta_{xr}^I}{\sigma_r^I}\right)\right]^{\mathbb{1}\left\{w_i^I > \underline{w}\right\}} \tag{5.3}$$

where $\theta_r^I = \left(\theta_{xr}^I, \sigma_r^I\right)'$. We specify X_i^I as a function of the individual's type and year

[17] We use the first recorded wage in each individual's work history. Similarly, for the job-to-job transitions we use the observed wages in each spell; see Appendix C.1.

indicators where the effects of the year indicators are fixed across types:

$$X_i^I \theta_{xr}^I = \theta_{1r}^I + \theta_2^I \mathbb{1}\{y_i = 2004\} + \theta_3^I \mathbb{1}\{y_i = 2005\} \tag{5.4}$$

Given these parameters and the reduced form parameters $\tilde{\theta}_r^E$ governing the job-to-job transitions, we can estimate the q_{ir}'s.

5.2 Step 2: employed-side parameters

With the estimated conditional type probabilities (the q_{ir}'s) in hand, we now proceed with the estimation of the employed-side parameters. Estimation proceeds as it would without unobserved heterogeneity but where the q_{ir}'s are used as weights. For each employment spell s, we observe its duration, t_{is}, and the wage, w_{is}. Let $w_{is+1} = 0$ when individual i transitions to unemployment during their s^{th} employment spell. Estimation of the type-r job separation rate δ_{0r} then directly follows as the weighted number of transitions to unemployment divided by the weighted time spent in employment:

$$\hat{\delta}_{0r} = \frac{\sum_{i=1}^{N} q_{ir} \sum_{s=1}^{S_i} \mathbb{1}\{w_{is+1} = 0\}}{\sum_{i=1}^{N} q_{ir} \sum_{s=1}^{S_i} t_{is}} \tag{5.5}$$

We estimate the other employed-side parameters via maximum likelihood. We express the type-r hazard from moving from a job that pays w to one that pays w' as follows:

$$h_{ww'r} = \lambda_r f_{ww'r} p_{ww'r} \tag{5.6}$$

The wage offer distribution, $f_{ww'r}$, is parameterized using an ordered logit that depends on current wages. First, we specify the wage cutoffs as having the following recursive structure:

$$\phi_w = \begin{cases} \theta_1^\phi & \text{for } w = \underline{w} \\ \phi_{w_-} + \exp(\theta_2^\phi + \theta_3^\phi \ln(w) + \theta_4^\phi \ln(w)^2) & \text{for } w > \underline{w} \end{cases} \tag{5.7}$$

where w_- denotes the preceding support point of the wage distribution. These cutoffs specify how large the latent index needs to be to reach a particular wage bin.

We then define the distribution of offered wages using the wage cutoffs as well as current wages w:

$$f_{ww'r} = \begin{cases} \Lambda(\phi_{\underline{w}} + X^f\theta^f_r) & \text{for } w' = \underline{w} \\ \Lambda(\phi_{w'} + X^f\theta^f_r) - \Lambda(\phi_{w'_-} + X^f\theta^f_r) & \text{for } \underline{w} < w' < \overline{w} \\ 1 - \Lambda(\phi_{\overline{w}_-} + X^f\theta^f_r) & \text{for } w' = \overline{w} \end{cases} \tag{5.8}$$

$$X^f\theta^f_r = \theta^f_1 \ln(w) + \theta^f_2 \mathbb{1}\{w = \underline{w}\} + \theta^f_{3r} \tag{5.9}$$

where $\Lambda(\cdot)$ denotes the logistic function. The log of the current wage then shifts the latent index and the unobserved types affect the offered wage distribution through the location shift θ^f_{3r}.

Finally, given the logistic distribution assumption on the instantaneous shock ε, the conditional choice probabilities that enter Equation (5.6) can be written as:

$$p_{ww'r} = \frac{\exp\left(V_{w'r} - V_{wr} - c\right)}{1 + \exp\left(V_{w'r} - V_{wr} - c\right)} \tag{5.10}$$

In practice, we use the Bellman equation for the value function of employment and solve for a fixed point in the differenced value functions that appears in Equation (5.10), for all the states.

We collect the employed-side parameters that remain to be estimated in $\theta^E_{2r} \equiv (\lambda_r, \theta^\phi, \theta^f_r, \alpha, c)'$. It follows that the likelihood contribution of a job spell s for a type-r worker i is given by

$$\mathcal{L}^E_{isr}(\delta_{0r}, \theta^E_{2r}) = \prod_{w,w'} \left[(h_{ww'r})^{\mathbb{1}\{w_{is}=w, w_{is+1}=w'\}} \exp(-h_{ww'r}t_{is}) \right]^{\mathbb{1}\{w_{is}=w\}} \tag{5.11}$$

We then estimate these parameters by maximizing the expected complete log-likelihood with respect to θ^E_{2r}:

$$\max_{\theta^E_{2r}} \sum_{i=1}^{N} \sum_{r=1}^{2} \sum_{s=1}^{S_i} q_{ir} \ln\left(\mathcal{L}^E_{isr}(\hat{\delta}_{0r}, \theta^E_{2r})\right) \tag{5.12}$$

5.3 Step 3: unemployed-side parameters

In the third and final step, we estimate the distribution of offered wages out of unemployment, $g_{wr}(t)$, and the offer arrival rates, $\lambda_r(t)$, using maximum likelihood. We then rely on our constructive identification strategy to estimate the flow payoff of unemployment.

Note that the type-r hazard of leaving unemployment at duration t to wage w is given by:

$$h_{wr}(t) = \lambda_r(t) g_{wr}(t) p_{wr}(t) \tag{5.13}$$

In the next subsections, we show how each of these terms are specified.

5.3.1 Specification of $p_{wr}(t)$

We focus first on expressing $p_{wr}(t)$ in a way consistent with the structure of the model. We introduce $\kappa_{ww'r} \equiv V_{wr} - V_{w'r}$, so that $\exp(\kappa_{ww'r}) = \exp(V_{wr})/\exp(V_{w'r})$. Using this identity, we can express the ratio of the conditional choice probabilities out of unemployment as:

$$\begin{aligned}\frac{p_{wr}(t)}{p_{w'r}(t)} &= \frac{\exp(V_{wr})/[\exp(V_{0r}(t)) + \exp(V_{wr})]}{\exp(V_{w'r})/[\exp(V_{0r}(t)) + \exp(V_{w'r})]} \\ &= \exp(\kappa_{ww'r}) \left[1 - p_{wr}(t)\{1 - \exp(-\kappa_{ww'r})\}\right]\end{aligned} \tag{5.14}$$

We can therefore express all conditional choice probabilities relative to one other conditional choice probability, say the one associated with the minimum wage $p_{\underline{w}r}(t)$, and the corresponding $\kappa_{w\underline{w}r}$ terms:

$$p_{wr}(t) = \frac{p_{\underline{w}r}(t)\exp(\kappa_{w\underline{w}r})}{1 - p_{\underline{w}r}(t)\left[1 - \exp(\kappa_{w\underline{w}r})\right]} \tag{5.15}$$

Furthermore, we express the CCPs of accepting an offer from the first wage bin in terms of a parameterized hazard rate out of unemployment to the first bin:

$$p_{\underline{w}r}(t) = \frac{h_{\underline{w}r}(t)}{\lambda_r(t) g_{\underline{w}r}(t)} \tag{5.16}$$

where[18]

$$h_{\underline{w}r}(t) = \exp(X^h \theta_r^h) \quad \text{with} \tag{5.17}$$

$$X^h = \begin{bmatrix} 1 & t^{-1} & t^{-1}\ln(t) & t^2 & t^3 \end{bmatrix} \tag{5.18}$$

It follows that we can express the CCPs as

$$p_{wr}(t) = \begin{cases} \frac{h_{\underline{w}r}(t)}{\lambda_r(t)\, g_{\underline{w}r}(t)} & \text{for } w = \underline{w} \\ \frac{h_{\underline{w}r}(t)\exp(\kappa_{w\underline{w}r})}{\lambda_r(t)\, g_{\underline{w}r}(t) - h_{\underline{w}r}(t)[1-\exp(\kappa_{w\underline{w}r})]} & \text{for } w > \underline{w} \end{cases} \tag{5.19}$$

5.3.2 Specification of $\lambda_r(t)$

We parametrize the offer arrival rates $\lambda_r(t)$ as

$$\lambda_r(t) = \exp(X^\lambda \theta^\lambda \nu_r^\lambda + \psi_r^\lambda) \quad \text{where} \tag{5.20}$$

$$X^\lambda = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \tag{5.21}$$

$$\nu_1^\lambda = 1 \quad \text{and} \quad \psi_1^\lambda = 0 \tag{5.22}$$

The type-specific parameters $(\nu_r^\lambda, \psi_r^\lambda)$ provide a parsimonious scale and location shift of the common Type 1 profile.

5.3.3 Specification of $g_{wr}(t)$

Finally, we parameterize the offered wages $g_{wr}(t)$ using a similar ordered logit structure to that used in the employed offer distribution. We take the wage cutoffs ϕ as given from the employed side in Equation (5.7), and add a type-specific variance-scale parameter β_r, a level shifter, γ_{1r}, and a duration shifter, γ_{2r}, specifying the wage offer

[18]We chose this polynomial because it fits the nonparametric Nelson–Aalen hazard estimates the best.

distribution out of unemployment as:

$$g_{wr}(t) = \begin{cases} \Lambda\left(\beta_r\phi_{\underline{w}} + \gamma_{1r} + \gamma_{2r}\ln(t)\right) & \text{for } w = \underline{w} \\ \Lambda\left(\beta_r\phi_w + \gamma_{1r} + \gamma_{2r}\ln(t)\right) - \Lambda\left(\beta_r\phi_{w_-} + \gamma_{1r} + \gamma_{2r}\ln(t)\right) & \text{for } \underline{w} < w < \overline{w} \\ 1 - \Lambda\left(\beta_r\phi_{\overline{w}_-} + \gamma_{1r} + \gamma_{2r}\ln(t)\right) & \text{for } w = \overline{w} \end{cases} \tag{5.23}$$

Note that the only (type-specific) parameters to estimate are β_r, γ_{1r}, and γ_{2r}.

5.3.4 Estimation of $p_{wr}(t)$, $\lambda_r(t)$, and $g_{wr}(t)$

Putting the three components together, the structural hazards are given by

$$h_{wr}(t) = \begin{cases} h_{\underline{w}r}(t) & \text{for } w = \underline{w} \\ h_{\underline{w}r}(t)\,\frac{g_{wr}(t)}{g_{\underline{w}r}(t)}\,\frac{\exp(\kappa_{w\underline{w}r})}{\lambda_r(t)\,g_{\underline{w}r}(t) - h_{\underline{w}r}(t)[1-\exp(\kappa_{w\underline{w}r})]} & \text{for } w > \underline{w} \end{cases} \tag{5.24}$$

We estimate these structural parameters in a maximum likelihood procedure, stratified by types. First, we estimate the parameters $\theta_1^U = (\theta^\lambda, \theta_1^h, \beta_1, \gamma_{11}, \gamma_{21})'$ for Type 1 individuals. Then, given these estimates we estimate the parameters $\theta_2^U = (\nu_2^\lambda, \psi_2^\lambda, \theta_2^h, \beta_2, \gamma_{12}, \gamma_{22})'$ for the second type. In both cases, we impose that the CCPs are non-decreasing in t.[19]

The likelihood contribution of a type-r individual i's spell $\tilde{s}$ is

$$\mathcal{L}_{i\tilde{s}r}^U(\theta_r^U) = \prod_w \left\{ \left[h_{wr}(t_{i\tilde{s}})\right]^{\mathbb{1}\{w_{i\tilde{s}}=w\}} \exp\left(-\int_0^{t_{i\tilde{s}}} h_{wr}(u)\,du\right)\right\} \tag{5.25}$$

We first estimate the parameters θ_1^U as follows:

$$\max_{\theta_1^U} \sum_{i=1}^{N} q_{i1} \sum_{\tilde{s}=1}^{\tilde{S}_i} \ln\left(\mathcal{L}_{i\tilde{s}1}^U(\theta_1^U)\right) \tag{5.26}$$

$$\text{s.t. } p_{\underline{w}1}(t) \le p_{\underline{w}1}(t+1) \quad \text{for } 1 \le t < T-1 \tag{5.27}$$

$$p_{\underline{w}1}(1) \ge \varepsilon \tag{5.28}$$

$$p_{\overline{w}1}(T) \le 1-\varepsilon \tag{5.29}$$

[19]Appendices C.2.1 and C.2.2 show how these constraints simplify.

for some small ε and where $T = 269$ denotes the end of our time window.[20]

Taking the shape of the offer arrival process as given, we then estimate the remaining type $r = 2$ parameters as follows:

$$\max_{\theta_2^U} \sum_{i=1}^{N} q_{i2} \sum_{\tilde{s}=1}^{\tilde{S}_i} \ln\left(\mathcal{L}_{i\tilde{s}2}^U(\theta_2^U)\right) \tag{5.30}$$

$$\text{s.t. } p_{\underline{w}2}(t) \leq p_{\underline{w}2}(t+1) \quad \text{for } 1 \leq t < T-1, \tag{5.31}$$

$$p_{\underline{w}2}(1) \geq \varepsilon \tag{5.32}$$

$$p_{\overline{w}2}(T) \leq 1 - \varepsilon \tag{5.33}$$

5.3.5 Estimation of flow payoff of unemployment $b_r(t)$

For the last remaining parameters, we first need to calculate the value function and its first derivative. Given the estimates of the employed and unemployed parameters, we calculate $V_{0r}(t)$ pointwise at each duration t using[21]

$$V_{0r}(t) = \begin{cases} \frac{\alpha \ln(w) - \sum_{w'} \lambda_r f_{ww'r} \ln(1-p_{ww'r})}{\rho} - \frac{\delta_{0r}+\rho}{\rho} \ln\left(\frac{p_{wr}(t)}{1-p_{wr}(t)}\right) & \text{for } t = 0 \\ \frac{\alpha \ln(w) - \sum_{w'} \lambda_r f_{ww'r} \ln(1-p_{ww'r}) + \delta_{0r} V_{0r}(0)}{\delta_{0r}+\rho} - \ln\left(\frac{p_{wr}(t)}{1-p_{wr}(t)}\right) & \text{for } t > 0 \end{cases} \tag{5.34}$$

From the time trajectory of the value function, we estimate its first derivative as

$$\dot{V}_{0r}(t) = \frac{V_{0r}(t+\Delta\tau) - V_{0r}(t)}{\Delta\tau} \tag{5.35}$$

where $\Delta\tau$ is an arbitrarily small time interval.[22]

We finally calculate the flow payoff of unemployment using the expression

$$b_r(t) = \rho V_{0r}(t) + \sum_{w} \lambda_r(t) g_{wr}(t) \ln(1 - p_{wr}(t)) - \dot{V}_{0r}(t) \tag{5.36}$$

where $\rho = 0.05$ and all of the other right-hand side parameters have been estimated in previous steps.

[20] In practice we set $\varepsilon = 5 \times 10^{-4}$.

[21] This expression would appear as though $V_{0r}(t)$ is heavily overidentified as the expression holds for all wages w. However, we have already imposed the structure of the model prior to this stage so that all values of w lead to the same value of $V_{0r}(t)$.

[22] We set $\Delta\tau = 10^{-5}$ in our application.

6 Results

We now discuss our estimation results. We begin with the employed-side parameters, showing substantial permanent heterogeneity (through the unobserved types) as well as showing the importance of current wages on future wage offers. We then turn to the unemployed side where the nonstationarities lie. Duration dependence affects both the rate at which offers are received as well as the size of the wage offers, both of which decline over time. As a result, workers become more willing to accept lower wage offers over time.

6.1 Employed-side results

Table 5 below shows the estimates of the employed-side parameters with the exception of the initial conditions and the wage offer distribution. Most workers (87%) are classified as Type 1. Relative to Type 2 workers, these workers receive offers at a lower rate and have higher job destruction rates. Type 1 workers expect to receive a job offer once in every 2.9 years (0.349 annually) and have a 25.9% chance of separating from their current job per annum; Type 2 workers receive offers slightly more frequently (one in every 2.5 years or 0.408 per annum) and separate from their jobs substantially less frequently (8.3% probability per annum). It follows that the index of search frictions, which corresponds to the average number of job offers received during any given employment spell (Ridder and van den Berg, 2003), is substantially higher for this group of individuals (4.9 vs. 1.3 for Type 1 individuals) who also tend to have higher initial wages. The mean index of search frictions across types is equal to 1.8, a value which fits in the range of the estimates based on the joint distribution of job durations and wages for French labor force survey data, but substantially lower than those for CPS data in the US (Ridder and van den Berg, 2003).

The estimated parameter associated with the flow utility of log wages is equal to 0.323, which is about a third in magnitude of the cost of switching jobs. The switching cost translates directly into the probability of switching to a job that pays the same wage, resulting in the probability of acceptance of a same-wage job of 27%.[23] The flow

[23] The probability of switching to a same-wage job conditional on receiving a same wage offer is $\exp(-c)/(1+\exp(-c)) = \exp(-.986)/(1+\exp(-.986)) = 0.27$.

utility parameter is sufficiently large as to produce substantial heterogeneity in the probability of accepting a job given the current and offered wage. For example, Type 2's employed in the highest wage bin who receive an offer from the lowest wage bin have an acceptance probability of less than 1%; those in the lowest wage bin who receive an offer from the highest wage bin have an acceptance probability of 96%.

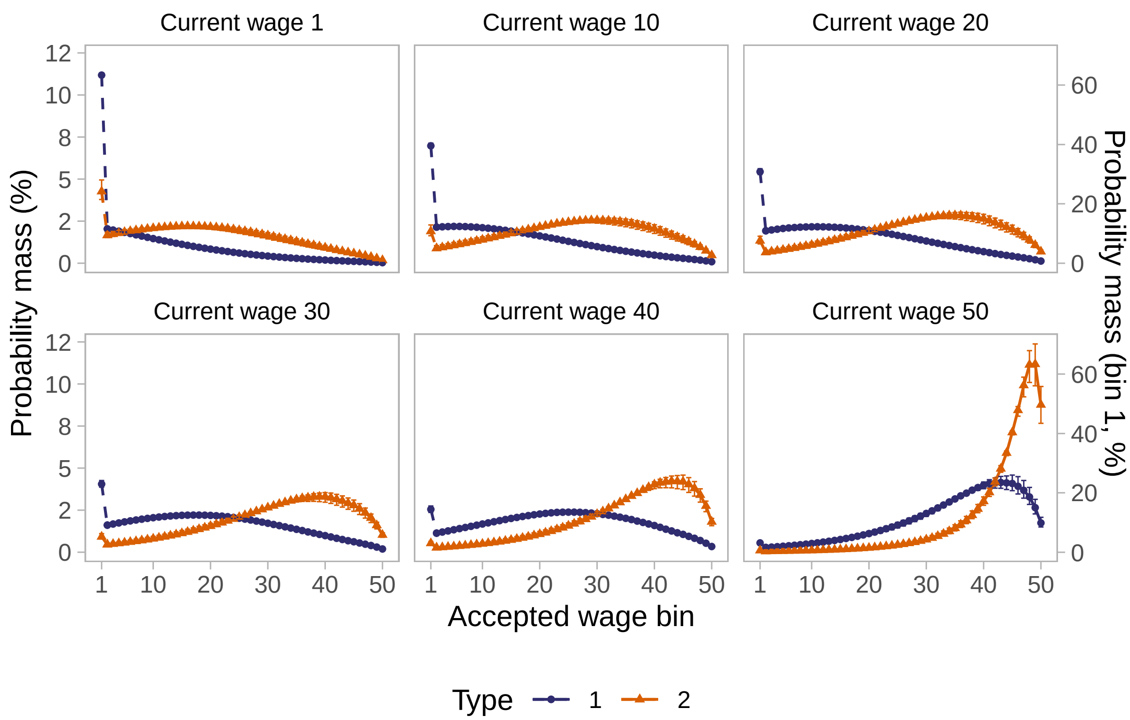
Table 5: Structural parameter estimates, employed side

	Parameter	**Estimate**	
		Type 1	*Type 2*
λ	Offer arrival rate	0.349	0.408
		[0.294, 0.421]	[0.363, 0.495]
δ	Job separation rate	0.259	0.083
		[0.258, 0.261]	[0.082, 0.085]
λ/δ	Search friction index	1.345	4.888
		[1.137, 1.610]	[4.345, 5.878]
α	Flow utility of log wages	0.323	
		[0.287, 0.370]	
c	Job switching cost	0.986	
		[0.730, 1.243]	
π	Type probability	0.867	0.133
		[0.864, 0.871]	[0.129, 0.136]

Notes: The offer arrival rate λ and the job separation rate δ are yearly rates. The flow utility of log wages α and the job switching cost c are fixed across heterogeneity types. 95% bootstrap confidence intervals in brackets (500 replications).
Source: CERS-HAS, authors' own calculations.

As wage offers are allowed to depend on the wage in the current job, Figure 2 shows the offer distributions for workers currently in wage bin 1, 10, 20, 30, 40, and 50. At any current wage, Type 1's face a worse wage offer distribution than Type 2's. However, as the current wage rises, the distribution of offered wages shifts to the right for both types. Hence a Type 1 worker currently working in the 40th wage bin faces a better offer distribution than a Type 2 worker currently making the minimum wage. As shown in Table 10 in Appendix D, Type 1 workers also have lower initial wages. Virtually all (over 99%) of workers in each of the first ten initial wage bins are Type 1 compared to less than 2% in the top initial wage bin.

Figure 2: Wage offer distribution, employed side

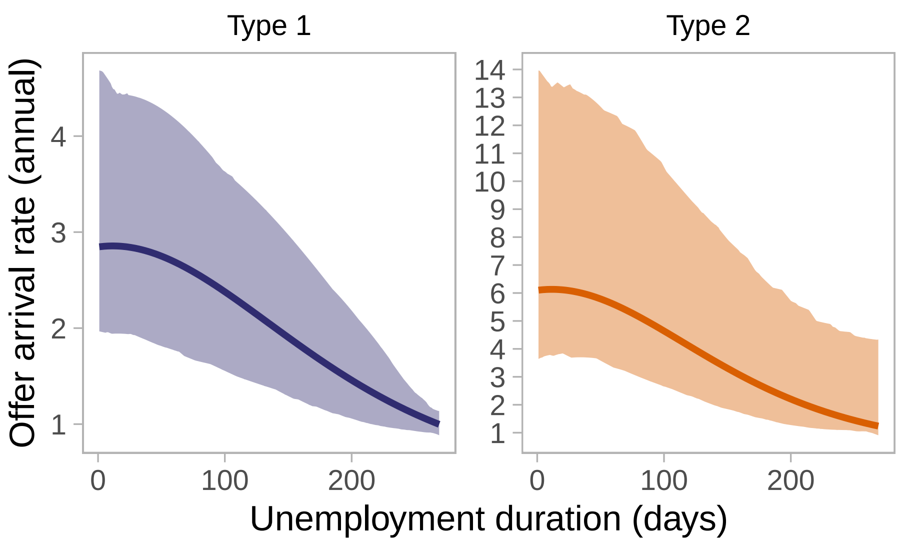


Notes: Distributions are conditional on the current wage bin. The probability mass of bin 1 offers are represented on the secondary vertical axis (right). Error bars represent 95% bootstrap confidence intervals (500 replications).
Source: CERS-HAS, authors' own calculations.

6.2 Unemployed-side results

We now turn to the unemployed-side results. Our model allows for nonstationarities along multiple dimensions. Figure 3 shows one of these dimensions, revealing how unemployed offer arrival rates evolve over time. For both types, increased unemployment durations are associated with fewer offers. For Type 1's, offers come in at a rate of 2.9 per year at the beginning of the unemployment spell but fall to a rate of 1 per year by the end of the time window. Type 2's, who already have better prospects on the employed side, receive offers at a much higher rate, beginning at a rate of 6.1 per year but falling to the same rate as Type 1 individuals by benefit expiration.

Figure 3: Offer arrival rates out of unemployment

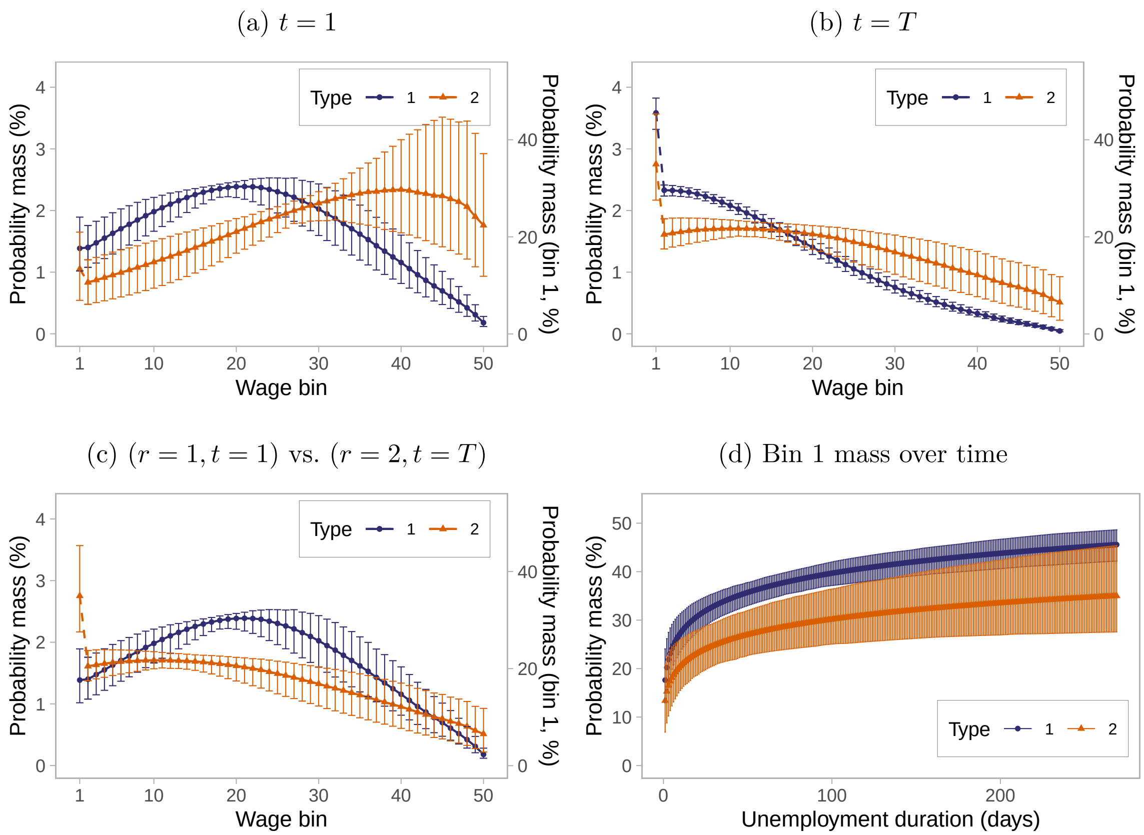


Notes: Annual rates. Shaded regions represent 95% bootstrap confidence band (500 replications).
Source: CERS-HAS, authors' own calculations.

A second source of nonstationarity, illustrated in Figure 4, is in the offered wage distribution for unemployed workers. Panels (a) and (b) show stark differences in the offer distributions between offers at the start of unemployment ($t = 1$) and at the end of our time window ($t = T = 269$ days). At $t = 1$, Type 2's face a much better offer distribution that Type 1's. However, this advantage vanishes near benefit expiration. As unemployment duration increases, the offer distributions for both types become substantially worse. Notably, Panel (c) shows that this deterioration of the offered wage distribution is larger than the initial wage offer differences across types: Type 1's at $t = 1$ face a better wage offer distribution than Type 2's at $t = 269$.

A third and last source of nonstationarity is the flow payoff of unemployment. The evolution of these are displayed in Figure 9 in Appendix D. The flow payoff drops sharply upon entering unemployment and then remains relatively flat. However, for both types of individuals, the flow value decreases again close to benefit expiration. In Figures 10 and 11 we show how these three sources of nonstationarity combined—offer arrival wages, wage offers, and the flow payoff of unemployment—affect the unemployment-to-job transitions and the value function of unemployment, respectively. As unemployment duration increases, the value function for unemployment

Figure 4: Wage offer distribution, unemployed side



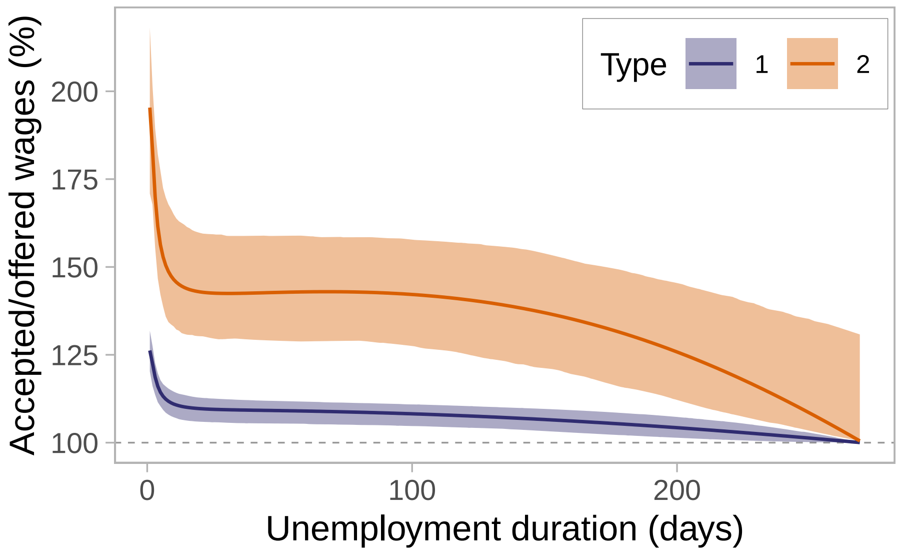
Notes: Distributions are conditional on unemployment duration t. The probability mass of bin 1 offers are represented on the secondary vertical axis (right) in Panels (a), (b), and (c). Panel (c) contrasts the Type 1 distribution at duration $t = 1$ to the Type 2 distribution at duration $t = T$. Panel (d) compares the evolution of the probability mass of wage offers from the first bin across types. Error bars represent 95% bootstrap confidence intervals (500 replications).
Source: CERS-HAS, authors' own calculations.

falls. Correspondingly, the job acceptance probabilities rise sharply over the course of unemployment (see Figure 12 in Appendix D).

With job acceptance probabilities rising, the ratio of average accepted wages to average offered wages falls over time. This is displayed in Figure 5 below. Like with the flow payoff of unemployment, we see a sharp drop in the accepted/offered wage ratio immediately after entering unemployment. As unemployment duration increases, workers gradually become less and less selective over which jobs they accept. By the time benefits are about to expire, job seekers find almost all jobs acceptable.

Finally, the combined effects of these different sources of nonstationarities are dis-

Figure 5: Offered vs. accepted wages out of unemployment



Notes: Shaded regions represent 95% bootstrap confidence band (500 replications).
Source: CERS-HAS, authors' own calculations.

played in Table 6. The first row points to a dynamic selection pattern whereby the relative share of Type 2 individuals declines over the course of unemployment. Namely, because Type 2's receive offers at a much higher rate than Type 1's, Type 2's exit unemployment faster than their Type 1 counterparts, making up to 12% of those who leave in the shortest durations but only 8.4% of those who leave in the longest durations. As shown in the first column, this translates to Type 2 individuals who exit to a job having unemployment durations that are on average eleven days shorter than their Type 1 counterparts.

Averaging across heterogeneity types illustrates how well our model matches the patterns displayed in Table 4 in Section 4.1. The model slightly underpredicts average unemployment duration. Importantly though, it matches the key patterns of declining accepted wages out of unemployment as duration of unemployment increases, both in terms of the mean accepted wage and the share of accepted job offers at the minimum wage.

Taken together, our estimation results provide evidence that nonstationarity in the offer arrival rates, the wage offer distribution, as well as the flow payoff of unemployment are central features of the job search environment over the course of unemployment.

Table 6: Summary statistics by type, unemployment-to-employment transitions

	Overall	By unemployment duration (days)				
		1–30	*31–60*	*61–90*	*91–180*	*181–269*
Pop. prob. of Type 2 (%)	10.0	12.0	11.4	10.3	8.7	8.4
Unconditional on type						
Mean U duration (days)	100.6	17.1	44.9	74.9	131.1	221.4
Mean acc. wage (HUF)	3,102	3,532	3,242	3,126	2,960	2,708
Share $\underline{w}$ (%)	30.9	21.7	27.1	30.1	34.0	40.5
Type 1						
Mean U duration (days)	101.7	17.1	44.9	74.9	131.2	221.2
Mean acc. wage (HUF)	2,892	3,227	2,988	2,905	2,789	2,605
Share $\underline{w}$ (%)	32.8	23.5	29.1	32.1	35.8	42.0
Type 2						
Mean U duration (days)	90.8	16.9	44.6	74.7	129.4	222.6
Mean acc. wage (HUF)	4,990	5,761	5,208	5,044	4,744	3,836
Share $\underline{w}$ (%)	13.9	8.3	11.4	13.2	15.5	24.0

Notes: The table shows summary statistics of simulated UE transitions, based on model estimates. The last row shows the share of UE transitions to the lowest wage bin (75 to 107% of the minimum wage). Accepted wages recoded as $w = \max(w, w_{\min})$. Summary statistics are weighted by type probabilities. Accepted wages are daily wage levels reported in Hungarian forints (200 HUF $\approx$ 1 USD in 2004).
Source: CERS-HAS, authors' own calculations.

Our findings also highlight the importance of allowing for worker-level unobserved heterogeneity. In particular, workers differ markedly in the wage offer distribution and job offer arrival rates they face, which constitutes a source of spurious duration dependence.

7 Conclusion

In this paper, we extend the canonical continuous-time job search model with on-the-job search to allow for preference shocks. Incorporating preference shocks and using the insights from conditional choice probability methods results in constructive identification of the model parameters, even in rich nonstationary settings. In terms of estimation, nonstationary search models typically require solving a nonlinear differential equation within the maximization routine. But in our setting no differential equation needs to be solved to estimate the parameters of the model. As a result, the computational costs are small for the class of nonstationary search models we consider.

We apply our methods to administrative data from Hungary. Nonstationarities when unemployed operate through three sources: the offered wage distribution, the offer arrival rates, and the flow payoff of unemployment. Our model estimates show that the wage offer distribution becomes worse and offer arrivals slow substantially as the duration of unemployment increases. Job seekers then become less selective in the jobs they are willing to accept over the course of unemployment, implying that the gap between accepted and offered wages shrinks with unemployment duration.

Beyond this particular application, our framework can be applied to a broad class of job search models which may include heterogeneous job types, involuntary wage changes, as well as aggregate labor market shocks. Our approach can also be extended in several other directions. Notably, a natural research avenue would be to extend the class of job search models considered in this paper to accommodate more general forms of nonstationarity. For instance, it would be interesting to explore the identification of a model where both the value of unemployment and the value of employment are allowed to vary as a function of calendar time, as a more flexible way to capture aggregate fluctuations. We leave this analysis for future research.

References

Agarwal, N., I. Ashlagi, M. A. Rees, P. Somaini, and D. Waldinger (2021). Equilibrium Allocations under Alternative Waitlist Designs: Evidence from Deceased Donor Kidneys. *Econometrica 89*(1), 37–76.

Aguirregabiria, V. and P. Mira (2010). Dynamic Discrete Choice Structural Models: A Survey. *Journal of Econometrics 156*, 38–67.

Arcidiacono, P., P. Bayer, J. R. Blevins, and P. B. Ellickson (2016). Estimation of Dynamic Discrete Choice Models in Continuous Time with an Application to Retail Competition. *Review of Economic Studies 83*, 889–931.

Arcidiacono, P. and P. Ellickson (2011). Practical Methods for Estimation of Dynamic Discrete Choice Models. *Annual Review of Economics 3*, 363–394.

Arcidiacono, P. and J. B. Jones (2003). Finite Mixture Distributions, Sequential Likelihood, and the EM Algorithm. *Econometrica 71(3)*, 933–946.

Arcidiacono, P. and R. A. Miller (2011). Conditional Choice Probability Estimation of Dynamic Discrete Choice Models with Unobserved Heterogeneity. *Econometrica 79(6)*, 1823–1868.

Cahuc, P., S. Carcillo, and A. Zylberberg (2014). *Labor Economics*. MIT Press.

Card, D., R. Chetty, and A. Weber (2007). The Spike at Benefit Exhaustion: Leaving the Unemployment System or Starting a New Job? *American Economic Review 97*(2), 113–118.

Cockx, B., M. Dejemeppe, A. Launov, and B. Van der Linden (2018). Imperfect Monitoring of Job Search: Structural Estimation and Policy Design. *Journal of Labor Economics 36*(1), 75 – 120.

DellaVigna, S., A. Lindner, B. Reizer, and J. F. Schmieder (2017). Reference-dependent Job Search: Evidence from Hungary. *Quarterly Journal of Economics 132*(4), 1969–2018.

Eckstein, Z. and G. J. van den Berg (2007). Empirical Labor Search: A Survey. *Journal of Econometrics 136*(2), 531 – 564.

Flinn, C. and J. Heckman (1982). New Methods for Analyzing Structural Models of Labor Force Dynamics. *Journal of Econometrics 18*(1), 115 – 168.

French, E. and C. Taber (2011). Identification of Models of the Labor Market. In O. Ashenfelter and D. Card (Eds.), *Handbook of Labor Economics, Volume 4A*, pp. 537—617. Elsevier.

Frijters, P. and B. van der Klaauw (2006). Job Search with Nonparticipation. *Economic Journal 116*(508), 45 – 83.

Gyetvai, A. (2021). Job Mobility Within and Across Occupations. *Working Paper*.

Harasztosi, P. and A. Lindner (2019). Who Pays for the Minimum Wage? *American Economic Review 109*(8), 2693–2727.

Heckman, J. J. and B. Singer (1984). The Identifiability of the Proportional Hazard Model. *Review of Economic Studies 51(2)*, 231–241.

Hotz, V. J. and R. A. Miller (1993). Conditional Choice Probabilities and the Estimation of Dynamic Models. *Review of Economic Studies 60(3)*, 497–530.

Johnston, A. C. and A. Mas (2018). Potential Unemployment Insurance Duration and Labor Supply: The Individual and Market-Level Response to a Benefit Cut. *Journal of Political Economy 126*(6), 2480–2522.

Krueger, A. B. and B. D. Meyer (2002). Labor Supply Effects of Social Insurance. In A. J. Auerbach and M. Feldstein (Eds.), *Handbook of Public Economics*, pp. 2327—2392. North Holland.

Krueger, A. B. and A. I. Mueller (2016). A Contribution to the Empirics of Reservation Wages. *American Economic Journal: Economic Policy 8*(1), 142–179.

Launov, A. and K. Walde (2013). Estimating Incentive and Welfare Effects of Nonstationary Unemployment Benefits. *International Economic Review 4*(54), 1159 – 1198.

Le Barbanchon, T., R. Rathelot, and A. Roulet (2017). Unemployment Insurance and Reservation Wages: Evidence from Administrative Data. *Journal of Public Economics*.

Llull, J. and R. Miller (2018). Internal Migration in Dual Labor Markets. *Working Paper*.

Lollivier, S. and L. Rioux (2010). An Empirical Examination of the Sources of Changes Over Time in the Job Finding Rate Using Reservation Wages and Rejected Wage Offers. *International Economic Review 51*(4), 1039–1069.

Magnac, T. and D. Thesmar (2002). Identifying Dynamic Discrete Decision Processes. *Econometrica 70(2)*, 801–816.

McFadden, D. (1978). Modelling the Choice of Residential Location. In A. Karlqvist, L. Landqvist, and J. Weibull (Eds.), *Spatial Interaction Theory and Residential Location*, pp. 75—96. North Holland.

Nekoei, A. and A. Weber (2017). Does Extending Unemployment Benefits Improve Job Quality? *American Economic Review 107*(2), 527–561.

Paserman, D. M. (2008). Job Search and Hyperbolic Discounting: Structural Estimation and Policy Evaluation. *Economic Journal 118*(531), 1418 – 1452.

Ransom, T. (2022). Labor Market Frictions and Moving Costs of the Employed and Unemployed. *Journal of Human Resources 57*(S), S137–S166.

Ridder, G. and G. J. van den Berg (2003). Measuring Labor Market Frictions: A Cross-Country Comparison. *Journal of the European Economic Association 1*(1), 224–244.

Robin, J.-M. (2011). On the Dynamics of Unemployment and Wage Distributions. *Econometrica 79*(5), 1327–1355.

Schmieder, J. F. and T. von Wachter (2016). The Effects of Unemployment Insurance Benefits: New Evidence and Interpretation. *Annual Review of Economics 8*, 547–581.

Sorkin, I. (2018). Ranking Firms Using Revealed Preference. *Quarterly Journal of Economics 133*(3), 1331 – 1393.

Sullivan, P. and T. To (2014). Search and Nonwage Job Characteristics. *Journal of Human Resources 49*(2), 472–507.

Sun, Y., P. B. Gilbert, and I. W. McKeague (2009). Proportional Hazards Models with Continuous Marks. *Annals of Statistics 37*(1), 394–426.

Taber, C. and R. Vejlin (2020). Estimation of a Roy/Search /Compensating Differential Model of the Labor Market. *Econometrica 88*(3), 1031–1069.

van den Berg, G. J. (1990). Nonstationarity in Job Search Theory. *Review of Economic Studies 57*(2), 255–277.

van den Berg, G. J. (2001). Duration Models: Specification, Identification and Multiple Durations. In J. J. Heckman and E. Leamer (Eds.), *Handbook of Econometrics*, pp. 3381—3460. Elsevier.

Verner, E. and G. Gyöngyösi (2020). Household Debt Revaluation and the Real Economy: Evidence from a Foreign Currency Debt Crisis. *American Economic Review 110*(9), 2667–2702.

Wolpin, K. I. (1987). Estimating a Structural Search Model: The Transition from School to Work. *Econometrica 55*(4), 801 – 817.

Online Appendix

A Mathematical appendix

A.1 Proof of Theorem 1

A.1.1 Proof of Lemma 2 (ii)

Akin to Equation (3.2), for any triple $(w, w', \tilde{w}) \in \Omega_w^3$:

$$\begin{pmatrix} \ln\left(\frac{h_{ww}^{ss'}}{\lambda^{ss'} f_w^{s'} - h_{ww}^{ss'}}\right) + \ln\left(\frac{h_{ww}^{s's}}{\lambda^{s's} f_w^{s} - h_{ww}^{s's}}\right) \\ \ln\left(\frac{h_{ww}^{ss'}}{\lambda^{ss'} f_w^{s'} - h_{ww}^{ss'}}\right) + \ln\left(\frac{h_{ww}^{s's}}{\lambda^{s's} f_w^{s} - h_{ww}^{s's}}\right) \end{pmatrix} = \begin{pmatrix} \ln\left(\frac{h_{w'w'}^{ss'}}{\lambda^{ss'} f_{w'}^{s'} - h_{w'w'}^{ss'}}\right) + \ln\left(\frac{h_{w'w'}^{s's}}{\lambda^{s's} f_{w'}^{s} - h_{w'w'}^{s's}}\right) \\ \ln\left(\frac{h_{\tilde{w}\tilde{w}}^{ss'}}{\lambda^{ss'} f_{\tilde{w}}^{s'} - h_{\tilde{w}\tilde{w}}^{ss'}}\right) + \ln\left(\frac{h_{\tilde{w}\tilde{w}}^{s's}}{\lambda^{s's} f_{\tilde{w}}^{s} - h_{\tilde{w}\tilde{w}}^{s's}}\right) \end{pmatrix} \tag{A.1}$$

Note that now we exploit transitions across job types s and s', thus we are able to use the same wage in the old and new jobs. This nonlinear system of two equations and two unknowns—$\lambda^{ss'}$ and $\lambda^{s's}$—can be rewritten as follows:

$$\begin{pmatrix} B_{w'}\lambda^{ss'} + C_{w'}\lambda^{s's} - A_{w'}\lambda^{ss'}\lambda^{s's} \\ B_{\tilde{w}}\lambda^{ss'} + C_{\tilde{w}}\lambda^{s's} - A_{\tilde{w}}\lambda^{ss'}\lambda^{s's} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{A.2}$$

where the A, B, C coefficients are defined in Lemma 2 (ii). Assuming $A_{w'} \neq 0$ (Condition (a) from Lemma 2 (ii)) and replacing $\lambda^{ss'}\lambda^{s's}$ in the second equation by its expression from the first equation identifies the ratio of the arrival rates, with:

$$\lambda^{s's} \quad = \quad \left(\frac{B_{w'}A_{\tilde{w}} - B_{\tilde{w}}A_{w'}}{A_{w'}C_{\tilde{w}} - A_{\tilde{w}}C_{w'}}\right)\lambda^{ss'}$$

where $A_{w'}C_{\tilde{w}} - A_{\tilde{w}}C_{w'} \neq 0$ from Condition (c). Finally, substituting for $\lambda^{s's}$ in the first equation identifies, under Condition (b), $\lambda^{ss'}$ and then $\lambda^{s's}$, which admit the following closed-form expressions:

$$\lambda^{ss'} = \frac{B_{w'}C_{\tilde{w}} - B_{\tilde{w}}C_{w'}}{B_{w'}A_{\tilde{w}} - B_{\tilde{w}}A_{w'}} \qquad \text{and} \quad \lambda^{s's} = \frac{B_{w'}C_{\tilde{w}} - B_{\tilde{w}}C_{w'}}{A_{w'}C_{\tilde{w}} - A_{\tilde{w}}C_{w'}} \tag{A.3}$$

Having identified the arrival rates $\lambda^{ss'}$ and the wage offer distribution f_w^s, identification of the CCPs $p_{ww'}^{ss'}$ follows. Then, we can identify $c^{ss'} + c^{s's}$, and together with the assumption that switching costs are symmetric (i.e., $c^{ss'} = c^{s's}$), $c^{ss'}$ is identified.

A.1.2 Proof of Lemma 3 (ii)–(iii)

(ii) Identification of CRRA preferences. We assume that workers are endowed with CRRA preferences, such that:

$$u(w) = \alpha \frac{w^{1-\theta}}{1-\theta}$$

From the prior identification result in Lemma 3 such that u_w is identified up to a constant, it follows that for $\tilde{w} > w' > w$, the following ratio is identified:

$$\frac{u_{w'} - u_w}{u_{\tilde{w}} - u_w} = \frac{w'^{1-\theta} - w^{1-\theta}}{\tilde{w}^{1-\theta} - w^{1-\theta}} \tag{A.4}$$

In order to establish identification of the risk aversion parameter θ, we show that the function $\theta \mapsto \frac{y^{1-\theta}-x^{1-\theta}}{z^{1-\theta}-y^{1-\theta}}$, where $z > y > x > 0$, is monotonically increasing on $(0, \infty)$.

$$f(\theta) = \frac{y^{1-\theta} - x^{1-\theta}}{z^{1-\theta} - y^{1-\theta}} \tag{A.5}$$

$$\begin{aligned} f'(\theta) = &\left(z^{1-\theta} - y^{1-\theta}\right)^{-2} \cdot \Big[\left(x^{1-\theta}\ln x - y^{1-\theta}\ln y\right)\left(z^{\theta} - y^{\theta}\right) \\ &- \left(y^{1-\theta} - x^{1-\theta}\right)\left(y^{1-\theta}\ln y - z^{1-\theta}\ln z\right)\Big] \end{aligned} \tag{A.6}$$

$$f'(\theta) > 0 \tag{A.7}$$

$$\begin{aligned} &\Leftrightarrow \left(x^{1-\theta}\ln x - x^{1-\theta}\ln y\right)\left(z^{1-\theta} - y^{1-\theta}\right) + \left(x^{1-\theta}\ln y - y^{1-\theta}\ln y\right)\left(z^{1-\theta} - y^{1-\theta}\right) \\ &> \left(z^{1-\theta}\ln y - z^{1-\theta}\ln z\right)\left(y^{1-\theta} - x^{1-\theta}\right) + \left(y^{1-\theta}\ln y - z^{1-\theta}\ln y\right)\left(y^{1-\theta} - x^{1-\theta}\right) \end{aligned} \tag{A.8}$$

$$\Leftrightarrow \left[x^{1-\theta}\ln(x/y)\right]\left(z^{1-\theta} - y^{1-\theta}\right) > \left[z^{1-\theta}\ln(y/z)\right]\left(y^{1-\theta} - x^{1-\theta}\right) \tag{A.9}$$

$$\Leftrightarrow \ln(y/x)\left[1 - (y/z)^{1-\theta}\right] < \ln(z/y)\left[(y/x)^{1-\theta} - 1\right] \tag{A.10}$$

$$\Leftrightarrow \quad (y/x)^{1-\theta}\ln(z/y) + \ln(y/x)(y/z)^{1-\theta} > \ln(y/x) + \ln(z/y) \tag{A.11}$$

The above condition holds if and only if $g(\theta) > g(1)$, where, for all $\theta > 0$, $g(\theta) \equiv (y/x)^{1-\theta}\ln(z/y) + (y/z)^{1-\theta}\ln(y/x)$. The derivative of $g(\cdot)$ is given by:

$$g'(\theta) = \ln(y/x)\ln(z/y)[(y/z)^{1-\theta} - (y/x)^{1-\theta}]$$

It follows that $g'(\theta) < 0$ on $(0,1)$ and $g'(\theta) > 0$ on $(1,\infty)$. Identification of θ follows.

Having identified θ, it follows that the utility coefficient α is identified and given by the following closed-form expression:

$$\alpha = \frac{u_{\tilde{w}} - u_w}{\tilde{w}^{1-\theta} - w^{1-\theta}} \tag{A.12}$$

which yields full identification of the flow utility of wages.

(iii) Identification of ϕ^s up to $V_0(0)$. We can express the log odds ratio in terms of the structural parameters using Equation (3.7):

$$\begin{aligned}
\ln\left(\frac{p^{s\tilde{s}}_{w\tilde{w}}}{1-p^{s\tilde{s}}_{w\tilde{w}}}\right) &= V^{\tilde{s}}_{\tilde{w}} - c^{s\tilde{s}} - V^s_w \\
&= \Big(u_{\tilde{w}} + \phi^{\tilde{s}} + \delta^{\tilde{s}}_0 V_0(0) - \sum_{w'}\sum_{s'} \delta^{\tilde{s}s'}_{\tilde{w}w'}\left[c^{\tilde{s}s'} + \ln\left(p^{\tilde{s}s'}_{\tilde{w}w'}\right) - \ln\left(1 - p^{\tilde{s}s'}_{\tilde{w}w'}\right)\right] \\
&\quad - \sum_{w'}\sum_{s'} \lambda^{\tilde{s}s'} f^{s'}_{w'} \ln\left(1 - p^{\tilde{s}s'}_{\tilde{w}w'}\right)\Big)\Big/(\rho + \delta^{\tilde{s}}_0) \\
&\quad - \Big(u_w + \phi^s + \delta^s_0 V_0(0) - \sum_{w'}\sum_{s'} \delta^{ss'}_{ww'}\left[c^{ss'} + \ln\left(p^{ss'}_{ww'}\right) - \ln\left(1 - p^{ss'}_{ww'}\right)\right] \\
&\quad + \sum_{w'}\sum_{s'} \lambda^{ss'} f^{s'}_{w'} \ln\left(1 - p^{ss'}_{ww'}\right)\Big)\Big/(\rho + \delta^s_0) - c^{s\tilde{s}}
\end{aligned} \tag{A.13}$$

Collecting all known terms on the left hand side, the equation can be rearranged as:

$$\kappa^{s\tilde{s}}_{w\tilde{w}} = \frac{1}{\rho + \delta^{\tilde{s}}_0}\,\phi^{\tilde{s}} - \frac{1}{\rho + \delta^s_0}\,\phi^s + \left(\frac{\delta^{\tilde{s}}_0}{\rho + \delta^{\tilde{s}}_0} - \frac{\delta^s_0}{\rho + \delta^s_0}\right) V_0(0) \tag{A.14}$$

where

$$
\begin{aligned}
\kappa_{w\tilde{w}}^{s\tilde{s}} &= \ln\left(\frac{p_{w\tilde{w}}^{s\tilde{s}}}{1-p_{w\tilde{w}}^{s\tilde{s}}}\right) + c^{s\tilde{s}} \\
&- \frac{u_{\tilde{w}} - \sum_{w'}\sum_{s'} \delta_{\tilde{w}w'}^{\tilde{s}s'} \left[c^{\tilde{s}s'} + \ln\left(p_{\tilde{w}w'}^{\tilde{s}s'}\right) - \ln\left(1-p_{\tilde{w}w'}^{\tilde{s}s'}\right)\right] - \sum_{w'}\sum_{s'} \lambda^{\tilde{s}s'} f_{w'}^{s'} \ln\left(1-p_{\tilde{w}w'}^{\tilde{s}s'}\right)}{\rho + \delta_0^{\tilde{s}}} \\
&+ \frac{u_{w} - \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'} \left[c^{ss'} + \ln\left(p_{ww'}^{ss'}\right) - \ln\left(1-p_{ww'}^{ss'}\right)\right] - \sum_{w'}\sum_{s'} \lambda^{ss'} f_{w'}^{s'} \ln\left(1-p_{ww'}^{ss'}\right)}{\rho + \delta_0^{s}}
\end{aligned} \tag{A.15}
$$

Now, since $\phi^1 = 0$, writing Equation (A.14) for $s = 1$ yields:

$$
\tilde{\kappa}_{w\tilde{w}}^{1\tilde{s}} = \frac{1}{\rho + \delta_0^{\tilde{s}}}\,\phi^{\tilde{s}} + \left(\frac{\delta_0^{\tilde{s}}}{\rho + \delta_0^{\tilde{s}}} - \frac{\delta_0^{1}}{\rho + \delta_0^{1}}\right) V_0(0) \tag{A.16}
$$

Thus, we can write $\phi^{\tilde{s}}$ as a known linear function of $V_0(0)$. Furthermore, note that when the job destruction rates are not specific to job types, i.e., $\delta_0^s = \delta_0$ for all s, the non-pecuniary payoffs ϕ^s are directly identified from Equation (A.16).

A.2 Extension: aggregate shocks

One can extend our identification strategy to accommodate aggregate shocks. Specifically, consider the case where the market economy can be in one of K different states, where the job offer arrival rates, the job destruction rates, the rates of involuntary wage mobility, the offered wage distributions, and the flow payoff of unemployment are allowed to depend on the state of the economy. We further assume that the econometrician perfectly observes the state of the economy. We denote the rate at which the economy transitions from state k to k' by $q_{kk'}$, which is identified from the observed transition rates across market states.

On the employment side, identification of the state-specific offer arrival rates, destruction and involuntary wage mobility rates, offered wage distribution and conditional choice probabilities, along with the switching cost all follow directly from the baseline case, leaving the flow payoff of employment as the only unknown parameters. The value function of employment V^s_{wk} is given by:

$$\Big(\rho + \sum_{k'} q_{kk'} + \delta^s_{0k} + \sum_{s'} \lambda^{ss'}_k\Big) V^s_{wk} = u_w + \phi^s + \delta^s_{0k} V_{0k}(0) + \sum_{k'} q_{kk'} V^s_{wk'}$$
$$+ \sum_{w'} \sum_{s'} \delta^{ss'}_{ww'k} \Big[V^{s'}_{w'k} - V^s_{wk}\Big] + \sum_{s'} \lambda^{ss'}_k \sum_{w'} f^{s'}_{w'k} \ln\Big(1 - p^{ss'}_{ww'k}\Big) \tag{A.17}$$

where $V^{s'}_{w'k} - V^s_{wk} = \ln\Big(p^{ss'}_{ww'k}\Big) - \ln\Big(1 - p^{ss'}_{ww'k}\Big) + c^{ss'}$.

Subtracting off the corresponding expression for $V^s_{\tilde{w}k}$ (with $\tilde{w} \neq w$) yields:

$$\Big(\rho + \sum_{k'} q_{kk'} + \delta^s_{0k} + \sum_{s'} \lambda^{ss'}_k\Big) \left[V^s_{wk} - V^s_{\tilde{w}k}\right] = u_w - u_{\tilde{w}} + \sum_{k'} q_{kk'} \left[V^s_{wk'} - V^s_{\tilde{w}k'}\right]$$
$$+ \sum_{w'} \sum_{s'} \Big(\delta^{ss'}_{ww'k} \Big[V^{s'}_{w'k} - V^s_{wk}\Big] - \delta^{ss'}_{\tilde{w}w'k} \Big[V^{s'}_{w'k} - V^s_{\tilde{w}k}\Big]\Big)$$
$$+ \sum_{s'} \lambda^{ss'}_k \sum_{w'} f^{s'}_{w'k} \Big(\ln\Big(1 - p^{ss'}_{ww'k}\Big) - \ln\Big(1 - p^{ss'}_{\tilde{w}w'k}\Big)\Big) \tag{A.18}$$

where the difference in value functions on the left and right-hand sides are given by the sum of the log odds ratio and the switching cost. This identifies the wage component of the flow payoff up to a constant. Identification of the non-pecuniary components ϕ^s then proceeds in a similar fashion, using instead the job-to-job transitions across job types.

Identification of the unemployment-side parameters then follows from similar arguments as in Section 3.3. The same strategy applies to a context with aggregate shocks, after conditioning the hazard rates out of unemployment on the (observed) states of the economy.

B Data appendix

B.1 Sample creation

We define our analysis sample as follows:

1. Flip primary and secondary work arrangements (PWAs, SWAs)

    - In the raw data, PWA is defined as the arrangement with the highest earnings in the month. This setup may result in PWAs and SWAs flipping in the raw data, e.g. when a worker works only a few days in their PWA.

- *Solution:* Looping through all worker-months, we flip variables related to PWAs and SWAs as follows:

month	firmid1	var1	firmid2	var2
$t-1$	A	x_{t-1}	B	y_{t-1}
t	B	x_t	A	y_t

$\rightsquigarrow$

month	firmid1	var1	firmid2	var2
$t-1$	A	x_{t-1}	B	y_{t-1}
t	A	y_t	B	x_t

2. Calculate durations

   (a) Employed: we calculate or infer spell-year durations in PWA. See Appendix B.2 for details.

   (b) Unemployed: we observe daily unemployment durations in the raw data. For spells that end after October 2005 (the end date of our sample), we flag spells as right-censored and shorten their durations by the out-of-sample portion. Therefore, our analysis sample includes U spells that are censored earlier than 269 days.

3. Define EE, EU, UE, EN, NE transitions

4. Calculate wages

   (a) Calculate counterfactual minimum wage earnings: how much the worker would have earned in a day working full time in a minimum-wage job

   (b) Calculate daily wages as total earnings in a spell-year, divided by spell-year durations

   (c) Discretize wages: see Appendix B.3 for details

   (d) Calculate accepted wages

5. Define covariates for population probabilities

6. Save analysis sample

B.2 Correcting employment spell durations

The raw data on employment spells are recorded at a monthly frequency. In each month, the total number of days worked (`days`) and total earnings are known. Furthermore, days worked and earnings at PWAs and SWAs (`days_1`, `days_2`) are known if the arrangement was ongoing on the 15th of the month. We focus on PWAs only. Table 7 summarizes the possible ways in which EE transitions show up in the raw data when observations on PWAs are not missing. When `days` equals `days_1`, we know with certainty that the transition happened on the boundary of the month: we label this as a clean EE transition (see Panel a). When `days` does not equal `days_1`, we need to make some assumptions about the uncovered days: Panels b-d illustrate these cases that we label fuzzy. The bottom tables summarize our assumptions on the number of days worked in each PWA.

Table 7: EE scenarios in raw data, no missing PWAs

(a) Clean EE

days	days_1	firmid1
31	31	A
30	30	A
31	31	B

⇓

no assumption needed

(b) Fuzzy EE 1

days	days_1	firmid1
31	31	A
30	16	A
31	31	B

⇓

31	A
16	A
14	B
31	B

(c) Fuzzy EE 2

days	days_1	firmid1
31	31	A
30	16	B
31	31	B

⇓

31	A
14	A
16	B
31	B

(d) Fuzzy EE 3

days	days_1	firmid1
31	31	A
30	16	B
31	31	C

⇓

31	A
$a < 14$	A
16	B
$30 - 16 - a$	C
31	C

Table 8 summarizes our assumptions when PWA data are missing.

Table 8: EE scenarios in raw data, missing PWAs

(a)

days	days_1	firmid1
31	31	A
25	.	.
31	31	A

⇓

31	A
25	A
31	A

(b)

days	days_1	firmid1
31	31	A
25	.	.
31	31	B

⇓

31	A
$d < 15$	A
$25 - d$	B
31	B

(c)

days	days_1	firmid1
31	31	A
10	.	.
7	.	.
30	30	B

⇓

31	A
10	A
7	B
31	B

(d)

days	days_1	firmid1
31	31	A
20	.	.
25	.	.
31	31	B

⇓

31	A
$a < 15$	A
$20 - a + 25 - b$	X
$b < 15$	B
31	B

Furthermore, we censor spells that spill over calendar years. We do so in order to track yearly wage changes observed in the raw data. Additionally, we censor spells at October 31st due to data limitations, as mentioned in the text. As an example, a continuous E spell from March 2003 until May 2005 that pays wage w and is followed by a EE transition to a job paying w' is represented as a right-censored spell of 8 months in w, a right-censored spell of 10 months in w, and a spell of 5 months with a EE transition from w to w'.

B.3 Discretizing wages

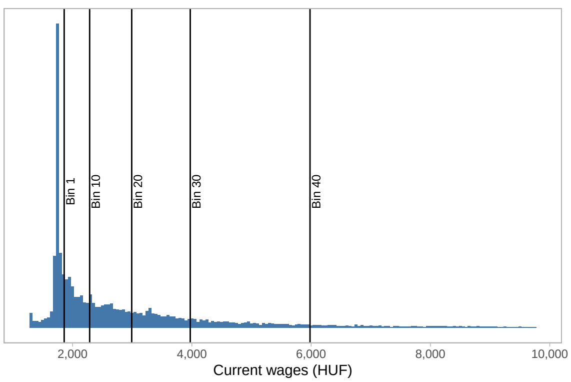
We discretize the continuously observed wages in the data into W bins, with $W = 50$ for our main results. First, we calculate the average daily wage for each worker in a given year across all months spent in employment. Then we categorize these continuous wages into discrete bins. The first bin contains wages between 75 and 107 percent of the effective minimum wage.[24] We drop wage observations below 75 percent of the effective minimum wage because we cannot distinguish between full-time and part-time earners in the data. Furthermore, we add a 7 percent padding to the right cutoff of the first bin to ensure that we include all minimum wage earners in the first bin. We then split the other wage observations, censored at the 99th percentile, evenly across the remaining $W - 1$ bins. We repeat the same discretization procedure for each calendar year: Figure 6 demonstrates our discretization method for 2004 for various groups.

Figure 7 plots the resulting discrete distribution of current wages. Current wages for employment spells that lead to a job-to-job transition, on the left panel, have a mean of 3,428 HUF (percentiles: 25th 1,738 HUF; 50th 2,347 HUF; 75th 3,685 HUF). Current wages for all employment spells, on the right panel, have a mean of 3,670 HUF (percentiles: 25th 1,738 HUF; 50th 2,557 HUF; 75th 4,249 HUF). Similarly, Figure 8 plots the discrete distribution of accepted wages for job-to-job and unemployment-to-employment transitions. Accepted wages for job-to-job transitions have a mean of 3,657 HUF (percentiles: 25th 1,738 HUF; 50th 2,516 HUF; 75th 4,056 HUF). Accepted wages out of unemployment are more right-tailed than those for job-to-job

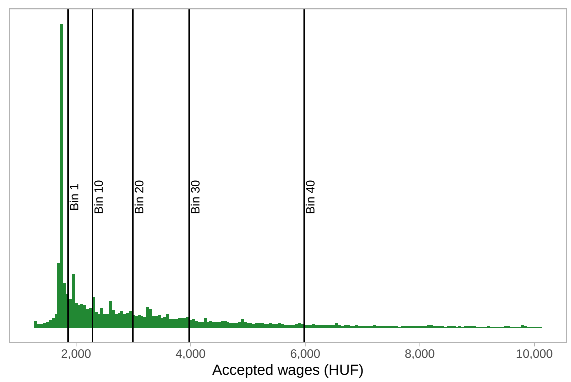
[24]During our sampling period, Hungary had a simple minimum wage policy: 50,000 HUF in 2003, 53,000 HUF in 2004, and 57,000 HUF in 2005 (200 HUF $\approx$ 1 USD in 2004).

Figure 6: Discretizing observed wages

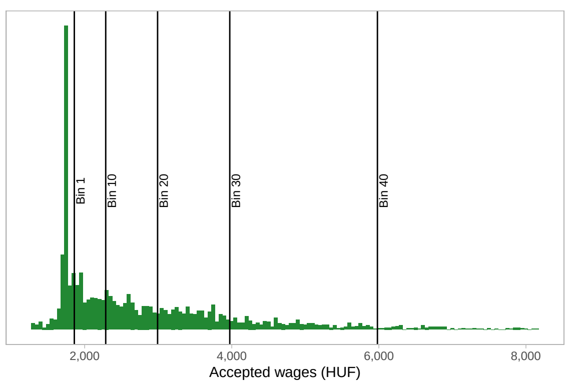
(a) Current wages



(b) Accepted wages (EE)



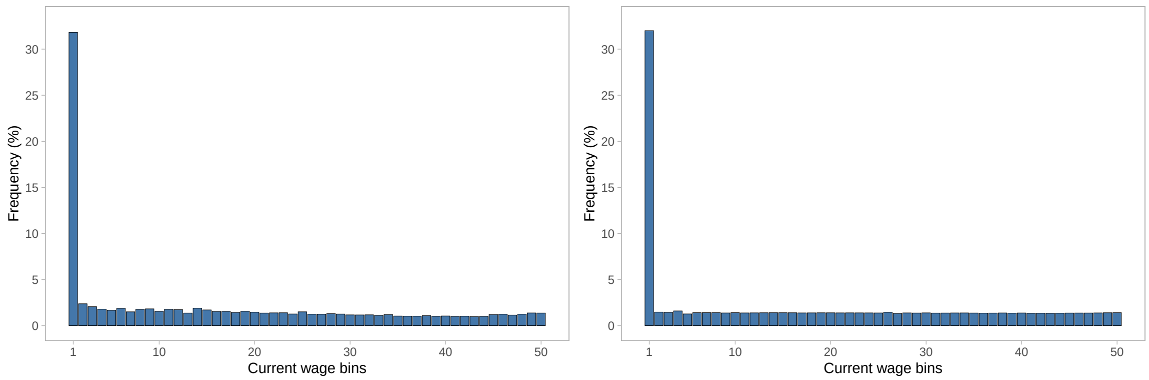
(c) Accepted wages (UE)



Notes: Histograms of daily wage rates in 2004 with 50 HUF bin width, truncated at the 95th percentile (200 HUF $\approx$ 1 USD in 2004). Vertical lines denote selected wage bin cutoffs. Panel (a): current daily wages for employment spells that lead to an EE transition. Panel (b): accepted daily wages for employment spells after an EE transition. Panel (c): accepted daily wages for unemployment spells after a UE transition.
Source: CERS-HAS, authors' own calculations.

Figure 7: Discrete distribution of current wages

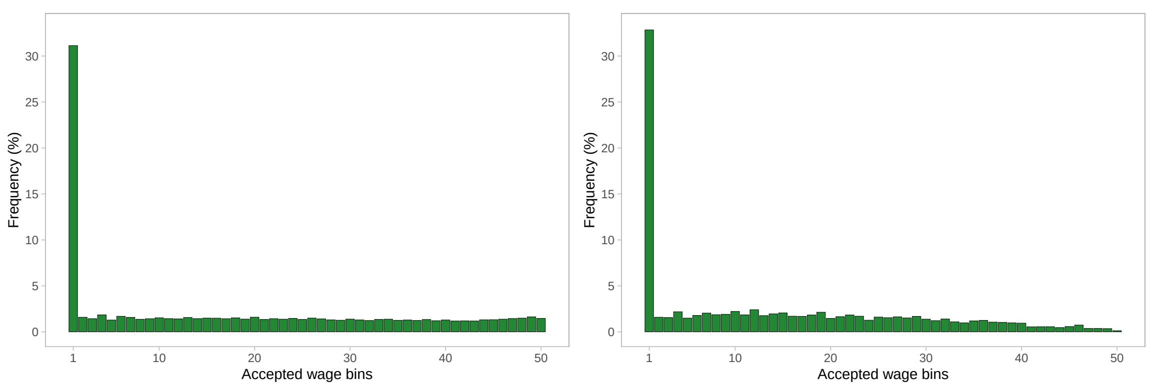
(a) Spells leading to EE transitions | (b) All spells



Notes: Panel (a): discrete distribution of current wages for employment spells that lead to an EE transition. Panel (b): discrete distribution of current wages for all employment spells.
Source: CERS-HAS, authors' own calculations.

Figure 8: Discrete distribution of accepted wages

(a) EE transitions | (b) UE transitions



Notes: Panel (a): discrete distribution of accepted wages for employment spells that lead to an EE transition. Panel (b): discrete distribution of accepted wages for unemployment spells that lead to an employment spell.
Source: CERS-HAS, authors' own calculations.

transitions, with a mean of 3,021 HUF (percentiles: 25th 1,802 HUF; 50th 2,427 HUF; 75th 3,543 HUF), in line with the notion that the unemployed tend to move to lower-paying jobs.

C Estimation appendix

This appendix details our estimation procedure, outlined in Section 5.

C.1 Posterior type distribution

Rather than imposing the structure of the model when classifying types, we instead choose a flexible functional form for the likelihood of job-to-job transitions. In particular, we obtain estimates of θ_r^I by maximizing an alternative objective function:

$$\sum_i \ln \left(\sum_r \pi_r \mathcal{L}_{ir}^I(\theta_r^I) \prod_{s=1}^{S_i} \tilde{\mathcal{L}}_{isr}^E(\tilde{\theta}_r^E) \right) \tag{C.1}$$

where $\mathcal{L}_{ir}^I(\theta_r^I)$ was defined in Equation (5.3) and we specify the reduced-form likelihood associated with employment spell s below.

We break the hazard of going from w to w' into two parts: (i) the hazard of leaving w-paying job for any other job, and (ii) the probability that the accepted job pays w'. These two parts are associated with the parameters $\tilde{\theta}_r^h$ and $\tilde{\theta}_r^w$, respectively. We specify the reduced-form hazard of leaving a w-paying job given the individual is of type-r as:

$$\tilde{h}_{wsr} = \exp(\tilde{\theta}_{1r}^h + \tilde{\theta}_{2r}^h \ln(w_s) + \tilde{\theta}_{3r}^h \mathbb{1}\{w_s = \underline{w}\} + \tilde{\theta}_{4r}^h \mathbb{1}\{y_s = 2004\} + \tilde{\theta}_{5r}^h \mathbb{1}\{y_s = 2005\}) \tag{C.2}$$

where y_s refers to the calendar year of spell s.

Conditional on moving to a new job, for the reduced form we model the accepted wage as a tobit like in Equation (5.3) but where one of the conditioning variables is the log of the current wage. Note that here we use the actual observed wage level in a given spell (unlike for the utility of wages where we use the mean wage in each bin). $\tilde{\mathcal{L}}_{isr}^E(\tilde{\theta}_r^E)$ is then given by:

$$\tilde{\mathcal{L}}^E_{isr}(\tilde{\theta}^E_r) = \left[\prod_w \tilde{h}_{wsr} \exp(-\tilde{h}_{wsr} t_s)\right]^{\mathbb{1}\{w_s = w\}} \tag{C.3}$$
$$\times \left[\Phi\left(\frac{\ln(\underline{w}) - \tilde{X}^w_s \tilde{\theta}^w_{xr}}{\tilde{\sigma}^w_r}\right)\right]^{\mathbb{1}\{w_{s+1} = \underline{w}\}} \cdot \left[\frac{1}{\tilde{\sigma}^w_r}\phi\left(\frac{\ln(w_{s+1}) - \tilde{X}^w_s \tilde{\theta}^w_{xr}}{\tilde{\sigma}^w_r}\right)\right]^{\mathbb{1}\{w_{s+1} > \underline{w}\}}$$

with $\tilde{\theta}^E_r = \left(\tilde{\theta}^h_r, \tilde{\theta}^w_{xr}, \tilde{\sigma}^w_r\right)'$, where $\tilde{X}^w_s \tilde{\theta}^w_{xr}$ is given by:

$$\tilde{X}^w_s \tilde{\theta}^w_{xr} = \tilde{\theta}^w_{1r} + \tilde{\theta}^w_2 \ln(w_s) + \tilde{\theta}^w_3 \mathbb{1}\{y_s = 2004\} + \tilde{\theta}^w_4 \mathbb{1}\{y_s = 2005\} \tag{C.4}$$

We then estimate the parameters $\left(\theta^I_r, \tilde{\theta}^E_r\right)$ using:

$$\max_{\theta^I_r, \tilde{\theta}^E_r} \sum_i \ln\left(\sum_r \pi_r \mathcal{L}^I_{ir}(\theta^I_r) \prod_{s=1}^{S_i} \tilde{\mathcal{L}}^E_{isr}(\tilde{\theta}^E_r)\right) \tag{C.5}$$

and recover the conditional type probabilities using:

$$q_{ir} = \frac{\pi_r \mathcal{L}^I_{ir}(\theta^I_r) \prod_{s=1}^{S_i} \tilde{\mathcal{L}}^E_{isr}(\tilde{\theta}^E_r)}{\sum_r \pi_r \mathcal{L}^I_{ir}(\theta^I_r) \prod_{s=1}^{S_i} \tilde{\mathcal{L}}^E_{isr}(\tilde{\theta}^E_r)} \tag{C.6}$$

C.2 Unemployed-side structural parameters

C.2.1 Optimization constraints for Type 1

The first set of constraints in Equation (5.27) simplify to the following nonlinear constraints:

$$p_{\underline{w}1}(t) \leq p_{\underline{w}1}(t+1) \tag{C.7}$$

$$\frac{h_{\underline{w}1}(t)}{\lambda_1(t)\, g_{\underline{w}1}(t)} \leq \frac{h_{\underline{w}1}(t+1)}{\lambda_1(t+1)\, g_{\underline{w}1}(t+1)} \tag{C.8}$$

$$\frac{\exp(X_{t+1}^{\lambda}\theta^{\lambda})}{\exp(X_t^{\lambda}\theta^{\lambda})} \frac{\Lambda\left(\beta_1\phi_{\underline{w}} + \gamma_{11} + \gamma_{21}\ln(t+1)\right)}{\Lambda\left(\beta_1\phi_{\underline{w}} + \gamma_{11} + \gamma_{21}\ln(t)\right)} \leq \frac{\exp(X_{t+1}^{h}\theta_1^{h})}{\exp(X_t^{h}\theta_1^{h})} \tag{C.9}$$

$$\begin{aligned} &\left(X_{t+1}^{\lambda} - X_t^{\lambda}\right)\theta^{\lambda} - \left(X_{t+1}^{h} - X_t^{h}\right)\theta_1^{h} \\ &+ \ln\left[1 + \exp\left(-\beta_1\phi_{\underline{w}} - \gamma_{11} - \gamma_{21}\ln(t)\right)\right] \\ &- \ln\left[1 + \exp\left(-\beta_1\phi_{\underline{w}} - \gamma_{11} - \gamma_{21}\ln(t+1)\right)\right] \leq 0 \end{aligned} \tag{C.10}$$

The second constraint simplifies to the following nonlinear constraint:

$$p_{\underline{w}1}(1) \geq \varepsilon \tag{C.11}$$

$$\frac{h_{\underline{w}1}(1)}{\lambda_1(1)\, g_{\underline{w}1}(1)} \geq \varepsilon \tag{C.12}$$

$$\frac{\exp(X_1^{h}\theta_1^{h})}{\exp(X_1^{\lambda}\theta^{\lambda})} \frac{1}{\Lambda\left(\beta_1\phi_{\underline{w}} + \gamma_{11}\right)} \geq \varepsilon \tag{C.13}$$

$$X_1^{h}\theta_1^{h} - X_1^{\lambda}\theta^{\lambda} - \ln\left[1 + \exp(-\beta_1\phi_{\underline{w}} - \gamma_{11})\right] \geq \ln(\varepsilon) \tag{C.14}$$

The third constraint simplifies as follows:

$$p_{\overline{w}1}(T) \leq 1 - \varepsilon \tag{C.15}$$

$$\frac{h_{\underline{w}1}(T)\exp(-\kappa_{\underline{w}\overline{w}1})}{\lambda_1(T)\, g_{\underline{w}1}(T) - h_{\underline{w}1}(T)[1-\exp(-\kappa_{\underline{w}\overline{w}1})]} \leq 1 - \varepsilon \tag{C.16}$$

$$\exp(X_T^h \theta_1^h)\left[1 + \frac{\varepsilon}{1-\varepsilon}\exp(-\kappa_{\underline{w}\overline{w}1})\right] \leq \exp(X_T^\lambda \theta^\lambda)\,\Lambda\left(\beta_1 \phi_{\underline{w}} + \gamma_{11} + \gamma_{21}\ln(T)\right) \tag{C.17}$$

$$X_T^h \theta_1^h + \ln\left[1 + \frac{\varepsilon}{1-\varepsilon}\exp(-\kappa_{\underline{w}\overline{w}1})\right] \leq X_T^\lambda \theta^\lambda - \ln[1+\exp(-\beta_1\phi_{\underline{w}} - \gamma_{11} - \gamma_{21}\ln(T))] \tag{C.18}$$

C.2.2 Optimization constraints for Type $r = 2$

The first set of constraints in Equation (5.31) simplify to the following nonlinear constraints:

$$p_{\underline{w}2}(t) \leq p_{\underline{w}2}(t+1) \tag{C.19}$$

$$\frac{h_{\underline{w}2}(t)}{\lambda_2(t)\, g_{\underline{w}2}(t)} \leq \frac{h_{\underline{w}2}(t+1)}{\lambda_2(t+1)\, g_{\underline{w}2}(t+1)} \tag{C.20}$$

$$\frac{\exp(X_{t+1}^\lambda \theta^\lambda \nu_2^\lambda + \psi_2^\lambda)}{\exp(X_t^\lambda \theta^\lambda \nu_2^\lambda + \psi_2^\lambda)} \frac{\Lambda\left(\beta_2\phi_{\underline{w}} + \gamma_{12} + \gamma_{22}\ln(t+1)\right)}{\Lambda\left(\beta_2\phi_{\underline{w}} + \gamma_{12} + \gamma_{22}\ln(t)\right)} \leq \frac{\exp(X_{t+1}^h \theta_2^h)}{\exp(X_t^h \theta_2^h)} \tag{C.21}$$

$$\begin{aligned} &\left(X_{t+1}^\lambda - X_t^\lambda\right)\theta^\lambda \nu_2^\lambda - \left(X_{t+1}^h - X_t^h\right)\theta_2^h \\ &+ \ln\left[1 + \exp\left(-\beta_2\phi_{\underline{w}} - \gamma_{12} - \gamma_{22}\ln(t)\right)\right] \\ &- \ln\left[1 + \exp\left(-\beta_2\phi_{\underline{w}} - \gamma_{12} - \gamma_{22}\ln(t+1)\right)\right] \leq 0 \end{aligned} \tag{C.22}$$

The second constraint simplifies as follows:

$$p_{\underline{w}2}(1) \geq \varepsilon \tag{C.23}$$

$$\frac{h_{\underline{w}2}(1)}{\lambda_2(1)\, g_{\underline{w}2}(1)} \geq \varepsilon \tag{C.24}$$

$$\frac{\exp(X_1^h \theta_2^h)}{\exp(X_1^\lambda \theta^\lambda \nu_2^\lambda + \psi_2^\lambda)} \frac{1}{\Lambda\left(\beta_2\phi_{\underline{w}} + \gamma_{12}\right)} \geq \varepsilon \tag{C.25}$$

$$X_1^h \theta_2^h - X_1^\lambda \theta^\lambda \nu_2^\lambda - \psi_2^\lambda - \ln\left[1 + \exp(-\beta_2\phi_{\underline{w}} - \gamma_{12})\right] \geq \ln(\varepsilon) \tag{C.26}$$

The third constraint which ensures that the CCPs are less than one simplifies to the following nonlinear constraint:

$$p_{\overline{w}2}(T) \leq 1 - \varepsilon \tag{C.27}$$

$$\frac{h_{\underline{w}2}(T) \exp(-\kappa_{\underline{w}\overline{w}2})}{\lambda_2(T)\, g_{\underline{w}2}(T) - h_{\underline{w}2}(T)[1 - \exp(-\kappa_{\underline{w}\overline{w}2})]} \leq 1 - \varepsilon \tag{C.28}$$

$$\exp(X_T^h \theta_2^h) \left[1 + \frac{\varepsilon}{1-\varepsilon} \exp(-\kappa_{\underline{w}\overline{w}2})\right] \leq \exp(X_T^\lambda \theta^\lambda \nu_2^\lambda + \psi_2^\lambda)\, \Lambda \left(\beta_2 \phi_{\underline{w}} + \gamma_{12} + \gamma_{22} \ln(T)\right) \tag{C.29}$$

$$X_T^h \theta_2^h + \ln\left[1 + \frac{\varepsilon}{1-\varepsilon} \exp(-\kappa_{\underline{w}\overline{w}2})\right] \leq X_T^\lambda \theta^\lambda \nu_2^\lambda + \psi_2^\lambda - \ln[1 + \exp(-\beta_2 \phi_{\underline{w}} - \gamma_{12} - \gamma_{22} \ln(T))] \tag{C.30}$$

Table 9: Computation time

Step	**Elapsed time**
Estimate posterior probabilities	24.13 min
Estimate job-to-job structural parameters	12.34 min
Estimate unemployment-to-job structural parameters	7.87 sec
Total	36.77 min

Notes: Computation time of the full three-step estimation procedure, using a random perturbation around the baseline estimates as starting values. Total includes, on top of the three estimation steps, reading in the data and estimating nonparametric unemployment-to-employment hazards. Benchmarked on a 32-core Intel® Xeon® Gold 6134 3.20GHz CPU with 96GB RAM, running MathWorks® MATLAB® R2018b (9.5.0.1033004).

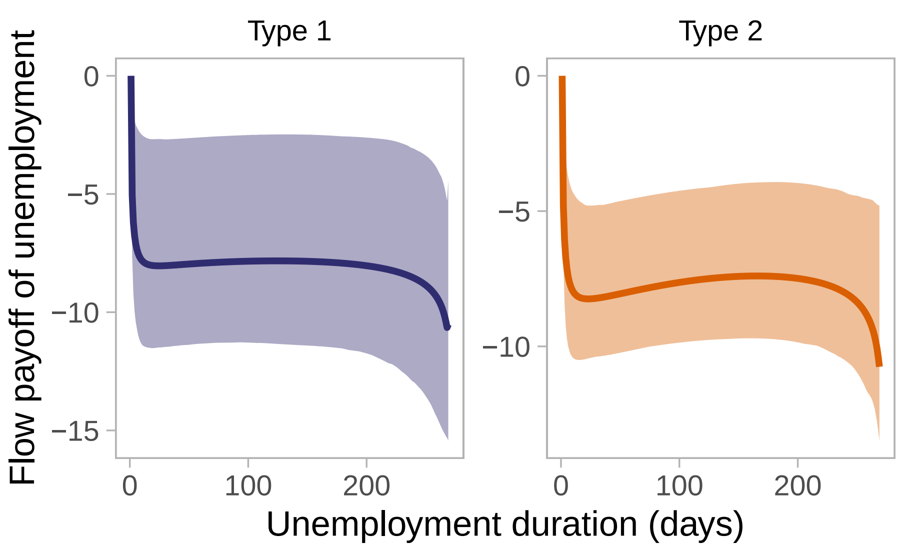
D Additional results

Table 10: Type probabilities

Initial wage bin	Type probability	
	Type 1	*Type 2*
1	99.8%	0.2%
	[99.8%, 99.8%]	[0.2%, 0.2%]
10	99.0%	1.0%
	[98.9%, 99.0%]	[1.0%, 1.1%]
20	96.2%	3.8%
	[95.9%, 96.4%]	[3.6%, 4.1%]
30	87.4%	12.6%
	[86.7%, 88.1%]	[11.9%, 13.3%]
40	61.8%	38.2%
	[60.0%, 63.0%]	[37.0%, 40.0%]
50	1.5%	98.5%
	[1.3%, 1.7%]	[98.3%, 98.7%]

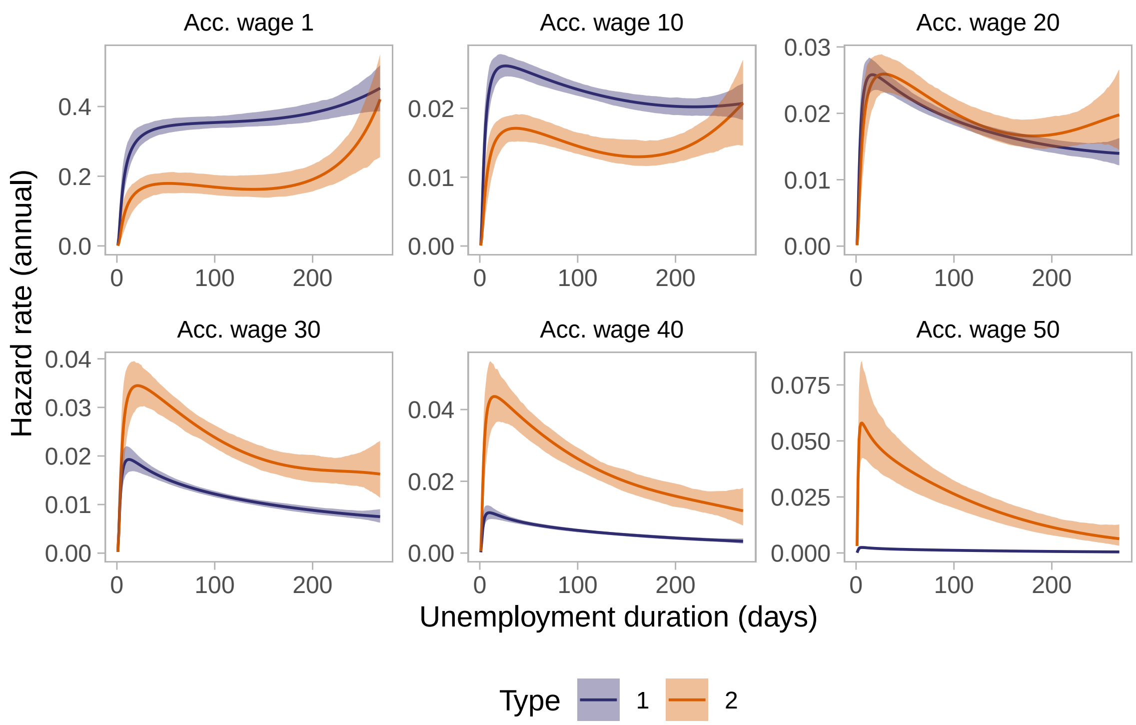
Notes: 95% bootstrap confidence intervals in brackets (500 replications).
Source: CERS-HAS, authors' own calculations.

Figure 9: Flow payoff of unemployment (normalized)



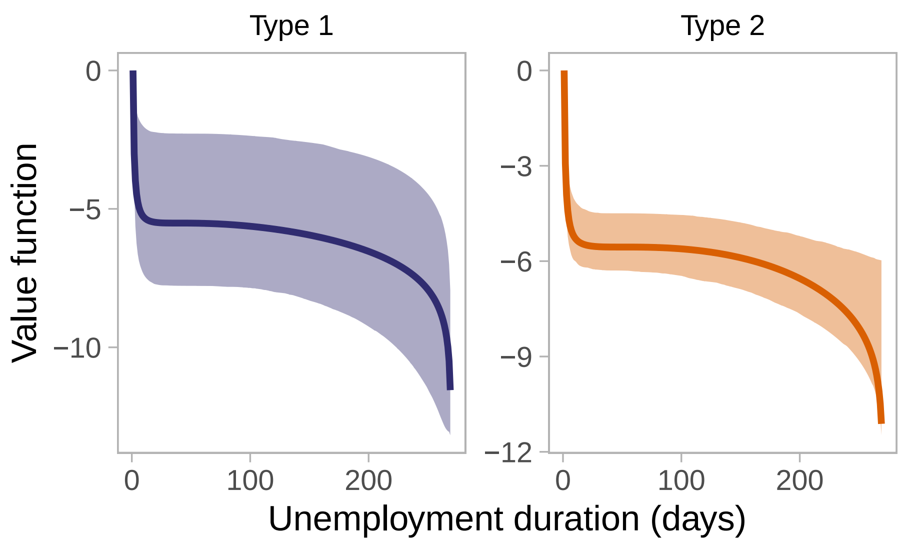
Notes: Flow payoff normalized w.r.t. $t = 0$ for each type. Shaded regions represent 95% bootstrap confidence band (500 replications).
Source: CERS-HAS, authors' own calculations.

Figure 10: Structural unemployment-to-employment hazards



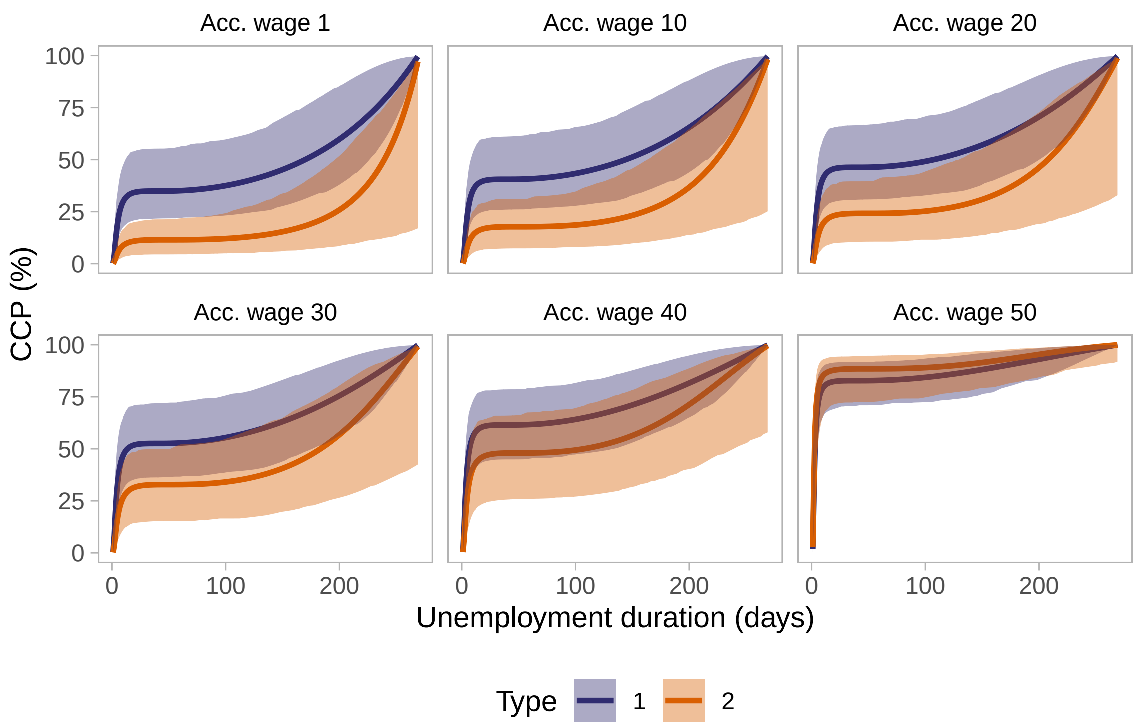
Notes: Annual hazard rates, conditional on exiting to a given wage bin. Shaded regions represent 95% bootstrap confidence band (500 replications).
Source: CERS-HAS, authors' own calculations.

Figure 11: Value function of unemployment (normalized)



Notes: Value function normalized w.r.t. the value of unemployment at $t = 0$ for each type. Shaded regions represent 95% bootstrap confidence band (500 replications).
Source: CERS-HAS, authors' own calculations.

Figure 12: CCPs, unemployment-to-employment transitions



Notes: Shaded regions represent 95% bootstrap confidence band (500 replications).
Source: CERS-HAS, authors' own calculations.