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ABSTRACT

Observing the increasingly important roles played by the creation and transmission of information and tacit knowledge, we construct an information-network model incorporating both information transmitters and information aggregators. Given information-processing roles in aggregation or transmission, we establish various general properties concerning the existence of a network equilibrium, its optimality and the patterns of equilibrium and optimal configuration. We then allow for endogenous choice of the information-processing roles. We prove the existence and show that, with sufficiently small link maintenance costs, the monocentric network with one aggregator connecting to all other agents as transmitters on a tree graph is the unique configuration. In general, a rich array of equilibrium configurations may emerge, including core-star, star-with-satellites and cycles. We further characterize an information-processing chain network with all information aggregators and transmitters linked along a chain and compute numerically the ranges of transmission decays and link costs within which a network equilibrium arises.

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"If one man starts a new idea, it is taken up by others and combined with suggestions of their own; and thus it becomes the source of new ideas." (Marshall, A., 1895, *Principles of Economics*, Macmillan and Co., London, p. 352).

1 Introduction

The importance of information and knowledge production and transmission has not gained wide attention until the development and growth of the ICT industry. Yet, several decades earlier, Hayek (1948) and Arrow (1969) have already started some valuable discussions on the economics of information and knowledge. Arrow (1969, p. 32) emphasized: "The observer of the outcome of an activity can be supposed to form new probability judgments ... [t]he transmission of the observation or of the revised probability judgments must take place over channels which have a limited capacity and are therefore costly." He elaborated later in his AEA Presidential Address on limited knowledge (1974, p. 7) that, despite the cost, "there is clearly a great incentive to acquire information of predictive value ... and an incentive to produce such information." He further claimed (1994, p. 8) that "knowledge and technical information have an irremovably social component, of increasing importance over time."

There are two distinct types of knowledge: codified (or formal) knowledge and tacit knowledge. While Hayek (1948, ch. 4) stressed the dispersed and tacit nature of knowledge, Gertler (2003) highlights tacit knowledge as a central component of a learning economy and a key to innovation. As pointed out by von Hipple (1994), it is more effective to transmit tacit knowledge through face-to-face interactions with frequent contacts (see also Saxenian, 1996). This is referred to as "sticky knowledge" whose transmission cost is found to rise with distance (cf. Feldman and Audretsch, 1999).

The costly but valuable transmission of information and tacit knowledge is a key focus of our paper. The localization of such knowledge and the costly nature of such transmission motivate us to model the transmission process in network games. Since the pivotal work by Aumann and Myerson (1988), increasing attentions have been paid to applying network games to economic environments wherein hierarchical organization of strategic interactions between individuals plays an important role (cf. Jackson and Wolinsky, 1996; Dutta and Mutuswami, 1997; Bala and Goyal, 2000).

While this literature has provided a useful framework for information processing or knowledge transmission, it is silent about another key ingredient – information aggregation – despite its crucial role in knowledge creation. In the context of political institutions, Piketty

(1999, p. 792) quoted Hayek (1948, p. 519), highlighting the role of information aggregation because "information pertinent to individual decisions never exists in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess." To take into account the role of information aggregation, we shall go beyond the existing setups of network games.

Specifically, we build upon the conventional network structure to incorporate not only the information transmission role but also the information aggregation role of networks. In the presence of the latter role, an efficient network structure not just simply minimizes decay losses but rather maximizes the benefits from information aggregation. Thus, central or "core" players are crucial beyond their "centrality" in position – which are thereby distinct from the typically defined key players in the literature (cf. Jackson and Zenou 2015). In our model, core players have an active role in information aggregation and whether to play such a role can be an endogenous choice. Core players are information aggregators who have a central role even in perfectly symmetric networks like circles and complete graphs where every node has the same weight according to all centrality measures. The feature of core-by-choice also implies more "oligopolistic" patterns in the sense that links will be centered around cores and the core can sustain remote information transmitters in the network where links would have been severed should there have been no cores. Our framework is especially useful when individual interactions involve the creation of new information and knowledge. To name but a few, these include research and collaboration networks, trade networks including the Sogo Shosha hierarchy, production and marketing networks including location-based services and information services via mobile commerce, as well as communication and other information networks whereby tacit knowledge transfers are active.

Upon constructing a network model allowing for distinctive roles played by a finite number of information transmitters and information aggregators, we establish various general properties of this information network on an arbitrary graph, where information-processing roles – aggregation or transmission – are exogenously given. First, a configuration of *geodesic-distance network* arises in equilibrium when the cost of link maintenance is sufficiently small, whereas a degenerate information-network with no information processed can be an equilibrium when decays in information transmission or costs of link maintenance are sufficiently large.

Second, we examine the case of only one aggregator. A configuration is an equilibrium or optimal network *only if it is a tree*. Moreover, with sufficiently small link costs, a network configuration is optimal if and only if it is a minimum geodesic-distance network. We establish sufficient conditions under which the *core-star* network is the unique and efficient equilibrium network. In the case of multiple aggregators on a tree network, we establish sufficient conditions under which a *star-with-satellites* network arises in equilibrium.

Third, *cycles* may arise in equilibrium and be optimal when there are more than one aggregator with sufficiently small link costs. We also establish sufficient conditions under which cycles will not occur in equilibrium: (i) it contains a cycle of transmitters with only one access node to aggregators outside the cycle, or a cycle of even number of transmitters with two access nodes that are linked together; (ii) it contains an aggregator and a reference transmitter linked together on a cycle, and another transmitter not linked but could link to this aggregator with a distance greater than the distance after the reference transmitter severs the link to the aggregator.

One may then inquire: (i) whether it is optimal to link to, instead of a closer aggregator, a farther aggregator which is connected to more transmitters, and (ii) whether an optimal network features the concentration of transmitters around few aggregators or a dispersion of many aggregators. We show by construction that *it may be optimal for a transmitter to link with a farther aggregator* which is directly connected to a large population of transmitters and that an optimal tree network features concentration.

We then turn to investigate information-network equilibrium with endogenous information-processing roles – that is, we allow each node to choose endogenously whether to play the role of an aggregator or a transmitter. We prove the existence of an information-network equilibrium with endogenous information-processing roles featuring active information processing in a network with a finite number of players on an arbitrary graph. We show that with sufficiently small link costs, the *monocentric* network with one aggregator connecting to all other agents as transmitters on a tree graph is the unique configuration of equilibrium network. With intermediate range of transmission decays, the *local-stars* (multicentric) configuration may arise in equilibrium. Moreover, cycles may also arise in equilibrium with larger decays in information transmission and smaller costs of link maintenance. Interestingly, when decays in information transmission are in the intermediate range, cycles may not arise if link maintenance costs are too low, because the aggregator would be better off to switch its role to transmitter. Furthermore, when transmission decays or link costs are sufficiently large, a degenerate information-network equilibrium with endogenous information-processing roles featuring no information processing exists.

To the end, we characterize an information chain network with all information aggregators and transmitters linked along a chain. Various sets of deviation-proof conditions for the information-chain networks are compared. We find that the efficacy of information aggregation does not affect a middle transmitter's (who is in between two aggregators) decision on whether to sever a link but makes a side transmitter (who is on either side of the two aggregators) less likely to sever a link and raises the incentives for all transmitters to switch role to aggregators. Thus, the equilibrium network, on balance, *need not feature more concentration*. Furthermore, we compute numerically the ranges of transmission decays and link maintenance costs within which a network equilibrium arises. A robust finding is that, in an information-processing chain network, aggregating efficacy is more effective than transmitting efficacy when the size of the network is not too large and knowledge delays are sufficiently strong.

It is worth noting that our framework is especially useful when individual interactions involve creation of new information and knowledge. To name but a few, examples are research and collaboration networks, trade networks including the Sogo Shosha hierarchy, production and marketing networks including location-based services, and information services via mobile commerce, as well as communication and other information networks whereby tacit knowledge transfers are active. In a symmetric information-processing chain network (with identical numbers of side and middle transmitters), aggregating efficacy is found to be more effective than transmitting efficacy when the size of the network is not too large and knowledge delays are sufficiently strong.

Now we return to real world practices. Consider some frequently observed types of information networks within which players may be human beings (researchers, businessmen, or government officers), legal entities (factories, shops, within-firm subdivisions, schools, or government units), or locations (cities or other economic or administrative districts). Our results suggest:

- With tacit and sticky knowledge, transmission decays and link costs are expected to be large, thereby more likely to support a monocentric equilibrium configuration. With intermediate range of transmission decays, local-stars (multicentric) configuration may arise in equilibrium – when there are two local stars, one may refer to it as a duocentric configuration. Both monocentric and multicentric configuration are often seen in location-related networks such as urban networks and production or marketing networks (single or multiple factories or sales posts in a given area).

- With better interface communication inclusive of internet, link costs are expected to be low, so an equilibrium network may feature an optimal geodesic-distance tree, as observed in mobile commerce businesses. With endogenous roles in information transmission and aggregating, this need not lead to market concentration because transmitters may have stronger incentives to become aggregators. As such, mobile commerce businesses appear to have highly competitive markets.

- With a strong leader – a core endowed with high aggregating efficacy, an equilibrium network may also feature a geodesic-distance tree, which is often seen in business organizations (think of Bill Gates and Steve Jobs) and governmental hierarchies (think of former Singapore Prime Minister, Kuan Yew Lee).

- There exists, in general, an information sharing advantage: transmitters may link to a distant aggregator if the aggregator is connected with many transmitters (think of Paul Erdős in a research network with many coauthors from around the world).

Related Literature

The equilibrium concept of network games varies from a weaker cooperative pairwise stability (cf. Jackson and Wolinsky, 1996; Dutta and Mutuswami, 1997), to a stronger version that is less dependent on the initial graphs (cf. Wang and Watts 2006), to a noncooperative version that is robust to all individual deviations (cf. Bala and Goyal, 2000). Jackson and Watts (2002) extends the static pairwise stability model to dynamic network formation using the concept of improving paths where individuals sever, or form links based on improvements of the resulting network offers over the current network. These games are particularly relevant to information-processing. On this specific subject, Radner (1993) and Bolton and Dewatripont (1994) provided the basic framework regarding networks as organizational hierarchies for firms to minimize information processing delays and to achieve most efficient within-the-firm communication.

Some recent studies focus mainly on strategic behavior regarding truthtelling and signaling. For example, Galeotti and Goyal (2010) establish the law of the few in that individuals rely on a few informational sources. While Galeotti, Goyal, and Jackson (2010) explore the strategic interactions of social networks, Acemoglu, Bimpikis and Ozdaglar (2014) analyze information exchange in communication social networks. Galeotti, Ghiglino, and Squintani (2013) further characterize strategic information transmission, whereas Dellarocas, Katona and Rand (2013) investigate the interplays between contents and links.

There is also a growing literature on the applications of network theory. For example, Walden (2019) documents evidence from stock trading to support predictions by network theory. While König, Liu, and Zenou (2019) study R&D networks, Michaeli and Spiro (2017) find network-induced biased norms. Galeotti and Goyal (2009) explore firms' marketing via social networks, whereas a recent work by Agha and Zeltzer (2022) investigates drug diffusion via peer networks.

For comprehensive surveys of the broad literature, the reader is referred to Jackson (2014), Jackson and Zenou (2015) and Jackson, Rogers, and Zenou (2017).

Takeaways

By contrasting with previous studies, we may now highlight the main takeaway of our paper: we examine not only the information transmission role but also the information aggregation role of networks. Moreover, which role to serve – and hence whether to become a core player – is also allowed to be endogenously determined by individual players. These issues, while potentially useful in various contents, remain unexplored, thus reaffirming the contribution of our paper. Having information transmission and information aggregation roles interacting with each other, we are able to obtain a rich array of equilibrium configurations, including core-star, star-with-satellites, overlapping core-stars, monocentric, and local-stars networks, as well as the possibility of cycles.

2 The Information Network

To develop an information network that incorporates both information transmission and information aggregation roles, we consider an economy featuring a pre-existing *geography* G which is a connected graph[1] consisting of potential links among two types of agents: *information-aggregating* agents and *information-transmitting* agents. Links are undirected; information can go both directions along a link. Let A denote the set of information-aggregating agents, or "aggregators," and T the set of information-transmitting agents, or "transmitters." The set of all nodes on the graph G is $I = A \cup T$. In the benchmark setting, aggregator-owned information does not go to another aggregator, whereas transmitter-owned information counts as "knowledge" and serves for "local production" as well. With exogenous transmission and information aggregation roles, we allow deviation of one link only, as pairwise stability. When such roles are endogenously determined with role-switching, agents

[1] Expositions of the network theory commonly assume that the potential links consist of a complete graph or consist of a specific geography such as a line or a circle.

can sever multiple links.

Specifically, denote cardinality of a set as $|.|$. All agents can be potentially connected on G, and agents choose whether to link with adjacent agents on G. Further denote agent i's set of links $l_i = \{ij \mid ij \in N\} \in 2^{\{ij|ij \in G\}}$. The profile of link choice $l = (l_i)_{i \in I}$ constitute a *network* $N \subseteq G$, $N = \cup_{i \in I} l_i$. Let $d(i,j)$ denote the distance between agents i and j on network N, which is the number of links on the shortest path connecting i, j. When i and j are not connected, $d(i,j) = \infty$. ($d(.)$ is distance on network N while a subscript N is committed to simplify notation). The role of geography G is similar to the real-world geography of possible traffic connections, and N is like the actual traffic network built on the geography. Let $d^G(i,j)$ denote the geodesic distance (shortest distance) between i and j on G.

We rule out the case where two aggregators link together since they enjoy no direct benefit from doing so. The cost of maintaining a link is $c > 0$; a transmitter shares half of the link cost if linked with another transmitter and pays the whole link cost if linked with an aggregator. Let p_t and q_t denote the number of aggregators and transmitters linked to transmitter t, respectively. The total link cost incurred by t is thus $c(p_t + q_t/2)$.[2]

Each agent is endowed with one unit of information. Information can be transmitted with a transmission rate δ, $\delta < 1$, per link along connected agents (δ is a decay discounting factor where a larger δ means a higher transmission rate and lower decays). While information-transmitting agents transmit information to connected agents, information-aggregating agents can, additionally, aggregate information obtained to synthesize new valuable information into *knowledge*. This knowledge can then be transmitted in the network N to transmitters from the aggregator creating it. Let K_a denote the amount of knowledge created by aggregator $a \in A$, which augments the amount of aggregated information by a factor:

$$K_a = \kappa \left[1 + \sum_{t \in T} \delta^{d(a,t)} \right], \tag{1}$$

where parameter $\kappa \geq 1$ measures the efficacy in information aggregation and knowledge creation. An aggregator uses this knowledge for production and also spills it over to transmitters. For simplicity, the output or benefit to an aggregator from a unit of knowledge is normalized to one. Knowledge transmits in the network with the same transmission rate δ

[2]The assumptions of aggregators not linking directly together and not paying for link costs are maintained throughout the paper. This is to reduce strategic considerations of the aggregators, which will accept any link, and focus on transmitter behaviors.

per link as information.[3]

A transmitter $t \in T$ receives knowledge from aggregators that are connected to it. A transmitter receives knowledge from all connected aggregators, and the available knowledge is thus given by the function,

$$K_t\left(\left(\delta^{d(t,a)}K_a\right)_{a\in A}\right). \tag{2}$$

Aggregated information created by aggregator a serves transmitter t with a transmission rate δ. If not connected to an aggregator and thus not receiving knowledge, a transmitter uses its own one unit of information, i.e. $K_t(0) = 1$. One unit of knowledge yields one unit of output. The general function K_t is weakly increasing in all arguments. For example, it can be the maximum, the average, or other types of functions. We take a pure public good view of information and knowledge. They are nonrival and repeatedly used by many agents. This is not far from the standard connections model of networks, for example, a la Jackson and Wolinsky (1996) and Ballester, Calvó-Armengol and Zenou (2006). In such models, every node generates external benefits to everyone else who are connected in the network, like widespread spillovers from pure public goods (similar to the matching model of Berliant, Reed and Wang, 2006). The difference is that our network benefits have a aggregation stage where information gathers at aggregators, then a diffusion stage where aggregators spill knowledge back to transmitters.

The net output of each agent is as follows:

$$V_a = \kappa\left[1+\sum_{t\in T}\delta^{d(a,t)}\right], \tag{3}$$

$$V_t = K_t\left(\left(\delta^{d(t,a)}K_a\right)_{a\in A}\right) - c\left(p_t+\frac{q_t}{2}\right). \tag{4}$$

The total net output on the network N is the sum of net outputs of all nodes,

$$\sum_{a\in A}\kappa\left[1+\sum_{t\in T}\delta^{d(a,t)}\right]+\sum_{t\in T}\left[K_t\left(\left(\delta^{d(t,a)}K_a\right)_{a\in A}\right) - c\left(p_t+\frac{q_t}{2}\right)\right].$$

A network is *optimal* or *efficient* if the total net output is maximized over all possible link profiles.

A simple but useful functional form is:

$$V_t = 1+\Sigma_{a\in A}\left(\delta^{d(t,a)}K_a\right) - c\left(p_t+\frac{q_t}{2}\right). \tag{5}$$

[3] Differences in roles can also be modeled as a mixed choice by every agent. For example, while transmitting information, an agent can invest in activities of information aggregation with a cost. Yet, as an early attempt to investigate this issue, we would keep the simpler setting of distinct roles.

which will be adopted in later analyses.

In the network, the link profile of a transmitter t is denoted by $l_t \in 2^{\{tj|ij \in G\}}$, and the link profile of an aggregator a is denoted by $l_a \in 2^{\{aj|aj \in G. j \notin a\}}$, seeking to maximize their net benefits. A strategic version of pairwise stability, which is commonly seen in network games, is employed as the equilibrium concept: A link is maintained when both nodes benefit from it, and not maintained when at least a node is worse off. The strategic interpretation is that an agent can sever a link unilaterally, but linking imposes costs on both agents and needs mutual consent:

- An agent can change the network configuration via severing or establishing a link. That is agent i changes her links from l_i to $l'_i \in \{l_i \backslash ij, l_i \cup ij\}$ for any $j \in I$.
- Agent i's severed links are $l_i \setminus l'_i$ and her established links are $l'_i \setminus l_i$. Links of other agents l'_j are hence updated according to i's new link profile: $l'_j = l_j \setminus ij$ if $ij \in l_i \setminus l'_i$, and $l'_j = l_j \cup ij$ if $ij \in l'_i \setminus l_i$.

Definition 1. An *information-network equilibrium* is a link profile $l = (l_i)_{i \in I}$ such that for any agent $i \in I$, there is no other link strategy $l'_i \in \{l_i \backslash ij, l_i \cup ij\}$ for some $j \in I$ such that

$$\begin{aligned} V_i(l') &> V_i(l), \\ V_j(l') &> V_j(l), \text{ if } ij \in l'_i \setminus l_i . \end{aligned}$$

That is, an information-network equilibrium rules out two possible deviations: (i) an agent i has a higher net benefit by sever her links and (ii) both agents i and j have higher net benefits when forming a new link.

2.1 Equilibrium and Optimality

Equilibrium and efficient information networks are characterized in this section in a general geography with multiple aggregators. We also narrow down the scope of investigation to special cases: when there is only one aggregator, when there are multiple aggregators on a complete geography, and when there is one aggregator on a complete geography.

The *geodesic-distance networks* $\hat{N}$ emerge as an important structure for equilibrium and optimality. Link all transmitters $t \in T$ to an aggregator $a \in A$ according to the following algorithm. Links are constructed around all aggregators from closest transmitters then add farther ones. Let $g^r(i,j)$ denote a geodesic path from i to j of distance r. Let

$\hat{d}_G = \max_{i \in A, j \in T} d^G(i,j)$ denote the longest geodesic distance between an aggregator and a transmitter on the geography G. Note that $\hat{d}_G = 1$ if G is complete.

- Transmitters of geodesic distance 1 to any aggregator a on G are denoted by set $I^1 = \{t \in T \mid d^G(a,t) = 1, \textit{ for some } a \in A\}$. Link I^1 to associated aggregators. Such links are denoted by the set L^1.
- Keep connecting transmitters of longer distances to every a as the geodesic distance increases (of distance 2, ..., $\hat{d}_G$), until there is no transmitter left.
- For each distance r, the set of transmitters are $I^r = \{t \in T \mid d^G(a,i) = r, \textit{ for some } a \in A\}$. If a pair of t and a already have a geodesic path $g^r(a,t)$ in the connected links $\cup_{s=1}^{r-1} L^s$, leave them out. For any pair $a \in A$ and $t \in I^r \setminus \{t \in T \mid g^r(a,t) \in \cup_{s=1}^{r-1} L^s\}$, take a geodesic path $g^r(a,t)$ starts with a link tt'. There must be a path $g^{r-1}(a,t')$ in $\cup_{s=1}^{r-1} L^s$ by construction. Establish link tt (one new link for transmitter t), and such new links are denoted by L^r. And $\cup_{s=1}^{r} L^s$ is the set of constructed links so far.

Thus, every transmitter is connected to every a on a unique geodesic paths. The resulting *geodesic-distance network* is

$$\hat{N} = \cup_{r=1}^{\hat{d}_G} L^r.$$

There may be multiple versions of $\hat{N}$, due to multiple geodesic-distance paths between nodes. Furthermore, a *minimum geodesic-distance network* is defined to be the one containing the lowest total number of links among all geodesic-distance networks. Note that when there is only one aggregator, network $\hat{N}$ is a *geodesic-distance tree.*

When geography G is a complete graph, transmitters have geodesic distance 1 to all aggregators. The geodesic-distance tree, therefore, reduces to the star network with an aggregator as the core, called the *core-star* network. When there are multiple aggregators, the geodesic-distance network becomes an *overlapping core-stars* network, where each aggregator links to all transmitters and each transmitter links to all aggregators, and it is the minimum geodesic-distance network (Figure 1).

In designing an optimal network, one faces the tradeoff between link costs and knowledge transmission. On the one hand, transmitters can be provided with geodesic paths to every aggregator, decays are at minimum. This results in a geodesic-distance network, but will certainly generate redundant links. On the other hand, link costs can be kept at minimum by linking nodes in a tree network, which requires only as many links as the number of nodes

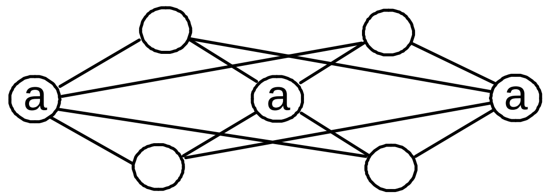

Figure 1: The overlapping-core-stars network.

minus one, $|A| + |T| - 1$. Thus, not all transmitters will be connected to aggregators via geodesic distances. Depending on the relative size of link costs and knowledge transmission rate, an optimal network is a trade-off between minimizing total links and minimizing connection distances on geography G.

Theorem 1 and Propositions 1 and 2 examine equilibrium and optimality regarding the geodesic distance network and the tree network respectively. To ease the analysis, we will use the simple sum of knowledge from aggregators defined in (5).

Theorem 1. (Equilibrium and Optimality) *In an information network with a finite number of agents on any geography G,*

(i) (Existence) *a geodesic-distance network is an equilibrium network when the cost of link maintenance is not too large, where a sufficient condition is given by,* $c < \kappa \left(\delta^{\hat{d}_G} - \delta^{\hat{d}_G+1}\right)\left(1 + \delta^{\hat{d}_G}\left(|T| - 1\right)\right)$;

(ii) (Degenerate Information-Network) *a degenerate information-network equilibrium with no information processed exists, when the cost of link maintenance is sufficiently larger compared with information transmission rate, where a sufficient condition is given by,* $c > 2\kappa\delta\,|A|\left(1 + \delta\,|T|\right)$;

(iii) (Optimality) *a minimum geodesic-distance networks is the unique efficient network when the cost of link maintenance is not too large, where a sufficient condition is given by,* $c < \kappa \left(\delta^{\hat{d}_G} - \delta^{\hat{d}_G+1}\right)\left(1 + \delta^{\hat{d}_G}\,|T|\right)$.

Proof.

(i) In a geodesic-distance network, no additional link will increase payoff since every transmitter is connected to all aggregators by paths of the geodesic distance. Severing a link is the only possible deviation. When a transmitter severs a link (it is still connected in the network though), its information travels longer to the aggregator and knowledge transmission from the aggregator take a longer distance and be reduced by a factor of at least, $\delta^{\hat{d}_G} - \delta^{\hat{d}_G+1}$. A lower bond of this knowledge loss is $\kappa\left(\delta^{\hat{d}_G} - \delta^{\hat{d}_G+1}\right)\left(1+\delta^{\hat{d}_G}(|T|-1)\right)$, which is the least possible impact on the transmitter. Transmitters are paying costs $c/2$ or c. A sufficient condition to maintain all links is $c < \kappa\left(\delta^{\hat{d}_G} - \delta^{\hat{d}_G+1}\right)\left(1+\delta^{\hat{d}_G}(|T|-1)\right)$.

(ii) The total knowledge transmitter t receives from all aggregators is $\Sigma_{a\in A}\left(\delta^{d(a,t)}K_a\right)$, which can be less than the link cost for one link with small δ or large c. The maximal knowledge that can transmit on a link is in an overlapping core-star network, $\kappa\delta|A|(1+\delta|T|)$. A sufficient condition for no link is $c/2 > \kappa\delta|A|(1+\delta|T|)$.

(iii) A geodesic-distance network has the maximal total knowledge and thus maximal outputs, since knowledge decays are kept at minimum. A minimum geodesic-distance network yields the highest net output among all geodesic-distance networks. Suppose another network configuration, not geodesic-distance, improves net output. It cannot increase knowledge creation and has to reduce the number of links. However, severing a link causes a transmitter to connect (if still) to an aggregator on longer paths. At least one transmitter will be affected by each link. Its knowledge reduced by a factor of at least $\delta^{\hat{d}_G} - \delta^{\hat{d}_G+1}$ (or larger). This knowledge reduction also goes to other transmitters, so a lower bond is $\kappa\left(\delta^{\hat{d}_G} - \delta^{\hat{d}_G+1}\right)\left(1+\delta^{\hat{d}_G}|T|\right)$. A sufficient condition is $c < \kappa\left(\delta^{\hat{d}_G} - \delta^{\hat{d}_G+1}\right)\left(1+\delta^{\hat{d}_G}|T|\right)$. This condition implies the total net output is positive. ∎

Notably, Theorem 1 refers to a general unspecified G. Exact conditions are hence impossible to come by. Weaker sets of sufficient conditions specifying the distance of transmitters in the network can be written, but do not offer much insight. Rather, we present stronger sufficient conditions in the above theorem to illustrate how c and δ affect equilibrium and optimality.

Proposition 1. (Information Network with One Aggregator on General G) *In an information network with a finite number of agents on an arbitrary geography, when there is only one aggregator, an equilibrium or an efficient network is a tree network.*

Proof. In a cycle, take one of the transmitters that has the longest distance to the aggregator. This transmitter has two links on the cycle. Either the two links lead to the same distance to the aggregator or one leads to a longer distance. In both cases, the transmitter can server a link without reducing knowledge. The other transmitter affected is not using this link either. There is still the same amount of processed knowledge for every node. Therefore, this transmitter has the incentive to sever the link, and it is optimal to sever the link. ∎

When G is a complete graph, every transmitter can link to aggregators. The core-star network offers a parallel comparison with the star network in Jackson and Wolinsky (1996), which is stable and efficient in a proper parameter range of c and δ.

Corollary 1. (Core-Stars Network with Multiple Aggregators on Complete G) *In an information network with a finite number of agents on complete G,*

(i) the overlapping core-stars network is an equilibrium network if $c \leq \kappa\left(1+\delta\left(|T|\text{-}1\right)\right)\left(\delta\text{-}\delta^3\right)+\kappa\left(\delta^2\text{-}\delta^4\right)$*;*

(ii) the overlapping core-stars network is an efficient network when c is not too large, where a sufficient condition is given by, $c < \kappa\delta\left(\delta-\delta^2\right)\left(1+|T|\right)$*.*

Proof. (i) In this network, transmitters have no incentive to establish new links. The knowledge a transmitter receives from an aggregator is $\kappa\delta\left(1+\delta|T|\right)$. When a transmitter severs a direct link, the path will go through another aggregator and a transmitter then to the affected aggregator, causing a factor of distance change $\delta-\delta^3$. After severing the link, knowledge from the affected transmitter is $\kappa\delta^3\left(1+\delta\left(|T|-1\right)+\delta^3\right)$. An exact sufficient condition can be derived.

(ii) It follows Theorem 1.iii. ∎

In Corollary 1, when $c > \kappa\left(1+\delta\left(|T|-1\right)\right)\left(\delta-\delta^3\right)+\kappa\left(\delta^2-\delta^4\right)$, an equilibrium network would not have all transmitters linking to all aggregators directly. As link cost c increases, links with low benefits cannot remain in equilibrium.

Proposition 2. (Core-Stars Network with One Aggregator on Complete G) *In an information network with a finite number of agents and one aggregator on complete G,*

(i) the core-star network is the unique equilibrium network if and only if $c \leq \kappa\delta\left(1+\delta|T|\right)$*, otherwise the no-link network is the unique equilibrium;*

(ii) the core-star network is the uniquely efficient network if and only if $c \leq \left(\kappa\delta+1/|T|\right)\left(1+\delta|T|\right)$*, otherwise the no-link network is uniquely efficient.*

Proof. (i) Transmitters have no incentive to establish new links, and a transmitter receives knowledge $\kappa\delta\left(1+\delta|T|\right)$ from the aggregator. This is also the maximal knowledge the aggregator can produce. If $c > \kappa\delta\left(1+\delta|T|\right)$, no link will form.

(ii) Every transmitter is linked to all aggregators with distance 1; the knowledge amount is at maximum. The number of links is minimal, $|T|$. It is optimal and there is no other configuration that can yield a higher total net output. The other possibility is that its total knowledge output, $\left(\kappa\delta|T|+1\right)\left(1+\delta|T|\right)-c|T|$, is negative. So, if $c > \left(\kappa\delta+1/|T|\right)\left(1+\delta|T|\right)$, the empty network is optimal. ∎

2.2 Optimal Tree Networks

Efficient networks can be investigated further. Suppose link costs are high or the transmission rate is low, but do not result in disconnected agents. So, it is optimal to restrict the number of links to its minimum, $|A| + |T| - 1$, which allows tree networks only. What is the efficient tree network? The benchmark model is a *local-area network*, which connects each transmitter to only one aggregator of the closest geodesic distance. This can be obtained by using the geodesic-distance network algorithm for all aggregators. Then, when a transmitter is connected to an aggregator, stop connecting it to other aggregators. After this operation, we have several disconnected local geodesic-distance trees (not connecting to all transmitters); each centers around an aggregator. Next, link these trees by geodesic paths among aggregators. This can be done by starting at any aggregator as the root and link it to other aggregators that can be connected without passing through another aggregator. Then, link to aggregators that can be connected passing one aggregator. Keep linking aggregators that can be connected to the root by passing 2, 3, ... and more aggregators. The resulting network is a tree network, composed of sub-networks where aggregators serve local transmitters in close distances.

Keeping links minimal, local-area networks seems to be good candidates for the optimum since transmitters are connected to the closest aggregator to alleviate decays in knowledge transmission. Would an optimal tree network feature the concentration of large numbers of transmitters around few aggregators or a more dispersed configuration with all aggregators link to equal numbers of transmitters? Conventional wisdom says that concentration of transmitters would be optimal. But would a longer path to a more popular aggregator, which connects directly to more transmitters, instead of a nearer aggregator, yields a higher output by reducing decays in knowledge? Corollary 2 examines these questions in simple case of one transmitter choosing links to one of two aggregators, which link to different numbers of other transmitters. An optimal tree network features concentration of large numbers of transmitters around few aggregators. Yet, benefits from concentration do not in general outweigh decays from a longer distance. It is not beneficial to connect a transmitter via a longer path to a more popular aggregator, instead of a nearer aggregator, unless under extreme conditions where the larger aggregator is sufficiently large.

Corollary 2. (Optimal Tree Network) *In a local area network with total number of links restricted to* $|A| + |T| - 1$*,*

(i) an optimal tree network always features concentration;

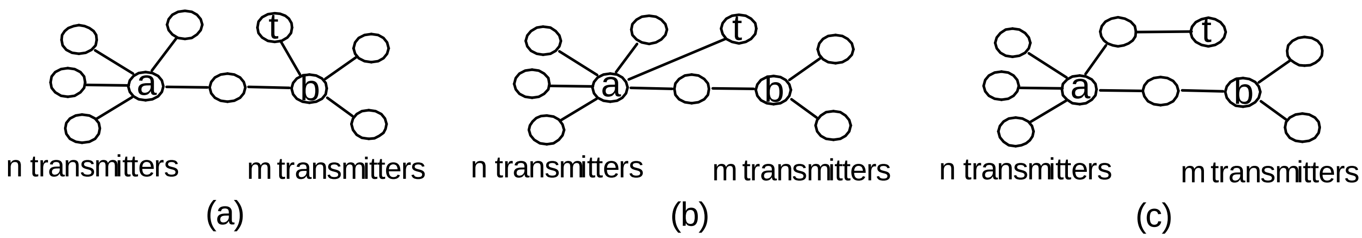

Figure 2: Local-area networks

(ii) it is optimal to link a transmitter to a near aggregator than to a farther aggregator in general, unless to a significantly larger population of transmitters.

Proof. (i) (To ease the description, the transmitter in between the two aggregators are called a middle transmitter and, transmitters on one side of the two aggregators are called side transmitters.) See Figure 2. There are two aggregators a and b which are connected through a common transmitter. Putting the middle transmitter and t aside, the aggregators are directly linked to another n and m transmitters respectively and $n > m$. In Figure 2.a, t is linked to aggregator b, not the more popular aggregator a. Aggregators generate knowledge $K_a = \kappa \left(1 + \delta \left(n + 1\right) + \delta^3 \left(m + 1\right)\right)$ and $K_b = \kappa \left(1 + \delta^3 n + \delta \left(m + 2\right)\right)$. Each of the n nodes on the left, as well as t, yields output $1 + \delta K_a + \delta^3 K_b$, and each of the m nodes on the right with t yields output $1 + \delta^3 K_a + \delta K_b$. And the total output in the network is

$$n \left(1 + \delta K_a + \delta^3 K_b\right) + \left(1 + \delta K_a + \delta K_b\right) + \left(m + 1\right) \left(1 + \delta^3 K_a + \delta K_b\right) + K_a + K_b,$$
$$= n + m + 2 + K_a^2/\kappa + K_b^2/\kappa.$$

Figure 2.b has t linked to a. Transmitter t's information is processed and transmitted through a with lower decays to more nodes. Aggregators generate knowledge $K_a' = \kappa \left(1 + \delta \left(n + 2\right) + \delta^3 m\right)$ and $K_b' = \kappa \left(1 + \delta^3 \left(n + 1\right) + \delta \left(m + 1\right)\right)$. And the total output on network is

$$\left(n + 1\right) \left(1 + \delta K_a' + \delta^3 K_b'\right) + \left(1 + \delta K_a' + \delta K_b'\right) + m \left(1 + \delta^3 K_a + \delta K_b\right) + K_a + K_b$$
$$= n + m + 2 + \left(K_a'\right)^2/\kappa + \left(K_b'\right)^2/\kappa.$$

Figure 2.b has a higher output if and only if

$$\left(1 + \delta \left(n + 1\right) + \delta^3 m\right) \left(\delta - \delta^3\right) + \left(1 + \delta^3 n + \delta \left(m + 1\right)\right) \left(\delta^3 - \delta\right)$$
$$= \left(n - m\right) \left(\delta - \delta^3\right)^2 > 0.$$

which holds true when $n > m$.

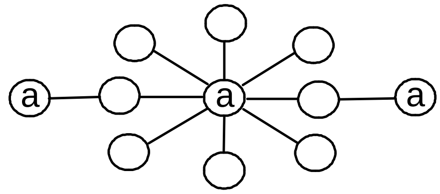

Figure 3: The star-with-satellite network.

(ii) Continue from Figure 2.a. Figure 2.c shows an alternative configuration with transmitter t linked to the more popular but farther aggregator a, instead of the nearer aggregator. Aggregators generate knowledge $K''_a = \kappa\left(1+\delta(n+1)+\delta^3 m+\delta^2\right)$ and $K''_b = \kappa\left(1+\delta^3 n+\delta(m+1)+\delta^4\right)$. Notice that t yields $1+\delta^2 K''_a+\delta^4 K''_b$. And the total output on the network is

$$
\begin{aligned}
&n\left(1+\delta K''_a+\delta^3 K''_b\right)+\left(1+\delta K''_a+\delta K''_b\right)\\
&+m\left(1+\delta^3 K''_a+\delta K''_b\right)+\left(1+\delta^2 K''_a+\delta^4 K''_b\right)+K''_a+K''_b,\\
&=n+m+2+\left(K''_a\right)^2/\kappa+\left(K''_b\right)^2/\kappa.
\end{aligned}
$$

Figure 2.a has a positive output difference if and only if:

$$
\begin{aligned}
&2\left(1+\delta(n+1)+\delta^3 m\right)\left(\delta^3-\delta^2\right)+\delta^6-\delta^4+2\left(1+\delta^3 n+\delta(m+1)\right)\left(\delta-\delta^4\right)+\delta^2-\delta^8\\
&=2(1+\delta)\left(\delta\text{-}\delta^2\text{+}\delta^3\text{-}\delta^4\right)+2\left[\left(\delta n\text{+}\delta^3 m\right)\left(\delta^3\text{-}\delta^2\right)+\left(\delta^3 n\text{+}\delta m\right)\left(\delta\text{-}\delta^4\right)\right]+\left(\delta^2\text{+}\delta^4\text{-}\delta^6\text{+}\delta^8\right)>0
\end{aligned}
$$

The first and third terms are positive and the middle term is

$$
\begin{aligned}
&2\left[\left(2\delta^4-\delta^3-\delta^7\right)n+\left(\delta^6+\delta^2-2\delta^5\right)m\right]\\
&=2\left[\left(\delta^2-\delta^3+2\delta^4-2\delta^5+\delta^6-\delta^7\right)n+\left(\delta^2-2\delta^5+\delta^6\right)(m-n)\right]
\end{aligned}
$$

The above is in general positive unless n is significantly larger than m and with a proper δ (a numerical example is $\delta=0.5$, $n=200$, $m=4$). ∎

Corollary 2 shows that when the total number of links is limited, optimal trees feature concentration of transmitters and linking to nearer aggregators. When geography G is complete, all transmitters have equal geodesic distance 1 to aggregators. Concentration leads to all transmitters linking to the same aggregator, as in a star network. The optimal use of the rest of aggregators is to link each of them to a transmitter, resulting in a *star-with-satellites* network (Figure 3).

Proposition 3. (Star-with-Satellites Network) *When G is complete, the star-with-satellites network is an equilibrium network if* $c < \kappa \left(1 + \delta + \delta^3 (|T| - 1)\right) \left(\delta - \delta^3\right)$.

Proof. The core aggregator yields knowledge $K_c = \kappa (1 + \delta |T|)$, and the satellite aggregators yields $K_s = \kappa \left(1 + \delta + \delta^3 (|T| - 1)\right)$. Each of the $|T| - 1$ double-linked transmitter receives knowledge $\delta (K_c + K_s) + \delta^3 K_s (|A| - 2)$, and each single-linked transmitter receives $\delta K_c + \delta^3 K_s (|A| - 1)$.

If a single-linked transmitter severs the link, its knowledge loss is $\delta K_c + \delta^3 K_s (|A| - 1)$. If a double-linked transmitter severs the link to the core, its knowledge loss is least $\delta K_c + \delta^3 K_s (|A| - 2)$. If a double-linked transmitter severs the link to the satellite, its knowledge loss is δK_s. No transmitters would link together since this will not reduce distances to any aggregator. If a single-linked transmitter links with a satellite, received knowledge increases by $\delta (K_c + K_s) + \delta^3 K_s (|A| - 2)$ - $\delta K_c + \delta^3 K_s (|A| - 1) = \left(\delta - \delta^3\right) K_s$. So, $c < \left(\delta - \delta^3\right) K_s$ prevents all the above deviations. ∎

2.3 Multiple Aggregators and Cycles

This section investigates the occurrence of cycles on equilibrium networks. The redundant cyclic links may provide transmitters shorter distances to some aggregators on other parts of the network. Though, an equilibrium or efficient network does not contain cycles when there is only one aggregator (Proposition 1). Cycles occur commonly, however, with multiple aggregators. Our paper thus contributes to the relatively thin literature on the formation of cyclic networks. For example, cycles are shown to emerge as stable or efficient communication networks when players are non-myopic (Watts 2002), when investments in link strength are endogenized (Bloch and Dutta 2009), or when individuals are allowed to choose their ranges of costly communication (Hong and Chun 2010). The role of information aggregation played by multiple aggregators facilitates the emergence of cycles in the network.

Proposition 4 presents some patterns of cycles that will not occur in equilibrium in our general networks. An *access node* of a cycle of transmitters is a node via which other transmitters connect to aggregators outside the cycle, or the node itself is an aggregator. First, it is not an equilibrium if there is a cycle of transmitters with only one access node, or a cycle composed of an even number of transmitters with two access nodes that are linked together. Second, if there is an aggregator and a transmitter linked together on a cycle, and there is another transmitter, not linked but could link to the aggregator, which is farther away from the aggregator on the network than the first transmitter after its link to the

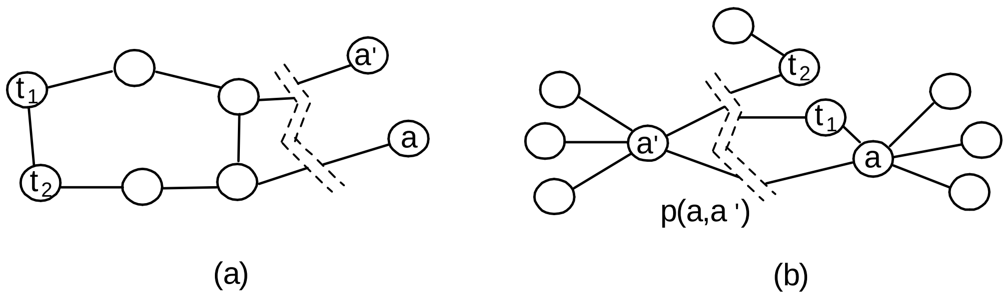

Figure 4: Non-occurrence of circles

aggregator is severed, the network is not in equilibrium.

Proposition 4. (Non-Occurrence of Cycles) *In an information network with more than one aggregator, a network N cannot be in equilibrium if*

(i) *it contains a cycle of transmitters with only one access node to aggregators outside the cycle, or a cycle of even number of transmitters with two access nodes that are linked together;*

(ii) *it contains aggregator a and transmitter t_1 linked together on a cycle, and another transmitter t_2 not linked but could link to a with distances $d'(a, t_2) \geq d'(a, t_1)$, where $d'(.)$ is the distance on $N' = N \backslash at_1$ after t_1 severs its link to a.*

Proof. (i) The case of one access node is similar to the case of one aggregator in the proof of Proposition 3. One of the nodes with the longest distance to the access node will sever a link. Suppose there is a cycle of an even number of transmitters (Figure 4.a). There are two access nodes, linked together, that have access to aggregators outside the cycle. Then there exist two nodes t_1 and t_2 on the cycle with the longest distance to the access nodes. The link between t_1 and t_2 is not needed by either of them and will be severed.

(ii) (See Figure 4.b) Transmitter t_1 can deviate by severing the link to a. The knowledge at a will be reduced at lease by $\kappa\left(\delta - \delta^{d'(a,t_1)}\right)$. This is the amount of knowledge at t_1 transmitting via a longer distance; if there are more nodes affected, the new knowledge at a will be even smaller. So, new knowledge at a is $K'_a \leq K_a - \kappa\left(\delta - \delta^{d'(a,t_1)}\right)$. The knowledge drop coming from a causes a change from δK_a to $\delta^{d'(a,t_1)} K'_a$ in the output of t_1. The total output decrease at t_1 could be larger. If N (with at_1) is in equilibrium, $\delta K_a - \delta^{d'(a,t_1)} K'_a > c$.

Transmitter t_2 can deviate by linking to a, thus increases the knowledge at a by at least $\kappa\left(\delta - \delta^{d'(a,t_2)}\right)$. New knowledge at a_2 will be larger if there are other nodes getting

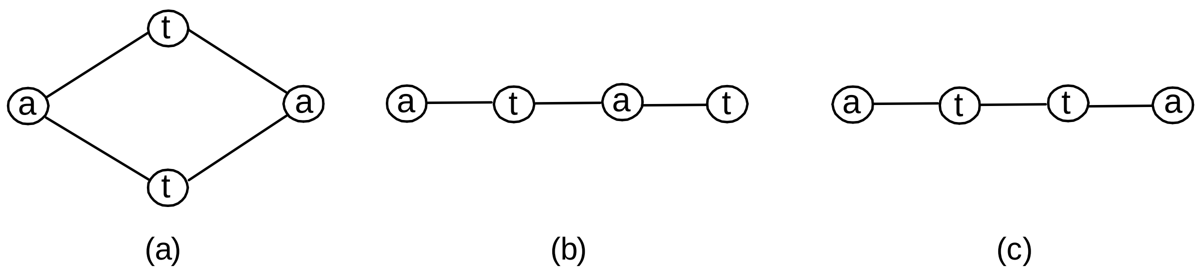

Figure 5: Configurations with two aggregators

shorter distances due to this new link. So, new knowledge $K''_a \geq K_a + \kappa \left(\delta - \delta^{d'(a,t_2)}\right)$. This knowledge hike coming from a causes a change from $\delta^{d'(a,t_2)} K_a$ to $\delta K''_a$ in the output of t_2. The total output increase at t_2 could be larger. If N (without at_2) is in equilibrium, $\delta K''_a - \delta^{d'(a,t_2)} K_a < c$. The above two gives

$$\delta K_a - \delta^{d'(a,t_1)} K'_a > c > \delta K''_a - \delta^{d'(a,t_2)} K_a;$$
$$\delta K_a - \delta^{d'(a,t_1)} \left(K_a - \kappa \left(\delta - \delta^{d'(a,t_1)}\right)\right) > \delta \left(K_a + \kappa \left(\delta - \delta^{d'(a,t_2)}\right)\right) - \delta^{d'(a,t_2)} K_a,$$
$$\left(\delta - \delta^{d'(a,t_1)}\right) K_a + \kappa \delta^{d'(a,t_1)} \left(\delta - \delta^{d'(a,t_1)}\right) > \left(\delta - \delta^{d'(a,t_2)}\right) K_a + \kappa \delta \left(\delta - \delta^{d'(a,t_2)}\right),$$
$$\left(\delta^{d'(a,t_2)} - \delta^{d'(a,t_1)}\right) K_a + \kappa \left[\delta^{d'(a,t_1)} \left(\delta - \delta^{d'(a,t_1)}\right) - \delta \left(\delta - \delta^{d'(a,t_2)}\right)\right] > 0.$$

Since $\delta^{d'(a,t_2)} \leq \delta^{d'(a,t_1)}$ and $\delta - \delta^{d'(a,t_1)} < \delta - \delta^{d'(a,t_2)}$, the above is a contradiction. ∎

When there are more than one aggregator, transmitters can obtain better access to aggregators via more links, cycles occur commonly in equilibrium and optimal networks. This is illustrated by examples of two aggregators and two transmitters.

Corollary 3. (Occurrence of Cycles) *In an information network with more than one aggregator,*

(i) *an equilibrium network may contain a cycle when link costs are not too large;*

(ii) *an optimal network may contain a cycle when link costs are not too large.*

Proof. (i) Figure 5.a shows a circular network with two aggregators, two transmitters, and four links. Each aggregator creates knowledge $\kappa (1 + 2\delta)$, and each transmitter's output is $1 + 2\delta\kappa (1 + 2\delta) - 2c > 0$. If one of the transmitters severs a link, the network becomes Figure 5.b. The transmitter after severing a link, and hence receiving knowledge via a longer path, has output $1 + \delta\kappa (1 + 2\delta) + \delta^3 \kappa \left(1 + \left(\delta + \delta^2\right)\right) - c$. The link will not be severed if the

output difference $\delta\kappa\left(1+2\delta-\delta^2-\delta^3-\delta^4\right)-c>0$. The first terms in the above is positive, so it holds when the link costs are not too large.
(ii) If there are two or more aggregators, an optimal network may contain cycles. We demonstrate this with an example of two aggregators and two transmitters. Figure 5 presents all of the three possible networks; notice that aggregators will not link together. Figure 5.a is a circular network, Figure 5.b and Figure 5.c show the two linear/tree networks. In Figure 5.a, total output in the network is $2+2\kappa\left(1+2\delta\right)^2-4c$.

In Figure 5.b, aggregators create knowledge $\kappa\left(1+2\delta\right)$ and $k\left(1+\delta+\delta^3\right)$ respectively. Transmitters have output $1+\delta\kappa\left(1+2\delta\right)+\delta k\left(1+\delta+\delta^3\right)$ and $1+\delta\kappa\left(1+2\delta\right)+\delta^3 k\left(1+\delta+\delta^3\right)$ respectively. Total output on the network is $2+\kappa\left(1+2\delta\right)^2+k\left(1+\delta+\delta^3\right)^2-3c$.

In Figure 5.c, each of the aggregators create knowledge $\kappa\left(1+\delta+\delta^2\right)$, then transmitters receive knowledge, and the output is $1+\kappa\left(\delta+\delta^2\right)\left(1+\delta+\delta^2\right)$. The total output on the network is $2+2\kappa\left(1+\delta+\delta^2\right)^2-3c$. The cycle network generates more output if the following two hold:

$$\begin{aligned}\kappa\left(\left(1+2\delta\right)^2-\left(1+\delta+\delta^3\right)^2\right)-c &> 0,\\ 2\delta\kappa\left(2+\delta-2\delta^2-\delta^3\right)-c &> 0.\end{aligned}$$

Since the first term is positive in each of the two inequalities, both of them hold for c not too large. ∎

2.4 Tree versus Cycles: A Numerical Example

With multiple aggregators and complete G, networks can be full of cycles. Cycles offer closer distances with redundant links, while trees utilize minimum number of links. When would a cycle be more efficient than a tree? Two extreme cases are parameterized and contrasted in the following simple example. The optimal number of cycles is examined.

The information network has two aggregators and $2n+1$ transmitters on a complete geography G. When G is complete, all transmitters are able to link directly with at least one aggregator. Proposition 4.ii eliminates equilibrium configurations with a cycle between two aggregators while some transmitters are not linked to both aggregators (Figure 6.a). So, in equilibrium, either all transmitters are in cycles or there is no cycle, which leaves the following two cases: (i) All transmitters link to both aggregators, as an *overlapping core-stars* network (introduced in Section 2.1; see Figure 6.b). (ii) One transmitter links to both aggregators and other transmitters link to one of the aggregators, as a *local-stars* network

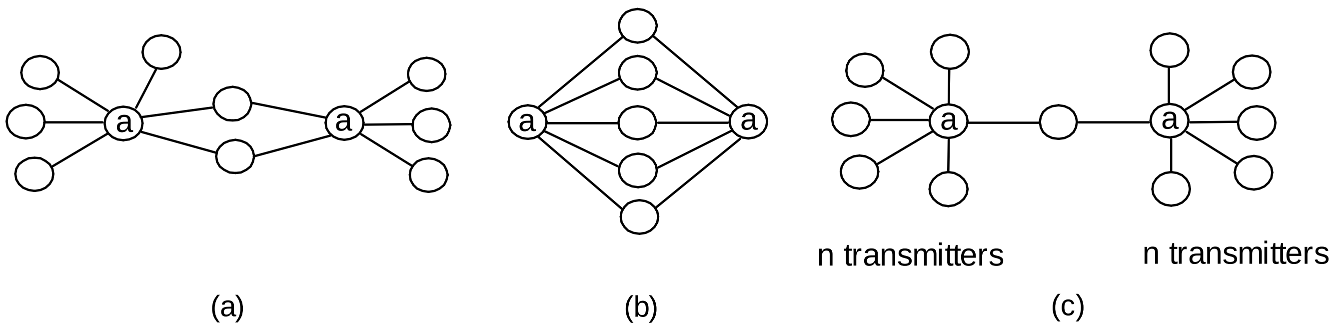

Figure 6: Overlapping-stars and local-stars

(Figure 6.c). In the latter, we narrow down to examine symmetric cases where two local stars are of the same size. Notice that if they are not of the same size, transmitters have incentives to switch to link with the larger star.

The parameter range for these networks to be in equilibrium are examined first:
(i) The overlapping-stars network (Figure 6.b) uses the maximum number of links available and there is no more link to be added; no transmitters want to link together as it does not reduce distance to any aggregator. The only deviation is that one of the middle transmitters severs a link, and the knowledge of the affected aggregator will be reduced from $\kappa(1+\delta(2n+1))$ to $\kappa\left(1+2\delta n+\delta^3\right)$. And this amount of knowledge will transmit to that transmitter with a smaller transmission rate δ^3, so equilibrium requires the knowledge decrease to outweigh link costs $c<\delta\kappa(1+\delta(2n+1))-\delta^3\kappa\left(1+2\delta n+\delta^3\right)$.
(ii) In the local-stars network (Figure 6.c), each aggregator links to a group of n transmitters separately forming two stars, and the remaining transmitter links to both aggregators. Each aggregator generates knowledge $K_0=\kappa\left(1+\delta(n+1)+\delta^3 n\right)$. If one of the $2n$ transmitters on the sides severs the link, she has knowledge loss is $\left(\delta+\delta^3\right)K_0$. If middle transmitter severs a link, she has knowledge loss δK_0. Requiring $c<\delta K_0$ prevents both deviations.

If a side transmitter adds a link to the other aggregator, knowledge at the linked aggregator increases to $K_2=\kappa\left(1+\delta(n+2)+\delta^3(n-1)\right)$. The linking transmitter receives $\delta(K_0+K_2)$, so the equilibrium needs $c<\delta(K_0+K_2)-\left(\delta+\delta^3\right)K_0=\delta K_2-\delta^3 K_0$.

If one of the side transmitters adds a link to a transmitter on the other side, each aggregator has knowledge $K_3=\kappa\left(1+\delta(n+1)+\delta^2+\delta^3(n-1)\right)$ and the linking transmitter receives $\left(\delta+\delta^2\right)K_3$. Thus, equilibrium requires $c>2\left(\left(\delta+\delta^2\right)K_3-\left(\delta+\delta^3\right)K_0\right)$.

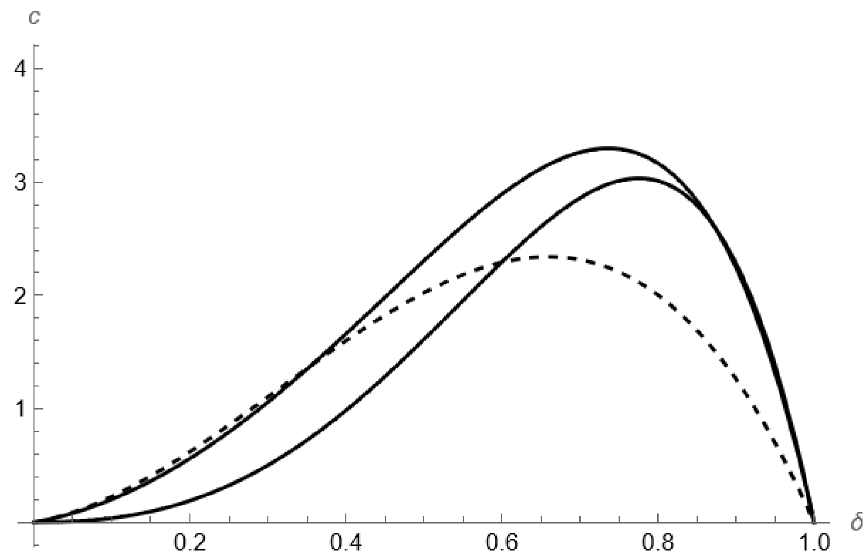

Figure 7: Equilibrium configurations over decay and link cost parameters (δ, c).

Figure 7 presents simulation results with $n = 5$, $\kappa = 1.2$, and $\delta \in [0.2, 1]$. Plotted curves are the upper and lower bounds of link cost c over δ.

The overlapping-stars network is in equilibrium for the range of c below the dashed curve. This configuration occurs in all range of $\delta \in [0, 1]$ given c not too large. The local-stars network is in equilibrium for the range of c between the two solid curves. This configuration occurs approximately in range $\delta \in [0, 0.87]$, given a proper range of c. In the lower range approximately $\delta \in [0, 0.6]$, these two configurations coexist with proper c values.

To examine the optimal network configuration between these two types of networks, we parameterize the number of double-linked transmitters to be $2k + 1$, $k \in [0, n]$. This also indicates the number of cycles. Thus, each aggregator has another $n - k$ transmitters single-linked to it. When $k = 0$, it is the local-star network; when $k = n$, it is the overlapping-stars network. Each of the aggregators has knowledge $K_a = \kappa \left(1 + \delta (n + k + 1) + \delta^3 (n - k)\right)$. The optimal number of middle links $2k + 1$ is determined in the following:

Network has total output

$$\begin{aligned} & 2K_a + 2(n-k)\left(1 + \left(\delta + \delta^3\right) K_a\right) + (2k+1)(1 + 2\delta K_a) - 2(n+k+1)c, \\ = \; & 2n + 1 + 2(K_a)^2/\kappa - 2(n+k+1)c. \end{aligned}$$

Its first derivative with respect to k is $4k\left(\delta - \delta^3\right)^2 k + 2\left(\delta - \delta^3\right)\left(\left(\delta + \delta^3\right) n + 1 + \delta\right) - 2c$. When c is small, this derivative can be positive in the range $k \in [0, n]$ and the network is optimal with maximal double linked transmitters, which means overlapping-stars. On the other hand, when c is large, this derivative can be negative in the range $k \in [0, n]$, and the network is optimal with minimal double linked transmitters, which means local-stars. Middle cases, for example, are presented in the following table with $n = 10$, $\kappa = 1.2$, $\delta = 0.8$. When $c = 11$ and 12, the optimal number of middle double links $(2k + 1)$ is 9 and 19 respectively. When $c = 10$, local-stars is optimal and when $c = 13$, overlapping-stars is optimal.

c	10	11	12	13
optimal k	0	4	9	10

3 Endogenous Aggregators

The basic model in Section 2 has exogenous information-processing roles assigned to agents. This is a realistic setting for more structured organizations such as firms, where job roles are pre-determined for individuals. In other network environments, such as research collaboration and trade, individuals can choose their information-processing roles and aggregators emerge endogenously. An agent can switch from an aggregator to a transmitter unilaterally. When an agent switches from a transmitter to an aggregator, however, she needs consent from linked transmitters since costs are incurred on linked partners.

An agent $i \in I$ in this generalized framework chooses whether to be an information aggregator (a) or an information transmitter (t). The choice of roles facing agent $i \in I$ is denoted by $\rho_i \in \{a, t\}$; if $\rho_i = a$ then $i \in A$, and if $\rho_i = t$ then $i \in T$. The set of strategies agents can choose are expanded to include role choice: one can switch the roles between an aggregator or a transmitter, while sever a few links. Severing links facilitates switching to a transmitter since it increases link costs. We can then modify Definition 1 to arrive at:

Definition 2. An *information-network equilibrium with endogenous information-processing roles* is a pair of link profile and information-processing roles $(l, \rho) = (l_h, \rho_h)_{h \in I}$ such that for any agent $i \in I$, (i) there is no strategy $(l'_i, \rho'_i) \in \{(l_i \backslash ij, \rho_i), (l_i \cup ij, \rho_i)\}$ for some $j \in I$, and (ii) no $(l'_i, \rho'_i) = (l_i \backslash Z_i, s)$ for any $Z \subseteq l_i$ if $\rho_i \neq s$, such that

$$
\begin{aligned}
V_i(l', \rho') &> V_i(l, \rho), \\
V_j(l', \rho') &> V_j(l, \rho), \text{ if } ij \in l'_i \setminus l_i.
\end{aligned}
$$

In addition to changing a link, agent i can switch to a role $s \neq \rho_i$, and at the same time, sever a set Z_i of links. We present two fundamental theorems below. For any node $i \in I$, construct the geodesic-distance trees for i as in Section 3.1. Take the geodesic-distance tree with the least links to be $\hat{N}^m(i)$. Suppose i is the only aggregator and all other transmitters are connected to i on $\hat{N}^m(i)$. The output at i is, thus, $\kappa\left(1+\sum_{j\in I\backslash i}\delta^{d_G(i,j)}\right)$. Let $\bar{d}_G = \max_{i,j\in I} d^G(i,j)$ denote the longest geodesic distance between any two nodes on G.

Condition S: $c < \min\{S1, S2, S3, S4, S5, S6\}$ where $S1 = \kappa\left(\delta^{\bar{d}_G} - \delta^{\bar{d}_G+1}\right)\left(1+\delta^{\bar{d}_G}(|I|-2)\right)$, $S2 = \left(1-\left(1-\delta^{\bar{d}_G}\right)\kappa\left(1+\delta\left(|I|-1\right)\right)\right)/\left(|I|-2\right)$, $S3 = \kappa\delta(1+\delta)$, $S4 = 2\kappa\delta^{2\bar{d}_G}$, $S5 = 4\left(1\text{+}\left(\delta\text{+}\delta^2\text{-}1\right)\kappa\delta^{\bar{d}_G}\left(1\text{+}\delta^{\bar{d}_G}\left(|I|\text{-}2\right)/2\right)\right)/\left(|I|\text{-}2\right)$, $S6 = 2\left(1\text{+}\kappa\delta^4\text{-}\kappa\left(1\text{-}\delta^2\right)\left(1\text{+}\delta\left(|I|\text{-}2\right)\right)\right)$.

Theorem 2. (Existence and Unique Pattern) *In an information network with a finite number of agents on an arbitrary geography, an information-network equilibrium with endogenous information-processing roles featuring active information process exists. The monocentric pattern, a geodesic-distance tree network, with one aggregator is the unique equilibrium network, if the transmission rate is high and link costs are not too large, where a sufficient condition is given by Condition S.*

Proof. (i) (Existence) Take agent $\hat{a} \in I$ whose position in G has the largest potential knowledge, that is $\arg\max_{a\in I}\kappa\left(1+\sum_{j\in I\backslash a}\delta^{d_G(a,j)}\right)$. Connect the geodesic-distance tree $N(\hat{a})$ for $\hat{a}$ as the aggregator. In this network, $A=\{\hat{a}\}$, $T = I\backslash\hat{a}$, and the distance between two nodes is denoted by $d(i,j)$. First, a transmitter will not change links. All agents are linked to $\hat{a}$ by geodesic distances, there is no way to reduce decays by adding a link. Moreover, severing a link gains the link cost, which can be smaller than the knowledge loss on the link. A sufficient condition is $c < \kappa\left(\delta^{\bar{d}_G} - \delta^{\bar{d}_G+1}\right)\left(1+\delta^{\bar{d}_G}(|T|-1)\right)$ (as in Theorem 1.i, replacing with the longest distance on G). Second, if $\hat{a}$ switches to be a transmitter, output will be 1 as there is no knowledge aggregation in the network. This is less than aggregator knowledge $K_{\hat{a}} = \kappa\left(1+\sum_{i\in T}\delta^{d(\hat{a},i)}\right)$.

Third, any transmitter t will not switch to be an aggregator, which would yield knowledge $\kappa\left(1+\sum_{i\in T\backslash t}\delta^{d(t,i)}\right) < K_{\hat{a}}$. The knowledge change for switching is less than $\left(1-\delta^{d(\hat{a},t)}\right)K_{\hat{a}}-1$. Take extreme cases when $K_{\hat{a}}$ has knowledge $\kappa(1+\delta|T|)$ and t has $(|I|-2)/2$ direct links, a sufficient condition is $\left(1-\delta^{\bar{d}_G}\right)\kappa(1+\delta|T|) - 1 + c(|I|-2)/4 < 0$.

(ii) (Uniqueness) We will show in the following steps that in equilibrium: there must be an aggregator, every transmitters is connected to an aggregator, every transmitter is connected to all aggregators, there is only one aggregator, and then the network is a tree.

(a) (There is an aggregator.) Suppose there is no aggregator. Any agent can deviate to be an aggregator and link with an adjacent transmitter. The aggregator's payoff is $\kappa(1+\delta) > 1$ and the transmitter payoff is $1+\delta\kappa(1+\delta)-c$, which will be larger than 1 when c is not too large. This requires $c < \kappa\delta(1+\delta)$.

(b) (Every transmitter is connected to an aggregator.) Suppose there are some transmitters not connected to any aggregator. Then, there is a transmitter t either adjacent to an aggregator a, or adjacent to another transmitter t' which is connected to a since G is complete. If t links to a, the gains in knowledge for t is $\delta K_a > \kappa\delta\left(1+\delta^{\bar{d}_G}(|I|-1)\right)$; a sufficient condition $c < \kappa\delta\left(1+\delta^{\bar{d}_G}(|I|-1)\right)$ works (this is implied by the sufficient condition in i). If t links to t'. The knowledge gain for connected end node t' is $\delta^{d(a,t')}\kappa\delta^{d(\hat{a},t)} > \kappa\delta^{2\bar{d}_G}$; a sufficient condition is $c/2 < \kappa\delta^{2\bar{d}_G}$, treating it as a far end in the network. The knowledge gain for t is $\delta^{d(a,t)}\kappa\left(1+\delta^{d(a,t)}K_a\right) > \kappa\delta^{\bar{d}_G}\left(1+\delta^{\bar{d}_G}(|I|-1)\right)$ ($c < 2\kappa\delta^{2\bar{d}_G}$ implies it outweighs link cost).

(c) (Every transmitter is connected to all aggregators.) Suppose there are transmitter t_1 and aggregator a_2 not connected. The above part (b) says that t_1 is connected to an aggregator a_1, and a_2 is connected to a transmitter t_2. These two pairs belong to two disconnected subgraphs. And there is no transmitter staying disconnected. Since the underlying geography G is connected, any disconnected subgraphs must have an adjacent potential link to another disconnected subgraph. The potential link can be between two transmitters, between an aggregator and a transmitter, or between two aggregators. Without loss of generality, suppose we have the above two adjacent disconnected subgraphs. First, suppose the potential link is t_1t_2. Linking t_1t_2 brings t_1 at least t_2's information aggregated by a_1 and a_2 and t_1's information aggregated by a_2 which is $\delta^{d(a_1,t_1)}\kappa\left(1+\delta^{d(a_1,t_2)}\right)+\delta^{d(a_2,t_1)}\kappa\left(1+\delta^{d(a_2,t_2)}\right)+\delta^{d(a_1,t_1)}\kappa\left(1+\delta^{d(a_2,t_1)}\right) > 3\delta^{\bar{d}_G}\kappa\left(1+\delta^{\bar{d}_G}\right)$ ($c < 2\kappa\delta^{2\bar{d}_G}$ implies it outweighs link cost). Similarly, this brings positive deviating payoff for t_2 too, with the same sufficient condition.

Second, suppose the potential link is t_1a_2. Linking t_1a_2 brings t_1 extra knowledge from a_1 and a_2 similar to the above case. A lower bound for this knowledge gain is $3\delta^{\bar{d}_G}\kappa\left(1+\delta^{\bar{d}_G}\right)$ ($2\kappa\delta^{2\bar{d}_G} > c$ implies it outweighs link cost).

Third, suppose the potential link is a_1a_2, and suppose $K_{a_1} \geq K_{a_2}$. If a_2 links with a_1 and becomes a transmitter, K_{a_2} will be passing through a_2 and going to be aggregated at a_1. The knowledge at a_2 is $\delta K_{a_1}+\delta^2K_{a_2}$. The output increases by $1+\delta K_{a_1}+\left(\delta^2-1\right)K_{a_2}$ minus changes in links costs. This output increase will be positive for large δ, and will be at the lowest when $K_{a_1}=K_{a_2}$. Each transmitter gives at least knowledge $\kappa\delta^{2\bar{d}_G}$ and at most link cost $c/2$. If $\left(\delta+\delta^2-1\right)\kappa\delta^{2\bar{d}_G} > c/2$, net gain is positive; if $\left(\delta+\delta^2-1\right)\kappa\delta^{2\bar{d}_G} < c/2$, take the extreme case of the two aggregators split the network; $1+\left(\delta+\delta^2-1\right)\kappa\delta^{\bar{d}_G}\left(1+\delta^{\bar{d}_G}(|I|-2)/2\right)-c(|I|-2)/4 > 0$ is sufficient. This concludes that the network is connected.

(d) (There is only one aggregator.) Suppose a_1 and a_2 are two of the aggregators and a_1 has the higher knowledge in the network $K_{a_1} \geq K_{a_2}$. If a_2 becomes a transmitter, the output gain is $1 + \delta^{d(a_2,a_1)} \left(K_{a_1} + \kappa \delta^{d(a_2,a_1)} \right) - K_{a_2}$ minus associated link costs. a_2 can sever all links but one. An extreme case is when they link equally to all transmitters $K_{a_1} = K_{a_2} - \kappa \left(1 + \delta \left(|I| - 2\right)\right)$, A sufficient condition is $1 + \kappa\delta^4 - \kappa \left(1 - \delta^2\right) \left(1 + \delta \left(|I| - 2\right)\right) - c/2 > 0$

(e) Finally, by Proposition 3, the above monocentric network is a tree network since it has only one aggregator. ∎

Cycles may also appear with endogenous role choices, and they face two types of deviation incentives: a transmitter severing a link to save costs; and an aggregator switching to be a transmitter. Intuitively, a larger link cost favors the former deviation, and a smaller link cost favors the latter; the transmission rate does not have a monotonic effect on these deviation incentives. We use the simplest circle network in Figure 5. a, evaluated at $\kappa = 1.2$. as an example, but the intuition holds in general. The circle is in equilibrium when decays in information transmission are large together with a sufficiently small costs of link maintenance ($\delta = 0.7$ and $c < 1.115$). When decays in information transmission are in the intermediate range (for instance, $\delta = 0.8$), cycle does not arise if link maintenance is highly costly ($c > 0.997$) or relatively negligible ($c < 0.368$). And when decays are sufficiently small (for $\delta > 0.87$), the cycle network is not in equilibrium. In the first case of large decays, aggregators will not switch to transmitters. In the second case, the lower bound is a result of endogenous aggregators – with such a low link cost but negligible information transmission decays, the aggregator would be better off switching to transmitter; and the upper bound is from transmitters severing links due to costly links.

Theorem 3 considers a degenerate information-network equilibrium due to high decays or high link costs.

Theorem 3. (Degenerate Information Network) *In an information network with a finite number of agents on an arbitrary geography, a degenerate information-network equilibrium with endogenous information-processing roles featuring no information aggregation exists if the transmission rate is low and link costs are large, where a sufficient condition is given by,* $c > 2 + \kappa\delta \left(1/\delta + |I|\right) \left(1 + \delta \left(|I| - 1/\delta\right)/2\right)$.

Proof. The maximal knowledge a transmitter t can receive from M aggregators is $\kappa\delta M \left(1+\delta \left(|I| \text{-} M\right)\right)$ in a complete graph. Take $M = \left(1/\delta + |I|\right)/2$ for maximal value. A sufficient condition is $c/2 > 1 + \kappa\delta \left(1/\delta + |I|\right) \left(1 + \delta \left(|I| - 1/\delta\right)/2\right)/2$. ∎

4 Information Chain Networks

In this section, we further investigate information-network equilibrium with endogenous information-processing roles, in an economy on a *chain* with two aggregators. This benchmark is simple but rich enough to provide additional insights towards understanding the emergence of information networks. It also captures interesting frameworks including the Hotelling model of market competition and the duo-centric model of location theory. The results in this section also hold for general cases with more than two aggregators.

Call the two information-aggregating agents a_1 and a_2, with a_1 locates on the left of a_2. There are n_L agents on the left of a_1, n_R agents on the right of a_2, and n agents in-between a_1 and a_2. Suppose $n_L \geq n_R$. The information network can be delineated as follows:

$$\begin{array}{ccccc} \cdots - t - \cdots & \cdots - \ a_1 \ - \cdots & \cdots - t - \cdots & \cdots - \ a_2 \ - \cdots & \cdots - t - \cdots \\ (n_L \text{ agents}) & & (n \text{ agents}) & & (n_R \text{ agents}) \end{array}$$

Let κ be the aggregation coefficient, the knowledge created by each of the two aggregators becomes:

$$\begin{aligned} K_1 &= \kappa \left(1 + \sum_{h=1}^{n_L} \delta^h + \sum_{h=1}^{n} \delta^h + \delta^{n+1} \sum_{h=1}^{n_R} \delta^h \right), \\ K_2 &= \kappa \left(1 + \sum_{h=1}^{n_R} \delta^h + \sum_{h=1}^{n} \delta^h + \delta^{n+1} \sum_{h=1}^{n_L} \delta^h \right). \end{aligned}$$

4.1 Equilibrium

Possible types of deviations are checked in the following. A middle transmitter at a distance of m links away from a_1, $n > m \geq 1$, receives benefit

$$b_1(m) = 1 + \delta^m K_1 + \delta^{n+1-m} K_2 - c.$$

A side transmitter m links, $n_L > m \geq 1$, to the left of a_1 receives

$$b_2(m) = 1 + \delta^m K_1 + \delta^{n+m+1} K_2 - c,$$

whereas a side transmitter m links to the right of a_2, $n_R > m \geq 1$, receives

$$b_3(m) = 1 + \delta^{n+m+1} K_1 + \delta^m K_2 - c.$$

And the transmitters at the very end on the left and right sides receives respectively,

$$b_2(n_L) + c/2 \text{ and } b_3(n_R) + c/2.$$

(i) Severing a link

(a) A middle transmitter may break the link to the right, towards a_2, deviation will not happen if and only if

$$1+\delta^{m}\kappa\left(1+\sum_{h=1}^{n_L}\delta^{h}+\sum_{h=1}^{m}\delta^{h}\right)-\frac{c}{2}\leq b_1\left(m\right).$$

She may break the link to the left, towards a_1, deviation will not happen if and only if

$$1+\delta^{n+1-m}\kappa\left(1+\sum_{h=1}^{n_R}\delta^{h}+\sum_{h=1}^{n+1-m}\delta^{h}\right)-\frac{c}{2}\leq b_1\left(m\right).$$

(b) A side transmitter may sever the link to the aggregators. The highest incentive happens at the last link at the far-right end. Deviation will not happen if and only if

$$b_3\left(n_R\right)+c/2\geq 0.$$

(c) A side transmitter may sever a link rendering other transmitters disconnected. The highest incentive happens at the last link at the two ends. If the second to last transmitter on the far left severs the link to the left, she loses knowledge from the end transmitter $\delta^{n_L-1}\kappa\left(\delta^{n_L}\right)+\delta^{n_L+n}\kappa\left(\delta^{n+1+n_L}\right)$. Deviation will not happen if and only if

$$\left(\delta^{2n_L-1}+\delta^{2n_L+2n+1}\right)\kappa-c/2\geq 0.$$

The second to last transmitter on the far right will not deviate if and only if

$$\left(\delta^{2n_R-1}+\delta^{2n_R+2+1n}\right)\kappa-c/2\geq 0.$$

(ii) A transmitter switches the role to play aggregator

(a) Consider a middle transmitter t who is m links away from a_1, $n>m>1$. If she plays an aggregator, deviation will not happen if and only if either of the following conditions holds:

$$\kappa\left(1+\delta^{m}\sum_{h=1}^{n_L}\delta^{h}+\sum_{h=1}^{m-1}\delta^{h}+\sum_{h=1}^{n-m}\delta^{h}+\delta^{n+1-m}\sum_{h=1}^{n_R}\delta^{h}\right)\leq b_1\left(m\right),$$

or adjacent transmitters get an additional benefit lower than extra link costs,

$$c/2\geq\delta\kappa\left(1+\delta^{m}\sum_{h=1}^{n_L}\delta^{h}+\sum_{h=1}^{m-1}\delta^{h}+\sum_{h=1}^{n-m}\delta^{h}+\delta^{n+1-m}\sum_{h=1}^{n_R}\delta^{h}\right).$$

(b) A side transmitter m links away from a_1 can switch to be an aggregator, deviation will not happen if and only if ether of the following holds:

$$\kappa\left(1+\sum_{h=1}^{n_L-m}\delta^h+\sum_{h=1}^{m-1}\delta^h+\delta^m\sum_{h=1}^{n}\delta^h+\delta^{n+1+m}\sum_{h=1}^{n_R}\delta^h\right)\leq b_2\left(m\right),$$

or adjacent transmitters get an additional benefit lower than the extra link costs,

$$c/2\geq\delta\kappa\left(1+\sum_{h=1}^{n_L-m}\delta^h+\sum_{h=1}^{m-1}\delta^h+\delta^m\sum_{h=1}^{n}\delta^h+\delta^{n+1+m}\sum_{h=1}^{n_R}\delta^h\right).$$

(c) A side transmitter m links away from a_2 can switch to be an aggregator, deviation will not happen if either of the following holds:

$$\kappa\left(1+\sum_{h=1}^{n_R-m}\delta^h+\sum_{h=1}^{m-1}\delta^h+\delta^m\sum_{h=1}^{n}\delta^h+\delta^{n+1+m}\sum_{h=1}^{n_L}\delta^h\right)\leq b_3\left(m\right),$$

or adjacent transmitters get an additional benefit lower than the extra link costs,

$$c/2\geq\delta\kappa\left(1+\sum_{h=1}^{n_R-m}\delta^h+\sum_{h=1}^{m-1}\delta^h+\delta^m\sum_{h=1}^{n}\delta^h+\delta^{n+1+m}\sum_{h=1}^{n_L}\delta^h\right).$$

(iii) An aggregator switches to transmitter

The switching incentive is higher for the smaller aggregator a_2. (a) a_2 maintains both links and, as a transmitter, only receives knowledge from a_1; deviation will not happen if and only if

$$1+\delta^{n+1}K_1-c\leq K_2.$$

(b) a_2 switches to be a transmitter and cut the link to the right; deviation will not happen if and only if

$$1+\delta^{n+1}\left(K_1-\delta^{n+1}\sum_{h=1}^{n_R}\delta^h\right)-c/2\leq K_2.$$

By examining the three sets of deviation-proof conditions for the information-chain networks, we find that the efficacy of information aggregation does not affect a middle agent's decision on whether to sever a link but makes a side agent less likely to sever a link and raises the incentives for all transmitters to switch role to aggregators. On balance, the equilibrium network *need not* feature more concentration.

4.2 Simulation Results

It remains to investigate the range of the two key parameters, transmission delays and link costs, within which the information-chain network forms in equilibrium (simulation of a chain network with endogenous note size can be found in Kung and Wang 2012).

Let $\kappa = 1.2$. We will fix either $\delta = 0.9$ or $c = 1$ and then the range of the other parameter within which a network equilibrium arises. We must check all conditions given above. For comparison purposes, we select the benchmark parameter values: $(\kappa, \delta, c) = (1.2, 0.9, 1)$, then derive the ranges of δ (by fixing $c = 1$) and of c (by fixing $\delta = 0.9$) that can support a network equilibrium. Results are summarized in the table below:

Value of (n_L, n, n_R)	Range of δ $c = 1$	Range of c $\delta = 0.90$
$(4, 4, 4)$	$(1, 0.86)$	$(0, 1.50)$
$(4, 4, 3)$	$(1, 0.86)$	$(0, 1.50)$
$(4, 2, 4)$	$(1, 0.85)$	$(0, 1.75)$
$(3, 4, 3)$	$(1, 0.82)$	$(0, 1.90)$
$(3, 3, 3)$	$(1, 0.82)$	$(0, 2.00)$
$(3, 2, 3)$	$(1, 0.81)$	$(0, 2.15)$

Consider the case with link cost $c = 1$. It is clear that when the number of side or middle agents reduces, we can accept larger values of information transmission decays (lower δ) to support an information-chain network in equilibrium. This is because such decays are less harmful in a shorter chain. Turn next to the case with information transmission decays $\delta = 0.90$. One may see that with fewer side or middle agents, we may still have an information-chain network equilibrium even with higher links costs. This is due to the greater benefit from maintaining the link as a result of stronger information aggregation.

To the end, consider a symmetry case with $n_L = n = n_R$ and rewrite:

$$K = K_1 = K_2 = \kappa \left[1 + \sum_{h=1}^{n} \delta^h \left(2 + \delta^{n+1} \right) \right]$$

By straightforward differentiation, one obtains:

$$\begin{aligned} \frac{\partial \ln K}{\partial \ln \kappa} &= 1 \\ \frac{\partial \ln K}{\partial \ln \delta} &= \frac{\delta \kappa}{K} \left\{ (n+1)\, \delta^{n+1} \frac{1-\delta^n}{1-\delta} + \frac{2+\delta^{n+1}}{(1-\delta)^2} \left[1 - \delta^n (n - n\delta + 1) \right] \right\} \\ &= \frac{\delta \left\{ (n+1)\, \delta^{n+1} (1-\delta^n) + \frac{2+\delta^{n+1}}{1-\delta} \left[1 - \delta^n (n - n\delta + 1) \right] \right\}}{1 - \delta + \left(2 + \delta^{n+1} \right) \delta (1 - \delta^n)} \end{aligned}$$

where in deriving the second expression we have used,

$$\begin{aligned}\frac{\partial}{\partial\delta}\sum_{h=1}^{n}\delta^{h}\left(2+\delta^{n+1}\right) &= \sum_{h=1}^{n}\delta^{h-1}\left[2h+(n+1+h)\,\delta^{n+1}\right] \\ &= (n+1)\,\delta^{n}\sum_{h=1}^{n}\delta^{h}+\frac{2+\delta^{n+1}}{\delta}\sum_{h=1}^{n}\delta^{h}h \\ &= (n+1)\,\delta^{n+1}\frac{1-\delta^{n}}{1-\delta}+\frac{2+\delta^{n+1}}{(1-\delta)^{2}}\left[1-\delta^{n}\left(n-n\delta+1\right)\right]\end{aligned}$$

Thus, it is said aggregating efficacy (κ) is more effective than transmitting efficacy (δ) (i.e., $\frac{\partial \ln K}{\partial \ln \kappa} > \frac{\partial \ln K}{\partial \ln \delta}$) if and only if

$$\frac{1-\delta}{\delta} > \left(n\delta^{n+1}-2\right)\left(1-\delta^{n}\right)+\frac{2+\delta^{n+1}}{1-\delta}\left[1-\delta^{n}\left(1+n\left(1-\delta\right)\right)\right].$$

Numerical analysis (see Figure 8) shows that, in an information-processing chain network, aggregating efficacy (vertical axis) is more effective than transmitting efficacy when the size of the network is not too large and knowledge delays are sufficiently strong.

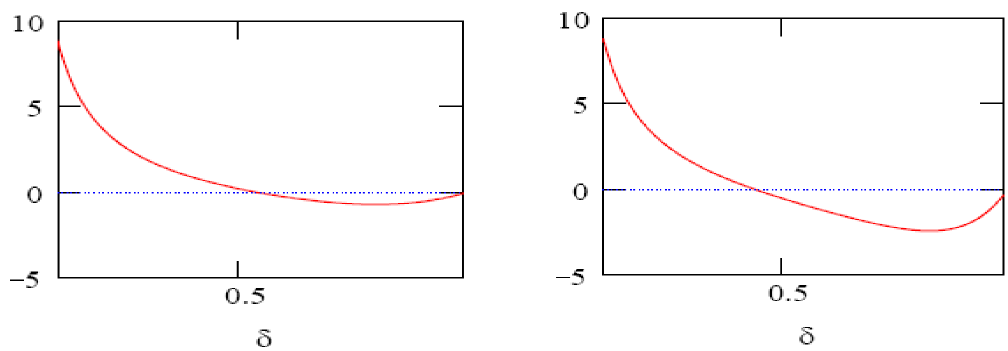

Figure 8: Aggregating efficacy more effective with stronger knowledge delays

5 Conclusions

We examined an information-network economy populated with not only information transmitters but also information aggregators. Several propositions are established under both exogenous or endogenous information-processing roles in aggregation or transmission. We also characterized an information-processing chain network with all information aggregators and transmitters linked along a chain. Of particular interest, we showed the uniqueness of the monocentric network when link costs are sufficiently small, and examined under what circumstances a network with cycles may arise in equilibrium.

Along these lines, a potentially interesting avenue for future study is to allow for asymmetric aggregators serving roles in a network. For example, in an information-chain network with two aggregators, the left-side agents may only be served by the left aggregator whereas the right-side agents by the right aggregator. Similarly, in a cycle with multiple aggregators, those only connected one aggregator may only be served by that aggregator – for illustrative purposes, let us also call those side agents (and those connecting to more than one aggregator middle agents). One may then investigate the differential roles of those side and middle agents played in the equilibrium configuration of the network. This may help understand the formation of subnetworks within a grand network. Of course, it should be acknowledged that such an extension would increase the level of complexity significantly. In order to carry out this task, it would require further simplification of the current structure of the model. This is beyond the scope of the present paper, so we leave it for future work.

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