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ABSTRACT

We develop a dynamic spatial growth model to explore the role of trade and internal migration in the process of spatial development and aggregate growth. Growth is shaped by the best global and local ideas that contribute to the local stock of knowledge. Global ideas diffuse more to locations that are relatively more exposed to international trade. Local ideas are diffused across space when workers move to another location. We embed the diffusion of ideas through trade and migration into a multi-country, multi-region framework with international trade, forward-looking dynamic migration decisions, and endogenous capital accumulation. We apply our framework to study the role of initial conditions, international trade, and internal migration on China's spatial development and aggregate growth during the 1990s and 2000s. We find that initial conditions across space, idea diffusion, and capital accumulation play an important role in understanding the process of spatial development and aggregate growth in China. Changes in international trade costs and mobility restrictions during the 1990s and 2000s also contribute to aggregate growth, with large heterogeneity across space.

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An online appendix is available at http://www.nber.org/data-appendix/w30579

1 Introduction

Understanding economic growth requires understanding how countries accumulate factors of production and increase the productivity of such factors. In recent decades, the world has witnessed the successful growth experiences of developing countries such as Vietnam, Laos, the Philippines, and China, among others, where high economic growth has occurred hand-in-hand with increased trade openness, more internal migration, and high productivity growth. Aggregate economic growth is shaped in part by the process of development across space within a country, namely, the dynamics of the distribution of economic activity in space, the extent to which locations have differential exposure to trade, the internal mobility of labor, the local evolution of productivity, and other local characteristics. In this paper we develop a tractable dynamic spatial growth model to study quantitatively the process of spatial development across locations in a country and how this process shapes aggregate growth.

We consider a world economy with multiple countries and multiple locations within a country. Growth in each location is shaped by the endogenous evolution of total factor productivity, which is the outcome of the diffusion of global and local ideas. In each location, a continuum of differentiated goods is produced, and there are many potential producers of each good. Producers have heterogeneous productivities (ideas) to produce goods and those who are actively producing contribute to the local pool of ideas that determines the local stock of knowledge, namely, the local fundamental productivity.

Ideas diffuse across locations because of trade in goods and the migration of workers. To make the process of diffusion tractable, we model the diffusion of ideas across space as a stochastic process. We consider that ideas arrive stochastically to each producer. The productivity of each idea is a combination of an random original component, a random insight drawn from the ideas of sellers to that location, and a random insight drawn from workers in that location. In particular, global ideas are embedded in imported intermediate goods and diffuse more to locations relatively more exposed to international trade. Workers learn about local ideas and diffuse them across space when they migrate and interact with local producers in the destination location. As a result, the local pool of insights from workers contains local ideas from non-migrants and ideas from migrants. Building on the results in Buera and Oberfield (2020), we show that under mild conditions, the distribution of productivities at each location follows a Fréchet distribution and that the evolution of the stock of knowledge at each location can be characterized by a system of difference equations. Therefore, the evolution of fundamental productivity in each location depends on how connected the particular location is to the other locations through trade and migration as well as the quality of insights from those locations. The idea diffusion process in our framework is endogenous since migration is shaped by the decisions of forward-looking workers, and trade is determined by the production decisions of firms.

Since the productivities at each location follow a Fréchet distribution, we apply the results in

Eaton and Kortum (2002) to model the trade and production structure in our framework. Producers with heterogeneous productivities (ideas) in each location source intermediate goods from the lowest-cost suppliers across countries and combine them with labor and capital to produce goods. The labor supply across locations is shaped by the forward-looking migration decisions of workers as in Caliendo, Dvorkin, and Parro (2019). At each moment in time, workers supply labor, purchase local goods, and sort into different locations. Workers carry ideas from their previous location with them as they migrate, and they provide insights to producers in the current location. The supply of capital in each location features forward-looking landlords making investment decisions in local capital as in Kleinman, Liu, and Redding (2021), which determines capital accumulation at each location.

Our framework allows for a rich heterogeneity across space in terms of trade openness, mobility frictions, factor endowments, and initial stock of knowledge. With all the margins previously described, our paper provides a tractable dynamic spatial growth model to study the role of spatial development on aggregate growth in general equilibrium. We show how to invert the model to uniquely characterize the initial stock of knowledge at each location. Given the initial stock of knowledge and data on initial allocations (production, trade, migration), we show how to compute the model without having to assume that the economy is in the balanced growth path at the initial period. We also show the existence and uniqueness of the balanced growth path equilibrium.

We use our dynamic spatial growth framework to study quantitatively the role of our spatial mechanisms in shaping aggregate and spatial growth. In particular, we study how spatial heterogeneity in initial conditions, as well as changes in fundamentals, impact aggregate and spatial growth, and in particular, the importance of internal migration and international trade to the process of spatial development. To do so, we look at an economy that is in transition, with heterogeneous locations in terms of stock of knowledge and initial supply of capital and supply of labor, and with locations having heterogeneous exposure to international trade and mobility frictions. We apply our dynamic spatial growth framework to the Chinese economy, which features such spatial heterogeneity, and study the mechanics of spatial growth in China in the 1990s and 2000s through the lens of our general equilibrium framework.

During the 1990s and far into the 2000s, China experienced fast economic growth, sustained capital accumulation, the relocation of factors and production across space, and increased trade openness. This successful growth experience is sometimes called the China shock in recent literature and has been primarily used to study the effects of import competition on labor markets and other outcomes in the United States and other countries. Less work has been devoted to understanding its internal spatial dynamics. As we subsequently describe, different factors contributing to the rise of China have been studied in the literature; these include China's trade openness, the fast increase in productivity, and the role of internal reforms that facilitate the movement of factors across space. Guided by this literature, we study the mechanics of spatial growth in China that led to fast aggregate growth during this period. We answer questions such as what was the role of international trade and internal migration on China's spatial growth during the 1990s and 2000s? We use our spatial growth framework to provide a quantitative answer to this question.

To do so, we take the model to the year 1990. We divide China into 30 provinces, and we group the rest of the countries in a rest of the world. We construct gross migration flows across provinces in China using census data. We condition gross flows by Hukou type. Hence, we take into account that migrants may make different migration decisions depending on their Hukou type. The Hukou system in China affects incentive to migrate and to return migrate and therefore contributes to uneven spatial growth, an aspect of China's development experience that we capture in our framework. We also collect data on production, expenditure, and trade between provinces and our constructed rest of the world. We also estimate elasticities that govern the rate of idea flows from trade and migration as well as the rate of innovation. We do so by obtaining cross-sectional measures of fundamental productivity using a model inversion and generating a set of time series moments that we use as moment conditions to estimate these elasticities by applying the generalized method of moments (GMM). With our data and these estimates in hand, we proceed to our quantitative assessment.

We first study how initial conditions and our mechanism shaped spatial growth in China in the 1990s and 2000s. When we take the model to the data, we do not assume that the economy is in a balanced growth path. Hence, the initial allocations in 1990—namely, trade openness, spatial mobility, factor endowments, and stock of knowledge across locations—are the result of all changes to the economic environment in China as a consequence of reforms and policies before the year 1990. We then use our framework to answer the following question: How would China have developed absent of the changes to trade and migration costs that occurred after 1990? We find an important role of initial conditions for the subsequent aggregate growth in China; that is, we find that reforms and policies before the 1990s explain a substantial fraction of aggregate growth in China in the subsequent decades. Importantly, we find that ideas from sellers contributed more to aggregate growth than ideas from migrants. This intuition comes from the fact that all provinces benefit from trade openness and access to better global ideas from the rest of the world. The contribution of ideas from people is smaller, as some locations benefited from relatively high-quality insights from migrants returning from high-productivity places but productivity growth also slowed in locations receiving migrants with relatively low-quality insights. We also find an important role of capital accumulation; aggregate growth would have declined by around half in the absence of capital accumulation.

Turning to the spatial growth effects, we find that initial conditions in 1990 played an important role in redistributing economic activity to the coastal provinces during the 1990s and 2000s. We quantify the real GDP growth rates across provinces and the contribution of each province to aggregate growth in China during different time frames. We find that aggregate growth is shaped by large heterogeneity in growth rates across space. During the 1990s, provinces located in coastal areas such as Shanghai, Guangdong, and Hainan benefited from access to better insights from the rest of the world and experienced higher growth rates. Over time, spatial growth moderated and tended to equalize as the economy moved closer to the balanced growth path. The engines of aggregate growth also changed over time; notably, Guangdong became the main contributor to aggregate growth in China while other provinces located in the central and eastern parts of China became less relevant engines for aggregate growth. We also study how the initial distribution of fundamentals across space shaped subsequent spatial growth in China. We find that provinces with a higher initial stock of knowledge experienced higher real GDP growth in the subsequent decades. While in our framework offsetting mechanisms can cause this correlation to go in any direction, we find that provinces with a higher initial stock of knowledge also tend to be more open to international trade and benefit relatively more from global ideas, more than offsetting some of the dampening forces from the idea diffusion from migrants coming from low-productivity places. We also find that provinces with more international trade openness experience higher subsequent growth, which is expected given that the initial stock of knowledge is higher in the rest of the world. The correlation between the initial mobility frictions and subsequent growth is less clear, as receiving migrants with high-quality insights has beneficial effect on economic growth, while migrants with low-quality insights could slow productivity growth.

While initial conditions seem to be important for understanding the process of spatial development and aggregate growth in China in the 1990s and 2000s, China also experienced reforms and policies that resulted in further changes to international trade and internal migration costs during the 1990s and 2000s, most notably the country's accession to the World Trade Organization and the elimination of some Hukou restrictions. We then study how observed changes in trade costs and internal migration frictions after 1990 contributed to growth. To do this, we first estimate changes in bilateral trade frictions over the period 1990-2010 between provinces in China and the rest of the world and then ask how spatial and aggregate growth in China would have looked if the only change in fundamentals over the period 1990-2010 had been bilateral international trade costs. We find that the change in trade costs contributed to extra aggregate growth by somewhat less than one percentage point annually and that the growth effects were very heterogeneous across space. In particular, coastal areas benefited the most from cheaper access to foreign goods, and this heterogeneous exposure also contributed to spatial growth inequality in the 1990s and 2000s. The aggregate effects of changes to migration frictions were also positive although they were smaller than the effects of changes to international trade costs. However, the small aggregate effects of changes to migration frictions mask large spatial heterogeneity as changes in mobility frictions benefit more open provinces that scale up production and provinces that receive migrants with good insights from high-productivity places, while they slow growth in provinces left behind by international trade and internal migration.

We also provide reduced-form evidence of idea diffusion through trade and migration. Measuring the local stock of knowledge in the data is a difficult task. To construct a proxy for it, we obtain province-level patent data and patent data for the rest of the world and use it along with our trade and migration data to provide empirical evidence of the roles played by trade and migration to diffuse ideas and to contribute to the local stock of knowledge. We find evidence of the mechanism for spatial growth through idea diffusion, namely provinces more open to trade and with more migrants from locations with a larger stock of knowledge experience a relative larger growth in their knowledge stock. In addition, guided by the structural relationship between the local knowledge stock and idea diffusion through trade and migration from our model, we run an instrumental variable regression and find evidence consistent with our reduced-form results.

Our research is related to different strands of existing work. While our paper contributes to a large body of quantitative spatial economics literature (see Redding and Rossi-Hansberg (2017) for a review), it mainly engages with recent work on dynamic spatial models. The general equilibrium trade structure and forward-looking migration decisions build on Caliendo, Dvorkin, and Parro (2019), where locations trade goods as in Eaton and Kortum (2002). We model workers' mobility decisions subject to frictions as a dynamic discrete-choice problem as in Artuc, Chaudhuri, and McLaren (2010). We introduce capital accumulation and spatial growth into a dynamic framework with labor market dynamics and trade. As described previously, capital accumulation in our framework features forward-looking atomistic landlords making investment decisions in local capital to maximize intertemporal utility following the structure in Kleinman, Liu, and Redding (2021). The distinction between landlords and workers also relates to the formulations in Angeletos (2007) and Moll (2014), and as discussed later on, adds tractability in the context of a dynamic spatial model with forward-looking mobile workers. Capital accumulation in our dynamic spatial framework also connects to dynamic models of capital accumulation and international trade (e.g., Eaton, Kortum, Neiman, and Romalis (2016), Alvarez (2017), Ravikumar, Santacreu, and Sposi (2019), Anderson, Larch, and Yotov (2019)), with the important difference that labor is assumed to be immobile across countries in that strand of the literature.

The distinctive feature of our dynamic spatial framework is the presence of spatial growth, as mentioned earlier. The process of innovation and diffusion that gives rise to the theory of total factor productivity in our model is a discrete-time version of Buera and Oberfield (2020). The model in Buera and Oberfield (2020) also relates to Kortum (1997) when there is no idea diffusion from insights, and to Jones (1995) and Atkeson and Burstein (2019) in a model with intertemporal knowledge spillovers that are not modeled explicitly as a function of insights. In addition, we extend Buera and Oberfield (2020) to a spatial setting where ideas diffuse in goods as well as in workers who move across space. Our paper also relates to Cai and Xiang (2022), who study technology diffusion through multinational production and global growth. In our context, ideas diffuse not only globally but also locally. Previous literature has developed dynamic spatial frameworks with innovation, local diffusion of technology, and spatial growth, most notably in Desmet and Rossi-Hansberg (2014) and Desmet, Nagy, and Rossi-Hansberg (2018), and an extension to study the dynamic effects of climate change in Cruz and Rossi-Hansberg (2022). Our framework shares some aspects with these papers such as the spatial heterogeneity in fundamentals and the geographic aspect of local idea diffusion. However, in our framework, technology diffuses spatially through trade and migration, both of which are endogenous, instead of being dictated by geographical distance; moreover, innovation comes from an exogenous arrival rate of ideas instead of being an investment decision by firms. Our framework also departs from these papers by introducing forward-looking migration and capital accumulation decisions.

Also related to our paper, Eaton and Kortum (1999) develops a model of idea diffusion across countries where the distribution of productivities in each country follows a Fréchet distribution and the evolution of the stock of knowledge is characterized by a system of differential equations. In their model, ideas diffuse across countries exogenously, and countries are assumed to be under autarky otherwise. Building on Eaton and Kortum (1999), Cai, Li, and Santacreu (2022) builds a trade and growth model with dynamics through innovation and technology diffusion across countries and sectors. In their model, ideas diffuse with exogenous and heterogeneous speeds across all sectors and countries. In contrast, in our model the speed of diffusion across locations is endogenous and mediated by trade and migration. Our framework departs from this paper's framework as it incorporates spatial growth with forward-looking migration and capital accumulation decisions. Idea diffusion through trade in our paper is also related to other recent frameworks modeling innovation and diffusion of technologies as stochastic processes to study the connection between trade and the diffusion of ideas (e.g., Lucas (2009), Perla, Tonetti, and Waugh (2021), Sampson (2016)).

The process of idea diffusion from migrants in our framework is motivated in part by a growing literature with empirical evidence on knowledge flows resulting from people interactions (e.g., Atkin, Chen, and Popov (2022)), and with empirical evidence on the impact of immigrants on ideas, innovation, and growth in the United States and in other countries (e.g., Kerr (2008), Hunt and Gauthier-Loiselle (2010), Lewis (2011), Akcigit, Grigsby, and Nicholas (2017), Bernstein, Diamond, McQuade, and Pousada (2018), Sequeira, Nunn, and Qian (2019), Arkolakis, Lee, and Peters (2020), Burchardi, Chaney, Hassan, Tarquinio, and Terry (2020), Prato (2021)). There is also recent evidence on how internal migrants impact productivity and other related outcomes in their destination location in countries like China that have experienced large internal migration episodes (e.g., Facchini, Liu, Mayda, and Zhou (2019), Imbert, Seror, Zhang, and Zylberberg (2022)).

Our paper also contributes to a strand of the literature that studies the spatial effects of the rise of China. The role of trade and migration in shaping spatial inequality and productivity in China in the 2000s has been studied through the lens of static spatial general equilibrium models, most notably in Tombe and Zhu (2019) and Fan (2019). We crucially depart from this line of research by studying growth in China in the 1990s and 2000s through the lens of a dynamic spatial growth model. Finally, our paper also relates to other strands of the literature that have pointed to different determinants of the rise of China. Caliendo and Parro (2022) provides a review of the recent literature on the origins of the China shock. More generally, the effects of China's trade expansion on U.S. labor markets as well as other outcomes in different countries has been the focus of an extensive body of literature (e.g., Autor, Dorn, and Hanson (2013), Acemoglu, Autor, Dorn, Hanson, and Price (2016), Pierce and Schott (2016), Caliendo, Dvorkin, and Parro (2019)). Our paper develops a dynamic spatial growth framework to speak about the internal dynamics of spatial growth in China during the 1990s and 2000s.

The rest of the paper is structured as follows. In Section 2 we develop our dynamic spatial growth model. We start by describing the process of idea diffusion for a single economy; we then introduce locations and present the dynamic spatial growth framework. We also characterize the equilibrium properties of the model. Section 3 describes how we take the model to the data. The section discusses data measurement and the data sources used to take the model to the Chinese economy at the province level, discusses our estimation strategy of the relevant elasticities, and describes the method we use to perform counterfactual analysis. Section 4 presents our quantitative results, and Section 5 provides reduced-form evidence on the contribution of idea diffusion through trade and migration to local knowledge. Section 6 concludes. We relegate all proofs, theoretical derivations, and detailed data descriptions to the online appendix.

2 Dynamic Spatial Growth Model

In this section we develop the dynamic spatial growth model. We begin with a description of technology diffusion in a single economy given a general source distribution in Subsection 2.1. We then introduce locations in the framework and describe the demand side of the model—that is, the production and trade structure—in Subsection 2.2. After doing so, we turn to specify the supply of factors in our framework. In Subsection 2.3 we describe the capital accumulation decisions by local landlords, and in Subsection 2.4 we specify the dynamic labor supply decisions by migrants. In Subsection 2.5 we endogenize the idea diffusion process, relate it to migration and trade, and derive the evolution of the stock of knowledge across space. We also define the balanced growth path equilibrium of the economy and establish the existence and uniqueness of the balanced growth path equilibrium.

2.1 Innovation and Idea Diffusion

We start by describing the process of innovation and diffusion that gives rise to the evolution of an economy's stock of knowledge. The building block in our framework is a discrete-time version of Buera and Oberfield (2020). To simplify the exposition, consider a single economy in which there is a continuum of intermediate varieties produced in the unit interval. For each variety, there is a large set of potential producers who have different technologies to produce the good. Each potential producer is characterized by the productivity of her idea, which we denote by q, to produce an intermediate variety. Between time t and time $t+1$, producers interact with other agents in the economy and are exposed to new ideas to produce a variety. The productivity of a new idea might or might not be higher than that of the ideas the producer already has so she only adopts a new idea if the new ideas' productivity is greater than q. Both the number of new ideas and the productivity of them are stochastic, which generates randomness in the usage of the new ideas. In particular, the number of new ideas to which a producer is exposed is stochastic and follows a Poisson distribution.

Each new idea corresponds to a new productivity to produce the variety and is given by zq'^{ρ}. This new idea has two random components: z is the original component of the producer, drawn from an exogenous distribution $H(z)$; and q' is an insight drawn from a source distribution $G_t(q')$ whose evolution we describe subsequently. The stochastic arrival of new ideas generates randomness in the exposure to new ideas that are originated by producers and by new insights. Producers generate new ideas originated from their internal source of ideas, drawn from their own distribution of original ideas. Diffusion is a component that is external to the producer and that allows her to be exposed to the ideas of other producers/sellers in the economy. These ideas diffuse at a rate that is captured by the parameter ρ. In this context, the original component of the producer's ideas can also be interpreted as randomness in the adaptation of insights from others to alternative uses.

To gain tractability, in Assumption 1 we specify the internal distribution of original ideas, the process for the arrival of ideas, and the parametric restrictions required to characterize the evolution of the knowledge frontier over time. We then impose these assumptions, and in Proposition 1 we characterize the frontier of knowledge in the economy and the evolution of the stock of knowledge over time. We present the main results and relegate all the detailed derivations to Online Appendix A. We also refer the reader to Buera and Oberfield (2020) for a continuous-time-version derivation of the equilibrium evolution of technology in the economy.

Assumption 1

a) The internal distribution $H(z)$ of original ideas is Pareto, namely

$$H(z) = 1 - \left(\frac{z}{\bar{z}}\right)^{-\theta},$$

where $\bar{z}$ is the lower bound of the support and $\theta > 1$ is the shape parameter of the distribution.

b) The strength of idea diffusion, $\rho \in [0, 1)$, is strictly less than 1.

c) The number of new ideas that arrive between t and $t+1$ follows a Poisson distribution with mean

$$\Lambda_t = \alpha_t \bar{z}^{-\theta}.$$

d) The source distribution has a sufficiently thin tail so that $\alpha_t \lim_{\bar{z}\to 0} \bar{z}^{-\theta} \left[1 - G_t\left(\left(\frac{q}{\bar{z}}\right)^{\frac{1}{\rho}}\right)\right] = 0.$

In what follows we impose Assumption 1 to solve for the distribution of productivity in the economy. The next proposition presents the result.

Proposition 1 *Under Assumption 1, between t and $t+1$, the probability that the best new idea has a productivity no greater than q—namely, $F_t^{best\ new}(q) = \Pr[$all new ideas are no greater than $q]$— is given by*

$$F_t^{best\ new}(q) = \exp\left(-\alpha_t q^{-\theta} \int_0^\infty x^{\rho\theta} dG_t(x)\right)$$

in the limiting case when $\bar{z} \to 0$.

Proposition 1 shows that the probability distribution of the best new ideas is Fréchet with scale parameter θ and a location parameter determined by $\alpha_t \int_0^\infty x^{\rho\theta} dG_t(x)$. Note that in order to obtain this result there is no need to specify the external source distribution. This is an important result that we will use when we impose more structure over the source distribution. In addition, note that we can use the result of Proposition 1 to characterize the frontier of knowledge and its evolution over time. In particular, we denote by $F_t(q)$ the fraction of varieties whose best producer has productivity no greater than q. In a probabilistic sense, $F_t(q)$ is also the probability that the best productivity for a specific variety is no greater than q at time t. We call this object the *frontier of knowledge*. We characterize the evolution of $F_t(q)$ between t and $t+1$ as new ideas arrive that might have better productivity than the current best ideas. Accordingly, at $t+1$ (see Online Appendix A for the derivation),

$$F_{t+1}(q) = F_0(q) \cdot \prod_{\tau=0}^{t} F_\tau^{best\ new}(q).$$

Proposition 2. *Assume that the initial frontier of knowledge at time 0 follows a* **Fréchet** *distribution given by $F_0(q) = \exp(-A_0 q^{-\theta})$.*

Imposing this assumption, it follows that $F_t(\cdot)$ is Fréchet at any t given by

$$\begin{aligned} F_t(q) &= \exp\left[-\left(A_0 + \sum_{\tau=0}^{t-1} \alpha_\tau \int_0^\infty x^{\rho\theta} dG_\tau(x)\right) q^{-\theta}\right] \\ &= \exp\left(-A_t q^{-\theta}\right), \end{aligned}$$

where the law of motion for the knowledge stock is given by

$$A_{t+1} = A_t + \alpha_t \int_0^\infty x^{\rho\theta} dG_t(x).$$

Proposition 2 establishes two results that we use in subsequent sections. First, the result indicating that at each moment in time the frontier of knowledge follows a Fréchet distribution allows us to specify the production and trade structure in our framework, as we describe in the next section. Second, we can see that both the arrival rate of new ideas α_t and the learning pool $G_t(\cdot)$ matter for the evolution of A_t. Later in the paper, after we describe the economic environment in our framework, we return to discuss how ideas diffuse over space and relate the learning pool $G_t(\cdot)$ to ideas from sellers and from migrants. Finally, note that it can also be shown that asymptotically, in the limit when $t \rightarrow \infty$, that $F_t(\cdot)$ converges to a Fréchet, even without imposing a Fréchet distribution on the initial frontier of knowledge. Hence, this assumption is not strictly needed to obtain this result.

2.2 Production, Factor Demand, and Trade

We now consider a world with N different geographical areas indexed by i and n. At each location i there are heterogeneous and perfectly competitive producers of varieties of intermediate goods.[1] The technology to produce these intermediate goods requires labor and capital, which are the primary factors of production, and material inputs. The efficiency of an intermediate good producer is given by $q_{i,t}$, where we now index efficiencies by location. The output for a producer of an intermediate variety with efficiency $q_{i,t}$ in location i is given by

$$y_{i,t} = q_{i,t} \left(L_{i,t}^{\xi} K_{i,t}^{1-\xi} \right)^{\gamma} M_{i,t}^{1-\gamma},$$

where $L_{i,t}$, $K_{i,t}$, and $M_{i,t}$ are the demands for labor, capital, and material inputs, respectively. The parameters γ and $1-\gamma$ are the shares of value added and material inputs in output, and ξ and $1-\xi$ are the shares of labor and capital in value added, respectively. It follows from the cost minimization problem of the producers that the unit price of an input bundle is given by

$$x_{i,t} = B \left(w_{i,t}^{\xi} r_{i,t}^{1-\xi} \right)^{\gamma} P_{i,t}^{1-\gamma},$$

where $w_{i,t}$, $r_{i,t}$, and $P_{i,t}$ denote the price of labor, rental rate of capital, and the price of materials, respectively, and where B is a constant.[2]

We now use the results from the previous section in which we derived the law of motion of the stock of knowledge in an economy. Firms purchase intermediate goods from the lowest-cost supplier in the world. The frontier of knowledge in each location at each t is described by a

[1] As explained later on, at the beginning of the period producers get insights from sellers and migrants, with randomness in the productivity of those insights for alternative uses in the destination location. At the end of the period, technology to produce a variety can be imitated, and therefore, producers decide to charge a price equal to the marginal cost. Alternatively, we could have assumed producers engage in Bertrand competition so that the lowest-cost supplier of a variety either charge the optimal markup or set a limit price to just undercut the second-lowest cost supplier of the variety. As shown in Bernard, Eaton, Jensen, and Kortum (2003) and Buera and Oberfield (2020), with Bertrand competition aggregate costs are a fraction $\theta/(1+\theta)$ of aggregate revenues in all locations, and under the assumption that profits from local producers are spent domestically, equilibrium conditions are isomorphic to those under perfect competition except for a constant in the price index.

[2] In particular, $B = \left[\xi^{\xi}(1-\xi)^{1-\xi}\right]^{-\gamma} \gamma^{-\gamma} (1-\gamma)^{\gamma-1}$.

Fréchet distribution with shape parameter θ and location-specific scale parameter $A_{i,t}$; namely, $F_{i,t}(q) = \exp(-A_{i,t} q^{-\theta})$.

Shipping goods across locations, from n to i, is subject to iceberg trade costs, $\kappa_{in,t}$, and therefore the cost of purchasing an intermediate variety with efficiency q from n in location i is given by $\kappa_{in,t} x_{n,t}/q$. Hence, we can now follow the Eaton and Kortum (2002) formulation and derive the fraction of goods purchased by location i from location n as (see Online Appendix B.1 for the derivation), which is given by

$$\lambda_{in,t} = \frac{A_{n,t} \left(\kappa_{in,t} x_{n,t}\right)^{-\theta}}{\sum_{h=1}^{N} A_{h,t} \left(\kappa_{ih} x_{h,t}\right)^{-\theta}}. \tag{1}$$

Similarly, we can solve for the price index in location i, which is given by

$$P_{i,t} = T \left(\sum_{n=1}^{N} A_{n,t} \left(\kappa_{in,t} x_{n,t}\right)^{-\theta} \right)^{-1/\theta}, \tag{2}$$

where T is a constant.[3] Given this environment, total expenditure in location i, which we denote by $X_{i,t}$, is given by

$$X_{i,t} = (1-\gamma) \sum_{n=1}^{N} \lambda_{ni,t} X_{n,t} + I_{i,t},$$

which reflects that the total expenditure on goods is firms' expenditure on intermediate goods plus households' expenditure where a household's income is given by $I_{i,t} = w_{i,t} L_{i,t} + r_{i,t} K_{i,t}$. The term $\sum_n \lambda_{ni,t} X_{n,t}$ is the total demand for goods produced in i from all locations. The trade balance condition is given by

$$\sum_{n=1}^{N} \lambda_{in,t} X_{i,t} = \sum_{n=1}^{N} \lambda_{ni,t} X_{n,t},$$

where the left-hand side is the total imports by location i and the right-hand side is the total exports from i (with domestic purchases entering on both sides of the expression). Combining the expenditure condition and the trade balance condition, we obtain that total expenditure in i can be expressed as a function of factor payments:

$$X_{i,t} = \frac{w_{i,t} L_{i,t} + r_{i,t} K_{i,t}}{\gamma}.$$

Finally, the factor market clearing conditions are given by

$$w_{i,t} L_{i,t} = \xi \gamma \sum_{n=1}^{N} \lambda_{ni,t} X_{n,t},$$

[3]Intermediate varieties are aggregated with a constant elasticity of substitution η, and T is a gamma function evaluated in the argument $T = \Gamma\left(1 + (1-\eta)/\theta\right)^{\theta/(1-\eta)}$.

which is the labor market clearing condition, and

$$r_{i,t}K_{i,t} = (1-\xi)\gamma \sum_{n=1}^{N} \lambda_{ni,t} X_{n,t},$$

which is the capital market clearing condition. Combining these expressions with trade balance, we obtain that $w_{i,t}L_{i,t} + r_{i,t}K_{i,t} = \sum_{n=1}^{N} \lambda_{ni,t}(w_{n,t}L_{n,t} + r_{n,t}K_{n,t})$ and using the relative demand for capital and labor, we note that $\frac{w_{i,t}L_{i,t}}{\xi} = \frac{r_{i,t}K_{i,t}}{1-\xi}$. It follows that the labor market clearing condition can be rewritten as

$$w_{i,t}L_{i,t} = \sum_{n=1}^{N} \lambda_{ni,t} w_{n,t} L_{n,t}. \tag{3}$$

2.3 Capital Accumulation Across Locations

We now turn to the supply side of the model. We start by describing capital accumulation decisions across space. At each location, we assume that there are atomistic landowners who consume local goods with logarithm preferences over consumption goods and whose source of income is from renting capital structures.[4] Landowners are forward-looking and seek to maximize the present discounted value of their utility by deciding how much to consume and invest at each moment in time. Landowners are geographically immobile, have access to an investment technology in local capital, and make their investment in units of consumption goods. We follow Kleinman, Liu, and Redding (2021) and interpret capital as buildings and structures that are geographically immobile once installed, and we specify the problem of a landowner in location i as

$$\max_{\{C_{i,t}, K_{i,t+1}\}_{t=0}^{\infty}} U = \sum_{t=0}^{\infty} \beta^t \log(C_{i,t}),$$
$$\text{s.t. } r_{i,t}K_{i,t} = P_{i,t}\left[C_{i,t} + K_{i,t+1} - (1-\delta) K_{i,t}\right] \text{ for all } t,$$

where δ is the depreciation rate and $K_{i,0}$ is taken as given. The solution to this dynamic programming problem can be characterized by the policy functions on consumption and investment,

$$C_{i,t} = (1-\beta)\left[r_{i,t}/P_{i,t} + (1-\delta)\right] K_{i,t},$$
$$K_{i,t+1} = \beta\left[r_{i,t}/P_{i,t} + (1-\delta)\right] K_{i,t}, \tag{4}$$

which give rise to the law of motion of capital accumulation across locations. In Online Appendix B.3 we provide the detailed derivation of these policy functions. Note that since capital structures are accumulated locally and used for local production, the evolution of capital struc-

[4]Our assumption regarding the logarithm preferences of landlords is consistent with the preferences we specify for workers in the next section.

tures in part shapes the evolution of economic activity across space.

Similar to Kleinman, Liu, and Redding (2021), the immobility of landlords allows us to introduce forward-looking capital accumulation decisions in dynamic spatial economies with workers' mobility in a tractable way, and it prevents the number of state variables from increasing exponentially over time.[5]

We now turn to describe the dynamic labor supply decisions made by workers and migrants across locations in the model.

2.4 Dynamic Labor Supply Decisions

There is a continuum of heterogeneous forward-looking workers in the economy. Each worker observes the economic conditions and optimally decides where to locate in each period subject to mobility frictions and idiosyncratic taste shocks. We model this migration decision as a dynamic discrete-choice problem. In particular, workers maximize the present discounted value of their utility by deciding at each moment in time where to live. They supply one unit of labor inelastically at where they live, and they consume given their labor income ($w_{i,t}$) and the local price of goods ($P_{i,t}$). We denote by $U_{i,t}(c_{i,t})= log(c_{i,t})$ the current utility of a worker living in location i, where $c_{i,t} = w_{i,t}/P_{i,t}$. We assume that the decision of where to live the next period is affected by idiosyncratic amenity shocks that vary across locations denoted by $\epsilon_{n,t}$ and by mobility frictions of going from location i to location n, denoted by $m_{in,t}$. The presence of migration costs and idiosyncratic shocks generates a gradual adjustment of labor supply in response to changes in the economic environment.

As a result, the value of a worker in region i at time t is given by

$$v_{i,t} = log(w_{i,t}/P_{i,t}) + \max_{\{n\}_{n=1}^{N}} \{\beta E_t[v_{n,t+1}] - m_{in,t} + \nu\epsilon_{n,t}\}, \tag{5}$$

where β is the discount factor, which is assumed to be the same as the discount factor of landowners.

We assume that the idiosyncratic shocks $\epsilon_{n,t}$ are *i.i.d.* realizations from a Gumbel Type I distribution with dispersion parameter ν. We denote by $E_t[v_{n,t+1}]$ the expectation at time t over the future realizations of the idiosyncratic shocks that shape the continuation value of each location. Using the properties of the Gumbel Type I distribution, we can integrate both sides of equation (5) over $\epsilon_{n,t}$. We then obtain the value of location i for a representative worker in that location at time t, denoted by $V_{i,t} = E_t[v_{i,t}]$. The value of location i is given by

$$V_{i,t} = log(w_{i,t}/P_{i,t}) + \nu log\left(\sum_{n=1}^{N} exp\left(\beta V_{n,t+1} - m_{in,t}\right)^{1/\nu}\right). \tag{6}$$

[5]As a result, this framework can accommodate alternative capital accumulation formulations such as assuming decreasing return to investment, as in Lucas and Prescott (1971) and Hercowitz and Sampson (1991).

We denote by $\mu_{in,t}$ the fraction of workers that moves from location i to location n, which using the properties of the Gumbel Type I distribution can be derived in closed form as

$$\mu_{in,t} = \frac{exp\left(\beta V_{n,t+1} - m_{in,t}\right)^{1/\nu}}{\sum_{h=1}^{N} exp\left(\beta V_{h,t+1} - m_{ih,t}\right)^{1/\nu}}. \tag{7}$$

This equilibrium condition determines the gross migration flows of workers across space (see Online Appendix B.2 for the derivation). It shows that individuals are forward-looking and decide where to supply labor tomorrow by evaluating the relative net future value of each location. The elasticity of the migration flow ($1/\nu$) shapes how changes to migration costs affect migration flows. This expression for gross migration flows determines the evolution of the labor supply at each location i over time. In particular, the supply of workers at location i at time $t+1$ is given by the workers who decide to migrate to location i from all other locations n (including stayers in i) at time t. Therefore, the stock of workers at each location evolves according to

$$L_{i,t+1} = \sum_{n=1}^{N} \mu_{ni,t} L_{n,t}. \tag{8}$$

Having described the demand and supply sides of the model, in the next subsection we return to the idea diffusion process to specify the evolution of the local stock of knowledge across space as a result of trade and migration.

2.5 The Stock of Knowledge and Diffusion with Trade and Migration

We now specify the innovation and diffusion process described in Section 2.1 to allow for migrants and sellers to contribute to the local pool of ideas. To do this, we consider an economy in which producers in location n obtain new insights from two sources. First, producers obtain insights from sellers; namely, ideas from producers in other locations are embedded in imported intermediate varieties. Second, we assume that migrants carry insights with them when they arrive in a new location but the quality of those insights do not directly affect their wages or their migration decisions. The interpretation is that a migrant becomes exposed to the local ideas in their previous location, and then as they move across locations, they randomly meet a local producer. When they meet, the migrant shares ideas from her previous location and provides new insights that can contribute to the local stock of knowledge. As a result, the productivity of a new idea that arrives can be generalized to

$$q = z q_{\ell}^{\rho_{\ell}} q_{m}^{\rho_{m}},$$

where q_{ℓ} is the insight drawn from a source distribution that is shaped by migration and q_m the insight drawn from a source distribution that is shaped by sellers. Note that under this functional form, having both the migrants and the foreign good makes the new insight more productive than only having one of them, provided that migrants arrive from their origin locations with good insights.

The parameters $\rho_\ell, \rho_m \in [0, 1)$ capture the learning intensity from both types of insights (trade and migration) with $\rho_\ell + \rho_m < 1$. After imposing Assumption 1 and following the same steps as in Section 2.1, extending the notation by indexing the location by n, and given the results from Propositions 1 and 2, we obtain that the frontier of knowledge at each location is

$$F_{n,t}^{best\ new}(q) = exp\left(-A_{n,t} q^{-\theta}\right),$$

and the stock of knowledge evolves over time as

$$A_{n,t+1} - A_{n,t} = \alpha_t \int_0^\infty \int_0^\infty \left(q_\ell^{\rho_\ell} q_m^{\rho_m}\right)^\theta dG_{n,t}(q_\ell, q_m).$$

We assume that since q_ℓ is drawn from people and q_m is drawn from goods, they represent two different sources of (independent) ideas. Formally, when a worker from i at the end of period t decides to move to n, she carries with her an insight q_ℓ, which is a random draw from the frontier distribution in i, whose cumulative distribution function is $F_{i,t}(q_\ell)$. At the end time t, in location n, producers randomly meet a worker currently living in n, and the insight from this individual is the insight component of the new idea. Hence,

$$G_{n,t}(q_\ell) = \sum_{i=1}^{N} s_{in,t} F_{i,t}(q_\ell),$$

where $s_{in,t} = \frac{\mu_{in,t} L_{i,t}}{\sum_{h=1}^{N} \mu_{hn,t} L_{h,t}}$ is the share of workers in location n that arrived from i at the end of period t (see the derivation in Online Appendix A.2).

In the case of the source distribution of goods, we assume that there is learning from sellers as in Buera and Oberfield (2020); namely, that diffusion opportunities are randomly drawn from the set of best practices across all goods sold locally. In this way the source distribution $G_{n,t}(q_m)$ is given by the fraction of goods for which the lowest-cost provider of the good to location n is a producer in i with productivity less than or equal to q_m. Under these mechanisms for idea diffusion, we obtain that the difference equation that determines the evolution of the stock of knowledge at each location is given by

$$A_{n,t+1} - A_{n,t} = \alpha_t \Gamma_{\rho_\ell, \rho_m} \underbrace{\left[\sum_{i=1}^{N} s_{in,t} \left(A_{i,t}\right)^{\rho_\ell}\right]}_{people} \underbrace{\left[\sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{A_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}\right]}_{goods}, \tag{9}$$

where $\Gamma_{\rho_\ell, \rho_m}$ is a constant given by $\Gamma(1 - \rho_\ell) \times \Gamma(1 - \rho_m)$ and where $\Gamma(x)$ is gamma function evaluated at x. In Online Appendix A.3 we present more details on the derivation of the law of motion of the stock of knowledge across locations with idea flows from people and goods.

Equilibrium condition (9) is quite intuitive. It shows that the local stock of knowledge evolves over time according to the arrival rate of new ideas α_t, according to how the location is connected and exposed to ideas from migrants, $s_{in,t}$, and according to how open the location is to trade, $\lambda_{ni,t}$. The diffusion of ideas from migrants and sellers is endogenous since both migration and trade patterns are equilibrium objects in our framework. The relative strength of idea diffusion, governed by the diffusion parameters ρ_ℓ and ρ_m, shapes the importance of learning from people or goods. The fact that there are diminishing returns to technological improvement from insights, given that the strength of idea diffusion is less than one, makes it harder to obtain insights that are good enough over time. Hence, if α_t is time-invariant, then as the knowledge frontier evolves over time, the growth rate of the stock of knowledge falls with a limiting value of zero. As a result, as the knowledge frontier evolves, ideas need to arrive faster over time in order to sustain a constant growth rate. This feature is shared with semi-endogenous growth models in Buera and Oberfield (2020), Jones (1995), Kortum (1997), and Atkeson and Burstein (2019). Given this, we make the following assumption about the arrival rate.

Assumption 2 *α_t has constant growth rate g_α given by*

$$\alpha_t = \alpha_0(1+g_\alpha)^t.$$

We now define formally the equilibrium of the dynamic spatial growth model.

Equilibrium *Given an initial distribution of the local stock of knowledge $\{A_{i,0}\}_{i=1}^N$, factor endowments $\{L_{i,0}, K_{i,0}\}_{i=1}^N$, evolution of fundamentals $\{\alpha_0, \kappa_{in,t}, m_{in,t}\}_{i=1,n=1,t=0}^{N,N,\infty}$, and the set of parameters and elasticities $(\rho_\ell, \rho_m, \theta, \nu, \gamma, \xi, \beta)$, the sequential competitive equilibrium of the dynamic spatial growth model is characterized by a sequence of values, factor prices, goods prices, labor allocations, capital stocks, and stock of knowledge, $\{V_{i,t}, w_{i,t}, r_{i,t}, P_{i,t}, L_{i,t}, K_{i,t}, A_{i,t}\}_{i=1,t=0}^{N,\infty}$, that satisfies the equilibrium conditions determined by the bilateral trade shares (1), the equilibrium location prices (2), the labor market clearing condition (3), the capital accumulation condition (4), the location value function (6), the worker gross flow condition (7), the law of motion of labor (8), and the evolution of the stock of knowledge (9).*

In the long run, as the economy evolves over time, it approaches a balanced growth path equilibrium in which all equilibrium variables grow at a constant long-run rate. We now characterize the balanced growth path of the model. We first formally define the balanced growth path. We then express all equilibrium variables in the model relative to their balanced growth rate (what we refer to as the detrended variables) and then show that the equilibrium conditions of the detrended model give rise to a unique solution. Namely, we show that there exists a unique balanced growth path of the dynamic spatial growth model.

Balanced Growth Path. *Along the balanced growth path all equilibrium variables grow at a constant rate. In particular, denote by g_y the growth rate of a generic variable y at the balanced growth path. At the* *balanced growth path the stock of knowledge grows at a rate* $1+g_A = (1+g_\alpha)^{\frac{1}{(1-\rho_l-\rho_m)}}$ *, capital grows at a rate* $1+g_k = (1+g_A)^{\frac{1}{\theta\xi\gamma}}$*, and values grow at a rate* $1+g_v = (1+g_A)^{\frac{1}{\theta\xi\gamma(1-\beta)}}$.

Online Appendix C solves for the equilibrium long-run growth rates of all variables along the balanced growth path. The appendix also shows how to detrend all the equilibrium variables and equilibrium conditions, namely, how to express them relative to their balanced long-run growth. We now show the equilibrium conditions of the detrended model. We refer to each variable with a "~" as the variable relative to its long-run growth. In particular, $\tilde{y}_t = y_t/\left(1+g_y\right)^t$.

The equilibrium conditions of the detrended model are given by

$$\tilde{V}_{i,t} = \beta log\left(1+g_v\right) + log\left(\frac{\tilde{w}_{i,t}}{\tilde{P}_{i,t}}\right) + \nu log\left(\sum_{n=1}^{N} exp\left(\beta\tilde{V}_{n,t+1} - m_{in,t}\right)^{1/\nu}\right), \tag{10}$$

$$\tilde{P}_{i,t} = T\left(\sum_{n=1}^{N}\tilde{A}_{n,t}\left(\kappa_{in,t}\tilde{x}_{n,t}\right)^{-\theta}\right)^{-1/\theta}, \tag{11}$$

$$\tilde{w}_{i,t}L_{i,t} = \sum_{n=1}^{N}\tilde{A}_{i,t}\left(\frac{\kappa_{ni,t}\tilde{x}_{i,t}}{\tilde{P}_{n,t}/T}\right)^{-\theta}\tilde{w}_{n,t}L_{n,t}, \tag{12}$$

$$\tilde{r}_{i,t}\tilde{K}_{i,t} = \sum_{n=1}^{N}\tilde{A}_{i,t}\left(\frac{\kappa_{ni,t}\tilde{x}_{i,t}}{\tilde{P}_{n,t}/T}\right)^{-\theta}\tilde{r}_{n,t}\tilde{K}_{n,t}, \tag{13}$$

$$L_{i,t+1} = \sum_{n=1}^{N}\mu_{ni,t}L_{n,t}, \tag{14}$$

$$\tilde{K}_{i,t+1} = \frac{\beta}{(1+g_k)}\left(\tilde{r}_{i,t}/\tilde{P}_{i,t} + (1-\delta)\right)\tilde{K}_{i,t}, \tag{15}$$

$$\tilde{A}_{n,t+1} - \frac{\tilde{A}_{n,t}}{(1+g_A)} = \frac{\alpha_0\Gamma_{\rho_\ell,\rho_m}}{(1+g_A)}\sum_{i=1}^{N}s_{in,t}\left(\tilde{A}_{i,t}\right)^{\rho_l}\sum_{i=1}^{N}\lambda_{ni,t}\left(\frac{\tilde{A}_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}, \tag{16}$$

where we note that since there is no population growth, employment does not have a long-run growth rate; namely, $\tilde{L}_{n,t} = L_{n,t}$. Since values grow at the same rate in the long run, it follows that $\tilde{\mu}_{ni,t} = \mu_{ni,t}$, as we show in Online Appendix C.

The next proposition establishes the existence and uniqueness of the balanced growth path equilibrium. First it presents the set of equilibrium conditions that determine the equilibrium allocation at the balanced growth path. Note that at the balanced growth path, all the detrended variables are not growing, and as a result the equilibrium variables of the detrended model reach a steady state once the model reaches the balanced growth path. Hence, at the balanced growth path, $\tilde{y}_{t+1} = \tilde{y}_t = \bar{y}$, and it remains constant for all t. We use an upper bar to express the detrended equilibrium variables at the balanced growth path.

Proposition 3 *Given parameters and elasticities $(\rho_\ell, \rho_m, \theta, \nu, \gamma, \xi, \beta)$, there exists a unique (up to scale) solution $\{\bar{w}_i, \bar{P}_i, \bar{L}_i, \bar{\phi}_i, \bar{A}_i\}_{i=1}^N$ that satisfies the equilibrium conditions of the detrended model at the balanced growth path that are given by*

$$\bar{w}_i^{1+\xi\gamma\theta} \bar{P}_i^{\theta(1-\xi\gamma)} \bar{L}_i \bar{A}_i^{-1} = (T\Psi)^{-\theta} \sum_{n=1}^{N} \bar{\kappa}_{ni}^{-\theta} \bar{w}_n \bar{P}_n^{\theta} \bar{L}_n, \tag{17}$$

$$\bar{P}_i^{-\theta} = (T\Psi)^{-\theta} \sum_{n=1}^{N} \bar{\kappa}_{in}^{-\theta} \bar{w}_n^{-\theta\xi\gamma} \bar{P}_n^{-\theta(1-\xi\gamma)} \bar{A}_n, \tag{18}$$

$$\bar{w}_i^{-\frac{\beta}{\nu}} \bar{P}_i^{\frac{\beta}{\nu}} \bar{L}_i \phi_i^{-\beta} \zeta^{-\frac{\beta}{\nu}} = \sum_{n=1}^{N} \widetilde{m}_{ni} \bar{L}_n \bar{\phi}_n^{-1}, \tag{19}$$

$$\bar{\phi}_i = \sum_{n=1}^{N} \widetilde{m}_{in} \zeta^{\frac{\beta}{\nu}} \bar{w}_n^{\frac{\beta}{\nu}} \bar{P}_n^{-\frac{\beta}{\nu}} \bar{\phi}_n^{\beta}, \tag{20}$$

$$\bar{w}_n^{-\frac{\beta}{\nu}} \bar{P}_n^{\frac{\beta}{\nu}-(1-\rho_m)\theta} \bar{\phi}_n^{-\beta} \bar{A}_n \bar{L}_n = \frac{\varpi}{\zeta^{-\frac{\beta}{\nu}}} \sum_{i=1}^{N} \widetilde{m}_{in} \bar{\phi}_i^{-1} \bar{A}_i^{\rho_l} \bar{L}_i \sum_{i=1}^{N} \kappa_{ni}^{-(1-\rho_m)\theta} \bar{w}_i^{-(1-\rho_m)\theta\xi\gamma} \bar{P}_i^{-(1-\rho_m)\theta(1-\xi\gamma)} \bar{A}_i, \tag{21}$$

where $\bar{\phi}_i = \sum_{n=1}^{N} exp\left(\beta \bar{V}_n - \bar{m}_{in}\right)^{1/\nu}$, $\widetilde{m}_{in} \equiv exp\left(\bar{m}_{in}\right)^{-1/\nu}$, $\zeta = (1+g_v)^{\beta}$, and where ϖ is a constant.[6]

Proposition 3 establishes that the model has a unique balanced growth path. Online Appendix D presents the proof of this result, which extends the results of Kleinman, Liu, and Redding (2021). We show that the spectral radius of the matrix of elasticities of the non-linear system at the balanced growth path is equal to one, which establishes the uniqueness of the balanced growth path equilibrium in our spatial growth model and the unique steady state in the detrended model up to a normalization.

In the proof we solve for the six eigenvalues that characterize the system of equilibrium conditions. The eigenvalues are $(1, 1, \frac{-b\pm\sqrt{b^2-4ac}}{2a}, 0, \rho_\ell + \rho_m)$ where $a = \beta + \nu + \theta\gamma\nu\xi$, $b = -\nu(1 + \beta - \gamma\xi(1 + \theta(1 - \beta)))$, $c = \beta(\nu - 1 - \gamma\xi\nu(1 + \theta))$. To gain further intuition of this result, Online Appendix D presents a series of partial results that helps us solve for the general model. For example, we present results for a version of the model with no idea flows. In this case we show that there are four eigenvalues, which are given by $(1, 1, \frac{-b\pm\sqrt{b^2-4ac}}{2a})$. We then consider the case of an economy with idea flows from sellers. We show that the equilibrium eigenvalues are $(1, 1, \frac{-b\pm\sqrt{b^2-4ac}}{2a}, \rho_m)$, where the additional eigenvalue compared to the model with no idea flows is exactly given by ρ_m, the strength of idea flows from trade. Similarly, in a model with only idea flows from migration, one obtains that the new eigenvalue is given by ρ_ℓ, the strength of idea flows from migration. Finally, the appendix also presents the results of the general model.

[6] In particular, $\varpi = \frac{\alpha_0 \Gamma_\rho (T\Psi)^{-(1-\rho_m)\theta}}{g_A}$, with $\Psi = B\left(\frac{(1-\xi)}{\xi}\right)^{(1-\xi)\gamma} \left(\frac{((1+g_k)-\beta(1-\delta))\xi}{(1-\xi)\beta}\right)^{(1-\xi)\gamma}$.

We now turn to quantitatively study the importance of our mechanisms to explain aggregate and spatial growth. To do so, we look at an economy that is in transition, with heterogenous locations in terms of stock of knowledge, initial supply of capital and supply of labor, and exposure to international trade. In addition, we consider internal migration of workers, noting that migration is costly and that migrants might have access to a given amount of amenities at their destination. In particular, we apply our dynamic spatial growth framework to China, an economy that features such spatial heterogeneity, and we study how, given a initial set of conditions, aggregate and spatial development is shaped by these initial conditions. We also study the roles of internal migration and international trade in spatial and aggregate growth.

3 Quantitative Analysis

During the 1990s and far into the 2000s, China experienced fast economic growth, considerable capital accumulation, shifts in the distribution of economic activity and factors of production across space, increased productivity, and trade openness. Caliendo and Parro (2022) reviews recent literature that describes the macroeconomic performance of China during the 1990s and 2000s and the different factors that contributed to China's economic growth.

We now turn to study spatial growth in China in the 1990s and 2000s through the lens of our dynamic spatial growth model developed in the previous section. We take the model to year 1990 in a world composed of 30 Chinese provinces and a constructed rest of the world. In doing so, we use migration, production, and value added data. We also use trade data between provinces and the rest of the world. Importantly in the case of China, where there are well-defined mobility frictions across provinces, we condition gross migration flows across provinces by Hukou status. To understand how the Hukou system works, think about a province-level "passport" that identifies an individual based on their province of origin and restricts non-locals' access to certain amenities.

Accordingly, in the quantitative analysis we extend our framework to take into account these considerations. In particular, we allow for workers with different Hukou statuses to value locations differently, as Hukou restrictions give them access to different amounts of amenities, and we also allow workers to face different mobility restrictions. In equilibrium, this implies different mobility rates across provinces for individuals with different Hukou status that we discipline in the data. Hence, the equilibrium conditions of the dynamic labor supply decisions of workers are now given by

$$V_{i,t}^{H} = log(\psi_{i}^{H} w_{i,t}/P_{i,t}) + \nu log\left(\sum_{n=1}^{N} exp\left(\beta V_{n,t+1}^{H} - m_{in,t}^{H}\right)^{1/\nu}\right), \tag{22}$$

$$\mu^{H}_{in,t} = \frac{exp\left(\beta V^{H}_{n,t+1} - m^{H}_{in,t}\right)^{1/\nu}}{\sum_{g=1}^{N} exp\left(\beta V^{H}_{g,t+1} - m^{H}_{ig,t}\right)^{1/\nu}}, \tag{23}$$

$$L_{i,t+1} = \sum_{H}\sum_{n=1}^{N} \mu^{H}_{ni,t} L^{H}_{n,t}, \tag{24}$$

where the H index denotes Hukou status and ψ_i^H is the amenity parameter of location i for an individual with Hukou status H. Once in the same location, workers with different Hukou status consume the same basket of goods and earn the same real wages although their levels of utility are different because they have access to different amenities. In this way, we aim to capture a characteristic of this economy in transition: that is, that migrants to a given province registered in a different province have access to different sets of amenities, face different mobility costs, and as a result, make different migration decisions compared with migrants registered in the destination province. We later provide some descriptive evidence of the importance on two-way migration across provinces in China in part due to the Hukou restrictions.

We now proceed to describe the data sources we use in our quantitative analysis. In Online Appendix G we describe in greater detail the data sources and data construction.

3.1 Data

As we subsequently show, to bring the model to the data, we need data across provinces in China and for the rest of the world on bilateral trade shares $\lambda_{in,t}$, total expenditure $X_{i,t}$, value added $w_{i,t}L_{i,t} + r_{i,t}K_{i,t}$, the distribution of employment $L_{i,t}$, and migration flows across provinces $\mu_{in,t}$, conditional on Hukou type. We also need to compute the share of value added in gross output γ, the share of labor in value added ξ, and the initial capital stock $K_{i,0}$. In addition, we need estimates of the trade elasticity θ, the migration elasticity $1/\nu$, the discount factor β, and the depreciation rate δ. We later describe how we discipline the elasticities that govern innovation and idea diffusion $(\alpha_0, \rho_l, \rho_m)$.

We consider a model in which each period represents five years. Hence, we use a discount factor β of 0.86, equivalent to an annual discount factor of 0.97, which implies a yearly interest rate of roughly 4 percent. The trade elasticity $\theta = 4.55$ is obtained from Caliendo and Parro (2015). We set a migration elasticity of $1/\nu = 0.15$, which is the value estimated by Cruz (2021) for a five-year period in a sample of developing countries. We set a depreciation rate $(1-\delta) = 0.95^5$, which corresponds to an annual depreciation rate of 5 percent. We compute the values of $\gamma = 0.38$ and $\xi = 0.54$, which correspond to the parameter values for the year 1990 from the world's aggregates in the Eora multi-region input-output table. Finally, we set a value of $g_\alpha = 0.013$ that matches the long-term productivity growth in the U.S. economy that during the great moderation period in the 1990s arguably was in a balanced growth path.

3.1.1 Gross Migration Flows

We obtain five-year mobility rates across provinces in China from the 1 percent sample of the 1990 census from IPUMS. The census data contains both the location (province) in 1990 and the location (province) five years ago. We take the working-age (15-64 years old) population as our sample. Furthermore, we keep respondents who are actively employed in 1990. To check the representativeness of our sample, we compute the employment share of each province out of the nationwide employment, and we compare it to the data counterpart provided in the 1991 China Statistics Yearbook.

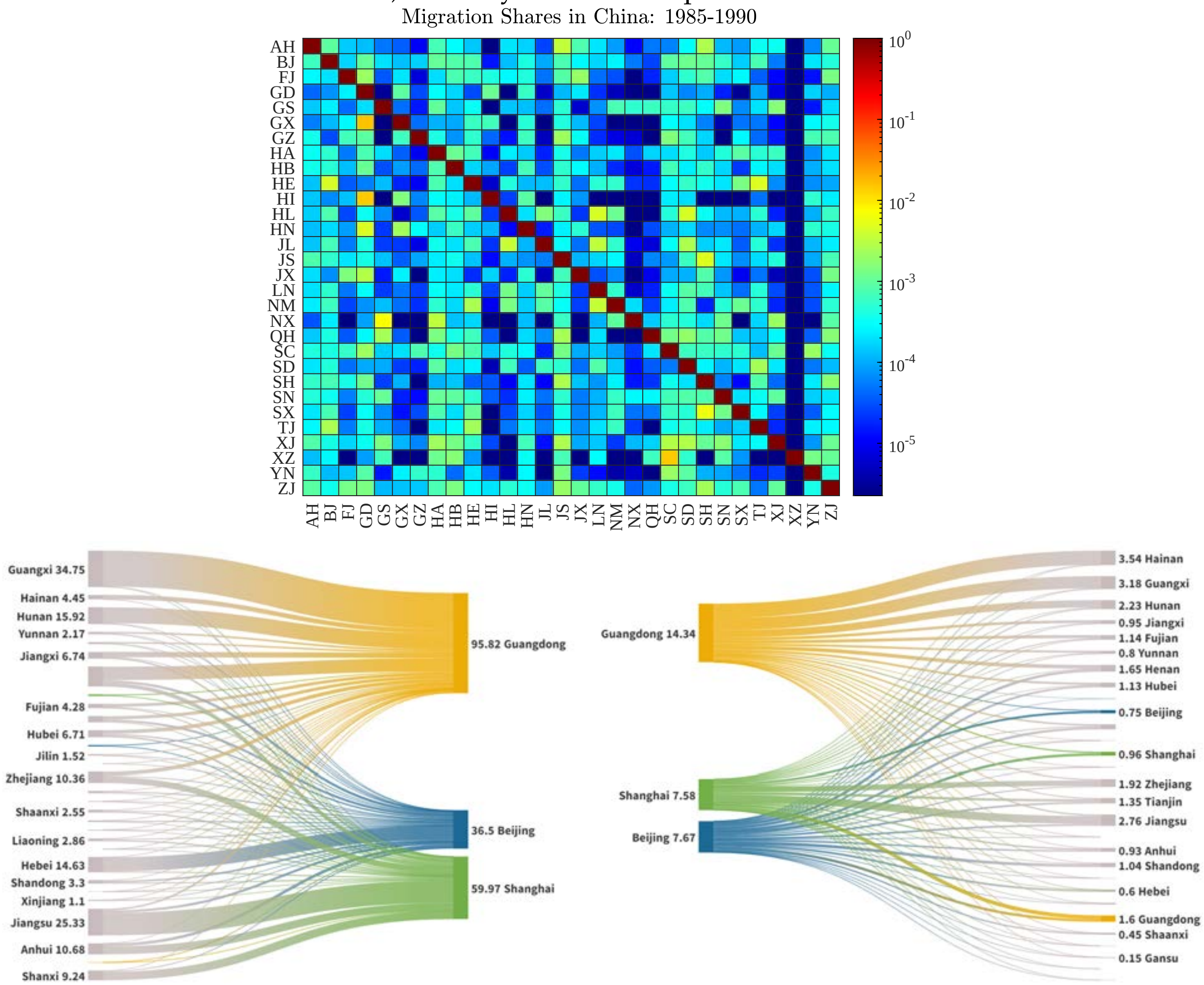
To condition the gross flows on Hukou type, we proceed as follows. We use information from the 1990 census on the status and nature of registration. In particular, if the individual has the status "residing and registered here", we use the current location in 1990 as the registration location. If the individual has the category "residing here over 1 year, but registered elsewhere", "living here less than 1 year and absent from registration place over 1 year", or "living here with registration unsettled", we use the person's location in 1985 as the registration location; otherwise, the individual was living abroad, and we drop such observations (less than 0.02 percent of the observations). For those who registered non-locally yet resided in the same province in 1985 and 1990, we assign their registration location with a probability given by the immigration share from that location.

As an illustration of the mobility patterns across provinces in China, Figure 1 presents the five-year mobility flows across provinces. In the upper panel we present a heat map with the migration shares across all provinces in China. We can see the heterogeneity of mobility patterns across provinces. As expected, the larger flows are stayers (the diagonal in the heat map); however, we see the importance of two-way migration across provinces and the heterogeneity in the number of origin provinces for each given destination province.

To see more clearly these patterns, consider the bottom panels of the figure, which display the mobility flows from all provinces to Beijing, Shanghai, and Guangdong (left-hand panel) and from these provinces to the rest of the provinces in China (right-hand panel). Origin provinces are on the left axis and destination provinces are on the right axis, and a thicker line in the figure means a larger flow. As we describe in the next section, these three provinces are the ones with higher initial measured productivity, and as expected, in the left-hand panel we see how they receive migrants from all provinces in China. In the right-hand panel, we also observe how migrants move from these high-productivity places to the rest of China, which is an indication of the importance of return migration in China due in part to the Hukou restrictions as well as how return migrants diffuse knowledge from high-productivity places.

Figure 1: Mobility across provinces in China (1985-1990)

a) Mobility flows across provinces



Note: The figure presents the mobility flows across provinces in China. The upper panel presents a heat map with the migration shares across all provinces in China, where the y-axis shows the origin provinces and the x-axis presents the destination provinces. The lower panels display the mobility flows (in 10,000 people) for the selected provinces, where the left axis presents the origin provinces and the right axis shows the destination provinces.

3.1.2 Trade and Production Data

We obtain export and import data between Chinese provinces and the rest of the world from the China Compendium of Statistics, 1949-2008. We also obtain GDP and employment data across provinces from the same source. The GDP for the rest of the world is obtained from the Penn World Table 10.0 (PWT). The PWT reports real GDP at constant 2017 national prices; hence, we convert real GDP for the rest of the world to 1990 prices using the world GDP deflator from the World Bank's World Development Indicators. To estimate the series of capital stock across provinces, we follow Shan (2008) and apply the perpetual inventory method, using fixed capital formation from the China Compendium of Statistics as the measure of investment and estimates of capital stocks at a base year from Young (2003). For the rest of the world, we obtain capital stock at constant 2017 national prices from the PWT, which we convert to 1990 prices using GDP deflators from the same source. Using our constructed series of capital stock and equation (4), we obtain the initial real rental rates across locations.

Finally, we point out that in the quantitative analysis we abstract from trade across provinces and sectoral heterogeneity given the lack of data along these dimensions in the Chinese statistics for the 1990s and even for more recent years. Still, mobility across provinces in part captures the mobility of workers between sectors (e.g, agriculture, non-agriculture) given the very uneven distribution of economic activity in China. The focus of our quantitative analysis on the role of local idea diffusion through internal migration and global idea diffusion through international trade is guided by the existing international trade data between China and the rest of the world that we described in this subsection.[7]

3.2 Initial Stock of Knowledge

To estimate the initial stock of knowledge across locations, we start with the definition of real GDP. In our model, real GDP in location n at $t = 0$ is given by

$$Real\ GDP_{n,0} = \frac{w_{n,0}L_{n,0} + r_{n,0}K_{n,0}}{P_{n,0}} = (\Upsilon)^{-\frac{1}{\gamma\theta}} \left(A_{n,0}/\lambda_{nn,0}\right)^{\frac{1}{\gamma\theta}} \left(K_{n,0}\right)^{(1-\xi)} \left(L_{n,0}\right)^{\xi}, \tag{25}$$

where $\Upsilon = (BT)^{\theta} (1-\xi)^{(1-\xi)\gamma\theta} (\xi)^{\xi\gamma\theta}$.[8] Real GDP in our model is determined by factor accumulation (capital, labor) and by measured productivity. In particular, measured productivity is captured by the term $\left(\frac{A_{n,0}}{\lambda_{nn,0}\Upsilon}\right)^{\frac{1}{\gamma\theta}}$. It has two main components: fundamental productivity $A_{n,o}$, and trade openness captured by the inverse of the domestic expenditure share $\lambda_{nn,0}$. The intuition is that in a closed economy— namely, when $\lambda_{nn,0} = 1$—measured productivity is the same as fundamental productivity $A_{n,o}$, which is the average efficiency of the set of goods produced and consumed in n. In an open economy, firms purchase a fraction of goods from abroad and produce only that set of goods of which they are the lowest-cost supplier in the world. Hence, a smaller domestic expenditure share $\lambda_{nn,0}$ results in firms in n producing a smaller set of goods with higher marginal efficiency.

Therefore, inverting equation (25), and solving for fundamental productivity $A_{n,0}$, we obtain

$$A_{n,0} = \Upsilon \left(\frac{Real\ GDP_{n,0}}{\left(K_{n,0}\right)^{1-\xi} \left(L_{n,0}\right)^{\xi}} \right)^{\gamma\theta} \lambda_{nn,0}. \tag{26}$$

Using the data described in the previous subsection, we compute the initial stock of knowl-

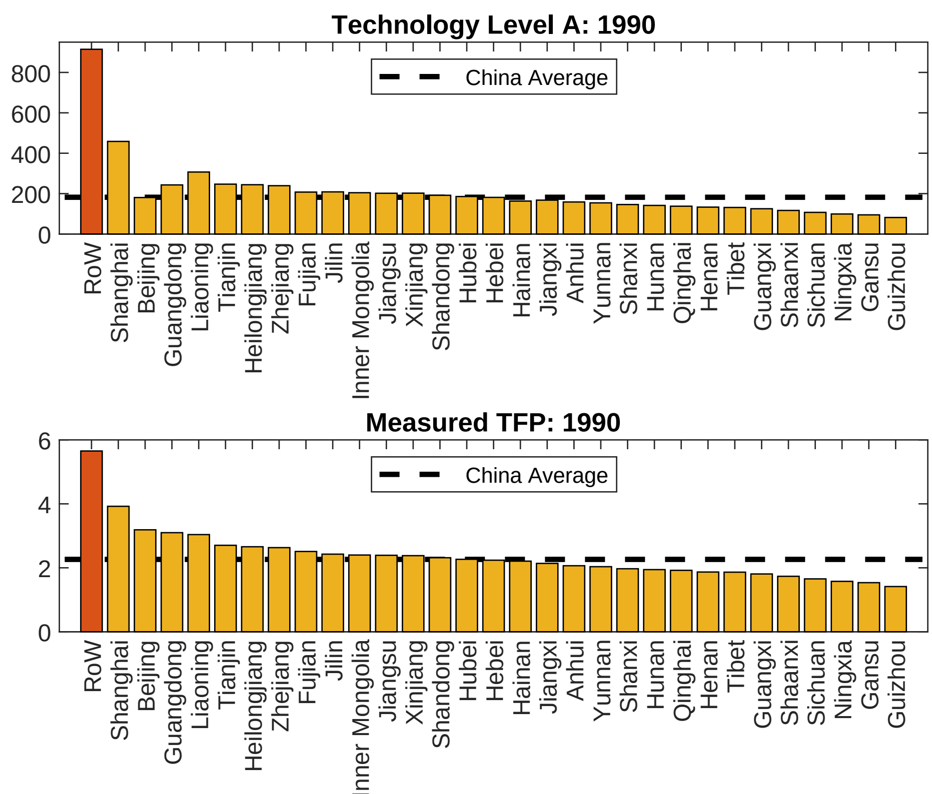
[7] Previous work that has incorporated internal trade across provinces has imputed these data in different ways (e.g., Tombe and Zhu (2019), Poncet (2003)).

[8] See Online Appendix G.1 for the details of this derivation.

edge across provinces in China as well as for the rest of the world.[9] Figure 2 presents the initial stock of knowledge (year 1990) across locations. In the upper panel, we see that the 1990 stock of knowledge for provinces in China is smaller than that for the rest of the world. Across provinces in China, the initial stock of knowledge is very heterogeneous, with Shanghai, Liaoning, and Guangdong being the top three provinces with the highest initial stocks of knowledge, and Gansu, Guizhou, and Ningxia the bottom three provinces. The bottom panel presents the 1990 measured productivity across locations, which corrects for the impact of trade as previously explained. Again we observe that the rest of the world has a higher measured productivity in 1990 than the provinces in China. Measured productivity is also heterogeneous across provinces, and we can see that the heterogeneous levels of trade openness have some impact on the ranking of measured productivities. In particular, in terms of measured productivity, Shanghai, Beijing, and Guangdong are the top three provinces, whereas Gansu, Guizhou, and Ningxia are the bottom three provinces.

Figure 2: Initial stock of knowledge and measured productivity across locations (1990)

a) Initial stock of knowledge



Note: The figures present the initial stock of knowledge (upper panel), computed as described in this section, and measured TFP (bottom panel), computed as $\left(\frac{A_{n,0}/\lambda_{nn,0}}{Y}\right)^{\frac{1}{\gamma\theta}}$.

[9] We set a value of $\eta = 2$ in the gamma function in equation (25).

3.3 Estimation of Idea Diffusion from Trade and Migration

In our dynamic spatial growth model, three parameters discipline productivity growth and idea diffusion across locations: the strength of idea diffusion from sellers ρ_m, the strength of idea diffusion from migrants ρ_l, and the arrival rate of insights α_0. There are no benchmark values for these parameters in the literature that we can use as points of comparison. Buera and Oberfield (2020) present an estimate of ρ_m by using aggregate cross-country data and obtain a value of $\rho_m = 0.6$, and calculate the arrival rate of ideas as the residual needed to explain the evolution of TFP backed out by their calculation. To discipline these parameters in the our dynamic spatial model, we proceed as follows.

We first measure fundamental productivity, $A_{n,t}$, by geography for different periods of time. We do this using equation (26), as described in Section 3.2. It is important to emphasize that our estimated fundamental productivities for the various periods of time are cross-sectional measures; we do not impose any structure on how each of these measures might be related over time. After obtaining cross-sectional estimates of fundamental productivity, we then create moment conditions related to the evolution of fundamental productivity over time and compute the same moments using the model-implied fundamental productivity using equation (9). We then use the GMM to estimate the parameters of interest following Hansen and Singleton (1982) and Newey (1985). One key advantage of this method is that we do not need to make assumptions about the statistical distribution of the data. Instead, we calibrate the parameters to match moment conditions.

We denote by $\Theta \equiv (\alpha_0, \rho_m, \rho_l)'$ the set of parameters that we want to estimate. We need at least three moments to estimate these parameters. We define the moment conditions or orthogonality conditions as $g_N\left(\hat{\Theta}\right) \equiv (1/N)\, h\left(\varpi_t, \hat{\Theta}\right)$, where $h\left(\varpi_t, \hat{\Theta}\right)$ is a vector of moment conditions and $\varpi_t \equiv (\lambda_{in,t}, s_{ni,t}, A_{i,t}, A_{i,t+1})'$ is the vector containing all the available information used for the estimation. We use fundamental productivity estimates for the period 1990-2000 and five moment conditions: the first moment is the average change in fundamental productivity levels across locations; the second moment is the average growth rate in fundamental productivities; the third moment is the variance in the time changes in fundamental productivity levels; the fourth moment is the covariance between the initial fundamental productivities and the change in fundamental productivity levels; and the fifth moment is the covariance between the initial fundamental productivities and the growth rate in fundamental productivities.

To provide further intuition on how these five moments help identify the innovation and diffusion parameters, we note that the first two moments help us identify α_0 since the initial arrival rate of ideas scales up productivity everywhere. The third moment helps us separate α_0 from the diffusion parameters ρ_m and ρ_l since they provide information about how heterogeneity in trade openness and mobility flows result in cross-province variations in the stock of knowledge over time. The last two moments provide information to disentangle ρ_m from ρ_l. The intuition is that provinces with a higher initial stock of knowledge tend to be more open to trade and therefore benefit more from the global diffusion of ideas from the rest of the world. Hence, ideas from sellers tend to generate a positive covariance between the initial stock of knowledge and subsequent changes in productivity. On the other hand, ideas from people are not necessarily associated with a positive covariance; this depends on whether locations receive migrants from places with relatively good insights.

GMM searches for the parameters that minimize all moment conditions by solving the following optimization problem,

$$\hat{\Theta} = \arg\min : g_N\left(\hat{\Theta}\right)' W g_N\left(\hat{\Theta}\right), \tag{27}$$

where W is a weighting matrix that weights how different linear combinations of moments account for the data. The weighting matrix is a function of the parameters to be estimated, so a priori we do not know the appropriate weighting matrix. The identity matrix is thus used in the first step. The parameters are then estimated following an iterative process. We start by solving first the minimization problem setting $W = I$, and after that, we construct the weighting matrix with the estimated parameters and solve the problem again until the estimation is approximately equal to the one from the previous iteration. Hansen (1982) shows that the optimal weighting matrix is given by the inverse of spectral density at frequency zero of the error terms, the inverse of the long-run variance-covariance of $h\left(\varpi, \hat{\Theta}\right)$. We use the Newey-West correction to estimate the long-run variance-covariance. Finally, in our GMM estimation we allow for an error term that together with α_0 captures the effects of determinants of the evolution of knowledge other than ideas from goods and ideas from people. In Online Appendix G.2 we display the empirical moment conditions and the model-implied moments predicted by the evolution of fundamental productivity using equation (9). Using this procedure, we obtain our preferred estimates of $\rho_l = 0.2$, $\rho_m = 0.61$ and $\alpha_0 = 0.18$.

3.4 Computing Counterfactuals

To compute the dynamic spatial growth model, we apply dynamic-hat algebra techniques developed in Caliendo, Dvorkin, and Parro (2019) and show that by expressing the equilibrium conditions in relative time differences, we are able to compute the model without needing to estimate the exogenous fundamentals of the economy or assuming that the economy is in the balanced growth path at the initial period. The intuition is that solving the model in relative time differences requires conditioning the model on observable allocations, which contain all the information about the fundamentals of the economy, and matching the cross-section of the actual economy at the initial year that does not need to be in a balanced growth path.

In particular, let us define $\hat{y}_{t+1}$ as the time difference in the detrended variable $\tilde{y}$; namely, $\hat{y}_{t+1} = (\tilde{y}_{t+1}/\tilde{y}_t)$. The equilibrium conditions in time differences of the detrended system are therefore given by (for simplicity, we omit the Hukou index in what follows)

$$\log(\hat{u}_{i,t+1}) = \log\left(\hat{w}_{i,t+1}/\hat{P}_{i,t+1}\right) + \nu \log\left(\sum_{n=1}^{N} \mu_{in,t}\left(\hat{u}_{n,t+2}\right)^{\beta/\nu}\left(\hat{m}_{in,t+1}\right)^{-1/\nu}\right), \tag{28}$$

$$\mu_{in,t+1} = \frac{\mu_{in,t}\left(\hat{u}_{n,t+2}\right)^{\beta/\nu}\left(\hat{m}_{in,t+1}\right)^{-1/\nu}}{\sum_{h=1}^{N}\mu_{ih,t}\left(\hat{u}_{h,t+2}\right)^{\beta/\nu}\left(\hat{m}_{ih,t+1}\right)^{-1/\nu}}, \tag{29}$$

$$L_{i,t+1} = \sum_{n=1}^{N} \mu_{ni,t} L_{n,t}, \tag{30}$$

$$\hat{x}_{i,t} = \left(\hat{w}_{i,t}^{\xi}\hat{r}_{i,t}^{1-\xi}\right)^{\gamma}\hat{P}_{i,t}^{1-\gamma}, \tag{31}$$

$$\hat{P}_{i,t+1} = \left(\sum_{n=1}^{N}\lambda_{in,t}\hat{A}_{n,t+1}\left(\hat{\kappa}_{in,t+1}\hat{x}_{n,t+1}\right)^{-\theta}\right)^{-1/\theta}, \tag{32}$$

$$\lambda_{in,t+1} = \lambda_{in,t}\hat{A}_{n,t+1}\left(\frac{\hat{\kappa}_{in,t+1}\hat{x}_{n,t+1}}{\hat{P}_{i,t+1}}\right)^{-\theta}, \tag{33}$$

$$\hat{w}_{i,t+1}\hat{L}_{i,t+1} = \frac{1}{\tilde{w}_{i,t}L_{i,t}}\sum_{n=1}^{N}\lambda_{ni,t+1}\hat{w}_{n,t+1}\hat{L}_{n,t+1}\tilde{w}_{n,t}L_{n,t}, \tag{34}$$

$$\tilde{K}_{i,t+1} = \frac{\beta}{(1+g_k)}\tilde{R}_{i,t}\tilde{K}_{i,t}, \tag{35}$$

$$\tilde{R}_{i,t+1} = 1-\delta + \frac{\hat{w}_{i,t+1}\hat{L}_{i,t+1}}{\hat{P}_{i,t+1}\hat{K}_{i,t+1}}\left[\tilde{R}_{i,t}-(1-\delta)\right], \tag{36}$$

$$\hat{A}_{n,t+1} = \frac{1}{(1+g_A)} + \frac{\alpha_0\Gamma_\rho}{\tilde{A}_{n,t}(1+g_A)}\sum_{i=1}^{N}s_{in,t}\left(\tilde{A}_{i,t}\right)^{\rho_\ell}\left[\sum_{i=1}^{N}\lambda_{ni,t}\left(\frac{\tilde{A}_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}\right], \tag{37}$$

where $\hat{u}_{i,t+1} = \exp(\tilde{V}_{i,t+1} - \tilde{V}_{i,t})$, $\hat{m}_{in,t+1} = \exp(m_{in,t+1} - m_{in,t})$, and $\tilde{R}_{i,t} = \tilde{r}_{i,t}/\tilde{P}_{i,t} + (1-\delta)$. Note that $L_{n,t} = \tilde{L}_{n,t}$ and $\mu_{in,t} = \tilde{\mu}_{in,t}$, as previously discussed, and also note that $\lambda_{in,t} = \tilde{\lambda}_{in,t}$. In Online Appendix E we derive these equilibrium conditions.

In the detrended balanced growth path, $\hat{A}_n = 1$, and therefore $\hat{y} = 1$ for all variables $\tilde{y}$. We use this property of the detrended model to develop an algorithm to compute counterfactuals in the dynamic spatial growth model, which is described in Online Appendix F. In addition, as the system of equations in time differences shows, solving the model in relative time differences requires conditioning the model on the initial observable allocations $\lambda_{in,0}$, $\tilde{w}_{i,0}L_{i,0} + \tilde{r}_{i,t}\tilde{K}_{i,0}$, $L_{i,0}$, $\mu_{in,0}$, and $\tilde{K}_{i,0}$, and elasticities θ, ν, β, δ, ρ_ℓ, ρ_m, and α_0. The previous sections have described our process for collecting these initial allocations and disciplining the parameters and elasticities in our spatial growth framework.

4 Mechanics of Spatial Growth in China

In this section we describe our quantitative analysis of the mechanics of spatial growth in China. In Subsection 4.1 we study quantitatively the role of initial conditions in shaping spatial and aggregate development in China in the 1990s and 2000s. Subsection 4.1.1 focuses on aggregate growth for China, Subsection 4.1.2 describes how initial conditions shaped the distribution of economic activity across space in China during the 1990s and 2000s, and Subsection 4.1.3 discusses the process of spatial growth in China. In Subsection 4.1.4 we study the role of the initial distribution of fundamentals in shaping subsequent spatial growth in China. Finally, in Subsection 4.2 we explore quantitatively the effects of changes in international trade costs during the period 1990-2010, and the reduction in migration restrictions in part due to the Hukou registration system.

4.1 Role of Initial Conditions

We first use our framework to study how initial conditions shaped spatial growth in China in the 1990s and 2000s. To do so, as we described before, we take the model to the data in the year 1990. The model does not assume that the economy is in a balanced growth path in the initial year since the model is taken to the data in the initial period conditioning on observable allocations. We then compute the economy with 1990 fundamentals and the endogenous evolution of productivity; namely, we answer the counterfactual question: How would the provinces in China and the rest of the world have looked if fundamentals (trade and migration costs) would have stayed at their 1990 levels and the changes in the stock of knowledge operate due to the mechanisms in the spatial growth model?

4.1.1 Aggregate Growth in China

We start by describing aggregate growth in China. Table 1 presents the annual real GDP growth in China for different time frames over the period 1990-2020 in the data as well as the real GDP growth over the same periods if fundamentals (trade and migration costs) would have stayed constant at the 1990 levels. We also quantify the importance of idea flows from migration and trade and from capital accumulation to aggregate growth in China.

Comparing the actual real GDP growth and that under the initial 1990 conditions in the first two rows of the table, we find that initial conditions play an important role in explaining subsequent growth in China during the 1990s and 2000s. As discussed in the review by Caliendo and Parro (2022), many reforms in China that involve changes to trade and industrial policy took place before the 1990s. In our model these reforms are captured by the initial conditions. As previously described, our methodology does not assume that the economy is in a balanced growth path at the initial period; the framework is taken to the actual data in 1990, and therefore, the actual initial allocations contain information about fundamentals and policies in the Chinese economy and the rest of the world up to that year. We find an important role of these initial conditions in aggregate growth in China in the decades after 1990.

Table 1: Annual GDP Growth Rate

	90-95	90-00	90-05	90-10	90-15	90-20
Actual GDP growth	12.3%	10.4%	10.2%	10.5%	9.9%	9.3%
With fundamentals in 1990	10.6%	10.1%	9.6%	9.2%	8.9%	8.6%
W/o ideas from people ($\rho_l = 0$)	8.7%	8.0%	7.4%	6.9%	6.5%	6.2%
W/o ideas from goods ($\rho_m = 0$)	6.4%	5.4%	4.7%	4.1%	3.7%	3.4%
W/o capital accumulation	5.0%	4.9%	4.7%	4.6%	4.5%	4.5%
W/o idea diffusion ($\rho_l = 0$, $\rho_m = 0$)	6.1%	5.1%	4.4%	3.8%	3.4%	3.0%

Note: Aggregate real GDP growth was obtained from the World Development Indicators. GDP growth with 1990 fundamentals was computed by solving the dynamic spatial growth model with constant fundamentals. The growth rate without idea flows from people was obtained by computing the model with $\rho_l = 0$, and the growth rate without idea flows from goods was obtained by computing the model with $\rho_m = 0$. The second to last row presents the aggregate GDP growth in the absence of capital accumulation. The last row presents the aggregate growth with capital accumulation and no idea diffusion, obtained by computing the model with $\rho_l = 0$ and $\rho_m = 0$.

In the next two rows of the table, we evaluate the contribution of idea flows from people and from goods to aggregate growth in China. In the third row, we compute the model assuming $\rho_l = 0$; namely, that productivity evolves endogenously only due to idea flows from goods. We find that without idea diffusion from people, aggregate growth in China would have been smaller but still important. In the fourth row of the table, we quantify aggregate growth in China with idea diffusion from people only; namely, in a model with $\rho_m = 0$. We find that without idea flows from goods, aggregate growth in China would have been even smaller. Intuitively, two factors explain the larger importance from idea flows from goods for aggregate growth in China. First, as described in Section 3.2, the initial stock of knowledge across provinces in China is lower than it is in the rest of the world; hence, international trade makes an important contribution to growth through the diffusion of good ideas from the rest of the world to all provinces in China. At the same time, the contribution of idea flows from people can have offsetting effects on growth since return migration from high-productivity places adds to the stock of knowledge in the destination province but receiving migrants from low-productivity locations slows down the process of knowledge accumulation. As we described in Subsection 3.1.1, both are relevant to the mobility patterns across provinces in China. Second, the estimated elasticity that governs the diffusion of ideas from people is smaller than the one that disciplines the diffusion of ideas from goods, which explains in part this differential contribution to aggregate growth.

In the second to last row of the table, we quantify the importance of capital accumulation for aggregate growth in China. We find that in a model without capital accumulation, initial conditions in 1990 would have resulted in aggregate growth of around half in subsequent decades. In the last row of the table, we compute the aggregate growth with capital accumulation and no idea diffusion. Comparing the last two rows of the table, we can see that capital accumulation played a more important role than idea diffusion as an engine for aggregate growth in the early 1990s, but idea diffusion became more important over time. In the absence of changes in fundamentals, capital accumulation had a relatively stable contribution to aggregate growth. However, as knowledge diffused and locations increased their stock of knowledge, people contributed with better insights in their locations and in other locations when moving, and provinces more opened to trade also benefited more from better global insights, increasing the role of idea diffusion in aggregate growth. We next turn to dig into the process of spatial development that shape the aggregate growth effects described in this section.

4.1.2 Regional Distribution of Economic Activity

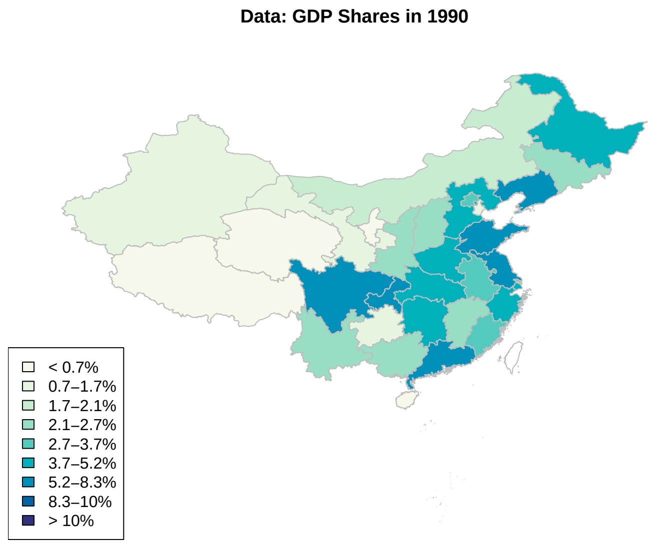
In this subsection, we describe the initial distribution of economic activity in China and its evolution in subsequent decades. We also explore the role of initial conditions in shaping the distribution of economic activity in China across the 1990s and 2000s.

Figure 3 presents the actual GDP shares across provinces in the year 1990 and the GDP shares twenty years later in 2010. The left column of the bottom panels shows the actual GDP shares across provinces in China in 2010, and the right panel presents the predicted GDP shares in 2010 under the initial conditions.

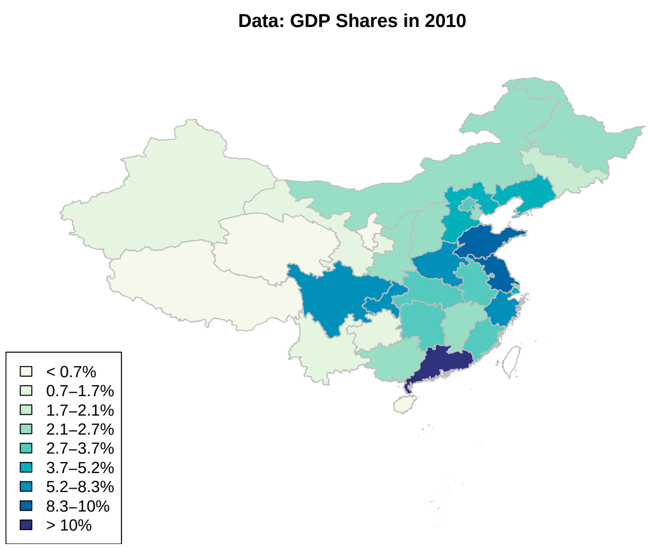
Starting with the initial GDP shares across provinces in China, we can see that economic activity tends to concentrate in the center and coastal areas of China, with Guangdong, Shandong, and Jiangsu being the largest provinces in terms of GDP in 1990. In 2010, in the bottom left panel, we can see a persistent concentration of economic activity in the same areas of China, but unlike in 1990, in 2010, activity tends to move from the central part of China to the coastal areas. For instance, Guangdong, Jiangsu, Zhejiang, Fujian, and Shandong are all provinces that increased their GDP shares, and these provinces are located in the coastal areas of China. In the bottom right panel, we see a similar pattern predicted by the model, pointing to the role of initial conditions in shaping the redistribution of economic activity across space during the subsequent two decades. In Online Appendix H.1 we present the evolution of the GDP shares across provinces in China every five years, which displays the same pattern as that described in this subsection.

Figure 3: Regional distribution of economic activity (GDP shares)

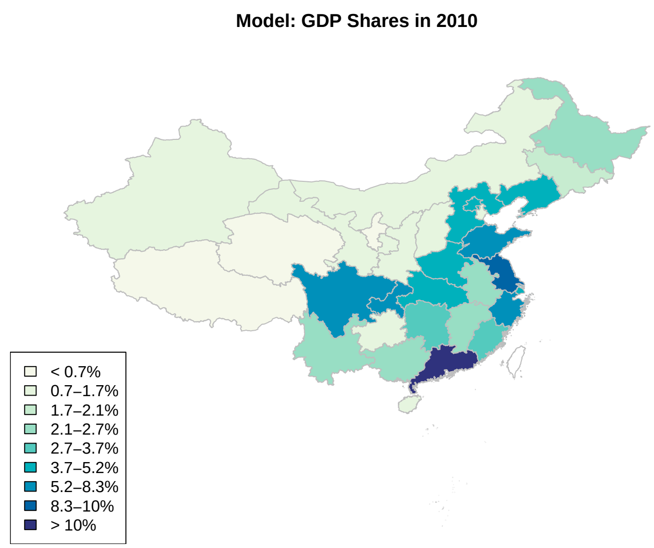
a) 1990



b) 2010 (Actual)



c) 2010 (1990 conditions)



Note: The figures show the distribution of economic activity across provinces in China, measured as GDP shares, in the data and with 1990 fundamentals over the period 1990-2010.

4.1.3 Spatial Growth

We now turn to describe spatial growth across provinces in China. In particular, in this subsection we study how the aggregate growth in China described in Section 4.1.1 was shaped by spatial growth. To do so, we turn to Figures 4 and 5, which display the real GDP growth across provinces in China during different time frames over the period 1990-2020 as predicted by our model under the initial conditions in 1990. In each panel, the upper figure presents the annual real GDP growth and the lower panel displays the contribution of each province to the aggregate real GDP growth in China during that period.

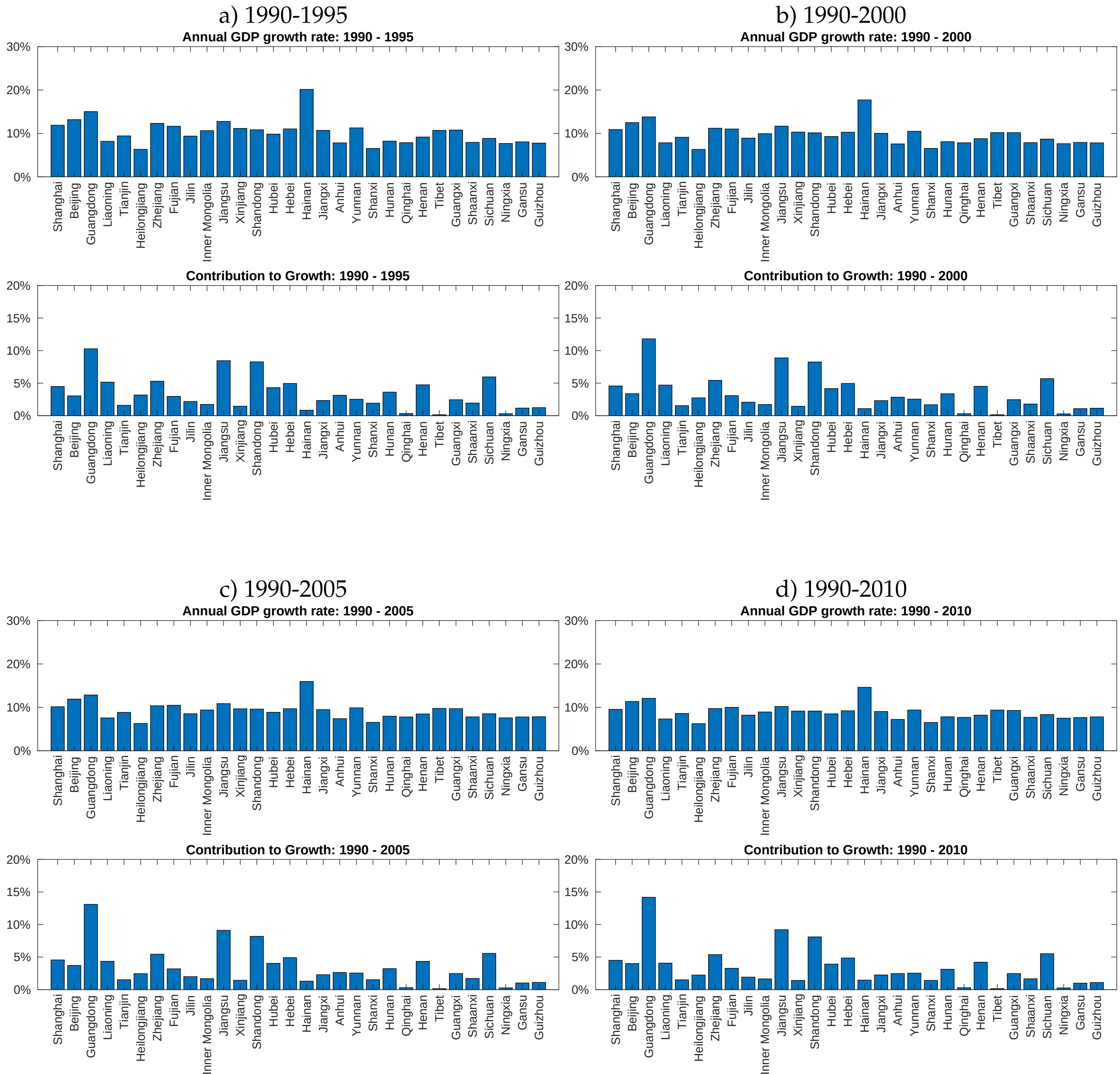
Several interesting results emerge from the figure. Looking first at the upper figures in each panel, in the 1990s we find large heterogeneity in spatial growth. In the period 1990-1995 (Panel (a)), we can see that Hainan, Shanghai, Beijing, and Guangdong are the provinces with the highest growth rates. Among them, the last three are the ones with the highest initial measured productivity, as described in Section 3.2. At the same time, these are provinces located in the coastal areas with better access to foreign goods. Hence, we observe some divergence after 1990 in the form of higher growth in provinces with better initial technology, which is in part shaped by the idea diffusion from knowledge from the rest of the world. Over the 2000s, we can see that growth rates tend to moderate and converge across provinces, as shown in the remaining upper figures, as the Chinese economy starts approaching the balanced growth path. In addition, ideas from people diffuse across space, and insights from migrants coming from provinces with a relatively low stock of knowledge slow growth in destinations with a relatively higher stock of knowledge, fostering some convergence in the knowledge stock across space.

In the lower figures of each panel, we present the contribution of each province to aggregate growth in China in each period of time. From these figures, we can see how Guangdong became a much more important engine of aggregate growth in China over time, as it benefits relatively more from idea diffusion from the rest of the world, especially given its advantaged location on the coast near Hong Kong. The figures also show that other provinces like Beijing became more important contributors to aggregate growth in China. At the same time, other large provinces like Shandong, Henan, and Hubei decreased in importance in aggregate growth over time.

In Online Appendix H.2 we present additional results about spatial growth. In particular, we compute the relative contributions of ideas from goods and ideas from people to growth across provinces in China. Consistent with the intuition explained in Section 4.1.1, we find on average a larger contribution of idea diffusion from trade than from migration but also spatial heterogeneity in their relative contributions across provinces.

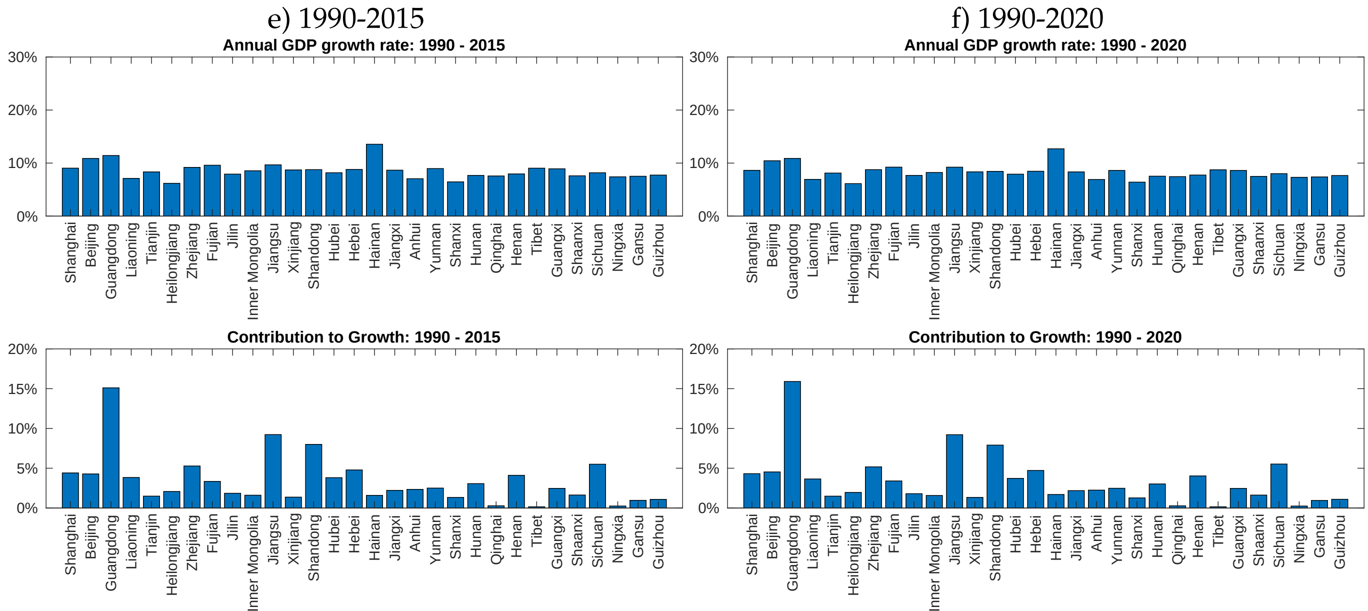
Overall, this subsection illustrates how aggregate growth is shaped by heterogeneous spatial growth across provinces in China over time. The mechanics of spatial growth across provinces is in turn shaped by initial conditions as well as the dynamics of productivity as a result of the idea diffusion from goods and people, changes in trade openness and migration, and the dynamics of labor markets and capital accumulation across space. In the next subsection, we study further the role of initial conditions in shaping spatial growth in China.

Figure 4: Spatial growth (annual, percent)



Note: The figures show the annual real GDP growth across provinces and the contribution of each province to the aggregate growth in China in different time frames over the period 1990-2020. Spatial growth in each figure is computed in the model under the initial 1990 conditions.

Figure 5: Spatial growth across provinces (annual, percent)



Note: The figures show the annual real GDP growth across provinces and the contribution of each province to the aggregate growth in China in different time frames over the period 1990-2020. Spatial growth in each figure is computed in the model under the initial 1990 conditions.

4.1.4 Initial Conditions and Spatial Development

Another interesting aspect that we can study with our framework is how the spatial growth across provinces during the 1990s and 2000s that we described in the previous section correlates with the initial distribution of fundamentals; namely, the initial stock of knowledge, the initial level of trade openness, and initial mobility frictions.

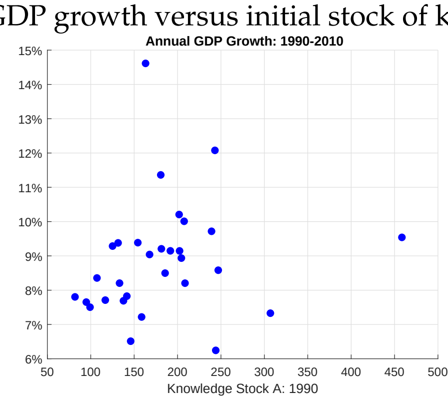
The upper panel of Figure 6 shows a scatter plot between the initial stock of knowledge and the real GDP growth across provinces over the period 1990-2010. We can see a somewhat positive correlation, meaning that provinces with a relatively higher initial fundamental productivity tend to grow more in the subsequent decades. It is important to emphasize that our dynamic spatial growth model allows for this correlation to have any sign. On the one hand, provinces with a higher initial stock of knowledge might also be more opened to trade and as a result, can benefit more from idea diffusion from the rest of the world relative to provinces with a lower initial stock of knowledge. On the other hand, provinces with a relatively higher fundamental productivity might be attractive for migrants from other provinces who bring relatively few good-quality insights, which might slow down growth in provinces with a higher initial stock of knowledge. Our finding of a positive relationship between the initial stock of knowledge and subsequent spatial growth is in line with the positive correlation between GDP growth and initial per capita GDP across provinces that we observe in the data during the same period.

Panel (b) presents a scatter plot between real GDP growth and the initial level of trade openness measured by domestic expenditure share λ_{ii}, where a smaller value of λ_{ii} means higher trade openness. We can see in the figure a clear negative correlation, meaning that a higher initial level of trade openness leads to higher growth in subsequent decades. This correlation is intuitive since provinces more open to trade benefit more from idea diffusion from goods from the rest of the world.

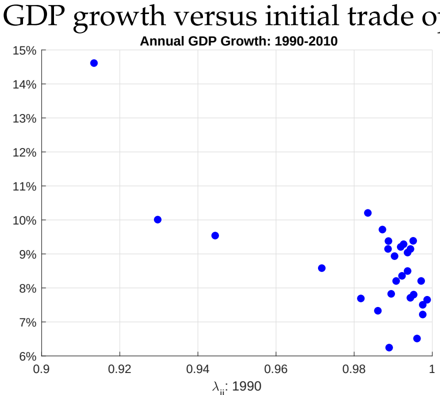
Finally, Panel (c) presents a scatter plot between growth and initial mobility measured by the fraction of stayers in a province denoted by s_{ii}, where a smaller value of s_{ii} means lower mobility frictions and therefore higher mobility. The correlation is slightly negative, pointing to higher growth in provinces with initially lower mobility frictions. However, the correlation is less clear than it is in the case of trade openness. As explained before, provinces that receive migrants experience faster growth in knowledge only if the insights from the provinces from which they migrate are sufficiently good quality. All these scatter plots deliver the same message when we repeat the analysis for the period 1990-2020.

Figure 6: Real GDP growth versus initial conditions

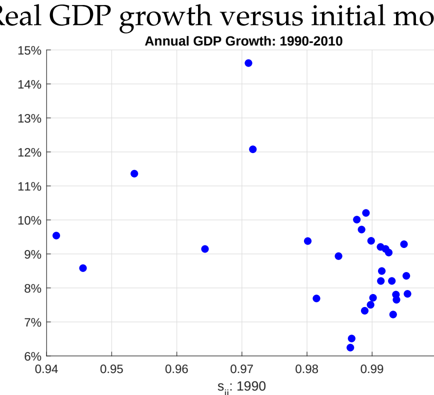
a) Real GDP growth versus initial stock of knowledge



b) Real GDP growth versus initial trade openness



c) Real GDP growth versus initial mobility



Note: The figures show scatter plots of annual real GDP growth across provinces in China over the period 1990-2010 against different initial conditions in 1990: initial stock of knowledge in Panel (a), initial level of trade openness, λ_{ii}, in Panel (b) (where two outlier provinces were trimmed from the figure), and initial mobility, s_{ii}, in Panel (c).

4.2 Changes in Fundamentals

Our quantitative analysis in the previous sections shows that initial conditions seem to be important for understanding and quantifying the process of spatial development and aggregate economic growth in China. It also shows the importance of the general equilibrium interactions of the mechanisms in our framework such as idea diffusion through trade and migration, labor market dynamics, and capital accumulation.

Over the 1990s and 2000s, China undertook other reforms related to changes in trade costs and migration frictions that might have also impacted spatial growth. In particular, when China joined the World Trade Organization, provinces more exposed to trade might have developed more relative to the less exposed provinces. Likewise, Hukou reforms might have fostered idea flows by increasing mobility across provinces. In this section, we explore quantitatively through the lens of our framework the impact of these reforms on spatial growth in China.

In terms of trade costs, we capture the changes in trade costs between China and the rest of the world using the time variation in bilateral trade shares relative to domestic expenditure shares across provinces. In other words, from our model we can back up changes in trade costs as $\frac{\hat{\lambda}_{in,t}\hat{\lambda}_{ni,t}}{\hat{\lambda}_{ii,t}\hat{\lambda}_{nn,t}} = \left(\hat{\kappa}_{in,t}\hat{\kappa}_{ni,t}\right)^{-\theta}$.[10]

We estimate changes in bilateral trade frictions over the period 1990-2010 between provinces in China and the rest of the world and then ask how spatial growth in China would have looked if the only change in fundamentals over the period 1990-2010 would have been bilateral international trade costs. To do so, we compare this counterfactual economy with the evolution of the economy with 1990 initial conditions described in the previous sections.

We also explore Hukou reforms in a simple way. We capture the changes in migration frictions across provinces in China as a consequence of the Hukou system using the cross-variation in five-year mobility rates from 1985-1990 to 1995-2000 as $\frac{\hat{\mu}_{in,t}\hat{\mu}_{ni,t}}{\hat{\mu}_{ii,t}\hat{\mu}_{nn,t}} = \left(\hat{m}_{in,t}\hat{m}_{ni,t}\right)^{-\frac{1}{v}}$, and apply this change in mobility frictions to all Hukou types.[11] We then ask the counterfactual question of how spatial growth in China would have looked if the only change in fundamentals after 1990 were the change in mobility frictions to Hukou types. To do so, we compare this counterfactual economy with the evolution of the economy with 1990 initial conditions described in the previous sections.

The spatial growth effects of changes in trade costs and mobility frictions are presented in Figure 7. The left-hand panels display the effects of changes in trade costs, and the right-hand panels present the effects of Hukou restrictions. In each panel, the upper figure shows the annual growth effects and the bottom figure illustrates the contribution of each province to aggregate growth, both relative to the baseline economy with 1990 trade and migration costs. We present the results for the periods 1990-2000 and 1990-2020, and in Online Appendix H.3 we present

[10] This statistic is known as the Head-Ries index (Head and Ries (2001)) and is widely used in the trade and spatial literature to measure bilateral trade frictions.

[11] Note that since Hukou type is assigned to either the origin or the destination province in the data, changes in mobility frictions are isomorphic to changes in amenities by Hukou type.

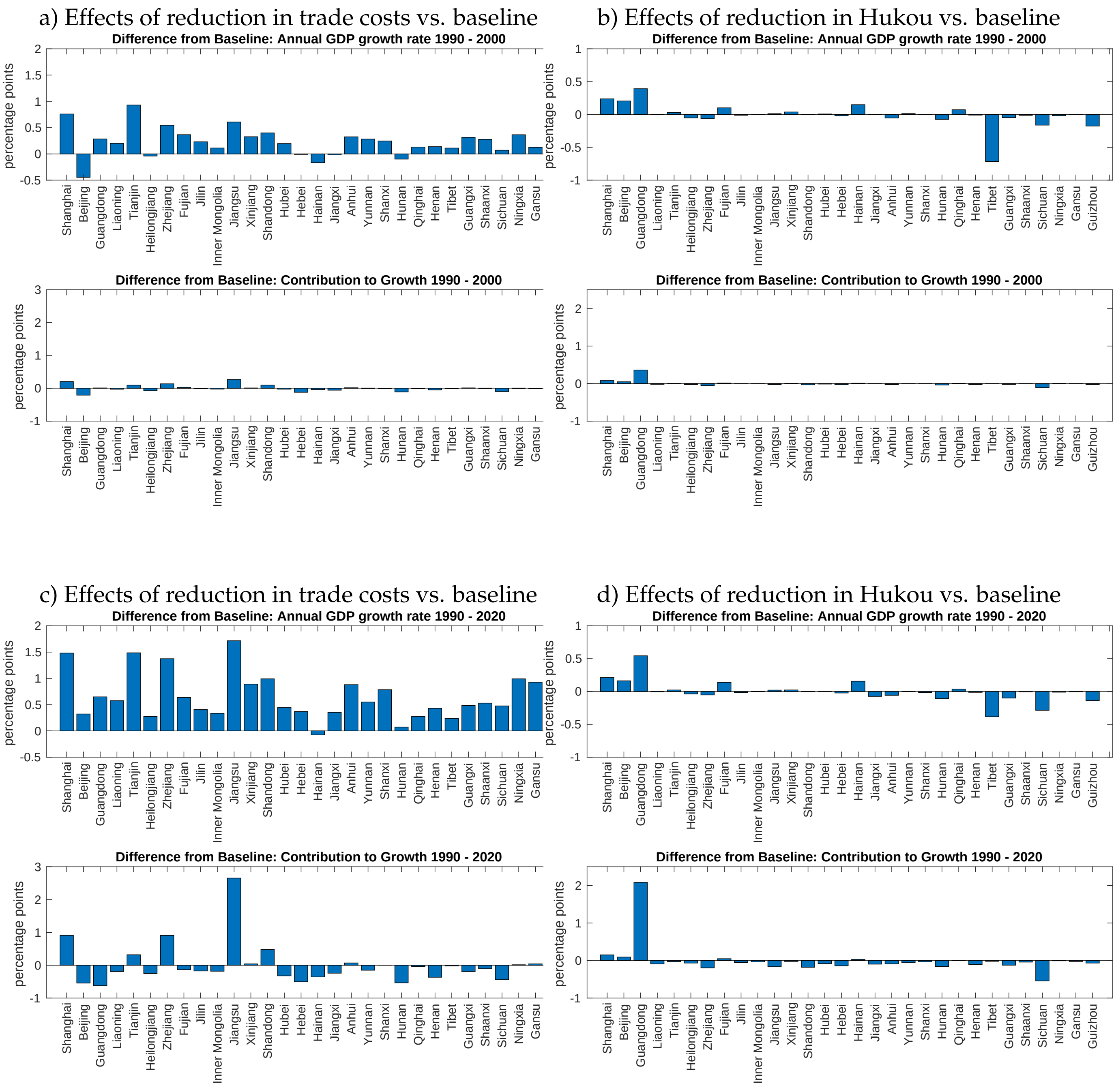
results for additional time frames.

We find that the change in trade costs contributed to extra aggregate growth by somewhat less than one percentage point annually and that the growth effects were very heterogeneous across space. Panel (a) shows the spatial growth effects of changes in trade costs over the 1990s. We can see that changes in trade costs increase growth in almost all provinces in China relative to the baseline economy with initial trade and migration costs. The coastal provinces of Shanghai, Tianjin, and Jiangsu experience the largest growth effects from changes in trade costs as they benefit relatively more from trade openness compared with provinces located farther away. Some provinces that initially are relatively more open to trade, such as Beijing, Hainan, and Hunan, see growth slightly decline in the 1990s, as they face increased competition from other provinces, especially in the coastal areas that benefit from trade with the rest of the world. Moving down to the bottom panels, we can see that changes in trade costs foster growth across all provinces over time as ideas from the rest of the world continue to diffuse to provinces in China.

In the right-hand panels, we present the spatial growth effects of changes to Hukou restrictions. Two points emerge from observing the spatial growth effects over time. First, changes in mobility restrictions lead to smaller growth effects than changes in trade costs. Second, the growth effects of changes in mobility restrictions are more heterogeneous than the growth effects of trade costs, with increases in growth in some provinces and decreases in growth in others relative to the baseline economy with 1990 conditions. Consistent with the intuition provided in previous sections, changes in mobility frictions benefit more open provinces that have scaled-up production and provinces that benefit from the ideas from migrants coming from places with a higher stock of knowledge, while they slow down growth in provinces left behind by international trade and internal migration. In Online Appendix H.3 we present the same figures with the spatial growth effects from changes in trade costs and Hukou restrictions together across provinces in China.

Finally, Table 2 presents the aggregate growth in China due to both 1990 fundamentals and the changes in international trade costs and migration restrictions over the 1990s and 2000s. To facilitate the analysis, the first two rows of the table reproduce the actual aggregate annual growth rate, and the growth rate with 1990 fundamentals displayed in Table 1. The rest of the rows presents the growth effects with the 1990 fundamentals and the changes in trade costs and migration restrictions. We can see from the table that both changes in trade costs and changes in mobility restrictions added about one percentage point of extra annual aggregate growth in China by the 2000s, mostly coming from the changes in international trade costs. The changes in migration restrictions did not have a significant impact on aggregate growth, although they had more significant effects in particular locations, as described previously.

Figure 7: Effects of trade and Hukou reforms on spatial growth (percentage points)



Note: The figures show the percentage point change in real GDP growth across provinces as consequence of the trade and Hukou restrictions in different time frames over the period 1990-2020. The left-hand panels present the effects of changes in trade costs and the right-hand panels show the effects of Hukou restrictions. All effects are computed relative to the baseline economy with 1990 trade and migration costs.

Table 2: Annual GDP Growth Rate: Changes in fundamentals

	90-95	90-00	90-05	90-10	90-15	90-20
Actual GDP growth	12.3%	10.4%	10.2%	10.5%	9.9%	9.3%
With fundamentals in 1990	10.6%	10.1%	9.6%	9.2%	8.9%	8.6%
Fundamentals in 1990 & change in trade cost	10.7%	10.3%	10.1%	9.9%	9.6%	9.4%
Fundamentals in 1990 & change in mig. restrictions	10.6%	10.1%	9.7%	9.3%	9.0%	8.7%
Fundamentals in 1990 & change in fundamentals	10.7%	10.4%	10.2%	10.0%	9.7%	9.4%

Note: The first two rows of the table reproduce the actual aggregate annual growth rate of China and the growth rate with 1990 fundamentals displayed in Table 1. The third row presents the annual growth rate with 1990 fundamentals and changes in international trade costs. The fourth row presents the annual growth rate with 1990 fundamentals and changes in migration restrictions. The last row presents the annual growth rate with 1990 fundamentals and changes in international trade costs and migration restrictions.

5 Empirical Evidence of Idea Diffusion

In the previous sections we highlighted the importance of the spatial mechanisms in our framework for shaping spatial development and aggregate growth, and in particular, the role of idea diffusion through trade and migration. We did so through the lens of the spatial dynamic growth model developed in Section 2 and the structural quantitative analysis in Section 3. In this section, we complement the structural analysis by providing reduced-form evidence of idea diffusion from trade and migration. To do so, we obtain province-level patent data as a proxy for the local stock of knowledge and use it along with our trade and migration data to provide empirical evidence of the role played by trade and migration in the diffusion of ideas.

We obtain province-level patent data from the China Statistics Yearbooks. To proxy the measure of knowledge stock, $A_{n,t}$, we obtain the cumulative approved patents at the province level for each year over the period 1985-2010. We then compute the change in the stock of knowledge every five years from 1985 to 2010. For the approved patents of the rest of the world, we obtain data from Google Patent from 1985-2010 following Liu and Ma (2021).[12] In Online Appendix I we provide more details on these patent data.

Using our patent data, as well as the migration flows and trade data described in Section 3.1 and in Online Appendix G, we run the following empirical specification:

$$\log\left(A_{n,t+1} - A_{n,t}\right) = \tau + \beta_m \log \lambda_{nn,t} + \beta_l \log\left(migration_{n,t}\right) + \tau_n + \tau_t + \epsilon_{n,t}.$$

The term $\lambda_{nn,t}$ is the domestic expenditure share and captures the level (inverse) of trade openness of province n. We expect the coefficient β_m to be negative, indicating that provinces more exposed to international trade benefit more from the global diffusion of ideas and experience higher growth in the stock of knowledge as a result. We define $\log\left(migration_{n,t}\right) = \log\left[\sum_{i=1}^{N} s_{in,t} A_{i,t}\right]$, which equals the weighted average of the stock of knowledge diffusing to

[12]We are grateful to Song Ma for sharing the Google Patent data.

location n at time t through both migrants and locals. This term in our model captures the idea diffusion from people, and as a result, we expect the coefficient β_l to be positive. Finally, the term τ_n controls for province fixed effects, τ_t is a year fixed effect, and $\epsilon_{n,t}$ is an error term following an i.i.d. standard normal distribution. Since individual-level population census data for the year 1995 do not exist, migration flows are unavailable for the period 1990-1995. Hence, we run the regression using data for five-year intervals from 1995 to 2010.

Table 3 reports the results. In Column (1) we first show that faster growth in the stock of knowledge is associated with a higher degree of trade openness, and Column (2) shows that the growth in knowledge stock positively correlates with idea diffusion from people, controlling for year and province fixed effects. In Column (3), we show that trade openness and idea diffusion through people together contribute to the growth in the stock of knowledge in the way that our theory suggests. In each of these specifications, the coefficients of interest, β_m and β_l, have the expected signs and are statistically significant.[13]

Table 3: Estimates of knowledge diffusion through trade and migration

	$\log\left(A_{n,t+1} - A_{n,t}\right)$		
	(1)	(2)	(3)
$\log \lambda_{nn,t}$	-6.138***		-5.782**
	(2.222)		(2.296)
$\log\left(migration_{n,t}\right)$		0.353*	0.274*
		(0.201)	(0.153)
Constant	8.692***	5.448**	5.960***
	(0.113)	(2.003)	(1.535)
Observations	90	90	90
R-squared	0.544	0.473	0.564
Year FE	✓	✓	✓
Province FE	✓	✓	✓

Note: Robust standard errors in parentheses.*** p<0.01, ** p<0.05, * p<0.1.

In the previous regression we documented the contribution of idea diffusion through people to the local stock of knowledge. One might wonder whether this contribution is entirely driven by local ideas, as a significant share of locals stay in the same location during each five-year window. To address this question, we distinguish between idea diffusion through the locals (stayers) and that through non-locals (immigrants) with the following specification,

$$\log\left(A_{n,t+1} - A_{n,t}\right) = \tau + \beta_m \log \lambda_{nn,t} + \beta_i \log\left(immigration_{n,t}\right) + \beta_s \log A_{n,t} + \tau_n + \tau_t + \epsilon_{n,t},$$

where we define $\log\left(immigration_{n,t}\right) = \log\left[\sum_{i \neq n} s_{in,t} A_{i,t}\right]$, which captures the weighted sum of the knowledge brought by migrants to n from locations other than n. In this specification we

[13] For completeness, in Online Appendix I we present scatter plots with simple correlations between the change in the local stock of knowledge and our measures of trade openness and idea diffusion through people.

control for $\log A_{n,t}$, which is the measure of local knowledge described previously. Therefore, the coefficient β_i captures the idea diffusion from non-locals (immigrants), and the coefficient β_s captures the knowledge diffusion from locals (stayers).

Table 4 reports the results. In Column (1), we start by showing the positive and significant coefficient of $\log A_{n,t}$, β_s, which reveals that idea diffusion from local knowledge contributes to growth in the local knowledge. Column (2) suggests that in addition to the local knowledge, higher knowledge growth tends to be seen in locations with more exposure to international trade. In Column (3), the positive and significant coefficient of the term $\log(immigration_{n,t})$, β_i shows that after controlling for the local knowledge stock, the knowledge brought by non-local immigrants also contributes to the growth in the stock of knowledge. Column (4) shows that international trade openness, ideas brought by migrants, and local knowledge stock all contribute to the growth in local knowledge stock, in line with the spatial mechanisms in our model. Alternatively, we could replace the dependent variable by the growth rate in the knowledge stock. In Column (5), the dependent variable is $\log\left(\frac{A_{n,t+1}-A_{n,t}}{A_{n,t}}\right)$, and the purpose of this alternative specification is to normalize the change in knowledge stock in each location by the local knowledge stock. Again, the results suggest that international trade and ideas diffused by non-locals contribute significantly to the growth in the stock of knowledge.

Table 4: Estimates of knowledge diffusion through trade and local and non-local knowledge

	$\log(A_{n,t+1}-A_{n,t})$				$\log\left(\frac{A_{n,t+1}-A_{n,t}}{A_{n,t}}\right)$
	(1)	(2)	(3)	(4)	(5)
$\log \lambda_{nn,t}$		-5.234**		-5.200**	-4.549*
		(2.412)		(2.399)	(2.591)
$\log(immigration_{n,t})$			0.133*	0.128*	0.170**
			(0.076)	(0.070)	(0.073)
$\log A_{n,t}$	0.655***	0.549***	0.703***	0.596***	
	(0.211)	(0.185)	(0.218)	(0.189)	
Constant	3.121*	3.825**	2.012	2.753	-0.817*
	(1.775)	(1.548)	(2.049)	(1.724)	(0.406)
Observations	90	90	90	90	90
R-squared	0.537	0.610	0.550	0.622	0.827
Year FE	✓	✓	✓	✓	✓
Province FE	✓	✓	✓	✓	✓

Note: Robust standard errors in parentheses.*** $p<0.01$, ** $p<0.05$, * $p<0.1$.

We have provided reduced-form evidence of the contribution of idea diffusion through both international trade and migration to the stock of local knowledge. In doing so, we do not aim at establishing a causal effect since our theory is consistent with two-way causality between knowledge and idea diffusion from trade and migration due to general equilibrium effects. Still, in Online Appendix I we provide further evidence of the role of idea diffusion by implementing an instrumental variable strategy to estimate the effect of idea diffusion on the local stock of knowledge. We also derive empirical specifications using the model's structure. Similar to the reduced-form evidence presented in this section, we find a statistically significant contribution of idea diffusion through trade and migration to the local stock of knowledge.

6 Concluding Remarks

In this study we have developed a dynamic spatial growth model to study, understand, and quantify the role of spatial growth on aggregate economic activity. In our spatial growth model, internal migration and trade provide the mechanics for spatial growth. Producers and migrants share ideas with other producers, and the flow of ideas across space and time serves as the main mechanism that generates spatial growth. We characterized the equilibrium properties of our model and showed that it has a unique balanced growth path. We also showed how to take the model to the data in order to perform quantitative exercises without assuming that the economy is initially in a balanced growth path.

As an application, we took our model to the data and studied the importance of trade and migration as engines of growth for the Chinese economy after 1990. Initial conditions and our spatial mechanisms that operate through international trade and internal migration played a considerable role in spatial development and aggregate growth during the 1990s and 2000s in China. The changes in fundamentals due to trade openness and Hukou restrictions also contributed to aggregate growth and heterogeneous spatial development in China.

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