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A MODEL OF THE GOLD STANDARD

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ABSTRACT

The gold standard emerged as the international monetary system by the end of the 19th century. We formally study its properties in a micro-founded model and find that the scarcity of the world gold stock not only results in a suboptimal output of goods that are purchased with money but also subjects the domestic economy of a country to external shocks. The creation of inside money in the form of private credit instruments adds to the money supply, usually resulting in a Pareto improvement, but opens the door to the international transmission of banking crises. These properties of the gold standard can explain the limited adherence by peripheral countries because of the potential risks to their economies. We argue that the gold standard can be sustainable at the core but not at the periphery.

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1 Introduction

A widespread view among monetary economists is that, throughout the monetary history of nations, money backed by precious metals usually implies a scarce supply of aggregate liquidity, which results in suboptimal production of goods that are purchased with money.[1] On the one hand, precious metals, such as gold and silver, possess many desirable properties that allowed them to emerge as universally accepted media of exchange. But, on the other hand, their supply tends to be inelastic. During periods of rapidly growing domestic and international exchanges, this low elasticity of supply can cause severe problems for the development of industry and commerce. For example, a common interpretation of the so-called Long Depression of 1873-1896 is that the gold standard could not accommodate the rapid growth of the world economy until the fortuitous gold discoveries of the 1880s-1890s and the development of the cyanidation process for extracting gold from low-grade ore. Indeed, Mitchell (1975, p. 737) reports a 42% drop in the British wholesale price index between 1873 and 1896.

An important related discussion in the literature is the transmission of shocks across countries that results from the adoption of a common commodity money system (Friedman and Schwartz, 1963). Under the "rules" of the gold standard, the issuance of money backed by the available gold stock links the domestic economy of a country to other economies by absorbing (or releasing) a fraction of the world gold stock.[2] Changes in aggregate productivity in one region, by increasing or decreasing the demand for money in that region, can precipitate substantial monetary movements in the international economy as gold flows from one country to another, with implications for the money supply, prices, and output in different areas. If these differences in the fundamentals of open economies that have adopted the gold standard are permanent, the operation of the international monetary system can result in a persistent loss of output, consumption, and welfare in the least productive countries, which are usually referred to in this context as the peripheral countries of the gold bloc (see Gallarotti, 1995, and Ögren and Øksendal, 2012).

In this paper, we argue that these features of commodity money systems, in general, and of the gold standard, in particular, are interconnected.[3] We show that the scarcity of gold in the world economy not only leads to suboptimal levels of production and exchange of goods that are purchased with money but also exposes the domestic economy of a country to the real effects of external shocks. Additionally, we show that when the world gold stock is scarce, the issuance of private credit instruments in the form of transferable debt claims increases aggregate liquidity

[1] A classical and eloquent exposition of this view is Marshall (1887).

[2] Bordo (1981), Redish (1990), Bordo and Kydland (1995), and Eichengreen (2019) provide a useful description of the operation of the international monetary system under the gold standard.

[3] Other interesting features of commodity money systems that we do not explore are the coexistence of different types of coins as media of exchange and whether the practice of debasement has any real effects. An excellent paper that discusses these issues is Velde et al. (1999). We do not study the positive and normative properties of a bimetallic monetary system (see our discussion below of Velde and Weber, 2000). During its existence, the gold standard was under the constant threat of the "lure of bimetalism" (Eichengreen, 2019, p. 11).

in world markets and can result in a Pareto improvement. However, the introduction of inside money opens the door to the international transmission of banking crises (i.e., shocks that cause fluctuations in the supply of inside money in one country). The occurrence of a banking crisis in one country in the gold bloc sets in motion specie flows in world markets that result in a decline of aggregate liquidity in all countries that have adopted the gold standard. From the perspective of the domestic economy, a banking crisis that originates in a foreign country causes a contraction of the domestic money supply and a decline in the output of goods that are purchased with money.

Our results can explain why countries that joined the gold standard at an early stage of economic development experienced difficulties in the operation of their monetary system and frequently imposed controls on the export of bullion. For instance, we show that the country with the least productive industries is better off if it unilaterally restricts the export of bullion to other countries after the realization of a permanent productivity shock in a foreign country. In other words, that country gains by going off the gold standard. Such a restriction on gold exports prevents the decline of the domestic money supply that would result from the ensuing specie movements under the gold standard. Because of the non-neutrality of money, abandoning the gold standard prevents a decline in output and social welfare in the peripheral country.

We conclude that these properties of the gold standard imply that the system can be sustainable at the core but not at the periphery because of the risks associated with the commitment to the convertibility of the money supply. A core country benefits from the potential gold inflow from peripheral countries in the event of a positive productivity shock or banking crisis in its domestic economy. The specie flows associated with the operation of the gold standard allow that core country to smooth out fluctuations in output and consumption, increasing the welfare of its citizens.

More speculatively, the unraveling of the gold standard in the periphery countries can unleash forces (i.e., sovereign debt defaults, protectionism, etc.) that might contribute to the suspension of the gold standard in the core countries as well. Many forces converged during the 1930s to doom the gold standard: the extension of the franchise to low-income voters that made the deflationary politics of previous decades politically unfeasible, the lack of international monetary cooperation, the unwillingness of the U.S. to rebalance the world economy after World War I, the trade-offs for central banks between maintaining convertibility and being lenders of last resort, etc. We do not deny the importance of any of them. More circumspectly, we argue that the tensions highlighted by our model were an additional mechanism behind the demise of the gold standard that has been overlooked. Interestingly, our model highlights a structural flaw in the gold standard that goes beyond concrete historical contingencies.

Surprisingly, there have been few attempts in the literature to model the gold standard in a modern dynamic general equilibrium framework, especially given the importance that contemporary authors attributed to specie flows and the associated mechanisms utilized to mitigate them.

In particular, we are unaware of any dynamic model that analyzes the welfare effects of the specie flows associated with the operation of the gold standard.

Sargent and Wallace (1983) provide an early analysis of commodity money in an overlapping generations model. In their framework, commodity money takes the form of a capital good that can be converted into the consumption good or accumulated over time (no such accumulation is possible for the consumption good). They find that a commodity money system is inefficient. Our analysis focuses on the role of durable assets as media of exchange and the welfare implications of international specie flows that originate from productivity and financial shocks. For our purposes, gold production in the short run is of second-order importance.

Another important contribution to the literature is Kiyotaki et al. (1993), who analyze the emergence of an internationally traded currency in a micro-founded model. The authors build on the model of Kiyotaki and Wright (1989) to study the existence of an equilibrium in which a fiat currency can serve as an international medium of exchange. Their work, however, does not provide an analysis of the welfare consequences of the transmission of shocks under an international monetary arrangement.

Velde and Weber (2000) provide a formal analysis of a bimetallic system in which gold and silver coexist as media of exchange. Their study focuses on the sustainability of the joint circulation of these metals in a single country. Specifically, their welfare analysis focuses on the welfare benefits of unilaterally moving from a bimetallic to a monometallic system. In contrast, we provide an analysis of the international monetary system under the gold standard (i.e., a monometallic arrangement), and we consider the welfare effects of shocks within the gold bloc to study its sustainability in the long run. Additionally, we study the welfare properties of inside money under the gold standard, which was an important development during the period of the classical gold standard.

The seminal work of Friedman and Schwartz (1963) documents the economic consequences of banking crises under the gold standard. They provide an analysis of the mechanism that leads to a contraction of the domestic money supply in the event of a banking crisis in the context of the international monetary system under the gold standard. Our model provides a modern framework for the analysis of the effects of banking crises and their international transmission via the gold standard. We find that a banking crisis that occurs in a core country in the gold bloc results in a contraction of aggregate liquidity in other countries in the bloc following the associated specie flows.

The rest of the paper is organized as follows. Section 2 introduces a simple model of the gold standard. Section 3 looks at the model when the total supply of gold in the world economy is scarce. Section 4 looks, instead, at the case when the gold stock is abundant. Section 5 explores the development of banking within the context of the gold standard. Section 6 concludes.

2 A Model of the Gold Standard

Our analytical framework is a variation of the model by Lagos and Wright (2005), a standard workhorse in monetary economics. The global economy comprises the gold bloc and the rest of the world. The gold bloc contains two countries or regions, indexed by $i \in \{a, b\}$. Each country is composed of a unit mass of buyers and a unit mass of sellers. The rest of the world plays a minor role in the model: It is a potential source of gold and dividend payments to the gold bloc.

Time is discrete and continues forever. All agents have the same discount factor $\beta \in (0, 1)$. Often, it will be more transparent to express results in terms of the agents' rate of time preference: $r \equiv (1 - \beta)/\beta$. Each period is divided into two subperiods, the first with a decentralized market (DM) and the second with a centralized market (CM). In the DM, a buyer is randomly matched with a seller of the same country with probability $\sigma \in (0, 1)$. The seller can produce a divisible and perishable good, referred to as the DM good, by exerting effort. Only the buyer derives utility from the consumption of this good. In the CM, agents trade a divisible and perishable good, referred to as the CM good. Any agent can supply labor in the CM to produce the CM good using a linear technology.

There exists a third commodity in the global economy, gold, which, for the moment, we assume is in fixed supply and does not depreciate. Let $Q \in \mathbb{R}_+$ denote the gold stock per buyer in the gold bloc. Gold entitles its owner to receive a dividend payment $\delta \in \mathbb{R}_+$ in terms of the CM good each period, which is to be interpreted as rents received from producers outside the gold bloc. For example, the use of gold as collateral may open investment opportunities in the rest of the world that are not available to agents without gold.[4] The CM good and gold can be traded internationally. Let $\rho_t \in \mathbb{R}_+$ denote the period-t value of gold in terms of the CM good. Because transportation costs are zero, the value of gold must be the same in all countries.

Since the DM meetings are anonymous, the seller is willing to produce the DM good for the buyer only if the latter offers something tangible that can be exchanged for goods in the following CM. In the context of our model, that tangible object is either gold or gold certificates fully backed by the available gold stock (we assume the existence of an enforcement technology that ensures the gold certificates are honored). From now on, when we refer to gold we will understand that it includes both gold and gold certificates. In contrast with the basic model of Lagos and Wright (2005), we do not allow the existence of fiat money.

All buyers in the gold bloc have identical preferences represented by:

$$U(q, x) = u(q) + x,$$

[4] An alternative interpretation is that gold provides direct services, for example, in the form of jewelry. Geromichalos et al. (2007) provide an analysis of the equilibrium properties of an economy in which a durable, interest-bearing asset serves as a medium of exchange.

where $q \in \mathbb{R}_+$ denotes consumption of the DM good and $x \in \mathbb{R}$ denotes net consumption of the CM good (i.e., total consumption minus production). The utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$ satisfies $u' > 0$, $u'' < 0$, $u'(0) = \infty$, $u(0) = 0$, and $-\frac{qu''(q)}{u'(q)} < 1$ for all $q > 0$.

The seller's preferences in country $i \in \{a, b\}$ are represented by:

$$V^i(q, x) = -\omega^i q + x,$$

where $q \in \mathbb{R}_+$ denotes production of the DM good and $x \in \mathbb{R}$ denotes net consumption of the CM good. The parameter $\omega^i > 0$ measures the seller's disutility of production and can be thought of as the inverse of the productivity level in country i. Let $q^*(\omega^i)$ denote the surplus-maximizing quantity, which is the unique solution to $u'(q^*(\omega^i)) = \omega$.

We now characterize this gold standard economy. Because the buyer's preferences are identical across borders and the law of one price implies a unique value of gold in the international market, we temporarily omit the superscript i denoting the buyer's country of residence in the derivation of the buyer's optimization problem.

The buyer's Bellman equation. Let $J(s, t)$ denote the value function of a buyer holding $s \in \mathbb{R}_+$ units of money (i.e., gold or gold certificates) at the beginning of period t. The buyer's Bellman equation is given by:

$$J(s, t) = \sigma \left[u\left(q_\omega(s, t)\right) + W\left(s - z_\omega(s, t), t\right)\right] + (1 - \sigma) W(s, t).$$

The value $q_\omega(s, t)$ gives the amount of DM goods traded in a bilateral meeting in exchange for $z_\omega(s, t)$ units of money. Both quantities depend only on the buyer's money holdings because of quasi-linear preferences with respect to the CM good. The CM value function $W(s, t)$ is given by:

$$W(s, t) = \max_{(x, s') \in \mathbb{R} \times \mathbb{R}_+} \left[x + \beta J(s', t + 1)\right]$$

subject to the budget constraint:

$$x + \rho_t s' = (\rho_t + \delta) s. \tag{1}$$

Because of quasi-linear preferences, we can write the value $W(s, t)$ as:

$$W(s, t) = (\rho_t + \delta) s + \max_{s' \in \mathbb{R}_+} \left[-\rho_t s' + \beta J(s', t + 1)\right].$$

Then, the beginning-of-the-period Bellman equation can be written as:

$$J(s, t) = \sigma \left[u\left(q_\omega(s, t)\right) - (\rho_t + \delta) z_\omega(s, t)\right] + (\rho_t + \delta) s + \max_{s' \in \mathbb{R}_+} \left[-\rho_t s' + \beta J(s', t + 1)\right].$$

This completes the description of the buyer's problem in our sequential markets economy.

The seller's Bellman equation. The seller's problem is the same in both countries, but the productivity of DM goods can take on different values across borders. Let $K^i(s,t)$ denote the value function of a seller in country $i \in \{a,b\}$ holding $s \in \mathbb{R}_+$ units of money at the beginning of the period. This seller's Bellman equation is given by:

$$K^i(s,t) = \sigma\left[-\omega^i q_{\omega^i}(\tilde{s},t) + B^i(s + z_{\omega^i}(\tilde{s},t), t)\right] + B^i(s,t).$$

The variable $\tilde{s} \in \mathbb{R}_+$ represents the money holdings of the buyer with whom the seller is matched in the DM. The CM value function $B^i(s,t)$ is given by:

$$B^i(s,t) = \max_{(x,s') \in \mathbb{R} \times \mathbb{R}_+} \left[x + \beta K^i(s', t+1)\right]$$

subject to the budget constraint (1).

If $\rho_t > \beta(\delta + \rho_{t+1})$, the seller will optimally choose *not* to hold gold across periods. The seller will, however, accept gold or gold certificates in the DM as a means of payment in exchange for his output, provided the proposed terms of trade are individually rational for him.

Bargaining. The terms of trade in the DM are determined by the generalized Nash bargaining solution. To eliminate the holdup problem, we assume that the buyer has all the bargaining power. In this case, the values $(q,z) \in \mathbb{R}_+^2$ are obtained by solving:

$$\max_{(q,z) \in \mathbb{R}_+^2} \left[u(q) - (\rho_t + \delta) z\right]$$

subject to the seller's individual rationality constraint,$-\omega q + (\rho_t + \delta) z \geq 0$, and the liquidity constraint, $z \leq s$.

The solution to this problem is given by:

$$q_\omega(s,t) = \begin{cases} \frac{(\rho_t+\delta)s}{\omega} \text{ if } s < \frac{\omega q^*(\omega)}{\rho_t+\delta} \\ q^*(\omega) \text{ otherwise,} \end{cases}$$

and

$$z_\omega(s,t) = \begin{cases} s \text{ if } s < \frac{\omega q^*(\omega)}{\rho_t+\delta} \\ \frac{\omega q^*(\omega)}{\rho_t+\delta} \text{ otherwise.} \end{cases}$$

These schedules imply that the surplus-maximizing quantity is traded if the buyer's money holdings are sufficiently large to compensate the seller for the disutility of production. Note that $\frac{d[\omega q^*(\omega)]}{d\omega} < 0$, so a higher productivity of DM goods (i.e., a lower value for ω) raises the threshold value for the buyer's money balances to induce the seller to produce the (now larger) surplus-maximizing quantity.

The relative price of the DM good is given by $\frac{\omega}{\rho_t+\delta}$. This price depends positively on the disutility ω of producing it (equivalently, the price depends negatively on the productivity of the seller) and negatively on the price of gold, ρ_t, and the dividend, δ. The intuition for this last result is simple: The higher the dividend of gold, the higher the opportunity cost for the buyer of purchasing the DM good and, therefore, the less the good is traded in equilibrium, lowering its relative price. Interestingly, the relative price of the DM good does not depend on whether the surplus-maximizing quantity is produced.

Optimal portfolio. Given the terms of trade in the DM, the buyer's portfolio problem in the CM can be written as:

$$\max_{s' \in \mathbb{R}_+} \left\{ -\rho_t s' + \beta \left\{ \sigma \left[u \left(q_{\omega^i} \left(s', t+1 \right) \right) - \left(\rho_{t+1} + \delta \right) z_{\omega^i} \left(s', t+1 \right) \right] + \left(\rho_{t+1} + \delta \right) s' \right\} \right\}.$$

If $\rho_t < \beta \left(\delta + \rho_{t+1} \right)$, the problem has no solution. Suppose that $\rho_t > \beta \left(\delta + \rho_{t+1} \right)$. Then, there is a unique interior solution characterized by the first-order condition:

$$-\rho_t + \beta \left(\delta + \rho_{t+1} \right) \left[\frac{\sigma}{\omega^i} u' \left(\frac{\left(\rho_{t+1} + \delta \right) s'}{\omega^i} \right) + 1 - \sigma \right] = 0.$$

Define the function $L_\omega : \mathbb{R}_+ \to [1, \infty)$ by:

$$L_\omega \left(A \right) = \begin{cases} \frac{\sigma}{\omega} u' \left(\frac{A}{\omega} \right) + 1 - \sigma \text{ if } A < \omega q^* \left(\omega \right) \\ 1 \text{ otherwise.} \end{cases}$$

Let $s_t^i \in \mathbb{R}_+$ denote the period-t optimal portfolio choice of a buyer in country i. Then, we can write the first-order condition as

$$\rho_t = \beta \left(\delta + \rho_{t+1} \right) L_{\omega^i} \left(\left(\delta + \rho_{t+1} \right) s_t^i \right).$$

This first-order condition implicitly defines the demand for money as a function of the current and future value of gold, provided $\beta \left(\delta + \rho_{t+1} \right) < \rho_t$. The demand for money is decreasing in ρ_t. The demand for money is increasing in ρ_{t+1} provided $L_w \left(A \right) + A L_w' \left(A \right) > 0$ holds for all $A < \omega q^* \left(\omega \right)$. Because $-\frac{q u''(q)}{u'(q)} < 1$ for all $q > 0$, it is straightforward to show that $L_w \left(A \right) + A L_w' \left(A \right) > 0$ holds.

Market clearing. Let $Q_t^i \in \mathbb{R}_+$ denote the gold supply per buyer in country i. The market-clearing condition in the gold bloc is given by:

$$2Q = Q_t^a + Q_t^b.$$

The market-clearing condition in the money market of each country implies $s_t^a = Q_t^a$ and $s_t^b = 2Q - Q_t^a$, together with the non-negativity condition $Q_t^a \leq 2Q$.

Equilibrium. We can define an equilibrium for the gold bloc as a non-negative stream

$\langle Q_t^a, \rho_t \rangle_{t \geq 0}$ satisfying $Q_t^a \leq 2Q$,

$$\rho_t = \beta \left(\delta + \rho_{t+1}\right) L_{\omega^a} \left(\left(\delta + \rho_{t+1}\right) Q_t^a\right),$$

and

$$\rho_t = \beta \left(\delta + \rho_{t+1}\right) L_{\omega^b} \left(\left(\delta + \rho_{t+1}\right) \left(2Q - Q_t^a\right)\right)$$

at all dates.

It is helpful to adopt the following normalization: Set $\omega^b = 1$ and $\omega^a = \omega \in [1 - \varepsilon, 1 + \varepsilon]$, with $0 \leq \varepsilon < 1$. We restrict our attention to stationary equilibria, given that the efficient allocation is necessarily stationary. Then, we can define a *stationary equilibrium* for the gold bloc as a non-negative vector $\langle Q^a, \rho \rangle$ satisfying:

$$\frac{(1 + r)\,\rho}{\rho + \delta} = L_\omega \left(\left(\rho + \delta\right) Q^a\right) \tag{2}$$

and

$$\frac{(1 + r)\,\rho}{\rho + \delta} = L_1 \left(\left(\rho + \delta\right) \left(2Q - Q^a\right)\right), \tag{3}$$

with $Q^a \leq 2Q$.

In the next sections, we characterize the stationary equilibrium of our economy, study the effects of an unanticipated and permanent gold inflow from the rest of the world, and characterize the international monetary movements, or specie flows, that result from an asymmetric productivity shock in the gold bloc.

3 Scarce Gold Stock

We start our investigation of the gold standard by characterizing the allocation when the available gold stock is scarce, that is, when $\rho > \frac{\delta}{r}$ holds in equilibrium. In this regime, the price of gold, ρ, is above its fundamental value: the discounted value of dividend flows, δ/r. There are two types of equilibrium under the scarce gold regime. If all countries in the gold bloc have the same productivity of DM goods, the equilibrium is symmetric. If the productivity of DM goods differs across countries, the equilibrium is asymmetric.

3.1 Symmetric equilibrium

If the productivity of DM goods is the same in all countries that are on the gold standard, a symmetric equilibrium emerges, which serves as a benchmark allocation for our analysis of specie flows that follows. Additionally, the symmetric equilibrium illustrates two fundamental properties of the model: (i) the non-neutrality of money and (ii) its tendency toward secular deflation.

Suppose that $\omega = 1$. If $\rho > \frac{\delta}{r}$ holds in equilibrium, we argue next that the gold supply per buyer is the same in all countries in the bloc, that is, $Q^a = Q$ holds at the equilibrium position. Because $(1+r)\rho > \delta + \rho$ and $L_1(\cdot)$ is strictly decreasing in the range $(0, q^*)$, we find that $Q^a = Q$ necessarily holds in a symmetric equilibrium (we denote $q^*(1)$ simply by q^*).

Given this property, the value of gold in a symmetric equilibrium can be characterized as a solution $\rho = \rho(1, Q)$ to:

$$\frac{(1+r)\rho}{\delta+\rho} = L_1((\delta+\rho)Q). \tag{4}$$

As the next proposition will show, the condition for the scarcity of the gold stock in the gold bloc (and hence, for the value of gold to be above its fundamental values) is:

$$Q < \frac{rq^*}{\delta(1+r)} = (1-\beta)\frac{q^*}{\delta}. \tag{5}$$

The following result establishes existence and uniqueness when $\omega = 1$.

Proposition 1 *If condition (5) holds, there exists a unique symmetric stationary equilibrium characterized by the solution $\rho(1, Q)$ to (4). In this equilibrium, we have $\rho(1, Q) > \delta/r$ and $\frac{\partial \rho}{\partial Q} < 0$.*

Proof. The left-hand side of (4) is increasing and strictly concave in ρ and equals 1 at the lowest point $\rho = \frac{\delta}{r}$. The right-hand side is decreasing in ρ provided (5) holds. To verify this claim, note that $(\delta+\rho)Q = \frac{(1+r)\delta Q}{r} < q^*$ when $\rho = \frac{\delta}{r}$, which implies $L_1\left(\frac{(1+r)\delta Q}{r}\right) > 1$. Because $L_1(A)$ is strictly decreasing in the range $(0, q^*)$, we find that $L_1((\delta+\rho)Q)$ is strictly decreasing in ρ in the range $\left(\frac{\delta}{r}, \frac{q^*}{Q} - \delta\right)$ and equals 1 in the range $[\frac{q^*}{Q} - \delta, \infty)$. Thus, there exists a unique value $\rho(1, Q) \in \left(\frac{\delta}{r}, \frac{q^*}{Q} - \delta\right)$ satisfying (4). It is straightforward to show that $\frac{\partial \rho}{\partial Q} < 0$. ■

Non-neutrality of money. Our model predicts that an unanticipated and permanent gold inflow from the rest of the world into the gold bloc (i.e., an increase in Q, for instance, due to a current account surplus in the gold bloc with the rest of the world or the discovery of new gold mines) leads to a decline in the exchange value of gold, moving it closer to its fundamental price, and to an expansion of DM output.

Using the equilibrium value of gold derived in the previous proposition, we obtain the quantity traded in the DM by solving:

$$\sigma u'(q(1, Q)) + 1 - \sigma = 1 + r - \frac{\delta}{\rho(1, Q)}, \tag{6}$$

where $q(1, Q)$ denotes DM output in the symmetric equilibrium. It is straightforward to show that

$$\frac{\partial q}{\partial Q} = \frac{\frac{\delta}{\rho^2}\frac{\partial \rho}{\partial Q}}{\sigma u''(q)} > 0,$$

which implies that DM output rises following an unanticipated and permanent gold inflow from the rest of the world.

Money is *not* neutral in this model because increases in the money supply allow for further trading in the decentralized market. Although the value of gold declines following an unanticipated and permanent expansion in Q, the value of the buyer's portfolio rises in each country in the bloc. Conversely, the model also predicts that an unanticipated and permanent gold outflow from the countries in the bloc to the rest of the world results in a decline in DM output in the gold bloc. Thus, all factors that contribute to the removal of precious metals from the system, such as a *decrease* in the current account surplus with the rest of the world, are a contractionary force under the gold standard.[5]

The non-neutrality of money in our model supports mercantilist policies aimed at increasing a country's supply of bullion through a combination of tariffs, export subsidies, and colonial expansion (Heckscher, 1935). Even if those policies might create distortions (not included in our model), the welfare cost of such distortions might, in principle, be smaller than the welfare cost of lower DM output. We will revisit this point momentarily when we analyze the effects of an asymmetric productivity shock in the gold bloc.

This result also accounts for a number of gold devices used by many central banks during the late 19th century to increase their holdings of bullion, such as interest-free loans to gold importers or locating branches at ports of entry to the country to buy gold and, thus, lowering transportation costs from private importers (Eichengreen, 2019, p. 18).

Secular deflation. Another interesting experiment in the model is to study the effects of symmetric changes on the productivity of DM goods. The next proposition documents these effects when the productivity level rises or falls by the same amount in all countries in the gold bloc.

Proposition 2 *Suppose that $\omega^a = \omega^b = \omega$. If condition (5) holds, there is $\varepsilon \in (0,1)$ such that a unique symmetric stationary equilibrium, characterized by $\rho(\omega, Q)$, exists provided $\omega \in (1-\varepsilon, 1+\varepsilon)$. In this equilibrium, we have $\frac{\partial \rho}{\partial \omega} < 0$ and $\frac{\partial q}{\partial \omega} < 0$.*

Proof. We can generically characterize a symmetric stationary equilibrium by finding a solution $\rho = \rho(\omega, Q)$ to

$$\frac{(1+r)\rho}{\delta+\rho} = L_\omega\left(\left(\delta+\rho\right)Q\right). \tag{7}$$

Because condition (5) holds, there exists a sufficiently small $0 < \varepsilon < 1$ such that, for any $\omega \in (1-\varepsilon, 1+\varepsilon)$, we have:

$$Q < \frac{r\omega q^*(\omega)}{\delta(1+r)}. \tag{8}$$

[5] One advantage of the Lagos and Wright (2005) framework is that all the transitions (here and in the following results) occur in one period because there is no other state variable except the holdings of gold, which adjust in just one round of DM and CM trading.

Then, there is a unique $\bar{\rho}(\omega, Q) > \delta/r$ such that $[\delta + \bar{\rho}(\omega, Q)] Q = \omega q^*(\omega)$. Additionally, $\bar{\rho}(\omega, Q)$ is strictly decreasing in ω. If condition (8) holds, there is a unique $\rho(\omega, Q) \in (\delta/r, \bar{\rho}(\omega, Q))$ satisfying (7).

The *Implicit Function Theorem* implies $\frac{\partial \rho}{\partial \omega} < 0$ provided $-\frac{q u''(q)}{u'(q)} < 1$ holds for all $q > 0$. This result, together with (6), yields $\frac{\partial q}{\partial \omega} < 0$. ■

A symmetric increase in the productivity of DM goods results in a higher equilibrium value of gold and a larger output of goods that are purchased with money. A higher productivity level implies that DM goods become cheaper relative to CM goods. Thus, a larger value of liquid assets is required to attain the (now larger) surplus-maximizing quantity. The only way to accommodate this higher demand for money, given the unchanged gold stock, is to allow the value of gold to rise. This result shows that an increase in the productivity level of domestic industries is, *ceteris paribus*, a deflationary force in the international monetary system under the gold standard.

Our analysis shows that deflation is associated with a symmetric increase in productivity in the gold bloc. It reflects the fact that gains in the productivity of DM goods raise the demand for money in the economy, but the gold standard does not provide an automatic mechanism for the expansion of the money supply. The increase in the price of gold is the only available mechanism to accommodate the increase in the demand for goods that are purchased with money. This inelasticity of the money supply under the gold standard was already identified in the 19th century as a major flaw of the system. We shall argue in Section 5 that private money creation in the form of negotiable credit instruments can provide elasticity to the money supply to accommodate productivity gains.

This property of the model suggests that the secular deflation experienced in the gold bloc during the last quarter of the 19th century (Mitchell, 1975, p. 737) can be the result of sustained productivity gains in the domestic industries during the period, which was characterized by slow growth in the gold supply (or, more precisely in the data, did not grow as much as productivity).

3.2 Asymmetric equilibrium

Next, we explore the welfare implications of monetary movements (or specie flows) that result from an unanticipated and permanent change in the productivity level of the domestic industries in one country *only*. To characterize these monetary movements, we initially describe the *asymmetric* equilibrium that emerges in the world economy when $\omega^a = \omega < 1$ and $\omega^b = 1$.[6] Let $\left\langle \widehat{\rho}(\omega, Q), \widehat{Q}^a(\omega, Q) \right\rangle$ denote the value of gold and the gold supply in country a, respectively, in an asymmetric equilibrium. Additionally, let $\widehat{q}^i(\omega, Q)$ denote DM output in country $i \in \{a, b\}$. The following proposition establishes the existence of a unique asymmetric equilibrium.

[6] Recall that since ω can be interpreted as a utility cost, changes in ω have an alternative, yet equivalent, interpretation as preference shocks.

Proposition 3 *Suppose that* $\omega^a = \omega < 1 = \omega^b$*. Assume that* $u(q) = (1-\alpha)^{-1} q^{1-\alpha}$ *with* $0 < \alpha < 1$*. If condition (5) holds, for any* $\omega \in (0,1)$*, there is a unique pair* $\left\langle \widehat{\rho}(\omega, Q), \widehat{Q}^a(\omega, Q) \right\rangle$ *satisfying (2) and (3), together with* $\widehat{Q}^a(\omega, Q) \leq 2Q$*. In this equilibrium, we have* $\widehat{\rho}(\omega, Q) > \rho(1, Q)$*,* $\widehat{q}^a(\omega, Q) > q(1, Q) > \widehat{q}^b(\omega, Q)$*, and* $\widehat{Q}^a(\omega, Q) > Q$*. Additionally, we have* $\frac{\partial \widehat{\rho}}{\partial \omega} < 0$*,* $\frac{\partial \widehat{Q}^a}{\partial \omega} < 0$*,* $\frac{\partial \widehat{q}^a}{\partial \omega} < 0$*, and* $\frac{\partial \widehat{q}^b}{\partial \omega} > 0$ *in an open neighborhood of* $\omega = 1$*.*

Proof. Because $\omega q^*(\omega)$ is strictly decreasing in ω, we have:

$$Q < \frac{r \omega q^*(\omega)}{\delta(1+r)}$$

for any $\omega < 1$. This condition implies

$$u'\left(\frac{[\widehat{\rho}(\omega, Q) + \delta]}{\omega} \widehat{Q}^a(\omega, Q)\right) = \omega u'\left([\widehat{\rho}(\omega, Q) + \delta]\left[2Q - \widehat{Q}^a(\omega, Q)\right]\right).$$

Then, we have $\widehat{Q}^a(\omega, Q) = \frac{2Q}{1+\widehat{\omega}}$, where $\widehat{\omega} \equiv \omega^{\frac{1-\alpha}{\alpha}}$.

We can describe an asymmetric equilibrium as the solution $\widehat{\rho} = \widehat{\rho}(\omega, Q)$ to

$$\frac{(1+r)\widehat{\rho}}{\delta + \widehat{\rho}} = L_1\left((\widehat{\rho} + \delta)\frac{2\widehat{\omega}}{1+\widehat{\omega}} Q\right) \tag{9}$$

when condition (5) holds. If $\widehat{\rho} = \frac{\delta}{r}$, then $(\widehat{\rho} + \delta)\frac{2\widehat{\omega}}{1+\widehat{\omega}} Q = \frac{2\widehat{\omega}}{1+\widehat{\omega}} \frac{\delta Q(1+r)}{r} < \frac{2\widehat{\omega}}{1+\widehat{\omega}} q^* < q^*$. Define $\tilde{\rho}(\omega, Q) \equiv \frac{1+\widehat{\omega}}{2\widehat{\omega}} \frac{q^*}{Q} - \delta > \frac{\delta}{r}$. The right-hand side of (9) is strictly decreasing in $\widehat{\rho}$ in the range $\left(\frac{\delta}{r}, \tilde{\rho}(\omega, Q)\right)$. The left-hand side of (9) is strictly increasing in $\widehat{\rho}$ in the range $\left(\frac{\delta}{r}, \infty\right)$ and equals 1 at $\widehat{\rho} = \frac{\delta}{r}$. Then, there exists a unique $\widehat{\rho}(\omega, Q) \in \left(\frac{\delta}{r}, \tilde{\rho}(\omega, Q)\right)$ satisfying (9). Moreover, it follows that $\widehat{\rho}(\omega, Q) > \rho(1, Q)$ because the right-hand side of (9) is strictly decreasing in ω. ■

The previous result shows that the country with the most productive domestic industries attracts a disproportionately large amount of gold in the bloc. The least productive country loses gold, which reduces DM output in that country. Most importantly, if these equilibrium differences across countries are permanent, they have persistent real effects under the gold standard.

Specie flows. The previous results allow us to construct the following experiment. Suppose the world economy is initially at the symmetric equilibrium with $\omega^a = \omega^b = 1$. Assuming that condition (5) holds, there is a liquidity premium on gold, and both countries have the same money supply, DM output, and CM income stream from the ownership of gold. Then, consider a one-time unanticipated and permanent increase in the productivity of DM goods in country a. After the realization of this asymmetric and permanent shock, the changes in the domestic money supplies are:

$$\Delta^a = \widehat{Q}^a(\omega, Q) - Q > 0$$

and

$$\Delta^b = \underbrace{2Q - \widehat{Q}^a(\omega, Q)}_{\text{final position}} - \underbrace{Q}_{\text{initial position}} = Q - \widehat{Q}^a(\omega, Q) < 0.$$

The asymmetric shock results in a gold *inflow* to country a and a gold *outflow* from country b. The model predicts a redistribution of the gold stock within the bloc.

The increase in productivity in country a makes DM goods cheaper relative to CM goods in that country, which leads to an increase in the demand for money. Because the international price of gold rises and gold flows to country a, the exchange value of liquid assets rises in that country, and so does DM output. Country b loses gold. Although the value of gold is higher at the final equilibrium position, the gold loss in country b results in the fall of the exchange value of liquid assets in that country, leading to a permanent decline in DM output.

The long-term welfare change in country a is given by:

$$\delta\left[\widehat{Q}^a(\omega, Q) - Q\right] + u\left(\widehat{q}^a(\omega, Q)\right) - u\left(q(1, Q)\right) + q(1, Q) - \omega\widehat{q}^a(\omega, Q) > 0,$$

so the citizens of country a are better off after the realization of the shock. The long-term welfare change in country b is:

$$\delta\left[Q - \widehat{Q}^a(\omega, Q)\right] + u\left(\widehat{q}^b(\omega, Q)\right) - u\left(q(1, Q)\right) + q(1, Q) - \widehat{q}^b(\omega, Q) < 0.$$

so the citizens of country b are worse off after the realization of the shock.

There is one last term that we need to consider in the welfare analysis. In the period in which the permanent shock occurs, country a receives the previously described gold inflow but exports CM goods in exchange, and the opposite trade occurs in country b. This means that the citizens of country a exchange $\widehat{\rho}(\omega, Q)\left[\widehat{Q}^a(\omega, Q) - Q\right]$ units of the CM good for the extra per capita amount $\widehat{Q}^a(\omega, Q) - Q$ of gold. In the short run, the citizens of country b are better off because they sell gold abroad at a price higher than the fundamental value. However, there is a permanent reduction in the money supply in that country, which results in a permanently lower DM output level. If the agents in both countries are sufficiently patient, the short-term effects become negligible relative to the permanent changes in welfare associated with the new equilibrium position.

Gold export controls. Our analysis so far has shown that permanent productivity differentials have long-lasting consequences for the domestic economy of a country that participates in the gold standard. The citizens of the country that lags behind in productivity are worse off as a result of foreign technological development. The international character of the gold standard results in a channel for the transmission of productivity shocks when the worldwide gold supply is scarce. This property of the model can explain why peripheral countries that joined the gold standard experienced difficulties in the operation of their monetary system and frequently imposed controls on the export of bullion. For instance, country b would be better off if it unilaterally restricted the export of bullion to country a after the realization of the productivity shock.

The country that is likely to have higher productivity relative to the other gold standard countries benefits from the adherence of peripheral countries. In our example, if only country a participated in the gold standard, the final position of its economy (after the realization of the unanticipated and permanent productivity shock) would be precisely that described in Proposition 2. The increase in the value of gold would be larger and the expansion of DM output smaller. Thus, country a is better off at the final position when country b remains on the gold standard.

4 Abundant Gold Stock

We now consider the equilibrium allocation in the world economy when the world gold stock is sufficiently abundant to eliminate any liquidity premium on gold. Formally, we say that the gold stock is abundant when

$$Q \geq \frac{r \max\left\{\omega q^{*}\left(\omega\right), q^{*}\right\}}{\delta\left(1+r\right)}. \tag{10}$$

In this situation, the equilibrium value of gold must be equal to its fundamental value, that is, $\rho\left(\omega, Q\right) = \delta/r$. The next proposition shows that DM output attains its efficient level and that the money supply in each country in the bloc becomes indeterminate in equilibrium.

Proposition 4 *If condition (10) holds, then the equilibrium value of gold attains its fundamental level: $\rho\left(\omega, Q\right) = \delta/r$. If (10) holds as a strict inequality, the per capita money supply in each country is indeterminate, with $Q^{a}\left(\omega, Q\right)$ satisfying*

$$\frac{r \max\left\{\omega q^{*}\left(\omega\right), q^{*}\right\}}{\delta\left(1+r\right)} < Q^{a}\left(\omega, Q\right) < 2Q - \frac{r \max\left\{\omega q^{*}\left(\omega\right), q^{*}\right\}}{\delta\left(1+r\right)}.$$

The production and consumption amounts are the same across all equilibria, attaining their efficient levels.

Proof. It is straightforward to show that DM output equals the surplus-maximizing quantity when the value of gold attains its fundamental level. Then, $\rho\left(\omega, Q\right) = \delta/r$ implies:

$$\frac{\delta\left(1+r\right)}{r} Q^{a} \geq \max\left\{\omega q^{*}\left(\omega\right), q^{*}\right\}$$

and

$$\frac{\delta\left(1+r\right)}{r}\left(2Q - Q^{a}\right) \geq \max\left\{\omega q^{*}\left(\omega\right), q^{*}\right\}.$$

Any value of Q^{a} satisfying both conditions is consistent with an equilibrium in which the gold supply is abundant. ■

Although the money supply is indeterminate when condition (10) holds as a strict inequality, the equilibrium allocation is such that production and consumption attain their efficient levels. Thus, a sufficiently abundant gold supply in the world economy necessarily leads to an efficient equilibrium in the gold bloc. Additionally, if we consider an unanticipated and permanent shock to the productivity of DM goods in country a, it is straightforward to show that there is an equilibrium in which the allocation in country b remains unchanged so long as condition (10) continues to hold after the realization of the shock. Thus, the abundance of gold in the world economy implies that the real effects of productivity shocks disappear in the gold bloc. In this case, we can say that the gold standard is both efficient and stable (i.e., capable of absorbing shocks within the bloc).

5 The Development of Banking

So far, we have shown that a scarce gold supply results in an inefficient international monetary system and subjects the peripheral economies in the gold bloc to shocks. In this section, we investigate whether the introduction of inside money can lead to a welfare improvement over the economy with a single payment instrument and whether it can be a mechanism for insulating countries from shocks.

We are motivated in our investigation by a common view in the literature that monetary systems based on precious metals invariably result in a scarce money supply and that privately issued monies, such as bank deposits and bank notes, can increase the quantity of money in the economy, possibly resulting in a Pareto improvement.[7] The development of private money was observed in reality. In their seminal contribution to monetary history, Friedman and Schwartz (1963) documented the evolution of privately issued monies in the United States after the Civil War. More recently, Davies and Connors (2016) have described the evolution of banking and credit arrangements in the United Kingdom and other countries during the gold standard era.

Entrepreneurs. We extend the benchmark model to include private credit instruments that circulate as media of exchange. To achieve this goal, we add a third type of agent to the economy, referred to as an entrepreneur. Assume that entrepreneurs live for two periods, participate only in the CM, and consume only in old age (specifically, they derive linear utility from the consumption of the CM good when old).

A new generation of entrepreneurs is born each period. Entrepreneur j is endowed at birth with an indivisible and non-tradable project that requires one unit of the CM good as input and pays off γ_j units of the CM good in the following period. Project returns are known in advance, publicly observable, and heterogeneous across entrepreneurs. The support of the distribution of

[7]Some early models describing the interplay between inside and outside money and the welfare properties of monetary arrangements with multiple media of exchange include Cavalcanti et al. (1999) and Cavalcanti and Wallace (1999)

project returns is $[0, \tau]$ with $\tau > \beta^{-1}$, which implies that some projects are socially efficient to operate, but others are not. There is a measure $\eta > 0$ of entrepreneurs with each return γ in the support, so the total measure of entrepreneurs in each country is $\eta\tau$.

Entrepreneurs have no endowments. They must fund their project by issuing a debt claim in the CM, which provides the entrepreneur with one unit of the CM good today and entitles the lender to receive $1 + \mu_t^i \in \mathbb{R}_+$ units of the CM good in the following period. A pledgeability constraint limits their ability to issue a debt claim: Only a fraction $\theta \in (0, 1)$ of the project's return can be pledged to creditors. To look at interesting cases, we assume that $\frac{1}{\beta\theta} < \tau < \frac{2}{\theta}$ (i.e., the upper limit of the project returns is neither too low nor too high).

Banks. The debt claims issued by the entrepreneurs can be transformed into a liquid payment instrument via financial intermediation. We assume that banks can be created in the CM by issuing their debt claims, referred to as *bank deposits*, which are a transferable debt instrument that sells for one unit of the CM good in period t and entitles the *bearer* to receive $1 + \phi_t^i \in \mathbb{R}_+$ units of the CM good in the following period. Banks invest the proceeds from deposits in the debt claims issued by the entrepreneurs. We further assume that an individual bank specializes in providing loans to entrepreneurs of a single type γ. Because there is free entry into banking in each submarket, we have $\mu_t^i = \phi_t^i$ in equilibrium. Because bank deposits are a transferable debt instrument, buyers can use them as a means of payment in the DM.

Efficiency of aggregate investment. To comprehend the implications of privately issued payment instruments for the efficiency of aggregate investment, suppose, to the contrary, that bank deposits cannot be used as a medium of exchange in the DM (i.e., deposits are an illiquid asset). In this case, the equilibrium net return on bank deposits is $\phi_t^i = r$.

Because the deposit holders are both the original and the final creditors, in this case, they are willing to hold an illiquid credit instrument over time only if its real return is the same as the rate of time preference. Given that only a fraction of the project's return can be pledged to secure a bank loan and $\mu_t^i = \phi_t^i$ must hold in equilibrium, all entrepreneurs with types $\gamma \in [\frac{1}{\beta}, \frac{1}{\beta\theta})$ own a socially productive project but are *credit constrained* in the equilibrium with illiquid bank deposits. As a result, there would be *underinvestment* from a social perspective if bank deposits were an illiquid credit instrument.

Liquid bank deposits. Suppose now that bank deposits can be used as a means of payment in the DM. We further assume that agents in country a do not recognize bank deposits issued in country b, and vice versa (for example, because of the absence of a joint clearing house or due to enforcement problems under different legal systems). Let $b_t^i \in \mathbb{R}_+$ denote the buyer's holdings of bank deposits in country i. The terms of trade in a bilateral meeting are now given by $(q, z, y) \in \mathbb{R}_+^3$, where $y \in \mathbb{R}_+$ denotes the amount of deposits the buyer transfers to the seller.

The solution to the bargaining problem is now given by:

$$q_\omega(s,b,t) = \begin{cases} \frac{(\rho_t+\delta)s+\left(1+\phi_{t-1}^i\right)b}{\omega} \text{ if } (\rho_t+\delta)\,s + \left(1+\phi_{t-1}^i\right)b < \omega q^*(\omega) \\ q^*(\omega) \text{ otherwise,} \end{cases}$$

$$k_\omega(s,b,t) = \begin{cases} (\rho_t+\delta)\,s + \left(1+\phi_{t-1}^i\right)b \text{ if } (\rho_t+\delta)\,s + \left(1+\phi_{t-1}^i\right)b < \omega q^*(\omega) \\ \omega q^*(\omega) \text{ otherwise,} \end{cases}$$

where $k_\omega(s,b,t) = (\rho_t+\delta)\,z_\omega(s,b,t) + \left(1+\phi_{t-1}^i\right)y_\omega(s,b,t)$. In this solution, only the value of the buyer's total assets transferred to the seller is determinate. The composition of the assets used as a means of payment is indeterminate.

Supply of bank deposits. Consider now the entrepreneur's funding problem in country i. Because an entrepreneur of type γ born in period t is subject to the pledgeability restriction $1+\mu_t^i \leq \theta\gamma$, he will receive funding only if γ is greater than or equal to $\left(1+\mu_t^i\right)/\theta$. Because $\mu_t^i = \phi_t^i$ must hold in equilibrium, the supply of bank deposits in country i is given by:

$$\eta\left(\tau - \frac{1+\phi_t^i}{\theta}\right).$$

For any given θ, a reduction in the interest rate on deposits leads to an increase in aggregate investment by allowing a larger number of entrepreneurs to issue debt claims. The market-clearing condition in the deposit market is:

$$b_t^i = \eta\left(\tau - \frac{1+\phi_t^i}{\theta}\right)$$

for each country $i \in \{a,b\}$.

Equilibrium. The equilibrium of the economy with multiple payment instruments can be defined as a non-negative stream $\left\langle Q_t^a, \rho_t, \phi_t^a, \phi_t^b\right\rangle_{t\geq 0}$ satisfying $Q_t^a \leq 2Q$ and:

$$\begin{aligned} \frac{(1+r)\,\rho_t}{\delta+\rho_{t+1}} &= L_\omega\left((\delta+\rho_{t+1})\,Q_t^a + \eta\,(1+\phi_t^a)\left(\tau - \frac{1+\phi_t^a}{\theta}\right)\right) \\ \frac{(1+r)\,\rho_t}{\delta+\rho_{t+1}} &= L_1\left((\delta+\rho_{t+1})\,(2Q-Q_t^a) + \eta\left(1+\phi_t^b\right)\left(\tau - \frac{1+\phi_t^b}{\theta}\right)\right) \\ 1+\phi_t^a = 1+\phi_t^b &= \frac{\delta+\rho_{t+1}}{\rho_t} \end{aligned} \quad (11)$$

at all dates. These conditions imply $\phi_t^a = \phi_t^b$ along the equilibrium path. Although bank deposits issued in one country can only circulate locally, the fact that, in equilibrium, their expected return must be the same as that of gold implies the equality of returns across borders. A positive mass of entrepreneurs will always issue debt claims and buyers will always find it optimal to hold gold. As a result, condition (11) must hold in equilibrium.

In what follows, we restrict attention to stationary equilibria. Let $1+\phi$ denote the interest rate on bank deposits in a stationary equilibrium. Then, condition (11) implies:

$$1+\phi = 1+\frac{\delta}{\rho} \equiv f(\rho).$$

Notice that $f(\delta/r) = 1+r$, $f' < 0$, and $f(\infty) = 1$. When the value of gold equals the fundamental value, the rate of return on bank deposits must be the same as the rate of time preference. Second, the higher the value of gold, the lower the rate of return on deposits. Third, there exists a lower bound on the interest rate on deposits; specifically, it *cannot* be negative (i.e., there is a zero lower bound on the *real* interest rate in this economy).

It is helpful to define the exchange value of bank deposits as a function of the value of gold. Define $\widehat{H} : [\delta/r, \infty) \times (0,1) \to \mathbb{R}_+$ by

$$\widehat{H}(\rho,\theta) \equiv f(\rho)\left[\tau - \frac{f(\rho)}{\theta}\right].$$

Because $\frac{1}{\beta\theta} < \tau < \frac{2}{\theta}$, it is straightforward to show that $\frac{\partial \widehat{H}}{\partial \rho} > 0$ and $\frac{\partial \widehat{H}}{\partial \theta} > 0$ hold at any interior point.

We can define a stationary equilibrium of the economy with multiple payment instruments as a non-negative vector $\langle Q^a, \rho \rangle$ satisfying $Q^a \leq 2Q$ and

$$\frac{(1+r)\rho}{\delta+\rho} = L_\omega\Big((\rho+\delta)Q^a + \eta\widehat{H}(\rho,\theta)\Big) \quad (12)$$

$$\frac{(1+r)\rho}{\delta+\rho} = L_1\Big((\rho+\delta)(2Q-Q^a) + \eta\widehat{H}(\rho,\theta)\Big). \quad (13)$$

Let $Q^{a,*}(\omega, Q)$ and $\rho^*(\omega, Q)$ denote the equilibrium values in this economy.

5.1 Scarce aggregate liquidity

Consider initially the symmetric position: $\omega = 1$. In this case, we have $Q^{a,*}(1,Q) = Q$. The equilibrium value of gold can be determined as the solution to:

$$\frac{(1+r)\rho}{\delta+\rho} = L_1\Big((\rho+\delta)Q + \eta\widehat{H}(\rho,\theta)\Big). \quad (14)$$

The condition for the scarcity of aggregate liquidity in the world economy is now given by:

$$\frac{\delta(1+r)Q}{r} + \eta(1+r)\left(\tau - \frac{1+r}{\theta}\right) < q^*. \quad (15)$$

If condition (5) holds, the gold stock is scarce in the case where we only allow gold certificates. The inequality in (15) implies that aggregate liquidity is also scarce in the case with multiple payment instruments, provided the mass of investment projects is not too large.

The benchmark allocation for studying the welfare properties of the economy with inside money is the symmetric stationary equilibrium of the economy with illiquid bank deposits. If condition (5) holds, the equilibrium values of ρ and Q^a are the same as those of Proposition 2, and the equilibrium interest rate is $\phi = r$, which implies that aggregate investment is given by $\eta\left(\tau - \frac{1+r}{\theta}\right)$. As we have seen, this investment level is inefficiently low when $\theta < 1$.

Welfare-improving inside money. Given the previously described benchmark allocation, the following proposition shows the existence of a unique symmetric equilibrium in the economy with multiple payment instruments and establishes the condition for a Pareto improvement over the economy with illiquid bank deposits.

Proposition 5 *Suppose that $\omega = 1$. If condition (15) holds, there exists a unique symmetric stationary equilibrium characterized by the solution $\rho^*(1, Q)$ to (14). In this equilibrium, we have $\frac{\partial \rho^*}{\partial Q} < 0$. The ensuing equilibrium allocation Pareto dominates the equilibrium allocation with illiquid bank deposits provided $f(\rho^*(1, Q)) \geq \theta(1 + r)$.*

Proof. Define $\tilde{H}(\rho, Q) \equiv (\delta + \rho) Q + \eta \widehat{H}(\rho, \theta)$, which is strictly increasing in both arguments. If (15) holds, there exists a unique $\tilde{\rho}(1, Q) > \delta/r$ such that $\tilde{H}(\tilde{\rho}(1, Q), Q) = q^*$. Because $\tilde{H}(\rho, Q) > (\delta + \rho) Q$ for all $\rho \geq \delta/r$, we have $\tilde{\rho}(1, Q) < \bar{\rho}(1, Q)$. Then, there exists a unique $\rho^*(1, Q) < \rho(1, Q)$ satisfying (14). It is straightforward to show $\partial \rho^*/\partial Q > 0$.

To verify that the unique equilibrium allocation Pareto dominates the allocation with illiquid bank deposits, note that $\theta(1 + r)$ is the interest rate that maximizes the present value of investment projects. Because $f(\rho^*(1, Q)) < 1+r$, all entrepreneurs with types $\gamma \in [f(\rho^*(1, Q)), 1+r)$ are credit constrained in the equilibrium with illiquid bank deposits. In the equilibrium with liquid bank deposits, they receive funding because of the lower interest rate on deposits.

If $f(\rho^*(1, Q)) \geq \theta(1 + r)$, the present value of investment projects necessarily increases from the initial position at $1 + r$, so no entrepreneur is worse off, and some of them are strictly better off. Finally, we can use condition (6) to show that the lower value of gold in the equilibrium with liquid bank deposits implies a larger surplus in the DM when (15) holds. Therefore, the equilibrium of the economy with multiple payment instruments Pareto dominates the equilibrium with illiquid bank deposits. ■

This proposition shows that the introduction of inside money in the form of transferable debt claims leads to a Pareto improvement over the allocation with illiquid bank deposits when gold is scarce in the world economy. An interesting property of the equilibrium is that the value of gold is lower than that of the economy with illiquid bank deposits, moving it closer to its fundamental value. In this respect, we can say that the rise of bank deposits as a medium of exchange eases the strains on a monetary system based exclusively on gold, expanding aggregate liquidity, and leading to a lower interest rate.

The sufficient condition for a Pareto improvement, $f(\rho^*(1,Q)) \geq \theta(1+r)$, deserves some attention. The term $\theta(1+r)$ gives the interest rate that maximizes the present value of investment projects. To verify this claim, notice that the welfare derived from the consumption of CM goods is given by:

$$\frac{\eta}{1+r}\int_{\frac{1+\phi}{\theta}}^{\tau}\gamma d\gamma - \eta\left(\tau - \frac{1+\phi}{\theta}\right) = \frac{\eta}{2(1+r)}\left[\tau - \left(\frac{1+\phi}{\theta}\right)^2\right] - \eta\left(\tau - \frac{1+\phi}{\theta}\right)$$

with $1+\phi = f(\rho)$. This expression is strictly concave in $1+\phi$ and attains its maximum at $1+\phi = \theta(1+r)$. If $f(\rho^*(1,Q)) \geq \theta(1+r)$ holds in the equilibrium with liquid bank deposits, then the welfare derived from the net consumption of CM goods in this equilibrium is strictly higher than that derives from the equilibrium with illiquid bank deposits. Because DM output is larger in the equilibrium with liquid bank deposits, the introduction of inside money results in a Pareto improvement.

Asymmetric equilibrium and specie flows. Our next step is to show that the real effects on the domestic economy of an unanticipated and permanent shock to the productivity of DM goods in a foreign country are similar to those documented for the economy with a single payment instrument. As before, we first demonstrate the existence of a unique asymmetric equilibrium for the economy with multiple payment instruments when $\omega < 1$.

Proposition 6 *Suppose that $u(q) = (1-\alpha)^{-1} q^{1-\alpha}$ with $0 < \alpha < 1$. If condition (15) holds, we find that, for any $\omega \in (0,1)$, there is a unique pair $\langle \rho^*(\omega,Q), Q^{a,*}(\omega,Q)\rangle$ that satisfies conditions (12) and (13), together with $Q^{a,*}(\omega,Q) \leq 2Q$. In this equilibrium, we have $\rho^*(\omega,Q) > \rho^*(1,Q)$, $q^{a,*}(\omega,Q) > q^*(1,Q) > q^{b,*}(\omega,Q)$, and $Q^{a,*}(\omega,Q) > Q$ if $\omega \in (0,1)$. Additionally, we have $\frac{\partial \rho^*}{\partial \omega} < 0$, $\frac{\partial Q^{a,*}}{\partial \omega} < 0$, $\frac{\partial q^{a,*}}{\partial \omega} < 0$, and $\frac{\partial q^{b,*}}{\partial \omega} > 0$.*

Proof. By following the same steps as those of Proposition 3, we can show that, when condition (15) holds, an asymmetric equilibrium can be described as a solution to

$$\frac{(1+r)\rho}{\delta+\rho} = L_1\left(\frac{2\widehat{\omega}}{1+\widehat{\omega}}\left[(\delta+\rho)Q + \eta\widehat{H}(\rho,\theta)\right]\right). \tag{16}$$

We can define $\tilde{H}(\rho,Q) \equiv (\delta+\rho)Q + \eta\widehat{H}(\rho,\theta)$ as before. Then, we can show that, for any $\omega \in (0,1)$, there is a unique $\rho = \rho^*(\omega,Q)$ satisfying (16). The other properties of equilibrium can be derived by following the same steps as those of Proposition 3. ■

Given this result, we can now perform the same kind of experiment as that described in the previous section. Suppose that condition (15) holds initially. If we consider the same type of unanticipated and permanent productivity shock in the foreign country (country a), we find that the domestic economy (country b) is subject to the same adverse effects of an external shock as those described in the previous section. Although the creation of inside money increases the supply of liquid assets in the world economy, such an increase in aggregate liquidity is still insufficient to insulate the domestic economy from the effects of external productivity shocks if condition (15) holds initially.

5.2 International transmission of banking crises

Although the introduction of inside money in the form of bank deposits leads to an expansion of aggregate liquidity in the world economy, many authors have argued that inside money can be a source of instability for the international monetary system. The reason is that a banking crisis in one country can precipitate specie flows in world markets that result in declining aggregate liquidity in other countries. In our model, fluctuations in the supply of inside money can be caused by an external shock to the pledgeability parameter θ in a foreign country. In particular, we consider the case of an unanticipated and permanent decline in the value of the pledgeable portion of a project's return in country a, which can be interpreted as a banking crisis in that country, and study its effects on the domestic economy (country b).

To characterize the effects of a banking crisis in country a, suppose that all countries have the same productivity level so that $\omega = 1$. Let $\bar{\theta} \in (0, 1)$ denote the initial value of the pledgeable portion of a project's return in all countries, and suppose that the final value of this parameter in country a is $\theta \in \left(\bar{\theta} - \varepsilon, \bar{\theta}\right)$, with $0 < \varepsilon < \bar{\theta}$. At the initial position, we assume that:

$$\frac{\delta (1 + r) Q}{r} + \eta (1 + r) \left(\tau - \frac{1 + r}{\bar{\theta}}\right) < q^*. \tag{17}$$

Thus, the initial equilibrium is characterized by a scarce supply of aggregate liquidity, with the same allocation of resources in all countries in the bloc.

In this environment, we can describe a stationary equilibrium for the gold bloc as a non-negative vector $\langle Q^a, \rho \rangle$ satisfying $Q^a \leq 2Q$ and

$$\frac{(1 + r) \rho}{\delta + \rho} = L_1 \left((\delta + \rho) Q^a + \eta \widehat{H} (\rho, \theta)\right) \tag{18}$$

$$\frac{(1 + r) \rho}{\delta + \rho} = L_1 \left((\delta + \rho) (2Q - Q^a) + \eta \widehat{H} \left(\rho, \bar{\theta}\right)\right). \tag{19}$$

Let $\rho^* (\theta, Q)$, $Q^{a,*} (\theta, Q)$, $q^{a,*} (\theta, Q)$, and $q^{b,*} (\theta, Q)$ denote the equilibrium value of gold, the money supply in country a, DM output in country a, and DM output in country b, respectively, as a function of θ and Q. Condition (17) implies $\rho^* \left(\bar{\theta}, Q\right) > \delta / r$, $Q^{a,*} \left(\bar{\theta}, Q\right) = Q$, and $q^{a,*} \left(\bar{\theta}, Q\right) = q^{b,*} \left(\bar{\theta}, Q\right) < q^*$ at the initial position.

The following proposition establishes the existence of a unique asymmetric equilibrium with respect to θ when $\theta \in \left(\bar{\theta}-\varepsilon,\bar{\theta}\right)$. This asymmetric equilibrium gives the final position of the economies in the gold bloc after a banking crisis occurs in country a.

Proposition 7 *If condition (17) holds, there is $\varepsilon \in \left(0,\bar{\theta}\right)$ such that, for any $\theta \in \left(\bar{\theta}-\varepsilon,\bar{\theta}\right)$, there exists a unique pair $\langle \rho^*(\theta,Q), Q^{a,*}(\theta,Q)\rangle$ that satisfies conditions (18) and (19), together with the feasibility condition $Q^{a,*}(\theta,Q) \leq 2Q$. In this equilibrium, we have $\rho^*(\theta,Q) > \rho^*\left(\bar{\theta},Q\right)$, $q^{a,*}(\theta,Q) = q^{b,*}(\theta,Q) < q^{a,*}\left(\bar{\theta},Q\right) = q^{b,*}\left(\bar{\theta},Q\right)$, $Q^{a,*}(\theta,Q) > Q$, $\frac{\partial\rho^*}{\partial\theta} < 0$, and $\frac{\partial Q^{a,*}}{\partial\theta} < 0$.*

Proof. If condition (17) holds, there is $\varepsilon > 0$ such that:

$$\frac{\delta(1+r)Q}{r} + \eta(1+r)\left(\tau - \frac{1+r}{\theta}\right) < q^*$$

holds for any $\theta \in \left(\bar{\theta}-\varepsilon,\bar{\theta}\right)$. If $\theta = \bar{\theta}$, we have $Q^{a,*}\left(\bar{\theta},Q\right) = Q$, and the equilibrium value of gold, $\rho^*\left(\bar{\theta},Q\right)$, is given by the unique solution to:

$$\frac{(1+r)\rho}{\delta+\rho} = L_1\left((\delta+\rho)Q + \eta\widehat{H}\left(\rho,\bar{\theta}\right)\right).$$

In this equilibrium, we have $\rho^*\left(\bar{\theta},Q\right) > \delta/r$ so that:

$$L_1\left(\left[\delta+\rho^*\left(\bar{\theta},Q\right)\right]Q + \eta\widehat{H}\left(\rho^*\left(\bar{\theta},Q\right),\bar{\theta}\right)\right) > 1.$$

There is $\varepsilon > 0$ sufficiently small such that, for any $\theta \in \left(\bar{\theta}-\varepsilon,\bar{\theta}\right)$, we have:

$$\left[\delta+\rho^*(\theta,Q)\right]Q^{a,*}(\theta,Q) = \left[\delta+\rho^*(\theta,Q)\right]Q + \frac{\eta}{2}\left[\widehat{H}\left(\rho^*(\theta,Q),\bar{\theta}\right) - \widehat{H}\left(\rho^*(\theta,Q),\theta\right)\right],$$

so (18) and (19) can be written as a single equation:

$$\frac{(1+r)\rho}{\delta+\rho} = L_1\left((\delta+\rho)Q + \frac{\eta}{2}\left[\widehat{H}\left(\rho,\bar{\theta}\right) + \widehat{H}(\rho,\theta)\right]\right). \tag{20}$$

It remains to be shown that this equation has a unique solution $\rho = \rho^*(\theta,Q)$.

Because $\partial\widehat{H}/\partial\theta > 0$, we have $\widehat{H}(\rho,\theta) < \widehat{H}\left(\rho,\bar{\theta}\right)$ for any $\theta \in \left(\bar{\theta}-\varepsilon,\bar{\theta}\right)$, which implies $\frac{\eta}{2}\left[\widehat{H}\left(\rho,\bar{\theta}\right) + \widehat{H}(\rho,\theta)\right] < \eta\widehat{H}\left(\rho,\bar{\theta}\right)$. Following the same steps as before, we find that there is a unique $\rho^*(\theta,Q) > \rho^*\left(\bar{\theta},Q\right) > \delta/r$ satisfying equation (20).

Finally, notice that:

$$Q^{a,*}(\theta,Q) = Q + \frac{\eta\left[\widehat{H}\left(\rho^*(\theta,Q),\bar{\theta}\right) - \widehat{H}\left(\rho^*(\theta,Q),\theta\right)\right]}{2\left[\delta+\rho^*(\theta,Q)\right]} > Q = Q^{a,*}\left(\bar{\theta},Q\right),$$

which establishes that $Q^{a,*}(\theta,Q)$ is strictly decreasing in θ. ■

A banking crisis in country a results in a decline in the supply of deposits and a gold inflow to that country. Money holdings increase in country a, but aggregate liquidity declines because the decline in the value of deposits is greater than the increase in money balances. The existence of an international market for gold and the coexistence of gold certificates and bank deposits as media of exchange imply that the interest rate on deposits declines by the same amount in all countries under the gold standard. The supply of deposits in country b increases because of the decline in the interest rate, but the gold supply falls in that country. The overall effect in country b is a decline in aggregate liquidity and a decrease in the output of goods purchased with money.

Thus, a banking crisis that originates in a foreign country sets in motion international specie flows that result in an increase in the price of gold and a decline in the interest rate in all countries under the gold standard. The lower interest rate expands aggregate investment in the domestic economy, but the ensuing gold loss results in a decline in money balances. The overall effect on the value of liquid assets in the domestic economy is such that aggregate liquidity falls and so does the output of goods purchased with money. Depending on the parameter values, the welfare level in the domestic economy can decline as a result of a foreign development. The possibility of such a permanent loss of welfare can be the reason for the adoption of restrictions on the export of bullion (i.e., going off the gold standard) as a means of limiting the exposure of a country to international banking crises.[8]

5.3 Abundant aggregate liquidity

Next, we show that a sufficiently abundant mass of investment projects in the countries in the gold bloc eliminates any liquidity premium on money-like assets and results in the efficient DM output. However, the equilibrium does not attain the first best because aggregate investment is suboptimal. Suppose that condition (5) holds so that bullion is scarce in the world economy. The condition for the abundance of aggregate liquidity in the world economy when multiple assets serve as media of exchange is given by:

$$\max\left\{q^*, \omega q^*\left(\omega\right)\right\} \leq \frac{\delta\left(1+r\right)Q}{r} + \eta\left(1+r\right)\left(\tau - \frac{1+r}{\theta}\right). \tag{21}$$

For a sufficiently large value for η, condition (21) necessarily holds. It is, then, straightforward to show the following result.

Proposition 8 *If condition (21) holds, then the equilibrium value of gold attains its fundamental level: $\rho^*\left(\omega, Q\right) = \delta/r$. The equilibrium interest rate on bank deposits equals the rate of time preference. The ensuing equilibrium allocation Pareto dominates the equilibrium allocation with*

[8]Another type of policy intervention is to sterilize the inflow of gold to the country where the crisis has occurred, as discussed in Friedman and Schwartz (1963). Our analysis has shown that such a policy intervention leads to a suboptimal outcome in country a.

illiquid bank deposits. However, it does not attain the first best because aggregate investment is suboptimal when $\theta < 1$.

Lagos and Rocheteau (2008) have studied the welfare properties of an economy with liquid capital (i.e., an economy in which claims on the neoclassical capital technology can be used as a medium of exchange in bilateral meetings). In their analysis, capital is a productive technology that requires units of the CM good in the current period to obtain a return in the following period. They find that when the capital technology is sufficiently productive, there is no liquidity premium on capital goods, and the equilibrium accumulation of capital goods coincides with the social optimum.

In our analysis, the pledgeability restriction on investment projects leads to an inefficiently low investment level if no liquidity premium emerges on claims backed by capital. When the world gold supply is scarce, and the mass of investment projects is relatively small so that condition (15) holds, the use of bank deposits as a payment instrument leads to a Pareto improvement over the allocation with illiquid bank deposits by increasing DM output *and* by increasing the utility flow associated with investment projects. When the world gold supply is scarce, and the mass of investment projects is sufficiently large so that condition (21) holds, the use of bank deposits as a means of payment leads to a Pareto improvement by increasing DM output to its efficient level, with aggregate investment unchanged. As previously mentioned, the first best is not attained because aggregate investment is below the social optimum when $\theta < 1$.

6 Conclusions

This paper has developed a model of the gold standard that illustrates the positive and normative properties of the equilibrium allocation under two distinct regimes: when the world gold supply is scarce and when it is abundant. In particular, we have characterized the specie flows associated with an asymmetric productivity shock in the gold bloc and with the occurrence of a banking crisis in one country in the bloc. The real effects of the international monetary movements that follow from the realization of either one of these shocks can result in a large welfare loss for peripheral countries under the gold standard. If the world gold stock is scarce, which seems to be the case throughout modern history, the risks associated with adherence to the gold bloc can be too large for the countries at the periphery, which are likely to lag in productivity growth relative to the other countries in the bloc. Our analysis suggests that the gold standard can be sustainable at the core, but its survival at the periphery is unlikely to occur because of the associated risks.

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