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ABSTRACT

To provide sharp answers to basic questions in international trade, a standard approach is to focus on a small open economy (SOE). Whereas the classic tradition is to define a SOE as an economy that takes world prices as given, in the modern trade literature a SOE is defined instead as one that takes foreign-good prices and export demand schedules as given. In this paper we develop a generalized gravity model that nests all of its standard microfoundations (e.g., Armington and Melitz-Pareto) and show how to take the limit so that an economy that becomes infinitesimally small behaves like a SOE. We then show how the resulting model of a SOE can be used to understand comparative statics and the optimal tariff in a way that is robust across the different microfoundations consistent with the gravity model.

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1 Introduction

How does an improvement in foreign productivity affect trade flows, prices, and wages? What are the welfare effects of import tariffs? What are optimal policies in open economies facing domestic distortions? To provide sharp answers to these and related questions in international trade, a standard approach has been to simplify the analysis by focusing on a small open economy (SOE). In the classical literature, a SOE is defined as an economy that takes world prices as given. In the context of modern trade theory, however, even infinitesimally small countries have pricing power, so a different conceptualization is needed.[1]

Flam and Helpman (1987) were the first to consider a SOE assumption in a new trade model, which they used to study the effects of various trade and industrial policies under monopolistic competition. Demidova and Rodríguez-Clare (2009) refined Flam and Helpman's definition of a SOE as one that takes foreign-good prices and export demand schedules as given, and further showed how to extend the assumption to a setting with heterogeneous firms and selection a la Melitz (2003).[2] This modern version of the SOE assumption has now been used to study the comparative statics of trade-cost shocks (e.g., Demidova and Rodríguez-Clare, 2013), optimal trade policy (e.g., Demidova and Rodríguez-Clare, 2009; Haaland and Venables, 2016), and optimal industrial policy in open economies (e.g., Bartelme et al., 2021), among several different applications.

In this paper we revisit the SOE assumption in a generalized gravity model of trade that nests all the standard microfoundations that have been provided for such a model. We show how one can obtain the SOE as the limit in which an economy becomes infinitesimally small, although one must simultaneously let trade costs go to infinity to avoid awkward implications in the limit. The finding that this limit yields the SOE assumptions is important to formally link the results derived for the SOE to those derived in the standard case with large economies. We illustrate the usefulness of the SOE model by studying its implications for comparative statics and the optimal tariff.

Rather than limiting the analysis to a particular gravity microfoundation, as, for example, Demidova and Rodríguez-Clare (2009) do with the Melitz-Pareto model, we consider

[1]The fact that even infinitesimally small countries retain market power is why Gros (1987) finds that the optimal tariff in a Krugman (1980) setting does not converge to zero as the economy's size becomes infinitesimally small. Alvarez and Lucas (2007) reach the same conclusion in the context of the Ricardian model developed by Eaton and Kortum (2002) with productivity assumed proportional to country size.

[2]In the Krugman (1980) model, Flam and Helpman's SOE takes as given the wage and the variety of goods in the rest of the world. The latter assumption implies that the export demand curve is fixed but not isoelastic. Demidova and Rodríguez-Clare (2009) instead take the wage and price index in the rest of the world as given, leading to an isoelastic export demand curve.

a general framework that nests the Armington and Eaton-Kortum models with external economies of scale (EES) and the Krugman and Melitz-Pareto models with nested preferences as in Kucheryavyy et al. (forthcoming). We also allow for the fixed trade costs in the Melitz model to be in terms of labor in the source or destination country. This generality is possible by allowing for a positive scale elasticity and three different trade elasticities: one with respect to trade costs, one with respect to tariffs, and one with respect to wages. Specific models are obtained from particular combinations of these elasticities. For example, the Krugman (1980) model corresponds to the case in which the three trade elasticities are the same and the scale elasticity is the inverse of this common trade elasticity. As another example, the Melitz-Pareto model with fixed trade costs paid in labor of the destination country is obtained by setting the scale elasticity equal to the inverse of the trade elasticity with respect to trade costs, and the trade elasticity with respect to trade costs equal to the one with respect to wages but lower than the one with respect to tariffs.

Simply letting an economy become infinitesimally small in such a framework implies that the domestic trade share tends to zero in the limit.[3] This not only makes it impossible to map the SOE to data, but it also leads to the awkward implication that the SOE would experience zero gains from optimal trade policy and infinite gains from trade.[4] To avoid this, we assume that inward and outward trade costs go to infinity. At one extreme, if the scale elasticity is zero – as in the Armington or Eaton-Kortum models with no EES – then we have the outward trade costs go to infinity; at the other extreme, if the scale elasticity is equal to the inverse of the trade elasticity – as in the standard Krugman and Melitz-Pareto models – then it is the inward trade costs that go to infinity. Between these extremes, both outward and inward trade costs go to infinity at a rate determined by the scale and trade elasticity with respect to trade costs.

The equilibrium conditions in the SOE are simple and intuitive. As illustrated in Figure 1, the equilibrium wage w is determined by the intersection of the downward sloping export demand curve $X(w) = D\left(AL^{\phi}\right)^{\varepsilon} w^{-\rho}$ and the upward sloping import demand curve $M(w) = \frac{1-\lambda(w)}{1+(t-1)\lambda(w)} wL$, with $\lambda(w) = \frac{\left(AL^{\phi}\right)^{\varepsilon} w^{-\rho}}{\left(AL^{\phi}\right)^{\varepsilon} w^{-\rho} + t^{-\zeta}\mathcal{P}^{-\rho}}$ being the domestic trade share. Here D and $\mathcal{P}$ are exogenous parameters that capture the SOE's access to foreign markets on the export and import sides, respectively; A and L are productivity and labor endowment in the SOE; ϕ is the scale elasticity; ε, ρ, and ζ are the trade elasticity with

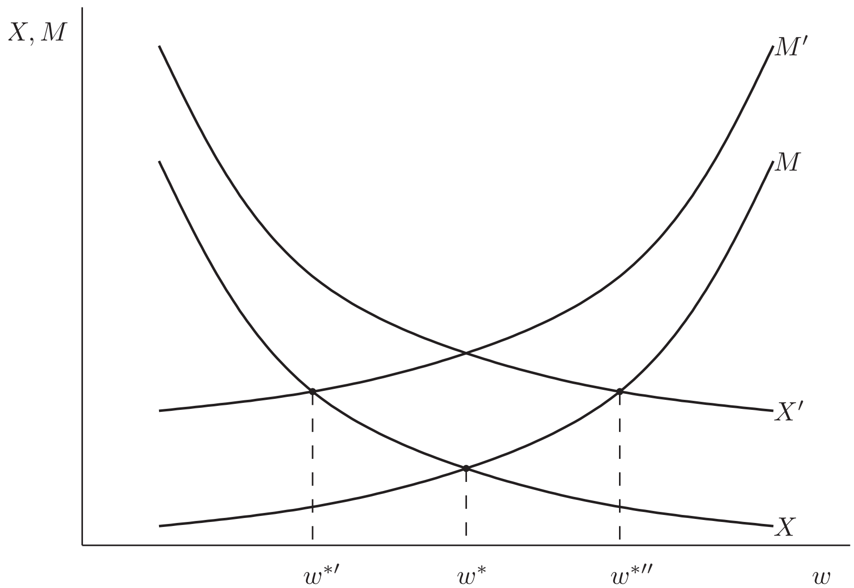
[3] See, for example, Caliendo and Parro (2022), who derive the SOE's optimal tariffs of Gros (1987), Alvarez and Lucas (2007), and Demidova and Rodriguez-Clare (2009) separately by taking labor of one of two countries to zero.

[4] Demidova and Rodríguez-Clare (2013) argue that they can obtain their SOE (with nonzero domestic trade share) in the Melitz-Pareto model by letting the economy become infinitesimally small, but their analysis is incorrect as the values of wage and productivity cutoffs in the limit were miscalculated due to wrongly assuming that the wage was strictly positive in the limit.

respect to trade costs, wages, and tariffs, respectively; and t is (one plus) the SOE's import tariff. In turn, the gains from trade (equilibrium welfare divided by counterfactual autarky welfare) are given by

$$\mathrm{GT} = \lambda^{-1/\varepsilon} \left(\lambda + (1-\lambda)/t \right)^{-\zeta/\varepsilon}.$$

This collapses to the expression for gains from trade in Arkolakis et al. (2012) – henceforth ACR – if there are no tariffs ($t = 1$). Even with tariffs ($t > 1$), GT is decreasing in λ.



We can now use a simple graphical analysis to understand how different shocks affect the wage, trade flows, and welfare. An improvement in foreign productivity or a decline in inward trade costs would correspond to a decline in $\mathcal{P}$, leading to an upward shift in the M curve and a decline in the equilibrium wage. An increase in export demand corresponds to an increase in D, which leads to an upward shift in the X curve and an increase in the equilibrium wage. While the wage moves in opposite directions, in both cases there is an increase in imports (or exports) evaluated at international prices. This leads to a decline in the domestic trade share and an increase in the gains from trade.

Maximizing GT with respect to t yields the optimal tariff, which depends intuitively on the values of the different elasticities, as implied by our formula

$$t^* - 1 = \frac{1}{(1+\rho)(\zeta/\rho) - 1}.$$

Except for the Melitz-Pareto model with fixed trade costs paid in destination-country labor, all the microfoundations nested by our generalized gravity model entail $\zeta = \rho$ and so the optimal tariff is equal to the inverse of the trade elasticity with respect to wages, $t^* - 1 = 1/\rho$. The Armington, Eaton-Kortum, and Krugman models (with or without EES, and with or without nested preferences) have $\rho = \varepsilon$ and so the optimal tariff is given by the inverse of the trade elasticity with respect to trade costs, as in Gros (1987) for the Krugman model and Alvarez and Lucas (2007) for the Eaton-Kortum model. In the Melitz-Pareto model with fixed trade costs paid in source-country labor we have $\rho > \varepsilon$. Thus, consistent with Demidova and Rodríguez-Clare (2009), the optimal tariff in this model is lower than the inverse of the trade elasticity with respect to trade costs, $t^* - 1 = 1/\rho < 1/\varepsilon$. Finally, the Melitz-Pareto model with fixed trade costs paid in destination-country labor entails $\zeta > \rho = \varepsilon$, and hence, an optimal tariff even lower than the trade elasticity with respect to wages, $t^* - 1 < 1/\rho = 1/\varepsilon$.

Our analysis is closely related to a contemporaneous paper by Caliendo and Feenstra (2022), in which they also study how to take a limit so as to achieve a SOE with a strictly positive domestic trade share. We highlight three differences between the two papers. First, we develop a generalized model that nests all standard microfoundations for the gravity model of trade and then take the limit as one economy's size falls to zero, whereas Caliendo and Feenstra (2022) take the limit separately for each of the different microfoundations. We view our approach as having the benefit of simplicity and highlighting sufficient statistics that are common across all models, in the spirit of ACR and Kucheryavyy et al. (forthcoming). Second, we develop a simple and intuitive graphical approach to comparative statics for the SOE, as illustrated in Figure 1. Third, our analysis for the optimal tariff is straightforward and yet valid across all the microfoundations nested in our generalized gravity model.

The rest of the paper is organized as follows. Section 2 presents the generalized gravity model, establishes that the equilibrium is unique, and describes how it nests the different microfoundations. Section 3 shows how to take the limit as one economy becomes infinitesimally small and describes the equilibrium of the resulting SOE. Section 4 studies comparative statics and the optimal tariff for the SOE, and Section 5 concludes.

2 A Generalized One-Sector Gravity Model

In this section we present a generalized one-sector and one-factor trade model exhibiting external economies of scale (EES) and satisfying a standard gravity equation. As shown in Appendix A, there are five different sets of microfoundations leading to the model equations that we present next: (i) an Armington model with technological EES; (ii) an Eaton-Kortum model with technological EES; (iii) a generalized Krugman model with nested CES preferences; (iv) a generalized Melitz-Pareto model with nested CES preferences and fixed trade costs paid in labor of destination countries; and (v) a generalized Melitz-Pareto model with nested CES preferences and fixed trade costs paid in labor of source countries. The nested CES preferences in the last three models allow for a different elasticity of substitution between varieties produced within the same country and those produced across different countries. In turn, this allows the scale elasticity (defined below) to be different than the inverse of the trade elasticity.[5]

2.1 Gravity, Price Index, and Trade Balance

There are $N+1$ countries indexed by $i, j, l = 0, 1, ..., N$. We let w_i and L_i denote the wage and labor endowment of i, A_i be a productivity shifter for i, τ_{ij} be the ad-valorem trade cost from i to j, and t_{ij} denote one plus the ad-valorem tariff that j imposes on imports from i. Without loss of generality, we set $\tau_{jj} = t_{jj} = 1$. Trade shares $\lambda_{ij} \equiv X_{ij} / \sum_l X_{lj}$, where X_{ij} is j's expenditure on varieties from i, are given by

$$\lambda_{ij} = \frac{t_{ij}^{-\zeta}[\tau_{ij}/(A_i L_i^{\phi})]^{-\varepsilon} w_i^{-\rho}}{\sum_l t_{lj}^{-\zeta}[\tau_{lj}/(A_l L_l^{\phi})]^{-\varepsilon} w_l^{-\rho}} = \frac{t_{ij}^{-\zeta}(\tau_{ij}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{\sum_l t_{lj}^{-\zeta}(\tau_{lj}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}}; \alpha \equiv \varepsilon\phi. \tag{1}$$

Parameter ε is the trade elasticity with respect to ad-valorem trade costs defined formally as $\varepsilon \equiv -\frac{\partial \ln(\lambda_{ij}/\lambda_{jj})}{\partial \ln \tau_{ij}}$; parameter ζ captures the trade elasticity with respect to tariffs, $\zeta \equiv -\frac{\partial \ln(\lambda_{ij}/\lambda_{jj})}{\partial \ln t_{ij}}$; and parameter ρ is the trade elasticity with respect to wages, $\rho \equiv -\frac{\partial \ln(\lambda_{ij}/\lambda_{jj})}{\partial \ln w_i}$. In the Armington, Eaton-Kortum, and Krugman models we have $\varepsilon = \zeta = \rho$, but this is no longer the case in the Melitz-Pareto model. First, in this model we have $\zeta > \varepsilon$ because of the additional effect of the tariff on trade flows through the extensive margin (see the Online Appendix of Costinot and Rodríguez-Clare, 2014; Felbermayr et al., 2015). Second, parameter ρ also differs from ε if fixed trade costs are paid in source countries' labor, in which case we have $\rho > \varepsilon$. Kucheryavyy et al. (forthcoming) refer to parameter ϕ as the scale elasticity while Breinlich et al. (2022) refer to parameter $\alpha \equiv \varepsilon\phi$ as the output elasticity. In the standard Krugman and Melitz-Pareto models we have $\alpha = 1$, in the generalized Krugman and Melitz-Pareto models with nested CES preferences we

[5]The analysis in this section follows closely the one in Kucheryavyy et al. (forthcoming), but restricting it to the case of a single sector while extending it to allow for tariffs and the case with fixed trade costs paid in source labor.

may have $\alpha \neq 1$ (see Kucheryavyy et al., forthcoming).

The price index in country j is given by

$$P_j = \delta w_j^{-(\rho/\varepsilon-1)} [\sum_i (\lambda_{ij}/t_{ij})/(L_j/f_{jj})]^{\zeta/\varepsilon-1} [\sum_i t_{ij}^{-\zeta} (\tau_{ij}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}]^{-1/\varepsilon}, \tag{2}$$

where δ is a model-specific constant. As discussed further below, the term $[\sum_i (\lambda_{ij}/t_{ij})/(L_j/f_{jj})]^{\zeta/\varepsilon-1}$ implies that, in the presence of tariffs, the Armington, Eaton-Kortum, and Krugman models will have different welfare implications than the Melitz-Pareto model.

Finally, trade balance (or, equivalently, labor market clearing) is given by

$$w_i L_i = \sum_j \Lambda_{ij} w_j L_j, \tag{3}$$

where

$$\Lambda_{ij} \equiv \frac{\lambda_{ij}/t_{ij}}{\sum_l \lambda_{lj}/t_{lj}} = \frac{t_{ij}^{-(\zeta+1)} (\tau_{ij}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{\sum_l t_{lj}^{-(\zeta+1)} (\tau_{lj}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}}$$

is the share of expenditure that j devotes to goods from i evaluated at pre-tariff import prices.

2.2 Equilibrium

An equilibrium is a wage vector $w \equiv (w_0, w_1, ..., w_N)$ such that (3) holds for all i.

Proposition 1. *There exists a unique equilibrium.*

Proof. See Appendix B. □

2.3 Microfoundations

We finish this section by describing in Table 1 how the five different models map into the generalized model corresponding to the previous equations:

Model	Armington–EES	EK–EES	Gen. Krugman	Gen. Melitz destination	Gen. Melitz source
ε	$\eta - 1$	ϑ	$\eta - 1$	$\theta\xi$	$\theta\xi$
ζ	$\eta - 1$	ϑ	$\eta - 1$	$\theta\xi[1+1/(\sigma-1)-1/\theta]$	$\theta\xi[1+1/(\sigma-1)-1/\theta]$
ρ	$\eta - 1$	ϑ	$\eta - 1$	$\theta\xi$	$\theta\xi[1+1/(\sigma-1)-1/\theta]$
ϕ	γ	γ	$1/(\sigma-1)$	$1/\theta$	$1/\theta$
$\alpha \equiv \varepsilon\phi$	$(\eta-1)\gamma$	$\vartheta\gamma$	$(\eta-1)/(\sigma-1)$	ξ	ξ
τ_{ij}	$\overline{\tau}_{ij}$	$\overline{\tau}_{ij}$	$\overline{\tau}_{ij}$	$(f_{ij}/f_{jj})^{1/(\sigma-1)-1/\theta}\overline{\tau}_{ij}$	$(f_{ij}/f_{jj})^{1/(\sigma-1)-1/\theta}\overline{\tau}_{ij}$
A_i	$\overline{A}_i$	$B_i^{1/\vartheta}$	$(f_i^e)^{-1/(\sigma-1)}a_i$	$(f_i^e)^{-1/\theta}b_i$	$(f_i^e)^{-1/\theta}b_i$
δ	1	δ^{EK}	δ^{K}	δ^{M}	δ^{M}

Table 1: Mapping the five different trade models into the general model

Parameter η is the elasticity of substitution across varieties from different countries (applicable in all models except the Eaton-Kortum model), while σ is the elasticity of substitution across varieties from the same country (applicable in the Krugman and Melitz-Pareto models). Parameter γ is the technological scale elasticity in the Armington and Eaton-Kortum models. Parameter ϑ is the shape parameter of the Frechet distribution in the Eaton-Kortum model while $\theta > \sigma - 1$ is the shape parameter of the Pareto distribution in the Melitz-Pareto model. Parameter ξ is given by

$$\xi \equiv \{1 + \theta[1/(\eta-1) - 1/(\sigma-1)]\}^{-1}.$$

Ad-valorem trade costs τ_{ij} are equal to the iceberg trade cost $\overline{\tau}_{ij}$ in all models except Melitz-Pareto, where instead the trade costs combine the iceberg and fixed trade costs, $\tau_{ij} \equiv (f_{ij}/f_{jj})^{1/(\sigma-1)-1/\theta}\overline{\tau}_{ij}$. The productivity shifter A_i is productivity in the Armington model ($A_i = \overline{A}_i$), the average productivity in the Eaton-Kortum model ($A_i = B_i^{1/\vartheta}$), the common firm-level productivity adjusted by the effect of entry costs through variety in the Krugman model ($A_i = (f_i^e)^{-1/(\sigma-1)}a_i$), and the lower bound of the support of the Pareto distribution adjusted by the effect of entry costs on average productivity of surviving firms in the Melitz-Pareto model ($A_i = \left(f_i^e\right)^{-1/\theta} b_i$). Finally, the δ^{EK}, δ^{K}, and δ^{M} are model-specific constants derived in Appendix A.

The first three rows of Table 1 highlight several points. First, we have $\varepsilon = \zeta = \rho$ in the Armington, Eaton-Kortum, and Krugman models. Second, the trade elasticity with respect to tariffs ζ is larger than the trade elasticity with respect to tariffs ε only in the Melitz-Pareto model. Third, the only difference between the two fixed cost specifications of the Melitz-Pareto model lies in the trade elasticity with respect to wages: $\varepsilon = \rho < \zeta$ when fixed trade costs are paid in labor of destination countries, while $\varepsilon < \rho = \zeta$ when fixed trade costs are paid in labor of source countries. The last point will create a difference in the SOE's optimal tariff between the two cases of the Melitz-Pareto model, as we will see in subsection 4.2.

Turning to welfare, from (1) for $i = j$ and (2) we obtain

$$w_j/P_j = [(L_j/f_{jj})/\sum_i(\lambda_{ij}/t_{ij})]^{\zeta/\varepsilon-1}\delta^{-1}A_jL_j^{\phi}\lambda_{jj}^{-1/\varepsilon}.$$

Since welfare (i.e., real expenditure per capita) is equal to its tariff multiplier times its real wage, we immediately obtain

$$W_j \equiv [1/\sum_i(\lambda_{ij}/t_{ij})](w_j/P_j) = (L_j/f_{jj})^{\zeta/\varepsilon-1}[1/\sum_i(\lambda_{ij}/t_{ij})]^{\zeta/\varepsilon}\delta^{-1}A_jL_j^{\phi}\lambda_{jj}^{-1/\varepsilon}.$$

Letting W_j^A denote welfare in the counterfactual corresponding to autarky and letting $\mathrm{GT}_j \equiv W_j/W_j^A$ denote the gains from trade, we then have

$$\mathrm{GT}_j \equiv W_j/W_j^A \equiv \lambda_{jj}^{-1/\varepsilon}[\sum_i(\lambda_{ij}/t_{ij})]^{-\zeta/\varepsilon}.$$

Interestingly, since $\varepsilon = \zeta$ in the Armington-EES, Eaton-Kortum-EES, and generalized Krugman models while $\zeta > \varepsilon$ in the Melitz-Pareto model, the previous result implies that in the presence of tariffs the gains from trade are higher in the Melitz-Pareto model than in the other models. The reason for this is that j's tariff multiplier $1/\sum_i(\lambda_{ij}/t_{ij}) > 1$, which is the ratio to j's total expenditure to its wage income (e.g., Felbermayr et al., 2015), directly affects the price index in the Melitz-Pareto model. This is because the tariff revenue transferred to the representative household increases j's total expenditure, which decreases j's cutoff productivity for domestic sales. This increases the mass of entrants surviving in j's domestic market, thereby, decreasing its price index.

3 Small Open Economy

We now use the generalized gravity model described in the previous section to obtain a well-behaved equilibrium with a SOE. We, henceforth, take labor of country N as the numeraire: $w_N \equiv 1$. Suppose that country 0's labor is expressed as $L_0 \equiv n\widetilde{L}_0$, where $\widetilde{L}_0$ is constant. As $n \to 0$, country 0 becomes a SOE in the limit.

3.1 A First Look

To understand potential problems of an equilibrium with a SOE, and get an idea of how to fix them, we first consider two popular examples with two countries ($i, j, l = 0, 1$): (i) the Armington or Eaton-Kortum model without EES (i.e., $\alpha = 0$); and (ii) the standard Krugman or Melitz-Pareto model with either fixed cost specification (i.e., $\alpha = 1$).

In example (i) equation (3) reduces to

$$\underbrace{\frac{A_0^\varepsilon t_{01}^{-1-\zeta} \tau_{01}^{-\varepsilon} w_0^{-\rho}}{A_0^\varepsilon t_{01}^{-1-\zeta} \tau_{01}^{-\varepsilon} w_0^{-\rho} + A_1^\varepsilon}}_{=\Lambda_{01}} L_1 = \underbrace{\frac{A_1^\varepsilon t_{10}^{-1-\zeta} \tau_{10}^{-\varepsilon}}{A_0^\varepsilon w_0^{-\rho} + A_1^\varepsilon t_{10}^{-1-\zeta} \tau_{10}^{-\varepsilon}}}_{=\Lambda_{10}} w_0 n \widetilde{L}_0.$$

This is country 0's trade balance condition, with exports on the left-hand side and imports on the right-hand side, both evaluated at pre-tariff import prices. The problem is that, as n approaches zero, w_0 approaches infinity. Intuitively, as n decreases to zero, country 0's imports fall to zero. For country 0's trade balance to hold, country 0's exports must then also fall to zero, which requires that country 0's wage increases to infinity. Moreover, as w_0 approaches infinity, Λ_{10} approaches one, implying that Λ_{00} approaches zero.[6]

The reason that w_0 increases to infinity as n decreases to zero is that country 0's wage is the only variable that can adjust to ensure equality between its exports and its shrinking imports. This requires country 0's exports to become more and more costly as n decreases. We formalize this idea by assuming that $\tau_{01} \equiv n^{-1/\varepsilon} \widetilde{\tau}_{01}$, where $\widetilde{\tau}_{01}$ is constant. Substituting this into the above equation, multiplying both sides by $1/n$, and taking $n \to 0$, country 0's trade balance reduces to

$$\frac{A_0^\varepsilon t_{01}^{-1-\zeta} \widetilde{\tau}_{01}^{-\varepsilon} \widetilde{w}_0^{-\rho}}{A_1^\varepsilon} L_1 = \frac{A_1^\varepsilon t_{10}^{-1-\zeta} \tau_{10}^{-\varepsilon}}{A_0^\varepsilon \widetilde{w}_0^{-\rho} + A_1^\varepsilon t_{10}^{-1-\zeta} \tau_{10}^{-\varepsilon}} \widetilde{w}_0 \widetilde{L}_0.$$

This equation determines a unique equilibrium wage $\widetilde{w}_0 \in (0, \infty)$, and we also obtain $\Lambda_{00} \in (0, 1)$.

In example (ii), (3) implies

[6] Alvarez and Lucas (2007) avoid the SOE's infinite wage by assuming that its absolute advantage parameter B_0 is proportional to its labor endowment. This implies $B_0 \equiv n\widetilde{B}_0$, and thus, $A_0^\varepsilon = (B_0^{1/\vartheta})^\vartheta = n\widetilde{B}_0$ (see Table 1). Substituting this into the above trade balance condition, multiplying both sides by $1/n$, and taking $n \to 0$, we obtain $(\widetilde{B}_0 t_{01}^{-1-\zeta} \tau_{01}^{-\varepsilon} \widetilde{w}_0^{-\rho} / A_1^\varepsilon) L_1 = \widetilde{w}_0 \widetilde{L}_0$, which is solved for a unique $\widetilde{w}_0 \in (0, \infty)$. However, since $A_0^\varepsilon = n\widetilde{B}_0$ approaches zero, Λ_{10} approaches one. This is why the SOE's domestic expenditure share remains zero in Alvarez and Lucas (2007).

$$\underbrace{\frac{A_0^\varepsilon t_{01}^{-1-\zeta}\tau_{01}^{-\varepsilon}w_0^{-\rho}n\widetilde{L}_0}{A_0^\varepsilon t_{01}^{-1-\zeta}\tau_{01}^{-\varepsilon}w_0^{-\rho}n\widetilde{L}_0 + A_1^\varepsilon L_1}}_{=\Lambda_{01}} L_1 = \underbrace{\frac{A_1^\varepsilon t_{10}^{-1-\zeta}\tau_{10}^{-\varepsilon}L_1}{A_0^\varepsilon w_0^{-\rho}n\widetilde{L}_0 + A_1^\varepsilon t_{10}^{-1-\zeta}\tau_{10}^{-\varepsilon}L_1}}_{=\Lambda_{10}} w_0 n\widetilde{L}_0.$$

The problem now is that, as n approaches zero, Λ_{10} approaches one, implying that Λ_{00} approaches zero even if w_0 is positive and finite. This is because country 0's mass of entrants is proportional to n in the standard Krugman or Melitz-Pareto model. To neutralize this effect, suppose now that country 0's imports become more and more costly as n decreases: $\tau_{10} \equiv n^{-1/\varepsilon}\widetilde{\tau}_{10}$, where $\widetilde{\tau}_{10}$ is constant. Substituting this into the above equation, multiplying both sides by $1/n$, and taking $n \to 0$, country 0's trade balance reduces to

$$\frac{A_0^\varepsilon t_{01}^{-1-\zeta}\tau_{01}^{-\varepsilon}\widetilde{w}_0^{-\rho}\widetilde{L}_0}{A_1^\varepsilon L_1} L_1 = \frac{A_1^\varepsilon t_{10}^{-1-\zeta}\widetilde{\tau}_{10}^{-\varepsilon}L_1}{A_0^\varepsilon \widetilde{w}_0^{-\rho}\widetilde{L}_0 + A_1^\varepsilon t_{10}^{-1-\zeta}\widetilde{\tau}_{10}^{-\varepsilon}L_1}\widetilde{w}_0\widetilde{L}_0.$$

Note that we have eliminated n in all terms of Λ_{10}. Since the above equation is solved for a unique equilibrium wage $\widetilde{w}_0 \in (0, \infty)$, we again obtain $\Lambda_{00} \in (0, 1)$.

3.2 General Case

The two previous examples reveal that, by appropriately adjusting ad-valorem trade costs, we can obtain a well-behaved equilibrium with a SOE, whose domestic expenditure share is positive, unlike Alvarez and Lucas (2007). To generalize this idea to our gravity model with $N + 1$ countries and a general α, we assume that the adjustments in general case are:

$$\begin{aligned}
L_0 &\equiv n\widetilde{L}_0, \\
\tau_{0j} &\equiv n^{-(1-\alpha)/\varepsilon}\widetilde{\tau}_{0j}, j = 1, ..., N, \\
\tau_{i0} &\equiv n^{-\alpha/\varepsilon}\widetilde{\tau}_{i0}, i = 1, ..., N.
\end{aligned}$$

This is a direct generalization of the two examples: as α increases from zero to one, we need to adjust country 0's import trade costs more and more, while adjusting country 0's export trade costs less and less.

The following proposition characterizes an equilibrium with a SOE:

Proposition 2. *As country 0 becomes arbitrary small (i.e., as $n \to 0$), the equilibrium converges to the one in which:*

1. *$(w_1, ..., w_N)$ solves (3) for all $i, j, l = 1, ..., N$ not including country 0;*

2. $\widetilde{w}_0$ *solves*

$$\sum_{j=1}^{N} \frac{A_0^{\varepsilon} t_{0j}^{-1-\zeta} \widetilde{\tau}_{0j}^{-\varepsilon} \widetilde{w}_0^{-\rho} \widetilde{L}_0^{\alpha}}{\sum_{i=1}^{N} A_i^{\varepsilon} t_{ij}^{-1-\zeta} \tau_{ij}^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}} w_j L_j = \frac{\sum_{i=1}^{N} A_i^{\varepsilon} t_{i0}^{-1-\zeta} \widetilde{\tau}_{i0}^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{A_0^{\varepsilon} \widetilde{w}_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{i=1}^{N} A_i^{\varepsilon} t_{i0}^{-1-\zeta} \widetilde{\tau}_{i0}^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}} \widetilde{w}_0 \widetilde{L}_0. \tag{4}$$

Proof. As shown in Proposition 1, for any $n > 0$ there exists a unique equilibrium. This implies that there is a well-defined sequence of equilibria associated with a sequence of n with $n \to 0$.

We first rewrite (3) using the above trade cost adjustments as

$$w_i L_i = \widetilde{\Lambda}_{i0} w_0 n \widetilde{L}_0 + \sum_{j=1}^{N} \widetilde{\Lambda}_{ij} w_j L_j, i = 1, ..., N, \tag{5}$$

$$w_0 \widetilde{L}_0 = \widetilde{\Lambda}_{00} w_0 \widetilde{L}_0 + \sum_{j=1}^{N} \widetilde{\Lambda}_{0j} w_j L_j, \tag{6}$$

where the expenditure shares evaluated at pre-tariff import prices with trade cost adjustments are given by (see Appendix C)

$$\widetilde{\Lambda}_{ij} \equiv \Lambda_{ij}, i, j = 1, ..., N, \tag{7}$$

$$\widetilde{\Lambda}_{0j} \equiv \Lambda_{0j}/n, j = 1, ..., N, \tag{8}$$

$$\widetilde{\Lambda}_{i0} \equiv \Lambda_{i0}, i = 1, ..., N, \tag{9}$$

$$\widetilde{\Lambda}_{00} \equiv \Lambda_{00}. \tag{10}$$

Note that (6) is obtained by multiplying (3) for $i = 0$ by $1/n$. In contrast to Λ_{0j}, which approaches zero as n approaches zero, $\widetilde{\Lambda}_{0j} \equiv \Lambda_{0j}/n$ stays positive even if n approaches zero.

We can now characterize the limit equilibrium as $n \to 0$. Suppose that $w_0 \in (0, \infty)$, which will be verified later. Then (5), (7), and (9) reduce to (3) for $i = 1, ..., N$, where summations are taken for $j, l = 1, ..., N$ not including country 0. Again, from Proposition 1, there exists a unique $(w_1, ..., w_N)$ solving (3) for $i = 1, ..., N$.

Moreover, as $n \to 0$, (6), (8), and (10) reduce to (4). Given the unique $(w_1, ..., w_N)$ that solves equation (3) for $i = 1, ..., N$, the left-hand side of (4) is decreasing in $\widetilde{w}_0$, approaches infinity as $\widetilde{w}_0 \to 0$, and approaches zero as $\widetilde{w}_0 \to \infty$, whereas its right-hand side is increasing in $\widetilde{w}_0$, approaches zero as $\widetilde{w}_0 \to 0$, and approaches infinity as $\widetilde{w}_0 \to \infty$. Thus, there exists a unique $\widetilde{w}_0 \in (0, \infty)$ solving equation (4). $\square$

Proposition 2 has two important implications. First, from (10) and $\widetilde{w}_0 \in (0,\infty)$, the SOE has a positive domestic expenditure share (evaluated at pre-tariff import prices) $\widetilde{\Lambda}_{00} \in (0,1)$. This allows for applications of our model with the SOE in quantitative analysis using actual production and trade data. Second, all variables within a group of large countries 1 to N are determined independently of country 0. This satisfies Demidova and Rodríguez-Clare (2009, p. 269) SOE assumptions that the rest of the world's cutoff productivity for domestic sales, mass of entrants, income, and price index are independent of variables related to the SOE. Our model then provides a limit-economy foundation for the small-country Melitz model assumption of Demidova and Rodríguez-Clare (2009, 2013) in a more general gravity framework.

Disposing of the tildes and the 0 subindex (i.e., we now use w instead of $\widetilde{w}_0$), using t_i instead of t_{i0} for import tariffs in the SOE, and using τ_{*j} and τ_{i*} instead of $t_{0j}^{(\zeta+1)/\varepsilon}\widetilde{\tau}_{0j}$ and $\widetilde{\tau}_{i0}$, respectively, we can now rewrite equation (4) as exports equal imports

$$X(w) = M(w), \tag{11}$$

with exports given by

$$X\left(w\right) = DA^{\varepsilon}L^{\alpha}w^{-\rho},$$

and imports given by

$$M(w) = \frac{\sum_{i=1}^{N}\lambda_i\left(w\right)/t_i}{1-\sum_{i=1}^{N}\left(1-1/t_i\right)\lambda_i\left(w\right)}wL,$$

with the share of expenditure (at domestic prices) devoted to imports from country i given by

$$\lambda_i\left(w\right) = \frac{t_i^{-\zeta}p_i^{-\rho}}{A^{\varepsilon}L^{\alpha}w^{-\rho}+\sum_{l=1}^{N}t_l^{-\zeta}p_l^{-\rho}}.$$

Here

$$D \equiv \sum_{j=1}^{N}\frac{\tau_{*j}^{-\varepsilon}w_jL_j}{\sum_{i=1}^{N}A_i^{\varepsilon}t_{ij}^{-(\zeta+1)}\tau_{ij}^{-\varepsilon}w_i^{-\rho}L_i^{\alpha}}$$

captures market access abroad and is exogenous to the SOE, and $p_i \equiv [\tau_{i*}/(A_iL_i^{\phi})]^{\varepsilon/\rho}w_i$ is a shifter of the SOE's price of imports from country i.[7]

[7] If we impose $t_i = t$ for all i and use $\mathcal{P} = \left(\sum_{i=1}^{N}p_i^{-\rho}\right)^{-1/\rho}$ then using $\lambda\left(w\right) = 1-\sum_i\lambda_i\left(w\right)$ in the

Turning to welfare, we use $\text{GT} \equiv W/W^A$ introduced in the last part of Section 2 as the SOE's welfare measure,

$$\text{GT} \equiv W/W^A \equiv \lambda^{-1/\varepsilon} \left(\lambda + \sum_{i=1}^{N} \lambda_i / t_i\right)^{-\zeta/\varepsilon}. \tag{12}$$

Without tariffs we would have $\lambda + \sum_{i=1}^{N}(\lambda_i/t_i) = \lambda + \sum_{i=1}^{N}\lambda_i = 1$ and so the previous expression leads directly to the ACR formula, $\text{GT}'/\text{GT} = (\lambda'/\lambda)^{-1/\varepsilon}$. With tariffs, however, the SOE's welfare depends not only on λ but also on λ_i and t_i through the tariff multiplier. The relative change in the SOE's welfare in this case is

$$\text{GT}'/\text{GT} = (\lambda'/\lambda)^{-1/\varepsilon}[\Lambda(\lambda'/\lambda) + \sum_{i=1}^{N} \Lambda_i (\lambda_i'/\lambda_i)/(t_i'/t_i)]^{-\zeta/\varepsilon}.$$

Thus, to obtain GT'/GT, we need to know relative changes in λ, λ_i, and t_i, as well as the initial values of $\Lambda = \lambda/[\lambda + \sum_{l=1}^{N}(\lambda_l/t_l)]$ and $\Lambda_i = (\lambda_i/t_i)/[\lambda + \sum_{l=1}^{N}(\lambda_l/t_l)]$, and the two trade elasticities ε and ζ.

4 Comparative Statics and Optimal Tariffs in the SOE

4.1 Comparative Statics

Figure 1 illustrates how the SOE's equilibrium wage is determined. Curve X is downward sloping while curve M is upward sloping, leading to a unique equilibrium wage w^* at which the SOE achieves trade balance.

Now imagine a shock that improves market access to the rest of the world, as captured by an increase in D. This shifts curve X up to curve X', leading to an increase in the equilibrium wage from w^* to $w^{*\prime\prime}$. On the other hand, a shock that improves the SOE access to foreign goods, as captured by a decline in p_i for some i, shifts curve M up to curve M', leading to a decline in the equilibrium wage from w^* to $w^{*\prime}$. In both cases the shock leads to an increase in trade.

The improvement in market access to the rest of the world clearly increases welfare. This follows immediately from the fact that the increase in the wage leads to an increase

previous expressions simplifies to $X(w) = D\left(AL^{\phi}\right)^{\varepsilon} w^{-\rho}$ and

$$M(w) = \frac{1 - \lambda(w)}{1 + (t-1)\lambda(w)} wL,$$

as in the Introduction.

in $\lambda_i(w)$ for all i, leading to a decline in both λ and

$$\lambda + \sum_{i=1}^{N} \lambda_i / t_i = 1 - \sum_{i=1}^{N} (1 - 1/t_i) \lambda_i.$$

The decline in p_i for some i also increases welfare if there is no variation in tariffs across source countries, $t_i = t$.[8] In this case GT is simplified to

$$\mathrm{GT} = \lambda^{-1/\varepsilon} [\lambda + (1-\lambda)/t]^{-\zeta/\varepsilon}.$$

Since this is decreasing in λ, all we need to show is that λ falls with the shock. Now, if $t_i = t$ then we have

$$M(w) = \frac{1-\lambda}{1 + (t-1)\lambda} wL,$$

(see footnote 7). Since the shock leads to a decline in w, the fact that $M(w)$ increases then necessarily implies that λ falls. In contrast, if t_i varies across countries then we could have a situation in which welfare falls with the decline in p_i for some i. This is because such a shock could increase the tariff-induced misallocation in the SOE's expenditures across different origins.

4.2 Tariffs

Let $\widehat{x} \equiv d \ln x \equiv dx/x$ represent the logarithmic change, or the rate of change, in x. To focus on the SOE's import tariffs t_i, we assume that $\hat{D} = 0$. From (11) we obtain (see Appendix D)

$$\frac{\partial \ln w}{\partial \ln t_i} = \frac{\zeta + 1}{\Delta} \frac{\Lambda}{1 - \Lambda} \Lambda_i > 0, \tag{13}$$

where $\Delta \equiv 1 + \rho(1 + \Lambda) > 1$. Thus, as with changes in trade costs τ_i, an increase in t_i leads to a trade surplus, and hence, a higher wage is needed to restore trade balance.

The effect of tariff changes on the SOE's welfare is more complicated compared to shocks in trade costs. Differentiating (12) and using (13), we obtain (see Appendix D)

[8] Since τ_i includes both variable and fixed trade costs in the Melitz-Pareto model, the analysis above can be applied to shocks in both types of trade costs in that model. This generalizes the results in Demidova and Rodríguez-Clare (2013) who study the case with no differences between elasticities of substitution within/across countries (i.e., $\eta = \sigma$) and no tariffs (i.e., $t_i = 1$ for all i).

$$\frac{\partial \ln \mathrm{GT}}{\partial \ln t_i} = \underbrace{(\rho/\varepsilon)[1-\lambda+\zeta(\Lambda-\lambda)]\frac{\zeta+1}{\Delta}\frac{\Lambda}{1-\Lambda}\Lambda_i}_{\text{terms of trade effect}} \underbrace{-(\zeta/\varepsilon)(\zeta+1)(\lambda_i-\Lambda_i)}_{\text{distortionary effect}}. \tag{14}$$

This highlights the well-known tradeoff associated with a tariff: a higher tariff reduces welfare by discouraging imports (the distortionary effect) but generates gains by increasing the wage (the terms of trade effect). Equating $\frac{\partial \ln \mathrm{GT}}{\partial \ln t_i}$ to zero yields the optimal tariff, which is the same across all source countries.

Proposition 3. *The SOE's optimal tariff is given by*

$$t^* - 1 = 1/[(1+\rho)(\zeta/\rho) - 1] \forall i = 1, ..., N. \tag{15}$$

Proof. Substituting $\Lambda - \lambda = \Lambda \sum_{l=1}^{N} \lambda_l (1 - 1/t_l)$ and $\lambda_i - \Lambda_i = \Lambda_i\{[\lambda + \sum_{l=1}^{N}(\lambda_l/t_l)]t_i - 1\}$ into (14), the first-order condition for welfare maximization with respect to t_i is given by

$$\begin{aligned}
&\frac{\partial \ln \mathrm{GT}}{\partial \ln t_i} = 0 \\
\Leftrightarrow & \rho[1-\lambda+\zeta\Lambda\textstyle\sum_{l=1}^{N}\lambda_l(1-1/t_l)]\dfrac{\Lambda}{1-\Lambda} \\
= & [1+\rho(1+\Lambda)]\zeta\{[\lambda+\textstyle\sum_{l=1}^{N}(\lambda_l/t_l)]t_i - 1\}\forall i = 1, ..., N.
\end{aligned} \tag{16}$$

Since (16) is symmetric across i, we can impose

$$t_i = t \forall i = 1, ..., N. \tag{17}$$

Using (17), (16) is solved for t as (15). $\square$

Table 2 shows what the SOE's optimal tariff formula in (15) implies for each of the five different microfoundations:

Model	Armington–EES	EK–EES	Gen. Krugman	Gen. Melitz destination	Gen. Melitz source
ε	$\eta - 1$	ϑ	$\eta - 1$	$\theta\xi$	$\theta\xi$
ζ	$\eta - 1$	ϑ	$\eta - 1$	$\theta\xi[1+1/(\sigma-1)-1/\theta]$	$\theta\xi[1+1/(\sigma-1)-1/\theta]$
ρ	$\eta - 1$	ϑ	$\eta - 1$	$\theta\xi$	$\theta\xi[1+1/(\sigma-1)-1/\theta]$
$t^* - 1$	$\frac{1}{\eta-1}$	$\frac{1}{\vartheta}$	$\frac{1}{\eta-1}$	$\frac{1}{(1+\theta\xi)[1+1/(\sigma-1)-1/\theta]-1}$	$\frac{1}{\theta\xi[1+1/(\sigma-1)-1/\theta]}$

Table 2: The SOE's optimal tariff for the five microfoundations

In the Armington-EES, Eaton-Kortum-EES, generalized Krugman, and generalized Melitz-Pareto model with fixed trade costs paid in labor of source countries, we have $\zeta = \rho$, and hence, the SOE's optimal tariff is the inverse of ρ, the trade elasticity with respect to wages. However, since in the first three models we have $\varepsilon = \rho$ while in the latter model we have $\varepsilon < \rho$, then if we equalize ε across the four models we would get that the optimal tariff is lower in the Melitz-Pareto model with fixed trade costs in source-country labor than in the Armington-EES, Eaton-Kortum-EES, and generalized Krugman models. Demidova and Rodríguez-Clare (2009) and Costinot et al. (2020) explain that this is because of the decline in the import price index associated with the increase in the variety of available foreign goods caused by higher overall imports. This weakens the terms of trade gains from the tariff, leading to a lower optimal tariff.

The generalized Melitz-Pareto model with fixed trade costs paid in labor of destination countries is the exception to the rule that the optimal tariff is the inverse of the trade elasticity with respect to wages. Mechanically, this happens because the trade elasticity with respect to tariffs is larger than the one with respect to wages, $\zeta > \rho$, and this leads to an optimal tariff that is lower than the inverse of ρ, $t^* - 1 < 1/\rho$ in (15). More specifically, since $\zeta > \rho$ then $t^* - 1 < 1/\rho$, implying that the optimal tariff in the standard Melitz-Pareto model is smaller if fixed trade costs are paid in destination countries rather than in source countries (assuming we equalize ρ across the two Melitz-Pareto models). To understand this result, note that if fixed trade costs are paid in destination countries then imports are associated with a higher demand for labor in the importing country. Thus, a higher tariff lowers labor demand and counteracts the improvement in the terms of trade arising from the standard channels. Since the terms of trade gains from the tariff are smaller, the result is a smaller optimal tariff.

Finally, we note that – as discussed in Demidova and Rodríguez-Clare (2009) and Costinot et al. (2020) – the effect of a tariff can be equally achieved by an export tax (a direct expression of Lerner symmetry) or a subsidy to consumption or production of domestic varieties.

5 Conclusion

Basic questions in the field of international economics can be more easily addressed by considering a small open economy. We have derived the equations characterizing the equilibrium of such an economy in a generalized gravity model as the limit in which the economy becomes infinitesimally small, provided we simultaneously let trade costs go to infinity at a rate determined by the magnitude of the scale and trade elasticities.

These equilibrium equations lead to a simple graphical analysis that can be used to study comparative statics, and the optimal tariff can be derived by differentiation of a simple function giving welfare in terms of the tariff. The comparative statics results show how the SOE's wage, trade flows, and welfare are affected by foreign shocks, while our new optimal tariff formula highlights the role of the trade elasticities with respect to wages and tariffs.

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Appendix A: Five Microfoundations

An Armington Model with External Economies of Scale

Consider an Armington model with a constant elasticity of substitution $\eta(>1)$ across varieties from different countries. Suppose that i's labor productivity exhibits external economies of scale: $\overline{A}_i L_i^{\gamma}$, where γ is the scale elasticity. Under perfect competition, the supply price of a firm producing variety i and selling from i to j is given by $p_{ij} = w_i/(\overline{A}_i L_i^{\gamma})$, and thus, j's demand price of variety i is expressed as $t_{ij}\overline{\tau}_{ij}p_{ij} = t_{ij}\overline{\tau}_{ij}w_i/(\overline{A}_i L_i^{\gamma})$.

The model is summarized as

$$\begin{aligned}
w_i L_i &= \textstyle\sum_j \Lambda_{ij} w_j L_j, \\
\Lambda_{ij} &= \frac{\lambda_{ij}/t_{ij}}{\sum_l (\lambda_{lj}/t_{lj})}, \\
\lambda_{ij} &= \frac{(t_{ij}\overline{\tau}_{ij}w_i/\overline{A}_i)^{-(\eta-1)} L_i^{(\eta-1)\gamma}}{\sum_l (t_{lj}\overline{\tau}_{lj}w_l/\overline{A}_l)^{-(\eta-1)} L_l^{(\eta-1)\gamma}}, \\
P_j &= [\textstyle\sum_i (t_{ij}\overline{\tau}_{ij}w_i/\overline{A}_i)^{-(\eta-1)} L_i^{(\eta-1)\gamma}]^{-1/(\eta-1)}.
\end{aligned}$$

The key in deriving the above system is j's tariff multiplier $X_j/(w_j L_j) = 1/\sum_i(\lambda_{ij}/t_{ij})$, where X_j is j's total expenditure. Noting that j's expenditure on goods from i evaluated at pre-tariff import prices is expressed as $X_{ij}/t_{ij} = \lambda_{ij}X_j/t_{ij}$, j's government budget constraint is given by $T_j = X_j \sum_i \lambda_{ij}(t_{ij}-1)/t_{ij}$, where T_j is the lump-sum transfer from j's government to its representative household. Substituting this into j's household budget constraint $X_j = w_j L_j + T_j$, and solving it for X_j, we obtain

$$X_j = [1/\textstyle\sum_i(\lambda_{ij}/t_{ij})] w_j L_j,$$

or j's tariff multiplier. Moreover, substituting the above expression for X_j into $X_{ij}/t_{ij} = \lambda_{ij}X_j/t_{ij}$, j's expenditure on goods from i evaluated at pre-tariff import prices is rewritten as

$$X_{ij}/t_{ij} = [(\lambda_{ij}/t_{ij})/\textstyle\sum_l(\lambda_{lj}/t_{lj})] w_j L_j = \Lambda_{ij} w_j L_j.$$

Substituting this into i's trade balance $\sum_j X_{ij}/t_{ij} = \sum_j X_{ji}/t_{ji}$, and noting that $\sum_j \Lambda_{ji} = 1$, we obtain $w_i L_i = \sum_j \Lambda_{ij} w_j L_j$. It is important to note that the above process of deriving j's tariff multiplier, expenditure shares evaluated at pre-tariff import prices, and trade balance is common to all five microfoundations.

An Eaton-Kortum Model with External Economies of Scale

Consider an Eaton-Kortum model with a constant elasticity of substitution $\sigma(>1)$ across a continuum of varieties $\omega \in \Omega_j \equiv [0,1]$, and without intermediate goods. Suppose that i's labor productivity exhibits external economies of scale: φL_i^{γ}, where the constant $\overline{A}_i$ in the Armington model with EES is replaced by a random variable φ. Under perfect competition, the supply price of a firm producing variety ω and selling from i to j is given by $p_{ij}(\omega) = w_i/(\varphi L_i^{\gamma})$, and thus, the corresponding CIF price is expressed as $t_{ij}\overline{\tau}_{ij}p_{ij}(\omega) = t_{ij}\overline{\tau}_{ij}w_i/(\varphi L_i^{\gamma})$.

Following Eaton and Kortum (2002), we assume that φ follows a Frechet distribution: $G_i(\varphi) \equiv \exp(-B_i\varphi^{-\vartheta}); B_i > 0, \vartheta > \sigma - 1$, where B_i is the scale parameter, and ϑ is the shape parameter. After the standard calculations, the equilibrium system is given by

$$
\begin{aligned}
w_iL_i &= \textstyle\sum_j \Lambda_{ij}w_jL_j,\\
\Lambda_{ij} &= \frac{\lambda_{ij}/t_{ij}}{\sum_l(\lambda_{lj}/t_{lj})},\\
\lambda_{ij} &= \frac{B_i(t_{ij}\overline{\tau}_{ij}w_i/L_i^{\gamma})^{-\vartheta}}{\Phi_j}; \Phi_j \equiv \textstyle\sum_l B_l(t_{lj}\overline{\tau}_{lj}w_l/L_l^{\gamma})^{-\vartheta},\\
P_j &= \delta^{EK}\Phi_j^{-1/\vartheta} = \delta^{EK}[\textstyle\sum_i B_i(t_{ij}\overline{\tau}_{ij}w_i/L_i^{\gamma})^{-\vartheta}]^{-1/\vartheta}; \delta^{EK} \equiv \Gamma(1+(1-\sigma)/\vartheta)^{1/(1-\sigma)},
\end{aligned}
$$

where $\Gamma(1+(1-\sigma)/\vartheta)$ is the Gamma function. Note that λ_{ij} is originally calculated as the probability that $t_{ij}\overline{\tau}_{ij}p_{ij}(\omega)$ is the lowest of all source countries, but it turns out to be equal to the fraction of varieties j buys from i, and also the expenditure share of varieties j buys from i.

A Generalized Krugman Model

Following Kucheryavyy et al. (forthcoming), consider a generalized Krugman model with nested CES preferences given by

$$
\begin{aligned}
U_j &= (\textstyle\sum_{i=0}^{N} C_{ij}^{(\eta-1)/\eta})^{\eta/(\eta-1)}; \eta > 1,\\
C_{ij} &= (\int_{\Omega_{ij}} c_{ij}(\omega)^{(\sigma-1)/\sigma}d\omega)^{\sigma/(\sigma-1)}; \sigma \geq \eta,
\end{aligned}
$$

where U_j is j's utility, C_{ij} is j's consumption index for varieties from i, $c_{ij}(\omega)$ is j's consumption of variety ω from i, η is elasticity of substitution across varieties from different countries (as in the Armington model with EES), and σ is the elasticity of substitution across varieties within each country (as in the Eaton-Kortum model with EES). The assumption that $\sigma \geq \eta$ is consistent with the empirical finding that: "varieties appear to be closer substitutes in more disaggregate product categories" (Broda and Weinstein, 2006, p. 542).

Suppose that all firms producing differentiated varieties in i have the same labor productivity a_i. Then each firm producing variety ω and selling from i to j sets its supply price as $p_{ij}(\omega) = w_i/(\mu a_i)$, where $\mu \equiv 1 - 1/\sigma \in (0, 1)$. Since all firms in i behave in the same way, we can omit ω from now on. The free entry condition is given by $w_i f_i^e = \sum_j \pi_{ij}$, where f_i^e is the fixed entry cost in terms of labor in i, and π_{ij} is the profit of a firm selling from i to j. Combining i's free entry condition with its labor market-clearing condition $L_i = M_i^e(\sum_j y_{ij}/a_i + f_i^e)$, where y_{ij} is the supply of a firm selling from i to j, the mass of entrants in i is solved as $M_i^e = L_i/(\sigma f_i^e)$.

The equilibrium system is given by

$$
\begin{aligned}
w_i L_i &= \textstyle\sum_j \Lambda_{ij} w_j L_j, \\
\Lambda_{ij} &= \frac{\lambda_{ij}/t_{ij}}{\sum_l (\lambda_{lj}/t_{lj})}, \\
\lambda_{ij} &= \frac{[t_{ij}\overline{\tau}_{ij} w_i/((f_i^e)^{-1/(\sigma-1)} a_i)]^{-(\eta-1)} L_i^{(\eta-1)/(\sigma-1)}}{\sum_l [t_{lj}\overline{\tau}_{lj} w_l/((f_l^e)^{-1/(\sigma-1)} a_l)]^{-(\eta-1)} L_l^{(\eta-1)/(\sigma-1)}}, \\
P_j &= \delta^K \{\textstyle\sum_i [t_{ij}\overline{\tau}_{ij} w_i/((f_i^e)^{-1/(\sigma-1)} a_i)]^{-(\eta-1)} L_i^{(\eta-1)/(\sigma-1)}\}^{-1/(\eta-1)}; \delta^K \equiv \sigma^{1/(\sigma-1)} \mu^{-1}.
\end{aligned}
$$

To obtain λ_{ij} and P_j, we first rewrite $P_{ij} \equiv [\int_{\Omega_{ij}} (t_{ij}\overline{\tau}_{ij} p_{ij}(\omega))^{1-\sigma} d\omega]^{1/(1-\sigma)}$, the price index corresponding to C_{ij}, to obtain $P_{ij} = \delta^K [t_{ij}\overline{\tau}_{ij} w_i/((f_i^e)^{-1/(\sigma-1)} a_i)] L_i^{-1/(\sigma-1)}$. Substituting this into $\lambda_{ij} = X_{ij}/\sum_l X_{lj} = (P_{ij}/P_j)^{1-\eta} = P_{ij}^{1-\eta}/\sum_l P_{lj}^{1-\eta}$, we obtain the above expression for λ_{ij}. Finally, P_j is obtained by substituting the expression for P_{ij} into $P_j^{1-\eta} = \sum_l P_{lj}^{1-\eta}$.

A Generalized Melitz-Pareto Model with Fixed Marketing Costs Paid in Labor of Destination Countries

Consider a generalized Melitz-Pareto model with the same nested CES preferences as the above Krugman model. Suppose that each firm producing a differentiated variety in i has a random labor productivity φ. Then the supply price of firm φ selling from i to j is given by $p_{ij}(\varphi) = w_i/(\mu\varphi)$.

We assume that, when a firm sells its variety from i to j, it has to incur a fixed marketing cost $w_j f_{ij}$ in labor of the destination country j. As mentioned in Arkolakis et al. (2012), in the Melitz-Pareto model, only this specification satisfies their macro-level restriction R3′, requiring that the elasticity of $\lambda_{ij}/\lambda_{jj}$ with respect to w_i is equal to $-\varepsilon$ from (1). The cutoff productivity of firms selling from i to j, denoted by φ_{ij}, is determined by the zero cutoff profit condition

$$\pi_{ij}(\varphi_{ij}) = 0 \Leftrightarrow r_{ij}(\varphi_{ij}) = t_{ij}^{-\sigma}[\overline{\tau}_{ij}w_i/(\mu\varphi_{ij}P_{ij})]^{1-\sigma}(P_{ij}/P_j)^{1-\eta}X_j = \sigma w_j f_{ij}, \tag{18}$$

where $\pi_{ij}(\varphi)$ and $r_{ij}(\varphi)$ are the profit and revenue of firm φ selling from i to j, respectively. Only firms with $\varphi \geq \varphi_{ij}$ survive, earning nonnegative profits by selling from i to j.

With a Pareto distribution $G_i(\varphi) \equiv 1 - b_i^{\theta}\varphi^{-\theta}; \varphi \in [b_i, \infty), \theta > \sigma - 1$, where b_i is the scale parameter, and θ is the shape parameter, and the corresponding probability density function $g_i(\varphi) \equiv G_i'(\varphi) = \theta b_i^{\theta}\varphi^{-\theta-1}$, the free entry condition in i is simplified to

$$\begin{aligned} w_i f_i^e &= \textstyle\sum_j \int_{\varphi_{ij}}^{\infty} \pi_{ij}(\varphi) g_i(\varphi) d\varphi = \textstyle\sum_j H_i(\varphi_{ij}) w_j f_{ij}; \\ H_i(\varphi_{ij}) &\equiv (\theta/\kappa - 1)(1 - G_i(\varphi_{ij})) = (\theta/\kappa - 1) b_i^{\theta}\varphi_{ij}^{-\theta}, \\ \kappa &\equiv \theta - (\sigma - 1) \in (0, \theta). \end{aligned} \tag{19}$$

Two things should be noted about the fixed marketing costs. First, since countries pay the fixed marketing costs to one another, in general we should use each country's current account balance (i.e., the sum of its trade balance and its net income from abroad) to characterize an equilibrium. However, we can show that the total fixed marketing cost from i to j is proportional to i's exports to j evaluated at pre-tariff import prices X_{ij}/t_{ij} with the proportionality constant $\nu \equiv \kappa/(\theta\sigma) \in (0, 1)$. Since this implies that i's zero current account balance is equivalent to i's zero trade balance (which, in turn, is equivalent to i's zero net income from abroad), we can still use each country's trade balance to characterize an equilibrium as in the other models.

Second, i's labor demand includes the fixed marketing costs paid by all source countries. However, since those payments are exactly offset by i's payment of the fixed marketing costs to all destination countries, i's labor market-clearing condition implies that its total wage income is equal to its total revenue evaluated at pre-tariff import prices as in the other models. Combining i's labor market-clearing condition with its free entry condition (19), we can solve for its mass of entrants as $M_i^e = L_i/\{[(\theta/\kappa)/(\theta/\kappa-1)]\sigma f_i^e\}$. The model is summarized as

$$
\begin{aligned}
w_i L_i &= \textstyle\sum_j \Lambda_{ij} w_j L_j, \\
\Lambda_{ij} &= \frac{\lambda_{ij}/t_{ij}}{\sum_l (\lambda_{lj}/t_{lj})}, \\
\lambda_{ij} &= \frac{[t_{ij}^{\iota}(f_{ij}/f_{jj})^{\iota-1}\overline{\tau}_{ij} w_i/((f_i^e)^{-1/\theta} b_i)]^{-\theta\xi} L_i^{\xi}}{\sum_l [t_{lj}^{\iota}(f_{lj}/f_{jj})^{\iota-1}\overline{\tau}_{lj} w_l/((f_l^e)^{-1/\theta} b_l)]^{-\theta\xi} L_l^{\xi}}; \qquad (20) \\
\xi &\equiv \{1+\theta[1/(\eta-1)-1/(\sigma-1)]\}^{-1} \in [0,1], \\
\iota &\equiv 1 + 1/(\sigma-1) - 1/\theta > 1, \\
P_j &= \delta^M [\textstyle\sum_i (\lambda_{ij}/t_{ij})/(L_j/f_{jj})]^{\iota-1} \{\textstyle\sum_i [t_{ij}^{\iota}(f_{ij}/f_{jj})^{\iota-1}\overline{\tau}_{ij} w_i/((f_i^e)^{-1/\theta} b_i)]^{-\theta\xi} L_i^{\xi}\}^{-1/(\theta\xi)}; \qquad (21) \\
\delta^M &\equiv (\theta/\kappa-1)^{-1/\theta}\delta^K.
\end{aligned}
$$

Deriving λ_{ij} and P_j requires much more calculations than the Krugman model. We rewrite $P_{ij} \equiv [\int_{\Omega_{ij}} (t_{ij}\overline{\tau}_{ij} p_{ij}(\omega))^{1-\sigma} d\omega]^{1/(1-\sigma)}$ using the expressions for $p_{ij}(\varphi)$ and M_i^e to obtain

$$P_{ij} = (\theta/\kappa-1)^{1/(1-\sigma)} \delta^K (f_i^e b_i^{-\theta} L_i^{-1})^{1/(\sigma-1)} t_{ij}\overline{\tau}_{ij} w_i \varphi_{ij}^{\kappa/(\sigma-1)}. \qquad (22)$$

so that

$$P_{ij}/P_{jj} = [(f_i^e/f_j^e)(b_i/b_j)^{-\theta}(L_i/L_j)^{-1}]^{1/(\sigma-1)} (t_{ij}\overline{\tau}_{ij} w_i/w_j)(\varphi_{ij}/\varphi_{jj})^{\kappa/(\sigma-1)}.$$

Using the zero cutoff profit condition (18), $\varphi_{ij}/\varphi_{jj}$ is expressed as

$$\varphi_{ij}/\varphi_{jj} = t_{ij}^{\sigma/(\sigma-1)} (\overline{\tau}_{ij} w_i/w_j)(f_{ij}/f_{jj})^{1/(\sigma-1)} (P_{ij}/P_{jj})^{(\eta-1)/(\sigma-1)-1}.$$

The above two equations are solved for $\varphi_{ij}/\varphi_{jj}$ and P_{ij}/P_{jj} as

$$\varphi_{ij}/\varphi_{jj} = \{(f_{ij}/f_{jj})^{1/(\eta-1)}[(f_i^e/f_j^e)(b_i/b_j)^{-\theta}(L_i/L_j)^{-1}]^{1/(\sigma-1)-1/(\eta-1)} t_{ij}^{\eta/(\eta-1)} \overline{\tau}_{ij} w_i/w_j\}^{\xi}, \tag{23}$$

$$P_{ij}/P_{jj} = [(f_{ij}/f_{jj})^{\kappa/(\sigma-1)}(f_i^e/f_j^e)(b_i/b_j)^{-\theta}(L_i/L_j)^{-1}(t_{ij}^{\iota}\overline{\tau}_{ij}w_i/w_j)^{\theta}]^{\xi/(\eta-1)}. \tag{24}$$

Combining (24) with $\lambda_{ij} = (P_{ij}/P_j)^{1-\eta} = (P_{ij}/P_{jj})^{1-\eta}/\sum_l (P_{lj}/P_{jj})^{1-\eta}$, we obtain (20).

Next, substituting X_j from j's tariff multiplier $X_j/(w_j L_j) = 1/\sum_i(\lambda_{ij}/t_{ij})$ into the zero cutoff profit condition (18) for $i = j$, φ_{jj} is expressed as

$$\varphi_{jj} = \sigma^{1/(\sigma-1)}\mu^{-1}\{L_j/[f_{jj}\textstyle\sum_i(\lambda_{ij}/t_{ij})]\}^{-1/(\sigma-1)} w_j P_{jj}^{(\eta-1)/(\sigma-1)-1} P_j^{-(\eta-1)/(\sigma-1)}. \tag{25}$$

Substituting (25) into (22) for $i = j$, we obtain

$$P_{jj}/P_j = \{(\theta/\kappa-1)^{-1/\theta}\delta^K\{L_j/[f_{jj}\textstyle\sum_i(\lambda_{ij}/t_{ij})]\}^{-(\iota-1)}[w_j/(((f_j^e)^{-1/\theta}b_j)]L_j^{-1/\theta}P_j^{-1}\}^{\theta\xi/(\eta-1)}. \tag{26}$$

Substituting (26) into $\lambda_{jj} = (P_{jj}/P_j)^{1-\eta}$, and combining it with (20) for $i = j$, we obtain (21).

A Generalized Melitz-Pareto Model with Fixed Marketing Costs Paid in Labor of Source Countries

In the generalized Melitz-Pareto model of the previous subsection, suppose, alternatively, that the fixed marketing cost of selling from i to j is paid in labor of the source country i: $w_i f_{ij}$. This is the specification used in Demidova and Rodríguez-Clare (2009, 2013) among others. Then the zero cutoff profit condition (18) and free entry condition (19) are replaced by, respectively,

$$\pi_{ij}(\varphi_{ij}) = 0 \Leftrightarrow r_{ij}(\varphi_{ij}) = t_{ij}^{-\sigma}[\overline{\tau}_{ij}w_i/(\mu\varphi_{ij}P_{ij})]^{1-\sigma}(P_{ij}/P_j)^{1-\eta}X_j = \sigma w_i f_{ij}. \tag{27}$$

$$w_i f_i^e = \sum_j \int_{\varphi_{ij}}^{\infty} \pi_{ij}(\varphi) g_i(\varphi) d\varphi = \sum_j H_i(\varphi_{ij}) w_i f_{ij}; \tag{28}$$
$$H_i(\varphi_{ij}) \equiv (\theta/\kappa - 1)(1 - G_i(\varphi_{ij})) = (\theta/\kappa - 1) b_i^\theta \varphi_{ij}^{-\theta},$$
$$\kappa \equiv \theta - (\sigma - 1) \in (0, \theta).$$

$\varphi_{ij}/\varphi_{jj}$ and P_{ij}/P_{jj} are solved as

$$\varphi_{ij}/\varphi_{jj} = \{(f_{ij}/f_{jj})^{1/(\eta-1)}[(f_i^e/f_j^e)(b_i/b_j)^{-\theta}(L_i/L_j)^{-1}]^{1/(\sigma-1)-1/(\eta-1)}(t_{ij}w_i/w_j)^{\eta/(\eta-1)}\overline{\tau}_{ij}\}^{\xi}, \tag{29}$$

$$P_{ij}/P_{jj} = \{(f_{ij}/f_{jj})^{\kappa/(\sigma-1)}(f_i^e/f_j^e)(b_i/b_j)^{-\theta}(L_i/L_j)^{-1}[(t_{ij}w_i/w_j)^{\iota}\overline{\tau}_{ij}]^{\theta}\}^{\xi/(\eta-1)}; \tag{30}$$
$$\xi \equiv \{1 + \theta[1/(\eta - 1) - 1/(\sigma - 1)]\}^{-1} \in [0, 1],$$
$$\iota \equiv 1 + 1/(\sigma - 1) - 1/\theta > 1.$$

The only difference between (23) and (24) is that t_{ij} and $\overline{\tau}_{ij}w_i/w_j$ are replaced by $t_{ij}w_i/w_j$ and $\overline{\tau}_{ij}$, respectively. This implies that λ_{ij} and P_j are given by, respectively,

$$\lambda_{ij} = \frac{[t_{ij}^{\iota}(f_{ij}/f_{jj})^{\iota-1}\overline{\tau}_{ij}w_i^{\iota}/((f_i^e)^{-1/\theta}b_i)]^{-\theta\xi}L_i^{\xi}}{\sum_l [t_{lj}^{\iota}(f_{lj}/f_{jj})^{\iota-1}\overline{\tau}_{lj}w_l^{\iota}/((f_l^e)^{-1/\theta}b_l)]^{-\theta\xi}L_l^{\xi}}. \tag{31}$$

$$P_j = \delta^M w_j^{-(\iota-1)}[\sum_i(\lambda_{ij}/t_{ij})/(L_j/f_{jj})]^{\iota-1}\{\sum_i[t_{ij}^{\iota}(f_{ij}/f_{jj})^{\iota-1}\overline{\tau}_{ij}w_i^{\iota}/((f_i^e)^{-1/\theta}b_i)]^{-\theta\xi}L_i^{\xi}\}^{-1/(\theta\xi)}; \tag{32}$$
$$\delta^M \equiv (\theta/\kappa - 1)^{-1/\theta}\delta^K.$$

There are only two differences between (20) and (21). First, w_i in (20) and (21) is replaced by w_i^{ι}. Second, the right-hand side of (21) is multiplied by $w_j^{-(\iota-1)}$, which keeps linear homogeneity of P_j with respect to wages.

Appendix B. Proof of Proposition 1 (Existence and Uniqueness of an Equilibrium)

The equilibrium system can be rewritten as

$$z_i(w) = 0, i = 0, 1, ..., N,$$

where $z_i(w)$ is country i's excess labor demand function implied by (3),

$$z_i(w) \equiv (\sum_j \Lambda_{ij}(w) w_j L_j - w_i L_i)/w_i.$$

We first show that the excess labor demand function $z(w) \equiv (z_0(w), z_1(w), ..., z_N(w))$ defined for all $w \gg 0$ satisfies the properties in Proposition 17.B.2 of Mas-Colell et al. (1995): (i) $z(w)$ is continuous; (ii) $z(w)$ is homogeneous of degree zero; (iii) $w \cdot z(w) = 0$ for all w (Walras' law); (iv) there exists $s > 0$ such that $z_i(w) > -s$ for all i and for all w; (v) if $w^m \to w^0$, where $w^0 \neq 0$ and $w_i^0 = 0$ for some i, then $\max\{z_0(w^m), z_1(w^m), ..., z_N(w^m)\} \to \infty$.

Properties (i) to (iii) are obvious, while property (iv) is obtained by letting s be larger than $\max_i L_i$. For property (v), consider a wage vector w^0, where $w_i^0 = 0$ for some i, and $w_l^0 > 0$ for all $l \neq i$. Then

$$\begin{aligned}
\lim_{w^m \to w^0} \Lambda_{ij}(w^m) &= \frac{t_{ij}^{-(\zeta+1)} (\tau_{ij}/A_i)^{-\varepsilon} (w_i^0)^{-\rho} L_i^{\alpha}}{t_{ij}^{-(\zeta+1)} (\tau_{ij}/A_i)^{-\varepsilon} (w_i^0)^{-\rho} L_i^{\alpha} + \sum_{l \neq i} t_{lj}^{-(\zeta+1)} (\tau_{lj}/A_l)^{-\varepsilon} (w_l^0)^{-\rho} L_l^{\alpha}} \\
&= \frac{1}{1 + t_{ij}^{\zeta+1} (\tau_{ij}/A_i)^{\varepsilon} (w_i^0)^{\rho} L_i^{-\alpha} \sum_{l \neq i} t_{lj}^{-(\zeta+1)} (\tau_{lj}/A_l)^{-\varepsilon} (w_l^0)^{-\rho} L_l^{\alpha}} \\
&= 1 \forall j.
\end{aligned}$$

This implies that

$$\begin{aligned}
\lim_{w^m \to w^0} z_i(w^m) &= \underbrace{\Lambda_{ii}(w^0)}_{=1} L_i + \sum_{j \neq i} \underbrace{\Lambda_{ij}(w^0)}_{=1} w_j^0 L_j / w_i^0 - L_i \\
&= \sum_{j \neq i} w_j^0 L_j / w_i^0 \\
&= \infty,
\end{aligned}$$

which verifies property (v). Since Proposition 17.B.2 constitutes a set of sufficient conditions for Proposition 17.C.1 of Mas-Colell et al. (1995), there exists an equilibrium wage vector $w \gg 0$ such that $z(w) = 0$.

We next show that $z(w)$ satisfies the gross substitute property in Definition 17.F.2 of Mas-Colell et al. (1995): if w' and w are such that $w'_l > w_l$ for some l and $w'_i = w_i$ for all $i \neq l$, then $z_i(w') > z_i(w)$ for all $i \neq l$. With differentiability, $\partial z_i(w)/\partial w_l > 0 \forall i, l, l \neq i$ ensures the gross substitute property. Direct calculation gives

$$\begin{aligned}\partial z_i/\partial w_l &= \{[\Lambda_{il} + (\partial \Lambda_{il}/\partial w_l) w_l] L_l + \textstyle\sum_{j \neq l} (\partial \Lambda_{ij}/\partial w_l) w_j L_j\}/w_i \\ &= [\Lambda_{il} L_l + \textstyle\sum_j (\partial \Lambda_{ij}/\partial w_l) w_j L_j]/w_i.\end{aligned}$$

However, noting that $w_l, l \neq i$, appears only in the denominator of

$$\Lambda_{ij} = \frac{t_{ij}^{-(\zeta+1)} (\tau_{ij}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{\sum_l t_{lj}^{-(\zeta+1)} (\tau_{lj}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}},$$

we immediately have $\partial \Lambda_{ij}/\partial w_l > 0 \forall i, j, l, l \neq i$. This implies that $\partial z_i(w)/\partial w_l > 0 \forall i, l, l \neq i$. Since the gross substitute property is a sufficient condition for Proposition 17.F.3 of Mas-Colell et al. (1995), there exists a unique equilibrium wage vector $w \gg 0$ such that $z(w) = 0$.

Appendix C. Transformed System

With the trade cost adjustments $\tau_{0j} \equiv n^{-(1-\alpha)/\varepsilon} \widetilde{\tau}_{0j}, j = 1, ..., N$, and $\tau_{i0} \equiv n^{-\alpha/\varepsilon} \widetilde{\tau}_{i0}, i = 1, ..., N$, corresponding to $L_0 \equiv n \widetilde{L}_0$, (1) and (3) are rewritten as

$$\widetilde{\lambda}_{ij} \equiv \frac{t_{ij}^{-\zeta}(\tau_{ij}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{n t_{0j}^{-\zeta}(\widetilde{\tau}_{0j}/A_0)^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{l=1}^{N} t_{lj}^{-\zeta}(\tau_{lj}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}} = \lambda_{ij}, i, j = 1, ..., N, \tag{33}$$

$$\widetilde{\lambda}_{0j} \equiv \frac{t_{0j}^{-\zeta}(\widetilde{\tau}_{0j}/A_0)^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha}}{n t_{0j}^{-\zeta}(\widetilde{\tau}_{0j}/A_0)^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{l=1}^{N} t_{lj}^{-\zeta}(\tau_{lj}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}} = n^{-1}\lambda_{0j}, j = 1, ..., N, \tag{34}$$

$$\widetilde{\lambda}_{i0} \equiv \frac{t_{i0}^{-\zeta}(\widetilde{\tau}_{i0}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{(1/A_0)^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{l=1}^{N} t_{l0}^{-\zeta}(\widetilde{\tau}_{l0}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}} = \lambda_{i0}, i = 1, ..., N, \tag{35}$$

$$\widetilde{\lambda}_{00} \equiv \frac{(1/A_0)^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha}}{(1/A_0)^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{l=1}^{N} t_{l0}^{-\zeta}(\widetilde{\tau}_{l0}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}} = \lambda_{00}; \tag{36}$$

$$n\widetilde{\lambda}_{0j} + \sum_{i=1}^{N} \widetilde{\lambda}_{ij} = 1, \widetilde{\lambda}_{00} + \sum_{i=1}^{N} \widetilde{\lambda}_{i0} = 1.$$

$$\begin{aligned}\widetilde{\Lambda}_{ij} &\equiv \frac{t_{ij}^{-(\zeta+1)}(\tau_{ij}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{n t_{0j}^{-(\zeta+1)}(\widetilde{\tau}_{0j}/A_0)^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{l=1}^{N} t_{lj}^{-(\zeta+1)}(\tau_{lj}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}} \\ &= \frac{\widetilde{\lambda}_{ij}/t_{ij}}{n\widetilde{\lambda}_{0j}/t_{0j} + \sum_{l=1}^{N}(\widetilde{\lambda}_{lj}/t_{lj})}, i, j = 1, ..., N,\end{aligned} \tag{37}$$

$$\begin{aligned}\widetilde{\Lambda}_{0j} &\equiv \frac{t_{0j}^{-(\zeta+1)}(\widetilde{\tau}_{0j}/A_0)^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha}}{n t_{0j}^{-(\zeta+1)}(\widetilde{\tau}_{0j}/A_0)^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{l=1}^{N} t_{lj}^{-(\zeta+1)}(\tau_{lj}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}} \\ &= \frac{\widetilde{\lambda}_{0j}/t_{0j}}{n\widetilde{\lambda}_{0j}/t_{0j} + \sum_{l=1}^{N}(\widetilde{\lambda}_{lj}/t_{lj})}, j = 1, ..., N,\end{aligned} \tag{38}$$

$$\widetilde{\Lambda}_{i0} \equiv \frac{t_{i0}^{-(\zeta+1)}(\widetilde{\tau}_{i0}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{(1/A_0)^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{l=1}^{N} t_{l0}^{-(\zeta+1)}(\widetilde{\tau}_{l0}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}} = \frac{\widetilde{\lambda}_{i0}/t_{i0}}{\widetilde{\lambda}_{00}/t_{00} + \sum_{l=1}^{N}(\widetilde{\lambda}_{l0}/t_{l0})}, i = 1, ..., N, \tag{39}$$

$$\widetilde{\Lambda}_{00} \equiv \frac{(1/A_0)^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha}}{(1/A_0)^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{l=1}^{N} t_{l0}^{-(\zeta+1)}(\widetilde{\tau}_{l0}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}} = \frac{\widetilde{\lambda}_{00}/t_{00}}{\widetilde{\lambda}_{00}/t_{00} + \sum_{l=1}^{N}(\widetilde{\lambda}_{l0}/t_{l0})}; \tag{40}$$

$$n\widetilde{\Lambda}_{0j} + \sum_{i=1}^{N} \widetilde{\Lambda}_{ij} = 1, \widetilde{\Lambda}_{00} + \sum_{i=1}^{N} \widetilde{\Lambda}_{i0} = 1.$$

$$w_i L_i = \widetilde{\Lambda}_{i0} w_0 n \widetilde{L}_0 + \sum_{j=1}^{N} \widetilde{\Lambda}_{ij} w_j L_j, i = 1, ..., N, \tag{41}$$

$$w_0 \widetilde{L}_0 = \widetilde{\Lambda}_{00} w_0 \widetilde{L}_0 + \sum_{j=1}^{N} \widetilde{\Lambda}_{0j} w_j L_j. \tag{42}$$

(37), (38), (39), (40), (41), and (42) correspond to (7), (8), (9), (10), (5), and (6), respectively. The expressions for $\widetilde{\lambda}_{ij}, \widetilde{\lambda}_{0j}, \widetilde{\lambda}_{i0}$, and $\widetilde{\lambda}_{00}$ in (33), (34), (35), and (36), respectively, will be used for welfare analysis.

Appendix D. Derivations of (13) and (14)

In starting comparative statics with respect to tariffs for the SOE, we note that the wage is given by the solution to (11)

$$X(w) = (1 - \Lambda(w))\, wL,$$

with

$$X(w) \equiv D A^{\varepsilon} L^{\alpha} w^{-\rho}$$

and

$$\Lambda(w) \equiv \frac{A^{\varepsilon} L^{\alpha} w^{-\rho}}{A^{\varepsilon} L^{\alpha} w^{-\rho} + \sum_{i=1}^{N} t_i^{-(\zeta+1)} p_i^{-\rho}},$$

while welfare is given by (12)

$$\mathrm{GT} = \lambda^{-1/\varepsilon} \left(\lambda + \sum_{i=1}^{N} \lambda_i / t_i\right)^{-\zeta/\varepsilon}.$$

Logarithmically differentiating (11), and using $\widehat{x} \equiv d \ln x$, we obtain

$$-\rho \widehat{w} = [-\Lambda/(1-\Lambda)]\widehat{\Lambda} + \widehat{w}.$$

Logarithmically differentiating the definition of $\Lambda(w)$, we obtain

$$\widehat{\Lambda} = -\rho(1-\Lambda)\widehat{w} + (\zeta+1)\sum_{i=1}^{N} \Lambda_i \widehat{t}_i, \tag{43}$$

where $\Lambda_i = t_i^{-(\zeta+1)} p_i^{-\rho} / (A^{\varepsilon} L^{\alpha} w^{-\rho} + \sum_{l=1}^{N} t_l^{-(\zeta+1)} p_l^{-\rho})$ is the SOE's import expenditure share from i evaluated at pre-tariff import prices. Substituting (43) into the above equation, $\widehat{w}$ is solved as

$$\widehat{w} = [(\zeta+1)/\Delta][\Lambda/(1-\Lambda)]\sum_{i=1}^{N} \Lambda_i \widehat{t}_i; \tag{44}$$
$$\Delta \equiv 1 + \rho(1+\Lambda) > 1.$$

This implies (13).

Next, logarithmically differentiating (35) and (36), we obtain

$$\widehat{\lambda}_i = -(\zeta\widehat{t_i} - \lambda\rho\widehat{w} - \textstyle\sum_{l=1}^N \lambda_l \zeta \widehat{t_l}), i = 1, ..., N, \tag{45}$$

$$\widehat{\lambda} = -(\rho\widehat{w} - \lambda\rho\widehat{w} - \textstyle\sum_{l=1}^N \lambda_l \zeta \widehat{t_l}). \tag{46}$$

Logarithmically differentiating (12) and using (45) and (46), the logarithmic change in the SOE's welfare is given by

$$\widehat{\mathrm{GT}} = (\rho/\varepsilon)[1 - \lambda + \zeta(\Lambda - \lambda)]\widehat{w} - (\zeta/\varepsilon)(\zeta + 1)\textstyle\sum_{i=1}^N (\lambda_i - \Lambda_i)\widehat{t_i}. \tag{47}$$

From (13) and (47), we obtain (14).