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INFLATION INSURANCE

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INFLATION INSURANCE

ABSTRACT

A contract to insure \$1 against inflation is equivalent to a European call option on the consumer price index. When there is no deductible this call option is equivalent to a forward contract on the CPI. Its price is the difference between the prices of a zero coupon real bond and a zero coupon nominal bond, both free of default risk. Provided that the risk-free real rate of interest is positive, the price of such an inflation insurance policy first rises and then falls with time to maturity. It is a decreasing function of the real interest rate and an increasing function of both the expected rate of inflation and the real risk premium on nominal bonds.

When a deductible is introduced, the insurance policy can no longer be priced like a CPI forward contract. The option feature has its greatest value when the deductible is close to the forward rate of inflation, defined as the difference between the risk-free nominal and real interest rates. Such inflation insurance contracts are priced using the model developed by Black-Merton-Scholes. Pricing an inflation insurance policy with a cap requires only a minor modification of the model.

The approach presented in this paper permits fairly precise quantification of the cost of implementing proposals to index pension benefits for inflation. It also gives us a way of estimating the savings to the Social Security system that would result from introducing a deductible.

Key words: Inflation, insurance, forward contract, call option, put option, contingent claim, deductible, cap, futures contract, CPI, dynamic hedging, portfolio rebalancing.

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## INFLATION INSURANCE

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## INFLATION INSURANCE

### 1. Introduction

Most economists would agree that inflation is inevitable in a market economy that wants to enjoy nearly full employment. If we could perfectly predict the rate of inflation, then it would not be much of a problem. Since we cannot, uncertainty about the rate of inflation is a major problem for large segments of our population.

The way risk-averse people cope with risk is to buy insurance. Not all risks, however, are insurable. Until recently there was no efficient way for people to insure against inflation risk in the U.S. In the past investors have sought protection from inflation risk by investing in real estate, precious metals, and even more esoteric asset classes. But all of these so-called inflation hedges are only imperfectly correlated with the consumer price level and expose the investor to other risks that may exceed the risk of inflation.<sup>1</sup>

Recent innovations in the U.S. now make it possible to hedge inflation risk quite efficiently. Several financial institutions have recently issued securities linked to the U.S. consumer price level. The new securities were issued first by the Franklin

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<sup>1</sup>Research by Bodie [1982] has shown that incorporating these other assets in one's portfolio does not help very much in reducing the variance of one's real rate of return.

Savings Association of Ottawa, Kansas, in January 1988 in two different forms. The first is certificates of deposit, called Inflation-Plus CDs, insured by the Federal Savings and Loan Insurance Corporation (FSLIC), and paying an interest rate tied to the Bureau of Labor Statistics Consumer Price Index (CPI). Interest is paid monthly and is equal to a stated real rate plus the proportional increase in the CPI during the previous month. As of this writing (May 1989), the real rate ranges from 3% per year for a one-year maturity CD to 3.2% per year for a ten-year maturity.

The second form is twenty-year noncallable collateralized bonds, called Real Yield Securities or REALs. These offer a floating coupon rate of 3% per year plus the previous year's proportional change in the CPI, adjusted and payable quarterly. A recent issue of similar bonds includes a put option.

Two other financial institutions have recently followed the lead of Franklin Savings.<sup>2</sup> If the trend continues, we have reached a milestone in the history of this country's financial markets. Consider that for years prominent economists at all points of the ideological spectrum have argued that the U.S. Treasury should issue such securities, and scholars have speculated why private markets for them have not hitherto

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<sup>2</sup>In August 1988 Anchor Savings Bank became the second U.S. institution to issue REALs, and in September 1988 JHM Acceptance Corporation issued modified index-linked bonds subject to a nominal interest rate cap of 14% per annum. The investment banking firm of Morgan Stanley and Company is the underwriter and market maker for REALs.

developed.<sup>3</sup> The current innovative environment in the U.S. financial markets appears to finally have put an end to this speculation by producing private indexed bonds in several forms.

The emergence of a market for virtually risk-free securities linked to the U.S. consumer price level makes possible a host of other financial products that offer a variety of inflation insurance features. The purpose of this paper is to describe some of the most likely of these inflation insurance products and to show how they can be created and competitively priced.

## 2. Inflation Insurance as a Forward Contract on the CPI

Let  $P(t)$  be the consumer price level at time  $t$ . With no loss of generality let the current price level be 1 (that is,  $P(0) = 1$ ). Let  $r(T)$  be the riskless real rate of interest on a default-free CPI-linked zero coupon bond maturing  $T$  years from now. By definition  $e^{-rT}$  dollars invested in such a bond now will pay  $P(T)$  dollars at time  $T$ .

Let  $R(T)$  be the riskless nominal rate of interest on a default-free zero coupon bond of the conventional kind (like a U.S. Treasury bill). By definition  $e^{-RT}$  dollars invested in such a bond now will pay \$1 at time  $T$ .

A forward contract on the CPI with a forward price of 1 and delivery date  $T$  pays  $P(T) - 1$  dollars at maturity. We can replicate the payoff from this forward contract by investing

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<sup>3</sup>See, for example, the analysis in Fischer (1986).

$e^{-rT}$  in the CPI-linked bond and borrowing  $e^{-RT}$  at the risk-free nominal rate of interest. At maturity we will receive  $P(T)$  from the investment and have to pay \$1 in principal and interest on the loan. The competitive price for the CPI forward contract,  $f$ , assuming zero transaction costs is therefore  $e^{-rT} - e^{-RT}$ .

In other words, a CPI forward contract is equivalent to a long position in a zero coupon real bond and a short position in a zero coupon nominal bond of the same maturity. By arbitrage the price of the contract is therefore the difference between the prices of these two bonds.

Figure 1 and Table 1 present the prices of real and nominal zero coupon bonds and the price of the CPI forward contract as a function of maturity. They assume a flat term structure for both real and nominal risk-free interest rates with  $r(T)$  equal to 3% per year and  $R(T)$  equal to 9% per year for all maturities.

The first and second derivatives of the price of the CPI forward contract with respect to  $T$  are given by the formulas:

$$\frac{\delta f}{\delta T} = -re^{-rT} + Re^{-RT}$$

$$\frac{\delta^2 f}{\delta T^2} = r^2 e^{-rT} - R^2 e^{-RT}$$

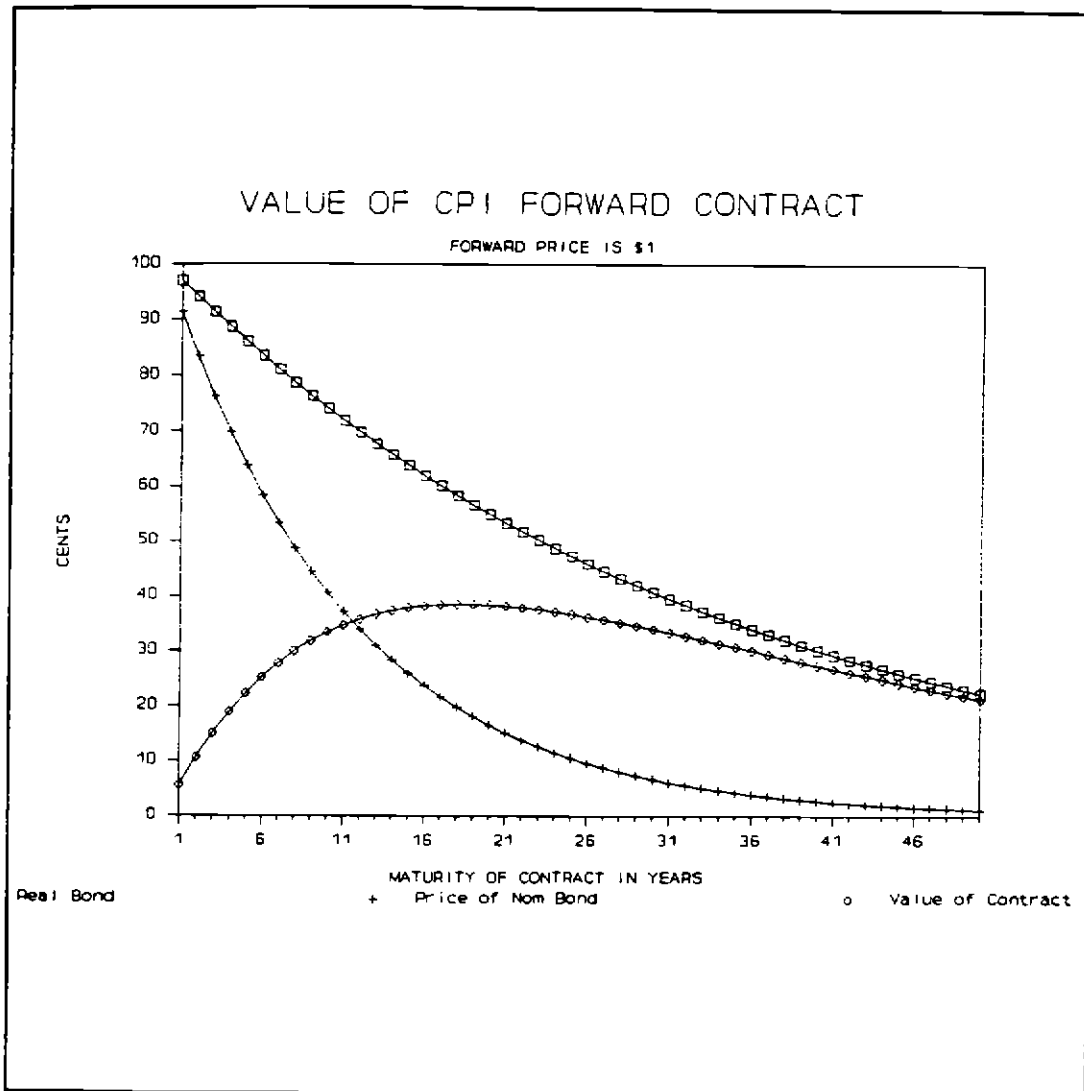
Note that the value of the CPI forward contract first rises and then falls with maturity. It is at its maximum when the maturity is  $\frac{\ln(R/r)}{R - r}$ .



TABLE 1. PRICES OF REAL BONDS, NOMINAL BONDS, AND CPI FORWARD CONTRACTS

MATURITY	REAL BOND	NOMINAL BOND	FORWARD CONTRACT
1	97.04455	91.39311	5.651434
2	94.17645	83.52702	10.64943
3	91.39311	76.33794	15.05516
4	88.69204	69.76763	18.92441
5	86.07079	63.76281	22.30798
6	83.52702	58.27482	25.25219
7	81.05842	53.25918	27.79924
8	78.66278	48.67522	29.98756
9	76.33794	44.48580	31.85214
10	74.08182	40.65696	33.42485
11	71.89237	37.15766	34.73470
12	69.76763	33.95955	35.80808
13	67.70568	31.03669	36.66899
14	65.70468	28.36540	37.33927
15	63.76281	25.92402	37.83878
16	61.87833	23.69277	38.18556
17	60.04955	21.65356	38.39599
18	58.27482	19.78986	38.48495
19	56.55254	18.08657	38.46596
20	54.88116	16.52988	38.35127
21	53.25918	15.10718	38.15199
22	51.68513	13.80692	37.87820
23	50.15760	12.61857	37.53902
24	48.67522	11.53251	37.14271
25	47.23665	10.53992	36.69673
26	45.84060	9.632763	36.20783
27	44.48580	8.803683	35.68212
28	43.17105	8.045960	35.12509
29	41.89515	7.353454	34.54170
30	40.65696	6.720551	33.93641
31	39.45537	6.142121	33.31324
32	38.28928	5.613476	32.67581
33	37.15766	5.130331	32.02733
34	36.05949	4.688769	31.37072
35	34.99377	4.285212	30.70856
36	33.95955	3.916389	30.04316
37	32.95589	3.579310	29.37658
38	31.98190	3.271243	28.71065
39	31.03669	2.989691	28.04700
40	30.11942	2.732372	27.38704
41	29.22925	2.497200	26.73205
42	28.36540	2.282269	26.08313
43	27.52707	2.085836	25.44124
44	26.71353	1.906311	24.80721
45	25.92402	1.742237	24.18178
46	25.15785	1.592285	23.56557
47	24.41432	1.455239	22.95908
48	23.69277	1.329988	22.36278
49	22.99254	1.215517	21.77703
50	22.31301	1.110899	21.20211

Assumptions: Real rate of interest is 3% per year and nominal rate is 9% for all maturities.  
The forward price is 1.



**Figure 1**

Figure 1. Prices of Real Bonds, Nominal Bonds, and CPI Forward Contracts as a Function of Time to Maturity

Notes: The figure assumes that the riskless real rate of interest is 3% per year and the riskless nominal rate 9% per year for all maturities. The forward price is assumed to be 1.

The rising cost for the shorter maturities is caused by the prices of nominal zero coupon bonds declining more rapidly with maturity at first than the prices of the corresponding real bonds. Eventually, however, this is reversed. The value of the forward contract approaches the price of the real bond asymptotically as an upper bound and must therefore decline with it.

Under our assumptions the cost of insuring a dollar against inflation reaches a maximum of 38.5 cents for a maturity of 18.3 years. The cost of insuring a dollar to be received one year from now is only 5.7 cents and of a dollar to be received 50 years from now 21 cents.

These costs are, however, quite sensitive to the assumptions that we have made. In the next section we examine this sensitivity.

## 2.1 Comparative Statics for the Price of a CPI Forward Contract.

The price of a CPI forward contract is a function only of its maturity and the riskless real and nominal rates of interest. By definition the relationship between these two interest rates is:

$$R(T) = r(T) + \text{expected inflation rate} + \text{risk premium}$$

$$\text{or } R(T) = r(T) + \pi(T) + \phi(T)$$

Let us define the expected real rate of interest on a nominal bond as the risk-free nominal rate minus the expected rate of inflation:  $E(r_N) = R - \pi$ . Then it follows that this

expected real rate on the nominal bond will exceed the risk-free real rate of interest ( $r$ ) by the risk premium ( $\phi$ ).<sup>4</sup>

Let us maintain the assumption that the term structures of real and nominal risk-free rates are both flat and consider the effect of an increase in the risk-free real rate, holding constant expected inflation and the risk premium on nominal bonds. Under this assumption, any increase in  $r$  will be matched by an equal increase in  $R$ .

The effect on the prices of CPI forward contracts is presented in Figure 2 and Table 2. We assume that the spread between the nominal and real risk-free interest rates is 6% per year. An increase in the real interest rate causes a downward shift in the prices of CPI forward contracts of all maturities.

The first and second derivatives of the price of the CPI forward contract with respect to  $r$  are:

$$\frac{\delta f}{\delta r} = -Te^{-rT} + Te^{-RT} = -Tf$$

$$\frac{\delta^2 f}{\delta r^2} = T^2 f$$

Thus the effect of an increase in the real interest rate is always to lower the value of a CPI forward contract.

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<sup>4</sup>For a discussion of the size of this risk premium see Bodie (1988).

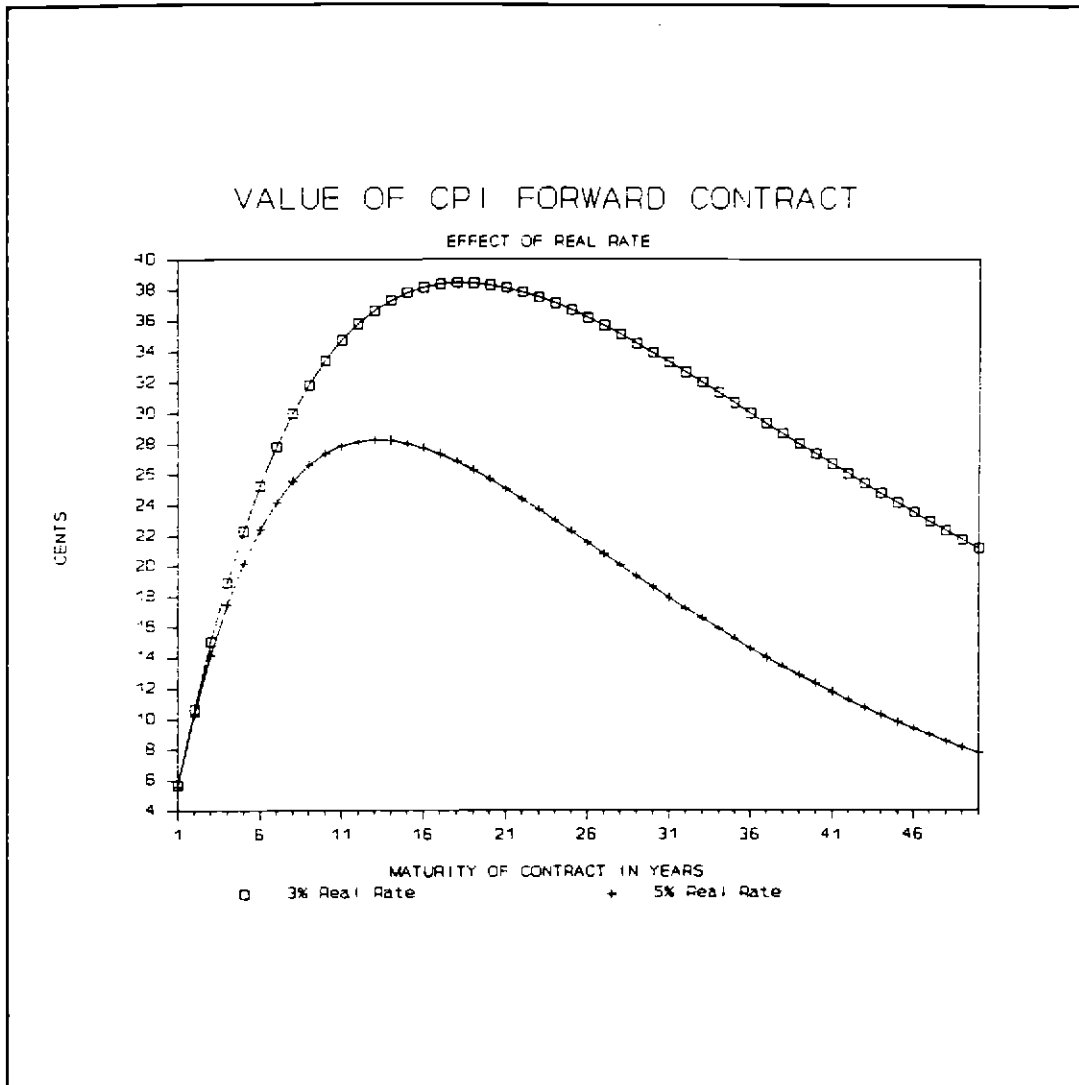


Figure 2

Figure 2. Price of a CPI Forward Contract as a Function of Maturity and Real Interest Rate

Notes: The figure assumes that the term structures of real and nominal interest rates are flat and that the nominal rate exceeds the real rate by 6% per year. The forward price is assumed to be 1.

TABLE 2. PRICE OF CPI FORWARD CONTRACT AS A FUNCTION OF MATURITY AND REAL INTEREST RATE

MATURITY -----	REAL INTEREST RATE -----										
	0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
0											
1	5.82	5.77	5.71	5.65	5.60	5.54	5.48	5.43	5.38	5.32	5.27
2	11.31	11.08	10.86	10.65	10.44	10.23	10.03	9.83	9.64	9.45	9.26
3	16.47	15.99	15.51	15.06	14.61	14.18	13.76	13.35	12.96	12.58	12.20
4	21.34	20.50	19.70	18.92	18.18	17.47	16.78	16.13	15.49	14.89	14.30
5	25.92	24.65	23.45	22.31	21.22	20.19	19.20	18.26	17.37	16.53	15.72
6	30.23	28.47	26.81	25.25	23.78	22.40	21.09	19.86	18.71	17.62	16.59
7	34.30	31.98	29.81	27.80	25.92	24.17	22.53	21.01	19.59	18.27	17.03
8	38.12	35.19	32.49	29.99	27.68	25.55	23.59	21.78	20.10	18.56	17.13
9	41.73	38.13	34.85	31.85	29.11	26.61	24.32	22.22	20.31	18.56	16.96
10	45.12	40.83	36.94	33.42	30.24	27.37	24.76	22.41	20.27	18.34	16.60
11	48.31	43.28	38.77	34.73	31.12	27.88	24.97	22.37	20.04	17.95	16.08
12	51.32	45.52	40.37	35.81	31.76	28.17	24.98	22.16	19.65	17.43	15.46
13	54.16	47.56	41.76	36.67	32.20	28.27	24.83	21.80	19.14	16.81	14.76
14	56.83	49.40	42.95	37.34	32.46	28.22	24.53	21.33	18.54	16.12	14.01
15	59.34	51.08	43.96	37.84	32.57	28.03	24.13	20.77	17.87	15.38	13.24
16	61.71	52.59	44.81	38.19	32.54	27.73	23.63	20.13	17.16	14.62	12.46
17	63.94	53.94	45.51	38.40	32.39	27.33	23.06	19.45	16.41	13.85	11.68
18	66.04	55.16	46.07	38.48	32.15	26.85	22.43	18.73	15.65	13.07	10.92
19	68.02	56.25	46.51	38.47	31.81	26.31	21.75	17.99	14.88	12.30	10.17
20	69.88	57.21	46.84	38.35	31.40	25.71	21.05	17.23	14.11	11.55	9.46
21	71.63	58.07	47.07	38.15	30.93	25.07	20.32	16.47	13.35	10.82	8.77
22	73.29	58.81	47.20	37.88	30.40	24.39	19.58	15.71	12.61	10.12	8.12
23	74.84	59.46	47.25	37.54	29.83	23.70	18.83	14.96	11.89	9.44	7.50
24	76.31	60.03	47.22	37.14	29.22	22.98	18.08	14.22	11.19	8.80	6.92
25	77.69	60.50	47.12	36.70	28.58	22.26	17.33	13.50	10.51	8.19	6.38
26	78.99	60.90	46.96	36.21	27.92	21.53	16.60	12.80	9.87	7.61	5.87
27	80.21	61.23	46.74	35.68	27.24	20.79	15.87	12.12	9.25	7.06	5.39
28	81.36	61.49	46.48	35.13	26.55	20.06	15.16	11.46	8.66	6.55	4.95
29	82.45	61.69	46.16	34.54	25.85	19.34	14.47	10.83	8.10	6.06	4.54
30	83.47	61.84	45.81	33.94	25.14	18.62	13.80	10.22	7.57	5.61	4.16
31	84.43	61.93	45.42	33.31	24.43	17.92	13.14	9.64	7.07	5.19	3.80
32	85.34	61.97	45.00	32.68	23.73	17.23	12.51	9.09	6.60	4.79	3.48
33	86.19	61.97	44.55	32.03	23.03	16.55	11.90	8.56	6.15	4.42	3.18
34	87.00	61.92	44.07	31.37	22.33	15.89	11.31	8.05	5.73	4.08	2.90
35	87.75	61.84	43.58	30.71	21.64	15.25	10.75	7.57	5.34	3.76	2.65
36	88.47	61.72	43.06	30.04	20.96	14.62	10.20	7.12	4.97	3.46	2.42
37	89.14	61.57	42.53	29.38	20.29	14.02	9.68	6.69	4.62	3.19	2.20
38	89.77	61.39	41.98	28.71	19.63	13.43	9.18	6.28	4.29	2.94	2.01
39	90.37	61.18	41.42	28.05	18.99	12.86	8.70	5.89	3.99	2.70	1.83
40	90.93	60.95	40.86	27.39	18.36	12.31	8.25	5.53	3.71	2.48	1.67
41	91.46	60.70	40.28	26.73	17.74	11.77	7.81	5.19	3.44	2.28	1.52
42	91.95	60.42	39.70	26.08	17.14	11.26	7.40	4.86	3.19	2.10	1.38
43	92.42	60.12	39.11	25.44	16.55	10.77	7.00	4.56	2.96	1.93	1.25
44	92.86	59.81	38.52	24.81	15.98	10.29	6.63	4.27	2.75	1.77	1.14
45	93.28	59.48	37.92	24.18	15.42	9.83	6.27	4.00	2.55	1.63	1.04
46	93.67	59.13	37.33	23.57	14.88	9.39	5.93	3.74	2.36	1.49	0.94
47	94.04	58.77	36.73	22.96	14.35	8.97	5.61	3.50	2.19	1.37	0.86
48	94.39	58.40	36.14	22.36	13.84	8.56	5.30	3.28	2.03	1.26	0.78
49	94.71	58.02	35.55	21.78	13.34	8.17	5.01	3.07	1.88	1.15	0.71
50	95.02	57.63	34.96	21.20	12.86	7.80	4.73	2.87	1.74	1.06	0.64

Assumptions: Nominal interest rate exceeds real rate by 6% per year.

For example, consider a CPI forward contract maturing in 10 years. By reading across the corresponding row of Table 2 we see that the price of the contract falls from a high of 45.12 cents when the real interest rate is 0 to a low of 16.6 cents when the real rate is 10% per year.

The first column in Table 2, which corresponds to a real interest rate of zero, deserves special comment. Note that the price of a CPI forward contract rises continuously with maturity as one reads down the column and approaches the value of \$1 asymptotically. The reason is that when the real interest rate is zero, the price of a zero coupon real bond is \$1 no matter what the maturity.

Now let us consider the effect of an increase in the expected rate of inflation, holding constant  $r$  and  $\phi$ . An increase in the expected inflation rate will cause the riskless nominal rate to rise. This will cause the prices of nominal bonds of all maturities to fall and the prices of CPI forward contracts to rise. Figure 3 and Table 3 show this effect.

The first and second derivatives of the price of the CPI forward contract with respect to  $\pi$  are:

$$\frac{\delta f}{\delta \pi} = T e^{-RT}$$

$$\frac{\delta^2 f}{\delta \pi^2} = -T^2 e^{-RT}$$

Thus the effect of an increase in the expected rate of inflation is always to raise the value of a CPI forward contract.

TABLE 3. PRICE OF CPI FORWARD CONTRACT AS A FUNCTION OF MATURITY AND EXPECTED INFLATION RATE

MATURITY -----	EXPECTED INFLATION RATE -----											
	0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	
0												
1	0.97	1.92	2.87	3.81	4.73	5.65	6.56	7.46	8.35	9.24	10.11	
2	1.86	3.69	5.48	7.24	8.96	10.65	12.30	13.92	15.51	17.07	18.60	
3	2.70	5.32	7.87	10.33	12.73	15.06	17.31	19.50	21.63	23.69	25.69	
4	3.48	6.82	10.03	13.11	16.08	18.92	21.66	24.29	26.81	29.24	31.57	
5	4.20	8.19	11.99	15.60	19.04	22.31	25.42	28.38	31.19	33.87	36.41	
6	4.86	9.45	13.76	17.82	21.65	25.25	28.65	31.84	34.85	37.69	40.36	
7	5.48	10.59	15.35	19.80	23.94	27.80	31.40	34.76	37.89	40.81	43.53	
8	6.05	11.63	16.78	21.54	25.93	29.99	33.73	37.18	40.37	43.32	46.03	
9	6.57	12.58	18.06	23.08	27.66	31.85	35.68	39.18	42.38	45.30	47.97	
10	7.05	13.43	19.20	24.42	29.15	33.42	37.29	40.79	43.96	46.83	49.42	
11	7.49	14.20	20.21	25.59	30.41	34.73	38.61	42.07	45.18	47.96	50.45	
12	7.89	14.89	21.09	26.60	31.48	35.81	39.65	43.05	46.07	48.75	51.13	
13	8.25	15.50	21.87	27.45	32.36	36.67	40.45	43.77	46.69	49.25	51.50	
14	8.58	16.05	22.53	28.17	33.08	37.34	41.04	44.27	47.07	49.50	51.62	
15	8.88	16.53	23.11	28.77	33.64	37.84	41.65	44.56	47.23	49.54	51.52	
16	9.15	16.95	23.59	29.25	34.07	38.19	41.69	44.67	47.22	49.39	51.23	
17	9.39	17.31	23.99	29.63	34.38	38.40	41.78	44.64	47.05	49.08	50.79	
18	9.60	17.62	24.32	29.91	34.58	38.48	41.74	44.47	46.74	48.64	50.23	
19	9.79	17.88	24.57	30.10	34.68	38.47	41.60	44.18	46.32	48.09	49.56	
20	9.95	18.09	24.76	30.22	34.69	38.35	41.35	43.80	45.81	47.45	48.80	
21	10.09	18.27	24.89	30.27	34.62	38.15	41.01	43.33	45.21	46.74	47.97	
22	10.21	18.40	24.97	30.25	34.48	37.88	40.60	42.79	44.55	45.96	47.09	
23	10.31	18.49	25.00	30.17	34.28	37.54	40.13	42.19	43.83	45.13	46.16	
24	10.39	18.56	24.98	30.04	34.01	37.14	39.60	41.54	43.06	44.26	45.20	
25	10.45	18.59	24.92	29.86	33.70	36.70	39.03	40.84	42.26	43.36	44.22	
26	10.50	18.59	24.83	29.64	33.35	36.21	38.41	40.11	41.42	42.44	43.22	
27	10.53	18.56	24.70	29.38	32.95	35.68	37.77	39.36	40.57	41.50	42.20	
28	10.54	18.51	24.53	29.09	32.53	35.13	37.09	38.58	39.70	40.55	41.19	
29	10.55	18.44	24.34	28.76	32.07	34.54	36.39	37.78	38.81	39.59	40.17	
30	10.54	18.34	24.13	28.41	31.59	33.94	35.68	36.97	37.92	38.63	39.16	
31	10.52	18.23	23.89	28.04	31.08	33.31	34.95	36.15	37.03	37.68	38.15	
32	10.49	18.10	23.63	27.64	30.56	32.68	34.21	35.33	36.14	36.73	37.16	
33	10.44	17.95	23.35	27.23	30.02	32.03	33.47	34.51	35.25	35.79	36.17	
34	10.39	17.79	23.06	26.80	29.47	31.37	32.72	33.68	34.37	34.86	35.20	
35	10.33	17.62	22.75	26.36	28.91	30.71	31.97	32.87	33.49	33.94	34.25	
36	10.27	17.43	22.43	25.91	28.35	30.04	31.23	32.05	32.63	33.03	33.31	
37	10.19	17.23	22.09	25.45	27.77	29.38	30.48	31.25	31.78	32.14	32.39	
38	10.11	17.03	21.75	24.99	27.20	28.71	29.74	30.45	30.94	31.27	31.49	
39	10.02	16.81	21.40	24.51	26.62	28.05	29.01	29.67	30.11	30.41	30.61	
40	9.93	16.59	21.05	24.04	26.04	27.39	28.29	28.89	29.30	29.57	29.75	
41	9.83	16.36	20.69	23.56	25.47	26.73	27.57	28.13	28.50	28.74	28.91	
42	9.73	16.12	20.32	23.08	24.89	26.08	26.87	27.38	27.72	27.94	28.09	
43	9.62	15.88	19.95	22.60	24.32	25.44	26.17	26.64	26.95	27.15	27.28	
44	9.51	15.63	19.58	22.12	23.75	24.81	25.49	25.92	26.20	26.39	26.50	
45	9.39	15.38	19.20	21.64	23.19	24.18	24.81	25.22	25.47	25.64	25.74	
46	9.28	15.13	18.83	21.16	22.64	23.57	24.15	24.52	24.76	24.90	25.00	
47	9.16	14.88	18.45	20.69	22.09	22.96	23.50	23.85	24.06	24.19	24.28	
48	9.03	14.62	18.08	20.22	21.54	22.36	22.87	23.18	23.38	23.50	23.57	
49	8.91	14.36	17.71	19.75	21.01	21.78	22.25	22.54	22.71	22.82	22.89	
50	8.78	14.10	17.33	19.29	20.48	21.20	21.64	21.90	22.07	22.16	22.22	

Assumptions: Real interest rate is 3% per year.  
Nominal interest rate exceeds real rate by 1% per year plus the expected inflation rate.



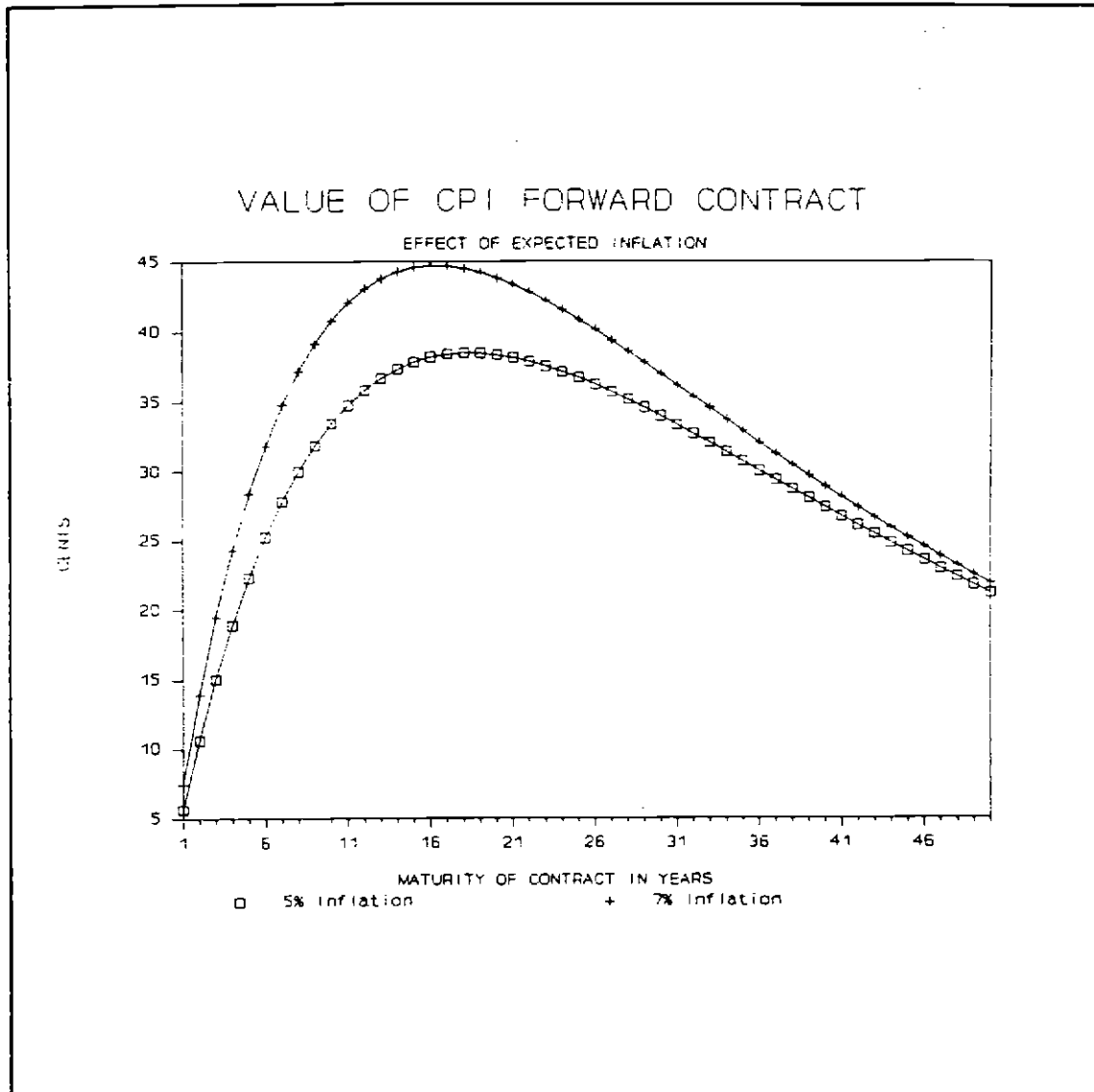


Figure 3

Figure 3. Price of a CPI Forward Contract as a Function of Maturity and Expected Inflation Rate

Notes: The figure assumes that the term structures of real and nominal interest rates are flat, that the real rate is 3% per year, and that the nominal rate exceeds the real rate by 1% plus the expected inflation rate. The forward price is assumed to be 1.

## 2.2 Forward Rates of Inflation and the Term Structure of Nominal and Real Interest Rates.

Let us now consider a forward contract with a forward price equal to  $e^{iT}$  where  $i$  is some contractually specified rate of inflation.

The payoff to the forward contract at maturity is:

$$P(T) - e^{iT}$$

We can replicate that payoff by going long one indexed bond and short  $e^{iT}$  nominal bonds. The cost of establishing this position is:

$$f = e^{-rT} - e^{iT}(e^{-RT}) = e^{-rT} - e^{(i-R)T} = e^{-rT} - e^{-(R-i)T}$$

Let us define the T-period forward rate of inflation as the value of  $i$  that makes the value of a CPI forward contract maturing at time T equal to zero. Equivalently, the forward inflation rate is that rate of inflation which would make the realized return on a real bond equal to the realized return on the corresponding nominal bond.

It follows that the forward rate of inflation,  $i^*(T)$ , is the difference between the risk-free nominal rate and the risk-free real rate of interest:

$$i^*(T) = R(T) - r(T)$$

It also follows that the forward rate of inflation is the sum of the expected rate of inflation plus the risk premium:

$$i^*(T) = \pi(T) + \phi(T)$$

We can directly measure the term structure of forward rates of inflation from the observed term structures of real and

nominal interest rates. But we cannot know without making further assumptions what fraction of the forward rate for any maturity is expected inflation and what fraction is the risk premium.

As of this writing (May 1989) the term structure of risk-free real interest rates is slightly upward sloping. The 1 year rate is 3% and the 10 year rate 3.2%.<sup>5</sup> The term structure of nominal rates as reflected in the U.S. Treasury yield curve is approximately flat at 9% per year. This implies a term structure of forward inflation rates that is slightly downward sloping: from 6% per year for a 1 year maturity to 5.8% per year for a 10 year maturity.<sup>6</sup> Throughout the rest of this paper, however, we will assume flat term structures for both real and nominal interest rates.

### 3. Inflation Insurance as a Call Option.

A forward contract is not what most people have in mind when they talk about insurance. Insurance is a contingent claim that pays the policyholder a specified amount if the contingency occurs and nothing otherwise, whereas a forward contract may entail the payment of money by the contract holder at the

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<sup>5</sup>These are the rates available on Franklin Savings Inflation Plus Certificates of Deposit, which are insured up to \$100,000 by FSLIC.

<sup>6</sup>Munnell and Grolnic [1986] provide information on the term structure of real interest rates on index-linked Treasury bonds in the U.K.

maturity of the contract.

In our example a more correct specification of the insurance contract would be that the payoff at T is:

$$\text{Max } [0, P(T) - 1]$$

This is the same as the payoff to a European call option on the CPI with an exercise price of \$1 maturing at time T.

From the put-call parity theorem we know that a European call option is equivalent to a forward contract plus a European put option. Here the put has the following payoff:

$$\text{Max } [0, 1 - P(T)].$$

In other words the put is deflation insurance.

Thus an inflation insurance policy (the call option) is equivalent to a forward contract on the CPI plus a deflation insurance contract (the put option).

By an arbitrage argument, the price of the call must equal the price of the forward contract plus the price of the put:

$$c = f + p$$

If the probability of deflation (that is,  $P(T) < 1$ ) over the term of the inflation insurance policy is virtually zero, then the put will be virtually worthless and the price of the call will equal the price of the forward contract.

### 3.1 Inflation Insurance with a Deductible.

But suppose that there is a deductible in the policy. The deductible might specify that the contract compensates the holder only for inflation above some specified rate of  $i$  per year. Then

the exercise price of the call option is  $e^i$ , and the put option is now insurance against inflation being less than  $i$ . It will be valuable if  $i$  gets close to  $i^*$ .

If we set the deductible equal to the forward rate of inflation, then the price of the forward contract will be zero and the price of the call will exactly equal the price of the put.

### 3.2 Black-Merton-Scholes Valuation of Inflation Insurance

The price of the inflation insurance will depend on the stochastic processes for the rate of inflation and the real and nominal risk-free interest rates. Since inflation insurance is isomorphic to a European call option the method developed by Merton [1973] and Black and Scholes [1973] can be applied here.

If we assume that the stochastic behavior of  $P(T)$  can be approximated by a diffusion process and that interest rates are nonstochastic, then we can apply a modified form of the Black-Scholes option pricing formula:<sup>7</sup>

$$C = N(d_1)e^{-rt} - N(d_2)Xe^{-Rt}$$

The exercise price on the option is determined by the inflation rate one establishes as the "deductible," (that is,

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<sup>7</sup>To be more precise, we assume that the CPI follows an Ito process of the form:  $dP/P = \alpha dt + \sigma dz$  where  $dz$  is an increment of a standard Wiener process with zero mean and variance of 1,  $\alpha$  is the instantaneous mean rate of inflation per unit time, and  $\sigma^2$  the variance per unit time.

$X = e^{iT}$ ). Table 4 and Figure 4 present the value of this call option and the value of a corresponding forward contract for a 10 year maturity as a function of the deductible, when the riskless real interest rate is 3% per year, the riskless nominal rate is 9% per year, and the standard deviation of the rate of inflation is 3% per year.

The forward contract is assumed to have a forward price equal to the exercise price on the option. For values of  $i$  greater than  $i^*$  the value of the forward contract is negative.

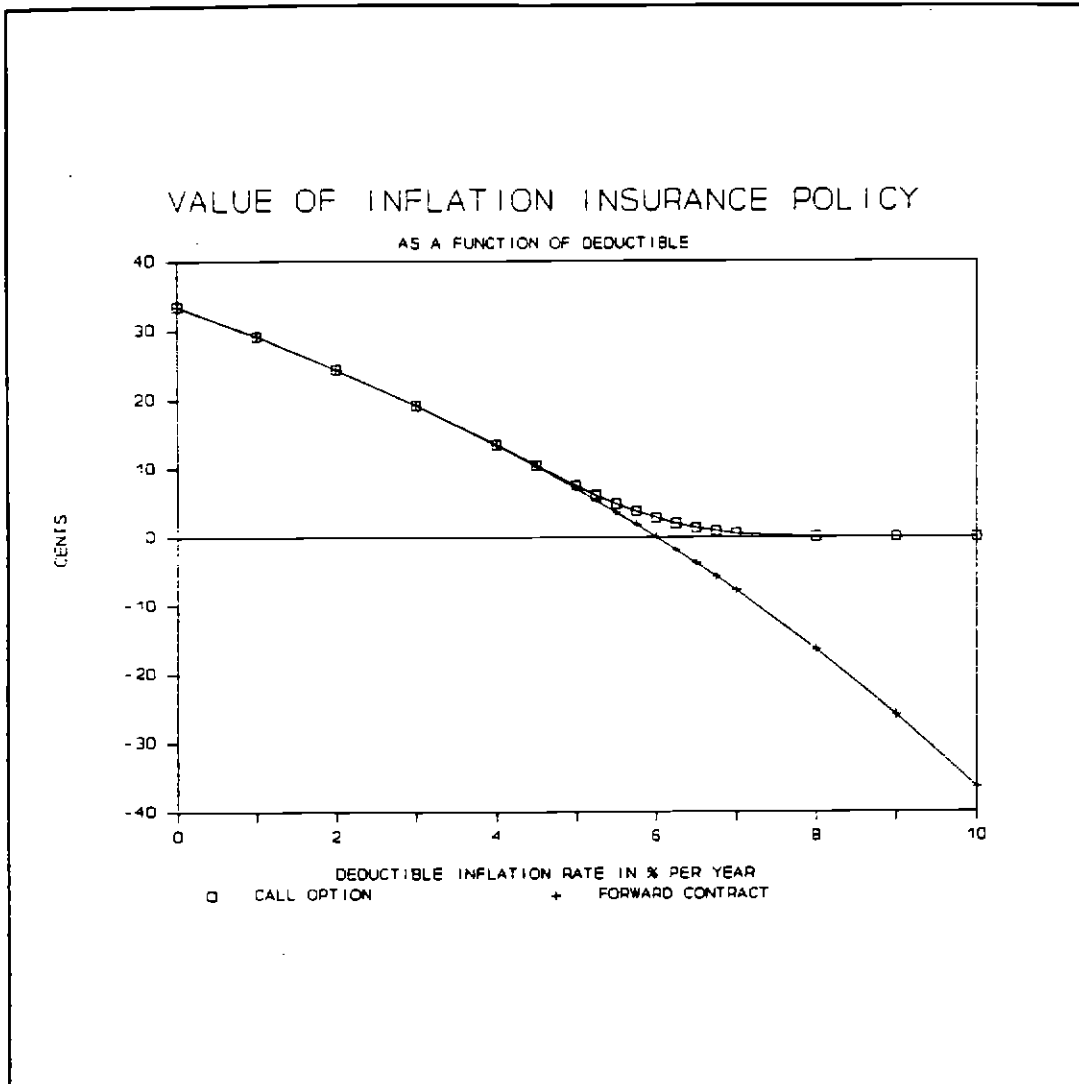
For low values of  $i$  the value of the call is virtually identical to the value of the forward contract. This is because the probability that the call will wind up out-of-the-money is virtually zero. (Equivalently, the value of the corresponding put option is virtually zero for low values of  $i$ .)

For values of  $i$  greater than 6% per year (the forward inflation rate), the value of the forward contract is negative. The value of the call option however, can never be negative. It approaches zero asymptotically as  $i$  increases.

Table 4. Price of a CPI Call Option as a Function of the Deductible Rate of Inflation

Deductible Rate of Inflation	Price of CPI Call	Value of CPI Forward Contract
-----	-----	-----
0%	33.425	33.425
1	29.149	29.149
2	24.423	24.423
3	19.202	19.201
4	13.469	13.429
5	7.550	7.050
6	2.811	0.000
7	0.553	-7.791
8	0.049	-16.402
9	0.002	-25.918
10	0.000	-36.435

Assumptions: The price is in cents per dollar insured.  
Maturity is 10 years; the risk-free real interest rate is 3% per year; the risk-free nominal rate is 9% per year; and  $\sigma$  is 3% per year.



**Figure 4**

**Figure 4. Price of Inflation Insurance as a Function of the Deductible Rate of Inflation**

**Assumptions:** Maturity is 10 years; the risk-free real interest rate is 3% per year; the risk-free nominal rate is 9% per year; and  $\sigma$  is 3% per year.



Consider the value of the inflation insurance policy that has a deductible equal to the forward rate of inflation. We know that for such a policy the corresponding forward contract has zero value. The value of the call in this case is:

$$C = [N(d_1) - N(d_2)]e^{-rT}$$

Table 5 and Figure 5 present the value of this inflation insurance policy as a function of maturity for two different values of  $\sigma$ , the volatility of the rate of inflation. Note that the value of the call first rises and then falls with maturity.

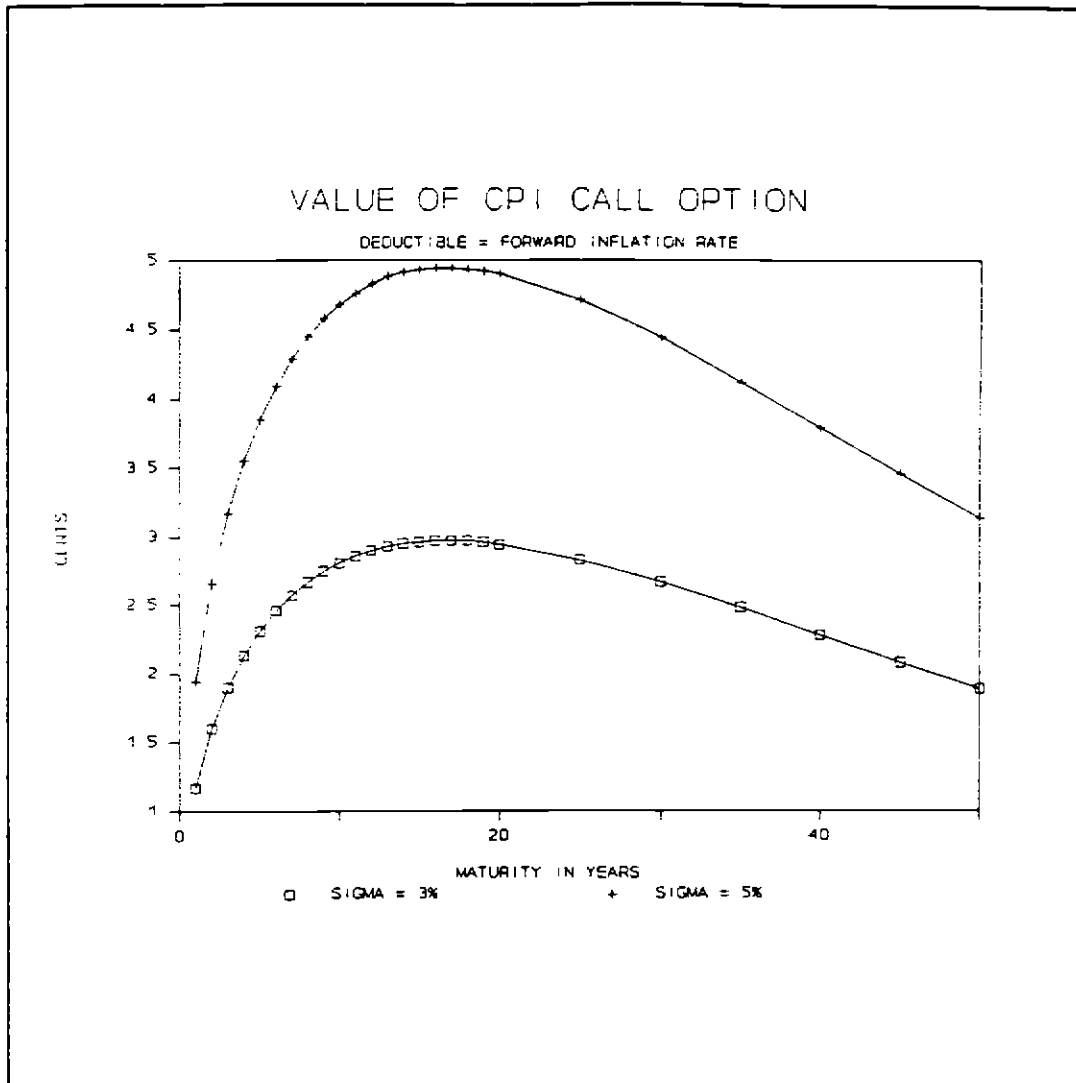
This pattern is the same as we observed for the CPI forward contract with a forward price of 1 in Figure 1. The reason for it is similar. As with forward contracts, the value of a call option can never exceed the value of the security on which it is written. Since the value of the real bond goes to zero as  $T$  increases (provided that the real interest rate is positive), so too must the value of the CPI call option.

Note that the value of the option increases with volatility. This is a well-known result in option pricing theory. It reflects the asymmetric payoff structure of the call option. Increasing the volatility increases the upside potential without increasing the downside risk.

TABLE 5. PRICE OF A CPI CALL OPTION AS A FUNCTION OF MATURITY AND VOLATILITY OF INFLATION

	VOLATILITY			
	3%	5%	10%	
	----	----	----	
	1	1.17	1.94	3.88
	2	1.60	2.66	5.32
	3	1.90	3.17	6.32
	4	2.13	3.55	7.08
MATURITY	5	2.31	3.85	7.68
	6	2.46	4.09	8.16
	7	2.57	4.29	8.55
	8	2.67	4.45	8.86
	9	2.75	4.58	9.12
	10	2.81	4.68	9.32
	11	2.86	4.76	9.49
	12	2.90	4.83	9.61
	13	2.93	4.88	9.70
	14	2.95	4.91	9.77
	15	2.96	4.93	9.81
	16	2.97	4.94	9.82
	17	2.97	4.94	9.82
	18	2.97	4.93	9.80
	19	2.96	4.92	9.77
	20	2.94	4.90	9.72
	25	2.83	4.71	9.34
	30	2.67	4.44	8.78
	35	2.48	4.12	8.15
	40	2.28	3.79	7.48
	45	2.08	3.46	6.82
	50	1.89	3.14	6.17

Assumptions:  $r = 3\%$  per year,  $R = 9\%$  per year, and the deductible equals the forward inflation rate,  $6\%$  per year.



**Figure 5**

Figure 5. Price of CPI Call Option as a Function of Maturity and Volatility of Inflation

**Assumptions:** The risk-free real rate is 3% per year; the risk-free nominal rate is 9% per year; the deductible equals the forward rate of inflation (6% per year).

### 3.3 Inflation Insurance with a Cap.

Often the cost-of-living adjustments that are promised under certain pension plans and life insurance company annuities are subject to a cap. Our previous analysis can easily be modified in order to price such an inflation insurance policy.

The only adjustment needed to the model presented in the preceding section is to subtract from the price of an inflation insurance policy with no deductible the price of a policy that has a deductible equal to the specified cap rate of inflation. The price of an inflation insurance policy with a cap is therefore equal to the price of a CPI call option with an exercise price of 1 minus the value of a CPI call option with exercise price  $e^{cT}$ , where  $c$  is the cap on the inflation rate.

For example, consider an inflation insurance policy that is capped at 4% per year. Assume that  $r = 3\%$ ,  $R = 9\%$ ,  $T = 10$  years, and  $\sigma = 3\%$  per year. The price of the CPI call option with no deductible is 33.42 cents. The price of the CPI call option with a deductible equal to 4% per year is 13.47 cents. The price of the capped inflation insurance policy is therefore 19.95 cents.

#### 4. How to Produce Inflation Insurance

In the preceding section we derived the prices of CPI call options using the Black-Merton-Scholes method. Now let us explain how that method allows us to synthesize CPI call options from real bonds and nominal bonds. In order to clarify the process it is useful to think of a financial intermediary that issues inflation insurance policies (CPI calls) and wants to hedge them by a suitable investment strategy.

In calculating the price of a CPI call option using the modified Black-Scholes formula, we computed  $N(d_1)$ . This is called the hedge ratio, and it is the number of real bonds that the intermediary must buy per CPI call option sold. Thus if  $N(d_1)$  is .5, the intermediary would invest in one half of a real bond per \$1 that it is insuring and would borrow  $N(d_2)Xe^{-RT}$  dollars at the risk-free nominal rate.

Once the intermediary issues an inflation insurance policy it must continuously rebalance its portfolio to remain perfectly hedged. The hedge ratio will usually change as a result of changes in  $T$  and the realized rate of inflation. The process of continuously rebalancing the portfolio is called dynamic hedging.<sup>8</sup>

Table 6 and Figure 6 present the hedge ratio as a function of the deductible for the case where  $r = 3\%$ ,  $R = 9\%$ ,  $T = 10$  years, and  $\sigma = 3\%$ .

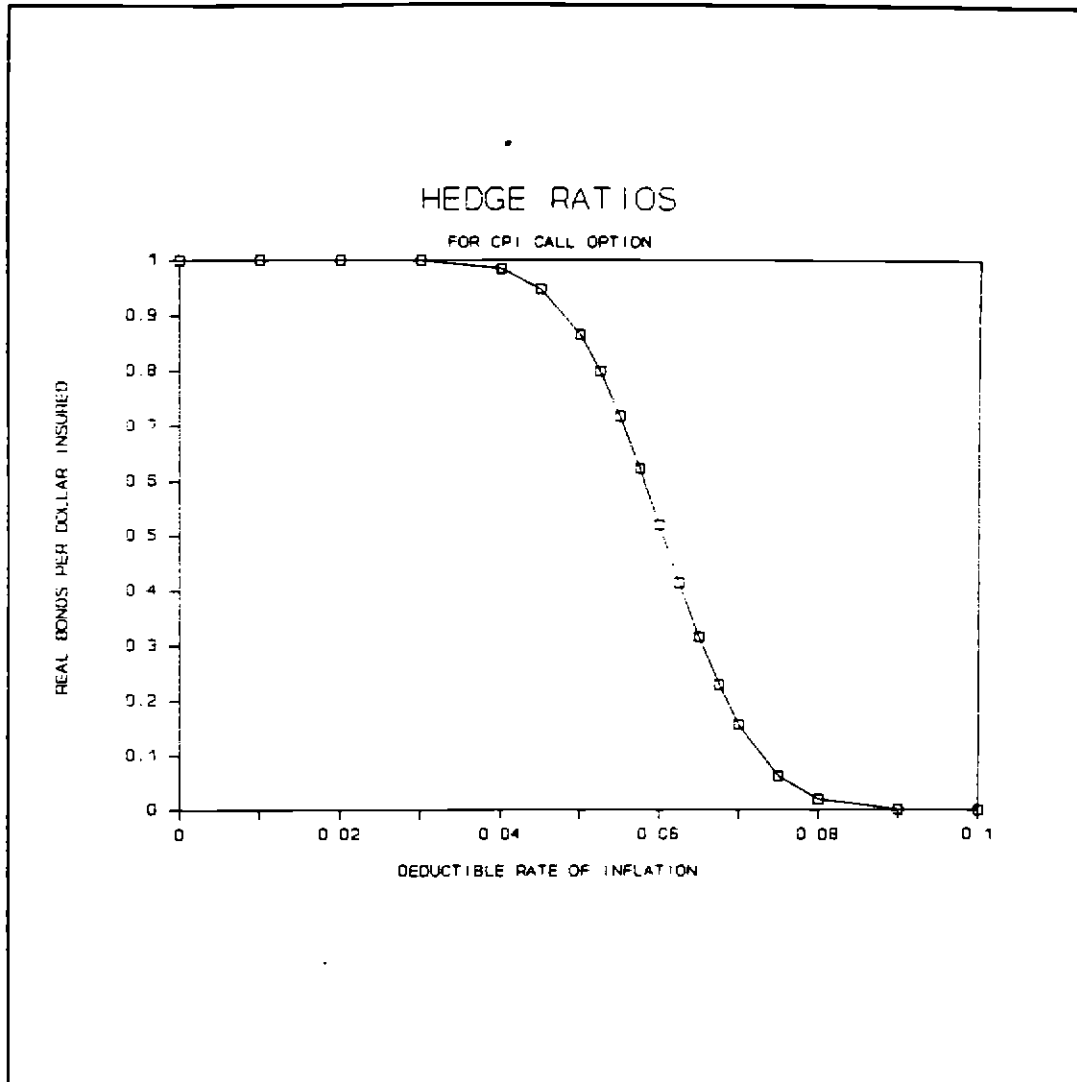
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<sup>8</sup>See Chapter 19 in Bodie, Kane, and Marcus [1989] for a discussion of dynamic hedging.

Table 6. Hedge Ratio for a CPI Call Option as a Function of the Deductible Rate of Inflation

Deductible Rate of Inflation	Hedge Ratio
-----	-----
0%	1.000
1	1.000
2	1.000
3	.999
4	.984
5	.865
6	.519
7	.157
8	.020
9	.001
10	.000

Assumptions: Maturity is 10 years; the risk-free real interest rate is 3% per year; the risk-free nominal rate is 9% per year; and  $\sigma$  is 3% per year.



**Figure 6**

Figure 6. Hedge Ratios for CPI Call Option as a Function of the Deductible Inflation Rate

Assumptions: Maturity is 10 years; the risk-free real interest rate is 3% per year; the risk-free nominal rate is 9% per year; and  $\sigma$  is 3% per year.

5. Inflation-Protected Annuities.

If inflation insurance became available, it is likely that the major demand for it would be to insure pension benefits.<sup>10</sup> The cost of insuring a stream of nominal payments against inflation is the sum of the costs of insuring each individual payment.

Table 7 and Figure 7 present the cost of insuring a 20 year nominal annuity of \$1 per year against inflation as a function of the deductible. Thus, with a zero deductible, the cost of inflation insurance is \$5.95. Since the price of the nominal annuity is \$8.86, this means that the cost of insuring it fully against inflation is 67% of its value.

The cost of inflation insurance with a deductible equal to the forward inflation rate (6% per year) is \$.52 or roughly 6% of the value of the nominal annuity. And the cost of "catastrophic" inflation insurance, defined as a policy with a deductible equal to 10% per year, is only \$.002 or .02% of the value of the bond.

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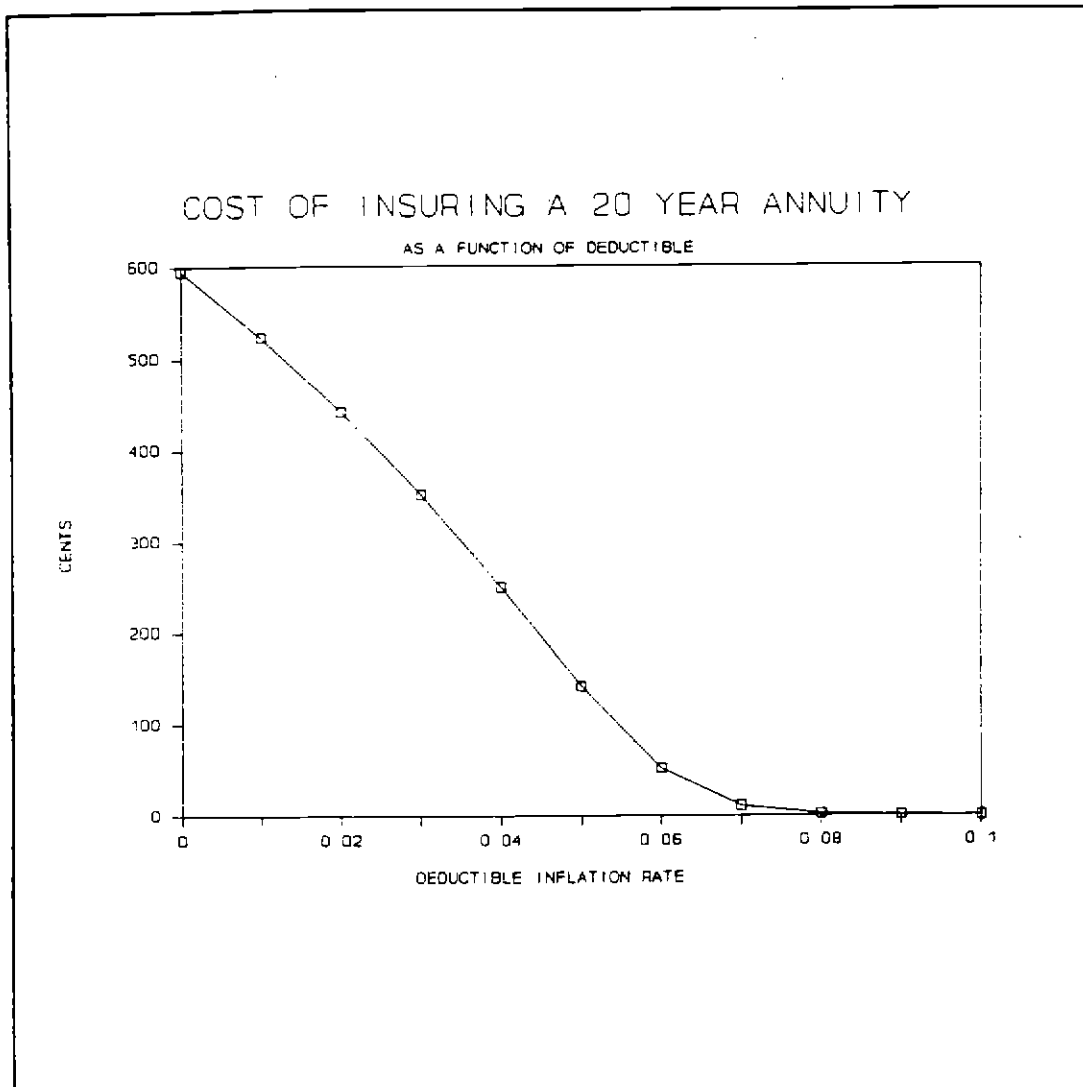
<sup>10</sup>For a discussion of this issue see Bodie [1989].



Table 7. Cost of Insuring a 20 Year Annuity Against Inflation

Deductible Rate of Inflation	Cost	Cost as a Fraction of Value of Annuity
-----	-----	-----
0%	\$5.95	.67
1	5.23	.59
2	4.42	.50
3	3.51	.40
4	2.51	.28
5	1.42	.16
6	.51	.06
7	.11	.01
8	.02	.002
9	.006	.0007
10	.002	.0002

Assumptions: The risk-free real interest rate is 3% per year; the risk-free nominal rate is 9% per year; and  $\sigma$  is 3% per year. The price of a \$1 nominal 20 year annuity is \$8.86.



**Figure 7**

**Figure 7. Cost of Insuring a 20 Year Nominal Annuity Against Inflation**

**Assumptions:** The risk-free real interest rate is 3% per year; the risk-free nominal rate is 9% per year; and  $\sigma$  is 3% per year.

## 6. The Role of the Government

For many years economists considered it desirable, if not downright essential, for the Federal government to issue CPI-linked bonds in order to lay the foundation for inflation insurance. Economists like Milton Friedman, Franco Modigliani, and James Tobin, who hold very different opinions on other issues, were united in their enthusiastic support for the idea of the U.S. Treasury's issuing CPI-linked bonds. Even now many people think that the only entity that can truly guarantee default-free inflation insurance is the Federal government.

While that proposition is strictly speaking true, it is equally true that private insurance can be almost free of default risk. A private insurance company can offer policies that are virtually free of default risk through a combination of three elements: (1) diversifying the risk of its liabilities through risk-pooling (as life insurance companies do with their life insurance policies), (2) hedging the risk of its liabilities through appropriate investment strategies, and (3) maintaining adequate equity capital so that the residual risk that is not diversified away or hedged away by the company's investment strategy is fully absorbed by the company's shareholders.

Inflation insurance is not a diversifiable risk in the aggregate, so the first of the three elements is not available. But diversifiability of risk is neither a necessary nor a sufficient condition for it to be insurable. An insurance company or other financial intermediary can use a combination of

the other two elements to provide nearly complete inflation insurance.

These observations make clear that what is necessary for there to exist inflation insurance is someone in the economy who is willing to bear some part of the risk of inflation at a fair market price. The natural candidates for doing this would be people or institutions who are "over-indexed" for inflation. Feldstein [1983] and Summers [1983] have maintained that substantial numbers of households at all stages of the life cycle may find themselves in this position. Fischer [1986] maintains that many non-financial business firms may be in such a position too.

#### 7. Private Supply of CPI-Linked Securities

To the extent that the demand for inflation insurance is small, the existing supply of CPI-linked securities can provide enough of a hedge asset. But what if the demand should grow in the future?

One promising source of CPI-linked investments for an inflation insurance intermediary is CPI-linked home mortgages. The U.S. Department of Housing and Urban Development (HUD) is about to certify a variety of price-level-adjusted mortgages (PLAMs) for Federal Housing Administration approval (FHA). There is reason to believe that once FHA mortgage insurance is available and the tax status of PLAMs is clarified, they could account for a significant portion of new lending in the home

mortgage market.<sup>11</sup>

## 8. Policy Implications

Proposals to index pension benefits and other nominal annuities in both the private and public sectors have a long history.<sup>12</sup> In the U.K. the government has gone so far as to mandate the indexation of the minimum level of employer-provided pension benefits, and the government of the Province of Ontario, Canada is on the verge of adopting similar measures.<sup>13</sup> The approach presented in this paper permits fairly precise quantification of the cost of implementing such proposals.

This approach also gives us a way of estimating the savings to the Social Security system that would result from introducing a deductible. Feldstein [ ] has advocated limiting the Social Security cost-of-living adjustment to the excess of the actual inflation rate over some deductible. Our approach can help to quantify the savings that would result from any deductible rate of inflation.

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<sup>11</sup>See Modigliani and Lessard [1975] for a discussion of these mortgage designs.

<sup>12</sup>See Bodie and Pesando [1983].

<sup>13</sup> See Friedland [1988] for the Canadian situation and Hemming and Kay [1982] for the U.K.

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