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CONSUMPTION AND SAVING AFTER RETIREMENT

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Abstract

The paper analyzes consumption decisions of retired workers, using Danish register data. A major puzzle, which motivates much of the analysis below, is that wealth actually increases for a large fraction of the people in our data. One would expect that wealth accumulated before retirement would be used to augment consumption in later life, with the implication that wealth should decline over time. The risk of large out-of-pocket medical expenditures is negligible in Denmark, so although explanations associated with such expenditures might explain similar patterns in U.S. data, these explanations are not plausible for Denmark (and therefore also questionable for the U.S.). Our analysis instead attempts to explain wealth paths using a model that emphasizes fluctuations in the marginal utility of consumption. The results show that a latent state variable extension of the standard life-cycle consumption model is quite successful in explaining the curious observed wealth patterns after retirement for singles.

1 Introduction

The basic economic forces affecting retirement decisions are still not well understood, although this has been an active area of research recently, and substantial progress has been made.[1] But the literature has largely ignored the rather obvious point that it makes little sense to try to understand retirement decisions without first understanding what happens after retirement. The main question is why many people choose to work full-time for about 40 years, and not work at all for the remaining 15 years or so, rather than taking more time off when they are younger, and working more when they are older. As life expectancy increases, people allocate the added years of life either to work or retirement, and how this allocation is made has a big effect on the financing of social security systems.

This paper analyzes register data from Denmark on consumption, saving, and wealth after retirement using a dynamic programming model. The model allows for a bequest motive, and for the possibility that the marginal utility of consumption depends on the level of a persistent state variable. The model can explain why some people accumulate wealth after retirement, and why they may retire without wealth. The point of the model is to go beyond descriptions of patterns seen in the data, and develop a theory of the decisions that generated these patterns, so that it becomes possible to predict decisions that would be made in other circumstances. Future retirement decisions will be made in an environment that involves substantial changes in life expectancy, and in the generosity of public pension schemes. Merely extrapolating from past data is unlikely to provide accurate predictions of future saving and retirement decisions, and such predictions are essential for informed public policy regarding social security systems.

The structure of our model is largely motivated by wealth change patterns in the data. Table 1 shows counts of wealth decreases and increases from age 68 to the year before death for people in the 1927 birth cohort who were

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[1] See French (2005), French and Jones (2011) and Fan et al. (2017), for example.

single and retired at age 68, and who survived at least five years. One might expect that people who retire with some accumulated wealth and a life annuity (such as the old-age pension that is universally available in Denmark) would use some of this wealth to finance increased consumption, perhaps retaining some portion to leave as a bequest. Thus, one would expect a gradual decrease in wealth as time goes on. But in the data we find that many people actually accumulate wealth after retirement, meaning that they save a portion of their annuity income.

Table 1: **Wealth Changes**

Age 68 to year before death						
Single, survived at least 5 years from age 68						
	Men			**Women**		
	Decrease	Increase	Total	Decrease	Increase	Total
	942	1,268	2,210	2,770	2,618	5,388
Property						
Yes	120	462	582	285	782	1,067
No	822	806 49.5%	1,628	2,485	1,836 42.5%	4,321

De Nardi et al. (2010) and De Nardi et al. (2021) have argued that consumption and savings decisions after retirement in the U.S. are heavily influenced by the risk of large medical expenses late in life, especially in the case of richer people. A great advantage of our data is that out-of-pocket medical expenses are negligible in the Danish welfare system, and so the analysis can focus on pure consumption choices, where the essential tradeoff is between current and future consumption (including the consumption of heirs).

For property owners, wealth increases might to some extent reflect increases in property values. McGee (2019) documents that house price increases to a large extent are passed on to future generations in the form of bequests, based on survey data from England. For this reason, our empirical analysis is focused on people who do not own property. Nevertheless, in Table 1, even among people with no property, wealth increases are remarkably common. One possible interpretation of the wealth increases in these data is that there are changes in bequest incentives due to unexpected events occurring after retirement (since anticipated bequests should presumably be covered by wealth accumulated before retirement). Still, on average, such changes should be zero.

We focus on single people without property. A full analysis of married couples would require a more complicated household model, and such a model would in any case necessarily include a component specifying the behavior of people who become single, because of a divorce or the death of a spouse.[2] We pursue the explanation that the marginal utility of consumption is state-dependent, with transitions in an underlying latent state. According to this view, savings can be motivated by currently low marginal utility of consumption, combined with an expectation of transition to higher marginal utility in the future. Reversals, with wealth initially increasing, then running down to zero, as also observed in many cases in the data, reflect such transitions in the latent state, and could not be rationalized by a bequest motive alone.

[2] De Nardi et al. (2021) have recently considered the case of couples, using a unitary household model to analyze data from the AHEAD component of the U.S. Health and Retirement Survey. The analysis focuses on the importance of late-life medical expenses, and the interaction between bequest motives and precautionary savings motives.

1.1 Literature

Here, we briefly mention a number of issues and studies that are relevant for our work. First, we study consumption and saving decisions by the elderly, and since most medical expenditures are insured by the government in the Danish welfare state, we can study these decisions in the absence of medical expenditure risks. Using the Danish registers, Christensen et al. (2016) find that out-of-pocket medical expenses amount to only 2.3% of personal health care expenditures, with the remainder covered by the government, and this fraction does not increase in the last year before death. Nevertheless, utility of consumption may depend on health, and this is a possible interpretation of the latent state variable in our model.

The effect of health on early retirement is studied by Christensen and Kallestrup-Lamb (2012), using the Danish register data and a pure duration model. Finkelstein et al. (2013) argue that people who are in bad health have a lower marginal utility of consumption, based on survey responses to questions about happiness. Blundell et al. (2020) present a more detailed analysis, based on survey responses to direct questions about health, augmented by objective measures of difficulties in activities of daily living as well as diagnoses of specific diseases. Blundell et al. (2020) find that bad health reduces the marginal utility of consumption, and more specifically that this effect is largely confined to the category of luxury goods.

Our paper is most closely related to the work of Laitner et al. (2018), which seeks to explain why the average wealth of a cohort might rise after retirement. There is just one health transition, from good to bad health, and this affects the marginal utility of expenditure, because the same level of real consumption may require additional expenditure in the bad health state. Wealth accumulation is explained by showing that there is an optimal level of wealth in the good health state, and if initial wealth after retirement is below this, it is optimal to consume less than current income, so as to build up savings for the bad health stage, when the marginal utility of expenditures will be higher. This begs the question of why someone in good health would choose to retire with insufficient wealth.

Gan et al. (2015) analyze the bequest motive for single people with children, using a quasi-linear specification in which the marginal utility of bequests depends on the number of children, but not on the size of the bequest. This implies that if there is a positive bequest, the marginal utility of consumption is fixed, so the consumption trajectory is independent of initial wealth. There is an initial wealth level, and a constant level of annuity income, and borrowing against future annuity income is not feasible. The focus of the paper is on how subjective mortality beliefs affect consumption in later life. Median regressions indicate that the bequest motive is negligible, although a few rich families leave large bequests, which can't be explained without a bequest motive.

1.2 This paper

We study the Danish register data with the purpose of estimating and assessing an optimizing intertemporal model, focusing on consumption and saving after retirement. Relative to previous work, we generalize in several ways. First, we allow for latent state transitions in the marginal utility of consumption, in both directions. Second, we consider subpopulations with and without bequest motives, letting the data tell us who belongs to which. Third, we include a "hand-to-mouth" extension of the basic dynamic programming framework, to capture limited consumption smoothing.

In our model, the agent derives utility from current consumption, discounts the future at a constant rate, and chooses optimal saving, taking into account current and future income, survival probabilities, and random utility shocks. For the implementation, we discretize the control and state variables, i.e., savings and wealth. We impose a maximum age of 101 years, and solve the dynamic programming problem by backward recursion for each individual in the data, and at each trial parameter value. We assume that for each person, the observed income in each year up to the year of death was known in advance, and since the income paths differ across people, we compute distinct value functions for each person. The model is estimated by maximum likelihood, nesting the parameter-dependent solution of the model in the optimization. Transitions in the marginal utility of consumption presumably might have occurred prior to retirement, as well, in which case different wealth levels are expected as of the retirement date. Therefore, we split the sample by initial wealth, considering that parameters are summaries of behavioral characteristics (including preferences) and constraints, and initial wealth differences might signal heterogeneity in these. We also split by gender, although in principle there is no reason why men and women should behave differently in our context, given that the model allows for differences in life-expectancy between men and women. To avoid modeling joint household decisions, and to focus on behavior least affected by possible (unobserved) existence of children, which would likely intensify bequest motives, we focus on singles. In the main analysis we disregard marital histories, but in Appendix A.3 we further split the sample into the never married versus the ever married (divorced, widow(er)s). We construct likelihood ratio tests of the sample splits by initial wealth, gender, and marital history. All are highly significant.

In the latent state model, an individual who is currently in the low marginal utility state may accumulate wealth, with a plan to run it down in the future if the high state is reached. Retirement without wealth is explained by past sojourns in the high marginal utility of consumption state. In the data, we observe many people who run wealth down over several years, then start saving in subsequent years, and also the converse: wealth accumulation for some years, followed by spending down to zero. In the latent state model, such savings reversals are explained by marginal utility transitions, from high to low marginal utility in the former case, and conversely from low to high. The latent state model is estimated using a hidden Markov likelihood function, including estimates of the transition probabilities as well as the probabilities of each person initially being in the high and low marginal utility states. In simulations of wealth levels and saving choices for each individual at estimated parameter values, the model explains a considerable portion of the observed wealth changes in Table 1.

We consider the possibility of a bequest motive, in order to ascertain its empirical importance relative to the latent state specification. In a model with no latent state, a bequest motive can explain the tendency to hold on to wealth, but not wealth increases, since saving for bequests could have been undertaken before retirement. Further, it cannot rationalize the savings reversals mentioned above. The identification of the bequest motive in the encompassing model including the latent state specification hinges on the existence of individuals holding on to wealth even in the high marginal utility state, as predicted by the hidden Markov analysis, and on an acceleration in the saving rate with age, as the bequest motive weighs relatively more heavily in the value function.[3] We allow for the possibility that not all individuals have a bequest motive, and estimate the probability of having one, along with proportionality and threshold parameters for the utility of bequest function, thus leading to a mixture distribution in the likelihood function.

In the data, a sizable fraction of the population does not experience significant wealth changes. This is consistent with the idea that these individuals have planned adequately for their retirement, and simply consuming their annuity income is a good approximation to the optimal policy. Our model accommodates this "hand-to-mouth" type of behavior via a single parameter, a reward added to current utility if consumption is sufficiently close to income (in the chosen discretization). In effect, this hand-to-mouth parameter acts as a penalty on the standard deviation of savings, building in a behavioral tendency to perform only limited consumption smoothing. We show that this novel device greatly improves the model fit.

The estimated model predicts considerable saving out of the annuity. Overall, we conclude that latent state transitions in marginal utility, the co-existence of subpopulations with and without bequest motives, and the hand-to-mouth extension are empirically important innovations for the analysis of consumption and saving behavior after retirement. The shifts in marginal utility explain the behavior of saving out of an annuity, as well as reversals in wealth paths, and the hand-to-mouth specification facilitates incremental modeling of decisions around a central,

[3]We show in Appendix A.1 that our model accommodates cases in which savings increase with age (specifically, the death rate), but that this requires a bequest motive. The general idea is that in case of survival, there is an annuity payment, y, so marginal utility is at most the value corresponding to consuming the annuity. An increase in death probability makes it more likely that savings will go to the heir, and the marginal utility of the heir may be higher than own marginal utility next period, conditional on survival.

approximately optimal rule.

The remainder of the article is laid out as follows. Section 2 describes the data. Section 3 presents the model of consumption after retirement, and the likelihood function. Empirical results appear in Section 4, and Section 5 concludes. Appendix A.1 illustrates some aspects of the model using a simple two-period example, Appendix A.2 contains further details on the data, and Appendix A.3 shows additional results.

2 Data

We use the register data from Statistics Denmark, covering the period 1995 through 2016. The registers contain annual individual level information on age, gender, marital status, income, and wealth, for the entire Danish population. The data are based on administrative registers and contain no survey element.

We analyze data for the 1927 birth cohort, for which the old-age pension (OAP) age was 67. We follow this cohort between the ages of 68 and 89, i.e., from 1995 to 2016.[4] This is a natural choice, as individuals in the 1927 cohort are mostly retired by 1995. We also use data for an earlier cohort, born in 1919, to estimate survival probabilities, in order to have sufficient information at older ages. Our analysis is focused on singles, meaning neither married nor cohabitating. Besides gender and age, we record financial indicators, namely, income and net wealth, based on tax data, and deflated to 2015 levels using the Consumer Price Index. Information on children in the register data is deficient for the 1927 cohort (and adjacent cohorts), so we do not use it.[5]

Starting with the entire population surviving to age 68, we select people who are single and fully retired. Because the registers contain no direct information on consumption, we must infer consumption and saving choices from the income and wealth data: consumption is computed as income minus the change in wealth. In order for this to work properly, income must include accurate measures of capital gains, since otherwise observed increases in wealth would be attributed to implausibly large savings. The main difficulty involves changes in house prices. Indeed, the Danish housing price index increased by 92% in real terms over the period $1995-2016$. We avoid having to deal with wealth increases generated by general increases in property or other asset values by excluding people who owned property, stocks, or mutual funds at any time during the period covered by our data.

Details of the subpopulations that we analyze are given in Table 2. From the table, 65% of people in the cohort were still alive in 1995 (at age 68). Of these, 32% were single in 1995, and did not remarry before the end of the sample period in 2016. For comparison, 63% were married in 1995, with the remaining 5% either cohabitating in 1995, or remarrying between 1995 and 2016. The requirement that individuals have no labor market attachment reduces the sample to 12,458. Further, we discard individuals who own property or stocks, reducing the sample to 5,460. Moreover, we restrict attention to people who never had negative measured wealth.[6] This is a conservative choice, and it reduces the sample to 3,169.[7] A number of individuals are discarded because they died in the first year,

[4] The reason for starting in 1995 is that until 1994, nursing home residents were not registered in the public records with their income, largely OAP, which was paid to the nursing home. Instead, residents were paid a smaller spending allowance by the nursing homes, and only this was registered as income. Starting in 1994, nursing home residents receive their OAP benefits and pay rents out of this, and benefits are registered as income. Based on aggregate data for 2014-2019, 16% of people aged 85 and above were in nursing homes.

[5] Determining the number of children for a given individual involves a lot of uncertainty. Prior to 1960, a large share of the people in the registers do not have a match to a parent. Thus, for the 1927 cohort a significant number of people are not matched with their children in the registers, unless the parents were 33 years or older when the children were born. Similarly, we have no way to measure bequests left to children.

[6] Possible explanations of negative measured wealth could be (i) bank loans (not private loans, as these would not show up as debt in the wealth measure), or (ii) private pensions that are not annuities. Our wealth measure does not include future income that is not conditional on survival, in which case true wealth might actually be positive, but we have no way to identify this situation in our data. According to Danish Ministry of the Interior and Housing (2013), around 10 percent of individuals above the age of 64 have negative wealth. For our sample, the fraction of people with negative wealth is comparable.

[7] Correspondingly, we impose a zero lower bound on wealth in the model. Alternatives would be to set negative wealth observations to zero, or to allow negative wealth, down to some limit, in the data and the model, and in the model impose a cost in each period that the

Table 2: **Sub-Populations**: 1927 Birth Cohort

	Male	Female	Total
Born in 1927	33,079	34,945	68,024
Alive in 1995	20,499	23,378	43,877
	62.0%	66.9%	64.5%
Married	15,066	12,629	27,695
	73.4%	54.0%	63.1%
Cohabitating/Remarried	1,156	941	2,097
Single	4,277	9,808	14,085
	20.9%	42.0%	32.1%
Retired (Married)			22,019
Retired (Single)	3,427	9,031	12,458
	80.1%	92.1%	88.5%
No property	2,058	5,301	7,359
No stocks	1,674	3,786	5,460
Died after 1996	1,415	3,461	4,876
Positive wealth	869	2,300	3,169
Other*	758	2,030	2,788
Widow(er)	216	1,154	1,370
Divorced	269	614	883
Never Married	273	262	535

* Missing observations, inconsistent consumption or income data, attrition (e.g., emigration).

1995/96, or had missing data or implausibly low consumption or income at some point during the sample period. The final analysis data set consists of 2,788 individuals.

2.1 Wealth, Income and Consumption

We have data on wealth, w_t, excluding pension wealth, and income after tax, y_t, excluding capital income. In the register data, income includes interest. A separate variable records interest income, so we subtract this, to arrive at y_t. As we exclude people with property, stocks, or mutual funds, w_t represents bonds and bank accounts, and should earn interest.

In terms of timing, w_t is an end-of-period measure. We assume that interest is earned on w_{t-1} at rate r before deciding consumption c_t. Thus, consumption in period t, as used in the likelihood function, is

$$c_t = y_t + Rw_{t-1} - w_t,$$

with $R = 1 + r$. In the dynamic programming model, the upper bound on the set of feasible consumption values c_t

wealth is negative. Imposing that debt must be paid back before death is not an option, since time of death in uncertain in the model.

corresponds to the corner solution,

$$c_t = y_t + Rw_{t-1}.$$

This reflects an incomplete markets assumption. We assume that these individuals cannot finance current consumption by borrowing against future retirement income. The relevant state variable for the consumption-saving decision in period t is beginning-of-period wealth, Rw_{t-1}. Wealth and income data are deflated by the CPI to begin with, to keep calculations in real terms.[8]

2.2 Some Descriptive Statistics

Income after retirement is dominated by annuity flows from the public pension system. For the single people in our data set, Figure 1 shows the distribution of annual income at age 68 in Danish Kroner (at 2015 prices, 6 DKK being roughly equivalent to one U.S. dollar). Clearly, the dispersion in income is quite low. The distributions are slightly different for women and men, with more concentration at low income levels for men.

In our model, we assume that the realized income values were already known to each individual in the initial period of the dynamic programming problem, as would be the case if the only source of income were a life annuity. This also means that people who died during the period of observation knew what their income would have been in each year if they had survived, but of course we only have income data for the years while they were alive. We handle this by assigning a constant income level to each person beyond the year of death, and we set this to the last observed income.[9]

Wealth at age 68 for singles is also shown in Figure 1. Distributions are similar by gender, and most of the mass is below $50,000$ DKK, i.e., relatively low, compared to annual income.[10]

3 Consumption After Retirement

3.1 The Model

We assume that an individual with (at most) n periods remaining chooses consumption by solving the dynamic programming problem

$$V_n^h(w) = \max_{x\in[0,w+y_n]} \left\{\theta^h u(w+y_n-x) + \beta(1-\delta_n)\left(\rho^h V_{n-1}^h(Rx) + \left(1-\rho^h\right)V_{n-1}^{h'}(Rx)\right) + \delta_n \tau u^b(x)\right\}.$$

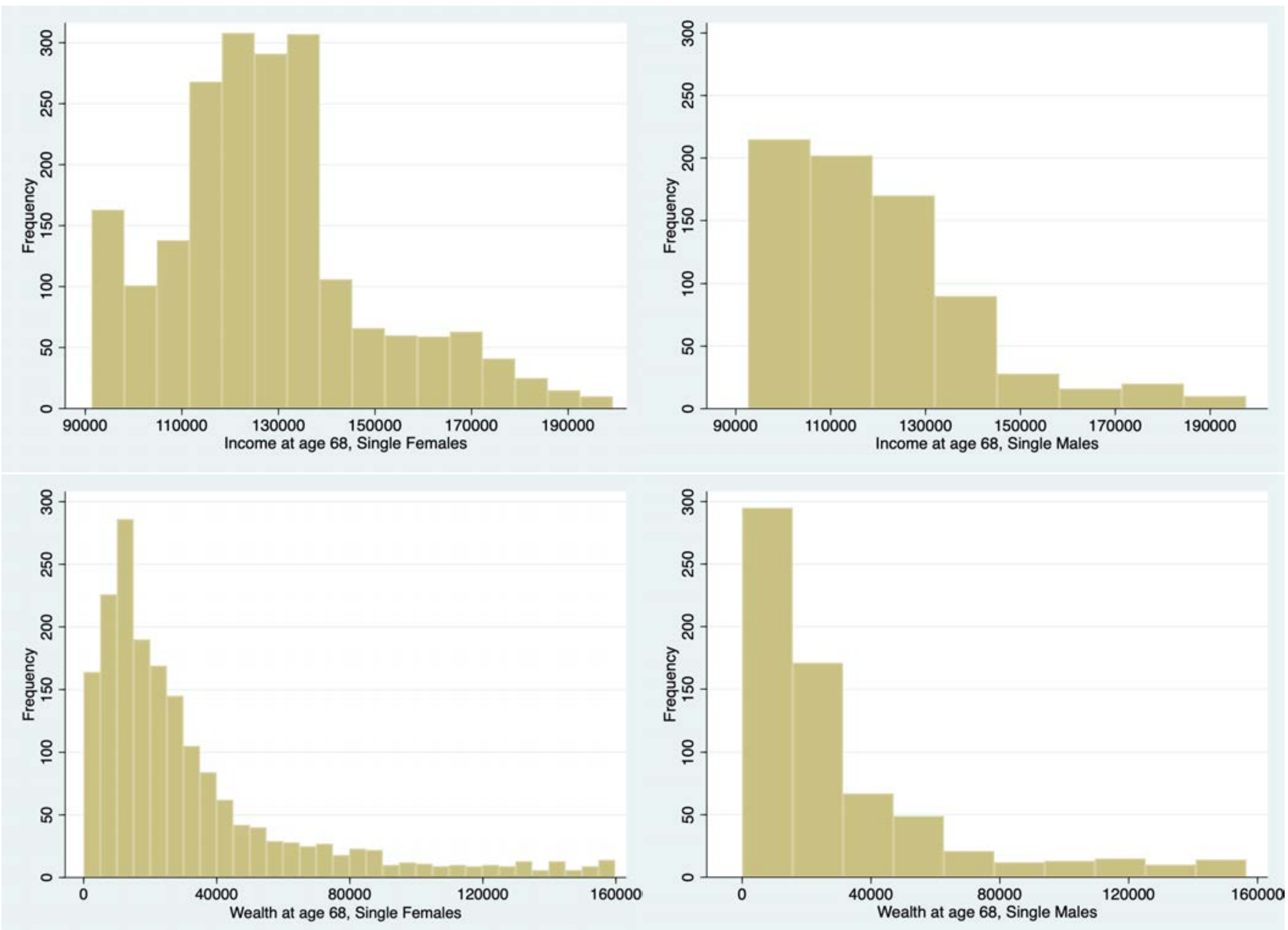
The notation is as follows. Annuity income in each period is y_n, and the wealth level before receiving this income is w. Consumption c cannot exceed current resources $w+y_n$, and $x = w+y_n-c$ is the amount carried over, yielding Rx next period. The discount factor is β. The marginal utility of consumption is shifted by a two-state Markov chain, taking values $h \in \{L, H\}$, with $\theta^h \in \left\{\theta^L, \theta^H\right\}$ and $\theta^L \leq \theta^H$. The probability of death is δ_n, increasing as the

[8] We ignore wealth taxation in the modeling. The wealth tax in Denmark was abolished in 1997. As our analysis data set starts in 1995, ignoring the wealth tax can be done without much loss.

[9] Figure 4 in Appendix A.3 shows the distribution across individuals of the standard deviation of real income over years of observation. Clearly, variation over time is typically small, with most of the mass at standard deviations below 10,000, consistent with the notion of the income stream as essentially an annuity, so treating income as known in advance is a reasonable approximation.

[10] Figure 5 in Appendix A.3 shows real wealth distributions over all observation years, including the $12,458$ people in the 1927 birth cohort who were single and retired at age 68. Distributions are similar across gender, and similar to the wealth distributions in Figure 1, but with some mass below zero, and longer right tails, in part due to ownership of property/stocks. The corresponding individuals were discarded from the analysis data set, cf. Table 2.

Figure 1: Income and Wealth at Age 68



number of remaining periods dwindles. In case of survival, ρ^h is the probability of state h next period, given the current state h. Thus, ρ^h is the persistence probability for state h, and we write h' for the opposite state. In the event of death, remaining resources are left as a bequest. We assume a utility of bequest function u^b and a strength τ of the bequest motive.

One interesting case is when there is a corner solution in the high marginal utility state, and an interior solution in the low state. In this case,

$$V_n^H(w) = \theta^H u(w + y_n) + \beta(1 - \delta_n)\left(\rho^H V_{n-1}^H(0) + \left(1 - \rho^H\right) V_{n-1}^L(0)\right) + \delta_n \tau u^b(0).$$

This corner solution plays a central role in the Laitner et al. (2018) model. The difference in our model is that neither state is absorbing. Thus, there may be positive saving even late in life. For example, if poor health is associated with a higher marginal utility of consumption expenditure, as Laitner et al. (2018) assume, someone currently in good health has an incentive to save, running wealth down to zero when there is an unfavorable transition, and starting to save again later if health improves.

The flow utility is specified as a constant relative risk aversion Stone-Geary function,

$$u(c) = \frac{(c - c_0)^{1-\gamma} - 1}{1 - \gamma},$$

with subsistence level c_0; the same functional form is used for the bequest utility, but with a threshold parameter κ that is presumed to be positive, following De Nardi (2004),

$$u^b(c) = \frac{(c + \kappa)^{1-\gamma_b} - 1}{1 - \gamma_b}.$$

As De Nardi (2004) points out, a positive threshold parameter, $\kappa > 0$, implies that the marginal utility of a bequest is bounded above, so that if the marginal utility of own consumption becomes very large at low consumption levels, it is optimal to leave no bequest. This allows the model to distinguish between people who have no interest in bequests, and people who would leave bequests only if they are sufficiently wealthy. We allow for the possibility that there are two types of agents, with $\tau = 0$ for one type, indicating no bequest motive, and $\tau > 0$ measuring the importance of the bequest motive for the other type.

3.2 "Hand-to-Mouth" Consumption

Several recent papers have analyzed the empirical observation that many consumers tend to spend a fluctuating income stream as it comes in, eschewing the benefits of consumption smoothing.[11] One interpretation of this behavior is that although large fluctuations in consumption may be inconsistent with expected utility maximization, it is plausible that relatively small fluctuations could be optimal if there are transaction costs associated with perfect consumption smoothing. In our context, a simple way to modify the optimization problem suffices to allow for this possibility. We are studying people who have no substantial wealth, and who have a predictable and fairly regular income stream. We simply introduce a utility bonus that attaches value to a consumption choice that is close to current income. Thus, the utility flow is specified as

$$\tilde{u}(c) = \theta^h u(c) + \psi \mathbb{I}(|y - c| \leq s_0),$$

where s_0 is a small threshold value, $\mathbb{I}$ is the indicator function, and ψ is a parameter to be estimated.

[11] See Kaplan and Violante (2014) and Attanasio et al. (2020), for example.

3.3 Saving After Retirement: Discrete Choice Version

We implement the model by discretizing the wealth and consumption variables, using the grids

$$\begin{aligned} C &= \{c_1, c_2, \ldots, c_J\}, \\ W &= \{w_1, w_2, \ldots, w_K\}, \end{aligned}$$

with $c_j = c_0 + j\Delta$ and $w_k = w_0 + k\Delta$.

Changes in the marginal utility of consumption may be associated with conditions that are not recorded in our data. Thus, it is reasonable to consider a model in which the state variables affecting consumption decisions include unobserved components, along with observed components, such as income and wealth. The choice problem is then

$$V_n^h (w_k, Y_n, \zeta_n) = \max_{\{j: c_j \leq w_k + y_n\}} \left\{ v_{nkj}^h (Y_n) + \zeta_n^j \right\}.$$

Here, ζ_n^j is an i.i.d. choice-specific payoff shock, drawn from the type I extreme value distribution, $\zeta_n = \left(\zeta_n^j\right)$, and v_{nkj}^h are the continuation values; k and j index the positions in the wealth and consumption grids, and Y_n is a vector that includes current (annuity) income y_n, along with future values, allowed to vary over time, but assumed known in advance. In the empirical implementation, observed income is used when available. In the event of death, the last observed income is treated as an annuity that would have continued at a constant level if the person had not died. Thus, the continuation values are given by

$$\begin{aligned} v_{nkj}^h (Y_n) &= \theta^h u (c_j) + \psi \mathbb{I} (|y_n - c_j| \leq s_0) \\ &\quad + \beta (1 - \delta_n) \left(\rho^h \mathbb{E} V_{n-1}^h \left(R (w_k + y_n - c_j), Y_{n-1}, \zeta_{n-1}^j \right) + \left(1 - \rho^h\right) \mathbb{E} V_{n-1}^{h'} \left(R (w_k + y_n - c_j), Y_{n-1}, \zeta_{n-1}^j \right) \right) \\ &\quad + \delta_n \tau u^b (w_k + y_n - c_j). \end{aligned}$$

The expectation on the right side is with respect to the distribution of the payoff shocks ζ_{n-1}, taking into account the maximization over next period consumption choices. We specify $s_0 = 1$, meaning that the hand-to-mouth effect is active whenever the consumption choice differs from current income by no more than one grid point. The implementation involves looking up $R (w_k + y_n - c_j)$ in the wealth grid. Let $w_{k'}$ be the closest point in the grid. The location of this depends on the chosen consumption, c_j, as well as on w_k and y_n, i.e., $k' = k' (k, n, j)$. The backward recursion in continuation values can be written as[12]

$$\begin{aligned} v_{nkj}^h (Y_n) &= \theta^h u (c_j) + \psi \mathbb{I} (|y_n - c_j| \leq s_0) \\ &\quad + \beta (1 - \delta_n) \rho^h \log \left(\sum_{j'} \exp \left(v_{n-1, k'(k,n,j), j'}^h (Y_{n-1}) \right) \right) \\ &\quad + \beta (1 - \delta_n) \left(1 - \rho^h\right) \log \left(\sum_{j'} \exp \left(v_{n-1, k'(k,n,j), j'}^{h'} (Y_{n-1}) \right) \right) \\ &\quad + \delta_n \tau u^b (w_k + y_n - c_j). \end{aligned}$$

[12] Here we use results for the extreme value distribution due to McFadden (1974) and Rust (1994); a simplified derivation can be found in Kennan (2008).

The choice probabilities are then given by[13]

$$\alpha_{nkj}^{h}\left(Y_{n}\right)=\frac{\exp\left(v_{nkj}^{h}\left(Y_{n}\right)\right)}{\sum_{i=1}^{J}\exp\left(v_{nki}^{h}\left(Y_{n}\right)\right)}. \tag{1}$$

3.4 The Likelihood Function

3.4.1 The Hidden Markov Model

For a given individual, write c_t for consumption observed in period t. Let p_{th} be the probability of this observation if the latent state in period t is h. The derivation is for a general latent state taking values $h \in \{1, \ldots, M\}$. Let π_h be the probability that the initial state is h, and let $A = (A_{ij})$ be the transition probability matrix (where A_{ij} is the probability that the state next period is j, given that the current state is i). The likelihood is

$$L = \pi D_1 \prod_{t=2}^{T} (AD_t)\,\iota = \pi D_1 \prod_{t=2}^{T} C_t \iota,$$

where D_t is a diagonal matrix with $(p_{t1}, \ldots, p_{tM})$ on the diagonal, $C_t = AD_t$, $\pi = (\pi_h)$ is a $1 \times M$ row vector, and ι is a column vector of ones. To see this, write P_{th} for the joint probability that the state in period t is h, and the observed history up to this period is $(c_1, \ldots, c_t)$. Then

$$P_{th} = \sum_{i=1}^{M} P_{t-1,i} A_{ih} p_{th}$$

for $t \geq 2$, and $P_{1h} = \pi_h p_{1h}$. Writing $P_t = (P_{t1}, \ldots, P_{tM})$, we have $P_1 = \pi D_1$ and

$$P_t = P_{t-1} A D_t = P_{t-1} C_t,\ t \geq 2.$$

Iterating on this, $P_T = P_1 \prod_{t=2}^{T} C_t$. Substituting for P_1, we have $P_T = \pi D_1 \prod_{t=2}^{T} C_t$, so the hidden Markov model (HMM) likelihood is

$$\begin{aligned} L = \text{Prob}\left(c_1, \ldots, c_T\right) = \sum_{h=1}^{M} P_{Th} = P_T \iota \\ = \pi D_1 \prod_{t=2}^{T} C_t \iota. \end{aligned} \tag{2}$$

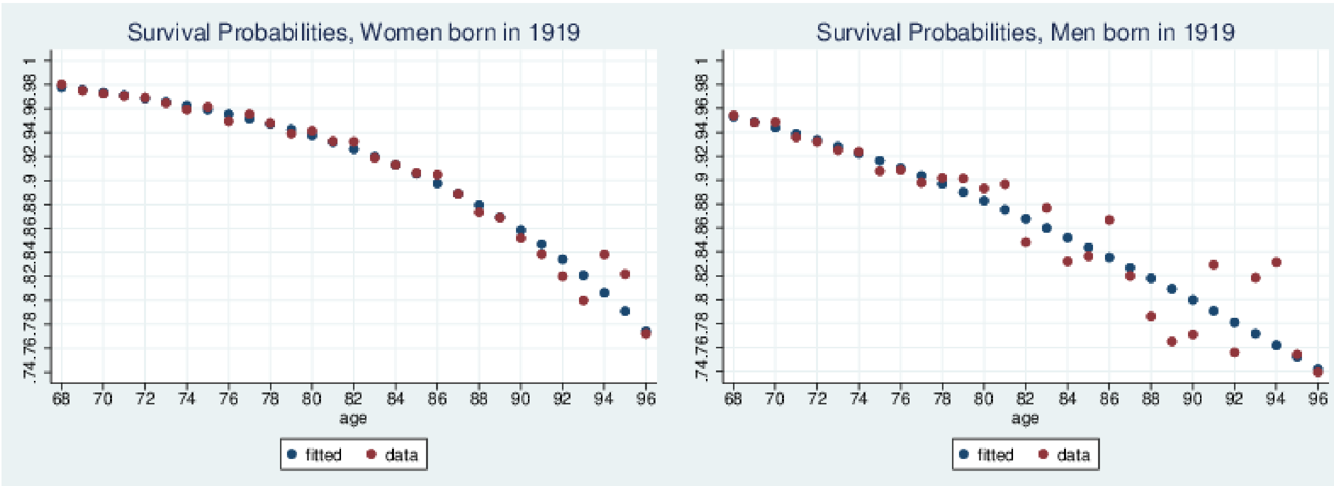
In the empirical implementation, $M = 2$, with $h \in \{L, H\}$, and if the observations at time t on consumption, wealth, and income correspond to the grid points j, k, and n, then p_{th} is the logit probability $\alpha_{nkj}^{h}\left(Y_n\right)$ from equation (1).

3.4.2 Sample Likelihood

The individual likelihood uses survival probabilities estimated from data for the 1919 birth cohort as a function of age, by fitting a logit model with age and its square and reciprocal as explanatory variables. This model fits quite well, as shown in Figure 2.

[13] We do not include a scale parameter for the payoff shocks, as each state is allowed a separate shift θ^h on marginal utility, $h \in \{L, H\}$.

Figure 2: Survival Probabilities



We estimate 10 parameters: the marginal utility coefficients associated with the two latent states, $\left(\theta^L, \theta^H\right)$, the persistence parameters associated with each state, $\left(\rho^L, \rho^H\right)$, the risk aversion or curvature parameter, γ, the proportion of people with a bequest motive, p^b, the threshold parameter and strength of the bequest motive for those who have one, (κ, τ), the hand-to-mouth parameter ψ and, finally, the proportion of people who are in the high marginal utility state initially, π^H, where $\pi = \left(\pi^H, 1-\pi^H\right)$. Taking L as the HMM likelihood from equation (2), and L^0 the restricted version imposing no bequest motive, i.e., $\tau = 0$, the individual likelihood is the mixture $L^b = p^b L + \left(1-p^b\right) L^0$. The sample likelihood is obtained by multiplication across individuals. The logarithm of this is maximized directly by the Newton-Raphson method with quadratic line search for step length. The value function is calculated by backward recursion for each individual and at each trial parameter value through the iterative maximization.[14] We fix the discount rate at $\beta = .95$, and assume that the real interest rate on savings is $r = 2\%$, i.e., $R = 1.02$. Finally, we set the subsistence level c_0 to zero, and $\gamma_b = \gamma$, as is standard in the literature.[15]

4 Empirical Results

The model was estimated by maximizing the sample likelihood described in Section 3.4.2, using separate datasets for men and for women, with several thousand (person-year) observations for men, and a substantially larger number for women.[16] We also distinguish between those who had essentially no retirement savings, and those who had at least

[14] This is a modified nested fixed point mixed hidden Markov model. The basic likelihood calculation was described in Harte (2006) for the portion of L with neither mixing $\left(p^b = 1\right)$ nor nested fixed point (value function iteration). The modification refers to the hand-to-mouth extension of the basic dynamic programming setup.

[15] We have estimated versions of the model with the last two restrictions relaxed, but we wish to focus instead on the more novel extensions in our model: the latent state, the mixture of bequest types, and the hand-to-mouth consumption specification.

[16] Even though distinct value functions must be computed for each person (because income histories differ across people), the maximum likelihood estimates can be computed within a day or so, on a single computer. This is perhaps surprising, given that Hidden Markov models are usually estimated using some version of the EM algorithm (the Baum-Welch algorithm, for example), on the grounds that more direct methods would be too slow.

some wealth at age 68.[17] The reason for splitting on initial wealth is the initial conditions problem. People with different bequest motives or latent state histories during working life, or other differences in behavioral characteristics and constraints, will have saved differently by the retirement age, and initial wealth may reflect such underlying differences. Summary statistics for the four datasets used in estimation are given in Table 3.

Table 3: **Summary Statistics**

Initial wealth	$> 25,000$DKK				$\leq 25,000$DKK			
	Women		**Men**		**Women**		**Men**	
Observations	11,748		2,996		13,483		3,524	
Right-Censored	24.3%		8.6%		22.0%		4.2%	
	Mean	*Std.*	Mean	*Std.*	Mean	*Std.*	Mean	*Std.*
Wealth	17.2	18.4	21.5	22.6	4.7	5.4	4.7	6.4
Income	29.1	5.3	26.9	5.1	27.9	4.6	25.7	4.5

The second row shows the percentage of people who were still alive at the end of the sample period.
Wealth and income are in units of $5,000$ DKK, at 2015 prices.

4.1 Parameter Estimates

Our main estimation results are shown in Table 4. We find that the utility coefficients in the two latent states are substantially different in all cases: the ratio of the coefficients $\left(\frac{\theta^L}{\theta^H}\right)$ ranges from .18 to .25. This is the main idea of the latent state variable model: the marginal utility coefficient moves between high and low levels. The initial high state proportion, π^H, estimates the fraction of individuals starting in the high marginal utility state at age 68. The estimated proportion is high in all four cases, i.e., most persons are in the high marginal utility state around the retirement age. Those with very low initial wealth exhibit an initial high state proportion at about 0.9, higher than for those with higher initial wealth. These results are consistent with persons without savings at the end of their working lives having previously enjoyed relatively high marginal utility of consumption. From age 68 on, both latent states are highly persistent but not permanent, with transition probabilities $\left(\rho^H, \rho^L\right)$ significantly positive but below unity, i.e., marginal utility changes matter for savings behavior in retirement, and neither state is absorbing.

The "hand-to-mouth" parameter, ψ, is highly significant, large in magnitude, and quite stable across the subpopulations. The standard model of consumption smoothing is embedded in our model, and strongly rejected in favor of an extended model that downplays the importance of consumption smoothing. This is a novel empirical result, using a large panel data set, which includes many people with negligible wealth.

As differences in life expectancy are controlled for, there are no economic reasons for differences in parameter values by gender. Nevertheless, likelihood ratio tests strongly reject common parameters across gender for both initial wealth subsets (and also across initial wealth level subsets within gender).[18] Therefore we continue to split on both gender and initial wealth in goodness of fit tests below.

[17] In Appendix A.3, we present a more complete set of results, in which we further distinguish between two categories of singles: those who never married, and those with previous marriages that ended in divorce or death of a spouse. Due to sample size issues, these results are for women only.

[18] Values of test statistics and degrees of freedom are shown in Table 4, and parameter estimates for the restricted models are in Table 9 in Appendix A.3. In all cases, value functions are computed with gender-specific survival probabilities, which presents no additional challenge, as individual-specific value functions are computed in any case, due to individual differences in income streams.

Table 4: **Parameter Estimates**

Initial wealth	$> 25,000$DKK				$\leq 25,000$DKK			
	Women		**Men**		**Women**		**Men**	
	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$
State H								
Persistence $\left(\rho^H\right)$	0.990	*0.001*	0.987	*0.004*	0.970	*0.002*	0.978	*0.004*
Utility coefficient $\left(\theta^H\right)$	1,666.8	*230.6*	5,543.1	*2,147.6*	849.8	*106.0*	1,502.9	*453.4*
Initial type H proportion $\left(\pi^H\right)$	0.755	*0.018*	0.673	*0.036*	0.896	*0.015*	0.917	*0.021*
State L								
Persistence $\left(\rho^L\right)$	0.950	*0.004*	0.922	*0.015*	0.973	*0.004*	0.942	*0.020*
Utility coefficient $\left(\theta^L\right)$	307.5	*34.7*	991.4	*325.8*	210.4	*23.0*	314.5	*83.2*
Utility curvature (γ)	2.229	*0.046*	2.823	*0.145*	1.877	*0.042*	2.189	*0.101*
Hand-to-mouth parameter (ψ)	1.040	*0.016*	1.069	*0.031*	1.142	*0.019*	0.924	*0.037*
Bequest coefficient (τ)	1,200.5	*183.7*	254,408.6	*243,191.2*	42.29	*3.38*	1544.9	*540.7*
Bequest type proportion $\left(p^b\right)$	0.722	*0.022*	0.646	*0.048*	0.786	*0.019*	0.442	*0.043*
Bequest threshold (κ)	9.48	*0.52*	82.26	*12.52*	0.77	*0.07*	15.37	*1.89*
Relative Utility coefficient $\left(\frac{\theta^L}{\theta^H}\right)$	0.18		0.18		0.25		0.21	
Loglikelihood	-29,508.0		-8,134.98		-26,038.4		-7,064.6	
observations (persons)	945		327		1,085		431	
observations (person-years)	11,748		2,996		13,483		3,524	
LR(10): initial wealth	560.0		193.6					
LR(10): gender	200.2				224.9			

Note: 1927 birth cohort, single people, no property
The columns labeled as $\hat{\vartheta}$ show estimated parameter values (with estimated standard errors in adjacent columns).

Table 5: **Estimated Marginal Utilities**

Initial wealth	$> 25,000$DKK				$\leq 25,000$DKK			
	Women		**Men**		**Women**		**Men**	
Marginal Utility of Consumption	State H	State L	State H	State L	State H	State L	State H	State L
$75,000$ DKK	3.99	0.74	2.60	0.49	5.27	1.30	4.00	0.84
$125,000$ DKK	1.28	0.24	0.62	0.12	2.02	0.50	1.31	0.27
$150,000$ DKK	0.85	0.16	0.37	0.07	1.43	0.35	0.88	0.18
Marginal Utility of Bequest (bequest type)								
$5,000$ DKK	6.39		0.96		14.45		3.40	
$25,000$ DKK	3.11		0.84		1.57		2.11	
$75,000$ DKK	0.96		0.62		0.24		0.88	
Note: 1927 birth cohort, single people, no property								

4.1.1 Latent States and Bequests

The model involves both latent states and a bequest motive. Indeed, savings can be motivated either by the possibility of a future transition from low to high marginal utility of consumption, or by the utility of leaving a bequest. Both of these model components are required to explain the data. Some individuals are observed at the corner, having run wealth to zero, and yet they start saving out of their annuity in subsequent periods. This can be explained by the individual experiencing high marginal utility of consumption initially, hence consuming all resources, then low marginal utility, with a chance of a return to high marginal utility in the future, which provides an incentive to save. With only a bequest motive, but no latent state, it would not be optimal to postpone saving for the bequest. At the same time, other individuals are observed to increase their saving rate over time. This is rational, given a bequest motive, as the utility of the bequest weighs more heavily in the value function over the remaining life time as the person gets older (see Appendix A.1). This change in saving rate with age is not predicted by the latent state model.

From Table 4, the estimated proportion p^b of individuals who have a bequest motive ($\tau > 0$) exceeds 40% within each subpopulation. One might expect that especially individuals with a strong bequest motive would save for bequests during their working life, and thus enter our analysis with positive initial wealth, and our estimates are consistent with this. For those with a bequest motive, Table 4 reports estimates of the threshold (κ) and the strength of the motive (τ). In addition, to facilitate interpretation, Table 5 reports estimates of the marginal utility of bequests (for those with a bequest motive) at three different bequest levels.[19] Marginal utility decreases quite substantially for women, while for men it starts at a lower level, and is less sensitive to the magnitude of the bequest. For comparison, the table shows the state-contingent marginal utilities of own consumption at levels below and around the typical values of the annual annuity. The comparison confirms that for those with a bequest motive, there is indeed an incentive to leave at least a small bequest, especially in the low marginal utility state.

From the results, the bequest motive is stronger for women than for men. First, in Table 4, the bequest type proportions $\left(p^b\right)$ are higher for women than for men, i.e., a greater share of women have a bequest motive, relative to men. Second, one can ask what the expected bequest would be, for someone in the last year of life, if consumption with no bequest would be at about the OAP level, closest to $125,000$DKK in Table 5. For the bequest type, this

[19] These bequest levels correspond to roughly 4%, 20%, and 50%, respectively, as fractions of average annual income. Note that although some of the coefficients in Table 4 might seem implausibly large, the implied marginal utilities in Table 5 are quite reasonable (in particular, this is true for high-wealth men with a bequest motive).

involves finding the bequest that equates the bequest marginal utility with the marginal utility of own consumption, and the expected bequest is this amount multiplied by the bequest proportion.[20] In all four cases (classified by initial wealth and latent state), the expected bequest thus defined is larger for women than for men.

From Table 4, the latent state transition probabilities are significantly positive and below unity, even in the presence of a bequest motive for a substantial fraction of the population. To further assess the empirical importance of the latent state specification, we estimate a special case of the model, with the two persistence parameters fixed at unity. This specification corresponds to (constant) unobserved heterogeneity in marginal utility, with a discrete (two-point) distribution across individuals. Results are reported in Table 6. The restricted model is strongly rejected by likelihood ratio tests (reported in the table) for all subpopulations, indicating that the moving latent state corresponds to an important feature in the data.

[20] The expected bequest is $p^b B$, where B solves the equation $\tau\,(B+\kappa)^{-\gamma} = \theta^h\,(y-B)^{-\gamma}$, i.e.,

$$B = \frac{\left(\frac{\tau}{\theta^h}\right)^{\frac{1}{\gamma}} y - \kappa}{1 + \left(\frac{\tau}{\theta^h}\right)^{\frac{1}{\gamma}}},$$

with income y.

Table 6: **Parameter Estimates: Permanent States**

Initial wealth	$> 25,000$DKK				$\leq 25,000$DKK			
	Women		**Men**		**Women**		**Men**	
	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$
Utility coefficient $\left(\theta^H\right)$	1,401.9	*180.8*	2,794.7	*956.4*	1,184.1	*207.5*	2,317.9	*627.4*
Utility coefficient $\left(\theta^L\right)$	228.1	*24.7*	599.0	*179.6*	220.8	*33.6*	574.5	*134.6*
Initial H proportion $\left(\pi^H\right)$	0.72	*0.02*	0.71	*0.03*	0.90	*0.01*	0.79	*0.03*
Utility curvature (γ)	2.19	*0.04*	2.68	*0.12*	2.09	*0.06*	2.40	*0.09*
Hand-to-mouth parameter (ψ)	1.10	*0.02*	1.15	*0.03*	1.29	*0.02*	1.08	*0.04*
Bequest coefficient (τ)	1,647	*247*	350,337	*217,882*	501.3	*79.8*	22,604	*9,902*
Bequest type proportion $\left(p^b\right)$	0.79	*0.02*	0.98	*0.03*	0.71	*0.02*	0.56	*0.05*
Bequest threshold (κ)	14.59	*0.64*	136.95	*14.90*	8.74	*0.57*	54.77	*5.86*
Relative Utility coefficient $\left(\frac{\theta^L}{\theta^H}\right)$	0.16		0.21		0.19		0.25	
Marginal Utility of Consumption								
75,000 DKK	3.76	0.61	1.95	0.42	4.08	0.76	3.49	0.86
125,000 DKK	1.23	0.20	0.50	0.11	1.40	0.26	1.02	0.25
150,000 DKK	0.83	0.13	0.30	0.07	0.96	0.18	0.66	0.16
Marginal Utility of Bequest (bequest type)								
5,000 DKK	4.06		0.63		4.27		1.45	
25,000 DKK	2.46		0.59		2.08		1.23	
75,000 DKK	1.00		0.49		0.66		0.85	
Loglikelihood	-29,712.4		-8,202.1		-26,203.8		-7,143.4	
observations (persons)	945		327		1,085		431	
observations (person-years)	11,748		2,996		13,483		3,524	
LR(2) statistic	408.9		134.3		330.8		157.7	

Note: 1927 birth cohort, single people, no property
The LR(2) statistic compares the restricted likelihood in this table with the corresponding likelihood in Table 4
(two persistence parameters are fixed at unity under the null).
For example, for the smallest subpopulation, high initial wealth men, the test statistic is $-2\log Q = 2\,(-8134.9 + 8202.1) =$134.3,
with a microscopic p-value in the asymptotic χ^2 distribution with two degrees of freedom.

4.1.2 Initial conditions

Again, the reason for splitting on wealth at age 68 is the initial conditions problem, and likelihood ratio tests confirm that parameters differ across initial wealth level subpopulations, as is shown in Table 4. As an alternative, to verify the robustness of our main results, we consider splitting by marital history prior to age 68, i.e., never married versus ever married (divorced or widow(er)), since people are likely to have arrived at the initial point of our analysis in different ways, and this can indicate differences in behavioral characteristics and constraints. From Table 2, the never-married share is 36% among men, and only 13% among women. Figure 6 in Appendix A.3 shows income and wealth distributions at age 68 by marital history. Distributions for the ever-married are similar to those in Figure 1, while the never-married have considerably lower incomes. To avoid small samples when we analyze the marital history subpopulations, we estimate the model only for women, and we split on initial wealth only for ever-married women. Estimation results are shown in Table 10 in Appendix A.3.

The estimates reveal some differences by marital history, and by initial wealth for the ever married: likelihood ratio tests strongly reject common parameters across marital history subpopulations for women, and across initial wealth levels for ever-married women. While this confirms that parameters differ by initial conditions, whether measured by initial wealth, marital history, or both, our main findings are confirmed, too. Thus, latent state persistence parameters are significantly below unity, i.e., changes between very different high and low marginal utility levels matter for savings behavior in retirement, and the bequest motive and hand-to-mouth parameters are all strongly significant, as well, across marital histories.

4.2 Goodness of Fit

We use the estimated model to generate simulated data that can be compared with the actual data, focusing on the extent to which the model helps explain the wealth change patterns shown in Table 1 above. The model is estimated using discretized data, with sample selection summarized in Table 2. Table 7 shows wealth changes in the simulated data, along with the corresponding changes in the data used to estimate the model. The simulations involve 20 replicas of each person in the actual data. Even though the data set used in estimation is a reduced version of the data in Table 1, the wealth increase proportions in the simulated data are similar to the corresponding proportions in Table 1 (between 40% and 50%). The model slightly under-predicts the proportion of wealth increases for those who have almost no retirement savings, but overall the propensity to save out of the annuity is captured reasonably well.

Our model includes a flexible specification of bequest motives. We allow for the possibility that some people are not at all interested in bequests, and our parameter estimates from Table 4 indicate that a substantial fraction of people are in this category, with substantial differences across the four sub-populations, ranging from 21% of the low-wealth women to 56% of the low-wealth men. In the absence of a bequest motive, the model implies that there is a strong tendency to consume the entire annuity flow in each period, unless the marginal utility of consumption is currently low. Moreover, even in the case of people who care about bequests, it may still be optimal to consume all available resources if the marginal utility of current consumption is higher than the (expected) marginal utility of a small bequest.

In Table 8, we show that such corner solutions occur frequently in the data, and also in our model simulations. Naturally, corner solutions are more common in the case of people who start with little or no retirement savings, and our model is quite successful in matching this feature of the data. But the model does tend to over-predict the frequency of corner solutions, particularly in the case of people who begin with nontrivial retirement savings.

Table 7: **Wealth Changes**

Age 68 to year before death				
Single, survived at least 5 years from age 68				
	Men		**Women**	
	Increase	Total	Increase	Total
All				
Data	141	314	604	1292
	44.9%		46.7%	
Model Simulation	2,668	6,280	11,944	25,840
	42.5%		46.2%	
Initial Wealth$>25,000$DKK				
Data	48	149	207	599
	32.2%		34.6%	
Model Simulation	1,105	2,980	4,449	11,980
	37.1%		37.1%	
Initial Wealth$\leq 25,000$DKK				
Data	93	165	397	693
	56.4%		57.3%	
Model Simulation	1,563	3,300	7,495	13,860
	47.4%		54.1%	

The table shows the proportion of people who had more wealth in the final period than they had at age 68, ignoring very small changes ($10,000$DKK or less). The unit of observation is a person (rather than a person-year).

Table 8: **Corner Solutions**

	Men			Women		
	Wealth		Total	Wealth		Total
Initial Wealth$>25,000$DKK	Zero	Negligible		Zero	Negligible	Total
Data	3	99	2,295	55	447	10,215
	0.1%	4.4%		0.5%	4.9%	
Model Simulation	1,440	5,585	45,900	5,809	22,917	204,300
	3.1%	12.2%		2.8%	11.2%	
Initial Wealth$\leq 25,000$DKK						
Data	205	730	2,374	573	3,348	11,692
	8.6%	39.4%		4.9%	33.5%	
Model Simulation	6,636	22,135	47,480	14,723	80,047	233,840
	14.0%	46.6%		6.3%	34.2%	

The table shows the frequency of cases where people consumed all available resources (accumulated wealth plus current income), meaning that nothing is carried over for future consumption or to provide for bequests. There is a hard version (exactly zero wealth) and a fuzzy version (wealth below some negligible amount, meaning $10,000$ DKK). The unit of observation is a person-year.

5 Conclusion

We have analyzed the post-retirement consumption and savings decisions of a single cohort, namely the people who were born in Denmark in 1927. To avoid modeling joint decisions made by couples, we have focused on people who were single at the beginning of our observation period, which begins at age 68. Also, to avoid having to deal with wealth increases generated by general increases in property values, we have considered only those who did not own property at age 68 or later (most people in fact did not own property at this point). The previous literature has relied heavily on the importance of anticipated medical expenses late in life as a central explanation for the rather surprising failure of older people to spend down their wealth, but because out-of-pocket medical expenses are negligible in our data, we can focus our analysis more narrowly on the basic tradeoff between consumption now and consumption later.

Our results show that an extended version of the standard life-cycle consumption model is quite successful in explaining the curious observed consumption patterns after retirement in our data, including periodic episodes during which people save a portion of pension annuity income for no obvious reason. The extension adds two additional features to the standard model. We show that a simple "hand-to-mouth" modification of the flow utility function generates robust evidence that the standard model substantially overstates the importance of exact consumption smoothing. Although this is an interesting and novel empirical result, the main point of our model is that it also includes a latent state variable with a Markov chain structure that affects the marginal utility of current consumption. The interpretation is that some individuals are saving because they are currently in a low marginal utility state, but looking ahead to the possibility of moving to a high marginal utility state in the future. The empirical importance of the latent state specification is shown while also allowing for a bequest motive. The bequest motive remains separately identified in the presence of the latent state specification as it increases in importance with age (namely, probability of death). From the results, the latent state specification and the bequest motive are jointly significant. We find that the bequest motive is stronger among women than among men.

The latent states in our model might be associated at least to some extent with health, but the model is more general than one tying the marginal utility of consumption to a stochastic health state. It may well be that marginal utility depends on health, but fluctuations in marginal utility could occur for other reasons, and assuming that marginal utility is constant conditional on the health state is a strong restriction. Indeed, it seems likely that an extension that retains the latent state while also allowing marginal utility to depend on an observed health state would provide better evidence on the relationship between health and consumption. Further research is required to investigate this issue.

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Appendix

A.1 A Two-period Model

In order to provide some intuition for the dynamic programming model, we consider a simple case with only two periods, and no utility shocks. As we show, if there is no bequest motive, then increasing the death probability cannot lead to an increase in savings. In contrast, with a bequest motive, there are cases in which increasing the death probability leads to an increase in savings.

Consider an individual who has at most two periods of life remaining, and who is deciding how much to save in the first period, knowing that savings will either be consumed in the second period or left as a bequest. Conditional on survival, there will be a further decision in the second period, determining the amount of the bequest. The optimization problem is

$$\max_{x\in[0,w+y]} \left\{\theta^h u\left(w+y-x\right) + \left(1-\delta\right)\left[\rho^h V^h\left(Rx\right) + \left(1-\rho^h\right) V^{h'}\left(Rx\right)\right] + \delta\tau u^b\left(x\right)\right\},$$

$$V^h\left(Rx\right) = \max_{B\geq 0}\left\{\theta^h u\left(Rx+y-B\right) + \tau u^b\left(B\right)\right\}.$$

The notation is as follows. The flow utility of consumption c is $\theta^h u\left(c\right)$, where $\theta^h > 0$ is the current state, and the utility of a bequest B is $\tau u^b\left(B\right)$, where $\tau > 0$ is the strength of the bequest motive; u^b is concave, and u is strictly increasing and strictly concave. The initial wealth level is w, and there is a life annuity yielding income y in each period, conditional on survival. The amount saved in the first period is $x \geq 0$, yielding Rx in the second period, and $B \geq 0$ is the bequest choice in the second period (conditional on survival). The death probability at the end of the first period is δ (and death is certain at the end of the second period). Conditional on θ^h, the expected next period state is $E^h\left(\theta\right) = E\left(\theta \mid \theta^h\right) = \rho^h\theta^h + \left(1-\rho^h\right)\theta^{h'}$.

With no bequest motive, i.e., $\tau = 0$, the last period value function reduces to the utility of consumption, and $B = 0$, i.e., $V^h\left(Rx\right) = \theta^h u\left(Rx+y\right)$. Substituting this in the first-period maximization problem yields

$$\max_{x\in[0,w+y]} \left\{\theta^h u\left(w+y-x\right) + \left(1-\delta\right)E^h\left(\theta\right)u\left(Rx+y\right)\right\}.$$

The first-order condition for x is then

$$-\theta^h u'\left(w+y-x\right) + \left(1-\delta\right)RE^h\left(\theta\right)u'\left(Rx+y\right) \quad \leq \quad 0,$$

and this holds with equality if $x > 0$ at an optimum.

If the corner solution, $x = 0$, is optimal in the first period (i.e., no savings), then the first-order condition requires

$$-\theta^h u'\left(w+y\right) + \left(1-\delta\right)RE^h\left(\theta\right)u'\left(y\right) \quad \leq \quad 0.$$

By positive marginal utility of consumption, $u'\left(y\right) > 0$, so increasing δ will only make the left side more negative, and $x = 0$ will still be optimal.

If the optimal solution for savings in the first period is interior, $x > 0$, then the first-order condition for x is

$$-\theta^h u'(w+y-x) + (1-\delta) R E^h(\theta) u'(Rx+y) \quad = \quad 0.$$

Writing this in the form $F(x,\delta) = 0$ and invoking the implicit function theorem, the savings response to a change in the death probability is

$$\begin{aligned} \frac{dx}{d\delta} &= -\frac{\partial F/\partial \delta}{\partial F/\partial x} \\ &= \frac{R E^h(\theta) u'(Rx+y)}{\theta^h u''(w+y-x) + (1-\delta) R^2 E^h(\theta) u''(Rx+y)}. \end{aligned}$$

By concavity, the denominator is negative, and by positive marginal utility, the numerator is positive, implying $dx/d\delta < 0$. Thus, without a bequest motive, savings x remain constant (at zero, the corner solution) or decrease as δ increases.

The argument shows that an increase in savings with age (here, the death probability) requires the presence of a bequest motive. It does not rule out the possibility that x can never increase with δ. We show next, by example, that cases in which savings increase with the death probability do exist.

Example

Consider the case with log utility, no state transitions, no initial wealth, no interest, but with a bequest motive,

$$\begin{aligned} u(x) &= \log(x) \\ u^b(B) &= \tau \log(B) \\ \theta^h &= 1 \\ R &= 1 \\ 0 &< \tau \leq 1 \\ w &= 0 \end{aligned}$$

In this example, the optimization problem is

$$\max_{x\in[0,y],\, B\in[0,x+y]} \left\{ \log(y-x) + (1-\delta)\left[\log(x+y-B) + \tau \log(B)\right] + \delta\tau \log(x) \right\}.$$

The log utility specification guarantees an interior optimum. The first-order conditions for x, B are

$$\begin{aligned} -\frac{1}{y-x} + \frac{1-\delta}{y+x-B} + \frac{\delta\tau}{x} &= 0, \\ -\frac{1}{y+x-B} + \frac{\tau}{B} &= 0. \end{aligned}$$

The latter condition implies

$$B = \frac{y+x}{1+\frac{1}{\tau}}.$$

When the optimal bequest B is substituted in the first-order condition for x, the equation determining x can be written as

$$\begin{aligned}\delta &= \frac{x\left(2x-\tau\left(y-x\right)\right)}{\left(y-x\right)\left(\tau y-x\right)} \\ &= \frac{\left(\frac{2}{\frac{y}{x}-1}-\tau\right)}{\tau\frac{y}{x}-1}.\end{aligned}$$

Suppose $x>\tau y$. Since δ is positive, this would imply that δ is the ratio of two negative numbers, so

$$\begin{aligned}2 &< \tau\frac{y}{x}-\tau \\ &< 1-\tau,\end{aligned}$$

a contradiction. Thus $x<\tau y$. Write the first-order condition for x as

$$-\frac{1}{y-x}+\frac{\left(1-\delta\right)\tau}{B}+\frac{\delta\tau}{x} \quad = \quad -\frac{1}{y-x}+\frac{\left(1-\delta\right)\left(1+\tau\right)}{y+x}+\frac{\delta\tau}{x}=0,$$

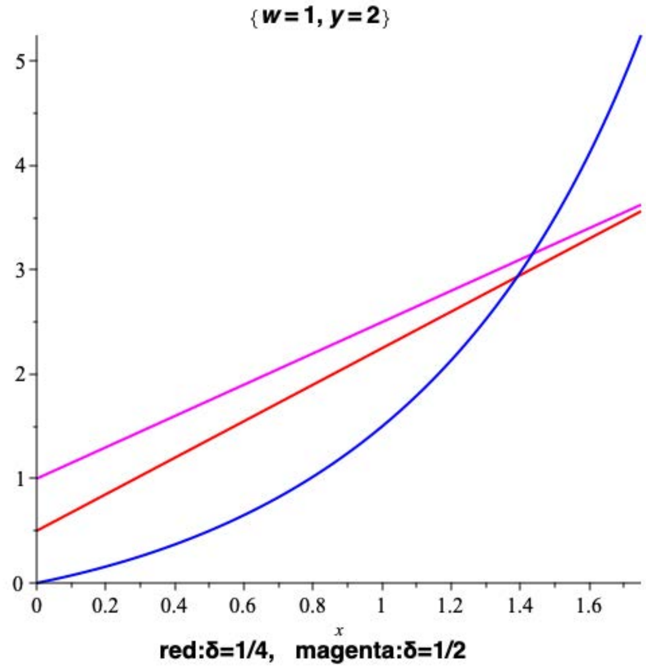
and rearrange this as

$$\delta\tau y+\left(1-\delta+\tau\right)x=\frac{x\left(y+x\right)}{y-x}. \tag{3}$$

The right side of this equation is a strictly increasing and strictly convex function of x, with zero intercept, and the left side is an affine function of x, with intercept $\delta\tau y>0$, so there is a unique solution for x. Since $x<\tau y$, an increase in δ shifts the affine function upward without affecting the convex function on the right, so x must increase.

Thus, the example provides a case of savings increasing with age (death probability). It assumes zero initial wealth, but the argument also works for small values of initial wealth: for example, Figure 3 shows the effect of an increase in the death probability for a case with positive initial wealth.

Figure 3: Two-Period Example



The plot shows the right side of equation (3) (in blue) and the left side for two alternative values of δ.

A.2 Data Details

The data are from the official Danish registers, accessed through Statistics Denmark, covering the period 1995 through 2016. We consider the 1927 birth cohort, for which the Old Age Pension (OAP) age was 67. Thus, individuals in our data reach the OAP age of 67 in 1994, and we follow them from age 68 through 89, or until death, whichever is first. We drop everyone who is ever in the interval 1995 to 2016 observed to be working, self-employed, married or cohabitating, or to have held stocks, mutual funds, or property. Stock and mutual fund holdings are identified using the register variable KURSAKT. We drop individuals who die in the first two years, and so are not present in 1997. The register variables are measured in November of each year. To avoid timing issues, we drop the year of death and the year before, for each individual. All financial variables are deflated to 2015 levels.

For w_t, wealth exclusive of pension wealth in year t, we use the register variable FORM. This represents end of year wealth. Before 1987, Denmark had a wealth tax, but even after its abolishment, people must report wealth to the tax authorities, and it is in the registers. The variable FORM is computed as total assets (QAKTIVF) minus total liabilities (QPASSIV). Since we have dropped people with stocks, mutual funds, and property, total assets primarily consist of bank deposits. In addition, some individuals hold bonds, but the extent of this is very limited in our data. Pension wealth (annuities, OAP) is not part of FORM. We assume that all private pension wealth has been converted to annuities, and so is not bequeathable. Total liabilities primarily consist of bank loans. Mortgages are generally not present, since property owners are discarded. Some individuals with very high wealth are discarded in order to have a reasonable wealth grid. In addition, following the discussion in footnote 6, we drop individuals if measured wealth in any year is negative.

For y_t, income after tax, we use the register variable DISPINDK, which represents disposable income for the year ending in November, including alimony income, OAP benefits, and other pension income. We drop people if measured income in any year falls below a lower bound based on the OAP benefit level (only 15 people are dropped for this reason). Future (real) income after death is assumed to continue at the level last seen in the data set.

Finally, consumption is computed as $c_t = y_t + Rw_{t-1} - w_t$, with $R = 1 + r$. We assume $r = 0.02$, i.e., a 2.0% after tax real interest rate on wealth in the model. Based on this, we apply an additional filter, because some information could be missing in the income and/or wealth data. In the register data, income is essentially taxable income. Missing income information includes bequests and gifts received. Missing wealth information includes loan proceeds/payments. Occasionally, large increases or decreases in wealth are seen, without anything in reported income justifying such changes. We regard this as a sign that some information is missing in the income data. Large increases in wealth tend to generate low consumption figures. If bequests and/or gifts received show up in recorded wealth (e.g., as increased bank deposits), but are left out of income, calculated consumption will be too low. Similarly, if private loan proceeds show up in recorded wealth (e.g., bank deposits), but are not countered by a registered debt, then wealth will appear to have increased, and calculated consumption will again be too low. To guard against this problem, we apply a floor on consumption, $c_t \geq \underline{c}$. A value $\underline{c} = 35,000$ DKK seems conservative. Thus, we eliminate people whose measured consumption level ever falls below this threshold.[21]

[21] Given the way we measure consumption, large decreases in wealth tend to generate large consumption figures, and may be caused by unregistered gifts given away or private loans repaid. Bank loans do not present a problem, since the change in the bank account generated by a new loan or repayment of an old loan should match the change in debt, for a neutral effect on wealth, and hence measured consumption. Gifts and private loan repayments could be considered as consumption, which is what we do. An alternative could be to estimate the portion of calculated consumption stemming from gifts given away, and value this via the bequest function rather than the flow utility function. Gifts given to close relatives (heirs) are tax-exempt below a certain threshold and not classified as income for recipients, and they could be considered a means of reducing tax payments (by heirs) on bequests.

A.3 Further Results

Figure 4: Standard Deviations of Real Income over Time

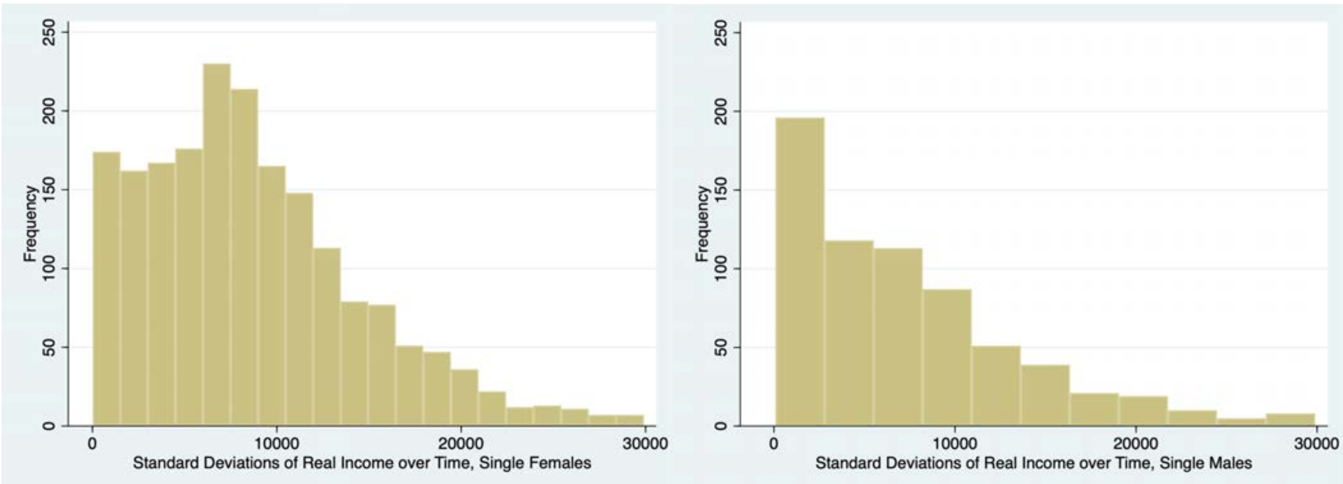


Figure 5: Real Wealth after Retirement

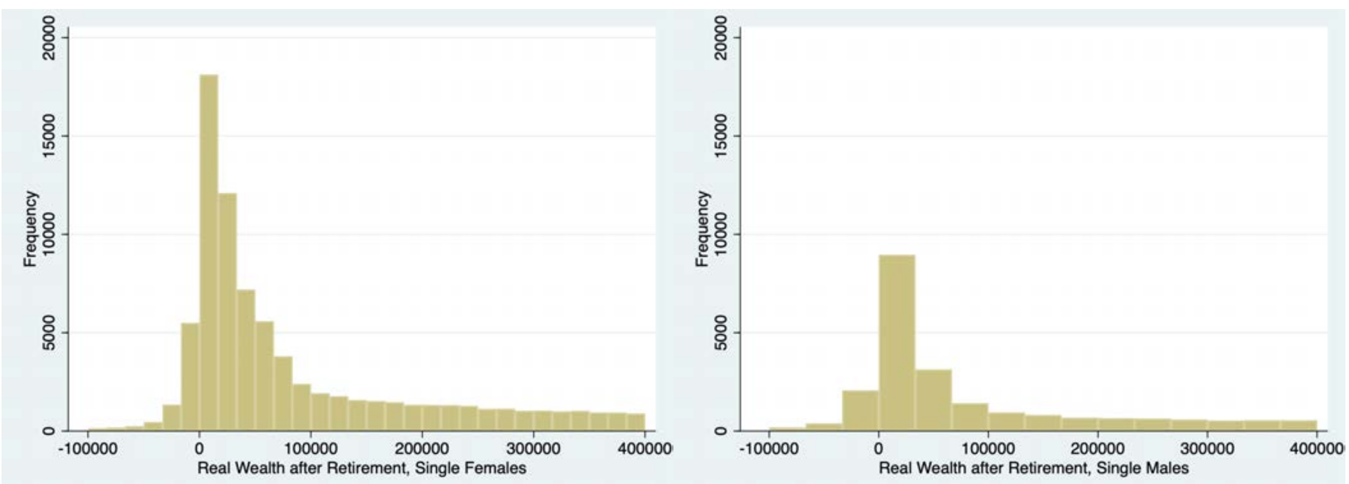


Table 9: **Parameter Estimates: Initial Wealth and Gender**

	All Wealth				$> 25,000$DKK		$\leq 25,000$DKK	
	Women		**Men**		**Women+Men**			
	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$
State H								
Persistence $\left(\rho^H\right)$	0.993	*0.001*	0.984	*0.003*	0.990	*0.001*	0.990	*0.001*
Utility coefficient $\left(\theta^H\right)$	2,635.9	*305.7*	5,723.0	*1,296.0*	2,301.1	*299.7*	1,704.3	*293.4*
Initial H proportion $\left(\pi^H\right)$	0.860	*0.010*	0.770	*0.020*	0.720	*0.016*	0.948	*0.008*
State L								
Persistence $\left(\rho^L\right)$	0.955	*0.003*	0.941	*0.009*	0.948	*0.004*	0.960	*0.006*
Utility coefficient $\left(\theta^L\right)$	441.0	*41.9*	983.4	*180.3*	387.2	*41.2*	282.3	*41.7*
Curvature (γ)	2.366	*0.039*	2.721	*0.077*	2.361	*0.043*	2.211	*0.059*
Hand-to-mouth parameter (ψ)	1.167	*0.012*	1.047	*0.023*	1.052	*0.014*	1.170	*0.016*
Bequest coefficient (τ)	2,539.5	*324.3*	119,084.2	*41,270.5*	2,170.2	*323.8*	743.7	*118.7*
Bequest type proportion $\left(p^b\right)$	0.649	*0.017*	0.493	*0.033*	0.709	*0.020*	0.583	*0.022*
Bequest threshold (κ)	14.15	*0.53*	63.68	*4.74*	12.34	*0.54*	8.97	*0.53*
Relative utility coefficient $\left(\frac{\theta^L}{\theta^H}\right)$	0.167		0.172		0.168		0.166	
Marginal Utility of Consumption	State H	State L	State H	State L	State H	State L	State H	State L
75,000 DKK	4.34	0.73	3.61	0.62	3.85	0.65	4.28	0.71
125,000 DKK	1.30	0.22	0.90	0.15	1.15	0.19	1.38	0.23
150,000 DKK	0.84	0.14	0.55	0.09	0.75	0.13	0.92	0.15
Marginal Utility of Bequest (bequest type)								
5,000 DKK	4.08		1.41		4.79		4.60	
25,000 DKK	2.35		1.20		2.58		2.18	
75,000 DKK	0.87		0.83		0.88		0.66	
Loglikelihood	-55,826.4		-15,296.2		-37,743.0		-33,215.4	
observations (persons)	2,030		758		1,272		1,516	
observations (person-years)	25,231		6,520		14,744		17,007	

Note: 1927 birth cohort, single people, no property

Figure 6: Income and Wealth at Age 68 − Marital History

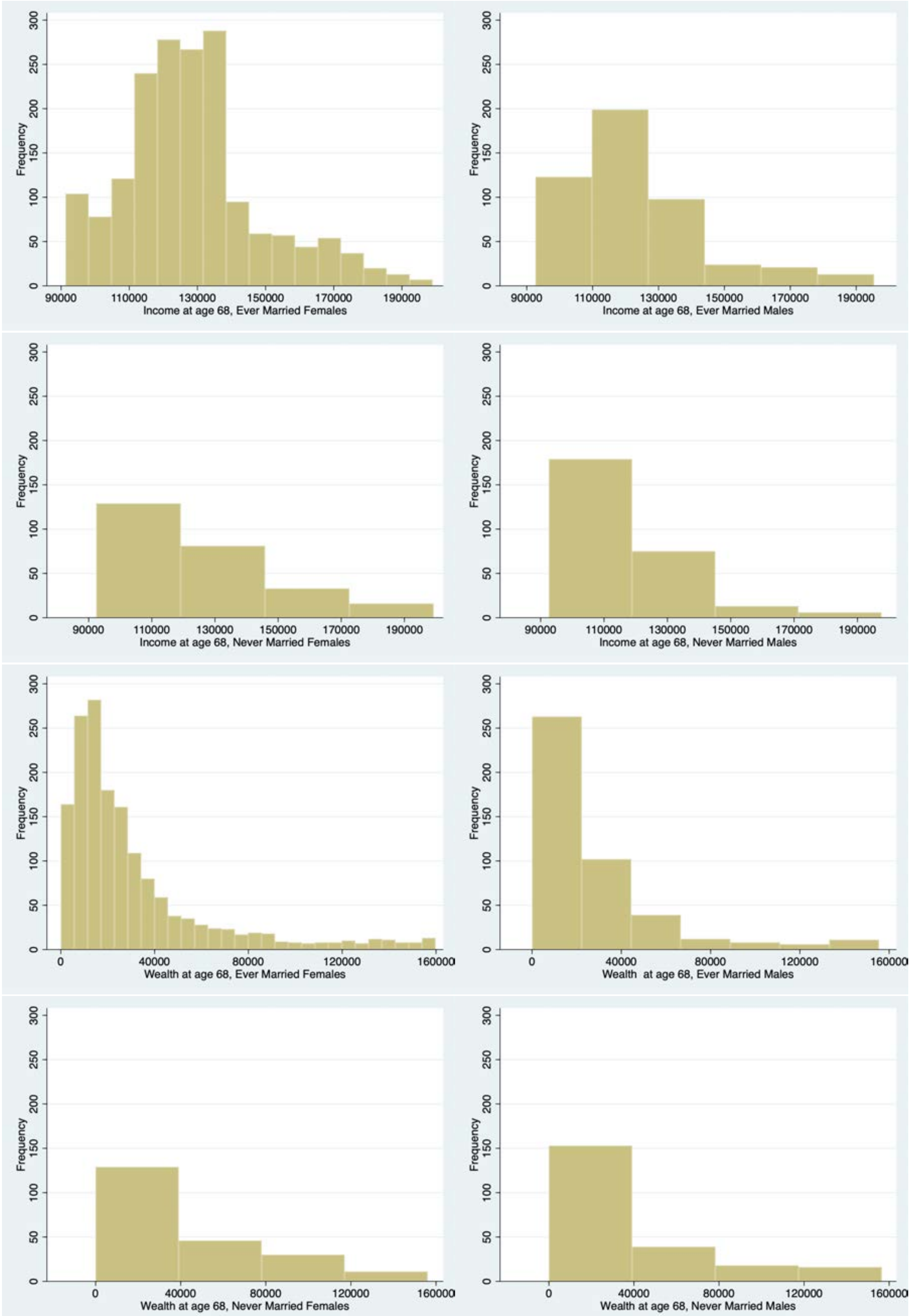


Table 10: **Parameter Estimates: Women, Marital History**

	Never Married		Ever Married					
			All Wealth		$> 25,000$DKK		$\leq 25,000$DKK	
	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$	$\hat{\vartheta}$	$\hat{\sigma}_{\vartheta}$
State H								
Persistence $\left(\rho^H\right)$	0.986	*0.003*	0.993	*0.001*	0.991	*0.001*	0.969	*0.003*
Utility coefficient $\left(\theta^H\right)$	1,630.6	*478.8*	2,776.6	*349.6*	1,969.4	*297.3*	718.9	*92.5*
Initial H proportion $\left(\pi^H\right)$	0.718	*0.034*	0.885	*0.010*	0.779	*0.019*	0.907	*0.016*
State L								
Persistence $\left(\rho^L\right)$	0.952	*0.007*	0.956	*0.004*	0.953	*0.004*	0.973	*0.004*
Utility coefficient $\left(\theta^L\right)$	293.2	*69.6*	459.2	*47.7*	363.1	*45.1*	190.5	*21.1*
Curvature (γ)	2.21	*0.10*	2.38	*0.04*	2.282	*0.051*	1.818	*0.042*
Hand-to-mouth parameter (ψ)	1.043	*0.033*	1.178	*0.013*	1.042	*0.018*	1.158	*0.020*
Bequest coefficient (τ)	1,782.5	*572.2*	2,388.8	*329.4*	1,334.0	*221.6*	37.7	*3.1*
Bequest type proportion $\left(p^b\right)$	0.630	*0.044*	0.657	*0.018*	0.720	*0.024*	0.803	*0.020*
Bequest threshold (κ)	11.93	*0.97*	13.50	*0.58*	9.47	*0.59*	0.717	*0.074*
Relative utility coefficient $\left(\frac{\theta^L}{\theta^H}\right)$	0.180		0.165		0.184		0.265	
Marginal Utility of Consumption	State H	State L	State H	State L	State H	State L	State H	State L
75,000 DKK	4.09	0.73	4.39	0.73	4.08	0.75	5.23	1.39
125,000 DKK	1.32	0.24	1.30	0.21	1.27	0.23	2.06	0.55
150,000 DKK	0.88	0.16	0.84	0.14	0.84	0.15	1.48	0.39
Marginal Utility of Bequest (bequest type)								
5,000 DKK	6.20		4.09		6.28		14.09	
25,000 DKK	3.42		2.29		3.00		1.58	
75,000 DKK	1.22		0.82		0.91		0.25	
Loglikelihood	-7,207.6		-48,560.8		-24,773.6		-23,543.3	
LR(10) (marital history)			116.0					
LR(10) (initial wealth)							487.6	
observations (persons)	262		1,768		781		987	
observations (person-years)	3,005		22,226		9,955		12,271	

Note: 1927 birth cohort, single people, no property