NBER WORKING PAPER SERIES

A NOTE ON TEMPORARY SUPPLY SHOCKS WITH AGGREGATE DEMAND INERTIA

Ricardo J. Caballero
Alp Simsek

Working Paper 29815
http://www.nber.org/papers/w29815

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 2022, Revised July 2022

Felipe Del Canto provided outstanding research assistance. We also thank Chris Ackerman, Jonathan Brownrigg, Marios Angeletos, William English, Giuseppe Moscarini, and seminar participants at TOBB ETU and the University of Glascow for their comments. Caballero acknowledges support from the National Science Foundation (NSF) under Grant Number SES-1848857. First draft: October 30, 2021. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

A Note on Temporary Supply Shocks with Aggregate Demand Inertia
Ricardo J. Caballero and Alp Simsek
NBER Working Paper No. 29815
March 2022, Revised July 2022
JEL No. E21,E32,E43,E44,E52,G12

ABSTRACT

We study optimal monetary policy during temporary supply contractions when aggregate demand has inertia and the central bank is concerned about future constraints on expansionary policy. In this environment, it is optimal to run the economy hot until supply recovers. However, the policy does not remain loose throughout the low-supply phase. Overall, when the initial aggregate demand is low, the goal is to frontload the rate cuts to raise demand in anticipation of the recovery of supply. If inflation also has inertia, the central bank still overheats the economy during the low-supply phase but gradually cools it down over time.

Ricardo J. Caballero
Department of Economics, E52-528
MIT
77 Massachusetts Avenue
Cambridge, MA 02139
and NBER
caball@mit.edu

Alp Simsek
Yale School of Management
Yale University
Edward P. Evans Hall
165 Whitney Ave
New Haven, CT 06511
and NBER
alp.simsek@yale.edu

Dynamic link to most recent version is available at
https://www.dropbox.com/s/x2i8a1o26ql69pa/SupplyShocks_public.pdf?dl=0

The U.S. headline inflation reached 7 percent during 2021, vastly exceeding the Fed's stated average inflation target. Similar inflation gaps were observed all around the world. These gaps emerged primarily from the clash between the brisk recovery in aggregate demand, supported by expansionary policies, and a weaker recovery in aggregate supply, due to Covid-related bottlenecks. The static picture was one of overheating, which triggered widespread concern that central banks were falling behind the curve. Throughout most of 2021, major central banks were reluctant to heed the advice to tighten monetary policy, arguing that the supply bottlenecks were only *temporary*, and hence unlikely to generate lasting overheating.

In this note, we characterize the *optimal* monetary policy response to a temporary supply contraction. As a benchmark, observe that in the standard New Keynesian (NK) model, the optimal policy in response to a supply shock is to *raise* interest rates by enough to bring aggregate demand down to the lower aggregate supply. Only once aggregate supply recovers, it is optimal to lower the interest rate and boost aggregate demand.

Set against this benchmark, we analyze the optimal policy with two realistic frictions. First, we assume *aggregate demand inertia*: past spending decisions affect future spending. This type of inertia can emerge from several frictions, e.g., habit formation or infrequent spending adjustments. Second, we assume *expansionary policy constraints:* when the output gap is negative, the central bank cannot instantly raise aggregate demand to its desired level. This might be because the central bank cannot cut the interest rate sufficiently (e.g., due to the zero lower bound) or because it prefers to adjust the interest rate gradually due to frictions such as policy uncertainty or concerns with financial stability (see, e.g., Bernanke (2004)).

Our main result is that the *interaction* between aggregate demand inertia and (anticipated) expansionary policy constraints implies that it is *optimal for the central bank to run the economy hot during a temporary supply contraction.* When aggregate demand has inertia, overheating the economy in the low-supply phase ensures that the economy has higher aggregate demand once aggregate supply recovers. Having a higher aggregate demand in the high-supply phase is useful because it alleviates the anticipated constraints on expansionary policy and accelerates the recovery (a form of "backward guidance"). The optimal policy balances the costs of positive output gaps during the low-supply phase with the benefits of faster recovery and less negative output gaps in the high-supply phase.

Our analysis does *not* suggest that monetary policy should remain loose *throughout* the low-supply phase. Tempering our main result, we find that the optimal policy quickly normalizes interest rates once the output gap reaches its desired (positive) level. This second result is also driven by the inertia in aggregate demand. With inertia, the initial expansionary monetary policy creates aggregate demand momentum, which keeps the output gap close to its desired (positive) level without the need for low interest rates. Keeping the interest rates "too low for too long" overheats the economy beyond the optimal output gap. Overall, when the initial aggregate demand is relatively low, the goal is to *frontload* the interest rate cuts to raise demand in anticipation of the recovery of supply.

While our baseline model assumes fully sticky prices, our main results also hold when prices are partially flexible and inflation responds to output gaps. When inflation is determined according to the standard New-Keynesian Phillips Curve (NKPC), our analysis is mostly unchanged. In a temporary supply contraction, the central bank (typically) induces positive inflation gaps along with positive output gaps. Once aggregate supply recovers, the inflationary pressure flips sign and the central bank fights disinflation and negative output gaps. As before, the central bank runs the economy hot in the low-supply phase, to mitigate the future negative gaps it expects in the high-supply phase.

When inflation is determined by an *inertial Phillips curve* (e.g., because price setters have backward-looking expectations), the optimal policy features richer dynamics. With inflation inertia, the central bank initially overheats the economy and gradually cools it down while it waits for the aggregate supply recovery. As the recovery is delayed, inflation gradually builds up and the central bank faces a more severe trade-off between inflation and output gaps. Running the economy hot becomes increasingly costly and the optimal policy "undoes" some of the overheating it has initially induced.

Literature. This note applies and extends our earlier analysis in Caballero and Simsek (2021). In that paper, we use a Calvo-type consumption adjustment model to generate the inertial behavior of aggregate demand that we assume in this paper. We characterize the optimal monetary policy with inertial aggregate demand, and show that a central bank facing a negative output gap frontloads interest rate cuts and "overshoots" asset prices. In an appendix, we also show that the central bank might *preemptively* overshoot asset prices when it expects aggregate demand to be below potential output in the future, e.g., because of a temporary supply shock. Here, we focus on temporary supply shocks and characterize the optimal policy in greater detail. We also use a more standard model (a minor modification of the textbook New-Keynesian model) and we focus on the optimal path of output, inflation, and interest rates—rather than on the path of asset prices.

Our note is related to a New-Keynesian literature that investigates the policy trade-offs induced by aggregate supply shocks. Blanchard and Galí (2007) focus on supply shocks in an environment with real wage rigidities. In this context, a contractionary supply shock reduces the second-best output (with real rigidities but no nominal rigidities) more than the first-best output (without any rigidity). The central bank can replicate the second-best output by stabilizing inflation, but this is not optimal. The central bank faces a trade-off between allowing for some inflation and stabilizing the "excessive" decline in output. More recently, and motivated by the Covid-19 episode, Guerrieri et al. (2021) show that reallocation shocks can also induce policy trade-offs similar to supply shocks. They study a multisector economy with downward wage rigidity subject to an asymmetric shock that shifts demand between sectors. The contracting sectors experience high unemployment but no wage or price decline, due to the downward rigidity, while the expanding sectors experience positive output gaps with high inflation. Under some conditions, the central bank might overheat the economy to accelerate the reallocation process.[1]

We also focus on supply shocks but we uncover a different policy trade-off and offer a complementary rationale for running the economy hot. In our model, the central bank faces an *intertemporal* trade-off between allowing for positive output gaps in the low-supply state and shrinking the negative output gaps that it expects to emerge once aggregate supply recovers. This intertemporal trade-off arises even in the limit case with extreme price stickiness, and implies that the central bank is more likely to accommodate supply shocks that it perceives to be more temporary.

A central feature of our model is aggregate demand inertia. This type of inertia emerges from various sources, such as infrequent adjustment of spending decisions or habit formation. An extensive literature documents the infrequent adjustment of durables consumption and investment (see Bertola and Caballero (1990) for an early survey). There is also a literature that emphasizes infrequent re-optimization for broader consumption categories—due to behavioral or informational frictions—and uses this feature to explain the inertial behavior of aggregate consumption (e.g., Caballero (1995); Reis (2006)) as well as asset pricing puzzles (e.g., Lynch (1996); Marshall and Parekh (1999); Gabaix and Laibson (2001)). Habit formation also introduces inertia into aggregate spending (see Woodford (2005) for an exposition). Fuhrer (2000); Amato and Laubach (2004) embed habit formation into standard business cycle models used for monetary policy analysis. We contribute to this line of work by analyzing the optimal monetary policy response to a temporary supply shock when there is demand inertia.

The rest of this note is organized as follows. Section 1 introduces our baseline model, with fixed prices. Section 2 characterizes the optimal monetary policy in this environment

[1] See Aoki (2001); Benigno (2004); Woodford (2005); Rubbo (2020); Fornaro and Romei (2022) for other analyses of how sectoral heterogeneity affects optimal monetary policy. The common theme in this literature is that monetary policy is also concerned with relative prices.

and establishes our main results. Section 3 extends our baseline model to add partially flexible prices and inflation. This section corroborates the monetary policy implications of the simpler model and establishes additional results when the inflation block of the model also features inertia. Section 4 provides final remarks. The online appendices contain the omitted proofs and results, the extensions of the baseline model, and the parameters used for the numerical examples.

1. A simple model with aggregate demand inertia

In this section, we describe our model's environment. It features a temporary supply shock, inertial aggregate demand, and constraints on expansionary policy in the high-supply state. We also characterize the equilibrium in a benchmark case with *no* reason for overheating the economy during the low-supply phase. For our baseline model, we assume that goods' prices are fixed. In this inflation*less* context, *overheating* simply means a positive output gap.

*IS curve with inertial aggregate demand***.** Consider a discrete time model and let $y_t = \log Y_t$ denote log output, which is determined by aggregate demand. Suppose the log-linearized IS curve is given by

$$y_t = \eta y_{t-1} + (1-\eta)\left(-\left(i_t - \rho\right) + E_t\left[y_{t+1}\right]\right), \tag{1}$$

where ρ is the households' discount rate and i_t is the interest rate at time t (the nominal and real interest rates are the same since prices are fixed). When $\eta = 0$, this reduces to the standard IS curve of the benchmark New Keynesian model. We assume $\eta > 0$, which captures inertia in spending decisions. This kind of inertia in the IS curve is broadly found in, e.g., models with consumption habits (see, e.g., Woodford (2005)), or in models with sluggish consumption adjustment (see, e.g., Caballero and Simsek (2021)).[2] Our IS curve is parsimonious and abstracts from many other factors that might affect aggregate demand (see the final remarks for how fiscal policy would affect our analysis).

Temporary supply shocks. There are two states $s_t \in \{L, H\}$ with potential outputs $y_L^* < y_H^*$. The economy starts in state L and transitions to state H with probability λ in each period. Once the economy is in state H, it stays there (i.e., H is an absorbing state).

[2] Large-scale New-Keynesian models, e.g., the Fed's FRB/US model, assume inertia because it helps match the observed gradual response of spending to a variety of exogenous shocks (see Brayton et al. (2014)).

Constraints on expansionary monetary policy. The second key ingredient in our model is a constraint on the central bank's ability to cut interest rates and implement expansionary policy. In Appendix B, we conduct our analysis assuming the central bank is subject to a zero lower bound (ZLB) constraint: that is, $i_t \geq 0$. However, this realistic constraint generates analytical complexity that is not central to our main points. Thus, in the main text we simplify the analysis by assuming an alternative lower bound constraint:

$$i_t \geq \underline{i}_t(y_t) = \rho + \phi(y_t - y_H^*). \tag{2}$$

Here, ρ is the long-run "rstar" for this economy. The central bank can lower the policy rate below ρ, but this is costly and the central bank will do so only if output falls below its long-run potential. The parameter $\phi > 0$ captures the sensitivity of the policy rate to the output gap when the constraint binds. This constraint simplifies the analysis because it implies a standard Taylor rule *after* the economy switches to the high-supply state.

Central bank's problem. We consider a central bank with a mandate to close the output (and inflation) gap. Formally, the central bank minimizes the present discounted value of quadratic output gaps, $E_t\left[\sum_{h=0}^{\infty} \beta^h \frac{\left(y_{t+h}-y^*_{s_{t+h}}\right)^2}{2}\right]$. We assume the central bank sets the policy interest rate *without commitment.* We can then formulate the policy problem recursively as

$$\begin{aligned} V_{s_t}(y_{t-1}) &= \max_{i_t, y_t} -\frac{\left(y_t - y^*_{s_t}\right)^2}{2} + \beta E_t\left[V_{s_{t+1}}(y_t)\right] \\ \text{s.t. } y_t &= \eta y_{t-1} + (1-\eta)\left(-(i_t - \rho) + E_t\left[Y_{s_{t+1}}(y_t)\right]\right) \\ i_t &\geq \underline{i}_t(y_t). \end{aligned} \tag{3}$$

Here, $Y_s(y_{-1})$ and $V_s(y_{-1})$ denote the output and the central bank's value, respectively, when the current state is $s \in \{H, L\}$ and the most recent output is y_{-1}. The central bank takes its future decisions as given and sets the current interest rate and output to minimize quadratic gaps, subject to the inertial IS curve and the constraint on expansionary policy.

Benchmark without constraints on expansionary policy. Let us start with a "first-best" benchmark case in which the central bank faces no constraints on expansionary policy ($\underline{i}_t(y_t) = -\infty$). In this benchmark, the central bank can achieve a zero output gap in every period and state, $y_t = y^*_{s_t}$, since there is always a feasible interest rate that ensures a zero output gap. Let us solve for these interest rates.

Consider the high-supply state H. Using $y_t = y_{t+1} = y_H^*$, the IS curve (1) implies

$$i_{t,H} = \rho - \frac{\eta}{1-\eta}\left(y_H^* - y_{t-1}\right). \tag{4}$$

If aggregate demand has recently been weak, $y_{t-1} < y_H^*$, *the interest rate needs to be cut below its steady-state level* to ensure the economy operates at its potential. In particular, for the first period in which the economy transitions to the high-supply state, we obtain

$$i_{tran,H} = \rho - \frac{\eta}{1-\eta}\left(y_H^* - y_L^*\right). \tag{5}$$

The central bank needs to *cut* the rate by a greater amount after the transition when aggregate demand has more inertia (higher η), and when the temporary supply shock is more severe (larger $y_H^* - y_L^*$).

Next consider the temporary supply shock state L. Using $y_t = y_L^*$ and $E\left[y_{t+1}\right] = \lambda y_H^* + \left(1-\lambda\right) y_L^*$, the IS curve (1) implies

$$i_{t,L} = \rho + \lambda\left(y_H^* - y_L^*\right) - \frac{\eta}{1-\eta}\left(y_L^* - y_{t-1}\right). \tag{6}$$

When recent output is equal to potential, $y_{t-1} = y_L^*$, the interest rate in state L is *above* its steady-state level, ρ. Since supply is temporarily low but is expected to recover (and this expectation raises current demand), the central bank raises the interest rate to ensure that current demand is in line with the reduced supply. When $y_{t-1} \neq y_L^*$, the interest rate also accounts for the inertia in aggregate demand.

These interest rate expressions hint that constraints on expansionary policy have the potential to cause problems (especially) during the transition from state L to state H. We next turn to our main case.

2. Overheating with inertia and constrained expansionary policy

In this section, we establish our main result that the optimal policy *overheats* the economy during the temporary supply shock state. When the initial demand is low, the central bank achieves this by *frontloading* interest rate cuts, which generates aggregate demand *momentum*. The reason for optimally overheating the economy in the supply-shock phase is to increase the starting level of aggregate demand once supply constraints dissipate and the expansionary policy constraints become binding (a form of "backward guidance").

We start by characterizing the equilibrium in the high-supply state $s = H$.

Lemma 1. *Suppose the economy has switched to the high-supply state, $s = H$, with past output $y_{t-1} < y_H^*$. Then, the expansionary policy constraint binds:*

$$i_t = \rho + \phi \left(y_t - y_H^* \right). \tag{7}$$

The output gap converges to zero at a constant rate:

$$Y_H \left(y_{t-1} \right) - y_H^* = \gamma_H \left(y_{t-1} - y_H^* \right), \tag{8}$$

where $\gamma_H \in (0, 1)$ is the smaller root of the polynomial $P(x) = x^2 - x \left(\frac{1}{1-\eta} + \phi \right) + \frac{\eta}{1-\eta}$. The central bank's value function is given by

$$V_H \left(y_{t-1} \right) = -\theta_H \frac{\left(y_{t-1} - y_H^* \right)^2}{2}, \qquad \text{where } \theta_H = \frac{\gamma_H^2}{1 - \beta \gamma_H^2}. \tag{9}$$

Current output is increasing *in past output, $\frac{dY_H(y_{t-1})}{dy_{t-1}} = \gamma_H > 0$. In addition, the parameters γ_H and θ_H are increasing in η: more inertia makes the output and the value function more sensitive to past output.*

In the high-supply state, the interest rate constraint binds and the policy effectively follows a Taylor rule. Output *eventually* reaches its potential level, y_H^*. However, the convergence is not immediate and output is influenced by demand. Importantly, output is increasing in past output. Intuitively, the recent decline in output along with inertia keeps current demand low. The policy is constrained and cannot immediately bring demand back to potential. A greater past output increases current demand, accelerates the recovery, and increases the central bank's value. These effects are stronger when aggregate demand has more inertia.

Next consider the equilibrium in the low-supply state $s = L$. Suppose past output y_{t-1} is not too low so that the expansionary policy constraint does not bind in the low-supply state (see Appendix A.1.2 for the case with the binding constraint). Using (9) (and assuming there is an interior solution), we can rewrite problem (3) as

$$V_L \left(y_{t-1} \right) = \max_{y_t < y_H^*} -\frac{\left(y_t - y_L^* \right)^2}{2} + \beta \left(\left(1 - \lambda \right) V_L \left(y_t \right) - \lambda \theta_H \frac{\left(y_t - y_H^* \right)^2}{2} \right). \tag{10}$$

We dropped the IS curve, which determines the interest rate the central bank needs to set to implement the optimal output level. Note that the value function does not depend on past output: $V_L(y_{t-1}) \equiv V_L$ is constant. The optimality condition is:

$$y_L - y_L^* = \beta \lambda \theta_H (y_H^* - y_L). \tag{11}$$

This leads to our main result. To state the result, let $y_L \in (y_L^*, y_H^*)$ denote the solution to (11) and assume the initial output satisfies:

$$y_{t-1} \geq \overline{y}_L = y_L - \frac{1-\eta}{\eta} (\lambda (1 - \gamma_H) + \phi) (y_H^* - y_L). \tag{12}$$

Proposition 1. *Suppose the economy is in the temporary supply shock state, $s = L$, with past output y_{t-1} that satisfies* (12)*. Then the expansionary policy constraint does not bind in $s = L$. The central bank implements the constant output level $y_L \in (y_L^*, y_H^*)$ that solves* (11)*. The central bank induces positive output gaps in the current low-supply state (current overheating), $y_L > y_L^*$, and negative output gaps after transition to the high-supply state (future demand shortages), $Y_H(y_L) < y_H^*$.*

The central bank targets the optimal output by setting the interest rate

$$i_{t,L} = \rho + \lambda (Y_H(y_L) - y_L) - \frac{\eta}{1-\eta} (y_L - y_{t-1}). \tag{13}$$

If $y_{-1} < y_L$, we have $i_{0,L} < i_{t,L} \equiv i_L$ for $t \geq 1$, where $i_L = \rho + \lambda (Y_H(y_L) - y_L)$. The central bank initially sets a relatively low interest rate and then normalizes the interest rate and keeps it at a constant level until the transition to the high-supply state.

The first part of the result characterizes the optimal output choice in the low-supply state. For intuition, observe that the left side of (11) captures the marginal cost of overheating and the right side of (11) captures the marginal benefit from overheating. When output is at its potential level, $y_L = y_L^*$, the marginal cost of overheating is zero but the marginal benefit is strictly positive. Therefore, the central bank optimally induces some overheating. Overheating *in the current period* mitigates the demand shortage and accelerates the recovery *in future periods* after the transition to high supply. Observe also that the marginal benefit from overheating declines as y_L rises toward y_H^* and it becomes zero when $y_L = y_H^*$. Therefore, there is a unique interior optimum $y_L \in (y_L^*, y_H^*)$. The central bank stops short of overheating to the point that the economy would have no (negative) output gaps after the transition to the high-supply state.

The second part of Proposition 1 shows that the central bank does *not* keep the interest rate low *throughout* the low-supply phase. Rather, the central bank frontloads the interest rate cuts and then quickly normalizes the interest rate once the output gap reaches its target level (y_L). This feature is also driven by the inertia in aggregate demand. Recall the IS curve (1)

$$y_t = \eta y_{t-1} + (1-\eta)\left(-\left(i_t - \rho\right) + E_t\left[y_{t+1}\right]\right).$$

With inertia ($\eta > 0$), a greater past output y_{t-1} supports a greater current output y_t for any given interest rate i_t (and expected output $E_t\left[y_{t+1}\right]$). Therefore, once the central bank raises output to its target level, it does not need to keep the interest rate low to keep the output at this level. The initial expansionary monetary policy creates aggregate demand *momentum*. This demand momentum keeps the output gap close to its desired (positive) level without the need for low interest rates. Keeping the interest rates "too low for too long" would overheat the economy beyond the optimal output gap.

Remark 1 (Overheating vs overlooking supply shocks)**.** *Proposition 1 considers the case in which the initial demand is relatively low, $y_{-1} < y_L$. This is arguably relevant for highly disruptive supply shocks (such as pandemics) that reduce output as well as potential output. For less disruptive supply shocks (such as oil shocks), the initial demand can be relatively high, $y_{-1} \in (y_L, y_H^*]$. In this case, the central bank initially sets a relatively high interest rate to bring aggregate demand down to y_L*—but not all the way down to *y_L^*. The central bank "partially overlooks the supply shock" as opposed to deliberately overheating the economy. The robust result that applies in both cases is that the central bank "runs the economy hot" and targets positive output gaps while the supply is temporarily low.*

Remark 2 (Backward guidance)**.** *The optimal policy features "backward guidance" in the sense that it resembles the "forward guidance" policies analyzed by a large literature. With forward guidance, the central bank promises to keep future output above its potential, which raises current output via forward-looking expectations. With backward guidance, the central bank keeps past output above its potential, which raises current output via demand inertia.*

Numerical illustration. Figure 1 illustrates the equilibrium. In this simulation, the economy starts in the temporary low-supply state with a relatively low level of initial demand ($y_{-1} < y_L$). The economy transitions to the high-supply state in period four.

The (blue) solid lines plot the equilibrium characterized in Proposition 1, where the central bank faces the constraint on expansionary policy. The policy induces *overheating* in the low-supply state. The policy achieves this by *cutting* the rate in the first period while the economy is in the low-supply state. Once the policy brings output in the low-supply state to the optimal level of overheating, $y_L > y_L^*$, it raises the interest rate to keep output

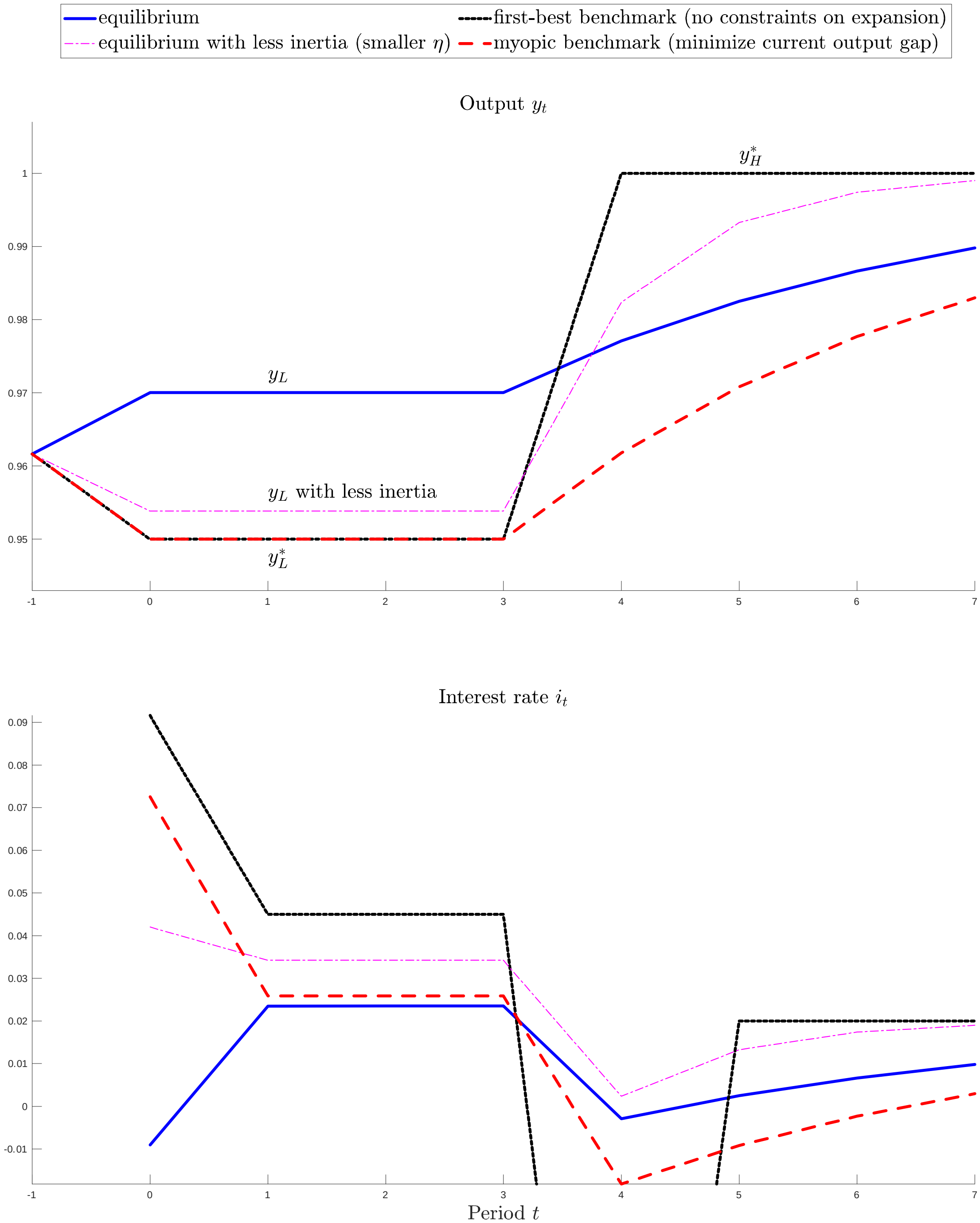

Figure 1: A simulation of the equilibrium starting in the low-supply state, $s_0 = L$, with initial output that satisfies $y_{-1} \in [\overline{y}_L, y_L)$. The solid lines correspond to the equilibrium with optimal policy. The dotted lines correspond to a first-best benchmark case without expansionary policy constraints. The dashed lines correspond to a myopic benchmark case in which the policy minimizes the current output gap. The dash-dotted lines correspond to the equilibrium with a smaller inertia parameter (η). See Appendix C for the parameters used.

constant until the economy transitions to the high-supply state. After the transition, the policy cuts the interest rate once again to raise aggregate demand toward the higher aggregate supply level. Due to the constraint on expansionary policy, the recovery in the high-supply state takes several periods to complete.

Why does the optimal policy cut the interest rates in the low-supply state and induce overheating? As Figure 1 illustrates, the central bank anticipates that the transition to the high-supply state will start with low aggregate demand. Because aggregate demand has inertia, the central bank recognizes that a greater aggregate demand in the low-supply state will accelerate the recovery after the economy transitions to the high-supply state. The optimal policy induces positive output gaps in the low-supply state, but it also shrinks the negative output gaps that emerge when the economy switches to the high-supply state.

The remaining lines in the figure correspond to alternative scenarios that illustrate various properties of the optimal policy. The (black) dotted lines plot the first-best benchmark case, where the central bank does not face any constraints on expansionary monetary policy. In this benchmark, as illustrated by Eqs. $(5-6)$, the policy sharply raises the interest rate in the initial period and sharply cuts the interest rate in the first period after transition to high supply. Compared to this benchmark, the (constrained) optimal policy *frontloads* the interest rate cuts and avoids large interest rate changes.

The (red) dashed lines plot a myopic benchmark case in which the central bank focuses on closing *current* gaps: formally, the central bank solves problem (3) with the period-by-period objective function $-\frac{\left(y_t-y_L^*\right)^2}{2}$. In this benchmark, the central bank keeps output in the low-supply state equal to its potential. Consequently, the economy transitions to high supply with a lower aggregate demand and the recovery takes longer. Compared to this benchmark, the optimal policy features overheating in the low-supply phase and a faster recovery after transition to high supply.

Finally, the (magenta) dash-dotted lines plot the equilibrium with less inertia (smaller η). In this case, after the supply recovers, output converges to its potential faster (see Lemma 1). Since the central bank anticipates smaller negative gaps after transition to high supply, it overheats the economy by a smaller amount in the low-supply phase. Compared to this scenario, the optimal policy with more inertia features greater output gaps (in absolute value) both before and after the supply recovery. These comparisons highlight that our results are driven by the *interaction* of aggregate demand inertia and expansionary policy constraints.

3. Overheating with inflation

In this section we extend our setup to allow for partially flexible prices and an inflation rate that is responsive to overheating. We start with the textbook case in which inflation is determined by a New-Keynesian Phillips Curve (NKPC) without *inflation* inertia. In this case, our substantive conclusion remains the same: the central bank overheats the economy in the temporary supply shock state to fight the negative output gaps and *the disinflation* that it expects to emerge after the supply recovers. We then assume that the inflation block of the model also features inertia. This case leads to richer dynamics within the temporary supply shock state: *With inflation inertia, the central bank initially overheats the economy, as before, but gradually cools it down as the supply contraction continues.*

We first modify the baseline setup in Section 1 to incorporate inflation. Let P_t denote the nominal price level and $\pi_t = \log(P_t/P_{t-1})$ denote (log) inflation. With inflation, the IS curve (1) becomes

$$\begin{aligned} y_t &= \eta y_{t-1} + (1-\eta)\left(-(r_t - \rho) + E_t[y_{t+1}]\right) \qquad (14) \\ \text{where } r_t &= i_t - E_t[\pi_{t+1}]. \end{aligned}$$

Here, r_t denotes *the real interest rate.*

To simplify the exposition, we also modify the expansionary policy constraint in (2). Recall that the constraint implied that the policy follows a Taylor rule *after* transition to the high-supply state (see Eq. (7)), but it is unconstrained in the low-supply state (except for a case we relegated to the appendix). With inflation, we directly assume the policy follows a (generalized) Taylor rule *only* in the high-supply state,

$$i_t = \rho + \phi_y (y_t - y_H^*) + \phi_\pi \pi_t \quad \text{if } s_t = H. \qquad (15)$$

The policy responds to both output and *inflation gaps*—the deviation of inflation from its target. We normalize the inflation target to zero so that the inflation gap is the same as inflation. The parameters $\phi_y, \phi_\pi > 0$ capture the sensitivity of the policy rate to the corresponding gap. For simplicity, the policy is unconstrained in the low-supply state.

Finally, we adjust the central bank's objective function to incorporate the costs of inflation gaps:

$$E_t\left[\sum_{h=0}^{\infty} \beta^h \left(-\frac{\left(y_{t+h} - y_{s_{t+h}}^*\right)^2}{2} - \psi \frac{\pi_{t+h}^2}{2}\right)\right]. \qquad (16)$$

Here, ψ denotes the relative welfare weight for the inflation gaps.

We next describe the inflation block and characterize the optimal policy. We consider two specifications that differ on whether the Phillips curve features inertia or not.

3.1. Overheating with the New-Keynesian Phillips curve

First, suppose inflation is determined by the standard NKPC (see Galí (2015) for a derivation):

$$\pi_t = \kappa \left(y_t - y_{s_t}^* \right) + \beta E_t \left[\pi_{t+1} \right]. \tag{17}$$

Inflation depends on the current output gap, $y_t - y_{s_t}^*$, as well as the expectations for future inflation. The coefficient κ captures the extent of price flexibility. The equation does not feature inertia because inflation expectations are rational and forward-looking.

Consider the first-best benchmark without any constraints on expansionary policy. As before, the central bank achieves zero output gaps throughout. In view of Eq. (17), this implies zero inflation throughout. In this benchmark, the central bank can simultaneously stabilize output and inflation—a result known as the "divine coincidence" of monetary policy. Therefore, introducing inflation does not change the analysis.

Next, suppose the central bank faces the expansionary policy constraint (15) in the high-supply state. The analysis closely parallels the baseline analysis in Section 2. Therefore, we relegate the formal results to Appendix A.2.1 and discuss the intuition.

In the high-supply state, $s = H$, the equilibrium is characterized by the IS curve (14), the NKPC (17), and the policy rule (15). Under appropriate parametric restrictions, output and inflation gaps *eventually* converge to zero (see Lemma 2 in the appendix). However, the convergence is not immediate. Starting with $y_{t-1} < y_H^*$, the economy experiences a period of negative output gaps and disinflation. As before, increasing y_{t-1} mitigates these gaps and increases the value function.

In the low-supply state, $s = L$, the central bank solves a modified version of problem (10). Proposition 4 in the appendix characterizes the solution and shows that our main result extends to this setup. *The central bank chooses a level of output that induces positive output gaps in the low-supply state* (current overheating), $y_{t,L} \equiv y_L > y_L^*$, *and negative output gaps and disinflation after transition to the high-supply state (future demand shortages),* $Y_H(y_L) < y_H^*, \Pi_H(y_L) < 0$.

Figure 2 illustrates this result in a numerical example. As before, the economy starts in the low-supply state and with a relatively low initial demand ($y_{-1} < y_L$). The solid

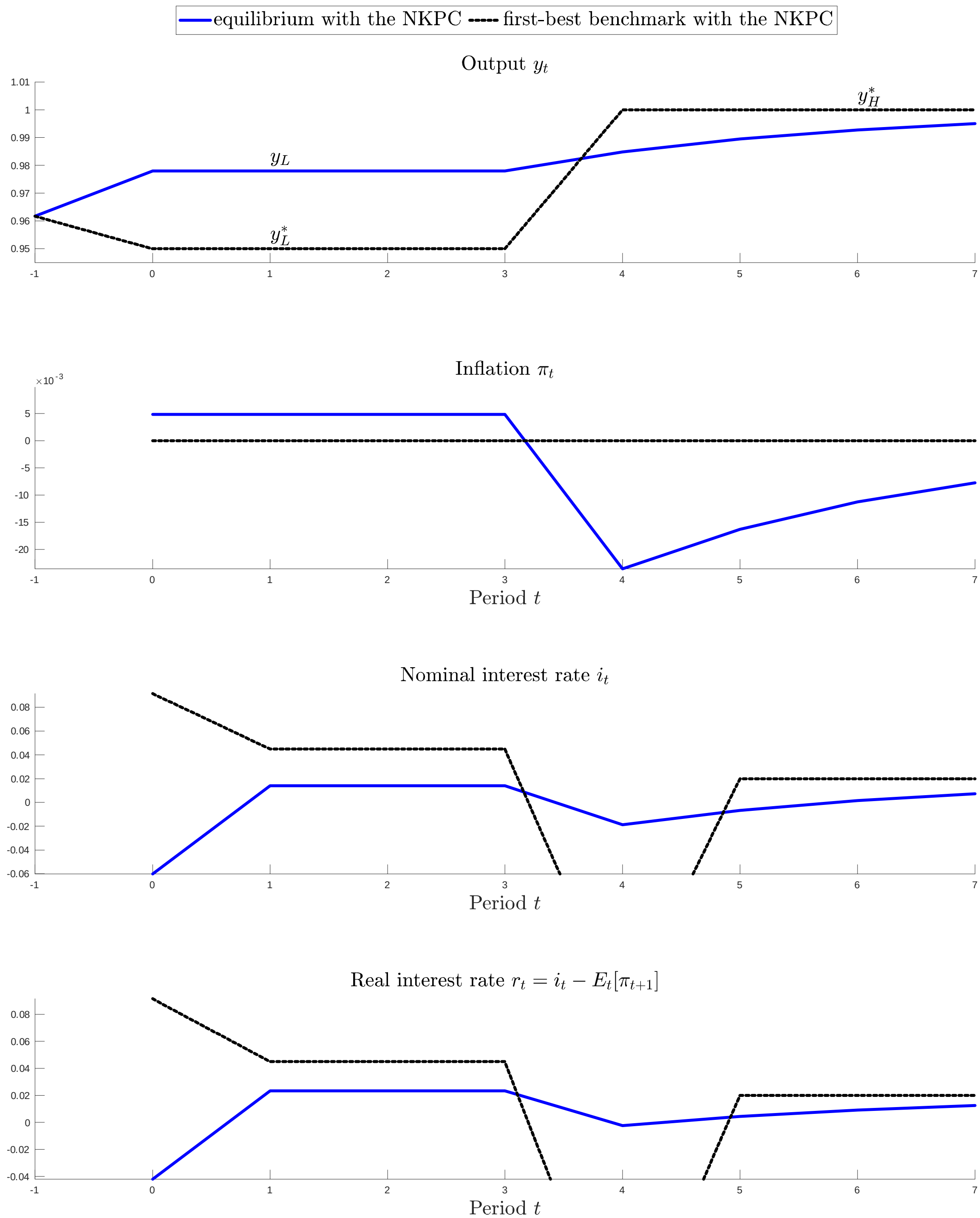

Figure 2: A simulation of the equilibrium in which inflation is determined acording to the NKPC. The solid lines show the equilibrium with optimal policy. The dotted lines illustrate a first-best benchmark case without expansionary policy constraints. See Appendix C for the parameters used.

lines show the equilibrium with the optimal policy. Compared to the first-best benchmark (the dotted lines), the optimal policy frontloads interest rate cuts to bring output above its potential level and inflation above its target level. Similar to before, the central bank anticipates that the recovery will start with low aggregate demand *and disinflation*. Therefore, the central bank temporarily overheats the economy to ensure that the recovery starts with a greater aggregate demand *and a smaller inflation gap*.

As the figure shows, the central bank implements a relatively low inflation despite the fact that it sets a positive output gap. This aspect is driven by the forward looking nature of the NKPC. Price setters recognize that the supply recovery will start with negative output gaps and disinflation. The expected disinflation puts downward pressure on current inflation. While this result makes overheating relatively less costly, empirical studies of the Phillips curve suggest that inflation is not influenced by the rationally expected future output gaps as much as predicted by the NKPC (see, e.g., Rudd and Whelan (2005)). We next turn to an alternative setup in which inflation is backward-looking.

3.2. Overheating with an inertial Phillips curve

Next suppose inflation is determined by an inertial Phillips curve:

$$\pi_t = \kappa \left(y_t - y_{s_t}^* \right) + b\pi_{t-1}. \tag{18}$$

Here, $b \in (0,1)$ is a parameter that captures the strength of inflation inertia. In theory, inflation inertia can emerge from several frictions, e.g., backward-looking indexation of prices or wages (e.g., Galı and Gertler (1999)) or adaptive inflation expectations (e.g., Blanchard (2016)). For analytical tractability, we assume inflation is fully backward-looking.

First, consider the first-best benchmark setup without constraints on expansionary policy. As long as the central bank does not inherit past inflation, $\pi_{-1} = 0$, it is easy to check that the equilibrium is the same as before. In particular, the central bank achieves zero output gaps and zero inflation throughout. As long as $\pi_{-1} = 0$, *the divine coincidence* still applies with a backward-looking Phillips curve.

Next, consider the main setup in which the policy is constrained to follow the Taylor rule (15) in the high-supply state. To simplify the exposition, suppose also that the Taylor rule coefficient on inflation satisfies $\phi_\pi = b$. With this assumption, Lemma 3 in the appendix shows that there is an equilibrium in which the output gaps converge to zero at a constant rate γ_H. Along this equilibrium path, the value function satisfies

$$V_H\left(y_{t-1},\pi_{t-1}\right) = -\frac{\theta_H}{2}\left(y_{t-1}-y_H^*\right)^2 - \frac{\Psi_H}{2}\pi_{t-1}^2 - \mathcal{I}_H\left(y_{t-1}-y_H^*\right)\pi_{t-1}.$$

Here, $\theta_H, \Psi_H, \mathcal{I}_H > 0$ are derived coefficients given by Eqs. $(A.18)$ in Appendix A.2.2. In this case, the value function depends on the past inflation, π_{t-1}, in addition to the past output gap, $y_{t-1}-y_H^*$ [cf. Eq. (9)].

In the low-supply state, $s = L$, the central bank solves a version of problem (10):

$$\begin{aligned} V_L\left(y_{t-1},\pi_{t-1}\right) &= \max_{y_t,\pi_t} -\frac{\left(y_{t-1}-y_H^*\right)^2}{2} - \psi\frac{\pi_{t-1}^2}{2} + \beta\left(\left(1-\lambda\right)V_L\left(y_t,\pi_t\right) + \lambda V_H\left(y_t,\pi_t\right)\right) \\ \pi_t &= \kappa\left(y_t-y_L^*\right) + b\pi_{t-1} \end{aligned} \quad (19)$$

Our main result characterizes the optimal policy and extends Proposition 1 to this setting. To state the result, we define the following derived parameters:

$$\begin{aligned} A &= 1+\beta\lambda\left(\theta_H+\kappa\mathcal{I}_H\right) \\ B &= \kappa\psi+\beta\lambda\left(\mathcal{I}_H+\kappa\Psi_H\right) \\ C &= \beta\left(1-\lambda\right)b\left(1+\beta\lambda\theta_H\right) \\ D &= \beta\left(1-\lambda\right)b\beta\lambda\mathcal{I}_H \\ E &= \beta\lambda\left[\theta_H+\kappa\mathcal{I}_H-\beta\left(1-\lambda\right)b\theta_H\right]. \end{aligned} \quad (20)$$

Proposition 2. *Consider the setup with an inertial Phillips curve. Suppose* $\phi_\pi = b$ *and* $\phi_y > \kappa$. *Let* $A, B, C, D, E > 0$ *given by (20). The polynomial* $P(x) = x^2 - \frac{A+B\kappa+bC}{C+D\kappa}x + \frac{Ab}{C+D\kappa}$ *has a single root in the interval* $(0,b)$, *which we denote with* γ_L. *As long as the economy remains in the low-supply state, the optimal choice of output and inflation,* (y_t,π_t), *converge to a steady-state,* $(\overline{y}_L,\overline{\pi}_L)$, *where*

$$\overline{y}_L = y_L^* + \frac{E}{A-C+(B-D)\frac{\kappa}{1-b}}\left(y_H^*-y_L^*\right) \in \left(y_L^*,y_H^*\right) \quad (21)$$

$$\overline{\pi}_L = \frac{\kappa}{1-b}\left(\overline{y}_L-y_L^*\right) > 0. \quad (22)$$

Along the transition path, the optimal output and inflation are given by

$$y_t-\overline{y}_L = -\frac{b-\gamma_L}{\kappa}\left(\pi_{t-1}-\overline{\pi}_L\right) \quad (23)$$

$$\pi_t-\overline{\pi}_L = \gamma_L\left(\pi_{t-1}-\overline{\pi}_L\right). \quad (24)$$

The associated real and nominal interest rates are given by $(A.21 - A.22)$ *in the appendix. Starting with zero past inflation,* $\pi_{-1} = 0$*, the central bank implements a relatively high initial output gap,* $y_0 > \overline{y}_L > y_L^*$*. Absent transition to the high supply state, the central bank gradually decreases the output gap toward its steady state value,* $y_t - y_L^* \downarrow \overline{y}_L - y_L^*$*, and increases the inflation toward its steady state value,* $\pi_t \uparrow \overline{\pi}_L$*.*

With inertia in the Phillips curve, the central bank still implements high output gaps in the low-supply state, as before, but it also reduces the output gaps as the supply recovery is delayed. For intuition, note that the output gaps increase inflation. With inflation inertia, past inflation shifts the Phillips curve and worsens the trade-off between increasing the output (to accelerate the *future* recovery) and raising inflation. As time passes and the recovery is delayed, it becomes increasingly costly to induce positive output gaps. The central bank optimally "undoes" some of the overheating it has initially induced.

Figure 3 illustrates the result in a numerical example. The (blue) solid lines show the equilibrium with optimal policy. As before, the central bank frontloads interest rate cuts and brings output above its potential level. As time passes and the recovery is delayed, this policy raises inflation. To keep inflation under control, the central bank gradually brings output closer to its potential—undoing some of the initial overheating.

The last two panels of the figure show the nominal and real interest rates the central bank targets. As before, after the initial interest rate cut, the central bank raises the interest rate. Unlike before, the nominal rate can exceed its long-run neutral level (cf. Figures 1 and 2). In this model, the level of the nominal interest rate is also influenced by *expected* inflation. As inflation increases over time, *expected* inflation increases (we assume *consumers'* inflation expectations are rational and forward looking). Thus, the central bank raises the nominal rate to keep the real rate relatively stable.

4. Final Remarks

Summary. In this note, we developed a model to address two substantive questions: *Should central banks tolerate some degree of overheating during a temporary supply contraction?* And if the answer is yes, as we find, *does this imply that optimal monetary policy should remain ultra-loose throughout the supply constrained phase?*

Our answers to these questions build on the realistic modeling ingredient that aggregate demand has inertia. Inertia implies that the level of aggregate demand in the future, once aggregate supply recovers, is increasing in the level of aggregate demand in the

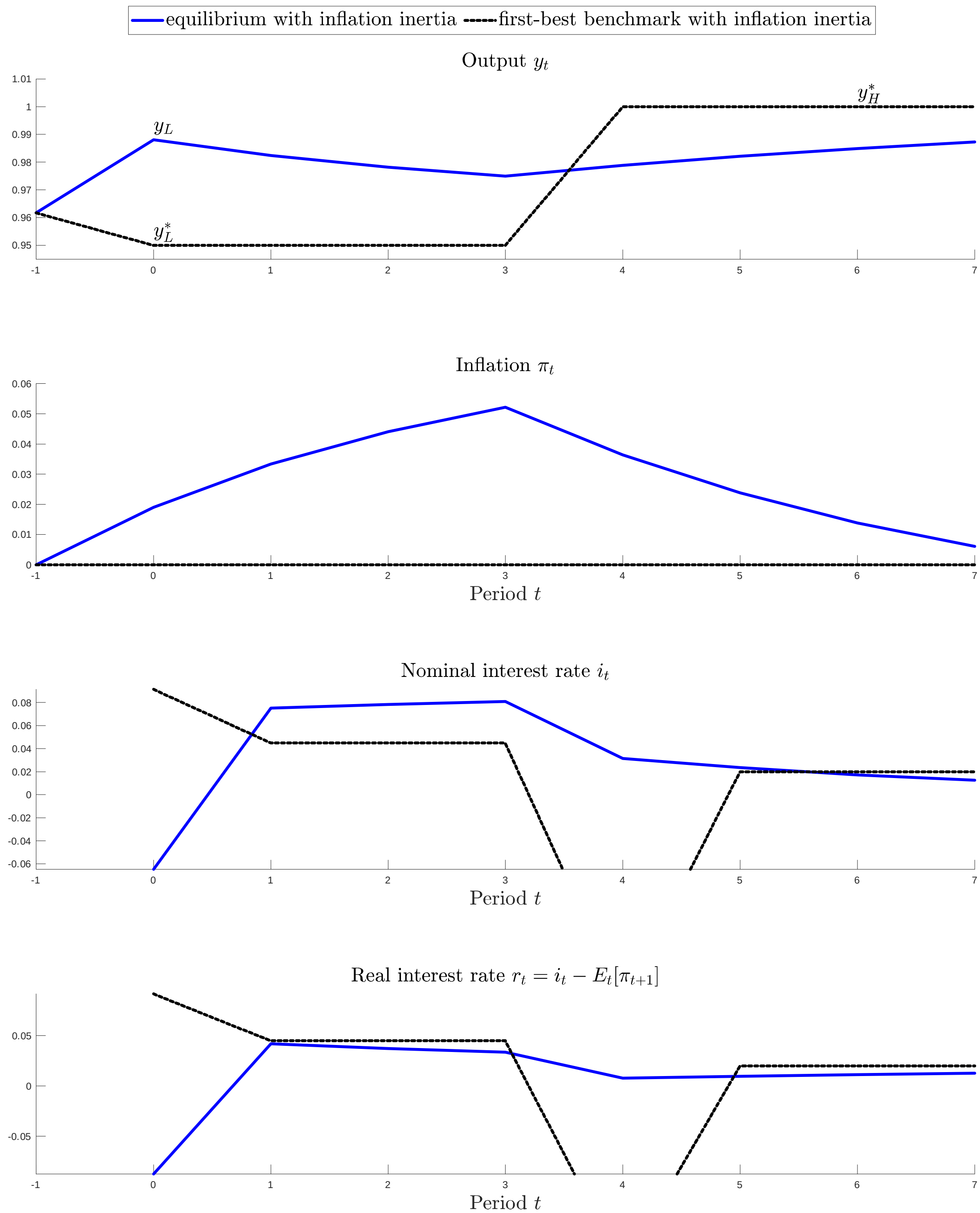

Figure 3: A simulation of the equilibrium in which inflation is determined acording to an inertial Phillips curve. The solid lines show the equilibrium with optimal policy. The dotted lines illustrate a first-best benchmark case without expansionary policy constraints. See Appendix C for the parameters used.

current low-supply state. This dynamic linkage across states implies that a policymaker that anticipates being constrained and facing a negative output gap in the future, once aggregate supply recovers, overheats the economy in the current low-supply state (a form of "backward guidance").

Aggregate demand inertia also implies that, within the low-supply phase, the optimal policy frontloads the interest rate cuts and then quickly normalizes the interest rate. The reason is that the initial expansion generates aggregate demand momentum. This momentum supports aggregate demand and ensures that output stays at the optimal level of overheating without the initially low interest rate. In this context, keeping the interest rate "too low for too long" overheats the economy beyond the optimal level.

If the inflation block of the model also features inertia, then the optimal policy features richer dynamics. The initial expansion in the low supply state gradually increases inflation, which makes it increasingly costly to run the economy hot. As the recovery is delayed, the central bank optimally "undoes" some of the initial overheating. The build-up of inflation also raises expected inflation, which might induce the central bank to raise the nominal interest rate above its long-run neutral level. In this context, adjusting the nominal interest rate too slowly lowers the real interest rate and overheats the economy beyond the optimal level.

Clarifications and extensions. We assumed that potential output immediately recovers to a high level once the temporary supply contraction ends. This feature is meant to capture a Covid-19 style shock, where supply remains depressed mainly due to virus-related developments (e.g., whether there will be a new variant) and can be expected to recover rapidly once the virus is under control. For other supply shocks, where the expected supply recovery is more gradual, the first-best benchmark implies smaller interest rate cuts during the recovery, which also reduces the need for frontloading interest rate cuts (see Figure 1). In this sense, our results are more relevant for temporary supply shocks driven by highly disruptive but short-lived events, such as epidemics, wars, or political conflicts.

We also assumed that potential output is exogenous. Our policy conclusions would be even stronger if potential output is endogenous to aggregate demand, as in Benigno and Fornaro (2018); Fornaro and Wolf (2021). In that setting, running the economy hot in the low-supply phase would increase future potential output, which would make future output gaps more negative and justify further overheating during the supply disruption.

We abstracted from fiscal policy and focused on the optimal path of monetary policy. However, fiscal and monetary policy are substitutes in terms of their impact on aggregate demand, which suggests that our results can also speak to the optimal timing of fiscal policy. In particular, aggregate demand inertia provides a rationale for frontloading fiscal policy, especially if monetary policy is constrained in the early stages of the low supply state, as in the Covid-19 episode. We leave a more complete analysis of the optimal fiscal policy response to temporary supply shocks for future work.

Finally, it is important to emphasize that the optimal policy in our model is driven by the *anticipation* of a binding policy constraint once supply recovers. We assumed there is no uncertainty, which implies that the constraint always binds after the supply recovery. In a more realistic model with uncertainty, our results on optimal policy would still hold *ex ante*, but the constraint might not bind *ex post*, e.g., if the supply recovery is weaker than expected, as in the Covid-19 episode.

References

Amato, J. D., Laubach, T., 2004. Implications of habit formation for optimal monetary policy. Journal of Monetary Economics 51 (2), 305–325.

Aoki, K., 2001. Optimal monetary policy responses to relative-price changes. Journal of monetary economics 48 (1), 55–80.

Benigno, G., Fornaro, L., 2018. Stagnation traps. The Review of Economic Studies 85 (3), 1425–1470.

Benigno, P., 2004. Optimal monetary policy in a currency area. Journal of international economics 63 (2), 293–320.

Bernanke, B., 2004. Gradualism. Remarks at an economics luncheon co-sponsored by the Federal Reserve Bank of San Francisco (Seattle Branch) and the University of Washington, Seattle, Washington, May 20, 2004.

Bertola, G., Caballero, R. J., 1990. Kinked adjustment costs and aggregate dynamics. NBER macroeconomics annual 5, 237–288.

Blanchard, O., 2016. The phillips curve: Back to the'60s? American Economic Review 106 (5), 31–34.

Blanchard, O., Galí, J., 2007. Real wage rigidities and the new Keynesian model. Journal of money, credit and banking 39, 35–65.

Brayton, F., Laubach, T., Reifschneider, D. L., et al., 2014. The FRB/US model: A tool for macroeconomic policy analysis. FEDS Notes 2014-04-03.

Caballero, R. J., 1995. Near-rationality, heterogeneity and aggregate consumption. Journal of Money, Credit & Banking 27 (1), 29–49.

Caballero, R. J., Simsek, A., 2021. Monetary policy and asset price overshooting: A rationale for the Wall/Main street disconnect. NBER working paper No. 27712.

Fornaro, L., Romei, F., 2022. Monetary policy during unbalanced global recoveries. working paper.

Fornaro, L., Wolf, M., 2021. Monetary policy in the age of automation. working paper.

Fuhrer, J. C., 2000. Habit formation in consumption and its implications for monetary-policy models. American economic review 90 (3), 367–390.

Gabaix, X., Laibson, D., 2001. The 6D bias and the equity-premium puzzle. NBER macroeconomics annual 16, 257–312.

Galí, J., 2015. Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications. Princeton University Press.

Galı, J., Gertler, M., 1999. Inflation dynamics: A structural econometric analysis. Journal of monetary Economics 44 (2), 195–222.

Guerrieri, V., Lorenzoni, G., Straub, L., Werning, I., 2021. Monetary policy in times of structural reallocation. University of Chicago, Becker Friedman Institute for Economics Working Paper (2021-111).

Lynch, A. W., 1996. Decision frequency and synchronization across agents: Implications for aggregate consumption and equity return. The Journal of Finance 51 (4), 1479–1497.

Marshall, D. A., Parekh, N. G., 1999. Can costs of consumption adjustment explain asset pricing puzzles? The Journal of Finance 54 (2), 623–654.

Reis, R., 2006. Inattentive consumers. Journal of monetary Economics 53 (8), 1761–1800.

Rubbo, E., 2020. Networks, phillips curves and monetary policy. working paper.

Rudd, J., Whelan, K., 2005. New tests of the new-keynesian phillips curve. Journal of Monetary Economics 52 (6), 1167–1181.

Woodford, M., 2005. Interest and Prices. Princeton University Press.

Online Appendices for: A Note on Temporary Supply Shocks with Aggregate Demand Inertia

Ricardo J. Caballero   Alp Simsek

A. Omitted Proofs

This appendix contains the results and the derivations omitted from the main text.

A.1. Omitted proofs and extensions for Section 2

We first present the proofs omitted from Section 2. We then characterize the equilibrium for the remaining case in which the expansionary policy constraint might bind also in the low-supply state.

A.1.1. Omitted proofs

Proof of Lemma 1. Suppose the economy switched to the high-supply state $s = H$ with $y_{t-1} < y_H^*$. We verify that the conjectured allocation is an equilibrium.

We first show that the expansionary policy constraint in (2) binds along the conjectured equilibrium path. Suppose the constraint does not bind. Then, the central bank would target a zero gap, $y_t = y_H^*$, by setting the interest rate in (4),

$$i_t = \rho - \frac{\eta}{1-\eta}\left(y_H^* - y_{t-1}\right).$$

Along the conjectured path, we have $y_{t-1} < y_H^*$ and the required interest rate satisfies, $i_t < \rho$. However, since the policy targets a zero output gap, $y_t = y_H^*$, the policy constraint implies $i_t \geq \rho$. This provides a contradiction and implies that the policy constraint binds. In particular, the policy effectively follows the Taylor rule in (7).

We next characterize the evolution of output. Combining the IS curve in (1) and the Taylor rule in (7), output follows the difference equation,

$$y_t = \eta y_{t-1} + (1-\eta)\left(-\phi\left(y_t - y_H^*\right) + y_{t+1}\right).$$

We drop the expectations since there is no (residual) uncertainty. Let $\tilde{y}_t = y_t - y_H^*$ denote the output gap. Then, we can rewrite the difference equation as,

$$\tilde{y}_t = \eta \tilde{y}_{t-1} + (1-\eta)\left(-\phi \tilde{y}_t + \tilde{y}_{t+1}\right).$$

In matrix notation, we have the system,

$$\begin{bmatrix} \tilde{y}_{t+1} \\ \tilde{y}_t \end{bmatrix} = M \begin{bmatrix} \tilde{y}_t \\ \tilde{y}_{t-1} \end{bmatrix} \text{ where } M = \begin{bmatrix} \frac{1}{1-\eta} + \phi & -\frac{\eta}{1-\eta} \\ 1 & 0 \end{bmatrix}.$$

The characteristic polynomial of the matrix M is given by

$$P(x) = x^2 - x\left(\frac{1}{1-\eta} + \phi\right) + \frac{\eta}{1-\eta}.$$

This polynomial has two roots that satisfy

$$0 < \gamma_1 < 1 < \gamma_2.$$

Since $\tilde{y}_{t-1}$ is predetermined and $\tilde{y}_t$ is not, this condition ensures the system is saddle path stable. Moreover, letting $\gamma_H \equiv \gamma_1 \in (0,1)$ denote the stable eigenvalue, the solution converges to zero at a constant rate:

$$\tilde{y}_{t+h} = \gamma_H \tilde{y}_{t+h-1} = \gamma_H^{h+1} \tilde{y}_{t-1}.$$

This proves (8).

We can then solve for the value function over the region $y_{t-1} < y_H^*$ as

$$V_H = \sum_{h=0}^{\infty} -\beta^h \frac{\left(\tilde{y}_{t+h}\right)^2}{2} = \sum_{h=0}^{\infty} -\beta^h \frac{\left(\gamma_H^{h+1} \tilde{y}_{t-1}\right)^2}{2} = -\frac{\gamma_H^2}{1-\beta\gamma_H^2} \frac{\left(\tilde{y}_{t-1}\right)^2}{2}.$$

This establishes (9).

Note that $\frac{d\theta_H}{d\eta} > 0$ as long as $\frac{d\gamma_H}{d\eta} > 0$. To establish the latter inequality, let $\tilde{\eta} = \frac{\eta}{1-\eta}$ and note that γ_H is the solution to the following equation over the range $(0,1)$:

$$P(x, \tilde{\eta}, \phi) = x^2 - x(1 + \tilde{\eta} + \phi) + \tilde{\eta} = 0.$$

Implicitly differentiating with respect to $\tilde{\eta}$ and evaluating around $x = \gamma_H$, we obtain

$$\frac{dx}{d\tilde{\eta}} = -\left.\frac{\partial P/\partial \tilde{\eta}}{\partial P/\partial x}\right|_{x=\gamma_H} = \frac{1-\gamma_H}{1+\tilde{\eta}+\phi-2\gamma_H} > 0.$$

Here, the inequality follows since $\gamma_H < 1$ and $2\gamma_H < \gamma_1 + \gamma_2 = 1 + \tilde{\eta} + \phi$ (since γ_H is the smaller of the two roots γ_1, γ_2). Since $\tilde{\eta} = \frac{\eta}{1-\eta}$ is increasing in η, we also have $\frac{dx}{d\eta} > 0$. This completes the proof.

For completeness, consider also the case in which the initial output is above its potential $y_{t-1} \geq y_H^*$. In this case, the expansionary policy constraint does not bind and output converges to its potential immediately. The central bank sets the policy rate, $i_t = \rho - \frac{\eta}{1-\eta}(y_H^* - y_{t-1})$, and implements $y_t = y_H^*$. The interest rate constraint does not bind because $i_t > \rho$ and $\underline{i}_t(y_t) = \rho + \phi(y_t - y_H^*) = \rho$. Over this range ($y_{t-1} \geq y_H^*$), the value function satisfies $V_H(y_{t-1}) = 0$. □

Proof of Proposition 1. Suppose y_{t-1} is sufficiently high that the expansionary constraint does not bind in state L. Then, we can write the central bank's problem as

$$\begin{aligned} V_L &= \max_{y_t} -\frac{(y_t - y_L^*)^2}{2} + \beta\left((1-\lambda)V_L - \lambda\theta_H V_H(y_t)\right) \qquad \text{(A.1)} \\ \text{where } V_H(y_t) &= \begin{cases} -\frac{(y_t - y_H^*)^2}{2} & \text{if } y_t < y_H^* \\ 0 & \text{if } y_t \geq y_H^* \end{cases}. \end{aligned}$$

The second line combines the two cases analyzed in Lemma 1. This is a concave optimization problem. Any y_t that satisfies the first order optimality condition is an optimum. In the main text, we show that an interior solution (with $y_L < y_H^*$) satisfies the optimality condition in (11). Solving this condition, we obtain

$$y_L = \frac{y_L^* + \beta\lambda\theta_H y_H^*}{1+\beta\lambda\theta_H} \in (y_L^*, y_H^*).$$

It follows that the optimum output is interior and given by y_L. This also implies that solving problem ($A.1$) is equivalent to solving problem (10) in the main text.

Next consider the interest rate that implements this output level. The IS curve (1) implies

$$i_{t,L} = \rho + \lambda\left(Y_H(y_t) - y_{t,L}\right) + (1-\lambda)(y_{t+1,L} - y_{t,L}) - \frac{\eta}{1-\eta}(y_{t,L} - y_{t-1}).$$

After substituting $y_{t+1,L} = y_{t,L} = y_L$, we obtain (13).

We also need to verify that this rate does not violate the expansionary policy constraint in (2). Using Lemma 1, we obtain $Y_H(y_L) = \gamma_H y_L + (1-\gamma_H) y_H^*$. Substituting this into (13), we have

$$i_t = \rho + \lambda (1-\gamma_H)(y_H^* - y_L) - \frac{\eta}{1-\eta}(y_L - y_{t-1}).$$

Since $y_t = y_L$, the policy constraint holds as long as:

$$i_t \geq \rho + \phi (y_L - y_H^*).$$

Combining these observations, we verify that the constraint holds as long as the past output gap satisfies the condition in (12),

$$y_{t-1} \geq \overline{y}_L = y_L - \frac{1-\eta}{\eta}(\lambda(1-\gamma_H)+\phi)(y_H^* - y_L).$$

This completes the proof of the proposition. □

A.1.2. Omitted extensions

Proposition 1 characterizes the equilibrium when the past output is not too low so that the expansionary constraint does not bind in the low-supply state. We next characterize the equilibrium in the other case in which the expansionary constraint binds for at least one period. In this case, the output gradually converges to the target level y_L after finitely many periods (absent transition to the high-supply state). Once the output reaches y_L, the equilibrium is the same as in Proposition 1.

Proposition 3. *Suppose the economy is in the temporary supply shock state, $s = L$, with past output y_{t-1} that violates (12), that is: $y_{t-1} < \overline{y}_L$.Then the expansionary policy constraint binds in $s = L$ for at least one period. The initial interest rate is constrained, $i_t = \rho + \phi(y_t - y_H^*)$, and the initial output is below its unconstrained level, $Y_L(y_{t-1}) < y_L$. The output function $Y_L(y_{-1})$ is continuous, piecewise linear, and strictly increasing. Absent a transition to the high-supply state, output converges to the target level y_L after finitely many periods.*

Proof of Proposition 3. Suppose $y_{t-1} < \overline{y}_L$. Then, the interest rate is given by $i_t = \rho + \phi(y_t - y_H^*)$. Using (1) and $Y_H(y_t) = y_H^* + \gamma_H(y_t - y_H^*)$ [see Lemma 1], output follows the recursive equation:

$$\begin{aligned} y_t &= \eta y_{t-1} + (1-\eta)\left(\phi\left(y_H^* - y_t\right) + \lambda Y_H\left(y_t\right) + (1-\lambda)\, y_{t+1}\right) \\ &= \eta y_{t-1} + (1-\eta)\left(\phi\left(y_H^* - y_t\right) + \lambda\left(y_H^* + \gamma_H\left(y_t - y_H^*\right)\right) + (1-\lambda)\, y_{t+1}\right) \end{aligned} \tag{A.2}$$

After rearranging terms, this implies

$$y_t = \frac{\eta y_{t-1} + (1-\eta)\left(\phi y_H^* + \lambda\left(1-\gamma_H\right) y_H^* + (1-\lambda)\, y_{t+1}\right)}{1 + (1-\eta)\left(\phi - \lambda\gamma_H\right)}. \tag{A.3}$$

Let $\overline{y}_{L,-1} = y_L$ and $\overline{y}_{L,0} = \overline{y}_L < y_L$. We recursively define a sequence of cutoffs $\left\{\overline{y}_{L,k}\right\}$ as follows: given $\overline{y}_{L,k-1}$ and $\overline{y}_{L,k}$, let $\overline{y}_{L,k+1}$ denote the unique solution to:

$$\overline{y}_{L,k} = \frac{\eta \overline{y}_{L,k+1} + (1-\eta)\left(\phi y_H^* + \lambda\left(1-\gamma_H\right) y_H^* + (1-\lambda)\, \overline{y}_{L,k-1}\right)}{1 + (1-\eta)\left(\phi - \lambda\gamma_H\right)}.$$

Using $(A.3)$, the output function maps a lower cutoff into the higher cutoff:

$$Y_L\left(\overline{y}_{L,k+1}\right) = \overline{y}_{L,k}. \tag{A.4}$$

By induction, we can also show that the cutoffs satisfy $\overline{y}_{L,k+1} < \overline{y}_{L,k} - \frac{1-\eta}{\eta}\phi\left(y_H^* - y_L\right)$. Therefore, there exists K_L such that $\overline{y}_{L,K_L} < 0$. Then, the cutoffs $\left\{\overline{y}_{L,k}\right\}_{k=-1}^{K_L}$ cover the entire region $[0, y_L]$.

We can then define the output function recursively over the intervals $\left[\overline{y}_{L,k}, \overline{y}_{L,k-1}\right]$. Let $Y_{L,0}\left(y_{-1}\right) = y_L$ and define a sequence of functions with:

$$Y_{L,k}\left(y_{-1}\right) = \frac{\eta y_{-1} + (1-\eta)\left(\phi y_H^* + \lambda\left(1-\gamma_H\right) y_H^* + (1-\lambda)\, Y_{L,k-1}\left(Y_{L,k}\left(y_{-1}\right)\right)\right)}{1 + (1-\eta)\left(\phi - \lambda\gamma_H\right)}. \tag{A.5}$$

These functions are uniquely defined, linear, and strictly increasing over $[0, \overline{y}_L]$. Then, Eq. $(A.4)$ implies that for each interval the output function agrees with the corresponding function in the sequence

$$Y_L\left(y_{-1}\right) = Y_{L,k}\left(y_{-1}\right) \text{ for } y_{-1} \in \left[\overline{y}_{L,k}, \overline{y}_{L,k-1}\right].$$

In particular, the output function is the piecewise-linear function that maps each interval $\left[\overline{y}_{L,k}, \overline{y}_{L,k-1}\right]$ into the higher interval $\left[\overline{y}_{L,k-1}, \overline{y}_{L,k-2}\right]$. This implies that, absent transition to the high-supply state, output converges to the target level y_L after finitely many periods (at most $K_L + 1$ periods). This completes the proof of the proposition. □

A.2. Omitted results and proofs for Section 3

We first consider the case with the NKPC and present the formal results omitted from Section 3.1 along with their proofs. We then consider the case with an inertial Phillips curve analyzed in Section 3.2 and present the omitted results and proofs.

A.2.1. Overheating with a New-Keynesian Phillips Curve

Suppose inflation is determined according to the NKPC (17)

$$\pi_t = \kappa \left(y_t - y^*_{s_t} \right) + \beta E_t \left[\pi_{t+1} \right].$$

Let $\Pi_s (y_{t-1}), Y_s (y_{t-1}), V_s (y_{t-1})$ denote the inflation, the output, and the value function level when the current state is $s \in \{H, L\}$, and the most recent output is y_{t-1}.

We first characterize the equilibrium in the high supply state $s = H$. To state the result, we define the polynomial:

$$P(x) = x^3 - x^2 \left(\frac{1}{1-\eta} + \phi_y + \frac{1+\kappa}{\beta} \right) + x \left(\left(\frac{1}{1-\eta} + \phi_y \right) \frac{1}{\beta} + \phi_\pi \frac{\kappa}{\beta} + \frac{\eta}{1-\eta} \right) - \frac{1}{\beta} \frac{\eta}{1-\eta}. \tag{A.6}$$

Lemma 2. *Consider the setup with inflation determined by the NKPC* (17)*. Suppose the polynomial in* (A.6) *has exactly one stable root that satisfies* $\gamma_H \in (0,1)$ *(a sufficient condition is* $\phi_y (1-\beta) + (\phi_\pi - 1)\kappa > 0$ *and* $\beta \phi_\pi \leq 1$*). Suppose the economy has switched to the high-supply state,* $s = H$*, with past output* y_{t-1}*. Then, the output gap and the inflation functions are given by:*

$$Y_H (y_{t-1}) - y^*_H = \gamma_H (y_{t-1} - y^*_H) \tag{A.7}$$

$$\Pi_H (y_{t-1}) = \boldsymbol{\pi}_h (y_{t-1} - y^*_H) \text{ where } \boldsymbol{\pi}_h = \frac{\kappa \gamma_H}{1 - \beta \gamma_H}. \tag{A.8}$$

The output gap and inflation both converge to zero at a constant rate γ_H*. The value function is given by*

$$V_H (y_{t-1}) = -\theta_H \frac{(y_{t-1} - y^*_H)^2}{2} \text{ where } \theta_H = \frac{\gamma_H^2}{1 - \beta \gamma_H^2} \left(1 + \psi \left(\frac{\kappa}{1 - \beta \gamma_H} \right)^2 \right). \tag{A.9}$$

In the high-supply state, the equilibrium is determined by the IS curve, the NKPC, and the Taylor rule in (15). Under appropriate parametric conditions, the Taylor rule ensures that the output and inflation gaps converge to zero. As before, the convergence is not immediate. Due to inertial demand, *past* output, y_{t-1}, affects the output and inflation gaps in the high-supply state.

Next consider the equilibrium in the low supply state $s = L$. Using Lemma 2, the central bank solves the following version of problem (10):

$$\begin{aligned} V_L(y_{t-1}) &= \max_{y_t,\pi_t} -\frac{(y_t - y_L^*)^2}{2} - \psi\frac{\pi_t^2}{2} + \beta\left((1-\lambda)V_L(y_t) - \lambda\theta_H\frac{(y_t - y_H^*)^2}{2}\right) \qquad \text{(A.10)} \\ \text{s.t. } \pi_t &= \kappa(y_t - y_L^*) + \beta((1-\lambda)\Pi_L(y_t) + \lambda\boldsymbol{\pi}_H(y_t - y_H^*)). \end{aligned}$$

Here, the functions, $V_L(y_{t-1})$ and $\Pi_L(y_{t-1}) \equiv \pi_L$, are also both independent of y_{t-1}. Using this observation, the optimality condition is given by

$$\begin{aligned} y_L - y_L^* + \psi\frac{d\pi_t}{dy_t}\pi_L &= \beta\lambda\theta_H(y_H^* - y_L) \\ \text{where } \frac{d\pi_t}{dy_t} &= \kappa + \beta\lambda\boldsymbol{\pi}_H \\ \text{and } \pi_L &= \frac{\kappa(y_L - y_L^*) + \beta\lambda\boldsymbol{\pi}_H(y_L - y_H^*)}{1-\beta(1-\lambda)}. \end{aligned}$$

Here, the last line uses the NKPC to solve for the inflation in the low-supply state. Combining these observations, the optimum is given by the unique solution to:

$$\left[1 + \frac{\psi(\kappa + \beta\lambda\boldsymbol{\pi}_H)\kappa}{1-\beta(1-\lambda)}\right](y_L - y_L^*) = \beta\lambda\left[\theta_H + \frac{\psi(\kappa+\beta\lambda\boldsymbol{\pi}_H)\boldsymbol{\pi}_H}{1-\beta(1-\lambda)}\right](y_H^* - y_L). \qquad \text{(A.11)}$$

This leads to the following result, which generalizes Proposition 1 to this setting.

Proposition 4. *Consider the setup with inflation determined by the NKPC* (17) *and the parametric conditions described in Lemma 2. Suppose the economy is in the temporary supply shock state, $s = L$, with past output y_{t-1}. The central bank implements the constant output level $y_L \in (y_L^*, y_H^*)$ that solves* (A.11) *along with the constant inflation*

$$\pi_{t,L} = \pi_L \equiv \frac{\kappa(y_L - y_L^*) + \beta\lambda\boldsymbol{\pi}_H(y_L - y_H^*)}{1-\beta(1-\lambda)}. \qquad \text{(A.12)}$$

The associated real and nominal interest rates are given by

$$\begin{aligned} r_{t,L} &= \rho + \lambda(Y_H(y_L) - y_L) - \frac{\eta}{1-\eta}(y_L - y_{t-1}) \qquad \text{(A.13)} \\ i_{t,L} &= r_{t,L} + \lambda\Pi_H(y_L) + (1-\lambda)\pi_L. \qquad \text{(A.14)} \end{aligned}$$

The central bank chooses a level of output that induces positive output gaps in the current low-supply state (current overheating), $y_L > y_L^*$, and negative output gaps and disinflation after transition to the high-supply state (future demand shortages), $Y_H(y_L) < y_H^*$ and $\Pi_H(y_L) < 0$.

Comparing $(A.11)$ and (11) shows that inflation affects the policy trade-off in two ways. On the one hand, positive output gaps in the low-supply phase increase current inflation. This raises the cost of overheating, captured by the second term inside the brackets on the left side of $(A.11)$. On the other hand, negative output gaps expected in the *future* high-supply phase reduce *current* inflation. Since overheating helps shrink future gaps, this effect raises the benefit of overheating, captured by the second term inside the brackets on the right side of $(A.11)$. It follows that inflation affects the cost as well as the benefit of overheating, but it does not change the *qualitative* aspects of optimal policy.

The equilibrium with the NKPC has one subtlety: The central bank does not *necessarily* induce positive inflation in the low-supply state: that is, π_L is not necessarily positive (even though $y_L > y_L^*$). This effect is driven by the forward-looking term in the NKPC, together with the fact that the economy experiences disinflation after transition to the high-supply state, $\boldsymbol{\pi}_H(y_L - y_H^*) < 0$ (see $(A.12)$). Nonetheless, in our simulations this effects is typically weak and the central bank implements $\pi_L > 0$ along with $y_L > y_L^*$.

Proof of Lemma 2. Combining the NKPC, the IS curve, and the Taylor policy rule, the dynamic system that characterizes the equilibrium is given by

$$\begin{aligned} y_t &= \eta y_{t-1} + (1-\eta)\left(-\phi_y\left(y_t - y_H^*\right) - \phi_\pi \pi_t + E_t\left[\pi_{t+1}\right] + E_t\left[y_{t+1}\right]\right) \\ \pi_t &= \kappa\left(y_t - y_H^*\right) + \beta E_t\left[\pi_{t+1}\right]. \end{aligned}$$

We drop the expectations since there is no (residual) uncertainty. Let $\tilde{y}_t = y_t - y_H^*$ denote the output gap. Then, we can rewrite the system as

$$\begin{aligned} \tilde{y}_t &= \eta \tilde{y}_{t-1} + (1-\eta)\left(-\phi_y \tilde{y}_t - \phi_\pi \pi_t + \pi_{t+1} + \tilde{y}_{t+1}\right) \\ \pi_t &= \kappa \tilde{y}_t + \beta \pi_{t+1}. \end{aligned}$$

In matrix notation, we have

$$\begin{bmatrix} \tilde{y}_{t+1} \\ \pi_{t+1} \\ \tilde{y}_t \end{bmatrix} = M \begin{bmatrix} \tilde{y}_t \\ \pi_t \\ \tilde{y}_{t-1} \end{bmatrix} \text{ where } M = \begin{bmatrix} \frac{1}{1-\eta} + \phi_y + \frac{\kappa}{\beta} & \phi_\pi - \frac{1}{\beta} & -\frac{\eta}{1-\eta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

The characteristic polynomial of the matrix M is

$$\begin{aligned}
P(x) &= -\det\left(\begin{bmatrix} \frac{1}{1-\eta}+\phi_y+\frac{\kappa}{\beta}-x & \phi_\pi-\frac{1}{\beta} & -\frac{\eta}{1-\eta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta}-x & 0 \\ 1 & 0 & -x \end{bmatrix}\right) \\
&= x^3-x^2\left(\frac{1}{1-\eta}+\phi_y+\frac{1+\kappa}{\beta}\right)+x\left(\left(\frac{1}{1-\eta}+\phi_y\right)\frac{1}{\beta}+\phi_\pi\frac{\kappa}{\beta}+\frac{\eta}{1-\eta}\right)-\frac{1}{\beta}\frac{\eta}{1-\eta}.
\end{aligned}$$

This is the polynomial we define in $(A.6)$. We assume the parameters are such that this polynomial has a single stable root that satisfies $\gamma_H \in (0,1)$. The conditions in the propositions are sufficient (but not necessary). To check sufficiency, note that we have $P(0) < 0$. We also have

$$P(1) = \frac{\phi_y(1-\beta)+(\phi_\pi-1)\kappa}{\beta} > 0$$

in view of the first part of the sufficient condition, $\phi_y(1-\beta)+(\phi_\pi-1)\kappa$. We also have

$$P\left(\frac{1}{\beta}\right) = -\frac{\kappa}{\beta^3}+\phi_\pi\frac{\kappa}{\beta^2} \leq 0$$

in view of the second part of the sufficient condition, $\beta\phi_\pi \leq 1$. Thus, with these conditions the roots of the polynomial satisfy

$$0 < \gamma_1 < 1 < \gamma_2 \leq \frac{1}{\beta} \leq \gamma_3.$$

In particular, the polynomial has exactly one stable root that satisfies $\gamma_H \equiv \gamma_1 \in (0,1)$.

Since $\tilde{y}_{t-1}$ is predetermined but $\tilde{y}_t, \pi_t$ are not, the system is saddle path stable. Moreover, the solution converges to zero at the constant rate $\gamma_H \in (0,1)$, that is:

$$\begin{aligned}
\tilde{y}_{t+h} &= \gamma_H\tilde{y}_{t+h-1} = \gamma_H^{h+1}\tilde{y}_{t-1} \\
\tilde{\pi}_{t+h} &= \gamma_H\tilde{\pi}_{t+h-1}.
\end{aligned}$$

This establishes $(A.7)$. To solve for the initial inflation, we use the NKPC to obtain

$$\pi_t = \sum_{h=0}^{\infty}\beta^h\kappa\tilde{y}_{t+h} = \sum_{h=0}^{\infty}\beta^h\gamma_H^h\kappa\gamma_H\tilde{y}_{t-1} = \frac{\kappa\gamma_H\tilde{y}_{t-1}}{1-\beta\gamma_H}.$$

This establishes $(A.8)$.

Finally, we calculate the value function as:

$$\begin{aligned}
V_H &= -\sum_{h=0}^{\infty} \beta^h \left(\frac{\tilde{y}_{t+h}^2}{2} + \psi \frac{{\pi_{t+h}}^2}{2} \right) \\
&= -\sum_{h=0}^{\infty} \left(\beta \gamma_H^2\right)^h \left(\frac{{\tilde{y}_t}^2}{2} + \psi \frac{{\pi_t}^2}{2} \right) \\
&= -\frac{1}{1-\beta\gamma_H^2} \left(\gamma_H^2 + \psi \left(\frac{\kappa \gamma_H}{1-\beta\gamma_H} \right)^2 \right) \frac{{\tilde{y}_{t-1}}^2}{2}.
\end{aligned}$$

Here, the second line uses the fact that inflation and the output gap converge to zero at rate $\gamma_H \in (0,1)$ and the last line substitutes $\tilde{y}_t$ and π_t in terms of the past output gap $\tilde{y}_{t-1}$. This establishes $(A.9)$ and completes the proof of the lemma. □

Proof of Proposition 4. The proof is mostly presented earlier in the section. To solve for the real interest rate, note that the IS curve (14) implies

$$r_{t,L} = \rho + \lambda \left(Y_H \left(y_t \right) - y_{t,L} \right) + (1-\lambda) \left(y_{t+1,L} - y_{t,L} \right) - \frac{\eta}{1-\eta} \left(y_{t,L} - y_{t-1} \right).$$

After substituting $y_{t,L} = y_{t+1,L} = y_L$, this implies $(A.13)$. The nominal interest rate is then

$$i_{t,L} = r_{t,L} + E_t \left[\pi_{t+1} \right] = r_{t,L} + \lambda \Pi_H \left(y_L \right) + (1-\lambda) \pi_L.$$

This establishes $(A.14)$ and completes the proof. □

A.2.2. Overheating with an inertial Phillips Curve

Suppose inflation is determined according to the inertial Phillips curve (18)

$$\pi_t = \kappa \left(y_t - y_{s_t}^* \right) + b \pi_{t-1}.$$

Suppose also that the parameters satisfy the simplifying assumptions described in the main text. We first state the lemma that characterizes the equilibrium in the high supply-state $s = H$. We then present the proof of Proposition 2, which characterizes the optimal policy in the low-supply state $s = L$.

Lemma 3. *Consider the setup with an inertial Phillips curve. Suppose the parameters satisfy $\phi_\pi = b$ and $\phi_y > \kappa$.*

Suppose the economy has switched to the high-supply state, $s = H$, with past output y_{t-1}. Let $\gamma_H \in (0,1)$ denote the smaller root of the polynomial $P(x) = (1+\kappa) x^2 -$

$\left(\frac{1}{1-\eta}+\phi_y\right)x+\frac{\eta}{1-\eta}$. *Then the output gap and the inflation functions are given by:*

$$Y_H\left(y_{t-1},\pi_{t-1}\right)-y_H^* = \gamma_H\left(y_{t-1}-y_H^*\right) \tag{A.15}$$
$$\Pi_H\left(y_{t-1},\pi_{t-1}\right) = \kappa\gamma_H\left(y_{t-1}-y_H^*\right)+b\pi_{t-1}. \tag{A.16}$$

The value function in the first period after transition (with $s_{t-1}=L$) is given by:

$$V_H\left(y_{t-1,}\pi_{t-1}\right)=-\frac{\theta_H}{2}\left(y_{t-1}-y_H^*\right)^2-\frac{\Psi_H}{2}\pi_{t-1}^2-\mathcal{I}_H\left(y_{t-1}-y_H^*\right)\pi_{t-1}, \tag{A.17}$$

where the coefficients $\Psi_H,\mathcal{I}_H,\theta_H$ are given by

$$\begin{aligned}\Psi_H &= \frac{b^2}{1-\beta b^2}\psi \\ \mathcal{I}_H &= \frac{\gamma_H b}{1-\beta\gamma_H b}\left(\psi+\beta\Psi_H\right)\kappa \\ \theta_H &= \frac{\gamma_H^2}{1-\beta\gamma_H^2}\left(1+\left(\psi+\beta\Psi_H\right)\kappa^2+2\beta\mathcal{I}_H\kappa\right).\end{aligned} \tag{A.18}$$

Proof of Lemma 3. Combining the inertial Phillips curve, the IS curve, and the Taylor policy rule, the dynamic system that characterizes the equilibrium is given by

$$\begin{aligned}y_t &= \eta y_{t-1}+(1-\eta)\left(-\phi_y\left(y_t-y_H^*\right)-\phi_\pi\pi_t+E_t\left[\pi_{t+1}\right]+E_t\left[y_{t+1}\right]\right) \\ \pi_t &= \kappa\left(y_t-y_H^*\right)+b\pi_{t-1}.\end{aligned}$$

We drop the expectations since there is no (residual) uncertainty. Let $\tilde{y}_t=y_t-y_H^*$ denote the output gap. Then, we can rewrite the system as

$$\begin{aligned}\tilde{y}_t &= \eta\tilde{y}_{t-1}+(1-\eta)\left(-\phi_y\tilde{y}_t-\phi_\pi\pi_t+\pi_{t+1}+\tilde{y}_{t+1}\right) \\ \pi_t &= \kappa\tilde{y}_t+b\pi_{t-1}.\end{aligned}$$

After rewriting the second equation and substituting the first equation, we obtain

$$\begin{aligned}\tilde{y}_{t+1} &= \frac{1}{1+\kappa}\left(\frac{\tilde{y}_t-\eta\tilde{y}_{t-1}}{1-\eta}+\phi_y\tilde{y}_t+\left(\phi_\pi-b\right)\pi_t\right) \\ \pi_t &= \kappa\tilde{y}_t+b\pi_{t-1}.\end{aligned}$$

This system is in general complicated, because there are two state variables $\tilde{y}_{t-1},\pi_{t-1}$. However, in the special case $\phi_\pi=b$, inflation drops out of the first equation and the system becomes block-recursive. In particular, the output gap satisfies the difference equation:

$$\tilde{y}_{t+1} = \frac{1}{1+\kappa}\left(\left(\frac{1}{1-\eta} + \phi_y\right)\tilde{y}_t - \frac{\eta}{1-\eta}\tilde{y}_{t-1}\right).$$

This is a standard difference equation with the characteristic polynomial given by

$$P(x) = (1+\kappa)x^2 - \left(\frac{1}{1-\eta} + \phi_y\right)x + \frac{\eta}{1-\eta} = 0.$$

Note that $P(0) > 0$ and $P(1) < 0$ in view of the parametric condition $\phi_y > \kappa$. Thus, the polynomial has a single stable root that satisfies $\gamma_H \in (0,1)$. It follows that the output gap converges to zero at a constant rate

$$\tilde{y}_{t+h} = \gamma_H \tilde{y}_{t+h-1} = \gamma_H^{h+1}\tilde{y}_{t-1}.$$

This establishes $(A.15)$. Substituting $\tilde{y}_t$ into the inertial Phillips curve, we solve for inflation as:

$$\pi_t = \kappa\tilde{y}_t + b\pi_{t-1} = \kappa\gamma_H\tilde{y}_{t-1} + b\pi_{t-1}.$$

This establishes $(A.16)$.

Finally, consider the value function. The value function satisfies the recursive relation

$$\begin{aligned} V_H(y_{t-1,}\pi_{t-1}) &= -\frac{1}{2}\tilde{y}_t^2 - \frac{\psi}{2}\pi_t^2 + \beta V_H(y_t,\pi_t) \\ \text{where } \tilde{y}_t &= \gamma_H\tilde{y}_{t-1} \\ \text{and } \pi_t &= \kappa\gamma_H\tilde{y}_{t-1} + b\pi_{t-1}. \end{aligned}$$

We conjecture that the value function has the quadratic functional form in $(A.17)$. After substituting the functional form, and dropping the H subscripts, we obtain:

$$\begin{aligned} -\theta\tilde{y}_{t-1}^2 - \Psi\pi_{t-1}^2 - 2\mathcal{I}\tilde{y}_{t-1}\pi_{t-1} &= -\tilde{y}_t^2 - \psi\pi_t^2 + \beta\left(-\theta\tilde{y}_t^2 - \Psi\pi_t^2 - 2\mathcal{I}\tilde{y}_t\pi_t\right) \\ &= -(1+\beta\theta)\tilde{y}_t^2 - (\psi+\beta\Psi)\pi_t^2 - 2\beta\mathcal{I}\tilde{y}_t\pi_t \\ &= \begin{bmatrix} -(1+\beta\theta)(\gamma\tilde{y}_{t-1})^2 \\ -(\psi+\beta\Psi)(\kappa\gamma\tilde{y}_{t-1} + b\pi_{t-1})^2 \\ -2\beta\mathcal{I}(\gamma\tilde{y}_{t-1})(\kappa\gamma\tilde{y}_{t-1} + b\pi_{t-1}) \end{bmatrix} \\ &= \begin{bmatrix} -(1+\beta\theta + (\psi+\beta\Psi)\kappa^2 + 2\beta\mathcal{I}\kappa)\gamma^2\tilde{y}_{t-1}^2 \\ -(\psi+\beta\Psi)b^2\pi_{t-1}^2 \\ -(2\beta\mathcal{I} + 2(\psi+\beta\Psi)\kappa)\gamma b\tilde{y}_{t-1}\pi_{t-1} \end{bmatrix}. \end{aligned}$$

Here, the third line substitutes $\tilde{y}_t, \pi_t$ in terms of $\tilde{y}_{t-1}, \pi_{t-1}$ and the last line collects terms. After matching the coefficients for the terms $\tilde{y}_{t-1}^2, \pi_{t-1}^2, \tilde{y}_{t-1}\pi_{t-1}$, we obtain

$$\begin{aligned}
\theta &= \left(1 + \beta\theta + (\psi + \beta\Psi)\kappa^2 + 2\beta\mathcal{I}\kappa\right)\gamma^2 \\
\Psi &= (\psi + \beta\Psi) b^2 \\
\mathcal{I} &= (\beta\mathcal{I} + (\psi + \beta\Psi)\kappa)\gamma b.
\end{aligned}$$

Solving these equations and substituting back the H subscripts, we establish $(A.18)$, completing the proof. □

Proof of Proposition 2. Consider problem (19), which we replicate here

$$\begin{aligned}
V_L(y_{t-1}, \pi_{t-1}) &= \max_{y_t, \pi_t} -\frac{(y_t - y_L^*)^2}{2} - \psi\frac{\pi_t^2}{2} + \beta\left((1-\lambda)V_L(y_t, \pi_t) + \lambda V_H(y_t, \pi_t)\right) \\
\pi_t &= \kappa(y_t - y_L^*) + b\pi_{t-1}
\end{aligned}$$

In this case, the value function $V_L(y_{t-1}, \pi_{t-1})$ depends on past inflation, π_{t-1}, but it is still independent of past output, y_{t-1}. Using this observation, we can write the problem as

$$\begin{aligned}
V_L(\pi_{t-1}) &= \max_{\pi_t} F(\pi_{t-1}, \pi_t) + \beta(1-\lambda)V_L(\pi_t) \\
\text{where } F(\pi_{t-1}, \pi_t) &= -\frac{(\pi_t - b\pi_{t-1})^2}{2\kappa^2} - \psi\frac{\pi_t^2}{2} + \beta\lambda V_H\left(y_L^* + \frac{\pi_t - b\pi_{t-1}}{\kappa}, \pi_t\right).
\end{aligned}$$

This is a standard dynamic optimization problem. The first order condition is given by the Euler equation:

$$\frac{\partial F(\pi_{t-1}, \pi_t)}{\partial \pi_t} + \beta(1-\lambda)\frac{\partial F(\pi_t, \pi_{t+1})}{\partial \pi_t} = 0. \tag{A.19}$$

We calculate the derivatives as:

$$\begin{aligned}
\frac{\partial F(\pi_{t-1}, \pi_t)}{\partial \pi_t} &= -\frac{(\pi_t - b\pi_{t-1})}{\kappa^2} - \psi\pi_t + \beta\lambda\left(\frac{\partial V_H(y_t, \pi_t)}{\partial y_t}\frac{1}{\kappa} + \frac{\partial V_H(y_t, \pi_t)}{\partial \pi_t}\right), \\
\frac{\partial F(\pi_t, \pi_{t+1})}{\partial \pi_t} &= \frac{b}{\kappa}\left(\frac{\pi_{t+1} - b\pi_t}{\kappa} - \beta\lambda\frac{\partial V_H(y_{t+1}, \pi_{t+1})}{\partial y_{t+1}}\right).
\end{aligned}$$

Combining these observations, and using $y_t - y_L^* = \frac{\pi_t - b\pi_{t-1}}{\kappa}$, the Euler equation $(A.19)$

implies

$$
\begin{aligned}
& y_t - y_L^* + \kappa\psi\pi_t - \beta\lambda\left(\frac{\partial V_H(y_t,\pi_t)}{\partial y_t} + \kappa\frac{\partial V_H(y_t,\pi_t)}{\partial \pi_t}\right) \\
= \;& \beta(1-\lambda)\,b\left(y_{t+1} - y_L^* - \beta\lambda\frac{\partial V_H(y_{t+1},\pi_{t+1})}{\partial y_{t+1}}\right).
\end{aligned}
$$

We next use Eq. $(A.17)$ to calculate the partial derivatives of $V_H(y_t,\pi_t)$ as follows:

$$
\begin{aligned}
\frac{\partial V_H(y_t,\pi_t)}{\partial y_t} &= -\theta_H(y_t - y_H^*) - \mathcal{I}_H\pi_t \\
&= -\theta_H(y_t - y_L^*) - \mathcal{I}_H\pi_t + \theta_H(y_H^* - y_L^*) \\
&\quad \text{and} \\
\frac{\partial V_H(y_t,\pi_t)}{\partial \pi_t} &= -\Psi_H\pi_t - \mathcal{I}_H(y_t - y_H^*) \\
&= -\Psi_H\pi_t - \mathcal{I}_H(y_t - y_L^*) + \mathcal{I}_H(y_H^* - y_L^*).
\end{aligned}
$$

Substituting these expressions into the Euler equation, we obtain

$$
\begin{aligned}
& y_t - y_L^* + \kappa\psi\pi_t + \beta\lambda\begin{pmatrix} (\theta_H + \kappa\mathcal{I}_H)(y_t - y_L^*) \\ +(\mathcal{I}_H + \kappa\Psi_H)\pi_t \\ -(\theta_H + \kappa\mathcal{I}_H)(y_H^* - y_L^*) \end{pmatrix} \\
= \;& \beta(1-\lambda)\,b\begin{pmatrix} (1+\beta\lambda\theta_H)(y_{t+1} - y_L^*) \\ +\beta\lambda\mathcal{I}_H\pi_{t+1} \\ -\beta\lambda\theta_H(y_H^* - y_L^*) \end{pmatrix}.
\end{aligned}
$$

Rearranging terms, we have

$$
\begin{aligned}
A(y_t - y_L^*) + B\pi_t &= C(y_{t+1} - y_L^*) + D\pi_{t+1} + E(y_H^* - y_L^*) \text{ where} \\
A &= 1 + \beta\lambda(\theta_H + \kappa\mathcal{I}_H) \\
B &= \kappa\psi + \beta\lambda(\mathcal{I}_H + \kappa\Psi_H) \\
C &= \beta(1-\lambda)\,b(1+\beta\lambda\theta_H) \\
D &= \beta(1-\lambda)\,b\beta\lambda\mathcal{I}_H \\
E &= \beta\lambda[\theta_H + \kappa\mathcal{I}_H - \beta(1-\lambda)\,b\theta_H].
\end{aligned}
$$

Here $A, B, C, D, E > 0$ are the derived parameters in (20). Note also that $A > C$ and $B > D$.

Combining the equation for y_t with the NKPC, we obtain the system:

$$\begin{aligned} A\left(y_t - y_L^*\right) + B\pi_t &= C\left(y_{t+1} - y_L^*\right) + D\pi_{t+1} + E\left(y_H^* - y_L^*\right) \\ \pi_t &= \kappa\left(y_t - y_L^*\right) + b\pi_{t-1}. \end{aligned} \tag{A.20}$$

We next calculate the steady-state, denoted by $(\overline{y}_L, \overline{\pi}_L)$. From the second equation, the steady-state inflation satisfies $\overline{\pi}_L = \frac{\kappa\left(\overline{y}_L - y_L^*\right)}{1-b}$. Substituting this into the first equation, we solve for the steady-state output as:

$$\overline{y}_L - y_L^* = \frac{E\left(y_H^* - y_L^*\right)}{A - C + (B - D)\frac{\kappa}{1-b}}.$$

Note that $\overline{y}_L > y_L^*$ (and thus $\overline{\pi}_L > 0$) since $E > 0, A > C$, and $B > D$. This establishes $(21 - 22)$.

We next characterize the transition dynamics away from the steady-state. Let $\tilde{y}_t = y_t - \overline{y}_L$ and $\tilde{\pi}_t = \pi_t - \overline{\pi}_t$ denote the deviations from the steady state (these variables are different than the output and inflation gaps). With this notation, we write $(A.20)$ as

$$\begin{aligned} A\tilde{y}_t + B\tilde{\pi}_t &= C\tilde{y}_{t+1} + D\tilde{\pi}_{t+1} \\ \tilde{\pi}_t &= \kappa\tilde{y}_t + b\tilde{\pi}_{t-1}. \end{aligned}$$

After substituting $\tilde{\pi}_{t+1} = \kappa\tilde{y}_{t+1} + b\tilde{\pi}_t$ and $\tilde{\pi}_t = \kappa\tilde{y}_t + b\tilde{\pi}_{t-1}$ in the first equation, we can write this system as

$$\begin{aligned} \left(C + D\kappa\right)\tilde{y}_{t+1} &= \left(A + \left(B - Db\right)\kappa\right)\tilde{y}_t + \left(B - Db\right)b\tilde{\pi}_{t-1} \\ \tilde{\pi}_t &= \kappa\tilde{y}_t + b\tilde{\pi}_{t-1}. \end{aligned}$$

In matrix notation, we have

$$\begin{bmatrix} \tilde{y}_{t+1} \\ \tilde{\pi}_t \end{bmatrix} = \begin{bmatrix} \frac{A+(B-Db)\kappa}{C+D\kappa} & \frac{(B-Db)b}{C+D\kappa} \\ \kappa & b \end{bmatrix} \begin{bmatrix} \tilde{y}_t \\ \tilde{\pi}_{t-1} \end{bmatrix}.$$

The characteristic polynomial is given by

$$\begin{aligned} P(x) &= \det\left(\begin{bmatrix} \frac{A+(B-Db)\kappa}{C+D\kappa} - x & \frac{(B-Db)b}{C+D\kappa} \\ \kappa & b-x \end{bmatrix}\right) \\ &= x^2 - \left(\frac{A+(B-Db)\kappa}{C+D\kappa} + b\right)x + \frac{Ab}{C+D\kappa} \\ &= x^2 - \frac{A+B\kappa+bC}{C+D\kappa}x + \frac{Ab}{C+D\kappa}. \end{aligned}$$

Note that $P(0) > 0$ and

$$\begin{aligned} P(b) &= b^2 - \frac{A+B\kappa+bC}{C+D\kappa}b + \frac{Ab}{C+D\kappa} \\ &= -\frac{(B-bD)}{C+D\kappa}\kappa b < 0. \end{aligned}$$

This implies there is a stable root that satisfies $\gamma_L \equiv \gamma_1 \in (0, b)$. We also claim that $P(1) < 0$, which holds iff

$$P(1) = \frac{(C-A)(1-b) + (D-B)\kappa}{C+D\kappa} < 0.$$

The inequality holds because $A > C$ and $B > D$. This inequality implies that there is also an unstable root that satisfies $\gamma_2 > 1$.

These observations prove that the system is saddle path stable. Starting with the inflation deviation $\tilde{\pi}_{t-1}$, both the output deviation and inflation deviation converge to zero at a constant rate γ_L

$$\tilde{y}_{t+1} = \gamma_L \tilde{y}_t \text{ and } \tilde{\pi}_t = \gamma_L \tilde{\pi}_{t-1} \text{ for each } t.$$

To characterize the output in terms of past inflation, note the Phillips curve implies

$$\tilde{\pi}_t = \gamma_L \tilde{\pi}_{t-1} = \kappa \tilde{y}_t + b\tilde{\pi}_{t-1} \Longrightarrow \tilde{y}_t = -\left(\frac{b-\gamma_L}{\kappa}\right)\tilde{\pi}_{t-1}.$$

This establishes $(23-24)$.

Finally, we calculate the interest rate the central bank needs to set to implement the optimal output and inflation path. First consider the real interest rate. Using the IS curve (14),

$$\begin{aligned} r_t &= \rho + E_t\left[y_{t+1}\right] - \frac{y_t}{1-\eta} + \frac{\eta}{1-\eta} y_{t-1} \\ &= \rho + \lambda Y_H\left(y_t\right) + \left(1-\lambda\right) y_{t+1} - \frac{y_t}{1-\eta} + \frac{\eta}{1-\eta} y_{t-1} \\ &= \rho + \lambda\left(Y_H\left(y_t\right) - y_t\right) + \left(1-\lambda\right)\left(y_{t+1} - y_t\right) - \frac{\eta}{1-\eta}\left(y_t - y_{t-1}\right). \end{aligned} \tag{A.21}$$

Here, y_{t+1} denote the future output if the economy stays in the low-supply state (characterized earlier). Likewise, the nominal interest rate is given by

$$\begin{aligned} i_t &= E_t\left[\pi_{t+1}\right] + r_t \\ &= \begin{array}{c} \lambda \Pi_H\left(y_t\right) + \left(1-\lambda\right)\pi_{t+1} + \\ \rho + \lambda\left(Y_H\left(y_t\right) - y_t\right) + \left(1-\lambda\right)\left(y_{t+1} - y_t\right) - \frac{\eta}{1-\eta}\left(y_t - y_{t-1}\right). \end{array} \end{aligned} \tag{A.22}$$

Here, π_{t+1} is the inflation if the economy stays in state L. This completes the proof. □

B. Alternative model with a ZLB constraint

In the main text, we formalize the expansionary policy constraints by assuming that the central bank is subject to a Taylor-rule type lower bound on the nominal interest rate (see (2)). In this appendix, we analyze an alternative model in which the central bank is subject to a zero lower bound (ZLB) constraint. We show that our main result holds also in this more realistic scenario. We relegate the proofs to the end of the appendix.

Environment with a ZLB constraint. Consider the setup in Section 1 with the difference that the lower bound on the interest rate is zero [cf. (2)]

$$i_t \geq \underline{i}_t\left(y_t\right) = 0. \tag{B.1}$$

As before, the central bank sets policy without commitment, and it minimizes the present discounted value of quadratic output gaps. We can then formulate the policy problem recursively as

$$\begin{aligned} V_{s_t}\left(y_{t-1}\right) &= \max_{i_t, y_t} -\frac{\left(y_t - y^*_{s_t}\right)^2}{2} + \beta E_t\left[V_{s_{t+1}}\left(y_t\right)\right] \\ \text{s.t. } y_t &= \eta y_{t-1} + \left(1-\eta\right)\left(-\left(i_t - \rho\right) + E_t\left[Y_{s_{t+1}}\left(y_t\right)\right]\right) \\ i_t &\geq 0. \end{aligned} \tag{B.2}$$

As in our main setup, $Y_s(y_{-1})$ and $V_s(y_{-1})$ denote the central bank's optimal output choice and optimal value, respectively, when the current state is $s \in \{H, L\}$ and the most recent output is y_{-1}. The central bank takes its future interest rate decisions and output choices as given and sets the current interest rate and output to minimize quadratic gaps, subject to the inertial IS curve and the ZLB constraint.

Overheating with a ZLB constraint. Recall that, in the first-best benchmark without expansionary policy constraints, the central bank sets a relatively low interest rate in the first period after transition to the high-supply state [see (5)]. We assume the parameters are such that this interest rate is negative: In the first-best benchmark, the ZLB constraint is violated in the first period after transition. Thus, a central bank that is subject to a ZLB constraint cannot achieve zero gaps in all periods and states.

Assumption 1. $\rho - \frac{\eta}{1-\eta}(y_H^* - y_L^*) < 0$.

Our first result characterizes the equilibrium after the economy transitions to the absorbing state $s = H$.

Lemma 4. *Suppose Assumption 1 holds and the economy has switched to the high-supply state, $s = H$, with past output $y_{-1} \equiv y_{t-1}$. Let $\overline{y}_H = y_H^* - \frac{1-\eta}{\eta}\rho \in (y_L^*, y_H^*)$.*

- *If $y_{-1} \geq \overline{y}_H$, then the ZLB constraint does not bind and the central bank can achieve zero gaps, $Y_H(y_{-1}) = y_H^*$ and $V_H(y_{-1}) = 0$. The interest rate is given by*

$$i_{t,H} = \rho - \frac{\eta}{1-\eta}(y_H^* - y_{t-1}). \tag{B.3}$$

- *If $y_{-1} < \overline{y}_H$, then the ZLB constraint binds and the output gap is negative for at least one period, $Y_H(y_{-1}) < y_H^*$ and $V_H(y_{-1}) < 0$. The output and the value functions are characterized in the proof and satisfy the following:*
  - *$Y_H(y_{-1}) \geq y_{-1}$ is continuous, strictly increasing, and piecewise linear (it is linear except for a finite number of kink points). Output converges to the efficient level y_H^* after finitely many periods.*
  - *$V_H(y_{-1})$ is continuous, strictly concave and increasing, and piecewise differentiable. At the ZLB cutoff, $y_{-1} = \overline{y}_H$, the value function is differentiable with a zero derivative, $\frac{dV_H(\overline{y}_H)}{dy_{-1}} = 0$.*

Lemma 4 says that, after the supply recovers, the ZLB constraint binds when output is sufficiently low relative to potential. Technically, the ZLB constraint introduces a finite number of kink points into the solution, but the optimal output and the value function satisfy intuitive properties. Starting with a sufficiently low output level, the output gradually recovers and eventually reaches its potential level, y_H^*. Similar to our baseline analysis in Lemma 1, a greater past output increases the current output as well as the value function (over the relevant range $y_{-1} < \overline{y}_H$).

We next establish the analogue of our main result (Proposition 1) in this alternative setup with a ZLB constraint. Consider the optimal policy in the temporary low-supply state, $s = L$. For now, suppose past output y_{-1} is high enough so that the ZLB constraint does not bind in the low-supply state (we consider the case with a binding ZLB in this state subsequently). Then, we can rewrite problem (3) as

$$V_L\left(y_{-1}\right) = \max_y -\frac{\left(y - y_L^*\right)^2}{2} + \beta\left(\left(1-\lambda\right) V_L\left(y\right) + \lambda V_H\left(y\right)\right). \tag{B.4}$$

The value function in the future low-supply state does not depend on past output, $V_L\left(y\right) \equiv V_L$ (as long as the ZLB does not bind, which we will verify). The value function in the future high-supply state $V_H\left(y\right)$ is concave. Therefore, the optimality condition is

$$y - y_L^* = \beta\lambda\delta; \quad \text{where } \delta \in \nabla V_H\left(y\right). \tag{B.5}$$

Here, δ is a subgradient of the value function. It is equal to the derivative, except possibly at kink points, where it lies in an interval between the left and the right derivatives. Let y_L denote the optimum that solves $(B.5)$.

Eq. $(B.5)$ establishes our main result with the ZLB constraint: the (unique) optimum satisfies $y_L \in \left(y_L^*, \overline{y}_H\right)$ and thus $y_L > y_L^*$ and $Y_H\left(y_L\right) < Y_H\left(\overline{y}_H\right) = y_H^*$. *In the temporary low-supply state, the central bank chooses a level of output that induces positive output gaps in the current low-supply state (current overheating), and negative output gaps after transition to the high-supply state (future demand shortages).* The intuition is the same as in Section 2. As before, the central bank overheats the current output to accelerate the recovery in future periods after transition to high supply.

We can now solve for the associated interest rate:

$$i_t = \rho + \lambda\left(Y_H\left(y_L\right) - y_L\right) - \frac{\eta}{1-\eta}\left(y_L - y_{t-1}\right). \tag{B.6}$$

Recall that $Y_H\left(y_L\right) > y_L$. This shows that the ZLB constraint does not bind in the low-supply state ($i_t > \rho > 0$) when past output is already equal to the target level, $y_{t-1} = y_L$. However, there is a sufficiently low level of past output (y_{t-1}) below which the ZLB constraint binds in the low-supply state for at least one period:

$$\overline{y}_L = y_L - \frac{1-\eta}{\eta}\left(\rho + \lambda\left(Y_H\left(y_L\right) - y_L\right)\right). \tag{B.7}$$

The following proposition summarizes the discussion in this appendix and completes the characterization of equilibrium in $s = L$.

Proposition 5. *Suppose Assumption 1 holds and the economy is in the temporary supply shock state, $s = L$, with past output $y_{-1} \equiv y_{t-1}$. Let $\overline{y}_L$ be given by (B.7).*

- *If $y_{-1} \geq \overline{y}_L$, then the ZLB constraint does not bind in $s = L$ and the central bank chooses the output level y_L that is the unique solution to (B.5). The output choice satisfies $y_L \in (y_L^*, \overline{y}_H)$. In the temporary supply shock state, the economy experiences overheating, $y_L > y_L^*$. At the transition to the high-supply state, the economy experiences demand shortages, $Y_H(y_L) < Y_H(\overline{y}_H) = y_H^*$. The interest rate in $s = L$ is given by (B.6).*

- *If $y_{-1} < \overline{y}_L$, then the ZLB constraint binds in $s = L$ for at least one period. The initial interest rate is zero, $i_t = 0$, and the initial output is below its unconstrained level, $Y_L(y_{-1}) < y_L$. The output function $Y_L(y_{-1})$ (characterized in the appendix) is continuous and strictly increasing. Absent a transition to the high-supply state, output converges to the target level y_L after finitely many periods.*

Numerical illustration. Figure B.1 simulates the equilibrium for a numerical example. The figure resembles Figure 1 in the main text. The solid lines plot the equilibrium with the ZLB constraint and illustrate the main result. As before, the optimal policy induces *overheating* in the low-supply state. The policy achieves this by *cutting* the rate aggressively in the earlier periods while the economy is in the low-supply state. In fact, in this simulation the policy runs into the ZLB constraint in the first period. Once the policy brings the output in the low-supply state to a target level above the potential (denoted by $y_L > y_L^*$ in the figure), it raises the interest rate to keep the output constant until the economy transitions to the high-supply state. After the transition, the policy cuts the interest rate once again to raise aggregate demand toward the higher aggregate supply level. However, the policy runs into the ZLB constraint. Due to the binding ZLB, the recovery in the high-supply state takes several periods to complete.

The figure illustrates several other cases to illustrate different properties of the equilibrium with the optimal policy. Compared to the first-best benchmark without the ZLB constraint (dotted lines), the optimal policy frontloads the interest rate cuts. Compared to the myopic benchmark where the central bank closes the current output gaps (the dashed line), the optimal policy generates some overheating in the low-supply phase but accelerates the recovery once the economy transitions to the high-supply phase. Finally, compared to a case with less inertia (dash-dotted line), the baseline case with higher inertia results in higher gaps both before and after transition to the supply recovery. These comparisons highlight that our results in this section (as in the main text) are driven by the *interaction* of the aggregate demand inertia and expansionary policy constraints.

Proof of Lemma 4. If $y_{-1} \geq \overline{y}_H$, then the central bank can achieve a zero gap, $Y_H(y_{-1}) = y_H^*$ and $V_H(y_{-1}) = 0$. Using the IS curve (1) with $y_t = y_{t+1} = y_H^*$, the interest rate is given by $(B.3)$. The interest rate is nonnegative, $i_{t,H} \geq 0$. In this case, the ZLB constraint does not bind.

In contrast, if $y_{-1} < \overline{y}_H$, then the ZLB constraint binds and the output gap is negative for at least one period, $Y_H(y_{-1}) < y_H^*$ and $V_H(y_{-1}) < 0$.

Consider the constrained range, $y_{-1} \leq \overline{y}_H$. In this range, the IS curve with $i_{t,H} = 0$ implies that output satisfies the recursive relation

$$Y_H(y_{-1}) = \eta y_{-1} + (1-\eta)\left(\rho + Y_H\left(Y_H(y_{-1})\right)\right). \tag{B.8}$$

We first solve this relation over a sequence of cutoff points for past output. Given $\overline{y}_{H,-1} \equiv y_H^*$ and $\overline{y}_{H,0} = \overline{y}_H$, we recursively define a sequence of cutoffs with:

$$\overline{y}_{H,k+1} = \overline{y}_{H,k} - \frac{1-\eta}{\eta}\left(\rho + \overline{y}_{H,k-1} - \overline{y}_{H,k}\right). \tag{B.9}$$

Using $(B.8)$, it is easy to check that the output function maps a lower cutoff into the higher cutoff:

$$Y_H\left(\overline{y}_{H,k+1}\right) = \overline{y}_{H,k}. \tag{B.10}$$

Note also that the cutoffs satisfy $\overline{y}_{H,k+1} \leq \overline{y}_{H,k} - \frac{(1-\eta)\rho}{\eta}$. Therefore, there exists K_H such that $\overline{y}_{H,K_H} < 0$. Then, the cutoffs $\left\{\overline{y}_{H,k}\right\}_{k=-1}^{K_H}$ cover the entire region $[0, y_H^*]$.

We next extend the solution to the intervals, $\left[\overline{y}_{H,k}, \overline{y}_{H,k-1}\right]$. Specifically, we claim that the output function is piecewise linear and strictly increasing. That is, there exist $\{a_k, b_k\}_{k=0}^{K_H}$ such that

$$Y_H(y_{-1}) = a_k y_{-1} + b_k \text{ for } y_{-1} \in \left[\overline{y}_{H,k}, \overline{y}_{H,k-1}\right]. \tag{B.11}$$

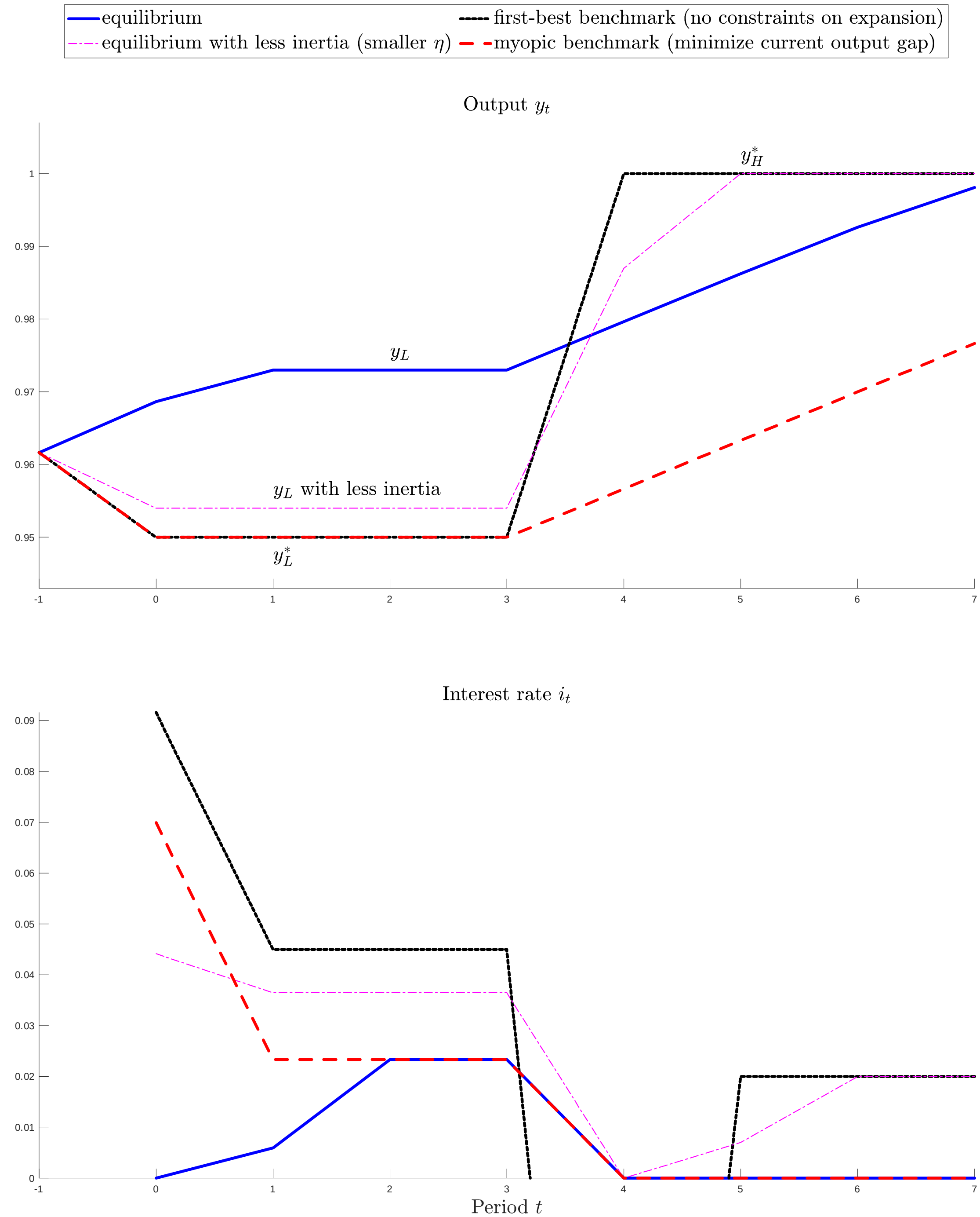

Figure B.1: A simulation of the equilibrium with a ZLB constraint starting in the low-supply state, $s_0 = L$, with the most recent output that satisfies $y_{-1} < y_L$. The solid lines correspond to the equilibrium with the optimal policy. The dotted lines correspond to a first-best benchmark case without the ZLB constraint. The dashed lines correspond to a myopic benchmark case in which the policy minimizes the current output gap. The dash-dotted lines correspond to the equilibrium with a smaller inertia parameter (η). See Appendix C for the parameters used.

We also claim that the slope coefficients satisfy $a_k > a_{k-1} \geq 0$ and $a_k < \min\left(1, \frac{\eta}{1-\eta}\right)$.

Using the characterization for the unconstrained region, the claim holds for $k = 0$ with the coefficients

$$a_0 = 0 \text{ and } b_0 = y_H^*. \tag{B.12}$$

Suppose the claim holds for $k-1$ and consider it for k. Using Eq. $(B.8)$, we have

$$a_k y_{-1} + b_k = \eta y_{-1} + (1-\eta)\left(\rho + a_{k-1}\left(a_k y_{-1} + b_k\right) + b_{k-1}\right).$$

After rearranging terms, we obtain a recursive characterization for the coefficients

$$\begin{aligned} a_k &= \eta + (1-\eta)\, a_{k-1} a_k \\ &\implies a_k = \frac{\eta}{1-(1-\eta)\, a_{k-1}} \\ b_k &= (1-\eta)\left(\rho + a_{k-1} b_k + b_{k-1}\right) \\ &\implies b_k = \frac{(1-\eta)\left(\rho + b_{k-1}\right)}{1-(1-\eta)\, a_{k-1}} = a_k \frac{1-\eta}{\eta}\left(\rho + b_{k-1}\right). \end{aligned} \tag{B.13}$$

Note that $a_{k-1} < 1$ implies $a_k = \frac{\eta}{1-(1-\eta)a_{k-1}} \in (0,1)$. Likewise, $a_{k-1} < \frac{\eta}{1-\eta}$ implies $a_k = \frac{\eta}{1-(1-\eta)a_{k-1}} < \frac{\eta}{1-\eta}$. We also need to check $a_k = \frac{\eta}{1-(1-\eta)a_{k-1}} > a_{k-1}$. Note that this is equivalent to $P\left(a_{k-1}\right) > 0$ where $P(x) = x^2 - \frac{1}{1-\eta}x + \frac{\eta}{1-\eta}$. This polynomial has roots $\frac{\eta}{1-\eta}$ and 1. Since $a_{k-1} < \min\left(1, \frac{\eta}{1-\eta}\right)$, we have $P\left(a_{k-1}\right) > 0$ and thus $a_k > a_{k-1}$. This proves the claim in $(B.11)$ by induction.

Eqs. $(B.10)$ and $(B.11)$ imply that the output function maps each interval $\left[\overline{y}_{H,k}, \overline{y}_{H,k-1}\right]$ into the higher interval $\left[\overline{y}_{H,k-1}, \overline{y}_{H,k-2}\right]$. This establishes the claim in the proposition that output converges to y_H^* after finitely many periods (at most $K_H + 1$ periods).

We next consider the value function $V_H\left(y_{-1}\right)$. Following similar steps, we can define the value function recursively over the intervals $\left[\overline{y}_{H,k}, \overline{y}_{H,k-1}\right]$. Let $V_{H,0}\left(y_{-1}\right) = 0$ and define a sequence of functions with:

$$V_{H,k}\left(y_{-1}\right) = -\frac{1}{2}\left(a_k y_{-1} + b_k - y_H^*\right)^2 + \beta V_{H,k-1}\left(a_k y_{-1} + b_k\right). \tag{B.14}$$

For each interval, the value function agrees with the corresponding function in the sequence:

$$V_H\left(y_{-1}\right) = V_{H,k}\left(y_{-1}\right) \text{ for } y_{-1} \in \left[\overline{y}_{H,k}, \overline{y}_{H,k-1}\right].$$

Note also that the functions in the sequence are differentiable with derivatives that satisfy:

$$\frac{dV_{H,k}\left(y_{-1}\right)}{dy_{-1}} = -\left(a_k y_{-1} + b_k - y_H^*\right) a_k + \beta \frac{dV_{H,k-1}\left(a_k y_{-1} + b_k\right)}{dy_{-1}} a_k. \tag{B.15}$$

Therefore, *inside* each interval, the value function is differentiable and its derivative agrees with the derivative of the corresponding function in the sequence:

$$\frac{dV_H\left(y_{-1}\right)}{dy_{-1}} = \frac{dV_{H,k}\left(y_{-1}\right)}{dy_{-1}} \text{ for } y_{-1} \in \left(\overline{y}_{H,k}, \overline{y}_{H,k-1}\right).$$

At each cutoff $\overline{y}_{H,k}$, the value function is left and right-differentiable with derivatives respectively given by $\frac{dV_{H,k+1}\left(\overline{y}_{H,k}\right)}{dy_{-1}}$ and $\frac{dV_{H,k}\left(\overline{y}_{H,k}\right)}{dy_{-1}}$.

We next prove that the value function, $V_H\left(y_{-1}\right)$, is strictly concave over the constrained range, $y_{-1} \leq \overline{y}_{H,0}$. For the interior points, $\left(\overline{y}_{H,k}, \overline{y}_{H,k-1}\right)$, it is easy to check that the derivative, $\frac{dV_H(y_{-1})}{dy_{-1}}$, is strictly decreasing. Consider the cutoff points, $\overline{y}_{H,k}$. It suffices to check that the left derivative is greater than the right derivative:

$$\frac{dV_{H,k+1}\left(\overline{y}_{H,k}\right)}{dy_{-1}} > \frac{dV_{H,k}\left(\overline{y}_{H,k}\right)}{dy_{-1}}.$$

This claim is true for $k = 0$. Suppose it is true for $k - 1$. Using Eq. $(B.15)$, we have

$$\begin{aligned} \frac{dV_{H,k+1}\left(\overline{y}_{H,k}\right)}{dy_{-1}} &= -\left(\overline{y}_{H,k-1} - y_H^*\right) a_{k+1} + \beta \frac{dV_{H,k}\left(\overline{y}_{H,k-1}\right)}{dy_{-1}} a_{k+1} \\ \frac{dV_{H,k}\left(\overline{y}_{H,k}\right)}{dy_{-1}} &= -\left(\overline{y}_{H,k-1} - y_H^*\right) a_k + \beta \frac{dV_{H,k-1}\left(\overline{y}_{H,k-1}\right)}{dy_{-1}} a_k. \end{aligned}$$

Since $\frac{dV_{H,k}\left(\overline{y}_{H,k-1}\right)}{dy_{-1}} > \frac{dV_{H,k-1}\left(\overline{y}_{H,k-1}\right)}{dy_{-1}}$ and $a_{k+1} > a_k$, we also have $\frac{dV_{H,k+1}\left(\overline{y}_{H,k}\right)}{dy_{-1}} > \frac{dV_{H,k}\left(\overline{y}_{H,k}\right)}{dy_{-1}}$. This proves the claim and shows that $V_H\left(y_{-1}\right)$ is strictly concave over the constrained range.

Finally, we prove that the value function is differentiable at the cutoff point at which starts to bind, $y_{-1} = \overline{y}_H = \overline{y}_{H,0}$, with derivative equal to zero, $\frac{dV_H\left(\overline{y}_{H,0}\right)}{dy_{-1}} = 0$. The right derivative is zero since $V_{H,0}\left(y_{-1}\right) = 0$. Recall that $Y_H\left(\overline{y}_{H,0}\right) = y_H^*$. Therefore, using Eq. $(B.15)$ for $k = 1$, we have

$$\frac{dV_{H,1}\left(\overline{y}_{H,0}\right)}{dy_{-1}} = -\left(Y_H\left(\overline{y}_{H,0}\right) - y_H^*\right) a_1 = 0.$$

This completes the proof of the proposition. Note also that Eqs. $(B.9 - B.15)$ enable a numerical characterization of equilibrium in the high-supply state. □

Proof of Proposition 5. The case $y_{-1} > \overline{y}_L$ is analyzed before the proposition. Suppose $y_{-1} < \overline{y}_L$ so that the ZLB constraint binds. In this case, the IS curve with $i_{t,L} = 0$ implies the output function satisfies the recursive relation

$$Y_L(y_{-1}) = \eta y_{-1} + (1-\eta)\left(\rho + \lambda Y_H(Y_L(y_{-1})) + (1-\lambda) Y_L(Y_L(y_{-1}))\right). \tag{B.16}$$

The analysis follows similar steps as in the proof of Lemma 4. Given $\overline{y}_{L,0} = \overline{y}_L$ and $\overline{y}_{L,-1} \equiv y_L$, we recursively define a sequence of cutoffs with:

$$\overline{y}_{L,k+1} = \overline{y}_{L,k} - \frac{1-\eta}{\eta}\left(\rho + \lambda Y_H\left(\overline{y}_{L,k}\right) + (1-\lambda)\overline{y}_{L,k-1} - \overline{y}_{L,k}\right). \tag{B.17}$$

Using $(B.16)$, it is easy to check that the output function maps a lower cutoff into the higher cutoff:

$$Y_L\left(\overline{y}_{L,k+1}\right) = \overline{y}_{L,k}. \tag{B.18}$$

Using $Y_H(y_L) > y_L$, we also obtain $\overline{y}_{L,k+1} < \overline{y}_{L,k} - \frac{(1-\eta)\rho}{\eta}$. Therefore, there exists K_L such that $\overline{y}_{L,K_L} < 0$. Then, the cutoffs $\left\{\overline{y}_{L,k}\right\}_{k=-1}^{K_L}$ cover the entire region $[0, y_L]$.

We can then define the output function recursively over the intervals $\left[\overline{y}_{L,k}, \overline{y}_{L,k-1}\right]$. Let $Y_{L,0}(y_{-1}) = y_L$ and define a sequence of functions with:

$$Y_{L,k}(y_{-1}) = \eta y_{-1} + (1-\eta)\begin{pmatrix} \rho + \lambda Y_H\left(Y_{L,k}(y_{-1})\right) \\ +(1-\lambda) Y_{L,k-1}\left(Y_{L,k}(y_{-1})\right) \end{pmatrix} \text{ for } y_{-1} \in \left[\overline{y}_{L,k}, \overline{y}_{L,k-1}\right]. \tag{B.19}$$

These functions are uniquely defined and increasing over $[0, \overline{y}_L]$ (since the output function in the high-supply state, $Y_H(\cdot)$, is piecewise linear with slopes strictly less than one, as we characterized earlier). Then, Eq. $(B.18)$ implies that for each interval the output function agrees with the corresponding function in the sequence

$$Y_L(y_{-1}) = Y_{L,k}(y_{-1}) \text{ for } y_{-1} \in \left[\overline{y}_{L,k}, \overline{y}_{L,k-1}\right].$$

In particular, the output function maps each interval $\left[\overline{y}_{L,k}, \overline{y}_{L,k-1}\right]$ into the higher interval $\left[\overline{y}_{L,k-1}, \overline{y}_{L,k-2}\right]$. This establishes the claim in the proposition that, absent transition to the high-supply state, output converges to the target level y_L after finitely many periods (at most $K_L + 1$ periods). This completes the proof of the proposition. Note also that Eqs. $(B.17 - B.19)$ enable a numerical characterization of equilibrium in the low-supply state. □

C. Parameters for the numerical examples

This appendix describes the parameters used for the numerical examples plotted in Figures 1-3 and B.1.

C.1. Parameters for Figure 1

We think of each period as a year. For the baseline model (analyzed in Section 2 and illustrated in Figure 1), we set the following parameters:

Discount rate:	$\beta = \exp(-0.02)$
Inertia:	$\eta = 0.8$
Potential output in states H, L:	$y_H^* = 1, y_L^* = 0.95$
Probability of transition to H:	$\lambda = 0.5$
Taylor rule coefficient:	$\lambda = 0.5$
Initial past output:	$y_{-1} = \overline{y}_L = 0.96.$

These parameters are relatively standard. We set the discount rate so that the long-run *real* interest rate ("rstar") is about 2%. To make our results stark, we set the inertia parameter to a relatively high level, $\eta = 0.8$. The (magenta) dash-dotted lines in Figure 1 plot the equilibrium for an alternative case with lower inertia where we set, $\tilde{\eta} = 0.5$. We set $\lambda = 0.5$, which corresponds to expected supply recovery in about two years. In Figure 1 (as well as in other figures), the actual recovery is delayed relative to expectations and takes place in year four. We set the output gap coefficient in the Taylor rule to a relatively high level, $\phi = 1$ (see (2)). Finally, we start the economy with past output equal to the threshold level below which the lower bound constraint binds, $y_{-1} = \overline{y}_L < y_L$ (see (12)).

C.2. Parameters for Figure 2

For the model with inflation determined by the NKPC (analyzed in Section 3.1 and Appendix A.2.1 and illustrated in Figure 2), we adopt the same parameters in the previous Section 1 (except for ϕ). For the parameters specific to this model, we set:

Inflation sensitivity to output gap:	$\kappa = 0.5$
Generalized Taylor rule coefficients:	$\phi_y = 1, \phi_\pi = 1$
Relative welfare weight on inflation gaps:	$\psi = 1.$

The inflation sensitivity to output gap is in line with the standard calibrations of the Phillips curve. For the Taylor rule, the coefficient on the output gap is the same as before, $\phi_y = \phi = 1$. The coefficient on inflation, $\phi_\pi = 1$, ensures the Taylor condition (marginally) holds. Finally, we assume the central bank puts the same welfare weight on inflation and output gaps, $\psi = 1$ (see (16)).

C.3. Parameters for Figure 3

For the model with inertial inflation (analyzed in Section 3.2 and Appendix A.2.2 and illustrated in Figure 3), we adopt the parameters in the previous Section 3, except for ϕ_π. We reset this parameter to satisfy the simplifying assumption in Lemma 3, $\phi_\pi = b$. For the parameters specific to this model, we set:

$$\begin{aligned} \text{Inflation inertia:} \quad & b = 0.9 \\ \text{Initial past inflation:} \quad & \pi_{-1} = 0. \end{aligned}$$

We set the inertia in the Phillips curve to a relatively high level, $b = 0.9$, to make our results stark (see (18)). We start the economy with past inflation equal to zero.

C.4. Parameters for Figure B.1

For the model with the zero lower bound constraint (analyzed in Section B and illustrated in Figure B.1), we adopt the same parameters in Section 1 for the baseline model with a Taylor rule constraint.