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SOVEREIGN-DEBT RENEGOTIATIONS REVISITED

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ABSTRACT

The sovereign-debt literature has often implicitly assumed that all the power in the bargaining game between debtor and creditor lies with the latter. An earlier paper provided a game-theoretic basis for this contention, in that all the subgame-perfect equilibria of the game modeled have an extreme form in which the game's surplus is captured by the creditor. Two related games are analyzed here. Equilibria in which the debtor captures some of the surplus are shown to exist in one of them but not the other, and the roles of various assumptions in all three games is examined.

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1. Introduction

The purpose of this paper is to gain further understanding of sovereign-debt renegotiations by continuing to explore those features of debt negotiations that can be explained by incentives to repay that come from sources external to the creditor-debtor relationship (e.g., the debtor's credit rating). A first step in this endeavor was undertaken in [FR] where we analyze a game-theoretic model of sovereign-debt renegotiations. There we argue that the knowledge that banks prefer countries to repay some portion of their debts rather than defaulting completely, and that countries prefer to have their credit records clear, makes for a kind of bargaining game.

In our version of that game, the two players, a creditor bank and its sovereign debtor, bargain implicitly over time by taking actions having economic consequences. The actions taken by the creditor are partial forgivenesses of the outstanding debt. The actions of the debtor are consumption, investment, and debt-service decisions. If and when the unforgiven part of the debt together with accrued interest is finally repayed, the game ends with the sovereign receiving a "bonus". An interpretation of the bonus is the value of the country's improved credit rating that results from satisfaction of the creditor's claims. This is over and above the present value to the (former) debtor of continuing optimally from the capital stock with which the game ends. If there is no final repayment, the game goes on forever and the debtor simply continues to accrue the utility it associates with its consumption decisions. Two other critical assumptions in [FR] are: (i) The creditor discounts future receipts at the same rate at which interest accrues on the debt. (ii) The value of the bonus is zero when there is zero capital at the end of the game.

The results of that paper are: that all the subgame-perfect equilibria of the game are similar in that they all result in the same payoffs to the players; that the payoffs are Pareto optimal (in particular, the creditor is ultimately satisfied, so a bonus is received); and that the creditor extracts all the surplus associated with the bonus (unless the debtor is able to repay the initial debt without forgiveness and prefers that to the option of ignoring the debt altogether; in this case the creditor cannot extract the full value of the bonus). Furthermore, the play of one of these subgame-perfect equilibria has the property that there is only one forgiveness, and this occurs at the first move of the game. Finally, in none of the plays of any subgame-perfect equilibrium does a forgiveness occur after a positive payment has been made.

In the present paper we analyze two models of sovereign-debt renegotiations which differ from that of [FR], hereafter called Model 1, in several of their features, thus permitting us to explore the sensitivity of our earlier results to these modifications. In one of these, Model 2, the form of the debt is altered so that instead of being a number from which payments and forgivenesses are subtracted and to which interest is added during the course of play, the debt is now, more realistically, a schedule of payments due, all of which the debtor is initially expected to meet. The creditor is free to suggest alternative payment schedules and the debtor to accept or reject the creditor's offer. Whenever the debtor fails to make a payment, however, the debt is no longer simply rescheduled automatically at the given fixed rate of interest, as in Model 1. The results for Model 2 are similar to those obtained for Model 1, despite the fact that assumption (i) in

Model 1 has no counterpart in Model 2. Thus, the multidimensionality of the debt schedule here serves to obviate the need for that assumption.

The other new model, Model 3, returns to the Model-1 specification of the debt as a scalar and abstracts away from the complications associated with capital accumulation and growth by assuming that the debtor country receives a constant sequence of endowments. This enables us to focus more on the role that the creditor's relative impatience plays in the bargaining game (cf. assumption (i)). Since capital accumulation has no counterpart in Model 3, assumption (ii) has no counterpart; there is, however, a different assumption about the bonus. The results for Model 3 turn out to be rather different. There are generally many subgame-perfect equilibria; when assumption (i) does not hold these all differ from the one emphasized for Model 1 in [FR]. Furthermore, there now exist subgame-perfect equilibria at which the debtor captures some of the surplus; and for some instances of Model 3 this is true of all the subgame-perfect equilibria.

It has often been assumed in the sovereign-debt literature (e.g. [EG], [EGS], [K], [S]) that the set of loans that will be fully repaid is the same as the set of loans for which the debtor's benefits from full repayment exceed the costs the debtor associates with default. As argued in [FR], this is tantamount to assuming that the creditor has all the power in the bargaining game. While the results of [FR] can be seen as providing a game-theoretic foundation for this assumption, Models 2 and 3 enable us to take a closer look at the factors that drive that result. The conclusions derived from these models, in particular the existence of equilibria with very different characteristics in Model 3, point to the fact that the results of Model 1 rest on subtle combinations of the assumptions.

The rest of the paper is organized as follows. Sections 2 and 3 present and analyze Models 2 and 3, respectively; and each section concludes with some remarks on the nature of the equilibria obtained as compared with those for Model 1. Section 4 follows with general remarks. All the proofs are relegated to an appendix.

2. Model 2

The creditor is named A and the debtor B. At the beginning of each time period $t \in \{0,1,\ldots\}$, the relevant part of the history of the game so far can be summarized by current values of the state variables K, D, and τ, where: K, a nonnegative real number, is the debtor's capital stock; $D = (D_0, D_1, \ldots)$, a sequence of nonnegative real numbers, is the sequence of payments currently scheduled for periods t, t+1, etc.; and $t+\tau$, τ being a nonnegative integer or ∞, denotes the time at which the debt will be considered repaid assuming B manages to pay $D_0, D_1, \ldots, D_\tau$ in periods $t, t+1, \ldots, t+\tau$, respectively. The pair (D,τ) is called a feasible schedule if each component of D is between 0 and M (M some large positive number) and if all components of D after τ are zero. (As the notation suggests, we shall be assuming a good deal of stationarity; this is solely to keep the notation as uncluttered as possible.) Initial values K^0 and feasible schedule (D^0,τ^0) are given as the last part of the definition of the game, called $G_2(K^0,D^0,\tau^0)$.

At the beginning of each period t production takes place, transforming K into g(K). Player A then makes the first move in the period: she selects an alternative feasible schedule (D',τ') to offer B. (Player A may set $(D',\tau')=(D,\tau)$ if she wishes to make no new offer.) Next, B chooses values for the variables x, c and p, where: $x \in \{Y,N\}$ is either "Yes" or "No" to the new offer, c is current consumption, and p is the amount of any payment B makes to A in the current period. Thus $c \geq 0$, $p \geq 0$, and $c+p \leq g(K)$. As a result of these moves, A enjoys the utility r(p) associated with her current-period receipt p, B enjoys the utility u(c) associated with her current-period consumption c, and g(K)−c−p becomes the capital stock in period t+1. If $\tau=0$, x=N, and $p \geq D_0$, or if $\tau'=0$, x=Y, and $p \geq D_0'$, then the debt is repaid and the game ends. Otherwise, period t+1 is entered with state variables $\bar{K}=g(K)-c-p$ and

$$(\bar{D},\bar{\tau}) = \begin{cases} ((D_1,D_2,\ldots),\tau-1) & \text{if } x=N \text{ and } p \geq D_0, \\ ((D_1',D_2',\ldots),\tau'-1) & \text{if } x=Y \text{ and } p \geq D_0', \\ ((M,M,\ldots),\infty) & \text{otherwise.} \end{cases}$$

For each player, a (pure) strategy is, as usual, a sequence of functions, one for each time period t, mapping the set of possible partial histories into the set of available moves. As a result of any pair of strategies, a sequence of consumptions, payment schedules, payments, and capital stocks, as well as a stopping time (if any) is determined. Possessing the discount factor α, Player A wishes to maximize

$$\sum_{t=0}^{T} \alpha^t r(p_t)$$

where T denotes the time final payment is received. For Player B, who discounts with factor β, the objective is to maximize

$$\sum_{t=0}^{T} \beta^t u(c_t) + \beta^{T+1} Z(K_{T+1}),$$

where Z(K) is the value of the future to B after the game ends with terminal capital stock K. (If the game never ends, $T=\infty$.) A subgame-perfect equilibrium is a pair of strategies, one for each player, such that for every possible partial history of actions, each player's strategy restricted to the continuation game (the subgame that follows the partial history) is optimal given the opponent's strategy.

The following assumptions will be maintained for the rest of this section: $0<\alpha<1$, $0<\beta<1$. u and r are continuous and increasing; u is also strictly concave. g is continuous, increasing and concave; and g is bounded above and below by M and 0, respectively. Z is continuous, increasing, concave and bounded below on [0,M] by the function v, which is defined by the Bellman equation:

$$v(K) = \max_{0 \le c \le g(K)} (u(c) + \beta v(g(K)-c)) \qquad \forall K \in [0,M]. \tag{1}$$

Also Z(0) is assumed equal to v(0). These assumptions ensure that:

<u>Lemma 2.1</u>: There is a unique solution v to (1), which is continuous, strictly concave and increasing. Furthermore, the value of c that maximizes the right-hand side in (1) is unique and continuous in $K \in [0,M]$.

Since v is the value of B's unique, optimal stand-alone policy, Z can be interpreted as this amount plus the value of the debt's retirement bonus.

Next we define $w(K,D,\tau)$ as the payoff to B from starting at state (K,D,τ) and repaying (D,τ) optimally when B hypothesizes that A will never make a revised offer. Thus

$$w(K,D_0,\ldots,D_\tau,\tau) = \max_{c_0,\ldots,c_\tau} \sum_{t=0}^{\tau} \beta^t u(c_t) + \beta^{\tau+1} Z(K_{\tau+1})$$

$$\text{subject to} \quad \left. \begin{array}{l} 0 \le c_t \le g(K_t)-D_t \\ K_{t+1} = g(K_t)-c_t-D_t \end{array} \right\} \text{ for } t = 0,\ldots,\tau$$

whenever the constraints are feasible, and $w(K,D_0,\ldots,D_\tau,\tau) = -\infty$ otherwise.

Lemma 2.2: For each τ, on the region where $w(\cdot,\tau) > -\infty$, w is continuous (jointly) in $(K,D_0,\ldots,D_\tau)$, increasing in K on $[0,M]$ and decreasing in each argument $D_0,\ldots,D_\tau$. Furthermore, the optimal value for c_0 is unique and varies continuously with $K,D_0,\ldots,D_\tau$.

We are now ready to specify a strategy b* for Player B. At each of her moves, B observes K,D,τ,D', and τ', and computes $w(K,D,\tau)$, $w(K,D',\tau')$, and $v(K)$. B sets x=Y if and only if $w(K,D',\tau') \ge \max\{w(K,D,\tau),v(K)\}$, in which case B sets c to be the maximizing c_0 for $w(K,D',\tau')$ and sets $p=D_0'$. In the other case (x=N) if $v(K)>w(K,D,\tau)$ she sets p=0 and c to maximize the right-hand side of (1); otherwise she sets c equal to the maximizing c_0 for $w(K,D,\tau)$ and $p=D_0$.

Next, to determine a strategy a* for A, consider first the Bellman equation:

$$\left. \begin{array}{l} y(K,D,\tau) = \max\limits_{D',\tau'} \ (r(p) + \alpha y(g(K)-c-D_0',(D_1',D_2',\ldots),\tau'-1)) \\ \text{where } c \text{ is the maximizing } c_0 \text{ for } w(K,D_0',\ldots,D_\tau',\tau') \\ \text{and subject to the constraint on } (D',\tau'): \\ \qquad w(K,D',\tau') \ge \max(w(K,D,\tau),\ v(K)). \end{array} \right\} \quad (2)$$

Lemma 2.3: There is a unique function y solving (2). Furthermore this y is continuous in $(K,D_0,\ldots,D_\tau)$ for each τ, and the constraint in (2) is binding for each (K,D,τ).

The strategy a* for A selects a maximizer of $y(K,D,\tau)$ at each A-move having state (K,D,τ).

The actual play of (a*,b*) has at most one rescheduling and that at the initial period followed by payments according to this schedule. The (Pareto optimal) payoffs from following (a*,b*) beginning at any initial state (K^0,D^0,τ^0) are $y(K^0,D^0,\tau^0)$ and $\max\{w(K^0,D^0,\tau^0),v(K^0)\}$ respectively.

Proposition 2.4: The strategy pair (a*,b*) constitutes a subgame-perfect equilibrium of the game $G_2(K^0,D^0,\tau^0)$ for every nonnegative K^0 and every feasible (D^0,τ^0).

Proposition 2.5: For each initial state (K^0,D^0,τ^0), every subgame-perfect equilibrium of $G_2(K^0,D^0,\tau^0)$ generates the same payoffs in every subgame as does (a*,b*).

Remarks

Here is some intuition concerning why we are able to dispense with assumptions like the one in [FR] that relates A's discount factor to the rate of interest on the debt. In Model 1, Player A's payoff function is assumed to be such that she is indifferent over all payment sequences that have the same present value, where the interest rate in the present-value calculation is that at which the debt grows. Her objective is therefore to maximize the current debt subject to respecting B's incentive-compatibility constraints — that B prefers paying to not paying. A does not care what pattern of payments

B chooses. If A's objective were something other, she would not be indifferent over B's payment sequence, and she would have a maximization problem to solve subject to incentive-compatibility constraints for B which in turn could feed back on A's objective function in a complicated way. The reason for such complications is that in Model 1 Player A has only one instrument at her disposal — the current debt level, and B can choose to pay it off in many ways. In Model 2, A selects the entire sequence of payments that B is to make, so A can simply maximize her utility over all the payment sequences that B would agree to repay. She has all the instruments she needs to avoid the feedback complications.

3. Model 3

The game of Model 3 is termed $G_3(e,D_0)$. It is played by the same two players over the same discrete time periods. There is no capital, production, or storage in Model 3, however. At the beginning of each period t, the debtor receives a new endowment e and inherits a debt D_t. The initial value of debt $D_0>0$ is specified exogenously, as is the constant value e. The first move at each t belongs to A, who selects f_t, the part of D_t to be forgiven currently. It is then B's move: she selects the current level of consumption c_t and debt-service payment p_t. If $p_t = D_t-f_t$, the game ends; otherwise, the next period is entered with

$$D_{t+1} = (1+r)(D_t-f_t-p_t)$$

where $r>0$ is the interest rate on the debt. Thus the following restrictions on the players' moves apply:

$0 \leq f_t \leq D_t$,

$0 \leq c_t, \quad 0 \leq p_t \leq D_t - f_t, \quad \text{and } c_t + p_t \leq e.$

Denote by T the first period at which $p_t = D_t - f_t$. If this never happens, $T = \infty$.

The creditor's maximization problem is unchanged from Model 1: A wishes to maximize the discounted sum of the debtor's payments $\Sigma_t \alpha^t p_t$, where $0<\alpha<1$ is the creditor's discount factor. The debtor wishes to maximize

$$\sum_{t=0}^{T-1} \beta^t u(c_t) + \beta^T u(c_T + z) + \beta^{T+1} Z \tag{3}$$

where $z \geq e$ is a one-period consumption bonus received by the debtor upon repayment of her debt, and βZ is the value of the future to the debtor at time T when ending the game at T with no debt. We assume that $\alpha \leq (1+r)^{-1}$, that u is concave and increasing, and that $u(z)+\beta Z \geq u(e)(1-\beta)^{-1}$. Thus, the difference $u(e)(1-\beta)^{-1} - [u(c_T+z)+\beta Z]$ can be thought of as the extra value to the debtor of regaining access to the capital market on debt-free terms. Note that, unlike in Models 1 and 2, B is assumed to gain some of the benefit from the bonus in consumption units immediately upon repaying the outstanding debt.

Consider the strategies <u>a</u> for A and <u>b</u> for B defined at any subgame at any time t defined by

$$\underline{a}: \text{ Set } \quad f_t = \begin{cases} D_t - e & \text{if } D_t > e \\ 0 & \text{otherwise} \end{cases}$$

$$\underline{b}: \text{ Set } (p_t, c_t) = \begin{cases} (0, e) & \text{if } D_t - f_t > e \\ (D_t - f_t, e - D_t + f_t) & \text{otherwise.} \end{cases}$$

Notice that when (<u>a</u>,<u>b</u>) is played, debt forgivenesses and payments occur only at time zero. Furthermore, the resulting payoffs are Pareto optimal with the creditor receiving the payoff $\min(e,D_0)$, which is less than the surplus due to the existence of the bonus when $u(z)+\beta Z>u(e)(1-\beta)^{-1}$.

Proposition 3.1: The strategy pair $(\underline{a},\underline{b})$ is a subgame-perfect equilibrium of $G_3(e,D_0)$.

There exist additional subgame-perfect equilibria that are not generally Pareto efficient, however. To illustrate this, we next construct one such strategy pair $(\hat{a},\hat{b})$ for the special case:

$$e=1,\ z=\beta^{-1},\ Z=[\beta(1-\beta)]^{-1},\ 1-\alpha>r,\ \text{and } u(c)=c. \qquad (4)$$

$$\hat{a}:\quad \text{Set } f_t = \begin{cases} 0 & \text{if } 0 \le D_t \le 1 \\ D_t - 1 & \text{if } 1 < D_t \le (1+r)(1-\alpha)r^{-1} \\ 0 & \text{if } (1+r)(1-\alpha)r^{-1} < D_t \le 1 + (1-\alpha)r^{-1} \\ D_t - 1 - (1-\alpha)r^{-1} & \text{if } D_t > 1 + (1-\alpha)r^{-1} \end{cases}$$

$$\hat{b}:\quad \text{Set } p_t = \begin{cases} D_t - f_t & \text{if } 0 \le D_t - f_t \le 1 \\ 0 & \text{if } 1 < D_t - f_t \le (1-\alpha)r^{-1} \\ D_t - f_t - (1-\alpha)r^{-1} & \text{if } (1-\alpha)r^{-1} < D_t - f_t \le 1 + (1-\alpha)r^{-1} \\ 0 & \text{if } D_t - f_t > 1 + (1-\alpha)r^{-1} \end{cases}$$

and $c_t = 1 - p_t$

In contrast with $(\underline{a},\underline{b})$, under $(\hat{a},\hat{b})$ when D_0 is sufficiently large repayment takes two periods. When $(1+r)(1-\alpha)r^{-1}<D_0<1+(1-\alpha)r^{-1}$, the play of

$(\hat{a},\hat{b})$ has $p_0=D_0-(1-\alpha)r^{-1}$ and $p_1=1$. If $\alpha<\beta$ this is not Pareto optimal, since B could be made better off increasing p_0 by $\epsilon<\min(1-D_0+(1-\alpha)r^{-1},\alpha)$ and decreasing p_1 by $\epsilon\alpha^{-1}$ without affecting A's payoff. When $D_0\geq 1+(1-\alpha)r^{-1}$, the play of $(\hat{a},\hat{b})$ has $f_0=D_0-1-(1-\alpha)r^{-1}$, $p_0=1$, $f_1=-1+(1+r)(1-\alpha)r^{-1}$, $p_1=1$. A's payoff is $1+\alpha$. B's payoff is $\beta z+\beta^2 Z-(1-\beta)^{-1}$.

Proposition 3.2: If (4) holds, $(\hat{a},\hat{b})$ is a subgame-perfect equilibrium of $G_3(e,D_0)$.

Using the two equilibria $(\underline{a},\underline{b})$ and $(\hat{a},\hat{b})$ and assuming (4), it is possible to construct a continuum of subgame-perfect equilibria for $G_3(e,D_0)$ that depend in more complicated ways on the history. For instance, there are history-dependent subgame-perfect-equilibrium strategies that support any payoff for A in the interval $(1,1+\alpha)$ when D_0 is sufficiently large. In such a strategy pair, A's strategy calls for an initial forgiveness of the debt down to $1+(1-\alpha)r^{-1}-\epsilon$, $0<\epsilon<\alpha$, and thereafter the strategy is given by $\hat{a}$. B's strategy is the same as $\hat{b}$ as long as A does not deviate from this initial forgiveness. Any such deviation by A, however, causes B to switch to $\underline{b}$ and A to switch to $\underline{a}$.

In [FR] we obtained a subgame-perfect equilibrium for Model 1 under the assumption $\alpha=(1+r)^{-1}$, the play of which had the creditor making one forgiveness in period 0 and the debtor thereafter repaying the debt optimally assuming that no further forgivnesses would be forthcoming. The debtor's payoff was her maximin payoff $v(K_0)$ (as given in (1)). When $\alpha<(1+r)^{-1}$, we were unable to compute subgame-perfect equilibria for Model 1. By way of contrast, we have here:

Proposition 3.3: If $\alpha<(1+r)^{-1}$, $D_0>e$, and $u(z)+\beta Z>u(e)(1-\beta)^{-1}$, there is no subgame-perfect equilibrium for $G_3(e,D_0)$ in the play of which the creditor makes a sole forgiveness in period 0 and in which the debtor obtains her maximin payoff.

Proposition 3.4: If $\alpha<(1+r)^{-1}$, $e<D_0<[(1-\alpha)+(1+r)^{-1}]e$, and $u(z)+\beta Z>u(e)(1-\beta)^{-1}$, at every subgame-perfect equilibrium of $G_3(e,D_0)$, B's payoff exceeds her maximin payoff.

Remarks

Here is some intuition as to why we are able to derive equilibria for $G_3(e,D_0)$ in which, unlike in Model 1, the debtor's payoff is greater than her maximin payoff. The assumptions that the debtor receives z immediately, that $z\geq e$, and that $u(z)+\beta Z\geq u(e)(1-\beta)^{-1}$ ensure that it is incentive compatible for B to repay 0 if $D_t-f_t>e$ and if she assumes that next period's forgiveness f_{t+1} will be $\max(D_{t+1}-e,0)$. Even when $\alpha=(1+r)^{-1}$, the analogue of the uniqueness proof in [FR] fails for Model 3 because B's payoff from repaying D optimally when she hypothesizes that there will be no future forgivenesses is discontinuous at $D=e$. The counterpart of any such discontinuity in Model 1 is avoided as a consequence of the assumption $Z(0)=v(0)$ and the assumed continuity of $Z(K)$.

4. Conclusions

This paper presents two models of sovereign-debt renegotiations that differ in various ways from Model 1. Model 2 preserves the spirit of the results obtained in [FR] in that there is an equilibrium in the play of which only one new debt schedule is offered (in period zero) and in that in all subgame-perfect equilibria the creditor is able to extract all the surplus associated with the bonus (unless the initial state is such that the initial debt can be repayed without any forgiveness). Since Model 2 allows us to dispense with the assumption relating the creditor's discount factor to the interest rate, the assumption that only the creditor is free to suggest new debt schedules might appear to be what is driving the results for both Models 1 and 2. That matters are not as simple as this, however, is illustrated in Model 3. Here, although the creditor is again the sole grantor of forgivenesses, when $\alpha<(1+r)^{-1}$ there is no subgame-perfect equilibrium like the one decribed in [FR]; moreover, there are initial conditions such that in all subgame-perfect equilibria B's payoff exceeds her maximin payoff. When $\alpha=(1+r)^{-1}$ although there is an equilibrium like the one described in [FR], there are generally many other subgame-perfect equilibria as well with payoffs that differ. This suggests that the bonus structure and the relative degree of impatience of the creditor interconnect with the various other assumptions in a complex fashion.

The existence of subgame-perfect equilibrium plays in Model 3 in which forgivenesses and payments alternate for more than one period seems to correspond better to the observed reality of repeated negotiations than the subgame-perfect equilibrium plays obtained for Models 1 and 2. The robustness of the Model 3 subgame-perfect equilibria is somewhat suspect, however, since these equilibria seem to rest on strong assumptions concerning the form and size of the bonus. Nonetheless, there may be interesting policy implications: the results suggest that if the debtor country could establish a sufficiently small, politically-credible, upper bound on its potential payments in every period, then such equilibria might obtain. (In fact, Peru has imposed such an upper bound on its yearly payments.)

As noted in [FR], the existence of uncertainty may be another avenue to obtaining repeated negotiations in equilibrium. Furthermore, without uncertainty it is hard to imagine how bad loans come about in the first place. Our attempts to introduce uncertainty concerning future production to this end have been unsuccessful so far, however. To illustrate the difficulties, suppose that in Model 2 there were some uncertainty about the outputs resulting from future production decisions. The analogue of a repayment plan for B that assumes no further forgivenesses in the deterministic model ought to be then a contingency plan which makes a satisfactory payment in the present period and then continues to make scheduled payments in the future as long as certain output targets are realized, but not in other cases. This means that the analogue here of the function w in the deterministic model would contain expressions that involved the maximum of two functions — one that described the value of the future if payments were to continue and the other if they were to stop. In such a model, B's objective would not be concave as a function of her consumption choice in the current period, and her optimal current consumption choice would not therefore be continuous generally in the components of the debt schedule she faced. This means in turn that the transition structure of the dynamic program that A faced would not be continuous generally and the existence of an optimal plan for A would become problemmatical.

Finally, it is well to review some of the additional hypotheses maintained throughout both papers:

1. The bonus is provided from outside the resources of the players of our games, and it accrues only when the creditor is ultimately satisfied—thus it has a rather discontinuous flavor.

2. No sanctions may be imposed by the creditor for failure to repay (cf. [BR]).

3. As contrasted with the recent bargaining literature (e.g. [R]), "offers" in Models 1 and 3 (and partially—only on the payment side—in Model 2) are real economic actions and do no have to be "accepted" before they are implemented.

4. Our games have only two players. We therefore ignore the roles of political constraints in the debtor country, the creditor's government, the World Bank and IMF, and interactions among the players of simultaneous debt-negotiation games.

Appendix

__Lemma 2.1__: There is a unique solution v to (1), which is continuous, strictly concave and increasing. Furthermore, the value of c that maximizes the right-hand side in (1) is unique and continuous in $K \in [0,M]$.

__Proof:__ All but monotonicity follows directly from Exercise 9.7 in [LPS]. For monotonicity, observe that if $K'>K$, $v(K')>v(K)$, since $g(K')-g(K)$ can be added to the consumption that maximizes (1).||

__Lemma 2.2__: For each τ, on the region where $w(\cdot,\tau) > -\infty$, w is continuous (jointly) in $(K,D_0,\ldots,D_\tau)$, increasing in K on [0,M] and decreasing in each argument $D_0,\ldots,D_\tau$. Furthermore, the optimal value for c_0 is unique and varies continuously with $K,D_0,\ldots,D_\tau$.

__Proof:__ Continuity of w follows from the continuity assumptions on u, g and Z. The monotonicity argument is analogous to that of the previous lemma. The conclusions about the optimizing c_0 follow from the strict concavity of the objective function and the maximum theorem.||

Lemma 2.3: There is a unique function y solving (2). Furthermore this y is continuous in $(K,D_0,\ldots,D_\tau)$ for each τ, and the constraint in (2) is binding for each (K,D,τ).

Proof: To establish existence and uniqueness of y we need only check that the hypotheses of Corollary 9.17.2 in [BS] hold (with the obvious change necessitated by replacing minimize with maximize). All the hypotheses of that corollary are immediate once we establish that

$$\{(K,D,\tau,D',\tau'):\ w_0(K,D_0',\ldots,D_\tau',\tau') \geq \max\{w(K,D,\tau),\ v(K)\}\}$$

is closed[1]; but this is an immediate consequence of Lemmas 2.1 and 2.2. Lower semicontinuity of y follows from the same corollary in [BS], for this only lower semicontinuity of (the analogue of) r needs to be assumed. The proof that y is continuous assuming r continuous follows analogously. That the constraint in (2) is binding follows from the continuity of w, the monotonicity of w in the arguments $D_0',\ldots,D_\tau'$, the monotonicity of r, and the assumption that $Z(0)=v(0)$.||

Proposition 2.4: The strategy pair (a^*,b^*) constitutes a subgame-perfect equilibrium of the game $G_2(K^0,D^0,\tau^0)$ for every nonnegative K^0 and every feasible (D^0,τ^0).

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[1]Denoting by I the nonnegative integers with the discrete topology and by $\tilde{M}$ the set of sequences of elements from $[0,M]$ with the product topology, the set in question lives in the product space $[0,M]\times\tilde{M}\times I\times\tilde{M}\times I$.

__Proof__: Assume Player B deviates at some node in the game tree: If she assumes that no revised schedule will be forthcoming, her payoff is not improved, by the construction of w. Now, since all subsequent offers by A under a* leave B no better off than if no new offer had been forthcoming, the deviation by B leaves her no better off. For A to deviate profitably from a*, she must offer a schedule that differs from an optimizing one in (2). Since b* calls for B to play as predicted in (2), such a deviation cannot improve A's payoff in the subgame.||

Consider a B-subgame characterized by (K,D,τ,D',τ') where (D,τ) is the previously accepted debt schedule and (D',τ') the new offer. Let $(\bar{a},\bar{b})$ be any strategy pair for $G_2(K^0,D^0,\tau^0)$ and let $H(K,D,\tau,D',\tau')$ denote B's payoff resulting from $(\bar{a},\bar{b})$ in the B-subgame.[2] Furthermore, let $(\tilde{K},\tilde{D},\tilde{\tau},\tilde{D}',\tilde{\tau}')$ denote the capital stock, accepted debt schedule, and new outstanding offer obtained under $(\bar{a},\bar{b})$ t periods after (K,D,τ,D',τ') (assuming no termination by t), and $\tilde{c}_s$ denote the consumption in each period s until t also resulting from $(\bar{a},\bar{b})$. Finally, let K* and c_s^* be similarly obtained from optimizing according to v for t periods starting from the same initial B-subgame. The following lemma is then a consequence of the optimal nature of the v program.

[2]From now on, we adopt the following simplifications concerning subgames: 1. Payoffs in a subgame do not include the partial payments that accrue before the subgame begins. From the separable payoff structure, this is an inessential abuse of language. 2. The discounting of such payoffs is to the time of the beginning of the subgame. 3. We identify different subgames that begin with the same state-variable values. This eases the notational burden and should cause no confusion.

Lemma A.1:

$H(\tilde{K},\tilde{D},\tilde{r},\tilde{D}',\tilde{r}')-v(\tilde{K})\geq\beta^{-t}(H(K,D,r,D',r')-v(K)) \quad \forall t<T.$

Proof:

$$H(K,D,r,D',r')=\sum_{s=0}^{t-1}\beta^{s}u(\bar{c}_{s})+\beta^{t}H(\bar{K},\bar{D},\bar{r},\bar{D}',\bar{r}') \text{ and}$$

$$v(K)=\sum_{s=0}^{t-1}u(c_{s}^{*})+\beta^{t}v(K^{*}) \geq\sum_{s=0}^{t-1}u(\bar{c}_{s})+\beta^{t}v(\bar{K}).$$

So $H(\tilde{K},\tilde{D},\tilde{r},\tilde{D}',\tilde{r}')-v(\tilde{K})\geq\beta^{-t}(H(K,D,r,D',r')-v(K))$. ||

Proposition 2.5: For each initial state (K^{0},D^{0},r^{0}), every subgame-perfect equilibrium of $G_{2}(K^{0},D^{0},r^{0})$ generates the same payoffs in every subgame as does (a^{*},b^{*}).

Proof: Note first that in any A-subgame characterized by (K,D,r), in every subgame-perfect equilibrium, B's payoff must be as least as great as $R(K,D,r)=\max(v(K),w(K,D,r))$ (B's payoff from (a^{*},b^{*}) in that subgame), since B has a strategy that guarantees her this payoff regardless of A's strategy in that subgame. Now let $(\bar{a},\bar{b})$ be any subgame-perfect-equilibrium strategy combination for $G_{2}(K^{0},D^{0},r^{0})$. In any B-subgame characterized by (K,D,r,D',r'), let $H_{Y}(K,D,r,D',r')$ be B's maximum payoff from accepting the new offer (D',r') and thereafter following $\bar{b}$ (against $\bar{a}$) and let $H_{N}(K,D,r,D',r')$ be B's maximum payoff from rejecting (D',r') and thereafter following $\bar{b}$ (against $\bar{a}$). Note that $H(K,D,r,D',r')$ must either equal $H_{Y}(K,D,r,D',r')$ or $H_{N}(K,D,r,D',r')$. Suppose that B receives more than $R(K,D,r)$ in some A-subgame under $(\bar{a},\bar{b})$. Let $q=\sup(H(K,D,r,D',r')-v(K))$ where the supremum is taken over all B-subgames which follow immediately after the proposal by A of an alternative feasible schedule (D',r') with

$H_Y(K,D,\tau,D',\tau')>H_N(K,D,\tau,D',\tau')$, as determined by $(\bar{a},\bar{b})$. Note that $q>0$ since, by hypothesis, under $(\bar{a},\bar{b})$ there is an A-subgame in which A proposes a debt-repayment schedule strictly better for B than $R(K,D,\tau)$.

Consider, therefore, an A-subgame characterized by (K,D,τ) in which $\bar{a}$ proposes (D',τ') such that $H_Y(K,D,\tau,D',\tau')>H_N(K,D,\tau,D',\tau')$ and such that $H(K,D,\tau,D',\tau')-v(K)>\beta q$. Strategy $\bar{b}$ must have B accept (D',τ'); moreover, along the equilibrium path of the subgame beginning with B's reply, $\bar{a}$ must propose no better debt schedule for B. To see this, note that otherwise, in the next B-subgame for instance, $H(\tilde{K},\tilde{D}',\tilde{\tau}',\tilde{D}'',\tilde{\tau}'')-v(\tilde{K})>q$ (with notation extending that in Lemma A.1), contradicting the definition of q; and, by Lemma A.1, the same reasoning iterated implies that in every period thereafter no new debt repayment schedule is both proposed and accepted under $(\bar{a},\bar{b})$. But then $H(K,D,\tau,D',\tau')=w(K,D',\tau')$, since, given that no better debt repayment schedule will be proposed, B cannot do better than to repay according to (D',τ'). Now, in this same A-subgame characterized by (K,D,τ), let A deviate by changing her proposal from (D',τ') to $(\hat{D},\tau')$, where $\hat{D}$ is identical to D' except for the last positive component which is increased by $\epsilon>0$ under $\hat{D}$ and where ϵ is such that $H(K,D,\tau,\hat{D},\tau')-v(K)$ is still greater than βq and $H_Y(K,D,\tau,\hat{D},\tau')>H_N(K,D,\tau,\hat{D},\tau')$. (That such ϵ exist follows from $Z(0)=v(0)$, $q>0$, and continuity of w.) By the previous argument, $\bar{b}$ must agree with b* at B's next move; i.e. accept and plan to repay according to $(\hat{D},\tau')$ under the assumption that no better debt schedules will be proposed. Hence, this is a profitable deviation for A in this subgame, and $(\bar{a},\bar{b})$ is therefore not subgame perfect.||

Proposition 3.1: The strategy pair ($\underline{a}$,$\underline{b}$) is a subgame-perfect equilibrium of $G_3(e,D_0)$.

Proof: Given $\underline{b}$, in any A-subgame beginning at t, A's maximum payoff is $\min(e,D_t)$ (since for all $k>0$: $\alpha^k e<e$ and $\alpha^k D_t(1+r)^k \le D_t$). Strategy $\underline{a}$, restricted to that subgame, achieves that payoff.

Given $\underline{a}$, at any B-subgame beginning at t, B's payoff from using $\underline{b}$ is

$$\begin{cases} u(e+z-D_t+f_t) + \beta Z & \text{if } D_t - f_t \le e \\ u(e) + \beta u(z) + \beta^2 Z & \text{otherwise} \end{cases}$$

If B follows an alternative strategy resulting ultimately in no final repayment of the debt, B's payoff is clearly smaller in both cases.

Suppose next that $D_t - f_t \le e$ at this B-subgame and that B follows an alternative strategy ending in final repayment at $T>t$, with last two payments p_{T-1} and p_T. For this case, we shall first argue that B can do at least as well by repaying finally at T-1 instead of repaying finally at T. B's payoff accruing in T-1 and T (discounted to T-1) is:

$$u(e-p_{T-1}) + \beta u(e-p_T+z) + \beta^2 Z \qquad (5)$$

Note first that since A follows $\underline{a}$, $D_{T-1}-f_{T-1} \le e$, and hence that

$$u(e+z-D_{T-1}+f_{T-1}) + \beta Z - [u(e) + \beta u(e+z-D_{T-1}+f_{T-1}) + \beta^2 Z] \ge 0,$$

since, by assumption,

$$u(e) \le (1-\beta)u(z) + \beta(1-\beta)Z \le (1-\beta)u(e+z-D_{T-1}+f_{T-1}) + \beta(1-\beta)Z.$$

Thus, it suffices to show that (5) is not greater than

$$u(e)+\beta u(e+z-D_{T-1}+f_{T-1})+\beta^2 Z, \qquad (6)$$

i.e. that

$$u(e) - u(e-p_{T-1}) \ge \beta[u(e+z-p_T) - u(e+z-D_{T-1}+f_{T-1})]. \qquad (7)$$

If $p_T \geq D_{T-1}-f_{T-1}$, (7) obviously holds since the left-hand side is non-negative and the right-hand side is non-positive. If $p_T < D_{T-1}-f_{T-1}$, then $p_T < e$; so since A follows <u>a</u>, $p_T = (D_{T-1}-f_{T-1}-p_{T-1})(1+r)$. Note that

$$(e+z-p_T)-(e+z-D_{T-1}+f_{T-1}) = p_{T-1}-r(D_{T-1}-f_{T-1}-p_{T-1}) \leq p_{T-1},$$

and that

$$e-(e-p_{T-1}) = p_{T-1}.$$

Hence, as a result of $\beta \leq 1$, concavity, and $z \geq e$, (7) holds in this case as well. Consequently, repaying the outstanding debt at T-1 is at least as good for B as repaying finally at T. So, instead of considering B's original alternative strategy, we might as well consider a modified version of it which exactly imitates it until period T-1, whereupon the outstanding debt is repaid. Repeating the argument as above for the new last two periods, T-2 and T-1, establishes that repaying the outstanding debt at T-2 is at least as profitable for B as repaying finally at T-1, and so on. By induction then, B's payoff from repaying the entire debt at t (i.e., from following <u>b</u> in the subgame) is at least as great as that obtained from any alternative strategy.

Suppose next that $D_t-f_t > e$ in this B-subgame. Then the following period, given that A follows <u>a</u>, $D_{t+1}-f_{t+1} = \min(e,(D_t-f_t-p_t)(1+r))$. In period t+1, as we have established above, B cannot do better than to repay all of this immediately. Thus, if a profitable deviation at t exists, it must involve a positive payment at t. B's payoff from such a deviation is at most $u(e-p_t)+\beta u(e+z-p_{t+1})+\beta^2 Z$, where $p_{t+1} = \min(e,(D_t-f_t-p_t)(1+r))$. But this is not greater than $u(e)+\beta u(z)+\beta^2 Z$ (to see this, replace $D_{T-1}-f_{T-1}$ by e in (7) and modify the subscripts)—which is precisely B's payoff from following <u>b</u>. Thus, there are no profitable deviations for B beginning with $p_t > 0$ either.||

Proposition 3.2: If (4) holds, $(\hat{a},\hat{b})$ is a subgame-perfect equilibrium of $G_3(e,D_0)$.

Proof: Suppose first that B uses $\hat{b}$. At any A-subgame beginning at t in which $D_t>1+(1-\alpha)r^{-1}$, if A forgives less than $D_t-1-(1-\alpha)r^{-1}$, B repays 0 that period and A's payoff cannot exceed $\alpha(1+\alpha)<1+\alpha$ (A's payoff from $(\hat{a},\hat{b})$ in such subgames). Forgiving more than $D_t-1-(1-\alpha)r^{-1}$ likewise yields a payoff smaller than $1+\alpha$. At any A-subgame in which $D_t\leq 1$, if A makes a positive forgiveness f, A obtains $D_t-f<D_t$ (A's payoff from $(\hat{a},\hat{b})$ in such subgames). If A makes a positive forgiveness f at any A-subgame in which $(1+r)(1-\alpha)r^{-1}<D_t\leq 1+(1-\alpha)r^{-1}$, A's payoff cannot exceed $D_t-f-(1-\alpha)r^{-1}+\alpha<D_t-(1-\alpha)r^{-1}+\alpha$ (A's payoff from $(\hat{a},\hat{b})$ in such subgames) if $D_t-f\geq(1-\alpha)r^{-1}$; is even less than this if $1<D_t-f<(1-\alpha)r^{-1}$; and cannot exceed $D_t-f\leq D_t-(1-\alpha)r^{-1}+\alpha$ if $D_t-f\leq 1$. Finally, at A-subgames in which $1<D_t<(1+r)(1-\alpha)r^{-1}$, if A forgives less than D_t-1 then: A's payoff cannot exceed $\alpha<1$ (A's payoff from $(\hat{a},\hat{b})$ in such subgames) if $1<D_t-f<(1-\alpha)r^{-1}$, and cannot exceed $D_t-f-(1-\alpha)r^{-1}+\alpha\leq 1$ if $(1-\alpha)r^{-1}\leq D_t-f$. If A forgives more than D_t-1, then A's payoff is $D_t-f<1$. Thus, A never profits by deviating from $\hat{a}$ in any subgame.

Now fix A's strategy at $\hat{a}$. At any B-subgame beginning at t in which $D_t-f_t>1+(1-\alpha)r^{-1}$, any positive payment p_t leaves B with an outstanding debt next period greater than $(1+r)(1-\alpha)r^{-1}$ and hence with a payoff no greater than

$$(1-p_t)+\beta(1-p_{t+1})+\beta^2 z+\beta^3 Z \tag{8}$$

where $p_{t+1}=\min(1,(D-f-p_t)(1+r)-(1-\alpha)r^{-1})$. Following $\hat{b}$ and paying zero in period t yields B a payoff of

$$1+\beta^2 z+\beta^3 Z. \tag{9}$$

If $p_{t+1}=1$, then (8) is no greater than (9). If $p_{t+1}=(D_t-f_t-p_t)(1+r)-(1-\alpha)r^{-1}<1$, then the upper bound on B's payoff from deviating (i.e.(8)) is monotonically increasing (decreasing) in p_t as $\beta(1+r)>(<)1$ and monotonically decreasing in D_t-f_t. Consequently, setting $p_t=1$ and $D_t-f_t=1+(1-\alpha)r^{-1}$ and subtracting (8) from (9) yields $1-\alpha\beta>0$; thus, in these subgames a positive payment at t cannot be part of a profitable deviation for B. Next, at any B-subgame in which $(1-\alpha)r^{-1}<D_t-f_t\leq 1+(1-\alpha)r^{-1}$, a payment of less than $D_t-f_t-(1-\alpha)r^{-1}$ at t implies that next period's debt is greater than $(1+r)(1-\alpha)r^{-1}$, and thus that B's payoff cannot exceed

$$(1-p_t)+\beta(1-p_{t+1})+\beta^2 z+\beta^3 Z, \tag{10}$$

where $p_{t+1}=\min(1,(D_t-f_t-p_t)(1+r)-(1-\alpha)r^{-1})$. Following $\hat{b}$ and setting $p_t=D_t-f_t-(1-\alpha)r^{-1}$, on the other hand, yields B a payoff of

$$1-(D_t-f_t-(1-\alpha)r^{-1})+\beta z+\beta^2 Z. \tag{11}$$

That (11) is greater than (10) can be seen by substituting the values of z and Z and manipulating the inequalities. Payment of more than $D_t-f_t-(1-\alpha)r^{-1}$ at t likewise yields B a smaller payoff than (11). In B-subgames in which $1<D_t-f_t<(1-\alpha)r^{-1}$, a positive payment at t yields B a payoff that cannot exceed

$$(1-p_t)+\beta(1-p_{t+1}+z)+\beta^2 Z, \tag{12}$$

where $p_{t+1}=\min(1,(D_t-f_t-p_t)(1+r))$. Following $\hat{b}$ and paying zero at t, on the other hand, yields B a payoff of

$$1+\beta z+\beta^2 Z. \tag{13}$$

Once again it is easy to show that if $p_{t+1}=1$ this deviation is not profitable. If $p_{t+1}=(D_t-f_t-p_t)(1+r)<1$, the upper bound on B's payoff from deviating is monotonically increasing (decreasing) in p_t as $\beta(1+r)>(<)1$. Setting $p_t=1$ and

$D_t - f_t = 1$ and subtracting (12) from (13) yields $1-\beta>0$, demonstrating that in these subgames a positive payment at t cannot be part of a profitable deviation for B. Lastly, in B-subgames in which $D_t - f_t \leq 1$, not repaying the entire debt immediately yields B a payoff of at most (12), whereas following $\hat{b}$ and repaying $D_t - p_t$ yields B a payoff of $z+\beta Z$. As in the proof of Proposition 3.1, repaying less than the outstanding debt is not a profitable deviation. This establishes that B never profits from deviating from $\hat{b}$ in any subgame.||

<u>Lemma A.2</u>: If $u(z)+\beta Z > u(e)(1-\beta)^{-1}$, for every subgame-perfect equilibrium of $G_3(e,D_0)$ and in every A-subgame characterized by D, B's payoff is no greater than $R(D) = u(e+z-\min(e,D))+\beta Z$.

<u>Proof</u>: Consider a subgame-perfect strategy combination $(\bar{\underline{a}},\bar{\underline{b}})$ in which there is an A-subgame characterized by D where B achieves more than R(D). Let H(D-f) denote B's payoff in a B-subgame characterized by D-f and let $q=\sup(H(D-f)-R(D))$, where the supremum is taken over all B-subgames immediately following a positive forgiveness determined by $\bar{\underline{a}}$. (Note that $q>0$ since, by hypothesis, in some A-subgame H(D-f) is greater than R(D)). Consider a B-subgame immediately following a positive forgiveness in which $H(D-f)-R(D)>\beta q$. Note first that along the equilibrium path of the subgame beginning with B's reply, $\bar{\underline{a}}$ calls for no future forgivenesses since, by analogy with Lemma A.1, if there is a next such positive forgiveness f_t at time t, $H(D_t-f_t)-R(D_t)>q$ (where D_t and f_t are determined by $(\bar{\underline{a}},\bar{\underline{b}})$), and consequently a positive forgiveness in any such period t would contradict the definition of q. Moreover, since $H>R$ and since no future forgivenesses are

expected, D-f must be strictly less than min(e,D). Also, $\bar{\underline{b}}$ must in turn have B paying the outstanding debt immediately, since, using the proof of Proposition 3.1 (but replacing the weak inequalities in the two lines following (5) by strict inequalities), this is the only payment that achieves B's maximum payoff in this subgame. Now let A deviate in the A-subgame by reducing f by $\epsilon>0$ such that $H(D-f+\epsilon)-R(D)$ is still strictly greater than βq (such ϵ exist since u is continuous and $D-f<e$). The same logic as before leads to the conclusion that B must repay immediately after the ϵ-deviation. Consequently, the ϵ-deviation is profitable for A, contradicting the hypothesis that $(\bar{\underline{a}},\bar{\underline{b}})$ is subgame perfect.||

<u>Corollary</u>: If $u(z)+\beta Z>u(e)(1-\beta)^{-1}$, at any subgame-perfect-equilibrium strategy combination for $G_3(e,D_0)$, A never forgives more than max(D-e,0) at any A-move and B always repays D-f immediately at any B-subgame with $D-f\leq e$.

<u>Proposition 3.3</u>: If $\alpha<(1+r)^{-1}$, $D_0>e$, and $u(z)+\beta Z>u(e)(1-\beta)^{-1}$, there is no subgame-perfect equilibrium for $G_3(e,D_0)$ in the play of which the creditor makes a sole forgiveness in period 0 and in which the debtor obtains her maximin payoff.

<u>Proof</u>: Suppose $(\bar{\underline{a}},\bar{\underline{b}})$ is a subgame-perfect strategy combination for $G_3(e,D_0)$ with $D_0>e$, in the play of which A makes one forgiveness in period 0, B repays optimally thereafter (under the assumption that there will be no further forgivenesses forthcoming), and B obtains her maximin payoff. Let repayment end in period T. Note that $T\geq1$ since $u(z)+\beta Z>u(e)(1-\beta)^{-1}$. Consider the last two payments in this sequence, T-1 and T. Since under $(\bar{\underline{a}},\bar{\underline{b}})$ B repays D_0-f_0

(with interest) without expecting any further forgivenesses, $\bar{b}$ must have $p_T=e$ and $p_{T-1}-D_{T-1}-e(1+r)\leq e$ (from concavity of u and $z\geq e$). Let B deviate by reducing p_{T-1} by $\epsilon(1+r)^{-1}$, where $0<\epsilon<e[1-\alpha(1+r)](1+r)^{-1}$, so that now $D_T=e+\epsilon$. If at the next A-move, $\bar{a}$ makes a forgiveness f_T smaller than ϵ, A's payoff against $\bar{b}$ cannot exceed the discounted value of what B would repay if B assumed no further forgivenesses were forthcoming (since, by the Corollary, the play of $(\bar{a},\bar{b})$ must have repayment end with a payment of e), i.e. B will make payments of at most $p_T=(1+r)^{-1}re+\epsilon(1+r)-f_T<(1-\alpha)e$ and $P_{T+1}=e$. Hence A's payoff (discounted to T) from $f_T<\epsilon$ against $\bar{b}$ is strictly smaller than e. By forgiving ϵ, on the other hand, A's payoff against $\bar{b}$ is e, since, by the Corollary, B repays e immediately. Consequently, the reduction of p_{T-1} by ϵ is a profitable deviation for B, and hence $(\bar{a},\bar{b})$ is not subgame perfect.||

<u>Proposition 3.4</u>: If $\alpha<(1+r)^{-1}$, $e<D_0<[(1-\alpha)+(1+r)^{-1}]e$, and $u(z)+\beta Z>u(e)(1-\beta)^{-1}$, at every subgame-perfect equilibrium of $G_3(e,D_0)$, B's payoff exceeds her maximin payoff.

<u>Proof</u>: If A makes an initial forgiveness f_0 such that $D_0-f_0>e$, then A's payoff cannot exceed $(D_0-f_0-e(1+r)^{-1})+\alpha e<e$, since by the Corollary at every subgame-perfect equilibrium the last payment made by B must be e whenever a B-subgame commences with $D-f>e$. By making a forgiveness f_0 such that $D_0-f_0=e$, on the other hand, A's payoff in every subgame-perfect equilibrium of the continuation game is e, again by the corollary. Hence A must forgive the debt down to e in every subgame-perfect equilibrium (when the initial condition is as given above), and B's payoff is thus greater than her maximin payoff. ||

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