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ABSTRACT

We study the question of whether there exist strategies whereby countries are able to sustain a cartel or collusive behavior when bargaining with a bank over the amount of debt to be repaid. We show that despite the existence of economies to scale in bargaining—if commitment were possible the countries would benefit from joint bargaining—a debtors' cartel will not emerge in equilibrium (in the absence of credible commitment mechanisms). A unique subgame-perfect equilibrium exists in which the bank is effectively able to isolate each country and extract from each the same payoff that it would obtain in the absence of economies to scale. Consequently, a country would be better off if another country declared default. We also show that if two countries of unequal size are bargaining with a bank, in equilibrium a decrease in the size of the smaller country implies a greater payoff to the large country although the payoff to the small country is invariant.

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1. Introduction

During the first couple of years after its inception, the debt crisis was treated, by both creditors and debtors alike, as a short-term liquidity problem requiring only case-by-case consideration. The last four years, in contrast, have been marked by a greater emphasis (on the part of the debtors) on the political and collective nature of the debt crisis. Commencing with a conference in Quito in January 1984, and as evidenced in the meetings in Cartagena 1984, Mar del Plata 1984, Santo Domingo 1985, La Habana 1985, Montevideo 1985, and Punta del Este 1986, Latin American countries have been much more willing to entertain the possibility of joint action regarding the debt problem. Nevertheless, a debtors' cartel has not emerged.

Although the more recent theoretical literature on sovereign debt has given greater emphasis to strategic analyses of various aspects of the debt crisis (e.g. Bulow and Rogoff (1986), Eaton and Gersowitz (1981), Fernandez and Kaaret (1987), Fernandez and Rosenthal (1988), Krugman (1985), Sachs (1983)), the question of why a debtors' cartel has not been formed has not been explored.

Our paper uses a strategic approach to explore the question of why a cartel may fail to emerge although it would be in all countries' interests for it to do so. We argue that the magnitude of the debt crisis implies that any delay in repayment of a country's debt to a bank imposes costs on the latter in addition to those directly associated with the loan itself, i.e. that there is also an indirect cost borne by the bank as a consequence of either greater uncertainty as to the bank's solvency or in the additional business that the bank must sacrifice. Hence, in a sense that will be made more rigorous below, a bank is more vulnerable before it has reached a repayment agreement with any one country than after. This greater "anxiety" on the part of the bank to reach an agreement implies that coordination of a group of countries' negotiating strategies may, by increasing the cost to the bank of not reaching an agreement, enable countries to better exploit the strategic "economies of scale" in the negotiating process. We show, however, that if countries are unable to credibly commit to joint bargaining, the unique equilibrium to our bargaining game is characterized by the bank achieving the same payoff as it would absent the strategic economies of scale. That is, the bank is effectively able to exploit each country's fear of betrayal by the others, i.e. that of being left isolated in the bargaining process, to reap the maximum benefit from negotiation. A natural result of this is that, in equilibrium, each country does worse than what it would achieve if the other country declared default and it was left as the sole bargaining party with the bank. We also examine the effect of different size countries on the bargaining problem. We show that, given two countries, one large and one small, the larger one benefits more from the equilibrium outcome than the smaller country.

The rest of the paper is organized as follows: In section 2 we describe the model and present the main result. We contrast the equilibrium that emerges if countries commit to act in the manner of a cartel, e.g. appoint only one negotiator, with the equilibrium obtained when commitment is not feasible. Section 3 is dedicated to the proof that the equilibrium obtained is subgame perfect and unique. Section 4 extends our results to the case of non-identical countries. Section 5 contains remarks and conclusions.

2. The Model

Our model has two countries (denoted by C_1 and C_2) and a Bank (denoted by B). Country i possesses a debt of size D_i, i=1,2. The Bank and countries are engaged in bargaining over the fractions of the debts that each country will service (and hence, on the fractions of the debts that will be forgiven). Once an agreement between a country and the Bank is reached, the remaining dedt is forgiven and payment is made. We assume that upon servicing its debt net of any forgiveness, a country receives a bonus of size $Z \leq D_i$ (the size of the bonus is the same for both countries). This bonus can be thought of as the utility derived from improved access to capital markets. More precisely, if F is the amount of debt forgiven by the Bank and X is the amount of debt serviced by a country, then the country's net payoff in the period in which the debt is serviced is $-X+Z$. (Note that everything is measured in units of the same single commodity.) By expressing the country's payoff in this manner, we are abstracting from any question associated with the tension between growth and repayment (e.g. the possible endogeneity of Z) and from any possibility for the country to smooth its payments over time (see Fernandez and Rosenthal (1988)). Moreover, by assuming the existence of only one bank, we are ignoring any of the problems that the banks may have in achieving collusion.

We will be examining the debt renegotiation problem in a bargaining model with complete (but imperfect) information that uses the solution concept of subgame-perfect equilibrium. Clearly, given that each country knows Z, a country will never pay the Bank more than Z (i.e., $X \leq Z$) since, if it does, its net payoff is negative while by not repaying at all it can guarantee itself a net payoff of zero. From here on we normalize Z to equal 1 and we let $0 \leq x_i \leq 1$ denote the fraction of Z paid by country i to the Bank.

Bargaining between the Bank and the two countries takes place over discrete time $t\epsilon\{1,2,\ldots\}$. In every odd period in which both countries are still negotiating, the Bank simultaneously announces a pair of offers x_1 and x_2, $0\le x_i\le 1$. Each country observes both offers and replies simultaneously (and independently) "Yes" or "No." When country i replies it does not know the reply of country j. We consider this to be the more realistic assumption over the alternative that one country is able to reply before the other. If C_i replies by "Yes", then it immediately pays the Bank the amount x_i (the Bank forgives $1-x_i$) and bargaining between the Bank and that country ends. If, however, C_i replies by "No" then time advances one period. In every even period in which both countries are still negotiating, the countries simultaneously (and independently) announce an offer y_i, $0\le y_i\le 1$. The Bank, after observing both offers, replies with either a "Yes" or a "No" to each offer. If the Bank says "Yes" to C_i, the country pays y_i to the Bank (who in turn forgives $1-y_i$) and the bargaining between C_i and the Bank terminates. If the Bank says "No" to either country, time advances one period. If the Bank and solely one country have reached an agreement, then thereafter bargaining between the remaining country and the Bank proceeds in the same alternating offers fashion, i.e. exactly as described in Rubinstein's (1982) model.

We can now discuss the possible outcomes of the game and the associated payoffs. If an agreement of size x_i is reached at some period T_i, C_i's payoff discounted to the beginning of the game is:

$$U_i(x_i,T_i) = \delta_C^{T_i-1}(1-x_i) \tag{1}$$

where δ_C is both countries' common discount factor, $0<\delta_C<1$.

The Bank's payoff from an agreement is a bit more complex. We find it reasonable to assume that as long as the debt problem exists, at least in its current size, the Bank's profit from activities not directly related to sovereign debt is smaller than what it would achieve absent the debt problem. A possible explanation of this is that the Bank's potential customers may believe that a large outstanding debt implies a high probability of bankruptcy (due, for example, to the existence of negative exogenous shocks to the Bank's profits) and that they therefore prefer to invest their money in other activities or institutions. Consequently, debt service by a country has two effects on the Bank's payoff: first, it increases the Bank's profits directly by allowing the Bank to use that money to offer new loans and to make new investments, and second, as a result of the decrease in the outstanding debt, there is an indirect effect on the Bank's profits due to the lower probability of bankruptcy and the concurrent increase in the number of customers willing to invest in the Bank. More formally, let $\bar{V}>0$ denote the difference between the Bank's per period profit in a state of the world in which the debt problem does not exist and the current state of the world (i.e., before the debt problem has been resolved with any of the two countries). For reasons that will be clear later, we prefer to rewrite $\bar{V}$ as αV. That is, $\bar{V}=\alpha V$, where $\alpha>0$ and $V>0$. Thus, a payment to the Bank (by either one or both countries) increases the Bank's per-period payoff via the "indirect effect" by an amount between zero and αV.

We introduce the following function:

$$A(x) = \begin{cases} \alpha x \text{ if } x \le V \\ \alpha V \text{ if } x > V. \end{cases} \tag{2}$$

$A(x)$ is the aforementioned "indirect effect" on the Bank's profit each period, as a result of total payments of size x. It is a strictly increasing function of x, the payments received by the Bank, as long as $x<V$. When $x\ge V$, it is

assumed that the Bank's potential depositors no longer fear the possibility of bankruptcy as a result of the debt crisis.

To simplify our presentation, we assume that the "direct effect" of an agreement of size x on the Bank's payoff is incurred solely in the period of agreement and that its size is exactly x. The Bank's stream of profits from an agreement x_i in period T_i and x_j in period T_j ($T_i \le T_j$) is therefore: zero in each period t $1 \le t < T_i$, $x_i + A(x_i)$ in period T_i, $A(x_i)$ in each period t $T_i < t < T_j$, $x_j + A(x_1 + x_2)$ in period T_j, and $A(x_1 + x_2)$ in each period $t > T_j$. The Bank's discounted payoff from a pair of agreements (x_i, T_i, x_j, T_j) is

$$V_B(x_i,T_i,x_j,T_j) = \delta_B^{T_i-1}[x_i+A(x_i)(1-\delta_B)^{-1}] + \delta_B^{T_j-1}[x_j+(A(x_1+x_2)-A(x_i))(1-\delta_B)^{-1}] \quad (3)$$

To illustrate the effect that the presence of two countries has on the bargaining game, we examine the game between C_i and the Bank given that the Bank has somehow already reached an agreement x_j with C_j. Using the method introduced by Shaked and Sutton (1984), it is easy to prove the following Lemma:

<u>Lemma 1</u>: Let $G_B(x_j)$ ($G_C(x_j)$) denote the sequential bargaining game between the Bank and C_i that commences with the Bank (C_i) making the first offer given that an agreement of x_j was previously reached with C_j. The unique subgame-perfect equilibrium for $0 < V < \delta_B(1-\delta_C)[1+\alpha-\delta_B\delta_C]^{-1}$ is an agreement reached in the first period of this game with the country's payment to the Bank given by:

$$R_B(x_j) = \begin{cases} \dfrac{1-\delta_C(1+\alpha(V-x_j))}{1-\delta_C\delta_B} & \text{if} \quad 0 \le x_j < V \\[2ex] \bar{R}_B = \dfrac{1-\delta_C}{1-\delta_C\delta_B} & \text{if} \quad V \le x_j \end{cases} \quad (4)$$

and

$$R_C(x_j) = \begin{cases} \dfrac{\delta_B(1-\delta_C) - \alpha(V - x_j)}{1 - \delta_C\delta_B} & \text{if } 0 \le x_j < V \\ \bar{R}_C = \dfrac{\delta_B(1-\delta_C)}{1-\delta_C\delta_B} & \text{if } V \le x_j \end{cases} \tag{5}$$

where $R_B(x_j)$, illustrated in Figure 1, denotes the amount paid by C_i to the Bank in $G_B(x_j)$ and $R_C(x_j)$, illustrated in Figure 2, is the amount paid by C_i in $G_C(x_j)$. Let R(x) denote the unique subgame-perfect equilibrium outcome to the Rubinstein game between solely one country and the Bank given that an agreement of x has previolsly been reached with the other country. Hence $R(x)=R_B(x)$ if bargaining commences with the Bank making the first offer, and $R(x)=R_C(x)$ if the country makes the first offer.

Throughout sections 2 and 3 we will assume that V lies in the range given in Lemma 1, i.e. that V is not very large. From (4) and (5) one can easily see that for any x_j the payment by country i to the Bank, in equilibrium, is larger if the Bank makes the first offer (i.e. $R_B(x_j)>R_C(x_j)$) and that in both cases the payment is increasing with x_j if $x_j<V$ and is constant if $x_j \ge V$. That is, the "better" the first agreement is for the Bank, the "tougher" it will be with the other country (i.e. its agreement with this country will also be "better"). The reasoning behind this is as follows. Suppose that the Bank has reached an agreement of size x_j with C_j and has not yet reached an agreement with C_i. As a result of this agreement the Bank's per-period profit increases by $A(x_j)$ from the period of that agreement onwards. Therefore, when bargaining with C_i, the Bank's fixed cost of delaying an agreement by one period is $\alpha V - A(x_j)$. This cost, however, is decreasing in x_j, the size of the

agreement reached with C_j. Note that $\bar{R}_B$, the maximum obtained by the Bank in this bargaining game, is also the unique subgame-perfect equilibrium outcome in Rubinstein's original bargaining game where the indirect effect (αV) is zero.

The discussion above provides intuition for why countries may do better by forming a cartel than by bargaining separately with the Bank. When each country bargains separately with the Bank, each faces the threat that the other country will reach an agreement with the Bank before it and thus place it in a weaker bargaining position. If, however, countries can form a cartel such that the Bank cannot reach an agreement first with one country and then with the other, i.e. if each country can commit to not accepting an offer unless the other country is also willing to accept the Bank's offer, the fear of isolation disappears. This may permit both countries to do better.

In order to establish a benchmark, before proceeding to derive our equilibrium we briefly discuss the results obtained in a bargaining game in which both countries are in cartel.

2.1 Cartel

Suppose that bargaining commences with both countries in a cartel and that each country's commitment to stay in the cartel is binding. We assume that the cartel simply appoints one negotiator who bargains on the amount of debt to be serviced by each of the two countries. Since the two countries have bonuses of equal size, we simply assume that the cartel bargains with the Bank over x, where $0 \le x \le 1$ is the amount to be paid by each of the two countries to the Bank (and 1-x is therefore the amount the bank forgives each country).

The bargaining process is one of alternating offers as described before, and it is assumed that the Bank makes the first offer. Solving for the subgame-perfect equilibrium of this game, one can show that an agreement is reached in the first period and that the amount paid by each country to the Bank is

$$R_k = \frac{1 - \delta_C - (\delta_C \alpha V)/2}{1 - \delta_C \delta_B} \quad . \tag{6}$$

2.2 The Equilibrium:

The main difference between the bargaining process in our model and that described in the previous Cartel section is that in our game, although the Bank bargains with both countries simultaneously, it is possible for the Bank to reach an agreement with only one country at a time. This seems to be the most reasonable assumption in the case of sovereign debt renegotiation, since it is difficult to think of a mechanism that could ensure commitment to solely joint bargaining.

Proposition: The bargaining game has a unique subgame-perfect equilibrium in which an agreement with both countries is reached in the first period and the Bank obtains $\bar{R}_B$ from each country.

Note that the proposition implies that the payoff obtained by each country when unable to precommit to cartel formation is less than in (6). Moreover, the inability to commit to a cartel condemns the countries to the same payoff that they would have received if there did not exist any economy to scale in negotiation. Specifically, they are reduced to the same payoff that they would receive in a bargaining game in which there is no indirect effect on the Bank's profit. The fundamental implication of this result is that there does not exist another equilibrium whereby countries are able to collude in some other fashion, for example, by bargaining sequentially and thereby allowing the first country to obtain $x > 1-\bar{R}_C$ and hence the second to obtain more than $1-R(\bar{R}_C)$. This sequential bargaining would not contradict the bargaining procedure. It would have the "second" country in the sequential bargaining process, say C_j, make a "ridiculously" small offer to the Bank and the "first" country make a more reasonable offer, but smaller than $\bar{R}_C$. The Bank would reject C_j's offer and accept C_i's, thus allowing each country to obtain a better outcome than $\bar{R}_B$. These strategies, however, as will be shown, are not subgame perfect.

Horn and Wolinsky (1988) use a strategic bargaining model to examine the conditions under which two distinct unions bargaining with the same firm are better off by appointing a sole negotiator to represent both unions or by bargaining separately. In their model, however, unions are assumed to be able to credibly commit to joint representation and, if unions bargain separately, bargaining is assumed to be sequential (in each period the firm can negotiate with only one of the two unions). Consequently, the main issue that we are interested in exploring—the stability of a collusive equilibrium—by assumption, does not arise in their work. The absence of credible commitment mechanisms cannot, in and of themselves, account for the absence of any collusive equilibrium is since results obtained from game theory show that collusive behavior can be sustained even without the possibility of binding commitments. For example, in infinitely repeated games, the threat of driving a player to his min max payoff is able, for high enough discount factors, to sustain any individually rational feasible payoff (see Aumann 1980 for a survey of the literature). This result also holds in bargaining games closely related to the one we study in our paper. In particular, in Sutton (1986) an example attributed to Shaked is given of a three person bargaining game in which two of the players are able to collude against the remaining player and extract all the surplus from the game.

Thus, unlike in the literature discussed above, in our model the two countries are unable to collude against the Bank. Furthermore, in equilibrium each country pays more than what it would pay if the other country declared default, i.e more than:

$$R_B(0) = \frac{1 - \delta_C(1 + \alpha V)}{1 - \delta_C\delta_B}$$

3. Proof of Proposition:

The proof is divided into two parts. In Part A we construct a subgame-perfect equilibrium that generates this outcome and in Part B we prove that this subgame-perfect equilibrium is unique.

Part A

The subgame-perfect equilibrium outcome of the bargaining process between only one country and the Bank, given that an agreement has already been reached with the other country, is given by Lemma 1. Therefore, we only discuss the strategies for the case where both countries are still in the game.

Let H_t denote the set of possible partial histories of play h_t through the end of period t-1; i.e., $H_1=\emptyset$ and for $t\geq2$, h_t specifies the sequence of offers and replies made by the players at every period $\tau<t$. Let h_{t+} denote a partial history of play up to and including the offer(s) made by the player(s) at the beginning of period t and let H_{t+} denote the set of all such histories. (If, for example, $h_{t+}\in H_{t+}$ and t is odd, then h_{t+} specifies the offers x_1 and x_2 made by the Bank to both countries in period t in addition to the partial history represented by h_t.)

A strategy for the Bank, denoted by σ_B, is a sequence of functions $\{B_t\}^{\infty}_{t=1}$ such that: for t odd, $B_t:H_t\to[0,1]^2$ and for t even, $B_t:H_{t+}\to\{Yes, No\}^2$
A strategy for C_i denoted by σ_i, is a sequence of functions $\{C_{it}\}^{\infty}_{t=1}$ such that: for t odd, $C_{it}:H_{t+}\to\{Yes, No\}$ and for t even, $C_{it}:H_t\to[0,1]$
Any strategy combination $(\sigma_B,\sigma_1,\sigma_2)$ generates an outcome and hence a payoff to each of the players. A Nash equilibrium is a strategy combination such that no player can benefit from a unilateral deviation to another strategy. A subgame-perfect equilibrium is a strategy combination that induces a Nash equilibrium in every subgame. That is, it helps to rule out those equiliria based on "incredible" threats.

Let $x_t = (x_{1t},x_{2t})$ denote the offers of the Bank at the beginning of period t, when t is odd. Let y_{it} denote the offer of country i at the beginning of period t, when t is even, and let $y_{mt} = \max\{y_{1t},y_{2t}\}$. The strategies described below depend only on the last offer and otherwise are history independent; hence we will drop the subscript t.

The following strategies constitute an equilibrium.

The Bank's strategy is:

$$B_t(h_t) = (\bar{R}_B, \bar{R}_B), \text{ for } t \text{ odd, and}$$

$$B_t(h_t) = \begin{cases} (\text{No, No}) & \text{if } y_m + A(y_m) + \delta_B[R_B(y_m) + (A(y_m + R_B(y_m)))/(1-\delta_B)] < \delta_B \bar{W} \\ (\text{Yes, No}) & \text{if } y_1 \geq y_2,\ y_1 + A(y_1) \\ & \quad + \delta_B[R_B(y_1) + (A(y_1 + R_B(y_1)))/(1-\delta_B)] \geq \delta_B \bar{W} \text{ and } y_2 < \bar{R}_C \\ (\text{No,Yes}) & \text{if } y_2 > y_1 \text{ and the same conditions as above with} \\ & \quad y_1 \text{ and } y_2 \text{ interchanged} \\ (\text{Yes,Yes}) & \text{otherwise} \end{cases} \tag{7}$$

for t even.

Each country i's strategy is:

$$C_{it}(h_{t+}) = \begin{cases} \text{Yes} & \text{if } x_i \leq R_B(x_j) \\ \text{No} & \text{otherwise} \end{cases}$$

for t odd, $j \neq i$ and (8)

$$C_{it}(h_t) = \bar{R}_C \text{ for } t \text{ even.}$$

Note that the equilibrium play that results from the above strategies has bargaining ending in the first period with each country paying the Bank $\bar{R}_B$, i.e. the Bank's payoff is:

$$\bar{W} = 2\bar{R}_B + \alpha V/(1-\delta_B) \tag{9}$$

In order to show that these strategies constitute a subgame-perfect equilibrium we need only consider the first two periods of the game (since the strategies repeat themselves every other period). Suppose, therefore, that the game has reached period 2, that the Bank has received offers (y_1, y_2), and assume that from period 3 on (if reached) the players will follow the strategies described above. Thus, by saying (No,No) the Bank can guarantee itself the payoff $\delta_B\bar{W}$.

In order to see that the Bank's strategy is indeed a best response to any offer (y_1,y_2) we make use of Figure 3. In this figure, the horizontal axis measures C_1's offer and the vertical axis C_2's offer. Any point below (above) the line $\delta_B\bar{W}$ represents a pair of offers (y_1,y_2) for which saying (Yes,Yes) will give the Bank a payoff less (greater) than $\delta_B\bar{W}$. The other two dark lines are $R_C(y_1)$ and $R_C(y_2)$. Note that the Bank is indifferent between receiving $R_C(x)$ in some period and $R_B(x)$ one period later. $R_C(y_1)$ gives the unique subgame-perfect equilibrium outcome (appropriately discounted, i.e. the Bank is indifferent between $R_C(y_1)$ this period and $R_B(y_1)$ next period) to the game between solely C_2 and the Bank, given that in the previous period an agreement y_1 was reached between the Bank and C_1. $R_C(y_2)$ does the same for C_1, given a previous agreement between the Bank and C_2. Consider, for example, the pair of offers represented by the point a. If the Bank says "Yes" to both countries it receives less than $\delta_B\bar{W}$. If, however, the Bank says "Yes" to C_1 and "No" to C_2 its payoff is identical to the payoff obtained by saying (Yes, Yes) to the pair of offers represented by point c, i.e. greater than $\delta_B\bar{W}$. If, on the other hand, the Bank says "No" to C_1 and "Yes" to C_2, its payoff is identical to the payoff obtained by accepting the pair of offers represented by point b, i.e., below $\delta_B\bar{W}$. Consequently, the Bank's best response is (Yes,No). In a similar way one can show that for any point in region A the Bank's best response is (No,No), in region B it is (Yes,No), in region C it is (No,Yes), and in region D it is (Yes,Yes). This is precisely the strategy described in (7).

Moving to the beginning of period 2, we must now show that if C_i offers $\bar{R}_C$, it is indeed a best response for C_j to also offer $\bar{R}_C$, given that the Bank responds according to (7). Suppose, therefore, that C_1 offers $\bar{R}_C$. If $y_2 \geq \bar{R}_C$, the Bank will accept y_2, which implies that C_2 should never offer more than $\bar{R}_C$. Can C_2 do better by offering $y_2 < \bar{R}_C$? In this case the Bank will accept only C_1's offer and C_2 will obtain $1-\bar{R}_B$ next period, which is clearly inferior. Hence, C_2 is better off offering $\bar{R}_C$.

We now turn to period 1. Here we must show that for any pair of offers (x_1,x_2), C_j cannot do better than responding according to (8) given that C_i responds according to (8). Suppose that x_j is such that $x_i \leq R_B(x_j)$. By (8), C_i says "Yes". If C_j also says "yes" it obtains $1-x_j$ this period, whereas if it says "No" it obtains $1-R_C(x_i)$ next period. If $x_j \leq R_B(x_i)$, then $1-x_j \geq 1-R_B(x_i)$, and since C_j is indifferent between $1-R_B(x_i)$ this period and $1-R_C(x_i)$ next period, C_j is better off by saying "Yes", (which is exactly what the strategy indicates). If, however, $x_j > R_B(x_i)$, C_j is better off saying "No". In a similar way it can be shown that if $x_i > R_B(x_j)$, C_j's response as indicated by her strategy is also optimal. The pair of offers $(\bar{R}_B,\bar{R}_B)$ has the property that if one country replies yes, it is also optimal for the other country to reply yes.

It is left to be shown that the Bank cannot do better than to offer $(\bar{R}_B,\bar{R}_B)$ in period 1, given that the countries respond according to (8) and given that if both countries say "No" the Bank obtains $\bar{R}_C$ from each country at the beginning of period 2. Figure 4 illustrates the responses of both countries, as given by (8), to any pair of offers (x_1,x_2). The horizontal axis measures the Bank's offer to C_1; the vertical axis measures the Bank's offer to C_2. The two dark lines are $R_B(x_1)$ and $R_B(x_2)$. The first gives the unique subgame-perfect equilibrium outcome (appropriately discounted, i.e. C_2

is indifferent between offering $R_B(x_1)$ that period and $R_C(x_1)$ one period later) to the game solely between C_2 and the Bank given that in the previous period an agreement x_1 was reached between C_1 and the Bank. $R_B(x_2)$ does the same for C_1 given a previous agreement between the Bank and C_2. In region A both countries say "Yes" and the Bank obtains x_1+x_2. In region B, C_1 says "No" and C_2 says "Yes", so the Bank obtains x_2 from C_2 this period and $R_C(x_2)$ from C_1 next period. In region C, C_1 says "Yes" and C_2 says "No", therefore the Bank obtains x_1 this period and $R_C(x_1)$ next period. In region D both countries say "No" and the Bank obtains $2\bar{R}_C$ next period.

We can now use Figure 4 to show that $(\bar{R}_B,\bar{R}_B)$ is the best the Bank can do. Since $(\bar{R}_B,\bar{R}_B)$ is in region A and given that it maximizes the Bank's payoff within this region, the Bank will certainly never deviate to any other pair in this same region. Similarly, since getting $2\bar{R}_B$ this period is better for the Bank than getting $2\bar{R}_C$ next period, the Bank will never deviate to any pair in D. The question remains, therefore, whether the Bank can do better by deviating to a pair in B or C? Suppose, for example, that the Bank deviates and offers the pair represented by the point b in region B. In such a case, the Bank's payoff will be the same as its payoff from offering the pair represented by the point a in region A, and this payoff is less than the payoff from $(\bar{R}_B,\bar{R}_B)$. The same can be shown for any other deviation to region B or C. This completes this part of the proof.

Part B

(The proofs in this section assume that the respective sups are actually attained. This is in no way essential to our results and is done only to avoid excess notation; otherwise, we would have to look at subgames in which the sup$-\epsilon$, $\epsilon>0$ and close to zero, is attained. The same proofs would then go through since they all rely on strict inequalities.)

<u>Lemma 2</u>: Let $\tilde{\sigma}=(\tilde{\sigma}_B,\tilde{\sigma}_1,\tilde{\sigma}_2)$ be any subgame-perfect-equilibrium strategy combination. Let Q be the sup of the sum of the shares obtained by the Bank in any subgame determined by $\tilde{\sigma}$ that commences with both countries still negotiating. The share obtained from country i when Q is obtained, Q_i, i=1,2 $(Q=Q_1+Q_2)$, is not greater than $\bar{R}_B$.

<u>Proof</u>: Suppose not. Hence (allowing without any loss of generality $Q_1 \geq Q_2$) either $Q_1 \geq Q_2 > \bar{R}_B$ or $Q_1 > \bar{R}_B \geq Q_2$. Suppose that in a subgame in which Q is obtained, Q_2 precedes Q_1, i.e. agreement with country 2 is obtained at least one period prior to that with country 1. Then, by Lemma 1, $Q_1 = R(Q_2) \leq R_B(Q_2) \leq \bar{R}_B$, which contradicts the assumption on the size of Q_1.

Suppose instead that in the above-mentioned subgame agreement is reached simultaneously with both countries. Note first that this cannot occur during C's (the countries') turn since there always exists an $\epsilon > 0$ that satisfies
$z_1 + Q_2 + \alpha V/(1-\delta_B) > \max(\delta_B(Q_1 + Q_2 + \alpha V/(1-\delta_B)),\ Q_2 + \delta_B R_B(Q_2) + \alpha V/(1-\delta_B))$
where $z_1 = Q_1 - \epsilon \geq \bar{R}_B$, such that C_1 profitably deviates and offers the Bank z_1 which the Bank accepts. If the Bank were to reject this offer then, by also rejecting C_2's offer, it would obtain at most Q_1+Q_2 next period (the payoff from this is given by the first expression in the parenthesis), or, if it rejected only C_1's offer and accepted C_2's offer, it would obtain $R_B(Q_2) < Q_1$ from C_1 next period (the payoff from following this strategy is given by the second expression). It is easy to verify that there exists $\epsilon > 0$ such that the Bank prefers to immediately accept C_1's offer of z_1.

If simultaneous agreement occurs during B's turn, a profitable deviation for C_1 is to reject the offer and in the following period to offer $R_C(Q_2)$. Since, as is clear in Figure 1, $\delta_C(1-R_C(Q_2)) \geq (1-\bar{R}_B) > 1-Q_1$, this constitutes a profitable deviation for C_1. This argument also establishes that the $\sup(x_1)$ in every simultaneous agreement (x_1,x_2) reached on B's turn must be no greater than $\bar{R}_B$ since otherwise one country will always find it to its advantage to reject the Bank's offer given that the other country accepts.

Lastly, suppose that in a subgame in which Q is obtained, Q_1 precedes Q_2 (hence $Q_2=R(Q_1)$). If this play begins during C's turn, then there always exists an $\epsilon>0$ that satisfies:

$$z_1 + \delta_B\bar{R}_B + \alpha V/(1-\delta_B) > \delta_B \max\{(2\bar{R}_B + \alpha V/(1-\delta_B)), (Q_1 + \delta_B\bar{R}_C + \alpha V/(1-\delta_B))\}$$

such that a profitable deviation for C_1 is to change its offer to $z_1=Q_1-\epsilon\geq\bar{R}_B$. If the Bank rejected this offer then, if next period it achieved a simultaneous agreement, the Bank could at most obtain $2\bar{R}_B$. If, instead, the Bank achieved a sequential agreement then again at most it could otain Q_1 followed by $R_C(Q_1)=\bar{R}_C$. It is easy to verify that there always exists an $\epsilon>0$ such that the Bank prefers to accept z_1 to either of the above alternatives.

If the play of the sequential agreement begins on B's turn, a profitable deviation by C_1 is to reject the Bank's offer and the following period to offer the Bank $y_1=1-\epsilon-(1-Q_1)/\delta_C>\bar{R}_C$, $\epsilon>0$ (with further restrictions on ϵ given below). Note that $y_1+\epsilon$ is simply the offer that makes the country indifferent between obtaining $1-Q_1$ this period and $1-y_1-\epsilon$ next period. To prove that this is a profitable deviation is rather more complicated than in the preceding cases, so we shall proceed in several steps. First note that $Q_1>\bar{R}_B$ implies that $y_1+\epsilon>\bar{R}_C$, $\forall\epsilon\geq0$ (a few algebraic manipulations establish this fact). We can now show that the Bank will accept an offer of $y_1+\epsilon$. Let us begin by examining the least favorable scenario for this conjecture. Suppose that this deviation leads to the Bank rejecting both countries' offers. Then next period the best that the Bank can do is to obtain the original sequential agreement of Q_1 followed by $R_C(Q_1)=\bar{R}_C$. Thus we must show that

$$y_1 + \epsilon + \alpha V/(1-\delta_B) + \delta_B\bar{R}_B > \delta_B[Q_1 + \alpha V/(1-\delta_B) + \delta_B\bar{R}_C].$$

Subtracting the expression on the RHS from both sides and recalling $\bar{R}_C=\delta_B\bar{R}_B-\delta_B(1-\delta_C)/(1-\delta_B\delta_C)$ yields, after a few manipulations, $\bar{R}_C(1-\delta_B{}^2)+\alpha V + [Q_1(1-\delta_B\delta_C)+\delta_C-1]/\delta_C > 0$. The first two terms are obviously positive and the last term can also be shown to be so by recalling that $Q_1>\bar{R}_B$ and manipulating this inequality. Thus if C_1 chooses an $\epsilon>0$ such that $y_1>\bar{R}_C$ and such that the above inequality (minus the ϵ) continues to hold, then z_1 is a profitable deviation for C_1. Suppose, however, that the Bank does not reject C_2's offer following the deviation by C_1. Then it must also accept C_1's offer of y_1 since by rejecting only C_1's offer, the most that the Bank can obtain the following period is $\bar{R}_B$ and, given that $y_1>\bar{R}_C$, the Bank strictly prefers y_1. This establishes that there always exists an $\epsilon>0$ such that a profitable deviation by C_1 is possible.||

Lemma 3: The sup of the share obtained by each country, $1-q_1$, is not greater than $1-\bar{R}_C$ in every subgame resulting from $\tilde{\sigma}$ that commences with both countries still negotiating.

Proof: Suppose not. Hence (allowing $1-q_1\geq1-q_2$) either $1-q_1\geq1-q_2>1-\bar{R}_C$ or $1-q_1>1-\bar{R}_C\geq1-q_2$. Examine the subgame in which the offer of q_1 is obtained. Let (q_1,x_2) be the respective shares obtained by the Bank from C_1 and C_2 in this subgame. Suppose the agreement with C_2 precedes that with C_1. Then, by Lemma 1, $q_1=R(x_2)$. But $1-R(x_2)\leq1-R_C(x_2)<1-x_2 \quad \forall x_2<\bar{R}_C$. Hence, if $x_2<\bar{R}_C$, then $1-x_2>1-q_1$ which contradicts the assumption on the size of q_1. If $x_2\geq\bar{R}_C$, then $1-q_1=1-\bar{R}\leq 1-\bar{R}_C$, another contradiction.

Suppose, instead, that in the above-mentioned subgame, agreement is reached simultaneously with both countries. If this occurs on B's turn, then $x_2=R_B(q_1)$. In this case a profitable deviation for the Bank is to offer $z_1=1-\delta_C(1-q_1)-\epsilon>q_1$ to C_1, $\epsilon>0$, and $R_B(z_1)$ to C_2. Both countries will accept since the utility from z_1 obtained this period is strictly greater for C_1 than achieving an offer of at most q_1 one period later (C_2 accepts by Lemma 1). If simultaneous agreement occurs on C's turn, then a profitable deviation for the Bank is to accept C_2's offer and reject that of C_1's and next period to offer $R_B(x_2)$ to C_1. As is clear from Figure 2, the Bank prefers to pay $R_B(x_2)$ next period to q_1 this period $\forall q_1 \leq \bar{R}_C$. This argument establishes that any simultaneous agreement (x_1,x_2) reached on a C turn must have $\min(x_1,x_2) \geq \bar{R}_C$.

Finally, suppose that in the subgame in which q_1 is attained, agreement with C_1 precedes that with C_2. Consequently, in the period in which C_1 is offering q_1, C_2 must be making an offer $\hat{x}_2 \leq q_1$ that is being rejected by the Bank. Note first that if the agreement with C_1 occurs in B's turn, a profitable deviation for the Bank is, as in the simultaneous case, to offer $z_1 = 1-\delta_C(1-q_1)-\epsilon < q_1$, $\epsilon>0$, to C_1 and $R_B(z_1)$ to C_2. Once again, both countries will accept these offers.

If instead the agreement with C_1 occurs in C's turn, then if (i) $q_1 \geq V$ a profitable deviation exists for C_2, and if (ii) $q_1 < V$ a profitable deviation exists for the Bank. If $q_1 \geq V$ then, by Lemma 1, C_2 receives $1-\bar{R}_B$. Consequently, a profitable deviation for C_2 is to change its offer to $\bar{R}_C$. The Bank will accept this offer since, by Lemma 2, the sup to the sum of shares that can be obtained by the Bank in any subgame determined by a subgame-perfect-equilibrium strategy combination is less than $2\bar{R}_B$. That C_2's offer dominates $2\bar{R}_B$ can be seen by noting that the Bank's payoff from accepting C_2's new offer is $\bar{R}_C+\delta_B\bar{R}_B+\alpha V/(1-\delta_B)$ which is strictly greater than $\delta_B(2\bar{R}_B+\alpha V/(1-\delta_B))$, the sup of the payoff to the Bank from rejecting this offer.

If $q_1<V$, there exists an $\epsilon>0$ such that a profitable deviation for the Bank is to reject C_1's offer and next period to offer $1-\delta_C(1-q_1)-\epsilon$ to C_1 and $\min(R_B(1-\delta_C(1-q_1)-\epsilon, \bar{R}_B)$ to C_2. Both countries will accept these offers since, by rejecting the offer, C_1 can obtain at best q_1 next period and it is indifferent between that and obtaining an offer of $1-\delta_C(1-q_1)$ this period (and hence strictly prefers an offer of $1-\delta_C(1-q_1)-\epsilon$). C_2 accepts since, by Lemma 1, it would obtain $\min(R_C(1-\delta_C(1-q_1)-\epsilon),\bar{R}_C)$ next period by rejecting the offer. Thus it remains to show that the Bank's payoff from following this deviation is greater than the payoff from obtaining q_1 the first period and $R_B(q_1)$ the following period, i.e. greater than

$$(1+\alpha)q_1 + \delta_B[R_B(q_1) + \frac{\alpha V}{1-\delta_B}]. \tag{10}$$

Note that the Bank obtains αV since $R_B(x)>V$ $\forall x\geq 0$. We divide the proof into two parts:

(i) $1-\delta_C(1-q_1) < V$ (i.e. $\min(R_B(1-\delta_C(1-q_1)-\epsilon),\bar{R}_B)=R_B(1-\delta_C(1-q_1)-\epsilon)$).

Thus, we must show that (10) is smaller than

$$\delta_B[1-\delta_c(1-q_1) + R_B(1-\delta_c(1-q_1)) + \frac{\alpha V}{1-\delta_B}]. \tag{11}$$

(11) - (10) yields, after a few manipulations

$$(\delta_B(1-\delta_C)(1-\delta_B\delta_C+\alpha\delta_C) - q_1[(1-\delta_B\delta_C)^2 + \alpha(1-\delta_B\delta_C{}^2)]) \frac{1}{1-\delta_B\delta_C}$$

Thus, in order for this deviation to be profitable we must have

$$\frac{\delta_B(1-\delta_C)(1-\delta_B\delta_C+\alpha\delta_C)}{(1-\delta_B\delta_C)^2 + \alpha(1-\delta_B\delta_C{}^2)} > q_1$$

$1-\delta_C(1-q_1)<V$ implies a maximum possible value for q_1, $\bar{q}_1$ (obtained when $V=\frac{\delta_B(1-\delta_C)}{1-\delta_B\delta_C+\alpha}$), given by

$$\bar{q}_1 < \frac{(1-\delta_C)[\delta_B(1+\delta_C)-(1+\alpha)]}{\delta_C(1-\delta_B\delta_C+\alpha)}$$

Thus it only remains to show that

$$\frac{\delta_B(1-\delta_C)(1-\delta_C+\alpha\delta_C)}{(1-\delta_B\delta_C)^2 + \alpha(1-\delta_B\delta_C^2)} > \frac{(1-\delta_C)[\delta_B(1+\delta_C)-(1+\alpha)]}{\delta_C(1-\delta_B\delta_C+\alpha)}$$

A few manipulations of this inequality yields:

$$(1-\delta_B)(1-\delta_B\delta_C)^2 + \alpha[(1-\delta_B\delta_C)^2 + (1-\delta_B+\alpha)] > 0$$

(ii) $1-\delta_C(1-q_1) \geq V$ (i.e. $\min\,(R_B(1-\delta_C(1-q_1)-\epsilon),\bar{R}_B) = \bar{R}_B$)

Now we must show that (10) is smaller than

$$\delta_B[1-\delta_C(1-q_1) + \bar{R}_B + \frac{\alpha V}{1-\delta_B}] \tag{12}$$

(12) - (10) yields, after a few manipulations

$$\frac{\delta_B(1-\delta_C)(1-\delta_B\delta_C) + q_1(\delta_B\delta_C-1-\alpha)(1-\delta_B\delta_C) + \alpha\delta_B\delta_C(V-q_1)}{1 - \delta_B\delta_C}$$

In order for the Bank's deviation to be profitable, we must show that the above expression is positive. Let us take the worst scenario possible for this case, i.e. $q_1=V$. Then we must show that

$$\delta_B(1-\delta_C)(1-\delta_B\delta_C) + V(\delta_B\delta_C-1-\alpha)(1-\delta_B\delta_C)$$

is strictly positive. But this is tantamount to requiring

$$\frac{\delta_B(1-\delta_C)}{1-\delta_B\delta_C + \alpha} > V$$

which holds by assumption.||

Lemma 4: In every B-subgame resulting from $\tilde{\sigma}$ in which both countries are still negotiating, the Bank offers both countries $\bar{R}_B$ simultaneously and both countries accept. In every C-subgame resulting from $\tilde{\sigma}$ in which both countries are still negotiating, each country offers the Bank $\bar{R}_C$ which the latter accepts.

Proof: By Lemma 3 the sup of the share that a country may receive in any C-subgame is no larger than $1-\bar{R}_C$. By Lemma 2 the sup of the sum of the shares obtained by the Bank is no greater than $2\bar{R}_B$. Consequently the Bank will simultaneously offer each country $\bar{R}_B$ and accept a simultaneous offer of $\bar{R}_C$; the countries will each make an offer of $\bar{R}_C$ and accept a simultaneous offer of $\bar{R}_B$.||

This establishes that there is a unique subgame-perfect equilibrium to our game. Q.E.D.

4. The Effect of Different Bonus Size:

In the previous sections we assumed that the two countries receive identical benefits from repaying their respective debts. We now relax this assumption in order to examine the effect of unequal bonus size on the equilibrium obtained. Consequently, when we perform the initial normalization, $Z_1 \neq Z_2$. Assume, therefore, that as before, the size of Z_1 is 1 but that C_2's bonus is now of size $Z_2<1$. If Z_2 is fairly close to 1, more specifically if:

$$V < Z_2 \frac{\delta_B(1-\delta_C)}{1 + \alpha - \delta_B\delta_C} ,$$

then the share of debt paid by each country to the Bank, in equilibrium, is $\bar{R}_B$. If, however, Z_2 is small, in particular, if

$$Z_2\bar{R}_B < V,$$

then the share paid by C_2 to the Bank is still $\bar{R}_B$ but the share paid by C_1 is now only $R_B(Z_2\bar{R}_B)<\bar{R}_B$. Thus, if country benefits are of different sizes, the smaller country pays a larger fraction of its debt to the Bank. Moreover, the larger (smaller) country pays a smaller (larger) fraction than it would have paid had the other country's benefit been of equal size.

The reason for the above result is as follows: suppose that the Bank has, somehow, reached an agreement of some fraction x with one country and is now bargaining with the other. The Bank's per-period payoff until reaching the second agreement is A(x) if the first agreement has been reached with the larger country and only $A(Z_2x)$ if the first agreement has been reached with the smaller one. This implies that if bargaining is sequential and the outcome of the first bargaining is a fixed fraction x, the gains from bargaining second are greater for the large country than for the small one. Note that in our model, in equilibrium, each country receives the same payoff as it would receive by waiting one more period and bargaining alone with the Bank. Consequently, it must be that the large country pays a smaller fraction of its debt to the Bank.

5. Remarks and Conclusions:

There may be several reasons why countries have not formed a debtors' cartel. Firstly, it is difficult to think of any debtor coalition succeeding without the participation of at least two of the three large debtors: Mexico, Brazil, and Argentina. The first two, however, may have important motives not to participate in a debtors' cartel; Mexico has a special relationship with the USA which it may not wish to jeopardize and, moreover, may believe that this can be more effectively exploited by remaining outside any coalition. Brazil's model of economic growth is considerably more outwardly oriented than the average Latin American economy and is consequently that much more susceptible to any trade reprisals that may be taken by the commercial banks.

On the whole, it would seem that Argentina has the most to gain by participating in a cartel, and indeed it has been Argentina which has most often instigated attempts to build coalitions.

A second hypothesis would relate the non-emergence of a cartel to the fragility of the governments in question or even to the non-representative character of the latter. A debtors' cartel which resulted in trade sanctions being imposed for some length of time may, in the short run, prove to be more costly than repaying the renegotiated debt. To the extent that the government is more impatient than society (it may lose elections or be overthrown), the government will discount the future by more than society. Furthermore, to the extent that governments are simply protecting the interests of a particular group or class, they may not enter into a cartel although it may be profitable from a societal viewpoint to do so.

Rather than focus on the asymmetries that may exist among countries or on the failure of the government to be a representative agent of society's interests, our paper concentrates on strategic reasons to explain the failure of a cartel to form despite the fact that all countries would benefit from doing so. The greater importance attached by the Bank to the first agreement reached implies that each country is afraid of being second in the chain of agreements. We have shown that despite the existence of benefits from a cartel, debtor countries are unable to successfully collude against the Bank in the absence of a commitment mechanism. (Moreover, it seems highly unlikely that in reality any credible commitment mechanisms are available - What penalties exist to enforce a contract among nations?) The Bank is effectively able to "isolate" each country so that the latter act as though they would each be second in reaching an agreement. The Bank obtains the payoff that it would receive in the absence of any indirect effect on profits.

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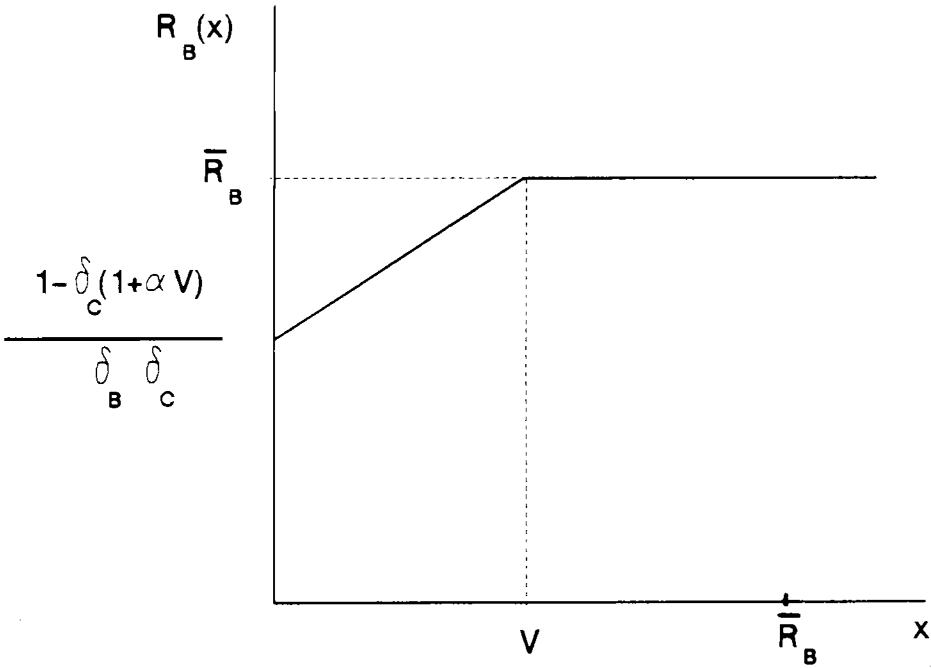

FIGURE 1

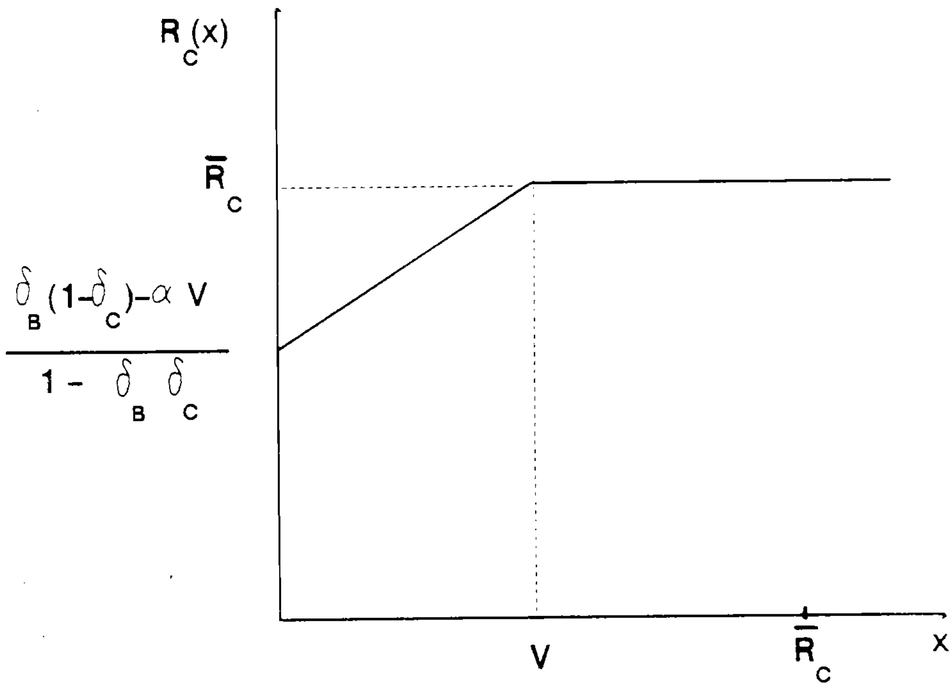
$R_C(x)$
$\bar{R}_C$
$\frac{\delta_B(1-\delta_C)-\alpha V}{1-\delta_B\delta_C}$
V
$\bar{R}_C$
x

FIGURE 2

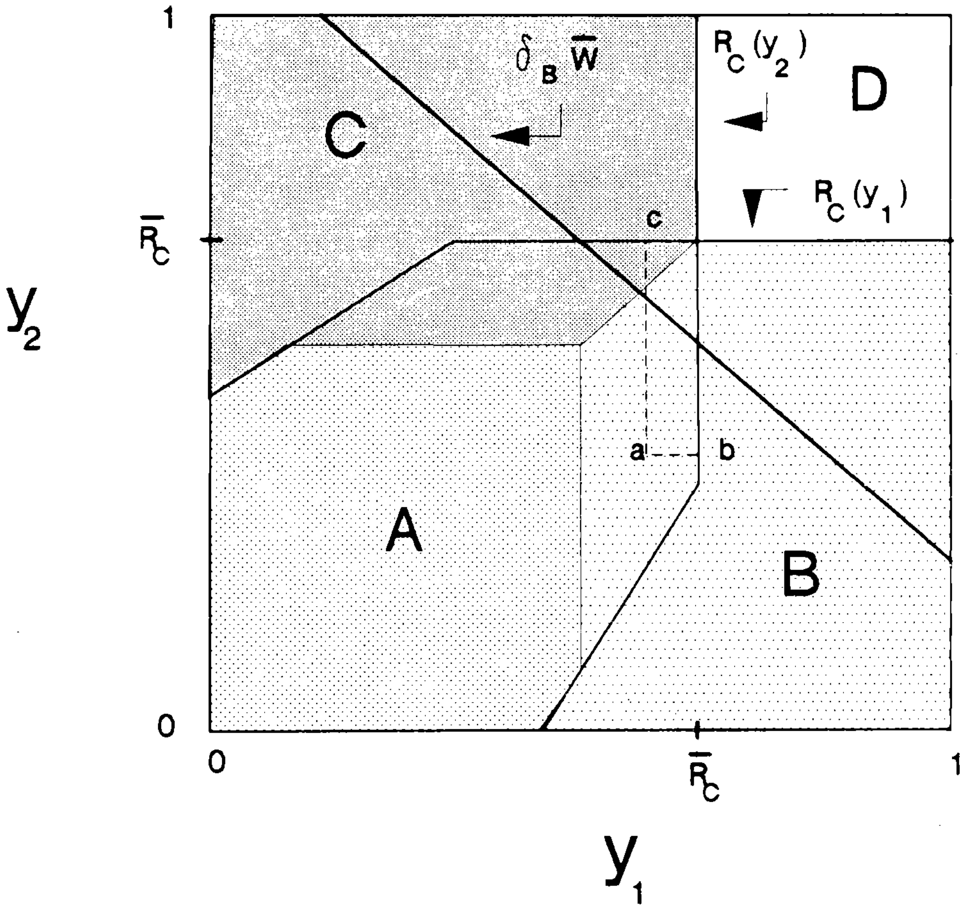

FIGURE 3

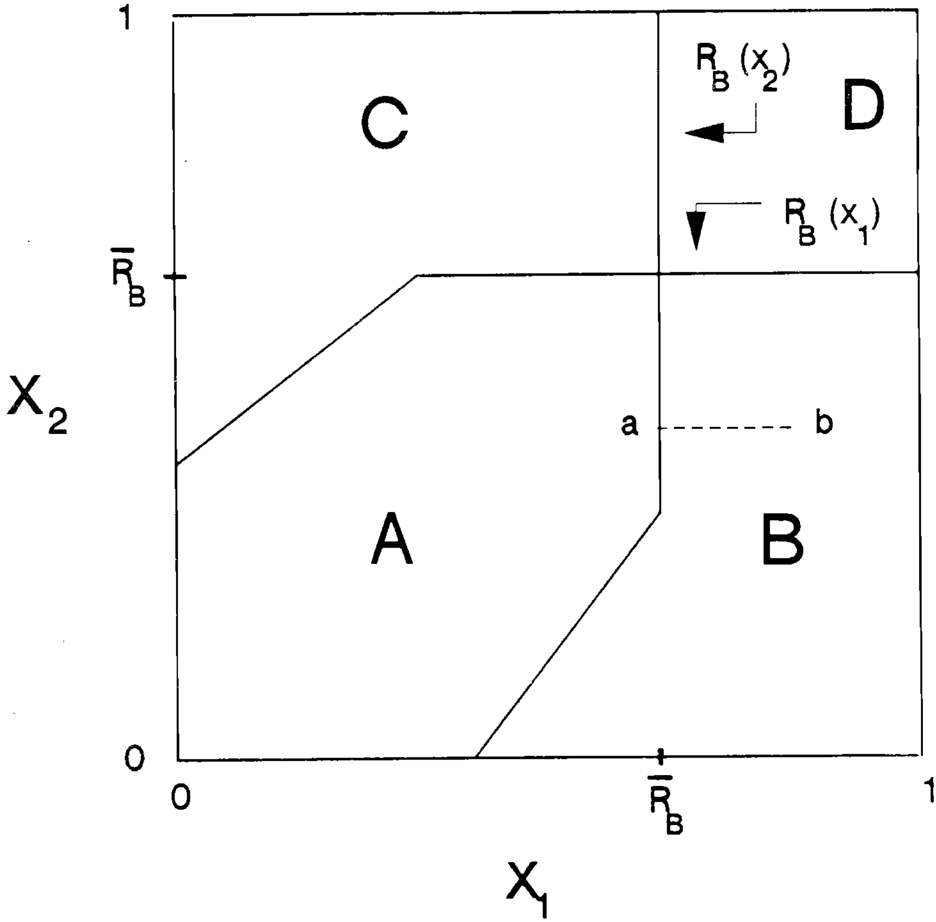

FIGURE 4