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ABSTRACT

Recent work has demonstrated that existing solutions of the unemployment volatility puzzle are at odds with the procylicality of the opportunity cost of employment, the cyclicality of wages, and the volatility of risk-free rates. We propose a model of business cycles that is immune to these critiques by incorporating two key features. First, we allow for preferences that generate time-varying risk over the business cycle to account for observed fluctuations in asset prices. Second, we introduce human capital acquisition consistent with the evidence on how wages grow with experience in the labor market. Our model reproduces the observed fluctuations in unemployment because hiring a worker is a risky investment with long-duration returns. As in the data, the price of risk in our model sharply increases in recessions. The benefit from hiring new workers therefore greatly declines, leading to a large decrease in job vacancies and an increase in unemployment of the same magnitude as in the data. We show that our results extend to versions of the model that include physical capital, a life cycle for workers, and alternative preference structures common in the asset-pricing literature.

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A data appendix is available at http://www.nber.org/data-appendix/w29794

1 Introduction

The most important theoretical contribution of search models of the labor market to the study of business cycles is that they interpret involuntary unemployment as an equilibrium phenomenon. The main insight of these models is that involuntary unemployment can arise even without any assumed inefficiencies in contracting, such as rigid wages. Despite its great promise, though, Shimer (2005) showed that the textbook search model cannot generate anywhere near the observed magnitude of the fluctuations in the job-finding rate and unemployment in response to shocks of plausible magnitude. A large body of work has attempted to address this *unemployment volatility puzzle*. Recent work, however, has demonstrated that existing attempts are inconsistent with key features of business cycles, namely, the procylicality of the opportunity cost of employment, the cyclicality of wages, and the volatility of risk-free rates. Hence, in this precise sense, the puzzle has not yet been solved.

In this paper, we solve this puzzle by proposing a model that reproduces these features of the data and respects the original promise of search models by accounting for involuntary unemployment without relying on inefficient contracting or wage rigidities. We do so by allowing for preferences that generate *time-varying risk* over the business cycle, consistent with observed fluctuations in asset prices, and for *human capital acquisition*, in line with the evidence on how rapidly wages grow with labor market experience. Throughout most of our analysis, we abstract from physical capital to help illustrate our mechanism in the most transparent way. When we extend our model to incorporate physical capital, we find that it successfully matches observed patterns of job-finding rates, unemployment, output, consumption, investment, and asset prices. In this exercise, we build on the work of Merz (1995) and Andolfatto (1996), which integrated search theory into quantitative business cycle models, and the work of Jermann (1998) and Tallarini (2000), which embedded asset-pricing preferences into quantitative business cycle models. Interestingly, in contrast to the classic separation result in Tallarini (2000), whereby introducing asset-pricing preferences into a standard real business cycle model has no effect on the fluctuations of real variables, we show that introducing such preferences in our model with human capital acquisition creates an important interaction between the real and financial sides of an economy that greatly amplifies fluctuations.

The main idea of our model is that hiring a worker is akin to investing in an asset with *risky* dividend flows that have *long* durations. In our model, as in the data, the *price of risk*—as captured by the ratio of the conditional standard deviation of the intertemporal price of consumption goods to its conditional mean—sharply rises in downturns. In this sense, risk is time-varying as it varies over the cycle. Because of human capital acquisition, the surplus flows from matches between firms and workers have long durations and so are sensitive to variation in the price of risk. The combination of time-varying risk and human capital acquisition then implies that the benefits of creating new matches between firms and workers greatly drop in downturns, which induces firms to substantially reduce the number of job vacancies they create and, correspondingly, leads unemployment to increase as much as it does in the data.

The two simple ingredients we add to the textbook search model make it consistent with two salient aspects of the data: asset prices fluctuate over the cycle, and wages increase with experience in the labor market. To reproduce the first feature, we augment the textbook model with preferences that generate time-varying risk, whereas to accommodate the second feature, we introduce human capital accumulation on the job and depreciation off the job. We choose parameters for preferences and technology that are consistent with the main properties of asset prices and wage-experience profiles, and we show that the resulting allocations display fluctuations in unemployment that are as large as those observed in the data.

We propose a model that generates involuntary unemployment without resorting to inefficiencies in wage contracting by focusing on labor market outcomes that arise from a competitive search equilibrium. We find this equilibrium concept appealing relative to common bargaining concepts such as Nash bargaining or alternating wage offer bargaining, since these bargaining schemes give rise to inefficient wage setting unless the parameters that characterize the bargaining process are suitably chosen. For instance, a well-known result is that equilibrium wage setting under Nash bargaining is efficient and, hence, leads to the same outcomes that arise under competitive search when the Hosios condition holds (see Hosios, 1990). In Kehoe, Lopez, Midrigan, and Pastorino (2021), we derive analogous conditions for efficiency for the alternative wage offer bargaining protocol. In light of these results, we can interpret our work as pertaining to economies with efficient wage setting, which can be achieved under any of three popular wage determination schemes: competitive search and, once appropriately parametrized, Nash bargaining with period-by-period contracting and alternating wage offer bargaining. In this sense, our results do not depend on the specific wage determination scheme chosen.

We argue that both of our two simple ingredients are necessary to account for the observed volatility of unemployment. In particular, we show that if we retain human capital acquisition but replace our asset-pricing preferences with standard constant relative risk aversion preferences, then the model generates no fluctuations in unemployment, regardless of the degree to which human capital accumulates with experience. Similarly, if we retain our asset-pricing preferences but abstract from human capital acquisition, then the model generates almost no fluctuations in unemployment.

We turn to providing further details about our two additional ingredients, starting with preferences. The literature on asset pricing has developed several classes of preferences and stochastic processes for exogenous shocks that give rise to large increases in the price of risk in downturns and, thus, reproduce key features of the fluctuations of asset prices. Indeed, as Cochrane (2011) emphasizes, all of these preferences and shocks generate variation in asset prices from variation in risk premia, consistent with the data.

In our baseline model, we use a variant of the original preferences in Campbell and Cochrane (1999), in which we eliminate the associated consumption externality arising from an external habit in consumption. We do so by replacing this external habit with a simple type of internal habit, which acts like an exogenous shock to the stochastic discount factor. We find these preferences a useful benchmark because they incorporate the idea that the price of risk and therefore risk premia rise in recessions in a transparent and intuitive way. Moreover, as we show, their implications for asset prices and unemployment fluctuations are nearly identical to those of the original preferences in Campbell and Cochrane (1999). To emphasize that our results are robust to the specific details of the preferences and shocks that achieve this variation in risk, we show in our extensions that our results hold for a wide range of the most common specifications in the macro-finance literature.

Consider now the process of human capital acquisition. In the model, a worker's human capital accumulates during employment and depreciates during unemployment. For simplicity, we assume constant rates of human capital accumulation and depreciation, and that market production, home production, and the cost of posting job vacancies are proportional to the stock of human capital. This formulation is particularly convenient because it implies that only the aggregate levels of human capital of employed and unemployed workers, rather than their distributions, need to be recorded as state variables.[1]

We then characterize the mechanism that generates our quantitative results. We show that the job-finding rate, which is the key determinant of unemployment, depends (log) linearly on the present value of the surplus flows from a match between a firm and a worker, scaled by aggregate productivity. This present value, in turn, can be expressed as a weighted average of the prices of claims to aggregate productivity at each future time horizon, referred to as *claims to future productivity* or, simply, *strips*. The weights of this weighted average are determined by the degree of human capital accumulation on the job and depreciation off the job, whereas the prices of claims to future productivity are determined by the preference and shock structure. Since the human capital process increases the duration of surplus flows, the greater is the rate of human capital accumulation on the job or depreciation off the job, the slower is the decay of the surplus flows from a match between a firm and a worker, and, hence, the larger are the weights attached to the prices of strips at longer horizons. As the prices of strips at longer horizons are more sensitive to aggregate shocks than those of strips at shorter horizons—a feature shared by all the asset-pricing setups we consider—the larger the weights on the prices of longer-horizons strips, the more sensitive is the job-finding rate to aggregate shocks.

Formally, we prove that the volatility of the job-finding rate can be well approximated by a single sufficient statistic: a weighted average, $\sum_n \omega_n b_n \sigma(s_t)$, over different horizons (n) of the elasticity of the price of a strip (b_n) with respect to the exogenous stochastic state of the economy (s_t), multiplied by the volatility of this state ($\sigma(s_t)$). As mentioned, the elasticity b_n increases with the horizon n so that the price of a strip becomes more sensitive to the state s_t as the maturity of the strip increases. Further, the weights $\{\omega_n\}$ decay more slowly the greater is the rate of human capital accumulation on the job or depreciation off the job. Hence, according to our model, the job-finding rate is volatile because it depends on the prices of long-maturity claims to future productivity that are highly sensitive to fluctuations in the aggregate state.

This sufficient statistic further allows us to characterize the roles of time-varying risk and human capital acquisition

[1] In Kehoe, Lopez, Midrigan, and Pastorino (2020, 2021), we consider a more general formulation of the human capital process, in which the rates of human capital accumulation on the job and depreciation off the job are stochastic and vary with the level of acquired human capital. This richer version of the model better reproduces the shape of empirical wage-experience profiles and yields results for the volatility of the job-finding rate and unemployment very similar to those implied by our baseline model. For an illustration of the amplification effect of human capital acquisition in the context of the Great Recession, see Kehoe, Midrigan, and Pastorino (2019).

for our results. First, we show that when time-varying risk is small, the elasticity b_n of the price of a claim to aggregate productivity in n periods with respect to the state s_t is small, regardless of the horizon n. Thus, the model cannot generate much volatility in the job-finding rate regardless of the weights $\{\omega_n\}$ on the prices of claims. Second, we show that when human capital acquisition is absent or of little importance, the weights $\{\omega_n\}$ are nearly all concentrated on the prices of short-horizon claims, which display little volatility under all of our asset-pricing specifications. Intuitively, these weights are small, because absent human capital acquisition, the duration of surplus flows from a match between a firm and a worker is very short. In this case, the problem of hiring a worker is nearly static, so variation in time-varying risk that affects the stochastic discount factor has little effect on the present value of surplus flows from a match. Therefore, the model cannot generate much volatility in the job-finding rate in this case either. Only when both features are present—namely, time variation in the price of risk and human capital acquisition—can our model produce sizable volatility in the job-finding rate and unemployment.

We conclude by considering three extensions. In the first and most important one, we augment our model with physical capital subject to adjustment costs along the lines of Jermann (1998) and construct a business cycle model in the spirit of the seminal work by Merz (1995) and Andolfatto (1996). As Shimer (2005) pointed out, though, these latter two papers missed an important feature of the data, namely, the strong negative correlation between vacancies and unemployment. Our model not only reproduces this feature but also matches salient patterns in the data for job-finding rates, unemployment, output, consumption, investment, and asset prices. We think of this version of the model as helping to reintegrate search models into quantitative business cycle models.

Second, we extend our model to a life-cycle setting with young and mature workers. In the data, both the growth rate of wages and the volatility of unemployment are higher for young workers than for mature ones. The simple life-cycle extension we propose can account for these patterns as well, since it implies that the greater the rate of human capital accumulation on the job, and so the greater the growth rate of wages, the higher the volatilities of the job-finding rate and unemployment.

Third, we show that our results hold for a variety of common preference structures. We first consider a version of the long-run risk framework of Bansal and Yaron (2004) with preferences as in Epstein and Zin (1989), modified along the lines suggested by Albuquerque, Eichenbaum, Luo, and Rebelo (2016) and Schorfheide, Song, and Yaron (2018) to allow for long-run risk and preference shocks in order to better reproduce observed features of asset prices like the volatility of risk-free rates and of the price-dividend ratio. Following the setup of Wachter (2013), we then consider Epstein-Zin preferences augmented with a time-varying risk of disasters, defined as episodes of unusually large decreases in aggregate consumption associated with marked declines in aggregate productivity. Finally, given the popularity of reduced-form asset-pricing models that simply specify a stochastic discount factor as a function of shocks, we explore a version of the affine stochastic discount factor model of Ang and Piazzesi (2003) as a representative model of this class.

We find that all these economies imply analogous results for the volatility of the job-finding rate and unemployment. As in the case of our baseline model, each of these models' implications for these volatilities can be summarized by our single sufficient statistic, which captures the volatility of the exogenous state of an economy $\sigma(s_t)$, the implied variation in the price of risk, as reflected in the variation of the prices of claims to future productivity captured by $\{b_n\}$, and the persistence of the returns to hiring workers, as captured by the weights $\{\omega_n\}$. All of these preference and shock structures also have broadly similar implications for asset prices. In the online appendix, we further show that the asset-pricing implications of our search model with endogenous production under each of these preference and shock structures are essentially identical to those of the original versions of the models that featured these structures, which were developed for pure exchange economies. In this sense, our various models' predictions for asset prices are simply inherited from existing asset-pricing models. Augmenting them with production and labor market search frictions makes them neither better nor worse.

Overall, we view our exercise as a promising step in developing a unified theory of real and financial business cycles.

2 Relation to the Literature

Ljungqvist and Sargent (2017) have extensively reviewed the work on the unemployment volatility puzzle of search models. All of these papers, including those by Hagedorn and Manovskii (2008), Hall and Milgrom (2008), and Pissarides (2009), are subject to at least one of the three critiques raised against this literature. Ours is immune to all three.

To elaborate, the first critique is by Chodorow-Reich and Karabarbounis (2016), who have argued that none of these papers is consistent with the measured cyclicality of the opportunity cost of employment. Specifically, these authors have documented that the opportunity cost of employment in the data is procyclical with an elasticity close to one—rather than zero, as assumed in these models. The authors have further demonstrated that once these models are made consistent with this aspect of the data, they cannot generate volatile unemployment. Our model can.

A second critique is by Kudlyak (2014) and concerns the literature that has addressed the unemployment volatility puzzle by introducing some form of wage rigidity. Kudlyak (2014) builds on the insight in Becker's (1962) classic paper that only the present value of the wages paid over the course of an employment relationship is allocative for employment. Kudlyak (2014) established that the appropriate measure of rigidity of the allocative wage for a large class of search models is the cyclicality of the user cost of labor, defined as the difference in the present value of wages between two firm-worker matches that are formed in two consecutive periods. As Kudlyak (2014) first estimated and Basu and House (2016) confirmed, the user cost of labor is highly cyclical in that it sharply falls when unemployment rises. Both of these papers have also argued that reproducing the cyclicality of the user cost of labor is the key litmus test for the cyclicality of wages implied by any business cycle model. As these authors discussed, early attempts to solve the unemployment volatility puzzle by introducing some form of wage rigidity fail this test. As we show, our model passes it.

Finally, a third critique of the literature on the unemployment volatility puzzle has been formulated by Borovicka and Borovickova (2019), who argued that the literature is grossly at odds with robust patterns of asset prices. In contrast to existing work, our model incorporates standard asset-pricing preferences, which generate movements in risk-free rates and risk premia in accord with the data. Our model thus overcomes this final critique as well.

The important contribution of Hall (2017) accounts for the observed volatility of unemployment within a model that features alternating wage offer bargaining, a reduced-form stochastic discount factor, and no human capital accumulation. This paper is immune to the critique by Chodorow-Reich and Karabarbounis (2016) but not to the critiques by Kudlyak (2014) and Borovicka and Borovickova (2019). In particular, as we show in Kehoe, Lopez, Midrigan, and Pastorino (2021), Hall (2017) relies on a parametrization of wage setting that yields highly (ex ante) inefficient allocations associated with a counterfactually low degree of cyclicality of the user cost of labor. Hence, in this precise sense, the wages in Hall (2017) are much more rigid than those in the data. Moreover, Borovicka and Borovickova (2019) discussed how in Hall's (2017) model, fluctuations in unemployment arise not from the time variation of the price of risk, as they do in our model, but rather from strongly countercyclical movements in the risk-free rate, which are counterfactual.

Also related to our paper is the work of Kilic and Wachter (2018). These authors embedded a reduced-form version of the wage-setting mechanism in Hall (2017) within a model with preferences as in Epstein and Zin (1989) and variable disaster risk. Although the resulting pricing kernel does not lead to a risk-free rate puzzle, the model generates volatile unemployment by relying heavily on a form of inefficient real wage stickiness as in Hall (2017). In contrast, we show that variable disaster risk can generate realistic fluctuations in the job-finding rate and unemployment under efficient wage setting without rigid wages, provided human capital acquisition is incorporated.

Fundamentally, our mechanism differs along two main dimensions from those in the large literature examined by Ljungqvist and Sargent (2017). First, this literature focuses on changes in unemployment in response to changes in aggregate productivity across steady states, because in this class of models, steady-state changes in aggregate productivity well approximate stochastic fluctuations in it. By contrast, in the models we propose, steady-state changes in aggregate productivity do not well approximate stochastic fluctuations in it. Second, the main result of Ljungqvist and Sargent (2017) on the conditions for unemployment to be volatile does not apply to our setting.

To demonstrate that steady-state changes in aggregate productivity in our class of models do not well approximate stochastic fluctuations in it, we contrast the implications of two classes of preferences: CRRA preferences and our baseline preferences with time-varying risk, in the presence of an aggregate productivity process with constant variance. Both classes of preferences lead to identical steady states and hence, according to the argument in Ljungqvist and Sargent (2017), should give rise to similar non-steady-state fluctuations. But, as we show, in response to productivity shocks, CRRA preferences lead to no fluctuations in unemployment, whereas our baseline model produces large ones.[2]

[2]In Kehoe, Lopez, Midrigan, and Pastorino (2021), we prove that in the models reviewed by Ljungqvist and Sargent (2017), the change in unemployment across steady states resulting from a change in aggregate productivity is identically zero, once we modify them to be consistent with both the critique by Chodorow-Reich and Karabarbounis (2016) and the insight of Shimer (2010) on the cost to firms of recruiting workers and bargaining over wages. Specifically, Shimer (2010) argued that if recruiting workers or bargaining takes time away from production, then the cost of doing so is proportional to the opportunity cost of a worker's time in production. We show that a similar result on the insensitivity of unemployment to steady-state changes in aggregate productivity holds in a large class of models like ours with asset-pricing preferences and either

Next, consider the main result of Ljungqvist and Sargent (2017), namely, that existing search models generate large fluctuations in unemployment only if they feature what they term a small *fundamental surplus fraction*, which is a scaled measure of the steady-state surplus from a match between a firm and a worker. This result does not hold in our class of models. For instance, as mentioned earlier, the version of our model with CRRA preferences and that with our baseline preferences have identical steady states. Hence, any statistics calculated from their respective steady states, such as the fundamental surplus fractions, are identical. But even so, the version with CRRA preferences generates no fluctuations in unemployment, whereas the version with our baseline preferences gives rise to large ones. So, the fundamental surplus fraction is not an informative statistic for our class of models. The reason is that fluctuations in unemployment in our models are driven by time-varying risk, which cannot be captured by statistics from a deterministic steady state.

In sum, our models fall outside of the large class considered by Ljungqvist and Sargent (2017).

3 Economy

We embed a Diamond-Mortenson-Pissarides (DMP) model of the labor market with competitive search in an economy with endogenously determined asset prices. The economy is subject to both aggregate shocks—namely, productivity shocks—and idiosyncratic shocks. We extend the DMP model to include two key features: asset-pricing preferences that generate time-varying risk and human capital acquisition with labor market experience. In our baseline model, we use a version of the preferences in Campbell and Cochrane (1999) with an internal consumption habit that is exogenously time-varying conditional on consumption—in short, an exogenously driven habit. In our extensions, we consider other popular preference structures.

The economy consists of a large number of firms and a measure of consumers such that a finite measure of consumers exists per firm.[3] Each consumer belongs to one of a large number of families that insure their members against idiosyncratic risk. Each consumer survives from one period to the next with probability ϕ. A measure $1 - \phi$ of new consumers is born each period so that the measure of consumers in the economy is constant over time and equal to one. Consumers' human capital accumulates during employment and depreciates during unemployment. Firms post vacancies to hire consumers with any desired level of human capital.

3.1 Technologies and Resource Constraints

Consumers are indexed by a state variable, z_t—referred to as *human capital*—that summarizes their ability to produce output. This variable captures a consumer's productivity and evolves with experience in the labor market. Specifically, in period t, a consumer with human capital z_t produces $A_t z_t$ units of output when employed and $bA_t z_t$ units of output when unemployed with $0 < b < 1$. Hence, the opportunity cost of employment is $bA_t z_t$ in period t with an elasticity to aggregate productivity of one, consistent with the findings in Chodorow-Reich and Karabarbounis (2016). Here we

productivity or preference shocks.

[3]Throughout, we maintain that the number of firms is sufficiently large that the assumption that each firm views aggregate allocations as unaffected by its individual decisions is a reasonable approximation.

follow Hall (2017), who incorporates these findings by assuming that the opportunity cost of employment is proportional to aggregate productivity; see the discussion in Hall (2017, p. 324). We assume that (log) aggregate productivity follows a random-walk process with drift g_a given by

$$\log(A_{t+1}) = g_a + \log(A_t) + \sigma_a \varepsilon_{at+1}, \tag{1}$$

where $\varepsilon_{at+1} \sim N(0,1)$. Newly born consumers draw their initial human capital from a distribution $\nu(z)$ with mean one and enter the labor market unemployed. After entry, when a consumer is employed, human capital evolves according to

$$z_{t+1} = (1+g_e)z_t, \tag{2}$$

and when a consumer is unemployed, it evolves according to

$$z_{t+1} = (1+g_u)z_t, \tag{3}$$

where $g_e \geq 0$ and $g_u \leq 0$ are constant rates of human capital accumulation on the job and depreciation off the job.[4]

Posting a job vacancy directed at a consumer with human capital z costs a firm $\kappa A_t z$ in lost production in period t. This specification of the cost of posting vacancies is consistent with the argument in Shimer (2010) that for firms to be able to recruit workers, employed workers must reduce their time devoted to production, thereby decreasing a firm's output. Under this view, the cost of hiring workers moves one-for-one with the productivity of a worker engaged in market production.[5] Assuming that home production and vacancy posting costs scale linearly with z_t is convenient, because as we show later, it implies that all value functions are linear in z_t. This scaling assumption, though, is unnecessary for our results and is motivated purely by analytical tractability and computational convenience. (In Kehoe, Lopez, Midrigan, and Pastorino, 2021, we consider a more general human capital process that does not scale with z_t and show that the resulting model works very similarly to our baseline model.)

The realization of the productivity innovation ε_t is the aggregate event. Let $\varepsilon^t = (\varepsilon_0, \ldots, \varepsilon_t)$ be the history of aggregate events at time t. An allocation is a set of stochastic processes for consumption $\{C(\varepsilon^t)\}$ and measures of employed consumers, unemployed consumers, and vacancies for each level of human capital z and history of aggregate events, $\{e(z,\varepsilon^t), u(z,\varepsilon^t), v(z,\varepsilon^t)\}$. For notational simplicity, we express these allocations in shorthand notation as $\{C_t, e_t(z), u_t(z), v_t(z)\}$ from now on, suppressing their explicit dependence on ε^t. The measures of employed and unemployed consumers satisfy

$$\int_z [e_t(z) + u_t(z)]dz = 1. \tag{4}$$

The timing of events in a period is as follows. At the beginning of period t, current productivity A_t is realized, firms

[4]We assume that the rate g_e of human capital accumulation on the job is small enough that the aggregate stock of human capital in the economy admits a stationary distribution.

[5]Note that since aggregate productivity follows a random-walk process with positive drift, if we were to assume that home production b and the vacancy posting cost κ are constant, then the ratios b/A_t and κ/A_t would (in a precise stochastic sense) converge to zero and all consumers would always work.

post vacancies and wage offers to attract consumers, and unemployed consumers from the end of period $t-1$ search for jobs. Then, new matches are formed, and employed consumers immediately begin to work. At the end of period t, a fraction σ of employed consumers separates from their firms, entering the pool of unemployed consumers of period $t+1$ who search for jobs at the beginning of period $t+2$, and consumption takes place.

To understand the law of motion for the measures of employed and unemployed consumers, consider unemployed consumers searching for a job at the beginning of period t with human capital z, denoted by $u_{bt}(z)$. These consumers were unemployed at the end of period $t-1$, had human capital $z/(1+g_u)$, which grew at rate $1+g_u$ to z between $t-1$ and t, and survived. Therefore,

$$u_{bt}(z) = \frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right). \tag{5}$$

The term $1/(1+g_u)$, which multiplies $u_{t-1}(z/(1+g_u))$ in (5), arises from the change of variable in the measure over $z/(1+g_u)$ to derive the measure over z.[6] At the beginning of period t, firms post a measure of vacancies $v_t(z)$ to target consumers with human capital z, thus creating a measure $m_t(u_{bt}(z), v_t(z))$ of matches, where $m_t(\cdot,\cdot)$ is a constant returns-to-scale matching function that is strictly increasing, strictly concave, and differentiable.

The transition laws for employment and unemployment for consumers with human capital z are then given, respectively, by

$$e_t(z) = \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + \lambda_{wt}(\theta_t(z))u_{bt}(z) \tag{6}$$

and

$$u_t(z) = \frac{\phi\sigma}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + [1-\lambda_{wt}(\theta_t(z))]u_{bt}(z) + (1-\phi)\nu(z), \tag{7}$$

where $\lambda_{wt}(\theta_t(z)) = m_t(u_{bt}(z), v_t(z))/u_{bt}(z)$ is the *job-finding rate* of an unemployed consumer with human capital z and $\theta_t(z) = v_t(z)/u_{bt}(z)$ is the *tightness* of the labor market for consumers with human capital z.

To understand these expressions, consider, for instance, (7). Observe first that new entrants into the unemployment pool include the measure $\phi\sigma e_{t-1}\left(z/(1+g_e)\right)/(1+g_e)$ of consumers with $z/(1+g_e)$ units of human capital in $t-1$ and z units of human capital in t, who worked in period $t-1$, separated from their firms at the end of the period (an event with probability σ), and survived (an event with probability ϕ). New entrants into unemployment also include all newborn consumers with human capital z of measure $(1-\phi)\nu(z)$. Finally, a proportion $1-\lambda_{wt}(\theta_t(z))$ of unemployed consumers at the beginning of period t remains unemployed.

For later use, it is convenient to define the *job-filling rate* of a vacancy for consumers with human capital z as $\lambda_{ft}(\theta_t(z)) = m_t(u_{bt}(z), v_t(z))/v_t(z)$. It follows that $\lambda_{wt}(\theta_t(z)) = \theta_t(z)\lambda_{ft}(\theta_t(z))$. We also define the (negative of the) elasticity of the job-filling rate with respect to $\theta_t(z)$ as $\eta_t(\theta_t(z)) = -\theta_t(z)\lambda'_{ft}(\theta_t(z))/\lambda_{ft}(\theta_t(z))$ so that $1-$

[6] Intuitively, this change of variable implies that the measure of consumers with human capital in the interval, say, $[\underline{z}_{t-1}, \bar{z}_{t-1}]$ in $t-1$ is assigned to the narrower interval $[(1+g_u)\underline{z}_{t-1}, (1+g_u)\bar{z}_{t-1}]$ in t since $1+g_u < 1$. Thus, the measure of consumers over z_{t-1} in $t-1$ must be scaled *up* by $1/(1+g_u)$ when changed to a measure over z_t. More formally, recall that if X and $Y = g(X)$, where $g(\cdot)$ is a strictly increasing function, are two continuous univariate random variables with densities $f_X(x)$ and $f_Y(y)$, then $f_Y(y) = f_X(x)(dx/dy)$. Here, the transformation is from $X = z/(1+g_u)$ to $Y = z$ with $dx/dy = 1/(1+g_u)$. See the online appendix for details.

$\eta_t(\theta_t(z)) = \theta_t(z)\lambda'_{wt}(\theta_t(z))/\lambda_{wt}(\theta_t(z))$.[7] Note that when we later assume a Cobb-Douglas matching function, the elasticity $\eta_t(\theta_t(z))$ is a constant, which we denote by η.

The aggregate resource constraint in period t is

$$C_t = A_t \int_z z e_t(z) dz + bA_t \int_z z u_t(z) dz - \kappa A_t \int_z z v_t(z) dz, \tag{8}$$

where the right side of this constraint adds the total output of the employed and the total output of the unemployed and subtracts the total cost of posting vacancies.

3.2 A Family's Problem

We represent the insurance arrangements in the economy by assuming that each consumer belongs to one of a large number of identical families, each consisting of a continuum of household members, that have access to complete one-period contingent claims against aggregate risk. Risk sharing within a family implies that each household member consumes the same amount C_t of goods at date t regardless of the idiosyncratic shocks that such a member experiences. (This type of risk-sharing arrangement is familiar from the work of Merz, 1995 and Andolfatto, 1996.)

Given this setup, we can separate a family's problem into two parts. The first part is solved at the level of the family and determines the family's choice of assets and the common consumption level of each member. The second part is solved at the level of individual consumers and firms in the family. The individual consumer problem determines the employment and unemployment status of each consumer in the family, whereas the individual firm problem determines the vacancies created and the matches formed by each firm that the family owns.

In our baseline model, we use a variant of the preferences in Campbell and Cochrane (1999). We replace their external consumption habit with an internal habit that fluctuates exogenously conditional on consumption—or, simply, an *exogenously driven habit*—to eliminate the consumption externality generated by their specification while retaining its desirable asset-pricing properties. (See Ljungqvist and Uhlig, 2015 for the implications of this externality.) We later show that our specification implies results nearly identical to those implied by their specification. Formally, with a consumption habit X_t, a family's utility is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - X_t)^{1-\alpha}}{1-\alpha}. \tag{9}$$

Let the habit X_t satisfy $X_t = (1 - \tilde{S}_t)C_t$ so that $\tilde{S}_t = (C_t - X_t)/C_t$ is the fraction by which a family's consumption exceeds its habit. In contrast to Campbell and Cochrane (1999), we assume that $\tilde{S}_t$ is *exogenous* rather than just *external* to a family's consumption choices. As will become apparent, using a scaled version of $\tilde{S}_t$, namely, $S_t = \tilde{S}_t^{(\alpha-1)/\alpha}$ with $(\alpha - 1)/\alpha > 0$, makes the pricing kernel for the economy and hence all asset-pricing formulae analogous to those that arise with an external habit. Note that in a symmetric equilibrium, C_t is equal for all families, and all families' common

[7] To see this, substitute $\lambda_{ft}(\theta_t(z)) = \lambda_{wt}(\theta_t(z))/\theta_t(z)$ and $\lambda'_{wt}(\theta_t(z)) = \lambda_{ft}(\theta_t(z)) + \theta_t(z)\lambda'_{ft}(\theta_t(z))$, rewritten as $\theta_t(z)\lambda'_{ft}(\theta_t(z)) = \lambda'_{wt}(\theta_t(z)) - \lambda_{wt}(\theta_t(z))/\theta_t(z)$, into the expression for $1 - \eta_t(\theta_t(z))$ with $\eta_t(\theta_t(z)) = -\theta_t(z)\lambda'_{ft}(\theta_t(z))/\lambda_{ft}(\theta_t(z))$.

marginal utility of consumption is $\beta^t \tilde{S}_t^{1-\alpha} C_t^{-\alpha} = \beta^t (S_t C_t)^{-\alpha}$ in any period t. Hence, without loss of generality, the law of motion for the surplus consumption ratio and the pricing kernel can be expressed in terms of S_t rather than $\tilde{S}_t$. We refer to S_t as the scaled surplus consumption ratio or, simply, the *surplus consumption ratio*.

We assume that $s_t = \log(S_t)$ is governed by an autoregressive process given by

$$s_{t+1} = (1 - \rho_s)s + \rho_s s_t + \lambda_a(s_t)[\Delta a_{t+1} - \mathbb{E}_t(\Delta a_{t+1})], \tag{10}$$

where $a_t = \log(A_t)$, $\Delta a_{t+1} = a_{t+1} - a_t$, and s denotes the mean of s_t. The *sensitivity function* $\lambda_a(s_t)$ is defined as

$$\lambda_a(s_t) = \frac{1}{S}\sqrt{[1 - 2(s_t - s)]} - 1 \tag{11}$$

when the right side of (11) is nonnegative and zero otherwise. As in Campbell and Cochrane (1999), the function $\lambda_a(s_t)$ is chosen so that in a downturn following a negative technology shock, risk aversion sharply rises, but the risk-free rate is relatively stable.[8] The *pricing kernel* for the economy is

$$Q_{t,t+1} = \beta \left(\frac{S_{t+1}C_{t+1}}{S_t C_t} \right)^{-\alpha}. \tag{12}$$

This kernel determines the intertemporal price of consumption goods and is the discount factor used by individual consumers and firms. Using similar notation, we let $Q_{t,r} = \beta^{r-t}[S_r C_r/(S_t C_t)]^{-\alpha}$ denote the discount factor for period $r \geq t + 1$ in units of the period-t consumption good.

Since each family is identical, has access to complete one-period contingent claims against aggregate risk, and faces prices of contingent claims that are related in the usual fashion to the marginal rate of substitution in (12), these claims in equilibrium are zero for each family. Thus, for notational simplicity, we do not explicitly include them in the budget constraint of a family, which can then be expressed as

$$C_t + I_t = W_t + \Pi_t + H_t. \tag{13}$$

In this expression, I_t is the total resources invested by a family to create new vacancies, W_t is the total wages of employed consumers of the family, Π_t is the profit flows of the firms that the family owns, and H_t is the total home production of unemployed consumers of the family. In equilibrium, $I_t = \kappa A_t \int_z z v_t(z)dz$, $W_t + \Pi_t = A_t \int_z z e_t(z)dz$, and $H_t = bA_t \int_z z u_t(z)dz$.

Note for later that the risk-free rate $R_{ft} = \exp(r_{ft})$—namely, the return on a claim purchased in t to one unit of consumption at all states in $t + 1$—satisfies $R_{ft} = 1/\mathbb{E}_t(Q_{t,t+1})$. More generally, the return R_{t+1} on any asset in $t + 1$ must satisfy the first-order condition $1 = \mathbb{E}_t(Q_{t,t+1}R_{t+1})$. By a standard argument in Hansen and Jagannathan (1991), this first-order condition implies that the (conditional) *Sharpe ratio* of any asset—namely, the ratio of the conditional

[8]As we show below in expression (42), derived when consumption is conditionally log-normally distributed, as is approximately the case in our model, the real risk-free rate is governed by two forces that move in opposite directions. Specifically, the function $\lambda_a(s_t)$ affects consumers' incentives to save to ensure that movements in the risk-free rate due to intertemporal substitution reasons are offset by corresponding movements in it due to precautionary saving reasons.

mean excess return on the asset, $\mathbb{E}_t(R_{t+1} - R_{ft})$, to the conditional standard deviation of the excess return, $\sigma_t(R_{t+1} - R_{ft})$—must satisfy

$$\left|\frac{\mathbb{E}_t(R_{t+1} - R_{ft})}{\sigma_t(R_{t+1} - R_{ft})}\right| \leq \frac{\sigma_t(Q_{t,t+1})}{\mathbb{E}_t(Q_{t,t+1})}. \tag{14}$$

The right side of this Hansen-Jagannathan bound is the highest possible Sharpe ratio in the economy, the *maximum Sharpe ratio*, which is a common measure of the *price of risk*. As Campbell and Cochrane (1999) show, a critical feature of preferences with a consumption habit is that the price of risk varies with the exogenous state s_t so that when the state is low, the price of risk is high and risky investments are unattractive. This feature of the price of risk will prove critical to the volatility of the job-finding rate and unemployment in our model.

3.3 Comparison with Original Campbell-Cochrane Preferences

Our preferences with an exogenously driven habit are very similar to those in Campbell and Cochrane (1999). The difference is that the habit X_t in the utility function in (9), whose law of motion in our formulation is determined by the process for the exogenous (scaled) surplus consumption ratio S_t, is replaced in Campbell and Cochrane (1999) by the habit $\bar{X}_t = (1 - \bar{S}_t)\bar{C}_t$, where $\bar{C}_t$ denotes aggregate consumption, $\Delta\bar{c}_{t+1} = \log(\bar{C}_{t+1}) - \log(\bar{C}_t)$, and the process for the (log) surplus consumption ratio $\bar{s}_t = \log(\bar{S}_t)$ is

$$\bar{s}_{t+1} = (1 - \rho_s)\,\bar{s} + \rho_s\bar{s}_t + \lambda(\bar{s}_t)[\Delta\bar{c}_{t+1} - \mathbb{E}_t(\Delta\bar{c}_{t+1})]. \tag{15}$$

The associated sensitivity function is given by $\lambda(\bar{s}_t) = \sqrt{[1 - 2(\bar{s}_t - \bar{s})]}/\bar{S} - 1$ when $\lambda(\bar{s}_t)$ is nonnegative and by zero otherwise. Each family chooses consumption C_t in order to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - \bar{X}_t)^{1-\alpha}}{1 - \alpha},$$

taking as given $\bar{S}_t$, $\bar{C}_t$, and $\bar{X}_t$. After we take first-order conditions and impose that in equilibrium $\bar{C}_t = C_t$, a family's marginal utility of consumption becomes $\beta^t(\bar{S}_t C_t)^{-\alpha}$ and so the pricing kernel for the economy is

$$Q_{t,t+1} = \beta \left(\frac{\bar{S}_{t+1}C_{t+1}}{\bar{S}_t C_t}\right)^{-\alpha}. \tag{16}$$

Comparing the pricing kernel in (16) with that in (12), we see that the only difference between them is that the shock to s_{t+1} in our setup is the innovation to the growth rate of log aggregate productivity, Δa_{t+1}, whereas the shock to $\bar{s}_{t+1}$ in the external habit setup is the innovation to the growth rate of log aggregate consumption, $\Delta\bar{c}_{t+1}$. The key implication of this difference is that allocations in our setup are efficient, unlike those in Campbell and Cochrane (1999). Recall that in the original pure exchange economy in Campbell and Cochrane (1999), consumption is exogenous with $\bar{c}_t = a_t$, where a_t can be thought of as exogenously specified output. Hence, $\Delta\bar{c}_{t+1} - \mathbb{E}_t(\Delta\bar{c}_{t+1}) = \Delta a_{t+1} - \mathbb{E}_t(\Delta a_{t+1})$ so that the specification of the consumption habit in Campbell and Cochrane (1999) and the specification we adopt are identical in a pure exchange economy. In our production economy, in contrast, since consumption does not equal productivity, these two specifications differ. In our model, however, consumption growth approximately equals productivity growth and these two specifications lead to nearly identical quantitative results.

3.4 Competitive Labor Market Search

We consider labor markets in which the matching of firms and consumers to create employment relationships is subject to search frictions. In particular, we set up a symmetric competitive search equilibrium in the spirit of the market-utility approach in Montgomery (1991); see also Moen (1997) and, for an extensive review of the literature, Wright, Kircher, Benoît, and Guerrieri (2021). Since we will show later that, under our assumptions, the equilibrium is unique and symmetric, from now on we refer to it simply as the *competitive search equilibrium*. Let $\mathbb{Z}_t$ be the set of human capital levels among the unemployed in period t. Given that we assume that firms can freely enter the labor market, we can think of there being a large number of firms that in period t search for consumers with any given level of human capital $z \in \mathbb{Z}_t$.

Each period t consists of two stages. In stage 1, any firm that searches for consumers with human capital z posts vacancies for such consumers and commits to a *wage offer* for a resulting match, $W_{mt}(z)$, defined as the present value of the wages to be paid over the course of a match with a consumer of type z. In stage 2, after having observed all offers, consumers of type z choose which market to search in. A market is defined by $(z, W_{mt}(z))$, namely, a skill level and a wage offer for that skill level.[9] These two stages should be thought of as occurring at the beginning of each period t, right after aggregate productivity is realized.[10] Then, matches are formed, output is produced, and at the end of the period, consumption takes place. We now turn to describing in greater detail our labor market setup as well as firms' and consumers' problems, starting from stage 2.

3.4.1 Stage 2: Consumers Choose Labor Markets

We start by considering symmetric histories in which all firms have made the same wage offers in stage 1 of period t, so that there is only one wage offer $W_{mt}(z)$ for each level of human capital z. We refer to $(z, W_{mt}(z))$ as the *common market*. We also refer to the present value of all payments to be received by a consumer with human capital z from future home production and future employment after a match formed in t dissolves as the *post-match value* in t, denoted by $W_{pt}(z)$, which is given recursively by

$$W_{pt}(z) = \phi\sigma\mathbb{E}_t[Q_{t,t+1}U_{t+1}(z')] + \phi(1-\sigma)\mathbb{E}_t[Q_{t,t+1}W_{pt+1}(z')], \tag{17}$$

with $z' = (1 + g_e)z$. The total value of a new match to a consumer is thus $W_t(z) = W_{mt}(z) + W_{pt}(z)$, since the current match pays $W_{mt}(z)$ and the consumer's post-match value is $W_{pt}(z)$. We decompose the total value of a match to a consumer into these two terms so as to clearly distinguish the portion that a firm chooses, namely, $W_{mt}(z)$, and the

[9] Rather than envisioning one large market with many firms that make the same wage offer, we find it useful to think of every firm as anticipating that it can create its own market when contemplating a wage offer and of consumers as freely flowing between these markets until the value of search $\mathcal{W}_t(z)$, defined later, is equated across them. Given a set of wage offers from all markets, the associated values of market tightness are determined by the equality of the value of search across markets. As a convention, we interpret two or more markets with identical human capital and wage offers as sub-markets of the same market.

[10] In a monthly model like ours, one might think of these two stages as all occurring early in the morning of the first day of a month. Then, on the same day, consumers and firms match and produce that day and for the rest of the month.

portion that a firm takes as given, namely, $W_{pt}(z)$. The value of unemployment $U_t(z)$ for a consumer with human capital z is

$$U_t(z) = bA_t z + \phi \mathbb{E}_t(Q_{t,t+1}\{\lambda_{wt+1}(\theta_{t+1}(z'))[W_{mt+1}(z') + W_{pt+1}(z')] \\ + [1 - \lambda_{wt+1}(\theta_{t+1}(z'))]U_{t+1}(z')\}), \tag{18}$$

with $z' = (1 + g_u)z$. The *value of search* for a consumer with human capital z in market $(z, W_{mt}(z))$ is

$$\mathcal{W}_t(z) = \lambda_{wt}(\theta_t(z))[W_{mt}(z) + W_{pt}(z)] + [1 - \lambda_{wt}(\theta_t(z))]U_t(z). \tag{19}$$

Since a firm that contemplates an arbitrary wage offer in stage 1 needs to anticipate consumers' behavior in stage 2, we need to determine consumers' responses to any such offer and the resulting outcomes in stage 2. Given that we focus on symmetric equilibria, it is sufficient to consider asymmetric histories at the beginning of stage 2 in which all firms but one have offered $W_{mt}(z)$ and one has offered, say, $\tilde{W}_{mt}(z)$. Consider then markets $(z, W_{mt}(z))$ and $(z, \tilde{W}_{mt}(z))$. The tightness $\theta_t(z)$ of market $(z, W_{mt}(z))$ satisfies the free-entry condition defined later and hence (23). The tightness $\tilde{\theta}_t(z)$ of market $(z, \tilde{W}_{mt}(z))$ is determined as follows. As long as the wage offer $\tilde{W}_{mt}(z)$ is sufficiently attractive, firms understand that consumers will flow between markets $(z, W_{mt}(z))$ and $(z, \tilde{W}_{mt}(z))$ until the value of search in the two markets is equated. In this case, $\tilde{\theta}_t(z)$ is determined by the *participation constraint* $\tilde{\mathcal{W}}_t(z) = \mathcal{W}_t(z)$, which can be expressed as

$$\lambda_{wt}(\tilde{\theta}_t(z))[\tilde{W}_{mt}(z) + W_{pt}(z)] + [1 - \lambda_{wt}(\tilde{\theta}_t(z))]U_t(z) \\ = \lambda_{wt}(\theta_t(z))[W_{mt}(z) + W_{pt}(z)] + [1 - \lambda_{wt}(\theta_t(z))]U_t(z), \tag{20}$$

with $\tilde{\mathcal{W}}_t(z)$ defined by the left side of this equality. Alternatively, if the wage offer $\tilde{W}_{mt}(z)$ is so low that the left side of (20) is less than the right side even with a job-finding rate $\lambda_{wt}(\tilde{\theta}_t(z))$ of one in that $\tilde{W}_{mt}(z) + W_{pt}(z) < \mathcal{W}_t(z)$, then $\tilde{\theta}_t(z) = 0$ and no consumers flow to market $(z, \tilde{W}_{mt}(z))$. In this case, although consumers can find jobs with probability one in market $(z, \tilde{W}_{mt}(z))$, they prefer to search in the common market $(z, W_{mt}(z))$.

We have derived consumers' participation constraint as the result of a firm's anticipation of consumers' optimal responses in stage 2 to a deviation from the symmetric equilibrium by such a firm in stage 1.[11] By the one-shot-deviation principle, the only deviations by firms in stage 1 of period t that we need to consider are one-time deviations. Hence, after period t, regardless of whether a consumer accepts the offer $W_{mt}(z)$ in market $(z, W_{mt}(z))$ or the offer $\tilde{W}_{mt}(z)$ in market $(z, \tilde{W}_{mt}(z))$, the consumer takes as given the same sequence of value functions $\{U_r(z)\}_{r=t}^{\infty}$ and so $\{W_{pr}(z)\}_{r=t}^{\infty}$ in any period $r \geq t$ resulting from future home production and employment. Note for later that if a firm makes the

[11] As is common in the market-utility approach of the directed search literature, the participation constraint can also be interpreted as specifying reasonable beliefs for a firm about the tightness $\tilde{\theta}_t(z)$ of market $(z, \tilde{W}_{mt}(z))$ for any wage offer $\tilde{W}_{mt}(z)$ that the firm makes in stage 1. According to this approach, given the values $W_{pt}(z)$, $U_t(z)$, and $\mathcal{W}_t(z)$, a firm's reasonable beliefs about the tightness of market $(z, W_{mt}(z))$ are summarized by $\tilde{\theta}_t(z) \equiv \theta^*(W_{mt}(z)|W_{pt}(z), U_t(z), \mathcal{W}_t(z))$, which solves $\lambda_{wt}(\tilde{\theta}_t(z))[\tilde{W}_{mt}(z) + W_{pt}(z)] + [1 - \lambda_{wt}(\tilde{\theta}_t(z))]U_t(z) = \mathcal{W}_t(z)$. Although this alternative approach does not specify the details of consumer behavior that justify such beliefs, it would yield equilibrium outcomes that are identical to ours.

symmetric wage offer $\tilde{W}_{mt}(z) = W_{mt}(z)$, then the tightness $\tilde{\theta}_t(z)$ of market $(z, \tilde{W}_{mt}(z))$ is the symmetric one $\theta_t(z)$ by the participation constraint in (20).

Finally, at the end of stage 2 of period t, each family consumes C_t.

3.4.2 Stage 1: Firms Choose Contingent Wage Offers and Post Vacancies

Consider the problem of a firm targeting a consumer with human capital z in stage 1 of period t when the state is ε^t and aggregate productivity is $A_t = A(\varepsilon^t)$. To set up this problem, given that the equilibrium is symmetric, we allow a firm to choose any possible wage offer $\tilde{W}_{mt}(z)$ when all other firms that search for consumers with human capital z make the *symmetric wage offer* $W_{mt}(z)$.

Consider market $(z, W_{mt}(z))$. Any firm targeting a consumer of type $z \in \mathbb{Z}_t$ incurs the cost $\kappa A_t z$ to post a vacancy. Denote by $Y_t(z)$ the present value of output produced by a match between a firm and a consumer of type z, and let $z' = (1 + g_e)z$. Since a match dissolves with exogenous probability σ, the present value $Y_t(z)$ can be expressed recursively as

$$Y_t(z) = A_t z + \phi(1 - \sigma)\mathbb{E}_t[Q_{t,t+1}Y_{t+1}(z')]. \tag{21}$$

Given a wage offer $W_{mt}(z)$ for consumers of type z, the value of a vacancy aimed at such consumers is

$$V_t(z) = -\kappa A_t z + \lambda_{ft}(\theta_t(z))[Y_t(z) - W_{mt}(z)] + [1 - \lambda_{ft}(\theta_t(z))]\mathbb{E}_t\{\max_{z'}[Q_{t,t+1}V_{t+1}(z')]\}. \tag{22}$$

Note that the last term in (22) captures the idea that if a firm is unsuccessful in hiring a consumer with human capital z in period t, then the firm can search again in period $t+1$ for a consumer with any human capital level z'. Free entry into market $(z, W_{ms}(z))$ implies that $V_s(z) = 0$ in any period s for any human capital z so that (22) yields

$$\kappa A_t z = \lambda_{ft}(\theta_t(z))[Y_t(z) - W_{mt}(z)]. \tag{23}$$

We refer to both $V_s(z) = 0$ for each s and z and (23) for each t and z as the *free-entry condition*.

Consider now the problem of a firm choosing an offer $\tilde{W}_{mt}(z)$ possibly different from $W_{mt}(z)$. We rely on the specification of consumers' search behavior in stage 2 to derive the tightness $\tilde{\theta}_t(z)$ of market $(z, \tilde{W}_{mt}(z))$ by restricting attention to *serious offers*, namely, offers that satisfy

$$\tilde{W}_{mt}(z) + W_{pt}(z) \geq \mathcal{W}_t(z) \tag{24}$$

and hence lead to a positive job-filling rate, as discussed earlier. When a firm makes a (serious) offer of $\tilde{W}_{mt}(z)$, the value of a vacancy is

$$\tilde{V}_t(z) = -\kappa A_t z + \lambda_{ft}(\tilde{\theta}_t(z))[Y_t(z) - \tilde{W}_{mt}(z)] + [1 - \lambda_{ft}(\tilde{\theta}_t(z))]\mathbb{E}_t\{\max_{z'}[Q_{t,t+1}V_{t+1}(z')]\},$$

where $\lambda_{ft}(\tilde{\theta}_t(z))$, determined from $\lambda_{wt}(\tilde{\theta}_t(z))$ in the participation constraint in (20), is the job-filling rate in market

$(z, \tilde{W}_{mt}(z))$. The problem of a firm that posts a measure of vacancies for consumers of type z is then

$$\max_{\{\tilde{W}_{mt}(z), \tilde{\theta}_t(z)\}} \tilde{V}_t(z), \tag{25}$$

subject to the participation constraint in (20) and the serious offer constraint in (24).[12]

Since a firm's value of any offer that is not a serious one is zero, it is without loss of generality to consider a relaxed version of this problem without the serious offer constraint. The first-order conditions for such a problem are $\lambda_{ft}(\tilde{\theta}_t(z)) = \tilde{\mu}_t(z)\lambda_{wt}(\tilde{\theta}_t(z))$ for $\tilde{W}_{mt}(z)$ and

$$\begin{aligned}\lambda'_f(\tilde{\theta}_t(z))(Y_t(z) - \tilde{W}_{mt}(z) - \mathbb{E}_t\{\max_{z'}[Q_{t,t+1}V_{t+1}(z')]\})\\ = \tilde{\mu}_t(z)\lambda'_{wt}(\tilde{\theta}_t(z))[\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z)]\end{aligned}$$

for $\tilde{\theta}_t(z)$, where $\tilde{\mu}_t(z)$ is the multiplier on the participation constraint. Rearranging terms, using the free-entry condition $V_{t+1}(z) = 0$, and exploiting the symmetry of equilibrium by setting $\tilde{W}_{mt}(z) = W_{mt}(z)$ and $\tilde{\theta}_t(z) = \theta_t(z)$ yields that the relationship

$$-\frac{\lambda'_{ft}(\theta_t(z))}{\lambda_{ft}(\theta_t(z))}[Y_t(z) - W_{mt}(z)] = \frac{\lambda'_{wt}(\theta_t(z))}{\lambda_{wt}(\theta_t(z))}[W_{mt}(z) + W_{pt}(z) - U_t(z)] \tag{26}$$

holds for all firms in equilibrium.

3.5 Competitive Search Equilibrium: Definition and Characterization

Given the stochastic processes for aggregate productivity A_t and the surplus consumption ratio S_t, we define a *competitive search equilibrium* as a collection of state-contingent sequences of allocations $\{C_t, e_t(z), u_t(z), v_t(z), \theta_t(z)\}$, pricing kernels $\{Q_{t,t+1}\}$, and values $\{W_{mt}(z), W_{pt}(z), U_t(z), \mathcal{W}_t(z), Y_t(z), V_t(z)\}$ such that *i*) for each t, taking as given the pricing kernels and the remaining allocations and values, market tightness $\theta_t(z)$ and the wage offer $W_{mt}(z)$ solve the firm's problem (25); *ii*) the values $W_{pt}(z)$, $U_t(z)$, $\mathcal{W}_t(z)$, $Y_t(z)$, and $V_t(z)$ satisfy the valuation equations (17), (18), (19), (21), and (22); *iii*) the laws of motion for employment and unemployment satisfy (6) and (7); *iv*) the free-entry condition (23) holds; *v*) the resource constraint (8) holds; *vi*) vacancies $v_t(z)$ are implied by the definition of $\theta_t(z)$ and (5); and *vii*) consumption C_t and the pricing kernel $Q_{t,t+1}$ satisfy (12).

We now turn to characterizing such an equilibrium, starting with the following result that we prove in the online appendix.

Proposition 1 (Uniqueness, Symmetry, and Efficiency of Competitive Search Equilibrium)**.** *The competitive search equilibrium is unique, symmetric, and efficient.*

That the competitive search equilibrium is efficient extends well-known results in the literature (see, for instance, the survey by Wright, Kircher, Benoît, and Guerrieri, 2021). One way to see this result is to observe that if we

[12]Since a firm posts a measure of vacancies to attract a measure of consumers, the resulting matches can be thought of as determined by the matching function $m_t(\cdot,\cdot)$, which is defined over measures. Hence, it makes sense to consider deviations by an individual firm in our setup. We thank Espen Moen for useful suggestions about this setup.

multiply both sides of (26) by $\theta_t(z)$ and use that $\eta_t(\theta_t(z)) = -\theta_t(z)\lambda'_{ft}(\theta_t(z))/\lambda_{ft}(\theta_t(z))$ and $1 - \eta_t(\theta_t(z)) = \theta_t(z)\lambda'_{wt}(\theta_t(z))/\lambda_{wt}(\theta_t(z))$, then this condition is equivalent to the Hosios condition for Nash bargaining, which in turn implies the efficiency of equilibrium.

We establish next that all equilibrium value functions are linear in z because market production, home production, and the cost of posting job vacancies are all linear in z. Thus, market tightness, job-finding rates, and job-filling rates are independent of z. In formalizing this result, also proved in the online appendix, we let W_{mt} denote $W_{mt}(1)$ and use similar notation for the remaining values.

Lemma 1 (Linearity of Competitive Search Equilibrium)**.** *In a competitive search equilibrium, labor market tightness $\theta_t(z)$, the job-finding rate $\lambda_{wt}(\theta_t(z))$, the job-filling rate $\lambda_{ft}(\theta_t(z))$, and the elasticity $\eta_t(\theta_t(z))$ of the job-filling rate with respect to $\theta_t(z)$ are independent of z, and values are linear in z in that $W_{mt}(z) = W_{mt}z$, $W_{pt}(z) = W_{pt}z$, $U_t(z) = U_tz$, $\mathcal{W}_t(z) = \mathcal{W}_tz$, and $Y_t(z) = Y_tz$.*

This result implies that to solve for equilibrium values and for the labor market and stock market variables of interest, we do not need to record the measures $e_t(z)$ and $u_t(z)$ of employed and unemployed consumers with each level of human capital z but rather only the aggregate human capital of employed and unemployed consumers given by $Z_{et} = \int_z ze_t(z)dz$ and $Z_{ut} = \int_z zu_t(z)dz$, respectively. As we show in the online appendix, multiplying (6) and (7) by z and integrating the resulting expressions with respect to z gives the transitions laws for the aggregate human capital of employed and unemployed consumers,

$$Z_{et} = \phi(1-\sigma)(1+g_e)Z_{et-1} + \phi\lambda_{wt}(\theta_t)(1+g_u)Z_{ut-1} \tag{27}$$

and

$$Z_{ut} = \phi\sigma(1+g_e)Z_{et-1} + \phi[1-\lambda_{wt}(\theta_t)](1+g_u)Z_{ut-1} + 1 - \phi, \tag{28}$$

which can be used to express the aggregate resource constraint as

$$C_t = A_tZ_{et} + bA_tZ_{ut} - \kappa A_t\phi\theta_t(1+g_u)Z_{ut-1}, \tag{29}$$

where we have used that aggregate vacancies in efficiency units satisfy $Z_{vt} = \int_z zv_t(z)dz = \phi\theta_t(1+g_u)Z_{ut-1}$, since $v_t(z) = \theta_tu_{bt}(z)$ and $u_{bt}(z) = \phi u_{t-1}(z/(1+g_u))/(1+g_u)$. Note that in light of Lemma 1, we have dropped the dependence of market tightness on z. From now on, then, we denote the job-finding rate, the job-filling rate, and the elasticity of the job-filling rate with respect to θ_t as $\lambda_{wt}(\theta_t)$, $\lambda_{ft}(\theta_t)$, and $\eta_t(\theta_t)$, respectively, or, simply, λ_{wt}, λ_{ft}, and η_t.

Proceeding similarly, we define aggregate employment as $e_t = \int_z e_t(z)dz$ and aggregate unemployment as $u_t = \int_z u_t(z)dz$. As we prove in the online appendix, integrating (7) with respect to z gives

$$u_t = \phi\sigma e_{t-1} + [1-\lambda_{wt}(\theta_t)]u_{bt} + 1 - \phi, \tag{30}$$

where $u_{bt} = \int_z u_{bt}(z)dz = \phi u_{t-1}$. Since consumers are of measure one, aggregate employment satisfies $e_t = 1 - u_t$. Hence, given some initial conditions for e_{-1} and u_{-1}, we can construct the processes for aggregate employment and unemployment just from the process for θ_t.

The next proposition establishes that for given initial conditions for the aggregate human capital Z_{e-1} and Z_{u-1} of employed and unemployed consumers, the competitive search equilibrium aggregate allocations $\{C_t, Z_{et}, Z_{ut}, \theta_t\}$ solve the *aggregate planning problem*, namely, the problem of maximizing (9) subject to (27) through (29), taking as given the exogenous processes for aggregate productivity A_t and the surplus consumption ratio S_t. The remaining competitive search equilibrium aggregate allocations of the employed e_t, the unemployed u_t, and vacancies $v_t = \int_z v_t(z)dz$ with $v_t(z) = \theta_t u_{bt}(z)$ are determined given some initial conditions for e_{-1} and u_{-1}.

Proposition 2 (Characterization of Aggregate Allocations)**.** *The competitive search equilibrium aggregate allocations* $\{C_t, Z_{et}, Z_{ut}, \theta_t\}$ *solve the aggregate planning problem.*

For later, it is useful to note that an immediate implication of Proposition 1 is that the solution to the aggregate planning problem is unique.

3.6 Characterization of the Job-Finding Rate

The first-order conditions for the solution to the aggregate planning problem imply that

$$\mu_{et} = A_t + \phi(1 + g_e)\mathbb{E}_t\{Q_{t,t+1}[(1 - \sigma)\mu_{et+1} + \sigma\mu_{ut+1}]\}, \tag{31}$$

$$\mu_{ut} = bA_t + \phi(1 + g_u)\mathbb{E}_t\{Q_{t,t+1}[\eta_{t+1}\lambda_{wt+1}\mu_{et+1} + (1 - \eta_{t+1}\lambda_{wt+1})\mu_{ut+1}]\}, \tag{32}$$

$$\kappa A_t = (1 - \eta_t)\lambda_{ft}(\mu_{et} - \mu_{ut}), \tag{33}$$

as derived in the appendix, where μ_{et} and μ_{ut} are the (scaled) multipliers associated with the transition laws for the aggregate human capital of employed and unemployed consumers in (27) and (28) and so describe the shadow values of augmenting their stocks of human capital by one unit. The discount factors $\{Q_{t,t+1}\}$ in (12) are defined from allocations. Note that conditions (31) to (33) are similar to those that arise in standard search models. In particular, equation (31) is analogous to the sum of the value of an employed consumer and the value of an employing firm, (32) is analogous to the sum of the value of an unemployed consumer and the value of a firm searching for a consumer, and (33) is analogous to the free-entry condition in those models. The key difference is that in our competitive search equilibrium, the planner takes into account the impact of vacancy creation on job-finding and job-filling rates and hence internalizes the search externality generated by firms posting vacancies to attract consumers in order to maximize their own profits.

Under the assumption of a Cobb-Douglas matching function of the form $m(u, v) = Bu^\eta v^{1-\eta}$, which we maintain in our quantitative analysis and implies that $\lambda_{ft} = B^{1/(1-\eta)}\lambda_{wt}^{-\eta/(1-\eta)}$, we can rewrite (33) as

$$\log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right)\log\left(\frac{\mu_{et} - \mu_{ut}}{A_t}\right), \tag{34}$$

where χ is a constant derived in the appendix. Expression (34) makes it clear that the job-finding rate λ_{wt} is determined by the value $\mu_{et} - \mu_{ut}$ of hiring a consumer scaled by aggregate productivity A_t, up to constants. Given $\{Q_{t,t+1}\}$, the multipliers μ_{et} and μ_{ut} are solutions to the dynamical system defined by (31) and (32). Here μ_{et} is the (expected) present (discounted) value of the sum of market and home production for a consumer with human capital $z_t = 1$ who is employed in period t, and μ_{ut} is the corresponding present value for a consumer with human capital $z_t = 1$ who is unemployed in period t. The difference in these present values, $\mu_{et} - \mu_{ut}$, is the value of hiring such a consumer, which is given by the present value of the incremental output produced by moving one consumer with one unit of human capital ($z_t = 1$) from unemployment to employment in t. We refer to the incremental output in period $t + n$ resulting from such a change in period t as the *surplus flow* in $t + n$ from a match created in t and denote it by v_{t+n}.

To develop intuition for the solution to this system, we consider an approximation to it, in which we ignore the variation in future job-finding rates $\lambda_{ws} = \lambda_w(\theta_s)$ by assuming that $\lambda_{ws} = \lambda_w(\theta)$, $s > t$, for a given θ, and so denote λ_{ws} by λ_w for simplicity. (In Kehoe, Lopez, Midrigan, and Pastorino, 2021, we show that the intuition we provide here and elsewhere continues to hold when we dispense with the assumption of constant future job-finding rates.) In our quantitative analysis, we solve this system through an accurate global nonlinear algorithm described later that involves no approximation. To determine the present value of surplus flows under our approximation of future job-finding rates, we impose the limit condition that the present values of future multipliers converge to zero, and we solve the dynamical system in (31) and (32) forward to obtain[13]

$$\begin{bmatrix} \mu_{et} \\ \mu_{ut} \end{bmatrix} = \sum_{n=0}^{\infty} \phi^n \begin{bmatrix} (1+g_e)(1-\sigma) & (1+g_e)\sigma \\ (1+g_u)\eta\lambda_w & (1+g_u)(1-\eta\lambda_w) \end{bmatrix}^n \begin{bmatrix} 1 \\ b \end{bmatrix} \mathbb{E}_t(Q_{t,t+n}A_{t+n}). \tag{35}$$

It follows from (35) that the value $\mu_{et} - \mu_{ut}$ of hiring a consumer on the right side of (34) depends on the present value of aggregate productivity and can be expressed as

$$\mu_{et} - \mu_{ut} = \sum_{n=0}^{\infty} \mathbb{E}_t(Q_{t,t+n}v_{t+n}), \tag{36}$$

where the surplus flow in period $t + n$ is given by

$$v_{t+n} = (c_\ell \delta_\ell^n + c_s \delta_s^n) A_{t+n}. \tag{37}$$

For this latter result, we have used that since μ_{et} and μ_{ut} are determined by the system in (35), so is their difference, which has the form shown in (36). In (37), δ_ℓ and δ_s are the large and small eigenvalues or *roots* of the two-by-two matrix in the vector difference equation in (35) with corresponding weights c_ℓ and c_s. For ease of interpretation, it is convenient to rewrite the present value of surplus flows as

$$\mathbb{E}_t(Q_{t,t+n}v_{t+n}) = (c_\ell \delta_\ell^n + c_s \delta_s^n) P_{nt}, \tag{38}$$

[13]We impose $\lim_{T\to\infty} \mathbb{E}_t(Q_{t,T}\Psi^{T-t}[\mu_{eT}, \mu_{uT}]^\intercal) = [0,0]^\intercal$ with $\Psi = \phi \begin{bmatrix} (1+g_e)(1-\sigma) & (1+g_e)\sigma \\ (1+g_u)\eta\lambda_w & (1+g_u)(1-\eta\lambda_w) \end{bmatrix}$.

where $P_{nt} = \mathbb{E}_t(Q_{t,t+n} A_{t+n})$ is the price of an asset that pays a one-time dividend of A_{t+n} in period $t+n$. We refer to this asset as a claim to productivity in n periods or simply a productivity *strip*. Note that the present value of surplus flows on the right side of (36) decays over time, as it represents the discounted difference between the value of being employed and the value of being unemployed and both states are transitory: an employed consumer can lose a job and an unemployed consumer can find one. That these surplus flows decay *slowly* over time will prove critical for our model's ability to generate volatile job-finding rates and unemployment.

Consider now the solution for $\mu_{et} - \mu_{ut}$ from the system in (35). To keep the algebra simple, we set the survival probability ϕ to one and maintain that g_u is zero; see the appendix for the general case. As we show there, the large $\delta_\ell > 1$ and small $\delta_s < 1$ roots of this solution are

$$\delta_\ell = 1 + \tfrac{1}{2}[\sqrt{(1-\lambda)^2 + 4\eta\lambda_w g_e} - \sqrt{(1-\lambda)^2}] \quad \text{and} \quad \delta_s = \lambda - \tfrac{1}{2}[\sqrt{(1-\lambda)^2 + 4\eta\lambda_w g_e} - \sqrt{(1-\lambda)^2}], \tag{39}$$

with associated weights

$$c_\ell = [(\lambda - \delta_s)(1-b) + g_e b]/(\delta_\ell - \delta_s) \quad \text{and} \quad c_s = 1 - b - c_\ell, \tag{40}$$

where $\lambda = (1-\sigma)(1+g_e) - \eta\lambda_w < 1$.[14] Combining these formulae with (34), (36), (37), and (38), we then obtain:

Proposition 3 (Job-Finding Rate)**.** *The job-finding rate approximately satisfies*

$$\log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right) \log\left[\sum_{n=0}^{\infty} (c_\ell \delta_\ell^n + c_s \delta_s^n) \frac{P_{nt}}{A_t}\right], \tag{41}$$

where χ is a constant and δ_ℓ, δ_s, c_ℓ, and c_s are given in (39) and (40).

This proposition shows that the (log) job-finding rate is a weighted average of the prices of claims to future productivity, up to the constants χ and η. Hence, movements in the job-finding rate are due solely to movements in these prices, relative to current productivity. Interestingly, this result applies as stated to all preference structures we examine. In particular, under our approximation of constant future job-finding rates, the weights $\{c_\ell \delta_\ell^n + c_s \delta_s^n\}$ remain fixed as we vary preferences, since they are determined solely by the labor market side of the economy—the roots (δ_ℓ, δ_s) and their weights (c_ℓ, c_s) do not depend on either the utility function or the aggregate productivity process. Then, the formula for the job-finding rate has this same form for all the preferences we consider and differs across them only in terms of the expression for P_{nt}/A_t.[15]

After we present our quantitative results, we use the formula in Proposition 3 to provide some intuition about the amplification mechanism for productivity shocks that our model gives rise to. As will become clear, the longer is the

[14] When ϕ is less than or equal to one and g_u is nonzero, $c_\ell = [(\phi\lambda - \delta_s)(1-b) + \phi(g_e - g_u)b]/(\delta_\ell - \delta_s)$, $c_s = 1 - b - c_\ell$, $\delta_\ell = \phi(1+g_u) + \frac{\phi}{2}[\sqrt{(1+g_u-\lambda)^2 + 4\eta\lambda_w(1+g_u)(g_e-g_u)} - \sqrt{(1+g_u-\lambda)^2}]$, and $\delta_s = \phi\lambda - \frac{\phi}{2}[\sqrt{(1+g_u-\lambda)^2 + 4\eta\lambda_w(1+g_u)(g_e-g_u)} - \sqrt{(1+g_u-\lambda)^2}]$ with $\lambda = (1-\sigma)(1+g_e) - \eta\lambda_w(1+g_u)$.

[15] If we drop our approximation of constant future job-finding rates, then when we solve (31) and (32) forward for μ_{et} and μ_{ut}, terms of the form $\mathbb{E}_t(Q_{t,t+n}\lambda_{wt+n}A_{t+n})$ appear that involve the covariance of $Q_{t,t+n}$ and $\lambda_{wt+n}A_{t+n}$. Since preferences affect this covariance, the resulting formulae for $\mu_{et} - \mu_{ut}$ and hence λ_{wt} depend on preferences. In Kehoe, Lopez, Midrigan, and Pastorino (2021), we show that the formulae in Proposition 3 can nonetheless be extended to this more general case.

horizon or maturity n of claims to future productivity, the more sensitive are the prices of these claims to fluctuations in productivity. Moreover, the larger is the growth of human capital during employment, and the larger is the decline of human capital during unemployment, the more slowly the present value of surplus flows decays over time. Correspondingly, by (38), the present value of the prices of claims to future productivity attributes large weights to long-maturity claims. That is, the persistence that the human capital process imparts to surplus flows implies that these flows have long durations, making the job-finding rate more dependent on the prices of long-maturity claims that fluctuate relatively more with productivity. This feature of our model will play a crucial role in amplifying the impact of aggregate shocks to the labor market.

4 Quantification

Here we describe how we choose parameters for our quantitative analysis. The model is monthly, and its parameters are listed in Table 1: six parameters, $\{B, b, \sigma, \eta, \phi, g_e\}$, have assigned values, and the remaining seven, $\{g_a, \sigma_a, \kappa, \beta, S, \alpha, \rho_s\}$, have values chosen to match seven moments from the data. We normalize the value of market tightness θ to one in the deterministic steady state, as in Shimer (2005), which pins down the efficiency of the matching function, B. Following Ljungqvist and Sargent (2017), we fix the home production parameter b to 0.6 and the matching function elasticity η to 0.5. We set the separation rate σ to 2.8%, which equals the Abowd-Zellner corrected estimate of the separation rate by Krusell et al. (2017), based on data from the Current Population Survey (CPS).[16]

We interpret each model year as corresponding to one year of (potential) labor market experience—measured in the data as age minus education minus six—and choose the survival probability ϕ to be consistent with an average working life of 30 years, which corresponds to the prime-age working horizon between the ages of 25 and 55 years. Below, we consider an alternative parameterization with ϕ set at a value consistent with an average working life of 40 years, as we assume in our later life-cycle extension of the baseline model. We select a growth rate of human capital during employment g_e of 3.5% per year, which, taking into account an aggregate productivity growth of 2.2% per year, matches the average annual growth rate of real hourly wages documented by Rubinstein and Weiss (2006, Table 2b), based on the National Longitudinal Survey of Youth (NLSY).[17] To make clear that our results do not rely on human capital depreciating during unemployment, we set g_u to zero in our baseline. We later explore the sensitivity of our findings to lower rates of human capital accumulation on the job and higher rates of human capital depreciation off the job. As we will discuss, our results hold for a wide range of values for g_e and g_u. In particular, a locus of pairs (g_e, g_u) exists with identical predictions for the volatility of the job-finding rate.

[16]This statistic is lower than the 3.4% monthly separation rate in Shimer (2005) owing to the correction for potential misclassification. We also experimented with a recalibration in which we used the higher separation rate in Shimer (2005) and found very similar results. As will become evident, employment responses in our model are determined mainly by the duration of surplus flows from a match rather than by the length of time a worker spends in any given match.

[17]Rubinstein and Weiss (2006, Table 2b) documented that annual wage growth is 7.7% over the first 10 years of labor market experience, 3.3% between 11 and 15 years of experience, and 4.9% between 16 and 25 years of experience, for an average annual growth rate of approximately 5.8%. Taking into account a growth rate of labor productivity of 2.2% per year (see footnote 18), we obtain an annual growth rate of wages of 3.5%, which we use in our baseline calibration. Despite our model's lack of specific predictions for flow wages, we later show that similar values for g_e are implied by our moment-matching exercise under different wage protocols for flow wages.

We turn now to the endogenously chosen parameters. We select the parameters g_a and σ_a of the aggregate productivity process to match the mean and standard deviation of labor productivity growth from the U.S. Bureau of Labor Statistics (BLS) for the period between January 1947 and December 2007.[18] We choose κ to reproduce an average quarterly unemployment rate of 5.9% based on data from the BLS between January 1948 and December 2007.

Consider next the preference parameters, $\{\beta, S, \alpha\}$. We choose the rate of time preference β and the mean S of the surplus consumption ratio S_t so as to match the mean and standard deviation of the real risk-free rate r_{ft} measured as $i_t - \mathbb{E}_t(\pi_{t+1})$, where i_t is the one-month Treasury bill rate and $\mathbb{E}_t(\pi_{t+1})$ is expected inflation.[19] To see how the process for the state S_t can be parametrized to generate a volatility of the risk-free rate in line with the data, consider the following argument adapted from Campbell and Cochrane (1999) to our economy. Under the approximation that consumption growth equals productivity growth in that $\Delta c_t \approx \Delta a_t$, consumption innovations are normally distributed and the risk-free rate, defined as $\exp(r_{ft}) = 1/\mathbb{E}_t(Q_{t,t+1})$, satisfies

$$r_{ft} \approx -\mathbb{E}_t[\log(Q_{t,t+1})] - \frac{1}{2}\sigma_t^2[\log(Q_{t,t+1})]. \tag{42}$$

Then, by using the law of motion for s_t, the definition of the sensitivity function $\lambda_a(s_t)$, and our approximation that $\mathbb{E}_t(\Delta c_{t+1}) \approx \mathbb{E}_t(\Delta a_{t+1}) = g_a$, simple algebra implies that

$$r_{ft} \approx \text{const} + \alpha \left[\frac{\alpha\sigma_a^2}{S^2} - (1-\rho_s)\right] s_t. \tag{43}$$

Hence, the model can reproduce the standard deviation of the risk-free rate in the data with a suitable choice of S. (For details, see the online appendix.)

We choose the inverse elasticity of intertemporal substitution α so that the (unconditional) Sharpe ratio of the stock market excess return $\mathbb{E}(R_{t+1} - R_{ft})/\sigma(R_{t+1} - R_{ft})$ in the model, measured by the return on consumption claims, equals the (unconditional) Sharpe ratio of the stock market excess return in the data.[20] This strategy is similar to that used by Campbell and Cochrane (1999) and Wachter (2006) in a related context. To pin down the persistence ρ_s of the log surplus consumption ratio s_t, we follow Mehra and Prescott (1985), Campbell and Cochrane (1999), and Wachter (2006) and interpret dividends as claims to aggregate consumption in the model and as claims to aggregate dividends in the data. Based on this strategy, we choose ρ_s to reproduce the annual serial correlation of the log price-dividend ratio in the data.[21]

[18]The actual variable is "Nonfarm Business Sector: Labor Productivity (Output per Hour) for All Employed Persons." We use data for labor productivity starting from 1947 to guarantee that the time series for productivity *growth* conforms to our time series for unemployment, which covers the period between 1948 and 2007.

[19]We calculate i_t using an updated version of the Fama and French's (1993) data for the thirty-day Treasury bill rate, available from Kenneth French's website, and $\mathbb{E}_t(\pi_{t+1})$ as the inflation predicted from a regression of monthly CPI (U.S. city average for all urban consumers) inflation on twelve of its lags, as is common in the literature.

[20]In the model, the consumption claim in t, $\{C_s\}_{s=t}^{\infty}$, is defined as a claim to a dividend stream equal to aggregate consumption today and in all future periods. We measure the Sharpe ratio of the stock market excess return in the data using the S&P composite stock price index available from Robert Shiller's website. Note that we would have obtained similar results by using data from, say, the Flow of Funds, since as shown by Larrain and Yogo (2008), the returns measured from broad-based stock price indices are highly correlated with the returns measured from aggregate stock market prices and dividends based on the Flow of Funds. In our sample, this correlation is 0.97.

[21]In the online appendix, we discuss with what degree of precision the data on asset prices pin down ρ_s and how our results on the volatility of

We conclude by describing the global algorithm that we use to solve the model; we provide more details in the online appendix. We found, as Wachter (2005) did in a related context, that only a global method proves an accurate solution. Accordingly, we use Chebyshev polynomials for policy rules and evaluate expectations by a Gauss-Hermite quadrature with a sufficiently large number of nodes so that results are not sensitive to changes in their number.[22] We follow Wachter (2005) in allowing for a large fine grid over the surplus consumption space that crucially places many grid points close to zero.

5 Findings

Shimer (2012) has argued that a key issue confronting existing search models is that they generate too little variation in the job-finding rate, which accounts for over two-thirds of the observed fluctuations in unemployment. Accordingly, our study is focused solely on a mechanism that increases the volatility of the job-finding rate. For this reason, we purposely abstract from fluctuations in the job-separation rate and compare, across the model and the data, statistics on the job-finding rate and a constant-separation unemployment rate implied by it, as in Shimer (2012), which we discuss next. We then turn to the model's implications for stock and bond returns.

5.1 Job-Finding Rate and Unemployment

As Table 1 illustrates, our model produces an average quarterly volatility of the job-finding rate of 6.60%, which is very similar to that in the data, 6.67%. Note, though, that even if our model exactly matched the empirical time series of the job-finding rate, it would not be able to match the empirical time series of the unemployment rate, because the separation rate varies in the data, whereas it is constant in our model. To address this issue, we follow Shimer (2012) and construct a monthly *constant-separation unemployment rate* series $\{\bar{u}_t\}$ based on data on unemployment from the BLS between 1948 and 2007 with law of motion $\bar{u}_{t+1} = \sigma(1 - \bar{u}_t) + (1 - \lambda_{wt+1})\bar{u}_t$, where σ equals 2.8%, which implies an average quarterly unemployment rate of 5.9%; see Shimer (2012) for details. For brevity, both in Table 1 and hereafter, we refer to this series simply as the *unemployment rate*.

Table 1 shows that our model closely matches the volatility of this constant-separation unemployment rate in the data (0.75% in the model and 0.79% in the data) and implies a serial correlation for it of 0.99, which is in line with that in data, 0.96. Table 1 further demonstrates that our model reproduces well the highly negative correlation between job-finding and unemployment rates, which is -0.99 in the model and -0.96 in the data. This result is consistent with

the job-finding rate vary across empirically relevant values of ρ_s. Specifically, in our baseline model, we choose ρ_s by targeting the autocorrelation of the log price-dividend ratio. In the online appendix, we first measure the variability of the estimate of this autocorrelation from the data. Then, we show how varying the estimated value of this autocorrelation in our moment-matching exercise affects the implied value of the habit persistence parameter and, in turn, the volatility of the job-finding rate. Finally, we investigate how the variability of other moments in the data that can alternatively be used to discipline ρ_s, such as the standard deviation of the excess return on stocks and the mean of the 20-year real bond yield, impacts the value of this parameter and the volatility of the job-finding rate. Overall, we find that our model's implications for the volatility of the job-finding rate are robust to different methods of determining the value of the habit persistence parameter and to a broad range of relevant values for it.

[22] In particular, we use Chebyshev polynomials of degree twenty in the transformation $g(s_t) = [1 - 2(s_t - s)]^{1/2}$ of the surplus consumption state s_t and of degree four in each of the human capital states Z_{et} and Z_{ut}.

Shimer's (2005) emphasis that unemployment rises in recessions because the job-finding rate falls owing to a decline in vacancy creation.

In light of all of these statistics, we conclude that our model solves the unemployment volatility puzzle.

5.2 Stocks and Long-Term Bonds

We now discuss our model's implications for stock market returns and long-term bond yields.

5.2.1 Stock Market Returns

In the data, flows of payments to equity or debt holders are mostly payments for physical and intangible capital, and depend on firm leverage. Our simple model without either physical or intangible capital features none of these payments and abstracts from leverage. Indeed, as the free-entry condition shows, equity flows in our model are simply payments for the up-front costs of posting job vacancies. For these reasons, we follow the simple approach in the asset-pricing literature that dates back at least to Mehra and Prescott (1985) and interprets stocks as claims to streams of aggregate consumption; see, for instance, Campbell and Cochrane (1999) and Wachter (2006). Following this approach, we price claims to streams of aggregate consumption in the model and contrast their prices to stock prices in the data.

In Table 1, we compare the mean and standard deviation of the excess return on stocks, their ratio—which corresponds to the Sharpe ratio of the stock market excess return—and the mean and standard deviation of the log price-dividend ratio computed from the S&P composite stock price index with the corresponding statistics on consumption claims implied by our baseline model. As is apparent from the table, the two sets of statistics are indeed close. For instance, the ratio of the mean to the standard deviation of the excess market return is 0.45 both in the model and in the data. The standard deviation of the log price-consumption ratio in the model is 86% of the standard deviation of the log price-dividend ratio from the S&P composite stock price index in the data, namely, 0.38 versus 0.44. In this sense, our model has predictions for the stock market broadly in line with the data.

5.2.2 Long-Term Bond Yields

One problem with the early habit model of, say, Jermann (1998), is that it implied excessively volatile interest rates. As Cochrane (2008, p. 295) explained, the next generation of habit models, such as Campbell and Cochrane (1999), were designed to overcome this limitation. As Table 1 illustrates, the standard deviation of the risk-free rate in our model is the same as that in the data. Hence, our baseline model does not imply excessively volatile short-term interest rates.

We now turn to examining our model's implications for long-term interest rates and show that our model does not generate a volatility puzzle for such rates either. Two basic approaches have been followed in the literature to analyze long-term interest rates. The first approach relies on Treasury Inflation-Protected Securities (TIPS) as a measure of real interest rates. The second approach consists of augmenting the economic model of interest with an inflation process and assessing the resulting model's implications for nominal interest rates. In terms of the data, which we take from Gurkaynak, Sack, and Wright (2007, 2010), we note that reliable measures of nominal 20-year bonds and TIPS are available only since 1981 and 1999, respectively.

Consider the first approach of directly constructing measures of real interest rates using TIPS. One issue with this approach, as discussed by D'Amico, Kim, and Wei (2018), is that in several well-known periods, these securities were quite illiquid, and thus their yields were distorted by sizable liquidity premia. We address this issue by following the strategy by D'Amico, Kim, and Wei (2018), who argued that a TIPS-specific liquidity premium can be filtered out from TIPS yields by regressing these yields on the first three principal components of nominal yields of up to 20-year maturity. To lengthen the sample, we backcast TIPS yields using predictions from this same regression on principal components of nominal yields estimated over the post-1999 period to obtain liquidity-adjusted TIPS yields for the period between 1981 and 1999, during which nominal 20-year bonds were actively traded but TIPS were not, as discussed by Gurkaynak, Sack, and Wright (2007).

In Table 1, we report the mean and standard deviation of real yields on 20-year bonds constructed using this strategy, as well as the corresponding statistics for real yields in our baseline model. The mean yield in the model, 3.55, is a bit lower than that in the data, 4.74, but the standard deviation of these yields in the model, 2.24, is quite close to that in the data, 1.95.

Consider next the second approach. We first solve for all real variables in our model. We then append to the model a purely exogenous process for inflation with shocks that are correlated with those to the real side of the economy. Specifically, following Wachter (2006), we specify inflation as an ARMA(1,1) process and estimate the process for CPI inflation allowing for the contemporaneous correlation of inflation with shocks to aggregate productivity. It is worth noting that, as in Wachter (2006), although inflation has no effect on real variables by construction, nominal bonds carry a positive risk premium over real bonds because the estimated process for inflation implies that inflation is high when surplus consumption is low, so that nominal bonds buy fewer goods when consumers have a high marginal utility of consumption. (See the online appendix for details.) In practice, this inflation-risk premium on nominal bonds is very small, about 25 basis points.

Given the estimated parameters of the inflation process, we can compare nominal yields in the model to those in the data. As reported in Table 1, the mean nominal yield on 20-year bonds is similar across the model and the data, 7.53 versus 7.71, and the standard deviation of these yields in the model is also close to that in the data, 2.30 versus 2.41.

In summary, our model does not display a risk-free rate puzzle at either short or long horizons.

6 Two Critical Forces: Time-Varying Risk and Human Capital

Here we demonstrate the critical roles played by time-varying risk and the process of human capital acquisition for our results. Without either ingredient, our model would not generate volatile job-finding rates or unemployment. To illustrate the role of time-varying risk, we study a model with CRRA preferences and show that such a model implies no volatility for either the job-finding rate or unemployment. To illustrate the role of human capital acquisition, we study a model in which consumers' human capital is constant over time and show that such a model generates negligible fluctuations in labor market variables.

6.1 Role of Time-Varying Risk

We investigate the importance of time-varying risk in our model by showing that as the degree of risk in the economy decreases, so does the volatility of the job-finding rate. We begin with a stark example in which we eliminate time-varying risk by removing the habit in consumption, so that our preferences reduce to preferences with constant relative risk aversion, and show that the job-finding rate is constant in this case. We then demonstrate, more generally, that as we reduce the degree of risk in the economy, the volatility of both the labor market and the stock market decreases.

6.1.1 Constant Relative Risk Aversion Preferences

Consider a version of our model in which we set $S_t = 1$ so that our baseline utility function reduces to the CRRA form

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\alpha}}{1-\alpha} \right). \tag{44}$$

For this specification of preferences and a general time-invariant constant returns-to-scale matching function, the fluctuations of the job-finding rate and unemployment are identically zero.

Proposition 4 (Constant Job-Finding Rate and Unemployment under CRRA)**.** *Starting from the steady-state values of the total human capital of the employed and the unemployed, Z_e and Z_u, with preferences of the form in (44), both the job-finding rate and the unemployment rate are constant.*

In interpreting this result, it is important to note that by construction, we have abstracted from the standard mechanism of *differential productivity across sectors* of search models, which implies that a decrease in aggregate productivity A_t reduces a consumer's productivity in market production but leaves a consumer's productivity in home production and the cost of posting job vacancies unaffected. Hence, by this mechanism alone, consumers find it less attractive to work in the market, and firms find it less profitable to hire workers in a downturn, so a reduction in aggregate productivity leads to an increase in unemployment. In our model, by contrast, a decrease in A_t equally decreases a consumer's productivity in market and home production as well as the cost of posting vacancies. Specifically, a consumer with human capital z produces $A_t z$ when employed and $bA_t z$ when unemployed, and it costs a firm $\kappa A_t z$ to post a vacancy for such a consumer. Therefore, the only effect of a change in aggregate productivity in our model is the resulting change in the present value of the surplus flows from a firm-consumer match, as (36) and (37) show. With CRRA preferences and random-walk productivity, this present value is constant relative to current productivity, and so are the job-finding rate by (34) and unemployment.

We now provide some intuition for this result using our approximation for the job-finding rate in (34). In the appendix, we show that the same logic applies to the formal proof of the result, which does not rely on any approximations. To start, note that by substituting into (36) the expression $(c_\ell \delta_\ell^n + c_s \delta_s^n) A_{t+n}$ for the surplus flow v_{t+n} by (37) and the CRRA pricing kernel $\beta^n (C_{t+n}/C_t)^{-\alpha}$ for the pricing kernel $Q_{t,t+n}$ by (12) and dividing by A_t, the key term on the right side of (34) becomes

$$\frac{\mu_{et}-\mu_{ut}}{A_t}=\sum_{n=0}^{\infty}\beta^n(c_\ell\delta_\ell^n+c_s\delta_s^n)\mathbb{E}_t\left\{\left(\frac{A_{t+n}}{A_t}\right)^{1-\alpha}\left(\frac{\tilde{C}_{t+n}}{\tilde{C}_t}\right)^{-\alpha}\right\}, \tag{45}$$

where $\tilde{C}_t = C_t/A_t$ is scaled consumption. With CRRA preferences and random-walk productivity, it is easy to show that consumption moves proportionately to productivity so that scaled consumption $\tilde{C}_t$ is constant. Moreover, given that productivity follows a random-walk process, its expected growth rate, $\mathbb{E}_t(A_{t+n}/A_t)^{1-\alpha}$, does not vary with A_t. Hence, the right side of (45) is constant, which implies that both the job-finding rate and unemployment are invariant to changes in A_t by (34).

6.1.2 Relation between Labor Market Volatility and Stock Market Volatility

We now consider two experiments in which we reduce the degree of risk in the economy and show that when we do so, both labor market volatility and stock market volatility fall. In this sense, our model generates volatility in the job-finding rate and unemployment only if it also generates volatility in stock prices.

Recall that time-varying risk arises in our model because of a large and, owing to fluctuations in the surplus consumption ratio S_t, volatile price of risk. To isolate the importance of time-varying risk, we then conduct two experiments. For the first experiment, note that at a steady state, the price of risk is approximately equal to $\alpha\sigma_a/S$.[23] In the first experiment, we thus reduce the price of risk by increasing S (for fixed α and σ_a) and adjust the vacancy posting cost κ so as to keep the mean unemployment rate fixed at its baseline value. In the left panel of Figure 1, we graph the resulting volatility of the job-finding rate, the volatility of the stock market as measured by the standard deviation of the log price-consumption ratio, and the equity premium $\mathbb{E}(r_{emt})$ as measured by the expected stock market excess return, defined as the difference between the mean return on the consumption claim and the risk-free rate. All three statistics are reported as a percentage of their level in the baseline. It is apparent from the figure that as we reduce the price of risk by decreasing $\alpha\sigma_a/S$, the risk in the economy, as captured by the volatility of the stock market and the equity premium, falls, and the volatility of the job-finding rate falls along with it.

In the second experiment, we consider the role of the variability in the consumption habit in generating time-varying risk. To isolate its importance, we vary the standard deviation of the process for the surplus consumption ratio, holding fixed the primitive volatility of the aggregate productivity process at its baseline value. Formally, recall that the standard deviation parameter σ_a affects both the process for aggregate productivity,

$$\log(A_{t+1}) = g_a + \log(A_t) + \sigma_a\varepsilon_{at+1}, \tag{47}$$

[23] To see this, note that assuming that returns $\{R_{t+1}\}$ are conditionally log-normally distributed, and using the approximation that $\Delta c_{t+1} \approx \Delta a_{t+1}$, we obtain that

$$\left|\frac{\log[\mathbb{E}_t(R_{t+1}/R_{ft})]}{\sigma_t[\log(R_{t+1}/R_{ft})]}\right| \leq \sigma_t[\log(Q_{t,t+1})] = \alpha[1+\lambda_a(s_t)]\sigma_a = \frac{\alpha}{S}\sqrt{1-2(s_t-s)}\sigma_a, \tag{46}$$

where the first inequality is the Hansen-Jagannathan bound for log-normal returns and the first equality follows from (10)-(12). The last term of this inequality is a measure of the price of risk for log-normally distributed returns, and it reduces to $\alpha\sigma_a/S$ in a steady state with $s_t = s$.

and the process for the log surplus consumption ratio,

$$s_{t+1} = (1 - \rho_s)s + \rho_s s_t + \lambda_a(s_t)\tilde{\sigma}\varepsilon_{at+1}, \tag{48}$$

since $\tilde{\sigma} = \sigma_a$ in the baseline. In this second experiment, we reduce the fluctuations in the surplus consumption ratio by lowering $\tilde{\sigma}$ while keeping the volatility σ_a of the aggregate productivity process in (47) unchanged. We proceed as we did above and adjust the vacancy posting cost κ to keep the mean unemployment rate fixed at its baseline value. In the right panel of Figure 1, we graph the same three variables implied by our model as in the left panel of the figure for different values of $\tilde{\sigma}$. As is apparent from the figure, if we reduce risk by reducing the variability of the surplus consumption ratio, then in this case too the equity premium falls together with the volatility of both the stock market and the job-finding rate.

These two experiments make it clear that our model with human capital acquisition produces labor market volatility through the same forces by which it produces stock market volatility and the equity premium.

6.2 Role of Human Capital

We explore the role of the human capital process in generating fluctuations in the job-finding rate and unemployment by examining the implications of alternative values for the rates of human capital accumulation on the job and depreciation off the job. In the next section, we develop further intuition for our findings by analytically characterizing the elasticity of the job-finding rate with respect to the exogenous state of the economy s_t and by showing how this elasticity is affected by the human capital process.

In Table 2, we compare our baseline model to one in which we set $g_e = g_u = 0$. In this latter model, which we refer to as *DMP model with baseline preferences*, as well as in other variants that we will consider in later sections, we maintain the same parametrization as in the baseline model, with the exception of the vacancy cost parameter κ, which is chosen in each instance to ensure that the model reproduces the average quarterly unemployment rate in the data reported in panel B of Table 1. As Table 2 shows, the volatility of the job-finding rate in the DMP model with baseline preferences drops to about 2% of that in the data (0.14%/6.67%). Thus, absent human capital, the unemployment rate barely moves. In the last column of Table 2, we consider the *baseline model with* $g_e = g_u = 3.5\%$ so that human capital grows at the same rate, regardless of whether a consumer is employed or unemployed. In this case too, the volatility of the job-finding rate is quite low, again approximately 2% of that in the data (0.14%/6.67%). These results illustrate that it is not the presence of human capital acquisition per se that is important for our results but rather the differential growth of human capital on and off the job, which makes hiring a worker an investment with long-duration payoffs.

The two panels of Figure 2 plot the impulse responses of the job-finding rate and the unemployment rate to a one-percent decrease in aggregate productivity, starting from the simulated ergodic means of the state variables S_t, Z_{et}, and Z_{ut}, for two versions of our model: the baseline model and the DMP model with baseline preferences.[24] Clearly, the

[24]Since our model is nonlinear, the response of a variable to a shock depends on the levels of the state variables and the size of the shock.

responses of both the job-finding rate and the unemployment rate are much larger in the presence of the human capital process than in the absence of it.

So far we have considered an extreme scenario in which all of the duration in surplus flows is due to the acquisition of human capital on the job, governed by the accumulation rate g_e, by setting the depreciation rate of human capital off the job g_u to zero. A variety of studies, though, have documented that the decrease in wages following a spell of nonemployment can be substantial (see, for instance, Davis and von Wachter, 2011). In light of this evidence, we now show that if we reduce the rate of human capital accumulation on the job and, in a manner consistent with the evidence on the wage losses due to nonemployment, correspondingly raise the rate of human capital depreciation off the job, then we obtain results that are identical to those implied by our baseline parametrization.

For instance, a conservative estimate of the degree of human capital depreciation off the job g_u is -7.9% per year, which matches the average wage loss after up to one year of nonemployment for workers with fewer than 35 years of labor market experience in the Panel Study of Income Dynamics (PSID).[25] If we set g_u equal to this value, then a value of g_e of just 1.6% per year is sufficient to generate the same volatility of the job-finding rate and unemployment as that under our baseline parametrization. Note that although such a value of g_e is much lower than our baseline value of 3.5%, it is actually in line with estimates of wage growth based on the PSID (see Rubinstein and Weiss 2006, Table 2b), as we discuss in Section 9. More generally, in the left panel of Figure 3, we graph the loci of values for (g_e, g_u) that give rise to a volatility of the job-finding rate that is equal to a given percentage of the volatility generated by our model under our baseline parametrization. We trace these loci by varying the values of g_e and g_u while keeping all other parameters fixed at their baseline values, except for κ, which, as discussed, we adjust to keep the mean unemployment rate unchanged.

Consider first the locus labeled *100%*, which implies the same volatility of the job-finding rate as in our baseline. Clearly, our amplification results hold for very modest rates of human capital accumulation on the job and depreciation off the job, including values of g_e below 1% and of g_u close to zero. In fact, when g_e is as low as 1%, even if g_u just equals -3.3%, the model produces 80% of the volatility in the baseline, whereas if g_u equals -0.6%, the model still produces 50% of the volatility in the baseline.

So far we have assumed that a consumer's average working life is 30 years. We now consider how our results change if we increase a consumer's average working life to 40 years by raising the value of ϕ from 0.9972 to 0.9979—we will use such a value of ϕ later in our extension of the baseline model to a life-cycle model with young and mature consumers. In the right panel of Figure 3, we graph loci of values for (g_e, g_u) constructed analogously to those in the left panel. Namely, we trace these loci by varying the values of g_e and g_u while keeping all other parameters fixed at their baseline values, except for κ, which we adjust to keep the mean unemployment rate unchanged, and ϕ, which is now set at a higher value. If we compare the panels for the 30-year working life and the 40-year working life, then it is clear that

As is standard, we compute the impulse response for, say, the job-finding rate in $t+n$ as $\mathbb{E}_t(\lambda_{wt+n}|\varepsilon_t = \Delta, S_t, Z_{ut}, Z_{et}) - \mathbb{E}_t(\lambda_{wt+n}|\varepsilon_t = 0, S_t, Z_{ut}, Z_{et})$, with S_t, Z_{ut}, and Z_{et} set to their simulated ergodic means.

[25]We computed this value of g_u using the sample used by Buchinsky, Fougère, Kramarz, and Tchernis (2010). See Kehoe, Midrigan, and Pastorino (2019) for details.

with the longer working life, for any given value of g_u, the model generates the same volatility of the job-finding rate as it does with the shorter working life for somewhat smaller values of g_e. For example, with a value of g_u of -7.9%, the model with the longer working life and g_e equal to 1.5% generates the same volatility of the job-finding rate as the model with the shorter working life and g_e equal to 1.6%. Likewise, with the longer working life, given a value of g_e as low as 1%, when g_u equals -2.9% (rather than -3.3% as in the case of the shorter working life), the model produces 80% of the volatility in the baseline, whereas when g_u equals -0.4% (rather than -0.6% as in the case of the shorter working life), the model produces 50% of the volatility in the baseline.

Overall, we conclude that our findings on the volatility of the job-finding rate are not knife-edged results that hold only for specific values of g_e, g_u, or the average working life. Rather, our results are robust to perturbations of the values of the parameters describing the evolution of human capital.

7 Our Mechanism

Here we explore the mechanism of our model in more detail. We first derive a closed-form solution for the job-finding rate and its dependence on the exogenous state of the economy s_t, based on the simple approximation for the multipliers μ_{et} and μ_{ut} in (35). We also identify a sufficient statistic for the volatility of the job-finding rate that will turn out to be common across all preference structures we explore. We then argue that our mechanism does not rely on wage rigidity and that the parametrization of the human capital process, which governs the dynamics of wages, is both consistent with evidence on wage growth untargeted in our moment-matching exercise and robust to alternative wage protocols to determine flow wages.

7.1 Inspecting the Mechanism

We now inspect our mechanism by deriving a closed-form solution for the job-finding rate and its dependence on the state of the economy. Using it, we next derive a sufficient statistic for the volatility of the job-finding rate that is robust across all preference structures we consider. To start, we simplify the calculation of the price P_{nt} in t of a claim to aggregate productivity in $t+n$ that appears in (41) by approximating the growth rate of consumption by the growth rate of aggregate productivity. Under this approximation, the pricing kernel becomes $Q_{t,t+1} = \beta[S_{t+1}A_{t+1}/(S_tA_t)]^{-\alpha}$. In the next lemma, we provide a risk-adjusted log-linear approximation to $P_{nt}/A_t = \mathbb{E}_t(Q_{t,t+n}A_{t+n}/A_t)$ around $s_t = s$, which is the price of a claim to the growth rate of aggregate productivity A_{t+n}/A_t in $t+n$ derived from (38). We focus on the *scaled* price P_{nt}/A_t because it is stationary, unlike the price P_{nt}, which grows over time since A_t is governed by a random-walk process with drift.

Lemma 2 (Price of Productivity Claim)**.** *The price P_{nt} of a claim to aggregate productivity in n periods approximately satisfies*

$$\log\left(\frac{P_{nt}}{A_t}\right) = a_n + b_n(s_t - s), \tag{49}$$

where $a_0 = b_0 = 0$*,* $a_n = \log(\beta) + (1-\alpha)g_a + a_{n-1} + [1 - b_{n-1} - (\alpha - b_{n-1})/S]^2\sigma_a^2/2$*, and*

$$b_n = (1-\rho_s)\alpha + \rho_s b_{n-1} + \left(1 - b_{n-1} - \frac{\alpha - b_{n-1}}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right)\sigma_a^2. \tag{50}$$

If $\alpha > 1$ *and* $1 - \rho_s + (1 - \alpha/S)\sigma_a^2/S > 0$*, then* b_n *grows monotonically from* 0 *to* α*.*

The proof of this lemma is in the appendix. To understand this result, observe that the constant a_n in (49), once recursively solved out, equals the log of the discount factor β^n up to an adjustment term for productivity growth and risk, captured by g_a and σ_a, and decreases with n as long as the drift rate g_a is large enough and $\alpha > 1$. The elasticity b_n of the (scaled) price P_{nt}/A_t with respect to the exogenous state s_t instead captures how this price moves with s_t and increases monotonically from 0 to α, provided that $1 - \rho_s + (1 - \alpha/S)\sigma_a^2/S > 0$, which is satisfied in our baseline parametrization.

Given that the elasticity b_n increases with the maturity n of a claim, (49) implies that the longer is the maturity of a claim, the more sensitive is its price at horizon n to the exogenous state s_t, and so the lower is the price of a long-maturity claim relative to that of a short-maturity one when the state is low ($s_t < s$). To see why the elasticity b_n increases with n, consider first an economy without risk ($\sigma_a = 0$) in which the dependence of the price of a claim to its horizon is governed purely by *intertemporal substitution* motives. In this case, the expression for b_n in (50) reduces to $b_n^{IS} = \alpha(1-\rho_s) + \rho_s b_{n-1}^{IS}$ with $b_0^{IS} = 0$, which implies that $b_n^{IS} = \alpha(1 - \rho_s^n)$. Hence, b_n^{IS} increases with n, since ρ_s^n decreases with it. Intuitively, when the exogenous state s_t is below its mean s, it is expected to revert back to it, as habits slowly adjust to the lower consumption level. Thus, consumers value current consumption more than future consumption. As a result, they are willing to pay relatively *less* for a claim to consumption in the far future, when the state is expected to be much closer to its mean, and relatively *more* for a claim to consumption in the near future, when the state is expected to be close to its current low value s_t. This intertemporal substitution logic explains why b_n increases with n in the absence of risk.

Now consider an economy with risk in which the last term in (50) is nonzero. This term is an offsetting adjustment factor for risk that captures both *precautionary saving* motives, which have a negative effect on b_n, and a *risk premium* effect, which has a positive impact on b_n and compensates for the risk resulting from the covariance between the future marginal utility of consumption and future productivity growth.[26] Although this offsetting adjustment factor is negative, as the precautionary saving motives dominate the risk premium effect, it does not reverse the monotonicity of b_n with n. Specifically, in the presence of risk, the elasticity b_n still increases with the maturity of a claim towards α, albeit more slowly in that $b_n \leq b_n^{IS}$. To see why b_n is smaller than b_n^{IS} but converges to it, note that in the short run, precautionary

[26]To see why precautionary saving motives have a negative effect on b_n whereas the risk premium effect has a positive impact on it, consider the price of a real risk-free bond that pays one unit of consumption goods in n periods, $P_{nt}^r = \mathbb{E}_t(Q_{t,t+n})$, which can be approximated as $\log(P_{nt}^r) \approx a_n^r + b_n^r(s_t - s)$. Since this bond pays off a constant amount of goods, its price moves with the state s_t only because of intertemporal substitution and precautionary saving motives. The sensitivity of this price to s_t is given by $b_n^r = (1-\rho_s)\alpha + \rho_s b_{n-1}^r + [-b_{n-1}^r - (\alpha - b_{n-1}^r)/S][(\alpha - b_{n-1}^r)/S]\sigma_a^2$. In this expression, precautionary saving motives are captured by the last term, which has an obvious negative effect on b_n^r, as b_n^r lies between 0 and α for any n under typical parametrizations. That the risk premium effect is positive is immediate by comparing the first term multiplying σ_a^2 in b_n^r and the corresponding term in b_n, since both $(\alpha - b_{n-1}^r)/S$ and $(\alpha - b_{n-1})/S$ are positive. See the online appendix for these derivations.

saving motives make consumers more willing to save, attenuating the incentives for intertemporal substitution. In the long run, though, as productivity risk cumulates, the risk premium effect becomes as important as precautionary saving motives so that the adjustment factor for risk converges to zero, implying that b_n converges to b_n^{IS} as n becomes large.

Since our baseline asset-pricing preferences imply that the prices of long-maturity claims to future productivity are more sensitive to shocks to s_t than those of short-maturity ones, our model can generate large movements in the price of an asset only if the asset assigns large weights to long-maturity claims.[27] For our purposes, then, as is apparent from (41), the key implication of the elasticity b_n increasing with n is that the response of the job-finding rate to a given shock to s_t is larger, the larger are the weights $\{c_\ell\delta_\ell^n + c_s\delta_s^n\}$ on the prices of long-maturity claims to productivity in the surplus flows from a match between a firm and a consumer by (37). It turns out that in the presence of human capital acquisition, surplus flows are indeed characterized by large weights on long-maturity claims, as we will discuss, and so their present value is sensitive to shocks to s_t. Hence, the combination of our asset-pricing preferences and human capital acquisition generates volatile job-finding rates and unemployment. We formalize these intuitions in the following proposition, in which $\sigma(s_t)$ denotes the standard deviation of s_t.[28]

Proposition 5 (Sufficient Statistic for Job-Finding Rate Volatility)**.** *Under the approximation in Lemma 2, the response of the job-finding rate to a change in s_t evaluated at the risky steady state is given by*

$$\frac{d\log(\lambda_{wt})}{ds_t} = \left(\frac{1-\eta}{\eta}\right)\sum_{n=0}^{\infty}\omega_n b_n \quad \textit{with} \quad \omega_n = \frac{e^{a_n}(c_\ell\delta_\ell^n + c_s\delta_s^n)}{\sum_{n=0}^{\infty} e^{a_n}(c_\ell\delta_\ell^n + c_s\delta_s^n)}, \tag{51}$$

where a_n and b_n are given in Lemma 2. The standard deviation of the job-finding rate satisfies

$$\sigma[\log(\lambda_{wt})] = \frac{d\log(\lambda_{wt})}{ds_t}\sigma(s_t). \tag{52}$$

As argued, since the elasticities $\{b_n\}$ of the prices of claims to future productivity with respect to the exogenous state s_t increase with the horizon of a claim, a change in the exogenous state s_t leads to a large change in the job-finding rate only if the weights $\{\omega_n\}$ on long-maturity claims to productivity are large. In the left panel of Figure 4, we graph the global solution for the prices of claims to productivity against the exogenous state s_t when the endogenous states (Z_{et}, Z_{ut}) are set to their simulated ergodic means—namely, we use neither the approximation that $\Delta c_{t+1} \approx \Delta a_{t+1}$ nor

[27] As shown in Kehoe, Lopez, Midrigan, and Pastorino (2021), most existing state-of-the-art asset-pricing models share this same property. More specifically, as we discuss, this property holds true for the other standard preference specifications we consider below, but for different reasons than those discussed here for our baseline model. For instance, in the frameworks with Epstein-Zin preferences and either long-run risk or time-varying disaster risk, long-duration claims are more exposed to the small low-frequency stochastic productivity components arising, respectively, from long-run risk and time-varying disaster risk. When the elasticity of intertemporal substitution is greater than one, as is typically assumed in this literature, the positive effect of higher productivity in the future on the prices of claims dominates the negative intertemporal substitution effect on them so that the prices of long-maturity claims are more responsive to changes in s_t than those of short-maturity ones.

[28] For this result, we compute the approximate solution to our model with market tightness θ set at its value at the *risky steady state*, defined as the steady state reached when agents use the decision rules computed in our stochastic equilibrium, but with all future shocks set to zero, following the method of Lopez, Lopez-Salido, and Vazquez-Grande (2017). This value of θ is very close to that in the deterministic steady state. A simple log-linear approximation around the deterministic steady state, however, is not close to the log-linear approximation around the risky steady state, which allows for terms in σ_a in the formulae for a_n and b_n and so in our sufficient statistic. See Kehoe, Lopez, Midrigan, and Pastorino (2021) for details, including approximate formulae for the case in which we dispense with the assumption of constant future λ_{wt} but retain the log-linear approximation for the prices of claims to future productivity. It turns out that these formulae are very accurate.

the risk-adjusted log-linear approximation of Lemma 2. Note that the prices of longer-maturity claims are indeed much more sensitive to changes in the state s_t than those of shorter-maturity ones. Moreover, as the figure makes clear, log prices are approximately linear in the state s_t. For a comparison, we plot in the right panel of Figure 4 the corresponding solution when we use both of the approximations of Lemma 2—namely, that $\Delta c_{t+1} \approx \Delta a_{t+1}$ and $\log(P_{nt}/A_t)$ is approximately linear in the state s_t. As is apparent from the two panels, the global and approximate solutions are quite close.

In the left panel of Figure 5, we plot the impulse responses of the prices of claims to a one-percent decrease in aggregate productivity based on the global solution to our model. Clearly, after this shock, the prices of short-horizon claims fall little, whereas those of long-horizon ones fall greatly. Thus, together with Proposition 5, these figures illustrate that our model generates large variations in the job-finding rate only when the weights $\{\omega_n\}$ are sufficiently large for large n.

We now turn to showing that without human capital acquisition, these weights decay very quickly over time. More generally, the greater is the rate of human capital accumulation during employment, and the greater is the rate of human capital depreciation during unemployment—namely, the more positive g_e is and the more negative g_u is—the slower these weights decay, and the larger is the response of the job-finding rate to changes in aggregate productivity.

DMP Model with Baseline Preferences. Consider the DMP model with our baseline preferences and $g_e = g_u = 0$. In this case, the constant c_ℓ on the large root is zero and the small root, referred to as the *DMP root*, is given by $\delta_{DMP} = 1 - \sigma - \eta\lambda_w$, where, as was the case before, σ is the separation rate, η is the elasticity of the job-filling rate with respect to market tightness, and λ_w is the job-finding rate. Thus, in the DMP version of our model, surplus flows follow a first-order difference equation with the surplus flow in n periods proportional to $\delta_{DMP}^n A_{t+n}$ by the analogue of (37). The weight in the corresponding expression for $d\log(\lambda_{wt})/ds_t$ is then $\omega_n = e^{a_n}\delta_{DMP}^n / \sum_{n=0}^{\infty} e^{a_n}\delta_{DMP}^n$. For standard parametrizations, the DMP root is substantially smaller than one so that surplus flows quickly decay over time. Specifically, with σ equal to 2.8%, η equal to 0.5, and λ_w equal to 45.35% as in our baseline, it follows that δ_{DMP} equals 74.5%, which amounts to a decay rate of over 25% per month. Hence, $(\delta_{DMP})^{24}$ is 0.09% after only two years. Accordingly, the weights on claims to future productivity with long maturities are practically zero, as the hiring problem is essentially static. These observations intuitively explain why the DMP model gives rise to an unemployment volatility puzzle.

Baseline Model. By our formula in (39) for the roots of the solution to the system in (35), the large root δ_ℓ is bigger than one, and the weight c_ℓ on this root is positive in the presence of human capital acquisition so that the present value of the surplus flows from a match between a firm and a consumer slowly decays over time. In turn, this fact implies that the formula for the job-finding rate in (41) assigns sizable weights to claims to future productivity with long maturities, which are very sensitive to the exogenous state s_t and hence lead to large fluctuations in the job-finding rate in response to productivity shocks. In the right panel of Figure 5, we report the cumulative weights for each maturity implied by the DMP model with baseline preferences and those implied by our baseline model, both without any approximation.

Clearly, the weights in the DMP model with baseline preferences decay very quickly relative to those in our baseline model—that is, cumulative weights converge to one very quickly in the former model relative to those in the latter model. For a sense of magnitudes, we compute the (Macaulay) *duration* of these weights, $\sum_{n=0}^{\infty} \omega_n n$, and compare it across models. The duration of the weights $\{\omega_n\}$ is 3.6 months in the DMP model with baseline preferences and 12.4 years in our baseline model under our parametrization. The expression for $d\log(\lambda_{wt})/ds_t$ in (51), however, implies that a more relevant measure of the duration of these weights is their average weighted by the elasticities $\{b_n\}$ of the prices of claims to future productivity with respect to the exogenous state s_t at different horizons, which is given by $\sum_{n=0}^{\infty} \omega_n b_n$ when η equals 0.5.[29] For the DMP model with baseline preferences, this elasticity is 0.04, and for our baseline model, it is 1.53.

7.2 Implications of Our Mechanism for Wages

Here we first discuss the implications of our model for the cyclicality of flow wages, which demonstrates that our findings are not predicated on wage rigidity. We then turn to examining additional evidence on wage growth that is consistent with our parametrization of the human capital process and to evaluating the robustness of this parametrization to alternative wage protocols that determine flow wages.

Cyclicality of Flow Wages. Note that our competitive search equilibrium determines the present value of wages paid to an employed consumer over the course of a match with a firm but not the flow wage received by such a consumer in each period. More generally, in any model with complete markets and commitment by both firms and workers to state-contingent employment contracts, many alternative sequences of flow wages give rise to the same present value of wages. Hence, in this precise sense, our model does not have specific predictions for flow wages.

Given this indeterminacy, we follow the approach popularized by Barlevy (2008) and Bagger, Fontaine, Postel-Vinay, and Robin (2014), who assume that when a match is formed in period t, a firm commits to pay a worker each period a share ϱ_t of the period output for the duration of the match. Thus, $w_{t,\tau} = \varrho_t A_\tau z_\tau$ is the wage in period $\tau \geq t$. Accordingly, we determine flow wages as follows. For any present value of wages $W_{mt}(z_t) = W_{mt} z_t$ implied by our model for a match between a firm and a consumer with human capital z_t that starts in t, we choose ϱ_t so that the present value of the wages $w_{t,t} = \varrho_t A_t z_t$, $w_{t,t+1} = \varrho_t A_{t+1} z_{t+1}$, and so on, calculated using our stochastic discount factor, exactly equals $W_{mt}(z_t)$.

Using this approach, we now argue that our model is robust to the critique by Kudlyak (2014) of the degree of rigidity of the wage process implied by prominent solutions to the unemployment volatility puzzle. Specifically, we find that our model is consistent with the degree of cyclicality of wages estimated by Kudlyak (2014) and Basu and House (2016). Hence, it does not rely on counterfactually rigid wages.

To elaborate, as Becker (1962) emphasized, in general the present value of the flow wages paid to a worker over the course of an employment relationship is allocative for employment, rather than the current flow wage. Kudlyak (2014)

[29]The term $\sum_{n=0}^{\infty} \omega_n b_n$ is a measure of duration that is different from the standard Macaulay one, in which instead of weighting the fraction ω_n of the present value of surplus flows accruing at horizon n at the risky steady state by the horizon length n, we weight the fraction ω_n by the elasticity of the (scaled) price of a claim to productivity at horizon n with respect to the state s_t, namely, b_n.

further proved that for a large class of search models, the appropriate allocative wage is the difference in the present value of wages between two matches that start in two consecutive periods, as captured by the *user cost of labor*. Intuitively, in a search model, hiring a worker is akin to acquiring a long-term asset subject to adjustment costs. Thus, by measuring the rental price of the services of a worker potentially employed for several years, the user cost of labor is a more relevant measure of the cost of hiring a worker than the flow wage.

Both Kudlyak (2014) and Basu and House (2016) measured the cyclicality of the user cost of labor as the semi-elasticity of the user cost with respect to the unemployment rate. Using NLSY data, these authors estimated the user cost of labor as $UC_t = PV_t - \beta(1-\sigma)PV_{t+1}$, where PV_t is an empirical measure of the present value of wages from a match that begins in t, defined as $PV_t = w_{t,t} + \sum_{\tau=t+1}^{T}[\beta(1-\sigma)]^{\tau-t}w_{t,\tau}$. In this expression, $w_{t,\tau}$ is the wage in period $\tau \geq t$, and $\beta(1-\sigma)$ is a fixed discount factor that takes into account the real interest rate and the job separation rate. (See Kudlyak, 2014 and Basu and House, 2016 for details.) Intuitively, the user cost measures the shadow wage that would make a risk-neutral firm indifferent between hiring a worker today at that wage, thus creating a match that survives to tomorrow with probability $1-\sigma$, and hiring a worker tomorrow at the present value of wages prevailing then. Importantly, as its formula shows, the user cost of labor in t captures not just the flow wage of a new hire in t but also the difference in the present value of wages from $t+1$ on between a worker hired in t and a worker hired in $t+1$. Hence, the user cost incorporates any potential extra cost or benefit of committing in t to a (possibly state-contingent) sequence of wage payments from $t+1$ on, relative to waiting and hiring an identical worker in $t+1$ at the relevant present value of wages in $t+1$. In particular, if recessions are times of scarring in that relative to workers hired in upturns, workers hired in downturns not only obtain a lower wage when hired but also in any subsequent period, then it is clear that the user cost of labor can be much more cyclical than the flow wage.

Kudlyak (2014) indeed estimated a semi-elasticity of the user cost of labor with respect to the unemployment rate of -5.2%, and Basu and House (2016) refined this estimate to -5.8% so that a one percentage point increase in the unemployment rate is associated with an approximately 6% decrease in the user cost of labor. Hence, the user cost is quite procyclical. In calculating the user cost of labor in our model, we treat the empirical measure of the user cost simply as a particular statistic of the allocative wage that takes as inputs a sequence of flow wages $\{w_{t,\tau}\}$ and the fixed discount factor $\beta(1-\sigma)$, according to the above formulae for UC_t and PV_t. Based on flow wages constructed as described, our model implies a cyclicality of the user cost of labor of -6.0%. Therefore, the comovement between the user cost of labor and unemployment implied by our baseline model is in line with that in the data, although untargeted, and shows that our mechanism for unemployment volatility does not rely on a counterfactual degree of wage rigidity.[30]

Sensitivity to Alternative Measurements of Wage Growth and Wage Protocols for Flow Wages. We begin by discussing additional evidence on how wages grow with experience, which is the moment we use to pin down the growth

[30] Intuitively, with human capital accumulation, the wage in any period τ of a worker with human capital z_t at time t is $w_{t,\tau} = \varrho_t A_\tau z_\tau = \varrho_t A_\tau (1+g_e)^{\tau-t} z_t$ so that the present value of wages in t used to calculate the user cost of labor in t for such a worker is $PV_t = \varrho_t z_t\{A_t + \sum_{\tau=t+1}^{T}[\beta(1-\sigma)(1+g_e)]^{\tau-t}A_\tau\}$. Then, for any given sequences of wages $\{w_{t,\tau}\}_\tau$ and $\{w_{t+1,\tau}\}_\tau$ from a match starting in t and a match starting in $t+1$, respectively, for such a worker, the difference between their present values increases with g_e. Specifically, $(1+g_e)^{\tau-t}$ "upweights" future terms in this difference of present values, thus magnifying the cyclicity of the user cost of labor.

of human capital on the job in our model. As noted, we have set the growth rate of human capital g_e so that the implied wage growth in our model matches the *longitudinal* wage growth with experience estimated by Rubinstein and Weiss (2006, Table 2b). We now argue that under our baseline parametrization discussed earlier, the wage process implied by our model is also consistent with the *cross-sectional* evidence on wage growth with experience documented by Elsby and Shapiro (2012). These authors report that the difference between the log real wages of workers with 30 years of experience and those just entering the labor market is 1.1 in the data between 1960 and 2007 for workers with 12 years of education, which is the historical mean and median number of years of education in the United States. The wage process implied by our baseline model is in accord with this untargeted statistic, as it implies a difference of 1.0.

We consider now how the parameters g_e and g_u of the human capital process that we infer in our moment-matching exercise vary with the protocol for flow wages, and we demonstrate that they are fairly insensitive to the details of the wage setting mechanism. We do so by showing that we obtain similar values for the yearly growth rate of human capital on the job, g_e, under very different wage protocols. A similar argument applies to the decay rate of human capital off the job, g_u. For this purpose, we consider two extreme wage protocols: a *constant wage protocol* over the duration of a match, which leads to the most rigid wages, and an efficient *recontracting Nash bargaining protocol*, in which wages are instead bargained every period under the Hosios condition, which gives rise to the most flexible wages. As we discuss in the online appendix, the value of g_e inferred under the constant wage protocol is 3.4% and the value of g_e inferred under the recontracting Nash bargaining protocol is 3.6%. These two values bracket the value of 3.5% in our baseline parametrization.

For an intuition about these results, note that in both our model and the data, some workers switch jobs, whereas others stay in their jobs over the course of a year. When wages are recontracted each period, they grow from one month to another as human capital is accumulated. When they are not recontracted each period, wages increase discretely only when a worker switches jobs. But if we measure wages yearly, these high-frequency differences tend to average out once aggregated across workers. Hence, the model's implications for annual wage growth are fairly insensitive to the specific details of the evolution of wages from one month to another, as they are driven primarily by the process of human capital acquisition that takes place throughout a year. Thus, effectively, by aggregating wages across months within a year and across workers with different employment experiences, we end up with an average wage growth with experience that simply reflects how much human capital increases from one year to another for employed workers and is therefore quite similar across protocols.

8 Extension to a Model with Physical Capital

The influential early business cycle work by Merz (1995) and Andolfatto (1996) integrated search theory into real business cycle models. Although ambitious, those papers did not attempt to make their models consistent with any asset-pricing patterns. Since those early contributions, most of the subsequent literature has focused on models without physical capital and ignored their asset-pricing implications. Here we embed our mechanism into a real business cycle framework with physical capital, retaining our baseline preferences. We thus construct a simple yet full-fledged real and financial business cycle model that solves the unemployment volatility puzzle and is in line with key patterns of job-finding rates, unemployment, output, consumption, investment, and asset prices in the data. Note that in contrast to the separation result between the real and financial sides of an economy in Tallarini (2000), the main result of this model with physical capital is that the presence of time-varying risk greatly amplifies the fluctuations of real variables.

Consider then the following extension of our baseline model. We assume that physical capital augments the production of goods in the market so that if a measure $e_t(z)$ of consumers with human capital z is paired with $K_t(z)$ units of physical capital, a firm produces $K_t^{\vartheta}(z)[A_t z e_t(z)]^{1-\vartheta}$, where $ze_t(z)$ are effective units of labor. We allow for costs of adjustment of the aggregate capital stock and maintain the same specifications of the technologies for producing goods at home and job vacancies as in our baseline model.

We characterize the competitive search equilibrium allocations by solving the aggregate planning problem for this economy. In such a problem, as we show in the online appendix, productive efficiency implies that the ratio of capital to effective labor is equalized across firm-consumer matches in that $\varphi_t = K_t(z)/[ze_t(z)]$ for all z. Hence, the ratio of aggregate capital to aggregate effective labor equals the same ratio, since

$$\frac{K_t}{Z_{et}} = \frac{\int_z K_t(z)dz}{\int_z ze_t(z)dz} = \frac{\int_z \varphi_t ze_t(z)dz}{\int_z ze_t(z)dz} = \varphi_t. \tag{53}$$

It is immediate that the economy aggregates in a similar fashion, as does the economy of our baseline model. In particular, aggregate production satisfies

$$\int_z K_t^{\vartheta}(z)[A_t z e_t(z)]^{1-\vartheta}dz = A_t^{1-\vartheta}\int_z \left[\frac{K_t(z)}{ze_t(z)}\right]^{\vartheta} ze_t(z)dz = \varphi_t^{\vartheta} A_t^{1-\vartheta} Z_{et} = K_t^{\vartheta}(A_t Z_{et})^{1-\vartheta},$$

where $K_t = \int_z K_t(z)dz$ is the physical capital used by employed consumers and we have used (53) in the second and third equalities. The aggregate resource constraint can be expressed as

$$C_t + I_t = K_t^{\vartheta}(A_t Z_{et})^{1-\vartheta} + b_h A_t Z_{ut} - \kappa A_t Z_{vt}.$$

As before, aggregate vacancies in efficiency units are given by $Z_{vt} = \int_z zv_t(z)dz = \phi\theta_t(1+g_u)Z_{ut-1}$. The aggregate capital stock follows the accumulation law $K_{t+1} = (1-\delta)K_t + \Phi(I_t/K_t)K_t$, where $\Phi(I/K) = \delta + e^{g_a} - 1 + \delta\{[I/(\delta K)]^{1-1/\xi} - 1\}/(1-1/\xi)$, as in Jermann (1998).[31]

We set ϑ equal to 0.26, set δ equal to 0.1/12, and choose the curvature parameter ξ of the adjustment cost function so that the standard deviation of investment growth relative to consumption growth in the model equals that in the data.[32] As

[31] In Jermann (1998), $\Phi(I/K) = b(I/K)^{1-a}/(1-a) + c$. Once we impose, as discussed in Jermann (1998), that the capital stock in steady state grows at the same rate as aggregate productivity and that adjustment costs are constant in the ratio of investment to capital near the steady state so that $\Phi(\delta) = e^{g_a} - 1 + \delta$ and $\Phi'(\delta) = 1$, the adjustment function in Jermann (1998) reduces to ours with $a = 1/\xi$, $b = \delta^{1/\xi}$ and $c = \delta + e^{g_a} - 1 - \delta/(1-1/\xi)$. See the online appendix for a derivation.

[32] This value of ϑ targets an average labor share of output of 70%, computed in the model as the mean ratio of total wages to output and in the data as the average ratio of wages to gross domestic income over the period between 1948 and 2007, where wages are measured by the BEA series "Gross Domestic Income: Compensation of Employees, Paid" and profits are measured by the BEA series "Gross Domestic Income: Net Operating Surplus."

is consistent with a model with physical capital, we choose the mean and standard deviation of the aggregate productivity process so as to match the corresponding moments of total factor productivity in the data (see Fernald, 2014). Observe that in our baseline model, the home production parameter b can be thought of as measuring the (expected) ratio of the amount of goods produced at home to those produced in the market in that $b = \mathbb{E}[b_h A_t z_t/(A_t z_t)] = b_h$ so b equals 0.6. To pursue this analogy in our model with physical capital, we need to specify the average amount of goods that a consumer with human capital z_t produces in the market. To do so, note that if a measure $e_t(z)$ of consumers with human capital z produces $K_t^{\vartheta}(z)[A_t z e_t(z)]^{1-\vartheta}$, then each such consumer produces

$$\frac{K_t^{\vartheta}(z)[A_t z e_t(z)]^{1-\vartheta}}{e_t(z)} = z A_t^{1-\vartheta}\left[\frac{K_t(z)}{z e_t(z)}\right]^{\vartheta} = z A_t^{1-\vartheta}\varphi_t^{\vartheta}.$$

We can then define b as the (expected) ratio of the amount of home-produced goods to the amount of market-produced goods for such a consumer,

$$b = \mathbb{E}\left(\frac{b_h A_t z}{z A_t^{1-\vartheta}\varphi_t^{\vartheta}}\right) = \mathbb{E}\left\{\frac{b_h}{[K_t/(A_t Z_{et})]^{\vartheta}}\right\},$$

where in the last equality, we have used (53). To keep this model parallel to our baseline, we set b_h so that b equals 0.6 in the model with physical capital as well.

As shown in Table 3, this augmented model gives rise to a volatility of the job-finding rate and a volatility of unemployment that are consistent with those in the data—in fact, both are slightly higher. Specifically, the volatility of the job-finding rate is 7.24% in the model and 6.67% in the data and that of unemployment is 0.84% in the model and 0.79% in the data. The model also successfully replicates important observed features of stock and bond returns. In particular, the model does not exhibit the excess volatility of either short-term or long-term bond yields of the early habit model of Jermann (1998). Indeed, the volatility of short-term bond yields is the same in the model and in the data, namely, 2.35%, and the volatility of 20-year nominal bond yields is 2.30% in the model and 2.41% in the data.

In sum, the model with physical capital fits both the real and financial data well.

9 Extension to a Life-Cycle Model

Our baseline model is a perpetual youth model in which all consumers face the same probability of survival and are characterized by the same human capital process on and off the job. In the data, though, the growth rate of wages tends to be higher for younger workers than for mature workers, which supports the notion that human capital grows faster for the former group than for the latter group—see Rubinstein and Weiss (2006) for a comprehensive review of the evidence on wage growth with labor market experience. At the same time, as we will discuss, the volatility of unemployment is higher for young workers than for mature workers. One question, then, is whether an extension of our model in which we explicitly account for differences in the human capital process between young and mature workers can reproduce the observed volatility of the job-finding rate and unemployment for these two groups. Intuitively, our mechanism can account for these aspects of the data, as it implies that the greater the rate of human capital accumulation on the job or depreciation off the job, the higher the volatilities of the job-finding rate and unemployment.

To address this question formally, we augment our model with the following simple life-cycle structure. We assume that consumers are born young and that at the end of each period, young consumers remain young with probability ϕ_y and become mature with probability $1-\phi_y$. Mature consumers survive with probability ϕ_m and die with probability $1-\phi_m$. Every period, dying mature consumers are replaced by a measure γ of unemployed newborn consumers with human capital z drawn from a distribution $\nu(z)$ with mean one. We assume that the measure of newborns each period is $\gamma = (1-\phi_y)(1-\phi_m)/(1-\phi_y+1-\phi_m)$ and the initial measures of young and mature consumers are $\gamma/(1-\phi_y)$ and $\gamma/(1-\phi_m)$ so that the measures of these two groups of consumers are constant over time and sum to one.[33]

We allow the rate of human capital accumulation on the job and depreciation off the job to differ across young and mature consumers. Specifically, when a consumer of age $i \in \{y, m\}$, where y denotes a young consumer and m denotes a mature consumer, is employed, human capital grows according to

$$z_{t+1} = (1+g_{ei})z_t. \tag{54}$$

When a consumer of age i is unemployed, human capital evolves according to

$$z_{t+1} = (1+g_{ui})z_t. \tag{55}$$

We let the cost of posting job vacancies $\kappa_i A_t z$ and the job separation rate σ_i vary with a consumer's age i. Each family now consists of a continuum of young and mature consumers. The definition of a competitive search equilibrium is the natural generalization of that in our baseline model, except that now a labor market is defined by the triple $(z, i, W_{mit}(z))$ of a given skill level, age group, and wage offer for a consumer of that skill level and age group.

It is immediate to show that the analogues of Lemma 1 and Propositions 1 and 2 hold. Now, though, we need to record the aggregate human capital of young and mature consumers by their employment status, namely, (Z_{eyt}, Z_{uyt}) and (Z_{emt}, Z_{umt}), as part of the state. The aggregate resource constraint is

$$C_t = A_t(Z_{eyt}+Z_{emt}) + bA_t(Z_{uyt}+Z_{umt}) - \kappa_y A_t \int_z z v_{yt}(z)dz - \kappa_m A_t \int_z z v_{mt}(z)dz, \tag{56}$$

where the first two terms on the right side of (56) are the total market and home output of young and mature consumers, and the last two terms are the costs of posting vacancies aimed at young and mature consumers. Note that the third term on the right side of (56), namely, the vacancy costs of recruiting young consumers, can be rewritten as

$$\kappa_y A_t \int_z z\theta_{yt} u_{byt}(z)dz = \kappa_y A_t \theta_{yt} \phi_y (1+g_{uy}) Z_{uyt-1}, \tag{57}$$

using that $v_{yt}(z) = \theta_{yt} u_{byt}(z)$ and $u_{byt}(z) = \phi_y u_{yt-1}(z/(1+g_{uy}))/(1+g_{uy})$. Similarly, the last term in (56), namely,

[33] If we let y_t and m_t denote, respectively, the measures of young and mature consumers in t, then the young and mature transition equations, together with the constancy of the measure of each group, imply that $y_t = \phi_y y_{t-1} + \gamma$, and so $y = \gamma/(1-\phi_y)$ with $y_t = y$ in each t, and $m_t = \phi_m m_{t-1} + (1-\phi_y) y_{t-1}$, and so $m = \gamma/(1-\phi_m)$ with $m_t = m$ in each t and $y = \gamma/(1-\phi_y)$. Since $y+m=1$, the expression for γ solves $\gamma/(1-\phi_y) + \gamma/(1-\phi_m) = 1$.

the vacancy costs of recruiting mature consumers, can be expressed as

$$\kappa_m A_t \int_z z\theta_{mt} u_{bmt}(z)dz = \kappa_m A_t \theta_{mt}[\phi_m(1+g_{um})Z_{umt-1} + (1-\phi_y)(1+g_{uy})Z_{uyt-1}], \tag{58}$$

using that $v_{mt}(z) = \theta_{mt} u_{bmt}(z)$ and

$$u_{bmt}(z) = \frac{\phi_m}{1+g_{um}} u_{mt-1}\left(\frac{z}{1+g_{um}}\right) + \frac{1-\phi_y}{1+g_{uy}} u_{yt-1}\left(\frac{z}{1+g_{uy}}\right). \tag{59}$$

As (59) indicates, unemployed mature consumers with human capital z at the beginning of period t, u_{bmt}, consist of unemployed mature consumers at the end of period $t-1$, u_{mt-1}, who survived to period t and whose human capital grew from $z/(1+g_{um})$ to z between $t-1$ and t, as well as of unemployed young consumers at the end of period $t-1$, u_{yt-1}, who became mature and whose human capital grew from $z/(1+g_{uy})$ to z between $t-1$ and t. Substituting (57) and (58) into (56) gives the resource constraint in terms of the aggregate states.

By a similar logic, we can generalize the transition laws for the aggregate human capital of employed and unemployed consumers from our baseline model as follows. The transition laws for the aggregate human capital of employed and unemployed young consumers are

$$Z_{eyt} = \phi_y(1-\sigma_y)(1+g_{ey})Z_{eyt-1} + \phi_y \lambda_{wt}(\theta_{yt})(1+g_{uy})Z_{uyt-1}$$

and

$$Z_{uyt} = \phi_y \sigma_y (1+g_{ey})Z_{eyt-1} + \phi_y[1-\lambda_{wt}(\theta_{yt})](1+g_{uy})Z_{uyt-1} + \gamma,$$

where γ is the measure of newborn consumers who start with a mean human capital of one. The transition laws for the aggregate human capital of employed and unemployed mature consumers are similar but also include terms that account for the human capital of young consumers who become mature at the end of period $t-1$,

$$\begin{aligned} Z_{emt} = {} & \phi_m(1-\sigma_m)(1+g_{em})Z_{emt-1} + (1-\phi_y)(1-\sigma_y)(1+g_{ey})Z_{eyt-1} \\ & + \lambda_{wt}(\theta_{mt})[\phi_m(1+g_{um})Z_{umt-1} + (1-\phi_y)(1+g_{uy})Z_{uyt-1}], \end{aligned}$$

and

$$\begin{aligned} Z_{umt} = {} & \phi_m \sigma_m (1+g_{em})Z_{emt-1} + (1-\phi_y)\sigma_y(1+g_{ey})Z_{eyt-1} \\ & + [1-\lambda_{wt}(\theta_{mt})][\phi_m(1+g_{um})Z_{umt-1} + (1-\phi_y)(1+g_{uy})Z_{uyt-1}]. \end{aligned}$$

Our strategy for choosing parameter values is nearly identical to that in the baseline model, so we remark here only on the main differences. We choose the probability ϕ_y that a young consumer remains young and the probability ϕ_m that a mature consumer survives in the market so that the mean duration of life as a young consumer is 10 years and the mean duration of life as a mature consumer is 30 years, in order to match an overall average working life of 40 years in the data.[34] Correspondingly, we pin down the key new human capital parameters $\{g_{ei}\}$ and $\{g_{ui}\}$ for young and mature consumers, respectively, based on estimates of the wage growth on the job and of the wage changes before and after a spell of nonemployment experienced by workers with fewer than 35 years of age (young) and more than 35 years of age (mature). Note that 35 years of age correspond approximately to 17 years of potential labor market experience at the historical mean and median number of years of education in the United States (12 years).

Specifically, we choose g_{ey} and g_{em} so as to reproduce, respectively, an average annual growth rate of real hourly wages, net of the growth rate of labor productivity, of 4.22% over the first 17 years of labor market experience and of 2.75% over the remaining working horizon, based on the estimates by Rubinstein and Weiss (2006) from the NLSY discussed earlier; see the online appendix for details. Similarly, we select g_{uy} and g_{um} so as to match the average percentage change between the first wage in the first employment spell *after* a nonemployment spell and the last wage in the last employment spell *before* a nonemployment spell for workers in the two age groups who experience a complete spell of nonemployment. Using data from the PSID at monthly frequency available between 1988 and 1997, we estimate that such an average percentage wage difference is -0.99% for workers with fewer than 17 years of labor market experience and -18.20% for workers with more than 17 years of labor market experience, net of the growth rate of labor productivity.[35]

Consider now job-finding and unemployment rates. Since unemployment rates from the BLS are available disaggregated by age group rather than by experience, we measure the job-finding rate for workers below and above the cutoff of 35 years of age, as discussed. The average quarterly job-finding rate over the period between June 1976 and December 2007 is 46.87% for workers with fewer than 35 years of age and 34.95% for workers with more than 35 years of age.[36] Using the monthly job-finding rates for the two age groups, we construct age-specific monthly constant-separation unemployment rate series, in which the age-specific monthly separation rates σ_y and σ_m are chosen so that the average of each constructed unemployment rate series reproduces the average unemployment rate of the relevant age group in the data. As the resulting average quarterly unemployment rate of young workers is much higher than that of mature workers (8.7% for the young versus 4.1% for the mature), the monthly separation rate of young workers is also higher (4.4% for the young versus 1.5% for the mature). Given these parameters, we choose the vacancy posting cost κ_i for each age group i so that the model replicates the average quarterly job-finding rates of the two groups in the data. Similarly to the baseline model, we normalize market tightness at the deterministic steady state, which pins down the efficiency of the matching function for each age group. We recompute the remaining relevant moments for this same period between

[34] Such a life cycle is consistent with a retirement age of approximately 55 to 60 years, depending on a worker's educational attainment, ranging from no high-school diploma to a college degree.

[35] If we alternatively choose g_{ey} and g_{em} so as to match the average annual wage growth of workers with fewer and more than 17 years of labor market experience, respectively, in the PSID based on the estimates of Rubinstein and Weiss (2006, Table 2b), we obtain a value of 2.51% for g_{ey} and of -0.43% for g_{em}. Even for such smaller degrees of human capital accumulation on the job, our model reproduces 78% and 74% of the volatility of the job-finding rates for young and mature workers and 83% and 78% of the volatility of unemployment for young and mature workers in the data.

[36] We focus on the period after June 1976 because the BLS reports short-term unemployment by age, which is necessary to compute the job-finding rate as we do in our baseline model following Shimer (2012) only from that date onward.

June 1976 and December 2007.

As Table 4 shows, the volatility of the average quarterly job-finding rate and that of the average quarterly unemployment rate, constructed as described, are both higher for the young than for the mature in the data. Specifically, in the data, the volatility of the job-finding rate is 4.98% for the young and 4.60% for the mature, and the volatility of unemployment is 0.86% for the young and 0.51% for the mature. Our life-cycle model reproduces this same pattern: the volatility of the job-finding rate is 5.46% for the young and 4.43% for the mature, and the volatility of unemployment is 0.97% for the young and 0.52% for the mature. In sum, the unemployment rate of younger workers is 1.69 times more volatile than that of older workers. Our model replicates this pattern well: it predicts that the unemployment rate of younger workers is 1.87 times more volatile than that of older workers.

In this simple exercise, we have examined whether our basic mechanism can account for the differential volatility of unemployment for young and mature workers in the data in light of differences in their wage and implied human capital acquisition patterns. To do so, we have highlighted the role of human capital accumulation on the job and depreciation off the job and kept other parameters, such as home production, identical across the two groups. If we further allowed, say, mature workers to be less attached to the labor market than young workers so as to capture shifting preferences for leisure with age, then the model could reproduce the data even more closely.

10 Robustness to Alternative Preferences

Here we show that our results for alternative preference structures are similar to those for the preference structure in our baseline. In the online appendix, we also demonstrate that the asset-pricing implications of these alternative preference structures in our production economy with frictional labor markets are nearly identical to those of these preference structures in pure exchange economies.[37] See Tables A.1-A.6 and Figure A.2 in the online appendix.

Campbell-Cochrane Preferences with External Habit. We adapt the setup of Campbell and Cochrane (1999) with external habit designed for a pure exchange economy, as discussed earlier, to our production economy. The only difference from the original specification in Campbell and Cochrane (1999) is that we replace their sensitivity function with

$$\lambda_t(\bar{s}_t) = \frac{\sigma(\varepsilon_{ct+1})}{\sigma_t(\varepsilon_{ct+1})} \frac{1}{\bar{S}} \sqrt{[1 - 2(\bar{s}_t - \bar{s})]} - 1. \tag{60}$$

Here $\sigma(\varepsilon_{ct+1})$ and $\sigma_t(\varepsilon_{ct+1})$ are, respectively, the unconditional and conditional standard deviations of the innovation to aggregate consumption growth, $\varepsilon_{ct+1} = \Delta\bar{c}_{t+1} - \mathbb{E}_t(\Delta\bar{c}_{t+1})$. The term $\sigma(\varepsilon_{ct+1})/\sigma_t(\varepsilon_{ct+1})$ in (60) adjusts for the time-varying conditional volatility of consumption in our production economy relative to its constant value in the pure exchange economy of Campbell and Cochrane (1999). This adjustment allows the model to reproduce the volatility of the risk-free rate in the data. Table 5 shows that this model generates results that are nearly identical to those generated by our baseline model.

[37] In Kehoe, Midrigan, Lopez, and Pastorino (2021), we further investigate the properties of these alternative preference structures.

Epstein-Zin Preferences with Long-Run Risk. We consider a model with Epstein-Zin preferences, a slow-moving predictable component in productivity as in Bansal and Yaron (2004), and discount factor shocks as in Albuquerque, Eichenbaum, Luo, and Rebelo (2016) and Schorfheide, Song, and Yaron (2018). In this case, preferences are given by

$$V_t = \left\{(1-\beta)S_t C_t^{1-\rho} + \beta[\mathbb{E}_t(V_{t+1}^{1-\alpha})]^{\frac{1-\rho}{1-\alpha}}\right\}^{\frac{1}{1-\rho}}.$$

Productivity growth now has a long-run risk component x_t in that $\Delta a_{t+1} = g_a + x_t + \sigma_a \varepsilon_{at+1}$ and $x_{t+1} = \rho_x x_t + \phi_x \sigma_a \varepsilon_{xt+1}$, where the shocks ε_{at} and ε_{xt} are standard normal i.i.d. and orthogonal to each other. The growth rate of the shock to the stochastic discount factor, $\Delta s_t = \Delta \log(S_t)$, follows an autoregressive process given by $\Delta s_{t+1} = \rho_s \Delta s_t + \phi_s \sigma_a \varepsilon_{st+1}$, where the innovation ε_{st} is standard normal i.i.d. and orthogonal to ε_{at} and ε_{xt}. The pricing kernel is

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \left(\frac{S_{t+1}}{S_t}\right) \left\{\frac{V_{t+1}}{[\mathbb{E}_t(V_{t+1}^{1-\alpha})]^{\frac{1}{1-\alpha}}}\right\}^{\rho-\alpha}. \tag{61}$$

We set the mean and the standard deviation of the aggregate productivity process to replicate those measured in the data. We choose the persistence ρ_x of the long-run risk state x_t so that the model generates the same standard deviation of the log price-consumption ratio as that generated by our baseline model. We assume that the parameter ϕ_x of the process for the long-run risk state x_t is such that its volatility $\sigma_x^2 = \phi_x^2 \sigma_a^2/(1-\rho_x^2)$ accounts for the same share of the volatility of aggregate productivity growth, $\sigma_x^2/(\sigma_a^2 + \sigma_x^2) = 0.0445$, as that in Bansal and Yaron (2004). We select the value of the parameter ϕ_s of the process for the discount factor shock to match the standard deviation of the risk-free rate. As for ρ_s, notice that the role of the surplus consumption ratio in our baseline model is similar to that of the discount factor shock in this model. Because of this feature, we set the persistence ρ_s of the process for the discount factor shock in the same way as we set the persistence of the process for the surplus consumption ratio in the baseline model.

We choose a risk-aversion parameter α of 4.35 to match a Sharpe ratio of the stock market excess return of 0.45 in the data and set ρ equal to 0.1 so that the elasticity of intertemporal substitution of consumption is 10. As is well-known, to reproduce the positive comovement between the price-dividend ratio and consumption in the data, the elasticity of intertemporal substitution must be greater than one (see Bansal and Yaron, 2004 and Kilic and Wachter, 2018). As noted by Kilic and Wachter (2018) in a related context, a large value for the theoretical elasticity of intertemporal substitution is nonetheless consistent with the low values for the elasticity of consumption growth with respect to interest rates commonly estimated. Indeed, when we estimate the contemporaneous elasticity of consumption growth with respect to interest rates based on data simulated from our model using powers of the lagged states s_t and x_t and lagged consumption growth as instruments in the spirit of Campbell (2003), we also find a coefficient of around 0.1—rather than 10—which is in line with the estimates in the literature (see, for instance, Hall, 1988 and Campbell, 2003). As Bansal and Yaron (2004) and Kilic and Wachter (2018) discussed, the reason why the parameter ρ that governs the elasticity of intertemporal substitution in a model like ours cannot typically be recovered through such an estimation exercise is that it is difficult to adequately control for the endogeneity of the conditional volatility of the future marginal utility of consumption.

As Table 5 shows, this model can produce 89% (70%/79%) of the volatility of unemployment in the data and has reasonable implications for asset prices.

Epstein-Zin Preferences with Variable Disaster Risk. We adopt a discrete-time version of the model in Wachter (2013) with Epstein-Zin preferences and a slow-moving probability of rare disasters. In this case, preferences are specified as

$$V_t = \left\{(1-\beta)C_t^{1-\rho} + \beta[\mathbb{E}_t(V_{t+1}^{1-\alpha})]^{\frac{1-\rho}{1-\alpha}}\right\}^{\frac{1}{1-\rho}}. \tag{62}$$

The process for aggregate productivity growth now includes a discrete-valued jump component j_{t+1} and is given by $\Delta a_{t+1} = g_a + \sigma_a \varepsilon_{at+1} - \omega j_{t+1}$, where the disaster component j_{t+1} is a Poisson random variable with intensity s_t, which evolves according to the process $s_{t+1} = (1-\rho_s)s + \rho_s s_t + \sqrt{s_t}\sigma_s \varepsilon_{st+1}$.

We choose the mean and the standard deviation of the aggregate productivity process to match those in the data, and a mean disaster intensity s of 3.55% per year as in Wachter (2013). We select the volatility σ_s of the disaster intensity to reproduce the standard deviation of the risk-free rate in the data, and the risk aversion coefficient α to match a Sharpe ratio of the stock market excess return of 0.45 in the data. We set the persistence ρ_s of the disaster intensity to generate the same standard deviation of the log price-consumption ratio as that generated by our baseline model. Like Wachter (2013), we choose a value of 0.26 for the disaster impact parameter ω and of 10 for the elasticity of intertemporal substitution ($\rho = 0.1$).[38]

As Table 5 shows, the model produces a volatility of unemployment of 0.78% that is similar to that in the data, 0.79%, and also has predictions for asset prices broadly in line with the data.

Affine Stochastic Discount Factor. Our results also hold for reduced-form stochastic discount factors of the type considered by Ang and Piazzesi (2003),[39]

$$\log(Q_{t,t+1}) = -(\mu_0 - \mu_1 s_t) - \frac{1}{2}(\gamma_0 - \gamma_1 s_t)^2 \sigma_a^2 - (\gamma_0 - \gamma_1 s_t)\sigma_a \varepsilon_{at+1}.$$

In this case, we assume that the exogenous state s_t evolves according to $s_{t+1} = \rho_s s_t + \sigma_a \varepsilon_{at+1}$ and is driven by shocks to aggregate productivity, ε_{at+1}. Aggregate productivity growth follows a random-walk process as in our baseline model. We keep the parameters for the mean and standard deviation of the aggregate productivity process, g_a and σ_a, at the same values as in our baseline model and choose the values of the four parameters $(\mu_0, \mu_1, \gamma_0, \gamma_1)$ so as to reproduce the mean and standard deviation of the risk-free rate, the Sharpe ratio of the stock market excess return, and the volatility of the stock market excess return in the data. We select the persistence ρ_s of the exogenous state to generate the same standard deviation of the log price-consumption ratio as that generated by our baseline model. Table 5 shows that this model produces 91% (0.72%/0.79%) of the volatility of unemployment in the data and also has predictions for asset prices consistent with the data.

[38]As in the version of our model with Epstein-Zin preferences and long-run risk, in this version of our model too, we estimate the elasticity of consumption growth with respect to interest rates on data simulated from our model using powers of s_t and lagged consumption growth as instruments. We find this elasticity to be around 0.1, despite assuming that the elasticity of intertemporal substitution $1/\rho$ equals 10.

[39]Here we simply posit a stochastic discount factor that is not derived from marginal utility, and so we define a competitive search equilibrium given $\{Q_{t,t+1}\}$. Hence, we drop condition *vii*) in the above definition of equilibrium.

11 Conclusion

We propose a new mechanism that allows search models to reproduce the observed fluctuations in the job-finding rate and unemployment at business cycle frequencies. Our model thus solves the unemployment volatility puzzle of Shimer (2005) and is immune to the critiques of existing mechanisms that address it, namely, those by Chodorow-Reich and Karabarbounis (2016) on the cyclicality of the opportunity cost of employment, by Kudlyak (2014) and Basu and House (2016) on the cyclicality of wages, and by Borovicka and Borovickova (2019) on the asset-pricing implications of these mechanisms.

To this purpose, we augment the textbook search model with two features: preferences from the macro-finance literature that are consistent with the observed variation in asset prices over the business cycle and a process of human capital acquisition that accounts for the longitudinal and cross-sectional evidence on wage growth with labor market experience. In such a framework, a firm's investment in hiring workers becomes a risky activity with long-duration flows of the surplus from a match between a firm and a worker, which are then sensitive to variation in the price of risk over the cycle. Hence, shocks to either aggregate productivity or, directly, to stochastic discount factors make the present value of these surplus flows sharply fluctuate. In turn, fluctuations in the present value of these surplus flows imply that investments in hiring workers are highly cyclical and thus that job-finding rates and unemployment are as volatile as in the data.

We further show that both new features we introduce play a critical role. That is, if we abstract from either preferences that generate time-varying risk or human capital acquisition, then the model gives rise to only negligible movements in unemployment. We demonstrate that the same intuition applies once we augment the model with physical capital or incorporate heterogeneity in the human capital process across workers. Overall, our results show that reintegrating search and business cycle theory is both a tractable and promising avenue of future research.

We have purposely kept our model simple so as to focus on our new mechanism. We have therefore abstracted from a host of potentially relevant sources of heterogeneity—for instance, in returns to experience across jobs, stochastic discount factors, households' exposure to time-varying risk, and labor market frictions. Including these multiple dimensions of heterogeneity and assessing their importance for labor market fluctuations would be an interesting extension of our framework for future work.

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Appendix

In this appendix, we provide some of the details and sketches of the proofs omitted from the main text. See the online appendix for complete arguments.

First-Order Conditions for the Aggregate Planning Problem and Expression for the Job-Finding Rate. By Proposition 2, the competitive search equilibrium aggregate allocations $\{C_t, Z_{et}, Z_{ut}, \theta_t\}$ solve the aggregate planning problem. Here, $C_t - X_t = \tilde{S}_t C_t = S_t^{\alpha/(\alpha-1)} C_t$, where S_t is governed by an exogenous stochastic process. Thus, we can express the aggregate planning problem as the problem of choosing $\{C_t, Z_{et}, Z_{ut}, \theta_t\}$, given the processes for A_t and S_t, to solve

$$\max \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{S_t^{-\alpha} C_t^{1-\alpha}}{1-\alpha} \tag{A.63}$$

subject to the following constraints for all periods t with multipliers ς_t, $\varsigma_t \mu_{et}$, and $\varsigma_t \mu_{ut}$, respectively,

$$\begin{aligned}
\varsigma_t: &\quad C_t = A_t Z_{et} + bA_t Z_{ut} - \kappa A_t \phi \theta_t (1+g_u) Z_{ut-1}, \\
\varsigma_t \mu_{et}: &\quad Z_{et} = \phi(1-\sigma)(1+g_e) Z_{et-1} + \phi \lambda_{wt} (1+g_u) Z_{ut-1}, \\
\varsigma_t \mu_{ut}: &\quad Z_{ut} = \phi \sigma (1+g_e) Z_{et-1} + \phi(1-\lambda_{wt})(1+g_u) Z_{ut-1} + 1 - \phi,
\end{aligned}$$

where we have omitted the explicit dependence of λ_{wt} on θ_t. The optimality conditions for the aggregate planning problem are

$$\begin{aligned}
C_t: &\quad \varsigma_t = \beta^t (S_t C_t)^{-\alpha}, \\
Z_{et}: &\quad \varsigma_t \mu_{et} = \varsigma_t A_t + \phi(1+g_e) \mathbb{E}_t \{\varsigma_{t+1}[(1-\sigma)\mu_{et+1} + \sigma \mu_{ut+1}]\}, \\
Z_{ut}: &\quad \varsigma_t \mu_{ut} = \varsigma_t b A_t + \phi(1+g_u) \mathbb{E}_t \{\varsigma_{t+1}[\lambda_{wt+1}\mu_{et+1} + (1-\lambda_{wt+1})\mu_{ut+1}]\} - \phi(1+g_u)\mathbb{E}_t(\varsigma_{t+1}\kappa A_{t+1}\theta_{t+1}), \\
\theta_t: &\quad \kappa A_t = \lambda'_{wt}(\mu_{et} - \mu_{ut}).
\end{aligned}$$

Using the definition of the stochastic discount factor $Q_{t,t+1} = \varsigma_{t+1}/\varsigma_t$ and the properties discussed that $\lambda_{wt} = \theta_t \lambda_{ft}$ and $1 - \eta_t = \theta_t \lambda'_{wt}/\lambda_{wt}$, we can rewrite the first-order conditions as equations (31) to (33).

To derive (34), note that with the Cobb-Douglas matching function $m(u,v) = Bu^\eta v^{1-\eta}$, it follows that $\eta_t = \eta$ and $\lambda_{ft} = B^{\frac{1}{1-\eta}} \lambda_{wt}^{-\frac{\eta}{1-\eta}}$, since

$$B^{\frac{1}{1-\eta}} \lambda_{wt}^{-\frac{\eta}{1-\eta}} = B^{\frac{1}{1-\eta}} \left(\frac{B u_t^\eta v_t^{1-\eta}}{u_t} \right)^{-\frac{\eta}{1-\eta}} = B \left(\frac{u_t}{v_t} \right)^\eta = \lambda_{ft}.$$

Substituting this expression into (33) gives

$$\kappa A_t = (1-\eta) B^{\frac{1}{1-\eta}} \lambda_{wt}^{-\frac{\eta}{1-\eta}} (\mu_{et} - \mu_{ut}).$$

By simple algebra, we then obtain that

$$\log(\lambda_{wt}) = \left(\frac{1-\eta}{\eta} \right) \log \left[\frac{B^{\frac{1}{1-\eta}}(1-\eta)}{\kappa} \right] + \left(\frac{1-\eta}{\eta} \right) \log \left(\frac{\mu_{et} - \mu_{ut}}{A_t} \right),$$

which is (34) with $\chi = (1-\eta)\log[B^{\frac{1}{1-\eta}}(1-\eta)/\kappa]/\eta$. □

Proof of Proposition 3. Recall the system in (35),

$$\begin{bmatrix} \mu_{et} \\ \mu_{ut} \end{bmatrix} = \sum_{n=0}^{\infty} \Psi^n \begin{bmatrix} 1 \\ b \end{bmatrix} \mathbb{E}_t (Q_{t,t+n} A_{t+n}), \tag{A.64}$$

where Ψ is the transition matrix given by

$$\Psi = \phi \begin{bmatrix} (1+g_e)(1-\sigma) & (1+g_e)\sigma \\ (1+g_u)\eta\lambda_w & (1+g_u)(1-\eta\lambda_w) \end{bmatrix}.$$

By performing a standard eigendecomposition, we can express Ψ as $\Psi = V \begin{bmatrix} \delta_\ell & 0 \\ 0 & \delta_s \end{bmatrix} V^{-1}$, where δ_ℓ and δ_s are the eigenvalues of Ψ and V is the corresponding matrix of eigenvectors. Hence, we can decompose Ψ^n as

$$\Psi^n = V \begin{bmatrix} \delta_\ell^n & 0 \\ 0 & \delta_s^n \end{bmatrix} V^{-1}. \tag{A.65}$$

With $\lambda = (1-\sigma)(1+g_e) - \eta\lambda_w(1+g_u)$, the eigenvalues of Ψ are given by

$$\begin{aligned} \delta_{\ell,s} &= \frac{\phi(1+g_u+\lambda)}{2} \pm \frac{\phi}{2}\sqrt{(1+g_u-\lambda)^2 + 4\eta\lambda_w(1+g_u)(g_e-g_u)} \\ &= \begin{cases} \phi(1+g_u) + \frac{\phi}{2}\left[\sqrt{(1+g_u-\lambda)^2 + 4\eta\lambda_w(1+g_u)(g_e-g_u)} - \sqrt{(1+g_u-\lambda)^2}\right] \\ \phi\lambda - \frac{\phi}{2}\left[\sqrt{(1+g_u-\lambda)^2 + 4\eta\lambda_w(1+g_u)(g_e-g_u)} - \sqrt{(1+g_u-\lambda)^2}\right] \end{cases}, \end{aligned} \tag{A.66}$$

where in (A.66) we have used that

$$\frac{\phi(1+g_u+\lambda)}{2} = \phi(1+g_u) - \frac{\phi}{2}(1+g_u-\lambda) \qquad \text{and} \qquad \frac{\phi(1+g_u+\lambda)}{2} = \phi\lambda + \frac{\phi}{2}(1+g_u-\lambda).$$

Now, note that premultiplying (35) by the row vector $\begin{bmatrix} 1 & -1 \end{bmatrix}$ yields $\mu_{et} - \mu_{ut}$. Then, by using our decomposition of Ψ^n, we obtain that $\mu_{et} - \mu_{ut} = \sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)\mathbb{E}_t(Q_{t,t+n}A_{t+n})$, where $c_\ell\delta_\ell^n + c_s\delta_s^n$ satisfies

$$\begin{bmatrix} 1 & -1 \end{bmatrix} V \begin{bmatrix} \delta_\ell^n & 0 \\ 0 & \delta_s^n \end{bmatrix} V^{-1} \begin{bmatrix} 1 \\ b \end{bmatrix} = c_\ell\delta_\ell^n + c_s\delta_s^n. \tag{A.67}$$

To solve for the constants c_ℓ and c_s, evaluate (A.67) at $n = 0$ and $n = 1$ using (A.65) to obtain a system of two equations in two unknowns,

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ b \end{bmatrix} = 1 - b = c_\ell + c_s \qquad \text{and} \qquad \begin{bmatrix} 1 & -1 \end{bmatrix} V \begin{bmatrix} \delta_\ell & 0 \\ 0 & \delta_s \end{bmatrix} V^{-1} \begin{bmatrix} 1 \\ b \end{bmatrix} = \phi\lambda(1-b) + \phi(g_e - g_u)b = c_\ell\delta_\ell + c_s\delta_s. \tag{A.68}$$

Solving these two equations for c_ℓ and c_s gives the expressions in footnote 14. □

Proof of Proposition 4. We prove that starting at the steady-state values of the aggregate human capital of employed and unemployed consumers, Z_e and Z_u, with CRRA utility and random-walk aggregate productivity, job-finding and unemployment rates are constant. We proceed by conjecturing and verifying that the solution to the aggregate planning problem is constant over time, which implies that the job-finding rate and the unemployment rate are also constant, as both are only functions of θ_t, which is constant. Recall that a solution to the aggregate planning problem is characterized by the first-order conditions (31) through (33) with $Q_{t,t+1} = \beta[\tilde{C}_{t+1}A_{t+1}/(\tilde{C}_t A_t)]^{-\alpha}$ and by the constraints (27) through (29). Define then $\tilde{C}_t = C_t/A_t$, $\tilde{\mu}_{et} = \mu_{et}/A_t$, and $\tilde{\mu}_{ut} = \mu_{ut}/A_t$. Since the competitive search equilibrium aggregate allocations $\{C_t, Z_{et}, Z_{ut}, \theta_t\}$ solve the aggregate planning problem and, as we show in the online appendix, the aggregate planning problem admits a unique solution, it suffices to show that there exists a solution to the aggregate planning problem with $\tilde{C}_t = \tilde{C}$, $Z_{et} = Z_e$, $Z_{ut} = Z_u$, $\theta_t = \theta$, $\tilde{\mu}_{et} = \tilde{\mu}_e$, and $\tilde{\mu}_{ut} = \tilde{\mu}_u$, when the initial conditions Z_{e-1} and Z_{u-1} equal the steady state values Z_e and Z_u.

To start, note that substituting $\mathbb{E}_t(Q_{t,t+1}) = \beta\mathbb{E}_t[(C_{t+1}/C_t)^{-\alpha}]$ in (31) and dividing both sides by A_t gives

$$\tilde{\mu}_{et} = 1 + \phi(1+g_e)\beta\mathbb{E}_t\left\{\left(\frac{A_{t+1}}{A_t}\right)^{1-\alpha}\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{-\alpha}[(1-\sigma)\tilde{\mu}_{et+1} + \sigma\tilde{\mu}_{ut+1}]\right\}, \tag{A.69}$$

where $(A_{t+1}/A_t)^{1-\alpha} = e^{(1-\alpha)(g_a+\sigma_a\varepsilon_{at+1})}$ by (1). Since ε_{at+1} has a standard normal distribution, it follows that $\mathbb{E}_t(A_{t+1}/A_t)^{1-\alpha} = e^{(1-\alpha)g_a+(1-\alpha)^2\sigma_a^2/2}$. Under our conjectured solution, $\tilde{C}_t$ is constant so that $\tilde{C}_{t+1}/\tilde{C}_t = 1$. If we let $\delta = \beta e^{(1-\alpha)g_a+(1-\alpha)^2\sigma_a^2/2}$, then (A.69) becomes

$$\tilde{\mu}_e = 1 + \phi(1+g_e)\delta[(1-\sigma)\tilde{\mu}_e + \sigma\tilde{\mu}_u]. \tag{A.70}$$

Proceeding in a nearly identical fashion with (32) gives that

$$\tilde{\mu}_u = b + \phi(1+g_u)\delta[\eta\lambda_w\tilde{\mu}_e + (1-\eta\lambda_w)\tilde{\mu}_u] \tag{A.71}$$

under our conjecture, since η_t, the job-finding rate, and the job-filling rate are constant if the matching function is time invariant and market tightness is constant. Finally, consider (33) and observe that

$$\kappa = (1-\eta)\lambda_f(\tilde{\mu}_e - \tilde{\mu}_u) \tag{A.72}$$

holds under our conjecture. Evaluated at our conjectured solution in which aggregate allocations are constant and the initial conditions satisfy $Z_{e-1} = Z_e$ and $Z_{u-1} = Z_u$, the transition equations for Z_{et} and Z_{ut} and the resource constraint are clearly satisfied. Thus, we have verified the conjecture. □

Proof of Lemma 2. We apply the log-linear approximation around a risky steady state described by Lopez, Lopez-Salido, and Vazquez-Grande (2017) to the pricing equation for a claim to aggregate productivity in n periods, $P_{nt} = \mathbb{E}_t(Q_{t,t+n}A_{t+n})$. To write this expression recursively, we start by noting that the price of a 0-period strip at time $t+n$ is $P_{0,t+n} = A_{t+n}$. Thus, the price of the 1-period strip at time $t+n-1$ is the expected present discounted value of this price in the previous period, $P_{1,t+n-1} = \mathbb{E}_{t+n-1}(Q_{t+n-1,t+n}P_{0,t+n})$. Proceeding with this backward recursion implies that the price of an n-period strip at time t is the expected present discounted value in t of the price of the $n-1$-period strip in $t+1$, that is, $P_{nt} = \mathbb{E}_t(Q_{t,t+1}P_{n-1,t+1})$. Letting $q_{t,t+1} = \log(Q_{t,t+1})$, $\Delta a_{t+1} = \log(A_{t+1}) - \log(A_t)$, and applying the same notational convention for similar expressions, we obtain that

$$\log(P_{nt}/A_t) = \log\{\mathbb{E}_t[\exp(q_{t,t+1} + \Delta a_{t+1} + \log(P_{n-1,t+1}/A_{t+1}))]\} \tag{A.73}$$

for $n \geq 1$, where $\log(P_{0t}/A_t) = 0$ since $P_{0t} = A_t$ for all periods t. Let $\hat{s}_t = s_t - s$ and use $\log(P_{nt}/A_t) \approx a_n + b_n\hat{s}_t$ to rewrite (A.73) as

$$a_n + b_n\hat{s}_t = \log\{\mathbb{E}_t[\exp(q_{t,t+1} + \Delta a_{t+1} + a_{n-1} + b_{n-1}\hat{s}_{t+1})]\}. \tag{A.74}$$

Note that $\log(P_{0t}/A_t) = a_0 + b_0\hat{s}_t = 0$ for all $\hat{s}_t$ gives the initial conditions $a_0 = b_0 = 0$. Now, the pricing kernel for our baseline preferences in (12) in log form is

$$q_{t,t+1} = \log(\beta) - \alpha\Delta s_{t+1} - \alpha\Delta c_{t+1}, \tag{A.75}$$

with $\Delta a_{t+1} = g_a + \sigma_a\varepsilon_{at+1}$. By (10), the law of motion for $\hat{s}_t$ is $\hat{s}_{t+1} = \rho_s\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1}$ so that

$$\Delta\hat{s}_{t+1} = \hat{s}_{t+1} - \hat{s}_t = -(1-\rho_s)\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1}. \tag{A.76}$$

Using (A.76), $\Delta s_{t+1} = \Delta\hat{s}_{t+1}$, and assuming that $\Delta c_{t+1} \approx \Delta a_{t+1}$, it follows that $q_{t,t+1} + \Delta a_{t+1} + a_{n-1} + b_{n-1}\hat{s}_{t+1}$ on the right side of (A.74) is approximately equal to

$$\begin{aligned}&[\log(\beta) - \alpha\Delta s_{t+1} - \alpha\Delta a_{t+1}] + \Delta a_{t+1} + a_{n-1} + b_{n-1}\hat{s}_{t+1}\\ &= \log(\beta) + (1-\alpha)g_a + [\rho_s b_{n-1} + (1-\rho_s)\alpha]\hat{s}_t + a_{n-1} + \{1 - \alpha[1+\lambda_a(s_t)] + b_{n-1}\lambda_a(s_t)\}\sigma_a\varepsilon_{at+1}.\end{aligned} \tag{A.77}$$

Next, we evaluate the right side of (A.74) using (A.77). Note that, except for the last term, all of the other terms on the right side of (A.77) involve variables that are known in t so that for these terms, we can drop the conditional expectation. As for the last term in (A.77), we further manipulate it using that the conditional expectation of a log-normal random variable with mean 0 and variance σ^2 is $\exp(\sigma^2/2)$ to obtain that

$$\begin{aligned}
&\log\{\mathbb{E}_t[\exp(\{1-\alpha[1+\lambda_a(s_t)]+b_{n-1}\lambda_a(s_t)\}\sigma_a\varepsilon_{at+1})]\} = \{1-\alpha[1+\lambda_a(s_t)]+b_{n-1}\lambda_a(s_t)\}^2\frac{\sigma_a^2}{2} \\
&\approx \{1-\alpha[1+\lambda_a(s)]+b_{n-1}\lambda_a(s)\}^2\frac{\sigma_a^2}{2} + (\sigma_a^2\,(b_{n-1}-\alpha)\,\lambda_a'(s)\{1-\alpha[1+\lambda_a(s)]+b_{n-1}\lambda_a(s)\})\hat{s}_t \\
&= \left[1-\frac{\alpha}{S}+b_{n-1}\left(\frac{1}{S}-1\right)\right]^2\frac{\sigma_a^2}{2} + \frac{(\alpha-b_{n-1})}{S}\left[1-\frac{\alpha}{S}+b_{n-1}\left(\frac{1}{S}-1\right)\right]\sigma_a^2\hat{s}_t \\
&= \left(1-b_{n-1}-\frac{\alpha-b_{n-1}}{S}\right)^2\frac{\sigma_a^2}{2} + \left(1-b_{n-1}-\frac{\alpha-b_{n-1}}{S}\right)\left(\frac{\alpha-b_{n-1}}{S}\right)\sigma_a^2\hat{s}_t, \qquad (A.78)
\end{aligned}$$

where in the second line, we have used a first-order Taylor expansion around $s_t = s$, and in the third line, we have used (11),

$$\lambda_a(s_t) = \frac{1}{S}\sqrt{[1-2(s_t-s)]} - 1, \qquad (A.79)$$

which implies that $1+\lambda_a(s) = 1/S$ and $\lambda_a'(s) = -1/S$ in a steady state. Using (A.77) and (A.78) and grouping together the constants and the terms in $\hat{s}_t$, we can express the right side of (A.74) as

$$\begin{aligned}
&\log\{\mathbb{E}_t[\exp(q_{t,t+1}+\Delta a_{t+1}+a_{n-1}+b_{n-1}\hat{s}_{t+1})]\} = \log(\beta)+(1-\alpha)g_a+a_{n-1}+\left(1-b_{n-1}-\frac{\alpha-b_{n-1}}{S}\right)^2\frac{\sigma_a^2}{2} \\
&+\left[(1-\rho_s)\alpha+\rho_s b_{n-1}+\left(1-b_{n-1}-\frac{\alpha-b_{n-1}}{S}\right)\left(\frac{\alpha-b_{n-1}}{S}\right)\sigma_a^2\right]\hat{s}_t. \qquad (A.80)
\end{aligned}$$

Matching up the constants and the coefficient of $\hat{s}_t$ on the right side of (A.80) gives the formula in (50).

We next show that the assumptions that

$$\alpha > 1 \qquad \text{and} \qquad 1-\rho_s+\left(1-\frac{\alpha}{S}\right)\frac{\sigma_a^2}{S} > 0 \qquad (A.81)$$

imply that the coefficients $\{b_n\}$ grow monotonically from 0 to α. To this end, it is useful to rearrange the formula for b_n as

$$b_n - b_{n-1} = (\alpha - b_{n-1})\phi(b_{n-1}), \qquad (A.82)$$

where

$$\phi(b_{n-1}) = 1-\rho_s+\left(1-\frac{\alpha}{S}\right)\frac{\sigma_a^2}{S}+b_{n-1}\left(\frac{1-S}{S}\right)\frac{\sigma_a^2}{S} \qquad \text{and} \qquad S = [(C-X)/C]^{(\alpha-1)/\alpha} \leq 1 \qquad (A.83)$$

using that $S = \tilde{S}^{(\alpha-1)/\alpha}$.

First, we claim that if $b_{n-1} \in [0,\alpha]$, then $\phi(b_{n-1}) \geq 0$ and $b_n \geq b_{n-1}$. To prove this claim, we note that $\phi(b_{n-1}) \geq 0$ if $b_{n-1} \geq 0$ by (A.81) and (A.83). Therefore, $b_n \geq b_{n-1}$ for any $b_{n-1} \in [0,\alpha]$ by (A.82).

Second, we claim that if $b_{n-1} \leq \alpha$, then $b_n \leq \alpha$. To establish this claim, we observe that the term $[(1-S)/S]\sigma_a^2/S$ is nonnegative since $S \leq 1$, which implies that the function $\phi(\cdot)$ is weakly increasing. Hence, if $b_{n-1} \leq \alpha$, then $\phi(b_{n-1}) \leq \phi(\alpha)$. Therefore, $b_n \leq \alpha$, since

$$b_n = b_{n-1}+(\alpha-b_{n-1})\phi(b_{n-1}) \leq b_{n-1}+(\alpha-b_{n-1})\phi(\alpha) = [1-\phi(\alpha)]b_{n-1}+\phi(\alpha)\alpha \leq \alpha, \qquad (A.84)$$

where in the first inequality, we have used that $\phi(b_{n-1}) \leq \phi(\alpha)$ and $\alpha - b_{n-1} \geq 0$. The last inequality follows because $[1-\phi(\alpha)]b_{n-1}+\phi(\alpha)\alpha$ is a convex combination of b_{n-1} and α, since $\phi(\alpha) = 1-\rho_s-(\alpha-1)\sigma_a^2/S$ belongs to $(0,1)$, and both b_{n-1} and α are less than or equal to α. To see that $\phi(\alpha) \in (0,1)$, note that $\phi(\alpha) < 1$ follows since ρ_s and $\alpha - 1$ are positive. That $\phi(\alpha) > 0$ follows because $S \leq 1$ and we have assumed that $1-\rho_s-(\alpha/S-1)\sigma_a^2/S > 0$. These facts imply that

$$\phi(\alpha) = 1-\rho_s-(\alpha-1)\sigma_a^2/S \geq 1-\rho_s-(\alpha/S-1)\sigma_a^2/S > 0.$$

Finally, we argue that b_n monotonically converges to α. To begin, recall that the sequence $\{b_n\}$ starts with $b_0 =$

$0 \in [0, \alpha]$. Recursively applying our first two claims implies that $b_n \in [b_{n-1}, \alpha]$ for all n. Since the sequence $\{b_n\}$ is monotone and bounded, it converges (monotonically) to some limit point b^*. To prove that this limit point is $b^* = \alpha$, note that b^* solves the quadratic equation $(\alpha - b^*)\phi(b^*) = 0$ by (A.82). This equation admits two roots, a positive one with $b^* = \alpha$ and, by (A.81) and (A.83), a negative one. The negative one is not relevant because, as argued above, b_n is a sequence that monotonically increases from $b_0 = 0$ to b^* so its limit point is positive. Hence, this limit point must be the positive root α. □

Proof of Proposition 5. Substitute $P_{nt}/A_t = e^{a_n + b_n(s_t - s)}$ from Lemma 2 into (41) in Proposition 3 to obtain

$$\log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right) \log\left[\sum_{n=0}^{\infty} (c_\ell \delta_\ell^n + c_s \delta_s^n) e^{a_n + b_n(s_t - s)}\right] \tag{A.85}$$

$$\approx \chi + \left(\frac{1-\eta}{\eta}\right) \log\left[\sum_{n=0}^{\infty} (c_\ell \delta_\ell^n + c_s \delta_s^n) e^{a_n}\right] + \left(\frac{1-\eta}{\eta}\right) \left\{\sum_{n=0}^{\infty} \frac{e^{a_n}(c_\ell \delta_\ell^n + c_s \delta_s^n)}{\left[\sum_{n=0}^{\infty} e^{a_n}(c_\ell \delta_\ell^n + c_s \delta_s^n)\right]} b_n\right\} (s_t - s)$$

$$= \text{const} + \left(\frac{1-\eta}{\eta}\right) \left(\sum_{n=0}^{\infty} \omega_n b_n\right) (s_t - s), \tag{A.86}$$

where the approximation sign follows from denoting the right-side of (A.85) by $f(s_t)$ and taking a first-order Taylor expansion of it around s using that $f(s_t) \approx f(s) + f'(s)(s_t - s)$. Differentiating (A.86) gives (51), from which (52) immediately follows. □

Table 1: Parametrization and results for baseline model

Panel A: Parameters		Panel B: Moments		
Endogenously chosen		*Targeted*	Data	Model
g_a, mean productivity growth (% p.a.)	2.28	Mean productivity growth (% p.a.)	2.28	2.28
σ_a, s.d. productivity growth (% p.a.)	1.84	S.d. productivity growth (% p.a.)	1.84	1.84
κ, hiring cost	1.00	Mean unemployment rate (%)	5.9	5.9
β, time preference factor	0.999	Mean risk-free rate (% p.a.)	0.74	0.74
S, mean of state S_t	0.21	S.d. risk-free rate (% p.a.)	2.35	2.35
α, inverse EIS	5.35	Sharpe ratio of market excess return (p.a.)	0.45	0.45
ρ_s^{12}, annualized persistence of state	0.935	Autocorrelation log price-dividend ratio (p.a.)	0.95	0.95
Assigned		*Labor market results*		
B, efficiency of matching technology	0.46	Mean job-finding rate (%)	45.35	50.14
b, home production parameter	0.6	S.d. job-finding rate (%)	6.67	6.60
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate (%)	0.79	0.75
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.96	0.99
g_e, human capital growth on job (% p.a.)	3.5	Correlation unemployment and job-finding rate	-0.96	-0.99
		Elasticity user cost labor to u (Basu and House, 2016)	-5.80	-6.00
		Asset market results		
		Mean excess return (% p.a.)	7.05	6.15
		S.d. excess return (% p.a.)	15.6	13.7
		Mean log price-dividend ratio	3.51	3.42
		S.d. log price-dividend ratio	0.44	0.38
		Mean 20-year real yield (% p.a.)	4.74	3.55
		S.d. 20-year real yield (% p.a.)	1.95	2.24
		Mean 20-year nominal yield (% p.a.)	7.71	7.53
		S.d. 20-year nominal yield (% p.a.)	2.41	2.30

Table 2: Role of human capital accumulation

	Data	Baseline	DMP model with baseline preferences $g_e = 0$ and $g_u = 0$	Baseline model with $g_e = 3.5\%$ and $g_u = 3.5\%$
S.d. job-finding rate (%)	6.67	6.60	0.14	0.14
Autocorrelation job-finding rate	0.94	0.99	0.99	0.99
S.d. unemployment rate (%)	0.79	0.75	0.02	0.02
Autocorrelation unemployment rate	0.96	0.99	0.99	0.99
Correlation u and job-finding rate	-0.96	-0.99	-1.00	-1.00

Table 3: Parametrization and results for model with baseline preferences and physical capital

Panel A: Parameters		Panel B: Moments		
Endogenously chosen		*Targeted*	Data	Model
g_a, mean productivity growth (% p.a.)	1.36	Mean productivity growth (% p.a.)	1.36	1.36
σ_a, s.d. productivity growth (% p.a.)	1.79	S.d. productivity growth (% p.a.)	1.79	1.79
κ, hiring cost	1.74	Mean unemployment rate (%)	5.9	5.9
β, time preference factor	0.999	Mean risk-free rate (% p.a.)	0.74	0.74
S, mean of state S_t	0.29	S.d. risk-free rate (% p.a.)	2.35	2.35
α, inverse EIS	7.15	Sharpe ratio of market excess return (p.a.)	0.45	0.45
ϑ, curvature of production function	0.26	Mean labor share of output	0.70	0.70
ξ, curvature of capital adjustment cost	0.26	Ratio s.d. invest. to consumption growth	4.5	4.5
ρ_s^{12}, annualized persistence of state	0.935	Autocorrelation log price-dividend ratio (p.a.)	0.95	0.95
Assigned		*Labor market results*		
B, efficiency of matching technology	0.46	Mean job-finding rate (%)	45.35	49.85
b, home production parameter	0.6	S.d. job-finding rate (%)	6.67	7.24
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate (%)	0.79	0.84
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.96	0.99
δ, physical capital depreciation rate	0.1/12	Correlation unemployment and job-finding rate	-0.96	-0.98
g_e, human capital growth on job (% p.a.)	3.5			
		Asset market results		
		Mean excess return (% p.a.)	7.05	5.52
		S.d. excess return (% p.a.)	15.6	12.2
		Mean log price-dividend ratio	3.51	3.27
		S.d. log price-dividend ratio	0.44	0.36
		Mean 20-year real yield (% p.a.)	4.74	3.68
		S.d. 20-year real yield (% p.a.)	1.95	2.25
		Mean 20-year nominal yield (% p.a.)	7.71	7.66
		S.d. 20-year nominal yield (% p.a.)	2.41	2.30

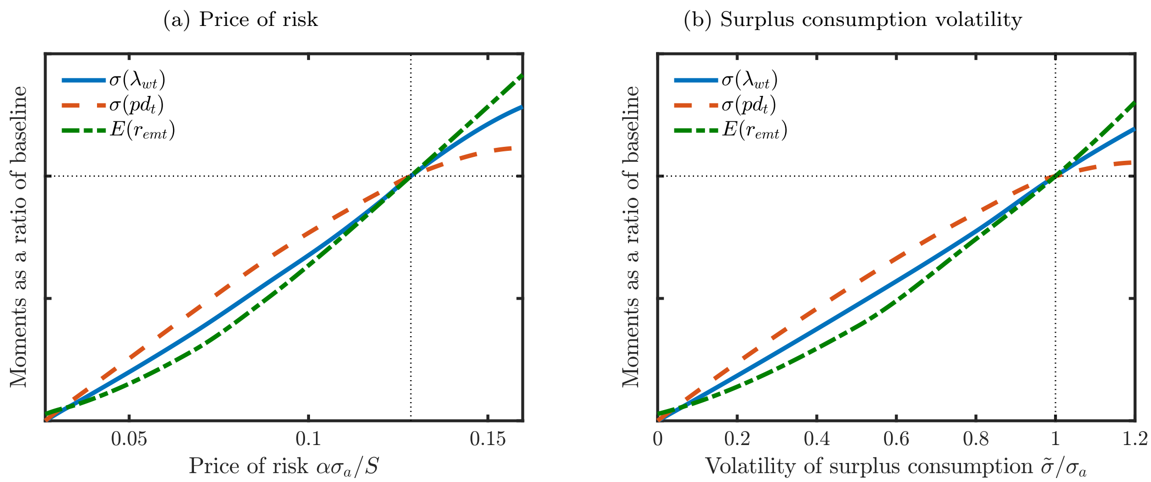
Table 4: Parametrization and results for lifecycle model with baseline preferences

Panel A: Parameters		Panel B: Moments		
Endogenously chosen		*Targeted*	Data	Model
g_a, mean productivity growth (% p.a.)	1.95	Mean productivity growth (% p.a.)	1.95	1.95
σ_a, s.d. productivity growth (% p.a.)	1.40	S.d. productivity growth (% p.a.)	1.40	1.40
κ_y, hiring cost for young	1.42	Mean unemployment rate for young (%)	8.7	8.7
κ_m, hiring cost for mature	6.01	Mean unemployment rate for mature (%)	4.1	4.1
β, time preference factor	0.998	Mean risk-free rate (% p.a.)	1.53	1.53
S, mean of state S_t	0.12	S.d. risk-free rate (% p.a.)	1.84	1.84
α, inverse EIS	4.00	Sharpe ratio of market excess return (p.a.)	0.45	0.45
ρ_s^{12}, annualized persistence of state	0.935	Autocorrelation log price-dividend ratio (p.a)	0.96	0.96
σ_y, probability of separation for young	0.044	Mean job-finding rate for young (%)	46.87	46.61
σ_m, probability of separation for mature	0.015	Mean job-finding rate for mature (%)	34.95	35.55
Assigned		*Labor market results*		
B, efficiency of matching technology	0.42	S.d. job-finding rate for young (%)	4.98	5.46
b, home production parameter	0.6	S.d. job-finding rate for mature (%)	4.60	4.43
η, matching function elasticity	0.5	Autocorr. job-finding rate for young	0.94	0.99
ϕ_y, survival probability for young	0.9917	Autocorr. job-finding rate for mature	0.89	0.99
ϕ_m, survival probability for mature	0.9972	S.d. unemployment rate for young (%)	0.86	0.97
g_{ey}, HK growth on job for young (% p.a.)	4.22	S.d. unemployment rate for mature (%)	0.51	0.52
g_{uy}, HK growth off job for mature (% p.a.)	-0.99	Autocorr. unemp. rate for young	0.95	0.99
g_{em}, HK growth on job for young (% p.a.)	2.75	Autocorr. unemp. rate for mature	0.95	0.99
g_{um}, HK growth off job for mature (% p.a.)	-18.2	Corr. unemp. and job-finding rate for young	-0.95	-0.99
		Corr. unemp. and job-finding rate for mature	-0.93	-0.99
		Asset market results		
		Mean excess return (% p.a.)	5.94	4.27
		S.d. excess return (% p.a.)	13.2	9.50
		Mean log price-dividend ratio	3.70	3.45
		S.d. log price-dividend ratio	0.49	0.30
		Mean 20-year real yield (% p.a.)	4.74	3.17
		S.d. 20-year real yield (% p.a.)	1.95	1.37
		Mean 20-year nominal yield (% p.a.)	7.71	7.45
		S.d. 20-year nominal yield (% p.a.)	2.41	1.46

Table 5: Results for other preferences

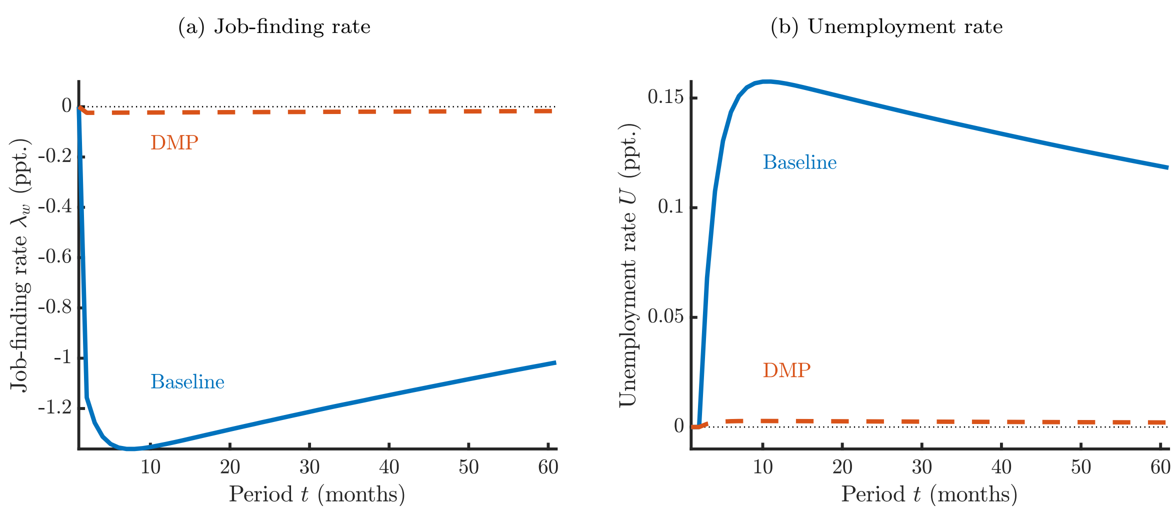
	Data	Baseline	Alternative preferences			
			CC	EZ w/ LRR	EZ w/ disasters	Affine SDF
Labor market results						
S.d. job-finding rate (%)	6.67	6.60	6.66	6.60	5.66	7.08
Autocorr. job-finding rate	0.94	0.98	0.99	0.99	0.99	0.99
S.d. unemployment rate (%)	0.79	0.75	0.77	0.70	0.78	0.72
Autocorr. unemployment rate	0.96	0.99	0.99	0.99	0.99	0.99
Correlation unemployment and job-finding rate	-0.96	-0.99	-0.98	-0.98	-0.98	-0.98
Asset market results						
Mean excess return (% p.a.)	7.05	6.15	6.32	4.26	4.77	6.92
S.d. excess return (% p.a.)	15.6	13.7	14.1	9.50	10.7	15.6
Mean log price-dividend ratio	3.51	3.42	3.41	3.85	3.76	3.29
S.d. log price-dividend ratio	0.44	0.38	0.39	0.31	0.36	0.38
Mean 20-year real yield (% p.a.)	4.74	3.55	3.70	2.79	-1.62	4.10
S.d. 20-year real yield (% p.a.)	1.95	2.24	2.40	1.26	2.15	2.07
Mean 20-year nominal yield (% p.a.)	7.71	7.53	7.67	6.48	2.04	8.19
S.d. 20-year nominal yield (% p.a.)	2.41	2.30	2.47	1.27	2.16	2.20

Figure 1: Sensitivity of key moments to preference parameters in baseline model



Note: λ_{wt} denotes the job-finding rate, pd_t denotes the log price-dividend ratio of the consumption claim, and r_{emt} denotes the market excess return, measured in the model as the excess return on a consumption claim. As the relevant parameter varies, the vacancy posting cost κ is adjusted so as to keep the mean unemployment rate constant at its baseline value.

Figure 2: Labor market responses to productivity shock in baseline model



Note: Impulse responses of the job-finding rate (left panel) and the unemployment rate (right panel) to a -1% productivity shock. Generalized impulse response functions are calculated based on 10,000 simulations.

Figure 3: Loci of human capital rates corresponding to fractions of job-finding rate volatility in baseline model

(a) Average working life of 30 years

(b) Average working life of 40 years

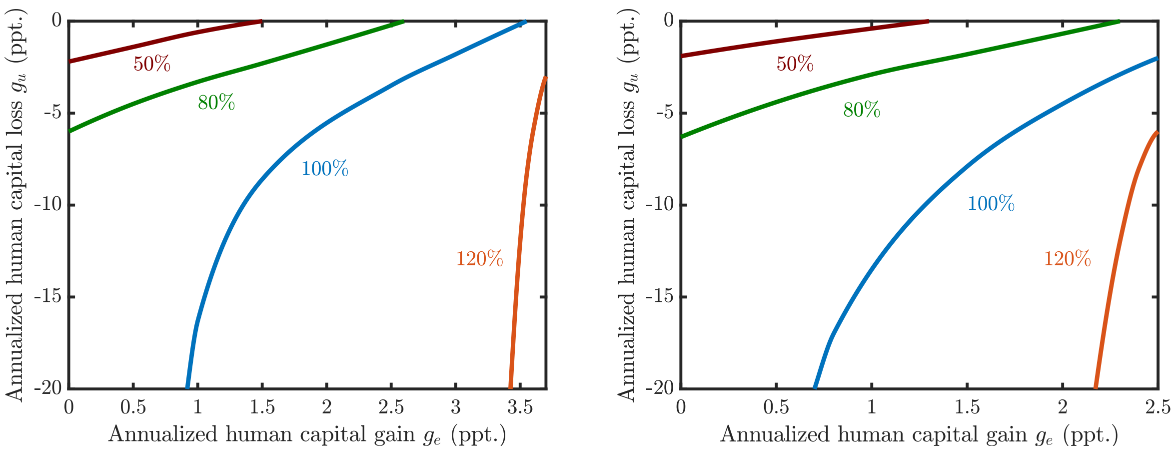


Figure 4: Prices of productivity strips in global and approximate solutions in baseline model

(a) Global solution

(b) Approximate solution

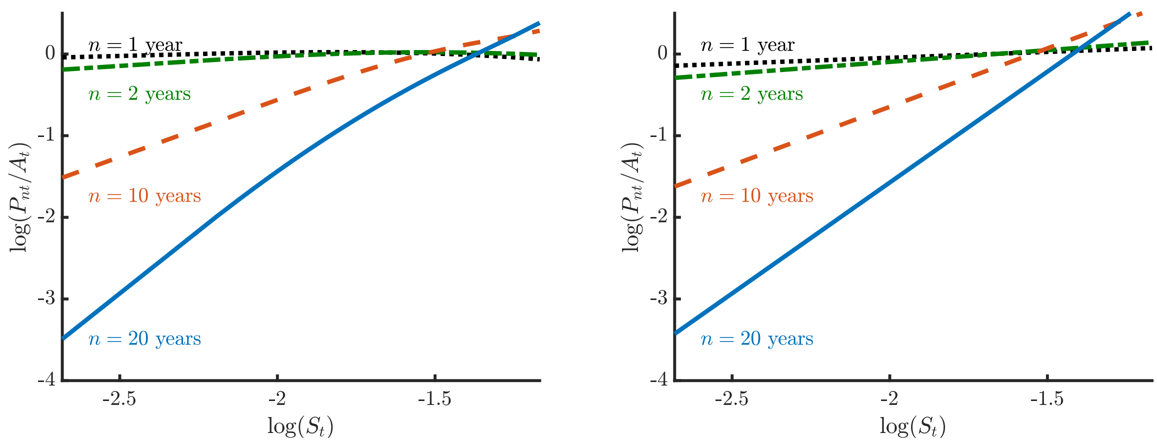
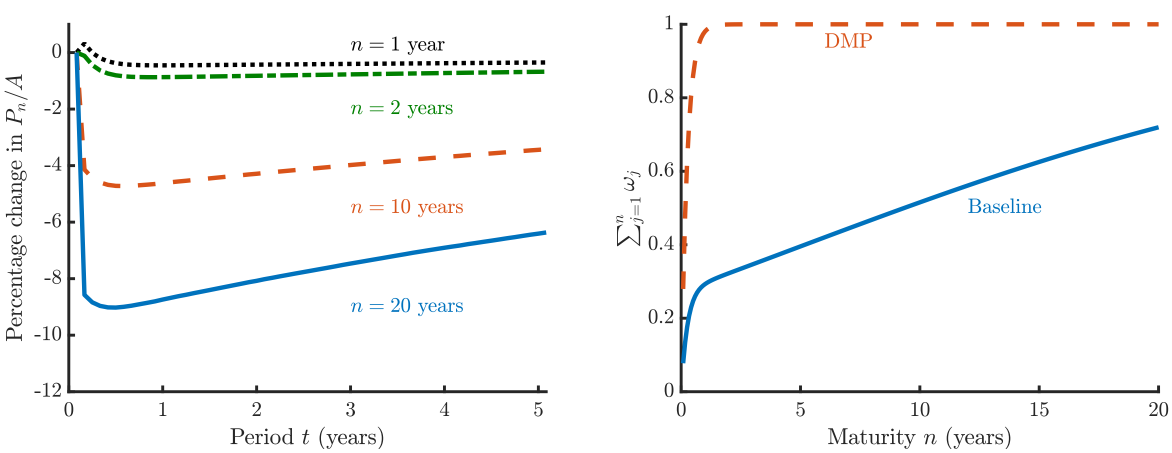


Figure 5: Asset market responses to productivity shock in baseline model and durations

(a) Prices of productivity strips by maturity

(b) Cumulative weights by maturity



Note: Impulse responses of the prices of productivity strips to a -1% productivity shock (left panel) and duration of surplus flows in the baseline model and DMP model with baseline preferences (right panel). Generalized impulse response functions are calculated based on 10,000 simulations.