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Russell Cooper

John Haltiwanger

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ABSTRACT

The literature on inventory holdings stresses their role in smoothing production when costs are convex. Existing empirical evidence suggests that output is more variable than consumption so that production smoothing is not apparently present. One way of explaining this finding is to allow for non-convex technologies. In this paper, we investigate some macroeconomic implications of the proposition that at least some firms in the economy produce with non-convex technologies.

We begin our analysis by studying a simple Robinson Crusoe economy with a single, storable good which is produced from a non-convex technology. The single agent can produce a finite amount of output simply by incurring a fixed production cost. We demonstrate that the efficient solution to this problem will entail periods of production followed by periods of inactivity: i.e. production will be bunched rather than smoothed. More importantly, inventories will be used to smooth consumption relative to this production path. Still, as long as the agent discounts the future or inventories depreciate over time, consumption will not be totally smooth. Instead, consumption will be highest in periods of production. Thus the non-convex technology will induce fluctuations in both production and consumption.

Using this analysis as a starting point, we then consider the implications of a non-convex technology in one sector of the economy for the behavior of other sectors through intersectoral technological linkages for both centralized and decentralized economies. For the centralized setting, the extent to which non-convexities spill over to other sectors depends on the degree to which intermediate and final goods can be inventoried and the nature of the technological interaction between factors. For the decentralized economy, the production of inputs which are strategic complements (substitutes) will be synchronized (staggered). Thus the presence of strategic complementarities (as in imperfectly competitive markets) will imply that non-convexities will have aggregate implications.

Russell Cooper
Department of Economics
University of Iowa
Iowa City, Iowa

John Haltiwanger
Department of Economics
University of Maryland
College Park, MD

I. Introduction

Macroeconomic models based on convex preferences and technologies generally have difficulty in explaining observed fluctuations in the economy. This is primarily a consequence of the smoothing implications of convex economies. In order to match observed fluctuations in a convex environment, it is necessary to either introduce large, aggregate stochastic elements into the analysis or large intertemporal substitution effects.

This tension between the smoothing effects of convex preferences and technologies and the presence of fluctuations is, perhaps, best exemplified in the literature on inventories. The initial models of inventories stressed their role in smoothing production relative to fluctuating sales when technologies are convex. Evidence on the time series of production and sales (see, e.g., Blinder [1986] and West [1986]) indicates that, in contrast to the predictions of the production smoothing model, the variance of sales appears to be significantly <u>less</u> than the variance of output. This observation has led to the development of alternative models of inventory holding whose predicted relative variances for production and sales are consistent with the data.[1]

One of these alternatives is to assume that production technologies are not convex. For example, the operations research literature on production, sales and inventories generally assumes that there is a start-up cost to a production run and a constant marginal cost of production leading to the bunching of production relative to sales, i.e. the (S,s) model.[2] These models generally have the implication that the variance of production exceeds the variance of sales at the level of the individual firm.

The goal of this paper is to investigate the aggregate implications of this microeconomic explanation for the observed behavior of inventory, sales and production. Given the role that inventories play in the business cycle, it seems natural to inquire whether or not an economy with non-convexities in technology at the firm level will display other properties of the cycle, particularly, the smoothing of consumption relative to production and the observed co-movement of sectors. Further, over the course of a cycle, variations in inventories are common across sectors of the economy. Finally, one tends to observe that the variance of production exceeds that of sales even for aggregate variables at business cycle frequencies.[3] For a theory of production bunching at the firm level to be consistent with this observation requires some synchronization in the production of goods economy-wide. Otherwise, the non-convexities at the firm level will be smoothed by the aggregation across firms.[4]

Section II of the paper focuses on the first issue, the correlations between consumption, production and inventories predicted by this production bunching model. Here we investigate the solution to the programming problem of a single agent with a non-convex technology. We find that the economy will quite naturally exhibit cycles in production and that, in the presence of depreciating inventories and/or discounting, consumption will be positively correlated with production. These results should not be totally surprising in that the bunching of production is almost an immediate consequence of the assumed technology. However, the fact that consumption is timed with production is of greater interest. The fixed cost of a production run creates a discontinuity in the cost of borrowing: in the period just before a production run inventories are completely exhausted so that it is quite costly to increase consumption in this period relative to others. Thus consumption fluctuates with production.

Based on these results, Section III focuses on the issue of aggregation and spillovers to other sectors linked through factor demand flows in a centralized setting.[5] We find that the extent to which the production bunching of final goods spills over to intermediate inputs depends on the cost of holding inventories. If, to take an extreme case, factors cannot be held in inventory, then the production of intermediate goods will be synchronized with the production of final goods. The predictions of this model are related to Blinder's [1981, 1986] argument that non-convexities are more important in retail trade than in manufacturing. The point is that the production bunching in the retail sector spills over to the manufacturing sector creating large fluctuations in production there as well.

We also investigate the implications of non-convex technologies in the production of intermediate goods. In this discussion, we focus on whether the production of intermediate goods will be synchronized or staggered and the resulting implications for final goods production. Here, we find that the extent to which production bunching in one intermediate goods sector spills over to other intermediate inputs and final goods depends on inventory holding costs <u>and</u> the degree of substitutability between factors within the production process.

Drawing on Section III, we then consider a decentralized environment in which sellers of inputs have non-convex technologies. In this setting, we investigate a timing game between these sellers. If the payoff functions for these sellers exhibits strategic complementarity (substitutability), then the equilibrium in this game is synchronization (staggering) of production runs. Thus the non-convexities at the firm level will have aggregate implications in the presence of strategic complementarities. This is of interest given the role of strategic complementarities in understanding multiplier effects and coordination difficulties in closely related macroeconomic models.[6]

Overall, we find that smoothing by aggregation need not occur when activities are sufficiently complementary and the holding of inventories is sufficiently costly. In these environments, the presence of non-convexities in a subset of the sectors of the economy can have interesting aggregate implications. Our conclusion outlines a number of extensions of this model to look at final demand linkages and provides a discussion of the implications of our results for macroeconomic fluctuations.[7]

II. Inventory, Consumption and Output Fluctuations with Non-Convex Technology

In this section, we consider the intertemporal choice problem of a representative agent. The agent is endowed with labor time in each period of his infinite lifetime which can be utilized in the production of a single commodity. Inventories of this commodity depreciate at rate δ each period. This good is produced from a non-convex technology: there is a fixed cost of initiating a production run, K, and zero marginal cost up to a capacity, Q. At this stage, we treat K and Q as exogenous.

The choice problem of the agent is:

$$(1)\quad \max_{\{c_t\},\{q_t\}} \sum_{t=0}^{\infty} \beta^t \, [u(c_t) - C(q_t)]$$

subject to:

$$(1.a)\quad c_t + I_t = I_{t-1}(1-\delta) + q_t,$$

$$(1.b)\quad I_t \geq 0,$$

$$(1.c)\quad C(q_t) = \begin{cases} 0 & \text{if } q_t = 0 \\ K & \text{if } q_t \in (0,Q] \\ \infty & \text{otherwise} \end{cases} \quad \text{and}$$

$$(1.d)\quad I_{-1}=0$$

In this problem, q_t refers to the level of production in period t, I_t is the level of inventory holdings at the <u>end</u> of the period and c_t is the level of consumption within a period. The accumulation equation for inventories is given by (1.a) with the stock of inventories at the start of period 0 set at 0. The agent discounts utility at rate $\beta \in (0,1]$ and goods depreciate at rate $\delta \in [0,1]$. The non-convexity of the technology is reflected in the cost function, $C(q_t)$.[8] Labor time is the only input into this production process so that the cost of production should implicitly be interpreted as labor costs, incorporating the technology and the disutility of work. The utility function for consumption, $u(\cdot)$, is strictly increasing and strictly concave and $u'(c) \to \infty$ as $c \to 0$ so that $c_t \geq 0$ is not included as a constraint.

The following two lemmas characterize the consumption of the agent for a given production plan. That is, we solve for the optimal $\{c_t\}$ given $\{q_t\}$.

<u>Lemma 1</u>: If, in the solution to (1), (1.b) is not binding for period t, then

$$(2)\quad u'(c_t) = \beta(1-\delta)u'(c_{t+1}).$$

<u>Lemma 2</u>: If, in the solution to (1), (1.b) is binding for period t, then

(3) $u'(c_t)>\beta(1-\delta)u'(c_{t+1})$.

The proofs of these lemmas and all propositions to follow are in the Appendix of this paper. Lemma 1 states that if the non-negativity constraint on inventories does not bind, then the agent will equalize the marginal rate of substitution over two adjacent periods to the return on the storage technology, as in (2).[9] If the constraint that $I_t \geq 0$ is binding, then the agent would prefer to increase consumption in period t but is unable to, as in (3). Note that when (2) holds and $\beta(1-\delta)<1$, consumption will monotonically decrease over time. Consumption may increase discontinuously between periods t and t+1 when (1.b) binds, as in (3).

We show that the solution to (1) entails production every T periods and consumption out of inventories between times of production. In the period before the next production run, inventories equal zero. This solution is similar to that of the (S,s) literature though our model is simplified by the fact that once production occurs, the agent always produces Q units and that inventories fall to zero prior to production.[10] Note that this allocation constitutes a stationary periodic allocation: consumption and inventory holdings can be determined solely by time relative to the production run, indexed by $\tau=1,\ldots,T$, where $\tau=1$ indicates a production run.

To understand the optimality of this solution, consider the sub-problem of deciding on consumption and inventories <u>given</u> that the agent produces Q units of the good every T periods. This is basically a T-period cake eating problem with discounting and depreciation. In our proposed solution to (1), the individual consumes according to (2) until the period just before a new delivery of goods and then eats all of the remaining inventories.

<u>Proposition 1</u>: If, in the solution to (1), there exists a T such that the agent produces every T periods, then c_τ will satisfy (2), I_τ will satisfy (1.a) and the condition that $I_\tau=0$ for $\tau=T$.

This proposition characterizes consumption and inventory behavior for a hypothesized production plan. The next step of characterizing the optimal value of T proceeds in two stages: first we show that the optimal production plan will imply production every T periods and second we find the optimal time between production periods, T^*.

To ensure that the analysis of production bunching is not vacuous, we assume production is costly enough that there will not be a production run each period yet is not so costly that production never occurs. The actual assumptions on the primitives of the problem to ensure an interior solution are stated below.

The discussion which follows uses the fact that the stock of inventories is the only state variable in this problem. Hence, by optimality, we know that the lifetime utility is dependent only on this state variable though this level of utility may be supported by more than one path of the choice variables. With this in mind, we also make use of the fact that given the state variable, we can choose among the various paths which yield the same level of lifetime utility.

Proposition 2: The solution to (1) entails production every T periods.

The final stage of characterizing T^* in general is a bit harder. Suppose that the agent produced every T periods and consumed all of the good by the period prior to a production run. Then the utility over the T periods, W(T), would be given by

$$W(T) = \max_{\{c_\tau\}} \sum_{\tau=1}^{T} \beta^\tau u(c_\tau)$$

subject to:

$$I_T = 0,$$

$$I_1 = Q - c_1 \quad \text{and}$$

$$I_\tau = (1-\delta)I_{\tau-1} - c_\tau \qquad \text{for } \tau = 1,\ldots,T-1$$

Here, the stock of inventories at the end of the period before a production run is constrained to equal zero.

The lifetime utility of an agent from producing every T periods is then

$$V(T) = \frac{W(T) - K}{1 - \beta^T}.$$

From this, an increase in T will have two effects. First, a given production run will be split over more periods. This is a cost in that increasing the span between production periods reduces $W(T)/1-\beta^T$.[11] Second, the cost of production will be delayed and this is beneficial to the agent. The optimal T trades off these costs and benefits.

At this stage, we can state more formally the assumptions which are sufficient for an interior solution. In particular, assume that $u(Q)<K$ and that there exists a $T<\infty$ such that $W(T)>K$. So, production will occur but is not profitable in each period.

While a full characterization of T^* is difficult to obtain, we can show that

<u>Proposition 3</u>: T^* is a non-decreasing function of K.

As a special and more tractable case, suppose that there is no discounting ($\beta=1$) and no depreciation ($\delta=0$). In the absence of discounting, we compare two programs based on the average utility they generate. For this specification, per period consumption <u>if</u> the agent produces every T periods will simply be Q/T.[12] With this in mind, one can characterize the optimal frequency of production through

<u>Proposition 4</u>: If $\beta=(1-\delta)=1$, then T^* solves $u'(Q/T^*) = K/Q$.

Thus, when there is no discounting and no depreciation, we obtain a clean characterization of T^*. Note that the time between production periods is an increasing function of K. Further, as capacity rises, T^* actually falls if $u'(c) + cu''(c)$ is positive.[13] This is somewhat surprising. An increase in Q has two effects: output per production run increases and cost per unit falls. The latter effect dominates the former if $u(\cdot)$ is not too concave.

The significant part of this solution is the predicted patterns of consumption, production and inventories. The model, due to the non-convex technology, implies production bunching and consequently that production will be much more volatile than sales (consumption). As discussed by Blinder [1981] and Ramey [1987], it is not very surprising that this non-convex technology can reproduce the observed relationship between the variances of production and sales. Of additional interest though is that consumption fluctuates <u>if</u> $\beta(1-\delta)<1$. From (2), consumption falls between production periods but is also a stationary function of the time since production, τ. Hence consumption in period 0 and T^* will be equal and higher than consumption in period T^*-1. The cost of initiating a new production run earlier creates an endogenous borrowing constraint in this economy which implies that consumption will fluctuate with production. If $\beta(1-\delta)=1$, then consumption will be smoothed over time and independent of production periods.

Is this model important for macroeconomics? While the model succeeds in reproducing the observed relative variances in production and sales, it does so by introducing a non-convexity in the technology of the representative agent. In light of the importance of inventories over the course of business cycles, the obvious question is whether this non-convexity will have any interesting aggregate implications. One immediate response is to appeal to a smoothing by aggregation argument that these non-convexities will be immaterial in the aggregate. This argument implicitly assumes that there are multiple producers in the economy who stagger their production periods. The sections that follow highlight circumstances in which the non-convexity at one firm will spillover to other activities by focusing on economies with multiple inputs and, in Section IV, many firms.

III. Factor Demand Linkages: Centralized Solution

In this section we investigate factor demand linkages across sectors to understand how production bunching induced by a non-convex technology in one sector affects other sectors linked by factor demands. We examine this issue by modifying the model of Section II to allow for intermediate goods production and the associated intersectoral linkages.

The representative agent's problem is altered to:

$$\max_{\{c_t\},\{n_t^y\},\ \{n_t^z\},\{I_t^j\}} \sum_{t=0}^{\infty} \beta^t\,[u(c_t) - C(n_t)], \qquad \beta < 1 \tag{4}$$

subject to:

$$c_t + I_t^q \le I_{t-1}^q\,(1-\delta^q) + q_t,\ 0 < \delta^q < 1 \tag{4.a}$$

$$q_t = f(y_t, z_t) \tag{4.b}$$

$$y_t + I_t^y \le g(n_t^y) + I_{t-1}^y(1-\delta^y),\ 0 \le \delta^y \le 1 \tag{4.c}$$

$$z_t + I_t^z \le h(n_t^z) + I_{t-1}^z(1-\delta^z),\ 0 \le \delta^z \le 1 \tag{4.d}$$

$$I_t^j \ge 0 \text{ for } j = q,y,z \quad \text{and} \tag{4.e}$$

$$n_t = n_t^y + n_t^z \le N. \tag{4.f}$$

As in Section II, the agent is endowed with N units of time in each period which can be allocated towards the production of either of two intermediate goods, y and z, where n^y and n^z are the amounts allocated to the two activities respectively. The function $C(\cdot)$ explicitly represents the disutility of labor and is assumed to be strictly convex. The two intermediate goods are combined to produce the final consumption good q. Inventories of each type of good as well as the associated depreciation rates are denoted by appropriate superscripts. All initial stocks are assumed to be zero.

In what follows, we consider the impact of nonconvex technologies at both the final goods and intermediate goods stage of production. We will be implicitly assuming throughout that the parameters are such that production bunching of the type analyzed in detail in Section II occurs in a sector specified to have a nonconvex technology. Accordingly, our focus in this section is on the spillover effects of production bunching in one sector on other sectors linked through factor demands.

A. Downstream Nonconvexities

We begin our analysis of this model by considering an environment in which the final goods production technology is non-convex, while the intermediate goods technologies are both assumed to be convex. Specifically,

$$(5) \qquad f(y_t, z_t) = \begin{cases} 0 \text{ for } k(y_t, z_t) < K \\ Q, \text{ otherwise.} \end{cases}$$

and $h'>0$, $h''\leq 0$, $g'>0$, and $g''\leq 0$. Assume that $k(\cdot)$ is differentiable, concave and increasing in its two arguments.

Based upon the logic from the analysis of Section II, we take as given that there exists a T* such that optimal production plans entail producing q_t every T* periods.[14] The following proposition characterizes the nature of the spillover effects of such downstream bunching on upstream production plans.

Proposition 5: Suppose q is produced every T^* periods in the amount Q. Then:

(i) $\delta^y = 1$ ($\delta^z = 1$) implies y (z) is produced every T^* periods synchronized with the production of q;

(ii) $0<\delta^{y}-\delta^{z}<1$ implies y_{τ} (z_{τ}) is increasing in τ where τ denotes the number of periods since the last production of q ($\tau = 1,\ldots,T^{*}$).[15] Further, the rate of increase is an increasing function of δ^{y} (δ^{z}) and a decreasing function of β.

Proposition 5 indicates the crucial role that inventories play in determining whether nonconvexities from downstream firms spillover to upstream firms. If intermediate goods cannot be held as inventories (and/or it is very costly to do so), production bunching generated from downstream nonconvexities will induce production bunching upstream for intermediate goods producers. If, however, intermediate goods can be held as inventories at zero or very low cost, then upstream firms facing convex costs will use inventories to smooth production even though orders from downstream firms are bunched.

It is important to note that the nature of the interaction between the alternative intermediate goods plays relatively little role in this environment. Whether y and z are complements or substitutes will have quantitative implications but this is not fundamental for the generation of spillover effects. The key is the tension created by production bunching by downstream firms and the desire for production smoothing by the convex producers upstream.

A potentially interesting empirical application of Proposition 5 is the interaction of the retail and manufacturing sectors of the economy. As noted by Blinder [1981], amongst others, the nature of the ordering and selling process in the retail sector likely involves nonconvexities in the cost structure. Proposition 5 can be interpreted as suggesting that manufacturing sectors supplying the retail sector may exhibit some production bunching due to spillovers from the retail sector even if convex technologies are present in the manufacturing sector.[16] The degree of spillover will depend on how inventoriable are the goods produced in the manufacturing sector.

B. Upstream Nonconvexities

We now turn our attention to nonconvexities present in the technology of intermediate goods producers. In particular, initially assume:

$$(6) \qquad g(n_t^y) = \begin{cases} 0 & \text{if } n_t^y < K \\ Q, & \text{otherwise.} \end{cases}$$

with $f(y_t, z_t)$ and $h(z_t)$ both increasing, concave functions of their arguments.

Following the logic of Section II again, we take as given that there exists a T* such that y is produced every T* periods in the amount Q.[17] Our interest is the consequences of such bunching for the production of the other intermediate good and the final good.

In this environment there are potentially numerous cases to consider, particularly in terms of the interaction of y_t and z_t. To simplify matters we focus on two polar assumptions regarding $f(y_t, z_t)$; in particular, we consider $f(y_t, z_t) = \min(y_t, z_t)$ and $f(y_t, z_t) = y_t + z_t$. The following propositions summarize optimal production plans for y_t and q_t under these alternative specifications.

<u>Proposition 6</u> : Suppose that $f(y_t, z_t) = \min(y_t, z_t)$ and that there exists a T^* such that y is produced every T^* periods in the amount Q. Then:

(i) $\delta^y = \delta^z = 1$ implies q and z are produced every T^* periods synchronized with the production of y;

(ii) $\delta^y=1$ but $0<\delta^z<1$ implies q is produced every T^* periods synchronized with the production of y and letting τ be the number of periods since the last production of y, z_τ is increasing in τ, for $\tau<T^*$;

(iii) $\delta^y<1$ but $\delta^z=1$ implies z_τ and q_τ are decreasing in τ, for $\tau<T^*$.

Proposition 7: Suppose $f(y_t,z_t) = y_t + z_t$ and that there exists a T* such that y is produced every T* periods in the amount Q. Then:

(i) the minimum production of z will be in periods of production of y. Further, letting τ be the number of periods since the last production of y, z_τ is non-decreasing in τ, for $\tau<T^*$;

(ii) $var(q) < var(y) + var(z)$.

Propositions 6 and 7 reveal that the nature of the spillover of nonconvexities in upstream firms depends critically on the technological interaction between intermediate goods. That is, Proposition 6 indicates that if there exists strong complementarities between intermediate goods in the production of the final good then the nonconvexity in one intermediate good sector tends to generate production bunching in the other intermediate goods sector and consequently in the production of the final good.

This tendency is mitigated by the ability to hold inventories of the intermediate goods. In particular, when z is storable at relatively low cost, there will be some tendency to smooth the production of z through building inventories of z prior to the production of y. In contrast, when y is storable at relatively low cost, there will again be some tendency to smooth the production of z but in this case by decumulating inventories of y in accordance with the production of z. In either case, even though the ability to hold inventories permits taking some advantage of production smoothing incentives for z, there is still an important spillover effect from the nonconvex y sector in that the production of z will tend to be bunched around the production runs of y.

Proposition 7 indicates further that when strong substitutibilities are present between intermediate goods there tends to be less spillover effects from nonconvexities present in one intermediate goods sector. This is because the technology permits substitution away from the production bunching occurring in the one sector. Proposition 7 indicates that this will be true as long as it is optimal to produce at least some of the intermediate good with a convex technology. This substitution away from the nonconvexity implies a negative covariance in the production of the two intermediate goods. This negative covariance implies that the production of final goods will be smoothed by this substitution.

Thus far in this section we have considered the question of whether nonconvexities in one sector will spillover to other convex sectors. An equally important issue is whether alternative sectors both facing nonconvexities should synchronize or stagger their respective bunched production plans. To consider this question, assume that both y and z involve symmetric nonconvex technologies given by:

$$(7) \qquad g(n_t^y) = \begin{cases} 0 & \text{if } n_t^y < K \\ Q, & \text{otherwise.} \end{cases}$$

$$(8) \qquad g(n_t^z) = \begin{cases} 0 & \text{if } n_t^z < K \\ Q, & \text{otherwise.} \end{cases}$$

Further, to maintain complete symmetry also assume that $\delta^y = \delta^z$. For ease of exposition, let δ^i be the common depreciation rate.

Using the logic of Section II again and given the completely symmetric specification of y and z under consideration, we take as given that there exists a T* such that y is produced every T* periods and z is produced every T* periods. Note, however, that the timing of the production runs could be either synchronized or staggered. This timing question is addressed in the following propositions.

Proposition 8 : Suppose that $f(y_t, z_t) = \min(y_t, z_t)$, $C''=0$, $\delta^i > \delta^q$ and there exists a T^* such that y and z are produced every T^* periods in the amount Q, then the production of y, z and q are perfectly synchronized.

Proposition 9: Suppose that $f(y_t, z_t) = y_t + z_t$, $\delta^i > \delta^q$ and there exists a T^* such that y and z are produced every T* periods. Then:

(i) y and z are produced every T^* periods but are completely staggered (i.e., the production of y occurs T*/2 periods after the production of z);

(ii) q is produced every $T^*/2$ periods synchronized with the staggered production of y and z;

Propositions 8 and 9 reveal that the nature of factor demand linkages is critical for whether alternative intermediate input sectors will stagger or synchronize production if both face nonconvex technologies. Specifically, Proposition 8 indicates that if the intermediate inputs are perfect complements, there is a tendency for synchronized production of these inputs. In contrast, Proposition 9 suggests that in the presence of strong substitutabilities between the intermediate inputs with nonconvex technologies there is a tendency to stagger production. Overall, whether aggregation over multiple inputs smooths out the bunching of production by individual sectors depends critically on whether the sectoral linkages involve complements or substitutes.

A few additional assumptions are made in Propositions 8 and 9 which deserve comment. In Proposition 8 it is assumed that $C''=0$. This assumption removes any disincentive to synchronize. If $C''>0$, however, the incentive to synchronize with perfect complements still dominates so that the intermediate goods would be produced either in the same period or in adjacent periods.

Further, in both Propositions 8 and 9 it is assumed that $\delta^i > \delta^q$. This assumption is made primarily to reduce the number and complexity of cases to consider. If this assumption is reversed then rather than converting intermediate goods into final goods immediately, there would be a tendency to convert intermediate goods into final goods at the time of consumption. Otherwise, the basic message of the two propositions would be the same. That is, there would be a tendency to synchronize the production of the intermediate goods when they are strong complements and a tendency to stagger the production of the intermediate goods when they are strong substitutes.

All of the results in this section pertain to a centralized environment in which the decisions on inputs are coordinated by a single agent. This is appropriate in a number of settings. First, one might argue that our results characterize the decisions of large integrated firms in which the different input sources are the various subsidiaries of these corporations. Equivalently, integrated firms can be reinterpreted as individual firms held together by explicit binding contracts with regard to their production activities. In that case, the optimization problems investigated in this section would yield the solutions to the contracting problem between these agents assuming that income effects are insignificant and ignoring any informational asymmetries. Finally, one might view this section as pertaining to a planned economy. As described, for example by Ickes [1986], planned economies do undergo fluctuations in economic activities which might be associated with non-convexities smoothed by the holding of inventories. Ericson [1983] presents a stochastic model of bottlenecks leading to fluctuations in planned economies which stresses the role of factor complementarities and inventories of intermediate goods.

Because of the non-convexities in the technology, we cannot argue that the allocations characterized in this section can be supported as competitive equilibria. For that reason, we now consider decentralized allocations with non-convex production functions.

IV. Factor Demand Linkages: Decentralized Solution

Consider an economy in which two intermediate producers have a non-convex technology and sell to a final goods producer which has a convex technology. We distinguish two important cases with regard to the interaction between the intermediate goods producers: strategic complements and strategic substitutes. These terms refer to the nature of the strategic interaction between players in the game. If there are two players each choosing a single dimensional strategy variable, then strategic complementarity (substitutability) implies that reaction curves are upward (downward) sloping. The main result reported below is that in the presence of strategic complementarities, agents will choose to synchronize production so that smoothing by aggregation will not arise and the non-convexities at the individual level will have aggregate implications.

Since strategic substitutability arises when many firms are producing inputs which are perfect substitutes, our analysis yields an interesting connection between market structure and fluctuations from non-convexities. The fluctuations induced by the non-convexities are more likely to be important at the aggregate level when markets are thin.

To understand this result, consider a game played by two agents, indexed by i=1,2, who live forever. Player 1's payoffs for period t are given by $\pi^1(y(t),z(t))$ where y(t) (z(t)) is agent 1's (2's) output or sales in period t. Player 2's preferences are defined analogously. Suppose that in this economy, agent i receives an endowment of good i in period t and then these endowments are traded thus generating, as a reduced form, the payoffs described by $\pi^i(\cdot)$. Agents are assumed to discount the future at rate β. Further, we assume that goods are not storable.[18]

We use this game to mimic the economy with non-convexities in technology while abstracting away from the choice of the frequency of production. In particular, suppose that the endowment process fluctuates so that each agent receives a high endowment in one period (H) followed by a low endowment in the next, (L). The game is extended to the choice of the frequency of production runs below.

The agents play a game of timing in which they choose whether to have their period of high endowment in even or odd periods. This is a simple device for modelling decisions to stagger or synchronize. To maintain symmet between these choices, Nature then flips a fair coin to determine whether the first period will be even or odd. If the Nash equilibrium entails both players receiving the high endowment in even or odd periods, then we term this a synchronized equilibrium. If the one player chooses a high endowment in even (odd) periods and his opponent chooses the high endowment in odd (even) periods, then a staggered equilibrium results.

This game is similar in some respects to that described by Maskin-Tirole [1988]. In that paper, agents were forced to commit to two period production plans. The moves in the game were staggered by assumption though Maskin-Tirole argue (see their Section 4) that staggering is the equilibrium of the appropriate timing game. Here we show

Proposition 10: If $\pi_{12} > 0$ ($\pi_{12}<0$), then the players will wish to synchronize (stagger) their endowment sequence.

The intuition behind this result is straightforward. Strategic complementarities ($\pi_{12}>0$) imply that each agent prefers to have a large value of their strategy variable when the other does as well: i.e. the marginal payoff from a high endowment increases with the quantity endowed to the other agent. So, each agent prefers to receive H when the other does. In contrast, strategic substitutability implies that the marginal gain from high endowment is lower when the other agent has high endowment as well. Thus, the equilibrium is to stagger in this case.

Interpreting the results of Proposition 10 requires a discussion of the conditions under which strategic complementarities may arise and an argument relating the exchange economy with fluctuating endowments to a production economy with a non-convex technology. We address these points in turn.

Cooper-John [1988] argues that strategic complementarities give rise to many interesting macroeconomic phenomena in static models including: the possibility of multiple Pareto-ranked equilibria and multiplier effects from shocks to the economy. Examples of these complementarities appear in the models of trading externalities of Diamond [1982] and Howitt [1985], the model of technological interactions by Bryant [1983] and models of imperfect competition by Hart [1982], Weitzman [1982] and many others.

For the game between two suppliers of inputs who sell to a price taking producer of final goods, whether or not the inputs are strategic substitutes or complements in the Cournot-Nash game between the two suppliers will depend on the structure of the technology.[19] For example, when the two inputs are perfect complements, as in Bryant [1983], then sychronization will be an equilibrium while staggering will arise if the inputs are perfect substitutes.

Proposition 10 thus provides an interesting extension of strategic complementarities to a dynamic setting. The main point is that if these complementarities are present, then agents will wish to synchronize their periods of high endowment and aggregate fluctuations will result. Thus smoothing by aggregation does not arise in the presence of complementarities. In contrast, strategic substitutes gives rise to staggering. Since, the leading example of strategic substitutes is that of firms producing identical products with Cournot-Nash interaction in product markets, Proposition 10 leads to the conjecture that the presence of fluctuations in non-convex economies may be related to market structure.

Proposition 10 concerns timing in a model with fluctuating endowments while our ultimate interest is in the timing of discrete production activities. We now turn attention to a model in which agents choose the timing of production runs. As in Proposition 10, the timing of production runs will be shown to depend on the nature of strategic interaction between the producers.

Assume there is a competitive final goods producer who purchases inputs from two suppliers and produces with a technology of $y = \min(z_1, z_2)$. The final good can be stored without depreciation. Let w_i be the price of input i in terms of the final good.

Both input suppliers produce using a non-convex technology: it costs K units of labor time to produce z units of the input. At this stage, assume that inputs cannot be stored. The period t utility of input suppliers is $u(c_t)$ where $u(\cdot)$ is strictly increasing and is strictly concave and c_t is consumption in period t. Lifetimes are infinite and utility is the sum of period utilities, i.e. there is no discounting. As in the Proposition 4 of Section II, allocations are evaluated by average utilities.

In each period, input suppliers simultaneously set their input prices, w_i.[20] The final goods producer takes these prices as given and determines its input demands. Sales of the input equal the minimum of the amount demanded and the amount that the suppliers have available. The input suppliers store the final goods they receive and consume optimally. Given that the final good does not depreciate and there is no discounting, the input suppliers will completely smooth their consumption. As a consequence, these agents will have no incentive to trade intertemporally.

To characterize an equilibrium, first note that since inputs cannot be held in inventory, trade will only occur in periods of production by <u>both</u> input suppliers. Thus there is clearly a strong incentive to <u>synchronize</u> periods of production of intermediate goods in this economy.

To understand the equilibrium frequency of production runs, consider first the cooperative solution in which the input suppliers jointly determine the frequency of production. This optimization problem, assuming that the input suppliers are treated identically and the output supplier has zero consumption, is the same as that solved in Section II of this paper with $Q=z/2$. Therefore, in the case of no discounting and no depreciation, Proposition 4 characterizes the optimal frequency of production, denoted by T^c.

Proposition 11: There exists a Nash equilibrium in which production of both inputs occurs every T^c periods, $w_i=1/2$ for $i=1,2$ in periods of production and $c_t=z/2T^c$ in all periods for both input producers.

Thus we see that the cooperative choice of the frequency of production can be sustained as a Nash equilibrium in which the production of the intermediate goods is synchronized. This result extends Proposition 8 to a non-cooperative setting. There are other equilibria in this game.[21]

Proposition 12: For $\lambda>1$, producing every λT^c periods is a Nash equilibrium in which $w_i=1/2$ for $i=1,2$ and $c_t=z/2\lambda T^c$ in all periods for both producers.

In contrast, there are no equilibria in which production occurs every λT^c periods where $\lambda<1$. Players could always do better by producing less frequently -- i.e. every T^c periods as in the joint optimization problem.

Thus we find that there are multiple equilibria in this games of the timing of production runs. As in the related paper of Murphy et. al. [1989], it is possible for the producers to become "stuck" at an inefficient equilibrium in which the time between production runs is too large. This is a type of coordination failure as the equilibrium with infrequent production is Pareto-dominated by one in which production occurs every T^c periods. Note that there is no limit to the span between production runs so that one equilibrium is never to produce.[22]

This model is extreme in a couple of important ways. First, the inputs are perfect complements. Suppose, in contrast, they were perfect substitutes so that the technology for producing the final good was $f(z_1+z_2)$ with $f'()>0$ and $f''()<0$. In this case, staggering of production runs is an equilibrium. Along the equilibrium path, the input suppliers exert full monopoly power in periods of production and then consume from their inventory of final goods. The alternative of producing in the same period is not more profitable since the producers of perfect substitutes would then compete leading each to earn lower profits.[23] This extends the results of Proposition 9 to a non-cooperative setting.

It is interesting to conjecture the implications of adding more firms to the model in which there are two inputs which are perfect complements. That is, suppose that there are two producers of z_1 and two producers of z_2 and that these inputs are perfect complements in the production of y. One equilibrium is for the four producers to split into two pairs with one producer of each input in each pair. Each pair might then act as described by Proposition 11 -- producing every T^c periods, charging $w_i=1/2$ for $i=1,2$ and consuming smoothly out of inventory. The pairs will produce in different periods to avoid competition between producers of identical products. Thus we see that adding more firms will reduce the amount of fluctuations in this economy -- as we add more and more firms, more and more pairs will be created and these pairs will wish to produce in different periods.[24] Eventually, the time slots will be filled so that no fluctuations will be observed. In this sense, there is a relationship between market structure and the amount of fluctuations this economy can exhibit.

Second, inputs depreciate completely in the example above. If inputs did not depreciate at all, then the complementarity between the sales of the inputs would have no implications for the timing of their production. However, if there is ε depreciation, then production will be synchronized since it is costly to the producers to carry inventories between periods given that final goods are completely storable. This extends the result reported in Proposition 8.

Finally, we could also relax the assumption that the final good does not depreciate. As long as the input are perfect complements, the equilibrium will still be for the producers to synchronize production and store their goods. Borrowing and lending will not be relevant in this case. However, if the goods are perfect substitutes, then borrowing and lending may arise to enable the input sellers to further smooth consumption.

V. Conclusion

The point of this paper has been to investigate the aggregate implications of production bunching at the microeconomic level. The second section of the paper focused on the optimal allocation of a single producer with a non-convex technology. The next two sections extended those results to focus on the centralized and decentralized allocations with multiple input suppliers. Our main results concern environments in which the non-convexity at the firm level is not smoothed by aggregation. This occurs when inputs are sufficiently complementary in the production process. Further, we found that competitive economies -- with many firms producing closely substitutable inputs -- are more likely to display smoothing by aggregation than are imperfectly competitive economies.

One extension of our analysis will be an investigation of final goods linkages. We anticipate that results similar to those described in Section IV will hold in that environment as well: strategic complementarities will imply the synchronization of production activities. This will arise either due to the "thick markets externalities" described by Diamond and Howitt or by the demand linkages across sectors stressed in the literature on imperfect competition and strategic complementarities.[25]

Further, we plan to link our results more closely with evidence on aggregate fluctuations. From this paper, we see that complementarities generate the temporal synchronization of activities but the frequency of these activities has been left unspecified. This frequency is determined by the relative sizes of Q and K which are exogenous in our analysis. Under what conditions will the fluctuations generated by non-convex technologies replicate the aggregate fluctuations observed in most economies?

In order to explain observed aggregate fluctuations it is likely that we must depart from the shockless environment characterized in the formal analysis. Allowing for shocks to, for example, tastes, technology, and resources in the context of the class of models we have considered clearly deserves further attention. The results we have derived thus far should point the way for understanding the likely consequences of such shocks.[26] In the face of nonconvexities, economic agents have incentives to bunch their respective individual responses to shocks. Whether agents will synchronize or stagger their responses presumably depends on the presence of strategic complementarities in much the same manner that we have characterized such synchronization decisions in this paper.

A related point concerns evidence on inventories. As noted earlier, there are two leading explanations for the observation that production is more volatile than consumption. One, explored here, is that non-convexities are important at the firm level. Second, is the importance of cost shocks to the economy. While empirical work has already begun to determine the relative strengths of these approaches (e.g. Ramey [1987] and Eichenbaum [1988]), it is useful to contrast the theoretical model proposed in Section II with a model of cost shocks.

Suppose that Robinson Crusoe produces with a convex technology in which output is given by $y_t = f(n_t, \theta_t)$ where $f_n > 0, f_{nn} < 0, f_\theta > 0$ and $f_{n\theta} > 0$. Interpret θ as a technology parameter. If Robinson Crusoe has preferences given by

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - g(n_t)]$$

and goods can be held in inventory with a depreciation rate of δ, then Robinson Crusoe's optimal allocation satisfies

$$u'(c_t) = \beta(1-\delta)u'(c_{t+1}) \text{ , for } I_t > 0 \quad \text{and}$$

$$u'(c_t)f_n(n_t,\theta_t) = g'(n_t) \qquad \text{for all t.}$$

From these conditions, we know that a temporary positive technology shock will induce: employment to increase (assuming the substitution effect dominates), consumption and savings to increase and future employment to fall. The point is to contrast these implications with those derived in Section II. Are there ways to discriminate between models with cost shocks and those with increasing returns to scale? At the level of an individual firm, the answer must be no. One way to generate the technology given by (1.c) is through variations in the costs of production which duplicate that specification. An open question is whether or not one can distinguish cost shock models from those with non-convexities in technology by looking at sectoral comovements of output and employment.

One means of distinguishing the models is to investigate time series in which fluctuations occur due to taste rather than cost shocks. Since the cost shock model assumes that the technology is convex, it predicts production smoothing when fluctuations are induced by changes in tastes. In contrast, the model with the non-convex technology model predicts production bunching and, we would conjecture, that the period of production would correspond to times in which the marginal utility of consumption is high.

Perhaps, the data on seasonal fluctuations reported by Barsky-Miron [1988] provide such an opportunity since the large increase in fourth quarter activity may be attributed to a demand shock. They argue that production smoothing is not revealed in the data since the fourth quarter is also a time of high production. We plan to investigate this further, by augmenting our model to include demand variations and explore more fully the different implications of the cost shock and production bunching models.[27]

Finally, we can consider other applications of the theme of this paper: the link between strategic complements (substitutes) and the synchronization (staggering) of discrete activities. One important application might concern the timing of price changes as discussed by Blanchard [1987]. We plan to think more formally about the relationship between our results and those on the choice between synchronized and staggered price changes.

Appendix

Proof of Lemma 1 and Lemma 2:

Directly from the first-order conditions. QED.

Proof of Proposition 1:

If inventories are drawn down just prior to the period of production, then consumption for all τ is given by Lemma 1. The pattern of inventories will then follow (1.a) subject to the condition that $I_\tau=0$ for $\tau=T$. These two sets of conditions determine a unique consumption path.

To see that this path of inventories is optimal, consider the alternatives. First, the agent could exhaust inventories in a period prior to $\tau=T$. However, since marginal utility goes to infinity as consumption goes to zero, this violates (2) so that (1.b) will never bind for $\tau<T$. The consumption pattern between the periods of production will therefore be given by (2).

Second, the agent could hold positive levels of inventories over production periods. In this case, there are two possibilities: either inventories reach zero at some point in time or are always positive. For the first of these sub-cases, suppose that production occurs with zero inventories in period t" and let t' denote that last time that a production run occurred with inventories equal to zero in the preceding period, i.e. $I_t=0$ for $t=t'-1$ and $t=t''-1$. By the hypothesis of this case, production occurs with positive inventories at least once between periods t' and t" so that $I_t>0$ for $t=t'+T-1$. Consumption between period t' and t" is given by (2) so that it never increases. In particular, note that consumption over the T periods preceding t" was smaller for each value of τ than consumption was, for each τ, in the T periods after t'. Yet inventories at the start of the production run preceding t" were positive ($I_t>0$ for $t=t"-T-1$) and thus exceeded the stock of inventories held at the end of period t'-1. This is a contradiction as it is not possible to consume less in each of T periods while $I_t>0$ for $t=t'+T$ and $I_t=0$ for $t=t"-1$.

Alternatively, if inventories were always positive, (2) implies that consumption would be falling through time and I_τ for $\tau=T$ increasing. This allocation is clearly dominated by a solution in which the stock of inventories is consumed at some point.

QED.

Proof of Proposition 2:

Suppose that the solution to (1) entails production in periods a,b,c,.... If inventories were equal to zero each time production occurred, then, by optimality, production must occur every T periods, i.e. b=a+T,c=b+T, since the state variable takes on the same value in each period of production.

If, alternatively, inventories never reached zero, then consumption would be monotonically falling. As before, the stock of inventories held just before a production run would be increasing over time which is sub-optimal. This then rules out any solution of erratic production in which inventories are always positive.

The final possibility is a periodic solution in which production occurs for some time without running inventories to zero and then, eventually, production occurs with zero inventories and the process repeats itself. In this solution, inventories would take on the same value at two distinct points in time with the path from those points onward not coinciding.[28] One can then construct a path giving equal utility in which production occurs only with positive inventories. Again, this violates optimality by the argument given above.

QED.

Proof of Proposition 3:

For two distinct values of K, $K_1 > K_2$ denote the corresponding optimal periods between production by T^*_1 and T^*_2. Optimality implies that when the cost of a production run is K_1 (K_2) the agent prefers to produce every T^*_1 (T^*_2) rather than every T^*_2 (T^*_1) periods. That is,

$$\frac{W(T_1^*) - K_1}{1 - \beta^{T_1}} \geq \frac{W(T_2^*) - K_1}{1 - \beta^{T_2}} \quad \text{and}$$

$$\frac{W(T_2^*) - K_2}{1 - \beta^{T_2}} \geq \frac{W(T_1^*) - K_2}{1 - \beta^{T_1}} .$$

For these two inequalities to hold, given that $K_1 > K_2$, it must be the case that

$$\frac{1}{1 - \beta^{T_1}} \leq \frac{1}{1 - \beta^{T_2}} \quad \text{which implies} \quad T_1^* \geq T_2^*.$$

QED.

Proof of Proposition 4:

For T^* to be optimal, it must be the case that this periodicity dominates producing either every $T^*+\lambda$ periods where λ is some finite integer. To compare these alternative periodicities, we consider the utility differential over $T^*(T^*+\lambda)$ periods.[29] Thus if T^* is optimal it must be the case that

$$\text{(A1)}\quad (T^*+\lambda)T^*[u(Q/T^*) - u(Q/T^*+\lambda)] \geq \lambda K.$$

Condition (A1) says that over $T^*(T^*+\lambda)$ periods, the agent prefers to produce every T^* rather than $T^*+\lambda$ periods. That is, the extra utility from the additional λQ units, split optimally over time, is not less than the cost of the extra λ production runs, λK.

The left side of (A1) can be rewritten as

$$\text{(A2)}\quad \lambda \left[\frac{u(Q/T^*) - u(Q/T^*+\lambda)}{(1/T^*) - (1/T^*+\lambda)} \right]$$

By strict concavity of $u(\cdot)$, this is strictly greater than $Qu'(Q/T^*)$ which, by hypothesis equals K. Thus $u'(Q/T^*)=K/Q$ implies that (A1) holds for all λ.

QED.

<u>Proof of Proposition 5</u>:

Given the separability of preferences in (4), the choice of the timing of the production of the intermediate goods can be reduced to the following cost minimization problem:

$$\text{(A3)} \qquad \min_{(n^y_\tau),\,(n^z_\tau)} \sum_{\tau=1}^{T*} \beta^\tau C(n_\tau)\}, \qquad \beta < 1$$

subject to:

$$\text{(A3.a)} \qquad k\left(\sum_{\tau=1}^{T^*} [(1-\delta^y)^{T*-\tau} g(n^y_\tau)],\ \sum_{\tau=1}^{T^*} [(1-\delta^z)^{T*-\tau} h(n^z_\tau)]\right) \geq K$$

$$\text{(A3.b)} \qquad n_\tau = n^y_\tau + n^z_\tau \leq N.$$

(i) Consider $\delta^y = 1$. This implies that inventories of intermediate good y will not be held so it is only optimal to produce y when the final good is produced. The same logic works for intermediate good z.

(ii) In this case, the relevant optimality conditions from (A3) for n^y_τ and n^z_τ are given by:

$$\text{(A4)} \qquad C'(n^y_\tau + n^z_\tau)/g'(n^y_\tau) = \beta(1-\delta^y)C'(n^y_{\tau+1} + n^z_{\tau+1})/g'(n^y_{\tau+1})$$

$$\text{(A4)}' \qquad C'(n^y_\tau + n^z_\tau)/h'(n^z_\tau) = \beta(1-\delta^z)C'(n^y_{\tau+1} + n^z_{\tau+1})/h'(n^z_{\tau+1})$$

Since $g'' \leq 0$, $h'' \leq 0$ and $C'' > 0$, C'/g' is an increasing function of n^y_τ in (A4) and C'/h' is increasing in n^z_τ in (A4)'. Given that $\beta < 1$ and $\delta^z = \delta^y < 1$, this implies with (A4) and (A4)' that n^y_τ and n^z_τ are increasing in τ and thus y_τ and z_τ are increasing in τ. Note, further, that (A4) implies that the rate of increase in n^y_τ is an increasing function of δ^y and a decreasing function of β. The same logic applies for the rate of increase in n^z_τ from (A4)'.

QED.

Proof of Proposition 6:

(i) Since neither intermediate good can be stored and the intermediate goods are perfect complements, z is produced only when y is produced. This in turn implies that q is produced only when y is produced.

(ii) Since the intermediate goods are perfect complements and y cannot be stored, q is produced only when y is produced. However, since z can be stored and C">0 and h"≤0, there is an incentive to smooth the production of z. The relevant optimality conditions from (4) for n^z_τ in this case are given by:

$$\text{(A5)} \qquad C'(n^z_\tau+n^y_\tau)/h'(n^z_\tau) = \beta(1-\delta^z)C'(n^z_{\tau+1}+n^y_{\tau+1})/h'(n^z_{\tau+1})$$

$$\text{(A6)} \qquad \sum_{\tau=1}^{T^*} (1-\delta^z)^{T^*-\tau}h(n^z_\tau) = Q$$

Here n^y_τ equals zero except at $\tau=T^*$. From (A6), observe that the agent will accumulate just enough inventories of z in order to produce Q units of q every T* periods. Given discounting and depreciation, since C'/h' is an increasing function of n^y_τ, (A5) implies that it will be optimal to produce z most just prior to periods of production of y.

(iii) Since inventories of y can be held but inventories of z cannot, the agent has an incentive to smooth the production of z through holding inventories of y. However, if $\delta^y > \delta^q$, the agent may prefer to convert the intermediate goods immediately to final goods in order to take advantage of the less expensive storage technology. Thus, there are two subcases here. First, if $\delta^y > \delta^q$ and the agent finds it optimal to convert to final goods immediately then q and z are synchronized with the production of y. Hence, in this subcase, q and z are trivially non-increasing in τ, for $\tau<T^*$.

Second, for sufficiently low δ^y, the agent will smooth the production of z through holding inventories of y. The relevant optimality conditions from (4) for n^z_τ are given by:

$$\text{(A7)} \qquad U'(h(n^z_\tau)) - C'(n^z_\tau + n^y_\tau)/h'(n^z_\tau) = \beta(1-\delta^y)[U'(h(n^z_{\tau+1}) - C'(n^z_{\tau+1} + n^y_{\tau+1})/h'(n^z_{\tau+1})]$$

$$\text{(A8)} \qquad \sum_{\tau=0}^{T^*-1} [h(n^z_\tau)/(1-\delta^y)^\tau] = Q$$

Note again that $n^y_\tau = 0$ except at $\tau = 0$ when y is produced. In this case, following each production run of y, the agent is producing z (and thus q) in a manner consistent with (A7) and exhausting the inventories of y just prior to the next production run. Since $U'(h(\cdot)) - C'(\cdot)/h'(\cdot)$ is a decreasing function of n^z_τ and given discounting and depreciation, (A7) implies that z and q will be decreasing in τ, for $\tau > 0$. Hence, in both subcases, z and q are non-increasing in τ, for $0 < \tau < T^*$.

QED.

Proof of Proposition 7:

(i) Given the linear final goods production technology, if the agent holds inventories then the agent will hold inventories of final goods if $\delta^q < \delta^y$ and inventories of the intermediate goods otherwise. Consider the case $\delta^q < \delta^y$. In this case, from (4) the time path of consumption of the final good and the production of z is governed by:

$$\text{(A9)} \qquad U'(c_\tau) = \beta(1-\delta^q)U'(c_{\tau+1}), \quad \text{if } I^q_\tau > 0$$

$$\text{(A10)} \qquad U'(c_\tau) = C'(n^z_\tau + n^y_\tau)/h'(n^z_\tau)$$

From (A9), as long as inventories are being held and given depreciation and discounting, consumption will be decreasing over time. Since positive inventories will be held at the end of periods of production of y, $n^y_\tau = K$ in periods of production runs and zero in other periods and given that C'/h' is an increasing function of n^z_τ, this implies with (A10) that the minimum production of z will be in periods of production of y. This also implies that as long as positive inventories are being held, z will be strictly increasing in τ. If inventories are exhausted prior to the next production run of y, then production and consumption of the final good will be equal to z and in this case (A10) implies a constant production of z. Thus, if inventories are exhausted, then z will be non-decreasing in τ, for $\tau < T^*$. The case of $\delta^y < \delta^q$ follows along similar lines.

(ii) The production path of y is Q every T* periods with zero production of y between production runs. The production path of z is characterized by minimal production of z during production periods of y. These two production paths imply a negative covariance between the production of y and z. Given the linear final goods technology, this implies var(q) < var(y) + var(z).

QED.

<u>Proof of Proposition 8</u>:

Since $\delta^i > \delta^q$, the agent will immediately convert available intermediate goods to final goods. Given the final goods production technology, this means that positive inventories of both intermediate goods will never be held contemporaneously. However, if the production of y and z are not synchronized, then positive inventories of one of the intermediate goods may be held until the next production run of the other intermediate good. Let λ

be the interval between the production of y and the production of z if the intermediate goods are not synchronized. Accordingly, let $T^*-\lambda$ be the interval between the production of z and the production of y. Further, without loss of generality, let $\lambda < T^*-\lambda$. In this environment this implies that if y and z are not synchronized then in the steady-state in periods of production of y, the agent will produce Q units of y and

$$\text{(A11)} \quad Q^y = Q[(1-\delta^i)^{T^*-\lambda} - (1-\delta^i)^{T^*}]/[1-(1-\delta^i)^{T^*}]$$

units of the final good. The remainder, $Q-Q^y$, will be held in inventory of the intermediate good until the next production of z. Similarly, in periods of production of z, the agent will produce Q units of z and

$$\text{(A12)} \quad Q^z = Q[(1-\delta^i)^{\lambda} - (1-\delta^i)^{T^*}]/[1-(1-\delta^i)^{T^*}]$$

units of the final good. The remainder will be held in inventory of the intermediate good until the next production of y. Observe that since $\lambda < T^*-\lambda$, $Q^z > Q^y$. More importantly, (A11) and (A12) imply that $Q > Q^z + Q^y$ for all strictly positive λ. Consider the implications of this inequality for any interval of length T^* beginning with a period in which both the final good and intermediate good z is produced. If y and z are synchronized then the production run of the final good is the amount Q in the first period of this interval. If y and z are not synchronized then the final good is produced in the amount Q^z in the first period of the interval and in the amount Q^y, $T^*-\lambda$ periods later within the interval. Since $Q > Q^z + Q^y$ and $\delta^i > \delta^q$, over any such interval synchronization leads to a greater available production of the final good. The key is that staggering leads to lost potential final goods production through the depreciation of the intermediate goods between intervals of the production of the intermediate goods.

Staggering not only leads to less production but also to higher costs of production. To see this, consider the present discounted cost of staggered production versus synchronized production. Under synchronized production, the present discounted cost from time 0 is given by (recall C"=0):

$$\text{(A13)} \qquad 2C(K) + 2C(K)\beta^{T*}/(1-\beta^{T*})$$

Under staggered production, the present discounted cost from time 0 is given by:[30]

$$\text{(A14)} \qquad 2C(K) + C(K)(1+\beta^{-\lambda})\beta^{T*}/(1-\beta^{T^*})$$

Given initial inventories are zero, joint production in time 0 necessarily occurs and this is reflected in the first term 2C(K) in both (A13) and (A14). After time 0, however, (A13) and (A14) indicate that staggered production implies higher costs. This is because, for example, in order to produce final goods in period T* with staggering, intermediate good y must have been produced λ periods earlier. This production of one the intermediate goods prior to its use in the production of the final good is what generates the higher cost under staggering.

Overall, then, staggering implies fewer goods, produced later and at higher cost which implies staggering is suboptimal. This implies the perfect synchronization of y, z and q.

QED.

Proof of Proposition 9:

Given the linear final goods production technology and $\delta^i > \delta^q$, the agent will immediately convert any intermediate goods into final goods. To establish (i) and (ii) we rule out alternative subcases successively. First, consider the possibility that the agent perfectly synchronizes y, z, and thus q. This is dominated by (i) and (ii) since the latter takes advantage of consumption smoothing and production smoothing incentives given u" < 0 and C" > 0. Alternatively, suppose the agent staggered the production of y and z but at unequal intervals. This is ruled out by the same arguments used in Proposition 2 since in this environment the two models are formally equivalent.

QED.

Proof of Proposition 10:

Suppose that both players synchronize their endowments, then the lifetime expected (because of nature's coin flip) discounted utility is given by

$$V^{sy} = \frac{1}{2(1-\beta)} \left[\pi(H\ ,H\ ) + \pi(L\ ,L\ ) \right].$$

Alternatively, if the players stagger, then the lifetime expected discounted utility for each is given by

$$V^{st} = \frac{1}{2(1-\beta)} \left[\pi(H\ ,L\ ) \ + \ \pi(L\ ,H\ ) \right]$$

The sign of $V^{sy}-V^{st}=\Delta$ is given by

$$\int_L^H \int_L^H \pi_{12}\ (x,z)\ dxdz\ .$$

Hence if $\pi_{12} > 0$ ($\pi_{12}<0$), $\Delta >$ (<) 0 and the players will synchronize (stagger) periods of high endowments.

QED.

Proof of Proposition 11:

For periods in which both inputs suppliers produce, the symmetric Nash equilibrium will be for each of the input suppliers to charge a price of 1/2. Because these agents do not discount and have strictly concave utility functions, consumption equals $z/2T^c$ in each period if they produce every T^c periods.

To see that producing every T^c periods is optimal, consider the alternatives open to one of the suppliers. An input producer has no incentive to produce more frequently since the final goods producer has no value for the inputs in periods where the other producer is inactive. Further, producing less frequently would imply that the input supplier would receive z/2 only in periods in which both suppliers are producing. This cannot be a profitable deviation since T^c is the jointly optimal timing between production runs. That is, if it was optimal for one producer to produce every λT^c periods where

$\lambda>1$, then that frequency of production would have solved the joint optimization problem.

QED.

<u>Proof of Proposition 12</u>:

As in the proof of Proposition 11, given that production occurs every λT^c periods, the input prices and consumption path follows immediately. Neither producer has an incentive to produce more freqently since, as before, production in other periods has no value and also has no incentive to produce less frequently since each production run yields positive utility.

QED.

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Footnotes

1. These include models with serially correlated demand shocks and/or stockout avoidance (e.g. Kahn [1987]) and cost shocks (e.g. Eichenbaum [1988]) as well as the models with non-convex technologies described further below.

2. Bertsekas [1976] provides a lengthy discussion of the (S,s) literature while Blinder [1981] and Caplin [1985] discuss some macroeconomic implications. Ramey [1987] presents a theoretical and empirical analysis of a firm producing in a range of decreasing marginal cost.

3. See the discussion in Blinder [1986].

4. In fact, the literature on non-convexities in general equilibrium models rests on smoothing by aggregation over a large number of agents.

5. That is, we expand the single agent model to allow for multiple inputs where the supply of these inputs is determined as the solution to a programming problem. Interpretations of this approach are provided at the end of Section III.

6. These terms and their macroeconomic implications are discussed in Cooper-John [1988] and Haltiwanger-Waldman [1988]. As discussed in Section IV, strategic complementarities (substitutes) implies that reaction curves in a game with a scaler strategic variable are upward (downward) sloping.

7. In a closely related paper, Murphy, Shleifer and Vishny [1989] stress the importance of demand linkages in a non-convex economy with durable goods. Cooper-Haltiwanger [1988] contains a discussion of final demand linkages in an economy with production runs and non-durable goods. In this paper, we have chosen to focus on factor demand linkages across sectors as stressed in the work of Long-Plosser [1983].

8. An alternative approach, not adopted here, would be to build the non-convexity into the preferences of the agent. Here the non-convexity in $C(\cdot)$ is derived from the technology.

9. By the term binding, we mean that the multipler on the constraint is not equal to zero.

10. Note that our problem includes it in the optimal choice of a consumption stream as well as the sub-problem of minimizing the cost of the stream. The (S,s) literature generally focuses on the cost minimization problem alone. Also, our problem lacks some of the stationarity of the traditional (S,s) problem in that our production lot is fixed at Q rather than the stock, after production, equalling S. It appears to be rather straightforward to reproduce many of the results in this section using a technology in which there is a positive marginal cost of production and no capacity.

11. It is easy to show that over T(T+1) periods one would prefer to have T+1 rather than T "cakes" of size Q.

12. Since goods do not depreciate and there is no discounting, the agent can produce in a very irregular pattern and simply hold goods as desired over time.

13. This is the condition used in many models of labor supply to guarantee that increases in the real wage induces greater labor supply.

14. In what follows, when we state that production of a good occurs every T* periods we are referring to the steady-state interval between production runs. Given the assumption of zero initial inventories, this does not become an issue until we consider the environments specified in Propositions 8 and 9.

15. Here $\tau=1$ is the period just after a production run in contrast to the notation used in Section II. Note that at $\tau=T^*$, another production run begins if production occurs every T^* periods.

16. Though the variance of production will not necessarily exceed the variance of sales for the manufacturing sector, production in that sector is much more volatile because of the non-convexity in the retail sector.

17. Since there are two intermediate goods, it may be that production of the final good is possible without the input of one of the intermediate goods. This possibility is particularly applicable for Proposition 7 which involves perfect substitutes. In what follows, we are implicitly assuming that the combination of K and Q are such that the agent finds it optimal to produce y every T* periods.

18. These results extend to the case in which goods can be held in inventory as long as some depreciation occurs as explained at the end of this section.

19. If f(y,z) is the final goods production function where y and z are the inputs, then the supplier of y will increase output when z increases iff $f_{yyz}(y,z)y + f_{yz}(y,z)$ is positive. This is equivalent to the requirement that marginal revenue associated with the production of y increases when z increases. There is an analogous condition for the supplier of z.

20. We consider the game in which prices are the strategy variable to avoid the multiplicity of equilibria that may arise in static games in which quantities are the choice variables, as in Bryant [1983]. This allows us to focus on the multiple equilibria that arise due to timing considerations across periods.

21. Our consideration of the possibility of mulitiple equilibria in this model was motivated by the findings of Murphy, Shleifer and Vishny [1989] of multiple equilibria in a model with durable goods.

22. This conclusion can be avoided by assuming that at some finite cost, input suppliers can produce the final good and that this yields positive utility.

23. The easiest way to see this is to note that even if the two producers could cooperatively set input prices, they could not earn more profits per firm than in the staggered solution due to the congestion effects of $f''()<0$. Given that they act non-cooperatively in periods where they both produce, the defection to producing in the same period cannot be more profitable than the equilibrium with staggering.

24. However, there may exist other equilibria we have not analyzed.

25. In Cooper-Haltiwanger [1989] we address related dynamic implications of strategic complementarities generated by final goods demand linkages. In particular, we demonstrated that strategic complementarities generated by final goods demand linkages together with inventory holding in at least some sectors provides an explanation of the observed persistent positive comovement in employment across sectors. A primary difference with the current analysis is that in the earlier paper we focused on economies with convex technologies subject to sectoral shocks. Murphy et. al. also concentrate on demand linkages in a model with durable goods.

26. Hall [1988] conjectures that in models with non-convexities and strategic complementarities, low frequency shocks may generate high frequency fluctuations.

27. For related work in this direction see Chatterjee-Ravikumar [1988] as well as the discussion in Miron-Zeldes [1988].

28. Implicitly we are neglecting integer problems by assuming here that the time intervals are small enough that the state variable takes on the same value twice in the solution we are arguing against. This problem arises elsewhere in the literature on non-convex programming problems, as in the money demand problem described in Tobin [1956] and in the Hadley-Whitin [1963] discussion of inventories.

29. To compare average utilities over the infinite horizon, we compare two programs over a finite interval of time in which both solutions complete cycles, i.e. $T^*(T^*+\lambda)$ periods. Given the periodic nature of these programs, domination of one over a common finite interval implies domination in terms of average utility over all time.

30. The present discounted cost given in (A14) presumes that when staggering begins y is produced before z. This is optimal as producing z first and then y would both be more costly and yield a lower discounted flow of final goods output.