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ABSTRACT

We model how accommodative policy can become trapped due to credit misallocation and its spillovers, as witnessed in Japan in the 1990s and in Europe in the 2010s. Following large negative shocks, the effective lower bound prevents stimulating bank lending through rate cuts. Unconventional policies that subsidize risk-taking such as regulatory forbearance can still expand credit, but excessive accommodation induces poorly-capitalized banks to lend to low-productivity “zombie” firms. Due to persistent congestion externalities of zombie lending on healthier firms, policymakers avoiding short-term recessions can get trapped into protracted low rates, excessive forbearance, and persistent output losses.

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1 Introduction

In this paper, we build a model with heterogeneous firms and banks to analyze how monetary and banking policies affect credit allocation and long-term economic outcomes. In particular, we explain why policy may get trapped into protracted low rates and excessive regulatory forbearance towards banks that are coincident with permanent output losses.

Since the housing and banking crisis in Japan in the early 1990s, regulatory forbearance towards banks—effectively allowing severely under-capitalized or insolvent banks to continue operating by backstopping bank creditors—has been increasingly used in conjunction with accommodative monetary policy in a bid to restore economic growth in the aftermath of aggregate shocks. This policy combination also found favor in the Eurozone following the global financial crisis of 2007–08 and especially after the European sovereign debt crisis in 2010–12. In both cases, despite an operative period exceeding in length the initial intentions and expectations, this policy script's impact on economic growth remained relatively muted. Starting with Peek and Rosengren (2005) and Caballero, Hoshi and Kashyap (2008), the literature has attributed this ineffectiveness of employed policies (at least in part) to credit misallocation and, in particular, to the phenomenon of zombie lending: the provision of subsidized credit to poorly-performing firms by weakly-capitalized banks.[1]

Section 2 puts the experiences of Japan and Europe into perspective, presenting stylized facts that document the proliferation of zombie lending in the aftermath of the crisis outbreaks, the negative externalities that zombie lending generated, and its feedback loop with policy interventions. Importantly, the evidence suggests that adverse effects of credit misallocation due to the proliferation of zombie lending persist and even compound over time.

Motivated by these facts, we build a tractable model that provides a unified framework simultaneously explaining and conforming to all of them. The economy is populated by heterogeneous firms that differ in their productivity and risk. Firms' investments require credit, which is provided by banks that are themselves heterogeneous in their level of capitalization. Banks face a portfolio problem, whose solution depends on their capital: they decide whether to invest in safe assets (meant to capture a wide range of non-loan assets, such as central bank reserves or safe government bonds and mortgage-backed securities) or lend to the corporate sector, and if so, to which type of firms.

Policies play a crucial role in banks' incentives, and thereby the equilibrium allocation of credit. We summarize all the components of policy that affect bank decisions into two simple

[1] Poorly-capitalized banks supply credit to poorly-performing firms either because they are incentivized to engage in risk-shifting (gambling for resurrection) or because they attempt to avoid reporting losses on distressed positions, as documented in Giannetti and Simonov (2013), Acharya et al. (2019), Blattner et al. (2023), Gropp et al. (2020), Faria E Castro et al. (2021), Acharya et al. (2021), and Schivardi et al. (2021), among others.

instruments: the risk-free rate R^f set by conventional monetary policy, and an unconventional "forbearance policy" p that determines the level of government guarantees granted to banks that are willing to lend. Accommodative conventional monetary policy makes lending to corporates more attractive relative to the alternative, by lowering the return on the safe assets R^f. This is a standard bank lending channel. Increasing forbearance also stimulates lending, by compressing the cost of funds associated with lending: a higher p lowers the cost of funds because a larger part of the bank loan risk is borne by public authorities (the government, the central bank, or the deposit insurance agency). However, excessive forbearance can shift banks' portfolios towards riskier loans to less productive firms, which we refer to as the "zombie lending channel".

The two-sided firm-bank heterogeneity opens the door to the "diabolical sorting" documented in the data: banks with low capital and high leverage end up lending to less productive firms, even though aggregate output would be raised by letting these firms exit and be replaced by more productive entrants. The reason is that the subsidy from forbearance increases with the interaction of banks' asset risk and leverage. This sorting between banks and firms leads to a delicate policy trade-off. While zombie lending and depressed creative destruction are the main perils on the side of poorly-capitalized banks, policymakers must also encourage well-capitalized banks to lend to the good firms. Well-capitalized banks are not tempted by zombie lending, but may simply invest in safe assets. The tension between inducing well-capitalized banks to lend and preventing poorly-capitalized banks from engaging in zombie lending is at the heart of our analysis of the optimal policy mix in response to exogenous shocks.

Suppose that an aggregate productivity or demand shock hurts firms' profitability. If both zombie loans and safe assets generate less output than loans to good firms, output reaches its potential if and only if all banks lend and there is no zombie lending. As long as the risk-free rate is not constrained, conventional monetary policy alone without any forbearance can achieve this objective. Without forbearance, there is no zombie lending by weak banks, while a sufficiently low risk-free rate (i.e., the "natural interest rate" in our economy) encourages banks to lend. However, larger negative shocks to healthy firms' profitability must be accommodated by lower interest rates. Hence, if the shocks are large enough, conventional monetary policy runs into an effective lower bound on interest rates (ELB), assumed exogenously in the model. This is where unconventional policy in the form of regulatory forbearance and its unintended consequences come into play.

We show that a small amount of forbearance is always beneficial, as it can substitute for the constrained conventional monetary policy and help lower banks' funding costs, thereby stimulating lending and output. Pushing on the forbearance string, however, eventually spurs zombie lending by weak banks. Zombie lending reduces aggregate output and productivity because of the misallocation of credit and, as documented in the empirical literature, because it can generate congestion externalities in input and output markets that ultimately impair the growth prospects of healthy firms in the economy.

Empirically, zombie lending induces persistent negative spillovers on the profitability of healthy firms in future periods, which can arise from, e.g., congestion externalities in customer markets, labor markets, and capital markets. A crucial parameter that determines whether the economy falls into such a trap is the horizon of policymakers. The optimal forbearance policy p depends on how policymakers trade off current and future output. "Myopic" policymakers are willing to preserve current incumbent firms at the expense of future output.[2] To do so, they focus on maximizing bank lending, ignoring concerns about the composition of lending since congestion externalities of zombie lending on healthy firms unfold over time. Larger shocks are met with more forbearance, at the cost of more zombie lending. In stark contrast, for "patient" policymakers who seek to avoid future output losses induced by congestion externalities from zombie lending, the optimal forbearance policy is *non-monotonic* in the size of the shocks. When shocks are moderate, forbearance is a useful tool whose use should increase with the size of the shock; but in the face of large shocks, we show that patient policymakers should actually backtrack and *reduce* forbearance to avoid triggering zombie lending, even though this entails letting some banks cut lending. Thus, there exists a "reversal" level of forbearance policy, a counterpart to the "reversal interest rate" below which conventional monetary policy turns contractionary (Abadi et al., 2023).

The dynamic consequences of myopic policies can be dire. The interaction of policy-induced risk-shifting and dynamic spillovers creates the possibility of Sisyphean *policy traps* whereby current accommodative policies lead to zombie lending, whose negative spillovers beget more accommodative policies and zombie lending in future periods. Even if the initial exogenous shock is purely transitory (i.e., firms' profitability would recover after one period), accommodative policies that allow zombie lending to emerge leave future policymakers with firms whose profitability remains endogenously low due to the congestion externalities. The future optimal myopic response is then to continue accommodating the inherited weak economy by keeping interest rates low and forbearance high. This keeps zombies alive for at least another period. At the very least, this negative dynamic feedback generates endogenous persistence.

In the extreme, for large enough initial shocks, the pattern repeats itself until the economy converges to a *sclerosis* steady state, defined as featuring a permanent combination of interest rates stuck at the ELB, high forbearance, zombie lending, and low output. In our theory, forward-looking policymakers should accept a "V-shaped" (i.e., sharp but transitory) recession precisely

[2]Myopic policymakers are not naive and realize the effect of their policies on equilibrium outcomes, but are subject to capture, term limits, or reputational concerns that shorten their effective horizon of decision-making. See, for instance, Alesina and Tabellini (1990) and Boot and Thakor (1993).

when fundamental shocks are large, which is exactly the opposite of what myopic policymakers do and what is often argued in practice.[3] We discuss two ways to exit the sclerosis steady state: a bank recapitalization, which improves banks' incentives at some fiscal cost, and an improvement in productivity. Both need to be sufficiently strong, as timid interventions only have a transitory effect before the economy and the policy become trapped again.

Finally, the central role of bank capital in our analysis raises important questions: Do under-capitalized banks have an incentive to issue more equity? And if not, can regulators eliminate the zombie lending problem by simply increasing capital requirements? We address these questions in an extension of our model, allowing for costly equity issuance, legacy lending, and capital requirements. We find that the same risk-shifting incentives affecting bank lending decisions also apply to capital structure decisions, preventing under-capitalized banks from raising enough capital to avoid zombie lending. Imposing high capital requirements can then deter zombie lending if the costs of breaking the relationship with a legacy zombie borrower (e.g., recognizing losses) are low enough. However, if these costs are high, we show that zombie lending becomes inevitable, in the sense that some banks will evergreen (lend to legacy zombie firms) for *any* level of capital requirement. Furthermore, there is a zombie-minimizing level of capital requirement and going beyond this level leads to more zombie lending.

Overall, our model consistently explains a set of empirical facts connecting bank capitalization, credit misallocation, policy choices, and aggregate growth and productivity, following adverse economic shocks. In particular, it makes three important contributions.

First, it helps understand why in the face of large shocks, the policy response to restore economic growth may feature a combination of conventional policy in the form of monetary accommodation and unconventional policy in the form of regulatory forbearance towards banks. Forbearance becomes useful in our model only when the conventional policy hits an effective or zero lower bound. This is a distinctive feature of our model relative to the banking literature that models regulatory forbearance as arising from a time-inconsistency problem of regulation (Mailath and Mester, 1994).

Secondly, our model derives the empirically documented phenomenon that regulatory forbearance leads to zombie lending and a diabolical sorting, whereby low-capitalization banks extend new credit or evergreen existing loans to low-productivity firms. It is this positive implication of the model that then allows for a meaningful normative analysis of the policies affecting bank incentives to engage in such lending.

Thirdly, by examining a dynamic setting in which zombie lending imposes congestion exter-

[3] While this argument resembles some of the classic "liquidationist" views of Hayek and Schumpeter, in our framework, this conclusion is contingent on many factors such as the size of the shock, the policy space available to address the shock with conventional tools, and the state of capitalization of the banking sector when the shock hits.

nalities in the form of adverse spillovers on the profitability of healthier firms, the model explains why economies facing large, but only transitory, shocks may jointly feature thereafter (i) a phase of delayed recovery and potentially permanent output losses, which we call economic sclerosis; and, (ii) a policy trap whereby monetary accommodation and regulatory forbearance aimed at avoiding short-term recessions become entrenched even as they persistently fail to restore long-term economic health. The possibility that a transitory shock turns into permanent stagnation when policymakers favor avoiding recessions in the short run is the most salient feature of our analysis. A key policy implication is that to avoid zombie lending and associated economic sclerosis, it is important to maintain a well-capitalized banking system but also that bank capital requirements need to be raised upfront rather than upon realization of economic shocks.

Related Literature

Our model builds on the seminal contribution of Caballero et al. (2008) (henceforth CHK) and extends it in two key dimensions. First, CHK features negative spillovers generated by zombie firms due to congestion in input and output markets, but it does not explicitly model the role of credit markets and financial intermediaries, and their incentives to extend credit to low productivity firms. By contrast, the credit market, banks and their capital structures are front and center in our framework. Second, our model stresses the nexus between policy interventions, credit allocation, and aggregate outcomes. To the best of our knowledge, ours is the first theoretical treatment emphasizing the zombie lending-policy feedback loop, where macro-financial policies dynamically affect and are affected by banks' credit supply choices. In this respect, our results on economic sclerosis and policy traps also speak to the stagnation traps analyzed by Benigno and Fornaro (2018), who highlight the ineffectiveness of conventional monetary policy alone in stimulating the economy.

Also related to our model are the theoretical contributions of Bruche and Llobet (2013), Hu and Varas (2021), and Begenau et al. (2021) investigating the role of bank incentives as driver of zombie lending, with a particular emphasis on the role of hidden losses and asymmetric information. We share with these papers the emphasis on developing a micro-founded model of bank lending, although we do not directly model asymmetric information frictions. Rather, we focus on developing a tractable general equilibrium framework to study how credit allocation and policy actions shape aggregate outcomes. In this regard, Tracey (2021) also shows that excessive forbearance may ultimately play a significant role in explaining the slow economic growth observed in the aftermath of aggregate shocks.

More broadly, our paper is related to the macroeconomic literature on financial frictions and misallocation. Gopinath et al. (2017), Banerjee and Hofmann (2018), Asriyan et al. (2024) and

Jafarov and Minnella (2023) stress how a low interest rate environment and financial frictions can induce capital misallocation and aggregate losses. Our focus is on the central role played by financial intermediaries and how their actions depend on and influence macro policies. Buera et al. (2013) study how policies aimed at stimulating output can lead to long-run productivity losses. Their focus is on targeted industrial policies such as credit subsidies directly aimed at firms. In contrast, we are interested in studying stabilization policies and bank lending incentives in response to macroeconomic shocks.

Finally, motivated by the policies adopted during the COVID-19 pandemic, Crouzet and Tourre (2021) and Li and Li (2025) also highlight how government interventions can have negative long-run effects, by exacerbating debt overhang or worsening the quality distribution of firms, respectively. Li and Li (2025) shows how public liquidity support might be able to preserve a country's production capacity in the short run but also dampen creative destruction in a self-perpetuating way. Complementary to these papers, our model focuses more on the zombie lending channel operating via weakly-capitalized banks.

The remainder of the paper is organized as follows. We first present stylized empirical facts about zombie lending in Section 2 and then develop our baseline model in Section 3. We turn to the dynamics in Section 4. Section 5 presents extensions around the role of bank capital and capital requirements. Section 6 concludes with some directions for future research.

2 Zombie Lending: Incidence, Consequences, and Policy Responses

To motivate our theoretical framework, we present some stylized facts on the incidence of zombie lending and its consequences for the real economy. We draw on two historical episodes: Japan's stagnation following the real estate crisis in the early 1990s and the experiences of Southern European economies—Italy, Spain, and Portugal—following the 2010–2012 sovereign debt crisis. In these countries, large aggregate shocks triggered a prolonged slowdown in real activity alongside an eventual surge in zombie lending. Table 1 summarizes salient aggregate statistics for both episodes.

Japan's stagnation is a textbook case of the phenomenon of zombie lending and its macroeconomic implications. Real GDP and total factor productivity (TFP) growth declined sharply between the early 1990s and the 2000s, falling by 3.2 and 1.2 percentage points, respectively, relative to the period that preceded the real estate collapse. A similar pattern emerged in Italy, Spain, and Portugal, which experienced pronounced post-crisis slowdowns in GDP and TFP growth relative to Germany, our benchmark country. In both Japan and Southern Europe, the crisis coincided

Table 1: Zombie lending, aggregate outcomes, and bank capital in Japan and Southern Europe

	pp. change between 1981–92 and 1993–02	pp. change relative to Germany during 2011–16		
Change in:	*Japan*	*Italy*	*Spain*	*Portugal*
Real GDP growth	-3.20	-2.13	-1.27	-2.13
Aggregate TFP growth	-1.24	-1.41	-0.92	-1.14
Share of zombie firms	9.26	7.90	2.22	3.65
Banking system capitalization	-2.63	-3.26	-2.83	-3.53

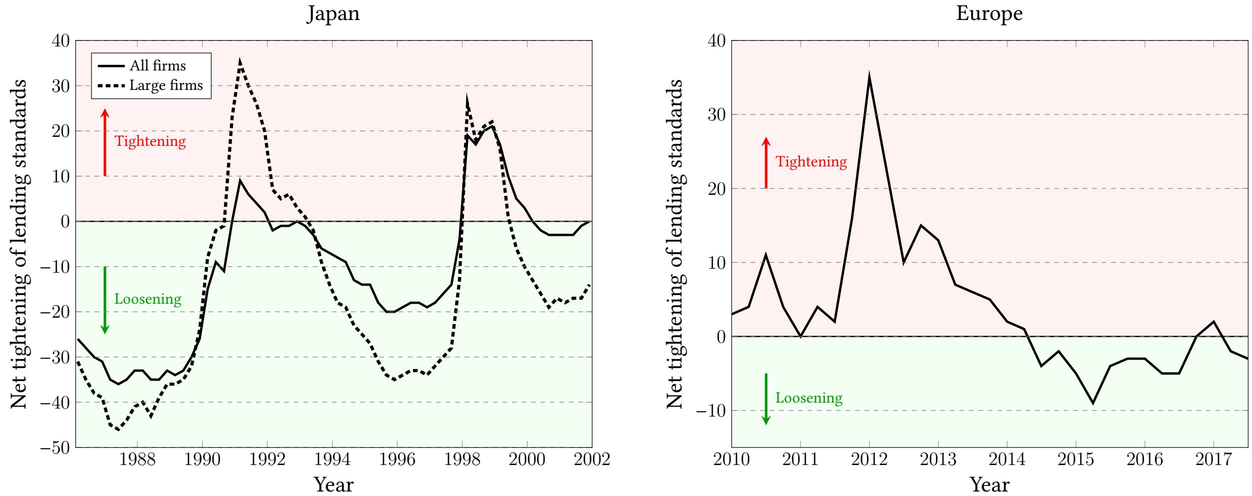
Note: This table reports aggregate statistics on changes in real GDP growth, total factor productivity (TFP), share of zombie firms, and banking system capitalization for Japan and Southern Europe (Italy, Spain, and Portugal). For Japan, we compute the difference between the average year-over-year change in the period 1993–2002 and that in 1981–1992. For Europe, we report the difference between the average year-over-year change in 2011–2016 for each country (Italy, Spain, or Portugal) and that for Germany. The zombie-firm share is defined as the asset-weighted fraction of active firms identified as zombies, following Caballero et al. (2008). Banking system capitalization is measured as the average capital ratio of the banking sector. All statistics are expressed as percentage-point changes. See Appendix B for details on data sources and variable definitions.

with a steady rise in underperforming firms receiving subsidized credit. The asset-weighted share of zombie firms increased by 9.3 percentage points in Japan relative to pre-crisis levels and by 7.9 percentage points in Italy relative to Germany. Table 1 also indicates that, during these episodes, we observe a weakening of the banking system capitalization, with the officially recorded capitalization of the banking system falling by 2.6 percentage points in Japan and by about 3 percentage points in Southern Europe relative to Germany.

Initial shocks and subsequent zombie lending. Figure 1 documents the credit tightening at the onset of both episodes and the subsequent easing as policies took hold. In Japan, banks tightened lending standards sharply in 1991, then eased through the mid-1990s before renewed stress around 1997–98 when non-performing loans were eventually recognized. In the euro area, bank lending standards tightened sharply in late-2011 and 2012. In both cases, while accommodative policies were successful in easing the initial aggregate credit supply shock, a strong rise in zombie lending followed (see, e.g., Peek and Rosengren 2005, Giannetti and Simonov 2013 for Japan and Acharya et al. 2019 for Europe).

Cross-industry variation helps uncover the mechanisms underlying zombie lending and its implications for the economy's fundamentals. The proliferation of zombie lending appears closely linked to the magnitude of the initial economic shock. To illustrate this relationship, we compute, for each country–industry pair in Italy, Spain, and Portugal, the change in asset-weighted EBITDA-to-assets during the initial phase of the sovereign debt crisis (2009–2011). We interpret this change as an industry-specific profitability shock at the onset of the crisis. Panel A of Figure

Figure 1: Bank lending standards in Japan and Europe.



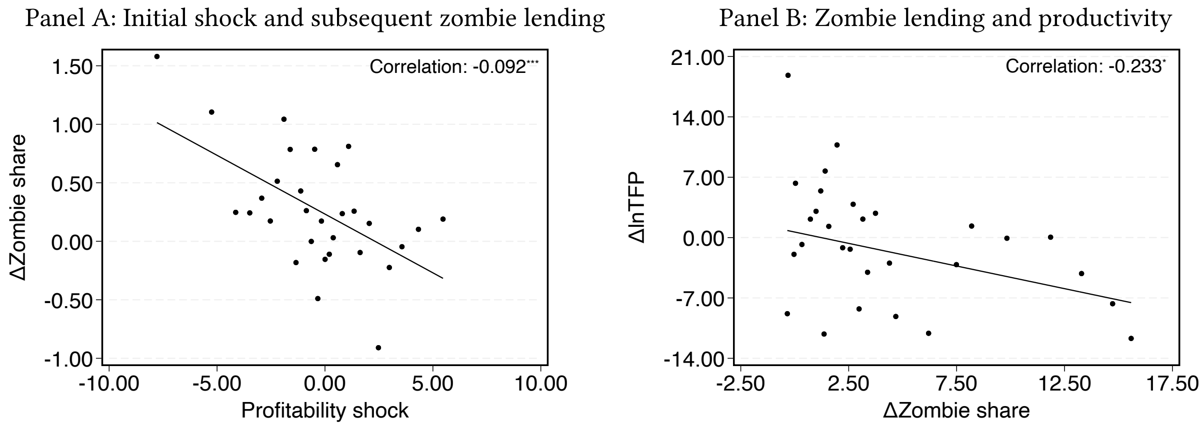
Notes: Left panel: Bank of Japan Tankan survey on "Lending attitudes of financial institutions" for all enterprises (solid line) and large enterprises (dotted line), inverted so that positive values denote net tightening. Right panel: ECB Bank Lending Survey on "Credit standards for loans to enterprises," plotted as the net percentage of banks reporting "tightened" minus "eased". Sources: Bank of Japan and ECB.

2 plots the relationship between this profitability shock and the subsequent change in the share of zombie firms (average 2012–2016 minus average 2009–2011). The strong negative correlation indicates that industries experiencing larger profitability declines subsequently exhibit a higher incidence of zombie lending.

Productivity distortions. Previous studies highlight several channels through which zombie lending can hinder economic growth (see, e.g., Hoshi and Kashyap 2015; Acharya et al. 2022). Zombie lending suppresses creative destruction by distorting both entry and exit margins: favorable lending terms keep unproductive firms afloat, tying up bank capital that could otherwise be allocated toward more productive newcomers. In Panel B of Figure 2, we provide additional evidence at the country–industry level consistent with long-run distortions induced by zombie lending. Specifically, for Japan, Italy and Spain (industry-level TFP is not available in KLEMS for Portugal), we plot the long-run change in the share of zombie firms (x-axis) against the corresponding long-run change in industry-level TFP obtained from Asia KLEMS and EU KLEMS (y-axis). The strong negative correlation suggests that sectors experiencing larger increases in zombie lending display weaker productivity performance, consistent with compositional shift towards low-productivity zombie firms.[4]

[4]Quantitatively, CHK find that in Japan, the presence of zombies reduced non-zombie firms' cumulative investment by 14–50 percentage points and employment by 5–19 percentage points. In the aftermath of the European sovereign crisis, Acharya et al. (2019) and Blattner et al. (2023) estimate that credit reallocated toward zombies accounted for a 3–11 percentage points employment loss among non-zombie firms and explained a substantial portion

Figure 2: Economic shocks, zombie lending, and productivity growth



Notes: The figure presents binned scatter plots summarizing relationships across country–industry pairs in Japan and Southern Europe. Panel A leverages cross-sectional variation in Italy, Spain, and Portugal to illustrate how aggregate shocks spur zombie lending. Each dot represents a bin of country–industry pairs grouped by similar changes in profitability during the initial phase of the sovereign debt crisis, measured as the percentage-point change in the asset-weighted EBITDA-to-assets ratio between 2009 and 2011. The horizontal axis reports the mean percentage-point change in profitability within each bin, and the vertical axis the corresponding mean change in the country–industry asset-weighted zombie share (average 2012–2016 minus average 2009–2011). Industries are defined at the 4-digit NACE Rev. 2 level. Panel B leverages cross-industry variation across Japan and Europe (Italy and Spain) to illustrate how zombie lending affects long-run productivity growth. Each dot represents a bin of country–industry pairs with similar changes in the share of zombie firms, expressed in percentage-point changes. The horizontal axis reports the mean change in the zombie share within each bin, and the vertical axis the mean long-run change in country–industry TFP. For Japan, the change in industry's zombie share is between 1981 and 2002 and the log change (×100) in industry's TFP is between 1992 and 2002. For Italy and Spain, the change in industry's zombie share is between 2012 and 2016 and the log change (×100) in industry's TFP is between 2012 and 2016. Industry definitions follow those in Asia KLEMS for Japan and EU KLEMS for Italy and Spain. See Appendix B for details on data sources and variable construction.

Spillovers on healthy firms. In addition, the presence of zombie firms can also harm the profitability of their non-zombie competitors, through congestion externalities that raise input costs (Caballero et al., 2008; Acharya et al., 2024) and distorted competition that lowers output prices (Acharya et al., 2024). In turn, the lower profitability makes non-zombies more likely to exit and less likely to enter, further amplifying the adverse compositional effects towards zombie firms.

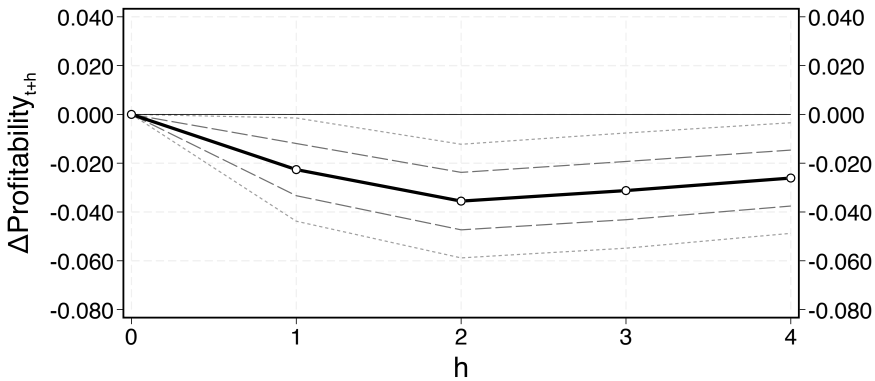
Figure 3 sheds light on the intertemporal nature of these negative spillovers. Drawing on data for Italy, Spain, and Portugal from Acharya et al. (2022), we estimate local projections to trace out how zombie lending affects the profitability of *non-zombie firms* within the same industry:

$$\text{Profitability}_{ci,t+h} - \text{Profitability}_{ci,t} = \alpha_i + \alpha_c + \beta_h \, \text{Zombie Share}_{ci,t} + \gamma_h \boldsymbol{X}_{ci,t} + \epsilon_{ci,t+h}, \qquad (1)$$

where α_i and α_c represent industry and country fixed effects, respectively. The dependent variable measures the change in the asset-weighted average EBITDA-to-assets ratio of non-zombie firms

of the observed decline in aggregate TFP.

Figure 3: Dynamic spillovers of zombie lending



Notes: This figure reports the estimates of the coefficients β_h from the local projection (1) for $h = 1, \ldots, 4$. The sample comprises country–industry pairs in Italy, Spain, and Portugal around the burst of the European sovereign debt crisis (2009–2016). The dotted and dashed lines denote the 95 and 68 percent robust confidence intervals of the estimates. See Appendix B for details on data sources and variable construction.

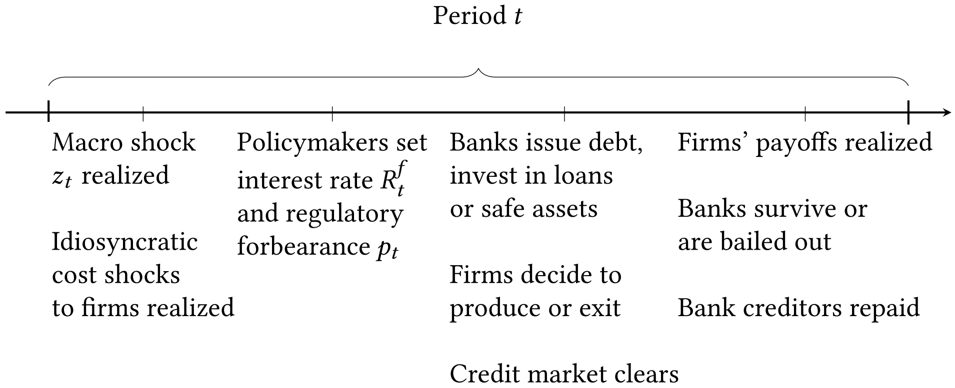
within a given country–industry pair (c, i) between years t and $t + h$ $(h = 1, \ldots, 4)$. The main regressor, Zombie Share$_{ci,t}$, denotes the asset-weighted share of zombie firms in the same country–industry pair in year t. To isolate the dynamic effects of this variable on non-zombies' profitability, the control vector $\boldsymbol{X}_{ic,t}$ includes two lagged values of the dependent variable (Profitability$_{ci,t-1}$, Profitability$_{ci,t-2}$) that control for pre-determined heterogeneity in profitability. The zombie share is standardized so that the coefficient of interest β_h captures the impact of a one-standard-deviation increase in zombie penetration.

Figure 3 plots the estimates of β_h over different horizons. A one-standard-deviation increase in an industry's zombie share leads to a 3 percentage point decline in the profitability of non-zombie firms after two years, a deterioration that unfolds and persists over time.

The role of policy interventions. Finally, these Japanese and European episodes also offer insights into the role played by policy interventions. In both cases, policymakers implemented a series of unprecedented macro-financial measures in an effort to shield the real economy from the adverse effects of the real estate and sovereign debt crises. These interventions (discussed in more detail in Section 3) featured limited capital injections into national banking systems as well as incisive packages of forbearance measures in the form of implicit or explicit government guarantees, central bank liquidity support facilities, and delayed loss-recognition schemes.[5]

[5]For instance, the ECB's Annual Report (ECB, 2013) directly relates the relaxation of credit standards apparent in Figure 1 to its policy response: "*From a supply-side perspective, the net tightening of credit standards declined in the course of 2013, as the ECB's standard and non-standard policy measures (in particular, the two three-year LTROs, as well as the changes in the collateral framework) and the announcements of the Outright Monetary Transactions, as well as the forward guidance, helped to reduce financial fragmentation and generally eased the funding difficulties faced by a number of banks.*"

Figure 4: Timeline of events within a period.



As shown in Table 1, however, interventions were not able to adequately recapitalize the banking sector. The forbearance policies did help lower banks' cost of capital but also allowed weakly-capitalized banks to extend new credit or evergreen existing loans to borrowers who should have otherwise been deemed insolvent (Peek and Rosengren 2005; Giannetti and Simonov 2013; Acharya et al. 2019), generating negative spillovers.

3 A Model of Zombie Lending

We begin with a static model building on the empirical evidence presented in Section 2. This static model can be viewed as one period of the dynamic model presented in Section 4.

3.1 Environment: Firms, Banks, and Policies

The economy is populated by heterogeneous firms that differ in their productivity and risk. These firms' investments require credit, which is provided by heterogeneous banks that differ in their level of capitalization. Figure 4 shows a timeline of the events in a period.

3.1.1 Firms

There are two types of firms, G or B. Initially, the economy is populated by a unit mass of incumbent firms. A mass $(1-\lambda)$ of incumbents are endowed with an indivisible project of type G, that yields revenues y^g in case of success and zero otherwise. A mass λ of incumbents are endowed with type B projects, yielding revenues y^b in case of success and zero otherwise. The success probability is θ^g for type G projects and θ^b for type B projects. Success is independent across firms, but the payoffs y^g and y^b are exposed to an aggregate shock z; we describe the dependence in z in Section 3.4. There are also potential entrants, each endowed with a type G project.

Without loss of generality, we assume the mass of potential entrants to be equal to λ to simplify expressions.

Both types of projects require \$1 in capital to be implemented. Firms have no wealth, and need to finance their project entirely via bank debt. Firm types are observable to banks. Therefore, the debt contracts feature type-specific interest rates: G firms borrow at a rate R^g and B firms borrow at a rate R^b.

In addition, all firms incur a production cost $\epsilon \in [0, \bar{\epsilon}]$ distributed according to the same c.d.f. H for both types of firms. The realization ϵ_i is known to the firm (but not to the bank) before production and financing decisions are made. Before their entry decision, potential entrants also observe their idiosyncratic cost ϵ, drawn from the same distribution H as incumbents.

Given the binary payoff structure, the project and the loan share the same risk: firms repay their loan entirely if their project succeeds, and default on the full loan if their project fails. We make the following assumption on payoffs:

Assumption 1. $\Delta\theta = (\theta^g - \theta^b) > 0$ *and* $0 \leq \theta^b y^b < \theta^g y^g - \bar{\epsilon}$.

In words, type B projects are riskier, which captures the fact that B firms have more outstanding debt and are thus more likely to default on their new loans. Moreover, even the least productive type G firms with a draw $\epsilon = \bar{\epsilon}$ are better than the most productive type B firms with a draw $\epsilon = 0$. The greater risk and lower profitability of type B projects mirror the characteristics of "zombie firms," which are shown empirically to be riskier due to their higher leverage, lower net worth, higher interest rate coverage ratio, and lower profitability ratios (Hoshi 2006; Acharya et al. 2019).

3.1.2 Banks

There is a unit mass of heterogenous financial intermediaries (hereafter, banks) with a balance sheet scale of \$1. Banks are indexed by their exogenous equity e, distributed in the interval $[e_{\min}, e_{\max}]$ according to the c.d.f. F, with $0 \leq e_{\min} \leq e_{\max} < 1$. In Section 5 we allow e to be chosen endogenously subject to equity issuance frictions and capital requirements.

Each bank can invest its entire \$1 in a single asset, which can be either a risky corporate loan or a safe asset. Banks can lend to a type $j \in \{b, g\}$ firm at rate R^j, earning an expected return equal to $\theta^j R^j$. Credit markets are competitive: loan rates R^j are taken as given by both firms and banks, and determined in general equilibrium. Alternatively, banks can invest in "safe assets," which serve to absorb any funds not deployed into loans, allowing us to model movements in aggregate lending caused by shocks and policies. We interpret safe assets as a broad class of assets held in banks' portfolios that are generally safer than corporate loans, such as central bank reserves and safe government bonds, mortgages or asset-backed securities. Safe assets are supplied elastically and pay a risk-free return R^f set by monetary policy. Investment in safe assets does not produce output; results are unchanged if investments in safe assets yield a positive output lower than what is produced by lending to firms.[6]

On the liability side, a bank with capital e needs to raise $(1-e)$ of debt in order to invest. In equilibrium, debt holders require an expected return equal to R^f. The actual contractual rate paid to debt holders by each bank, $\tilde{R}^j$, depends on the riskiness of banks' asset choice j and on the degree of regulatory forbearance indexed by a parameter p set by policy, as we describe next. Specifically, we assume that, under the forbearance policy, debt holders are able to recover their principal with probability $p \in [0,1]$ if the bank defaults. Thus the contractual rate $\tilde{R}^j$ on the debt of a bank that invests in asset $j \in \{g, b, f\}$ needs to satisfy

$$R^f = \theta^j \tilde{R}^j + \left(1-\theta^j\right) p. \tag{2}$$

A key observation is that a positive p makes riskier investments more attractive. The expected payoff from choosing investment of type i bank with capital e is

$$\theta^j[R^j - \tilde{R}^j(1-e)] = \theta^j R^j - R^f(1-e) + \underbrace{p(1-\theta^j)(1-e)}_{\text{subsidy}}, \tag{3}$$

where the last term is the policy-induced subsidy to type-j investments. Note that the subsidy is positive only if banks have some leverage *and* if they take positive risk; thus there is no subsidy for safe investments ($\theta^j = 1$) or for fully equity-funded banks ($e = 1$). Outside these extreme cases, the subsidy is increasing in p, in leverage $(1-e)$ and in risk $(1-\theta^j)$.

Relationship Lending and Evergreening. In the baseline model we focus on risk-shifting as a driver of zombie lending, and abstract from the role of relationship lending, and in particular from weak banks willing to "extend and pretend" by rolling over loans at subsidized rates to *legacy* borrowers that should be declared non-performing. We incorporate this important "evergreening" channel in Section 5.

3.1.3 Policy Instruments: R^f and p

Policymakers affect banks' decisions through the choice of the two variables R^f and p. They directly control the level of the risk-free rate R^f through conventional monetary policy, but are

[6] The assumption that banks invest in a single asset captures more broadly bank specialization, which is common in the data, see e.g. Berger et al. (2017), Paravisini et al. (2020), and Blickle et al. (2023). Loans can be reinterpreted as being portfolios of loans to the sector in which individual banks have acquired information and competences. What matters is that banks with different leverage end up with different portfolios, instead of all investing in the same diversified loan portfolio. The assumption of full specialization could be relaxed by allowing banks to hold a portfolio of projects with correlated risks a la Vasicek (1977) without affecting the key message of the model.

subject to an "effective lower bound" (ELB):

Assumption 2. *There is an effective lower bound* $R^f_{\min} > 0$ *on the risk-free rate:*

$$R^f \geq R^f_{\min}. \tag{4}$$

The ELB can be interpreted as a standard zero lower bound due to the presence of cash.[7] It can arise more broadly as a constraint on monetary policy due to conflicting objectives. For instance, the central bank may be unable to further lower R^f to stabilize banks when it is already busy fighting inflation. Here we focus on a "bank lending channel" of monetary policy working through portfolio rebalancing and abstract from other channels, such as the aggregate demand channel emphasized in New Keynesian models. We will show that the interesting regime for zombie lending is when shocks z are large enough to make the ELB bind, hence other channels of monetary policy also become inoperative.

Policymakers also set the parameter p, which influences banks' cost of capital through the debt pricing equation (2): a higher degree of insurance p encourages risky lending by decreasing the associated cost of funds. We focus on the case of a guarantee p that applies independently of banks' choice of investment, and discuss alternative policies below.

Assumption 3. p *is independent of banks' portfolio choices.*

The level of p captures the extent of explicit and implicit government guarantees to banks, which are often reinforced when bad macroeconomic or financial shocks hit. It can also be viewed as indexing the leniency of bank closure policy: higher p means more regulatory forbearance. Closure can be avoided by rescuing a distressed bank, which in turn reduces the ex-ante risk-sensitivity of the bank's creditors. The relevant common thread of these unconventional policies is that a higher p undermines the market discipline imposed by bank creditors by decoupling the cost of funds from asset risk, which can become a valuable tool to stimulate lending once the constraint (4) starts binding, consistent with the empirical findings in Acharya et al. (2019).

We use guarantees as a convenient modeling device for a range of interventions meant to support lending by absorbing risk away from the private sector, which we broadly refer to as "regulatory forbearance". One standard example is the state-contingent enforcement of capital requirements. Suppose that capital requirements are binding and bank equity is given by $e = \hat{e}$, where $\hat{e}$ is set by regulators (as we will model in Section 5). Holding the capital requirement $\hat{e}$ fixed, increasing p in bad times allows policymakers to stimulate risky corporate lending relative to investments in safe assets by increasing the subsidy $p(1-\hat{e})(1-\theta^j)$ defined in (3). Alternatively, suppose that there is a fixed level of guarantees (i.e., not state-contingent), but policymakers lower

[7] The lower bound on nominal rates translates into a lower bound on real rates if inflation expectations are sticky.

the capital requirement $\hat{e}$ in bad times. This is an equivalent way to increase the effective subsidy earned by banks, as a higher leverage $(1-\hat{e})$ makes risky assets more attractive and thus stimulates lending just like an increase in p. Since the *product* $p \times (1-\hat{e})$ is what matters for the subsidy and hence bank incentives, framing the model in terms of state-contingent guarantees allows us to illustrate the main mechanism without having to model banks' equity issuance decisions, which we leave for Section 5.[8]

Both forms of forbearance are widely used in practice. Governments and regulators indeed expand the coverage and depth of guarantees in bad times, without conditioning on the asset risk of individual banks. In the U.S., in the run-up to the Savings and Loans (S&L) crisis in the 1980s, the Depository Institutions Deregulation and Monetary Control Act increased deposit insurance from \$40,000 to \$100,000 per account with the purpose of curbing deposit outflows. The cap increased to \$250,000 in 2008 and the FDIC introduced the Transaction Account Guarantee Program guaranteeing corporate checking accounts without limit, and the Debt Guarantee Program providing debt guarantees to banks, until 2012. In the recent 2023 banking crisis, the FDIC effectively guaranteed all uninsured deposits of distressed banks and the Federal Reserve introduced the Bank Term Funding Program allowing banks to borrow for a term of up to one year against eligible securities at par.

3.2 Bank Portfolio Choice

A bank with equity e chooses among the three investment options (safe assets, lending to type G firms, lending to type B firm) to maximize expected profits. Taking as given p, the loan rates R^g, R^b and the risk-free rate R^f, the bank solves

$$\max_{i \in \{g,b,f\}} \theta^i \left[R^i - \tilde{R}^i (1-e)\right] \quad \text{s.t. } \tilde{R}^i = \frac{R^f - \left(1-\theta^i\right) p}{\theta^i}. \tag{5}$$

The subsidy defined in (3) is not only increasing in risk and leverage but also *supermodular* in these two variables, meaning that risk-shifting incentives induced by regulatory forbearance increase with bank leverage. When banks are heterogeneous in leverage, this complementarity between leverage and risk implies a natural "diabolical" sorting between poorly capitalized banks and riskier firms, as captured by the following lemma (proved in Appendix D) characterizing the solution of banks' problem as a function of their level of capitalization:

[8] There we also show how the simple equivalence between guarantees and regulatory forbearance is altered when banks face large costs of switching from old zombie borrowers to new healthy borrowers, including the costs of loss recognition.

Lemma 1. *Define the following equity levels:*

$$e^* = 1 - \frac{(\theta^g R^g - \theta^b R^b)}{p\Delta\theta} \quad \text{and} \quad e^{**} = 1 - \frac{(R^f - \theta^g R^g)}{p(1-\theta^g)}.$$

With parameters such that $e^* < e^{**}$, banks invest as follows:[9]

(i) Banks with equity $e < e^*$ lend to a type B borrower at rate R^b.

(ii) Banks with equity $e \in (e^*, e^{**})$ lend to a type G borrower at rate R^g.

(iii) Banks with equity $e > e^{**}$ do not lend and invest in safe assets at rate R^f.

The lemma takes loan rates as given, but it continues to hold once rates are determined in general equilibrium. Figure A.1 offers a graphical intuition for the result, showing the expected profits from the three available investments as a function of bank capital e.

Heterogeneity in bank capital is not essential for our main dynamic result on policy traps in Section 4, and Lemma 1 also applies in the special case of homogeneous banks. Allowing for heterogeneous banks allows us to connect to the evidence cited in the introduction that poorly-capitalized banks are more likely to extend credit to risky and unproductive borrowers. In that sense, Lemma 1 provides indirect support for the kind of guarantees we assume. In this regard, it is useful to contrast the effects of a uniform guarantee p with alternative policies, such as (i) subsidies that do not reward risk-taking, and (ii) more targeted subsidies that seek to stimulate lending while avoiding zombie lending.

Risk-Insensitive Subsidies. Equation (3) shows that the uniform guarantee p implies a "risk-based" subsidy, in the sense that the subsidy increases with the risk of bank assets. Consider instead a policy that subsidizes banks at a rate s that does *not* depend on risk, so that an asset of type i yields an expected payoff

$$\Pi^i(s) = \theta^i R^i - R^f(1-e) + s \tag{6}$$

where the subsidy s is the same across assets i. Such a policy would not induce risk-shifting, because banks' portfolio choice would be the same as without subsidy. The problem, however, is that a blanket subsidy s would also fail to encourage lending.[10] Thus *some* level of risk-dependence is required in order to stimulate good lending.

[9] Condition (A.2) in the Appendix ensures $e^* < e^{**}$. The only difference if $e^* \geq e^{**}$ is that region (ii) does not exist, i.e., no bank lends to a G firm. This leads to an even more extreme diabolical sorting: low equity banks lend to type B borrowers, and high equity banks invest in safe assets.

[10] Over time, however, the direct transfers s could potentially increase equity and decrease risk-shifting, but only if banks are forced to retain those earnings instead of paying them out as dividends. If the goal is to recapitalize banks, however, we show in Section 4.4 that a rapid and sufficiently large direct intervention would be more effective.

This point is consistent with, e.g., the evolution of the ECB's long-term refinancing operations (LTRO) policies since the Great Financial Crisis. The early design in 2008-2009 simply allowed banks to borrow at favorable terms against eligible collateral such as government bonds. Since the loan terms were not directly tied to corporate loans and those loans were not eligible as collateral, the policy was closer to the broad-based subsidy in (6) from the perspective of business lending. Muted lending effects led to a shift towards more risk-based subsidies, closer in spirit to our model's (3). In 2012, the ECB introduced the "Additional Credit Claims" framework allowing banks to pledge risky corporate loans as collateral. ECB-eligibility makes such loans more attractive, and the implicit subsidy is larger for riskier and more illiquid loans.[11] In 2014, the ECB introduced *targeted* longer-term refinancing operations (TLTRO), which make the subsidized rate at which each bank can borrow a decreasing function of how much the bank lends to firms and households, without penalizing riskier lending, but excluding mortgage lending which is considered safer and less productive.

Risk-Sensitive Guarantees and Capital Requirements. Risk-based regulation is a natural solution to the risk-shifting induced by government guarantees. Making p or capital requirements $\hat{e}$ a function of θ could allow policymakers to target more precisely their interventions. For instance, setting

$$p(\theta^g) > 0, \quad p(\theta^b) = 0$$

would allow policymakers to stimulate lending to G firms without subsidizing loans to B firms. An equivalent implementation would be to set a uniform guarantee p, but require banks to pay fairly priced deposit insurance premiums equal to the subsidy $p(1-\theta^b)(1-e)$ for loans to B firms, while waiving the premium for loans to G firms. Alternatively, regulators could impose capital requirements $\hat{e}(\theta)$ (i.e., requiring banks to raise sufficient equity to meet the constraint $e \geq \hat{e}$) such that the resulting subsidy $p(1-\hat{e}(\theta))(1-\theta)$ does not incentivize zombie lending at the expense of healthy lending. As stated in our Assumption 3, our results rely on policymakers being unable to

[11]See https://www.ecb.europa.eu/ecb-and-you/explainers/tell-me-more/html/acc_frameworks.en.html for a description of the ACC framework. The stated effect is to further stimulating lending: "*ACC frameworks have incentivised the acceptance of loans to smaller businesses and self-employed and private individuals as Eurosystem collateral for years. The temporary extension of ACC frameworks now allows the further easing of certain requirements for the acceptance of such loans. This can help banks to provide loans to the real economy.*" Individual countries also had ways to replicate this outcome on their own through "Government Guaranteed Bank Bonds." Carpinelli and Crosignani (2021) study how in 2011, "*right after the LTRO announcement, the Italian government offered banks a guarantee on securities otherwise ineligible at the ECB by paying a fee. As the ECB accepts all government-guaranteed assets as collateral, the program effectively gave banks a technology to "manufacture" ECB-eligible collateral.*" Italian banks used this guarantee to essentially pledge their entire illiquid assets "*by issuing and retaining unsecured bank bonds [...] banks could then obtain a government guarantee on these newly created bonds (called Government Guaranteed Bank Bonds) so that they became eligible to be pledged at the LTRO.*"

design fully risk-adjusted policy instruments. Put differently, the forces in our paper strengthen the case for risk-sensitive regulation.

From a theoretical perspective, Chan, Greenbaum and Thakor (1992) show that risk-sensitive guarantees may not be incentive-compatible in a general environment with private information about assets and/or moral hazard in monitoring. In fact, our model highlights an additional challenge relative to the standard setting in which guarantees lead to over-investment in a single risky asset and regulators "only" need to find a way to tax unobserved risk. Our setting features two closely related risky assets, with under-investment in one (loans to G firms) and over-investment in the other (loans to B firms). Inducing risk-taking in G loans is the intended effect of the guarantees, but the key tension is that the government's objective is *non-monotonic* in terms of risk: the goal is to subsidize an intermediate level of risk-taking (lending to good firms), while avoiding activities that are "too safe" (safe assets) or "too risky" (zombie lending). Thus simply taxing risk is too blunt, and subsidizing good loans creates strong incentives for banks to make zombie loans appear performing.

In our view, the empirically salient assumption is that informational or institutional frictions prevent regulators from tailoring policies to bank assets.[12] The information available to regulators to monitor a bank's risk-taking is mostly backward-looking, and the risk metrics available are often coarse. Moreover, information is often not available in a timely manner, or is too costly to collect at a high frequency.[13] Institutional frictions also play a role: p could be constrained to be the same across U.S. states or European countries in spite of large observable differences in the quality of local banks' balance sheets, as discussed previously in the case of the ECB's policies, and studied in the case of U.S. conforming mortgages by Hurst et al. (2016).

Remark 1 (Pricing of risk by bank debt holders.). Equation (2) implies that banks' funding costs increase with risk, albeit less so in the presence of government guarantees. Risk-sensitive pricing relies on debt holders being able to observe banks' asset risk. One explanation is that the market for uninsured deposits and subordinated debt can incorporate information unavailable to regulators in real time (Flannery 1998, Berger, Davies and Flannery 2000). In any case, the assumption that bank debt holders can discriminate between risky and safer banks is not crucial, as removing risk-sensitive pricing of bank debt would in fact strengthen banks' risk-shifting motive.[14]

[12] While a large number of countries have established deposit insurance funds, banks frequently do not pay an explicit premium (see Saunders et al. 2021 for a detailed discussion and international comparison). In the U.S., banks do pay a premium, but contributions cease once the Deposit Insurance Fund reaches a statutory cap set by Congress and seldom revised. Even when premiums are collected, they are only weakly linked to the risk of banks' assets or liabilities. Moreover, under the Dodd-Frank Act, the cost of FDIC forbearance toward failing systemically important institutions may ultimately be borne by the healthier surviving institutions.

[13] Stress tests are only conducted on a semi-annual basis and significant implementation costs restrict the number of scenarios considered (Parlatore and Philippon, 2024), whereas on-site inspections are typically randomized to economize supervisory resources (Passalacqua et al., 2020).

[14] To see this, suppose that banks' funding costs become completely insulated of their asset risk, that is $\tilde{R}^g =$

3.3 Equilibrium

We need to determine both the equilibrium allocation of bank capital (aggregate lending and aggregate investment in safe assets) and the composition of lending (good versus bad types of firms). As we explain below, the highest level of aggregate output is achieved when there is maximal *creative destruction.* That is, all the type B incumbent firms exit, and are replaced by more productive type G entrants. We model the entry and exit process building on CHK, with the additional layer of banks' portfolio choices. Equilibrium loan interest rates are the variables that adjust to bring about, or hinder, creative destruction.

Firms' Entry and Exit Decisions. Given the realization of production costs ϵ and the borrowing rates offered by banks, incumbent firms decide whether to produce or exit, and potential entrants decide whether to enter or not. Incumbents remain in business and undertake their project if and only if they expect positive profits, which happens if and only if the idiosyncratic cost realization ϵ is lower than a type-specific threshold $\tilde{\epsilon}^i$, $i = g, b$. A type i incumbent drawing ϵ produces if

$$\epsilon \leq \tilde{\epsilon}^i = \theta^i \left(y^i - R^i\right) \tag{7}$$

and exits otherwise. The masses of active firms of type G and B are respectively

$$\begin{aligned} m^g &= \underbrace{(1-\lambda) H(\tilde{\epsilon}^g)}_{\text{incumbents}} + \underbrace{\lambda H(\tilde{\epsilon}^g)}_{\text{entrants}} = H\left(\theta^g\left(y^g - R^g\right)\right), \\ m^b &= \lambda H\left(\tilde{\epsilon}^b\right) = \lambda H\left(\theta^b\left(y^b - R^b\right)\right). \end{aligned} \tag{8}$$

m^g and m^b are the aggregate loan demands from each type of firm. There is no intensive margin adjustment as projects are all of unit size, but higher loan rates decrease aggregate loan demand at the extensive margin.

Equilibrium. Given policies $\left(R^f, p\right)$, the static general equilibrium of the model is characterized by loan rates $\left(R^g, R^b\right)$ such that agents optimize and the two market clearing conditions hold:

$$F(e^*) = m^b = \lambda H\left(\theta^b\left(y^b - R^b\right)\right), \tag{9}$$

$$F(e^{**}) - F(e^*) = m^g = H\left(\theta^g\left(y^g - R^g\right)\right), \tag{10}$$

$\tilde{R}^b = \tilde{R}$ with $R^f \leq \tilde{R} \leq R^f/\theta^g$. Comparing to expression (3) shows that this is equivalent to an implicit guarantee $p(\theta^i) = R^f - \frac{\theta^i}{1-\theta^i}(\tilde{R} - R^f)$ which is risk-sensitive, but *in the wrong direction*, as $p(\theta^i)$ decreases with θ^i. Thus funding costs that are insensitive to risk imply an even stronger subsidy towards riskier assets. This configuration, which is empirically plausible, would reinforce risk-shifting towards type-B loans relative to our benchmark with a common p.

where the thresholds e^* and e^{**} are defined in Lemma 1.

Equation (9) equalizes the supply of zombie loans, by banks with equity $e < e^*$, to the demand from type B firms with idiosyncratic shocks $\epsilon \leq \tilde{\epsilon}^b = \theta^b \left(y^b - R^b\right)$. Similarly, equation (10) describes market-clearing for good loans.

Aggregate Output. Given equilibrium loan rates, aggregate output can be written as

$$Y = \int_0^{\theta^g(y^g - R^g)} [\theta^g y^g - \epsilon] dH(\epsilon) + \lambda \int_0^{\theta^b\left(y^b - R^b\right)} [\theta^b y^b - \epsilon] dH(\epsilon) . \tag{11}$$

The first term in (11) captures the net contribution of type G firms (both incumbents and entrants). The second term is the net contribution of type B firms. By Assumption 1 both terms are positive, thus lower lending rates R^g and R^b increase aggregate output by stimulating the entry and continuation of productive firms. Note that type B firms are not negative-NPV from a partial equilibrium perspective. However, they are relatively worse firms that prevent scarce resources (such as bank loans but also other inputs, as we discuss next) from going to more efficient firms, with a negative impact in general equilibrium.

Zombie lending will make credit supply scarcer for good firms and thus the equilibrium loan rate R^g higher. The threshold characterizing the marginal active good firm $\tilde{\epsilon}^g = \theta^g (y^g - R^g)$ is thus lower, meaning that good firms need a better idiosyncratic productivity draw to survive in an environment with zombies. While the average output produced by remaining good firms $\theta^g y^g - \mathbf{E}[\epsilon | \epsilon \leq \tilde{\epsilon}^g]$ is higher, the misallocation of lending necessarily reduces total output Y.

Potential Output. We define *potential output* Y^* as the highest possible aggregate output the economy can achieve given its fundamentals, given by

$$Y^* = \theta^g y^g - \mathbf{E}[\epsilon] .$$

According to equation (11), the economy attains Y^* when all bank capital is used to finance the productive sector (i.e., there is no investment in bonds) and, within the productive sector, the most productive firms (i.e., there is no zombie lending).

3.4 Policies Implementing Potential Output

The two policy variables R^f and p impact banks' decisions—and therefore credit allocation and equilibrium output—through two different channels. The first channel is a standard *bank lending channel*, that is, the choice between investing in safe assets versus lending to the productive sector. A lower R^f stimulates lending to both types of firms by decreasing the return of investing in safe assets relative to loans. Government guarantees subsidize riskier investments, thus a higher p also stimulates lending to both types of firms, by lowering the cost of funds.

The second channel is the *zombie lending channel*, operating through the choice between lending to different types of borrowers. A higher p not only makes lending in general more appealing, but it also increases the profits from loans to B firms relatively more. These loans are riskier, thus a given subsidy p lowers the cost of funds by more when lending to B firms, through the term $\left(1-\theta^{b}\right)p$ in (2).

When and how can policies (R^{f}, p) implement potential output? Our next result shows that the answer depends on the magnitude of the shocks affecting fundamentals.

Suppose that the economy is hit by an aggregate profitability shock z (due to, e.g., lower demand or productivity), with the convention that firm outputs y^{g} and y^{b} are both decreasing functions of z:

$$y^{g}(z)=\bar{y}^{g}(1-z), \tag{12}$$

$$y^{b}(z)=\bar{y}^{b}(1-\eta z), \tag{13}$$

and z lies between 0 and $z_{\max}$ such that Assumption 1 holds even for $z=z_{\max}$. Therefore potential output $Y^{*}(z)$ also decreases with z. We assume that η, the relative incidence on type B firms, is between 0 and an upper bound $\frac{\theta^{g}\bar{y}^{g}}{\theta^{b}\bar{y}^{b}}$. Since this upper bound is above 1, the shock structure (12)-(13) nests the case of symmetric shocks $\eta=1$. The shock z corresponds to the empirical aggregate profitability shock in Figure 2 (left plot) in Section 2.

The following result characterizes when and how policies can implement $Y=Y^{*}(z)$:

Proposition 1. *Given a shock size z, there exist a threshold $\bar{p}(z)>0$ and a function $\bar{R}^{f}(p;z)$ increasing in p and decreasing in z, such that output reaches its potential, $Y=Y^{*}(z)$, if and only if the following two conditions hold:*

$$\begin{cases} R^{f} \leq \bar{R}^{f}(p;z) & \textit{(full lending)} \\ p \leq \bar{p}(z) & \textit{(no zombie lending)} \end{cases}$$

For any $p_0>\bar{p}(z)$, zombie lending necessarily emerges in equilibrium.

Moreover, there exist thresholds $\underline{z}>0$ and $\bar{z}>\underline{z}$ (defined in the proof) such that:

(i) *For small shocks $z \leq \underline{z}$, $Y=Y^{*}(z)$ can be achieved by setting an interest rate $R^{f}(z)$ that decreases with the size z of the shock, together with $p=0$.*

(ii) *For moderate shocks $z \in (\underline{z}, \bar{z}]$, $Y=Y^{*}(z)$ can be achieved by setting $R^{f}=R^{f}_{\min}$ together with a positive $p(z)$ that increases with the size z of the shock.*

(iii) *For large shocks, $z > \bar{z}$, Y is strictly below $Y^*(z)$ for any policies (R^f, p).*

The threshold $\bar{z}$ increases with $e_{\min}$, hence an improvement in the health of weak banks leads to a more resilient economy, in the sense that policy can achieve Y^* in response to a larger range of shocks $z \in [0, \bar{z}]$.

The "full lending" condition $R^f \leq \bar{R}^f$ ensures that the return on safe assets is sufficiently low to make lending attractive to the least leveraged banks ($e = e_{\max}$), who benefit the least from government guarantees. The "no zombie lending" condition $p \leq \bar{p}$ ensures that the most leveraged banks ($e = e_{\min}$), who benefit the most from guarantees, still prefer to lend to type G firms.

If $\bar{R}^f(0; z)$ is above the ELB $R^f_{\min}$, then setting $R^f(z) = \bar{R}^f(0; z)$ together with $p(z) = 0$ achieves potential output while keeping the mass of zombie firms m^b_0 at zero. Thus $\bar{R}^f(0; z)$ provides a notion of the "natural interest rate", that is the interest rate required to achieve Y^* without subsidy. The natural rate fluctuates with fundamentals: larger z shocks must be accommodated by a lower risk-free rate, as in standard macroeconomic models.

Proposition 1 defines two thresholds $\underline{z}$ and $\bar{z}$. Following small shocks $z \leq \underline{z}$, an accommodative conventional monetary policy can achieve Y^* at no costs ($p = 0$). Moderate shocks $z \in [\underline{z}, \bar{z}]$, however, require combining conventional monetary policy and forbearance policy in order to keep the economy at its full capacity. Specifically, a positive p helps stabilize output once conventional monetary policy is constrained by the lower bound ($R^f = R^f_{\min}$). In this region, the optimal forbearance increases in response to more severe shocks. The increase in p subsidizes bank lending as much as possible, but all the lending is to type G firms. Thus if shocks are moderate, some forbearance $p > 0$ is sufficient to attain Y^*.

Once the shock is severe enough, $z > \bar{z}$, stimulating aggregate lending necessarily triggers some zombie lending by banks at the bottom of the equity distribution. Therefore, Proposition 1 especially highlights the role of *large shocks*. A lack of profitable investment opportunities for good firms is not only detrimental per se, but it also makes zombie lending more attractive to banks. Thus zombie lending tends to emerge after large shocks that hit economies with a weak banking sector.[15]

Complementarity between bank capital and stabilization policy. The threshold $\bar{z}$ is itself endogenous, as it increases with the minimal level of equity $e_{\min}$. Our model thus highlights that the capitalization of the banking system mediates the effectiveness of policy interventions following real economic shocks. This result links the effectiveness of accommodative policy to the level of

[15] Note that our results abstract from the frictions in the bankruptcy system that may also follow such large shocks. A massive wave of bankruptcies may lead to court congestion, fire sales, and widespread financial stress, calling for a richer set of policies than those we consider, such as those analyzed in Gourinchas et al. (2020) and Greenwood, Iverson and Thesmar (2020).

capitalization of the banking system, consistent with the evidence in Acharya et al. (2020). Our emphasis on widespread zombie lending as a central constraint on policy complements other contexts where weak bank balance sheets undermine policy effectiveness (e.g., Bernanke and Gertler 1995, Kashyap and Stein 2000, Van den Heuvel 2002, Bolton and Freixas 2006, Gambacorta and Shin 2018).

Remark 2 (Modeling aggregate lending). The central mechanism is that negative shocks can lead to an aggregate bank credit contraction, as seen in Figure 1 for Japan in 1991–92 and Europe in 2011–12, and that policies can partly restore lending, albeit by potentially distorting the composition of credit. In general, a negative shock to aggregate loan supply must occur through a mix of (i) reduced total bank assets and (ii) a shift away from lending towards other assets. Given our simplifying assumption of fixed bank sizes and simple portfolios, we implement the aggregate lending contraction by letting residual safe assets absorb the reallocation at the extensive margin, that is, via a downward shift in the cutoff e^{**} in response to the shock z. Conversely, policies such as lowering R^f and increasing p counteract the shock by shifting e^{**} in the opposite direction, thereby raising aggregate lending. As a result, the lending contraction may be muted in practice: even when negative shocks would cause aggregate lending to contract significantly absent policy interventions, the realized aggregate lending cut and shift into safe assets may be much smaller precisely because policymakers intervene to mitigate the contraction.

While we model this chain through the extensive margin, the same logic is consistent with richer environments that also allow for changes in bank size and intensive-margin portfolio adjustments between loans and other assets. For instance, a fall in lending due to deposit or wholesale funding outflows into domestic or foreign safe assets is equivalent, for aggregate lending, to the banking sector reallocating funds towards these assets.

To summarize, the single-period theoretical framework reproduces some key empirical findings relating the allocative efficiency of credit markets, policy actions, and the capitalization of the banking system. Next, we study the dynamic implications of zombie lending in the presence of negative externalities imposed by unproductive firms on the other firms in the economy.

4 Dynamics: Policy Traps and Sclerosis

Zombie lending is far from being a temporary problem. As discussed in Section 2, a substantial body of empirical evidence from the historical experiences of Japan and Southern European countries suggests that credit misallocation due to the proliferation of zombie lending may have persistent adverse effects on healthy firms.

We now turn to a dynamic version of our model that emphasizes how the interplay of accommodative policies and spillovers from zombie lending can lead to persistent output losses and policy traps. Our main result shows that in response to transitory shocks and policies seeking to avoid a recession in the short run, the economy can get stuck in a state of permanent low potential output (which we call "sclerosis") with policymakers forced to maintain a combination of low interest rates and high forbearance (which we call a "policy trap"). We then discuss how the economy can exit such a trap, through a large bank recapitalization or an improvement in the productivity of good firms.

4.1 Dynamic Environment

To capture that the full cost of keeping zombie firms alive materializes over time, we assume the presence of type-B firms hurts the profitability of healthy firms in the next period:

Assumption 4. *For $t \geq 0$, type-G firms' output follows $y^g_{t+1} = \bar{y}^g(1 - z_{t+1})$, where z_{t+1} increases with the extent of zombie lending in the previous period:*

$$z_{t+1} = \alpha m^b_t \quad \text{for } t \geq 0. \tag{14}$$

A substantial body of empirical evidence, discussed in Section 2, highlights that zombie firms can impact the performance of healthier firms in the economy through various channels, such as congestion in labor and input markets, congestion in output markets due to price competition, or reduced innovation incentives. The parameter $\alpha \geq 0$ captures the strength of these congestion externalities.[16]

The key point is that zombie lending has a *persistent* effect on healthy firms.[17] Several complementary mechanisms could generate persistence, through different notions of "capital", including customer bases, labor forces, and intellectual capital. Zombie lending also allows type-B firms to make date-t investments whose impact on other firms only materializes at $t + 1$, as in time-to-build models (Kydland and Prescott, 1982), or persists due to other forms of slow equilibrium adjustment. For example, Asriyan et al. (2024) show that relaxing unproductive firms' financial constraints allows them to bid up the price of capital, which can ultimately crowd out investment by more productive firms. If the supply of capital responds slowly to the higher price (due to, e.g., standard adjustment costs), then the misallocation of scarce capital propagates the initial misallocation of scarce bank lending over time. Secondly, as in the literature on customer markets (Phelps and Winter 1970, Ravn, Schmitt-Grohé and Uribe 2006, Gourio and Rudanko 2014) or,

[16]Congestion externalities should be viewed as decreasing healthy firms' profitability. As equation (11) shows, the aggregate effect on output depends on both the direct spillovers and resulting endogenous equilibrium adjustments. For instance, in equilibrium, the lower profitability caused by the prevalence of zombies through (14) must also raise the productivity of the marginal good firm that remains active (i.e., the idiosyncratic cost threshold $\tilde{\epsilon}^g$ in (7) is lower).

[17]Remark 3, following our results, explains how Assumption 4 could be relaxed.

relatedly, customer switching costs (Klemperer, 1987), if customers become "attached" to type-B firms at date t, it becomes more difficult for type-G firms to compete in future periods. Recognizing the persistence of customer bases, type-B firms could even take advantage of zombie loans to charge especially low prices to attract more customers at t, consistent with the evidence in Acharya et al. (2024) on the disinflationary effects of zombie lending on product prices. Similarly, zombie lending could facilitate labor hoarding or low-quality matches between firms and workers, with lasting negative effects in the presence of labor market frictions (Bertola and Caballero, 1994; Barlevy, 2002). Lastly, Schmidt et al. (2023) find that zombie lending reduces patent applications and R&D spending, particularly in R&D-intensive and highly competitive sectors.

Empirically, Caballero et al. (2008) and Acharya et al. (2019) find evidence of spillover effects attributable to the presence of zombie firms. The motivating evidence we present in Section 2 underscores the intertemporal nature of these effects, which appear to be persistent and gradually compound over time, as shown in Figure 3.

Bank and firm dynamics. To close the model, we need to specify the dynamics of the banking sector. Bank returns are stochastic, with some banks failing and others making large profits. In general, accounting for bank entry and exit and tracking the evolution of the full distribution of bank equity presents significant technical challenges, similar to the ones encountered in macroeconomic models with heterogeneous households and incomplete markets. We thus make the following assumptions to make the dynamic model tractable:

Assumption 5 (Bank dynamics)**.** *There are overlapping generations of bankers: bank managers at t are replaced after one period and earn a fraction ρ of the income accruing at $t+1$. The manager of a bank with date-t equity e_t chooses project $i \in \{b, g, f\}$ to maximize*

$$\rho\theta^i \left[R_t^i - \tilde{R}_t^i (1 - e_t)\right].$$

At the beginning of each period $t+1$, after date-t bank managers have been paid and replaced, failing banks are replaced by new banks and the profits of all surviving banks are pooled together and redistributed to all banks equally and banks raise equity $\iota > 0$.

This simplification allows us to keep track of the evolution of the aggregate capitalization of the banking system, rather than the entire distribution of bank equity. Lemma 1 continues to apply as we collapse the distribution of banks: even though the portfolio of individual banks is indeterminate in the limit of homogeneous banks, the aggregate portfolio of the banking system is well-defined, which is all we need to study the output effects of zombie lending.

The short-term nature of bank managers' contracts implies that banks' franchise value does not enter the bank investment problem, therefore banks' portfolio choice is the same as in Section

3. In particular, given date-t equilibrium rates, the optimal portfolio choice is characterized by the same thresholds e_t^* and e_t^{**} stated in Proposition 1. In a more general setting, banks would have to consider their franchise value when choosing their portfolios, which would then feed back into the equilibrium thresholds e_t^* and e_t^{**}. Accounting for the effect of the franchise value on banks' portfolio choices is an interesting extension that we leave for future research.[18]

Equilibrium. Given an initial shock z_0 and a path of policies $\left\{R_t^f, p_t\right\}$, a dynamic equilibrium is a sequence of masses $\left\{m_t^b, m_t^g, m_t^f\right\}$, equity e_t, and loan rates $\left\{R_t^g, R_t^b\right\}$ such that z follows (14), banks sort optimally, bank equity e follows Assumption 5, and markets clear at all times.

Next, we describe how policies are determined depending on policymakers' objectives, and characterize the resulting equilibria.

4.2 Policymakers' Horizon and Optimal Policy

We assume that policies $\{p_t, R_t^f\}_{t\geq 0}$ are set to maximize the present discounted value of aggregate output

$$\max_{\left\{p_t, R_t^f\right\}} \sum_t \beta^t Y_t.$$

The intertemporal congestion externalities (14) lead to an intertemporal policy trade-off: policies that stimulate current lending have short-term benefits, but possible long-term costs caused by zombie lending. The discount factor β determines the relative weights on current and future output, and reflects policymakers' horizon. We assume that the fiscal costs of the forbearance subsidy are small and thus only relevant to break ties: if different combinations of p and R^f can achieve the same level of output, policymakers choose the combination with the lowest forbearance p.

We interpret a low policy horizon as arising from term limits, regulatory capture by incumbents, or reputational concerns that create a wedge between the public and regulatory objectives, as analyzed, for example, by Boot and Thakor (1993). Another interpretation is to think of the short policy horizon as a reflection of policymakers' inability to implement policies that have immediate fiscal costs. Fiscally constrained governments tend to help financial institutions in distress by deploying guarantees and/or engaging in some form of forbearance, rather than promptly intervening with capital injections or restructuring and resolution measures (Acharya et al., 2021).

While Proposition 1 characterizes which equilibrium allocations are *feasible* as policies (R^f, p)

[18] We also assume that firms are focused on short-term profits, hence their entry and exit decisions are the same as in the static model; unlike the assumption on the bank side which simplifies the dynamic model considerably, the assumption on firms is mostly for exposition and can be relaxed to allow for forward-looking firms, see Appendix C.4.

vary, the following result derives the *optimal* policy as a function of policymakers' horizon. We focus on the following two polar policy regimes, corresponding to high or low policy discount factors β, that are sufficient to illustrate the economic mechanisms. Between these two extremes, intermediate values of β would lead to interior policy choices that trade off the marginal congestion externality incurred in future periods against the current output loss from reducing bank lending.

Long policy horizon: No Zombie lending policy. We first consider a policymaker that places a sufficiently high weight β on future outcomes so that avoiding any congestion externality represents the optimal policy. Given state variables z_t and e_t, the *No Zombie lending (NZ) forbearance policy*

$$p_t = p^{NZ}(z_t, e_t)$$

is defined as the smallest p that maximizes bank lending subject to the no-zombie lending constraint $m_t^b = 0$.

Short policy horizon: Myopic policy. Conversely, a policymaker with a sufficiently low weight β on future outcomes will choose to minimize the short-term output loss at each point in time by maximizing total bank lending. This might require allowing zombie lending in equilibrium, even if doing so jeopardizes future output. Specifically, the *myopic forbearance policy*

$$p_t = p^m(z_t, e_t)$$

is the smallest p that maximizes total bank lending $(m_t^g + m_t^b)$.

The NZ and myopic interest rate policies are identical, as both objectives lead policymakers to set $R_t^f = \bar{R}^f(0; z_t)$ for small shocks $z_t \leq \underline{z}$, and $R_t^f = R_{\min}^f$ for moderate and large shocks $z_t > \underline{z}$ where the threshold $\underline{z}$ and the function $\bar{R}^f$ are defined in Proposition 1, which nests this dynamic environment. However, the NZ and myopic forbearance policies differ dramatically in the case of large shocks.

Proposition 2 (Optimal policy)**.** *For small and moderate shocks $z_t \leq \bar{z}$, the myopic forbearance policy p^m and the NZ forbearance policy p^{NZ} are equal, both weakly increasing in z_t.*

For large shocks $z_t > \bar{z}$, the myopic forbearance policy p^m increases with z_t, while the NZ forbearance policy p^{NZ} ***decreases*** *with z_t.*

As in Proposition 1 (i)–(ii), for shocks $z_t < \bar{z}$, the economy can achieve its potential $Y = Y^*(z)$. If the economy is hit by severe aggregate shocks $z > \bar{z}$, conventional monetary policy is still constrained by the effective lower bound, but now the optimal forbearance needs to balance two opposite forces. On the one hand, an increase in regulatory forbearance (higher p) spurs lending at the expense of investment in safe assets. On the other hand, if forbearance p is too high, poorly-capitalized banks engage in zombie lending, which creates congestion externalities.

The key distinction between policy regimes becomes visible following large shocks. The optimal myopic forbearance p^m increases with the size of the shock z_t: larger shocks are accommodated with a higher p, until p reaches its upper bound of 1.

By contrast, the No Zombie lending forbearance policy p^{NZ} is *non-monotonic* in the size of the shock. As a result, for large enough shocks, avoiding zombie lending requires policymakers to *reduce* the degree of regulatory forbearance p as shock size z increases. Aggregate output Y necessarily falls short of its potential $Y^*(z)$. Put differently, when severe aggregate shocks hit the economy, policy should allow healthy banks to cut lending and, e.g., rebalance towards non-loan safe assets, rather than "pushing on a string": more accommodation would only trigger more zombie lending by the poorly-capitalized banks. The key point is that policy aiming at containing zombie lending should not just keep $p(z)$ constant at $p(\bar{z})$ for shocks $z > \bar{z}$, but that even holding p fixed at this high level would trigger more zombie lending, hence p should actually be reduced when z is larger. Our result thus shows that there exists a "reversal" level of forbearance p above which further accommodation becomes harmful, a counterpart to the "reversal interest rate" for conventional monetary policy (Abadi et al., 2023).

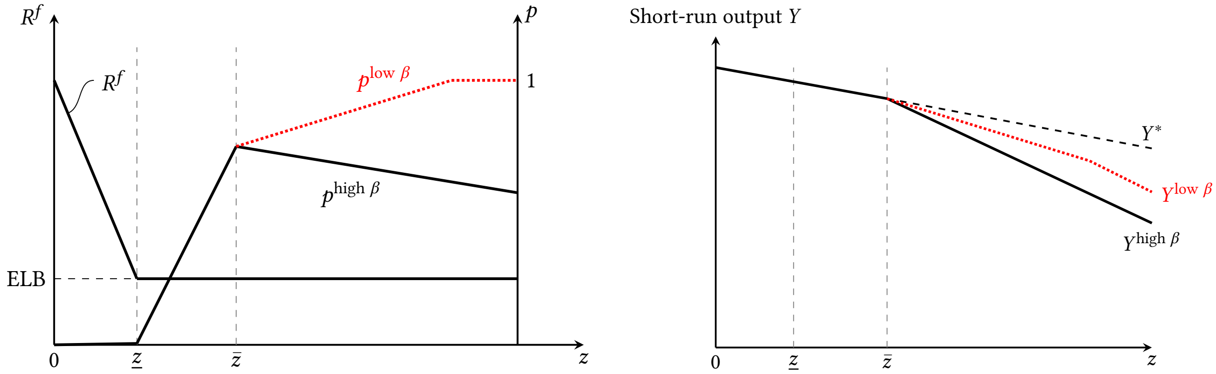
Figure 5 illustrates the result by contrasting the two policy regimes and the resulting levels of *short-run* output. The left panel shows how the high-β policy backtracks and reduces forbearance p as shocks grow larger, whereas the low-β policy keeps accommodating more and more until reaching the upper bound $p = 1$. The right panel shows the short-run cost of preventing zombie lending: it lowers current output Y_t relative to the myopic policy, as some healthy banks end up investing in safe assets instead of lending. Policymakers with a high enough discount factor β are willing to bear this cost, in order to avoid congestion externalities.

4.3 Persistence of Output Losses under Different Policy Regimes

We now turn to our main dynamic experiment and result: transitory shocks can generate permanent output losses and policy traps due to the dynamic externalities imposed by zombie lending. Suppose the economy starts in a "good" steady state in which the zero lower bound is not binding: $R^f = (\theta^g \bar{y}^g - \bar{\epsilon}) > R^f_{\min}$. Thus no forbearance is needed ($p = 0$), there is no zombie lending, aggregate output is $Y = Y^*$, and equity is $e_0 = \frac{\iota}{1-(1-\rho)R^f}$.

At date-0 a transitory shock $z_0 > 0$ hits healthy firms: $y^g_0 = \bar{y}^g\,(1 - z_0)$. We contrast the paths of the economy under the No Zombie lending and myopic policy rules. We restrict attention to large shocks $z_0 > \bar{z}$, the only ones for which the NZ and myopic policies differ. Under both policy

Figure 5: Optimal policy as a function of shock z under high-β and low-β policy regimes.



Note: The left panel illustrates the optimal joint policy response (R^f and p) when β is high (solid black line) and when β is low (red dotted line) as a function of the size of the shock z. The right panel illustrates the aggregate output Y under the policy regimes and potential output Y^* (dashed line).

stances, the optimal conventional policy implies setting the minimal risk-free rate $R_t^f = R_{\min}^f$ as long as $z_t > \bar{z}$.[19] However, the paths of p_t will differ across policy stances markedly. In fact, we show that seemingly small within-period differences between the NZ and myopic policies can lead to completely different long-run outcomes.

No Zombie Lending Policy: Transitory Recession and Full Recovery. Under the NZ policy (high β), congestion externalities never materialize since there is no zombie lending in any period in equilibrium. Following a large shock $z_0 > \bar{z}$, policymakers set $R_0^f = R_{\min}^f$ and $p_0 > 0$, and output falls below its potential, that is $Y_0 < Y^*(z_0)$.

Since there are no further exogenous shocks and no congestion externalities induced by zombie lending, z reverts immediately to zero starting from date-1. For all $t \geq 1$, the interest rate recovers to its pre-shock level $R_t^f = \theta^g \bar{y}^g - \bar{\epsilon} > R_{\min}^f$, there is no forbearance ($p_t = 0$), and output equals $Y_t = Y^*(z_0 = 0)$.

The date-0 recession is "V-shaped": it can be quite deep, but remains short-lived. Output recovers immediately from the transitory aggregate shock. This is in stark contrast with what happens under the optimal myopic policy.

Myopic Policy: Policy Trap and Sclerosis. Under a myopic policy regime (low β), policymakers accommodate using regulatory forbearance, and allow some zombie lending in spite of

[19] Recall that we abstract from the aggregate demand channel of monetary policy, by which a lower rate and higher aggregate demand could dampen congestion externalities, thus making α lower in states such that the ELB is not binding. Accordingly, the parameter α should be interpreted as the strength of congestion externalities *conditional on the ELB binding*, which is necessary for zombie lending to emerge and thus for α to be relevant. The binding ELB also prevents monetary policy from dampening congestion through the aggregate demand channel.

the potential long-term costs on healthy firms.

From (14), z follows a dynamical process $z_{t+1} = \alpha\lambda H\left(\theta^b\left(\bar{y}^b - R_t^b\right)\right)$, where the date-$t$ equilibrium loan rate R_t^b is determined by the myopic policy $p^m(z_t, e_t)$. In particular, since $z_0 > \bar{z}$, the date-0 mass of zombies m_0^b will be positive, which hurts the profitability of good firms at date-1 through $z_1 > 0$, and so on. The myopic policy creates an endogenous "reverse hysteresis" channel: current accommodation leads to endogenous persistence of the initial shock, that worsens when congestion externalities α are larger. If α is high enough, the myopic policy response to a sufficiently severe transitory shock z_0 pushes the economy to a steady state with *permanently* lower output, defined as follows:

Definition 1 (Sclerosis steady state)**.** A sclerosis steady state is a steady state equilibrium with the interest rate at the ELB ($R^f = R^f_{\min}$), permanent forbearance ($p > 0$), and permanent output losses ($z > 0$).

Sclerosis is associated with a *policy trap*: present policies aimed at minimizing short-term losses tie the hands of future policymakers through persistent congestion externalities. As a result, the economy may be stuck at the ELB forever even though the natural interest rate would recover to a positive level under a different policy rule. We can now express our main dynamic result:

Proposition 3 (Myopic policy and sclerosis)**.** *Suppose that congestion externalities are large enough, $\alpha \geq \bar{\alpha}$, for some positive $\bar{\alpha}$ (given in equation (A.6) in Appendix D . Then,*

1. *There exists a unique stable sclerosis steady state. It features maximal forbearance $p = 1$ and permanent output losses $z_\infty > 0$ such that*

$$z_\infty = \alpha\lambda H\left(\theta^b\bar{y}^b - R^f_{\min} + (1 - e_\infty)\left(1 - \theta^b\right)\right)$$

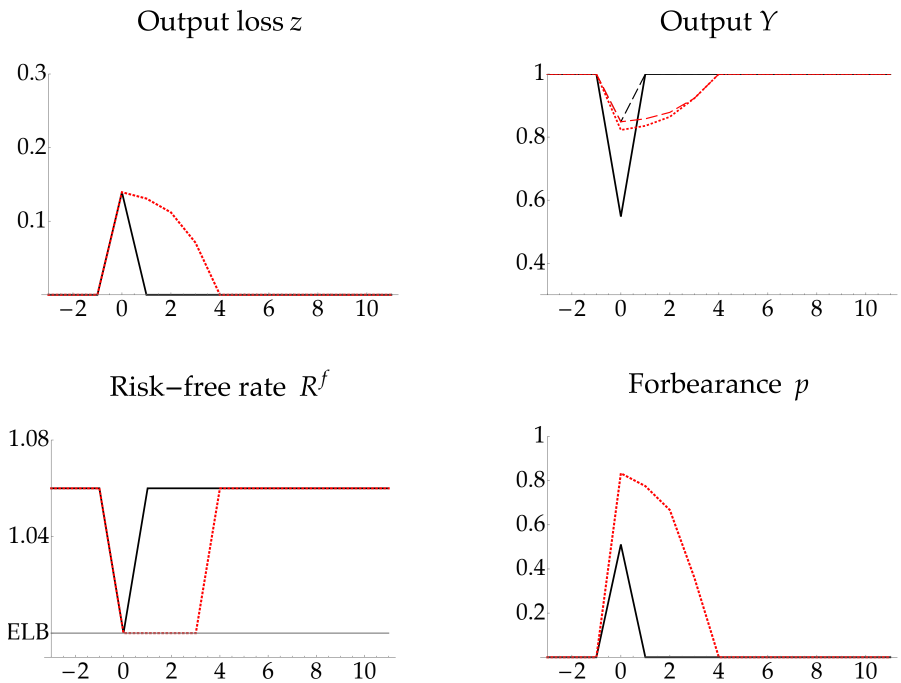
where $e_\infty = \frac{\iota}{1-(1-\rho)R^f_{\min}} < e_0$ denotes steady state bank equity.

2. *There exists a threshold $z^*(\alpha)$ decreasing in α such that for initial shocks $z_0 < z^*(\alpha)$, the economy converges to the no-zombie steady state, while for initial shocks $z_0 > z^*(\alpha)$ the economy converges to the stable sclerosis steady state.*

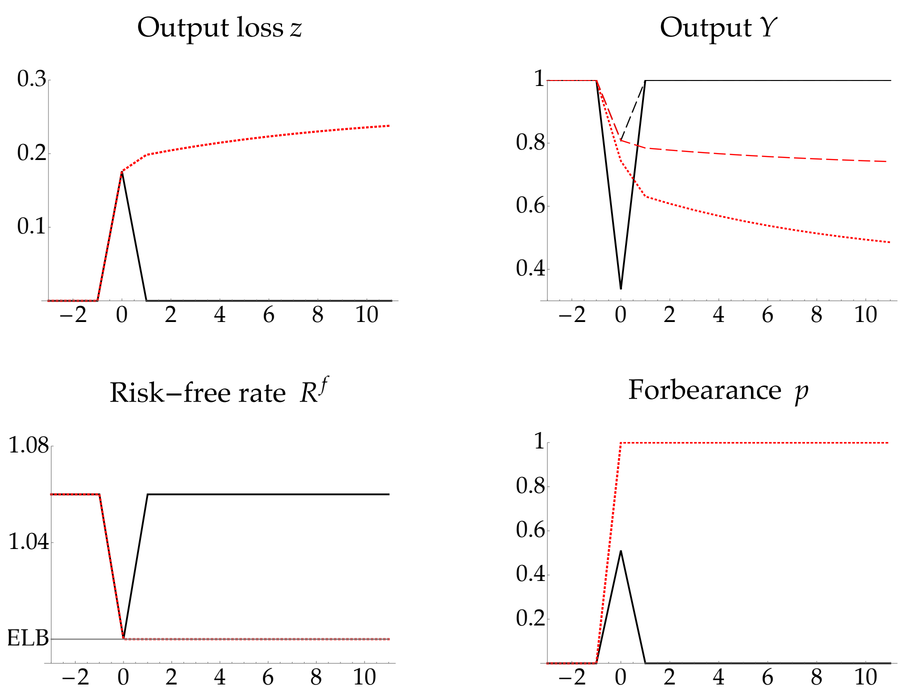
Figure 6 displays the impulse responses of output losses z_t, aggregate output Y_t, and the optimal policies R_t^f and p_t under the two policy regimes (NZ policy, in black, and myopic policy, in red). Panel A shows equilibrium paths following a shock z_0 that is above $\bar{z}$ but below the threshold $z^*(\alpha)$ defined in Proposition 3. The ELB binds at the time of the shock under both policy regimes. Forbearance also increases in both cases, but by much more under the myopic policy.

Figure 6: Impulse responses under the NZ policy (black) and the myopic policy (red dotted).

Panel A: Small initial shock $z_0 < z^*(\alpha)$



Panel B: Large initial shock $z_0 > z^*(\alpha)$



Note: Output losses z_t, aggregate output Y_t and potential output Y_t^* (dashed lines), and the optimal policies R_t^f and p_t under the two policy regimes (No Zombie lending, in solid black lines, and the myopic policy, in red dotted lines). Panel A: small initial shock $z_0 < z^*(\alpha)$. Panel B: large initial shock $z_0 > z^*(\alpha)$.

As a result, output drops sharply under the NZ policy, but recovers immediately to its pre-shock level at $t = 1$. The interest rate also recovers after the initial shock. By contrast, the myopic policy succeeds in stabilizing date-0 output at a higher level thanks to the more generous forbearance policy that keeps some zombie firms alive. However, the stabilization of short-term output comes at the cost of a protracted output loss for multiple periods, with interest rates stuck at the ELB and forbearance p at a high level. While this path features endogenous persistence of the initial shock, the economy eventually converges back to its pre-shock steady state.

Panel B shows the equilibrium paths following a large initial shock $z_0 > z^*(\alpha)$. While initially the paths under the two policy regimes are similar to the ones following a smaller initial shock, they soon start diverging from each other. Like before, the economy experiences a sharp but short-lived output loss under the No Zombie lending regime. But under the myopic policy, the date-1 output loss z_1 stemming from congestion externalities is even larger than the initial shock z_0. This puts the economy on a dangerous path: at $t = 1$, the endogenously weaker fundamentals induce myopic policymakers to accommodate even further, by keeping interest rates as low as possible and allowing even higher forbearance ($p_1^m > p_0^m$), which, in turn, hurts healthy firms' profitability at $t = 2$, and so on. For a while, this myopic policy manages to stabilize output Y_t close to potential output Y_t^*, albeit with a major side effect: potential output itself (dashed red line) falls subsequently because the presence of zombie firms reduces healthy firms' output. Moreover, once zombie lending becomes a permanent feature of the economy, all policymakers can do is exert maximal accommodation to stimulate output ($R^f = R^f_{\min}$, $p^m = 1$), which however is not sufficient to prevent a large gap between output and its potential. The economy snowballs towards sclerosis and monetary policy is trapped.

Remark 3. Our specification (14) assumes that the date-t zombie share only affects healthy firms at date $t + 1$. This is the simplest configuration that leads to policy traps, and highlights the most relevant economic forces. More generally, the key aspect is that zombie lending has *persistent* congestion externalities. Clearly, the risk of falling into a policy trap would be reinforced if zombie lending at t also affected healthy firms at $t+2, t+3, \ldots$ as myopic policymakers would put an even lower welfare weight on these future periods. But more interestingly, zombie lending at t could also hurt healthy firms in the current period t. Adding such contemporaneous congestion externalities would not change the logic behind our results, as long as the effects are not purely contemporaneous. Contemporaneous congestion externalities lower the immediate net benefit of using forbearance p, by hurting aggregate output in a similar way as the contemporaneous misallocation between G and B firms that is already included in our analysis. As long as zombie lending also has an effect on *future* profitability, there remains a trade-off between current and future output.

4.4 Exiting the Policy Trap

Proposition 3 characterizes the steady state for given fundamentals. Can an economy exit a policy trap and recover from sclerosis? An obvious way to exit the trap is to appoint a more conservative or long-horizon policymaker, as in the literature on inflation bias (Rogoff, 1985). In our context, this would correspond, for instance, to switching from a myopic policy regime to a NZ policy regime. This is isomorphic to our earlier example; the only difference is that the initial level of z_0 is not due to an exogenous shock, but caused by the congestion externalities in the sclerosis steady state (that is, $z_0 = \alpha m^b_\infty$ where m^b_∞ is the steady state mass of zombie firms). At date-0, the NZ policy reduces forbearance p sufficiently to induce all zombie firms to exit. This causes a sharp but transitory recession, and allows a clean start at $t = 1$.

More interestingly, suppose we maintain the myopic policy regime but change the initial conditions. We consider two experiments: an improvement in fundamentals and a bank recapitalization. In each case the economy starts from a policy trap with $R^f = R^f_{\min}$ and $p = 1$, hence from the associated sclerosis steady state with output losses z_∞.

Improvement in fundamentals $\theta^g \bar{y}^g$. Fundamentals affect the threshold $z^*(\alpha)$ in Proposition 3. For instance, z^* is increasing in $\theta^g \bar{y}^g$ and decreasing in the churn parameter λ. A low growth environment is thus particularly dangerous: not only is potential output already low, but the economy is also more fragile and output is more susceptible to fall below potential due to zombie lending. Conversely, an improvement in $\theta^g \bar{y}^g$ can help the economy exit the policy trap and sclerosis; but once the economy is in a trap it needs a large shift in fundamentals. Figure A.2, panel A, shows an example with a sufficiently large increase in $\bar{y}^g$. Lending to good firms becomes relatively more attractive, which again sets the economy on a virtuous path towards a good steady state.

Bank recapitalization. Suppose next that at $t = 0$ the government recapitalizes the banking sector. In our model this corresponds to an exogenous increase of bank equity from e_∞ to a higher level e_0. A small intervention will only have a transitory effect. But a large recapitalization can help the economy exit the policy trap. Figure A.2, panel B, shows such an example. Output falls at the time of the recapitalization: z_0 is still high initially, hence lending opportunities are still weak and the higher equity induces a subset of banks to invest in safe assets. However, a better capitalized banking sector implies that risk-shifting incentives and zombie lending fall, which triggers a virtuous feedback loop: congestion externalities are lower in the next period, which makes lending to good firms more attractive, and so on. Over time, the economy can recover and converge back to the "good" steady state with no zombie lending, high interest rate, no forbearance, and high output ($z = 0$).

Historically, recapitalizations of the banking sector by the government—either directly through capital injection or indirectly at times through the establishment of "bad banks"—have been the most effective antidote to the proliferation of zombie lending. Noteworthy cases of successful recapitalization efforts through asset purchases by a "bad bank" are the establishment of the Korea Asset Management Corporation (KAMCO) in South Korea following the 1997–1998 financial crisis and the establishment of the Resolution Trust Corporation (RTC) in the U.S. following the Savings and Loans crisis in the 1980s.

Despite their efficacy, decisive interventions have been more the exception than the norm. In both Japan and southern Europe, for example, despite policymakers' recapitalization efforts the capitalization of the banking system effectively shrunk (see Table 1) or did not increase enough to cope with the aggregate shocks hitting the economy. Furthermore, as shown empirically by Peek and Rosengren (2005) and Giannetti and Simonov (2013) in Japan, and Acharya et al. (2019) in Europe, the timid recapitalization measures put in place were unable to prevent the spread of zombie lending, as they were unable to effectively recapitalize the weakest financial institutions.[20]

5 Extensions: Evergreening and Capital Requirements

In this section, we consider two natural extensions of the model and how they interact. We first analyze how the distribution of bank equity itself responds to policies. How do the previous conclusions change when banks can choose their capital structure? We then show that costs of switching from old to new borrowers create an additional and complementary "evergreening" motive for zombie lending. Finally, we ask whether regulators can solve the misallocation of credit by forcing banks to raise more capital. The answer turns out to depend on the strength of the evergreening channel: capital requirements can prevent zombie lending when switching costs are low, but can backfire and amplify zombie lending when switching costs are high.

5.1 Bank Equity Issuance

Our framework highlights how an undercapitalized banking sector constrains policymakers, thereby making the economy more fragile in response to fundamental shocks. In the baseline model, we made this point taking the distribution of bank equity as given. The static environment described in Section 3 can be extended by allowing banks to issue equity.

[20] The government may lack the ability or the willingness to recapitalize the banks sufficiently in the short run, due to fiscal constraints or from the fact that just like the instruments we already considered, bank recapitalizations are subject to policy myopia. Hence a government may optimally delay injecting equity if the costs of doing so (e.g., political backlash, heightened sovereign credit risk) are borne immediately while the benefits only materialize over time. Studying the optimal mix of policies as a function of the government's fiscal capacity is an important extension for future research.

Suppose banks start with a pre-issuance equity level e before deciding jointly how much equity they want to issue ($\Delta \geq 0$) and which asset class $j \in \{g, b, f\}$ to invest in. Bank e solves:

$$\max_{j \in \{g,b,f\},\Delta} \theta^j \left(R^j - \tilde{R}^j (1 - e - \Delta)\right) - \kappa(\Delta)$$

where the equity issuance cost κ is quadratic. Conditional on choosing project j, the optimal equity issuance is

$$\Delta^j = (\kappa')^{-1} \left(\theta^j \tilde{R}^j\right). \tag{15}$$

Accounting for their optimal equity issuance decisions, banks sort themselves into projects j. The optimal equity issuance policy does not depend directly on a bank's pre-issuance equity e because the cost κ is additive. Yet, in equilibrium, the amount of issuance issued by different banks does vary with e. Intuitively, e determines banks' asset choices, which in turn affect the optimal equity issuance. Hence risk-shifting acts as a "double whammy": banks with a lower initial level of capitalization also issue less equity, anticipating that they will be the ones lending to relatively riskier borrowers. By contrast, banks that start with high capital internalize that they will be the ones lending to safer borrowers or even investing in safe assets, and thus have incentives to issue more equity.[21]

5.2 Relationship Lending and Evergreening

Empirically, an important source of zombie lending stems from weak banks willing to "extend and pretend", by rolling over loans at subsidized rates to *legacy* borrowers that should be declared non-performing.[22] We can incorporate this element by breaking the symmetry between old and new borrowers in a parsimonious way. Banks start the model matched with a legacy borrower. A random fraction λ of banks have an outstanding B borrower, and the remaining $(1 - \lambda)$ banks have an outstanding G borrower.

Assumption 6. *If a bank switches from its legacy B borrower to a new borrower, its equity falls from e to $(e - \delta)$, for some switching cost $\delta \geq 0$.*

The presence of a positive switching cost will prolong some borrower-lender relationships. The switching cost δ captures first and foremost the loss provisions that banks must put aside

[21]Equity issuance leads to an additional effect of interest rates. In Appendix C.1, we show that conventional monetary policy accommodation can increase zombie lending, by reducing banks' incentives to issue equity relative to debt.

[22]The empirical literature documents that the credit extended by under-capitalized banks to poorly-performing firms is granted at rates lower than justified by the credit risk of these borrowers. The subsidized nature of these credit transactions is one of the quintessential features of zombie lending. For this reason, Caballero et al. (2008) and most of the following literature use subsidized bank credit as a criterion to empirically identify zombie firms, and finds that their borrowing rates are often as low as those charged to the safest borrowers (Acharya et al. 2019; Schivardi et al. 2021). For incentives to postpone loss recognition, see also Blattner et al. (2023).

when declaring loans as non-performing; but δ is also meant to include the screening effort that the bank must spend when creating a relationship with a new borrower.[23] Indeed, banks will never want to switch from a legacy B borrower to a new B borrower, so the only switches that could be observed in equilibrium are towards a new G borrower. This presumes some costly information gathering to learn which borrowers are indeed good. Our results extend to a more general switching cost structure, with costs δ_{ij} depending on both the legacy match i and the new match j.[24]

The distinction between legacy and new borrowers requires us to model lending relationships. First, we need to determine which outstanding borrower-lender pairs are continued, and which of them are broken so that the bank can lend to a new borrower. Second, we must specify the loan rates offered to legacy borrowers, as those can differ from the rates offered to new borrowers due to the hold-up problem. We assume that the borrower-lender pair separates if and only if the joint surplus of remaining matched is lower than the joint surplus outside the relationship, in which case the bank lends to a new borrower and the borrower seeks to borrow from a new bank. In the case of continuation, the bank makes a take-it-or-leave-it offer to the firm, hence legacy and new B borrowers pay the same rate R^b. Positive bargaining power for the firm would decrease the rate to legacy borrowers to $\bar{R}^b < R^b$.[25]

Lemma 2 in the Appendix extends our sorting result (Lemma 1) to show that relationship lending induces a second source of sorting, in addition to risk-shifting: a positive switching cost δ increases zombie lending at the bottom of the bank equity distribution. Some banks with capitalization above e^* choose to roll over the loan to their legacy B borrower in order to economize the cost δ, even though given their capital they would lend to a new G borrower absent this pre-existing lending relationship. The most interesting implications of this "evergreening" channel, in the next section, arise when we consider how it interacts with capital requirements.

[23]The efficiency of the debt resolution system affects the cost of insolvencies and the magnitudes of loan loss provisions. Therefore, bankruptcy reforms may alleviate the incidence of zombie lending (Becker and Ivashina, 2022). However, the benefits of such reforms depend on the level of bank capitalization, which determines the strength of banks' zombie-lending incentives (Kulkarni et al., 2021).

[24]Darmouni (2020) estimates the impact of switching costs due to adverse selection and finds that switching costs are lower for borrowers whose bank is impaired. Absent switching costs, type-G borrowers who were previously matched with a weakly-capitalized bank (with equity below e^*) become part of the loan demand that must be met by remaining well-capitalized banks (with equity between e^* and e^{**}), as in the baseline model's market-clearing condition (10). This reallocation mitigates the decline in borrowing and activity relative to a counterfactual in which these type-G firms would be simply unable to switch while their impaired relationship bank shifts its lending to zombies. However, in equilibrium, the scarcity of well-capitalized banks relative to loan demand by good firms is reflected in a higher equilibrium rate R^g , which induces some of the type-G borrowers to exit the market even absent switching costs.

[25]Low loan rates are also a way to ensure repayment of the zombie loans when the default probability depends on loan rates, as in the literature on credit rationing (Stiglitz and Weiss, 1981). Faria E Castro et al. (2021) focus on this mechanism and find empirical support in U.S. data.

5.3 Capital Requirements

A key policy question in the face of prevalent zombie lending is whether tightening capital requirements is a good remedy. In light of our sorting result, improving the distribution of bank capital appears to be a natural solution to tilt credit allocation towards safer and more productive lending. The counterargument is that tighter regulation may backfire, by generating incentives for banks to extend and pretend out of fear of having to recapitalize to satisfy the requirement. We now allow for both equity issuance and positive switching costs $\delta > 0$ as described in the two previous subsections. In addition, the regulator can impose a capital requirement: post-issuance equity e' must remain above a floor $\hat{e}$. Consistent with Assumption 3, the capital requirement does not depend on banks' asset risk. Our main result is that if switching costs δ are high enough, and capital requirements are already strict, then tightening regulation further (increasing $\hat{e}$) can *increase* zombie lending through the evergreening channel.

Throughout this section we keep other policies R^f and p fixed (for instance, because the economy has already fallen into a dynamic policy trap) to focus on the effect of capital requirements. It is convenient to define

$$\sigma(e') = \theta^g \left[R^g - \tilde{R}^g(1-e')\right] - \theta^b \left[R^b - \tilde{R}^b(1-e')\right]$$

which represents the payoff difference between lending to a G firm and a B firm (ignoring any equity issuance costs) for a bank with post-issuance equity e'. We restrict attention to parameters such that if the regulator sets a capital requirement low enough that it does not bind even for the bank with the lowest capital $e = e_{\min}$ then that bank prefers to lend to a type-B firm. Formally,

$$\sigma(\hat{e}) < \kappa(\hat{e} - e_{\min} + \delta) - \kappa(\hat{e} - e_{\min}). \tag{16}$$

for all $\hat{e} \leq \min\left\{e_{\min} + \Delta^b, e_{\min} + \Delta^g - \delta\right\}$. Condition (16) means that there is indeed some zombie lending absent capital requirements. This is the only interesting case to consider, as otherwise capital requirements would be irrelevant for credit allocation and aggregate output, and introducing them would only create a deadweight loss in terms of equity issuance costs.

In the absence of any switching costs ($\delta = 0$), it is straightforward to deter zombie lending completely: the regulator can just impose a capital requirement $\hat{e}$ that is sufficiently high, and more precisely, above the equity threshold e^* in an equilibrium without zombie lending. Intuitively, the case of low enough switching costs must be similar to when there are no switching costs at all. Indeed, we find that for low enough δ, there always exists a sufficiently tight capital requirement $\hat{e}^{NZ}$ (where NZ stands for No Zombie lending) that suppresses zombie lending altogether ($m^b = 0$). Does this mean that we can always solve the zombie lending problem using capital regulation? We find that the answer is no. Surprisingly, when the switching cost δ is high enough, no capital requirement can deter zombie lending completely: some positive equilibrium zombie lending is inevitable. In fact, the stronger result is that increasing capital requirements beyond some level can even backfire, by further encouraging zombie lending:

Proposition 4. *Let $\hat{e}^{NZ}$ solve $\sigma\left(\hat{e}^{NZ}\right) = \kappa\left(\hat{e}^{NZ} - e_{\min} + \delta\right) - \kappa\left(\hat{e}^{NZ} - e_{\min}\right)$.*

- *If $\delta < \Delta^g - \Delta^b$, then any capital requirement above $\hat{e}^{NZ}$ suppresses zombie lending.*
- *If $\delta > \Delta^g - \Delta^b$, no capital requirement can suppress zombie lending completely. There exists a capital requirement $\hat{e}^*$ given by (A.10) that minimizes zombie lending, and such that increasing capital requirements above that level strictly increases zombie lending.*

The case of a high δ captures the evergreening motive of zombie lending. The intuition is as follows. A bank compares two options: recognizing the loss at a cost δ, which allows a fresh start with a new G borrower, or rolling over the loan to the legacy B borrower. The second option allows to economize the switching cost δ, and becomes especially attractive with a high δ. Switching to a new borrower brings an additional cost if the bank is already poorly-capitalized: its equity will drop to $e - \delta$, which forces the bank to undertake a costly recapitalization to satisfy the requirement $\hat{e}$. Thus there is a set of banks for which the cost of recapitalization acts as an additional motive to roll over the zombie loan, and the set of such banks expands as the capital requirement $\hat{e}$ increases.[26]

Proposition 4 highlights a subtle link between capital requirements and zombie lending. In particular, both cases are likely to be relevant because the switching cost δ and the threshold $\bar{\delta}$ depend on the country and industry of the borrower, and the history of the lending relationship. For instance, δ will be higher when there is more asymmetric information between banks and potential new borrowers, and when zombie debt has been accumulating for a longer time (as this increases the losses that banks would eventually recognize). Just like in the dynamic model, the longer policymakers wait before tackling the zombie lending problem, the harder it becomes to solve it. The case of high switching costs δ is consistent with some of the empirical evidence on the unintended consequences of capital requirements, for instance following the increase in capital requirements by the European Banking Authority in 2011, as documented by Blattner, Farinha and Rebelo (2023). Relatedly, Chopra, Subramanian and Tantri (2020) show that other regulatory actions such as ex-post bank cleanups can also trigger zombie lending if they are not accompanied by ex-ante bank recapitalization.

[26]In this analysis we are holding p fixed. Proposition 4 shows that the mapping between an increase in p and forbearance in the form of lower capital requirements $\hat{e}$, discussed in Section 3, is more complex once we introduce relationship lending. If δ is low, then both a higher p and a lower $\hat{e}$ increase zombie lending. If δ and the initial capital requirement $\hat{e}$ are sufficiently high, however, lowering $\hat{e}$ decreases zombie lending, hence it becomes more important to distinguish between government guarantees and forbearance in capital regulation.

6 Conclusion

In this paper we develop a theoretical framework with heterogeneous firms and banks to study the complex feedback loop between bank under-capitalization, credit misallocation due to zombie lending, accommodative monetary policy and regulatory forbearance, and adverse aggregate outcomes such as permanent losses in growth and productivity. Our model generates linkages that are consistent with several features of aggregate and banking sector data characterizing the "lost decade" of Japan following its real estate crisis, and more recently, the aftermath of the sovereign debt crisis in southern Europe. Viewed through the lens of our model, policymakers should avoid excessively "pushing on a string" of forbearance towards banks precisely when economies are hit by large shocks, as this can lead to delayed recoveries and persistent output losses.

Our results have salient policy implications and suggest several directions for further research. A focal point of our model is the interaction between monetary and banking policy with the fundamentals of firms and banks in the economy, potentially converting transitory shocks into accommodative policy traps and lost decades. This risk is receiving increasing attention in the aftermath of the recent pandemic, especially in the case of China, where the adoption of lenient regulatory stances toward financial intermediaries has inevitably raised the specter of long-term economic stagnation from a zombification of the economy. Further empirical work is needed to better inform policy makers coping with large shocks on how to optimally resolve the trade-off between short-term versus long-term losses.

Finally, our study also suggests how properly designed capital injections in the banking sector can effectively tackle the incentive problems at the root of zombie lending. However, it is assumed in our framework, as in the real world, that governments lack the willingness or the ability to recapitalize the banks sufficiently in a timely fashion. This can be due to policy myopia or to binding fiscal constraints. Studying the optimal mix of macro-financial policies as a function of the government's fiscal capacity also remains an open question and an important extension for future research.

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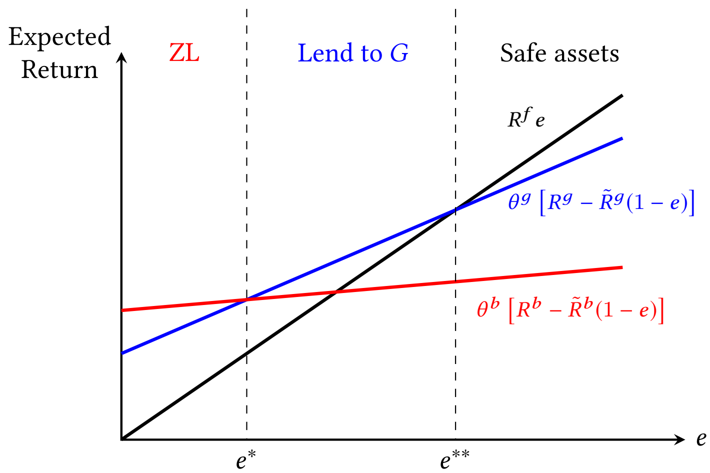
Online Appendix

A Additional Figures and Tables

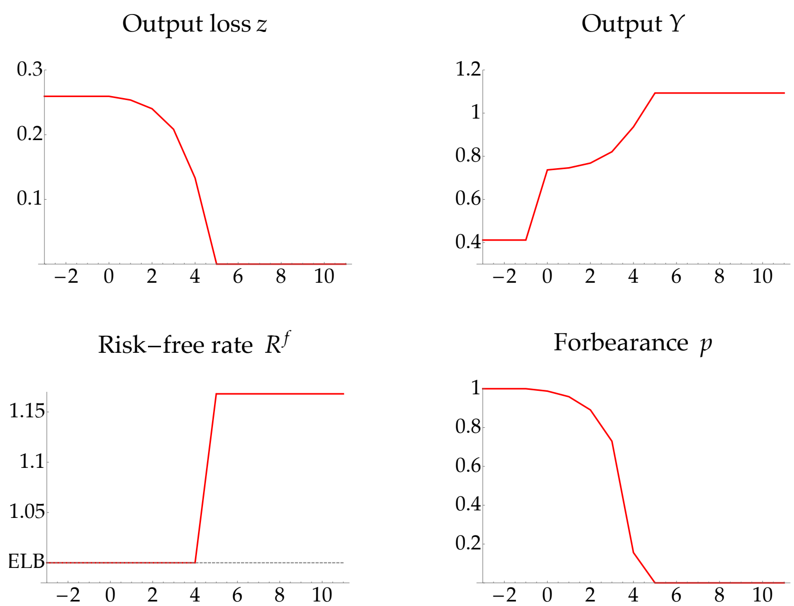
Figure A.1: Optimal asset choice as a function of bank capital e.



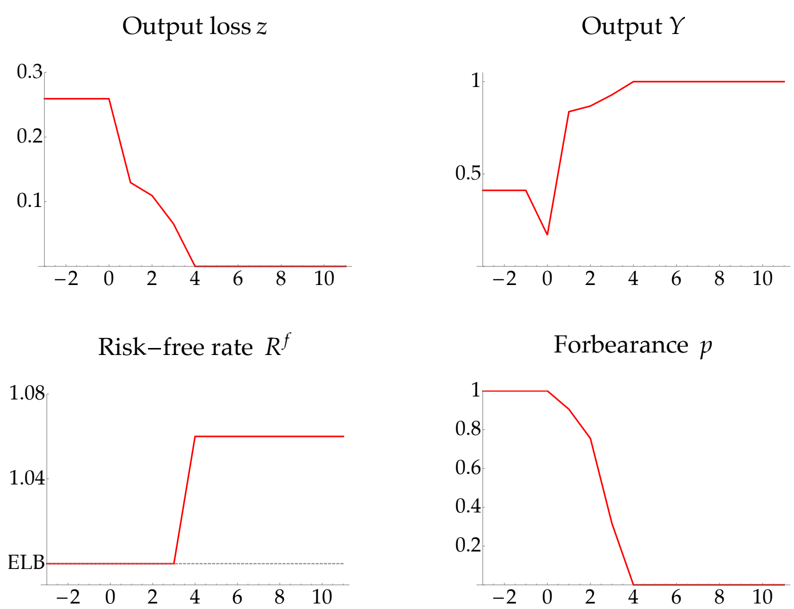
Note: Each line shows the expected profit from investing in asset i, $\theta^i \left[R^i - \tilde{R}^i (1-e)\right]$, as a function of e. The red line shows $i = b$ (lending to a type B firm). The blue line shows $i = g$ (lending to a type G firm). The black line shows $i = f$ (investing in safe assets).

Figure A.2: Impulse responses under the myopic policy.

Panel A: Improvement in $\bar{y}^g$ at $t = 0$.



Panel B: Bank recapitalization at $t = 0$.



Note: Output losses z_t, aggregate output Y_t, and the optimal policies R_t^f and p_t under the myopic policy regime. Panel A: permanent increase in $\bar{y}^g$ at $t = 0$. Panel B: bank recapitalization at $t = 0$.

B Data Sources

Japan. Data on the aggregate and industry-level zombie share (defined as the asset-weighted share of firms classified as zombies) are taken from Caballero et al. (2008). Aggregate real GDP data come from the Penn World Table, and aggregate and industry-level total factor productivity (TFP) data are sourced from Asia KLEMS. The capitalization of the banking system is defined as the ratio of total adjusted core capital to total assets of Japanese banks. We construct this variable using data from Fukao (2003, 2007). Adjusted core capital is defined as (Core capital + Unrealized capital gains and losses − Estimated under-reserves − Deferred Tax Assets). Core capital is Tier 1 capital. The computation of unrealized capital gains and losses uses the calculations suggested by Fukao (2003, 2007) and is equal to 0.6×(Market Value Shares − Book Value Shares). Estimated under-reserves are defined as the difference between loan-loss reserves and estimated loan losses. Deferred tax assets are calculated according to the methodology in Fukao (2003, 2007).

Italy, Spain, and Portugal. Data on the aggregate, industry-, and country–industry-level zombie share (defined as the asset-weighted share of firms classified as zombies) and on country–industry profitability are taken from Acharya et al. (2022). Profitability is measured as the asset-weighted average EBITDA-to-assets ratio within each country–industry pair. To produce Panel A of Figure 2, this average is computed across all firms in a given country–industry pair; to estimate the local projections in Figure 3, this average is computed only across non-zombie firms in a given country–industry pair. Aggregate real GDP data are sourced from the Penn World Table, and aggregate and industry-level TFP data come from EU KLEMS. Bank capitalization is measured as the Tier 1 ratio (Tier 1 capital over risk-weighted assets) obtained from the ECB Statistical Data Warehouse.

Data harmonization and cleaning. To construct the dataset used to produce Figure 2 panel A and Figure 3, industries are defined as in Acharya et al. (2021) (4-digit NACE Rev. 2 codes). To construct the dataset used for Panel B of Figure 2, we reconciled the industry classifications from Caballero et al. (2008) (Japan) and Acharya et al. (2021) (Europe) with the industry classifications in Asia KLEMS (2-digit ISIC Rev. 3 codes or aggregations thereof) and EU KLEMS (2-digit NACE Rev. 2 codes or aggregations thereof). This process required grouping and harmonizing industry codes across sources and excluding sectors with ambiguous or inconsistent mappings. To mitigate the influence of outliers, all variables were winsorized at the 5 percent level in each tail prior to constructing the binned scatter plots in Figure 2 and estimating the local linear projections reported in Figure 3.

C Additional Results

C.1 Equity Issuance and Monetary Policy

In the extension with equity issuance described in Section 5, there is again a diabolical sorting, whereby poorly-capitalized banks engage in risk-shifting and zombie lending. But now the equity thresholds e^* and e^{**} depend on the equity issuance margin. In order to focus on e^*, suppose that $e^{**} > e_{\max}$. We have the following result:

Proposition A.1. *A decrease in R^f raises e^* and thus zombie lending. An increase in p raises e^* and zombie lending more than without equity issuance.*

Proposition A.1 uncovers a new relationship between zombie lending and conventional monetary policy when bank equity is endogenous. As previously discussed, when banks cannot choose their leverage—or, equivalently, when equity issuance costs are infinitely high—the level of R^f has no bite on banks' relative returns from lending to good versus bad types of firms. Once equity issuance costs are introduced, however, a reduction in the monetary policy rate R^f increases the threshold e^*, thereby increasing zombie lending.

A higher interest rate increases the returns on all assets and therefore encourages banks to issue more equity to take advantage of these higher returns. Our reduced-form formulation in which equity is limited by an issuance cost function κ makes this point particularly stark and simple. More generally, higher interest rates will increase equity issuance if the required return on bank equity does not adjust fully with the risk-free rate, as is the case empirically, so that higher interest rates make the cost of equity relatively lower.[27]

The following result generalizes Proposition 1 and characterizes the optimal policy, in the case of quadratic equity issuance costs $\kappa(x) = \frac{1}{a}\frac{x^2}{2}$ that allow for closed-form solutions:

Proposition A.2 (Optimal policy with equity issuance)**.** *Output reaches its potential ($Y = Y^*$) if and only if*

$$\underline{R}^f(p) \leq R^f \leq \bar{R}^f(p)$$

and

$$p \leq \bar{p}$$

where $\underline{R}^f(p)$ and $\bar{R}^f(p)$ are given in the Appendix.

[27] An important implication of this result is that the endogenous response of banks' capital structure imposes an additional constraint on monetary policy. Moderate interest rates are needed to prevent banks from investing in safe assets instead of lending, as in the baseline model with exogenous equity. But there is a new force: lowering interest rates "too much" makes zombie lending more likely, by deterring equity issuance. Hence achieving potential output Y^* requires, as in Proposition 1, to set p and R^f low enough, together with a novel restriction that the risk-free rate R^f cannot be set too low either. Proposition A.2 in the Appendix formalizes this result.

The limit case $a \to 0$ recovers the no-issuance benchmark from Proposition 1. Under quadratic issuance costs, the optimal policy is characterized by the thresholds

$$\underline{R}^f(p) = p\left(1 - \frac{\theta^g + \theta^b}{2}\right) - \frac{1}{a}(1 - e_{\min})\left[\frac{\bar{p}}{p} - 1\right]$$
$$\bar{R}^f(p) = \frac{1}{1 + ap(1-\theta^g)}\bar{R}^f_{\text{no issuance}}(p) + \frac{ap^2(1-\theta^g)^2}{2(1 + ap(1-\theta^g))}$$

and $\bar{p}$ and $\bar{R}^f_{\text{no issuance}}(p)$ are as defined in Proposition 1.

C.2 Evergreening

C.2.1 Sorting with $\delta > 0$

With $\delta > 0$, Lemma 1 generalizes as follows:

Lemma 2 (Bank-firm sorting with evergreening)**.** *Let $v = \frac{R^f - p(1-\theta^g)}{p\Delta\theta}$ and suppose that $e^* \leq e^{**}$.*[28] *Banks matched with a legacy B borrower invest as follows:*

(i) Banks with equity $e < e^* + \delta v$ lend to a type B borrower at rate R^b.

(ii) Banks with equity $e \in (e^* + \delta v, e^{**})$ lend to a type G borrower at rate R^g.

(iii) Banks with equity $e > e^{**}$ do not lend and invest in safe assets at rate R^f.

Other banks follow the policies (i)-(iii) with a threshold e^* instead of $e^* + \delta v$ as in Lemma 1.

C.2.2 Static equilibrium conditions with $\delta > 0$

The market clearing condition for new B loans

$$(1-\lambda)F(e^*) + \lambda F(e^*)\left[1 - H\left(\theta^b\left(y^b - \bar{R}^b\right)\right)\right] = \lambda H\left(\theta^b\left(y^b - R^b\right)\right)[1 - F(e^* + \delta v)]$$

so indeed for $\delta = 0$ we have the simple form

$$F(e^*) = \lambda H\left(\theta^b\left(y^b - R^b\right)\right)$$

as in the main text.

[28] As before this is the case if condition (A.2) in the Appendix holds.

The market clearing condition for new G loans is

$$
\begin{aligned}
H\left(\theta^{g}\left(y^{g}-R^{g}\right)\right)= & (1-\lambda)\left[F\left(e^{**}\right)-F\left(e^{*}\right)\right] \\
& +\lambda\left[F\left(e^{**}\right)-F\left(e^{*}+\delta \nu\right)\right] \\
& +\lambda\left[1-H\left(\theta^{b}\left(y^{b}-\bar{R}^{b}\right)\right)\right]\left[F\left(e^{*}+\delta \nu\right)-F\left(e^{*}\right)\right]
\end{aligned}
$$

which also specializes to the simple form in the main text

$$
H\left(\theta^{g}\left(y^{g}-R^{g}\right)\right)=F\left(e^{**}\right)-F\left(e^{*}\right)
$$

for $\delta=0$.

C.3 Dynamic Equilibrium

Given a path of policies $\left\{R_{t}^{f}, p_{t}\right\}_{t \geq 0}$ and fundamentals $\left\{y_{t}^{g}, y_{t}^{b}\right\}_{t \geq 0}$, a dynamic equilibrium is a sequence of masses $\left\{m_{t}^{b}, m_{t}^{g}, m_{t}^{f}\right\}_{t \geq 0}$, equity e_{t}, and loan rates $\left\{R_{t}^{g}, R_{t}^{b}\right\}$ such that for all t, banks sort optimally:

$$
\begin{aligned}
m_{t}^{b}>0 \Rightarrow e_{t} \leq e_{t}^{*} & =1-\frac{\theta^{g} R_{t}^{g}-\theta^{b} R_{t}^{b}}{p_{t}\left(\theta^{g}-\theta^{b}\right)}, \\
m_{t}^{g}+m_{t}^{b}<1 \Rightarrow e_{t} \geq e_{t}^{**} & =1-\frac{R_{t}^{f}-\theta^{g} R_{t}^{g}}{p_{t}\left(1-\theta^{g}\right)},
\end{aligned}
$$

bank equity e_{t} follows

$$
e_{t}=\iota+(1-\rho)\left[m_{t-1}^{f} R_{t-1}^{f} e_{t-1}+m_{t-1}^{g} \theta^{g}\left[R_{t-1}^{g}-\tilde{R}_{t-1}^{g}\left(1-e_{t-1}\right)\right]+m_{t-1}^{b} \theta^{b}\left[R_{t-1}^{b}-\tilde{R}_{t-1}^{b}\left(1-e_{t-1}\right)\right]\right], \tag{A.1}
$$

where m_{t-1}^{i} is the mass of banks investing in asset class $i \in\{b, g, f\}$ at $t-1$, markets clear

$$
\begin{aligned}
F\left(e_{t}^{*}\right) & =m_{t}^{b}=\lambda H\left(\theta^{b}\left(y_{t}^{b}-R_{t}^{b}\right)\right), \\
F\left(e_{t}^{**}\right)-F\left(e_{t}^{*}\right) & =m_{t}^{g}=(1-\lambda) H\left(\theta^{g}\left(y_{t}^{g}-R_{t}^{g}\right)\right), \\
1-F\left(e_{t}^{**}\right) & =m_{t}^{f}=1-m_{t}^{b}-m_{t}^{g},
\end{aligned}
$$

and productivity follows (14).

C.4 Forward-looking firm dynamics

Incumbent firms draw a cost shock ϵ in each period. If they do not exit they earn current expected profit

$$\pi_t^i(\epsilon) = \theta^i \left(y_t^i - R_t^i\right) - \epsilon$$

Assume firms exit when their project fails. A forward-looking incumbent firm's value function if it does not exit is

$$\Pi_t^i(\epsilon) = \pi_t^i(\epsilon) + \beta\theta^i \underbrace{\mathbf{E}_t\left[\left(1-\lambda^i\right)\max\left\{\Pi_{t+1}^i(\epsilon_{t+1}), 0\right\} + \lambda^i \max\left\{\Pi_{t+1}^{-i}(\epsilon_{t+1}), 0\right\}\right]}_{=W_{t+1}^i}$$

where with a probability λ^i the firm can change type to $-i$ next period. Then the firm does not exit if and only if

$$\Pi_t^i(\epsilon) \geq 0 \Leftrightarrow \epsilon \leq \bar{\epsilon}_t^i = \theta^i\left(y_t^i + \beta W_{t+1}^i - R_t^i\right)$$

A myopic firm ignores the W_{t+1}^i part, hence does not exit if and only if $\pi_t^i(\epsilon) \geq 0$, i.e., $\epsilon \leq \theta^i\left(y_t^i - R_t^i\right)$.

Potential entrants are all of the $i = g$ type, and have cost $\epsilon - \gamma$. If they enter they must pay an entry cost κ, hence they earn current expected profit

$$\pi_t^n(\epsilon) = \theta^g\left(y^g - R_t^g\right) + \gamma - \epsilon - \kappa$$

in the first period. After one period they become incumbents and lose their productivity advantage γ (it is straightforward but inconvenient to generalize to γ lasting multiple periods). Thus a potential entrant enters if and only if

$$\epsilon \leq \bar{\epsilon}_t^n = \bar{\epsilon}_t^g + \gamma - \kappa$$

Incumbents' value functions satisfy

$$\Pi_t^i(\epsilon) = \pi_t^i(\epsilon) + \beta\theta^i\left[\left(1-\lambda^i\right)\int_0^{\bar{\epsilon}_{t+1}^i} \Pi_{t+1}^i(\epsilon')\, dH(\epsilon') + \lambda^i \int_0^{\bar{\epsilon}_{t+1}^{-i}} \Pi_{t+1}^{-i}(\epsilon')\, dH(\epsilon')\right]$$

Since ϵ is additive and iid, $\Pi_t^i(\epsilon) = \Pi_t^i(0) - \epsilon$ and by definition (in the case of an interior solution which we will check)

$$\Pi_t^i(0) = \bar{\epsilon}_t^i$$

Thus we need only keep track of the two paths of the two thresholds $\left\{\bar{\epsilon}_t^g, \bar{\epsilon}_t^b\right\}_t$. Rearranging the

Bellman equation, they solve

$$\bar{\epsilon}_t^i = \pi_t^i(0) + \beta\theta^i \left[\left(1-\lambda^i\right) \int_0^{\bar{\epsilon}_{t+1}^i} \left(\bar{\epsilon}_{t+1}^{i,o} - \epsilon'\right) dH(\epsilon') + \lambda^i \int_0^{\bar{\epsilon}_{t+1}^{-i}} \left(\bar{\epsilon}_t^{-i,o} - \epsilon'\right) dH(\epsilon') \right]$$

If H is uniform between 0 and 1, this simplifies to two quadratic equations.

D Proofs

D.1 Proof of Lemma 1

There are two cases to consider:

Case 1. A bank prefers lending to a type G borrower at rate R^g instead of lending to a type B borrower if:

$$\theta^g \left(R^g - \tilde{R}^g (1-e)\right) \geq \theta^b \left(R^b - \tilde{R}^b (1-e)\right).$$

Using the definition of $\tilde{R}^j$, $j = g, b$, this condition is met for banks with level of capitalization above the following threshold:

$$e \geq e^* = 1 - \frac{\theta^g R^g - \theta^b R^b}{p\Delta\theta}.$$

When $\delta > 0$ and a bank has a legacy B borrower, the bank prefers to switch to a new G borrower if

$$\theta^g \left(R^g - \tilde{R}^g (1-e+\delta)\right) \geq \theta^b \left(R^b - \tilde{R}^b (1-e)\right)$$

which is equivalent to

$$e \geq e^* + \delta\nu$$

where $\nu = \frac{\theta^g \tilde{R}^g}{p\Delta\theta} = \frac{R^f - p(1-\theta^g)}{p\Delta\theta}$.

Case 2. A bank prefers investing its capital in safe assets rather than lending to a type G borrower at rate R^g if:

$$R^f e > \theta^g \left(R^g - \tilde{R}^g (1-e)\right)$$

Using the definition of $\tilde{R}^g = \frac{R^d - (1-\theta^g)p}{\theta^g}$, this condition is met for banks with level of capitalization above the following threshold:

$$e > e^{**} = 1 - \frac{R^f - \theta^g R^g}{p(1-\theta^g)}.$$

As long as $e^{**} > e^*$, a bank that prefers investing in safe assets over lending to type G firms *a fortiori* prefers investing in safe assets over lending to type B firms. The following conditions ensure that $e^* < e^{**}$:

$$\frac{R^f - \theta^g R^g}{1-\theta^g} < \frac{\theta^g R^g - \theta^b R^b}{\theta^g - \theta^b},$$

or, equivalently,

$$R^f \Delta\theta < \theta^g R^g (1-\theta^b) - \theta^b R^b (1-\theta^g). \tag{A.2}$$

D.2 Proof of Proposition 1

$Y = Y^*$ is achieved when all banks lend and there is no zombie lending, hence $m^g = 1$ and $m^b = 0$. Relative to this composition of firms, any substitution towards bonds decreases output, and any increase in zombie lending decreases output by Assumption 1.

In an equilibrium with $Y = Y^*$ loan rates are given by

$$\begin{aligned} R^b &= \bar{y}^b(1-\eta z) \\ R^g &= \bar{y}^g(1-z) - \frac{\bar{\epsilon}}{\theta^g} \end{aligned}$$

Given these equilibrium loan rates, we verify that all banks lend, that is $e^{**} \geq e_{\max}$, and that there is indeed no zombie lending, that is $e^* \leq e_{\min}$.

These conditions can be rewritten respectively as

$$1 - \frac{R^f - \theta^g R^g}{p\,(1-\theta^g)} = 1 - \frac{R^f + \bar{\epsilon} - \theta^g \bar{y}^g (1-z)}{p\,(1-\theta^g)} \geq e_{\max} \Leftrightarrow R^f \leq \bar{R}^f(p,z)$$

where $\bar{R}^f(p,z) = \theta^g \bar{y}^g (1-z) - \bar{\epsilon} + (1-e_{\max})(1-\theta^g)\,p$, and

$$1 - \frac{\theta^g R^g - \theta^b R^b}{p\left(\theta^g - \theta^b\right)} = 1 - \frac{\theta^g \bar{y}^g - \theta^b \bar{y}^b - z\left(\theta^g \bar{y}^g - \theta^b \bar{y}^b \eta\right) - \bar{\epsilon}}{p\left(\theta^g - \theta^b\right)} \leq e_{\min} \Leftrightarrow p \leq \bar{p}(z)$$

where

$$\bar{p}(z) = \frac{\theta^g \bar{y}^g - \theta^b \bar{y}^b - z\left(\theta^g \bar{y}^g - \theta^b \bar{y}^b \eta\right) - \bar{\epsilon}}{(1-e_{\min})\left(\theta^g - \theta^b\right)}.$$

If R^f is lower than the type G project with the lowest net present value, i.e. $R^f < \theta^g \bar{y}^g (1-z) - \bar{\epsilon}$, then all banks lend and with $p \leq \bar{p}$ the economy reaches Y^* because there is also no zombie lending. Finally, if $p > \bar{p}$ then there is necessarily zombie lending in equilibrium and $Y < Y^*$, regardless of the level of R^f.

When the shock z is small, an accommodating conventional monetary policy alone can achieve $Y = Y^*$ at no costs ($p = 0$), without violating the ELB constraint. The monetary policy rate that achieves $m^g = 1$ with $p = 0$ is

$$R^f(z) = \theta^g \bar{y}^g (1 - z) - \bar{\epsilon}.$$

This interest rate satisfies the ELB constraint if $\theta^g \bar{y}^g (1 - z) - \bar{\epsilon} \geq R^f_{\min}$ or

$$z \leq \underline{z} = 1 - \frac{R^f_{\min} + \bar{\epsilon}}{\theta^g \bar{y}^g}.$$

For moderate shocks, $z_t > \underline{z}$, a combination of conventional and a lax forbearance policy, $p(z)$, can still achieve $Y = Y^*$ even if the ELB binds. Given the loan rates in an equilibrium without zombie lending, this requires

$$R^f(z) = \theta^g \bar{y}^g (1 - z) - \bar{\epsilon} + (1 - e_{\max})(1 - \theta^g) p(z).$$

Exhausting the stimulus from conventional monetary policy, the optimal policy sets $R^f(z) = R^f_{\min}$, so p must satisfy $R^f_{\min} = \theta^g \bar{y}^g (1 - z) - \bar{\epsilon} + (1 - e_{\max})(1 - \theta^g) p(z)$, or

$$p(z) = \frac{R^f_{\min} + \bar{\epsilon} - \theta^g \bar{y}^g (1 - z)}{(1 - e_{\max})(1 - \theta^g)}$$

which is an increasing function of z. The conjectured equilibrium loan rates are correct as long as $p(z) \leq \bar{p}(z)$ or

$$\frac{R^f_{\min} + \bar{\epsilon} - \theta^g \bar{y}^g (1 - z)}{(1 - e_{\max})(1 - \theta^g)} \leq \frac{\theta^g \bar{y}^g - \theta^b \bar{y}^b - z\left(\theta^g \bar{y}^g - \theta^b \bar{y}^b \eta\right) - \bar{\epsilon}}{(1 - e_{\min})\left(\theta^g - \theta^b\right)}. \tag{A.3}$$

The left-hand side is increasing in z and the right-hand side is decreasing in z since $\eta < \frac{\theta^g \bar{y}^g}{\theta^b \bar{y}^b}$, hence there exists a unique $\bar{z}$ such that the inequality holds for $z \leq \bar{z}$. Moreover, the right-hand side of inequality (A.3) is increasing in $e_{\min}$, hence an improvement in $e_{\min}$ relaxes the range of shocks z such that the inequality holds, or equivalently increases $\bar{z}$.

The threshold $\bar{z}$ is given by

$$\bar{z} = \frac{\left(\theta^g \bar{y}^g - \theta^b \bar{y}^b - \bar{\epsilon}\right)(1 - e_{\max})(1 - \theta^g) - \left(R^f_{\min} + \bar{\epsilon} - \theta^g \bar{y}^g\right)(1 - e_{\min})\left(\theta^g - \theta^b\right)}{\theta^g \bar{y}^g (1 - e_{\min})\left(\theta^g - \theta^b\right) + \left(\theta^g \bar{y}^g - \theta^b \bar{y}^b \eta\right)(1 - e_{\max})(1 - \theta^g)}. \tag{A.4}$$

D.3 Proof of Proposition 2

For large shocks, $z > \bar{z}$, conventional monetary policy is constrained by the lower bound and increasing the level of forbearance induces credit misallocation.

- Under the No Zombie lending policy, the optimal forbearance policy p solves:

$$F\left(1-\frac{(1-e_{\min})\left(\theta^g-\theta^b\right)}{1-\theta^g}-\frac{R^f_{\min}-\theta^b y^b}{p\left(1-\theta^g\right)}\right)=H\left(\theta^g\bar{y}^g\left(1-z\right)-\theta^b y^b-p\left(1-e_{\min}\right)\Delta\theta\right),$$

which implies that the optimal $p\left(z\right)$ is decreasing in the size of the shock.

- Under the myopic policy, the minimal forbearance policy p that maximizes bank lending and thus output is such that equilibrium loan rates $R^b(z,p)$ and $R^g(z,p)$ solve

$$\begin{aligned}\lambda H\left(\theta^b\left(y^b-R^b\right)\right) &= F\Bigg(\overbrace{1-\frac{\theta^g R^g-\theta^b R^b}{p\Delta\theta}}^{=e^*}\Bigg)\\ \lambda H\left(\theta^b\left(y^b-R^b\right)\right)+H\left(\theta^g\left(\bar{y}^g(1-z)-R^g\right)\right) &= F\Bigg(\underbrace{1-\frac{R^f_{\min}-\theta^g R^g}{p\left(1-\theta^g\right)}}_{=e^{**}}\Bigg)\end{aligned}$$

If e^{**} evaluated when $p=1$ is strictly below $e_{\max}$, i.e., $1-\frac{R^f_{\min}-\theta^g R^g(z,1)}{(1-\theta^g)}<e_{\max}$, then the optimal p is the maximal possible forbearance $p=1$. Otherwise the optimal p is the lowest p ensuring that all banks lend, solving

$$1-\frac{R^f_{\min}-\theta^g R^g(z,p)}{p\left(1-\theta^g\right)}=e_{\max}$$

which yields a solution $p(z)$ that is increasing in z. To see this, we rewrite the equilibrium system under the optimal policy as

$$\begin{aligned}D_b(R^b) &= S_b(R^b,R^g,p)\\ D_b(R^b)+D_g(R^g,z) &= 1\\ (1-e_{\max})(1-\theta^g)p+\theta^g R^g &= R^f_{\min}\end{aligned}$$

where we define the demand for type B loans $D_b=\lambda H\left(\theta^b\left(y^b-R^b\right)\right)$, the demand for type G loans $D_g=H\left(\theta^g\left(\bar{y}^g(1-z)-R^g\right)\right)$, and the supply of type B loans $S_b=F\left(1-\frac{\theta^g R^g-\theta^b R^b}{p\Delta\theta}\right)$. Suppose that the optimal forbearance is not everywhere non-decreasing with z, i.e., there exist $z_1<z_2$ such that $p(z_1)>p(z_2)$. Then from the third line of the system we have $R^g(z_1)<R^g(z_2)$. The second line then implies that $D_g(R^g(z_1),z_1)>D_g(R^g(z_2),z_2)$ hence $D_b(R^b(z_1))<D_b(R^b(z_2))$ and $R^b(z_1)>R^b(z_2)$. But this is incompatible with the first line given that S_b is decreasing in R^g, increasing in R^b, and increasing in p.

D.4 Proof of Proposition 3

We look for sufficient conditions ensuring an equilibrium such that at all $t \geq 0$, the ELB binds and $p^m(z_t, e_t) = 1$. Loan rates for $i = b, g$ satisfy:

$$\theta^i R_t^i = R_{\min}^f - (1 - e_t)\left(1 - \theta^i\right).$$

Therefore z follows the process:

$$\begin{aligned} z_{t+1} &= \alpha \lambda H\left(\theta^b\left(\bar{y}^b - R_t^b\right)\right) \\ &= \alpha \lambda H\left(\theta^b \bar{y}^b - R_{\min}^f + (1 - e_t)\left(1 - \theta^b\right)\right). \end{aligned}$$

Total date-t lending is indeed below 1 even under the most accommodative forbearance policy $p_t = 1$ (hence the optimal myopic policy sets $p^m(z_t, e_t) = 1$) if

$$H\left(\theta^g \bar{y}^g (1 - z_t) - R_{\min}^f + (1 - e_t)(1 - \theta^g)\right) + \lambda H\left(\theta^b \bar{y}^b - R_{\min}^f + (1 - e_t)\left(1 - \theta^b\right)\right) < 1,$$

which can be written as

$$z_t > \zeta(e_t)$$

where we define the function

$$\zeta(e_t) = 1 - \frac{R_{\min}^f - (1 - e_t)(1 - \theta^g) + H^{-1}\left(1 - \lambda H\left(\theta^b \bar{y}^b - R_{\min}^f + (1 - e_t)\left(1 - \theta^b\right)\right)\right)}{\theta^g \bar{y}^g}.$$

Therefore, for any t, the optimal myopic forbearance policy is indeed $p^m(z_t, e_t) = 1$ if

$$z_t > Z(e_t) = \max\left\{\bar{z}, \zeta\left(e_t\right)\right\}.$$

Given $R_t^f = R_{\min}^f$, equity follows

$$e_{t+1} = \iota + (1 - \rho) R_{\min}^f e_t.$$

Suppose now that in period t we have $z_t > Z(e_t)$. In the next period, we have again $z_{t+1} > Z(e_{t+1})$ if congestion externalities are strong enough:

$$\alpha > \frac{Z(e_{t+1})}{\lambda H\left(\theta^b \bar{y}^b - R_{\min}^f + (1 - e_t)\left(1 - \theta^b\right)\right)}. \tag{A.5}$$

The function Z is decreasing in e and the denominator is also decreasing in e_t. Along the desired equilibrium path, we have

$$\frac{\iota}{1-(1-\rho)\,R^f_{\min}} = \underline{e}_\infty \le e_t \le e_0 = \frac{\iota}{1-(1-\rho)\,[\theta^g \bar{y}^g - \bar{\epsilon}]}$$

hence the right-hand side of (A.5) is bounded above by

$$\bar{\alpha} = \frac{Z\left(\underline{e}_\infty\right)}{\lambda H\left(\theta^b \bar{y}^b - R^f_{\min} + (1-e_0)\left(1-\theta^b\right)\right)}. \tag{A.6}$$

Therefore, a sufficient condition for permanent sclerosis to happen is $z_0 > Z(e_0)$ and $\alpha \ge \bar{\alpha}$.

For any given α, the myopic policy $p^m(z,e)$ is weakly increasing in z and the law of motion

$$z_{t+1} = \alpha\lambda H\left(\theta^b \bar{y}^b - R^f_{\min} + p^m(1-e_t)(1-\theta^b)\right)$$

is weakly increasing in both p^m and α. Hence a larger initial shock z_0 leads to a weakly larger path $(z_t)_{t\ge 0}$, and a larger α shifts the whole path up. For small shocks z_0, the economy converges to the steady state without zombie lending, while for sufficiently large $z_0 > Z(e_0)$ and $\alpha \ge \bar{\alpha}$ the economy converges to the sclerosis steady state, as shown above. Monotonicity therefore implies that there exists a threshold date-0 shock size $z^*(\alpha)$ such that for $z_0 < z^*(\alpha)$ the economy converges to the steady state without zombie lending, whereas for $z_0 > z^*(\alpha)$ the economy converges to the sclerosis steady state. Moreover, as α increases, the set of initial shocks that lead to sclerosis expands, so $z^*(\alpha)$ is decreasing in α.

D.5 Proof of Proposition 4

When $\delta < \Delta^g - \Delta^b$, the intermediate region $\left[\hat{e}-\Delta^b, \hat{e}-\Delta^g+\delta\right]$ is empty. For banks in the constrained region, the derivative

$$\frac{\partial e^*}{\partial \hat{e}} = 1 - \frac{\kappa'\left(\Delta^g\right) - \kappa'\left(\Delta^b\right)}{\kappa'\left(\hat{e}-e^*+\delta\right) - \kappa'\left(\hat{e}-e^*\right)}$$

is negative when $\delta < \Delta^g - \Delta^b$. Thus increasing $\hat{e}$ decreases e^*, and setting $\hat{e} \ge \hat{e}^{NZ}$ pushes $e^* \le e_{\min}$, eliminating zombie lending.

When $\delta > \Delta^g - \Delta^b$, there are three relevant regions for banks initially matched with a bad firm. If $e < \hat{e}-\Delta^b$, then the capital requirement is binding even if the bank remains with its legacy B borrower. If $e > \hat{e}-\Delta^g+\delta$, the capital requirement is never binding, whether the bank switches or not. For intermediate equity $e \in \left[\hat{e}-\Delta^b, \hat{e}-\Delta^g+\delta\right]$, the capital requirement is binding only if the bank switches.

We start with the banks matched to a borrower that turns B.

1. Suppose that $\hat{e}$ is high enough that the bank $e = \hat{e} - \Delta^b$ prefers to switch to a new G borrower and thus issue $\hat{e} - e - \delta = \Delta^b + \delta$, that is

$$\sigma(\hat{e}) \geq \kappa\left(\Delta^b + \delta\right) - \kappa\left(\Delta^b\right) \tag{A.7}$$

or

$$\delta \leq \kappa^{-1}\left(\sigma(\hat{e}) + \kappa\left(\Delta^b\right)\right) - \Delta^b$$

Therefore, all the banks above $e = \hat{e} - \Delta^b$ will prefer to switch, and the only potential for zombie lending is for banks below $\hat{e} - \Delta^b$. In that case, banks lending to zombies are those with pre-issuance equity e below the indifference threshold e^* solving

$$\sigma(\hat{e}) = \kappa(\hat{e} - e^* + \delta) - \kappa(\hat{e} - e^*)$$

Note that $\sigma(\hat{e}) > 0$ implies $\hat{e} > E^*$. From the implicit function theorem, when $\hat{e}$ increases (holding loan rates fixed in this partial equilibrium first step) we have

$$\frac{\partial e^*}{\partial \hat{e}} = 1 - \frac{\sigma'(\hat{e})}{\kappa'(\hat{e} - e^* + \delta) - \kappa'(\hat{e} - e^*)} = 1 - \frac{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}{\kappa'(\hat{e} - e^* + \delta) - \kappa'(\hat{e} - e^*)}$$

This can be rewritten as

$$\frac{\partial e^*}{\partial \hat{e}} = 1 - \frac{\kappa'(\Delta^g) - \kappa'\left(\Delta^b\right)}{\kappa'(\hat{e} - e^* + \delta) - \kappa'(\hat{e} - e^*)}$$

Since $\delta > \Delta^g - \Delta^b$ and $\hat{e} - e^* \geq \Delta^b$, we necessarily have

$$\frac{\partial e^*}{\partial \hat{e}} > 0$$

and thus in this region, increasing capital requirements worsens legacy zombie lending.

2. Suppose then that (A.7) doesn't hold:

$$\delta > \kappa^{-1}\left(\sigma(\hat{e}) + \kappa\left(\Delta^b\right)\right) - \Delta^b$$

which implies that the bank with $e = \hat{e} - \Delta^b$ prefers to stay matched with its legacy B borrower.

   (a) If the bank with $e = \hat{e} - \Delta^g + \delta$ prefers to switch to a new G borrower, that is

$$\sigma(\hat{e}) > \kappa(\Delta^g) - \kappa\left(\Delta^b\right) + \theta^b \tilde{R}^b\left(\delta - \Delta^g + \Delta^b\right) \tag{A.8}$$

holds, then all banks with even higher e also switch. Thus the indifference threshold e^* is in the intermediate region $\left[\hat{e} - \Delta^b, \hat{e} - \Delta^g + \delta\right]$ and solves

$$\theta^b \left[R^b - \tilde{R}^b \left(1 - e^* - \Delta^b\right)\right] - \kappa \left(\Delta^b\right) = \theta^g \left[R^g - \tilde{R}^g (1 - \hat{e})\right] - \kappa (\hat{e} - e^* + \delta)$$

or

$$\sigma (\hat{e}) = \theta^b \tilde{R}^b \left(e^* - \hat{e} + \Delta^b\right) + \kappa (\hat{e} - e^* + \delta) - \kappa \left(\Delta^b\right)$$

By the implicit function theorem,

$$\frac{\partial e^*}{\partial \hat{e}} = 1 - \frac{\sigma' (\hat{e})}{\kappa' (\hat{e} - e^* + \delta) - \theta^b \tilde{R}^b} = \frac{\kappa' (\hat{e} - e^* + \delta) - \theta^g \tilde{R}^g}{\kappa' (\hat{e} - e^* + \delta) - \theta^b \tilde{R}^b} > 0$$

which follows from $\hat{e} - e + \delta \geq \Delta^g > \Delta^b$. Therefore, in this region as well, increasing capital requirements worsens legacy zombie lending.

(b) The last case is when $\hat{e}$ is so low that even the bank with $e = \hat{e} - \Delta^g + \delta$ prefers to lend to its legacy B borrower, that is

$$\sigma (\hat{e}) < \kappa (\Delta^g) - \kappa \left(\Delta^b\right) + \theta^b \tilde{R}^b \left(\delta - \Delta^g + \Delta^b\right) \tag{A.9}$$

holds, and so all the banks with lower equity also rollover the B loan. Then the indifference threshold e^* is above $\hat{e} - \Delta^g + \delta$ and is the same as in the absence of a capital requirement:

$$e^* = \underbrace{1 - \frac{\theta^g R^g - \theta^b R^b}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}}_{E^*} - \frac{\varphi\left(\theta^g \tilde{R}^g\right) - \varphi\left(\theta^b \tilde{R}^b\right)}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b} + \frac{\theta^g \tilde{R}^g}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b} \delta$$

so does not vary with $\hat{e}$. Low enough capital requirements become irrelevant for legacy zombie lending.

For banks matched with a good firm, since we abstract from switching costs δ, they will switch to a new zombie borrower if their post-issuance equity is below

$$E^* = 1 - \frac{\theta^g R^g - \theta^b R^b}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}$$

hence capital requirements have a knife-edge effect: either $\hat{e} \leq E^*$ and the capital requirement is irrelevant, or $\hat{e} \geq E^*$ and the capital requirement prevents all these banks (matched with a G firm) from switching to a new B borrower. Since we just showed that increasing $\hat{e}$ can never decrease legacy zombie lending, the only potential benefit is to prevent "new" zombie lending.

Next, note that the point $\hat{e}^*$ such that (A.9) holds with equality, that is

$$\hat{e}^* = E^* + \Delta^g - \frac{\varphi\left(\theta^g \tilde{R}^g\right) - \varphi\left(\theta^b \tilde{R}^b\right)}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b} + \frac{\theta^b \tilde{R}^b}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}\delta \tag{A.10}$$

is strictly above E^* since $\sigma(\hat{e}) = \kappa(\Delta^g) - \kappa\left(\Delta^b\right) + \theta^b \tilde{R}^b\left(\delta - \Delta^g + \Delta^b\right) > 0 = \sigma(E^*)$.

D.6 Proof of Proposition A.1

Following the same steps as without equity issuance costs we find:

$$e^* = 1 - \frac{\theta^g R^g - \theta^b R^b}{(\theta^g - \theta^b)p} - \frac{\varphi\left(\theta^g \tilde{R}^g\right) - \varphi\left(\theta^b \tilde{R}^b\right)}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}$$

$$e^{**} = 1 - \frac{R^f - \theta^g R^g}{p(1-\theta^g)} - \frac{\varphi\left(R^f\right) - \varphi\left(\theta^g \tilde{R}^g\right)}{R^f - \theta^g \tilde{R}^g}$$

where $\varphi(x) = x(\kappa')^{-1}(x) - \kappa\left((\kappa')^{-1}(x)\right)$. The function φ inherits the properties of κ, as $\varphi'(x) = (\kappa')^{-1}(x)$ and $\varphi''(x) = \frac{1}{\kappa''\left((\kappa')^{-1}(x)\right)}$. Since $\theta^g \tilde{R}^g - \theta^b \tilde{R}^b = (\theta^g - \theta^b)p > 0$, it follows from the convexity of φ that the slope of $\frac{\varphi\left(\theta^g \tilde{R}^g\right) - \varphi\left(\theta^b \tilde{R}^b\right)}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}$ is increasing with R^f and decreasing with p.

D.7 Proof of Proposition A.2

Banks choose borrower type based on their post-issuance equity $e' = e + \Delta e$. Define the function $\varphi(x) = x(\kappa')^{-1}(x) - \kappa\left((\kappa')^{-1}(x)\right)$. There are two cases to consider:
Case 1. A bank with pre-issuance equity e prefers lending to a type G borrower at rate R^g instead of lending to a type B borrower if:

$$\theta^g\left(R^g - \tilde{R}^g(1 - e - \Delta^g)\right) - \kappa(\Delta^g) \geq \theta^b\left(R^b - \tilde{R}^b\left(1 - e - \Delta^b\right)\right) - \kappa\left(\Delta^b\right)$$

which can be rewritten as

$$e > e^* = 1 - \frac{\theta^g R^g - \theta^b R^b}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b} - \frac{\varphi\left(\theta^g \tilde{R}^g\right) - \varphi\left(\theta^b \tilde{R}^b\right)}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}.$$

Case 2. A bank with pre-issuance equity e prefers investing its capital in safe assets rather than lending to a type G borrower at rate R^g if:

$$R^f\left(e + \Delta^f\right) - \kappa\left(\Delta^f\right) \geq \theta^g\left(R^g - \tilde{R}^g(1 - e - \Delta^g)\right) - \kappa(\Delta^g)$$

which can be rewritten as

$$e > e^{**} = 1 - \frac{R^f - \theta^g R^g}{R^f - \theta^g \tilde{R}^g} - \frac{\varphi\left(R^f\right) - \varphi\left(\theta^g \tilde{R}^g\right)}{R^f - \theta^g \tilde{R}^g}.$$