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CHARTER SCHOOL PRACTICES AND STUDENT SELECTION: AN EQUILIBRIUM ANALYSIS

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ABSTRACT

We provide a model to analyze charter school educational practices. Students differ in cognitive ability, motivation, and household income. Student achievement depends on ability, match of their school's curriculum to their ability, and effort. Charter schools choose curriculum to maximize achievement gains, optimally setting curriculum to attract lower ability students, in some cases induced by strategic public-school competition. We also investigate "no excuses" charter schools. These charters enforce an effort minimum that attracts highly motivated students. We find, consistent with the evidence, that these charters are highly effective in increasing achievement, with the largest gains accruing to lower ability students.

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A Data Appendix is available at http://www.nber.org/data-appendix/w29529

1. Introduction

Charter schools are playing an increasingly important role in US education. The first charter school opened in Minnesota in 1992. Charter schools now operate in 44 states. From fall 2009 to fall 2018, the percentage of public-school students attending charter schools increased from 3 to 7 percent.[1] The charter share is much higher in many, especially urban, areas, exceeding 15 percent in 20 cities in the 2010-11 school year.[2] Moreover, more than 10% of public-school students attend charter schools in 6 states. Charters are tuition-free public schools and are not permitted to impose selective criteria for admission. If oversubscribed, a charter must admit by lottery from its applicant pool. Charter schools are, however, permitted to adopt curriculum and educational practices different from those of traditional public schools. Much research has been devoted to comparing the performance of charter and public schools and to documenting educational approaches of charter schools. This research has established that large differences across charter schools exist in the extent to which they improve educational outcomes relative to the public schools their students would otherwise attend.

In contrast to this impressive body of empirical research, there has been little formal modeling of the decisions about curriculum and educational practices of a prospective charter school entrant, how those decisions vary with the educational environment the charter would enter, or the impacts on students of differing capabilities and socioeconomic background. There is, to the best of our knowledge, no existing framework that might be employed to address these fundamental issues.

In this paper, we develop such a framework, and apply it to analyze decisions about curriculum and educational practices made by a charter entrant seeking to maximize the educational gains of the students it attracts. We investigate how such choices vary with the characteristics of the student population from which the charter school draws its students. This in turn requires modeling the school choice decisions of students of differing capabilities and socioeconomic backgrounds. It also requires comparison of student outcomes in the equilibrium with charter entry relative to student outcomes in the equilibrium without charter entry. While charter schools cannot impose selective criteria for admission, and so must randomly admit students among their applicants if oversubscribed, their curriculum choice maximizes the achievement gains of their students, both by targeting their capabilities and by affecting student self-selection into the school. We find higher gains in schools that target lower ability students, in some cases induced by strategic public-school competition. Those students benefit more from a curriculum adapted to their abilities than do higher ability students with curriculum adapted to their abilities.

In addition to considering traditional charter schools, a key part of our analysis models "no-excuses charter schools." Such schools have been quite successful in increasing student achievement.[3]

[1] https://nces.ed.gov/programs/coe/indicator/cgb
[2] See Figure 4, panel C of Epple, Romano, and Zimmer (2016).
[3] The evidence is discussed in Section 2.

In addition to their curriculum choice, these charters enforce student work standards with parental support. We find these charters attract highly motivated students with a wider range of abilities.

Our model features some elements that, while being central to the educational process, have not received much attention in the theoretical literature. Students differ by cognitive and non-cognitive skills, which can be interpreted as, respectively, academic ability and motivation. Students also differ by income of their household. A student's achievement depends on the quality of and match to the school attended, the student's academic ability, and the (costly) effort the student exerts. Costs of effort in turn depend on motivation. Three key educational variables shape a school's educational process: academic standards, school curriculum and, in the case of a no-excuses charter school, the required effort minimum. Standards, set by the educational authority, determine how demanding it is for a student to graduate. Higher standards may push some students to exert greater effort than they would otherwise choose, but may discourage others from exerting any effort at all. A school's curriculum establishes the academic ability the school targets with its teaching, and thereby affects the quality of education received by every student. The model thereby captures an important feature of the educational process; the quality of the educational services provided by a school varies across students and depends on the alignment between a student's ability and the school's curriculum.

We follow with a computational counterpart to our theoretical model. For calibration, we utilize recent econometric evidence regarding the roles of different types of skills in determining educational outcomes. We couple this with information about the distributions of student ability and student achievement. The model makes predictions about charter school entry, their student characteristics, and achievement and attainment (graduation) that can be compared to empirical evidence. The model performs fairly well in making a variety of predictions. Such comparisons provide supportive evidence for the validity of our model.

Our paper proceeds as follows. Section 2 reviews related literature and discusses further our contribution. Sections 3 presents the baseline theoretical model and some preliminary analysis. Section 4 develops the theory of a "traditional charter school" that differentiates itself by choice of curriculum. Section 5 provides the computational model and applies it to competition between a public school and a charter school as well as a case including a magnet school. Section 6 pursues the analysis of a no-excuses charter school. Section 7 compares model predictions to empirical evidence and considers robustness of the model. Section 8 discusses some additional issues and potential extensions (e.g., to consider peer effects). Section 9 concludes. Some omitted analysis is collected is in an online appendix.

2. Literature Discussion and Our Contribution.

The relative newness and on-going growth of charter schools has inspired an extensive body of research. The focus has been on their performance in increasing student achievement relative to traditional public schools, with the primary identification challenge being student selection into charter schools on unobservables.[4] Here we provide a brief summary and refer the reader to Epple, Romano, and Zimmer (2016) for a comprehensive review of this research up through about 2015, and to Chabrier, Cohodes, and Oreopoulos (2016) for review of lottery studies of charter schools.[5] The thrust of the literature is that: (i) Overall, charter schools have mixed performance in increasing achievement, but with an upward trend and average positive gains in urban areas (see especially National Charter School Study, CREDO 2015[6]). (ii) Charter schools that follow the "No-Excuses" approach dramatically increase student achievement, especially for students with lower prior scores and for black students.[7] For example, Abdulkadiroglu, et.al (2011), exploiting the lottery setting, find local-average-treatment-effect estimates of attending a no-excuses school equal to .198σ and .359σ in middle school ELA and math and .265σ and .364σ in high school ELA and math[8].

We next discuss the research most closely related to our theoretical modelling of charter schools and student choices. In Section 7, we discuss the predictions of our model in light of the empirical research. We present a model of the micro-foundations of charter school differentiation based on curriculum focus, with an explicit charter objective. Students choose between the charter and traditional public school. Preference across schools varies with student ability, motivation, and socioeconomic background. Glomm, Harris, and Lo (2005) appeal to horizontal charter school differentiation, generally defined, to explain why, in Michigan and California, charter schools entered districts that are characterized by more varied parental and more racially diverse populations. Exogenous differentiation of charter schools plays a role in several empirical papers on charter schools. Singleton (2019) estimates a structural model of charter school entry allowing costs of educating students to vary with their demographics and across profit, not-for-profit, and no-excuses charter schools. He provides evidence that charter schools are more likely to enter localities where they attract student types with lower educational costs. Lower cost students are then selected by the locality of entry, this related to our model's emphasis on student selection, but here selected by educational practices.[9] Gilraine, Petronijevic, and Singleton (2019) investigate how achievement

[4] Empirical research on charter schools also investigates the impact on educational attainment and non-educational outcomes, where charter schools locate and what students they serve, and competitive effects on neighboring traditional public schools. See Epple, Romano, and Zimmer (2016) for citations and discussion, as well as Section 7 of this paper.
[5] Lottery studies exploit over-subscribed charter schools that are required to admit students randomly, essentially comparing the educational outcomes of lottery winners and losers, thus controlling for selection (though then able to use only over-subscribed charter schools).
[6] https://credo.stanford.edu/publications/national-charter-school-study. Another very recent national study finding average gains and a positive trend is Shakeel and Peterson (2021).
[7] See, for example, Abdulkadiroglu et al. (2011), Angrist et al. (2012), Dobbie and Fryer (2013), Angrist, Pathak, and Walters (2013), and Walters (2018).
[8] These LATE estimates are per year spent in the charter school and control for demographics and baseline scores. While dramatic, these are typical among studies of no-excuses schools.
[9] Other evidence that charter schools select easier-to-educate students exists. Bergman and McFarlin (2020) conduct an experiment showing that charter schools may discourage attendance of high-cost students by being less responsive to (fictitious) emails inquiring about the school that signal special needs, as well as low

gains vary among charter schools whose applications specify that "learning is experiential or project-based as opposed to focused on core skills through traditional instruction." Bau (2022), though studying private schools in Pakistan, has schools specialize to serve rich or poor students, this research related to our curriculum specialization to student abilities.[10] Geographic locational choice is central to several empirical models of charter school entry, these taking residences of racially and socio-economically diverse populations as given, with students facing travel costs to attend schools. This includes Singleton (2019), Ferreyra and Kosenok (2018), and Mehta (2017). Ferreyra and Kosenok (2018) also have schools differentiated by whether they follow a core, language-oriented, arts, vocational, or "other" curriculum. Mehta (2017) has schools dictate student effort, which is similar to what we do in our model of no-excuses schools. No-excuses charters differ from other schools by their educational practices. Those practices, which vary somewhat among such schools, include: (i) a longer school day and year; (ii) frequent testing; (iii) an ethos of comportment and a strong student work ethic;[11] (iv) extensive tutoring; (v) emphasis on core math and reading; and (vi) selective teacher hiring and professional development. Dobbie and Fryer (2013) construct an index of teacher feedback, use of data to guide instruction, tutoring, instructional time, and high expectations of students that explains 45 percent of the variation in charter school effectiveness. Angrist, Pathak, and Walters (2013) identify emphasis on discipline, school uniforms, cold-calling, adherence to school-wide standards, and employment of Teach-for-America teachers as the five practices that best explain effectiveness of no-excuses schools. Our contribution is to provide a theoretical and computational model of key charter school practices, chosen endogenously to maximize achievement gains of students strategically drawn from a given educational market. This contrasts with the focus in the structural models on charter choice of location, choice that is likely to make a charter school

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achievement and poor behavior. Charter schools in states that reimburse for a large share of special-needs costs did not so behave.

[10] It bears noting in the international context that publicly funded, privately run schools, akin to charter schools, are present in many countries, including Chile, England, the Netherlands, Spain, and Sweden. There is variation across these countries in private school autonomy (see Figure 1 in Eyles, Hupkau and Machin, 2016, and the discussion therein), practices that are afforded autonomy, funding models, and market shares. For instance, Chile only partially subsidizes private schools (called *escuelas particulares subvencionadas*), with parents contributing the rest of the school's funding. In Spain, the network of *escuelas concertadas* educate about a quarter of the population of students. Like US charter schools, these schools are not permitted to charge fees or to select students but, unlike in the US, they cannot modify the curriculum, and there is controversy as to whether or not, contrary to the law, some do charge fees (see, for instance, El País, 6th October 2021). In Sweden, the so-called free schools (*friskolor*), are not allowed to charge fees, select students, or modify the curriculum. Closest schools to American charter schools are, perhaps, English *academy schools*: these cannot charge fees or be selective and have autonomy to modify the curriculum and other aspects of their educational model. Some English academies adopt similar practices to "no excuses" US charter schools. There are differences, however: *some* English academies are religious and can select pupils by their faith (this not allowed in the US). That is also the case in the Netherlands; publicly funded private schools can be religious. There, as in our model, the Dutch government sets the graduation standards for a set of compulsory subjects (so-called attainment targets), while schools choose how to reach those targets when they design the curriculum.

[11] In Knowledge is Power Program (KIPP) charter schools, parents and students sign contracts including a pledge of timely attendance, hard work, and completion of assignments. KIPP is the largest charter management organization in the nation, with 255 schools at the time of this writing.

viable. Our model makes predictions about student characteristics that are drawn into charter schools from a given area complementing, but also quite distinct, from the existing literature. These predictions are consistent with empirical literature discussed in Section 7.

Central to our model is the dichotomous nature of student capability shown to be important to predict educational outcome sby Cunha, Heckman, and Schennach (2010), Heckman and Kautz (2012), Heckman Stixrud, and Urzua (2006), Knudsen, Heckman, Cameron, and Shonkoff (2006), Borghans, Golsteyn, Heckman, and Humphries (2016), and Cotton, et.al. (2020). These foundational contributions play a central role both in the formulation of our theoretical model and in the counterpart quantitative model. We are aware of no prior theoretical modelling of this skill dichotomy and its relation to student effort and schooling strategy.[12] Cotton, et.al. (2020) independently developed and estimated a model closely related to ours that *does* relate dichotomous skills to student effort and learning, though does not model school's targeting strategies. We discuss this paper further below. While different focuses of schools are elements of some of the structural models, as noted above, our model stresses curriculum targeting to ability as an explicit choice. Charter schools frequently stress such targeting in their marketing.[13] Regarding this element, Duflo, Dupas, and Kremer (2011) similarly examine curriculum targeting in their modelling and experimental evaluation of tracking in Kenya. They provide evidence of targeted teaching to the abilities of students (linked to teacher rewards), supporting this component of the model we adopt.[14] While student effort is not considered in their model, they state: "Student effort might also respond endogenously to teacher effort and the target teaching level. In such a model, ultimate outcomes will be a composite function of teacher effort, teacher focus level, and student effort (p. 1747)." Our model includes the interaction between student effort choice and the target. In addition to the pace and level of instruction, targeting can entail hiring teachers better matched to student capabilities. Ahn, Aucejo, and James (2023) estimate significant and large variation among teachers in math and reading achievement gains across student characteristics (black, gender, prior achievement, free-and-reduced lunch eligible, and of limited English proficiency), with the most variation in teacher comparative advantage regarding prior

[12] To study the impact of immigration on the school system, Albornoz et al. (2018) develop a model where parents (as well as schools) differ by the strength of their motivation to educate their child (or their pupils), and where adult labour market outcomes depend on learning effort exerted by the child at school. In contrast to our model, their model has students who lack an intrinsic motivation to learn. Instead, their model has parents and schools provide costly incentives that reward students' learning efforts.

[13] Here are just a few examples: From the Seacoast Charter Academy under their curriculum tab (https://seacoastcharteracademy.org/curriculum): "Lessons planned around the student body." From the Duval Charter School at Southside (https://www.southsidehcarter.org/apps/pages/index.jsp?uREC_ID=398200&type=d): "Teachers have the ability to use various resources to deliver curriculum in a way that best meets the needs of students.). From the Urban Academy (https://www.urbanacademypgh.org/academics/): "Urban Academy is free to design curricula and co-curricular activities that re mission-based and responsive to the needs of our students …" From the Advantage Academy (https://www.advantageacademy.org/Page/1901): "Foundational strategies … [a]t an appropriate level of rigor and complexity designed to facilitate deep knowledge acquisition."

[14] Evidence on gains from teaching at the student's level from India, Kenya, and Ghana is discussed at https://poverty-action.org/teaching-level-child , Innovations for Poverty Action.

achievement.[15] Efficient matching of teachers to students are estimated to have large effects.[16] Finally, our model takes account of potential divergence between parental and student preferences, and we find conflict in equilibrium in the case of no excuses charter schools. Students do not like to study as much as their parents prefer, consistent with some evidence on no excuses charter schools (Golann, 2015).

The theoretical school choice literature has focused on vouchers (e.g., Epple and Romano, 1998 and Nechyba, 2000) and public-school choice (e.g., Epple and Romano, 2003 and Avery and Pathak, 2021). These models typically have one-dimensional (ability) or two-dimensional (ability and income) student differentiation, some with a Tiebout-type locational element. Differences we highlight in our modelling are two-dimensional student capability differentiation and related student sorting among schools, the student-curriculum match determining idiosyncratic school quality, and competition between schools on this dimension. Our model also highlights student effort and that this is a potentially important margin for school policy. Related to curriculum differentiation is the Hotelling (1929) inspired literature. Hotelling studied duopoly equilibrium with product differentiation, illustrated by firm locational differentiation with prospective buyers facing travel costs, while stressing the generality of the equilibrium implications ("there will be many causes leading particular classes of buyers to prefer one seller to another, … here symbolized by transportation costs," p. 45). His main conclusion was that competition for buyers excessively reduces differentiation ("… cider is too homogenous," p. 57). His model spawned a subfield of study of oligopoly competition with endogenous product differentiation.[17] The other main application (see Downs 1957) considers location of politicians' policy platforms with voters differing in their policy preferences. Curriculum "location" in our model is relative to given student ability. As in the standard applications, differentiation results from locational (curriculum) choice, appeal to clientele (students) decreases with distance, and providers (schools) compete on the locational dimension. In addition to the application, important differences here are the objectives of the competitors and clients and the positive and normative consequences of the competition.

3. Model and Preliminary Results

3.1 The Model.

Households, Education Production, and Achievement. We study a continuum of households with mass normalized to 1. Each household consists of one school-aged child and a parent or parents. Our model reflects and synthesizes the latest available evidence on skill formation and the determinants of school achievement in the literature cited above. Households differ along three dimensions: parental

[15] See their paper for discussion of related research on matching of teachers to students.

[16] One example counterfactual they compute based on their estimates has teachers optimally reassigned given their comparative advantages within a district-grade holding constant classroom student demographics with gains close to three percent of a standard deviation in math and reading scores. Gains would be substantially larger if classes could be reconstructed too. See Scenarios II and III in their Table 7 (pp. 36).

[17] See Graitson (1982) for a review of Hotelling spatial competition models.

income (y), the child's intellectual or cognitive ability (b), and the child's motivation (m). The characteristics (b,m) of the child can be more broadly conceptualized as cognitive and non-cognitive skills, respectively. Students of greater cognitive ability have higher educational achievement for given school characteristics and effort ($e \geq 0$) in school. Students who are more motivated are more efficient at or more inclined to study, which we capture by assuming they face lower costs of exerting schooling effort. For expositional ease, we refer to b as just "ability." We assume parents know their own child's ability and motivation.

The probability density function of student types in the population is denoted with f(b,m,y), and is positive on its support $[b_m, b_x]\times[m_m, m_x]\times[y_m, y_x]$. In our computational analysis, we study cases where the household population is representative of the US population, a poverty population, and a suburban population. This is to investigate charter school practices if entering a school district with a representative population (e.g., a typical town) or if entering a poorer or richer district that arises from Tiebout sorting (e.g., an inner-city district or suburban district).

School quality varies among students depending on the match of the student's ability to the level of ability targeted in the school's instruction or curriculum (B). Specifically, the school's quality to student of ability b is given by:

(1) $q = Q\Gamma(|b-B|)$, with Γ twice differentiable and having $\Gamma' < 0$;

where Q is a common quality component (e.g., per student expenditure) and Γ a student-school match component. We generally assume functions introduced below are continuous and twice differentiable without repetition.[18] Curriculum targeting to ability regards the pace and level of instruction (as well as complementary inputs, especially teachers), assuming higher ability students have more capacity to grasp concepts in a given amount of time (see e.g,. Cotton, et.al. 2020, discussed above and in footnote 29). Education production combines the student's study effort with ability and school quality to produce achievement (a), with[19]

(2) $a = a(q,b,e)$, with $a(\cdot)$ quasi-concave, and all of $\frac{\partial a}{\partial q}, \frac{\partial a}{\partial b}, \frac{\partial a}{\partial e}, \frac{\partial^2 a}{\partial q \partial b}, \frac{\partial^2 a}{\partial q \partial e}, \frac{\partial^2 a}{\partial b \partial e}$ positive.

To ensure that achievement increases with ability, even if the distance to the school's curriculum target increases with ability (i.e., for $b > B$), we assume:

Assumption 1 (A1): $\frac{da}{db} = \frac{\partial a}{\partial b} + Q\frac{\partial a}{\partial q}\frac{\partial \Gamma}{\partial b} > 0.$

Our initial analysis will have one (traditional) public school and one charter school, with limited capacity in the charter school denoted by κ. Mehta (2017) indicates a charter school competing with only one public school is supported by the (North Carolina) data, but we realize this is not always the

[18] The reader will see that we make a series of curvature assumptions to keep problems relatively well behaved and thus avoid much tedium. When we turn to computational analysis in Section 5, we specify functional forms.
[19] De Fraja et al. (2010) constitutes an elegant piece of evidence supporting the notion that effort affects education outcomes.

case and we consider charter competition with a public and magnet school in Section 5.3. While charter schools are legally public, we refer to the non-charter school as the public school. We assume the charter needs to fill to capacity to cover its fixed costs. Parents choose the school to maximize their child's achievement, or the school the student prefers if either school would lead to the same achievement. Parental and child school preferences can diverge as discussed presently.[20] Parents apply to the charter school if attending would strictly increase achievement, or lead to the same achievement and is preferred by the child. If applications exceed capacity, the charter conducts a lottery. We assume that only a fraction $\mu \in (0,1]$ will accept an offer and that the matriculation probability (μ) is constant among students. This allows parents to change their minds about sending their child to the charter and is empirically motivated. We discuss heterogeneity of this probability in the Conclusion.

While parents choose the school, the child chooses effort. Children care about achievement, but also about their effort costs (c). We assume effort costs are positive and given by c(e, m), satisfying $c(0,m) = 0$, $\partial c/\partial e > 0$, $\partial c^2/\partial e^2 > 0$, $\partial c/\partial m < 0$, and $\frac{\partial^2 c}{\partial e \partial m} < 0$. Thus, total effort costs and marginal effort costs decline with motivation. Student utility is given by:

(3) $$U = a - c.$$

Assumption 2 (A2): U is assumed to be strictly quasi-concave in e with an interior maximum.

An achievement standard for graduation will constraint some students' effort choices and, in Section 6 when we examine "no excuses charter schools," an effort minimum will be enforced. Consequently, the child may not prefer the school that maximizes achievement and, if schools lead to the same achievement, will prefer the school requiring less effort.

Schools. In addition to adopting a curriculum, a school has a "standard" S, the minimum achievement necessary for the student to pass or to "graduate." We normalize the achievement of non-graduates to zero (see footnote 26). The constancy of achievement for non-graduates conforms to assuming utility does not vary with academic performance if failing, e.g., such individuals work in jobs that only require physical labor. The utility a student of type (b,m) derives from attending school j is therefore given by:

(4) $$U^j = \left\{ \begin{array}{ll} a\left(q^j, b, e\right) - c(e,m), & \text{if } a \geq S^j \\ -\ c(e,m), & \text{if } a < S^j \end{array} \right\}$$ where the superscripts denote values for school j.

Conditional on not graduating, students optimally choose to exert zero effort and "drop out."[21] The utility a dropout receives is therefore equal to zero. We consider three types of schools: a public

[20] Gibbons and Silva (2011) provide evidence from survey data in England that "school value added dominates school unhappiness [of students] in explaining parents' school satisfaction (p. 323)."

[21] The student may just fail, rather than actually drop out. However, we refer to these students as dropouts. No students will apply to a charter school where they would drop out.

school (P), a traditional charter school (TCS) and later a no-excuses charter school (NEC). All schools must adopt the same standard, which we assume the state sets.[22] Generally, states set minimum academic skill requirements for passing grades and graduating, with 43 states having adopted the Common Core Standards.[23] For now we assume all schools receive the same funding per student, which is becoming more common. Equating Q to per student expenditure, it is then identical across schools.

The public school targets its curriculum to maximize aggregate achievement of its students, this including the (zero) achievement of all students that drop out. Specifying a univariate objective for the public-school decision maker is for tractability, though we discuss variation in this in Section 7. Specifying aggregate achievement maximization of public students, including dropouts, is a defensible objective given school accountability systems that have been adopted under The Every Students Succeeds Act (ESSA).[24] Perusing "Table 5.1: State-level accountability and reporting on ESSA plans, 2018," one finds that student achievement and its growth dominant state accountability indices, but also with large weights on graduation rates for high schools.[25] Given our static model, growth measures fold into achievement itself, and the attention to graduation is consistent with public schools weighing significantly dropouts. Note that the weight on graduation in our objective is implicit, and we examine robustness of our treatment of dropouts in Section 7 and in the appendix.[26] The public school is not capacity constrained and accepts all students that do not apply to the charter, or that apply but do not attend either because they lose the lottery or have a change of mind.

A charter school chooses its curriculum target to maximize *achievement gains* relative to what its students would achieve in the public school. This objective and the notion of curriculum targeting is supported by states' legislation authorizing charter schools. It is typical for legislation to stress the goal of improved achievement as well as use of a targeted curriculum.[27] Aggregate and average achievement gains of charter school students are denoted with Δ and $\overline{\Delta}$, respectively. A charter is

[22] An extension would consider charter schools that adopt a different standard, perhaps at least as high as the public school's.
[23] Information on the Common Core Standards is found at: www.thecorestandards.org/standards-in-your-state .
[24] ESSA became federal law in 2015, replacing No Child Left Behind, which requires every state to measure and report performance measures in reading, math, and science. States have flexibility in implementation of ESSA and adopt their own school accountability measures, but state accountability measures use similar metrics.
[25] https://nces.ed.gov/programs/stateform/tab5_1-2020.asp
[26] As detailed in the appendix, the key assumption we are making is that the achievement of dropouts is constant, not that it is equal to zero. One can simply interpret a(•) in our model as achievement beyond that of dropping out.
[27] Here are just a few typical (!) examples of language in these statutes: For Florida: "Charter schools shall fulfill the following purposes: 1. Improve student learning and academic achievement." For New Mexico: "The Charter Schools Act is enacted to structure their educational curriculum to … address the needs of all student, including those at risk; … to improve student achievement …" For West Virginia: "Public Charter Schools … (b) Public charter schools are intended to empower new, innovative, and more flexible ways of education all children within the public-school system to: (1) Improve student learning … (2) … resulting in higher achievement … (3) Enable schools to establish a distinctive curriculum, specialized academic or technical theme or method of instruction."

subject to a size and viability constraint: it must attract enough students to fill its capacity κ but cannot exceed it.[28] We assume the charter capacity is not large. Charter schools must accept all applicants if they have enough seats, or run a lottery if over-subscribed as discussed above. When we consider the NEC charter school in Section 6 we introduce more latitude in charter strategies. For now, let a j superscript identify non-common values over schools, with j ∈ {P, T} indicating, respectively, the public school and a TCS.

Timing and Information. Schools know the relevant population distribution f(b,m,y). The standard S is given. The public school first sets its target, B^P. The charter school observes B^P and then chooses its curriculum ability target B^T. Parents observe (S,B^P,B^T) and apply to the charter school if their child's attendance would (strictly) increase achievement or would lead to the same achievement and the child prefers the school, also knowing their child's characteristics (b,m). If oversubscribed, the charter school runs a fair lottery, and each winner decides whether to matriculate or not. Finally, once assigned to schools, children decide how much effort to exert and payoffs result. The schools and parents anticipate all decision makers' optimization in the subgame perfect equilibrium.[29] Specifying the public school as the first mover in curriculum choice arguably makes sense because it is likely to know if a charter school is planning to enter.[30]

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[28] Having a simple capacity constraint for school viability avoids introducing a cost function and strikes us as a reasonable approximation.

[29] Having described the model, we can better relate our paper to Cotton, et.al. (2020). They develop *and estimate* a model stemming from research in the psychology of education (see their paper for references) with students characterized by learning "efficiency" and learning "perseverance." The former measures the time to complete a learning task and the latter the time-costs of doing so. These align with ability and motivation in our model. Employing rewards to students from completing math tasks, they estimate the joint distribution on these student characteristics conditional on observable student characteristics (e.g., race and gender). They estimate cost functions of study time that vary with the perseverance characteristic analogous to our student-cost specification. They go on to estimate the effects of the two characteristics' relevance in (math) skill development both in the short and long run, both important and significant. They find school quality directly impacts math proficiency and the evolution of these skills. This is, of course, quite an incomplete discussion of their important findings. Their estimation is very supportive of our (independent) modelling. In addition to the obvious differences that we study competition between a public and charter school and our paper is purely theoretical-computational, other central differences are: (i) our modelling of curriculum targeting to one of the two skills; (ii) our analysis of no-excuses schools' enforcement of an effort minimum to somewhat overcome lack of motivation; (iii) student-type sorting given charter and public schools; and (iv) equilibrium effort choices given type and school. Regarding (i), Cotton et.al. do estimate that "high-performing school districts have higher TFP and lean more heavily on [the] academic efficiency [student characteristic], whereas middle- and low-performing schools have lower TFP and lean more heavily on a student's motivation level [here using 'motivation' to describe the perseverance characteristic] to generate improvements in math score." Note that these findings are consistent with school targeting because the students in their data that attend the high-performing school overall are of higher efficiency, while many attending the lower-performing schools have higher perseverance. To clarify (iv), while they are explicit about study-time choice of students in their experiment used in identification, their model and estimates of the effects of efficiency and perseverance on achievement is reduced form.

[30] Simultaneous choice of curricula leads to existence problems, because the public school's best response will typically be to make the charter school non-viable, which is inconsistent with a pure strategy response of the charter school. Relatedly, if the charter school must first commit to their curriculum, then the public school could drive it out with their choice of curriculum, implying no entry.

3.2 Preliminary Results: Single Public School. As a baseline and to clarify the model, we begin with the case of only a public school. The school's curriculum B^P then maximizes aggregate achievement of the entire population of students.

Student Effort Choice. We start by defining and identifying three effort levels of a student: (i) the optimal (unconstrained) effort choice (e^*); (ii) the effort necessary to reach the school's standard (e_S); and (iii) the effort cutoff above which the student would rather drop out (e_0). The unconstrained effort choice maximizes the objective in (3), giving $e^*(q^P, b, m)$, where $q^P = Q\Gamma(|b - B^P|)$, the latter from (1). The effort $e_S(q^P, b)$ necessary to just meet the standard satisfies a = S. The effort cutoff for dropping out $e_0(q^P, b, m)$, satisfies a – c = 0. Note that $e_0 > e^*$ under A2.

Now letting a^* denote the achievement of the student if choosing e^*, Lemma 1 identifies three types of students, with Figure 1 illustrating the partition of students in the (b,m)-plane into the three groups. This partition uses two loci in this plane, denoted $\hat{m}_S(b)$ and $\hat{m}_0(b)$, the upper one satisfying $e^*(q^P, b, \hat{m}_S) = e_S(q^P, b)$, and the lower satisfying $e_S(q^P, b) = e_0(q^P, b, \hat{m}_0)$. Note that these loci would shift with (Q, B^P), but these values are here fixed and then suppressed as arguments. It is straightforward to show that both loci are downward sloping and $\hat{m}_S(b) > \hat{m}_0(b)$. [31] We have:

Lemma 1. (i) *High achievers* are students who achieve above the graduation standards: $a^* > S$. They choose $e = e^*$, satisfying $e_0 > e^* > e^S$ and are types (b,m) such that $m > \hat{m}_S(b)$.

(ii) *Effort-constrained students* are students who exert the minimum effort needed to graduate and so obtain achievement equal to S. For them, $e_0 \geq e_S \geq e^*$, and are types (b,m) such that $m \in [\hat{m}_0(b), \hat{m}_S(b)]$.

(iii) *Dropouts* are students who drop out of school and exert no effort. For them, $e_S > e_0 > e^*$ and are types (b,m) such that $m < \hat{m}_0(b)$.

Proof. The proof is simple and in the online appendix.

Figure 1 adopts the functional forms we use in our computational analysis in Section 5, where we have denoted by $\tilde{a}$ a student's achievement. The student partition is very intuitive. Students with high ability or motivation can meet the standard with their preferred effort choice. Students with low ability or motivation are unwilling or unable to put in enough effort to graduate, and then drop out. Students "between" choose higher effort than their unconstrained preferred effort, but are willing to do so to graduate. Overall, ability and motivation are substitutes in producing achievement. For example, a lower ability student has to put in more effort to meet a graduation standard, but if more

[31] See the online appendix.

motivated is willing to do so. Note that achievement increases as one moves northeast in the space where achievement equals a^*.

4. Traditional Charter Schools

Here we study competition between the public school and a traditional charter school. The analysis proceeds by backward induction. First, we establish which households apply to the charter school as a function of the curriculum targets (B^P,B^T). Second, we determine the charter school's optimal curriculum choice. The TCS selects its ability target to maximize the achievement gains of its students, subject to the viability constraint and given the public school target. Last, we examine the public school's choice that will maximize achievement of its students (accounting for all dropouts).

4.1 Parental School Choice. The educational quality a student enjoys at a particular school is higher the closer is the student's ability to the school's curriculum target. Using (1) and that Q and S are the same in each school, $\tilde{b} \equiv \frac{B^P + B^T}{2}$ identifies a cutoff ability, with parents having students of ability on opposite sides preferring one school over the other unless they would drop out of both. We assume any students that would drop out of the charter school do not apply to it (even if they would also drop out of the public school). Taking an example with $B^T < B^P$, all students with $b < \tilde{b}$ who would not drop out of the charter school apply to it.

For students that obtain higher quality from attending the charter school, analogously defined loci partition this subset of students into high achievers, effort-constrained students, and those that would still drop out. Staying with an example having $B^T < B^P$, Figure 2 illustrates how the $\hat{m}_S^j(b)$ and $\hat{m}_0^j(b)$ rotate at $b = \tilde{b}$, with $j \in \{P, T\}$ identifying the school. In addition, Figure 2 shows the achievement *gains* of those who apply and would benefit if getting into the charter school (using obvious notation). The results we discuss are provided formally in the appendix, but are very intuitive. For students that would benefit from the charter school, for given ability, the motivation necessary to be a high achiever is lower than at the public school. Similarly, the motivation threshold for dropping out is lower. Three sets of students that would benefit by attending the charter gain strictly in achievement. The lower set in the figure would drop out of the public school but not the charter school where they just meet the standard, thus gaining S. The most talented students (in ability and motivation) that gain would exceed the standard at both schools, but gain in achievement at the charter school because it is a better match. Below this group, are students who would exceed the standard in the charter school while just meeting it in the public school, thus gaining $a_T^* - S$, where a_T^* denotes their unconstrained achievement at the charter school. A fourth remaining group just meet the standard at both schools so do not gain. However, these students have higher utility in the charter school since they can put in less effort to meet the standard, and their parents then apply. A similar partition of students results if $B^T > B^P$, here with higher ability students applying to the charter school.

It is useful to define the charter school applicant set, given by:

(5) $$A^T(B^T, B^P) = \{(b,m) \big| \, |b - B^T| < |b - B^P|, \; m > \hat{m}_0^T(b)\}.$$

These are all students that would have higher quality at the charter school and would not drop out. Observe that this set is empty if $B^T = B^P$. *To be viable, the TCS must differentiate its curriculum from the public school's one.*

Also, interestingly, parental and *student preferences between the public school and the TCS coincide, though student preferences include their effort costs.* On the one hand, for student types whose achievement differs among schools, the school that maximizes achievement also maximizes student utility since it offers them greater quality. On the other hand, for those whose achievement is identical in the two schools, recall that parents prefer the school the child prefers.

4.2 Charter Curriculum Choice. The charter school chooses its curriculum to maximize the aggregate achievement *gains* relative to attending the public school of the students it attracts, taking the public-school curriculum as given. The charter school must differentiate its curriculum to attract students, but could choose a higher or lower curriculum target and engender achievement gains to some students. Thus, assuming the charter school can attract enough students to be viable with both a higher and lower curriculum target, it will have a local maximum with both of a higher or lower curriculum. While the public school can squeeze out charter entry on either side of its own curriculum if it wants to do so, we ignore this until we examine the public school optimum. That is, assume for now that the charter school can enter with either a higher of lower curriculum target. Still we must deal with corner optima for the charter school. Suppressing functional arguments that are fixed or obvious (which we continue to do) and taking the example with $B^T < B^P$, the charter school viability constraint is:

(6) $$\mu N^T \equiv \mu \int_{y_m}^{y_x} \int_{b_m}^{\tilde{b}(B^T)} \int_{\hat{m}_0^T(b,B^T)}^{m_x} f \, db \, dm \, dy \geq \kappa,$$

where we use N^T to denote the mass of applicants to the charter school. Recall that μ is the matriculation probability assumed to be constant among student types. In the analysis here, we include B^T as an explicit argument in the $\hat{m}^T(\cdot)$ loci because we find the optimal B^T. Let $B_M^T(B^P) < B^P$ denote the minimum B^T such that (6) holds with equality. Assume for simplicity:

Assumption 3 (A3): $N^T \geq \kappa / \mu$ for $B^T \in [B_M^T(B^P), B^P)$.

The assumption is that demand to attend the charter school does not drop below capacity (adjusted for the matriculation probability) as the charter curriculum rises toward the curriculum of the public school. This is a weak assumption because all students with $b < \tilde{b}$ who would not drop out prefer the charter school and $\tilde{b}$ is increasing in B^T.[32] We make the analogous assumption when considering

[32]Demand to attend the charter need not increase throughout the range of B^T in A3 to satisfy the assumption and in fact it does not in general because more students can drop out. But we have verified the assumption in our

ability targets above B^P. This is the minimum B^T consistent with viability. For consideration of entry with $B^T > B^P$, let $B_X^T(B^P)$ denote the maximum B^T consistent with viability.

Given the charter school must be of size κ, a constant matriculation probability, and random admissions if over-subscribed, maximizing aggregate achievement gains of its students is tantamount to maximizing *average* achievement *of its applicant set*. The local maximum with $B^T < B^P$ is then the solution to:[33]

$$(7)\qquad \underset{B_M^T \le B^T \le B^P}{\text{Max}}\ \bar{\Delta} = \left[\begin{array}{l} \int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\int_{\hat{m}_0^T(b,B^T)}^{\hat{m}_0^P(b)} \text{Sfdmdbdy} + \int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\int_{\hat{m}_S^T(b,B^T)}^{\hat{m}_S^P(b)} \left[a^*(B^T)-S\right]\text{fdmdbdy} + \\ \\ \int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\int_{\hat{m}_S^P(b)}^{m_x} \left[a^*(B^T)-a_P^*\right]\text{fdmdbdy} \end{array}\right] / N^T,$$

$$\text{s.t.}\quad N^T = \int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\int_{\hat{m}_0^T(b,B^T)}^{m_x} \text{fdmdbdy}.$$

The integrands in the objective function correspond to the achievement gain amounts for the three groups discussed above (see Figure 2), where a_P^* denotes achievement of high-achieving students if attending the public school.

Proposition 1. (TCS Curriculum Choice.) Assuming viable entry with B^T on both sides of B^P is feasible, the traditional charter school problem has at least two (local) maxima, at least one with entry at the low end, $B^{T*} < B^P$, and at least one with entry at the high end, $B^{T**} > B^P$. To avoid tedium, we assume a unique local maximum on both sides of B^P. We have:

1. Low-end entry: Assuming $B^T < B^P$, the local maximum satisfies:

$$(8)\qquad \left[\begin{array}{l} \int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^{T*})} (\bar{\Delta}-S)\frac{\partial \hat{m}_0^T(b,B^{T*})}{\partial B^T} f(b,\hat{m}_0^T(b,B^{T*}),y)\text{dbdy} \\ \\ +\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^{T*})}\int_{\hat{m}_S^T(b,B^{T*})}^{m_x} \frac{\partial a^*(B^{T*})}{\partial B^T} f(b,m,y)\text{dmdbdy} - \bar{\Delta}\cdot\frac{\partial \tilde{b}}{\partial B^T}\int_{y_m}^{y_x}\int_{\hat{m}_S^T(\tilde{b},B^{T*})}^{m_x} f(\tilde{b},m,y)\text{dmdy} \end{array}\right] \le 0;$$

with equality if $B^{T*} > B_M^T$.

2. High-end entry: Assuming $B^T > B^P$, the local maximum satisfies:

quantitative analysis, which amounts to verifying that, relative to $B^T = B_M^T$, the demand enhancing effects of a higher B^T dominate increases in drop outs that sometimes arise.

[33] By setting up the problem this way, the viability constraint in (6) can be ignored. It is satisfied for any $B^T \in [B_M^T, B^P]$.

$$
(9)\quad \left[\begin{array}{l} \displaystyle\int_{y_m}^{y_x}\int_{\tilde{b}(B^{T**})}^{b_x} (\bar{\Delta}-S)\frac{\partial \hat{m}_0^T(b,B^{T**})}{\partial B^T} f(b,\hat{m}_0^T(b,B^{T**}),y)dbdy \\ \\ \displaystyle +\int_{y_m}^{y_x}\int_{\tilde{b}(B^{T**})}^{b_x}\int_{\hat{m}_s^T(b,B^{T**})}^{m_x} \frac{\partial a^*(B^{T**})}{\partial B^T} f(b,m,y)dmdbdy - \bar{\Delta}\cdot\frac{\partial \tilde{b}}{\partial B^T}\int_{y_m}^{y_x}\int_{\hat{m}_s^T(\tilde{b},B^{T**})}^{m_x} f(\tilde{b},m,y)dmdy \end{array}\right] \geq 0;
$$

with equality if $B^{T**} < B_x^T$.

The complete proof is in the appendix. Taking the case of $B^T < B^P$, the proof shows in part that the sign of the LHS of (8) coincides with $\text{Sign}\left[\frac{\partial \bar{\Delta}}{\partial B^T}\right]$. The expression in (8) contains the effects of changing B^T on the charter objective. To interpret it, consider an increase in B^T and refer to Figure 2. There are three effects on average achievement gains. The increase in B^T shifts marginally to the right the vertical lines at B^T and $\tilde{b}$, i.e., these values both increase. The third term in the expression is the *loss* of bringing in students in the vicinity of $\tilde{b}$. These students add essentially 0 to the average achievement gain, while taking away seats, then reducing the average achievement gain $(\bar{\Delta})$ by an amount proportional to the average achievement gain (multiplied by the magnitude of their increased numbers). The second term corresponds to the effect on all students who are unconstrained in the charter, those who have achievement equal to $a^*(B^T)$. These are students in the charter with $b < \tilde{b}$ and $m > \hat{m}_s^T(b, B^T)$. Among them, those with $b >$ (<) B^T have increased (decreased) achievement and so an achievement gain (loss) results from the increase in B^T. Thus, the second term in (8) is of ambiguous sign. The first term captures the last effect on achievement gains. This stems from a steepening of $\hat{m}_0^T$ at (approximately) point $b = B^T$ (not shown in Figure 2). (The $\hat{m}_0^T$ locus also shifts marginally, this having second-order effects.) Some of those with $b < B^T$ drop out, resulting in reduced achievement gain of S. The reverse is true for some with $b > B^T$. The change in $\bar{\Delta}$ is the gain or loss of S relative to $\bar{\Delta}$. It is likely that $S > \bar{\Delta}$. Therefore, students in this group attracted to the school increase the average gain, while dropouts decrease it. Thus, the first term is of ambiguous sign too. We note that $\hat{m}_s^T$ will also steepen at (approximately) $b = B^T$, but this has no first-order effect on $\bar{\Delta}$ since these students have $a^* \approx S$.

To summarize, the first two terms on the LHS of (8) are of ambiguous sign, while the third is negative. The corner solution with $B^T = B_M^T$ arises if the expression is not positive there, which we find typically in our computational analysis. The intuition is that reducing B^T decreases dropouts so the charter school lowers it as much as feasible while attracting enough applicants to be viable. The local maximum for $B^T > B^P$ has analogous explanation.

Whether the global maximum has charter entry targeting lower or higher ability students depends overall on the public-school curriculum target, the achievement function and impact of the curriculum, and the type distribution. We do not have general results here, rather examine this computationally. Intuitively, a lower (higher) public curriculum target favors entry by the charter school to serve higher (lower) ability students since the public school provides a bad match for those students. The curriculum effect can be stronger for lower or higher ability students, obviously providing stronger incentives for the charter school to target those that benefit the most from a better match.[34] Regarding the type distribution, for example, the presence of numerous students near an ability target obviously implies higher achievement gains if targeted, though this just speaks to one element of distribution effects. We will find TCS entry targeting lower ability students arises for several student populations later, *but this is in part because the public school strategically chooses their target to induce this.*

4.3 Public School Curriculum Choice. The public school selects their curriculum in sequential equilibrium to maximize achievement of its students including all dropouts. As noted, we find in the computational analysis the public school chooses a curriculum that induces entry by the charter school that targets lower ability students. Here we examine the incentives that lead to this.

The public school could "force entry" by the charter school such that it targets students of lower ability than the public school or of higher ability. Let B^P_X denote the minimum B^P such that entry by the charter to attract higher ability students (this requiring $B^T > B^P$) is infeasible because there are not enough students that would prefer the charter school for it to be viable (see the appendix for formal definition). Likewise, define B^P_M as the maximum B^P precluding entry by the charter school with $B^T < B^P$ to attract lower ability students. Entry on either side of B^P is feasible for $B^P \in (B^P_M, B^P_X)$. While the public school can preclude entry at either end with choice of B^P outside of the latter range, more usefully one can show (see the appendix) that there exists tighter thresholds $\underline{\tilde{B}}^P, \tilde{\tilde{B}}^P \in (B^P_M, B^P_X)$, $\tilde{\tilde{B}}^P \geq \underline{\tilde{B}}^P$ such that the charter school would choose to enter with $B^T < B^P$ if $B^P > \tilde{\tilde{B}}^P$ and with $B^T > B^P$ if $B^P < \underline{\tilde{B}}^P$. In the following discussion, we assume a unique threshold $\underline{\tilde{B}}^P = \tilde{\tilde{B}}^P$, then just denoted $\tilde{B}^P$.[35] Then the charter school enters with $B^T < (>)\ B^P$ if $B^P > (<) \tilde{B}^P$. *Our intention here is to clarify the forces that determine the character of equilibrium rather than cataloging a set of cases for poorly behaved functions.*

What B^P is optimal for the public school? Refer to Figure 3 that shows the three possible cases of equilibrium. Let $\underline{a}^P(B^P)$ denote aggregate achievement of public-school students assuming

[34] The fact that Γ changes symmetrically as b moves away from B does not imply symmetric effects on achievement because Γ does not enter achievement linearly.
[35] The appendix provides a condition for this uniqueness.

entry by the charter school at their achievement-gain-maximizing curriculum for $B^T < B^P$. This function has domain $B^P > B^P_M$ so that it is feasible for the charter school to viably enter to attract lower ability students. Similarly define $\overline{a}^P(B^P)$ to equal aggregate achievement of public-school students given conditionally optimal (thus viable) entry by the charter school with $B^T > B^P$, having domain $B^P < B^P_X$. Figure 3 is drawn assuming both of these functions are single peaked with maxima on (B^P_M, B^P_X), though one can obtain similar results more generally. Denote these conditional maxima by $\underline{B}^{P*}$ and $\overline{B}^{P*}$.[36] Figure 3 also entails the property that $\underline{a}^P(B^P) > \overline{a}^P(B^P)$ for all B^P. The latter says that aggregate achievement of public-school students is higher when the charter school enters with $B^T < B^P$ and thus draws away lower ability students than if entering at the high end drawing away high ability students. This obviously holds if motivation does not vary among students or if it is perfectly positively correlated with ability.[37]

The three equilibrium possibilities are determined by the relative value of the threshold $\tilde{B}^P$: (i) If $\tilde{B}^P \leq \underline{B}^{P*}$, then $B^P = \underline{B}^{P*}$ and the charter school enters targeting lower ability students. (ii) If $\tilde{B}^P \in (\underline{B}^{P*}, \hat{B}^P]$, for $\hat{B}^P$ the upper value satisfying $\underline{a}(\hat{B}^P) = \overline{a}(\overline{B}^{P*})$, then $B^P = \tilde{B}^P$ and the charter school enters targeting lower ability students. (iii) If $\tilde{B}^P > \hat{B}^P$, then $B^P = \overline{B}^{P*}$ and the charter school enters targeting upper ability students.[38] Intuitively, the public school prefers entry by the charter school targeting lower ability students so long as it can induce this without having to adopt too high a curriculum itself. This happens in Cases (i) and (ii), respectively illustrated in Figure 3 for the thresholds $\tilde{B}^P_1$ and $\tilde{B}^P_2$. In Case (i), the public school's preferred curriculum given lower end entry by the charter school also induces lower end entry. This is what we find computationally below for the several student populations we examine. In Case (ii), the public school must increase its curriculum above that otherwise preferred to induce entry at the low end. In Case (iii), to induce lower-end entry would require too high a curriculum target to be optimal (e.g., for $\tilde{B}^P_3$ in Figure 3), and then the public school chooses the optimal target given there will be upper end entry by the charter school.

5 Computational Model and Analysis.

5.1 <u>Calibration of the Computational Model</u>. The building blocks of our model are the joint distribution of income, cognitive, and non-cognitive skills, the achievement production function, and

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[36] It is irrelevant to the results that follow that $\underline{B}^{P*} > \overline{B}^{P*}$, though this is realistic: If lower ability students are drawn off by the charter school, then the optimal B^P is higher than if higher ability students are drawn off by the charter school.

[37] It must also be that some students exceed the standard in the public school when the charter school enters at the low end, or that the ability distribution has sufficiently high upper bound to imply this.

[38] More generally, entry by the charter school with $B^T < (>)\ B^P$ arises if $\text{Max}_{B^P \geq \tilde{B}^P}\ \underline{a}(B^P) > (<) \text{Max}_{B^P \leq \tilde{B}^P}\ \underline{a}(B^P)$. With the restrictions in Figure 3, one gets the first outcome, sometimes at the corner solution, or otherwise the second outcome with interior solution for B^P to the second maximization.

the cost function for student effort. Education policy establishes curriculum and sets standards for graduation. These policies influence students' effort decisions which, coupled with the distribution of skills and the achievement function, determine the realized distribution of achievement and the realized dropout rate. Our approach to the calibration is to combine "direct" matching of some parameters to empirical counterparts when straightforward and choice of other parameters such that equilibrium predictions conform to empirical findings. The model we fit is of a small nationally representative area with one public high school and no charter school. The area has household distribution of income corresponding to that in the US, and with a student population for which the joint distribution of cognitive and non-cognitive skills is representative of the student population in the US and for whom attending the public high school yields an achievement distribution representative of the 12th grade achievement distribution in the US.

We specify simple functional forms for the achievement, school quality, and student effort-cost functions that economize on parameters. Consistent with (1) and (2) in the general development, the achievement function and school quality functions are given by:

(11) $$a = b^{\beta} qe;\; q = Q\Gamma_m Z(b,B) = Q\Gamma_m[\alpha + (1-\alpha)\exp\{-\left(\frac{b-B}{\tau}\right)^2\}],\ \alpha \in [0,1];$$

with Z(b,B) compact notation for the bracketed term in the school quality function. The curriculum element of school quality includes a weighting parameter, α, with lower value implying the ability targeting has greater effect on a student's achievement. The parameter τ permits further flexibility in the magnitude of curriculum effect, though we set it equal to one in the main analysis. The student-effort cost function is:

(12) $$c(e,m) = \frac{e^2}{2m}.$$

The parameters of our model are then the following:

μ_y, σ_y : mean and standard deviation of the distribution of the ln(y).

μ_b, σ_b : mean and standard deviation of the distribution of the ln(b).

μ_m, σ_m : mean and standard deviation of the distribution of ln(m).

ρ_{by} : correlation of ln(b) and ln(y).

ρ_{bm} : correlation of ln(b) and ln(m).

ρ_{my} : correlation of ln(y) and ln(m).

β: exponent of b in the achievement function.

Q: expenditure per student in the public school.

α, Γ_m, τ : parameters of the curriculum function.

S: standard in the public school.

B^P: Curriculum target

In addition to the dropout rate and an R^2-ratio discussed below, seven targeted values are:

μ_a, σ_a : mean and standard deviation of the distribution of ln(a).

ω_q: Five quantiles of NAEP mathematics achievement.

We first present parameters and relationships among parameters determined by direct matching.

- We estimate the mean and standard deviation of the distribution of the logarithm of income as follows. The US income distribution is well approximated by a lognormal distribution. Then $\ln(y) \sim N(\mu_y, \sigma_y)$. Mean and median household income in 2014 were, respectively $72,641 and $53,657.[39] Matching these empirical values implies $\mu_y = \log(53{,}657) = 10.89$ and $\sigma_y = .78$.[40]

- In 2014, 14.8% of households were below the poverty line.[41] Hence, when we analyze equilibrium with a poverty population, as when charter schools commonly enter in a central city, we take that population to be households below the 14.8th percentile of the income distribution using the implied conditional distribution. Relatedly, when we analyze equilibrium with a "suburban population," we take that population to be the national one with the poverty population removed.

- We take the joint distribution of the logarithms of ability and income to be normally distributed. Solon (1992) and Zimmerman (1992) estimate the correlation between father's income and son's income and both find .4. Hence, in modeling the relationship of income and ability, we take the correlation of ln(y) and ln(b), ρ_{by}, to be .4. This in turn implies that a household with ability one-tenth of a standard deviation above the median will have earnings 3.5% higher than a household at the median. This corresponds closely to the 3.3% estimate of the gain in annual income at age 50 from a one-tenth standard deviation increase in cognitive ability in Table 3 of Lin, Lutter, and Ruhm (2018).

- The distribution of the logarithm of ability is calibrated as follows. As noted above, we take ability to be lognormally distributed, and we take the logarithm of ability to be IQ. The standard measure of IQ has mean 100 and standard deviation 15. The ratio of the latter to the former is .15. Hence, in our model, the ratio of the standard deviation of log ability to the mean of log ability is $\sigma_b / \mu_b = .15$.[42]

- Findings of Cunha, Heckman, and Schennach (CHS) (2010) and Borghans, Golsteyn, Heckman, and Humphries (BGHH) (2016) play a central role in our calibration of non-cognitive skills, the relationship of non-cognitive to cognitive skills, and the roles of the two types of skills in determining achievement. From CHS, we obtain $\rho_{bm} = .2165$ and $\sigma_m / \sigma_b = \sqrt{.75}$. The details are presented in the accompanying appendix.

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[39] https://en.wikipedia.org/wiki/Household_income_in_the_United_States#Mean_household_income

[40] An example of a representative locality with closely matching income distribution, including proportion in poverty (see next bullet point in text), is Kokomo, IN, population 59,604. Its income-distribution values are $\mu_y^K = 10.89$, $\sigma_y^K = .734$, and poverty rate equal to 14.8%. The first and third values are exact matches to our national calibration and the standard deviation is just a little lower.

[41] https://www.census.gov/library/publications/2015/demo/p60-252.html

[42] Quantiles of a normal distribution are invariant to the choice of mean as long as the ratio of the standard deviation to the mean is preserved. For example, IQ can be measured with a mean of 1 and standard deviation of .15, a mean of 2 and standard deviation .3, etc. We exploit this feature at several points in our calibration.

- There is scant and mixed evidence to inform calibration of ρ_{ym}. Fletcher and Wolfe (2016) provide evidence of positive correlation, Li, Zhang, and Zhou (2023) provide mixed evidence, including finding negative correlation between perseverance and household income, and Khanam and Nghiem (2016) provide evidence of zero correlation. We then set $\rho_{ym} = 0$ but perform some sensitivity analysis that show small effects (see the appendix).
- We set Q = 14.8, equal to expenditure per student in thousands in public schools in 2013-14.[43]

We next present the parameters and relations among parameters that are targeted in solving for equilibrium of our model.

- BGHH Appendix Table 7.8 columns (1) and (2) provide regressions of achievement on IQ and achievement on non-cognitive skills respectively. The ratio of the R^2 statistic in column (2) to that in column (1) is .173/.489 = .35. Our calibration requires the counterpart regressions in our simulated sample of 20,000 public-school students in equilibrium to produce the same R^2 ratio. In particular, let R^2_{am} be the R^2 obtained in our model from regressing ln(a) on ln(m) and let R^2_{ab} be the R^2 obtained from regressing ln(a) on ln(b). Our calibration targets equilibrium that has $(R^2_{am}/ R^2_{ab})=.35$.
- In their regression including both cognitive and non-cognitive skills, (Table 7.8, column 3), BGHH obtain an $R^2 = .53$. As they note, measurement errors and/or unmeasured factors account for the unexplained variance. In our calibration, we assume the R^2 would be .75 if all relevant elements on the right-hand-side were measured without error. This implies that the variance of achievement, σ^2_a, can be obtained from the variance of measured achievement, σ^2_x, as follows: $\sigma^2_a = .75\sigma^2_x$.
- The mean (μ_a) and standard deviation of the logarithm of achievement (σ_a) are targeted in our calibration along with measurement error in achievement testing. To obtain empirical counterparts, we use the relationship from the preceding paragraph along with data for the distribution of 12th grade scores from the National Assessment of Educational Progress, NAEP.[44] Median NAEP scores and standard deviations are reported for mathematics, reading, and writing. As further detailed in the appendix, using these NAEP data and $\sigma^2_a = .75\sigma^2_x$, we calibrate $\mu_a = 2.89$ and $\sigma_a = .48$.
- We also target the high school dropout rate. From the National Center for Education Statistics[45], we obtain the high school dropout rate of 5.4%.
- With some trial and error, we found that our calibration targets could be met with $\tau = 1$ in the curriculum function $\Gamma(b,B)$. We then choose α such that, other things constant, students would on average gain a modest .05 standard deviations in the log of average achievement from a two standard

[43] https://nces.ed.gov/fastfacts/display.asp?id=66. In current dollars, this is about $20,000.
[44] We obtained this NAEP information from: https://www.nationsreportcard.gov/ndecore/xplore/NDE
[45] This was the dropout rate in 2017 reported in Table 2.1 of https://nces.ed.gov/pubs2020/2020117.pdf

deviation curriculum adjustment from their optimal curriculum.[46] We conduct sensitivity analysis below regarding this part of the calibration.

- To calibrate β in the achievement function (11), it is useful to consider a student (b,m) not constrained by the standard. Using (11) an (12), optimal student effort is given by:

$e^* = b^{\beta} Q\Gamma_m Z(b,B) m.$

Hence, achievement for an unconstrained student is: $a^* = b^{\beta} Q\Gamma_m Z(b,B) e^* = [b^{\beta} Q\Gamma_m Z(b,B)]^2 m.$

Taking logs, we obtain the following[47].

$$\text{(13)} \qquad \ln(a^*) = 2\beta \ln(b) + 2\ln(Q\Gamma_m) + 2[\ln(Z(b,B))] + \ln(m)$$

Taking expectations we then obtain:

$$\text{(14)} \qquad \begin{aligned} E\ln(a^*) &= 2\beta E\ln(b) + 2\ln(Q\Gamma_m) + 2E[\ln(Z(b,B))] + E\ln(m) \\ &= 2\beta\mu_b + 2\ln(Q\Gamma_m) + 2E[\ln(Z(b,B))] + \mu_m \end{aligned}$$

The above expression shows that $\beta\mu_b$ scales the relationship between ln(b) and ln(a*). Thus, $\beta\mu_b$ can be treated as a single parameter. Hence, without loss of generality, we set β=1 while preserving in our calibration the requirement derived above that specifies $\sigma_b / \mu_b = .15$.

- To complete our calibration, we do a search for parameters that such that equilibrium in a simulated sample of 20,000 students satisfies the following criteria. In addition to fixing the already calibrated values of the joint income-ability distribution, our calibration objective is to have the following conditions set forth above be satisfied in equilibrium:

$\rho_{bm} = .2165$, $\sigma_m = \sigma_b\sqrt{.75}$, $\sigma_b / \mu_b = .15$, $\mu_a = 2.89$, $\alpha = .85$, $\sigma_a / \mu_a = .167$, $(R^2_{am}/ R^2_{ab}) = .35$, and dropout rate=5.4%, In addition, we seek parameter values such that the 10th, 25th, 50th, 75th, and 90th percentiles of achievement in the NAEP mathematics examination correspond closely to those in the model when the public school chooses its curriculum target to maximize aggregate achievement of all students. We proceed as follows. We impose the first five of the above conditions. We choose the public school curriculum, B^P, to maximize the aggregate achievement of the student population. This yields $B^P = \exp(\mu_b) \approx$ 50th percentile of ability. We then search for values of the remaining parameters that approximate the three remaining conditions enumerated above and the mathematics achievement

[46] The appendix provides details. It is assumed that the curriculum is two standard deviations away from the student's optimum and then adjusted to it. If we let students adjust their efforts optimally, the gains in achievement would be double.

[47] Earlier, we chose the mean and standard deviation of the logarithm of achievement, μ_a and σ_a, by benchmarking those parameters to the NAEP. In doing so, we did not impose the requirement that the logarithm of achievement in our model be normally distributed (see the appendix for the detail). As the preceding equation demonstrates, the logarithm of achievement is not normally distributed. While both ln(b) and ln(m) are normally distributed, ln(Z(b,B)) is not. The above equation is for students not constrained by the curriculum standard. There are also students who are constrained by the standard, including some who drop out. These factors also give rise to a departure of the achievement distribution from lognormality. We emphasize that these departures from lognormality are not of concern. It is inevitable that a realistic model with curriculum standards and dropouts will have an achievement distribution that is not lognormal.

quantiles reported in the NAEP. A fine-grained grid search yields the following remaining parameter values: S = 7.85, Γ_m = .042, μ_b = 1.8, μ_m = .023, σ_b = .27, and σ_m = .234.

This concludes presentation of the calibration. To assess the quality of the calibration, we compare model outcomes to the three targets above and to the targeted quantiles of mathematics achievement in the NAEP, normalizing the latter.[48] We see that the model provides a very good fit to the targeted values, with the exception of the 25th percentile of achievement. This is because the standard in the model induces higher achievement of lower ability students than observed in the data. Finally, we also perform sensitivity analysis in the appendix on (μ_b, μ_m), key parameters of the student-skills distribution, having some moderate effects on equilibrium predictions, but not affecting the structure of equilibria that we find.

Table 1: Model Fit

Measure	Target	Model
R^2_{am}/ R^2_{ab}	.35	.369
Dropout Rate	.054	.055
σ_a / μ_a	.167	.172
$\omega_{.10}$	-1.282	-1.288
$\omega_{.25}$	-0.674	-0.986
$\omega_{.50}$	0.00	0.00
$\omega_{.75}$	0.674	0.650
$\omega_{.90}$	1.282	1.241

5.2 Analysis of Traditional Charter School. We compute equilibria for three populations of students competing with one public school, and for the cases with the public school choosing their curriculum strategically and non-strategically. The next subsection considers an equilibrium with a public sector having two tracks or two schools. Table 2 collects key model predictions. The first three columns are for the nationally representative population as when a charter school enters in a representative town. Column 1 is the baseline of comparison with just the public school. Interestingly, the stand-alone public school targets the 49.9th percentile ability to maximize aggregate achievement, with 5.51% dropouts to which we have calibrated. Targeting the 49.9th percentile ability would be about the majority choice of households too, though this congruence to our public-school objective does not carry over to the other populations. With non-strategic behavior by the public school, i.e., retaining B^P equal to the 49.9th percentile (see Column 2), the charter school would optimally enter targeting low-

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[48]The NAEP composite score gives 50% weight to mathematics and 25% weight each to reading and writing. The distributions of the scores on the components are provided by the NAEP, but the distribution of the composite score is not. Hence, we use mathematics for our calibration because it has the largest weight of the three components. We obtained from the NAEP web page the following: The mean and standard deviation of 12th grade math in 2013 were 153 and 34 respectively. The 10th, 25th, 50th, 75th, and 90th percentiles of 12th grade math in 2013 were 110, 130, 153, 176, 196. Hence, expressed as standard deviations from the mean, the 10th, 25th, 50th, 75th, and 90th percentiles of NAEP 12th grade math in 2013 were (-1.265, -0.676, 0, 0.676, 1.265). The 10th, 25th, 50th, 75th, and 90th percentiles from our model, expressed as standard deviations from the mean, are (-1.261, -0.986, 0, 0.665, 1.279). https://www.nationsreportcard.gov/ndecore/xplore/NDE

ability students, specifically with ability at the 4.41th percentile. This is a corner solution, implying just enough students apply to take-up all seats (given their .5 matriculation probability), the lowest 20 percent of the ability distribution. As reported in Table 2 these are poorer students. The students that attend have quite large achievement gains, about .47σ of mean population achievement (with no charter school). We use this standard deviation (σ) throughout to report gains to have a constant benchmark, rather than for example the standard deviation of the endogenous population of public-school students (or other reasonable measures). Achievement gains are also reported as a proportion of the charter school's students' average achievement in the charter school (.167). As a result of charter school entry, dropouts decline substantially to 3.91%, this due to the targeting of low-ability students. As such, aggregate achievement of all students rises (see 'Total Achiev' entry). Anticipating the charter entry, the public school optimally adjusts its curriculum up at bit (Column 3), this slightly increasing the equilibrium achievement of its students (always counting dropouts) and in fact students in aggregate, though dropouts rise slightly. The charter school continues to be at a corner optimum, surprisingly decreasing its curriculum target, this revealing they would have preferred to do so with the non-strategic B^P, but could not lower it while retaining enough applicants. Overall, the charter entry benefits education, as further discussed below. We also compare predictions to empirical findings in Section 7 once we have reported our set of predictions.

Columns 4-6 regard the poverty population, empirically more important given most charter school entry occurs in the inner city. Column 4 is the optimum with no charter school, which has the public school adopt a much lower curriculum target, equal to the 4.7th ability percentile of the national population and the 12.5th percentile ability of the poverty population. Dropouts rise to 6.43% in spite of the lower ability target. If the public school maintains its ability target, a charter school would enter with curriculum targeting a much higher ability student (see Column 5), at about the 80th percentile of the population, appealing to a large proportion of students with 50.9% applying, then needing to run a lottery that random selects 20% of which half matriculate. Anticipating such entry, the public school would then increase the ability it targets with its curriculum to induce optimal entry by the charter school targeting very low ability students, again at a corner optimum with binding charter capacity constraint. The combination of curricula maximizes aggregate achievement relative to the previous two allocations, but has the highest dropout rates. This is because the public option worsens for very low ability students, some of whom will not matriculate to the charter school and then drop out. Given the public curriculum target, note that the charter school significantly reduces dropouts, which would be 11.5% with no charter school (not shown in table). For this population, the strategic behavior of the public school has interesting and quite significant effects.

Columns 7-9 regard a "suburban population," recall that removes the poverty population. Without a charter school (Column 7), the optimal public-school curriculum target is somewhat higher, reflecting a stronger ability distribution. The dropout rate is the lowest among the public-school only cases. Charter school entry would be at the low end with markedly reduced drop outs and substantial achievement gains for their students, whether the public school acts strategically or not.

In all cases that allow the public school to act strategically, entry targets low ability students with large achievement gains.

Table 2: Traditional Charter School (TCS)									
	Nat. Rep. Population			***Pov. Population***			***Sub. Population***		
	Public Only	**TCS Pub Non-Strategic**	**TCS Pub Strategic**	**Public Only**	**TCS Pub Non-Strategic**	**TCS Pub Strategic**	**Public Only**	**TCS Pub Non-Strategic**	**TCS Pub Strategic**
B^P **Per.***	49.9	49.9	52.8	4.70 (12.5)	4.70 (12.5)	26.4 (49.9)	55.0 (51.2)	55.0 (51.2)	55.0 (51.2)
B^C **Per.***	n/a	4.41	3.39	n/a	56.2 (79.8)	2.81 (7.90)	n/a	4.23 (2.84)	4.23 (2840)
Prop Apply Chart	n/a	.20	.20	n/a	.509	.20	n/a	.20	.20
Prop Attend Charter	n/a	.10	.10	n/a	.10	.10	n/a	.10	.10
Med. Ch. Inc./Med Pop. Inc.	n/a	.671	.670	n/a	1.04	.935	n/a	.778	0.778
Total Achiev	49,830	50,530	50,537	42,964	43,287	43,435	51,106	51,717	51,717
Achiev Gain Charter (SD)**	n/a	.468	.481	n/a	.224	.738	n/a	.409	.409
Achiev Gain Charter/ Mean***	n/a	.167	.171	n/a	.064	.267	n/a	.146	.146
Prop Charter Exceed Standrd	n/a	22.7%	19.1%	n/a	91.4%	7.73	n/a	24.1%	24.1%
Drops****	5.51%	3.91%	3.92%	6.43%	6.42%	8.80%	4.23%	2.83%	2.83%

*Entries equal the percentile targeted ability using national population ability distribution, with relevant population ability distribution in parenthesis for poverty and suburban populations.
**Entries equal the expected average log achievement gain of charter applicants reported as the ratio of the achievement gain to the standard deviation of log achievement of students calculated for the national population with the public school only (which optimally targets the 50th pct of ability, column 1).
***Entries equal the expected mean log achievement gain of charter applicants reported as the ratio of the achievement gain to the mean achievement of the charter students.
****Entries are the percentage of students that are dropouts.

5.3 Traditional Charter School Competing with Two-Track Public Sector. Here we investigate entry of a charter school facing a public sector with two educational tracks, one directed at top students. We describe the public sector as having a regular public school and a magnet school, though two completely separate tracks in the public school is analogous. This analysis assumes that the top 10% of ability students attend the magnet school, with the public sector setting the magnet curriculum (B^M) to maximize that subset's achievement. Given this, the lower-track public then sets its curriculum to maximize achievement of public sector students (again counting all dropouts), given the anticipated entry of the charter school. Thus, this is a version of the Stackelberg model.

Table 3 collects results, with columns for the alternative student populations. Table 3 reports the same results as in Table 2, but adds a row with the relative income of the magnet students and a row with the proportion of magnet students that exceed the standard. Key findings are as follows. First, the presence of the magnet has no detectible effect on the strategic behavior of the public sector with regard to the curriculum that is targeted to the lower-track public students, with then no effect on the charter choices. To see this, compare B^P in the strategic cases of Table 2 to its counterpart (also denoted B^P) in Table 3, and then one can see how most other values are unaffected as well (e.g., the charter curriculum). One would guess that having a magnet that serves the top ten percent ability students would induce the public school to lower the target to their "regular" students, but this does not occur significantly. This is because marginal curriculum changes have a very small effect on students with ability far distant from it, these being the magnet-school students. Note that dropouts are unaffected, this because no (detectible magnitude of) magnet-school students dropped out without the magnet school and then do not with it. The magnet school does, though, benefit its students and this is associated with a modest increase in aggregate achievement.

6. No-Excuses Charter School.

6.1. Theoretical Analysis of No-Excuses Charter School. Now we examine competition between an NEC school and the public school. In addition to selecting its curriculum, denoted B^N, the NEC can enforce a minimum effort of its students, denoted e^m. NEC schools require significant parental involvement, and we assume they can successfully enlist parental help in enforcing the minimum effort. Characterizing NEC's as enforcing an effort minimum is arguably a tractable way to capture several of the documented practices of these schools discussed in Section 2, examples being a longer school day, frequent testing, and having an ethos of comportment and a strong work ethic. We allow costs of enforcing the effort requirement, the cost per student increasing in the effort minimum. Specifically, let $Q^N = Q^P \cdot (e^m)^{-\phi}$, where Q^P is expenditure per student in the public school, Q^N net expenditure in the NEC having subtracted out the enforcement costs, and ϕ a non-negative parameter. Note that $\phi = 0$ implies no enforcement costs. Beyond the effort minimum, the NEC school is modeled like the TCS. It maximizes achievement gains of its students, its students must meet the

Table 3: Magnet School			
	Traditional Charter School vs Strategic Public School with Magnet Track		
	Nat. Rep. Population	***Pov. Population***	***Sub. Population***
B^P Per.*	52.8	26.4 (49.9)	55.0 (51.2)
B^M Per.*	96.1	85.5 (96.2)	96.8 (96.1)
B^C Per.*	3.39	2.81 (7.90)	4.23 (2.84)
Prop Apply Chart	.20	.20	.20
Prop Attend Charter	.10	.10	.10
Med. Ch. Inc./Med Pop. Inc.	.670	.935	.778
Med. Magnet Inc./Med Pop. Inc.	1.73	1.08	1.56
Total Achiev	51,1817	44,078	52,360
Achiev Gain Charter (SD)**	.482	.738	.409
Achiev Gain Charter/ Mean***	.172	.267	.146
Prop Charter Exceed Standrd	19.1%	7.73%	24.1%
Prop Magnet Exceed Standrd	100%	100%	100%
Drop*****	3.92%	8.80%	2.83%

*Entries equal the percentile targeted ability using national population ability distribution, with relevant population ability distribution in parenthesis for poverty and suburban populations.

**Entries equal the expected average log achievement gain of charter applicants reported as the ratio of the achievement gain to the standard deviation of log achievement of students calculated for the national population with the public school only (which optimally targets the 50^{th} pct of ability, column 1).

***Entries equal the expected mean log achievement gain of charter applicants reported as the ratio of the achievement gain to the mean achievement of the charter students.

****Entries are the percentage of students that are dropouts.

same graduation standard, it faces the same size-viability constraint, and it must admit students randomly if over-subscribed. In short, the NEC is the same as the TCS but can enforce a minimum effort requirement.

Effort Choice. Assuming attendance at an NEC, a student chooses the largest of three effort levels: the unconstrained optimal effort, e^*, the effort needed to graduate, e_S, or the minimum effort enforced by the school, e^m. This is provided the latter effort is no greater than the maximum the student is willing to exert, e_0. If it is, the student would rather drop out and exert zero effort. Formally, let e^N denote the effort choice of the student. Using that $e_0 > e^*$, we have:

$$(15)\qquad e^N = \begin{cases} \text{Max}\left[e^*, e_S, e^m\right] & \text{if } \text{Max}[e_S, e^m] \le e_0 \\ \\ 0 & \text{if } \text{Max}[e_S, e^m] > e_0. \end{cases}$$

The minimum-effort policy of the NEC school would constrain the choices of student types (b,m) for whom $e^m > \text{Max}[e^*, e_S]$. Among them, those that satisfy $e^m \le e_0$ choose to exert the minimum enforced effort, while the rest would experience negative utility if they did, preferring to drop out and then would not apply to the charter school.

Applicant Set to the NEC. Applicants to the NEC are students that would have higher achievement there given the NEC policy (B^N,e^m) and the public alternative characterized by B^P. In general, this will combine students for whom the NEC offers higher quality and who are not deterred by the effort minimum (i.e., would not drop out), and students for whom the effort minimum would increase their achievement in spite of lower quality. Either of these student subsets might be empty.

To find the applicant set, it is useful to first identify the set of students that would be effort constrained assuming they attend the charter school. In developing the results here, we assume the functional forms in the calibrated model for the school quality function, the curriculum function ((11) with $Q = Q^N$), and the student effort cost function (i.e., (12)). Refer to Figure 4, which shows iso-effort loci for students in the NEC, ignoring the minimum effort requirement. The locus $\hat{m}_S^N$ separates students for whom effort choice e^* suffices to meet the achievement standard, from those who would need to choose higher effort to meet the standard. This locus is as above for the other schools we have studied, here given by:

$$(16)\qquad \hat{m}_S^N \equiv \frac{S}{\left[b^{\beta} Q^N \Gamma\left(b, B^N\right)\right]^2}.$$

The locus $\hat{m}_0^N$ separates students willing to choose effort to meet the standard from those that would prefer to drop out, also analogous to this locus above, here given by $\hat{m}_0^N = \hat{m}_S^N / 2$. An iso-effort locus for effort level e_i has a vertical portion between these loci satisfying $e_i = e_s = S / [b_i^\beta Q^N \Gamma(b_i, B^N)]$, above the b that satisfies the latter. These students all optimally choose e_i to just meet the standard. The other student types on the locus choose $e_i = e^*$ with achievement exceeding the standard. Straightforward calculation yields that their (m,b)-type satisfies:

$$(17) \qquad \hat{m}_e^N = \frac{e_i}{b^\beta Q^N \Gamma(b, B^N)}.$$

The latter is downward sloping and lies above the $\hat{m}_S^N$ locus.[49] Note, too, that the loci shift rightward as e_i declines. Those student types choosing to attend the NEC to the right of and below the iso-effort locus $e_i = e_m$, and who would not drop out, would be effort constrained by the minimum. We denote the ability of the student with minimum ability who is constrained by the effort minimum with b_N, i.e., the ability corresponding to the vertical portion of the e^m iso-effort locus.

A number of cases of the exact partition of student types into applicants and non-applicants arise depending on Q^N relative to Q^P (and thus ϕ), B^N relative to B^C, and e^m. To illustrate, the simplest case has $Q^N = Q^P$ ($\phi = 0$) and $B^N = B^P$. Then the quality of the NEC and public school is the same for all students. The set of applicants to the NEC would consist of all students that would be constrained by the effort minimum and not drop out because they would all gain in achievement and no others would. Using the iso-effort loci, this set is easily identified.[50]

While we do not find choice of $B^N = B^P$ to be optimal in our computations for any ϕ, *it bears emphasis that the effort minimum itself serves to increase achievement of students and to select them*. We find that the charter school could be viable by choosing $B^N = B^P$, but it also chooses to differentiate its curriculum.[51] It is further implied (in our computations following) that the public school cannot induce the charter school to select low ability students by itself targeting high ability students with its curriculum because the charter school can still attract high ability students using the effort minimum. In the sequential equilibria we find, for a range of minimum effort enforcement costs, the charter school chooses $B^N > B^P$ along with a "high" effort minimum. The NEC then attracts both high ability students and some lower ability students that are highly motivated. *But we also find*

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[49] This follows because $e_i = e^*$ along $\hat{m}_e^N$ which exceeds $e_s = S / \left[b^\beta Q^N \Gamma(B^N, b) \right]$.

[50] The set of applying students is given by: $\{(m,b) \mid b \geq b_N, m \in [\frac{e_m}{2b^\beta Q^N \Gamma(b,B^N)}, \frac{e_m}{b^\beta Q^N \Gamma(b,B^N)})\}$.

[51] Perhaps surprisingly, for $\phi = 0$, choice by the NEC of $B^N = B^P$ and a binding effort minimum is a local maximum. This is because marginal deviations of B^N from B^P would induce a set of students to apply who obtain higher quality from the NEC, but are not effort constrained, with these students barely gaining in achievement, thus finitely reducing achievement gains of matriculates. This is formally shown in an earlier version (2022) of this paper, available on request. But we do not find this local maximum to be a global maximum in our computations, though did in the earlier calibration used in the latter paper.

that high enough enforcement costs induce no effort minimum, and the equilibrium conforms to that with a TCS. Rather than cataloguing a variety of theoretical cases, we now turn to the computational analysis to see and clarify the nature of achievement gains.

6.2 Computational Analysis of No-Excuses Charter School. We use the same calibration as above but must calibrate the minimum effort enforcement cost parameter ϕ. The enforcement cost function implies $\phi = \frac{1}{\ln(e^m)}\ln(\frac{Q^P}{Q^N})$. We combine predictions of our model regarding the optimal effort minimum with estimates of Q^P/Q^N to guide calibration of ϕ, but examine a range of values. If there were no enforcement costs and thus $Q^P = Q^N$, our model predicts the NEC would choose an effort minimum about equal to 8. Using this value for e^m, a value of ϕ is then implied by Q^P/Q^N. Two alternative approaches yield similar estimates of Q^P/Q^N and thus ϕ. The largest no-excuses charter network in the US is KIPP, which served 119,761 students in the 2022-2023 school year. KIPP reported 'Total Program Expenses' of $86,234,822.[52] Dividing this by their enrollment implies expenses of about $720 per student. If administration and fundraising expenses are included, then the expenditure per student is $95,843,939/119,761=$800. Census data[53] reported a preliminary estimate of $16,340 K-12 spending per pupil in the US in FY2022. Hence, KIPP expenses relative to public school per pupil expenditure were 720/16340 =.044 to 800/16340 =.049, implying a value of ϕ on the order of .05. An alternative calibration can be obtained from Roland Fryer's (2014) experiment that introduced no-excuses practices in public schools in Houston in 2014. For his experiment, Fryer reports expenditures of $1,837 per secondary student with tutoring, $1,100 without tutoring, and $355 per elementary student. In 2014-2015, the budget per student in Houston was $9,798.[54] The ratios of the above three expenditures to $9,798 are .187, .112, and .036. These imply ϕ values of .10, .057, and .0175. We then compute equilibria for values of ϕ ranging from 0 to .06. We conduct the analysis for the poverty population.

Table 4 reports model predictions and Figure 5a the partition of students for the case of no enforcement costs (Column 2 of Table 4). Column 1 is the baseline for comparison, the same as Column 4 in Table 2, the equilibrium for the poverty population with no charter school. Columns 2-3 are the Stackelberg equilibria with an NEC entrant, with no effort enforcement costs and with the lower cost having ϕ = .02. Columns 4-5 have the highest enforcement costs with ϕ equal to .04 and .06, but are not actually equilibria because the public school can strategically induce the charter school to enter with no effort minimum. Ignore the latter for the moment. To understand the

[52] https://www.kipp.org/faq/
[53] https://www.census.gov/data/tables/2022/econ/school-finances/secondary-education-finance.html
[54] https://www.houstonisd.org/cms/lib2/TX01001591/Centricity/domain/4/documents/2015/06/TaxRate2015_FxnalSummary.pdf.

equilibria, begin with the case of no enforcement costs. The public school continues to choose the same curriculum as with no charter school, B^P at the 12.5th ability percentile, with the NEC then entering with a high ability target at the 78.5th percentile along with a high effort minimum that constrains almost all the students that attend. Note that public school choice of B^P equal to the 12.5th ability percentile persists in all the cases we examine (to at least two decimal points).[55] The strategy of the NEC selects highly motivated students along with a fairly wide range of high ability students. It bears emphasis that the public school cannot induce the charter school to target low ability students because the effort minimum itself serves as an effective selection device. Figure 5a identifies the student partition and achievement gains of the groups that apply to and attend the NEC (recall that we assume half that would gain actually choose to attend). In Figure 5a, $\tilde{b}$ is the ability half way between B^P and B^N who finds the public and charter school to be of the same quality. A small group of students gain in achievement from attending the NEC in spite of lower quality ($b < \tilde{b}$), because the effort minimum offsets the lower quality, these being the highly motivated 'triangle' of students identified in the upper middle of the graph. They must be highly motivated to not give up and choose zero effort (drop out). Their parents then make them attend, as they would prefer to attend the public school.[56] A larger set of students gain from the charter both due to it being a better curriculum match and due to the effort minimum, this the bulk of the students identified in the upper right of Figure 5a. Many of these students would also prefer to attend the public school. A small set of about 1% of the NEC students that are very highly motivated and with very high ability gain from the charter without the effort constraint being binding, these identified in the upper right corner. The effort minimum is high, a multiple of 2.54 of the average effort of public-school students in the baseline equilibrium. Achievement gains of charter students are large, .9 standard deviation of the baseline public equilibrium achievement distribution.

Figure 5b shows the partition of students and the nature of charter student gains with positive enforcement costs corresponding to $\phi = .2$. Column 3 of Table 4 presents equilibrium values. While the public strategy is (essentially) unchanged, introducing positive enforcement costs leads the charter school to reduce slightly both the ability targeted and effort minimum. Interestingly, the lower common quality Q^N implies there are two values of ability for whom the charter school provides the same quality, denoted $\tilde{b}_L$ and $\tilde{b}_H$ in Figure 5b, with students of ability between (outside) receiving

[55] The impact of curriculum changes on the achievement of students with ability sufficiently far from the curriculum is negligible. Thus, when the standalone public school optimally targets low abilities, the change in its optimal curriculum after an NEC enters that targets high abilities is very small. In our sensitivity analysis (see Section 7 and especially Table A1 in the appendix), we show NEC entry induces curriculum change by the public school for higher values of τ for which the distribution of achievement changes is flatter.

[56] Gibbons and Silva (2011) provide evidence from survey data in England that "school value added dominates school unhappiness [of students] in explaining parents' school satisfaction (p. 323)." Several press articles document student unhappiness in no-excuses charter schools, though focused more on comportment and discipline than study requirements (e.g., Golann and Debs, 2019).

higher quality from the charter (public) school. As seen in Figure 5b, then two groups arise that gain from the charter only due to the effort minimum. Also, a group with very high ability and motivation opt for the public school. Still charter school students are overall highly motivated and with a fairly wide range of ability. Achievement gains are still large for charter students, but lower than without enforcement costs.

	Table 4: No Excuses Charter (NEC) **(Poverty Population)**					
	Public Only	**Pub/NEC No Enf. Costs (ϕ = 0)**	**Pub/NEC Enf. Costs (ϕ = .02)**	**Pub/NEC Enf. Costs (ϕ = .04)**	**Pub/NEC Enf. Costs (ϕ =.06)**	**Pub/TCS Pub Strategic**
$\mathbf{B^P}_{Per}$*	12.5	12.5	12.5	12.5	12.5	49.9
$\mathbf{B^N}_{Per}$*	n/a	78.5	76.8	77.3	79.2	7.90
Prop Apply Charter	n/a	.20	.20	.20	.20	.20
Prop Attend Charter	n/a	.10	.10	.10	.10	.10
Med. Charter Inc./Med Pop. Inc.	n/a	1.06	1.06	1.06	1.06	.935
Total Achiev	42,964	44,309	44,160	44,018	43,884	43,435
Achiev Public School	42,964	37,362	37,402	37,431	37,462	39,296
Achiev Gain Charter/SD**	n/a	.900	.800	.705	.616	.738
Achiev Gain Charter/ MEAN***	n/a	.194	.177	.160	.143	.267
Prop Charter Exceed Standard	n/a	1	1	1	1	.077
Effort Min NEC/ Avg.*****	n/a	2.54	2.44	2.33	2.23	n/a
Prop Effort Const. NEC	n/a	.991	.999	.999	.999	n/a
Dropout rates	6.43%	6.43%	6.43%	6.43%	6.43%	8.80%

*Entries equal the percentile targeted ability using poverty population ability distribution.

**Entries equal the expected average log achievement gain of charter applicants reported as the ratio of the achievement gain to the standard deviation of log achievement of students calculated for the national population with the public school only.
***Entries equal the expected average log achievement gain of charter applicants reported as the ratio of the achievement gain to the mean achievement of the charter students.
****Entries are the effort minimum over the average effort in the case with only a public school (Column 1).

For higher enforcement costs corresponding to ϕ equal to .4 or .6, it is a local optimum for the public school to continue to target the same ability, with continuation equilibria similar to the case of $\phi = .2$, the corresponding values reported in Columns 4 and 5 of Table 4. These allocations are not equilibria however (though the charter school is making optimal choices), because the public school can now target higher ability and induce the charter school to change their strategy, *with no effort minimum*. Note that the allocations have lower achievement gains to charter students. By setting the same curriculum as in the Stackelberg equilibrium in the TCS case, $B^P = 49.9$, the charter school prefers no effort minimum, implying the exact allocation as in this TCS Stackelberg equilibrium for the poverty population. The values are again reported in Column 6 of Table 4, where we have also included separately the public student aggregate achievement. The public school does better here than in the allocations in Columns 4 and 5. This is the Stackelberg equilibrium for these values of ϕ. This reversion to the equivalent of the TCS case is, of course, driven by the "sufficiently high" enforcement costs, which the charter school escapes by not enforcing an effort minimum. *We then have a TCS arising endogenously.* This does not contradict the empirical evidence we used in calibrating the enforcement costs because the no-excuses schools received some additional funding in these cases. If the charter school has access to funding that covers enforcement costs, then our model predictions correspond to the case of $\phi = 0$.

7. Model Predictions, the Empirical Evidence, and Sensitivity Analysis[57]

The computational model has predictions that can be compared to empirical evidence on charter schools. These regard students that attend charter schools and effects on achievement and attainment. These comparisons do not, of course, provide an empirical test of the model, rather help to assess how informative the model might be in understanding charter schools.

Overall, the evidence is that more charter schools enter urban areas.[58] Table 2 shows the highest achievement gains to charter students from a poverty population, assuming strategic public response. The model of the TCS then provides some support for such entry though predicted gains are likely too large serving any of the three populations as we discuss.

The model of the TCS predicts selection of poorer and lower-achieving students, with the exception of the TCS serving the poverty population without a strategic public school. For example, in the case of Column 6 of Table 2, the ratio of achievement if attending the public school of charter

[57] We already referenced the appendix for sensitivity analysis on the moments of the student skill distribution.
[58] See Epple, Romano and Zimmer (2016) and Raymond, et.al. (2023), pp. 42-43.

non-applicants to applicants is 1.52. This "negative cream skimming" is consistent with the evidence (see Booker, Zimmer, and Buddin, 2005 and Raymond, et.al., 2023, pp. 41-42). The NEC schools are, however, predicted to attract higher-achieving and slightly higher-income students. This is found e.g., by Abdulkadiroglu, et.al. (2011), Angrist, et.al. (2012), and Angrist, Pathak, and Walters (2013), and Walters (2018).[59]

Turning to predicted effects on achievement, our model's predictions are consistent with NEC's having high and higher-than TCS effects on achievement gains, but the predicted effects would seem higher than the empirical evidence indicates. Walters (2018) provides empirical estimates that no-excuses charter schools serving poverty population increase achievement for mathematics and reading by .71 and .52 standard deviations respectively. From Table 4, our model prediction ranges from .616 - .900 standard deviation gains depending on any additional costs the NEC bears. While the model is perhaps not too far off here, it predicts achievement gains for the TCS as high as .738 standard deviations (for the poverty population with strategic public school), while the empirical evidence typically predicts no or much smaller achievement gains (see e.g. the survey of Epple, Romano, and Zimmer, 2016). We have also conducted sensitivity analysis regarding the value of parameter τ (with details in the appendix), which recall impacts the magnitude of the curriculum effect (see Equation 11). A higher value of τ lowers achievement gains in the TCS-public school equilibrium, with much smaller effect in the NEC-public school equilibrium because the key margin for gains there is through effort effects. In fact, we can match much better the predictions to the empirical evidence by doing this. Nevertheless, we prefer the current calibration because of its nice match to the baseline achievement distribution.[60]

Also of interest in assessing achievement gains is the relevance of effects on dropouts. For the TCS serving the poverty population, 99% of the achievement gains are from reducing dropouts (see Table A2 in the appendix), while the NEC does not reduce dropouts and thus has no gains from this margin.[61] This TCS application predicts that 26.5% of charter students would drop out if not attending the charter, all else constant, a prediction of interest in its own right to which we return. If gains from reducing dropouts are down-weighted or even not counted at all, obviously, TCS predicted achievement gains are lower or much lower. We have conducted sensitivity analysis of this, with details in the appendix. We find that equilibrium predictions about curriculum choice are unaffected by the weighting, this because the TCS continues to prefer to target lower ability students (and dropouts are irrelevant to the NEC).

[59] Another well-established characteristic of no excuses charter students is that they have higher proportion black students, though race is not an element of our model.

[60] In an earlier version of the paper we used a calibration with higher value of τ that well-matched empirical predictions about achievement gains, also with some other parameter values relative to our current calibration. Having" improved" the calibration (match to the achievement distribution in the public-school baseline), we feel most comfortable sticking with it.

[61] Because the vast majority of TCS students just meet the standard both with TCS entry and without, their achievement gains are 0, though these students have higher utility.

Related empirical evidence of interest regards estimated effects of charter schools on dropping out of high school. In their analysis of charter schools in Florida, Sass, et.al. (2016) estimate a 6% increase in graduation probability from charter attendance, a bit lower than Booker, et.al.'s (2011) methodologically similar analysis that also included charter schools in Chicago. On the other hand, Angrist, et.al. (2013) find no significant impact on graduation of Boston's no-excuses charter schools. While our prediction on graduation effects of the TCS schools is again high, our prediction for NEC's aligns with Angrist, et.al. and is in the right direction for the Florida (and Chicago) schools. As discussed earlier, there is no doubt that many charter schools focus on at-risk students.

Another model prediction is that achievement gains in the no excuses charter school is higher for the lower scoring students. This is also consistent with estimates (see Abdulkadiroglu, et.al., 2011 and Angrist, et.al., 2012, and Walters (2018)). In his analysis, Walters (2018) finds that the gains in achievement in no-excuses charter schools are greatest for students who were previously relatively low achievers. For a random sample of 400 charter lottery applicants from our model, we simulated their charter school scores if they won the charter lottery and their public-school scores if they had lost the lottery. This sample size is comparable to that in Walters (2018). The regression in (17) below using this simulated data is the counterpart to the regressions presented by Walters in Table 7. As in Walters empirical estimates, we find that that the gains to charter school attendees are greatest for those who would otherwise have had the lowest achievement scores if they were in public school. In this simulated sample, these coefficients are both highly significant ($p<.001$). Our coefficient of -.668 is larger in absolute value than the estimates obtained by Walters. In practice, of course, an achievement test measures scores with error, which would tend to attenuate the magnitude of the estimated coefficient. In addition, Walters includes, quite appropriately, indicator variables for demographic groups. This may also reduce the magnitude of the coefficient estimate. Thus, while differences in magnitude are to be expected, our model delivers the kind of inverse relationship between achievement gains and public-school achievement scores that is found by Walters in his empirical analysis.[62] This is another important respect in which the implications of our no-excuses model are supported by empirical evidence.

(17) $\ln(\text{Achievement Gain}) = 2.54 - .668 \cdot \ln(\text{Achievement in Public School})$

8. Additional Issues and Extensions

8.1 Efficiency. Efficiency implications of charter schools are discussed here. In the economics of education literature, efficiency is commonly equated to student achievement, here also corresponding

[62] Walters (2018) also makes the point that the finding that marginal students that select into no excuses charter schools gain the most implies potential for higher gains: "charter expansion is likely to be most effective when targeted to students who are currently unlikely to apply (p. 2182)." In our model, students just below the margin of application to the charter are unwilling to put in the required effort and would then fail at the charter school. To induce them to select in and succeed (i.e., to target them) would require a lower effort minimum, this adversely affecting the performance of more motivated students. An interesting question is whether several charter schools with different effort minimum requirements would improve average achievement gains in light of student selection.

to parental utility. Student utility can, however, differ here from parental utility so we consider two notions of efficiency.[63] Proposition 2 provides some baseline results.

Proposition 2.

1. Assuming non-strategic behavior by the public school. Entry of a TCS implies a Pareto Improvement including both parents (achievement) and students. Entry of an NEC implies a Pareto Improvement ignoring student utility, but not so counting their utility.

2. In any cases of equilibrium with excess demand for the charter school, aggregate achievement would be increased if the charter school (TCS or NEC) were able to directly admit students on their type (b,m).

Formal proof is in the appendix, though Proposition 2-1 is immediate. The results pertaining to achievement or parental utility hold simply because the charter school makes choices to maximize achievement gains of their students, who select in only when their gains are positive (with no change in achievement for some students in the case of the TCS). With no spillovers on the public school, the achievement results are clear for both types of the charter school. That the Pareto Improvement carries over for the TCS including student utility is because students attend the charter school only if quality increases to them, implying (strictly) increased utility. This is not so for the NEC, with an example provided below.

Focusing on achievement, allowing strategic adjustment of the curriculum by the public school, winners and losers will arise if the public school adjusts its curriculum. Losers will consist of students that remain in the public school but have lower achievement because of a degraded match to the public curriculum as well as some students that attend the charter school because the public alternative is worse for them. Note that those that continue to just meet the standard will not experience an achievement loss. Consider the example of the TCS case for the poverty population, comparing the equilibrium in Columns 4 with only the public school to the equilibrium in Column 6 with the TCS having entered. In this example, the public school substantially increases its curriculum target to induce entry by the charter school targeting low ability students. Here 6% of the population of students experience lower achievement, mainly those who would gain from the charter school but do not matriculate, but also with some in the public school before and after the entry with a worse curriculum match. The proportion of achievement winners is 60.3%, mainly students attending the public school before and after the change, though with a few achievement winners in the charter school. Most of the charter school students continue to just meet the standard, but are able to do so more easily. The appendix provides graphs in student-type space identifying those that win, lose, or have the same achievement.

[63] The argument for focusing on parental utility or achievement is that parents make choices in their children's long-term interest. Children being forced to substitute study time for leisure activities (e.g., playing video games) is in the child's interest.

We do the same calculations for the example of the NEC with no enforcement costs (and serving the poverty population), here comparing equilibrium in Columns 1 and 2 of Table 4. Because the public school does not change its curriculum as a result of charter entry, no achievement losers arise and all charter-school students have strictly higher achievement. The strict gains of charter students are because none of them just meet the standard in the charter school, i.e., none of those that choose the charter school do so to make it easier to just meet the standard. Obviously, then, the gainers correspond to the set that attend shown in Figure 5a, 10% of the student population.

Turning to student utility and Proposition 2-1, for the case of TCS entry without strategic behavior of the public school, not only do students that gain in achievement have higher utility but those that have the same achievement also gain in utility since they have lower costs. Thus, all charter school students gain strictly and no other students are affected. However, with strategic behavior and a change in curriculum of the public school, there will be utility winners and losers among students. The appendix examines this for the same example of TCS entry. Overall, the percent of students that have higher utility is 77.5, the percent with lower utility is 16.36, and the percent with the same utility is 6.18. The detail is provided below, but utility winners consist of most of the matriculates to the charter school and, more so, those in the public who get higher quality as a result of the increased curriculum. Most of the losers are students that would gain from the charter school but do not matriculate. All those indifferent are students that drop out before and after charter school entry. Graphs in the appendix identify the characteristics of the winners, losers, and those indifferent.

Taking the same example of NEC entry as above, student utilities change only for charter students. Among them, 87% have lower utility by being made to attend the charter school and having to put in high effort. The remaining 13% of charter-school students gain, these highly motivated students who benefit from the better curriculum match and are unaffected or hardly affected by the effort minimum. See the appendix for a graph identifying the student winners and losers.

Turning to Proposition 2-2, it indicates that requiring an open enrollment policy reduces achievement generally. This holds because all students that gain in achievement apply and the continuum of student types implies some that apply will gain only marginally. Given excess demand and a fair lottery for admissions, some that barely gain will attend, taking slots from some that have higher gains and lose the lottery. By not admitting the former students, it follows that aggregate achievement gains will increase (though obviously with winners and losers).

This argument relies on one margin of gain from direct admission, with another margin being "bias" in choice of curriculum by the TCS and both the effort minimum and curriculum by the NEC to select students under open enrollment. Policy choice driven in part by selection implies deviation from policy choice that maximizes achievement gains of attending students. Gains with direct admissions arise because, under open enrollment, there will generally be students that gain only marginally that attend (some will win the lottery) and then the charter will trade off policy that is ideal for its students against selecting more marginal students. With direct admissions, the charter school could adjust its policy (e.g., curriculum) while not admitting marginal students. This, however, requires in general that the charter school faces excess demand to be sure to be able to meet its capacity under such adjustments. Such gains may or may not be feasible if a charter school is at a corner solution that just meets its capacity. In formalizing the intuitive argument in the appendix, the selection effect is characterized mathematically.

The efficiency gains summarized in Proposition 2 have nothing to do with potential competitive effects on school productivities. To examine seriously potential competitive effects would require a model extension that entails distortions underlying inefficiencies. Without pursuing this, one thing interesting to observe that could play a role is the nature of "cream skimming" by the charter. As discussed, the TCS typically targets lower ability students implying "negative cream skimming." Scores in the public school would increase, absent competitive effects. In contrast, the NEC charter does draw off from the public-school high achievers. The NEC is then more likely to spur improvement of the public school. Another potential competitive effect on the public school, though, is loss of students if per student funding exceeds marginal cost for both types of charters.

8.2 Endogeneity of No-Excuses vs. Traditional Charter School Entry. Given the success of no-excuses practices in increasing achievement, why do most charter schools not so operate? We have seen that uncompensated costs of enforcing the minimum can lead to it being abandoned and the equivalent of our TCS. In our robustness analysis, we have also shown that a very strong curriculum effect along with value in reducing dropouts can lead to an "endogenous TCS." Additionally, sufficient weight on reducing dropouts itself implies preference for not enforcing an effort minimum. Of course, these results arise because an effort minimum makes students more inclined to dropout. How much schools value reducing dropouts is, of course, an empirical question though it seems clear from charter school authorization practices (and charter school marketing) that many providers care about reducing dropping out. Realistically, objectives of charter school founders are likely to differ as well. Another factor likely to be important is parental support for subjecting their children to and helping to enforce student study effort. These issues warrant further, especially empirical, investigation, where our model might provide a foundation.

8.3 Peer Effects. Peer effects among students have been extensively studied, with somewhat mixed findings as to their effects.[64] Here we outline how the model could be extended to their consideration and make some conjectures about equilibrium effects.

Adapting Blume et.al. (2015) and Liu et.al. (2014) modelling of peer effects, suppose that augmented achievement of student i in school j is given by:

[64]Surveys of the vast and growing literature are Epple and Romano (2011) and Sacerdote (2011). While the extent of peer effects on achievement remains controversial, household demand for "good" peers in their schools is not (see e.g., Rothstein, 2006).

$\hat{a}_{ij} = \left[b_i^{\beta} Q^j \Gamma(b_i, B^j) + \delta \hat{b}_i + \alpha \hat{e}_i \right] e_i$, $\delta \geq 0$, $\alpha \geq 0$; where $\hat{b}_i$ and $\hat{e}_i$ are, respectively, mean ability and mean effort of student i's classmates. In addition, suppose that augmented student costs are given by: $\hat{c}_{ij} = e_i^2 / 2m + \varphi \cdot \left[e_i - \hat{e}_i \right]^2 / 2.$ Student utility is then $U_{ij} = \hat{a}_{ij} - \hat{c}_{ij}$. This extension has three potential peer-effect channels, with positive spillovers on achievement from peer students' abilities (if $\delta > 0$), positive spillovers from peer students' efforts (if $\alpha > 0$), and a conformance cost that is increasing in own effort deviation from average peer efforts (if $\varphi > 0$). The rest of the model is "the same," meaning the public sector maximizes aggregate augmented achievement and charter schools maximize their students' augmented achievement gains, with again a graduation threshold. As above, students choose effort last taking as given the school selected by their parents and the school policies. Here effort would be chosen in Nash equilibrium among classmates, most simply assuming students have complete information about their peer students.[65]

We have not developed the extension, which we view as a topic for future research, but will bravely (if not foolishly) make some conjectures about likely effects on the analysis. We assume existence and uniqueness of equilibrium in the discussion that follows.[66] A central element of the augmented model is that student effort choices are "strategic complements," meaning one student's higher effort increases incentives of other students to increase effort through both the direct peer effect channel and the conformity channel. On the other hand, higher effort by a student makes weaker students more likely to drop out. A public school in isolation would likely have a stronger incentive to increase the curriculum target. This is because effort is complementary with $b_i^{\beta}\Gamma$ and with others' efforts. Increasing any student's effort also increases incentives of all students to increase effort and thus achievement. This would be tempered by increased incentives of weaker students to drop out.

Consider competition between a TCS and a public school, these schools differentiated by their curricula as above. Here differentiated curricula implies a student partition that depends on motivation as well as ability, this because motivation effects effort which effects all students. Here a higher curriculum will attract both higher ability and more motivated students. A TCS would appear to have a competitive advantage because some students that would prefer it do not choose to matriculate and continue to attend the public school, this implying a wider distribution of student types and efforts in the public school, which is here a disadvantage. The TCS will have a stronger incentive to target higher ability and motivated students because achievement gains will be higher for these students. On the other hand, the public school will have stronger incentive to induce entry at the

[65] An issue is whether students that drop out care about conforming in their effort choice, i.e., prior to actually dropping out. Our inclination would be to assume they continue to prefer zero effort, which would also be consistent with their having other students that will eventually drop out as their effective peer group.

[66] The graduation standard implies a discontinuity in student payoffs, which creates complications in showing existence for public sector students. The complementarities of effort choices raise uniqueness issues as well. The latter could be dealt with using selection of equilibrium with higher effort choice if an issue.

low end. Given the noted competitive advantage of a TCS, what equilibrium would emerge is an interesting question.

Most interesting perhaps are the implications for an NEC school. The complementarities in the utility function imply that enforcing an effort minimum has a higher payoff through all the peer-effect channels. Consider a simpler model without direct peer effects and without complementarity between own effort and ability or school quality ($a_{ij} = b_i^{\beta} + Q^j\Gamma(b_i, B^j) + e_i$), but with the above augmented cost function, i.e., with the student conformity incentive. This itself would reinforce NEC incentives to enforce an effort minimum to facilitate conformance. We find the notion that getting classmates to work hard reduces peer costs of working hard interesting. Our discussion is speculative and incomplete, again a topic for future research.

9. Concluding Remarks

We have developed a model to analyze charter school educational practices and entry. Charter schools aim to maximize achievement gains of their students, drawn from a student population differentiated by cognitive ability, motivation (or non-cognitive ability), and income. Students must put forth study effort to achieve, with achievement dependent on their cognitive ability, how well their school's curriculum matches their capability, and how hard they study. More motivated students are more inclined to study. We capture the autonomy of charter schools by allowing differentiation in the curriculum match to student ability. An entering charter school selects students by choosing the level of difficulty of its curriculum with the objective of maximizing student achievement gains subject to the requirement that it draw enough students to be viable. In a calibrated computational counterpart model, the global optimum of an entering charter school targets lower ability students, for some student populations this induced by a competing public school who wants to retain higher ability students. Achievement gains are predicted to be substantial, mainly driven by reducing drop outs. Charter effects on reducing drop outs is consistent with evidence from Florida and Chicago.

Motivated by the practices of the no-excuses segment of charter schools, we then also investigate charter schools that enforce an effort minimum. These charter schools employ the effort minimum to increase achievement but also to induce selection into the charter by highly motivated students along with curriculum targeting to their students. A competing public school cannot strategically prevent this by altering their curriculum. Consistent with the evidence on such charter schools, we find in our computational model that the no excuses approach is highly effective in increasing student achievement, especially the weaker students they attract. No excuses charter school students must be willing to study a lot. For some students, this occurs at the expense of student utility, which triggers a disagreement in the preferences over schools of those children and their parents.

Several extensions of the model for potential future research might be pursued, some already discussed. Our model has a graduation or retention standard for achievement, which we assume is dictated by the educational authority and must be adhered to by both the public and charter school. Varying this standard, if allowed, is another potential dimension on which a charter school might pursue its objective. We have not investigated variation in school funding out of space considerations. The maximum policy variation of a charter school in the model is to have one curriculum and an effort minimum, which might be generalized. The former is targeted to ability and the latter effectively to motivation. In as much as a school can vary its policies among its students, e.g., require after-school work (more effort) for a subset of students, the school could further increase achievement. Such more individualized targeting is likely to be expensive and thus limited. Our model assumes households choose schools to maximize achievement, but with a proportion failing to do so and with that proportion independent of household-student characteristics. Hastings, Kane, and Staiger (2006) provide evidence that lower income households are less likely to choose schools to maximize achievement and Hastings and Weinstein (2008) provide evidence that this is associated with access to information. Considering heterogeneity in efficacy of school choice is well motivated with potentially interesting effects on "biases" in charter school curriculum targeting. Our model considers competition between just one charter school and one or two public schools, one a magnet school in the latter case. Consideration of equilibrium with more schools is of interest. Finally, cost variation in educating less motivated and/or lower ability students is also of interest to study.

Finally, we have argued (see footnote 10) that U.S. charter schools are quite close to (secular) English academies and Dutch publicly funded private schools. We thus believe that our model might also do a good job at capturing essential elements of education provision in these two school systems, and that it could be readily adapted to study English and Dutch school markets.

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Figure 1. Student Partition in Public School

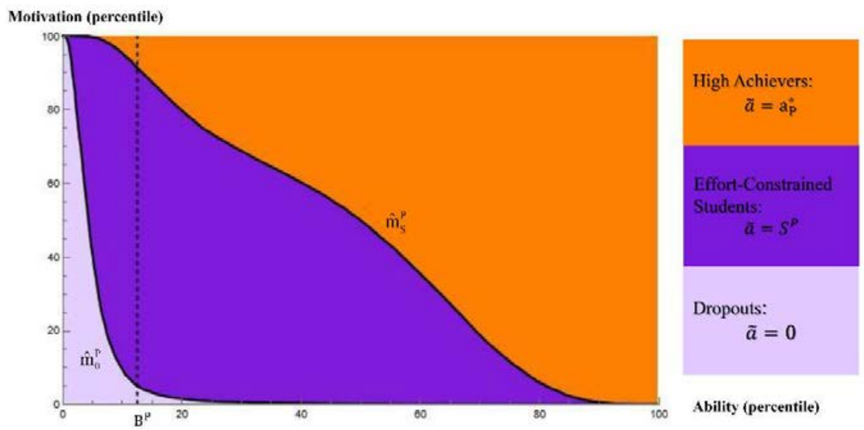

Figure 2. TCS Achievement Gains

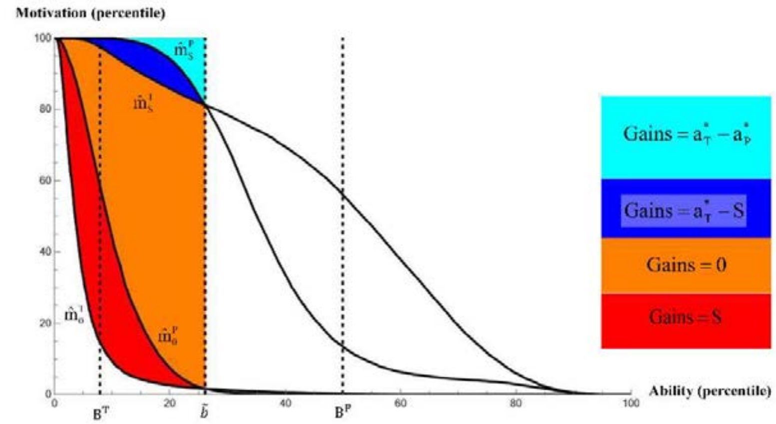

Figure 3. Public School Curriculum Choice

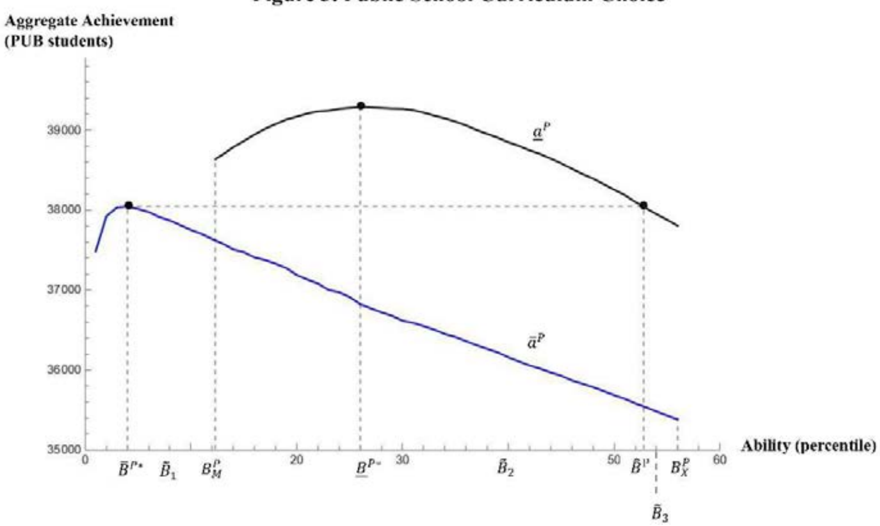

Figure 4. Iso-Effort Loci

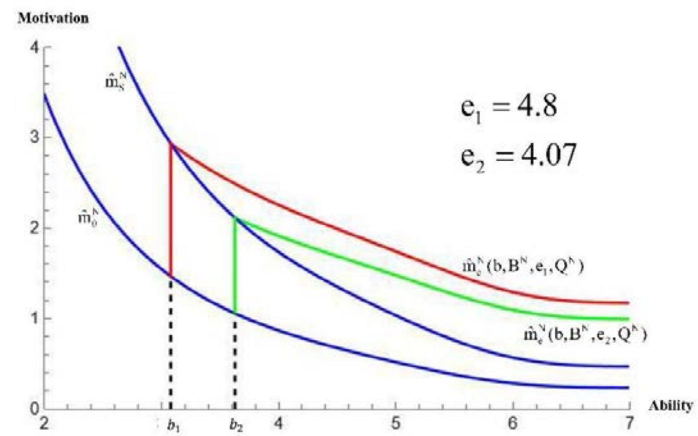

Figure 5A. NEC Applicants and Achievement Gains ($\varphi = 0$)

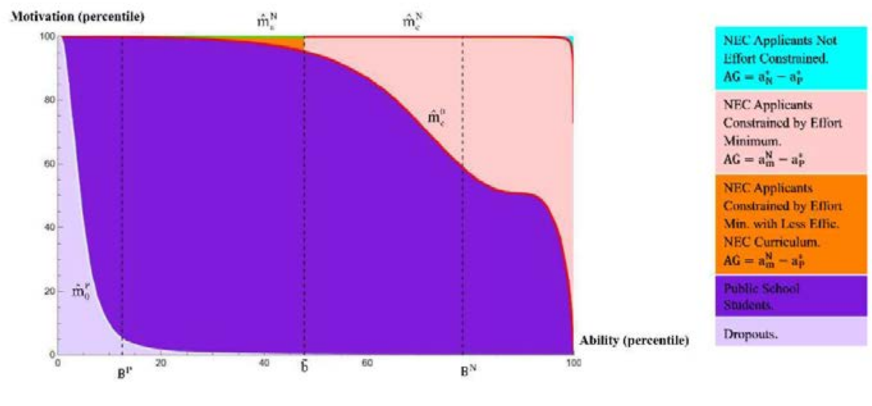

Figure 5B. NEC Applicants and Achievement Gains ($\varphi = 0.02$)