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LONG-RUN INCOME AND INTEREST ELASTICITIES OF MONEY DEMAND IN THE UNITED STATES

Dennis Hoffman

Robert H. Rasche

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ABSTRACT

This study investigates the stability of long-run log-linear demand functions for narrowly defined monetary aggregates (M1, Monetary Base) in the U.S. during the post World War II period. The hypotheses that the individual time series which appear in such equations (real M1, real Monetary Base, real Personal Income and short-term and long-term nominal interest rates) all have unit roots cannot be rejected. The primary conclusion of this study is that with proper attention to the time series properties of the available data, there exists strong evidence in support of a stable equilibrium demand function for real balances in the post-World War II U.S. economy. The hypothesis of a unitary equilibrium real income elasticity (a velocity function) cannot be rejected. Further, the estimates of equilibrium interest elasticities are approximately -.5 to -.6 for real M1 and -.4 to -.5 for real monetary base. The estimated interest elasticities are significantly different statistically depending on whether long- term or short-term interest rates are used, but the observed differences in these estimates are not of economic significance.

Dennis Hoffman
Department of Economics
Arizona State University
Tempe, AZ 85287-3806

Robert H. Rasche
Department of Economics
Michigan State University
East Lansing, MI 48824-1038

Long-Run Income and Interest Elasticities of Money Demand in the United States

> As a rule, we can suppose that the schedule of liquidity-preference relating the quantity of money to the rate of interest is given by a smooth curve which shows the rate of interest falling as the quantity of money is increased.
>
> J. M. Keynes [1936], p. 171

The assumption of a stable long-run relationship between the stock of real money balances and an interest rate, given some measure of real economic activity or volume of transactions has been an integral part of all macroeconomic theories and applied analyses of the effect of monetary policies for over fifty years. Over the past fifteen years statistical analyses of data from the United States are not particularly favorable toward this hypothesis. It is our view that much of the recent confusion in the discussion of empirical money demand functions is the result of a research focus on short-run relationships compounded by inherent difficulties in estimating and drawing useful inference from integrated time series data. The primary conclusion of this study is that with proper attention to the time series properties of the available data, strong evidence is available in support of a stable equilibrium demand function for real balances in the post-World War II U.S. economy. The hypothesis of a unitary equilibrium real income elasticity (a velocity function) cannot be rejected. Further, the estimates of equilibrium interest elasticities are approximately -.5 to -.6 for real M1 and -.4 to -.5 for real monetary base. The estimated interest elasticities are significantly different statistically depending on whether long- term or short-term interest rates are used, but the observed differences in these estimates are not of economic significance.

This study is organized as follows. In section I we present a short review of the empirical literature on long-run demand function for real balances in the U.S. In section II, the evidence in support of unit roots in the U.S. data for real balances, real income, and interest rates is presented.[1] In section III we review recent developments in the statistical theory of testing for cointegration. In the application at hand, this involves testing for a stationary log-linear combination of real balances, real income and interest rates (a equilibrium demand function for the stock of real cash balances as a function of the level of economic activity and the level of interest rates as postulated by Keynes), under the conditions where each of the individual time series in the specification is nonstationary. In section IV the evidence in support of a stable equilibrium demand function for real cash balances in the post-war U.S. economy is presented. Finally, we conclude with suggestions for the application of the results derived here in the investigation into the stability of a short-run demand for real cash balances.

I. A short history of estimates of long-run money demand function for the U.S.

In the late 1950s and early 1960s empirical studies of the demand for money in the U.S. focused on trying to determine the long-run income and interest elasticities. Typically these

[1]In terms of the terminology introduced by Engle and Granger [1987], these time series are integrated of order 1 [I(1)].

studies used annual observations over the fifty to sixty years, and generally found that a unitary long-run income elasticity could not be rejected, and that the long-run interest elasticity was on the order of -.6 to -.7 (e.g. Meltzer [1963]; Chow [1966]) for a narrow definition of money and a long-term rate of interest. Unfortunately, a close review of these studies suggests that these conclusions are not very robust to the choice of sample period (e.g. Meltzer [1963], Table 2; Laidler [1966], Table 4B).

Since that time, empirical studies of the demand for money generally have focused on estimating "short-run" demand functions, frequently using the Koyck lag form proposed by Chow [1966], short-term rates of interest patterned after Laidler's [1966] conclusion that "there is little question of the superior explanatory power of the shorter interest rate" (p. 547), and samples of quarterly or monthly data from the post-war history.[2] The results of this extensive literature are disappointing. The post-war short-run money demand functions exhibit the same subsample instabilities that characterized the early long-run demand functions, a problem that remains unresolved in the face of numerous ingenious repair attempts. Further, the implied long-run income elasticity in these estimates is usually much less than unity. Poole [1970, 1988] argues that the typical data in these studies do not meaningfully discriminate long-run income and interest elasticities over a wide range of values for such

[2]Extensive reviews of this literature are available in Boorman [1985] and Judd and Scaddings [1982].

elasticities. These and other problems have led some economists (e.g. Gordon [1984], Laidler [1982]) to conclude that successful identification and estimation of aggregate short-run money demand functions is impossible.

Other researchers (Gould and Nelson [1974], Nelson and Plosser [1982]) have noted that velocity (the ratio of income to a measure of the money stock) is well characterized as a random walk. A number of studies have recognized this by reformulating money demand equations in first difference form (Hafer and Hein [1982], Rasche [1987, 1988]) and found that with one notable exception, the subsample instability of post-war money demand functions is eliminated.[3] The problem with this solution is that it provides no evidence on the existance of or stability of a long-run relationship between the levels of real balances, real income and interest rates: Keynes' liquidity preference function. Further, as noted by Poole [1988], if a long-run relationship does exist among these three variables, there is no reason to assume that the sum of the estimated coefficients on the interest rate variables in the first difference equations measures the interest elasticity in the long-run levels relationship.

Poole attempts to resolve these problems by 1) using an updated sample of the annual data such as used by Meltzer [1963], Chow [1966] and Laidler [1966], 2) constraining the annual (and equilibrium) real income elasticity of the demand for real balances to unity, 3) carefully choosing sample periods during

[3]The exception to the stability of the first difference specifications of narrowly defined money is a shift in the constant term around the end of 1981 (Rasche [1987]).

which the interest rate has approximately zero net change from the beginning to the end of the sample, and 4) estimating a relationship between the level of velocity and the level of an interest rate variable. Under all of these restrictions he concludes that a long-term interest rate is more appropriate than a short-term interest rate, and that the equilibrium interest elasticity of the demand for real balances is about -.6.

We believe that the set of assumptions that Poole imposes in order to derive his estimate is a weak foundation upon which to base the argument for a stable equilibrium relationship between the stock of real cash balances, the level of economic activity and the level of interest rates. Yet we believe that his conclusion is fundamentally correct. In the following sections we construct an analysis from post-war U.S. data that provides strong support for the existance of a stable "liquidity preference" function during this period, for which the estimated real income elasticity is not significantly different from unity and the estimated interest elasticity is on the order suggested by Poole, Meltzer, and Chow.

II. Tests for Unit Roots in Real Balances, Real Income and Interest Rates

Six monthly data series are used in the following analysis for the sample period 1953-87 inclusive. These series are the Treasury Bill rate (RTB); the 10 year constant maturity U.S. government bond rate (R10); real Personal Income (Y/P); the deflator for Personal Income (P), constructed as the ratio of current dollar personal income to real personal income in 1982 dollars; the Adjusted Monetary Base (B), constructed by the Federal Reserve Bank of St. Louis; and the narrowly defined money stock (M1). The latter series is constructed from the currently published data from January, 1959, and from adjusted data for the period 1953-58 as described in Rasche [1987]. These data are not shift adjusted. The data for the monetary aggregates and personal income are seasonally adjusted; no seasonal adjustment is applied to the interest rate series.

Engle and Granger [1987] define an order of integration (say k or I(k)) of a time series variable as the number of times the vraiable must be differenced to achieve stationary. Ordinarily, linear combinations of a set of I(k) variables will also be I(k). However, if a linear combination of several I(k) variables is I(l); l < k, then the variables are said to be cointegrated. In testing for cointegration it is imperative that the variables under consideration each maintain the same order of integration.

We identify the order of integration maintained by each of the variables in our study using a series of univariate unit root tests. All of the tests used in this univariate analysis are described in detail in Schwert [1987]. The τ_μ and τ_τ tests refer to augmented Dickey-Fuller tests with and without time trends. The $T(\hat{\rho}_\mu-1)$ and $T(\hat{\rho}_\tau-1)$ refer to the "normalized bias" tests also discussed by Dickey and Fuller [1979]. The "c" correction refers to $Tc(\hat{\rho}-1)$ where c is the reciprocal of one minus the sum of the distributed lag coefficients in an augmented Dickey-Fuller representation. Lag length values of four and twelve are used for all tests.

In addition to the Dickey-Fuller tests, we calculate test statistics suggested by Phillips [1987]. These tests are designed to cope with potential serial correlation in models used to obtain the Dickey-Fuller tests discussed above. An arbitrary number of lagged residual autocovariances is used to modify the τ_μ test to obtain τ_{α^*}, τ_τ to obtain $\tau_{\hat{\alpha}}$, $T(\hat{\rho}_\mu-1)$ to obtain Z_{α^*} and $T(\hat{\rho}_\tau-1)$ to obtain $Z_{\hat{\alpha}}$. We chose both four and twelve residual autocovariances to compute the adjusted tests suggested by Phillips.

Schwert examines the .05 critical values of these unit root tests in simulated ARIMA(0,1,1) structures. Since our application is based on 420 monthly data points, we are concerned with Schwert's results for 444 observations. The critical values he obtains for moving average parameter values of: $\theta = -.5, 0,$ and .5 are listed at the top of Table 1 along with the indicated tests. Schwert finds, as revealed by these critical values, that the original Dickey-Fuller "t-statistics" are less affected by the presence of a nonzero moving average parameter that are the Phillips corrected analogs. Similarly, the "c" corrected normalized bias tests are most resilient to alternative moving average parameterizations.

We subjected the series used in our analysis to all of the univariate unit root tests discussed above. Our purpose is to identify the order of integration maintained by each series. The results appear in Table 1 with the values of the test statistics in the top half of the table obtained from the "levels" or "log-levels" of each series and the values of the test statistics obtained in the bottom half from the first differences of each series. With a very few exceptions, the tests fail to reject the unit root hypothesis at the 5% level. There is some evidence, though not conclusive, that the log of the Treasury bill rate, log(RTB) is stationary about a deterministic trend. The τ_τ values are significant at both lag lengths. The "c" adjusted normalized bias tests with trend are also significant at the .05 level for this series regardless of the lag length. However, the remaining tests fail to reject the hypothesis that this series is nonstationary about a trend.

The tests reveal convincingly that the first difference of each series is stationary about its mean. This conclusion remains regardless of the size of the moving average parameter, lag length, or the test considered. In sum, it seems safe to conclude that each series we consider is integrated of order one over the sample period used in our study.

III. Estimation and Testing Methodology

The notion of cointegration is formalized for the general case by Engle and Granger [1987]. For the purpose at hand, a set of variables which are integrated of order one are said to be cointegrated if there exists one or more linear combinations (cointegrating vectors) of these variables that are integrated of order zero. Natural logarithms of real balances, real income, and interest rate time series for the U.S. economy in the post-war period are integrated or order one as demonstrated in the previous section. If these variables are cointegrated, and if there exists a unique cointegrating vector, that linear combination may be interpreted as a equilibrium demand for the stock of real cash balances -- a "liquidity preference" function.[4]

Several recent innovations in the estimation of cointegrating vectors and accompanying error correction representations serve as the basis for our results. While Engle and Granger [1987] focus exclusively on bivariate systems, Engle and Yoo [1987] run simulations designed to test the performance of the simple "two-step" OLS tests for cointegration proposed by Engle and Granger in higher order systems of equations. They find that augmented Dickey-Fuller tests have critical values that increase with the number of variables in the system. Engle [1987] suggests that this occurs since linear combinations of increasing numbers of nonstationary variables "tend to look stationary" regardless of the presence of cointegration.

The two-step procedure essentially ignores the possibility that several cointegrating vectors are possible when more than two variables are encountered. Stock and Watson [1989] propose a test for the number of cointegrating vectors, say r, in an n dimensional vector process characterized by variables that are integrated of order one. The Stock-Watson test is formed from the n-r principal components (or common trends) that are the

[4]The critical assumption behind this assertion is that the equilibrium demand for real balances is identified in the economic model that is hypothesized to generate the observed data. It is evident from the estimation and testing methodology that is outlined below, that the esimated coefficients of the cointegrating vector are extracted from reduced form (multivariate time series) data. The parameters of an economic relationship can be inferred from reduced form parameters if and only if that relationship is identified by the model structure.

underlying or driving stochastic trends under the null hypothesis of r cointegrating vectors. The test is actually calculated from the n-r smallest eigenvalues of a serial correlation matrix formed from these principal components. Engle [1987] finds that the procedure offers potential over the OLS approach but "is not based on likelihood theory". Our own experiments with the Stock-Watson test suggest that inference is highly susceptable to arbitrary lag length specifications in the structures used to estimate the principal components. Also, the available critical values are numerically simulated and may not be applicable to a wide range of variables.

Recent papers by Johansen [1987] and Johansen and Juselius [1988] develop tests for both the number of cointegrating vectors and tests of hypotheses regarding elements of the cointegrating vectors. The basic idea (using Johansen's notation) is to rewrite a p-dimensional vector autoregression:

$$(1) \qquad X_t = \sum_{i=1}^{k} \Pi_i X_{t-i} + \epsilon_t$$

as:

$$(2) \qquad \Delta X_t = \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} - \Pi X_{t-k} + \epsilon_t$$

where:

$$(3) \qquad \Gamma_i = -I + \Pi_1 + \ldots + \Pi_i \qquad i = 1, 2, \ldots, k-1$$

and:

$$(4) \qquad \Pi = I - \Pi_1 - \Pi_2 - \ldots - \Pi_k.$$

so that the matrix Π conveys the long-run information in the data. When $0 < \text{rank}(\Pi) = r < p$ we can express $\Pi = \alpha\beta'$ where β may be interpreted as a $p \times r$ matrix of cointegrating vectors and α a $p \times r$ matrix of vector "error correction" parameters.

Johansen [1987] shows that estimates of β can be obtained from the eigenvectors associated with the r largest eigenvalues obtained by sloving the eigenvalue problem:

$$(5) \qquad |\lambda S_{kk} - S_{ko}S_{oo}^{-1}S_{ok}| = 0$$

where S_{ij}; $i,j = o,k$ represent residual moment matricies formed from least squares regressions of ΔX_t and X_{t-k} on ΔX_{t-i}, $i = 1 \ldots, k-1$. Hence the eigenvalues in this problem are the squared canonical correlations of the "levels" regression residuals with respect to those in the "differenced" regressions. The concentrated likelihood is also formed from these eigenvalues ($\hat{\lambda}_i$; $i = 1, \ldots, p$) so that a test statistic for the hypothesis that there are <u>at most</u> r cointegrating vectors is:

$$(6) \qquad -2 \log(Q) = - T \sum_{i=r+1}^{p} \log(1-\hat{\lambda}_i)$$

where $\hat{\lambda}_{r+1} > \ldots > \hat{\lambda}_p$ are the p-r smallest eigenvalues. This statistic has a nonstandard distribution and Johansen [1987] develops appropriate critical values.

Johansen and Juselius [1988] develop tests of hypotheses regarding individual elements of α and β. The likelihood test statistic suggested for H_0: $\beta = H\theta$ where H is an arbitrary $p \times s$ matrix, is:

$$(7) \qquad -2 \log(Q) = T\sum_{i=1}^{r} \log\{(1 - \lambda^*_i)/(1 - \hat{\lambda}_i)\}$$

where λ^*_i are the r largest eigenvalues obtained by solving the eigenvalue problem under the s linear restrictions conveyed by H_0. Similarly $\hat{\lambda}_i$ are the r largest eigenvalues obtained without the restriction. Johansen proves that this statistic is distributed as χ^2 with r(p-s) degrees of freedom. Johansen and Juselius develop a comparable Wald test that is essentially based on estimates of the asymptotic covariance matrix of $\hat{\beta}$.

IV. Estimates of Long-Run Interest and Income Elasticities of the Demand for Real Cash Balances

We examined the hypothesis that there is a stationary linear combination of real cash balances and real income as a first step in the investigation of a long-run liquidity preference function. If these variables are cointegrated, then an additional question is whether the coefficients of the two variables in the stationary vector are equal in absolute value. If neither of these hypotheses are rejected, then the hypothesis that velocity is stationary cannot be rejected. We regard this as an appropriate question, since many investigations of U.S. data have concluded that the velocity of narrowly defined monetary aggregates is a random walk, or that income and narrowly defined money are not cointegrated (Engle and Granger [1987]). The results of our tests for cointegration of the logs of real M1 and real personal income or the logs of the real monetary base and real personal income are given in Table 2. Three different sample periods are tabulated: 1953 - 74 which represents the sample period prior to the "missing money" phenomenon; 1953 - 81 which represents the period prior to the break in the "drift" in the velocity of these aggregates; and 1953 - 87 which represents the full sample period. The results presented in Table 2 uniformly fail to reject the hypothesis that there is no cointegrating vector involving either measure of real money balances and real income, regardless of the sample period examined.[5] This is consistent with the established literature that narrowly defined velocity is a random walk.[6]

This conclusion does not eliminate the possiblity of a stationary linear combination among an expanded set of variables. The hypotheses that stationary linear combinations exist among the logs of real M1 or the real monetary base, real personal income and interest rates are tested in Tables 3 and 4. The tests in top part of these tables use a short-term interest rate (RTB) and the tests in the bottom portion of the tables use a long-term interest rate (R10). Among three variables there is the possibility of zero, one, or two cointegrating vectors. Regardless of the concept of real money balances, the choice of

[5]This conclusion is robust with respect to the choice of the lag length in the construction of the regression residuals for the test statistic as illustrated by the comparative computations for lag lengths of k = 4 and 7.

[6]We have also examined a results for the samples reported in Table 2-4 after omitting observations for February-June of 1980 and January-April of 1981 and including a dummy variable that is zero before January, 1982 and one thereafter in the regressions that generate the estimated residuals used in constructing the Johansen test statistic. The 1980 observations are excluded because they cover the period during which credit controls were imposed. The 1981 observations are excluded because this is the period of massive portfolio shifts immediately following the nationwide legalization of NOW accounts. The dummy variable is included because it captures the significant instability in first difference specifications of money demand functions estimated over this period (Rasche [1987,1988]). These adjustments have no substantive impact on the results reported here.

interest rate, or the sample period investigated, the test results in these tables uniformly reject the hypothesis of zero cointegrating vectors and uniformly fail to reject the hypothesis of one or fewer cointegrating vectors. The computed test statistics also consistently fail to reject the hypothesis of two or fewer cointegrating vectors. Thus the evidence is consistent with the existence of a single cointegrating vector among each set of three variables considered. Wald tests suggest that each variable is significant, which insures that the cointegrating relation includes the entire menu of variables.[7]

The estimated coefficients of the unique cointegrating vector in each of the cases considered in these tables are of considerable interest. In each case, the absolute value of the estimated coefficient on the log of real income is of the order of the estimated coefficient on the log of real balances, and the estimated coefficient on the log of the interest rate is of the same sign as that on real balances, though smaller than the latter coefficient. The values of the χ^2 test statistic defined in (7) for the hypothesis that the coefficients of the log of real balances and the log of real personal income sum to zero fail to reject the maintained hypothesis. Therefore it is appropriate to interpret the cointegrating vector as a stationary linear combination of velocity (Y/M) and interest rates or

[7]We have constructed the log-linear combinations of real balances, real personal income, and interest rates using the point estimates of the coefficients of the cointegrating vectors. We subjected these constructed time series to the battery of unit root tests described in Section II. In every case, we failed to reject the hypotheses that the linear combinations of the three I(1) variables are stationary (I(0)).

alternatively that the equilibrium real income elasticity of the demand for real balances is unity. It should be noted that this conclusion is not rejected for any of the sample periods considered.

The implied equilibrium interest elasticities of the demand for real balances in the estimated cointegrating vectors are of particular interest. These are obtained from the restricted velocity models. First, the estimated elasticities of real M1 with respect to the long-term interest rate are of the same magnitude as that found by Poole [1988] using annual data and similar to the early estimates constructed from annual data by Meltzer [1963] and Chow [1966]. In contrast to the Poole estimate, which was constructed from a very carefully chosen sample period and which appears to be unstable as the sample period is altered, this estimate appears robust with respect to changes in the sample. In particular, there are no differences in the estimate for the long-term interest rate from adding the post 1974 "missing money" observations or adding the post 1981 "change in velocity drift" observations, and relatively small differences across sample periods when the Treasury bill rate is used. The precision of the interest elasticity is obtained from the Johansen and Juselius [1988] test defined in Corrollary 3.17 of their paper.

Second, the estimated equilibrium elasticities using short-term interest rates are always somewhat smaller than those estimated using long-term interest rates. Ideally, as suggested by Poole [1988], a well specified demand function for real balances should exhibit the same equilibrium interest elasticity, regardless of whether short-term or long-term interest rates are used, at least under the joint hypothesis that the term structure of interest rates is driven by expectations of future short-term interest rates. A test of the hypothesis that the equilibrium interest elasticity of velocity with respect to short-term interest rates is identical to the equilibrium elasticity with respect to long-term rates is constructed as follows:

Note from Tables 3 and 4 that we find:

$z_{rtb} = a_1 \ln(Y/M) - b_s \ln RTB$ and

$z_{r10} = a_2 \ln(Y/M) - b_l \ln R10$

are stationary vectors. Since the sum of two stationary variables is itself stationary, we note that $z = z_{rtb} + z_{r10}$ is also stationary. We can test the equality of the coefficients on the short and long-term rates from the trivariate menu: ln(Y/M), lnRTB and lnR10. We know from Tables 3 and 4 that two distinct cointegrating vectors prevail in this menu. The test for equality of interest elasticities is obtained by comparing the eigenvalues in the Johansen analysis with and without the restriction $b_s = b_l$. Technically this requires that the relative effect of the two interest rates is identical in each cointegrating vector, as is the case if the equilibrium interest elasticities are the same for long-term and short-term interest rates.

The results of this test are presented in Table 5. We first test equality of the estimated coefficient of ln(Y/P) and the negative of the coefficient on ln(M/P) [or ln(B/P)] in the four variate system ln(M/P) [or ln(B/P)], ln(Y/P), lnRTB and lnR10. Under the assumption of two cointegrating vectors among these four variables, the test statistic (from equation (7)) for this restriction presented in column 8 is distributed as χ^2 with 2 df. In all cases the computed values of the test statistic fail to reject the hypothesis at the five percent level that these coefficients are equal in absolute value. This confirms the unitary income elasticity tests in Tables 3 and 4 for the broader menu of variables.

Next we test the restriction that the coefficients on lnRTB and lnR10 are equal in both cointegrating vectors, while maintaining the unitary income elasticity restriction. In all cases the computed values of the test statistic reject the equality restriction on the interest elasticities at the five percent level as reported in column 11 of Table 5. Thus we conclude that the estimated equilibrium interest elasticity of M1 and monetary base velocity is statistically significantly smaller when the cointegrating vector is estimated using short-term interest rates that when it is estimated using long-term interest rates during these sample periods.

However, the magnitude of these differences is sufficiently small that there are no obvious consequences of economic significance that would result from using the estimate of the equilibrium interest elasticity derived from the short-term interest rate specification rather than the estimate derived from the long-term interest rate specification.

A third noteworthy result in Tables 3 and 4 is that the estimated equilibrium interest elasticities of base velocity are always smaller than the estimated interest elasticity in the corresponding specification for M1 velocity. The difference in the interest elasticities of the two velocities is always in the range .15 to .20, regardless of the interest rate or sample period chosen.

A test for equality of the interest elasticities of M1 and the monetary base velocities can be constructed along the lines of the test for equality of the long-term and short-term rate elasticities discussed above. First the estimated coefficients in the cointegrating vectors in the four variate menu ln(M1/P), ln(B/P), ln(Y/P), and lnRTB [or lnR10] can be tested to determine if the equilibrium income elasticity equals unity in the real M1 and the real base models. If this restriction is <u>not rejected</u> the hypothesis that the interest elasticities are the same for M1 velocity and base velocity may be examined under the maintained hypothesis of unitary income elasticity.

The results of these tests are reported in column 8 of Table 6. In 10 of the 12 cases examined, the hypothesis that the cointegrating vectors in the four variate menu can be written as:

$$a_1 \ln(M1/Y) + a_2 \ln(B/Y) + a_3 \ln RTB[\text{or } \ln R10]$$

is not rejected. Hence, the four variate menu also fails to reject the unitary income elasticity hypothesis. In all 10 of these cases, the additional restriction $a_1 = a_2$ is strongly rejected as indicated in column 11 of Table 6. Thus the data are <u>not consistent</u> with the hypothesis that the equilibrium interest elasticities of the demand for real M1 and the demand for the real base are equal.

The point estimates of the M1 and base equilibrium interest elasticities are inconsistent with a positive equilibrium interest elasticity of the M1-monetary base multiplier. An explanation for this result is that during these sample periods the equilibrium interest elasticity of the M1-monetary base multiplier is dominated by the equilibrium interest elasticity of the demand for reserve absorbing nontransactions deposits. The conventional wisdom is that the total equilibrium interest elasticity of the demand for such assets is positive, so that under conditions where all interest rates are changing, the own interest elasticity dominates the cross interest elasticities. Thus as interest rates change the ratio of such assets to transactions deposits changes in the same direction, and the partial effect is a change in the opposite direction of the M1-monetary base multiplier. If this effect dominates the equilibrium interest elasticities of all the other components of the M1-monetary base multiplier, then the equilibrium interest elasticity of monetary base velocity should be smaller than the equilibrium interest elasticity of M1 velocity. If this is the source of the observed difference in the interest elasticities of the M1 and adjusted monetary base equations, then there is reason to question whether this difference will continue in the future. Under the Monetary Control Act of 1980, among nontransactions deposits only nonpersonal time deposits, rather than all time deposits at commercial banks, remain subject to reserve requirements. Since the former are only a small fraction of the latter, the interest elasticity of the M1-monetary base multiplier originating from the source should be reduced in absolute value.[8]

The implicit equilibrium real cash balances and real monetary base for each month are shown in Figures 1 and 2 along with actual real M1 and real monetary base. The equilibrium estimates are constructed from the normalized estimated coefficients of the estimated cointegrating vector adjusted so that the sample mean of the equilibrium values equals the sample mean of the actual values.[9] The estimated interest elasticities from the sample periods ending in December, 1987 are used in the construction of these figures. Figures 1a and 2a are constructed using the Treasury bill rate and Figures 1b and 2b are constructed using the 10 year government bond rate. In Figures 1a and 2a the estimates of equilibrium real balances are "backforecasted" to January, 1948, five years prior to the beginning of the sample period.

For both monetary aggregates the estimated equilibrium real balances constructed from the Treasury bill rate exhibit greater volatility than the estimates constructed from the bond rate. This is evident from the similarity of the estimated equilibrium interest elasticities, since the bill rate is much more volatile

[8]An offsetting effect would occur if the net interest elasticity of nonpersonal time deposits is larger than the net interest elasticity of personal time deposits.

[9]If the vector $-\ln(Y/M) + b*\ln(R)$ is stationary, then the vector $-\ln(Y/M) + b*\ln(r) + a$ is also stationary for any value of a.

than the bond rate in the sample period. Equilibrium real base balances are less volatile than the corresponding real M1 balances. This also follows from the size of the estimated interest elasticities.

A second characteristic of the data in Figures 1 and 2 is that sizable differences between actual and estimated equilibrium real balances persist for considerable periods of time. However, these differences frequently disappear or even reverse in sign over very short time intervals. This behavior contrasts sharply with the assumption of the standard stock adjustment (lagged dependent variable) specification of the short-run demand for real balances that differences between actual and equilibrium real balances are slowly eliminated. We believe that this supports the conclusion that the typical estimated adjustment coefficient in short-run specifications of the demand for real balances reflects the nonstationarities in the data series rather than an economic behavioral parameter.

Finally there is no evidence in any of these figures that the behavior of real balances relative to estimated equilibrium real balances is any different in the 80s than during the previous 25 years. Deviations between the two corresponding series are no larger, nor are they more persistent during the most recent years than they are in the earlier years. Indeed, based on the estimated equilibruim real balances using the 10 year bond rate, actual real balances (both M1 and the adjusted monetary base) are about equal to estimated equilibrium real balances at the beginning of the 80s and that equality is reestablished by the end of 1987.

V. Conclusions

This research finds evidence that is consistent with the hypothesis of a stable long-run aggregate "liquidity preference" function in the U.S. economy throughout the post-accord period. The income elasticity of this function is not significantly different from unity, and the absolute value of the interest elasticity is of the order of .5 to .6 for M1 and .3 to .4 for the adjusted monetary base, depending on whether long-term or short-term interest rates are used in the estimation. These results suggest that many of the apparent instabilities noted in previous studies of demand functions for real balances reflect specification errors in the estimated equations. Further, they provide a foundation from which research on short-run demand functions for real balances can proceed and can shed light on short-run movements of interest rates and real income as variables adjust to equilibrium. The short-run dynamics of adjustment to the equilibrium demand for real balances are implied by an error correction model (Engle and Granger [1987]). The parameters of this error correction process can be derived from the estimated cointegrating vector and the estimated parameters of the VAR (equation (2)) (Johansen [1988]). Systematic examination of these issues is currently under investigation.

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Table 1
Unit Root Test Statistics

	τ_μ		τ_τ		$T(\rho_\mu-1)$			
	AR-4	AR-12	AR-4	AR-12	AR-4	AR-4[c]	AR-12	AR-12[c]
$\alpha=.05,\theta=-.5$	-2.94	-2.83	-3.54	-3.39	-29.1	-16.3	-29.9	-19.2
$\alpha=.05,\theta=0$	-2.88	-2.84	-3.42	-3.40	-14.3	-15.2	-14.9	-18.6
$\alpha=.05,\theta=.5$	-2.84	-2.84	-3.37	-3.38	-9.5	-14.7	-10.1	-19.2
R10	-1.28	-1.59	-2.27	-3.27	-2.30	-2.90	-2.79	-4.77
RTB	-2.06	-1.82	-2.85	-2.12	-6.79	-8.17	-5.77	-5.96
ln(R10)	-1.28	-1.71	-2.52	-2.84	-1.80	-2.52	-2.36	-3.43
ln(RTB)	-2.33	-2.56	-3.99	-3.57	-6.19	-10.55	-6.64	-9.91
ln(M1/P)	.31	-1.39	-.72	-2.34	.52	.88	-2.68	-10.03
ln(Y/P)	-.93	-1.36	-1.13	-1.68	-.26	-.32	-.37	-.58
ln(B/P)	1.41	.29	-1.23	-2.89	1.02	1.58	.22	.61
ΔR10	-7.50	-4.67			-273.1	-194.3	-250.3	-127.8
ΔRTB	-9.14	-5.61			-347.1	-440.4	-363.8	-533.7
Δln(R10)	-8.25	-5.36			-285.8	-258.4	-288.2	-312.8
Δln(RTB)	-8.94	-6.00			-283.8	-392.7	-315.5	-1012.5
Δln(M1/P)	-6.37	-3.61			-211.7	-121.1	-148.5	-44.5
Δln(Y/P)	-7.94	-4.53			-322.8	-238.8	-254.3	-87.5
Δln(B/P)	-6.38	-3.21			-236.2	-119.7	-139.1	-30.1

	$T(\rho_\tau-1)$				$t_{\alpha*}$		$t_{\tilde{\alpha}}$	
	AR-4	AR-4[c]	AR-12	AR-12[c]	AR-4	AR-12	AR-4	AR-12
$\alpha=.05,\theta=-.5$	-45.3	-26.5	-48.4	-37.9	-4.81	-5.97	-6.36	-8.04
$\alpha=.05,\theta=0$	-22.3	-24.1	-24.2	-37.2	-2.91	-2.95	-3.48	-3.54
$\alpha=.05,\theta=.5$	-14.6	-23.0	-16.0	-36.4	-2.77	-2.78	-3.25	-3.20
R10	9.03	-11.95	-14.04	-37.92	-1.39	-1.44	-2.14	-2.31
RTB	15.56	-19.92	-12.10	-14.21	-2.14	-1.91	-2.86	-2.41
ln(R10)	10.49	-15.86	-13.30	-28.47	-1.62	-1.62	-2.25	-2.26
ln(RTB)	-20.47	-40.27	-20.15	-46.25	-2.13	-1.99	-3.22	-2.90
ln(M1/P)	-1.51	-2.59	-5.49	-23.09	-2.51	.40	.03	-.59
ln(Y/P)	-2.75	-3.47	-4.08	-7.08	-1.10	-1.01	-.97	-1.18
ln(B/P)	-2.45	-3.81	-5.97	-21.20	2.27	1.43	-.67	-1.11
ΔR10					-11.48	-12.16		
ΔRTB					-11.50	-12.05		
Δln(R10)					-11.27	-11.73		
Δln(RTB)					-10.43	-10.37		
Δln(M1/P)					-14.32	-16.69		
Δln(Y/P)					-18.34	-22.98		
Δln(B/P)					-19.26	-24.71		

Table 1 Continued
Unit Root Test Statistics

	Z_{α^*}		$Z_{\hat{\alpha}}$	
	AR-4	AR-12	AR-4	AR-12
α=.05,θ=-.5	-41.0	-66.8	-70.4	-119.8
α=.05,θ=0	-14.4	-14.7	-22.2	-23.1
α=.05,θ=.5	-12.9	-12.8	-19.2	-18.7
R10	-3.10	-1.44	-10.32	-11.88
RTB	-8.44	-6.51	-17.98	-13.12
ln(R10)	-3.00	-3.03	-12.00	-12.02
ln(RTB)	-8.01	-6.90	-22.43	-18.51
ln(M1/P)	2.52	1.08	.08	-1.97
ln(Y/P)	-.34	-.35	-2.60	-3.53
ln(B/P)	1.91	1.64	-1.53	-1.11
ΔR10	-70.51	-33.16		
ΔRTB	-81.47	-54.32		
Δln(R10)	-79.51	-33.55		
Δln(RTB)	-87.82	-30.06		
Δln(M1/P)	-70.55	-30.07		
Δln(Y/P)	-80.66	-31.84		
Δln(B/P)	-75.67	-30.05		

Note:

Specifically τ_μ and τ_τ refer to tests of Ho: $\rho_\mu = 1$ or $\rho_\tau = 1$ in $y_t = \alpha + \rho_\mu y_{t-1} + \sum_{i=1}^{q}\phi_i \Delta y_{t-i} + \epsilon_t$ or $y_t = \alpha + \beta t + \rho_t y_{t-1} + \sum_{i=1}^{q}\phi_i \Delta y_{t-i} + \epsilon_t$. The normalized bias tests $T(\rho_\mu-1)$ and $T(\rho_\tau-1)$ use the estimated values for ρ_μ or ρ_τ in the structures defined above. Again these are tests of H_0: $\rho_m = 1$ or $\rho_\tau = 1$. The Phillips corrected normalized bias tests are formed by weighting these statistics by $c = 1/(1-\sum_{i=1}^{q}\phi_i)$. The t_{α^*} and $t_{\hat{\alpha}}$ defined as Z_{τ_μ} and Z_{τ_τ} by Schwert are adjusted Dickey-Fuller tests suggested by Phillips. The adjustments are designed to cope with potential ARMA errors in the augmented Dickey-Fuller equations. Similarly Z_{α^*} and $Z_{\hat{\alpha}}$ values are Phillips corrected normalized bias statistics. Again the corrections allow for potential ARMA errors in the Dickey-Fuller equation. We set the Phillips "lag truncation" value equal to the number of lags in the Dickey-Fuller specification (either 4 or 12). Schwert provides the details of the Phillips correction. Alternatively, a complete development of Phillips' argument can be found in Phillips and Perron [1986].

Table 2

Real Money -- Real Personal Income Cointegration Tests

Log Specifications

Johanson Test Statistic

Sample Period	k	r=0	r<=1
53,4 - 74,12	4	2.67	.04
	7	3.37	.15
53,4 - 81,12	4	3.39	1.09
	7	2.13	.51
53,4 - 87,12	4	5.27	.01
	7	5.08	.25

Real Monetary Base -- Real Personal Income Cointegration Tests

Johanson Test Statistic

53,4 - 74,12	4	6.38	.11
	7	7.42	.40
53,4 - 81,12	4	2.90	.85
	7	3.92	1.14
53,4 - 87,12	4	8.81	.002
	7	8.08	.32

Critical Values of Test Statistic*

97.5 %	13.8	5.0
95 %	12.1	3.9
90 %	10.4	2.8
50 %	5.4	.6

*Source: S. Johansen, "Statistical Analysis of Cotintegration Vectors", Journal of Economic Dynamics and Control, 1988, Table 1

Table 3

Cointegration Tests for Real M1, Real Personal Income and Interest Rates

Log Specifications

Sample	k	Johanson Test Statistic[a]			Unconstrained Cointegrating Vector			Test for Velocity Restriction	Implied Interest Elasticity of Velocity[b]
		r=0	r<=1	r<=2	M/P	Y/P	R		
		(23.2)	(12.1)	(3.9)				$\chi^2_{(1)}$	
					Treasury Bill Rate (RTB)				
53,1-	4	64.9	4.81	.02	16.13	-13.46	5.82	.23	.45 (.001)
74,12	7	28.4	4.50	.03	14.83	-15.06	6.98	.01	.46 (.006)
53,1-	4	35.4	3.39	.86	5.71	-8.11	4.62	.26	.55 (.006)
81.12	7	21.0	2.35	.03	4.54	-8.78	5.18	.43	.56 (.009)
53,1-	4	35.2	4.75	.02	10.44	-8.56	4.45	.45	.53 (.011)
87,12	7	23.5	6.23	.45	10.84	-9.10	4.94	.18	.56 (.005)
					10 Year Government Bond Rate (R10)				
53,4-	4	43.9	5.54	.06	18.49	-22.24	15.68	.26	.68 (.004)
74,12	7	24.2	6.27	.27	11.57	-22.10	17.32	.82	.69 (.013)
53,4-	4	24.7	4.78	1.70	20.46	-18.36	12.66	.11	.70 (.004)
81,12	7	15.6	2.56	.41	19.34	-20.12	14.11	.01	.70 (.005)
53,4-	4	28.9	5.23	.27	19.78	-16.84	10.83	.76	.66 (.011)
87,12	7	23.0	6.65	.63	18.29	-16.55	11.34	.15	.69 (.034)

[a] numbers in parantheses are the 95 percent critical values

[b] numbers in parentheses are estimated asymptotic standard errors

Table 4
Cointegration Tests for Real Monetary Base, Real Personal Income and Interest Rates
Log Specifications

Sample	k	Johanson Test Statistic[a]			Unconstrained Cointegrating Vector			Test for Velocity Restriction	Implied Interest Elasticity of Velocity[b]
		r=0 (23.2)	r<=1 (12.1)	r<=2 (3.9)	M/P	Y/P	R	$\chi^2_{(1)}$	
Treasury Bill Rate (RTB)									
53,1-	4	67.6	8.82	.69	17.49	-16.48	5.46	.03	.33 (.010)
74,12	7	42.7	13.19	1.22	21.45	-20.22	6.57	.05	.33 (.011)
53,1-	4	47.8	3.83	1.79	15.57	-13.34	4.85	.41	.39 (.009)
81.12	7	29.3	6.05	2.33	18.18	-15.39	5.54	.30	.39 (.010)
53,1-	4	53.5	8.52	.10	17.73	-13.98	4.59	2.47	.37 (.011)
87,12	7	35.3	10.93	.68	21.39	-16.37	5.30	2.13	.38 (.013)
10 Year Government Bond Rate (R10)									
53,4-	4	62.8	9.90	.79	23.45	-28.47	15.89	1.42	.51 (.019)
74,12	7	48.8	17.60	1.71	29.33	-33.90	18.33	.52	.50 (.032)
53,4-	4	46.3	4.95	1.80	24.69	-24.95	12.80	.01	.51 (.024)
81,12	7	34.4	7.76	2.46	30.11	-29.29	14.44	.03	.50 (.040)
53,4-	4	40.5	7.88	.30	20.94	-20.06	9.05	.09	.46 (.029)
87,12	7	32.4	12.00	.98	24.10	-22.49	9.95	.18	.46 (.008)

[a]numbers in parantheses are the 95 percent critical values
[b]numbers in parentheses are estimated asymptotic standard errors

Table 5
Tests for Equality of Long-term and Short-term Interest Elasticities

lnRTB, lnR10, ln(Y/P), ln(M1/P)

Sample	k	Eigenvalues - Unrestricted 4 - element vector				Eigenvalues Velocity Restriction			χ^2	Eigenvalues Velocity and Interest Restrictions		χ^2
53,4-74,12	4	.1383	.0371	.0115	.0002	.1382	.0320	.0005	1.38	.1380	.0008	8.34
	7	.0803	.0381	.0140	.0004	.0802	.0320	.0019	1.68	.0788	.0029	8.13
53,4-81,12	4	.0826	.0211	.0055	.0041	.0824	.0209	.0041	.15	.0802	.0065	5.86
	7	.0662	.0212	.0041	.0019	.0647	.0212	.0019	.55	.0062	.0051	6.54
53,4-87,12	4	.0707	.0338	.0149	.0003	.0693	.0324	.0146	1.23	.0678	.0148	8.19
	7	.0515	.0335	.0180	.0001	.0479	.0327	.0177	1.92	.0469	.0177	6.85

lnRTB, lnR10, ln(Y/P), ln(B/P)

Sample	k	Eigenvalues - Unrestricted 4 - element vector				Eigenvalues Velocity Restriction			χ^2	Eigenvalues Velocity and Interest Restrictions		χ^2
53,4-74,12	4	.1502	.0439	.0218	.0020	.1492	.0425	.0132	.69	.1465	.0199	6.92
	7	.0958	.0685	.0271	.0028	.0942	.0685	.0146	.46	.0938	.0399	8.01
53,4-81,12	4	.1232	.0421	.0055	.0025	.1230	.0421	.0034	.08	.1187	.0103	12.95
	7	.0840	.0587	.0081	.0023	.0840	.0587	.0023	.00	.0819	.0160	16.10
53,4-87,12	4	.1077	.0421	.0173	.0002	.1015	.0418	.0171	3.02	.1014	.0214	8.83
	7	.0655	.0486	.0241	.0013	.0607	.0474	.0228	2.64	.0606	.0312	7.08

Table 6

Tests for Equality of M1 and Adjusted Monetary Base Interest Elasticities

lnRTB, ln(Y/P), ln(M1/P), ln(B/P)

Sample	k	Eigenvalues - Unrestricted 4 - element vector				Eigenvalues Income Restriction			χ^2	Eigenvalues Income and Interest Restrictions		χ^2
53,4-74,12	4	.1330	.0689	.0459	.0015	.1330	.0591	.0099	2.73	.0934	.0628	10.63
	7	.0981	.0562	.0261	.0031	.0698	.0555	.0060	8.25	.0845	.0335	1.85
53,4-81,12	4	.1067	.0357	.0057	.0032	.1052	.0329	.0054	1.58	.0660	.0081	23.53
	7	.0667	.0292	.0069	.0021	.0651	.0288	.0047	.73	.0342	.0079	18.56
53.4-87,12	4	.0944	.0329	.0107	.0000	.0898	.0329	.0105	2.11	.0532	.0190	22.39
	7	.0566	.0377	.0107	.0010	.0524	.0377	.0106	1.85	.0342	.0214	14.94

lnR10, ln(Y/P), ln(M1/P), ln(B/P)

Sample	k	Eigenvalues - Unrestricted 4 - element vector				Eigenvalues Income Restriction			χ^2	Eigenvalues Income and Interest Restrictions		χ^2
53,4-74,12	4	.1164	.0657	.0509	.0020	.1136	.0547	.0106	3.88	.0753	.0291	18.01
	7	.1042	.0849	.0206	.0045	.1029	.0367	.0083	13.78	.0995	.0144	6.96
53,4-81,12	4	.1033	.0239	.0088	.0038	.1020	.0212	.0088	1.45	.0386	.0104	27.42
	7	.0871	.0161	.0080	.0042	.0850	.0159	.0078	.86	.0243	.0102	24.15
53,4-87,12	4	.0699	.0275	.0109	.0001	.0692	.0275	.0105	.31	.0188	.0135	27.95
	7	.0536	.0368	.0096	.0006	.0527	.0367	.0080	.44	.0289	.0105	21.54

Figure 1a

Actual and Equilibrium Real M1 Balances

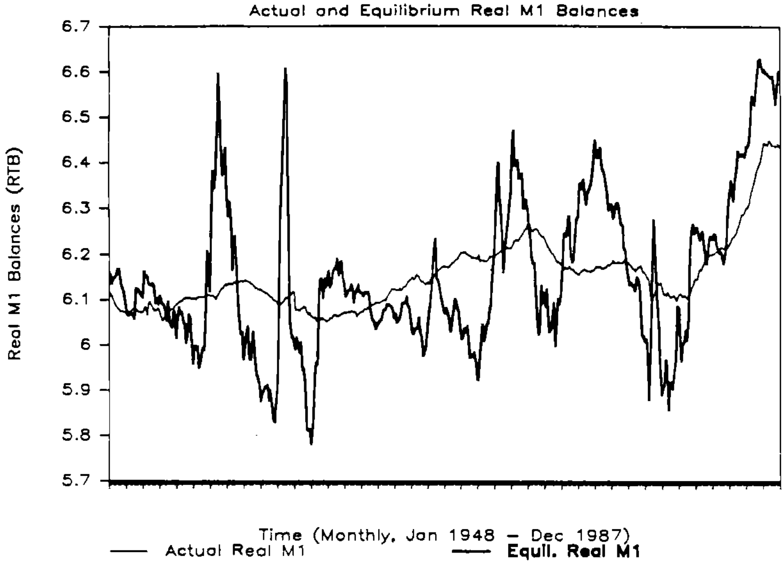


Figure 1b

Actual and Equilibrium Real M1

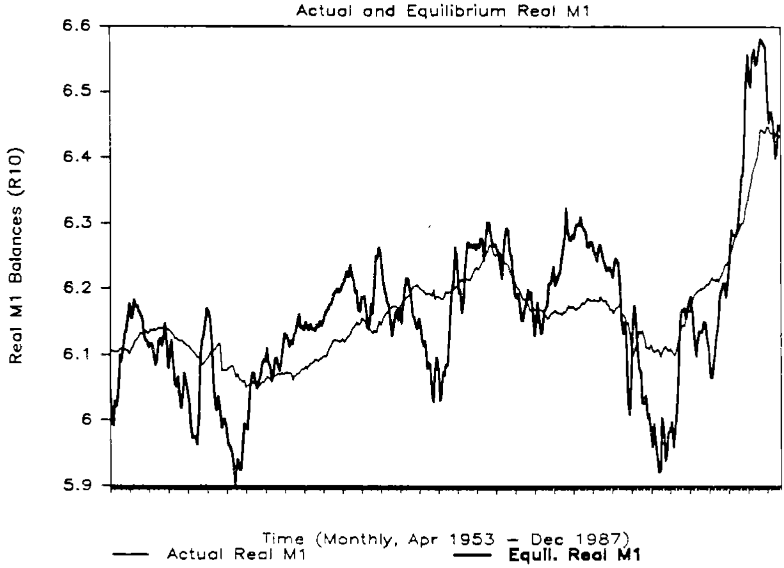
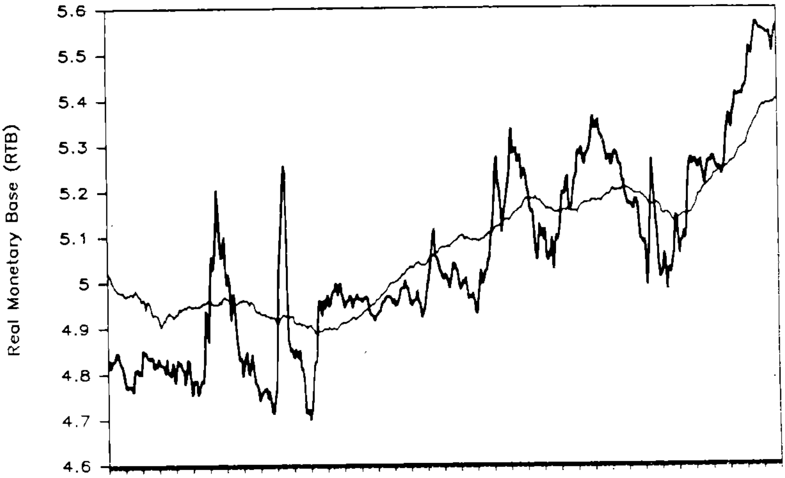


Figure 2a

Actual and Equilibrium Real Base

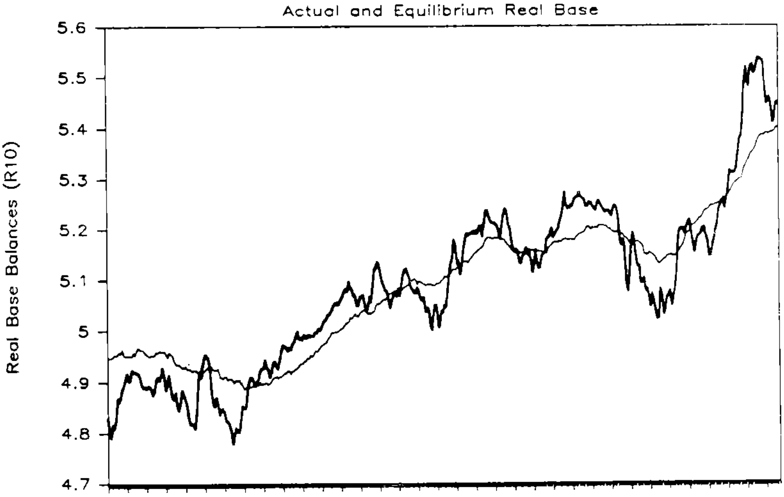


Time (Monthly, Jan 1948 – Dec 1987)

—— Actual Real Base —— **Equil. Real Base**

Figure 2b

Actual and Equilibrium Real Base



Time (Monthly, Apr 1953 – Dec 1987)

—— Actual Real Base —— **Equil. Real Base**