NBER WORKING PAPER SERIES

INTERMEDIATION AND VOLUNTARY EXPOSURE TO COUNTERPARTY RISK

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Working Paper 29467
http://www.nber.org/papers/w29467

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2021

I am indefinitely grateful to Amin Jafarian who made this paper possible, and to Doteck for always being there. I also thank Fernando Alvarez, Ana Babus, Simcha Barkai, Thomas Chaney, Rui Cui, Douglas Diamond, Rebekah Dix, Alex Frankel, Micheal Gofman, Lars Hansen, Zhigou He, Matthew Jackson, Gregor Jarosch, Amir Kermani, Roger Myerson, Laura Pilossoph, Raghuram Rajan, Adriano Rampini, Roberto Robatto, Amit Seru, Ali Shourideh, Fabrice Tourre, Harald Uhlig, Wei Xiong, Rasool Zandvakil and Luigi Zingales. Research support from the Deutsche Bank, Bradley Foundation, the Stevanovich Center and MFM group is gratefully acknowledged; any opinions expressed herein are the author's. The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

Intermediation and Voluntary Exposure to Counterparty Risk
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NBER Working Paper No. 29467
November 2021
JEL No. D85,G20,G21

ABSTRACT

I study a model of the financial sector in which intermediation among debt financed banks gives rise to an endogenous core-periphery network – few highly interconnected and many sparsely connected banks. Endogenous intermediation generates excessive systemic risk in the financial network. Financial institutions have incentives to capture intermediation spreads through strategic borrowing and lending decisions. By doing so, they tilt the division of surplus along an intermediation chain in their favor, while at the same time reducing aggregate surplus. The network is inefficient relative to a constrained efficient benchmark since banks who make risky investments "overconnect", exposing themselves to excessive counterparty risk, while banks who mainly provide funding end up with too few connections.

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1 Introduction

The years following the financial crisis resulted in an intense scrutiny of the architecture of financial markets. Many prominent economists have argued that the existing financial structure was socially suboptimal due to high systemic risk that emerged from excessive interconnectedness between financial intermediaries.[1] A relatively new, but fast growing, body of work tries to understand the optimal regulatory response to such financial structure. This literature mostly takes the financial structure as given, and assesses appropriate policy responses which minimize the systemic risk.[2] However, any policy which is implemented to mitigate the risk in the current financial architecture could feed back into bank decisions and influence the choice of inter-linkages. Alternative policy should account for endogenous changes to the financial structure. In this paper, I develop a new model where the bilateral exposures of financial institutions emerge endogenously from their profit maximizing decisions. In doing so, I generate the underpinnings of interconnectedness in the financial sector, which allows me to evaluate formally the efficiency of the current financial architecture.

I develop a model of the financial sector in which endogenous intermediation among debt financed banks gives rise to a *core-periphery network* – few highly interconnected and many sparsely connected banks. In other words, my model predicts that there is a small number of very interconnected banks that trade with many other banks and a large number of banks that trade with a small number of counterparties. Moreover, endogenous intermediation generates excessive systemic risk, which is measured as the distribution of total value lost due to bank failures. Financial institutions have incentives to capture intermediation spreads through strategic borrowing and lending decisions. By so doing, they tilt the division of surplus along an intermediation chain in their favor, while at the same time reducing aggregate surplus.

There is overwhelming recent evidence that interbank markets exhibit a core-periphery structure.[3] Moreover, banks at the core have high gross exposures and low net exposures among themselves. My model not only provides a theoretical framework that jointly explains these empirical stylized facts, but its main contribution is to do so by explicit modeling of

[1]A high degree of interconnectedness among financial institutions has been frequently recognized by policy makers. Federal Reserve chairman Ben Bernanke and undersecretary of finance Robert Steel, in their senate testimony on April 3, 2008, alluded to a potential risk of system wide failure due to mutual interconnections of financial institutions in defending Bear Stearns bailout.

[2]Notable examples are stress tests designed by the Fed. See Fed (2012), Fed (2013) for more detail.

[3]See Bech and Atalay (2010), Allen and Saunders (1986), Afonso and Lagos (2015) and Afonso et al. (2013) for evidence on the federal funds market, Boss et al. (2004), Chang et al. (2008), Craig and Von Peter (2014) and van Lelyveld and in 't Veld (2014) for interbank markets in other countries, Hollifield et al. (2017) and Peltonen et al. (2014) for OTC derivatives, and Di Maggio et al. (2017) for the corporate bond inter-dealer market.

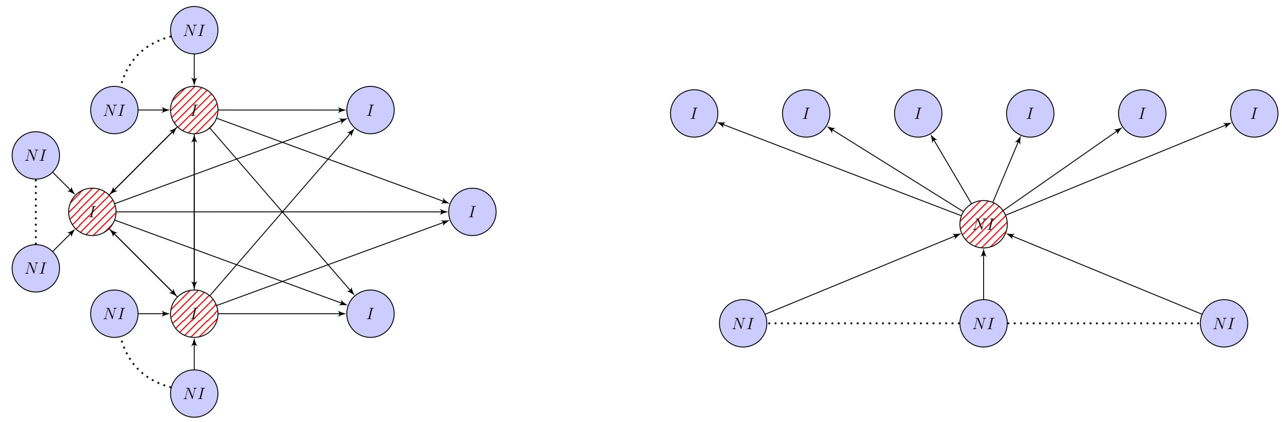
intermediation among banks and its frictions.

In the model, the financial network consists of banks and their interconnections. Banks need to raise resources for investment either from households or from other banks, via debt contracts. My model endogenously generates indirect lending and borrowing in the interbank market, which is a prominent feature of both the federal funds market and over-the-counter market for derivatives.[4] If the investment fails and the borrowing bank does not have sufficient funds to pay back her lender(s), it fails and potentially triggers a cascade of failures to the lenders, lenders of lenders and so on.

Banks are profit maximizers. There are two groups of banks in the model: those who have access to a risky investment opportunity, and those who do not. Each bank chooses its lending and borrowing relationships to get the highest expected possible rate on the funding it lends out and the investment it undertakes, net of cost of failure. When there are positive intermediation rents in the system, profit maximization creates private incentives to provide intermediation, which in turn leads to a particular structure for the equilibrium financial network. Since intermediation is profitable per-se, in equilibrium, competition implies that the banks who are able to offer the highest expected returns become intermediaries. These banks are exactly the ones who have access to the risky investment technology. On the other hand, a bank who is not an intermediary still wants to earn the highest possible returns, thus opting for the shortest connecting path to investing banks to avoid paying intermediation spread as often as possible. These two forces give rise to a core-periphery equilibrium network in which (a subset of) banks with risky investment opportunities constitute the core (Proposition 2).

The interbank network generated by the model is socially inefficient. Banks who make risky investments "overconnect", exposing themselves to excessive counterparty risk, while banks who mainly provide funding end up with too few connections. In other words, when default is costly, efficiency requires reaching the optimal scale of investment while minimizing the loss of failure, which leads to a different structure from the one which arises in equilibrium (Proposition 3). Figure 1 depicts the equilibrium and efficient interbank networks. This finding is in contrast to Gale and Kariv (2007) and Blume et al. (2009) who suggest that the financial architecture does not matter for efficiency. The main driving force behind this difference is the presence of intermediation rents which prevent social and private incentives from being aligned.

[4] Bech and Atalay (2010), Hollifield et al. (2017), and Di Maggio et al. (2017).



(a) Equilibrium

(b) Efficient

Figure 1: Equilibrium and efficient interbank networks [6]

1.1 Model Implications

The model predicts that multiple banks can be at the core of the financial system, with high gross and low net exposures among core banks. Consistent with this prediction, there is direct evidence from the financial crisis on substantial exposure among large financial institutions, which entailed runs and subsequent failure of one entity following its counterparty's failure.[5]

Equilibrium intermediaries are exposed to excessive risk since they do not contribute to the investment except through intermediation. The social planner prefers leaving such intermediaries out of the chain, replacing them with intermediaries who take minimal extra risk by intermediating. This minimizes the systemic risk without hurting the scale of investment. Thus social planner balances the net gain from investment with the expected loss of default. In contrast, private incentives compare rents, partially in the form of intermediation spreads, with the cost of default. The cost of default is a real cost while intermediation spreads are a mere redistribution of surplus. Consequently, I illustrate that the social and private incentives for intermediation diverge in several situations. The intuition can be obtained by focusing on Figure 1 that compares the equilibrium interbank network with the efficient one. Banks who intermediate are hatched in red in each structure.

One can also interpret the implications of the model in terms of *contagion*. In the model,

[5] A prominent example, as reported in the FCIC report on the financial crisis, is the immediate run on holders of Lehman unsecured Commercial Paper (CP) and lenders to Lehman in tri-party repo, such as Wachovia's Evergreens Investment and Reserve Management Company's Reserve Primary Fund, after Lehman failed on September 15, 2008. The first wave of runs was followed by a second wave of withdrawal from Lehman OTC counterparties, most notably UBS and Deutche Bank. Fore more details see FCIC (2011).

[6] The labels I and NI refer to banks with and without potential risky investment, the latter solely raising funds from households and intermediating them to investing banks. See the model for the detail. The dots represent more NI banks.

investment and funding opportunities arise at different banks, which requires funding to be channeled from banks with liquidity surplus to the ones with investment opportunities. This decentralized distribution of resources and investment opportunities gives rise to endogenous interbank intermediation. Moreover, the return to risky investment is not contractible, so all the bank liabilities are in the form of debt, which leads to failure if obligations are not met. As a result, lenders and intermediaries are exposed to counterparty risk. Because investment is positive NPV, there is an *optimal level* of contagion, due to counterparty risk exposure, in order to provide funding for the projects. In other words, even the financial structure chosen by a social planner involves a certain level of contagion when risky investment fails. The important prediction of the model is that the equilibrium interbank network involves *excessive* contagion, more than what is necessary to support the optimal level of investment.

The core-periphery structure implies that many banks are connected to each other only indirectly, a similar notion to weak ties as defined in Granovetter (1973). In the context of the model, the weak ties are intermediary's borrowing and lending relationships. As these relationships are associated with rents, every bank prefers to have many weak ties. In equilibrium, banks who are able to pledge the highest return to their creditors have many weak ties and are in the core.

The model not only provides predictions on the global structure of the interbank network, but also has implications about the bilateral interbank rates. My model predicts that core dealers charge higher average prices to the peripheral dealers than to other core ones, consistent with the empirical findings of Di Maggio et al. (2017) in the inter-dealer market for corporate bonds.

Finally, I use the model to shed light on several policies related to the architecture of the financial networks. The model provides a framework to assess bailouts, as well as policy proposals to impose a cap on the number of counterparties and swaps. Moreover, it provides a new rationale for introduction of a Central Clearing Party (CCP).

1.2 Literature Review

As a model of interbank networks, my paper is closely related to application of networks in economics (three early seminal papers are Jackson and Wolinsky (1996), Bala and Goyal (2000) and Aumann and Myerson (1988)). Jackson (2005), Jackson (2010) and Allen and Babus (2009) provide excellent reviews of the existing work.

There is also a fast growing literature on contagion and systemic risk in financial networks, started by the seminal work of Allen and Gale (2000) who studies the propagation of negative shocks in simple financial networks. A large part of this literature either focuses on properties of large networks, or take the structure of the network as given.[7] More recent work in this area focuses on strategic link formation among financial institutions.[8] Acemoglu et al. (2015a), by locating banks on a ring, predicts that the equilibrium network can exhibit both under and over connection. Zawadowski (2013) uses the same ring network to provide a rational for under-insurance due to the high market price of insurance. Related to this literature is Kiyotaki and Moore (1997), who is one of the first papers that look at the formation of credit networks. Although the modeling assumptions of this paper are more closely related to supply chain networks, the implications for contagion and under-insurance can be interpreted in the context of financial networks.

Also related to this paper are Hojman and Szeidl (2006), Hojman and Szeidl (2008) and Babus and Hu (2017), which predict minimally connected star equilibrium structures, based on costly link formation.[9] Moreover, unlike mine, these papers focus on undirected networks which is less suitable to model interbank, often asymmetric, relationships. My model contributes to this literature by providing rich predictions consistent with stylized facts about global structure of interbank networks missing from the previous work, and does that by underpinning a microfoundation for endogenous cost and benefit of interbank relationships.

Another strand of work on financial market structure uses a search framework to approximate the interbank network. Among these paper, Üslü (2019) take the approximate interbank network as given , while Farboodi et al. (2021), Farboodi et al. (2019) and Chang and Zhang (2019) endogenize the market structure. These papers are different from the current paper as by construction, they can not represent the interbank interconnections as persistent linkages.

There is also an emerging literature on bargaining and intermediation in (financial) networks (Gale and Kariv (2007), Manea (2018), Gofman (2011) and Babus and Hu (2017)). In all of these models except Babus and Hu (2017) intermediaries are determined exogenously. In my model, certain banks endogenously assume the role of intermediaries, which can lead to welfare losses in equilibrium.

Finally, my paper is also related to the extensive literature that studies the role of banks as intermediaries, their balance sheet structure and issues related to insolvency.[10] In this

[7]See Acemoglu et al. (2015b), Eisenberg and Noe (2001), Elliott et al. (2014), Gofman (2011), Gai and Kapadia (2010) and Caballero and Simsek (2013).

[8]See Acemoglu et al. (2015a), Blume et al. (2011), Babus (2016), Allen et al. (2012), Moore (2011), Rotemberg (2008), Zawadowski (2013),Bluhm et al. (2013) and Cabrales et al. (2017).

[9]Babus and Hu (2017) can have an equilibrium which is an interlinked star network as well.

[10]An incomplete list includes Diamond (1984), Rochet and Tirole (1996), Kiyotaki and Moore (1997), Moore (2011), Lagunoff and Schreft (2001), Leitner (2005), Cifuentes et al. (2005), Dang et al. (2010), Dasgupta (2004), Acharya et al. (2012), Acharya and Yorulmazer (2008) Bhattacharya and Gale (1987),

literature, banks are intermediaries between investors and entrepreneurs. I add to this literature by specifically modeling the role of banks as intermediaries among each other, and study the corresponding implications for the structure and efficiency of financial sector, as well as systemic risk.

The rest of the paper is organized as follows. Section 2 lays out the environment. Section 3 provides a simplified version of the economy with four banks and solves for the equilibrium and constrained efficient structure. Section 4 specifies further detailed required to address general network structures. Section 5 provides the general results. Section 6 discusses policy implications of the model. Section 7 concludes.

2 Model

There are three dates, $t = 0, 1, 2$, and one good which I refer to as funding. There are two types of agents, banks and households, who are both risk neutral and do not discount the future. There are K banks in the economy, k_I banks of type I who randomly get risky investment opportunities, and k_{NI} banks of type NI who do not, with $k_{NI} \geq k_I$ and $k_I + k_{NI} = K$. Let $\mathbb{I}$ and $\mathbb{NI}$ denote the set of I and NI banks, respectively, and let $\mathbb{N} = \mathbb{I} \cup \mathbb{NI}$. Banks maximize their net expected profit.

The investment opportunity is risky and linearly scalable. Each I bank receives the opportunity to invest in a risky asset with probability q. Let $\tilde{\mathbb{I}}_R$ denote the random variable corresponding to the subset of $\mathbb{I}$ that receives the opportunity, and let $\mathbb{I}_R$ denote the corresponding realization.[11]

Let $\tilde{R}_i \in \{0, R\}$ denote the per-unit random return of bank i's investment in the risky asset. Each investment succeeds with probability p and fails with probability $1 - p$, i.i.d. across banks, so that

$$\tilde{R}_i = \begin{cases} R & \text{with probability } p \\ 0 & \text{otherwise.} \end{cases}$$

Let V_i denote the value of each bank i's other businesses, assets, and services besides the risky investment opportunity. Bank i loses V_i if it defaults.[12] For simplicity, I assume

Bolton and Scharfstein (1996), Diamond and Rajan (2005), Farhi and Tirole (2013) and Gorton and Metrick (2012).

[11] Throughout the paper, I will use the following convention: $\tilde{x}$ denotes a random variable, and x denotes the realization of that random variable.

[12] This value accrues to the bank itself. James (1991) finds that losses due to bank failure are substantial, losses on assets and direct expenses averaging 30% and 10% of the failed bank's assets, respectively. This

$V_i = V_I$ for every $i \in \mathbb{I}$ and $V_i = V_{NI}$ for every $i \in \mathbb{NI}$.

Bankers do not have any wealth. They can raise financing from two sources in the form of debt. First, each bank NI_j raises funding from a unit measure of households, hh_j, at $t = 0$. Each household is endowed with one unit of funding. Households are captive and behave competitively. Second, any bank, I or NI, can borrow from other banks at $t = 1$. In order to do so, the borrower and lender banks must establish an *interbank agreement* at $t = 0$. An interbank agreement is a bilateral contract. It is a *commitment* by the lender bank to deliver at least one unit of funding to the borrower bank at $t = 1$ if two conditions are satisfied: First, the lender bank does not have a realized investment opportunity itself. Second, the borrower bank either receives an investment opportunity or has committed via another interbank agreement to lend to a bank who has received an investment opportunity.

From the borrower bank's perspective, it is *eligible* to draw (at least) one unit on its interbank agreement if it has a direct or indirect access to a realized investment opportunity, and its lender does not have a realized investment opportunity. With some abuse of language, I use *eligible* for interbank agreements as well. I assume that lender banks can commit to their interbank agreements credibly, thus for any realization of random investment opportunities, each lender must have sufficient funds to lend at least one unit to each eligible borrower. This implies an endogenous limit on the number of interbank agreements that a bank can participate in as a lender, and is equivalent to an opportunity cost of forming interbank agreements.[13] As such, a bank can enter arbitrarily many interbank agreements as a lender, as long as it enters sufficiently many agreements as a borrower, to guarantee that it can satisfy its commitments to its own borrower banks even if all of them are simultaneously eligible to draw on their interbank agreements.

For a concrete example, consider Figure 2. In Panel 2a, NI_1 has secured two units of funding, one from households and one through the interbank agreement $NI_2 \to NI_1$. Alternatively, in Panel 2b, NI_1 has the one unit it has raised from households but no other source of funding. In particular, interbank agreement $NI_2 \to NI_1$ does not exist. In both structures, NI_1 has committed to lend to both I banks, i.e. both interbank agreements $NI_1 \to I_1$ and $NI_1 \to I_2$ exist. Now consider the state at $t = 1$ when both I banks receive investment opportunities. We say that the network structure depicted in Panel 2a is *feasible*, as NI_1 can fulfill both of its commitments. However, the network structure in 2b is *infeasible*

model is isomorphic to one in which (negative) bankruptcy costs are borne by the banks in the event of default.

[13] Alternatively, the opportunity cost can be modeled with appropriately chosen upfront fixed or declining cost of link formation levied on the lender. The cost should be such that with j units of available funds, the expected marginal gains from $j + 1^{\text{th}}$ lending agreement is below the cost, while it covers the cost with $j + 1$ units. However, the motivation for this assumption is not to capture a fixed cost.

Figure 2: Feasibility of interbank relationships

as NI_1 has to violate one of its commitments.

The financial system consists of banks and their interbank agreements. I model the financial system as a network. The financial network is a directed graph $G = (\mathbb{N}, \mathbb{E})$, where $\mathbb{N} = \{1, 2, \cdots, K\}$ is the set of nodes and $\mathbb{E} = \{e_{ij}\}$ is the set of edges. Each node is a bank, and edge $e_{ij} \in \mathbb{E}$ is an interbank agreement between banks i and j. $e_{ij} \in \mathbb{E}$ if and only if at $t = 1$ funding is lent along this interbank agreement with strictly positive probability. An *intermediation chain* is the realization of a set of interbank agreements among consecutive banks. Let *chain length* denote the number of banks involved in the intermediation chain.

There is perfect information. Every bank knows the sets $\mathbb{I}$ and $\mathbb{NI}$, the structure of the interbank agreements, the realization of the investment opportunities, and the realization of final returns. However, the realization of final returns is not contractible, while the network structure at $t = 1$ is. Thus, the bilateral contracts are *contingent debt*. Note that in the real world, the interbank exposures are quite complex and can be through multiple channels such as secured and unsecured debt, derivative contracts, and holdings of common assets. However, for the purpose of this paper, I restrict interbank exposures to debt contracts. Moreover, holding precautionary liquidity is ruled out at $t = 1$ and banks lend or invest as much resources as they are able to raise.

Division of surplus $\mathcal{L}(.)$ There is imperfect competition among banks. To model the imperfect competition in a reduced form way, I assume an exogenous division of surplus among banks. For a given intermediation chain of length n and realization of investment opportunities $\mathbb{I}_R$, let $\mathcal{L}(n, \mathbb{I}_R)$ and $\mathcal{L}(i; n, \mathbb{I}_R)$ denote the rule for the division of surplus, and the share of expected surplus that accrues to bank i, respectively.

$\mathcal{L}(.)$ satisfies the following properties. The expected surplus from each unit of investment is divided only among the banks in the corresponding intermediation chain, as a function of the length of the chain and the bank position. For every unit of funding, every member of the corresponding intermediation chain receives strictly positive shares of expected surplus. Eliminating an intermediary from an intermediation chain weakly increases the share of every other bank along the chain, and strictly increases the share of the initial lender. The rule is anonymous. Lastly, renegotiation and side payments are ruled out.[14]

In order to keep the exposition transparent, throughout the paper, I use a specific rule for division of surplus which I call α*-rule*. Proposition 6 shows that the key characterization results of the paper generalize to the broader class of rules specified above.

α**-rule** Consider a chain of length $n \geq 2$. Let i_j denote the bank in position j of the intermediation chain, where i_1 is the first lender and i_n is the final borrower bank. Furthermore, assume that the bank i_n is an I bank who has received an investment opportunity, $i_n \in \mathbb{I}_R$. For a unit of funding that originates at bank i_1 and is intermediated to i_n through this intermediation chain, α-rule specifies the expected share of surplus that accrues to the bank in position j of the chain to be,

$$\begin{aligned} \mathcal{L}_\alpha(i_j; n, i_n \in \mathbb{I}_R) &= \alpha^{n-j}(1-\alpha)(pR-1), \qquad\qquad j \neq 1 \\ \mathcal{L}_\alpha(i_1; n, i_n \in \mathbb{I}_R) &= \alpha^{n-1}(pR-1). \end{aligned} \tag{1}$$

It is most intuitive to consider this division of surplus as defined recursively from the end of the chain. Bank i_n who invests, receives $(1-\alpha)$ fraction of the expected surplus, $pR-1$. Moving from bank i_n towards the start of the chain where the unit funding is originated, bank i_j receives $(1-\alpha)$ fraction of the remaining surplus, α^{n-j}, and the rest of the surplus accrues to banks $\{i_1, \cdots, i_{j-1}\}$, recursively in the same fashion. This rule is motivated by a moral hazard friction that endogenously generates the same division of surplus, as explained in Online Appendix A.

The face value of debt is set such that given the network structure, each bank along each intermediation chain receives its pre-specified share, in expectation. In other words, the rule for division of surplus determines the return on debt contracts among banks, given the structure of the financial network. At time $t=0$, each bank chooses its interbank agreements to maximize its expected profit net of cost of default.

Lastly, in order to highlight the role of intermediation, I make the following simplifying assumption to rule out diversification.

Assumption 1 *Assume more than one counterparty of bank i is eligible to borrow on their interbank agreement at $t=1$. If i does not have a realized investment opportunity itself,*

[14] Investment happens at $t=1$ and the non-contractible return is realized at $t=2$, thus the borrower cannot commit to pay the lender a side payment above and beyond the face value of debt enforceable by the contract. Note that in the period during which actual lending happens, no extra funding is available to make an early side payment. As a result, ruling out side payments is a reasonable assumption.

all of its funding is allocated randomly, with equal probability, to a single one of the eligible counterparties. If i has a realized investment opportunity, it does not lend out any funding.

I abstract away from diversification in order to keep the argument focused on the role of intermediation in network formation, as diversification is a relatively well-studied phenomena. I relax this assumption later in Online Appendix B and show that the equilibrium structure remains the same when intermediation and diversification interact.

The equilibrium concept is *group stability.* A group stable interbank network is a network structure that is not blocked by any coalition of banks.[15] Group stability is the appropriate equilibrium concept to study interbank intermediation chains, as these chains generically involve more than two banks and this equilibrium concept allows for group deviations. A group stable network is one where there is no coalition of banks who can jointly deviate to an alternative feasible financial network G' where they all get *strictly* higher net profits.[16] Note that the resulting network G' has to be feasible, i.e. the only viable group deviations are those in which every edge $e_{ij} \in G'$ is eligible and traversed with positive probability at $t = 1$. Finally, this equilibrium concept is different from β-core, in which the blocking coalition goes to autarky.

The timing of the model is as follows: At $t = 0$, banks raise funding from households and form interbank agreements with other banks. At $t = 1$, investment opportunities are realized and actual lending happens along (some of) the interbank agreements formed at $t = 0$. At $t = 2$, random returns are realized and banks repay their debt. The banks who are not able to pay back their creditors default.

3 Economy with Four Banks

I start by demonstrating the main mechanism of the model through a simplified version with four banks. I then generalize the results to an unrestricted economy.

Assume there are two I and two NI banks, $\mathbb{I} = \{I_1, I_2\}$ and $\mathbb{NI} = \{NI_1, NI_2\}$. Each bank I needs to secure funding on the interbank market at $t = 0$ to be able to invest in its project at $t = 1$.

A direct lending from an NI to an I bank is socially desirable if and only if $pR - 1 > (1 - p)(V_I + V_{NI})$. That is, net expected return of the project covers its expected cost of bank default. As both the lender and borrower banks fail if the project fails, the expected

[15] Group stability is defined in Roth and Sotomayor (1990) and is commonly used in the matching literature. It is a generalization of pairwise stability defined in Jackson and Wolinsky (1996).

[16] I use strict deviations so that the equilibrium structure is robust to introduction of small link formation costs.

cost is the expected sum of loss of outside value of the two banks.

Consider a set of parameters for which the following three inequalities are satisfied: participation constraint of a final borrower $i \in \mathbb{I}$, $(1-\alpha)(pR-1) > (1-p)V_I$, and participation constraint of a first lender $i \in \mathbb{N}\mathbb{I}$ in chains of length two and three, $\alpha(pR-1) > (1-p)V_{NI}$ and $\alpha^2(pR-1) > (1-p)V_{NI}$, respectively. Note that the first and second inequality ensure that a direct lending relationship is socially efficient.

With only two banks, individual participation constraints are sufficient but not necessary for efficiency. As a result, an economy with only two banks can only be inefficient due to trade breakdown and under-exposure, leading to under-investment. In contrast, I show that with more banks, the equilibrium is often inefficient for the opposite reason, i.e. that a certain group of banks are over-exposed to each other.

.

3.1 Equilibrium and Optimum

I first characterize prices, i.e. the face value of debt contracts, in intermediation chains of length two and three. I will later show that in an economy with four banks, every intermediation chain has length either two or three. Thus, it is sufficient to determine the face values of debt along these chains.

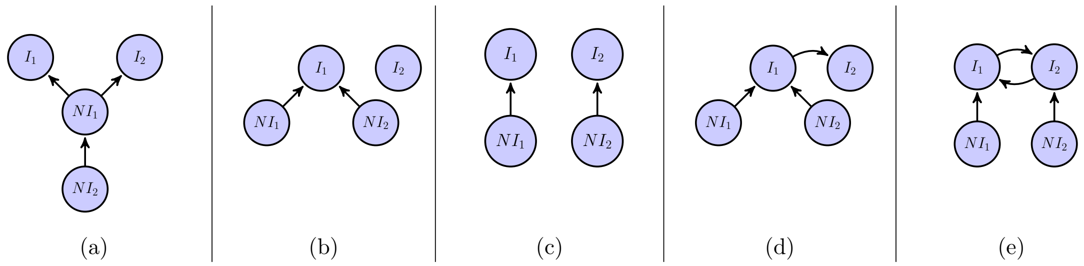
In a chain of length two, bank i lends one unit at face value D, to bank j who invests the unit. In a chain of length three, bank i lends one unit at face value D_1 to bank k, who in turn lends the unit at face value D_2 to bank j, who invests the unit. The face values of debt are set to ensure that in expectation each party receives its share of expected surplus implied by α-rule,

$$D_1 = \frac{\alpha^2(pR-1)+1}{p} < D_2 = D = \frac{\alpha(pR-1)+1}{p}.$$

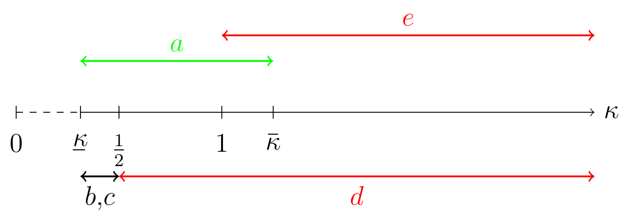
$D_2 - D_1$ represents the *intermediation spread*. The intermediation spread is the rent captured by the intermediary bank k if the investment succeeds. However, if the investment fails, borrower bank j will not be able to pay back the intermediary bank k, who in turn will not be able to repay its lender bank i, and thus suffers the cost of default. As such, intermediation rents come at the cost of risk of default.

The first result of the paper characterizes all the equilibria in an economy with four banks.

Proposition 1 *Let* $\kappa = \frac{\alpha(1-\alpha)(pR-1)}{(1-p)V_I}$. *There exist* $\underline{\kappa}$ *and* $\bar{\kappa}$ *such that an equilibrium exits if* $\kappa \geq \underline{\kappa}$. *Furthermore,*



Structure of Equilibria



Range of Euilibria

Figure 3: The equilibria for an economy with two I and two NI banks as a function of parameters. $\kappa = \frac{\alpha(1-\alpha)(pR-1)}{(1-p)V_I}$, $\bar{\kappa} = 1 + \frac{q}{2-1}$, and $\underline{\kappa} = \max\{\alpha, (1-\alpha)\frac{V_{NI}}{V_I}\}$. If $\underline{\kappa} > \frac{1}{2}$, network structures b and c are never an equilibrium.

(a) If $\min\{\underline{\kappa}, \frac{1}{2}\} \leq \kappa \leq \bar{\kappa}$*, an efficient equilibrium coexists with inefficient equilibria.*

(b) If $\kappa > \bar{\kappa}$*, every equilibrium is core-periphery and is inefficient.*

Finally, if $\underline{\kappa} < \frac{1}{2}$ *and* $\kappa < \frac{1}{2}$*, all inefficient equilibria display under-exposure, while if* $\kappa \geq \frac{1}{2}$ *all inefficient equilibria display over-exposure among banks.*

The proof of the proposition provides a more detailed characterization of all the equilibrium structures and the range of parameters for which each of them exists, as depicted in Figure 3.

To gain some intuition about the structure of the equilibria, consider Figure 3. The top row depicts all the equilibrium interbank networks, and the bottom row illustrates the range where each interbank network is an equilibrium. It is instructive to examine the deviation from interbank network 3a to 3e. This deviation involves the two I banks and the peripheral NI bank, $\{I_1, I_2, NI_2\}$, and is illustrated in Figure 4.

The key to this deviation is understanding the incentives of each bank in the coalition. The peripheral NI bank prefers to directly lend to an I bank and avoid paying intermediation rent as often as possible to maximize its expected return. On the other hand, I banks face a trade-off in being exposed to each other. Consider a given $\hat{I} \in \mathbb{I}$. Being connected to other I banks allows $\hat{I}$ to intermediate the funding it has secured on the interbank market to other I

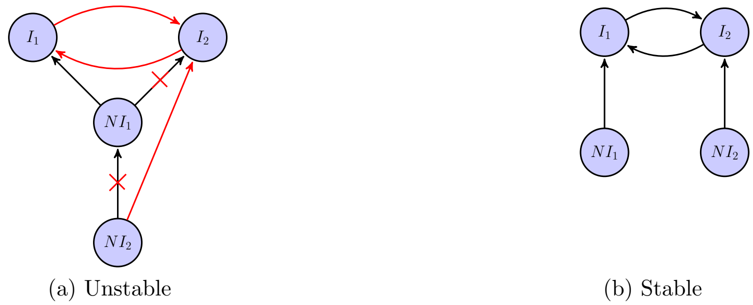


Figure 4: Joint deviation by $\{I_1, I_2, NI_2\}$

banks with investment opportunities, and collect the intermediation rent. However, lending to other I banks exposes $\hat{I}$ to their counterparty risk: if the borrower I bank defaults and does not pay back, $\hat{I}$ cannot pay back its lenders and will suffer the cost of default itself. As such, I banks bear both the cost and benefit of intermediation. If the intermediation spreads are high enough to compensate them for their default risk, I banks form a coalition with the peripheral NI bank, voluntarily expose themselves to counterparty risk, and move to a core-periphery equilibrium. In this network, the banks who are able to pledge the highest expected return to their creditors, i.e. the I banks, constitute the core.

To be specific, consider Figure 4. In Panel 4a, NI_1 serves as the intermediary for NI_2 when I_1 and/or I_2 receive(s) an investment opportunity. If the undertaken project succeeds, NI_1 collects the intermediation spread from NI_2. If the project fails, both NI banks, as well as the I bank who has undertaken the project fail and suffer the cost of default. The other I bank remains intact. In Panel 4b instead, each bank I_i who has a realized investment opportunity, directly borrows from bank NI_i. In such states, NI_i avoids paying an intermediation spread since it lends directly to I_i. If the project fails in any state that a single I bank invests, all four banks default.

It follows that NI_2, the *peripheral* NI bank in 4a, prefers the interbank network 4b to 4a.[17] Alternatively, I banks face a trade-off. Consider the states of the world when only one of the I banks receives an investment opportunity. Without loss of generality assume it is bank I_2. In this state, I_1 serves as the intermediary for NI_1 and captures the intermediation rents. However, if I_2 fails, besides NI_1 and NI_2 failing, I_1 fails as well. It follows that if $p(D_2 - D_1)$, the expected intermediation spread, covers $(1-p)V_I$, the expected cost of default contagion due to intermediation to an I bank, $\{I_1, I_2, NI_2\}$ jointly perform the deviation depicted in Figure 4a and go to 4b.

To complete the discussion of equilibria, it is useful to briefly consider the rest of the

[17] NI_1 has the opposite preference. This is irrelevant for the deviation as NI_1 is not part of the deviating coalition.

equilibrium interbank networks, depicted in Panels 3b, 3c, and 3d. In Panel 3d, only one of the I banks act as an intermediary, namely I_1. Thus, it collects more intermediation spreads, whereas its cost of default stays the same. As such, this interbank network is sustainable as an equilibrium at lower levels of intermediation spread compared to the interbank network 3e. This is evident in the bottom panel of Figure 3, which illustrates the range of equilibria. Finally, in Panels 3b and 3c, intermediation cannot be sustained. Intermediation spreads are too low to cover the cost of default due to contagion, so that no bank is willing to intermediate. As a result, some positive NPV investment opportunities are not efficiently funded.

I now proceed to investigate the constrained efficiency of equilibrium interbank networks. The constrained efficient network structure maximizes the equally weighted sum of expected return net of expected cost of bank default, subject to feasibility of interbank agreements and banks' participation constraints.

First, note that a project financed via direct lending is efficient and the investment opportunity exhibits constant return to scale, thus increasing the scale of projects is desirable. Second, as financing any project requires the investing bank to borrow on the interbank market, when a project fails, *contagion* also occurs: the I bank who has undertaken the investment cannot pay back its lender banks and fails, the lenders cannot pay back their own lenders and fail, and the chain of failures continues. As such, even the constrained efficient interbank network involves a certain level of contagion. However, the social planner wants to maximize the expected return to investment while minimizing the cost of contagion.

These observations imply that in the constrained efficient network, one of the NI banks serves as the sole intermediary. It borrows from the other NI bank and lends to both I banks, as illustrated in Figure 3a. This is quite intuitive. Consider one of investment opportunities. It is efficient for both NI banks to fund that opportunity at the risk of potential failure. Furthermore, intermediation is necessary to fully finance both of the investment opportunities. Designating one of the NI banks as the intermediary enables fully financing both projects without increasing the overall expected default cost of the interbank network. It follows that the unique constrained efficient interbank network is the one depicted in Figure 3a.

Note that in the efficient interbank network 3a, three banks are at the risk of default for each investment opportunity. However, in equilibrium core-periphrey networks with I banks at the core, Panels 3d and 3e, four banks are at risk. As such, these equilibrium core-periphrey networks involve excessive systemic risk.

Next, I examine the range of parameters for which each network structure exists. Consider the bottom panel of Figure 3. Let $\kappa = \frac{\alpha(1-\alpha)(pR-1)}{(1-p)V_I}$ denote the ratio of the intermediation spread per unit of intermediated funding over the expected cost of default for an I bank. κ is the key determinant of whether an interbank structure is an equilibrium or not. The green, red, and black regions indicate the existence region for three types of equilibria: efficient, inefficient due to over-exposure, and inefficient due to under-exposure, respectively. The most important observation is that if the intermediation spreads are substantial, $\kappa > \bar{\kappa}$, all the equilibria are inefficient due to over-exposure, as the I banks in the core choose to expose themselves to each others' counterparty risk excessively (red ranges). On the other hand, an efficient equilibrium exists only if the intermediation spreads are not too high (green range). Finally, if intermediation spreads are so low that intermediaries are not able to cover the extra cost of default that they have to bear if they intermediate, there exist equilibria in which intermediation breaks down (black range).

Finally, note the role of α-rule. α-rule implies that the expected surplus accrued to any borrower bank, final or intermediate, does not depend on the source of the funding. As such, keeping constant a borrower's surplus when it does not intermediate, its only gain to a change in the network structure is to become an intermediary and collect intermediation rents. This feature greatly clarifies the exposition, but it is not essential for the main results, as shown in Proposition 6.

4 General Specification

In a complex interbank network, for every realization of banks with investment opportunities, $\mathbb{I}_R$, a given bank i can be connected to each $I \in \mathbb{I}_R$ through multiple intermediation chains of different lengths. As such, to characterize the equilibrium outcomes for an unrestricted number of banks, the terms of contracts have to specify how the funds flow in a general network. To this end, I introduce the following concept from graph theory, which is helpful in expressing the flow of funds in a network given a particular realization of investment opportunities.

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Definition 1 **[Shortest Path]** *$SP(i,j)$, the shortest path from bank i to bank j in financial network $G = (\mathbb{N}, \mathbb{E})$, is the sequence with the minimum number of banks $\{l_1, \cdots, l_m\} \in \mathbb{N}$ such that $l_1 = i$, $l_m = j$, and $e_{l_d l_{d+1}} \in \mathbb{E}$ for $\forall d = 1, \cdots, m-1$.*

Let $\mathbb{S}$ denote the set of banks $\{j_1, \cdots, j_k\}$. We use $SP(i, \mathbb{S})$ to denote the collection of shortest paths of bank i to all the banks $j \in \mathbb{S}$. The following definition formalizes concepts of feasibility and eligibility introduced in Section 3.

Definition 2

[**Eligibility**] *Consider* $(G, \mathbb{I}_R)$*, banks* $i, j \in \mathbb{N}$*, and an interbank agreement* $e_{ij} \in \mathbb{E}$*. For each* $I \in \mathbb{I}_R$ *such that* $j \in SP(i, I)$*, bank* j *is eligible to receive at least one unit of funding from bank* i*.*

[**Feasibility**] *An interbank network* G *is feasible if for any* $\mathbb{I}_R$*, every lender has sufficient funds to lend at least one unit to each eligible borrower along the corresponding interbank agreement.*

Eligibility implies that if at $t = 1$, bank i can lend to bank $I \in \mathbb{I}_R$ through multiple routes, its funding is intermediated to I through the shortest possible intermediation chain. This ensures that i pays out minimum intermediation spreads. The intuition is that at $t = 1$, when presented with the option to lend through multiple chains already established, i chooses the option that provides it with the highest possible rate of return. What the lender and borrower cannot do, however, is establishing a new interbank agreement at $t = 1$. After the realization of investment opportunities, for bank j to borrow from bank i, the interbank agreement e_{ij} has to be already established at $t = 0$, and $j \in SP(i, \mathbb{I}_R)$.

An intermediary $j \in SP(i, \mathbb{I}_R)$ who receives a unit of funding from bank i, has to lend the unit along (one of the) $SP(i, \mathbb{I}_R)$ paths with the shortest length on which it lies. Within $SP(i, \mathbb{I}_R)$, j allocates i's funding to satisfy its own commitments as a lender.

Lastly, note that $e_{ij} \in \mathbb{E}$ implies that there exists $I \in \mathbb{I}$ such that $j \in SP(i, I)$. However, if there also exists $\hat{I} \in \mathbb{I}$ such that $j \notin SP(i, \hat{I})$, then if $\mathbb{I}_R = \{\hat{I}\}$, bank j does not receive any funding from bank i for this particular realization of investment opportunities.

4.1 Bank Optimization Problem

At $t = 0$, each bank i chooses its interbank agreement to maximize its expected profits net of expected cost of default. The collection of banks and their interbank agreements constitute the interbank network G. At $t = 1$, given G and the realization of investment opportunities, $\mathbb{I}_R$, the contracts determine the number of units lent along each interbank agreement, as well as the face value of debt corresponding to each debt contract. At $t = 2$, given any realization of project returns $\{R_k\}_{k \in \mathbb{I}_R}$, each borrower bank pays back its lender banks if it has enough resources, and defaults otherwise.

In order to formally represent banks' optimization problem, first consider date $t = 1$ objects, taking the interbank network, G, and the realization of investment opportunities, $\mathbb{I}_R$, as given. Let $m_{ij}(G, \mathbb{I}_R)$ denote the size of the loan from bank i to j, and let $D_{ji}(G, \mathbb{I}_R)$ denote the per-unit face value corresponding to this loan, averaged over the different loan units. Lastly, let $D^h_j(G, \mathbb{I}_R)$ be the face value of debt from j to households hh_j.

Next consider date $t = 2$ objects which depend on the realization of project returns $\{R_k\}_{k\in\mathbb{I}_R}$ as well as $(G, \mathbb{I}_R)$. Let $d_{ji}(G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R})$ and $d_j^h(G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R})$ denote the per-unit repayment of bank j to bank i, averaged over the different loan units, and households hh_j, respectively. As a convention, $D_j^h = d_j^h = 0$ if j has not borrowed from households. By construction, $d_{ji} \in [0, D_{ji}]$. Finally, let $L_i(G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R})$ and $A_i(G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R})$ denote the total liabilities and assets of bank i at date $t = 2$ after the realization of investment returns,

$$L_i(G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) = \sum_{j\in\mathbb{N}} m_{ji} d_{ij} + d_i^h$$
$$A_i(G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) = \mathbb{1}[i \in \mathbb{I}_R]\Big(R_i \sum_{j\in\mathbb{N}} m_{ji}\Big) + \sum_{j\in\mathbb{N}} m_{ij} d_{ji},$$

where $\mathbb{1}[i \in \mathbb{I}_R]$ is an indicator function that takes value one if i has a realized investment opportunity and zero otherwise. Considering limited liability, the expost profit of bank i can be written as

$$\Pi(i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) = \max\big\{0, A(i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) - L(i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R})\big\}.$$

If for a borrower bank, total assets does not cover total liabilities, each lender will be (partially) paid back pro-rata.[18] As such, the per-unit (partial) repayment from bank j to bank i in each state of the world can be written as

$$\begin{cases} d_{ji}(G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) = \min\big\{D_{ji}, D_{ji}\frac{A_j}{L_j}\big\} & \forall i, j \\ d_j^h(G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) = \min\big\{D_j^h, D_j^h\frac{A_j}{L_j}\big\} & \forall j \end{cases} \tag{2}$$

The face values of debt contracts at $t = 1$, D_{ji} and D_j^h, are determined by backward induction to satisfy the division of surplus specified by $\mathcal{L}(G, \mathbb{I}_R)$. In particular, given the solution to the system of (partial) debt repayments at $t = 2$ characterized by (2), the face value of each debt contract is set such that each bank i receives its share of surplus according to $\mathcal{L}(i; G, \mathbb{I}_R)$, and each household breaks even, in expectation.

Let $\mathbb{1}[i \text{ defaults}; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}]$ be the indicator function that is equal to one if bank i defaults at $t = 2$ and zero otherwise. The exante probability that bank i defaults given the

[18]This definition implies that all debt is pari passu. Junior household debt can be interpreted as capital and be used to study the effect of capital requirements.

financial network G formed at $t = 0$ is:

$$P_D(i; G) = \underset{\tilde{\mathbb{I}}_R, \{\tilde{R}_k\}_{k \in \mathbb{I}_R}}{\mathbb{E}} \Big[\mathbb{1}[i \text{ defaults}; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}] \Big],$$

where the expectation is taken over $\mathbb{I}_R$, date $t = 1$ realization of investment opportunities, and $\{R_k\}_{k \in \mathbb{I}_R}$, date $t = 2$ realization of investment returns.

With this notation, bank i's optimization problem at date $t = 0$ can be written as:

$$\begin{aligned} \max_{\{e_{im}, e_{mi}\}_{m \in \mathbb{N}, m \neq i}} \quad & \underset{\tilde{\mathbb{I}}_R, \{\tilde{R}_k\}_{k \in \mathbb{I}_R}}{\mathbb{E}} \Big[\Pi(i; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}) \Big] + \big(1 - P_D(i; G)\big) V_i \qquad (3) \\ \text{s.t.} \quad & \text{Feasibility}, \\ & \text{Participation Constraint}. \end{aligned}$$

Bank i's interconnections allow it to participate in risky investment(s), and since the banking sector is non-competitive, it receives part of the surplus generated by the investment(s) that it engages in. At the same time, taking part in the investment process exposes the bank to risk of default due to failure of the investment(s). Each bank chooses the structure of its interconnections to balance the benefits and costs of exposure to risky investment.

As such, in equilibrium, banks choose their interbank agreements to maximize their expected profit net of expected cost of default, such that the resulting interbank is group stable. Definition 3 formalizes the notion of equilibrium in the interbank market.

Definition 3 [Equilibrium] *An equilibrium interbank network is a group stable feasible network. In the equilibrium network each bank maximizes its expected profit net of cost of default by choosing its interbank agreements, and there is no group of banks* $\{i_1, \cdots, i_k\}$ *who can jointly deviate to an alternative feasible network in which all of them receive higher expected profit net of cost of default.*

5 Results

I next demonstrate how the findings in the interbank network with a small number of banks, in Section 3, generalize to large interbank networks. In particular, I characterize the endogenous equilibrium and optimum interbank network structure with unrestricted number of banks in this section.

I start with a lemma that provides an endogenous bound on the length of the intermediation chain in any equilibrium network.

Lemma 1 *In any financial network where the participation constraint of all banks are satisfied, the length of any intermediation chain is bounded by a number* $n_{\max}$*, such that* $Z < n_{\max} \leq Z + 1$ *for* $Z = \frac{1}{|\log \alpha|} \log \frac{(1-p)V_{NI}}{pR-1}$.

This lemma uses the participation constraint of the initial lender in the longest intermediation chain that is possible in an interbank network, to provide a bound on the length of any intermediation chain. Consider interbank network G. Let $n_{\max}$ denote the length of the longest intermediation chain in G, and let i_1 denote the first bank in the chain. As i_1 is the first lender, it only borrows one unit of funding from the households, and α-rule implies that it receives $\alpha^{n_{\max}-1}(pR-1)$ in expectation. This expected profit has to cover its expected cost of default due to contagion, $(1-p)V_{NI}$, thus $\alpha^{n_{\max}-1}(pR-1) \geq (1-p)V_{NI}$. On the other hand, an intermediation chain of length $n_{\max}+1$ is not possible, and the lending benefit is decreasing geometrically in distance, which in turn requires $\alpha^{n_{\max}}(pR-1) < (1-p)V_{NI}$. Along with $\alpha < 1$, these inequalities imply the upper and lower bounds provided in Lemma 1, respectively. It is worth noting that this maximum length is non-binding, and the exact bound depends on the exact structure of the interbank network.

Before describing the main results of the paper, I state the definition of a *core-periphery financial network* used in this paper formally.

Definition 4 [Core-Periphery Financial Network] *A core-periphery financial network has the following structure. The "core" of the financial network consists of a subset of the* I *banks,* $\mathbb{C} \in \mathbb{I}$*. The core is a complete digraph. Each* NI *bank lends to exactly one bank* $I \in \mathbb{C}$*, such that each* $I \in \mathbb{C}$ *has at least* k_I *lenders among* NI *banks. Every* $I \in \mathbb{C}$ *lends to every other* I *bank, and every* $I \notin \mathbb{C}$ *does not lend to any bank.*

Figure 1b depicts a representative core-periphery financial network where the "core" is represented by the hatched red banks. This network has a core of size 3, the I banks with hatched red shading. Every NI bank lends directly to a single bank in the core, thus the core banks collectively borrow from every NI bank. Each core bank lends to every other I bank, in and out of the core. As such, the core banks have a dual role in the interbank network: they invest when they receive an investment opportunity, and act as an intermediary and channel funding to other investment opportunities when they do not.

The next proposition uses Definition 4 to present the first characterization result of the paper. It describes the region where members of the core-periphery financial network family are an equilibrium.

Proposition 2 [Core-Periphery Equilibrium] *Assume* $k_{NI} \geq k_I$ *and surplus is divided via* α*-rule. There exists a sequence* $\{M_s\}_{s=1,\cdots,k_I}$ *such that for each* s*, if* $\frac{k_{NI}}{k_I} \geq s$ *and*

$\kappa = \frac{\alpha(1-\alpha)(pR-1)}{(1-p)V_I} \geq M_s$, every core-periphery financial network with core size $g \leq s$ is an equilibrium.

Why is this structure stable? Recall that intermediation is necessary to enable the flow of funds from where they are raised to where investment opportunities arise. However, intermediation is costly as well: it is costly for the lenders since they have to pay the intermediation spread, and it is costly for the intermediaries as they have to bear the cost of exposure to the default risk of their borrower counterparties.

From the perspective of lender banks, they would like to avoid paying intermediation spreads. This implies that lender banks prefer to lend to I banks through fewest possible intermediaries. An NI bank can best achieve this goal if it directly lends to an I bank who is connected to every other I bank itself. This structure allows each NI bank to refrain from paying any intermediation spread when its direct borrower receives an investment opportunity, and pay minimal spreads when the investment opportunity arises elsewhere in the financial network.

On the other hand, in order for this configuration to be sustainable, the I bank(s) must be able and willing to intermediate. In order to be able to intermediate to every $I \in \mathbb{I}$, an individual I bank has to secure sufficient funding from its peripheral lenders on the interbank network, to credibly commit to lend to all of its borrowers. Thus the periphery has to be sufficiently dense, i.e. $\frac{k_{NI}}{k_I}$ has to be sufficiently large. Furthermore, I banks have to be willing to intermediate. This requires sufficiently high intermediation spreads to cover I banks' cost of default contagion, i.e. $\frac{\alpha(1-\alpha)(pR-1)}{(1-p)V_I}$ has to be sufficiently high.

Thus in a core-periphery equilibrium, I banks, i.e. the banks who are able to pledge the highest return to their creditors, constitute the core. Core banks not only obtain profits because they invest, but also because they act as an intermediary in the interbank market and collect intermediation spreads. As such, they choose to voluntarily expose themselves to default due to contagion from their borrowers who invest. As such, these networks are sustainable as an equilibrium when the intermediation rents are sufficiently high to justify the exposure to counterparty risk.

To formalize the above intuition, Proposition 2 proves that when the financial interbank network is core-periphery and the conditions of the proposition are satisfied, no blocking coalitions exist and thus the network is stable.

g and s denote the size and maximum size of the core in Proposition 2, respectively. The proposition argues that the maximum size of the core is $\lfloor \frac{k_{NI}}{k_I} \rfloor$. To understand how this is determined, consider the case where $k_{NI} \geq k_I^2$. In this case, it is possible to have a network where k_I distinct NI banks connect to each $I \in \mathbb{I}$, in which case every I bank is able to commit to all other $I \in \mathbb{I}$. Thus $\mathbb{C} = \mathbb{I}$, and the interbank network has the largest possible core. Alternatively, when there are fewer NI banks, it is only possible to guarantee fewer I banks to be able to raise sufficient funding on the interbank network, to credibly commit to lend to every $I \in \mathbb{I}$. Thus the maximum sustainable size of the core will be lower. The next important observation is that when a core of size s is sustainable, cores of size $g < s$, consisting of I banks only, are also sustainable, as no coalition exists that blocks them. As a result there are multiple equilibria, all of them sharing the same principal properties.

An additional interesting implication of Proposition 2 is that financial networks with smaller core sizes are equilibria for a wider range of parameters. It is so because when the core is smaller, each I bank in the core collects intermediation spreads from more lender NI banks, which in turn cover a higher expected cost of default.

The next proposition characterizes the efficient interbank network structure in this framework.

Proposition 3 [Efficient Interbank Network] *Assume $k_{NI} > k_I$. Every efficient network has only NI bank(s) as intermediaries. The following interbank network is efficient: a single NI bank, NI_c, borrows from every other NI bank, directly or indirectly, and lends directly to every I bank. Furthermore, every core-periphery equilibrium is inefficient.*

Let NI-star denote the constrained efficient interbank network in which one NI bank, NI_c, directly borrows from every other NI bank and lends to every I bank, depicted in Figure 1b. NI_c is shaded hatched red bank.

Recall that both the funding and the investment opportunities are dispersed throughout the financial network. As such, determining the constrained efficient network relies on the resolution of the *intermediation trade-off*. On the one hand, intermediation is necessary to channel all the funding from where they are raised to where the investment opportunities arise. On the other hand, every bank in an intermediation chain, including the intermediary(ies), is exposed to risk of failure if the ivestment fails. Thus intermediation has its costs and benefits, and the constrained efficient network is the structure that most effectively settles this trade-off.

The interbank network that resolves the intermediation trade-off has two defining features. First, there is a path from every source of funding to every investment opportunity, i.e. every NI bank is connected to every I bank. Second, no I bank intermediates in the efficient network, to avoid excessive defaults that are unnecessary.

The first feature simply maximizes the scale of investment, noting that direct lending is efficient. That is, it is optimal for every NI bank to be exposed to every I bank. To understand the second feature, it is useful to first divide the aggregate welfare into two components: the expected surplus from investment, and the expected cost of bank default due to investment risk. Recall that the investment opportunities are identical and constant return to scale, and all banks are risk neutral. Thus, among the network structures that achieve the same aggregate scale of investment, total expected surplus from the projects is independent of the distribution of surplus, or alternatively from the structure of intermediation. Put differently, keeping the scale of investment constant, who is an intermediary only translates into a change in the division of surplus in favor of intermediaries, without any aggregate implications for the total investment surplus. However, there is a cost associated with intermediation as well, namely the risk of default due to exposure to counterparty risk. Importantly, this cost is determined by who intermediates in the financial network. It follows that efficiency requires organizing intermediation to minimize the cost of default due to contagion without hurting the scale of investment.

As such, designating any of the NI banks as the intermediary simultaneously serves two purposes. First, the designated NI intermediary borrows from all the other NI banks, which enables it to credibly commit to lending to all I banks. Moreover, there is a path from every unit of funding to every investment opportunity, as every NI bank is connected to every I bank through the intermediary NI bank. Thus, the maximum scale of investment is achieved. Second, no extra cost of default is imposed on the interbank network. Thus the intermediation trade-off is resolved optimally, and efficiency is achieved.

It is worth pointing out that NI-star structure is an equilibrium network only if intermediation spreads are sufficiently low, a generalization of Proposition 1. Otherwise, all the I banks and and peripheral NI banks form a coalition together, and block NI-star structure from being an equilibrium. This observation is formalized in Proposition 5, which provides a set of sufficient conditions under which core-periphery equilibria are unique.

The analogous argument implies that every core-periphery network structure is inefficient, as it features I banks as intermediaries. It is efficient for an I bank to be exposed to default only if it undertakes the investment itself. However, in a core-periphery network structure, I banks intermediate only when they do not have an investment opportunity, and thus by intermediating expose themselves to excessive cost of default. In any such instance, replacing the intermediary I bank with any of its peripheral NI banks improves the welfare. This observation also implies that core-periphery equilibria with larger cores are more inefficient, as more I banks are exposed to the excessive cost of default.

Lastly, note that in the NI-star network, bank NI_c can be interpreted as a central clearing house, in that all of the lending goes through this particular bank. What makes the existence of the central clearing party (CCP) optimal is that it optimally channels all of the funding to all of the investment opportunities, while preventing the excessive exposure to counterparty risk in the core of the financial network simultaneously.

In the next two propositions I first provide a general existence result, and then characterize the set of parameters for which the core-periphery equilibrium family is unique, implying that every equilibrium is inefficient. I start with the existence result.

Proposition 4 **[Existence]** *An equilibrium exists.*

The proof of existence proceeds in two steps. First I show that taking network structure as given, for any resolution of uncertainty, the interbank repayments, characterized by the system of equations (2), has a unique solution (à la Acemoglu et al. (2015b)). Then I provide a constructive proof of the equilibrium structure in the network formation stage.

More interestingly, one can show that when the periphery is sufficiently large and the intermediation rents are sufficiently high, all equilibria have a core-periphery structure. The next proposition formalizes this result.

Proposition 5 **[Uniqueness of Core-Periphery Equilibrium]** *There exist constants $\bar{M}$ and $\bar{K}$ such that if $\kappa = \frac{\alpha(1-\alpha)(pR-1)}{(1-p)V_I} > \bar{M}$ and $k_{NI} > \bar{K}$, every equilibrium is a core-periphery financial network.*

To gain intuition, recall that for a given financial network, Lemma 1 bounds the length of any intermediation chain from above. If there are enough NI banks, this means that sufficiently many of them are in the extreme periphery, i.e. do not have any lenders themselves. In other words, the network has numerous "leaf" nodes. These leaf NI banks do not intermediate for any other banks and so do not collect any intermediation spread. Thus on the one hand, if these extreme peripheral banks do not directly lend to an I bank, they prefer to do so, in order to pay a lower intermediation spread. On the other hand, when intermediation spreads are sufficiently high, the I banks are happy to intermediate these peripheral banks and collect intermediation rents. Consequently, all sustainable equilibrium financial networks have a core-periphery structure, which do not allow for such deviation.

Thus under these conditions, all equilibrium financial networks have the common feature that I banks intermediate in them. As such, there is excessive exposure to counterparty risk and the financial network is inefficient. The next corollary formally states this result.

Corollary 1 *There exist pair of constant $\bar{M}$ and $\bar{K}$ such that if $\kappa = \frac{\alpha(1-\alpha)(pR-1)}{(1-p)V_I} > \bar{M}$ and $k_{NI} > \bar{K}$, every equilibrium is inefficient.*

To recapitulate, in this framework the equilibrium interbank network exhibits a core-periphery structure. The critical force behind this structure is intermediation. When the intermediation spreads are positive, banks' incentives to maximize their return implies that on one hand, they prefer to lend through the shortest possible intermediation chain to avoid paying rents. On the other hand, they would like to be the intermediary if necessary to absorb the intermediation rents. When these spreads are substantial, resolution of this trade-off leads to the emergence of a core-periphery network in equilibrium. In this equilibrium, banks who are able to pledge the highest return to their lender banks are in the core, i.e. banks who have investment opportunities.

From a social perspective, financial intermediation is beneficial as it allocates funds from parts of the financial sector with excess liquidity to parts with profitable investment opportunities. However, it can also be socially costly if banks expose themselves to counterparty risk excessively in order to capture intermediation spreads. I show that when intermediation spreads are high, the same rent-seeking behavior that leads to emergence of core-periphery equilibria, mis-aligns private and social incentives. It follows that the equilibrium is inefficient and features excessive systemic risk. The main source of inefficiency is that the gains from intermediation are purely redistributional, whereas the loss is incremental.

I call this an *upside externality*. This is in contrast to the *downside externality* emphasized in the existing literature on interbank interconnectedness, in which a bank does not internalize its negative impact on its counterparties when making its decisions. Furthermore, this explanation is distinct from the existing explanations such as bailouts and ignoring tail risk. These alternative forces amplify banks' incentives to expose themselves to their counterparties, but neither is necessary to explain the excessive exposure to counterparty risk, prevelantly observed in the financial crisis of 2008.

The final proposition extends the previous results beyond the division of surplus determined by α-rule. Consider $\mathcal{L}(.)$, a general rule for division of surplus. Recall that $\mathcal{L}(i_j; n, i_n \in \mathbb{I}_R)$ denotes the share of expected surplus accrued to the bank in position j of an intermediation chain of length n, where the surplus is generated by one unit of investment, financed by a unit of funding intermediated from initial lender i_1 to final borrower i_n. The following result generalizes Propositions 2-5 from α-rule to any general $\mathcal{L}(.)$ which satisfies the properties specified in Section 2.

Proposition 6 *Redefine* $\kappa = \frac{\mathcal{L}(i_2;3,i_3\in\mathbb{I}_R)}{(1-p)V_I}$. *Then Propositions 2-5 hold for any* $\mathcal{L}(.)$ *satisfying the conditions of Section 2.*

This generalization is intuitive. Among the rules for division of surplus that satisfy the conditions of Section 2, α-rule has two additional properties that make it analytically tractable. First, starting from the second bank in any intermediation chain, the bank's surplus is exponentially decreasing along the chain. Second, the final borrower's share is invariant to the length of the chain. Proposition 6 shows that none of these properties are crucial for Propositions 2-5. In particular, the invariance of the final borrower's share to the length of the chain, implied by α-rule, requires the tighter conditions in Proposition 5.

5.1 Robustness

The most fundamental results in this paper are Propositions 2 and 3, the emergence of core-periphery equilibria, and the disparity between equilibrium and efficient financial networks. To underscore the generality of these results, I next offer some discussion on the robustness of these results to my modeling assumptions.

Exogenous $\mathcal{L}(.)$ with positive intermediation spreads Strictly positive intermediation spreads are consistently documented in numerous interbank markets, see Li and Schürhoff (2019) for the municipal bond market, Di Maggio et al. (2017) for the corporate bond market, and Bech and Atalay (2010) for the federal funds market. An exogenous rule for division of surplus implies that banks can affect their return only through strategically positioning themselves in the network, which keeps the discussion focused on the role of intermediation in the formation of the financial network. However, the predictions of the model are consistent with other empirical evidence about interbank rates of return as well. Di Maggio et al. (2017) document that in the inter-dealer market for corporate bonds, core dealers charge higher prices to the periphery than to other core dealers on average, consistent with the pricing implications of the current model. Furthermore, in Online Appendix A, I derive a rule for division of surplus from first principles, in a restricted version of the model. This microfoundation generates α-rule endogenously, and establishes the robustness of the main findings to endogenous determination of price in the interbank market.

Diversification In order to focus on the role of intermediation on formation of financial networks, I have assumed diversification away, ensured by Assumption 1. I have made this choice because diversification is a well studied mechanism in the financial sector. In Online Appendix B, I allow for diversification in a restricted version of the model to establish the robustness of the main findings of the paper with respect to this assumption.

Asymmetry between I and NI banks I have assumed that I banks raise no funding from the households. This assumption hints to an asymmetry between I and NI banks. It is straightforward to see that the structure of the core-periphery equilibrium is insensitive to this assumption. The efficiency result of the model is robust to weakening this assumption, in order to reduce the asymmetry between I and NI banks. In particular, allow each I bank to raise ϵ funding from households, where ϵ is such that $\epsilon\alpha(pR-1) < (1-p)V_I$. This implies that a unit of investment at bank I', financed by a unit of funding raised at I and lent directly to I' over the edge $e_{II'}$, is not efficient. In other words, it is efficient for each I bank to invest the funding that it raises from households only in its own investment opportunity. As such, the structure of efficient interbank networks remains intact as well.

There is an alternative assumption to make I and NI banks more symmetric, namely that NI banks probabilistically get access to household funding. Online Appendix C shows that the implications of the model are robust to this alternative assumption as well.

Precautionary saving I have assumed that banks do not hold any precautionary saving. Note that in this paper banks are profit maximizers and they do not maximize household utility. They borrow from the households using debt contracts, at an interest rate for which households break even. Thus banks make positive profits only when the investment upside is realized. This naturally leads to under-insurance and inefficiently low precautionary saving, which makes this assumption natural. It is different from Allen and Gale (2000) as in that framework, banks maximize household utility. In Online Appendix D, I allow for precautionary saving in a restricted version of the model, to establish the robustness of the main findings of the paper with respect to this assumption.

Exante lending commitment I have assumed that banks are restricted to borrow and lend over their exante interbank agreements and cannot spontaneously establish new interbank interconnections and reallocate their funding at $t = 1$. This assumption implies that intermediation is necessary to channel funding to investment opportunities. The persistent nature of interbank connections assumed here is consistent with the evidence suggesting that banks interacts through long term relationships. Afonso et al. (2013) document that in the federal funds market, approximately 60% of the funding an individual bank borrows in one month persistently comes from the same lender. Di Maggio et al. (2017) find that in the inter-dealer market, banks with longer term relationships get access to better terms.

If I allow banks to establish new interbank connections at no cost, a bank with funding can always lend directly to a bank with a realized investment opportunity and so the motive for intermediation disappears. Thus some irreversibility in interbank connections is important for my results. But as long as changing the interbank connections incurs an irreversible cost, the current connections are relevant for future lending opportunities and so some lending will involve intermediation. And because there is intermediation, a core-periphery network structure arises in equilibrium.

Liquidity risk I have abstracted away from liquidity risk by assuming that lenders have commitment power and do not default on their funding promises. As a result, contagion spreads only from borrowers to lenders in my model. An interesting extension of the model is to allow lenders to commit liquidity to several borrowers and default on some of their commitments if many borrowers demand liquidity at once, at some cost. This extension enriches the model and opens the possibility of contagion from lenders to borrowers.

6 Policy Implications

In this section, I use the model to investigate a number of policies that target the financial sector. The following proposition provides a set of comparative statistics results that are useful to discuss the implications of different policies on the network structure.

Proposition 7 *An increase in the investment return and probability of success, as well as a decrease in the cost of default born by the I banks, weakly increases the size of the core and the expected number of defaults in the family of core-periphery equilibria. It can also make the equilibrium more inefficient.*

Proposition 7 demonstrates that this framework is well-suited to study the welfare effects of bailouts. In the context of the model, bailout can be interpreted as a wedge between the true V_i, and the loss borne by the bank when it defaults. In other words, a bailout leaves the social cost of bank failure unchanged, at V_i, while the bank only bears βV_i, for some $\beta < 1$, and the difference $(1-\beta)V_i$ is borne by the government.

Let $\mathbb{C}$ denote the core of the interbank network. First, note that the size of inefficiency in a core-periphery equilibrium is uniquely determined by the expected number of defaults by I banks, which is in turn determined by the size of the core. However, the commonly used measure of bailout cost in the literature, $\sum_{i \in \mathbb{C}}(1-\beta)V_i$, overlooks the enodogeneity of the financial network. Proposition 7 shows that a bailout policy weakly enlarges the core of the financial sector, as banks optimally change their interconnections in expectation of being bailed out and expose themselves to more counterparty risk. The reason is that lower cost of default implies that less intermediation rent is required to justify the risky interconnections. This in turn implies that not only more banks with cost of default similar to V_i choose to take on the risk of intermediation, but also banks with larger default costs choose to excessively expose themselves to counterparty risk by intermediating, and join the core of the financial network. The crucial observation is that absent a bailout, these banks would not take this extra risk because their opportunity cost was too large. A bailout decreases the opportunity cost of default by shifting part of it to the government, inducing more risk-taking behavior by banks and elevating the level of systemic risk in the financial sector.

Put differently, taking banks' endogenous choice of interconnections into account uncovers an important amplification effect associated with bailouts. Expectation of a bailout not only makes the highly interconnected core of the financial sector larger, but also encourage banks with larger default costs to join the core. As such, in a downturn more (and larger) financial institutions are at risk, which worsens the systemic risk and deepens the financial crisis. This implies that once the endogenous choice of bank interconnections is taken into account, a bailout policy always leads to emergence of equilibiria that are (weakly) more inefficient than the equilibria with the old interbank structure. Finally, a higher degree of correlation among risky investment opportunities further amplifies this effect.

Second, the model provides a new rationale for introduction of a Central Clearing Party (CCP), different from those identified by Duffie and Zhu (2011) and Bond (2004). Designating a non-investing bank as the CCP and enforcing all the lendings to go through the CCP prevents excessive bilateral exposure among banks and enhances welfare particularly as investment returns become more correlated. The model predicts that such a structure is not an equilibrium when the intermediation spreads are sufficiently high, so intervention is necessary to implement it.

Third, consider Title VII of the Dodd-Frank Wall Street Reform and Consumer Protection Act, a proposed cap on the number of counterparties and swaps, which was later eliminated from the finalized rules.[19] The model suggests that this policy is likely inefficient. It either leads to under-investment, or encourages financial networks with larger cores which involve higher systemic risk. In the context of the model, network structures that allow the optimal scale of investment without entailing an excessive risk of failure, require intermediaries with many connections, prohibited by this policy. On the other hand, a cap on the number of bank interconnections shifts the family of core-periphery equilibria toward the ones with larger cores, which in turn increases the aggregate cost in the event of failure, particularly when investment outcomes are highly correlated.

Finally, the model implies that higher probability of success of risky investment both increase the intermediation spread and decrease the expected cost of default, because investments fail less frequently. Both channels give rise to financial networks with larger cores, which are more inefficient, and thus lead to arbitrarily large expost realized losses in a downturn. This mechanism is even more salient when the investment outcomes are more correlated. With this interpretation, during the run-up to the financial crisis of 2008, the large financial institutions did not ignore the tail risk. Alternatively, they voluntarily exposed themselves to the tail risk to capture intermediation spreads.

[19] See CFTC/SEC (2012) and Stroock Special Bulletin for more detail.

7 Conclusion

I develop a model of the financial sector in which endogenous intermediation among debt financed banks gives rise to a *core-periphery network* – few highly interconnected and many sparsely connected banks. The central feature of the model is that financial institutions have incentives to capture intermediation spreads through strategic borrowing and lending decisions. By doing so, they tilt the division of surplus along an intermediation chain in their favor, while at the same time reducing aggregate surplus. As such, endogenous intermediation generates excessive systemic risk. The network is inefficient relative to a constrained efficient benchmark since banks who make risky investments "overconnect", exposing themselves to excessive counterparty risk, while banks who mainly provide funding end up with too few connections.

The paper shows that explicitly modeling the interaction between banks' rent seeking behavior and intermediation, which is necessary to allocate liquidity within the financial system, jointly explains multiple prominent stylized features of the financial networks: the global core-periphery structure of the interbank networks, interbank interconnectedness, and gross and net exposures among financial institutions. Moreover, by providing sharp predictions about the sources of inefficiency in the interbank network, the paper contributes to the heated policy debate on how to regulate financial markets.

Finally, the model can be extended to incorporate other interesting aspects of the financial system. A fruitful avenue of future research is to incorporate liquidity risk, and how it can lead to unraveling of the interbank network. Moreover, one can think of specialization by banks in the context of the model. Whether banks should specialize, and if so in which activities, has long been a topic of debate among economists. The current model cannot answer these questions because it takes the existence of different types of banks, and their numbers, as given. Assessing the changes in positive and normative trade-offs as a result of endogenous bank specialization is worth further investigation.

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Appendix

A Proofs

I first prove the auxiliary Lemma A.1 below to show that the network structures depicted in Figure 3 are the only possible equilibria of the economy with four banks. I will then use Lemma A.1 to prove proposition 1.

Lemma A.1 *Network structures depicted in Figure 3 are the only possible equilibria with four banks.*

Proof. Any structure in which an NI does not lend to any other bank is trivially not an equilibrium. Aside from those, all the feasible structures with four banks are depicted in A.1. Each structure consists of the four banks and credit lines among them depicted in black.

Finally, the deviations which rule out the other structures (A.1d, A.1g and A.1h) are depicted as red or crossed out edges. For instance in A.1h, NI_1 has two units pledged to him but is only lending to a single I bank. NI_1 and I_2 strictly prefer to jointly deviate together. NI_1 saves on the intermediation rent payed to I_1 when only I_2 has an investment opportunity, while post deviation I_2 gets to invest 50% of time when both I_1 and I_2 get the investment opportunity and prior to deviation I_2 would not invest.[20] $e_{I_1 I_2}$ is removed since nothing is ever lent over that credit line and we move from A.1h to A.1a.

In A.1c, adding the $e_{I_1 I_2}$ and $e_{I_2 I_1}$ is not always a viable deviation because if $\alpha(1-\alpha)X < (1-p)V_I$, in the resulting network, lending over $e_{I_i I_j}$ always violates the participation constraint of I_i, so it would happen with probability zero. So this is not a valid coalitional deviation and A.1c is a possible equilibrium. ■

Proposition 1.

Equilibrium Let $\kappa = \frac{\alpha(1-\alpha)}{(1-p)V_I}$ and $\tilde{\kappa} = \max\{\alpha, (1-\alpha)\frac{V_{NI}}{V_I}\}$. The participation constraints of the direct lender and direct borrower banks jointly imply that $\kappa \geq \tilde{\kappa}$ is the relevant range of parameters. Assume the economy is in Figure 3e. The face values of debt are set as

[20] If the bargaining rule is such that both final lender and initial borrower save on intermediation rents when an intermediary is removed the second part of argument is redundant as I_2 also saves on intermediation rents when only he gets the investment opportunity and lending goes through I_1. However, in α-rule borrower does not care for the source of funds so the second part of argument is necessary.

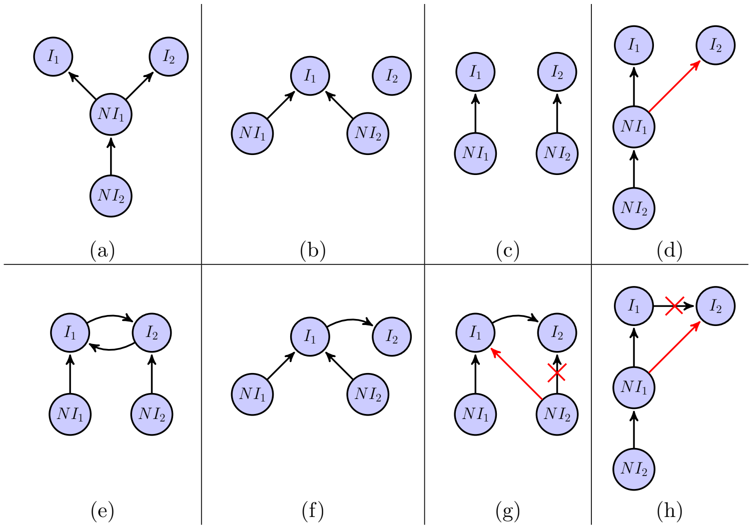


Figure A.1: Feasible lending structures for an economy with two I and two NI banks. The black edges are in the feasible structure. The red and crossed-out edges are the deviations which rule out each particular structure as an equilibrium.

explained in Section 3.1. In expectation, an I bank and NI_2 get the following, respectively:

$$\mathcal{V}_I^e = (1-q)^2 V_I + q^2[p(V_I + R - D)] + q(1-q)[p(V_I + 2(R-D))] + (1-q)q[p(V_I + D - D_1)]$$
$$\mathcal{V}_{NI_2}^e = (1-q)^2 V_{NI} + q^2[p(V_{NI} + D) - 1] + q(1-q)[p(V_{NI} + D) - 1] + q(1-q)[p(V_{NI} + D_1) - 1]$$

Now consider 3a:

$$\mathcal{V}_I^a = (1-q)^2 V_I + q^2\big[\frac{1}{2}V_I + \frac{1}{2}[p(V_I + 2(R-D))]\big] + q(1-q)[p(V_I + 2(R-D))] + (1-q)qV_I$$
$$\mathcal{V}_{NI_2}^a = (1-q)^2 V_{NI} + q^2[p(V_{NI} + D_1)) - 1] + 2(1-q)q[p(V_{NI} + D_1) - 1]$$

where I have substituted D for D_2. Note that $D - D_1 = \alpha(1-\alpha)X$ and it represents the intermediation spread. Substitute D and D_1 and compare what either bank gets in 3a and 3e to see that NI_2 always prefers to deviate to 3e while I bank would deviate if:

$$\frac{\alpha(1-\alpha)X}{(1-p)V_I} > 1 + \frac{q}{2(1-q)}.$$

Let $\bar{\kappa} = 1 + \frac{q}{2(1-q)}$ and take the joint deviation of the two I banks along with NI_2 to see that 3a is not an equilibrium if $\kappa > \bar{\kappa}$. Is 3e an equilibrium when $\kappa < \bar{\kappa}$? Counter intuitively, the answer is yes. Although both I bank prefer to deviate back to 3a, they need both NI banks to join the deviation and no NI bank agrees to be a leaf who is always intermediated, when in the current structure he gets to lend anytime there is an investment opportunity and with positive probability he gets un-intermediated rent. 3e ceases to be an equilibrium when intermediation rents do not cover the cost of default anymore and each I_i would prefer to unilaterally break $e_{I_iI_j}$ link. This happens when $\kappa < 1$. Finally, is 3e an equilibrium if $\kappa > \bar{\kappa}$? Yes since none of the I bank can improve on what either NI bank gets in this structure, so there is no way to convince NI banks to join any deviation.

Now assume the economy is in 3d. I_1, I_2 and each NI bank receive:

$$\begin{aligned}
\mathcal{V}^d_{I_1} &= (1-q)^2 V_I + q^2[p(V_I + 2(R-D))] + q(1-q)[p(V_I + 2(R-D))] \\
&\quad + (1-q)q[p(V_I + 2(D-D_1))] \\
\mathcal{V}^d_{I_2} &= (1-q(1-q))V_I + q(1-q)[p(V_I + 2(R-D))] \\
\mathcal{V}^d_{NI} &= (1-q)^2 V_{NI} + (q^2 + q(1-q)[p(V_{NI} + D_1)) - 1] + (1-q)q[p(V_{NI} + D_1) - 1]
\end{aligned}$$

In 3b I_1 and each NI get:

$$\begin{aligned}
\mathcal{V}^b_{I_1} &= (1-q)V_I + q[p(V_I + 2(R-D))] \\
\mathcal{V}^b_{NI} &= (1-q)V_{NI} + q[p(V_{NI} + D) - 1]
\end{aligned}$$

Although NI does not want to deviate from 3b to 3d but I_1 will unilaterally deviate and break $e_{I_1I_2}$ link if that increases its expected profit, which happens if $\kappa < \frac{1}{2}$.

Next consider 3b. Two type of deviations are perceivable: first, the two I banks jointly deviate and add $e_{I_1I_2}$, which happens when $\kappa > \frac{1}{2}$.[21] Thus, for 3b to ever be an equilibrium it should be that $\tilde{\kappa} < \frac{1}{2}$, so $\alpha < \frac{1}{2}$ and $(1-\alpha)\frac{V_{NI}}{V_I} < \frac{1}{2}$ which implies $V_{NI} < V_I$. A second possible deviation is for the two NI banks to jointly deviate with I_2 to go to 3a. This deviation requires NI_2 to be better off in 3a. A necessary condition is $\alpha > \frac{1}{2-q} > \frac{1}{2}$, which in turn implies that $\underline{\kappa} > \frac{1}{2}$ and 3b does not exist. As such, similar to the baseline economy, 3b is an equilibrium at most in the range $\underline{\kappa} \leq \kappa < \frac{1}{2}$.

Finally, consider 3c. The difference with 3b is that now both banks lose scale when they have an investment opportunity, and they could exante be better off adding $e_{I_1I_2}$ and $e_{I_2I_1}$ even when $\kappa < 1$. However, this is not a viable deviation when $\kappa < 1$, because in the interim period, when only investment opportunity i is realized, lending over $e_{I_jI_i}$ violated the participation constraint of I_j and will not happen, so $e_{I_jI_i}$ is never traversed and the above is not a viable deviation when $\kappa < 1$. The second candidate deviation in case of 3b is ruled out by the same argument as above. There is a third possible deviation: NI_2, I_2 and I_1 jointly deviate, break $e_{NI_2I_2}$, and add $e_{NI_2I_1}$ and $e_{I_1I_2}$. The first necessary condition is that adding

[21] Deviating to 3e is also possible but the former deviation is viable whenever the latter is, so there is no need to consider the latter.

$e_{I_1 I_2}$ must be a viable deviation, which requires $\kappa < \frac{1}{2}$. If so, I_1 and NI_2 gain. I_2 incentives are ambiguous because in 3a he does not get to invest when I_1 get an opportunity, but gets to invest 2 units when I_1 does not. I_2 bank's value of being in 3c is

$$\mathcal{V}^c_{I_2} = (1-q)V_I + q[p(V_I + (R-D))]$$

For the latter deviation to be viable, it must be that $\mathcal{V}^c_{I_2} < \mathcal{V}^e_{I_2}$, which holds if $q < \bar{q} = \left(2 - \frac{(1-p)V_I}{(1-\alpha)X}\right)^{-1}$. Note that $\frac{1}{2} < \bar{q} < 1$.

Efficiency Network structures 3a, 3d, and 3e all attain the same scale of investment, while the expected cost of bank default is lower in 3a. Thus among these three structures, 3a is the most efficient, and the other two equilibria exhibit over-exposure and excessive cost of default. Network structures 3b and 3c reach a lower scale of investment compared to 3a, and in return to each lower unit of investment, i.e. a surplus loss of $pR-1$, they gain at most $(1-p)V_{NI}$. The latter is smaller than the former, thus network structure 3a is also more efficient than 3b and 3c, and the latter two exhibit under-exposure and too little risk-taking by banks.

Lastly, I need to prove that there is no feasible network structure that is more efficient than 3a. Every investment opportunity receives 2 units of financing, so the maximum scale of investment is achieved in 3a. Furthermore, since one of the NI banks is the intermediary, investing the funding of each NI bank at every investment opportunity only exposes that NI bank to the cost of default contagion, and each direct lending is efficient. Thus it is not possible to improve the expected surplus net of cost of default relative to 3a. ■

Lemma 1. Consider a bank b who is the leaf lender along the longest possible intermediation chain of the network with probability non zero, noting that b can lend over shorter paths to other banks I as well. Let $n_{\max}$ denote the length of this chain.

Bank b has a single unit of funding, raised from the households, as it is a leaf bank. There is no diversifiation so if the ultimate borrower I fails every bank who has lent to him through any chain fails. As a result when bank NI lends directly or to indirectly to a bank I then he fails with probability $(1-p)$ regardless of the length of the intermediation chain. However, when he lends through its longest chain of length $n_{\max}$ in expectation he gets $\alpha^{n_{\max}-1}X$ which has to cover $(1-p)V_{NI}$. Un the other hand, a longer chain should not be feasible, thus $\alpha^{n_{\max}}X < (1-p)V_{NI}$. As $\alpha < 1$, $\log\alpha < 0$. This implies

$$\frac{1}{|\log\alpha|}\log\frac{(1-p)V_{NI}}{pR-1} < n_{\max} \leq \frac{1}{|\log\alpha|}\log\frac{(1-p)V_{NI}}{pR-1} + 1.$$

■

Proof of Proposition 2. I will show that there is no feasible deviation for the relevant set of parameters. Let $\mathbb{C}(G)$ and s denote the core and the size of the core, respectively, so there are s I banks in $\mathbb{C}(G)$ and $k_I - s$ out of the core. First consider the unilateral deviation of $I_1 \in \mathbb{C}(G)$. Note that with sufficiently many peripheries (as described in the statement of the proposition), if an I lends to one other I he would lend to as many I's as he can, since everything is linear; and similarly if he drops a lending he drops every lending. So I_1's relevant unilateral deviation is to drop all of its links to I banks and stop intermediating. That is the case if intermediation rents that I_1 captures is not sufficient to cover its cost of default. With a core of size s, the division of peripheries which maximizes the profit of the worst-off member of the core is the equal division of NI peripheries, so that each $I \in \mathbb{C}(G)$ gets $\frac{k_{NI}}{s}$ lending to him. So I_1 deviates if $\frac{k_{NI}}{s}\alpha(1-\alpha)X < (1-p)V_I$ which determines a lower bound on κ: $M_s = \frac{s}{k_{NI}}$.

Next, consider other possible deviations. The first coalition consists of only $I \in \mathbb{C}(G)$. Each I who is in the core has maximum possible lending relationships so I's at the core can not form a blocking coalition alone. Second, there can be a coalition of a (proper) subset of I's in the core $\mathbb{C}_D$, and NI banks lending to $I \in \mathbb{C}(G) \setminus \mathbb{C}_D$. In the current network, every NI gets an expected return of αX with probability q and $\alpha^2 X$ with probability $(1-q)(1-(1-q)^{k_I-1})$, and every single lending generates positive expected profits net of cost of default, so this is the maximum possible expected profit any bank can get without having any funds pledged from the interbank network. Simply becoming a periphery to a different core bank does not increase this payoff, so this is not a valid blocking deviation either.

Third, can a combination of I's outside the core and NI's form a profitable deviation? With the exact same argument as the last paragraph there is no such feasible deviation because it is not possible to make any NI_j better off than what they are without making some NI_k worse off (peripheral to NI_j). In this case, it is not even possible to make them as well of as before because the $I \in \mathbb{C}(G)$ bank(s) whose peripheral NI's are part of the suggested deviation never agree to join the deviation and add links to borrow from the I banks who are part of the suggested deviation (currently out of the core). So NI banks who join such deviation would get intermediated spreads strictly less often that current structure (and the exact same unintermediated spreads), so they would be strictly worse off.

Forth, can $I \notin \mathbb{C}(G)$ deviate alone? It cannot add any links, and only loses by severing links, so there is no such deviation either.

Finally, can (a subset of) NI's jointly deviate without any I's in the coalition? Again the answer is no, for the following reason: Any such deviation implies that there is some NI

at distance 2 to its closest I bank without any improvement in probability of being involved in the investment opportunity, which will be rejected by that NI.

The converse is simple. Assume $\kappa < M_s$. Then $\frac{k_{NI}}{s}\alpha(1-\alpha)X < (1-p)V_I$. Moreover, in any s-core network, at least one of $I \in \mathbb{C}(G)$ has $\frac{k_{NI}}{s}$ or less peripheries. This I bank would unilaterally deviate and severe all its potential lending contracts to all other I banks and strictly increase its expected surplus. ■

Proof of Proposition 3.

Efficiency First note that in this structure feasibility as well as the participation constraint of every bank are satisfied. Regardless of which bank receives the investment opportunity, all the funding will be channeled to some investment opportunity. Moreover, since every NI bank is lending to all I banks only through the same common intermediary, maximal concentration of risk is achieved. In other words, when multiple I banks receive investment opportunities, one and only one of them invests, which given the no diversification assumption 1 improves the welfare, since it concentrates risk as much as possible and saves on expected cost of default of some I's, while reaching the same scale of investment. Finally, for any realization of investment opportunities, aside form the single I bank who does the investment, every other bank with a realized lending and/or borrowing relationship provides funding for the investment, so removing him from the set of active lenders decreases the scale of investment by one while also decreasing the expected cost of default by $(1-p)V_{NI}$. As a project financed by a direct unit lending is efficient, the former is larger and this removal will be welfare destroying.

Core-periphery equilibrium inefficient Take any core-periphery network structure G. Recall that each I bank has at least k_I peripheral NI banks. Construct network G' from network G in two steps. Take and I bank, I_1, and one of its peripheral lenders, NI_1. First, replace I_1 with NI_1. The resulting intermediate network remains feasible. Second, add the edge $E_{NI_1 I_1}$ to the intermediate network, to get G'. G' is still feasible, as NI_1 has k_I lending commitments and at least k_I units of funding secured on the interbank network. G' reaches the exact scale of investment as G as there is a path from every unit of funding to every investment opportunity in G'. Furthermore, it has strictly less expected cost of default. It is so because in G, whenever $I_1 \notin \mathbb{I}_R$, I_1 intermediates and fail with probability $1-p$, which entailed an expected cost of default of $(1-p)V_I$. In G', I_1 does not participate in the process of investment when $I_1 \notin \mathbb{I}_R$, so its expected cost of default is saved. Thus G' is more efficient on G, i.e. G was inefficient.

■

Proposition 4. The proof is done in two steps. First I show that given any network G, realizations $\mathbb{I}_R$, and $\{R_k\}_{k\in\mathbb{I}_R}$, and face values of debt $\{D_{ij}\}_{i,j\in\mathbb{N}}$ and $\{D_i^h\}_{i\in\mathbb{N}_F}$ set at date $t = 1$, the system of interbank repayments (2) has a unique solution. This part of the proof is very similar to that of Acemoglu et al. (2015b), proposition 1. The proof proceeds in multiple steps. First define the total liabilities of bank i to bank j by multiplying the per-unit payment by number of units lent and then define the share of each bank j in bank i liabilities. Then I define an appropriate mapping function $\Phi(.)$ which maps the min of partial and full payments to itself. It is straight forward to show that this mapping is a contraction which maps a convex and compact subset of Euclidean space to tself. As a result by Brouwer fixed point theorem, this contraction mapping has a fixed point which is the set of feasible interbank face values of debt and their relevant partial payments. For detail of generic uniqueness see Acemoglu et al. (2015b).

Next, I focus on network formation stage, and show that (at least) one of following three networks is an equilibrium for any parameter set: smallest member of the core-periphery family (single-I-core network), the star structure with an NI core (NI-star network), or a structure where every NI banks lend to a (potentially multiple) I bank(s) but I banks are not connected to each other (island network). Assume the NI-star is not an equilibrium. Either k_{NI} times intermediation spread is larger than $(1-p)V_I$ (case 1) or it is smaller (case 2). The single-I-core is an equilibrium in case 1 (proof of Proposition 2). Now consider case 2. Since NI-star is not an equilibrium, there is a coalitional deviation to block it. The deviation cannot be only breaking links since every banks is getting strictly positive expected net surplus from every transaction at $t = 1$, and solely breaking the link gives it zero net surplus. So the deviation involves adding links. For a peripheral NI to deviate, he needs to get strictly closer than one intermediary away, to at least one I bank, as in NI-star he is one-intermediary away from every I. So any deviation requires (at least) adding a link between a peripheral NI_j and one of the I banks, I_i.

Consider a potential deviation which is only NI_j breaking its link from the core NI and adding a link to I_i. In this deviation, NI_j trades off the spread he had to always pay the core NI with the lower probability of getting it only when I_i receives an investment opportunity. There are two possible cases: when this deviation is profitable for NI_j (case 2-1) and when it is not (case 2-2). First consider the former. Assume we start in the island network where every NI bank lends to I_i (single island). As we are in case 2-1, NI banks have no incentive to deviate and become peripheral to one of the NIs, and (at best) create the NI-star network in order to get the lower, intermediated rate of return, more often. I_i

has no incentive to start intermediating as we are in case 2. The only remaining deviation is if (a subset of) other I's deviate with (a subset of) NI's and create a multi-core structure where I_i is completely left out. Note that I_i would not agree to be part of any deviating coalition. In the current structure he gets all the funding when he has a project and he is not willing to intermediate, so he cannot be better off than what he is in any other network). This structure is preferred by NI's because they get the same high rate that they get in the single island, plus they sometimes get an intermediated rate of return, so they would be willing to join such deviation. However, any link between two I banks, $e_{I_l I_k}$, will never be traversed because it is not individually rational for an I bank (I_j) to intermediate, which rules out this latter class of deviations. So the single-island network is an equilibrium in case 2-1.

Finally consider case 2-2. NI_j is only willing to deviate if he becomes peripheral to I_i who himself has a potential lending relationship to at lease some other I_j. However by the exact same argument as above, such deviations are rules out because traversing any link $e_{I_i I_k}$ violates individual rationality of I_i with probability one, so such links cannot be added in a coalitional deviation. So no NI_j bank would ever join a coalition, case 2-2 never happens, and NI-star is an equilibrium itself, which completes the existence proof.

■

Proof of Proposition 5.

The proof uses Lemma 1 and constructs a blocking deviation for any network which is not core-periphery.

First, assume $\bar{M}$ is sufficiently large for an I bank to be be willing to intermediate for any number of other I banks.

Second, Lemma 1 shows that the length ff any intermediation chain in financial network G is bounded by $n_{\max}$. Thus for any $\bar{k}_{NI}$, we can choose a sufficiently large $\bar{K}$ such that the number of leaf NI banks exceeds $\bar{k}_{NI}$.

Consider a bank NI_1 who lends directly or indirectly to bank I_1. Note that for I_1 the intermediation chain through which he borrows from NI_1 is irrelevant.

Next, let $\mathbb{I}_c$ denote the set of all I banks who lend to every $I \in \mathbb{I}_R$, i.e. the edge between them is traversed with positive probability.

Case I $\mathbb{I}_c = \emptyset$.

Assume $\bar{K}$ is sufficiently large such that $\bar{k}_{NI} \geq k_I^2$. Consider the following joint deviation by at least k_I^2 of leaf NI bank and every I bank: k_I leaf NI banks connect to each I bank, and edge $e_{I_1 I_2}$ is established $\forall I_1, I_2 \in \mathbb{I}_R$. The resulting network is feasible. Every NI bank is intermediated to every I bank, thus profits of no I bank decreases. Furthermore, no I bank was intermediating before and each of them do intermediate now, so their expected net profits have strictly increases. Finally, the leaf NI banks lend directly to one I bank. Thus they pay less expected intermediation spreads while still lending to every investment opportunity, so they are better off too. As such, this is a blocking coalition and G is not an equilibrium.

Case II $\mathbb{I}_c \neq \emptyset$

case II.1 $\exists NI_a$ bank who lends to $I \notin \mathbb{I}_c$.

There is a profitable deviation by NI_a and some $I_c \in \mathbb{I}_c$ by adding $e_{NI_a I_c}$. The resulting network is feasible and both NI_a and I_c get strictly more expected net profit. Thus this deviation blocks G and G is not an equilibrium.

case II.2 $\forall NI$ lends to some $I_c \in \mathbb{I}_c$.

In this case, for any $\hat{I} \notin \mathbb{I}_c$ and any $I_c \in \mathbb{I}_c$, the edge $e_{\hat{I} I_c}$ does not exist. In other words, $\hat{I} \notin \mathbb{I}_c$ does not lend to any bank and borrows from every $I_c \in \mathbb{I}_c$.

case II.2.a $\forall NI$ bank lends directly or indirectly, through a chain consisting of only NI bank, to a single $I_c \in \mathbb{I}_c$. Then network G is a core-periphery network.

case II.2.b $\exists NI^*$ bank who intermediates funds to more than one I_c bank, i.e. lends directly or indirectly, through a chain consisting of only NI banks, to at least 2 banks $I_c \in \mathbb{I}_c$.

First note that it is impossible for every unit of funding in the financial network to be intermediated to every I in every state through an intermediation chain of only NI bank, as that would imply $\mathbb{I}_c = \emptyset$.

Consider any such NI^* bank. Let NI_1 and NI_2 denote a pair of the leaf NI banks who lend directly or indirectly to NI^*, and I_{c1}, I_{c2} two of banks $I_c \in \mathbb{I}_c$ who NI^* was intermediating to. Consider the joint deviation of $\{I_{c1}, I_{c2}, NI_1, NI_2\}$ which consists of removing the NI_1 and NI_2 edges in the NI^* branch, and establishing edges $e_{NI_1 I_{c1}}$ and $e_{NI_2 I_{c2}}$. The resulting network is feasible. Both I_{c1} and I_{c2} are strictly better off as they are both on the shortest path to each other for one unit of funding where they were not before, so they both capture intermediation spreads. NI_1 and NI_2 are both strictly better off as they avoid paying intermediation spreads more often. Finally, it is possible that NI^* has to severe one of the edges in which he was a lender. This does not hurt neither I_{c1} nor I_{c2} as NI^* used to be connected to set $\mathbb{I}_c$ at least through two paths, thus he still is through at least one path, and $\mathbb{I}_c$ is a complete digraph thus neither I_{c1} nor I_{c2} lose any of their lenders. Thus this is a joint profitable deviation, set $\{I_{c1}, I_{c2}, NI_1, NI_2\}$ is a blocking coalition, and G is not an equilibrium.

■

Proof of Corollary 1.

The corollary directly follows from Propositions 3 and 5, as every core-periphery interbank network is inefficient.

■

Online Appendix

In the Online Appendix I use a restricted version of the framework to show that the predictions of the model are robust to a number of alternative assumption. Section G includes the proofs of the results in this appendix.

A Endogenous Rule for Division of Surplus

This section provides a microfoundation for α-rule, which is an example of a rule for division of suprlus with strictly positive intermediation spreads. This microfoundation enables the interbank network structure and interbank returns to be jointly determined endogenously.

Instead of assuming that there is an exogenous rule for division of surplus, assume that every bank in the financial sector faces a moral hazard problem. In particular, if bank i lends L units to bank j, bank j can appropriate a fraction $1-\alpha$ of the loan costlessly. Thus a borrower can only credibly commit to repay the proceeds on a loan size αL to the lender.[22]

As such, from the expected surplus generated by each unit of a bilateral loan, fraction $1-\alpha$ and α accrues to the borrower and the lender, respectively. As such, this moral hazard friction endogenously implies a division of surplus identical to α-rule. Furthermore, it implies that there is no room for any credible price renegotiation, as borrowers lack the commitment not to appropriate a fraction $1-\alpha$ of the loan.

B Diversification

In this section I relax assumption 1 to allow banks to hold diversified portfolios, and study the equilibrium structures. I find that the same structure of equilibria emerges, albeit with a twist. I focus on an economy with two I banks, $k_I = 2$, and k_{NI} NI banks, $k_{NI} > 4$. Restricting the number of I banks keeps the problem tractable while incorporating the main intuition associated with diversification.

I make the following bargaining assumption:

Assumption B.1 *Consider a realization of $\mathbb{I}_R$. If bank b has access to multiple $I \in \mathbb{I}_R$ through intermediation chains of different lengths, it can use the shortest chain to bargain its share in other chains up to what he gets in the shortest one. b's (direct and indirect) borrowers in each longer chain divide the remaining share pro-rata.*

[22] There are a number of different settings that generate this moral hazard friction. One is the presence of an outside option, an alternative investment opportunity, available to every bank within the financial sector, with a return that is fraction α of the return to the investment opportunity available to I banks.

Consider the following simple structure. $NI_0 \to NI_1 \to I_1$, and $NI_1 \to NI_2 \to I_2$. When both I banks have investment opportunities, NI_1 has direct access to one and indirect access to the other. The above assumption says that NI_1 can bargain up its share in the chain $NI_1 \to NI_2 \to I_2$ to α. I_2 and NI_2 divide the remaining $(1-\alpha)$ share with proportions $\frac{1}{1+\alpha}$ and $\frac{\alpha}{1+\alpha}$, respectively.[23]

The above assumption has an important implication for behavior of banks. It implies that all else equal, between two intermediaries, i cannot be worse off if the intermediary to which it lends is connected to an extra I banks, even if through longer chains. The following lemma formalizes this intuition.

Lemma B.1 **[Dominance]** *Consider two banks j_1 and j_2. Let $SPL_i = \{l_1^i, l_2^i \cdots, l_{z_i}^i\}$ be the set whose elements are lengths of paths in $SP(j_i, \mathbb{I})$, $i = 1, 2$. Assume elements of each set are sorted in increasing order. Also, without loss of generality, assume j_1 has more shortest paths to $\mathbb{I}_R$, $z_1 > z_2$. A leaf bank b prefers to lend to j_1 if*

$$\forall\, k \leq z_2 : \; l_k^1 \leq l_k^2$$

independent of l_k^1 for $k > z_2$.

Assume parameters are such that, absent diversification, an I bank chooses to intermediate (even) with a single peripheral lender. Consider the 2-I core-periphery structure that is an equilibrium without diversification. Assume each I_i has credit lines from Y_i of NI banks, where $Y_1 + Y_2 = k_{NI}$.

Consider the date $t = 1$ event where both I_1 and I_2 have investment opportunities (probability q^2). As described in section 4, I_i lends $\frac{Y_i}{2}$ to I_j. Let D_{ii} denote the face value of debt promised by I_i to each of its NI lenders. Moreover, let D_{ij} denote the face value of the debt payable to I_j by I_i.

I assume banks net out their payments at date $t = 2$. As a result, when $\frac{Y_i}{2} D_{ji} > \frac{Y_j}{2} D_{ij}$, j owes i the difference, namely, $\frac{Y_i}{2} D_{ji} - \frac{Y_j}{2} D_{ij}$.[24] So I_j is the *net borrower* and I_i is the *net lender*.

Without loss of generality, let $i = 1$ and $j = 2$ in the above discussion, so that I_1 is the net lender. Assumption B.1 is extremely useful in determining D_{12} and D_{21}. Each I_i has access to two investment opportunities: its own investment, which provides it with all the return (out of which he has to pay its lenders); as well as I_j investment opportunity. By

[23] The parameter are such that individual rationality is maintained.

[24] Although both banks lend to each other, and face values of debt are determined in equilibrium.

Assets	Liabilities
$\frac{Y_1+Y_2}{2}\tilde{R}$	$Y_1 D_{11}$
$\frac{Y_1-Y_2}{2}D_{21}$	

(a) Net Lender (I_1)

Assets	Liabilities
$\frac{Y_1+Y_2}{2}\tilde{R}$	$Y_2 D_{22}$
	$\frac{Y_1-Y_2}{2}D_{21}$

(b) Net Borrower (I_2)

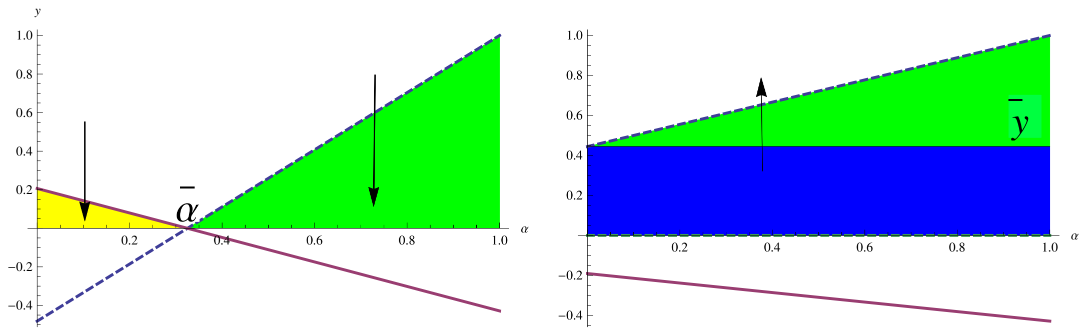
Figure B.1: Balance sheet of banks I_1 and I_2 when banks net out their payments. There are two I banks and k_{NI} NI banks. Y_i NI banks lend to I_i such that $Y_1 > Y_2$, so I_1 is the net lender and I_2 is the net borrower. In equilibrium, $D_{21} = R$ and $D_{22} = (1 + \alpha X)/p$.

assumption B.1, each I_i receives all the return from investment for each unit it lends to I_j.[25] This argument pins down both inter-I face values to be exactly R, $D_{12} = D_{21} = R$. So I_1 being the net lender implies $Y_1 > Y_2$. Consequently, at $t = 2$, bank I_2 owes I_1 a net payment of $\frac{Y_1-Y_2}{2}R$.

The balance sheets of I_1 and I_2 are depicted in Figure B.1. The critical observation is that survival of the net borrower solely depends on its own investment, while for the net lender, it also depends on whether the net borrower pays back. As a result, when both I banks invest, the net borrower survives exactly with probability p, whereas net lender's survival probability depends on other parameters of the model as well as the structure of the network, and is determined in equilibrium. Here I provide the main ingredients of the argument, and exact details are provided in the appendix.

I show that depending on the value of R, there can be two cases, as depicted in Figure B.2. Panel B.2a and B.2b correspond to high and levels of return, respectively. In each plot, the horizontal axis is α, the share of surplus that goes to a direct lender in a chain of length two, and the vertical axis is the ratio of the number of peripheries of the net borrower to the net lender, $y = \frac{Y_2}{Y_1}$. Note that $0 \leq y \leq 1$ and $0 \leq \alpha \leq 1$, so only the unit square in the first quadrant is relevant. Within this area, below the solid red line (yellow region), the liabilities of I_1 are low, so having more peripheries increases the gain to diversification, and I_1 survives with probability $1 - (1 - p)^2$. That is when $\alpha < \bar{\alpha}$. The reverse situation happens below the dashed blue line (green region). Here the liabilities are so high that I_1 fails unless all of its assets pay, so having many direct lenders increases its liabilities and leads to a higher probability of default, and I_1 survives only with probability p^2 . In the intermediate region, above both lines, I_1 survives exactly when its investment survives and fails exactly when its investment fails; that is, with probability p. On the horizontal axis, $y = 0$, I_1 fails with

[25] Note that I_j accepts as long as it has funding pledged to it directly by NI banks and the share of that investment covers its expected cost of default.



(a) $R > \bar{R}$

(b) $R < \bar{R}$

Figure B.2: Possible Equilibira with two I banks and k_{NI} NI banks and diversification. The x-axis is the share of expected net surplus that goes to the lender in a direct lending, α, and the y-axis is the ratio of the number of NI peripheries of I_2 to I_1, y. The arrows show the direction of the deviation of the NI banks.

probability p. So incentives of bank I_1 depends on level of α, which governs the amount of its liabilities.

Incentives of NI banks are more complicated. First note that they are purely driven by minimizing the probability of default, and default probability of NI banks who are peripheral to the net borrower I_2 is p. The complexity stems from the fact that NI bank liabilities are independent of α, and consequently its default probability is determined at $\alpha = 0$. Here is the relevant intuition for B.2a: the reason I_1 fails more often in certain regions compared to others, with the same successful assets, is that its liabilities are higher, i.e. α is high. However, NI banks have to pay the households only one unit in expectation, regardless of what α is. As a result, α is not relevant in determining failure probability of the NI banks. As a result in B.2a all NI banks *migrate* and lend to I_1, although at $\alpha > \bar{\alpha}$ this increases I_1's probability of default.

Given the above discussion, the next proposition characterize the equilibrium.

Proposition B.1 *Let y denote the ratio of the number of NI peripheries of net borrower to net lender I bank. There is a constant $\bar{R}$ such that*

- *When $R > \bar{R}$, there are two core-periphery equilibria with I banks at the core: $y = 0$ with I_1 at the core, and $y = \frac{1}{k_{NI}-1}$ with both I_1 and I_2 at the core.*
- *When $R < \bar{R}$, the single-core equilibrium is still an equilibrium. There are multiple two-core equilibria, one for each $y > \bar{y}$, where $\bar{y} = \frac{2}{p^2 R} - \frac{2-p}{p}$.*

Moreover, there are constants $\bar{\alpha}_l$, $\hat{\alpha}_l < \hat{\alpha}_h$ and $\hat{q} < \bar{q}$, all in $(0, 1)$, such that

- *$R > \bar{R}$ and $\hat{\alpha}_l < \alpha < \hat{\alpha}_h$: 2-I core-periphery equilibrium is inefficient when $\alpha > \bar{\alpha}_l$. It is also inefficient when $\alpha < \bar{\alpha}_l$ and $q < \bar{q}$.*
- *$R < \bar{R}$: 2-I core-periphery equilibrium is inefficient when $q < \hat{q}$.*

A detailed argument is provided in the appendix, but there are a few points worth mentioning here. First, diversification creates coordination problems between lenders an borrowers, which can in turn lead to inefficiencies in the financial network. In B.2b, for equilibria with y between the $y = \bar{y}$ and the dashed blue line, there are two sources of inefficiency: first, I_1 is exposed to the risk of default of I_2 when he only intermediates. Second, I_1 is not diversified in the best possible way when he also invests.

Second, this proposition shows that adding diversification does not alter the incentives to intermediates. Even when the gains from diversification are larger in the core-periphery network with two I banks at the core compared to the NI-star network, they can be dwarfed by the extra cost of I banks' failure due to excessive exposure to counterparty risk, and the core-periphery structure remains inefficient. Note that I have used the NI-star network to find sufficient conditions under which the 2-I core-periphery structure is not efficient, but these conditions are not necessary. There are more parameter regions where the above equilibria are dominated by NI-stare. Moreover, with diversification, the efficiency benchmark, which is necessary to characterize the necessary conditions, is more complicated to compute, and is left for future research.

Finally, adding diversification enables me to study the interesting question of under-insurance in the context of the model. Consider the $y = 0$ equilibrium, and assume $R > \bar{R}$ and $\alpha < \bar{\alpha}$. Imagine I_1 was able to offer the following deal to I_2 when both have investment opportunities: I_1 lends half of its funds to I_2 in order to fully diversify, and it pays I_2 exactly enough to cover I_2's expected cost of default, $(1-p)V_I$. Such an offer increases I_1 and all of NI's probability of survival from p to $1-(1-p)^2$, whereas it imposes some extra cost of default (that of I_2) on the economy. One can show that if $k_{NI} > \frac{V_I}{V_{NI}}\frac{(1-p)}{p}$, the above strategy improves welfare. However, I_1 would not make such an offer even if it could, because its individual gain to diversification, $p(1-p)V_I$, is lower than the price that it has to pay, $(1-p)V_I$. This means that I_1 does not internalize the positive externality of it buying insurance on its lenders. In other words, the price of insurance is too high for I_1, which leads to voluntary under-insurance and contagion.

C Random Funding to NI Banks

Consider the baseline economy with four banks, with the following adjustment. At $t = 1$ each bank $NI_j \in \mathbb{NI}$ receives a *fuding opportunity*, i.e. access to hh_j, with probability ζ, at the same time that the investment opportunities are realized at $I \in \mathbb{I}$ banks. Let $\mathbb{NI}_F$ denote the realization of the funding opportunities.

The borrowers do understand that each bank i's commitment to lend over an eligible interbank agreement is conditional on bank i (and its lenders, recursively) receiving funding from households, and that they can randomly be excluded from the set of recipients of i's funding. The exclusions are decided recursively, as follows.

Consider bank NI_j who did not receive funding from households. For a given realization of investment opportunities, track the hypothetical path of NI_j's funding towards $\mathbb{I}_R$. If every bank along the path is able to respect its lending commitments, because at $t = 0$ it has more funding secured than it has committed to, then the flow of funding happens as it would in the baseline model. Alternatively, moving from NI_j towards $\mathbb{I}_R$, consider the first intermediary, i, who does not have sufficient funding to service all of its commitments because it did not receive funding from NI_j, directly or indirectly. In this case, consider all the $I \in \mathbb{I}_R$ banks who have minimum shortest path from NI_j with $i \in SP(NI_j, I)$. Furthermore, consider all borrowers of bank i, b_i, who are on one of these shortest paths. One bank among the set of b_i banks is chosen at random, and it will not receive funding from i. The same algorithm is implemented recursively until we reach $I \in \mathbb{I}_R$ and no more lending happens. This algorithm couples conveniently with Assumption 1.

Thus in this extension, the face value of each unit of debt is conditional on the interbank network, the realization of investment opportunities, and the realization of funding opportunities, i.e. the tuple $(G, \mathbb{I}_R, \mathbb{NI}_F)$.

I will next prove the equivalent of Proposition 1, which shows the existence and uniqueness of core-periphery equilibria when κ is sufficiently large, as well as the constrained efficient interbank network. Similar to Proposition 1, I consider the set of parameters in which a single NI to I lending is both socially desirable and individually rational.

Proposition C.2 *Assume I and NI banks receive investment opportunities with probability q and funding opportunities with probability η, respectively. Let $\kappa = \frac{\alpha(1-\alpha)(pR-1)}{(1-p)V_I}$. There exist $\underline{\kappa}$ and $\bar{\kappa}$ such that an equilibrium exits if $\kappa \geq \underline{\kappa}$. Furthermore,*

(a) If $\min\{\underline{\kappa}, \frac{1}{2}\} \leq \kappa \leq \bar{\kappa}$, an efficient equilibrium coexists with inefficient equilibria.

(b) If $\kappa > \bar{\kappa}$, every equilibrium is core-periphery and is inefficient.

Finally, if $\underline{\kappa} < \frac{1}{2}$ and $\kappa < \frac{1}{2}$, all inefficient equilibria display under-exposure, while if $\kappa \geq \frac{1}{2}$ all inefficient equilibria display over-exposure among banks.

D Precautionary Saving

In order to incorporate precautionary saving in the model with minimal adjustments, assume that at date $t = 0$, the network has to be feasible with the same definition as the benchmark model. However, at $t = 1$, each lender bank is allowed to perform a portfolio optimization and hold precautionary saving as a function of the realized investment opportunities, and this is common knowledge among all the interbank market participants. This extension generates precautionary saving while still requiring intermediation to reach the optimal scale of investment.

The constrained social planner can choose a feasible network structure and determine the level of precautionary saving, but has to still ensure that the division of surplus is governed by α-rule and interbank payments are of the form of debt.

In what follows I prove that in any feasible network structure, either the individual and social level of precautionary saving is the same, or there is under-insurance in equilibrium. In particular, I argue that there is a range of parameters for which banks who either undertake the investment or intermediate, under-insure compared to what a constrained planner would do in the same network.

Consider an I bank who invests. For each unit of investment, an I bank has to pay D and receives $R - D$ if the project succeeds, with probability p, and nothing if the project fails. Furthermore, it suffers the cost of default V_I if the project fails, with probability $1 - p$.

Holding precautionary saving is only useful if it is sufficient to prevent default. This level of precautionary saving both hurt and helps the I bank. It hurts the I bank because holding precautionary saving decreases the scale of investment, which in turn decreases the surplus generated by investment, and decreases bank I's return. On the other hand, it saves the I bank $(1 - p)V_I$. This trade off determines the range of parameters for which an I bank is willing to hold sufficient precautionary saving to prevent default. In particular, this choice will only happen for low values of R.

Next, consider the constrained social planner in the same network. As the network structure is constant, the required per unit precautionary saving to prevent default is constant at each I bank. However, the planner weights the social cost of lower investment not only against $(1 - p)V_I$, but also the added expected cost of default of any initial lender and/or intermediary. As such, while the benefit of precautionary saving is the same, the cost considered by the planner is higher, because planner takes the contagion cost of default as well into account, not internalized by the I bank. This implies that the planner chooses to hold precautionary saving at an investing I bank for higher values of R. Put differently, there is a range of parameters where every I bank who invests under-insures itself in equilibrium.

Next, consider any intermediary bank who can hold part of the funding it has secured on the interbank market as precautionary saving. Similar to the bank who invests, in choosing the level of insurance, the bank who intermediates, whether I or NI, only internalizes its own cost of default and not the cost of default born by its lenders on the interbank market. As a result, similar to banks who invest, cost of default contagion is not internalized by intermediaries, which in turn leads to under-insurance for a range of parameters.

To put this together, there exists $\bar{R}$ such that for $R > \bar{R}$, every bank who invests and intermediates in the interbank network under-insures. Thus with precautionary saving, every network structure in Figure 3 leads to lower total expected total surplus net of expected cost of default, in equilibrium compared to constrained optimum, which in turn implies that in this case, equilibrium is always inefficient.

E Cost of Default in the Rule for Division of Surplus

Here I solve the 4 bank model of section 3 with a variation of α-rule which incorporates the default cost of banks along the intermediation chain. In this variation, the net surplus divided between the members of an intermediation chain is net of expected cost of default, and each bank receives its expected default cost plus its share. Let L, B, and In denote lender, borrower and intermediary respectively and let V_k be the cost of default of bank $k \in \{L, B, In\}$. Let X_k be the expected net surplus associated with a unit of investment intermediated through a chain of length k.[26]

$$X_1(V_B, V_L, V_{In}) = X - (1-p)(V_B + V_L)$$
$$X_2(V_B, V_L, V_{In}) = X - (1-p)(V_B + V_L + V_{In})$$

I suppress arguments to simplify the notation to X_1 and $X_2(V_{In})$, as the rest of the arguments do not change ($V_B = V_I$ and $V_L = V_{NI}$. Note that in each chain, each bank is compensated for the risk he takes as if this unit was the only unit he is involved in. This rule does not satisfy anonymity. Nevertheless, considering it reveals more insight from the model.

The new rule implies that banks are always compensated for the risk that they take (and maybe over-compensated). Now consider the deviation analogous to the one depicted in

[26] k is the number of edges along the chain.

Figure 4. Let $\hat{x}$ denote variables in the right panel, i.e. the core-periphery structure.

$$\hat{\mathcal{V}}_{NI_2} = q\alpha X_1 + (1-q)q\alpha^2 X_2(V_I) + V_{NI}$$
$$\hat{\mathcal{V}}_I = q^2(1-\alpha)X_1 + q(1-q)(1-\alpha)[X_1 + X_2(V_I)] + q(1-q)\alpha(1-\alpha)X_2(V_I) + V_I$$

while

$$\mathcal{V}_{NI_2} = (1-(1-q)^2)\alpha X_2(V_{NI}) + V_{NI}$$
$$\mathcal{V}_I = (q(1-q) + \frac{1}{2}q^2)2(1-\alpha)X_2(V_{NI}) + V_I$$

Let $\Delta\mathcal{V}_j = \hat{\mathcal{V}}_j - \mathcal{V}_j$, $j = I, NI$. With some algebra we get

$$\Delta\mathcal{V}_{NI} = q\alpha^2\Big[\frac{1-\alpha}{\alpha}X_1 + (1-p)V_{NI}\Big] + q(1-q)\alpha^2(1-p)(V_{NI} - V_I)$$
$$\Delta\mathcal{V}_I = q^2(1-\alpha)(1-p)V_{NI} + q(1-q)(1-\alpha)\alpha X_2(V_I) + q(1-q)(1-\alpha)(1-p)(2V_{NI} - V_I)$$

The sign of the last term in both expressions is ambiguous. The first observation is that if $V_I = V_{NI}$, both the peripheral lender and the I banks want to *unconditionally* deviate: I bank is now compensated for the excessive risk that he can take, and the cost is born by NI_1 (recall that the expected length of chains is *the same* in both network structures). Moreover, $\forall V_I\ \exists\bar{C}$ such that for $X > \bar{C}$, both $\Delta\mathcal{V}_{NI} > 0$ and $\Delta\mathcal{V}_I > 0$ even if $V_I > 2V_{NI}$. This condition is similar to what we have in section 3: if surplus of a unit investment is sufficiently large, the share of it which used to go to the NI intermediary before the deviation, and post deviation is divided between the peripheral NI and the new intermediaries, I banks, is sufficiently large to cover the extra cost that they hove to each bear by deviating. The higher cost is due to the fact that a *costlier* I banks intermediates in the new network, which is directly incorporated in the rule of division of surplus.

Now let me make an even more extreme assumption, and assume $V_{NI} = 0$, so if an NI intermediates it is costless. Then we have

$$\Delta\mathcal{V}_{NI} = q\alpha^2\Big[\frac{1-\alpha}{\alpha}X_1\Big] - q(1-q)\alpha^2(1-p)V_I$$
$$\Delta\mathcal{V}_I = q(1-q)(1-\alpha)\Big[\alpha X_2(V_I) - (1-p)V_I\Big]$$

comparing the two pair of expressions, it is clear that it is more difficult to satisfy the latter two. However, still $\exists\ddot{C} > \bar{C}$ for which the same argument goes through.

This appendix shows that the intuition for role of intermediation in formation of financial networks is general beyond the sufficient conditions provided for $\mathcal{L}(.)$ in Section 2, and it leads to a core-periphery interbank equilibrium. The crucial assumption is that there are positive intermediation spreads, and longer intermediation chains are associated with lower spreads per bank involved.

F Perfectly Correlated Project Returns

In this section I solve a version of the model, with unrestricted number of banks, where the project returns are perfectly correlated across banks, and lenders lend to all eligible borrowers. This extension exhibits the extreme opposite case of having iid return realizations for projects, and shows how social planner and individual incentives to intermediate and diversify vary as a function of this degree of correlation across projects.

Perfectly correlated project returns implies that there is no room for diversification. All active investment opportunities fail or succeed together. However, as 1 is relaxed a lender has to lend at least one unit to each of its eligible borrowers. The first implication is that from the social planner's perspective, gains from lending to one extra I bank is decreasing while the cost is constant. To see this, assume a bank is lending to x I banks. The net benefit from lending to the $x+1^{\text{th}}$ bank is that the lender is now able to lend as much funds as he is able to raise, when bank $x+1$ receives an investment opportunity while none of the first x banks did, net of cost of default of the borrower and lenders. However, there is an extra cost. Everything else equal, when any (subset) of the first x I banks, as well as bank $x+1$ receive an investment opportunity, the scale of investment remains fixed, but bank $x+1$ also invests and is now exposed to failure (of its own project). In other words, with multiple realized investment opportunities there is gain to *concentrating* the risk, which is lost here. Let $Z(x;K)$ denote the total net surplus from an NI bank, with K unites of funds (raised from its households and $K-1$ other NI banks), lending to x I banks ($K > x$).

$$Z(x;K) = (1-(1-q)^x)K\Big((pR-1)-(1-p)V_{NI}\Big) - (1-p)qV_I x$$
$$\Delta(x;K) = Z(x+1;K) - Z(x;K) = q\Big[(1-q)^x K\Big((pR-1)-(1-p)V_{NI}\Big) - (1-p)V_I\Big]$$

Let $c = V_I\Big(K(\frac{pR-1}{1-p} - V_{NI})\Big)^{-1}$. Note that from the assumption that one unit $NI \to I$ is efficient we know $c < 1$. The marginal gain turns negative when

$$x > x^* = \frac{\log(c)}{\log(1-q)}$$

First assume $k_I < x^*$, so the social planner prefers to lend to every I bank. The efficient solution requires investing every unit of funding whenever there is at least one realized investment opportunity, i.e. there should be a path from every NI bank to every I bank. Note that there is no room for concentration as Assumption 1 is relaxed. Moreover, all the intermediation must be done by NI banks, so no I bank lends.

In terms of equilibrium structure, the analogue of Proposition 2 holds here, with the exact same proof. This is the case because as long as there are no diversification effects, a lender only cares about level of rents, not where (or from how many borrowers) they come from, or what risk is undertaken to generate them. Moreover, the efficient structure is not an equilibrium when intermediation spreads are sufficiently high.

More interestingly, assume $k_I > x^*$. Now the social planner prefer to keep some investment opportunities unfunded because the marginal benefit is too small. In other words, reaching optimal scale of investment in one low-probability state requires destroying surplus in many states. This is the case when q is large, while k_{NI} is not too large. However, the same family of equilibria as defined in 2 still exist. A lender and/or intermediary wants to get as high a return as possible, as often as possible, so he prefers to be connected (directly or indirectly) to as many I banks as possible. Each I bank wants to invest as often as he gets an investment opportunity, so he would want to be connected to all units of funding. In this case, not only redistributional effects within a state are not internalized by individual players, but also redistributional effect across states are ignored.

This appendix manifests that incentives of banks to intermediate are robust to Assumption 1. In the extreme case where project returns are perfectly correlated, the core-periphery equilibrium remains inefficient because there is no gain to diversification. As section B shows, even with iid projects the core-periphery structure is inefficient under certain parameter restrictions. As the correlation across project returns rises,[27] the gain to diversification falls but gain to intermediate remains the same, so the space of parameters for which the core-periphery equilibrium is inefficient grows.[28]

G Proofs

Proof of Lemma B.1. j_1 is connected to at least z_2 of $I \in \mathbb{I}$, through "pointwise" weakly shorter paths, as defined in the lemma. Call this set $\mathbb{I}_{j_2}^{z_2}$. When any $I \in \mathbb{I}_{j_2}^{z_2}$ is in $\mathbb{I}_R$, the expected rate that j_1 (and consequently any lender to j_1) receives on their (indirect) lending is independent from distance of any $I \notin \mathbb{I}_{j_2}^{z_2}$ but $I \in \mathbb{I}_R$ to whom j_1 is connected.

[27] keeping project expectations the same.

[28] Solving for the most efficient structure with interim levels of return correlation is not straightforward, and is left for future work.

As a result the expected return that j_1 (and his lenders) receive conditional on realization of an investment opportunity at $I \in \mathbb{I}_{j_2}^{z_2}$ is larger that what j_2 (and his lenders) receive when what of the I banks j_2 is connected to is in $\mathbb{I}_R$. The above two events happen with exactly same probability (equal to at least one out of z_2 binomial random variables being one). Conditional the former event not happening j_1 still earns positive rents when $I \in \mathbb{I}$ $\mathbb{I}_{j_2}^{z_2}$ is in $\mathbb{I}_R$ which more than covers his expected cost of default[29], while j_1 earns no rents. So in expectation over all realizations of investment opportunities, j_1 and his lenders are better off than j_2 and his lenders, respectively. ■

Proof of Proposition B.1.

Equilibrium

All the references to figures in this proof are to Figure B.2.

First, solve for the face values payable to NI peripheries, D_{11} and D_{22}. Failure probability of I_2 determines the face value payable to its NI peripheries to be $D_{22} = \frac{1+\alpha X}{p}$. As a result, the only remaining equilibrium object is D_{11}. D_{11} depends on the share of surplus that goes to a direct lender, the endogenous probability of (partial) repayment by I_1, as well as Y_1 and Y_2.

The structure of equilibrium and the face value of debt from I_1 to his NI peripheries are jointly determined in equilibrium, based on which of the following regions the total liabilities of the net lender I_1 lies in:

$$\begin{cases} Y_1 D_{11} \geq \frac{Y_1+Y_2}{2} R & I_1 \text{ survives with probability } p^2 \\ \frac{Y_1-Y_2}{2} R \leq Y_1 D_{11} < \frac{Y_1+Y_2}{2} R & I_1 \text{ survives with probability } p \\ Y_1 D_{11} < \frac{Y_1-Y_2}{2} R & I_1 \text{ survives with probability } 1-(1-p)^2 \end{cases}$$

First note that liabilities can be high for two reasons: either α is high so that a large share of surplus goes to the lenders, or default probability of borrower is high. In the first region above liabilities are so high that unless both assets pay, I_1 fails. In the middle region I_1 fails if his asset investment fails and survives otherwise, and in the last region I_1 survives unless both assets fail. In the first two regions there will be partial payments. Let $\hat{D} = D_{22} = \frac{1+\alpha X}{p}$, which is the face value of debt which corresponds to the case where a bank fails exactly when his own investment fails.

[29] Because I assume participation constraint must be satisfied for each realization of lending.

Region One ($Y_1 D_{11} > \frac{Y_1+Y_2}{2}R$).

$$
\begin{aligned}
&p^2 D_{11} + p(1-p)\frac{Y_1+Y_2}{2Y_1}R + (1-p)p\frac{Y_1-Y_2}{2Y_1}R = \alpha X + 1 \\
&D_{11} = \frac{1}{p}(\hat{D} - (1-p)R)
\end{aligned}
$$

In order for the total liabilities with the above face value to be in region one it must be that

$$
\frac{Y_2}{Y_1} < \frac{2}{pR}\hat{D} - \frac{2-p}{p}
$$

Region Two ($\frac{Y_1-Y_2}{2}R \leq Y_1 D_{11} < \frac{Y_1+Y_2}{2}R$).

$$
\begin{aligned}
&pD_{11} + (1-p)p\frac{Y_1-Y_2}{2Y_1}R = \alpha X + 1 \\
&D_{11} = \hat{D} - (1-p)\frac{R}{2}(1 - \frac{Y_2}{Y_1})
\end{aligned}
$$

In order for the total liabilities with the above face value to be in region two it must be that

$$
\frac{Y_2}{Y_1} > \frac{2}{pR}\hat{D} - \frac{2-p}{p} \tag{4}
$$

$$
\frac{Y_2}{Y_1} > 1 - \frac{2}{R(2-p)}\hat{D} \tag{5}
$$

Region Three ($Y_1 D_{11} < \frac{Y_1-Y_2}{2}R$).

$$
\begin{aligned}
&(1-(1-p)^2)D_{11} = \alpha X + 1 \\
&D_{11} = \frac{1}{2-p}\hat{D}
\end{aligned}
$$

In order for the total liabilities with the above face value to be in region two it must be that

$$
\frac{Y_2}{Y_1} < 1 - \frac{2}{R(2-p)}\hat{D}
$$

Let $y = \frac{Y_2}{Y_1} \leq 1$ denote the ratio of the NI peripheries of I_2 to I_1. The inequality holds because I_1 is assumed to have more peripheries. The two inequalities defined in 4 characterize the three regions in which I_1 fails with different probabilities; where each region characterizes the set of (α, y) for which the probability of I_1 failure is the same.

The two lines cross each other and zero, if they do so, at $(\bar{\alpha}, 0)$ such that

$$1 = \frac{2}{R(2-p)} \frac{1+\bar{\alpha}X}{p}$$

However, the two lines will not cross zero (and each other) at any $\alpha \geq 0$ if even at $\alpha = 0$ I_1's own investment must survive for him to survive. This happens if

$$\frac{2}{pR}\frac{1}{p} - \frac{2-p}{p} > 0$$

Let $\bar{R} = \frac{2}{p(2-p)}$. The above inequality holds if

$$R < \bar{R} \tag{6}$$

This happens in panel B.2b. Recall that $R > \frac{1}{p}$ for the project to be positive NPV. The intuition is that if the project is positive NPV but the upside is not sufficiently high, I_1 fails if its own project, i.e. its larger asset, does not pay off. In other words, there are different combinations of (p, R) with the same NPV, that is, constant pR. I_1 prefers the combinations with higher R because it provides I_1 with sufficient resources to be able to pay its lenders, even if only I_1's smaller asset pays back. In this case $\bar{\alpha} < 0$.

In the left panel, B.2a, $\bar{\alpha} > 0$. When $0 \leq \alpha < \bar{\alpha}$, I_1 bank prefers to have many peripheries to lie below the red line, which would imply an unbalanced core-periphery structure, while for $\bar{\alpha} < \alpha \leq 1$ it prefers to have similar number of peripheries as I_2 has, which will be a more balanced core-periphery structure.

So the equilibria in the two case defined by 6 should be studied separately. For now ignore the constraint that α should be such that intermediation rents are high enough so that either one or both of the I banks agree to intermediate, i.e. ignore the participation constraint.[30]

When 6 does not hold, the two lines defines in 4 cross at $\alpha = \bar{\alpha}$ in B.2a. Recall that peripheries of net borrower fail with probability p and we need to consider incentives of NIs peripheral to net lender. These incentives are not necessarily aligned with that of the I banks. NI incentives about which I bank to lend to is purely driven by their default probability, and are determined at $\alpha = 0$, as explained in the text. Here at $\alpha = 0$ there is a range of positive y for which NI banks (and I_i) survive as often as possible, i.e. unless

[30] Note that I have assumed participation constraint must be satisfied case by case. When only one bank get the investment opportunity diversification does not come in, so this argument does not affect the range of α for which either one or both Is are willing to intermediate. The final equilibria are the ones which are consistent with both sets of conditions.

both projects fail. So at those y's NI's will survive at higher values of α as well, since the (partial) payments they receive from I_i only increases in α.

To see this, consider two different economies; L and H, with two different levels of α; $\alpha_L = 0$ and $\alpha_H > \bar{\alpha}$.[31] Denote the NI banks in economy L and H, NI_L and NI_H, respectively. First consider economy L and assume Y_1 and Y_2 are such that y lies below the solid red line. For this level of y, if at least one of the assets held by I_1 pays back (probability $(1-(1-p)^2)$, NI_L peripheries of I_1 are payed back in full. They pay all of what they get to households[32], and they survive with probability $(1-(1-p)^2)$, the same probability as I_1 survives.

Now consider economy H. Here I_1 survives only if both of its assets pay back, that is, if both investments are successful, because its liabilities are too high. This happens with probability p^2. However, when I_1 fails it makes partial payments if either of his assets pay back. As a result, for every state of the world, what each NI_H bank gets in the H economy, is at least as high as what each NI_L bank gets in the L economy. As NI_L and NI_H banks have the same expected liabilities, NI_H cannot fail more often than NI_L. This implies that for each (p, R, V_I, V_{NI}), and each level of y, the probability of default for an NI periphery of I_1, for any α, is the same as probability of default of an NI with $\alpha = 0$.

For $\alpha < \bar{\alpha}$, every NI lenders of I_2 prefers to instead lend to I_1 and save on the expected cost of default. I_1 likes that too. So every NI periphery of I_2 deviates to I_1 as long as I_2 has one periphery. If I_2 loses its last periphery, when both I banks have an investment opportunity, even if I_1 lends to I_2 and I_2 invests, I_2 does not receive a share of his own investment's net surplus, because I_1 absorbs all the returns. However, I_2 still incurs the expected cost of default. As a result, participation constraint of I_2 is violated and $I_1 \to I_2$ will not happen when both banks have the investment opportunity. Consequently, I_1's probability of default would rise to p, and I_2's last periphery would be indifferent between deviating or not, which by definition of equilibrium implies it does not deviate.[33]

On the other hand, when $\alpha > \bar{\alpha}$, I_1 fails more often below the dashed blue line while NI lenders to I_1 still fail less often. As a result, NI peripheries of I_2 want to deviate and lend to I_1. Interestingly, I_1 does agree to this deviation although it increases its probability of default. The reason is that the return it gets from investing this extra unit, more than covers the incremental cost of default, $\alpha(1-\alpha)X > (1-p)V_I > p(1-p)V_I$.

[31] This example is purely for illustration, so ignore the fact that NI_L's participation constraint is violated at $\alpha = 0$.

[32] Because $\alpha = 0$.

[33] The fact that I_2 remains with a single NI periphery is simply because I assumed intermediation rents are high enough so that intermediating a single unit of funding covers I's extra cost of default. If intermediating c units is necessary to keep I_2 intermediating, then it will end up with c peripheries.

The above argument requires a minor adjustment. Note that the 2-I core-periphery equilibrium never features $y = 0$, instead $y = \frac{1}{k_{NI}-1}$, which must be in the Region Three at $\alpha = 0$ for the above argument to work. As a result $\bar{R}$ needs to be updated to adjust for this:

$$\bar{R} = \frac{2}{p(2-p)} z \tag{7}$$

where $z = \frac{k_{NI}-1}{k_{NI}-2}$. Note that $\bar{R} \to \frac{2}{p(2-p)}$ as $k_{NI} \to \infty$. Moreover, instead of $\bar{\alpha}$ there are two relevant thresholds, $\bar{\alpha}_l$ and $\bar{\alpha}_h$, one on each line defining the borders of the three regions, which replace α

$$\bar{\alpha}_l = \Big(\frac{p(2-p)R}{2}(1 - \frac{1}{k_{NI}-1}) - 1\Big)(pR-1)^{-1}$$
$$\bar{\alpha}_h = \Big(\frac{pR}{2}(\frac{p}{k_{NI}-1} + 2 - p) - 1\Big)(pR-1)^{-1}$$

Note that as $k_{NI} \to \infty$, $\bar{\alpha}_l \to \alpha$ and $\bar{\alpha}_h \to \alpha$.

In the region where 6 does not hold(with adjusted $\bar{R}$ defined in 7), if $\alpha < \bar{\alpha}_l$, then I_1 survives with probability $1-(1-p)^2$. If $\bar{\alpha}_l < \alpha < \bar{\alpha}_h$, then I_1 survives with probability p. If $\alpha > \bar{\alpha}_h$, then I_1 survives with probability p^2. So a small region is added in the middle for I_1. All NIs who lend to I_1 still survive with probability $1-(1-p)^2$.

Next consider the case where 6 holds. As a result, Region Three disappears. Here the realized return of the project, R, is so low that even at $\alpha = 0$, regardless of level of y, I_1 fails if its larger asset, namely, its own investment, does not pay back. However, depending on the level of y and α, I_1 may need its second asset to also pay back in order to survive. Specifically, if α is high I_1 survives only if both assets pay back.

Now consider default probability of NI banks who are peripheral to I_1. Again, the relevant range of the parameters for NI peripheries, to prefer one borrower to the other, is determined only at $\alpha = 0$, but for different reasons. First note that the highest (partial) payments that an NI bank receives is at $\alpha = 1$, where NI receives R for the proportion of his portfolio invested in the successful project(s), and has to pay lenders who only have to break even, i.e. they in turn have $\alpha = 0$ effectively. This is the exact same problem that I_1 faces when his NI lenders have $\alpha = 0$.

Two different scenarios must be considered separately. First, can NI fail only with probability $1-(1-p)^2$, given that we know this is not possible for I_1? As I argued, the best an NI can do is at $\alpha = 1$, and for him to survive unless the two projects fail we should have

$$\frac{1}{Y_1}\frac{Y_1 - Y_2}{2}R > \frac{1}{1-(1-p)^2}$$

which boils down to the boundary of Region three at $\alpha = 0$ as argued above, which we know is negative when 6 holds. So this case never happens (regardless of how often I_1 survives).

When I_2 survives with probability π, NI does also survive with probability at least as high as π. So the only remaining case is when I_1 survives with probability p^2 but his peripheries survive with probability p.

Let D_{1h} denote the face value of debt payable to households lending to an NI bank peripheral to I_1. The trick is to realize that when I_1 fails, he pays all the proceeds from his project as partial payment, *as if* NI has $\alpha = 1$, and when NI fails himself he pays all of those proceeds to his households. As a result the equation which defines D_{1h} boils down to the same equation which defines D_{11} in Region two, at $\alpha = 0$:

$$pD_{1h} + (1-p)p\frac{Y_1 - Y_2}{2Y_1}R = 1$$

Which in turns implies that the boundary for this case is the same as the boundary in Region two at $\alpha = 0$, $\bar{y}$ in B.2b. So in this case when $y > \bar{y} = \frac{2}{p^2 R} - \frac{2-p}{p}$, NI peripheries of either I bank are indifferent between moving around since they have no room to improve on their default probability. However, when $y < \bar{y}$, NI peripheries of I_1 deviate to I_2 until $y \geq \bar{y}$. Such deviation pushes y up and above $\bar{y}$. Any $y > \bar{y}$ is an equilibrium because NI peripheries of I_1 has no incentive to deviate to I_2, because they fail with the same probability in both places.

Finally, one should consider $y = 0$, where only I_1 lends to I_2, separately. As long as intermediation rents are sufficiently high, $y = 0$ is also an equilibrium. The reason is that NIs would not benefit from any joint deviation with I_2 unless I_1 agrees to the deviation and adds the $e_{I_2 I_1}$ potential relationship, which would require I_1 to lose at least one of its peripheries to I_2, and I_1 does not agree to be part of such deviation even if it improves his survival probability, as explained above.

Efficiency

I will show that in the range provided in the proposition, the 2-I core periphery equilibrium is dominated by NI-star, and cannot be efficient. This does not necessarily means NI-star itself is efficient.

Consider NI-star, and let NI_c be the NI who lends to all I banks. NI_c survives either with probability p^2 or $1-(1-p)^2$ because his two assets are symmetric. Assume NI_c fails only of both projects fail. So if each of his assets pay back, he must be able to pay his liabilities in full

$$\frac{k_{NI}}{2}\frac{\alpha X+1}{p} \geq (k_{NI}-1)\frac{\alpha^2 X+1}{p(2-p)}+\frac{1}{p(2-p)}$$

The first term on right hand side is his total liabilities from other NIs assuming he pays back with probability $1-(1-p)^2$, and the second term to his households. With some algebra we get

$$k_{NI}\Big[\frac{2-p}{2}(\alpha X+1)-(\alpha^2 X+1)\Big] > -\alpha^2 X$$

This is a similar condition to what I_1 faces, with a few adjustments. Unlike I_1, total assets available to NI_c when an investment survives is lower than full value, R. His liabilities are also lower, and are not fully symmetric. A sufficient condition for the above inequality is

$$\alpha^2 X+1 < \frac{2-p}{2}(\alpha X+1) \tag{8}$$

This is now very similar to the condition for I_1, except that both assets and liabilities decrease with α, so for instance at $\alpha = 0$, NI_c fail: his liabilities are low, but the same with his assets. It does not hold at $\alpha = 1$ either. So the corresponding quadratic equation has two roots, $0 < \hat{\alpha}_l < \hat{\alpha}_h < 1$, and the above inequality holds if $\hat{\alpha}_l < \alpha < \hat{\alpha}_h$.

In this case, every NI survives with probability $1-(1-p)^2$. NI_c diversifies the risk that NIs face very well, but not the risk that I_1 faces.

Note that

$$\alpha^2 X+1 < \frac{p(2-p)}{2}\frac{(\alpha X+1)}{p} < \frac{p(2-p)}{2}R$$

where the last inequality holds simply because face value paid to NI_c is less than R as final borrower gets positive share of surplus. As a result when 6 holds, the total assets are too low and NI_c survives only if both asset pay.

When $\alpha > \hat{\alpha}_l$, all peripheral NIs can still survive with probability $1-(1-p)^2$ if their partial payment, when only one project pays off, is sufficiently large

$$\frac{\alpha^2 X+1}{(k_{NI}-1)(\alpha^2 X+1)+1}\frac{k_{NI}}{2}\frac{\alpha X+1}{p} > \frac{1}{p(2-p)}$$

The left hand side is increasing in α, so there is a constant $\tilde{\alpha}$ such that for $\alpha > \tilde{\alpha}$ it holds.

Next I compare the difference between the expected loss in NI-star and core-periphery equilibria. Let Δ denote the difference, so $\Delta > 0$ implies that the core-periphery network is inefficient (but not the reverse).

- $R > \bar{R}$, $\hat{\alpha_l} < \alpha < \hat{\alpha_h}$, $\alpha > \bar{\alpha}_l$

$$\Delta = q^2[(1-p)pV_{NI} + \mathbb{1}[\alpha > \bar{\alpha}_h]p(1-p)V_I] + 2q(1-q)(1-p)V_I > 0$$

The first term is when there are two investment opportunities. In NI-star, all NIs and one I survive if only one project pays off. In the core-periphery if only project of I_1 pays off, I_2 and his periphery fail. If $\alpha > \bar{\alpha}_h$, I_1 fails unless both projects payoff. The last term corresponds to states where only one I get the investment. So the NI-star is strictly better.

- $R > \bar{R}$, $\hat{\alpha_l} < \alpha < \hat{\alpha_h}$, $\alpha < \bar{\alpha}_l$

$$\Delta = q^2[(1-p)pV_{NI} - p(1-p)V_I] + 2q(1-q)(1-p)V_I$$

Here if only I_1's project survive there is a gain of one extra NI being saved in NI-star, but if only I_2's project survive there is a cost of I_1 failing in NI-star. Intermediation costs are the same. If $q < \hat{q} = \frac{V_I}{V_I + 0.5p(V_I - V_{NI})}$, $\Delta > 0$.

- $R < \bar{R}$: Here project payoff in case of success is low, so NI-star does poorly in terms of diversification.

$$\Delta > q^2[-p(1-p)k_{NI}V_{NI}] + 2q(1-q)(1-p)V_I$$

the first inequality comes from the fact that there are equilibria here where I_1 fail unless both projects payoff, in which case NI-star saves on that. However, no matter which project fail all NIs fail, which is not the case in the core-periphery equilibrium as NI banks reorganize themselves to improve on survival probability. This is the first term on right hand side. A sufficient condition for the above is

$$q < \hat{q} = \frac{V_I}{V_I + 0.5\frac{p}{1-p}k_{NI}V_{NI}}$$

■

Proposition C.2. Let $X = pR - 1$, $\kappa = \frac{\alpha(1-\alpha)X}{(1-p)V_I}$ and $\tilde{\kappa} = \max\{\alpha, (1-\alpha)\frac{V_{NI}}{V_I}\}$. The participation constraints of the direct lender and direct borrower banks jointly imply that $\kappa \geq \tilde{\kappa}$ is the relevant range of parameters. Furthermore, the face values of debt are identical to those in Section 3.1. The difference is that when banks calculate their exante expected profit net of cost of default, they have to take into account that some lending is not realized because the funding opportunity is not realized.

Network structure 3a Assume the economy is in Figure 3a. This is the only network in which an NI bank intermediate. To analyze this network, I will assume $\alpha(1-\alpha)X > (1-p)V_{NI}$, which insures that an NI bank is willing to intermediate even if it does not receive a funding opportunity itself. The complementary conditions can be solved for in an identical fashion.[34] In expectation, an I bank and NI_2 get the following, respectively:

$$\begin{aligned}\mathcal{V}_I^a =& \left((1-\zeta)^2 + (1-(1-\zeta)^2)\left((1-q) + \frac{1}{2}q^2\right)\right) V_I + \\ & (1-(1-\zeta)^2)(\frac{1}{2}q^2 + q(1-q))pV_I + (q^2 + 2q(1-q))\zeta p(R-D) \\ \mathcal{V}_{NI_2}^a =& (1-\zeta)V_{NI} + \\ & \zeta\left((1-q)^2 V_{NI} + (1-(1-q)^2)[p(V_{NI}+D_1)) - 1]\right)\end{aligned}$$

In words, in any network structure, if either no investment opportunity or no funding opportunity is realized, each I bank keeps its V_I. If NI_2 receives a funding opportunity, it will lend. Each I bank invests or intermediates according to the network structure and the realized funding and investment opportunities.

In this network, NI_1 receives maximum expected return net of cost of default, so it will not join any deviation. Thus any deviation requires at least one I bank and NI_2. As the two I banks are identical, the only possible coalition is $\{I_1, I_2, NI_2\}$. Furthermore, at least one of the I banks and at least one of NI banks are not willing to deviate to one of the networks 3b, 3c, 3d, because they either lose some funding or access to some investment opportunity. Thus the only possible deviation is to network structure 3e.

NI_2 always prefers to deviate to 3e, while I bank would deviate from 3a to 3e if:

$$\frac{\alpha(1-\alpha)(pR-1)}{(1-p)V_I} > \frac{1}{\zeta}\left(1 + \frac{\zeta(6-2q-\zeta(2-q)) - 2}{2(1-q)}\right),$$

which when $\zeta = 1$ reduces to

$$\frac{\alpha(1-\alpha)X}{(1-p)V_I} > 1 + \frac{q}{2(1-q)}.$$

[34] I refrain from including the solution under the complementary condition as it requires considering more cases and makes the section unnecessarily long, without adding to the intuition. The calculations are available upon request.

Let $\bar{\kappa} = \frac{1}{\zeta}\left(1 + \frac{\zeta(6-2q-\zeta(2-q))-2}{2(1-q)}\right)$, and consider the joint deviation of the two I banks along with NI_2 to see that 3a is not an equilibrium if $\kappa > \bar{\kappa}$.

There is no deviation to 3b or 3d as I_2 would never participate in those deviations. There is also no deviation to 3c since neither I bank would participate on it, as in 3c both I banks strictly lose scale without gaining on cost of default, compared to 3a. As such, similar to the baseline economy, 3a is an equilibrium when $1 \leq \kappa \leq \bar{\kappa}$.

Network structure 3e In this network:

$$\begin{aligned}
\mathcal{V}_I^e = &\left((1-\zeta)^2 + (1-(1-\zeta)^2)(1-q)^2 + (1-\zeta)q\right) V_I + \\
&\left((1-(1-\zeta)^2)(1-(1-q)^2) - (1-\zeta)q\right) pV_I + (q^2 + 2q(1-q))\zeta p(R-D) + (1-q)q\zeta p(D-D_1) \\
\mathcal{V}_{NI_2}^e = &(1-\zeta)V_{NI} + \\
&\zeta\left((1-q)^2 V_{NI} + q^2[p(V_{NI}+D)-1] + q(1-q)[p(V_{NI}+D)-1] + q(1-q)[p(V_{NI}+D_1)-1]\right)
\end{aligned}$$

First, assume $\kappa < \bar{\kappa}$. In order for 3e not to be an equilibrium, there should be a joint deviation that blocks it. When $\kappa < \bar{\kappa}$, although both I bank prefer to deviate back to 3a, they need both NI banks to join the deviation to deviate to that network. However, no NI bank agrees to be a leaf who is always intermediated, when in 3e it gets to lend anytime there is an investment opportunity, and with positive probability it gets un-intermediated rent. As such, 3e ceases to be an equilibrium when intermediation spread does not cover the cost of default for an I bank anymore, and each I_i bank prefers to unilaterally break $e_{I_iI_j}$ link. This happens when $\kappa < 1$.

Next, consider $\kappa > \bar{\kappa}$. Is 3e an equilibrium? Yes since there is no deviation that simultaneously improves on what both NI banks get in 3e, so there is no way to convince NI banks to join any deviation.

Network structure 3d Now assume the economy is in 3d. I_1, I_2 and each NI bank receive:

$$\begin{aligned}
\mathcal{V}_{I_1}^d = &\left((1-\zeta)^2 + (1-(1-\zeta)^2)\left((1-q)^2 + (1-(1-q)^2)p\right)\right)V_I + \\
&2q\zeta p(R-D) + 2(1-q)q\zeta p(D-D_1) \\
\mathcal{V}_{I_2}^d = &\left(1 - q(1-q) + q(1-q)(1-\zeta)^2 + q(1-q)(1-(1-\zeta)^2)p\right)V_I + q(1-q)2\zeta p(R-D) \\
\mathcal{V}_{NI}^d = &(1-\zeta)V_{NI} + \\
&\zeta\left((1-q)^2 V_{NI} + q^2[p(V_{NI}+D)-1] + q(1-q)[p(V_{NI}+D)-1] + q(1-q)[p(V_{NI}+D_1)-1]\right)
\end{aligned}$$

Using network 3b below, neither NI banks nor I_2 want to deviate from 3d to 3b. NI_2 would not participate in a deviation to 3a, and NI banks are indifferent between 3d and 3e, thus that deviation does not exist either. The only deviation is when I_1 unilaterally deviates and break $e_{I_1 I_2}$ link if that increases its expected profit. This happens if $\kappa < \hat{\kappa} = \frac{2-\zeta}{2}$, which reduces to $\kappa < \frac{1}{2}$, as shown in Proposition 1.

Network structures 3b In 3b I_1 and each NI get:

$$\begin{aligned}\mathcal{V}^b_{I_1} =&\Big((1-\zeta)^2 + (1-(1-\zeta)^2)\left((1-q)+qp\right)\Big)V_I + 2q\zeta p(R-D)\\ \mathcal{V}^b_{NI} =&(1-\zeta)V_{NI} + \zeta\left((1-q)V_{NI} + q[p(V_{NI}+D)-1]\right)\end{aligned}$$

Two types of deviations are perceivable: first, the two I banks jointly deviate and add $e_{I_1 I_2}$, which is profitable for them when $\kappa > \hat{\kappa} = \frac{2-\zeta}{2}$, and they deviate to 3d.[35]

A second possible deviation is for the two NI banks to jointly deviate with I_2 to go to 3a. This deviation requires NI_2 to be better off in 3a. A necessary condition is $\alpha > \frac{1}{2-q} > \frac{1}{2}$, which in turn implies that $\underline{\kappa} > \frac{1}{2}$ and 3b does not exist. As such, similar to the baseline economy, 3b is an equilibrium at most in the range $\underline{\kappa} \leq \kappa < \frac{1}{2}$.

Network structures 3c In 3c both I banks lose scale when they have an investment opportunity. There are three possible deviations. First, both I banks could exante be better off adding $e_{I_1 I_2}$ and $e_{I_2 I_1}$ even when $\kappa < 1$. However, this is not a viable deviation when $\kappa < 1$, because in the interim period, when only investment opportunity i is realized, lending over $e_{I_j I_i}$ violates the participation constraint of I_j and will not happen, so $e_{I_j I_i}$ is never traversed and the above is not a viable deviation when $\kappa < 1$.

The second candidate deviation to 3a is ruled out by the same argument as above.

There is a third possible deviation: NI_2, I_2 and I_1 jointly deviate, break $e_{NI_2 I_2}$, and add $e_{NI_2 I_1}$ and $e_{I_1 I_2}$. The first necessary condition is that adding $e_{I_1 I_2}$ must be a viable deviation, which requires $\kappa < \frac{1}{2}$. If so, I_1 and NI_2 gain. I_2 incentives are ambiguous because in 3d, I_2 does not get to invest when I_1 get an opportunity, but gets to invest 2 units when I_1 does not.

$$\mathcal{V}^c_{I_2} = (1-q)V_I + q[p(V_I + (R-D))]$$

For the latter deviation to happen we need $\mathcal{V}^c_{I_2} < \mathcal{V}^d_{I_2}$, which holds if $q < \bar{q} = \left(2 - \frac{(1-p)V_I}{(1-\alpha)X}\right)^{-1}$. Note that $\frac{1}{2} < \bar{q} < 1$.

[35] Beyond 3d, deviating to 3e is also possible but the former deviation is viable whenever the latter is, so there is no need to consider the latter.

Efficiency Efficiency of network structure 3a follows the exact same argument as Proposition 1. ■