NBER WORKING PAPER SERIES

POPULATION GROWTH AND FIRM DYNAMICS

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Working Paper 29424
http://www.nber.org/papers/w29424

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
October 2021

The author declares that he has no material financial interests that relate to the research described in the paper "Population Growth and Firm Dynamics". The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 29424
October 2021
JEL No. J11,L11,O3,O4,O44

ABSTRACT

Population growth has declined markedly in almost all major economies since the 1970s. We argue this trend has important consequences for the process of firm dynamics and aggregate growth. We study a rich semi-endogenous growth model of firm dynamics, and show analytically that a decline in population growth reduces creative destruction, increases average firm size and concentration, raises market power and misallocation, and lowers aggregate growth in the long-run. We also show lower population growth has positive effects on the level of productivity, making the short-run welfare impacts ambiguous. In a quantitative application to the U.S, we find that the slowdown in population growth since the 1980s and the projected continuation of this trend accounts for a substantial share of the fall in the entry and exit rates and the increase in firm size. By contrast, the impact on markups is modest. The effect on aggregate growth is positive for around two decades, before turning negative thereafter.

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A data appendix is available at http://www.nber.org/data-appendix/w29424

1 Introduction

Almost all major economies have experienced a substantial decline in population growth in recent decades. Figure 1 shows historical population growth for a group of major world economies from 1960 to 2020. Despite different political systems, cultures, and levels of development, a clear downward trend is evident for all of them. Moreover, according to the UN, this trend is projected to continue for at least the first half of the 21st century, driven largely by continuing declines in fertility.[1] A world of low and falling population growth looks like it is here to stay.

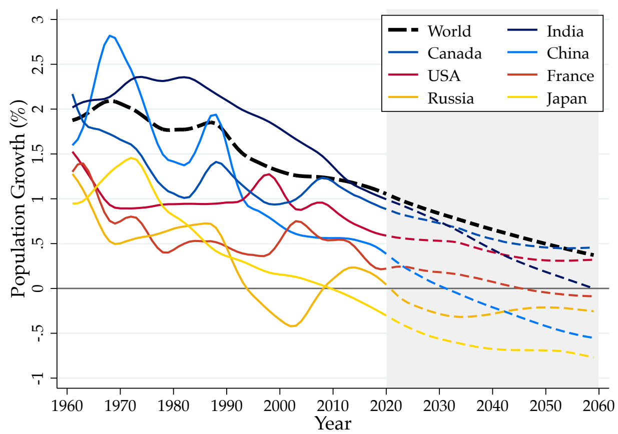
In this paper, we show this phenomenon is likely to have important implications for the process of firm dynamics and aggregate economic performance. We do so in the context of a firm-based model of semi-endogenous growth. Our baseline model is an enhanced version of Klette and Kortum (2004), augmented by the possibility of population growth, new-variety creation, own-innovation, and a demand elasticity that exceeds unity. The model is rich enough to rationalize many first-order features of the process of firm dynamics, yet has an analytic solution that allows us to express the process of firm dynamics, the firm-size distribution and the aggregate growth rate directly as a function of population growth.

This theory makes tight predictions for the effects of falling population growth on the process of firm dynamics: a slow-down in population growth reduces creative destruction and entry and increases concentration and average firm size. The reason is the following. In the long-run, the number of products available to consumers has to grow at the same rate as the population. If that was not the case, firm profits would either grow without bound or converge to zero, both of which are inconsistent with free entry. Falling population growth thus goes hand in hand with a decline in product innovation, reducing both the creation of new varieties and the rate of creative destruction. Importantly, we show that in equilibrium the decline in product innovation is only accommodated through a decline in entry - incumbent firms' optimal innovation policies are unaffected by changes in population growth. This change in the composition of creative destruction implies lower population growth raises firm growth conditional on survival and reduces incumbent firms' exit hazards. As a consequence, concentration and firm size rises, and both entry and exit rates fall.

In addition to firm-dynamics, our theory also makes clear predictions about the relationship between population growth and per-capita income growth. As in many aggregate models of semi-endogenous growth, the long-run equilibrium growth rate declines as the rate of population growth falls. However, we show that an important countervailing effect makes the relationship between population growth and welfare a priori ambiguous. By reducing creative destruction, falling population growth increases the value of firms, because future profits are discounted at a lower rate. Free entry therefore requires an increase in the economy-wide level of varieties to increase competition. Because additional varieties raise income per-capita, the welfare consequences of declining population growth therefore hinge on the importance of these static variety gains relative to the dynamic

[1] See Section SM-9 in the Supplementary Material, where we show that birth rates are falling and projected to continue to decline. Contributions from net migration are expected to be stable, and are mostly small in most major economies.

Figure 1: Population Growth across Major Economies



Notes: Solid lines plots historical population growth from the UN World Population Prospects 2019 for several major economies. Dashed lines plot the UN projections for population growth in the "Medium" scenario out to to 2060.

losses from lower growth.

We then show that these results are robust to a variety of changes in the environment. Most importantly, we extend our model to a setting where firms compete a la Bertrand and market power is endogenous. Declining population growth interacts with firms' ability to charge markups in an interesting way. In our theory, more productive firms post higher markups, and productivity increases over the firms' life cycle. Because creative destruction reduces firms' chances of survival, it hinders incumbents from accumulating market power and prevents the emergence of dominant producers. In short, creative destruction is pro-competitive. Declining population growth, by lowering creative destruction, reduces competition and increases markups and misallocation.

To quantify the strength of this mechanism, we calibrate our model to data for the population of US firms. In addition to targeting standard moments such as the entry rate, average size, and life-cycle growth, we also link firm-level information on sales to the US Census. We can therefore explicitly target a measure of the life-cycle of firm-level markups for a majority of firms in the US. Exploiting information on the evolution of both markups and size at the firm-level is an important aspect of our empirical methodology, and allows us to separately identify own-innovation and variety creation at the firm-level.

With the calibrated model in hand, we ask a simple question: what are the implications of the observed and projected decline in the rate of labor-force growth since 1980? Empirically, labor-force growth almost halved from 2% to 1% between 1980 and 2015, and the BLS projects labor-force growth will continue to decline to 0.24% after 2050. Our theory is tractable enough that we can solve for the transitional dynamics induced by this path, treating the projections of the BLS as the rational expectations of the agents in our theory. We find this decline has quantitatively large effects. Our model can explain almost the entirety of the decline in the entry and exit rate, the increase in average firm size, and the degree of concentration. However, markups change little; our calibrated model implies markups increase by around 1%. The effect on income growth is more subtle. Whereas growth will inevitably decline in the long-run, the static effect of variety creation can increase income growth during the transition. We find that it does, for about two decades. However, overall the welfare consequences of falling population growth are negative.

Throughout the paper, we often speak of population growth and labor-force growth interchangeably. We take these to be exogenous to market concentration and firm dynamics. Across the developed world, decreases in fertility in the 1960s and 1970s manifested in slower rates of growth in the labor force in the 1980s and 1990s - see De Silva and Tenreyro (2017, 2020). In the US in particular, slowing labor-force growth also reflects an end to increasing female participation and declining prime-age male participation. Whereas a declining labor share and rising market power may themselves have implications for worker participation, here the simplicity of taking these movements as given yields substantial insight into the changing patterns of firm dynamics we see in the data.

Related Literature. We are not the first to connect the decline in the growth rate of the labor force to changes in firm dynamics. Karahan et al. (2016) and Hathaway and Litan (2014) are early contributions that use geographic variation to provide direct support that a lower rate of population growth reduces the start-up rate. Recently, Hopenhayn et al. (2018) document the relationship between changes in demographics and firm dynamics in a quantitative model. Both Karahan et al. (2016) and Hopenhayn et al. (2018) perform their analysis in a model in the spirit of Hopenhayn (1992), where firm productivity is exogenous and markets are competitive. By contrast, our theory builds on models with endogenous firm dynamics, and highlights that a declining rate of population growth also affects the extent of market power and aggregate productivity growth. Engbom (2017, 2020) studies the implications of population aging in the context of a search model.

Our theory builds on firm-based models of Schumpeterian growth in the tradition of Aghion and Howitt (1992) and Klette and Kortum (2004). We augment these models by allowing for efficiency improvements of existing firms as in Atkeson and Burstein (2010), Luttmer (2007), Akcigit and Kerr (2015), or Cao et al. (2017), the creation of new varieties as in Young (1998), and endogenous markups as in Peters (2020) or Acemoglu and Akcigit (2012). Our model is thus akin to a version of Garcia-Macia et al. (2019) or Klenow and Li (2021), augmented by endogenous markups and endogenous innovation choices, and incorporating changes in the long-run growth in the labor force. To the best of our knowledge, our paper is the first that focuses squarely on how demographic changes are likely to affect the equilibrium firm-size distribution in the context of firm-based models of growth.

The relationship between economic growth and population growth has been been subject to an extensive literature. Many models of endogenous growth share the feature that economic growth depends on the population level (e.g., Aghion and Howitt (1992), Romer (1990) or Grossman and Helpman (1991)). By contrast, models of semi-endogenous growth imply income growth is determined by the rate of population growth (e.g., Jones (1995), Kortum (1997), Young (1998) or Jones (2021)). In our model, growth is tied to the micro process of firm dynamics. This link puts tight restrictions on the relationship between economic growth and population growth. If growth depends on the level of the population, so does the firm size distribution. By contrast, if economic growth depends on population growth, the firm size distribution is also independent of the size of workforce and only a function of its growth rate. In order for the firm size distribution to be stationary in the presence of a growing population, growth thus (generically) needs to be semi-endogenous.[2] Moreover, the relationship between economic growth and population growth is governed by parameters that we can tightly discipline from firm-level data.

In our quantitative application, we focus on the case of the US. A growing literature highlights the decline of dynamism in the US. This literature shows that the entry rate has fallen substantially (Karahan et al., 2015; Alon et al., 2018; Decker et al., 2014), that broad measures of reallocation have declined (Haltiwanger et al., 2015; Davis and Haltiwanger, 2014), that industries are becoming more concentrated (Kehrig and Vincent, 2017; Autor et al., 2020), and that markups and profits are rising (Edmond et al., 2018; De Loecker et al., 2020; Van Vlokhoven, 2021). See also Akcigit and Ates (2019a) for a summary. In terms of explanations, the literature has proposed improvements in IT technology (Aghion et al. (2019); Lashkari et al. (2019)), a rise in the use of intangible capital (De Ridder (2019)), or changes in the process of knowledge diffusion (Akcigit and Ates (2019b); Olmstead-Rumsey (2020)). Our paper is complementary to these studies by highlighting that all these phenomena occurred within an environment of declining population growth and are key implications of the theory we propose. Falling population growth might therefore be an important secular determinant of firm-dynamics and aggregate growth in the decades to come.

2 The Baseline Model

Time is continuous and there is a mass L_t of identical individuals, each supplying one unit of labor inelastically. The rate of population growth $\dot{L}_t/L_t = \eta_t$, which we take as exogenous, is the crucial parameter of this paper. Households have preferences over a final consumption good c_t:

$$U = \int_0^\infty e^{-(\rho-\eta)t} \ln(c_t)\, dt,$$

where $\rho > \eta$.

Production and Market Structure. The final consumption good is composed of a continuum of differentiated varieties, that (as in Klette and Kortum (2004)) may be produced by multiple firms. The production of the final good takes place in a competitive sector that combines the differentiated varieties according to

$$Y_t = \left(\int_0^{N_t} \left(\sum_{f \in S_{it}} y_{fit} \right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}.$$

[2] In our model, population growth is not the only determinant of the equilibrium growth rate. In particular, our economy features growth even in the presence of a stable population. However, the growth rate is independent of the level of the population.

Here, N_t is the mass of available varieties indexed by i. This mass evolves endogenously through the creation of new and the destruction of old products.

S_{it} is the set of firms with the knowledge to produce product i, which likewise evolves endogenously. Firms can be active in multiple product markets. Each firm f is characterized by a set of the products it produces, denoted by Θ_f, and an efficiency of producing these products, indexed by $\{q_{fi}\}_{i\in\Theta_f}$. We denote the number of products firm f produces by n_f. Production of each good uses only labor, and is given by

$$y_{fi} = q_{fi} l_{fi},$$

where l_{fi} is the amount of labor hired by firm f to produce product i, and q_{fi} denotes the efficiency of firm f in producing product i.

Because the output of firms producing the same product i is considered to be perfectly substitutable, each product is only produced by the most efficient firm. Suppose to begin with that the producing firm charges a constant monopoly markup over marginal cost $\mu = \frac{\sigma}{\sigma-1}$.[3] With constant markups, aggregate output Y_t and equilibrium wages w_t are given by

$$Y_t = Q_t N_t^{\frac{1}{\sigma-1}} L_t^P \quad \text{and} \quad w_t = \mu^{-1} Y_t / L_t^P, \tag{1}$$

where $Q_t \equiv \left(\int q_i^{\sigma-1} dF_t\left(q\right)\right)^{\frac{1}{\sigma-1}}$ is a measure of average efficiency, F_t is the distribution of product efficiency, and L_t^P is the total amount of labor devoted to the production of goods. Equilibrium profits per product are given by

$$\pi_t\left(q\right) = \left(\mu - 1\right)\left(\frac{q}{Q_t}\right)^{\sigma-1} \frac{L_t^P}{N_t} w_t. \tag{2}$$

Hence, profits are high if the product's efficiency q is large relative to average efficiency Q_t and if average employment per products, L_t^P / N_t, is large.

2.1 Product Innovation, Entry and Aggregate Growth

Suppose first that entry is the sole source of innovation and aggregate growth. Entrants engage in product innovation and have access to a linear entry technology, where each worker generates a flow of φ_E new products. Conditional on successfully creating a new product, this product can either be a new variety, or it can improve upon an existing product from another firm. After entry, the firm's productivity q remains constant. We also assume that product lines die at an exogenous rate of δ. Product-line death can be interpreted as a taste shock in which consumers no longer value a product line for exogenous reasons. Doing so helps ensure stationarity at low or negatives levels of population growth, but is otherwise inconsequential.[4]

[3] This can either be the case if the producer's relative efficiency advantage exceeds μ or if firms have to pay an infinitesimal sunk cost before producing, in which case, the less productive firm will not enter (see Garcia-Macia et al. (2019)).

[4] See Jones (2020) for a recent analysis of the implications of negative population growth.

In the baseline we assume innovation is "undirected", so that entrants cannot target new or existing varieties.[5] With probability α, the new product represents a technological advance over a (randomly selected) incumbent firm's product, increases the product's efficiency by a factor $\lambda > 1$, and forces the current producer to exit ("creative destruction"). With the complementary probability $1 - \alpha$, the product will be new to the society as a whole, and the mass of available products N_t grows ("variety creation"). The efficiency of new varieties is given by $q' = \omega Q_t$, where ω is drawn from a fixed distribution $\Gamma(\omega)$. Hence, as in Buera and Oberfield (2020), the efficiency of new varieties is determined both by the existing knowledge embedded in Q_t and by novel ideas. It is useful to define $\overline{\omega} \equiv \left(\int \omega^{\sigma-1} d\Gamma(\omega)\right)^{\frac{1}{\sigma-1}}$, which we also refer to as the mean efficiency of new products. As we show below, the equilibrium allocations depend only on $\overline{\omega}$.

Let Z_t denote the aggregate flow of entry and $z_t = Z_t/N_t$ be the entry intensity per product. It then follows that the rate of new variety creation ν_t and the rate of creative destruction τ_t are given by

$$\nu_t = (1-\alpha) z_t \quad \text{and} \quad \tau_t = \alpha z_t. \tag{3}$$

Creative destruction τ and variety creation ν are closely linked. Our formulation of undirected innovation makes this link particularly stark. However, as we show in Section 2.7, the optimal level of creative destruction and variety creation positively co-move even in a more general setting where α is a choice variable and the direction of innovation is endogenous.

The rate of variety and efficiency growth are given by

$$g_t^N = \frac{\dot{N}_t}{N_t} = \nu_t - \delta = (1-\alpha) z_t - \delta \quad \text{and} \quad g_t^Q = \frac{\dot{Q}_t}{Q_t} = \frac{\lambda^{\sigma-1}-1}{\sigma-1}\tau_t + \frac{\overline{\omega}^{\sigma-1}-1}{\sigma-1}\nu_t. \tag{4}$$

Because $\lambda > 1$, creative destruction is a source of aggregate efficiency growth. Whether the creation of new varieties raises or lowers efficiency growth depends on their initial efficiency $\overline{\omega}$. If new products are, on average, as productive as existing products (that is, $\overline{\omega} = 1$), the growth rate of average efficiency Q_t is independent of the rate of product creation ν_t. If new products are, on average, worse ($\overline{\omega} < 1$), faster product creation has a negative effect on efficiency growth. Using (4) to substitute for ν_t and τ_t, we can write g_t^Q as a function of the entry rate z_t:

$$g_t^Q = \frac{\overline{q}^{\sigma-1}-1}{\sigma-1} z_t, \quad \text{where} \quad \overline{q} = \left(\alpha\lambda^{\sigma-1} + (1-\alpha)\overline{\omega}^{\sigma-1}\right)^{\frac{1}{\sigma-1}}. \tag{5}$$

Hence, $\overline{q}$ parametrizes the average efficiency gains of a product innovation and is simply a CES-weighted average of the efficiency improvement of creative destruction λ and the relative efficiency of new varieties $\overline{\omega}$. Average efficiency thus increases in z_t as long as $\overline{q} > 1$, i.e. as long as α and $\overline{\omega}$ are sufficiently large (recall that $\lambda > 1$).

Finally, the overall growth of labor productivity $Y_t/L_t^P = Q_t N_t^{\frac{1}{\sigma-1}}$, which we denote g_t^y, depends on

[5] In Section 2.7 we discuss endogenizing this choice.

both efficiency growth g_t^Q and variety growth g_t^N;[6]

$$g_t^y = g_t^Q + \frac{1}{\sigma-1} g_t^N = \frac{\lambda^{\sigma-1}-1}{\sigma-1}\tau_t + \frac{\overline{\omega}^{\sigma-1}}{\sigma-1}\nu_t - \frac{\delta}{\sigma-1} = \frac{\overline{q}^{\sigma-1}-\alpha}{\sigma-1} z_t - \frac{\delta}{\sigma-1}.$$

Note variety growth ν_t is always a source of aggregate growth, even if $\overline{\omega} < 1$. This also implies that aggregate productivity growth is increasing in the rate of entry because $\overline{q}^{\sigma-1} > \alpha$ (recall that $\lambda > 1$ and $\overline{\omega} > 0$).

To solve for the equilibrium rate of entry, we have to characterize the value of product creation. Let $V_t(q)$ denote the value of producing a product with quality q at time t. This value function solves the HJB equation:

$$r_t V_t(q) - \dot{V}_t(q) = \pi_t(q) - (\tau_t + \delta) V(q). \tag{6}$$

The value of the firm $V_t(q)$ is increasing in the current flow profits $\pi_t(q)$ and decreasing in the risk of exit, which happens at the endogenous rate of creative destruction τ_t and the exogenous rate of product loss δ.

Given $V_t(q)$, we can compute the value of entry. With probability α, the new product improves on a randomly selected product with efficiency $\tilde{q}$, yielding the value $V_t(\lambda\tilde{q})$. With the complimentary probability, a new variety with quality $\tilde{\omega}Q_t$ and the associated value $V_t(\tilde{\omega}Q_t)$ is created. Integrating over the existing quality distribution $F_t(q)$ and the exogenous distribution of the quality of new varieties $\Gamma(\omega)$ implies that the free entry condition is given by

$$\frac{1}{\varphi_E} w_t = V_t^{Entry} = \alpha \int V_t(\lambda\tilde{q})\, dF_t(\tilde{q}) + (1-\alpha)\int V_t(\tilde{\omega}Q_t)\, d\Gamma(\tilde{\omega}) \equiv \alpha V_t^{CD} + (1-\alpha) V_t^{NV}. \tag{7}$$

Equation (7) allows us to solve for $V_t(q)$ explicitly, both on and off a balanced growth path. Because profits are homogeneous in $q^{\sigma-1}$, so is the value function $V_t(q)$. This implies that $V_t^{CD} = V_t(\lambda Q_t)$, $V_t^{NV} = V_t(\overline{\omega}Q_t)$ and that $V_t^{Entry} = (\overline{q}Q_t)$, where $\overline{q}$ is given in (5). Hence, the value of entry is simply the value of a product with quality $\overline{q}Q_t$. And because (7) requires this value to be tied to the equilibrium wage, the solution to the differential equation in (6) is given by (see Section A-1.1.3 in the Appendix):

$$V_t(q) = \frac{\pi_t(q)}{r_t + \tau_t + \delta + (\sigma-1) g_Q - g_w}. \tag{8}$$

The value of a firm is the present discounted value of profits, where the appropriate discount rate reflects four distinct considerations: the interest rate (r_t), the risk of firm death $(\tau_t + \delta)$, the fact that a higher growth rate of average quality Q_t reduces the firms' relative competitiveness $((\sigma-1) g_Q)$, and the rate of wage growth (g_w).

[6] Along a BGP, where the share of production workers L_t^P / L_t is constant, income per capita also grows at the rate of labor productivity. .

2.2 Equilibrium

To characterize the equilibrium, define the two aggregate statistics:

$$\mathcal{N}_t \equiv \frac{N_t}{L_t} \quad \text{and} \quad \ell_t^P \equiv \frac{L_t^P}{L_t}.$$

We refer to $\mathcal{N}_t$ as the economy's *variety intensity* and to ℓ_t^P as the *production share*. These two aggregate statistics are sufficient to characterize the entire equilibrium path. To determine the path of $\{\mathcal{N}_t, \ell_t^P\}_t$, note first that labor market clearing requires that $L_t = L_t^P + N_t \frac{1}{\varphi_E} z_t$. Using that $z_t = \frac{1}{1-\alpha}\nu_t$, this can be written as

$$\frac{1-\ell_t^P}{\mathcal{N}_t} = \frac{1}{\varphi_E}\frac{\nu_t}{1-\alpha}. \tag{9}$$

Holding the variety intensity $\mathcal{N}_t$ constant, a higher production share ℓ_t^P reduces the creation of new varieties ν_t, as fewer resources are allocated toward research. Equation (9) is the first key equation to characterize the equilibrium.

The second key equation is the free-entry condition. The free entry condition in (7) and the solution for V_t in (8) imply that

$$\frac{1}{\varphi_E} = \frac{V_t(\bar{q}Q_t)}{w_t} = \frac{(\mu-1)\,\bar{q}^{\sigma-1}}{r_t+\tau_t+\delta+(\sigma-1)\,g_Q-g_w}\frac{\ell_t^P}{\mathcal{N}_t}. \tag{10}$$

Together with the consumer Euler equation $g_y = r_t - \rho$, the expressions for τ_t and g_Q given in (3) and (5) and the fact that $g^w = g^y - g^{\ell^P}$ (see (1)), these equations fully characterize the equilibrium path of our economy.

Consider first a BGP where the interest rate and the economy-wide growth rate is constant. This implies both variety creation ν and creative destruction τ are constant. Equations (9) and (10) then require that $\mathcal{N}_t$ and ℓ_t^P are constant. This has the important implication that the number of varieties N_t has to grow at the rate of population growth:

$$\eta = g_N = \nu_t - \delta = (1-\alpha)\,z - \delta. \tag{11}$$

The aggregate quantities of product innovation and entry are thus directly tied to the growth rate of the labor force η. Economically, this link between population and product growth is a consequence of the free entry condition. If the number of products was growing faster than the population, profits per product would be declining. Eventually, entry would stop as the equilibrium wage would exceed the value of product creation. Conversely, if population growth was higher than the rate of new product creation, flow profits would perpetually rise. The free entry condition would require a steady increase in the rate at which future profits are discounted. This, however, would eventually violate the economy's resource constraint.

With equation (11) at hand, we can analytically characterize the allocations along the BGP as a function of population growth:

Proposition 1. *On a BGP, the following holds:*

1. *The rates of variety creation ν, creative destruction τ, and entry z are given by*

$$\nu = \eta + \delta \qquad \tau = \frac{\alpha}{1-\alpha}(\eta+\delta) \qquad z = \frac{\eta+\delta}{1-\alpha}. \tag{12}$$

2. *Aggregate productivity growth g^y is given by*

$$g^y = \left(\frac{\overline{q}^{\sigma-1}-\alpha}{\sigma-1}\right)\frac{\eta}{1-\alpha} + \left(\frac{\overline{q}^{\sigma-1}-1}{\sigma-1}\right)\frac{\delta}{1-\alpha}, \tag{13}$$

   where $\overline{q}^{\sigma-1} = \alpha\lambda^{\sigma-1} + (1-\alpha)\,\overline{\omega}^{\sigma-1} > \alpha$ (see (5)).

3. *The production share ℓ^P and the variety intensity $\mathcal{N}$ are given by*

$$\mathcal{N} = \frac{\varphi_E(\mu-1)\,\overline{q}^{\sigma-1}}{\rho + \frac{\mu\overline{q}^{\sigma-1}}{1-\alpha}\delta + \left(\frac{\mu\overline{q}^{\sigma-1}}{1-\alpha}-1\right)\eta} \quad \text{and} \quad \ell^P = \frac{\rho + \frac{\overline{q}^{\sigma-1}}{1-\alpha}\delta + \left(\frac{\overline{q}^{\sigma-1}}{1-\alpha}-1\right)\eta}{\rho + \mu\frac{\overline{q}^{\sigma-1}}{1-\alpha}\delta + \left(\mu\frac{\overline{q}^{\sigma-1}}{1-\alpha}-1\right)\eta}. \tag{14}$$

Proof. See Section A-1.1.2 in the Appendix. □

Proposition 1 contains three key theoretical results of this paper. First, a decline in population growth reduces variety creation, creative destruction and entry. In particular, because each firm only produces a single product, the entry rate $\mathcal{E}$, that is the share of firms that enter at each point in time, is trivially given by $\mathcal{E} = z$ and hence declines as population growth falls.

Second, the rate of population growth directly affects the rate of growth in two ways. First, population growth determines variety creation, which is itself a form of growth. Second, population growth also affects creative destruction and hence the rate of efficiency growth g^Q. Although the effect of population growth on variety growth is always positive, its effect on efficiency growth depends on the average efficiency of newly created products $\overline{\omega}$ and the increment of creative destruction λ. The overall effect on income growth, however, is unambiguous: falling population growth reduces long-run income growth, as is typical in models of semi-endogenous growth. By contrast, changes in the cost of entry φ_E do not affect the growth rate, but only the level of varieties (see (14)).

Crucially, the relationship between population growth and income growth is determined by $\frac{\overline{q}^{\sigma-1}-\alpha}{(\sigma-1)(1-\alpha)}$ and hence governed by parameters that can be disciplined from firm-level data. To see this more clearly, consider for example a version of the canonical semi-endogenous growth model of Jones (2021) or Bloom et al. (2020) where $Y_t = A_t\overline{s}L_t$, $\dot{A}_t/A_t = A_t^{-\beta}((1-\overline{s})L_t)$ and $\overline{s}$ denotes the share of the population working as researchers. The parameter β governs the extent to which ideas are getting harder to find. Along a BGP, income per capita grows at rate $g^y = \frac{1}{\beta}\eta$. Hence, as far as aggregate income growth is concerned, our model implies that $\frac{1}{\beta} = \frac{\bar{q}^{\sigma-1}-\alpha}{(\sigma-1)(1-\alpha)}$. A lower average quality $\bar{q}^{\sigma-1}$ or a higher elasticity of substitution σ (for a given $\bar{q}^{\sigma-1}$) are akin to a large value of β. Intuitively, if product innovations are on average of low quality and products are very substitutable so that variety gains are limited, the growth implications of changes in population growth are *as if* it is technologically difficult to generate new ideas.

Third, the *level* of varieties relative to the population, $\mathcal{N}$, and the share of workers allocated to research ℓ^P are also functions of population growth η. As seen in equation (14), a decline in population growth increases the variety intensity $\mathcal{N}$ if and only if

$$\frac{\mu\bar{q}^{\sigma-1}}{1-\alpha} > 1, \tag{15}$$

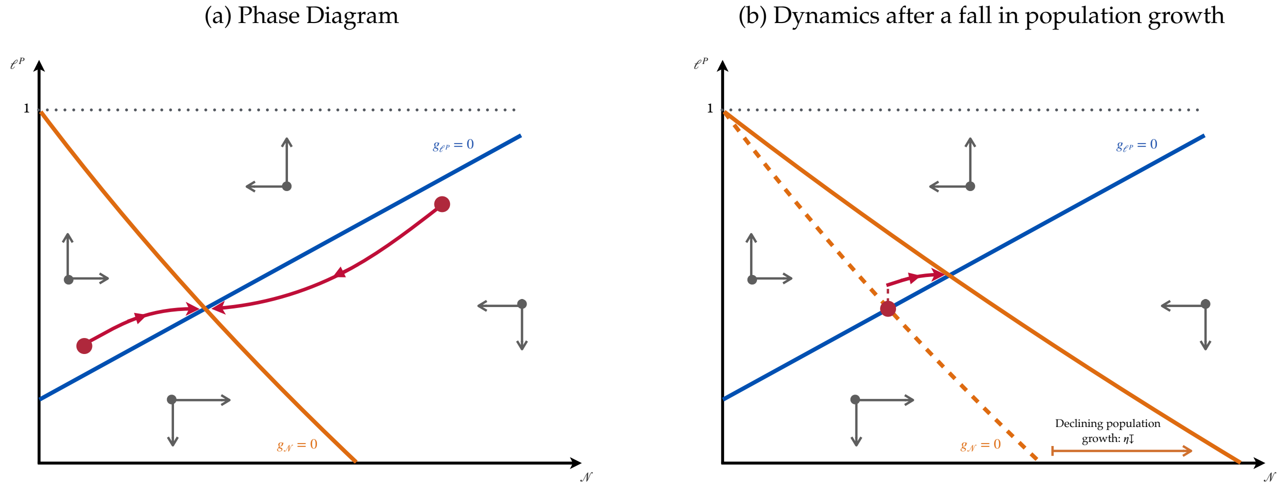
that is if λ, α, μ and $\overline{\omega}$ are sufficiently large. To understand the role of this restriction, note that $\tau + (\sigma-1)\,g_Q$ appears in the equilibrium discount rate of corporate profits (see (10)). Lower population growth reduces creative destruction. This channel increases the value of entry because firms live longer. At the same time, lower population growth also reduces variety creation ν, which could increase average quality growth g_Q if the average quality of new products $\bar{q}^{\sigma-1}$ is sufficiently low. This channel would lower the value of entry because firms face more competition during their life-time. As long as (15) is satisfied (which is the case for our estimated parameters), the creative-destruction effect always dominates the efficiency-growth effect, and falling population growth increases the value of entry through a higher rate of discounting.[7] Free entry therefore requires the *level* of flow profits to go down, which is achieved through an increase in the number of varieties per capita.

This increase in the variety intensity is a countervailing force to the negative growth implications of falling population growth. Hence, lower population growth has positive welfare consequences through a higher level of varieties (a "static" effect) but negative consequences via a decline in the growth rate (a "dynamic" effect). Similarly, ℓ^P is decreasing in η, so that falling population growth reallocates workers from the research to the production sector. Like the increase in the variety intensity, this reallocation increases income per capita statically.

Equations (9) and (10) not only describe the BGP, but the entire equilibrium path. We can characterize this path with a phase diagram akin to the neoclassical growth model, depicted in the left panel of Figure 2. The orange schedule depicts the locus of a stable variety intensity ($g_{\mathcal{N}} = 0$). This locus follows from the resource constraint: if ℓ_t^P is too high (low), there is too little (much) production innovation and the variety intensity falls (rises). The blue schedule shows the locus of a constant production share ($g_{\ell^P} = 0$) and summarizes the free entry condition. If the variety intensity is too high (low), there is little creative destruction and the production share, and with it flow profits, has to fall (rise) to satisfy free entry. Hence, there is a unique stable arm (shown in red), that takes the economy to the BGP characterized in Proposition 1: if the initial variety intensity is lower (higher) than its BGP value, initially, many (few) resources are allocated to research. During the transition,

[7] Note (15) is weaker than $\bar{q} > 1$, that is average efficiency growth can be declining in research expenditure. (15) requires that it cannot decline fast enough to outweigh the effect on creative destruction.

Figure 2: Equilibrium path of $\{\ell_t^P, \mathcal{N}_t\}$



Note: The left panel shows the phase diagram for the equilibrium path of $\left(\ell^P, \mathcal{N}\right)$. The right panel shows the response to a fall in population growth.

varieties are rising (falling) and workers are reallocated.

This analysis is not only useful to establish the stability and uniqueness of the equilibrium path, but also to analyze the dynamic impact of a fall in population growth. This experiment in shown in the right panel of Figure 2. A fall in population growth shifts the orange locus to the right: for a given variety intensity there are now too many workers employed in the research sector, given that the entry rate has to fall eventually. Hence, on impact, the production share ℓ_t^P jumps up in response to a fall in population growth. During the transition to the new BGP, there is continual reallocation out of the research sector and a rise in the variety intensity. This initial rise in the production of goods and the number of products per worker constitutes a source of welfare gains, especially in the short-run.

2.3 Adding Incumbent Innovation

So far, we assumed that entry is the sole source of innovative investment. We now extend our model to allow firms to engage in purposeful innovation spending during their life-cycle. Doing so is crucial to understand the link between population growth and firm-dynamics. To see why, let $S(A)$ denote the share of firms that survive until age a. In the absence of incumbent innovation, firms exit at rate $\tau + \delta$, so that

$$S(a) = e^{-(\tau+\delta)a} = e^{-\left(\frac{\alpha\eta+\delta}{1-\alpha}\right)A}. \tag{16}$$

Lower population growth reduces creative destruction, and therefore increases firms' chances of survival with age.[8] However, if firms do not experience any productivity growth after birth, average employment conditional on survival declines, because average quality Q_t (and the productivity of

[8] In fact, the share of firms of age a relative to new entrants is given by $\frac{zN_{t-a}e^{-(\tau+\delta)a}}{zN_t} = e^{-(\tau+\delta+\eta)a} = e^{-\frac{\delta+\eta}{1-\alpha}}$, and thus higher, the lower population growth.

younger firms) rises on average.[9] This is, of course, empirically counterfactual, and suggests that lower population growth would increase the aggregate importance of old and *small* firms. By allowing for incumbent innovation, we can capture the fact that firms, on average, grow as they age. As we will show, this implies that lower population growth increases the aggregate importance of old and *large* firms. At the same time, we also show that the aggregate implications of population growth contained in Proposition 1 survive almost unchanged.

We allow firms to grow both vertically, by increasing the efficiency of products they already produce, and horizontally by adding new products to their product portfolio. We model the vertical dimension as in Atkeson and Burstein (2010) or Luttmer (2007): incumbent firms increase the efficiency with which they produce their existing products deterministically at rate I, that is $\dot{q}_{it} = Iq_{it}$.[10] The horizontal dimension is modeled as in Klette and Kortum (2004): firms choose the Poisson rate X with which they expand into new product lines. Such expansion activities are costly, and we denote these costs (in units of labor) as

$$c_t^X (X, n) = \frac{1}{\varphi_x} X^\zeta n^{1-\zeta} = \frac{1}{\varphi_x} x^\zeta n, \tag{17}$$

where $\zeta > 1$, n denotes the number of products the firm is currently producing and $x = X/n$ is the firms' innovation intensity.[11] Conditional on successfully creating a product innovation, we treat incumbent firms entirely symmetrically to entrants, so that a fraction α of new ideas improve upon an exiting product and lead to creative destruction and a fraction $1 - \alpha$ yields a new variety, whose average efficiency is $\overline{\omega} Q_t$.

Hence, as in Garcia-Macia et al. (2019), our theory allows for own-innovation by existing firms, creative destruction (by both incumbents and entrants) and the creation of varieties (again both by incumbents and entrants). Letting $x_t = \frac{1}{N_t} \int x_{it} di$ denote the average expansion intensity by incumbent firms, the aggregate amount of variety creation ν and creative destruction τ are given by

$$\nu_t = (1 - \alpha)(z_t + x_t) \quad \text{and} \quad \tau_t = \alpha (z_t + x_t), \tag{18}$$

and reflect the activities of both entrants and incumbents. Moreover, aggregate efficiency growth is given by $g_t^Q = \frac{\overline{q}^{\sigma-1}-1}{\sigma-1}(z_t + x_t) + I$, which is similar to (5), except that the vertical dimension of life-cycle growth emerges as an additional source of aggregate efficiency gains.

Allowing for active innovative behavior by incumbent firms naturally changes the value function. Because firms can now produce multiple products with different qualities, let $\{q_{fi}\}_{i \in \Theta_f}$ be the state variables at the firm-level and $V_t\left(\{q_{fi}\}_{i \in \Theta_f}\right)$ be the corresponding value function. As we show

[9] Formally, let $\mathcal{L}(a)$ denote average employment of a firm of age a, conditional on survival. Then, $\mathcal{L}(a) = \mathcal{L}(0) e^{-(\sigma-1) g_Q a}$, i.e. average employment declines at rate g_Q as firms age.

[10] For simplicity, we start by assuming I is exogenous and constant over time. Below, in Section 2.7, we show how to endogenize this rate and how it depends on population growth.

[11] The particular functional form of the innovation cost function in (17) is not essential. All our results equally apply as long as $c_t^X (X, n)$ is homogeneous of degree one in both arguments.

in Section A-1.1 in the Appendix, this value function is additively separable across products, i.e. $V_t\left(\{q_{fi}\}_{i\in\Theta_f}\right) = \sum_{i=1}^{n_f} V_t(q_i)$. Moreover, the product-level value function $V_t(q)$ is given by the HJB equation

$$r_t V_t(q) - \dot{V}_t(q) = \pi_t(q) - (\tau_t + \delta) V_t(q) + \frac{\partial V_t(q)}{\partial q} Iq + \max_x \left\{ x\left(\alpha V_t^{CD} + (1-\alpha) V_t^{NV}\right) - \frac{x^\zeta w_t}{\varphi_x} \right\}, \tag{19}$$

where, as before, $V_t^{CD} = \int V_t(\lambda\tilde{q})\, dF_t(\tilde{q})$ and $V_t^{NV} = \int V_t(\tilde{\omega} Q_t)\, d\Gamma(\tilde{\omega})$.

Compared to the value function in the entry-only model (6), two additional considerations appear. First, product efficiency is now rising over the life-cycle. Second, the possibility of horizontal expansion carries an option value. As with entering firms, the ex-ante value of a successful product innovation is given by the weighed average $\alpha V_t^{CD} + (1-\alpha) V_t^{NV}$. Because product-innovation requires resources, (19) also takes into account the expansion costs $\frac{1}{\varphi_x} x^\zeta w_t$. However, despite these complications, the solution to (19) is again very similar to the case without incumbent innovation.

Proposition 2. *Consider the value function $V_t(q_i)$ in (19). On a BGP, $V_t(q)$ is given by*

$$V_t(q) = \underbrace{\frac{\pi_t(q)}{\rho + \tau + \delta + (\sigma - 1)(g^Q - I)}}_{\text{Production value}} + \underbrace{\frac{1}{\rho + \tau + \delta} \frac{\zeta - 1}{\varphi_x} x^\zeta w_t}_{\text{Innovation value}}, \tag{20}$$

where the optimal rate of product innovation x satisfies

$$x = \left(\frac{\varphi_x}{\zeta} \frac{\alpha V_t^{CD} + (1-\alpha) V_t^{NV}}{w_t}\right)^{\frac{1}{\zeta-1}} = \left(\frac{1}{\zeta}\frac{\varphi_x}{\varphi_E}\right)^{\frac{1}{\zeta-1}}. \tag{21}$$

Proof. See Section A-1.1 in the Appendix. □

Proposition 2 contains two important results. First, a comparison of (20) and the value function in the entry-only model (8) concisely highlights the consequences of allowing for incumbent innovation.[12] The value function in (20) is the sum of the net present value of flow profits (the "production value") and the option value of product innovation (the "innovation value"). The production value is almost the same as in the entry-only model above. The only difference is that the relative efficiency of a product $(q/Q_t)^{\sigma-1}$ changes at rate $(\sigma-1)\left(I - g^Q\right)$ rather than $-(\sigma-1)\, g^Q$ if q is constant. The innovation value is, of course, absent in the entry-only model. If incumbent innovation gets prohibitively expensive, i.e. $I = 0$ and $\varphi_x = \infty$, the solution in (20) coincides with (8) in the entry-only specification.

The second important implication of Proposition 2 is that the equilibrium rate of incumbent product innovation x is constant and a function of technology only. It is independent of *any* general equilibrium variables and, in particular, does not depend on the rate of population growth η. The reason is that the free entry condition in (7) still applies, which ties the ex-ante value of product innovation

[12] Note that, along a BGP, the discount rate in (8) is given by $\rho + \tau + \delta + (\sigma - 1)\, g_Q$.

$\alpha V_t^{CD} + (1-\alpha) V_t^{NV}$ to the entry costs $\frac{1}{\varphi_E} w_t$. Economically, it follows from the fact that incumbents' innovation technology has decreasing returns at the firm level, whereas entry, which operates at the aggregate level, has constant returns.[13] Hence, the free-entry condition pins down the value of product creation, and incumbent firms optimally choose the rate of product creation to equalize the marginal cost and the marginal benefits. This also implies that equation (21) holds both on and off the BGP and relies only on the free-entry condition to be binding.[14]

With the results of Proposition 2 at hand it is also immediate why most results of Proposition 1 still apply even in the presence of incumbent innovation. Labor market clearing and free entry (that is, the analogues of (9) and (10)) are now given by

$$\frac{1-\ell^P}{\mathcal{N}} = \frac{z}{\varphi_E} + \frac{1}{\varphi_x} x^\zeta \quad \text{and} \quad \frac{1}{\varphi_E} = \frac{V(\bar{q} Q_t)}{w_t} = \frac{(\mu-1)\,\bar{q}^{\sigma-1}}{\rho+\tau+\delta+(\sigma-1)\left(g^Q - I\right)} \frac{\ell^P}{\mathcal{N}} + \frac{(\zeta-1)\frac{1}{\varphi_x} x^\zeta}{\rho+\tau+\delta}, \tag{22}$$

and thus still require the number of products to grow at the same rate as the population. This in turn implies that variety creation ν and creative destruction τ are still given by the same expressions as in Proposition 1 and that the aggregate growth rate also takes the same form, augmented by the vertical dimension of own-innovation I:

$$g^y = I + \left(\frac{\bar{q}^{\sigma-1}-\alpha}{\sigma-1}\right)\frac{\eta}{1-\alpha} + \left(\frac{\bar{q}^{\sigma-1}-1}{\sigma-1}\right)\frac{\delta}{1-\alpha}. \tag{23}$$

Note again that the efficiencies of horizontal product innovation, φ_x or φ_E, do not determine the rate of growth but only the level of varieties and the allocation of labor (22).

Finally, the share of production workers ℓ^P and the variety intensity $\mathcal{N}$ are uniquely determined from (22) and the condition in (15) is still a sufficient condition for $\mathcal{N}$ to increase as population growth declines.[15]

The important new insight of allowing for innovative activity by incumbent firms is that the *composition* of product innovation between entrants and incumbent firms is endogenous, and depends on population growth. In particular, equations (18) and (21) imply that

$$z = \frac{\nu}{1-\alpha} - x = \frac{\eta+\delta}{1-\alpha} - x,$$

where x is constant and given in (21). Hence, the entirety of the decline in population growth is absorbed by the economy's extensive margin - entrants do all the work and incumbents' rate of product creation is insulated from demographics. This compositional change, whereby falling popu-

[13] Note that incumbent product creation also has constant returns in the aggregate: if the number of incumbent firms were to double, the amount of aggregate product creation performed by incumbents would also double.

[14] In Section 2.7 below, we generalize our results to the case in which the entry process has decreasing returns in the aggregate. In that case, x also depends on general equilibrium variables and is affected by population growth.

[15] The reason that (15) is now a sufficient and no longer a necessary condition, is the presence of the innovation value. Because falling population growth reduces creative destruction, the innovation value increases. This increases the value of entry. Free entry thus requires a stronger increase in $\mathcal{N}$ compared to the entry-only model.

lation growth increases incumbent innovation relative to entrants is a key aspect of how population growth changes the process of firm dynamics and the firm-size distribution.

2.4 Population Growth and Firm Dynamics

Above we already highlighted the impact of population growth on firms' chances of survival: lower population growth leads to less creative destruction and hence older firms - see (16). If firms grow over their life-cycle, this shift in the age distribution causes concentration. To see this formally in our model, define firms' *net* rate of product accumulation $\psi = x - (\tau + \delta)$, which is exactly the difference between the rate of product loss $\tau + \delta$ and the accumulation of products x. Using (12) to express τ in terms of the rate of population growth η yields $\psi = x - \frac{\alpha\eta+\delta}{1-\alpha}$, that is, a decline in the rate of population growth increases the net rate of product accumulation, as firms face less of a threat of creative destruction.

This net accumulation rate ψ emerges as the key determinant for the process of firm dynamics. Let $S(a)$ again denote the survival function, that is, the probability that a given firm survives until age a. Moreover, let $\overline{n}(a)$ denote the average number of products of a firm of age a. As we show in Section A-1.1.8 in the Appendix, $S(a)$ and $\overline{n}(a)$ are given by

$$S(a) = \frac{\psi e^{\psi a}}{\psi - x\left(1 - e^{\psi a}\right)} \quad \text{and} \quad \overline{n}(a) = 1 - \frac{x}{\psi}\left(1 - e^{\psi a}\right). \tag{24}$$

In Figure 3, we display $S(a)$ and $\overline{n}(a)$ graphically. Naturally, $S(a)$ is declining and satisfies $\lim_{A\to\infty} S(a) = 0$, because all firms exit eventually. Similarly, $\overline{n}(a)$ is increasing because surviving firms are selected on having had many successful product innovations and little creative destruction.[16] More importantly, lower population growth *increases* firms' survival rates and their life-cycle profile of product growth; that is, firms are becoming bigger *conditional* on age, because their expansion incentives do not change, and they lose products less often. Lower population growth therefore raises market concentration by shifting the age distribution towards older (and hence large) firms, and by increasing firm size conditional on age.[17]

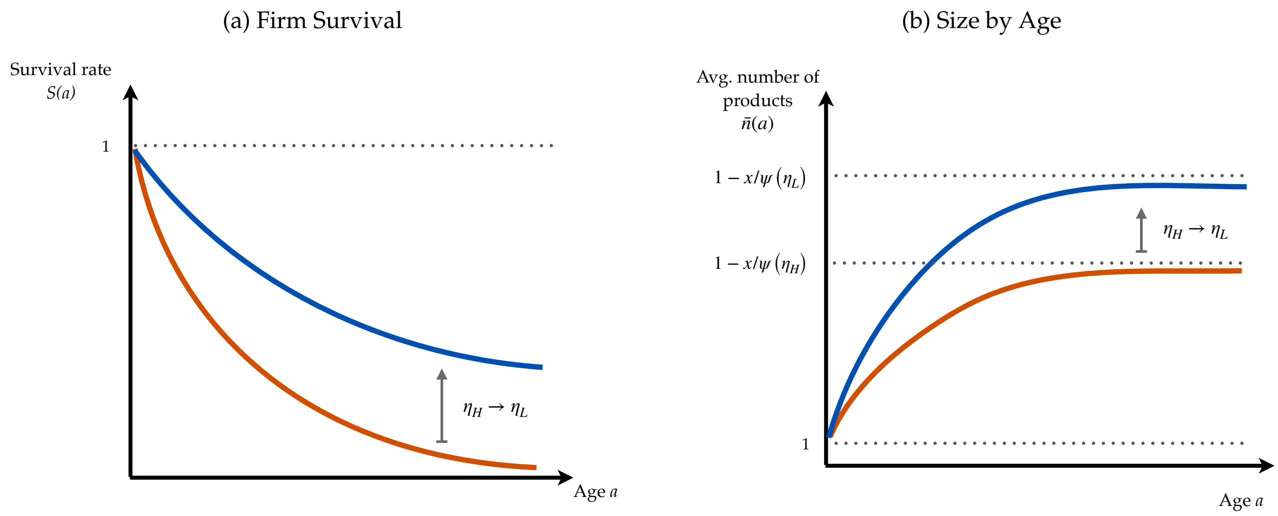
Interestingly, and in contrast to the entry-only model, these increases in concentration and firm size go hand in hand with an *increase* in the aggregate variety intensity $\mathcal{N}_t = N_t/L_t$. This is due to the multi-product nature of our theory: whereas population growth reduces the number of firms per worker, it increases the number of products per worker because each existing firm offers a larger product portfolio. Hence, higher concentration can coexist with an expansion of product variety. This potential positive welfare effect is absent in theories without multi-product firms.

Another way to analyze the effect of population growth on concentration is to consider the right tail

[16] Note that if incumbents firms do not engage in product innovation, that is $x = 0$, the expressions in (24) reduce to $\overline{n}(a) = 1$ and $S(a) = e^{-\left(\frac{\alpha\eta+\delta}{1-\alpha}\right)a}$ as in (16).

[17] In fact, one can show the average age of firms is given by $\mathbb{E}[\text{Age}] = \frac{1}{x}\ln\left(\frac{\alpha\eta+\delta}{\alpha\eta+\delta-(1-\alpha)x}\right)$, which is decreasing in η.

Figure 3: Falling Population Growth and Rising Concentration



Note: The figure shows the relationship between population growth η and firms' survival probabilities $S(a)$ in the left panel and the relationship between population growth η and the average number of products $\overline{n}(a)$ in the right panel.

of the size distribution. Because firm-level employment is given by

$$l_{ft} = \sum_{i=1}^{n_f} \left(\frac{q_i}{Q_t}\right)^{\sigma-1} \times \frac{L_t^P}{N_t} = \sum_{i=1}^{n_f} \left(\frac{q_i}{Q_t}\right)^{\sigma-1} \times \frac{\ell^P}{\mathcal{N}}, \tag{25}$$

the firm-size distribution depends on the distribution of the number of products n and of scaled efficiency q/Q. As we show in Section A-1.1.8 in the Appendix, both of these distributions may have a Pareto tail. The right tail of the employment distribution is thus given by

$$\varrho_l = \min\left\{\varrho_n, \frac{1}{\sigma-1}\varrho_q\right\},$$

where ϱ_n is the tail of the product distribution and ϱ_q is the tail of the scaled efficiency distribution. Intuitively, firms can be large in two ways: by having many products, or by having an extraordinarily good product.

As as long as $\eta > \psi > 0$, the results of Luttmer (2011) imply that the distribution of the number of products n_f has a Pareto tail ϱ_q, which is given by

$$\varrho_n = \frac{\eta}{\psi} = \frac{(1-\alpha)\eta}{x(1-\alpha)-\delta-\alpha\eta}. \tag{26}$$

Hence, the Pareto tail of the product distribution is a closed-form expression of the rate of population growth η, and a decline in population growth increases concentration, that is, lowers ϱ_n toward unity. Equation (26) highlights that population growth affects the product distribution through two channels. Holding firms' net expansion rate ψ constant, lower population growth increases concentration because it reduces the rate at which new firms, which are, on average, small by virtue of being young, enter. In addition, lower population growth endogenously increases the net accumulation rate ψ and concentration.[18]

The distribution of relative efficiency also has a Pareto tail. In particular, as long as the entrant efficiency distribution Γ has a thin tail, the tail parameter ϱ_q is implicitly defined by (see Section A-1.1.8 of the Appendix):

$$\varrho_q \left(\frac{\overline{q}^{\sigma-1} - 1}{\sigma - 1} \right) = -1 + \alpha \lambda^{\varrho_q}, \tag{27}$$

and hence depends on λ, α, σ, and $\overline{q}$. Interestingly, and in stark contrast to (26), the tail of the efficiency distribution ϱ_q is independent of population growth η. As $\lambda \to 1$, ϱ_q approaches $\varrho_q = (\sigma - 1) / \left(1 - \overline{\omega}^{\sigma-1}\right)$. Hence, if creative destruction does not contribute to efficiency growth, the tail of the efficiency distribution will be thicker the lower the relative efficiency of new varieties $\bar{\omega}$.

To summarize, declining population growth always increases concentration, because incumbent firms expand at a faster rate and survive longer. Whether this increase in concentration also shows up in the tail of the size distribution depends on the comparison of ϱ_n and ϱ_q . If $\varrho_n < \varrho_q$, lower population growth reduces the tail of the employment distribution. If $\varrho_q < \varrho_n$, the right tail of the employment distribution is unaffected by population growth. Which of these tail coefficients dominates is a quantitative question.

2.5 Growth and Firm-Dynamics without Scale Effects

One key implication of our theory is the absence of scale effects, both for the aggregate growth rate and the equilibrium firm size distribution. For the aggregate growth rate, this is immediately apparent from (23): as in the semi-endogenous growth model of Jones (1995), the rate of growth depends on the rate of population growth and is independent of the population level.[19] For the firm-size distribution this follows because along a BGP, both the distribution of the number of products and the distribution of scaled qualities are stationary. They are fully determined from the entry flow z, the rate of product innovation by incumbents x, and the rate of own-innovation I, all of which are independent of the level of the population L_t.[20]

[18] Note (26) can also be written as $\varrho_n = \frac{\eta}{\eta - z}$; that is, concentration is large if the flow of new entrants z is small *relative* to population growth η. Our theory implies that a decline in η reduces both z and $\frac{\eta}{\eta - z}$. This also implies the product distribution does not have a Pareto tail in the absence of population growth (as is the case in Klette and Kortum (2004)).

[19] In contrast to Jones (1995), however, our model features the possibility of positive growth even if the population is stationary, that is $\eta = 0$ - see (13). If there is no vertical growth through own-innovation ($I = 0$) and the average quality of product innovation coincides with the average quality in the population of existing products ($\overline{q}^{\sigma-1} = 1$), (13) reduces to $g^y = \left(\frac{1}{\sigma - 1}\right) \eta$, where the elasticity of substitution σ takes the role of the aggregate returns to scale.

[20] This result does not hinge on taking I to be exogenous, which we assumed for purely expositional purposes. In Section 2.7 we treat I as endogenous and show that it is independent of level of the population.

The reason why our theory does not generate strong scale effects for the firm size distribution is that the *level* of varieties is endogenous: a larger population increases the mass of products and the number of firms but leaves the distribution of firm-size unchanged. This is in contrast to firm-based models of growth that feature strong scale effects for the aggregate growth rate. As a case in point, consider for example Klette and Kortum (2004), which features strong scale effects and implies that the firm size distribution depends on the size the population: an increase in the level of the population increases the entry flow z but leaves incumbent innovation unchanged.[21] Hence, larger countries have faster growth, more creative destruction, higher entry and smaller firms. By contrast, our theory features weak scale effects: countries with higher population growth have faster income per capita growth, more creative destruction, higher entry and smaller firms. However, differences in the level of the population simply change the number of varieties N_t, leaving the process of firm-dynamics and aggregate growth unchanged.[22]

The reason for these differences is embedded in the free entry condition. If the mass of products is taken as exogenous, free entry requires that a larger population, which comes with higher profits for potential entrants, leads to higher discounting via creative destruction. In our model, the adjustment operates through the level of flow profits. Importantly, changes in creative destruction lead to changes in the growth rate and the firm size distribution, whereas changes in the variety intensity $\mathcal{N}$ lead to changes in the level of productivity while keeping the rest of the economy stationary. In that sense, our model is akin to Young (1998), augmented with a full endogenous process of firm dynamics.

The same intuition also applies to understand the consequences of perpetual population growth instead of changes in the size of the population. In models with a fixed product space, population growth leads to rising growth and a non-stationary firm size distribution. In our model, population growth is consistent with a BGP and a stationary firm size distribution. By contrast, *changes* in the rate of population growth change both the aggregate growth rate and the firm size distribution. Given that empirically the distribution of firm size is reasonably stable when compared to the large changes in the size of the population, firm-based models of growth would seem to point towards a world of semi-endogenous growth.

[21] The Klette and Kortum (2004) model is nested in our framework. It is a parametrization where the population is constant (i.e. $\eta = 0$), there is no own-innovation (i.e. $I = 0$) and the mass of varieties is exogenous (i.e. $\alpha = 1$ and $\delta = 0$) and equal to $\overline{N}$. Incumbents' innovation efforts are still constant and given by (21). The rate of entry is given by $z = \tau - x$. The rate of efficiency growth is thus given by $g_t^Q = \frac{\lambda^{\sigma-1}-1}{\sigma-1}\tau$ because creative destruction is the only source of growth so that $\overline{q} = \lambda$. The rate of creative destruction τ and the production share ℓ^P are determined from the free entry condition and the resource constraint as

$$\frac{1}{\varphi_E} = \frac{(\mu-1)\,\lambda^{\sigma-1}\frac{L_t}{\overline{N}}\ell^P}{\rho+\lambda^{\sigma-1}\tau} + \frac{1}{\rho+\tau}\frac{\zeta-1}{\varphi_x}x^{\zeta} \quad \text{and} \quad \frac{L}{\overline{N}}\left(1-\ell^P\right) = \frac{1}{\varphi_E}\left(\tau - \frac{\zeta-1}{\zeta}x\right).$$

It is easy to show that an increase in L increases τ. And because x is constant, an increase in L also increases z.

[22] Naturally, a larger population still has positive welfare effects, because the rise in the number of varieties increases aggregate productivity. This, however, is a level effect, not a growth effect.

2.6 The Mechanism: Demand or Supply?

Falling population growth impacts the economy both through a decline in the growth rate of labor supply and via aggregate demand. It is thus natural to ask whether the resulting changes in firm dynamics and aggregate growth reflect supply, demand or both.

To distinguish the supply and the demand channel, we can extend our model to a multi-sector environment. In particular, suppose there is a second sector that has access to a production technology $Y_t = \mathcal{A}_t H_t$, where H_t denotes the number of workers in sector 2 and $\mathcal{A}_t$ grows at an exogenous rate $g^{\mathcal{A}}$. The mass of sector-2-workers H_t grows at rate η^H and they can only provide labor to sector 2. As before, the mass L_t of sector-1-workers grows at rate η. All individuals have identical intra-temporal Cobb Douglas preferences $c_t = c_{1t}^{\vartheta} c_{2t}^{1-\vartheta}$, and spend a share ϑ on goods of sector 1. This extended model can either be thought of as an open economy extension, where sector 2 is a foreign country with its own productivity and population growth, or as a second industry in a closed economy where workers' skills (and hence labor supply) is sector-specific. This allows us to independently vary aggregate demand and labor supply for sector 1. Changes in population growth of sector-1-workers, η, still have supply and demand effects. By contrast, population growth of sector-2-workers, η^H, or changes in productivity growth $g^{\mathcal{A}}$ have no effect on labor supply in sector 1.

In Section SM-2 in the Supplementary Material we characterize this model in detail. In particular, we derive the analogue of Proposition 1 in this more general environment. This analysis delivers two striking results. First, we show that the process of firm-dynamics and the resulting firm-size distribution are *exactly* the same as in our baseline economy. The rate of creative destruction τ, variety creation ν and entry and incumbent innovation z and x are still given by (12), the rates of quality and variety growth g^Q and g^N are still given by (13) and the variety intensity $\mathcal{N}$ and the production share ℓ^P are still determined from (22). Crucially, neither $g^{\mathcal{A}}$ nor η^H have any effect.

By contrast, real consumption growth of workers in sector 1 now takes a different form. In particular, we show that

$$g^c = \vartheta \left(g^Q + \frac{1}{\sigma - 1} g^N \right) + (1 - \vartheta) \left(g^{\mathcal{A}} + \eta^H - \eta \right).$$

Hence, real consumption growth is an expenditure-share weighted average between productivity growth in sector 1, $g^Q + \frac{1}{\sigma-1} g^N$, and productivity and relative population growth in sector 2, $g_{\mathcal{A}} + \eta^H - \eta$. An increase in foreign productivity growth $g^{\mathcal{A}}$ raises domestic consumption growth through cheaper relative prices. Similarly, faster population growth abroad, $\eta^H > \eta$, is a source of welfare gains as relative wages fall, and sector 2 goods become relatively cheaper.

These results highlight how population growth operates in our model. The relationship between population growth, firm-dynamics and local productivity growth is a *supply side* phenomenon, and faster demand or population growth abroad is unable to counteract these trends. To what extent falling local population growth has adverse welfare effects, however, depends on the importance of the local economy in a consumer's consumption basket. In an open economy, domestic consumers can be somewhat isolated from falling population growth by importing global productivity or population growth.

2.7 Discussion of Assumptions

Four assumptions make our theory particularly tractable. First, we assume the economy has access to a linear entry technology. Second, product creation is undirected: a constant share α of product innovation results in creative destruction rather than new-variety creation. Third, we took the rate of own-innovation I to be exogenous. Forth, innovation and entry only require labor, and hence scale proportionally with aggregate productivity Q_t. In this section, we show why these assumptions are attractive, and that our main results qualitatively do not hinge on them.

Decreasing Returns in the Entry Technology. Assume the productivity of entrant labor hired to produce new ideas is given by

$$\varphi_E(z_t) = \tilde{\varphi}_E z_t^{-\chi} \text{ where } \chi \geq 0. \tag{28}$$

Here, z_t is the aggregate entry rate that each entrant takes as given. For $\chi = 0$, this specification yields the constant-returns case analyzed above. For $\chi > 0$, the cost of entry rises with the aggregate entry rate. We refer to χ as the strength of congestion.

Under (28), free entry requires that $V_t(\bar{q}Q_t) = \frac{1}{\varphi_E(z_t)} w_t = \frac{1}{\tilde{\varphi}_E} z_t^{\chi} w_t$. Hence, to the extent that there is congestion, that is $\chi > 0$, the average value of product creation (relative to the wage) is increasing in the aggregate entry rate. Alternatively, the aggregate entry supply curve is increasing in the value of entry with an elasticity $1/\chi$. For our baseline case of $\chi = 0$, entry is infinitely elastic.

Irrespective of the entry technology, the rate of variety growth is still tied to the rate of population growth, i.e. $\nu = \eta + \delta$. This directly implies that two important results of Proposition 1 still apply: the rate of creative destruction is still given by $\tau = \frac{\alpha}{1-\alpha}(\eta + \delta)$ (see (12)) and the aggregate growth rate g^Y is still given in (13).

By contrast, the composition of product innovation depends on the strength of congestion χ. The policy function of incumbents and the congestion-adjusted free-entry condition imply

$$\tau = \alpha(z + x) = \alpha\left(z + \left(\frac{\varphi_x}{\zeta}\right)^{\frac{1}{\zeta-1}} \left(\frac{V_t(\bar{q}Q_t)}{w_t}\right)^{\frac{1}{\zeta-1}}\right) = \alpha\left(z + \left(\frac{\varphi_x}{\zeta\tilde{\varphi}_E}\right)^{\frac{1}{\zeta-1}} z^{\frac{\chi}{\zeta-1}}\right).$$

Using $\tau = \frac{\alpha}{1-\alpha}(\eta + \delta)$, the equilibrium entry flow z is thus uniquely determined from the equation

$$\frac{\eta + \delta}{1 - \alpha} = z + \left(\frac{\varphi_x}{\zeta\tilde{\varphi}_E}\right)^{\frac{1}{\zeta-1}} z^{\frac{\chi}{\zeta-1}}.$$

It easy to see z is declining in η; that is falling population growth still reduces the entry flow z. Given z, the rate of incumbent product creation x is given by $x = (\varphi_x/(\zeta\tilde{\varphi}_E))^{\frac{1}{\zeta-1}} z^{\frac{\chi}{\zeta-1}}$. For the case of no congestion, $\chi = 0$, the solution is exactly as in Proposition 1 and x does not depend on population growth. If $\chi > 0$, x is increasing in z and hence also declining in population growth. Whether changes in population growth affect entrants or incumbents relatively more depends on the congestion elasticity χ relative to the convexity of the cost function ζ. In particular, $z/x \propto z^{\frac{\zeta-1-\chi}{\zeta-1}}$ so that entrants respond relatively more to changes in population growth if $\zeta - 1 > \chi$, that is, if the entry cost elasticity χ is smaller than the incumbent cost elasticity $\zeta - 1$. Hence, qualitatively, all the results derived above hold true as long as $\chi < \zeta - 1$. The case of $\chi = 0$ makes the "entry dependence" particularly salient.

Endogenizing the Direction of Innovation α. Our second assumption concerns the direction of innovation α. In Section A-1.2 in the Appendix, we present a detailed analysis of an extension of our model, where entrants and incumbents can directly choose the flow rate at which they want to creatively destroy products (x_{CD} and z_{CD}) and at which they want to create new varieties (x_{NV} and z_{NV}). Hence, $\tau = z_{CD} + x_{CD}$ and $g_N = x_{NV} + z_{NV}$. For entrants, we assume, as in the baseline model, that each worker can generate φ_E new business ideas. To turn a business idea into a viable product, new firms have access to the same innovation technology as incumbent firms. This structure maintains both the symmetry between entrants and incumbents and the linear entry technology, but endogenizes the direction of innovation α.

This extension of our model is still very tractable. First, the value function takes a similar form to (20) in Proposition 2: the value of having a product with efficiency q is given by

$$V_t(q) = \left(\frac{(\mu - 1)\left(\frac{q_i}{Q_t}\right)^{\sigma-1} \frac{L_t^P}{N_t}}{\rho + \tau + \delta + \left(g^Q - I\right)(\sigma - 1)} + \frac{\zeta - 1}{\rho + \tau + \delta}\left(\frac{x_{NV}^{\zeta}}{\varphi_{NV}} + \frac{x_{CD}^{\zeta}}{\varphi_{CD}}\right)\right) w_t.$$

Second, the optimal rates of incumbent innovation are given by

$$x_{NV} = \left(\frac{\varphi_N}{\zeta}\frac{V_t(\overline{\omega} Q_t)}{w_t}\right)^{\frac{1}{\zeta-1}} \text{ and } x_{CD} = \left(\frac{\varphi_{CD}}{\zeta}\frac{V_t(\lambda Q_t)}{w_t}\right)^{\frac{1}{\zeta-1}}. \tag{29}$$

Third, because entering firms have the same innovation technology as incumbents, $z_{NV} = x_{VN} z$ and $z_{CD} = x_{CD} z$, where z is the aggregate flow of entry (per product N_t). Fourth, free entry requires that

$$\frac{1}{\varphi_E} = \frac{\zeta - 1}{\varphi_{NV}} x_{NV}^{\zeta} + \frac{\zeta - 1}{\varphi_{CD}} x_{CD}^{\zeta}. \tag{30}$$

Equation (29) highlights why variety creation and creative destruction are tightly linked: both depend on the same value function $V_t(q)/w_t$. In fact, in a special case, this model is exactly isomorphic to our baseline model. Suppose $\lambda = \overline{\omega}$; that is, new varieties and creatively destroyed products have, on average, the same initial efficiency. Equation (29) then implies

$$\alpha = \frac{x_{CD}}{x_{CD} + x_{NV}} = \frac{\varphi_{CD}^{1/(\zeta-1)}}{\varphi_{CD}^{1/(\zeta-1)} + \varphi_N^{1/(\zeta-1)}}.$$

As in our baseline model, we can thus write $x_{CD} = \alpha x$ and $x_{NV} = (1 - \alpha)x$, where $x = x_{CD} + x_{NV}$

is the total quantity of incumbent innovation. The free-entry condition (30) then implies x is again fully determined from parameters and insulated from demographics. Finally, the rate of creative destruction and the amount of entry z are given by

$$(1-\alpha)\,x\,(1+z) = \eta + \delta \quad \text{and} \quad \tau = \alpha x\,(1+z) = \frac{\alpha}{1-\alpha}\,(\eta+\delta)\,.$$

Hence, as in our baseline model, falling population reduces creative destruction, and all the adjustment is achieved through a reduction in entry. In Section A-1.2 in the Appendix, we also analyze the general case of $\lambda \neq \overline{\omega}$, which implies α is no longer constant. However, falling population growth still reduces both creative destruction and the relative importance of entrants z/x.

Endogenous Own-Innovation I. Finally, consider our choice of treating the rate of own-innovation I as exogenous. As we show in Section A-1.2 in the Appendix, all of our results extend to a case where I is endogenous in a straight-forward way. In particular the expressions for τ and g_N are exactly the same as in Proposition 1 and so is the expression for the equilibrium growth rate g^y, except that I is no longer a parameter but a choice variable. This endogenous rate of own-innovation is in turn implicitly defined by

$$I = \left(\frac{(\sigma-1)\,(\mu-1)\,L_t^P/N_t}{\rho+\tau+\delta+\left(g^Q - \frac{\zeta-1}{\zeta} I\right)(\sigma-1)} \frac{\varphi_I}{\zeta} \right)^{\frac{1}{\zeta-1}}. \tag{31}$$

Note that I depends on the rate of population growth both through the discount rate (i.e. τ and g^Q) and the level of flow of profits (i.e. L_t^P/N_t) but is independent of the population level. Because lower population growth reduces τ and g^Q and increases N_t/L_t^P, it seems that the effect of population growth on own-innovation is ambiguous. This, however, is not the case. Using the free entry condition, one can show that the optimal rate of own-innovation I is given by

$$I = \varsigma \left(1 - \frac{\left(\frac{\zeta-1}{\zeta}\right)\left(\frac{1}{\zeta}\frac{\varphi_x}{\varphi_E}\right)^{\frac{1}{\zeta-1}}}{\rho+\tau+\delta} \right)^{\frac{1}{\zeta-1}}, \tag{32}$$

where ς is a collection of structural parameters. Note that (32) expresses I directly as a function of parameters and a single endogenous variable - the rate of creative destruction. And because I is increasing in the rate of creative destruction, a decline in population growth *reduces* the rate of own-innovation. This endogenous response of incumbents' own-innovation efforts thus amplifies the negative growth consequences of falling population growth.

The fact that I is increasing in the rate of creative destruction might at first seem surprising. After all, a higher rate of creative destruction reduces the expected life-span, which should reduce firms' incentives to invest in productivity improvements. Equation (31) shows that this intuition is indeed correct: holding the market size L_t^P/N_t fixed, a higher rate of creative destruction reduces the rate of own-innovation. However, once the change in L_t^P/N_t is taken into account, the general equilibrium effect of a higher rate of creative destruction becomes positive. The reason is that free entry requires the average production value *plus* the innovation value to be equal to the entry costs. A lower rate of population growth *increases* the innovation value because creative destruction declines. Hence, for the free entry condition to be satisfied, the production value has to *decrease.* And as the returns to own-innovation scale with the production value but not the innovation value, the returns to own-innovation are lower in an environment with lower population growth.

Rising Innovation Costs. We have also assumed innovation costs scale with overall productivity Q_t. In particular, the entry cost is a fixed amount of labor, and hiring the same amount of labor generates a constant flow of useful ideas regardless of how advanced the economy is. Doing so is crucial to generate a stationary firm size distribution. We can modify the theory to incorporate rising costs of entry as the economy grows. Formally, suppose that the cost of entry and incumbent innovation was given by $\frac{1}{\varphi_E} Q_t^{\varsigma}$ and $\frac{1}{\varphi_x} x^{\zeta} n Q_t^{\varsigma}$. Hence, if $\varsigma > 0$, a higher level of productivity Q_t increases the costs of further product innovation. We analyze this case in Section A-1.2.3 in the Appendix. The main implication is that average firm size is no longer constant, but must rise forever on the balanced growth path. Intuitively, if the cost of entry rise faster than aggregate productivity, free entry requires that aggregate profits must also rise. This is achieved though a decline in competition and a continual increase in the size of firms.

3 Extension for the Quantitative Analysis: Endogenous Market Power

So far, we have assumed markups were constant and equal to the standard CES markup. We now generalize our model by assuming firms compete *a la* Bertrand within product lines. Doing so makes the distribution of markups endogenous, and allows us to study the effects of falling population growth on market power.

Given the CES structure of demand, each firm would like to charge a markup of $\frac{\sigma}{\sigma-1}$ over marginal cost. However, the presence of competing firms within their product line implies the most efficient producer might have to resort to limit pricing. If they are unable to price at the optimal markup without inviting competition, they will set their price equal to the marginal cost of the next most efficient producer of that good, who is then indifferent between producing or not. The markup charged in product i, μ_i, is thus given by

$$\mu_i = \min\left\{\frac{\sigma}{\sigma-1}, \frac{q_i}{q_i^C}\right\} \equiv \min\left\{\frac{\sigma}{\sigma-1}, \Delta_i\right\}, \tag{33}$$

where q_i denotes the efficiency of the current producer in product i, q_i^C is the efficiency of the next best competitor, and $\Delta_i \equiv q_i/q_i^C > 1$ is the firm's efficiency advantage relative to it competitors (we also refer to this as the "gap"). Markups are increasing in the gap Δ because higher efficiency shields the firm from competition.

The static equilibrium allocations generalize in a straightforward way and aggregate output and equilibrium wages are now given by

$$Y_t = \mathcal{M}_t Q_t N_t^{\frac{1}{\sigma-1}} L_t^P \quad \text{and} \quad w_t = \Lambda_t Y_t / L_t^P = \Lambda_t \mathcal{M}_t Q_t N_t^{\frac{1}{\sigma-1}},$$

where

$$\mathcal{M}_t = \frac{\left(\int \mu^{1-\sigma} \left(q/Q_t\right)^{\sigma-1} dF_t\left(q,\mu\right)\right)^{\frac{\sigma}{\sigma-1}}}{\int \mu^{-\sigma} \left(q/Q_t\right)^{\sigma-1} dF_t\left(q,\mu\right)} \quad \text{and} \quad \Lambda_t = \frac{\int \mu^{-\sigma} \left(q/Q_t\right)^{\sigma-1} dF_t\left(q,\mu\right)}{\int \mu^{1-\sigma} \left(q/Q_t\right)^{\sigma-1} dF_t\left(q,\mu\right)},$$

and $F_t(q, \mu)$ denotes the joint distribution of efficiency and markups. The two aggregate statistics $\mathcal{M}_t$ and Λ_t fully summarize the static macroeconomic consequences of monopoly power. Market power reduces both production efficiency (the misallocation term $\mathcal{M}_t$) and lowers factor prices relative to their social marginal product (the labor share Λ_t). Because our model generates the joint distribution of markups and efficiency $F_t\left(q, \mu\right)$ endogenously and this distribution is a function of the rate of population growth, declining population growth affects allocative efficiency via $\mathcal{M}_t$ and has distributional consequences through Λ_t.

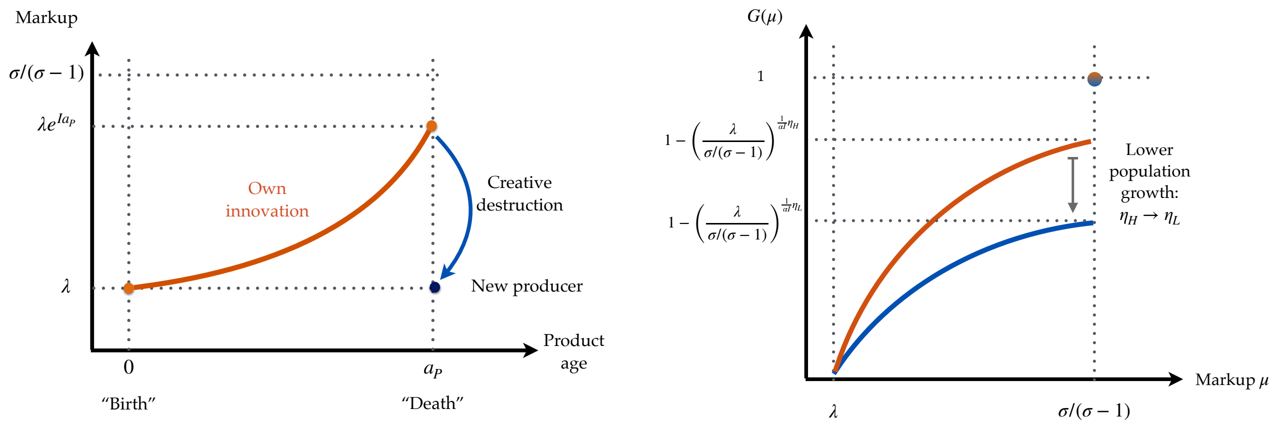
Most of our theoretical results carry over to this more general environment. Although the value function is more involved, we show in Section A-1.3 in the Appendix that we can still derive an analytic expression that has a similar form to the one derived in the constant markup case. More importantly, the key results of Proposition 1 *exactly* hold in the model with Bertrand competition, that is the equilibrium rate of creative destruction τ, the entry rate z, the rate of incumbent expansion x and the equilibrium growth rate g_y are still given by equations (12) and (13). Hence, our findings that lower population growth increases concentration and lowers growth directly carries over to the environment with Bertrand competition.

Allowing for imperfect competition, however, yields additional insights. Our model features a crucial asymmetry between productivity growth due to creative destruction and own-innovation. Suppose the current producer of product i has an efficiency gap of Δ_i. If this firm is replaced by another producer, the efficiency gaps *reduces* to λ as the new firm's efficiency exceeds the one of the previous producer by the creative-destruction step size λ. Hence, churning through creative destruction reduces markups. By contrast, if the existing firm successfully increases its efficiency through own-innovation, the efficiency gap and hence the markup increase at rate I (as long as $\Delta_i \leq \frac{\sigma}{\sigma-1}$). Therefore, own-innovation is akin to a positive drift for the evolution of markups, whereas creative destruction is similar to a "reset" shock, which lowers markups and keeps the accumulation of market power in check. This process is displayed in the left panel of Figure 4. When a firm adds a product to its portfolio, the initial markup is λ. Conditional on survival, markups increase at rate I. A faster rate of creative destruction lowers the expected time a given firm produces a particular product and limits the opportunities for incumbent firms to accumulate market power.

This stochastic process gives rise to a stationary distribution of efficiency gaps Δ and hence markups.

Figure 4: Falling Population Growth and Rising Market Power

(a) The Life-Cycle of Product Markups (b) The Distribution of Markups



Notes: This left panel shows a stylized example of how markups evolve at the product level. When a firm takes over a product, markups increase through own-innovation. Once the product is lost to another firm, markups are reset to the baseline level of λ. The right panel shows what happens to the distribution of markups when population growth falls.

Newly created varieties do not face any competitor and charge a markup of $\frac{\sigma}{\sigma-1}$. Products that have been creatively destroyed at some point in the past are subject to Bertrand competition and the markup depends on Δ. Let N_t^{NC} denote the mass of products without any competitor, and let N_t^C be the mass of products that are subject to competition. Consistency requires that $N_t = N_t^{NC} + N_t^C$. In Section A-1.3.5 in the Appendix, we prove two results. First, we show that, along a BGP, the share of product without any competitor is given by $N_t^{NC}/N_t = 1-\alpha$; that is, it is simply given by the share of product creation that results in new varieties (rather than creative destruction).[23] Second, the distribution of efficiency gaps among products with a competitor is given by

$$F^C(\Delta) = 1 - \left(\frac{\lambda}{\Delta}\right)^{\frac{\tau+\delta+\eta}{I}} = 1 - \left(\frac{\lambda}{\Delta}\right)^{\frac{1}{1-\alpha}\frac{\eta+\delta}{I}}; \tag{34}$$

the marginal distribution of efficiency gaps is a Pareto distribution with a tail parameter of $\frac{\tau+\delta+\eta}{I}$. As such, slower population growth increases the equilibrium distribution of efficiency gaps in a first-order stochastic dominance sense. First, slower population (and hence product) growth shifts the product distribution toward old products, which, on average, have higher markups. In addition, because slower population growth also reduces creative destruction, this effect is amplified; that is, the average product age is increasing even for a given cohort of firms.

To translate the distribution of efficiency gaps into the distribution of markups, recall from (33) that $\mu(\Delta) = \min\left\{\frac{\sigma}{\sigma-1}, \Delta\right\}$. For the case in which markups are below the "unconstrained", monopolistically competitive markup $\frac{\sigma}{\sigma-1}$, the distribution of markups is a truncated Pareto. Among products

[23] Let $N_t^{NC}(a)$ be the number of products without a competitor that have been around for a years at time t. Because $(1-\alpha)(z+x)N_t e^{-\eta a}$, such products entered at time $t-a$, receive a competitor at the rate of creative destruction τ, and exit at rate δ, $N_t^{NC}(a) = (1-\alpha)(z+x)N_t e^{-(\eta+\tau+\delta)a}$. Using that $\tau = \frac{\alpha}{1-\alpha}(\eta+\delta)$ yields $N_t^{NC} = \int_{a=0}^{\infty} N_t^{NC}(a)\,da = (1-\alpha)N_t$.

without a competitor, the markup is given by $\frac{\sigma}{\sigma-1}$. The cross-sectional distribution of markups across products is given by

$$G(\mu) = \begin{cases} \alpha F^{C}(\mu) & \mu < \frac{\sigma}{\sigma-1} \\ 1 & \mu = \frac{\sigma}{\sigma-1} \end{cases}.$$

A reduction in population growth therefore increases markups along the whole distribution and shifts more mass towards the maximum CES markup. In the right panel of Figure 4, we depict how the distribution of markups changes in response to a decline in population growth from η_H to η_L.

The macroeconomic consequences of misallocation are summarized by $\mathcal{M}$ and Λ, which depend on the joint distribution between efficiency gaps Δ and efficiency q. To derive this distribution, define relative efficiency $\hat{q} = \ln (q/Q_t)^{\sigma-1}$ and let $\hat{\lambda} = \ln \lambda^{\sigma-1}$. Denote $F_t^C(\Delta, \hat{q})$ as the joint distribution of efficiency gaps and relative efficiency for products that have a next-best competitor. Similarly, denote $F_t^{NC}(\hat{q})$ as the distribution of relative efficiency for products that do not have a competitor. We show in Section A-1.3.4 in the Appendix that these objects evolve according to differential equations given by

$$\frac{\partial F_t^C(\Delta,\hat{q})}{\partial t} = \underbrace{-\frac{\partial F_t^C(\Delta,\hat{q})}{\partial \Delta} I\Delta - (\sigma-1)(I-g_t^Q)\frac{\partial F_t^C(\Delta,\hat{q})}{\partial \hat{q}}}_{\text{drift from own innovation}} \underbrace{-(\tau_t+\delta+\eta)\,F_t\left(\Delta,\hat{q}\right)}_{\text{product loss}}$$
$$+ \underbrace{\lim_{s\to\infty} \tau_t F_t^C\left(s,\hat{q}-\hat{\lambda}\right)}_{\text{creative destruction of } C \text{ products}} + \underbrace{\tau_t \frac{N_t^{NC}}{N_t^C} F_t^{NC}(\hat{q}-\hat{\lambda})}_{\text{creative destruction of } NC \text{ products}},$$

$$\frac{\partial F_t^{NC}(\hat{q})}{\partial t} = \underbrace{-\frac{\partial F_t^{NC}(\hat{q})}{\partial \hat{q}}(\sigma-1)\left(I-g_t^Q\right)}_{\text{drift from own innovation}} \underbrace{-(\tau_t+\delta+\eta)\,F_t^{NC}(\hat{q})}_{\text{product loss}} + \underbrace{\frac{(1-\alpha)}{\alpha}\tau_t\Gamma\left(\frac{\exp(\hat{q})}{\sigma-1}\right)}_{\text{new products}}.$$

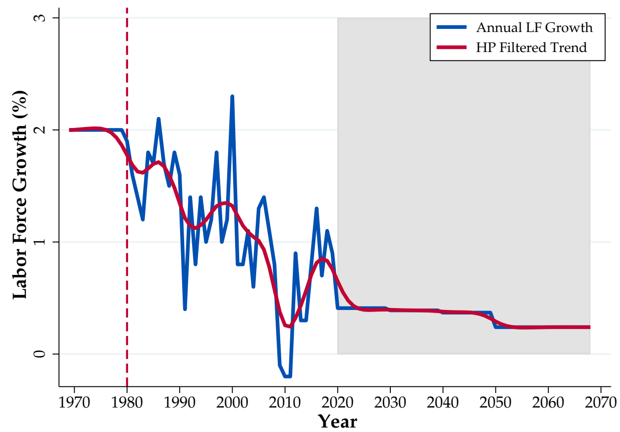
These expressions highlight the separate roles of own-innovation and creative destruction in influencing the evolution of efficiency and markups. Own-innovation causes both the production efficiency and the gap to drift upwards at the deterministic rate I, whereas creative destruction "resets" the mass in the distribution above Δ to have a gap of λ. Also note that there is a one-way flow of products from the non-competitive mass to the competitive through creative-destruction events, whereas the entrant distribution Γ only directly affects the non-competitive mass.

Though these distributions do not have a closed-form solution on the BGP, they can easily be computed. And given $F^C(\Delta, \hat{q})$ and $F^{NC}(\Delta, \hat{q})$, the economy-wide joint distribution is given by

$$F(\Delta,\hat{q}) = (1-\alpha)\,F^C(\Delta,\hat{q}) + \alpha F^{NC}(\Delta,\hat{q}),$$

because α is exactly the steady-state fraction of products that have a competitor. Given $F(\Delta, \hat{q})$ we can then quantify the aggregate consequences of market power. Because higher markups reduce

Figure 5: Labor Force Growth in the US



Notes: The figure shows the growth rate of the labor force in the US, with the raw series in blue and a HP-filtered trend component in red. The data is sourced from the BLS, accessed through FRED. Grey shading indicates projections.

the labor share Λ and more dispersed markups reduce allocative efficiency $\mathcal{M}$, lower population growth tends to increase profits relative to factor payments and has adverse effects on static allocation efficiency. Below, we quantify the strength of these forces and solve for the joint distribution $F_t^C(\Delta, \hat{q})$ computationally, both along the BGP and during the transition.

4 Quantitative Analysis: Calibration

To quantify the importance of declining population growth, we now calibrate our model to data from the US. In Figure 5 we display the historical growth rate of the labor force since 1980 and the official projections of the BLS. Our exercise to quantify the aggregate impact of this actual and projected decline is conceptually simple. We parametrize the model to a balanced growth path matching key moments of the data in 1980, when labor force growth was approximately 2%. We then study the aggregate impact of the path of labor force growth shown in Figure 5 by computing the dynamic response in our model. To do so, we treat the projections of the BLS as agents' rational expectations, and also assume that the projected labor force growth rate after 2050 persists in the long-run.

4.1 Data

Our main dataset is the US Census Longitudinal Business Database (LBD). The LBD is an administrative dataset containing information on the universe of employer establishments since 1978. It contains information on the age, industry, employment, and payroll of each establishment, along with identifiers at the firm level that allow us to track the ownership of each establishment over time. We define the age of the firm in the LBD as the age of the oldest establishment that the firm owns.

The birth of a new firm requires both a new firm ID in the Census and a new establishment record. We also modify the Census firm ID's to deal with some issues involving multi-establishment firms in the same way as developed in Walsh (2019).

To measure firms' markups, we require information on sales. We therefore augment the LBD data with information on firm revenue from administrative data contained in the Census Bureau's Business Register, following Moreira (2015) and Haltiwanger et al. (2016). The Business Register is the master list of establishments and firms for the US Census, and we are able to match approximately 70% of the records to the LBD.

In Table A-1, we provide some basic summary statistics on the firms in our dataset. In total our data comprises about 3.61 million firms in 1980 and 4.95 million firms in 2010. During that time period, average firm employment increased by around 10%, from 20 to 22 employees. The aggregate employment share of firms with less than 20 employees declined from 21.5% to 18.8%, and that of very large firms (with more than 10,000 employees) increased from 25.7% to 27%. Furthermore, firms became substantially older: the employment share of firms less than five years old declined from 38% to 30%. Qualitatively, all these facts are implications of our theory. Below we show the observed decline in population growth goes a long way toward replicating these patterns quantitatively.

4.2 Calibration

Our model is parsimoniously parametrized and rests on 11 parameters:

$$\Psi = \left\{ \underbrace{\alpha, \zeta, \varphi_E, \varphi_x, I, \bar{\omega}, \lambda}_{\text{Innovation \& Entry technology}}, \underbrace{\delta}_{\text{Exog. exit}}, \underbrace{\eta}_{\text{Pop. growth}}, \underbrace{\rho, \sigma}_{\text{Preferences}} \right\}.$$

We set three of them exogenously - the demand elasticity σ, the discount rate ρ, and the convexity of the innovation cost function ζ. We fix the elasticity of substitution between products σ at 4, following Garcia-Macia et al. (2019), set the discount rate ρ to 0.95, and assume a quadratic innovation cost function (i.e. $\zeta = 2$) as in Acemoglu et al. (2018).

The rate of labor force growth η is directly observed in the data (see Figure 5) and is our key parameter for the comparative statics. The remaining seven parameters are calibrated internally. First, we target three moments from the cross-sectional firm-size distribution in 1980: the entry rate, average firm size and the Pareto tail of the employment distribution. Second, we utilize two moments of firm-growth, namely the dynamics of sales and markups over firms' life-cycle.[24] Finally, we rely on two aggregate moments: the growth rate and the average markup.In Table 1 we report the parameters and the main moments we target.

[24]The LBD data do not contain direct information on products. Argente et al. (2019) use data from Nielsen to provide direct evidence on the process of life-cycle growth at the product-level. Akcigit et al. (2021) analyze a related model and show that, calibrated to employment data, it replicates the product-level distribution well. Cao et al. (2017) identify products (in the theory) with plants (in the data). For an early analysis of product-level data, see Bernard et al. (2011).

Table 1: Model Parameters

	Structural Parameters Description	Value	Moments	Data	Model
η	Labor force growth in 1980	0.02	Data from BLS	2%	2%
λ	Step size on quality ladder	1.11	Aggregate poductivity growth	2%	2%
I	Rate of own innovation	0.023	Markup growth by age 10 (RevLBD)	10.2%	10.2%
φ_X	Cost of inc. product creation	0.04	Sales growth by age 10 (RevLBD)	58%	58%
φ_E	Cost of entry	0.12	Avg. firm size (BDS)	20.7	20.7
δ	Exogenous rate product death	0.06	Entry rate in 1980 (BDS)	11.6 %	11.6 %
α	Share of creative destruction	0.59	Average profit share	25%	25%
$\bar{\omega}$	Relative efficiency of new products	0.45	Pareto tail of LBD employment distribution in 1980	1.1	1.1
ζ	Curvature of innovation cost	2	Set exogenously		
σ	Demand elasticity	4	Set exogenously		
ρ	Discount rate	0.05	Set exogenously		

Note: This table reports the calibrated parameters for the full model. Data for the firm lifecycle comes from the US Census Longitudinal Database, augmented with revenues from tax-information using the Census Bureau's Business Register. Data for average firm size and the firm entry rate in 1980 are taken from the public-use Business Dynamics Statistics.

While all moments are targeted simultaneously, there is nevertheless a tight mapping between particular moments and particular parameters which highlights how the different parameters are identified.

Innovation efficiency of incumbent firms: I and φ_x. We identify the relative efficiency of the different sources of innovation from the life-cycle profiles of sales and markups. Because markup growth is driven by incumbents' own-innovation activities (see Figure 4), this moment is informative about the rate of efficiency improvement I. Sales growth is in addition also affected by the rate of incumbent product creation, which depends directly on the cost of product expansion φ_x.

As we show in detail in Section A-2.3 in the Appendix, we can derive the life-cycle profiles of sales and markups (essentially) explicitly. This is not only convenient from a quantitative standpoint but also clarifies our identification strategy. The main insight in deriving these moments is to first express markups and sales of a given product as a function of the product age a_P. Average relative sales as a function of a product age a_P are given by

$$s_P(a_P) \equiv E\left[\frac{p_i y_i}{Y}\middle| a_p\right] = E\left[\mu_i^{1-\sigma}\left(\frac{q_i}{Q_t}\right)^{\sigma-1}\middle| a_p\right] = \mu\left(a_p\right)^{1-\sigma} e^{(\sigma-1)\left(I-g^Q\right)a_p}\overline{q}^{\sigma-1},$$

where $\mu(a_P) = \min\left\{\frac{\sigma}{\sigma-1}, \Delta(a_P)\right\} = \min\left\{\frac{\sigma}{\sigma-1}, \lambda e^{Ia_P}\right\}$, and the remaining terms are the average relative quality . Because own-quality q increases at rate I while average quality Q increases at rate g^Q, $e^{(\sigma-1)\left(I-g^Q\right)a_p}$ is the relative drift of these random variables. The last term reflects the initial average quality when the firm adds the product to its portfolio (see (5)).

With this expression for relative product sales $s_P(a_P)$ in hand, we can calculate the life-cycle of sales and markups at the *firm*-level. In particular, average sales and markups as a function of firm age a_f are given by[25]

$$s_f(a_f) = E\left[\sum_{n=1}^{N_f} s_P(a_P)\middle| a_f\right] \quad \text{and} \quad \mu_f(a_f) = E\left[\left(\sum_{i=1}^{N_f} \mu(a_p)^{-1} \frac{s_P(a_P)}{\sum_{i=1}^{N_f} s_P(a_P)}\right)^{-1}\middle| a_f\right],$$

where the expectations are taken with respect to the conditional distribution of N_f and a_P, conditional on a_f. Note the conditional distribution of product age will, in general, depend on the age of the firm a_f, as will the conditional distribution of the number of products N. We can, however, calculate these conditional distributions of product age a_P and the number of products N_f given firm age a_f essentially explicitly (see the Supplementary Material for details). This allows us to compute the life-cycle profiles of sales and markups without having to simulate the model.

Empirically, we measure markups at the firm level by the inverse labor share$\mu_f = \frac{py_f}{wl_f}$,where py_f is the total revenue of the firm, and wl_f is the total wage bill. We calculate the total wage bill by aggregating establishment payroll. While this approach allows us, in principle, to measure markups for the population of US firms, we only use firms' markup *growth* to calibrate our model. More specifically, letting $\mu_{f,t}$ be the markup of firm f at time t, we run a regression of the form

$$\ln \mu_{f,t} = \sum_{a=0}^{20} \gamma_a^{\mu} \mathbb{I}_{Age_{ft}=a} + \theta_f + \theta_t + \epsilon_{f,t}, \tag{35}$$

where $\mathbb{I}_{Age_{ft}=a}$ is an indicator for whether the firm is of age a and θ_f and θ_t are firm and time fixed effects respectively. Hence, γ_a^{μ} provides a non-parametric estimate of the rate of markup growth. We calibrate our model to the growth rate at the 10-year horizon, γ_{10}^{μ}. Because we explicitly control for a firm fixed effect when estimating (35), we do not have to take a stand on firms' output elasticities as long as they are constant with age.[26]

We follow the same approach when we estimate the life-cycle of sales; that is, we also estimate (35) using log sales as the dependent variable and target γ_{10}^{py} in our quantitative model. In the LBD, firms increase their average markup by roughly 10 percentage points and grow in size by about 80% by age 10.

Entry costs and product loss: φ_E and δ. We choose φ_E and δ to jointly match the entry rate and average firm size. The free condition determines market size L_t^P/N_t as a function of entry efficiency

[25] Equivalently, the firm-level markup μ_f can also be expressed as a cost-weighted average of product-level markups, $\mu_f = \sum_{i=1}^{N_f} \mu_i \frac{wl_i}{\sum_{i=1}^{N_f} wl_i}$, as in Edmond et al. (2018).

[26] If, for example, firms within sectors had different production functions with different output elasticities, neither the level nor the dispersion of markups could be distinguished from such differences in technology (see De Loecker and Warzynski (2012) and Peters (2020)). Also, by targeting markup growth, we avoid estimating output elasticities for labor, which is not feasible with the data we have as it does not contain data on capital or material inputs. Doing so would also complicate the mapping from model to data, since in our model labor is the only factor of production.

φ_E. This in turn is a key component of average firm employment. We thus choose φ_E to match an average firm employment of 20.04 in 1980 from the BDS. The higher the entry efficiency, the lower the market size, and the smaller the average size of firms. The exogenous rate of product loss δ directly influences the exit and hence - in a BGP - the entry rate of firms. We target the entry rate in 1980 of 11.6%.

Productivity growth through innovation: λ and $\overline{\omega}$. The parameters λ and $\overline{\omega}$ determine the relative quality of creatively destroyed products (λ) and newly generated varieties ($\overline{\omega}$). We infer these parameters from the aggregate rate of growth and the tail of the firm size distribution. That λ and $\overline{\omega}$ directly affect the growth rate is apparent from Proposition 1. For the tail of the firm size distribution, we find in our calibration that $\varsigma_n > \frac{1}{\sigma-1}\varsigma_q$, i.e. the tail of the employment distribution is given by $\varsigma_l = \frac{1}{\sigma-1}\varsigma_q$, where ς_q is given in (27). Given α and σ, this tail only depends on λ and $\overline{\omega}$. For our calibration, we chose λ and $\bar{\omega}$ to target a rate of productivity growth of 2% and a tail parameter of the firm size distribution of 1.1 (close to Zipf's law). See Section A-2.2 in the Appendix for the details of how we estimate the tail of the size distribution.

While none of the moments we target depend on higher moments of the entrant distribution $\Gamma(\omega)$, the shape of the final joint distribution of productivity and markups $F(\hat{q}, \Delta)$ is affected by this choice. We choose Γ to be a Pareto with mean $\overline{\omega}$ and a tail index of 4, which rationalizes the relatively low dispersion of entrant size in the LBD.

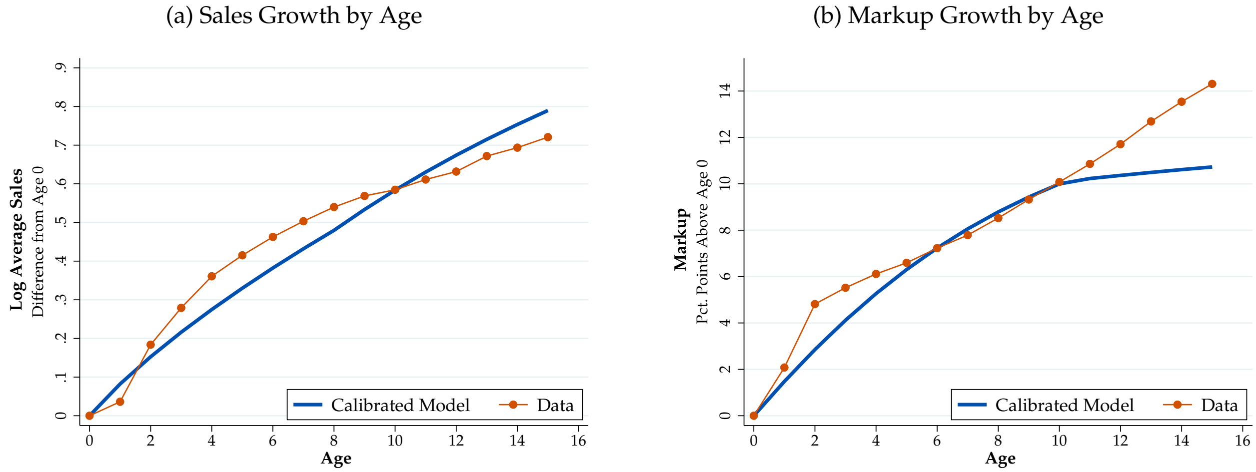
New varieties vs. creative destruction: α. The share of new products in innovation, $1-\alpha$, plays an important role for the level of markups in the economy. The higher α, the lower the economy-wide markup, because the higher the share of products that are subject to Bertrand competition. We target an economy-wide profit share of 25%.

4.3 Estimates and Model Fit

As seen in Table 1, our model is able to match the targeted moments perfectly. To match the fact that firm-level markups grow by around 10 percentage points at the 10 year horizon, our model implies a rate of own-innovation of around 2.3%. For a creative destruction event, we estimate a productivity increase of 11%. This is required to match an annual aggregate growth rate of 2%. The initial production efficiency of new products is estimated to be low, about 50% of the average product in the economy. This relatively low value is required to match the thickness of the tail of the employment distribution. These estimates imply that average quality $\overline{q}$ is given by 0.95, i.e. the average product innovation leads to a product which is 5% less productive than the average product in the economy. They also imply that the long-run relationship between economic growth and population growth is given by

$$dg_y/d\eta = \frac{\overline{q}^{\sigma-1} - \alpha}{(\sigma-1)(1-\alpha)} \approx 0.2.$$

Figure 6: Lifecycle Growth in Firm Sales and Markups



Note: Panel (a) in this figure compares the lifecycle of firm sales in the model to the estimated lifecycle in the data. The data lifecycle plots the age coefficients from estimating equation (35) in the LBD. The sample size is $35,300,000$, where this number has been rounded to accord with Census Bureau disclosure rules. Panel (b) does the same for relative markups.

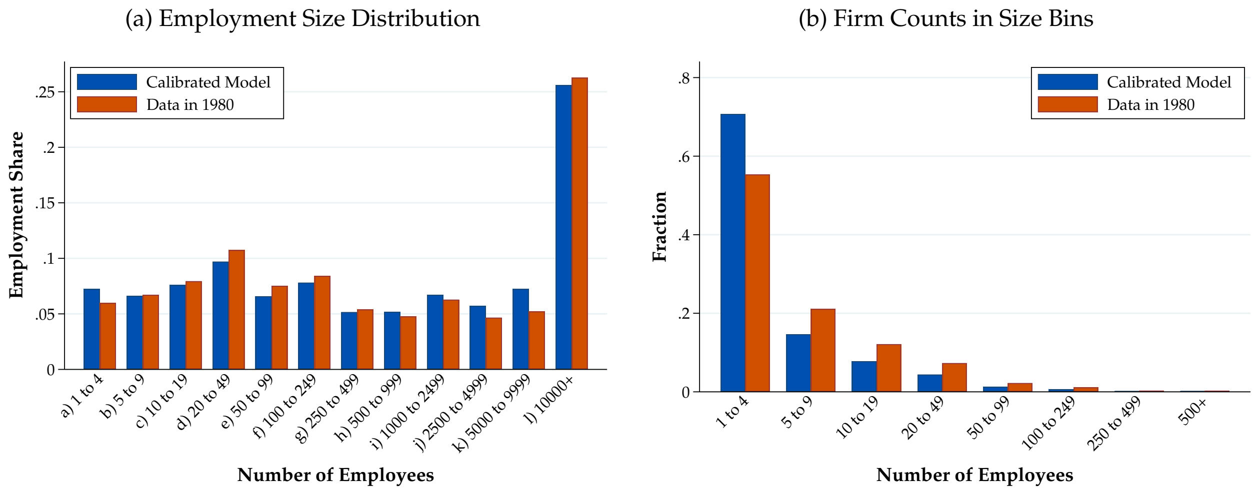
This estimate is qualitatively consistent with the findings of Bloom et al. (2020). In their model, $dg_y/d\eta = 1/\beta$ where β is the "degree of diminishing returns", that is the elasticity of productivity growth with respect to the level of productivity (i.e. $\dot{A}_t/A_t = A_t^{-\beta} L_t^R$). They report estimates of β between 0.4 for semi-conductors, 3 for the aggregate economy and 7 for agricultural technologies, suggesting, for example, that the average quality of the innovations in semi-conductors is large relative to the agricultural sector.

In addition to the targeted moments, our model, despite its parsimonious parametrization, also matches a variety of additional non-targeted moments. Consider first the sales and markup life cycle. In Figure 6 we show the model's performance by plotting the estimated coefficients γ_a^{μ} and γ_a^{py} from specification (35) estimated in the model and in the data. As highlighted in Table 1, we calibrate our model to match γ_{10}^{py} and γ_{10}^{μ}. Figure 6 shows that the model's implication for the whole age profile of sales (in the left panel) and markups (in the right panel) is quite close to what is observed in the data.

For the case of sales, the model replicates the slight concavity of log sales well. In the model, this shape reflects survivorship bias; small firms either grow or are destroyed, whereas large firms can have products stolen and shrink without exiting. As such, average growth conditional on survival is declining with age for young firms before, eventually, becoming log-linear for large old firms, matching Gibrat's law. The fit for markups in Panel (b) is also relatively good, even though in the data markups appear more linear with age than emerge from the model. Empirically, markups are increasing almost linearly by 1% each year. In the model, the rate of markup growth is much more concave, reflecting the fact that markups are bounded from above by $\frac{\sigma}{\sigma-1}$.[27]

[27] In Figure A-3 in the Appendix we show the joint density of markups and efficiency at the product level, illustrating the positive correlation between markups and efficiency induced by survival and own-innovation.

Figure 7: Size Distribution in Model and Data



Notes: Panel (a) of this figure plots the employment shares by firm size in the calibrated model (blue bars) and the data (orange bars). Panel (b) shows the shares of the firm counts in model and data. The data is from the BDS release of 1980.

In Figure 7, we confront our model's predictions for the size distribution with the data. Whereas we have explicitly targeted average size and the Pareto tail, our model matches the full non-parametric firm size and employment distribution very well. We plot the distribution of employment (left panel) and the number of firms (right panel) for both the model and the data in 1980.[28] Our model successfully matches both of these margins. Note, in particular, that it replicates the aggregate importance of very large firms with more than 1000 employees, which account for 25% of aggregate employment.

A central reason our model successfully replicates the firm-size distribution is that it provides a good fit for the empirically observed exit hazards. In the left panel of Figure 8, we depict the exit rate by age from the micro-data in the LBD for the 1980 cohort of firms.[29] Our model is remarkably successful in replicating theses exit rates, despite the fact that we do not target them in the estimation. In our theory, exit rates are declining in age because older firms have more product lines, and owning more products progressively lowers the likelihood that they will all be destroyed within a particular year.[30]

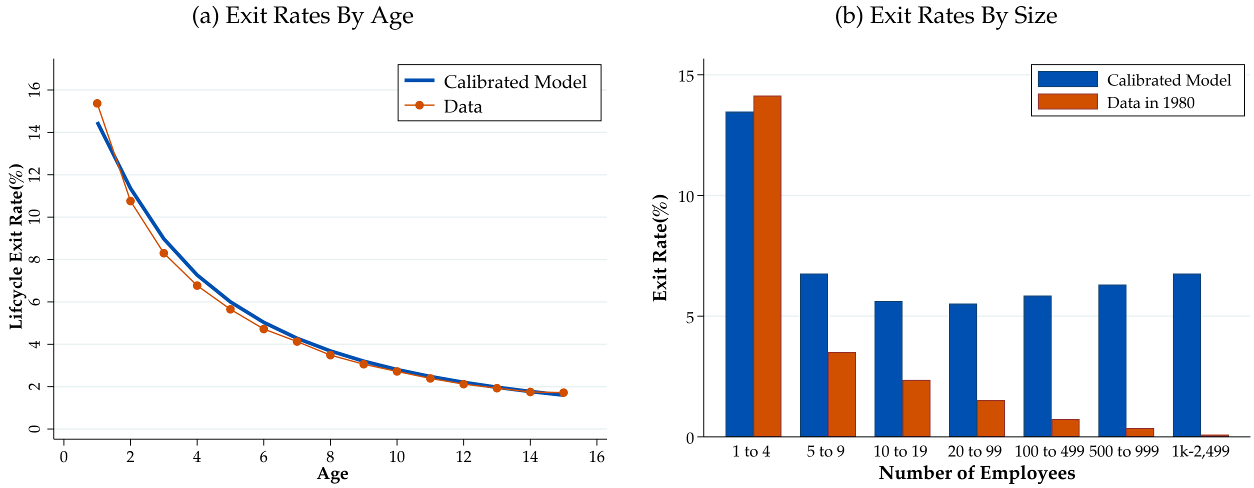
In the left panel, we depict the exit rate for different size categories. Empirically, these exit rates are declining. Our model implies that this exit rate is initially declining but essentially independent of size for firms with more than 10 employees. The reason our model has this counterfactual prediction

[28]For replicability we chose size bins that are also available in the publicly available data from the BDS.

[29]To construct exit rates by age, we estimate a non-parametric Kaplan-Meier survival function by age for firms in the LBD. We select the cohort of firms born between 1980 and 1990, and follow them until 2015. We then take the exit rates to be the increments of the estimated survival functions. Each estimate is essentially the fraction of the sample that exits at age a (though the estimator accounts for the truncation from ceasing to observe firms after 2015).

[30]Data on the age distribution in the target year of our calibration is unavailable both in the public BDS and the administrative LBD, since the Census only begins tracking age for new establishments in 1978 (the first year the data is available).

Figure 8: Firm Exit Rates: Model and Data



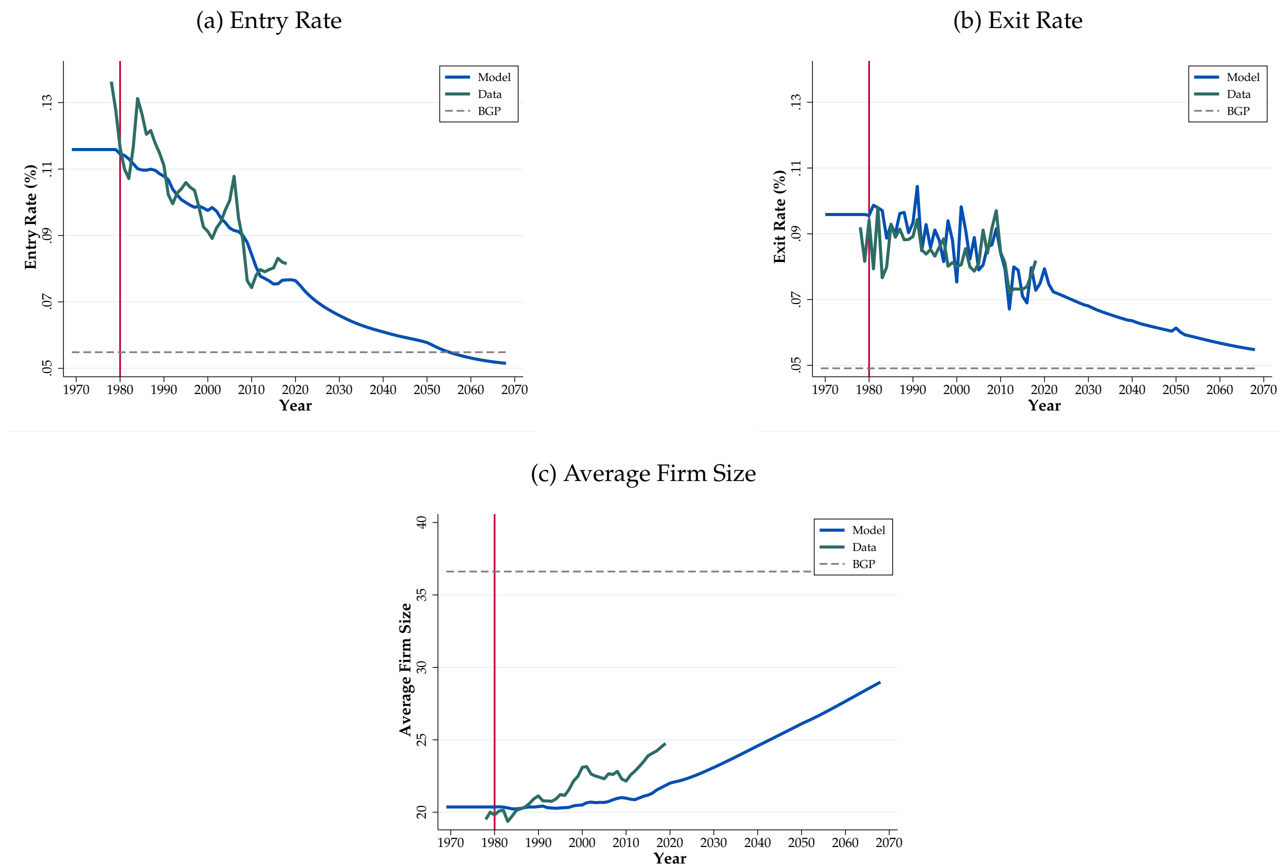
Notes: This figure presents a comparison of lifecycle exit rates between the model and data. The exit rates in the data are taken from the increments in a Kaplan-Meier survival function estimated on all firms in the LBD born between 1980 and 1990. Age of a on the horizontal axis indicates that the firm exited between age $a-1$ and age a. The sample size is $70,000,000$, where this count has been rounded to accord with US Census disclosure rules.

is that (in our calibration) the thick tail of the employment distribution is driven by the distribution of product quality q and not the extensive margin of product creation. Hence, large firms are firms with a few superstar products, not those with many products. And because creative destruction is independent of product quality, such firms are as likely to exit as other firms. However, because - as seen in Figure 7 - the mass of large firms is relatively small, this prediction does not interfere greatly with our models' ability to provide a good fit to the firm size distribution. To address this counterfactual prediction, in Section SM-3 in the Supplementary Material, we extend our model to allow for type heterogeneity, whereby some young firms (sometimes described as "rockets" or "gazelles", see Pugsley et al. (2019)) grow systematically at a faster rate. This extension allows a subset of firms to become large by adding a large number of products, an outcome that is unlikely in the baseline model. This extension improves the model's fit along this dimension substantially, because some large firms have many products and are thus unlikely to exit, but changes little else in the theoretical analysis.

5 The Aggregate Impact of Falling Population Growth

We now use our calibrated model to quantify the effects the implications of the observed and projected decline in labor force growth shown in Figure 5. To do so, we start with the calibrated BGP in 1980 and then feed the path displayed in Figure 5 into the model. In the context of our theory, we assume all agents have rational expectations about this path. All other parameters are held constant.

Figure 9: Declining Population Growth and Changing Firm Dynamics



Note: The figure displays the dynamic response of the entry rate (panel (a)), the exit rate (panel (b)), and average firm size (panel (c)) to the path of population growth shown in Figure 5. The same objects from the BDS data are shown in green.

5.1 Declining Population Growth and Changing Firm Dynamics

We start by considering the impact on firm dynamics. We focus first on the entry rate, the exit rate and average firm size. In Figure 9, we plot both the data and the implications of our theory.

Consider first the data, shown in green. The entry rate,shown in panel (a), declined markedly in the last 30 years from around 12% in the 1980s to around 8% in the mid 2000s. Note this series of the entry rate tracks the evolution of population growth shown in Figure 5 closely, and the contemporaneous correlation is 0.74.[31] Similarly, the exit rate, shown in panel (b), declined from 9% in 1980 to almost 7% in 2015. Average firm size, shown in panel (c), rose from 20 to 23 employees, increasing by around 15%.

In blue, we superimpose the predictions of our theory. Recall that we used both the entry rate and average size in 1980 as a calibration target, and hence match these numbers by construction. The exit

[31] Karahan et al. (2016) and Hathaway and Litan (2014) study this link directly in the geographic cross-section, showing that states with slower labor force growth, as predicted by lagged birth rates in previous decades, see lower rates of firm entry.

rate, by contrast, is not targeted. The subsequent fall in entry and exit and the rise in average size are the sole consequence of the observed and projected decline in population growth.

Figure 9 shows that the decline in population growth goes a long way toward explaining the observed changes in the entry and exit rate average size. For the entry and exit rates, our model matches the US experience almost perfectly. For average size, our model also predicts an increase in average employment. However, our model implies a somewhat slower increase than what is observed in the data, and that the long-run increase will take many decades to settle at a higher value once labor force growth stabilizes. The increase in concentration is also similar to what is observed in the data, with the employment share of large firms (defined by the BDS to be 10,000 employees or more) increasing by 1% by 2015, roughly in line with the data (see Table A-1).

In Figure 9, we only display the implications of our theory until 2070. Given the population growth path shown in Figure 5, our model has not reached a new BGP at this point. Hence, we also plot the long-run implications for the entry rate and average size as dashed lines. The entry and exit rates adjust relatively quickly and are already quite close to their long-run BGP values by 2070. By contrast, our model predicts that average size has some way left to run due to the slow-moving firm size distribution and will increase substantially in the long-run.

As also highlighted in Hopenhayn et al. (2018), the three series shown in Figure 9 and the rate of population growth are linked via a fundamental accounting equation: population growth is the sum of the entry rate $\mathcal{E}_t$ and the change in average firm size $\mathcal{S}_t$ minus the exit rate $\mathcal{X}_t$ [32]

$$\eta_t = \dot{\mathcal{S}}_t / \mathcal{S}_t + \mathcal{E}_t - \mathcal{X}_t. \tag{36}$$

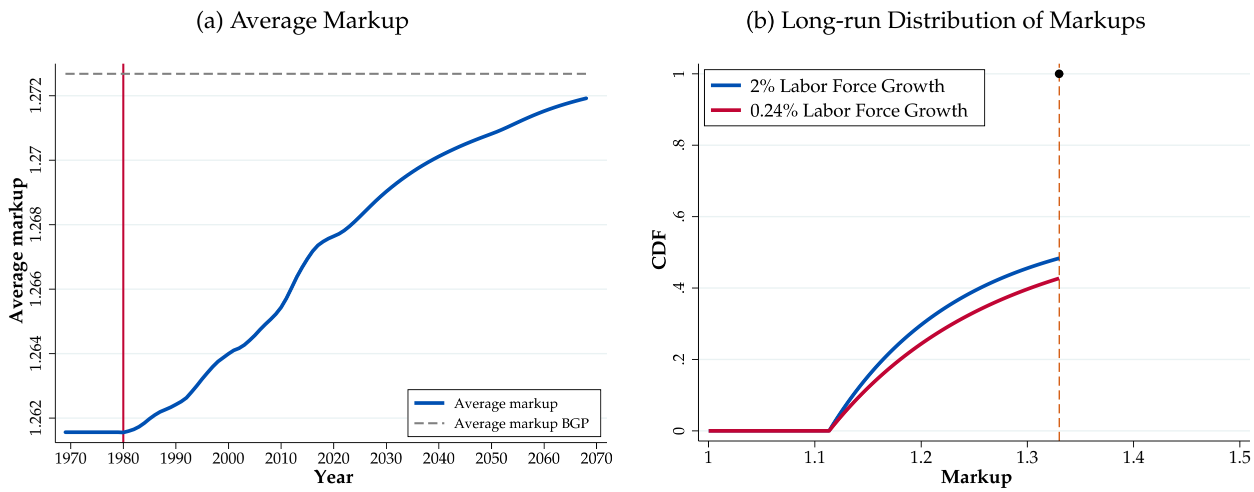
Because our theory correctly predicts an increase in average size and a fall in the exit rate, the decline in the entry rate is substantially larger then the decline in population growth. Along a BGP, where average size is constant, $\eta = \mathcal{E} - \mathcal{X}$. The fact that, empirically, the gap between the entry rate and the exit declined markedly since 1980, is a further justification that changes in population growth are a key aspect of the changes in the US firm size distribution in the last decades.

Our model predicts that average size is increasing both because of a shift in the age distribution towards older firms and because lower population growth increases firm size conditional on age. Quantitatively, however, much of the increase depicted in Figure 9 is due to shifts in the age distribution. In Figure A-4 in the Appendix, we show the change in markup growth, sales growth and exit by age.These objects do change as population growth declines, but only modestly. This dominant role of the age distribution is consistent with the data, where size or exit rates by age also changed little (see Karahan et al. (2016) and Hopenhayn et al. (2018)).

In Figure 10, we report the implied changes in product-level markups. We display both the evolution of the cost-weighted average markup (left panel) and the change in the distribution of markups in the BGP (right panel). As implied by our theoretical results, the decline in population growth increases

[32] Letting $\mathcal{F}_t$ denote the number of firms, $\mathcal{S}_t = L_t / \mathcal{F}_t$. Noting that $\dot{\mathcal{F}}_t / \mathcal{F}_t = \mathcal{E}_t - \mathcal{X}_t$ yields (36).

Figure 10: Declining Population Growth and Rising Market Power



Notes: The left panel shows the transition path of the average product markup as labor force growth changes according to the path in Figure 5. The right panel shows the markup distribution in the BGP before and after the transition.

markups in a first-order stochastic dominance sense. Quantitatively, the increase in market power is modest: the average product markup increases by about 1%. The markup distribution in the right panel highlights where this increase stems from. Declining population growth lowers creative destruction, and hence increases markups among products that have a competitor. By contrast, the markup for products without a competitor is - by construction - $\sigma/(\sigma-1)$ and hence independent of η. Moreover, the share of non-competitive products is given by $1-\alpha$ and hence also independent of η.

Our model implies the increase in markups shown in Figure 10 occurs mostly across firms and is a reflection of the fact that firms become older. Within firms, products tend to become older because products are destroyed less frequently. On its own, this fact would tend to raise average markups. However, firms also tend to accumulate more products, which are, on average, younger, and hence have lower markups (see Figure A-4). Quantitatively, these two forces almost exactly offset one another, so that the rise in markups reflects compositional changes whereby large and old firms with high markups increase their market share. This pattern is qualitatively consistent with the findings reported in Kehrig and Vincent (2017) and Autor et al. (2020).

In Table 2 we summarize the long-run effects of declining population growth on the process of firm dynamics. Average firm size increases substantially to 36 employees and both entry and exit rates decline. Note again that the gap between the entry and exit rate shrinks as this gap is exactly the rate of population growth. In the last two columns we also show that falling population growth is a source of concentration: the employment share of large firms with more than 10,000 employees rises, the one of small firms declines.

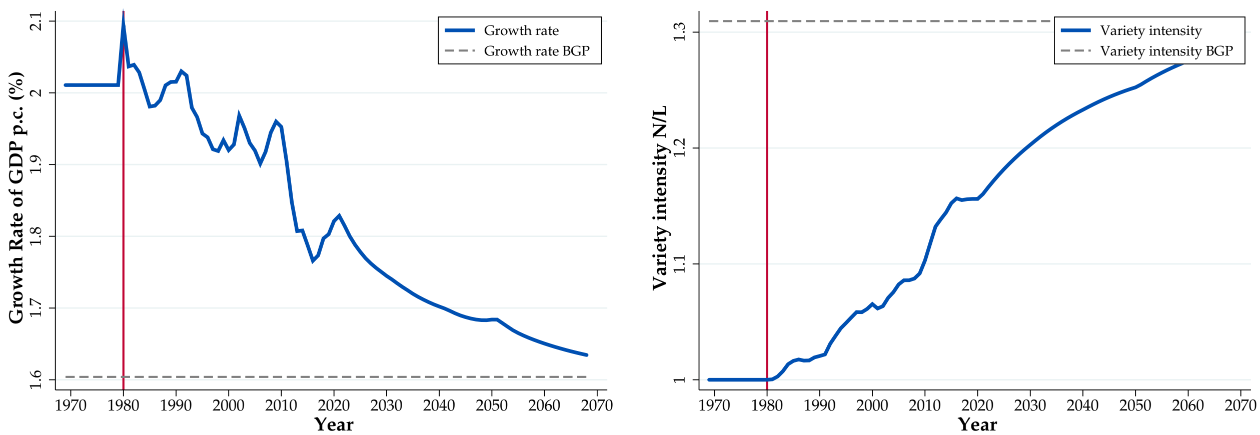
Table 2: Population Growth and Firm-level Moments

	Avg. Firm Size Emp. per Firm	Entry Rate %	Exit Rate %	Avg. Markup %	Employment Share >10000 Emp, %	Employment Share <20 Emp, %
Baseline	20.04	11.60	9.6	26.4	25.6	37.8%
η=0.24%	36.61	5.42	5.18	27.5	27.1	37.6%

Note: The table reports several firm level moments computed in the model. The first row refers to the BGP of the calibrated model in which population growth is 2%. The second row is the counterfactual BGP where we reduce population growth by 1.76% to 0.24%.

Figure 11: Declining Population Growth and Income per Capita

(a) Income per Capita Growth g_t^y (b) Variety Intensity $\mathcal{N}_t$



Note: The figure displays the dynamic response of the aggregate growth rate (left panel) and the variety intensity $\mathcal{N}_t = N_t/L_t$to the path of population growth shown in Figure 5.

5.2 Declining Population Growth, Aggregate Growth and Welfare

We now turn to the normative implications of the decline in population growth. In Figure 11 we depict the growth rate of income per capita (left panel) and the change in the variety intensity $\mathcal{N}_t$ (right panel). As in Figure 9 above, we trace out the model's implication until 2070 and indicate the long-run levels of the respective variables in the new BGP as dashed lines.

Interestingly, the effect of population growth on output growth is not monotone. On impact, a population growth decline *increases* output growth for about one decade. This is due to an increase in the variety intensity $\mathcal{N}_t$, which is a source of variety gains. These variety gains at the aggregate level coexist with rising average firm size, because firms produce multiple products and the number of products per firm increases. Hence, rising concentration does not necessarily go hand in hand with falling variety. The mass of products available to consumers actually goes up in response to falling population growth. Because the increase in variety is only a transitory phenomenon, output growth

Table 3: Population Growth and Economic Growth

	Growth				Production labor	Variety intensity
	g^y	Variety (N_t)	Efficiency (Q_t)	τ	$\ell^P = L_t^P / L_t$	$\mathcal{N} = N_t / L_t$
Calibrated Baseline	0.02	0.007	0.013	0.196	0.86	1
1.76% Decline in η	0.016	0.001	0.015	0.170	0.88	1.21

Note: The table reports the aggregate growth rate (g^y), the growth stemming from variety gains $\left(\frac{1}{\sigma-1}g_N\right)$ and efficiency growth $\left(g^Q\right)$, the rate of creative destruction (τ), the share of workers employed in researcher $\left(L_t^P/L_t\right)$ and the variety intensity (N_t/L_t). The first row refers to BGP of the calibrated model. The second row is the counterfactual BGP where we reduce population growth by 1.76 to 0.24%. For ease of comparison we normalize N_t/L_t to 1 in the calibrated baseline.

eventually declines and stabilizes at a lower level. In the long-run, declining population growth will reduce the growth rate of income per capita, as in most models of semi-endogenous growth. In our calibration, the long-run growth rate declines from around 2% to 1.6%.

These competing forces of rising variety and declining growth make the welfare consequences of falling population growth in principle ambiguous. We measure welfare in consumption equivalent terms, asking by how much would we need to change the level of consumption per capita in each period in the old BGP to achieve the same level of welfare as the transition for a member of the representative household who discounts the future at rate ρ. We find a reduction of 1.9%.

In Table 3 we use the model to decompose the long-run growth rate into its different components. As seen in Figure 11, a decline in population growth reduces the long-run equilibrium growth rate from 2% to 1.6%.[33] Furthermore, this decline stems almost entirely from falling variety growth. In fact, efficiency growth rises slightly in response to the decline in population growth. The reason is that we estimate the efficiency of production of new varieties $\overline{\omega}$ to be relatively low. The declining rate of variety creation therefore impacts average efficiency growth positively. Creative destruction declines from about 20% to 17%.

In the remaining two columns we also report the equilibrium allocation of labor and the long-run variety intensity, which is constant along a BGP. The number of available products per capita increases by about 20%. At the same time, the share of production workers increases slightly as lower population growth reduces the extent of innovative activity. Both of these are consistent with our theoretical analysis of the transitional dynamics in the phase diagram shown in Figure 2 and they increase income per capita for the given level of productivity. However, the long-run decline in productivity growth is still the dominating force to reduce per capita income and welfare in the long-run.

[33] We also conducted our analysis for the case of endogenous own-innovation I. Recall that we showed in Section 2.7 that this amplifies the effect of of population growth on productivity growth. Quantitatively, we find that long-run growth declines to 1.2% instead of 1.6%. At the same time, the increase in the variety intensity is more pronounced. Overall, we find the adverse welfare implications to be more pronounced compared to our baseline model.

6 Conclusion

Most countries have experienced declining rates of fertility and a slowdown in population growth in recent decades. There is little reason to think this trend is going to reverse any time soon; a world of low and falling population growth looks like it is here to stay.

In this paper we have shown that this trend is likely to have important implications for the process of firm dynamics and for aggregate productivity. We proposed a firm-based model of semi-endogenous growth, that is rich enough to rationalize many first-order features of the micro-data but nevertheless lends itself to an analytical characterization of the effects of population growth. We derived two main results: First, declining population growth reduces creative destruction and entry and increases average firm size, markups and market concentration. Second, lower population growth reduces economic growth in the long-run, but has positive effects on productivity in the short-run.

In our application to the US, we draw three main quantitative conclusions. First, the population growth channel can account for a large share of the change in entry rates and firm size since the 1980s. Hence, changes in population growth are likely to be an important contributor for the decline in dynamism in the US and the rest of the developed world. Second, even though the decline in population growth is predicted to lower economic growth in the long-run, economic growth remains higher for almost two decades. Finally, even though lower population growth increases market power and markups, we estimate this effect to be quantitatively small. Hence, the rise in markups and the fall in the labor share are unlikely to be driven by falling fertility, but rather by technological or institutional changes.

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