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ABSTRACT

We present a tractable dynamic general equilibrium model of self-fulfilling bank runs, where banks trade capital in competitive and liquid markets but remain vulnerable to runs due to a loss of creditor confidence. We characterize how the vulnerability of an individual bank depends on its leverage position and the economy wide asset prices. We study the effect of credit easing policies, in the form of asset purchases. When a banking crisis is generated by runs, credit easing can reduce the number of defaulting banks and enhance welfare. When the crisis is driven by fundamentals, credit easing may have adverse consequences.

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Most financial crises involve bank runs. Often, the runs occur simultaneously in multiple financial institutions and emerge after a deterioration of banks' balance sheets. The Great Depression and the 2008 Global Financial Crisis are two notable examples (Friedman and Schwartz, 1963; Bernanke, 2013).

Diamond and Dybvig (1983) spurred a vast literature analyzing whether a fundamentally solvent bank may be subject to a self-fulfilling run. During a run, investors rush to withdraw deposits from the bank, anticipating that others will do so as well. The run may thus cause a severe liquidity problem and leave the bank unable to meet the withdrawals, making the run self-fulfilling. As highlighted by Gorton (1988), bank runs are not isolated events. They tend to happen in many banks at the same time and are more likely when aggregate fundamentals are weak. This observation suggests that self-fulfilling bank runs may be the result of general equilibrium forces and that runs in turn may affect general equilibrium outcomes. Understanding this feedback and the potential implications for policy requires a dynamic general equilibrium model.

In this paper, we present a tractable dynamic macroeconomic model of financial crises in which banks may be subject to self-fulfilling runs. We analytically characterize how a bank's vulnerability depends on individual and aggregate fundamentals and how the number of banks facing a run affects aggregate fundamentals in turn. Our normative analysis shows that the interplay between self-fulfilling beliefs and general equilibrium feedback has distinct implications for policy. We establish that the desirability of credit easing depends on whether a financial crisis is driven by fundamentals or self-fulfilling bank runs. While credit easing helps reduce fragility in a run-driven crisis—as banks facing a run benefit from the rise in asset prices—we show that it may backfire in a fundamentals-driven crisis.

We build a dynamic model in which banks have limited commitment and trade capital in perfectly liquid and competitive markets. The possibility of default gives rise to an endogenous borrowing limit, which depends on future asset returns and the tightness of future borrowing limits. In turn, asset prices are determined in general equilibrium and are themselves affected by banks' current and future borrowing limits.

In our model, a bank may default because of fundamental reasons about the ex-post returns on its assets. But it may also default because of a run. We introduce runs following the formulation of Cole and Kehoe (2000): short-term creditors to a bank may panic and refuse to roll over their debts. In this case, the bank must repay its maturing debts by either cutting equity payouts or selling some of its assets holdings. If the costs of these actions is sufficiently high, it becomes optimal for the bank to default, making the run a self-fulfilling equilibrium outcome.

A distinctive feature of our model, relative to the Diamond and Dybvig model, is that runs occur even though the bank can sell its assets in a liquid market, a feature that resonates with the recent March 2023 banking turmoil.[1] Crucial for the possibility of runs is the existence of a positive spread between the return on capital and the cost of borrowing. A positive spread is associated with a positive franchise value for the bank, which implies that the inability to leverage because of a run reduces the value of the bank. When the reduction in the bank value is sufficiently strong, the bank becomes exposed to a self-fulfilling run, in which investors run because they expect others to run as well, thus preventing the bank from obtaining the intermediation profit. On the other hand, we show that when the spread is zero, individual banks are not vulnerable to runs. In this case, access to a spot liquid market for capital renders the presence of runs irrelevant.

In general equilibrium, transitional dynamics can be separated into three regions. When aggregate leverage is low, the economy converges to a stationary equilibrium in which all banks repay at all times. In this region, asset prices are high, reflecting banks' high productivity and collateral values. When aggregate leverage is high, all banks default, and asset prices are depressed. For intermediate values of leverage, we have an interior share of banks defaulting. The presence of runs increases the number of banks that default, generating an increase in financial fragility.

Finally, we turn to our normative analysis, which examines the role of credit easing policies in the form of asset purchases. The key question we tackle is how credit easing affects the share of defaulting banks and the level of welfare.

We show that the effects of credit easing are different depending on whether a crisis is driven by fundamentals or by runs. Namely, we show that credit easing reduces fragility in a crisis driven by runs, but may backfire in the absence of runs. The logic for this result can be understood by tracing which banks are the net sellers of capital and which banks are the net buyers, depending on the origin of the crisis.

Consider first a crisis driven by runs. In this situation, the marginal bank (i.e., the bank indifferent between repaying and defaulting) is a net seller of assets—it needs to sell assets to meet repayments of deposits. Thus, by increasing asset prices, credit easing raises the value of repaying for banks facing a run and reduces investors' incentives to run in the first place. The outcome is that fragility is reduced. In contrast, in a crisis driven by fundamentals, repaying banks tend to be net buyers in the model, as they absorb the assets sold by the defaulting banks. Thus, the government's purchase of assets, this has a negative impact on their profitability, potentially pushing more banks to default in equilibrium.

Literature. This paper is related to the literature on the role of financial factors in macroeconomic fluctuations. Building on the seminal contributions by Bernanke and Gertler (1989) and

[1] This is also a departure from Cole and Kehoe (2000), in which the borrower has a claim to a fixed stream of income, and there is also no spot market for such claim.

Kiyotaki and Moore (1997), many studies have presented models in which balance sheet losses on firms or financial intermediaries can trigger contractions of output and asset prices.[2] Unlike this literature, our paper considers a source of financial fragility induced by liquidity factors and self-fulfilling runs.

Our paper belongs to an extensive literature on bank runs. One strand of the literature, starting with Diamond and Dybvig (1983), considers bank runs that are the outcome of a self-fulfilling prophecy in the presence of a liquidity mismatch. A different strand of the literature studies models of runs based on fundamentals, following Bryant (1980). In this alternative paradigm, individual investors who have a sudden need for liquidity find it optimal to run, even if nobody else does. Allen and Gale (2000) and Uhlig (2010) are notable examples in this class of models studying contagion through interbank market linkages and asset prices.[3] The interplay between runs and asset prices is also at the heart of our analysis, but we consider self-fulfilling runs, as in the first strand of the literature. Overall, a contribution of our paper is to analyze the role of credit easing and to show that its desirability depends on whether a crisis is driven by fundamentals or self-fulfilling beliefs. We also differ from much of this literature by taking a dynamic macroeconomic perspective.

Gertler and Kiyotaki (2015) develop a macroeconomic model of systemic bank runs in which a good equilibrium with financial intermediation may coexist with a bad equilibrium in which asset prices are low, aggregate banks' net worth turns negative and banks are forced into liquidation.[4] In their model, when an individual bank's net worth turns negative, it is unable to continue operations. This implies that an individual investor would not roll over the deposits, regardless of whether other investors are rolling over.[5] By contrast, we present a model with self-fulfilling runs on individual banks. In our model, the condition for an individual bank to default is dynamic and depends critically on whether investors are willing to roll over the deposits. This feature leads to distinctive implications for the effectiveness of policies such as lender of last resort.[6] In

[2]A few examples include Gertler and Kiyotaki (2010), Mendoza (2010), Jermann and Quadrini (2012), He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), and Bianchi and Mendoza (2018).

[3]See also Angeloni and Faia (2013) for a dynamic model with two-period lived banks and Allen and Gale (2009) for a review of much of this literature.

[4]An active literature builds on their framework to study quantitative policy counterfactuals (see, e.g., Gertler, Kiyotaki and Prestipino (2016, 2020*a*, 2020*b*) and Robatto, 2019). A related literature studies financial fragility and multiplicity in different contexts (e.g., Gu et al., 2013; Benhabib and Wang, 2013; Brunnermeier and Sannikov, 2015; Boissay, Collard and Smets, 2016; Bocola and Lorenzoni, 2020; Ben-Ami and Geanakoplos, 2020; Schmitt-Grohé and Uribe, 2021; and Boissay et al., 2022).

[5]To the extent that the value of the bank is finite, the bank has incentives to divert assets when its net worth is negative. There is no solution to the bank problem that satisfies the incentive compatibility constraint, even if investors were willing to roll over the deposits.

[6]For example, a policy of liquidity provision or freezing deposits is effective in our setup to prevent a run, but it does not rule out defaults in Gertler and Kiyotaki (2015). In their model, because these policies do not alter banks' net worth, banks remain prone to diverting funds for personal use and default.

addition to many other differences in the modeling setups, we conduct a normative analysis of the desirability of credit easing.

Keister and Narasiman (2016) also tackle the question of how policy prescriptions differ depending on the origin of the crisis. They focus on ex-ante prudential policies in an environment featuring moral hazard due to bailouts, and they conclude that prudential policies are optimal regardless of whether crises are caused by self-fulfilling beliefs. Farhi and Tirole (2012) show how ex-post non-targeted interventions can lead to an excessive leverage equilibrium. In our paper, credit easing can be welfare reducing in the absence of runs, even from an ex-post point of view.

The bank run literature has considered several ex-post policy interventions, including deposit insurance, deposit freezes, bailouts and lender of last resort (e.g., Diamond and Dybvig, 1983; Cooper and Ross, 1998; Ennis and Keister, 2009; Dávila and Goldstein, 2020). These studies show how these policies can be desirable to avoid a run in a single bank. While we also emphasize how policies can have different implications depending on the source of the crisis, the mechanism in our model operates entirely through a general equilibrium channel involving asset prices. By affecting hidden trades, general equilibrium effects also play a crucial role in the analysis regulation by Farhi, Golosov and Tsyvinski (2009).[7]

Our paper also speaks to historical studies on the origins of banking crises, especially the debate on whether banking crises occur because of fundamentals or self-fulfilling prophecies (see, among others, Friedman and Schwartz, 1963; Gorton, 1988; Calomiris and Mason, 2003; Baron, Verner and Xiong, 2021). Our theory predicts credit easing has opposite effects on bank failures depending on the origin of the crisis, thereby providing a testable implication that can be used to distinguish empirically whether crises are driven by fundamentals or self-fulfilling runs.

Our paper is also related to a literature on credit easing that has flourished since the 2008 financial crisis (see, e.g., Gertler and Karadi, 2011; Curdia and Woodford, 2011, Kiyotaki and Moore, 2019). A common theme in this literature is how a central bank that is not balance sheet constrained can reduce excess returns by purchasing private assets when there are asset fire sales. In our model, there are adverse effects from this intervention if the portfolio return for the government does not exceed the one for investors. However, credit easing can become desirable when a crisis is driven by runs.

Our environment without runs is related to the literature on investment under limited commitment and, in particular, the papers of Thomas and Worrall (1994) and Alburquerque and Hopenhayn (2004). Using an optimal contract approach, those papers solve the investment problem of an individual firm (or government) that lacks commitment to repay its debts.[8] Our general

[7] See also Di Tella (2019).

[8] This optimal contract approach is followed by several other papers in this area that also focus on investment under limited commitment (e.g., Aguiar, Amador and Gopinath, 2009 and Kehoe and Perri, 2002).

equilibrium characterization of an economy with limited commitment frictions has direct antecedents in the work of Kehoe and Levine (1993) and the solvency constraints introduced by Alvarez and Jermann (2000) in particular.[9] For the environment with runs, we build on the formulation of rollover crises by Cole and Kehoe (2000), which has been used to study the individual problem of a government.[10] We adopt the canonical game, extend it with investment, embed it into a general equilibrium model, and draw implications for macroeconomic policy.

Outline. Section 1 presents the environment and characterizes the individual bank problem in partial equilibrium. Section 2 analyzes the general equilibrium. Section 3 conducts the normative analysis. Section 4 discusses extensions of the baseline model and Section 5 concludes. All proofs are collected in the online appendix.

1 Model

Time is discrete and infinite, $t \in \{0, 1, 2, ...\}$. There is a single final consumption good, and there are no aggregate shocks. The economy is populated by a continuum of financial institutions, which we refer to as banks, and creditors, both of measure one. In what follows, we use lowercase letters to denote individual variables and capital letters to denote aggregate variables.

Technology. Production of the final consumption good uses capital, k, as a single input. We assume that banks have direct access to the production technology, in line with the most recent strands of macro-finance models. Capital does not depreciate, and it is in fixed aggregate supply, equal to $\bar{K}$.

Preferences. Banks' preferences over a stream of dividend payments, c_t are given by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t),$$

where $\beta \in (0, 1)$, $u = \log$, and $\mathbb{E}$ is the expectation operator. Banks' creditors are risk neutral and discount payoffs at a rate R.

[9]See Jeske (2006) for another paper that studies limited commitment and external borrowing in decentralized environments.

[10]See, for example, Aguiar et al. (2016), Roch and Uhlig (2018), Bocola and Dovis (2019), and Bianchi and Mondragon (2022) for models in sovereign debt using that formulation.

1.1 Banks' problem and borrowing limits

We describe now the problem of an individual bank in partial equilibrium. That is, for a given sequence of capital prices $\{p_t\}$, banks choose bond issuances, investment, dividend payments, and whether to repay the existing creditors.[11]

Banks issue one-period bonds to creditors that promise a payment of R next period. A bank starts a period t with k_t units of capital and b_t units of maturing bonds, and decides whether to repay or to default.

If the bank chooses to repay, it produces using a linear technology and chooses its new holding of capital for the next period $k_{t+1} \geq 0$, the new amount of bonds to issue, b_{t+1}, and how many dividends to pay, c_t. The bank faces a price schedule $q_t(b_{t+1}, k_{t+1})$ for its bonds, which depends on its individual choices for new bonds and capital, as well as other aggregate variables that we summarize in t. These variables determine the incentives to default in the next period and hence alter the price at which creditors are willing to lend today.

If the bank chooses to default, it is permanently excluded from bond markets and can only invest in capital.[12] It also suffers a permanent productivity loss.

We will allow for the possibility of bank runs, but will do so only at period $t = 0$. But first, we describe the value of default to a bank. Determining this value allows us to solve for the equilibrium borrowing limits that banks face.

1.2 The value of default

The bank's productivity after defaulting is permanently equal to z^D. The budget constraint for a bank that has defaulted and has capital holdings equal to k_t is

$$c_t = (z^D + p_t)k_t - p_t k_{t+1}.$$

We define the return to capital when the bank defaults as

$$R^D_{t+1} \equiv \frac{z^D + p_{t+1}}{p_t},$$

for all $t \geq 0$. Note that this value is common across all defaulting banks.

We can solve for the value of default, exploiting the log-utility and the linearity of production. To guarantee the boundedness of the value function, we introduce the following condition.

[11] In the rest of the paper, we will use the notation $\{x_t\}$ to refer to the sequence $\{x_t\}_{t=0}^{\infty}$ for some variable x.

[12] The restriction that the bank cannot hold bonds after default is without loss of generality if the rate of return to capital in equilibrium for a bank that has defaulted is higher than R.

Condition 1. *The sequence of (strictly positive) prices* $\{p_t\}_{t=0}^{\infty}$ *is such that*

$$\lim_{t\to\infty} \beta^t \log\left(R_{t+1}^D\right) = 0.$$

Let us define the net worth of a defaulting bank to be

$$n_t^D = (z^D + p_t)k_t.$$

We have the following result:

Lemma 1 (The value of default). *Suppose that Condition 1 holds. Then the value of default,* $V_t^D(n_t^D)$*, in period* t *is finite and such that*

$$V_t^D(n_t^D) = A + \frac{1}{1-\beta}\log(n_t^D) + \frac{\beta}{1-\beta}\sum_{\tau\geq t}\beta^{\tau-t}\log\left(R_{\tau+1}^D\right), \tag{1}$$

with

$$A \equiv \frac{1}{1-\beta}\left[\log(1-\beta) + \frac{\beta}{1-\beta}\log(\beta)\right].$$

The value function is log-linear in wealth and the discounted future returns on capital. The associated policy function for capital, $\mathcal{K}_{t+1}^D(n_t^D)$, and dividend payout, $C_{t+1}^D(n_t^D)$, are given by

$$\mathcal{K}_{t+1}^D(n_t^D) = \beta\frac{n_t^D}{p_t},$$

$$C_t^D(n_t^D) = (1-\beta)n_t^D.$$

Because of log preferences, the optimal policy is independent of future returns. Under this investment policy, the evolution of net worth is given by

$$n_{t+1}^D = \beta R_{t+1}^D n_t^D.$$

1.3 The value of repayment

In case of repayment in period t, a bank with capital k_t and debt b_t can issue new debt, b_{t+1}, and purchase new capital, k_{t+1}, according to its budget constraint:

$$c_t = (z_t + p_t)k_t - Rb_t + q_t(b_{t+1}, k_{t+1})b_{t+1} - p_t k_{t+1}.$$

For all $t \geq 1$, we assume that the productivity under repayment is constant, $z_t = z$. At $t = 0$, we assume that z_0 is drawn from a cumulative distribution function F with support $[\underline{z}, \overline{z}]$. Such a draw is i.i.d. across banks.[13]

Let us define the net worth of a repaying bank at time t:

$$n_t = (z_t + p_t)k_t - Rb_t.$$

The value of the repaying bank at t as a function of its net worth n is

$$\begin{aligned} V_t^R(n_t) = &\max_{k_{t+1}\geq 0, b_{t+1}, c_t} \log(c_t) + \beta V_{t+1}(b_{t+1}, k_{t+1}) \\ &\text{subject to} \\ &\quad c_t = n + q_t(b_{t+1}, k_{t+1})b_{t+1} - p_t k_{t+1}, \end{aligned} \tag{2}$$

where V_{t+1} is the continuation value function, which incorporates the possibility of default.

Given that for $t \geq 1$, we have assumed that there are no runs, this continuation value is just given by the optimal choice between repayment and default next period. That is, for $t \geq 1$,

$$V_t(b_t, k_t) = \max\{V_t^R((z + p_t)k_t - Rb_t), V_t^D((z^D + p_t)k_t)\}.$$

Using $d_t = 0$ to represent a repayment decision at t and $d_t = 1$ a default, we have that the optimal default rule for $t \geq 1$ is

$$d_t(b_t, k_t) = \begin{cases} 1 & \text{if } V_t^R((z + p_t)k_t - Rb_t) < V_t^D((z^D + p_t)k_t), \\ 0 & \text{if } V_t^R((z + p_t)k_t - Rb_t) \geq V_t^D((z^D + p_t)k_t), \end{cases}$$

where we assume, without loss of generality, that the bank repays if indifferent.[14]

The lenders price the bonds taking into account this default rule, and thus $q_t(b_{t+1}, k_{t+1}) = d_{t+1}(b_{t+1}, k_{t+1})$. That is, creditors purchase bonds at a zero price when they expect a certain default and purchase bonds at a price of 1 when they expect certain repayment.

The value of repayment at time t is strictly decreasing in b_t for a given k_t, which implies that the optimal default rule can be expressed with a debt threshold that is determined by the equality of default and repayment values, $V_{t+1}^D((z^D+p_{t+1})k_{t+1}) = V_{t+1}^R((z+p_{t+1})k_{t+1} - Rb_{t+1})$. The pricing of the bonds inherits this threshold property, switching from 1 to 0 when debt exceeds the threshold.

We now guess that the value function under repayment (if finite) will be log-linear in net worth. Specifically, we guess that $V_{t+1}^R(n_{t+1}) = \frac{1}{1-\beta}\log(n_{t+1}) + \text{constant}$. This then implies that

[13] This initial productivity shock could capture different portfolio exposures to an aggregate shock.

[14] The reason why assuming that the bank pays if indifferent for $t > 0$ is without loss of generality is as follows. If banks were to randomize when indifferent for $t > 0$ (with some arbitrary probability), it would be strictly optimal for the bank to choose a level of debt ϵ below the indifferent point and borrow at a price of 1.

there exists a γ_t such that for $q_t(b_{t+1}, k_{t+1}) = 1$, $b_{t+1} \leq \gamma_t p_{t+1} k_{t+1}$ and zero otherwise.[15] A bank will not find it optimal to borrow above this threshold, as the revenue it receives from its bond issuances is zero. Thus, the bank is effectively subject to a borrowing constraint:

$$b_{t+1} \leq \gamma_t p_{t+1} k_{t+1},$$

where γ_t is an equilibrium object.[16]

We define the return to capital when the bank repays as

$$R^k_{t+1} \equiv \frac{z + p_{t+1}}{p_t},$$

for all $t \geq 0$. We assume that there is a productivity loss after default, $z^D < z$, which implies that $R^k_{t+1} > R^D_{t+1}$.

Let us define the levered return on equity, R^e_t, as

$$R^e_{t+1} \equiv R^k_{t+1} + (R^k_{t+1} - R)\frac{\gamma_t p_{t+1}}{p_t - \gamma_t p_{t+1}}, \tag{3}$$

which corresponds to the sum of the return on capital plus the excess return (of capital over bonds) times a leverage factor.[17] We need to impose as well a condition on $\{R^e_t\}$ to guarantee the value of repayment for an individual bank is bounded, similar to Condition 1 for the case of a defaulting bank. Anticipating the general equilibrium, we restrict attention to sequences of prices and borrowing limits that satisfy the following.

Condition 2. *The sequences of prices $\{p_t\}$ and $\{\gamma_t\}$ are such that*

(i) $R^k_{t+1} \geq R$ *for all* $t \geq 0$,

(ii) $\gamma_t p_{t+1} < p_t$ *for every* $t \geq 0$ *such that* $R^k_{t+1} > R$,

(iii) $\lim_{t\to\infty} \beta^t \log\left(R^e_{t+1}\right) = 0$.

Part (i) makes sure that capital demand is not zero. Part (ii) makes sure that capital demand is not infinite. Note that part (iii) of this condition implies Condition 1 is $R^e_{t+1} \geq R^k_{t+1} > R^D_{t+1} > 0$.

[15] These constraints are the equivalent of the “not too tight” solvency constraints introduced by Alvarez and Jermann (2000). In comparison with their environment, ours features the presence of capital, production, and default costs, as well as an exogenous risk-free rate R. In our environment without risk, the borrowing constraints also coincide with the endogenous borrowing constraints used by Zhang (1997).

[16] This also implies that equilibrium default occurs only in the initial period.

[17] The intuition for the expression for the leverage factor is as follows. Starting with one unit of net worth, the bank can use it to purchase $1/p_t$ units of capital, enabling it to borrow $\gamma_t p_{t+1}/p_t$ bonds. In turn, the additional borrowing allows the bank to purchase more capital and obtain further borrowing. If $\gamma_t p_{t+1} < p_t$, the bank's borrowing capacity per unit of net worth becomes $\gamma_t p_{t+1}/(p_t - \gamma_t p_{t+1})$. The return per unit of borrowing is $R^k_t - R$, thus leading to (3).

We can now solve for the value function of repayment (confirming that it is log-linear in net worth) as well as characterize the associated policy functions.

Lemma 2 (The value of repayment)**.** *Consider a sequence of (strictly positive) prices, $\{p_t\}$, and (non-negative) borrowing limits, $\{\gamma_t\}$, that satisfy Condition 2. Then, the repayment value, $V_t^R(n_t)$, for a bank with net worth n_t at time t, along with its corresponding policy functions, is as follows:*

(i) Value function:

$$V_t^R(n_t) = A + \frac{1}{1-\beta}\log(n_t) + \frac{\beta}{1-\beta}\sum_{\tau \geq t}\beta^{\tau-t}\log(R^e_{\tau+1}),$$

with constant A as in Lemma 1.

(ii) Policy functions:

$$C_t^R(n_t) = (1-\beta)n_t,$$

for all $t \geq 0$ and where $\mathcal{K}^R_{t+1}(n_t)$ and $\mathcal{B}^R_{t+1}(n_t)$, satisfy

$$p_t\mathcal{K}^R_{t+1}(n_t) - \mathcal{B}^R_{t+1}(n_t) = \beta n_t, \quad \mathcal{B}^R_{t+1}(n_t) \leq \gamma_t p_{t+1}\mathcal{K}^R_{t+1}(n_t), \quad \mathcal{K}^R_{t+1}(n_t) \geq 0$$

for all $t \geq 0$. And

$$\mathcal{K}^R_{t+1}(n_t) = \frac{\beta n_t}{p_t - \gamma_t p_{t+1}}, \qquad \mathcal{B}^R_{t+1}(n_t) = \gamma_t p_{t+1}\left(\frac{\beta n_t}{p_t - \gamma_t p_{t+1}}\right)$$

for all $t \geq 0$ such that $R^k_{t+1} > R$.

Under repayment, the problem also features a value function that is log-linear in net worth and future returns, thus confirming our previous guess that the borrowing constraint is linear.

The value of the bank (ignoring the constant A) can be split between the market value of its assets minus its liabilities, and the bank's return on equity. The return on equity term can be further decomposed as follows:

$$\frac{\beta}{1-\beta}\sum_{\tau\geq t}\beta^{\tau-t}\log(R) + \frac{\beta}{1-\beta}\sum_{\tau\geq t}\beta^{\tau-t}\log(R^k_{\tau+1}/R) + \frac{\beta}{1-\beta}\sum_{\tau\geq t}\beta^{\tau-t}\log(R^e_{\tau+1}/R^k_{\tau+1}),$$

where the first term captures the market return on savings (available to all agents), and where the last two terms capture the "franchise value" of the bank. The first of these two is the excess return earned by a bank from its production technology, $R^k_{t+1} \geq R$. And the second term is the excess return that arises from the bank's ability to leverage, $R^e_{t+1} \geq R^k_{t+1}$. When $R^k_{t+1} = R$ for all $t \geq 0$, these last two terms are zero; that is, there is no franchise value.[18] As we will see below, these terms play an important role for the existence of bank runs.

Regarding the portfolio choice, the solution distinguishes between the case in which $R^k_{t+1} = R$ and $R^k_{t+1} > R$. If the return on capital is equal to the return on debt at date t, the bank is indifferent between bonds and capital and chooses any portfolio as long as it is consistent with the dividend policy and the leverage constraint. If the return on capital exceeds the one on debt, the bank borrows to the limit.

Using the results of Lemma 2, we can express the evolution of net worth under repayment as

$$n_{t+1} = \beta R^e_{t+1} n_t$$

for all $t \geq 0$. Hence, next-period net worth is given by the amount of net worth that is not consumed, βn, times the return on equity.

Default thresholds at $t \geq 1$. Having characterized the values of repayment and default, we can now examine the default thresholds for $t \geq 1$. Using Lemmas 1 and 2, a bank that borrows to the maximum of its borrowing constraint is indifferent between repayment and default for $t \geq 1$ if the following holds:

$$((z + p_t)k_t - R\gamma_{t-1}p_t k_t) \prod_{\tau=t+1}^{\infty} (R^e_\tau)^{\beta^{\tau-t}} = (z^D + p_t)k_t \prod_{\tau=t+1}^{\infty} (R^D_\tau)^{\beta^{\tau-t}}.$$

The left-hand side is the (exponential of the) value of repaying in period t after borrowing $b_t = \gamma_{t-1}k_t$ in the previous period. The right-hand side is the (exponential of the) value of default in period t. Note that the value of k_t cancels, and the value of γ_{t-1} is determined by the remaining indifference. The following proposition rewrites the values of $\{\gamma_t\}$ recursively.

Proposition 1 (Default decision)**.** *Consider a sequence of (strictly positive) prices, $\{p_t\}$, and a sequence of (non-negative) borrowing limits, $\{\gamma_t\}$, that satisfy Condition 2. The sequence of $\{\gamma_t\}$ consistent with indifference between repayment and default for all $t \geq 1$ is such that*

$$\frac{z + p_{t+1}(1 - \gamma_t R)}{z^D + p_{t+1}} = \left(1 - \gamma_{t+1}\frac{p_{t+2}}{p_{t+1}}\right)^{\beta} \qquad \textit{for all } t \geq 0. \tag{4}$$

[18] In general equilibrium, this franchise can remain strictly positive because the limited commitment constraint prevents banks from competing away the arbitrage gap between R^k and R. An alternative source for a positive franchise value is imperfect competition (see Corbae and D'Erasmo, 2021, for an example of this in the context of a macroeconomic model).

The sequence for default thresholds $\{\gamma_t\}$ depends on preference and productivity parameters, as well as the sequence for $\{p_t\}$. One can see, in particular, that a higher γ_{t+1} in the future implies a higher γ_t today. Because a higher γ_{t+1} increases the continuation value of repayment, this also makes the bank more willing to repay today.

The above suggests that there could be potentially many sequences of borrowing limits, $\{\gamma_t\}$, that would be consistent with a partial equilibrium given a sequence of capital prices. However, for an equilibrium to be consistent with creditors' optimality, we also require a no-Ponzi game condition. That is,

$$\lim_{t\to\infty} R^{-t} b_t \leq 0,$$

where $\{b_t\}$ is a feasible sequence of debt issuances. As we show in Amador and Bianchi (2024), we can establish a uniqueness result for the sequence of $\{\gamma_t\}$ that is consistent with (4) once we impose this condition. Using the fact that $b_{t+1} \leq \gamma_t p_{t+1} \frac{\beta n_t}{p_t - \gamma_t p_{t+1}}$, together with the evolution of net worth, we therefore impose the following condition as an additional restriction to the sequence of $\{\gamma_t\}$:

Condition 3. *The sequence of prices $\{p_t\}$ and $\{\gamma_t\}$ is such that*

$$\lim_{t\to\infty} \left[\prod_{\tau=0}^{t} \left(\frac{\beta R_t^e}{R} \right) \right] \left(\frac{\beta \gamma_t p_{t+1}}{p_t - \gamma_t p_{t+1}} \right) \leq 0.$$

With this, we can characterize the sequence of γ_t that is consistent with banks' and creditors' optimality conditions, given a sequence of prices:

Definition 1. *Given a sequence of (strictly positive) prices $\{p_t\}_{t=0}^{\infty}$, we say a sequence of (non-negative) borrowing limits $\{\gamma_t\}_{t=0}^{\infty}$ is* equilibrium consistent *if Conditions 2 and 3 hold and equation* (4) *is satisfied for all $t \geq 0$.*

1.4 Initial period: Fundamental defaults and runs

In the above, we have described the default behavior for $t \geq 1$, in the absence of any future runs. We now proceed to study the default decision at $t = 0$ and introduce the possibility of self-fulfilling runs.

Recall that in period $t = 0$, a repaying bank has a productivity drawn from a c.d.f. F. Depending on the realization of this draw, a bank may choose to repay or default in period $t = 0$. We will consider two different situations. In the first situation, a bank is able to continue borrowing as long as it decides to repay. When a bank defaults in this case, we refer to it as a "fundamental" default. In the second situation, investors refuse to roll over the deposits. When a bank defaults in this case, we refer to it as a “run-driven” default. We describe next how the two default thresholds are determined in each case.

Fundamental default threshold. Consider a bank in period $t = 0$ that can roll over the debt. The default decision of such a bank is the same as in later periods: it compares the value of repaying, $V_0^R((z_0 + p_0)k_0 - Rb_0)$, with the value of defaulting, $V_0^D((z^D + p_0)k_0)$. Given that V_0^R is increasing in z_0 (for $k_0 > 0$), there exists a threshold $\hat{z}^F$ such that banks with a realization of z_0 below $\hat{z}^F$ default, while those with a realization above repay (in the absence of a run).

Using again Lemmas 1 and 2, we can obtain that the default threshold $\hat{z}^F$ is given by

$$\hat{z}^F = (z^D + p_0) \prod_{t=1}^{\infty} \left(\frac{R_t^D}{R_t^e}\right)^{\beta^t} - p_0 \left(1 - R\frac{b_0}{p_0 k_0}\right). \tag{5}$$

Given the threshold, the probability that a bank defaults (in the absence of runs) at $t = 0$ is $F(\hat{z}^f)$. One can immediately see the role of “leverage”: a higher $b_0/(p_0 k_0)$ increases this threshold and thus increases the probability of a fundamental default.

Run threshold. The analysis of the previous section tells us that any bank with a productivity draw $z_0 < \hat{z}^F$ will default at $t = 0$. We now incorporate the possibility of runs. Following the work of Cole and Kehoe (2000), a run will be the outcome of a coordination failure by the creditors of the bank.[19] In this case, a bank with $z_0 > \hat{z}^F$ may be forced to default, even though it is fundamentally sound.

Recall that $n_0 = (z_0 + p_0)k_0 - Rb_0$ and $n_0^D = (z^D + p_0)k_0$ denote the bank’s net worth under repayment and default. Let $V_0^{Run}(n_0)$ denote the repaying value for a bank with net worth n_0 if it is unable to issue new debt (that is, it suffers a run), but still decides to repay its existing creditors. This value is obtained as the solution to the following problem:

$$V_0^{Run}(n_0) = \max_{k_1 \geq 0, c_0 > 0} \log(c_0) + \beta V_1^R\left((z + p_1)k_1\right),$$

$$\text{subject to}$$

$$c_0 = n_0 - p_0 k_1.$$

The constraint set in the above problem is non-empty as long as $n_0 > 0$. Note that at $t = 1$, this

[19] The sovereign debt literature distinguishes between fundamental defaults and self-fulfilling defaults (runs), according to the timing of the play. In fundamental defaults, the sovereign first chooses to repay and then decides how much debt to issue. This is referred to as the Eaton-Gersovitz timing (Eaton and Gersovitz, 1981). In the run scenario, the sovereign issues the debt first and then chooses to repay. This second timing, the Cole-Kehoe timing, introduces the possibility that a default may be triggered because of a coordination failure in the financial markets that refuse to absorb newly issued debt. See Aguiar and Amador (2021).

bank starts without any debt, and as a result, the continuation value is given by the repaying value function $V_1^R((z+p_1)k_1)$ (as a bank with no liabilities does not default).

A bank suffering a run defaults if $V_0^{Run}(n_0) < V_0^D(n_0^D)$. Given that the repayment value $V_0^{Run}(n_0)$ is strictly increasing in n_0, defaults under a run occur following a threshold rule as before. Let $\hat{z}^{Run}$ denote the threshold value such that when facing a run, a bank with this productivity is indifferent between defaulting or not. Using Lemmas 1 and 2, we have that

$$((\hat{z}^{Run}+p_0)k_0 - Rb_0)\left[(R_1^k)^{\beta}\prod_{t=2}^{\infty}(R_t^e)^{\beta^t}\right] = (z^D+p_0)k_0\prod_{t=1}^{\infty}(R_t^D)^{\beta^t}.$$

The right-hand side represents the value of default, and it is the same as in the case without runs. The left-hand side is different: it incorporates that during a run, the return for a repaying bank is reduced from R_1^e to R_1^k, as the bank is unable to leverage during the run.

Solving out for the threshold, we have that

$$\hat{z}^{Run} = (z^D+p_0)\left(\frac{R_1^D}{R_1^k}\right)^{\beta}\times\prod_{t=2}^{\infty}\left(\frac{R_t^D}{R_t^e}\right)^{\beta^t} - p_0\left(1-R\frac{b_0}{p_0k_0}\right). \tag{6}$$

We say that a bank is "safe" in period $t=0$ if even under a run, it chooses to repay its debts rather than default. That is, a bank is safe if $V_0^{Run}(n_0) \geq V_0^D(n_0^D)$. We use the term "safe" because if a bank does not find it optimal to default upon a run, then investors do not have incentives to run. On the other hand, such a bank is "vulnerable" if it finds optimal to default under a run; that is, if $V_0^{Run}(n_0) < V_0^D(n_0^D)$.

Inspection of the value functions shows that $V_0^R(n_0) \geq V_0^{Run}(n_0)$, as a bank that does not suffer a run but repays is weakly better off than one that suffers one and repays. If $V_0^{Run}(n_0) = V_0^R(n_0)$ for all $n \geq 0$, then $\hat{z}^{Run} = \hat{z}^F$; the fundamental default and run thresholds coincide. Comparing the thresholds (5) and (6), we see that the thresholds are different if and only if $R_1^k < R_1^e$. In other words, runs precipitate a default only when there is a profit loss from the bank's inability to leverage in a run. Using (3), we can re-express this in terms of prices $\{p_t\}$ and borrowing limits $\{\gamma_t\}$:

Lemma 3 (Comparison of thresholds)**.** *Consider a sequence of prices $\{p_t\}$ and borrowing limits $\{\gamma_t\}$ that satisfy Condition 2. We have that $\hat{z}^F < \hat{z}^{Run}$ if and only if $\gamma_0 > 0$, and $R_1^k > R$. Otherwise, $\hat{z}^F = \hat{z}^{Run}$.*

This lemma tells us that defaults due to runs can occur in our model only if two conditions are met: $\gamma_0 > 0$, and $R_1^k > R$. If $\gamma_0 = 0$, then a repaying bank cannot borrow, and thus whether or not it suffers a run does not alter its default decision. If $R_1^k = R$, a bank that suffers a run could also optimally have chosen to reduce its debts to zero and scale down its capital. This is so because when $R_1^k = R_1^e = R$, and such a bank *is indifferent between capital and bonds.*[20]

To understand the nature of runs in our model, it is helpful to contrast with the Diamond and Dybvig model. In that setup, banks hold illiquid assets, which cannot be sold in the event of a run (or can be sold but face a liquidation cost). When a run occurs, a bank may be forced to default even though the bank would be solvent if depositors were to wait until the assets mature. In our model, the assets that the bank holds (capital) are perfectly liquid and can be sold at no cost at the market price p_t, which for an individual bank is a given. In equilibrium, if there is a gap between R_1^k and R and $\gamma_0 > 0$, then banks leverage and make profits. In this situation, a bank that does not suffer a run receives an excess return $R_1^e > R$ on its equity next period. When the bank suffers a run, it can indeed liquidate its assets at no cost, but it loses this excess return. It is this "illiquid" component of the bank technology that makes a bank vulnerable to a run.[21]

1.5 The marginal bank and demand for capital: Fundamentals vs. runs

Banks that default because of fundamentals or face a run have different demands for capital. As we will see below, this difference will have crucial implications for the effects of policies.

Let us begin by examining the bank with a productivity exactly at the fundamental default threshold, $\hat{z}^F$. Assuming that $R_1^k > R$, the demand for capital of this marginal bank in period $t = 0$, if it repays its debts, is

$$k_1^R \equiv \beta \frac{(\hat{z}^f + p_0)k_0 - Rb_0}{p_0 - \gamma_0 p_1}. \tag{7}$$

If this marginal bank were to default, its demand for capital would be

$$k_1^D = \beta \frac{(z^D + p_0)k_0}{p_0}. \tag{8}$$

[20] This result from our model is quite different from that of Cole and Kehoe (2000) in the sovereign debt literature, in which the possibility of a run always affects the default threshold. In that model, the government has an endowment stream that cannot be sold. By contrast, in our model the bank has access to a spot liquid market for capital. When $R_1^k = R$, the ability to sell assets in the market renders the presence of runs irrelevant.

[21] This feature resonates with the March 2023 turmoil in commercial banks. As long-term rates increase, banks face losses in their long-term Treasuries. At the same time, deposit rates did not increase one-to-one with interest rates, and so excess returns went up. The run on SVB and other banks, however, implied that banks could not leverage to exploit the excess return, thus effectively lowering the franchise value and making them vulnerable to the runs. See Jiang et al. (2023) and Drechsler, Savov and Schnabl (2023) for a discussion of these issues.

Using the threshold value in (5) together with Proposition 1, we can rewrite k_1^D as

$$k_1^D \equiv \frac{\beta}{\left(1 - \gamma_0 \frac{p1}{p_0}\right)^{\beta}} \frac{(\hat{z}^F + p_0)k_0 - Rb_0}{p_0}.$$

And it follows that

$$\frac{k_1^R}{k_1^D} = \left(1 - \gamma_0 \frac{p1}{p_0}\right)^{-(1-\beta)} \geq 1.$$

Equilibrium consistent borrowing limit $\{\gamma_t\}$ guarantees $1 - \gamma_0 \frac{p1}{p_0} \geq 0$. If $\gamma_0 > 0$, it thus follows that the demand for capital of the marginal bank if it were to default is strictly *lower* than the demand for capital if it were to repay.

Let us consider next the bank at the run threshold, $\hat{z}^{Run}$. The demand for capital of this marginal bank if it repays its debts *when facing a run* is

$$k_1^{Run} \equiv \beta \frac{(\hat{z}^{Run} + p_0)k_0 - Rb_0}{p_0}. \tag{9}$$

Note the distinction between (9) and (7): *the bank under a run cannot leverage*, and thus its demand for capital is lower than if the same bank repays but faces no run.

If this marginal bank were to default, its demand for capital is the same as it was in (8). Using the indifference (6), we can rewrite this as

$$k_1^D = \left(\frac{z + p_1}{z + p_1(1 - R\gamma_0)}\right)^{\beta} \beta \frac{(\hat{z}^{Run} + p_0)k - Rb}{p_0}.$$

And it follows that

$$\frac{k_1^{Run}}{k_1^D} = \left(1 - \frac{R}{R_1^k}\frac{p_1}{p_0}\gamma_0\right)^{\beta} \leq 1.$$

If $\gamma_0 > 0$ and $R_1^k > R$, equilibrium consistent $\{\gamma_t\}$ guarantees $1 \geq \gamma_0 \frac{p1}{p_0} > \gamma_0 \frac{R}{R_1^k}\frac{p1}{p_0}$. That is, the demand for capital of the marginal bank $\hat{z}^{Run}$ if it were to repay is *strictly lower* than the demand if it is subject to a run and it were to default. This is the opposite of the case at the fundamental default threshold. That is, the marginal bank at the fundamental threshold demands more capital than a defaulting bank, while the marginal bank at the run threshold demands less capital than a defaulting bank. It thus follows that the marginal bank at the run threshold sells more capital than the marginal bank at the fundamental threshold.

Notice that at this point, we have not established in which case banks are net sellers or net buyers. We will examine this once we study the general equilibrium.

To summarize the results of this subsection, for a given sequence of prices, a bank with $z_0 = \hat{z}^{Run}$ *that faces a run* (and decides to repay its debts) sells more capital than a bank with the same productivity that decides to default and a repaying bank at the fundamental default threshold. As we will see below, this distinction with regard to the demand for capital plays a role in understanding the effects of a policy that affects the equilibrium price of capital.

1.6 Discussion of modeling choices

Before we turn to the general equilibrium and policy analysis, let us discuss the modeling assumptions we made.

Curvature and production. First, we model banks as agents with concave utility that directly produce the final consumption good. The assumption of curvature in the utility function over dividends (or equity payouts) captures the fact that issuing equity is costly and delivers smooth dividend payments, as observed in the data. The assumption that banks make production decisions is also standard in the macro-finance literature (see, e.g., Gertler and Kiyotaki, 2015) and allows us to capture in a simple way the financial channel by which banks' capital affect output.

Short-term debt and deposit insurance. One crucial feature of the model that makes banks vulnerable to runs is that they issue short-term bonds. Given the reliance of banks on demand deposits in practice, we think this is a central institutional feature. We take the nature of the short-term non-state contingent deposits as a primitive in our model.[22] To the extent that a large fraction of depositors of commercial banks are insured, one can map the model more easily into investment banks, which indeed played a key role during the collapse of the financial system in 2008 (see, e.g., Brunnermeier, 2009; Bernanke, 2013). While a large fraction of depositors of commercial banks may be insured, this insurance is often limited or imperfect in practice. In fact, the banking turmoil in March 2023 has revealed significant uninsured deposits among commercial banks and shown that even insured deposits may be prone to runs.[23]

Default decision. We have also assumed that banks default strategically (i.e., default is a choice of the bank not to repay its depositors). To the extent that banks face limited liability and that

[22] Standard reasons why issuing short-term debt may be optimal have to do with liquidity benefits (Stein, 2012) or incentive reasons under asymmetric information (Diamond and Rajan, 2000; Calomiris and Kahn, 1991).

[23] Even though insured depositors may recover the totality of their depositors, the bureaucratic cost may prompt depositors to run, especially given the low costs of switching bank accounts. Another reason why insured deposits may choose to run is that they may be concerned about the solvency of the insurance funds.

equity holders can choose in practice whether to capitalize the bank or let it fail, we see this as a desirable feature of the model. Our assumptions about the cost of defaulting—namely, permanent exclusion from the bond market and lower productivity—deserve additional comments. Clearly, in practice, there is a bankruptcy procedure that details specific costs for banks from defaulting. Our assumptions in this regard are due, for the most part, to tractability reasons, as they allow us to scale the value of default and derive a tractable representation of the value of the bank under repayment facing a leverage constraint; however, we will see below that the results of our policy analysis do not hinge on the specific assumptions on default costs. At the same time, it is worth noting that the assumption accounts for certain realistic features. For example, equity holders often perceive positive payoffs even around bankruptcy. For example, investment banks that failed or were bailed out in the 2008 financial crisis, such as Lehman Brothers or Bear Stearns, paid almost as many dividends in the run-up to the crisis as in the years preceding the crisis. As observed by Acharya et al. (2022) and Acharya, Le and Shin (2017), paying dividends in such circumstances constitutes a transfer of resources from bondholders to shareholders. More recently, Credit Suisse shareholders in March 2023 perceived positive payoffs, while some of the bondholders did not.[24]

Finally, our assumption about productivity losses upon default is related to the fact that we are, in effect, consolidating financial and non-financial firms into a single entity by allowing banks to manage the capital stock directly. In turn, the empirical evidence shows that bank failures cause dislocations for firms that hold lending relationships with the failing banks (see, e.g., Fukuda, Kasuya and Akashi, 2009; Chodorow-Reich, 2014; May, 2014). As we saw, these modeling assumptions generate endogenously a borrowing constraint on banks similar to those in the literature, in which firms or banks can walk away from their obligations and abscond with funds from creditors or shareholders (e.g., Gertler and Kiyotaki, 2010).

Overall, these modeling assumptions allow us to embed self-fulfilling runs in a tractable dynamic general equilibrium model and to transparently analyze the effect of macroeconomic policies. We turn next to characterize the general equilibrium properties of the model.

[24]Historically, when a bank is close to bankruptcy, the government often intervenes to sell the bank so that shareholders recover a positive amount that is increasing in the value of the asset holdings. Two examples are Bear Stearns and Merrill Lynch in 2008. The former was acquired by JP Morgan in the face of extensive conflicts between bondholders and shareholders about who would face the burden of the losses (see Landon Thomas Jr., "It's Bondholders vs. Shareholders in a Race to Buy Bear Stearns Stock," *New York Times*, March 19, 2008). In the case of Merrill Lynch, investors lost confidence in its sustainability during the same week Lehman filed for bankruptcy, and Bank of America acquired it through the active intervention of the Federal Reserve (see, e.g., Gretchen Morgenson, "The Reckoning: How the Thundering Herd Faltered and Fell," *New York Times*, Nov 8, 2008). Extensive cross-country evidence about the resolution of banking crises is collected in the series of case studies in the *Journal of Financial Crises*.

2 General Equilibrium

In the previous section, we described the problem of an individual bank in partial equilibrium for a given price of capital $\{p_t\}$. We showed how the borrowing limits $\{\gamma_t\}$ are determined and discussed the differences between fundamental defaults and runs. In this section, we close the model by clearing the capital market.

As mentioned above, the economy is populated by a measure one of banks, which are assumed to be identical at the beginning of time. That is, each bank starts with $k_0 = \overline{K}$ units of the capital stock and a debt level $b_0 = B_0$ in period $t = 0$.

The occurrence of equilibrium default is limited to the initial period, $t = 0$. To reiterate, any bank with a productivity level, z_0, below $\hat{z}^F$ will inevitably default at $t = 0$. However, for banks with productivity levels between $\hat{z}^F$ and $\hat{z}^{Run}$, there are multiple potential equilibrium outcomes. Within this range, if a creditor anticipates that other creditors will withdraw their funds and the bank will default, it is optimal for that creditor also to withdraw its funds, leading to the bank's default. On the other hand, if a creditor expects other creditors to extend their loans, they will continue to lend to the bank, preventing its default. Consequently, the presence of a non-empty interval $(\hat{z}^F, \hat{z}^{Run})$ allows for multiplicity.[25]

We focus on threshold equilibria.[26] That is, our general equilibrium definition (provided below) requires a default threshold, denoted as $\hat{z}$, such that all banks with a productivity level below $\hat{z}$ at time $t = 0$ default, and all those with a productivity level above, it repay. This means that the proportion of banks defaulting at time $t = 0$ is given by $F(\hat{z})$. We consider two polar cases:

- In the first case, there are no runs, and we set $\hat{z} = \hat{z}^f$.
- In the second case, any bank that is susceptible to a run defaults at time $t = 0$, meaning $\hat{z} = \hat{z}^{Run}$.

Let N_t denote the total net worth of all repaying banks, and N_t^D the total net worth of all defaulting banks. In period $t = 0$, we have that

$$N_0 = \int_{\hat{z}}^{\overline{z}} ((z_0 + p_0)\overline{K} - RB_0) dF(z_0) \tag{10}$$

$$N_0^D = F(\hat{z})(z^D + p_0)\overline{K}. \tag{11}$$

[25] In Amador and Bianchi (2024) we showed that in the absence of runs, the model features unique equilibrium asset prices given a share of defaulting banks in period $t = 0$ (see Proposition 4). A similar result carries over here. In that paper, we also showed that the stationary equilibrium is unique in the absence of runs. This result contrasts with with the framework of Gertler and Kiyotaki (2015), which has multiple equilibrium asset prices.

[26] In Section 4, we explore an extension to this equilibrium selection procedure.

Using the linearity of the policy rules from Lemmas 1 and 2, we can trace out the aggregate net worth evolution:

$$N_{t+1} = \beta R^e_{t+1} N_t \tag{12}$$

$$N^D_{t+1} = \beta R^D_{t+1} N^D_t \tag{13}$$

for all $t \geq 0$ (where we exploit that the return to equity is the same for all banks).

For every $t \geq 0$, let K^D_t denote the total capital holdings of defaulting banks, and let K^R_t denote the total capital holdings of the banks that repay. Let δ_{t+1} and κ_{t+1} denote optimal policies for debt and capital in period t for a repaying bank that starts with net worth equal to 1. Let κ^D_{t+1} be an optimal capital policy for a defaulting bank with (defaulted) net worth equal to 1. Then, the linearity of the policy functions implies that total debt and capital levels are given by

$$B_{t+1} = \delta_{t+1} N_t \tag{14a}$$

$$K^R_{t+1} = \kappa_{t+1} N_t \tag{14b}$$

$$K^D_{t+1} = \kappa^D_{t+1} N^D_t \tag{14c}$$

for all $t \geq 0$.

Finally, market clearing requires that at all times,

$$K^D_t + K^R_t = \bar{K} \tag{15}$$

for all $t \geq 1$. With this, we can define a general equilibrium, encompassing both the case without runs and the case with runs:

Definition 2 (General Equilibrium). *A competitive equilibrium with default threshold $\hat{z}$ is a sequence of prices of capital, $\{p_t\}_{t=0}^{\infty}$, a sequence of borrowing limits, $\{\gamma_t\}_{t=0}^{\infty}$, and a sequence of net worths, debt and capital holdings, $\{N_t, N^D_t, B_t, K^R_t, K^D_t\}_{t=0}^{\infty}$, such that*

(i) the evolution of net worth follows (12) *and* (13) *with initial conditions given by* (10) *and* (11)*;*

(ii) for all $t \geq 0$, total debt and capital holdings follow equations (14a), (14b), *and* (14c) *with $\delta_{t+1} = \mathcal{B}_{t+1}(1)$, $\kappa_{t+1} = \mathcal{K}^R_{t+1}(1)$, and $\kappa^D_{t+1} = \mathcal{K}^D_{t+1}(1)$, and where $\mathcal{B}_{t+1}$, $\mathcal{K}^R_{t+1}$ $\mathcal{K}^D_{t+1}$ are optimal policy functions that solve the banks' problem in repayment and default, respectively, given $\{\gamma_t\}$ and $\{p_t\}$;*

(iii) the borrowing limits are equilibrium consistent; that is, Definition 1 is satisfied;

(iv) markets clear; that is, equation (15) *holds for all $t \geq 1$; and*

(v) the threshold $\hat{z}$ is defined by (5) *in the case of only fundamental defaults; and by* (6) *in the case of runs.*

Equilibrium characterization.

At $t = 0$, the state variable is the aggregate amount of debt B_0 (recall that all banks start with the same amount of capital and debt). Depending on the initial level of debt, there are three possible equilibrium scenarios.

When the debt level is very low, all banks repay. That is, at the market clearing sequence of asset prices, the default thresholds (5) and (6) with fundamentals and runs, respectively, are such that even the bank with the lowest productivity still finds it optimal to repay. In the case of $\beta R < 1$, the economy features a transition towards a stationary equilibrium in which aggregate debt and asset prices remain constant at $p^R = \frac{\beta z}{1-\beta-(1-\beta R)\gamma^R}$ and γ^R satisfies (4) given the constant price p^R. Given that policies are linear in individual net worth, all banks' net worth evolves at the same rate independent of their initial productivity, and aggregate dynamics can be characterized in terms of the aggregate net worth. As we showed in Amador and Bianchi (2024), the dynamics are as follows: For T periods, the return to capital is exactly R, aggregate net worth decreases at rate βR, and the borrowing constraint does not bind. In period T, the borrowing constraint binds, the return to capital is higher than R, and the economy remains at a stationary repayment equilibrium thereafter.[27]

When the debt level is very high, all banks default. In this case, the economy transitions immediately to a stationary equilibrium in which the market clearing price is $p_t = \frac{1}{1-\beta} z^D$. The economy falls into this stationary default equilibrium whenever the debt level is such that the highest productivity bank exceeds the thresholds (5) and (6) with fundamentals and runs, respectively.

When the debt is an intermediate region, we have a fraction of banks defaulting. Specifically, those banks with productivity below the corresponding threshold default, and those with productivity above it repay. As shown above, repaying banks buy more capital than defaulting banks, implying that in general equilibrium, repaying banks are on average net buyers of capital, while defaulting banks are net sellers. Over time, this means that defaulting banks shrink while repaying banks grow, and thus the economy converges to the same stationary equilibrium analyzed above (i.e., one in which $p^R = \frac{\beta z}{1-\beta-(1-\beta R)\gamma^R}$ and γ^R satisfies (4)).

Let us illustrate these results by simulating the model numerically for different initial values of aggregate debt B_0. In Figure 1, we present three key variables in three panels as a function of

[27] In the case of $\beta R = 1$, the return on capital equals the interest rate for all t and the portfolios of individual banks is undetermined. Moreover, net worth of individual banks is constant and thus, the aggregate amount of debt remains constant for all t.

the initial debt level: (a) the fraction of banks that default $F(\hat{z})$; (b) the initial price of capital p_0; and (c) the initial share of capital held by repaying banks for a range of initial values of debt $K_1^R/\overline{K}$. The red dashed line corresponds to the economy with runs, and the blue solid line corresponds to the economy in which we shut down the possibility of runs.

As Figure 1 shows, the fraction of defaulting banks increases continuously with the debt level until the point at which all banks default. In addition, the price of capital and the share of capital held by repaying banks fall monotonically with the level of debt. One can also see that as the share of banks defaulting approaches one, the price of capital becomes constant at the stationary level p^D, and repaying banks hold zero capital.

The figure also shows that for intermediate values of debt, the share of defaulting banks is higher in the presence of runs. In line with the above characterization, runs reduce the default threshold for given asset prices. Moreover, in general equilibrium, the fact that more banks default in the presence of runs implies that capital prices are lower. This reflects that the demand for capital is higher for non-defaulting banks.

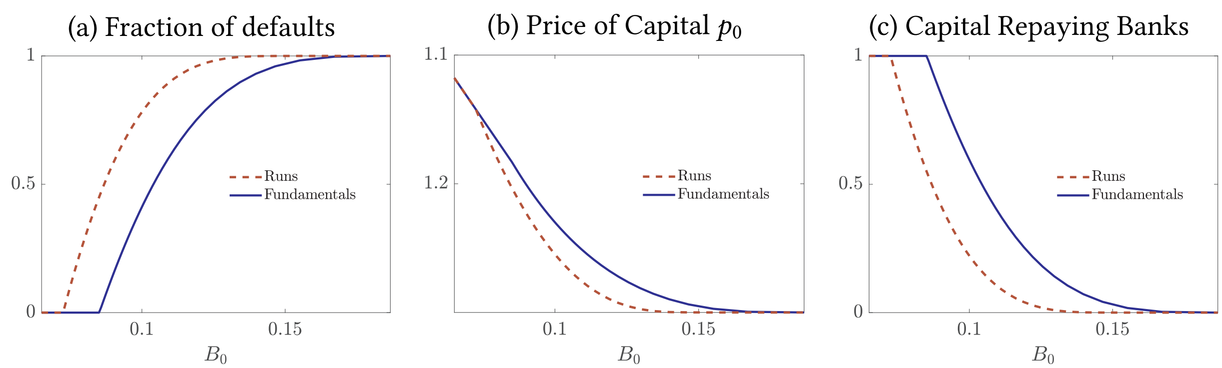

Figure 1: Transitional dynamics for a range of values of B_0

Notes: The simulation was generated using $R = 1.06$, $\beta = 0.8/R$, $z^D = \beta/(1-\beta)$, $z = 1.02z^D$, $\overline{K} = 1$, and a uniform distribution of z_0 between $[0.98z, 1.02z]$.

Figure 2 zooms in on the model simulations by focusing on an intermediate value of debt for which we have positive defaults in equilibrium. The figure presents the evolution of the price of capital, the leverage threshold, and the share of capital held by repaying banks over time. As the figure shows, both economies (with and without runs) converge in the long run to the stationary equilibrium in which the price of capital is p^R and all the capital is held by repaying banks.[28]

[28] To understand the evolution of the price of capital, recall that in period $t = 0$, we have a distribution of bank productivities under repayment. Given that we set a distribution of z_0 centered on z, the average of productivity conditional on repayment is higher in period $t = 0$ than at $t = 1$. On the other hand, from $t \geq 1$, productivity is constant and, therefore the evolution of the price of capital is monotonic from $t = 1$ onward.

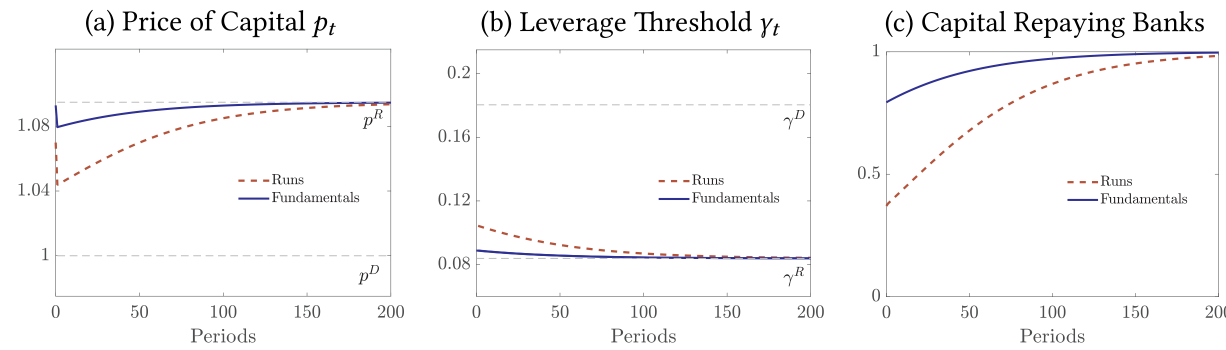

Figure 2: Transition dynamics

Notes: The simulation was generated using $R = 1.06,\ \beta = 0.8/R,\ z^D = \beta/(1-\beta),\ z = 1.02z^D,\ \bar{K} = 1,\ B_0 = 0.092$, and a uniform distribution for z_0 between $[0.98z, 1.02z]$.

Recall that the net worth of defaulting banks evolves at a rate βR^D, which is less than one in general equilibrium, and so asymptotically, they hold zero capital. Importantly, repaying banks hold a lower share of capital in the transition in the economy where banks face runs. Again, this reflects that more banks are defaulting, which in turn implies lower demand for capital and lower prices of capital.

Banks' welfare in equilibrium.

Given the default threshold $\hat{z}$, the ex-ante payoff of a bank at the beginning of $t = 0$ (before the realization of z_0), $W(\hat{z})$, is

$$W(\hat{z}) \equiv \int_{\hat{z}}^{\bar{z}} V_0^R((z_0 + p_0)\bar{K} - RB_0) dF(z_0) + F(\hat{z}) V_0^D((z^D + p_0)\bar{K}).$$

Keeping all other equilibrium objects constant, we can compute the effect on the ex-ante payoff of a change in the threshold $\hat{z}$:

$$W'(\hat{z}) = -f(\hat{z})(V_0^R((\hat{z}_0 + p_0)\bar{K} - RB_0) - V_0^D((z^D + p_0)\bar{K})) \tag{16}$$

Note that when evaluated at $\hat{z}^F$, $W'(\hat{z}^F) = 0$, the marginal bank is indifferent between repaying or defaulting, and thus an exogenous change in the default threshold has no first-order effect on the bank's payoff.[29]

Under runs, when $\hat{z} = \hat{z}^{Run}$, if $\gamma_0 > 0$ and $R_1^k > R$, we have that

$$V_0^R((\hat{z}^{Run} + p_0)\bar{K} - RB_0)) > V_0^D((z^D + p_0)\bar{K})).$$

[29]Of course, such a change will have equilibrium effects on prices and γ's, but we are ignoring them here.

Even though the marginal bank that defaults under a run is indifferent between defaulting and repaying, it is strictly better off if it does not face a run and repays. This reflects the costs of the coordination failure of runs: *a solvent bank at the margin is defaulting because of a run.* According to equation (16), $W'(\hat{z}^{Run}) < 0$. That is, an exogenous reduction in the default threshold strictly increases the bank's ex-ante payoff.

Regarding creditors, for a given size of the initial debt B_0, a decrease in the default threshold raises creditors' payoffs as they are repaid with higher probability, while an increase in the threshold lowers their payoff.

3 A Credit-Easing Policy

In this section, we turn to government policies. Our objective is to compare the effects of policy interventions when equilibrium defaults are driven by fundamentals or runs. We focus attention on ex-post policies—that is, policies that take place at $t = 0$ for a given initial level of aggregate debt that is maturing at that period.[30]

One inefficiency at play in our model emerges from the presence of an equilibrium price (the price of capital) in the determination of a bank's default option. As shown in Kehoe and Levine (1993), this can lead to inefficiencies in the market equilibrium. As we will see below, however, the presence of runs introduces another reason for policy intervention (coordination failures), which is the main focus of the analysis in this section.

3.1 Equilibrium with credit easing policy

We consider a "credit-easing" policy in which the government purchases capital at $t = 0$, holds it for one period, and sells it back at $t = 1$.[31] After $t > 1$, the government does not intervene. We assume that the government is less productive than a defaulting bank: a unit of capital in the hands of the government has a productivity of $z^g < z^D$; thus, holding capital entails a cost. To finance the purchases of capital, the government taxes banks in period $t = 0$ and borrows at the interest rate R. Note that in this exercise, the government is not taxing banks in any period after $t = 0$: we are not granting the government the ability to bypass the borrowing constraint of banks through its taxation power.[32]

[30] As mentioned in the literature review, many studies in the banking literature examine policies to deal with the coordination failure driving runs, such as lender of last resort, freezing of deposits, or deposit insurance. There are generally well-known trade-offs associated with these policies. Our focus is on government policies that operate through general equilibrium effects. We also leave the issue of how policies affect the ex-ante borrowing decisions and welfare for future work.

[31] Bernanke (2009) describes the program of asset purchases in the 2008 financial crisis as "credit easing."

[32] This ability can be beneficial, as shown in Woodford (1990).

We assume that the tax takes the form of a proportional tax on net worth in period $t = 0$.[33] Let τ_0 denote the tax that the government imposes on banks in period $t = 0$, and let K^g denote the units of capital that the government purchases. The post-tax net worth of banks in period $t = 0$ is then

$$N_0 = (1 - \tau_0) \int_{\hat{z}}^{\bar{z}} ((z_0 + p_0)\bar{K} - RB_0) dF(z_0), \tag{17}$$

$$N_0^D = (1 - \tau_0)(z^D + p_0)F(\hat{z})\bar{K}. \tag{18}$$

And using that the government repays the debt in period $t = 1$ by selling its holdings of capital, we can write the government intertemporal budget constraint as

$$p_0 K^g - \frac{\tau_0}{1 - \tau_0}(N_0 + N_0^D) = \frac{1}{R}(z^g + p_1)K^g, \tag{19}$$

where the right-hand side is the discounted value of the revenue from using and selling the capital in period $t = 1$.

The values for repaying and defaulting banks are analogous to those obtained before in the case without the policy intervention, but with the difference that the initial net worth now incorporates the taxes needed to finance the purchases of capital by the government. Given prices and borrowing limits, the value functions remain as before, but now using these post-tax net worth values. It thus follows that the default thresholds $\hat{z}^F$ and $\hat{z}^{Run}$ remain unaltered given a sequence of prices and borrowing limits. That is, they are given by equations (5) and (6).

The market clearing condition for capital at time $t = 0$ now becomes

$$K_1^D + K_1^R + K^g = \bar{K} \tag{20}$$

and remains as before for all $t \geq 1$.

We can now define a general equilibrium with the policy:

Definition 3. *A competitive equilibrium with a credit easing policy* (τ_0, K^g) *and default threshold* $\hat{z}$ *is a sequence of prices of capital,* $\{p_t\}_{t=0}^{\infty}$, *a sequence of borrowing limits,* $\{\gamma_t\}_{t=0}^{\infty}$, *and a sequence of net worth, debt and capital holdings,* $\{N_t, N_t^D, B_t, K_t^R, K_t^D\}_{t=0}^{\infty}$ *such that*

(i) the evolution of net worth follows (12) *and* (13)*, with initial conditions given by* (17) *and* (18)*;*

(ii) for all $t \geq 0$, *total debt and capital holdings follow equations* (14a), (14b), *and* (14c) *with* $\delta_{t+1} = \mathcal{B}_{t+1}(1)$, $\kappa_{t+1} = \mathcal{K}_{t+1}^R(1)$, *and* $\kappa_{t+1}^D = \mathcal{K}_{t+1}^D(1)$, *and where* $\mathcal{B}_{t+1}$, $\mathcal{K}_{t+1}^R$ $\mathcal{K}_{t+1}^D$ *are optimal*

[33] An alternative approach is to use a lump-sum tax instead. The main results regarding the effects of changes in policy are not sensitive to this choice, but the proportional tax on net worth has the property that (given prices and borrowing limits) the default thresholds are not directly affected by the policy.

policy functions that solve the banks' problem in repayment and default, respectively, given $\{\gamma_t\}$ *and* $\{p_t\}$*;*

(iii) the borrowing limits are equilibrium consistent; that is, Definition 1 is satisfied;

(iv) markets clear, that is, equation (20) *holds at* $t = 1$, *and* (15) *holds for all* $t > 1$*;*

(v) the threshold $\hat{z}$ *is defined by* (5) *in the case of only fundamental defaults; and by* (6) *in the case of runs;*

(vi) the government budget constraint, equation (19), *holds.*

As a final detail, in this definition of equilibrium, we are granting the government the ability to hold the capital stock (albeit unproductively). Yet, we have not allowed creditors to do the same. Assuming that the productivity of creditors is the same as the government's, creditors will not hold capital if their return, R_1^g, is lower than their discount factor, R. So, if

$$R_1^g \equiv \frac{z^g + p_1}{p_0} \leq R, \tag{21}$$

then creditors will not hold capital, even if allowed. We are going to focus attention to equilibria in which the above condition holds. This condition allows us to evaluate whether the government may want to purchase capital when creditors are able but unwilling to do so.

Note that inequality (21) implies that the government loses resources by intervening, and as a result, it needs to tax banks in order to finance its capital purchases. We can see this by noticing that the government's budget constraint can be rewritten as

$$\frac{\tau_0}{1-\tau_0}(N_0 + N_0^D) = \frac{R - R_1^g}{R} p_0 K^g \geq 0, \tag{22}$$

for $K^g \geq 0$, where the inequality inherits the strictness of (21). We have then narrowed our attention to a policy that is unprofitable for the government (and undesirable for creditors), requires the taxation of banks at time $t = 0$, and may entail an efficiency loss.

3.2 The effect of credit easing

In this section, we provide a theoretical approach for assessing the effects of a credit easing policy in the general equilibrium of the model. We then complement this analysis with numerical simulations.

An important component of a general equilibrium is the default threshold, which determines the share of defaulting banks and affects the dynamics that follow. As stated previously, the proportional tax on net worth used by the government to finance its purchases of capital leaves the default thresholds $\hat{z}^F$ and $\hat{z}^{Run}$ determined by the same equations, (5) and (6). Thus, the thresholds are influenced by the policy *only through general equilibrium effects*–, that is, only through changes in prices (and the corresponding equilibrium borrowing limits).

To make headway in this subsection, we are going to analyze the general equilibrium effects that are induced by the change in the price of capital at the moment of the policy, p_0, *while keeping constant all future prices* $\{p_t\}_{t=1}^{\infty}$.[34] The latter implies that the sequence of borrowing limits $\{\gamma_t\}_{t=0}^{\infty}$ also remains unchanged, given Proposition 1. So we narrow the question to how the policy, K^g, affects the default thresholds through its impact on p_0.

An increase in the price of capital. Recall that the default thresholds are determined according to the following indifference condition:

$$\hat{V}((1-\tau_0)((\hat{z}+p_0)\bar{K}-RB_0)) = V_0^D((1-\tau_0)(z^D+p_0)\bar{K}),$$

with $\hat{V} = V_0^R$, $\hat{z} = \hat{z}^F$, in the case of of the fundamental threshold, and $\hat{V} = V_0^{Run}$, $\hat{z} = \hat{z}^{Run}$, in case of the run threshold. The log-linear functional forms of the value functions, characterized in Lemmas 1 and 2, imply that the tax can be canceled in the above equation, and the indifference condition becomes $\hat{V}((\hat{z}+p_0)\bar{K}-RB_0) = V_0^D((z^D+p_0)\bar{K})$.

Assuming that the default threshold $\hat{z}$ is interior, we can differentiate the above expression with respect to p_0 and obtain

$$\hat{V}'(n(p_0))\frac{d\hat{z}}{dp_0} = \frac{dV_0^D(n^D(p_0))}{dp_0} - \frac{d\hat{V}(n(p_0))}{dp_0},$$

where $n(p_0) = (\hat{z}+p_0)\bar{K}-RB_0$ and $n^D(p_0) = (z^D+p_0)\bar{K}$. The two total derivatives on the right-hand side capture two effects: the change in the value because of the change in returns and the change in the values because of the changes in net worth.

The equation tells us that the response of the default threshold to a (marginal) increase in the price depends on whether the repaying or the defaulting bank is hurt relatively more by the price increase. This is intuitive: if the value of defaulting increases at the margin more than the value of repaying, then the default threshold increases (and more banks default in equilibrium). The opposite occurs if the value of repaying increases at the margin more than the value of defaulting.

Consider the marginal bank. Let k_1 denote the amount of capital it purchases if it repays. Specifically, $k_1 = k_1^R$ in case of a fundamental threshold, and $k_1 = k_1^{Run}$ in case of a run threshold.

[34] In a general equilibrium, all prices respond to the policy, but we leave the discussion of these additional effects to the numerical simulations.

Let k_1^D denote the amount of capital the bank purchases if it defaults. These capital demand values are given by equations (7), (8), and (9), setting $k_0 = \overline{K}$ and $b_0 = B_0$.

Differentiating the value functions (1) and (2) with respect to p_0 and replacing in the expression above, we obtain

$$\hat{V}'(n(p_0))\frac{d\hat{z}}{dp_0} = -\left(\frac{k_1^D}{\overline{K}} - 1\right) V_0^{D\prime}(n^D(p_0)) + \left(\frac{k_1}{\overline{K}} - 1\right) \hat{V}'(n(p_0)), \tag{23}$$

where the first term in parenthesis is the net purchases of capital of a marginal bank if it defaults, and the second term is its net purchases if it repays. The formula conveys a straightforward intuition that arises from the envelope condition: an increase in the price of capital hurts a bank if it is a net buyer, while it benefits the bank if it is a net seller. The increase in the price reduces the value of the bank, depending on the net amount of capital it purchases. The formula simply reflects the difference of these effects of the price on the value of repaying and defaulting, each weighted by their marginal valuations.

Note that marginal valuations are equal to the inverse of marginal utilities. Exploiting the logarithmic utility, we have that

$$V_0^{R\prime}(n) = V_0^{Run\prime}(n) = \frac{1}{(1-\beta)n} \quad \text{, and} \quad V_0^{D\prime}(n^D) = \frac{1}{(1-\beta)n^D},$$

For the run threshold, $\hat{z}^{Run}$, equation (23) then implies

$$\frac{d\hat{z}^{Run}}{dp_0} = \frac{k_1^{Run}}{k_1^D} - 1 \leq 0,$$

with strict inequality if $\gamma_0 > 0$ and $R_1^k > R$. This inequality follows from the discussion in Section 1.5, which showed that $k_1^D \geq k_1^{Run}$. Thus, the increase in the price *reduces* defaults under the run scenario. Under a run, marginal repaying banks facing runs are *selling* more capital than if they were to default and thus benefit from an increase in the price.

The case under fundamental defaults is as follows. On the one hand, a repaying bank at the fundamental threshold is *buying* more capital than if it were to default (the opposite of the above). So, a price increase hurts the repaying bank relatively more and is a force towards increasing the threshold. But on the other hand, a repaying bank at the threshold has a lower marginal utility, generating a force towards the other direction if $k_1^R < \overline{K}$. Note that if $k_1^R > \overline{K}$, so the marginal bank that repays is a net buyer, then $d\hat{z}^F/dp_0 > 0$; that is, the increase in the price *leads to more defaults* in the case of fundamental defaults. Although $k_1^R > \overline{K}$ is sufficient for this adverse effect of the increase in p_0, it is not necessary.

We summarize below the key takeaway regarding the impact that asset prices have on the marginal bank, depending on whether it is at the fundamental default threshold or at the run threshold.

Remark. *An increase in the capital price in period 0 (while maintaining borrowing limits and all other prices constant) reduces the share of defaulting banks in the presence of runs. If the marginal bank at the fundamental threshold is a net buyer of assets, the share of defaulting banks increases in the absence of runs.*

The policy, the price, and market clearing. The final element to discuss before moving on to the numerical analysis is the effect of the credit easing policy on the market clearing price.

Whether we are in a run scenario or not, the capital demand that is relevant for the market clearing condition is the "no-runs" capital demand (i.e., the demand for a bank that repays and *does not suffer* a run). In our model, as in Cole and Kehoe (2000), a bank that repays while suffering a run is an out-of-equilibrium event. The market clearing condition for capital in period $t = 0$ can be rewritten as

$$(1-\tau_0)\left[\int_{\hat{z}}^{\overline{z}} k_1^R(z_0)dF(z_0) + k_1^D F(\hat{z})\right] = \overline{K} - K^g,$$

where $k_1^R(z_0)$ represents the demand for capital for a bank that does not suffer a run and repays with a net worth $(z_0 + p_0)\overline{K} - RB_0$. We explicitly write the dependence of this capital demand on the period 0 productivity draw, z_0, as it encompasses not just the marginal bank at the relevant default threshold but all banks above that threshold. As before, we let k_1^D denote the demand for capital for a defaulting bank with net worth $(z^D + p_0)\overline{K}$. The $(1-\tau_0)$ factor that pre-multiplies capital demand captures the effect of the tax on net worth.

Assuming again that the threshold is interior and continuing to ignore all the general equilibrium effects on prices other than p_0, we can differentiate the market clearing condition above to obtain

$$(1-\tau_0)\left[\int_{\hat{z}}^{\overline{z}} \frac{dk_1^R(z_0)}{dp_0}dF(z_0) + \frac{dk_1^D}{dp_0}F(\hat{z}) - (k_1^R(\hat{z}) - k_1^D)f(z_0)\frac{d\hat{z}}{dp_0}\right]\frac{dp_0}{dK^g} - \frac{\overline{K} - K^g}{(1-\tau_0)}\frac{d\tau_0}{dK^g} = -1, \quad (24)$$

The right-hand side of the above expression shows we must have a reduction in the aggregate capital demand by banks to accommodate the government purchases. On the left hand-side, the terms in square brackets capture the effect of an increase in the price p_0 on the aggregate capital demands. It contains three terms: the inframarginal effect on the demand from repaying banks, the inframarginal effect on the demand from defaulting banks, and the marginal effect due to the change in the threshold. The final term on the left-hand side captures the reduction in demand due to the increase in the proportional tax required to finance the capital purchases.

It is straightforward to see, by inspecting (8), that dk_1^D/dp_0 is negative. The effect on the inframarginal demand from repaying banks can also be shown to be negative (starting from $K^g = 0$):

Lemma 4. *Consider a general equilibrium without a credit easing policy ($\tau_0 = 0, K^g = 0$) and associated prices $\{p_0\}$, borrowing limits, $\{\gamma_t\}$, and default threshold $\hat{z} \in (\underline{z}, \overline{z})$. Then,*

$$\int_{\hat{z}}^{\overline{z}} \frac{dk_1^R(z_0)}{dp_0} dF(z_0) \leq 0,$$

where the derivative represents a marginal change in p_0 keeping all other prices $\{p_{t+1}\}$ and borrowing limits $\{\gamma_t\}$ unchanged.

Let us consider a situation like the lemma, starting from a no credit easing policy, but where the prices are such that $R_1^g = R$. In that case, a marginal increase in K^g from 0 has a zero first-order effect on the tax τ_0, as can be seen from the budget constraint (22). And thus, the last term on the left-hand side of (24) is zero. In the case of fundamental defaults, we have that $\hat{z} = \hat{z}^F$. Recall again that $k_1^R(\hat{z}^F) \geq k_1^D$, and thus if $d\hat{z}^F/dp_0 \geq 0$; (as expected given our previous discussion), then all terms inside the square brackets of equation (24) are negative. This implies that $dp_0/dK^g > 0$, that is, the credit easing policy requires an increase in the price of capital at $t = 0$, p_0.

For the run threshold, $\hat{z}^{Run}$, we have argued previously that $d\hat{z}^{Run}/dp_0 \leq 0$. However, $k_1^R(\hat{z}^{Run}) \geq k_1^R(\hat{z}^F)$, given that $\hat{z}^{Run} \geq \hat{z}^F$, so it is no longer clear that the sum of all the terms within the square brackets of equation (24) is negative. If this were the case, then it would follow that $dp_0/dK^g > 0$, and again, the credit easing policy would induce an increase in the price of capital at $t = 0$ (starting from $R_1^g = R$).

Takeaway. To the extent that government losses from holding assets are not too large and that credit easing increases asset prices, the above analysis suggests that credit easing has positive effects on welfare in the presence of runs. In summary, the conclusion follows from three points highlighted above: (i) marginal banks facing a run have lower net purchases of capital than defaulting banks, as shown in Section 1.5; (ii) an increase in asset prices reduces the default threshold; and (iii) a decrease in the default threshold increases banks' welfare. On the other hand, in the economy without runs, to the extent that the marginal bank is a net buyer, credit easing increases the number of defaulting banks.

3.3 Numerical results

In the preceding analysis, we have taken an important short-cut: we have assumed that only the initial price, p_0, was affected by the policy. General equilibrium will potentially require changes in all subsequent prices, $\{p_{t+1}\}$, and, as a result, changes in the borrowing limits $\{\gamma_t\}$ as well. To see whether the above theoretical results generalize, we move on to study a numerical simulation of our model.

Figure 3 illustrates how the credit easing policy affects the default threshold depending on whether banks are subject to runs. Panel (a) presents the economy where banks are subject to runs. The figure presents the difference between the value function of repaying while facing a run and the value of defaulting at $t = 0$, as a function of productivity z_0, for $K^g = 0$ and $K^g = 1\%$ (labeled in the plot as "no policy" and "credit easing" respectively). The solid line indicates the difference in values when there is no policy intervention, and the dashed line indicates the difference in values when there is a credit-easing policy. The two solid dots represent the respective productivity threshold of the marginal bank (i.e., the productivity that makes a bank indifferent between repaying and defaulting). As one can see, the intervention shifts the curve to the left. The implication is that the default threshold is reduced, and fewer banks default. As highlighted above, the mechanism is that credit easing raises asset prices at time 0, increasing the value of repaying, because banks facing a run are net sellers of capital and benefit from the rise in asset prices. Thus, as argued in the theoretical analysis above, credit easing contributes to reducing defaults in an economy facing runs.

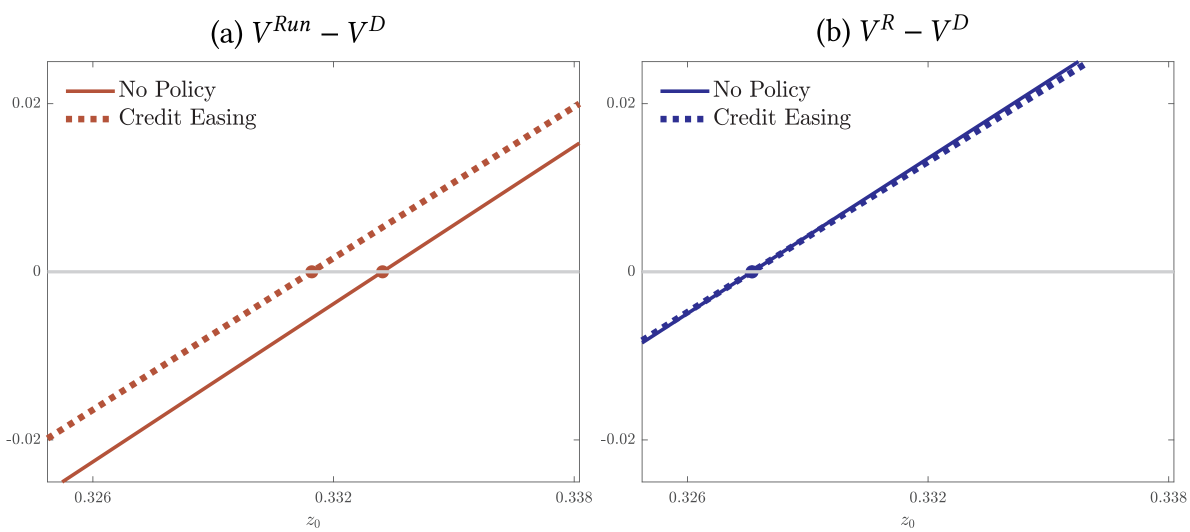

Figure 3: Credit Easing and Default Thresholds

Notes: The simulation was generated using $R = 1.06$, $\beta = 0.8/R$, $z^D = \beta/(1-\beta)$, $z = 1.02z^D$, $\bar{K} = 1$, $B_0 = 0.092$, $z^g = \frac{1}{5}z^D$, and a uniform distribution for z_0 between $[0.98z, 1.02z]$. Panel (a) corresponds to the economy with runs and panel (b) corresponds to the economy without runs.

Panel (b) shows the economy when banks are not subject to runs. The figure presents again the difference between the value of repayment and defaulting as a function of initial productivity z_0, but this time the value from repaying corresponds to the value of repaying while being safe. In contrast to the case with runs, the curve now rotates slightly to the right while the threshold remains about the same.[35]

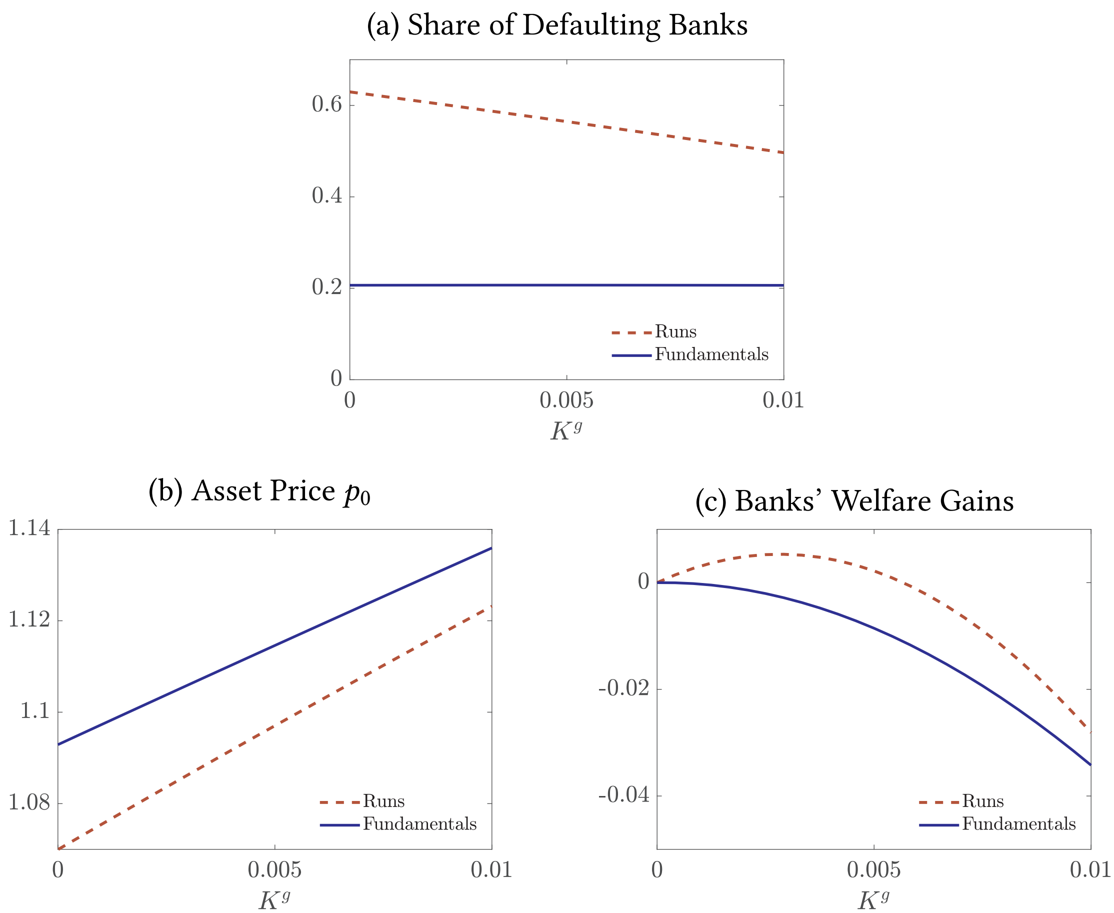

Figure 4: Credit Easing Policies

Notes: The simulation was generated using $R = 1.06$, $\beta = 0.8/R$, $z^D = \beta/(1-\beta)$, $z = 1.02z^D$, $\bar{K} = 1$, $B_0 = 0.092$, $z^g = \frac{1}{5}z^D$, and a uniform distribution for z_0 between $[0.98z, 1.02z]$.

[35]The higher is the productivity z_0, the larger the net buyer position. Thus, the rotation to the right indicates that the value of repaying falls relative to the value of default for relatively high productivity and increases for low productivity.

Figure 4 presents the results of credit easing for different levels of K^g. The three panels show the share of defaulting banks (panel [a]), the level of initial asset prices p_0 (panel [b]), and banks' welfare as a function of K^g (panel [c]). The red dashed line represents the economy with runs, and the blue solid line shows the economy in which defaults are due only to to fundamentals. In line with the results above, panel (a) shows that the share of defaulting banks falls monotonically with the level of asset purchases in the economy with runs, whereas in the case with only fundamental defaults, the share of defaulting banks remains almost the same. In both cases, we can see in panel (b) that the policy generates an increase in asset prices, as asset purchases raise the overall demand for capital.

In addition, panel (c) shows the different implications of credit easing for welfare, depending on whether defaults are due to runs or fundamentals. The simulations show that welfare is maximized for strictly positive asset purchases in the economy with runs, whereas welfare is decreasing in asset purchases in the economy without runs. As highlighted in equation (16), the reduction in the share of defaulting banks has a first-order positive welfare effect in the economy with runs. The overall effect depends in general on the balance between this effect and the losses from intervention, in addition to the other general equilibrium effects and the changes on $\gamma's$. In the parametrization considered, we obtain that the gains from reducing the share of defaulting banks in the presence of runs outweigh the government losses.

4 Discussion

In this section, we take stock on the main policy implications of our analysis and discuss several extensions of our baseline model.

4.1 Policy remarks

A distinctive implication of our model is that the desirability of credit easing depends on whether a crisis is driven by fundamentals or self-fulfilling runs.

Our result that credit easing may backfire in the case of fundamentals-driven crisis contrasts with much of the literature on unconventional policies, which attributes a stabilizing role to asset purchases during financial crises (e.g. Gertler and Karadi, 2011). A key feature that explains our results is that we allow for the possibility of endogenous default. Thus, as repaying banks are net buyers of assets, an increase in asset prices from credit easing depresses their value, pushing more banks into default.

On the other hand, we show that credit easing may be desirable during runs. The key difference with a fundamentals-driven crisis is that repaying banks facing a run are net sellers of assets and thus benefit from increases in asset prices. As a result, more banks become less vulnerable to runs because even if investors were to run, the bank would still be able to continue operations. Because defaults driven by runs are inefficient, credit easing generates a strictly positive effect on banks' welfare, while also preserving the value of creditors' bonds and raising their welfare.

Putting these findings together suggests that in a financial crisis, the policy response of using asset purchases may not necessarily be desirable. While it may indeed be difficult for policymakers to infer whether a crisis is driven by fundamentals or by self-fulfilling beliefs, a key takeaway is that the effectiveness of credit easing cannot be taken for granted in general and may depend on the source of the crisis.

Let us highlight also that for credit easing to be effective during a self-fulfilling runs, the government must hold assets. This policy implication contrasts with models in which an intervention occurs off the equilibrium path, such as that of Bocola and Dovis (2019).

4.2 Extensions

The model we presented can be extended in several directions. Below, we discuss a few of such extensions.

Sunspots. In our baseline general equilibrium definition, we consider a scenario in which the equilibrium is characterized by a default threshold, denoted by $\hat{z}$, causing all banks with $z_0 < \hat{z}$ to default at time $t = 0$. We studied the cases for $\hat{z} \in \{\hat{z}^F, \hat{z}^{Run}\}$, meaning the threshold could only take one of two specific values (part (v) of Definition 2). This restriction was helpful for highlighting the differences between the case with runs and the case with only fundamental defaults. However, the restriction is arbitrary. For instance, any value $\hat{z} \in (\hat{z}^F, \hat{z}^{Run})$ could have also been used to define a general equilibrium with a default threshold. We could have also introduced uncertainty. For example, in the sovereign debt literature, it is common to assume the existence of a sunspot variable, whose realization determines whether a run occurs or not, as in the work of Cole and Kehoe (2000).

Incorporating this alternative selection criteria into our model is not too complicated. To illustrate this, let's consider a scenario in which at the beginning of period $t = 0$, each bank is assigned an idiosyncratic random variable, denoted by π^i, which is drawn from a uniform distribution on the interval $[0, 1]$. We define equilibria based on a threshold value, denoted by $\overline{\pi}$, such that any bank with a realization $\pi^i < \overline{\pi}$ and with a productivity draw $z_0 \in [\hat{z}^F, \hat{z}^{Run})$ defaults because of a run. Given this, the only condition that needs to be changed in the general equilibrium definition is the net worth definitions (10) and (11). Specifically, we now have

$$
\begin{aligned}
N_0 &= \int_{\hat{z}^{Run}}^{\bar{z}} ((z_0+p_0)\bar{K} - RB_0)dF(z_0) + (1-\overline{\pi}) \int_{\hat{z}^F}^{\hat{z}^{Run}} ((z_0+p_0)\bar{K} - RB_0)dF(z_0) \\
N_0^D &= \left[F(\hat{z}^F) + (F(\hat{z}^{Run}) - F(\hat{z}^F))\overline{\pi} \right] (z^D + p_0)\bar{K}.
\end{aligned}
$$

The equilibrium definition in Definition 2 remains otherwise unchanged, except for requirement, (v) which needs to be modified to allow for a fraction $\overline{\pi}$ banks to default if $z \in (z^F, z^{Run})$. The cases $\overline{\pi} = 0$ and $\overline{\pi} = 1$ represent our two baseline cases without runs and with runs, respectively.

The policy analysis with sunspots is not too different from our baseline model. Note, however, that now one would need to take simultaneously into account the effects of the policy on both thresholds, $\hat{z}^F$ and $\hat{z}^{Run}$, as they both appear in the market clearing condition.

The initial debt level. Up until now, our analysis has assumed the initial debt level B_0 as given. Under the assumption that there is no uncertainty for $t \geq 1$, this implies that defaults occur solely at $t = 0$, contingent on the value of B_0. In this section, we expand the model to include a period $t = -1$, during which banks make a leverage decision while considering the potential occurrence of a default at $t = 0$. This allows us to determine the initial debt level by incorporating the dynamics of banks' actions and expectations in pricing the bond.

A bank in period $t = -1$ starts the period with some initial net worth, n_{-1}. The bank then chooses a level of capital for the following period, k_0, as well as a leverage choice $l_0 \equiv b_0/(p_0 k_0)$. The creditors anticipate the default probability in period $t = 0$, and thus the bond price in period $t = -1$ is

$$
q_{-1}(l_0) = 1 - F(\hat{z}(l_0)),
$$

where $\hat{z}(l_0)$ is given by (5) in case of fundamental defaults and by (6) in case of runs, and where we have made the dependence of the threshold on leverage explicit, as it is now a bank choice at $t = -1$. In period $t = -1$, the bank realizes that its own leverage choices affect the price of its bonds. The bank repayment problem at $t = -1$ is then

$$
\begin{aligned}
V_{-1}^R(n_{-1}) = \max_{c_{-1} \geq 0, l_0, k_0} \; & \log(c_{-1}) + \\
& \beta \left[\int_{\hat{z}(l_0)}^{\infty} V_0^R((z_0 + p_0 - Rl_0 p_0)k_0) f(z_0) dz_0 + F(\hat{z}(l_0)) V_0^D((z^D + p_0)k_0) \right]
\end{aligned}
$$

subject to

$$c_{-1} = n_{-1} + (1 - F(\hat{z}(l_0)))p_0 k_0 l_0 - p_{-1} k_0,$$

The log utility continues to imply that consumption at $t = -1$ remains a fraction $(1 - \beta)$ of the bank's net worth. Let $u'_{-1} \equiv 1/c_{-1}$ and $\mathbb{E}u'_0 \equiv \int_{\hat{z}(l_0)}^{\infty} \frac{1}{c_0} \frac{dF(z_0)}{1-F(z_0)}$. These correspond to the marginal utility out of a unit of consumption in period $t = -1$ and the expected marginal utility in period $t = 0$ conditional on repayment.

In the case in which $\hat{z} = \hat{z}^F$, assuming $\hat{z}^F$ is interior, the first-order condition for leverage is

$$\frac{1}{\mathbb{E}u'_0} - \frac{\beta R}{u'_{-1}} = \frac{1}{\mathbb{E}u'_0} \frac{f(\hat{z}^F)}{1 - F(\hat{z}^F)} \frac{\partial \hat{z}^F}{\partial l_0} l_0, \tag{25}$$

This simple formula has antecedents in the sovereign debt literature and provides a clear intuition for the optimal leverage choice.[36] If there was no default risk in period $t = 0$, the right-hand side would be zero, and the optimal policy would equalize marginal utility in period $t = -1$ to βR times the expected marginal utility in period $t = 0$, as usual. When there is default risk (and leverage is positive), the right-hand side reflects a positive wedge between the marginal utility in the initial period $t = -1$ and the expected marginal utility (conditional on repayment) in the second period $t = 0$, introducing an incentive for banks to reduce leverage. The left-hand side captures the balance of resources required to maintain a constant level of utility for the bank. When $\beta R < 1$, the bank is impatient relative to the creditors, and this is a force for additional leverage. Even though prices in equilibrium are actuarially fair, *default is costly* in the model. The right-hand side reflects the resources lost at the margin because of default risk, and it is a force for reducing leverage. The optimal choice of leverage balances these two forces, and which of these two dominates is a matter of quantitative analysis.

An additional term appears in the case of $\hat{z} = \hat{z}^{Run}$. Again, assuming that the threshold is interior, we have that the first-order condition for leverage is now

$$\frac{1}{\mathbb{E}u'_0} - \frac{\beta R}{u'_{-1}} = \frac{1}{\mathbb{E}u'_0} \frac{f(\hat{z}^{Run})}{1 - F(\hat{z}^{Run})} \frac{\partial \hat{z}^{Run}}{\partial l_0} \left[l_0 + \frac{\beta}{p_0 k_0 u'_{-1}} (\underline{V}^R - \underline{V}^D) \right],$$

where $\underline{V}^R$ and $\underline{V}^D$ are the $t = 0$ values of repayment and default at the run threshold.

Note the novel second term in the square brackets , a change from (25). We can use the threshold definition, as well as the shape of the value functions to obtain:

$$\underline{V}^R - \underline{V}^D = \frac{\beta}{1 - \beta} \log \left(\frac{R_1^e}{R_1^k} \right)$$

[36] See, for example, equation (16) in Aguiar et al. (2019).

That is, the difference between the value of repaying and defaulting at the run threshold is driven by the difference between the rate of return on equity, R_1^e, and the (unlevered) rate of return to capital, R_1^k, reflecting how during a run, the bank loses the ability to leverage.

The inequality $\underline{V}^R - \underline{V}^D > 0$ holds whenever $R_1^e > R_1^k$, providing an additional factor that reduces leverage. When runs can occur, a bank in the model exercises additional caution regarding its borrowing levels. This is because a higher debt level increases the probability of a run, which generates a discrete drop in the bank's payoff (as the bank is no longer indifferent between defaulting and repaying when facing a run). This additional motive for deleveraging disappears in the case of fundamental defaults, where $\underline{V}^R = \underline{V}^D$ at the threshold.

The default outside option. A key ingredient in the model is the endogeneity of bank's decision to default. Specifically, a bank compares the value of repaying against its outside option, which is the value of default. Moreover, we assumed that a bank that defaults can continue operating the capital (at a lower productivity) and trading it with other banks, and is excluded from the bond market. This assumption implies that the value of default for a bank is affected by equilibrium prices, and thus by the credit easing policy. In particular, since the defaulting bank is a net seller of capital, it benefits from the increase in asset prices resulting from credit easing.

In this section, we provide an alternative specification for the default costs that does not have this feature and show how the results carry over to this environment. To keep the analysis simple, let us assume now that repaying banks always have constant productivity; that is, $z_t = z$ for all t, including $t = 0$. We assume that once a bank defaults, it cannot longer borrow or save and cannot trade in the capital market. We also assume that the defaulting bank keeps a fraction of its capital while the remaining is lost. The outside option of default is then

$$V_t^D(k_t) = v_t^D + \frac{1}{1-\beta}\log(k_t).$$

where v_t^D encapsulates the fraction of capital kept after default as well as the productivity during default. We treat v_t^D as exogenous and unaffected by prices. In particular, for all periods $t \geq 1$, $v_t^D = v^D$. And for period $t = 0$, v_0^D is drawn from some cdf with support in $[\underline{v}, \overline{v}]$. The key difference with our baseline model is that defaulting banks are no longer affected by equilibrium prices, as v_t^D is exogenous.

Note that with this specification of the outside option for default, the linearity that we exploited in the baseline is maintained. In particular, banks are subject to a linear borrowing constraint, with the difference that $\{\gamma_t\}$ reflects the different outside option.

Suppose that $R_1^k > R$. Then, the demand for capital for a repaying bank in period $t = 0$ is

$$k_1^R = \frac{\beta n_0}{p_0 - \gamma_0 p_1}.$$

and the demand for capital for a bank that repays subject to a run is

$$k_1^{Run} = \frac{\beta n_0}{p_0},$$

where $n_0 = (z + p_0)\bar{K} - RB_0$. Note that as long as there is borrowing, that is, $\gamma_0 > 0$, we have that $k_1^{Run} < k_1^R$: banks that are subject to a run demand less capital than those that repay.

General equilibrium requires market clearing in the capital market at $t = 0$. Given that defaulting banks keep their capital, this means that $k_1^R = \bar{K}$. That is, repaying banks are neither net sellers nor net buyers. The above then implies that banks facing a run are necessarily net sellers of capital.

Consider now a marginal increase in p_0 (which could be generated by a credit easing policy). This increase has no first-order effect on the value of a repaying bank that does not face a run, as this bank is neither a net seller nor a net buyer. In the presence of losses from the credit easing policy, one would expect that the value of a repaying bank decreases with the policy, therefore increasing the share of defaulting banks in an economy without runs.

In the case of a run, however, an increase in p_0 has a first-order positive effect on the value of a bank that decides to repay subject to a run, as this bank *is a net seller*. In this case, we would expect that a credit easing policy that raises p_0 reduces the share of defaulting banks in an economy where banks are subject to runs.

The analysis, therefore, suggests that our perspective on how the desirability of credit easing depends on whether crises are triggered by fundamentals or self-fulfilling runs does not hinge on having a default outside option for the bank that is affected by asset prices.

5 Conclusion

We developed a tractable dynamic general equilibrium model of self-fulfilling bank runs. The model features banks that face limited commitment and optimally choose portfolios, equity, and default. These decisions are dynamic and depend on the entire sequence of asset prices, which are endogenously determined in equilibrium. We provide an analytical characterization of when an individual bank defaults because of fundamentals, when it defaults because of a self-fulfilling run, and when it is solvent and liquid and continues to operate. We then characterize the evolution of asset prices and the fraction of banks that default in general equilibrium.

Our analysis shows that the interplay between bank runs and general equilibrium has distinctive policy implications. A policy insight is that the effectiveness of credit easing during a crisis depends on whether the crisis is driven by fundamentals or by self-fulfilling runs. When a crisis is triggered by fundamentals, credit easing may lead to more banks defaulting in equilibrium, as the increase in asset prices reduces the value of repaying banks that are net buyers of the assets. When a crisis is instead triggered by self-fulfilling runs, credit easing becomes stabilizing. Repaying banks facing a run benefit from the increase in asset prices and therefore become less vulnerable to a run, because they are net sellers of assets.

The results suggest several avenues for future research. A first avenue is quantitative and requires enriching the model to provide a more complete description of the banking system. A second avenue is to consider the anticipation effects of future credit easing policies. Finally, while we have used the framework to explore the effects of credit-easing policies, it is possible to extend the model to consider other types of government policies, such as monetary and macroprudential policies.

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Online Appendix for
"Bank Runs, Fragility, and Credit Easing"

Manuel Amador and Javier Bianchi

A Proofs

A.1 Proof of Lemma 1

The problem of a bank under default facing a sequence of prices $\{p_t\}_{t=0}^{\infty}$ is given by

$$V_t^D(k) = \max_{k',c} \log(c) + \beta V_{t+1}^D(k') \tag{A.1}$$
$$\text{subject to: } c = \; (p_t + z^D)k - p_t k'.$$

We conjecture that

$$V_t^D(k) = \; \mathbb{B}_t^D + \frac{1}{1-\beta}\log((z^D + p_t)k). \tag{A.2}$$

Replacing this conjecture into (A.1) and substituting out consumption from the budget constraint, we have that

$$V_t^D(k) = \max_{k'} \log(z^D k + p_t(k - k')) + \beta\left[\frac{1}{1-\beta}\log(k'(p_{t+1} + z^D)) + \mathbb{B}_{t+1}^D\right]. \tag{A.3}$$

The first-order condition with respect to k' is given by

$$\frac{p_t}{z^D k + p_t(k-k')} = \left(\frac{\beta}{1-\beta}\right)\frac{1}{k'} \quad \Rightarrow \; k' = \frac{\beta(z^D + p_t\,)}{p_t}k. \tag{A.4}$$

By the method of undetermined coefficients, we can now verify the conjecture and solve for $\mathbb{B}_t^D$. We substitute (A.4) into the right-hand side of (A.3) and replace the conjectured guess for $V_t^D(k)$ on the left-hand side of (A.3):

$$\mathbb{B}_t^D + \frac{1}{1-\beta}\log((z^D + p_t)k) = \log\Big((1-\beta)(z^D + p_t)k\Big) + \beta\left[\frac{1}{1-\beta}\log\Big(\beta R_{t+1}^D(z^D + p_t)k\Big) + \mathbb{B}_{t+1}^D\right].$$

where we have used the definition of R_{t+1}^D. Rearranging this equation, we can observe that the terms multiplying $\log(k)$ cancel out. After simplifying, we obtain that the conjectured value function is verified when $\mathbb{B}_t^D$ satisfies

$$\mathbb{B}_t^D = \log(1-\beta) + \frac{\beta}{1-\beta}\log(\beta) + \frac{\beta}{1-\beta}\log\Big(R_{t+1}^D\Big) + \beta\mathbb{B}_{t+1}^D. \tag{A.5}$$

Iterating forward on this equation and imposing $\lim_{\tau\to\infty}\beta^\tau \log\left(R^D_{\tau+1}\right) = 0$, as in Condition 1, we have

$$\mathbb{B}^D_t = \frac{1}{1-\beta}\left[\frac{\beta}{1-\beta}\log(\beta) + \log(1-\beta)\right] + \frac{\beta}{1-\beta}\sum_{\tau\geq t}\beta^{\tau-t}\log\left(R^D_{\tau+1}\right). \tag{A.6}$$

Replacing (A.6) in (A.2), we obtain that the value under default is given by

$$V^D_t(k) = A + \frac{1}{1-\beta}\log((z^D + p_t)k) + \frac{\beta}{1-\beta}\sum_{\tau\geq t}\beta^{\tau-t}\log\left(R^D_{\tau+1}\right),$$

where $A = (\log(1-\beta) + \frac{\beta}{1-\beta}\log(\beta))/(1-\beta)$. We thus arrived at the value of V^D, as stated in the lemma. □

A.2 Proof of Lemma 2

We conjecture that the value function is

$$V^R_t(n) = \frac{1}{1-\beta}\log(n) + \mathbb{B}^R_t. \tag{A.7}$$

The borrowing constraint must be such that the bank does not default at $t+1$. That is,

$$\mathbb{B}^R_{t+1} + \frac{1}{1-\beta}\log(n') \geq \mathbb{B}^D_{t+1} + \frac{1}{1-\beta}\log((z^D + p_{t+1})k').$$

Replacing n' for the law of motion and manipulating this expression, we arrive at

$$b' \leq \frac{\left[(z + p_{t+1}) - (z^D + p_{t+1})e^{(1-\beta)(\mathbb{B}^D_{t+1} - \mathbb{B}^R_{t+1})}\right]}{R}k'.$$

Therefore, the borrowing constraint takes a linear form, as conjectured. In particular,

$$b' \leq \gamma_t p_{t+1} k',$$

where γ_t is the leverage parameter and is given by

$$\gamma_t = \frac{(z + p_{t+1}) - (z^D + p_{t+1})e^{(1-\beta)(\mathbb{B}^D_{t+1} - \mathbb{B}^R_{t+1})}}{Rp_{t+1}}. \tag{A.8}$$

We establish next that if $R^k_{t+1} > R$, the borrowing constraint binds at time t.

Lemma A.1. *If $R^k_{t+1} > R$, then the bank is against the borrowing constraint.*

Proof. The proof is by contradiction. Denote by $(c^*_t, k^*_{t+1}, b^*_{t+1})$ the solution to the bank problem with $b^*_{t+1} < \gamma_t p_{t+1} k^*_{t+1}$. Consider the following alternative policy: $(c^*_t, \tilde{k}_{t+1} + \Delta, \tilde{b}_{t+1} + \Delta p_t)$, with

$0 < \Delta < \frac{\gamma_t p_{t+1}\tilde{k}_{t+1} - \tilde{b}_{t+1}}{p_t - \gamma_t p_{t+1}}$. The alternative allocation is feasible and delivers higher net worth, since

$$\begin{aligned}\tilde{n}_{t+1} &= (\tilde{k}_{t+1} + \Delta)(z + p_{t+1}) - R\tilde{b}_{t+1} + \Delta p_t) \\ &= \tilde{k}_{t+1}(z + p_{t+1}) - R\tilde{b}_{t+1}) + \Delta(R^k_{t+1} - R) \\ &> \tilde{k}_{t+1}(z + p_{t+1}) - R\tilde{b}_{t+1} = n^*_{t+1},\end{aligned}$$

where $\tilde{n}_{t+1}$ and n^*_{t+1} are respectively the net worth under the alternative and original allocations. Since the alternative allocation delivers the same consumption and higher net worth, this contradicts that the original allocation with a slack borrowing constraint is optimal. □

We now proceed to finish the proof of Lemma 2. Consider first the case with $R^k_{t+1} > R$. From Lemma A.1, we know that borrowing constraint binds, and hence we can use $b' = \gamma_t p_{t+1} k'$. Replacing this in the law of motion for net worth and consumption, we obtain

$$n' = k'(z + p_{t+1}) - \gamma_t p_{t+1} k' R$$

and $c = n - k'(p_t - \gamma_t p_{t+1})$. Replacing these two expressions and the conjectured value function (A.7) in the right-hand side of equation (2), we have

$$V^R_t(n) = \max_{k'} \log(n - k'(p_t - \gamma_t p_{t+1})) + \beta \left[\frac{1}{1-\beta} \log(k'(z + p_{t+1}(1 - \gamma_t R)) + \mathbb{B}^R_{t+1} \right], \tag{A.9}$$

The first-order condition with respect to k' is

$$\frac{p_t - \gamma_t p_{t+1}}{n - k'(p_t - \gamma_t p_{t+1})} = \left(\frac{\beta}{1-\beta} \right) \frac{1}{k'}$$

and yields

$$k' = \frac{\beta n}{p_t - \gamma p_{t+1}}, \quad c = (1-\beta)n, \tag{A.10}$$

and

$$n' = \frac{\beta n}{p_t - \gamma_t p_{t+1}} (z + p_{t+1}(1 - \gamma_t R)).$$

Notice that by definition of R^e_{t+1}, we have that

$$R^e_{t+1} = \frac{z + p_{t+1}(1 - \gamma_t R)}{p_t - \gamma_t p_{t+1}}. \tag{A.11}$$

If we use (A.10) and (A.11) and replace (A.7), on the left-hand side of (A.9)

$$\mathbb{B}^R_t + \frac{1}{1-\beta} \log(n) = \log((1-\beta)n) + \beta \left[\frac{1}{1-\beta} \log(\beta R^e_{t+1} n) + \mathbb{B}^R_{t+1} \right].$$

Rearranging this equation, we can observe that the $\log(n)$ terms cancel out. We therefore obtain that the conjecture is verified when the $\mathbb{B}_t^R$ satisfies

$$\mathbb{B}_t^R = \frac{\beta}{1-\beta}\log(\beta) + \log(1-\beta) + \frac{\beta}{1-\beta}\log(R_{t+1}^e) + \beta\mathbb{B}_{t+1}^R. \tag{A.12}$$

Iterating forward and imposing $\lim_{t\to\infty}\beta^t\mathbb{B}_t^R = 0$, we have

$$\mathbb{B}_t^R = \frac{1}{1-\beta}\left[\frac{\beta}{1-\beta}\log(\beta) + \log(1-\beta)\right] + \frac{\beta}{1-\beta}\sum_{\tau\geq t}\beta^{\tau-t}\log\left(R_{\tau+1}^e\right), \tag{A.13}$$

so the value under repayment is given by

$$V_t^R(n) = \frac{1}{1-\beta}\log(n) + \mathbb{B}_t^R,$$

where $\mathbb{B}_t^R$ is given by (A.13). Equivalently, using the definitions of R^e and A, we arrive at the expression for V^R in the Lemma.

Notice also from (A.10) and (A.10) and the fact that $b' = \gamma_t p_{t+1} k'$ that we have also verified the policies in item (ii) of the lemma for the case of $R_{t+1}^k > R$.

Finally, it is straightforward to verify that in the case of $R_{t+1}^k = R$, the conjectured value function (A.7) solves the Bellman equation, and the bank is now indifferent across b', k', while consumption remains given by (A.10). This completes the proofs of the three items in the lemma.

□

A.3 Proof of Proposition 1

Rearranging (A.8), we obtain

$$\frac{\beta}{1-\beta}\log\left(\frac{z + p_{t+1}(1-\gamma_t R)}{z^D + p_{t+1}}\right) = \beta(\mathbb{B}_{t+1}^D - \mathbb{B}_{t+1}^R). \tag{A.14}$$

To obtain an expression for the right-hand side of (A.14), we use (A.5) and (A.12), and obtain the result that the difference in the intercepts in the value functions is given by

$$\mathbb{B}_t^D - \mathbb{B}_t^R = \beta(\mathbb{B}_{t+1}^D - \mathbb{B}_{t+1}^R) + \frac{\beta}{1-\beta}\left[\log(R_{t+1}^D) - \log(R_{t+1}^e)\right]), \tag{A.15}$$

Using the definition of R_{t+1}^D and R_{t+1}^e and replacing (A.14), we get that

$$\mathbb{B}_t^D - \mathbb{B}_t^R = \beta(\mathbb{B}_{t+1}^D - \mathbb{B}_{t+1}^R) - \frac{\beta}{1-\beta}\left[\log\left(\frac{z + p_{t+1}(1-\gamma_t R)}{p_t - \gamma_t p_{t+1}}\right) - \log\left(\frac{z^D + p_{t+1}}{p_t}\right)\right].$$

Using that using that $\log(p_t - \gamma_t p_{t+1}) = \log\left(1 - \gamma_t\frac{p_{t+1}}{p_t}\right) + \log(p_t)$, simplifying, and replacing (A.14), we arrive at

$$\mathbb{B}_t^D - \mathbb{B}_t^R = \frac{\beta}{1-\beta}\left[\log\left(1 - \gamma_t\frac{p_{t+1}}{p_t}\right)\right]. \tag{A.16}$$

If we update (A.16) one period forward and replace in (A.14), we arrive at

$$\frac{z + p_{t+1}(1 - \gamma_t R)}{z^D + p_{t+1}} = \left(1 - \gamma_{t+1}\frac{p_{t+2}}{p_{t+1}}\right)^{\beta},$$

which is the expression in the proposition. □

A.4 Proof of Lemma 4

The capital demand of a repaying bank with productivity z_0 can be written as

$$k_1^R(z_0) = \beta\frac{(z_0 + p_0)\overline{K} - RB_0}{p_0 - \gamma_0 p_1} = \beta\left(\frac{(z_0 + \gamma_0 p_1)\overline{K} - RB_0}{p_0 - \gamma_0 p_1} + \overline{K}\right).$$

We know from before that $k_1^R(z^F) \geq k_1^D$. We also have that $k_1^R(z^{Run}) \geq k_1^D$, as $z^{Run} \geq z^F$. So, independently of the default threshold, $\hat{z}$, we have

$$\int_{\hat{z}}^{z} (k_1^R(z_0) - k_1^D) dF(z_0) > 0,$$

where the inequality follows as the demand for capital is strictly increasing in z_0, and the threshold is interior. Market clearing at $t = 0$ requires that

$$\int_{\hat{z}}^{z} k_1^R(z_0) dF(z_0) + k_1^D F(\hat{z}) = \overline{K}.$$

Subtracting the previous inequality, we have that

$$\int_{\hat{z}}^{z} k_1^R(z_0) dF(z_0) + k_1^D F(\hat{z}) - \int_{\hat{z}}^{z} (k_1^R(z_0) - k_1^D) dF(z_0) < \overline{K}.$$

And thus, $k_1^D < \overline{K}$. It follows then that $\int_{\hat{z}}^{z} (k_1^R(z_0) - \overline{K}) dF(z_0) > 0$. The capital demand inequality implies

$$\int_{\hat{z}}^{z} \left(\beta\frac{(z_0 + p_0)\overline{K} - RB_0}{p_0 - \gamma_0 p_1} - \overline{K}\right) dF(z_0) > 0$$

$$\Rightarrow \beta \int_{\hat{z}}^{z} \left(\frac{(z_0 + \gamma_0 p_1)\overline{K} - RB_0}{p_0 - \gamma_0 p_1}\right) dF(z_0) > (1 - \beta)\overline{K}(1 - F(\hat{z})) > 0,$$

which delivers

$$\int_{\hat{z}}^{z} ((z_0 + \gamma_0 p_1)\overline{K} - RB_0) dF(z_0) > 0,$$

as $p_0 > \gamma_0 p_1$, an equilibrium requirement. We can then rewrite the capital demand of repaying banks as:

$$\int_{\hat{z}}^{z} k_1^R(z_0)dF(z_0) = \beta\left[\frac{\int_{\hat{z}}^{z}((z_0+\gamma_0 p_1)\overline{K} - RB_0)dF(z_0)}{p_0-\gamma_0 p_1} + \overline{K}(1-F(\hat{z}))\right].$$

Given what we have just shown, the numerator of the first term inside the square brackets is strictly positive, and thus it follows that an increase in p_0 strictly reduces demand from infra-marginal repaying banks. □