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#### TOWARD A THEORY OF RIGIDITIES

### ABSTRACT

This paper presents a theory of rigidity, or more properly inertia, in the responses of economic variables to changing environments. The theory rests on three fundamental assumptions: (1) that firms are risk averse, (2) that firms are uncertain of the impacts of changing decision variables and (3) that this uncertainty increases with the size of deviations in decision variables from appropriately defined past level. Under these circumstances an optimal portfolio of incremental decision variable adjustments exists which (a) takes variance minimizing adoptions to environmental change as a point of departure and then (b) is weighted in favor of changes in variables whose effects are less uncertain. In considering price and quantity adjustments, this implies that price and wage adjustments should largely incorporate expected inflation and, from that point, should be small relative to quantity adjustments, since in most situations the uncertainties associated with the consequences of quantity adjustment should be smaller than those associated with price adjustments.

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It has been widely noted of business cycles that, while quantities vary dramatically, prices vary only slightly, if at all. Applied to the labor market, this observation - that employment is far more variable over the cycle than wages - is one of the cornerstones of Keynesian theory. At the same time, real business cycle theorists accept it as one of the key-business-cycle-facts-to-be-explained. Yet attempts to provide a theoretical justification of these price rigidities have been largely unsuccess-The early Keynesian and later fix-price literatures simply took them for ful. granted, assuming that they were economic facts of life that could be assumed to hold. The problem with this approach is two-fold. Theoretically, it has never been clear (especially in the fix-price literature) why economic agents, who are otherwise highly sophisticated, choose to ignore the possibilities of price or wage changes. Empirically, wages (and prices) do, in fact, change and over long periods of time change substantially. A second more recent set of explanations appeals to small fixed costs of adjusting prices (menu costs). Imperfectly competitive firms which maximize profits should obtain only second order gains in profits from small changes in prices about their optimal levels. Since these small gains may be insufficient to offset fixed costs of adjustment, prices may well be rigid. However, since the fixed cost of quantity adjustments (lay-offs, etc.) are widely regarded as being greater than the costs of price adjustment, this basic approach argues as or more strongly for quantity rigidities than for price rigidities. What needs to be explained is why, in spite of greater adjustment costs, output and employment are more variable than prices and wages.

This paper provides an explanation based on three simple hypotheses: firms act in a risk averse manner; they are uncertain about the consequences of their

actions and the greater the change from the status quo, the greater the uncertainty;<sup>2</sup> and there is often greater uncertainty associated with pricing and wage decisions than with output and employment decisions. The first of these hypotheses is supported by ample empirical evidence and can be derived from more primitive assumptions related to either capital market imperfections or the impact of performance based compensation schemes on risk averse managers. The second, that firms are uncertain about the consequences of the actions, seems uncontroversial. The third hypothesis is, however, the critical one and it will be discussed further below.

The reason that rates of adjustment concerning the impacts of different decision variables are related to their relative uncertainties can be seen intuitively as follows. If firms are risk averse, then they will consider both the mean and the variance of the returns yielded by different combinations of changes in decision variables. As firms make adjustments, the change in the expected value and the variance of profits increase together. However, if uncertainty concerning the impact of one decision variable A (a price) is greater than uncertainty concerning the impact of another decision variable B (a quantity), then, other things being equal, the optimal portfolio of adjustments will contain less movement in A than B. (Where uncertainty here is appropriately defined in terms of the covariance matrix of uncertainties concerning the impacts of the several decision variables.) Following such initial changes which are greater in B than A, the expected returns to further changes in A are likely to rise relative to the expected returns to changes in B (since B will now be closer to its new optimal value). Thus, ultimately A may adjust as extensively as B, but in the short run A will exhibit inertia relative to B.

One important qualification must, however, be made to this simple description. When the consequences of actions are particularly uncertain, and firms are particularly risk averse, it is sometimes suggested that firms will simply maintain the status quo. But what does it mean to continue doing what you were doing before? Does it mean keeping absolute prices fixed, or relative prices? absolute wages, or relative wages? We provide here an answer: very risk averse firms will take those actions which minimize the variability of their profits. Thus, in speaking in the previous paragraph of the magnitude of changes in A relative to B, these must be interpreted as changes from the minimum variance point not as changes from preexisting levels. If the economic environment is one in which the variance of profit is related to relative wages or prices, firms will keep relative wages or prices fixed. Our model is thus able to provide a theory of nominal as well as real rigidities. (It should also be noted that the framework developed below provides a natural origin and set of coordinates in contrast to the Akerlof-Yellin model which does not.) In environments in which firms (believe that other firms) do not fully adjust their wages and prices to changes in the money supply, then the equilibrium policy of each firm is not to adjust fully its prices and wages.

## A Simple Model: The Portfolio Theory of Adjustment

Firms are assumed to maximize the expected utility of profits,  $\pi_t$ , where profits are assumed to be a (random) function of a vector of n decision variables,  $x_t$  (e.g. own prices or outputs), and a vector of m exogenous factors,  $z_t$  (e.g. competitor prices). To capture the idea the greater the change in the decision variables and in the exogenous factors, the greater the uncertainty, we write

$$\pi = \pi(x^*, z^*, \tilde{\mu}_t(x_t - x^*), \tilde{\eta}_t(z_t - z^*))$$

where  $x^*$ ,  $z^*$  are normal or, in a dynamic context, pre-existing levels of  $x_t$  and  $z_t$  respectively and  $\tilde{\mu}_t$  and  $\tilde{\eta}_t$  are random variables. At the beginning of period t, the firm sets  $x_t$  based on a forecast of  $z_t$ ,  $\tilde{z}_t$ . For simplicity, we assume that the actual levels of the environmental variables,  $z_t$ , are the sum of the forecast,  $\tilde{z}_t$ , and a random error,  $\tilde{e}_t$ , with E ( $\tilde{e}_t$ ) = 0,

$$z_t = \bar{z_t} + \tilde{e}_t . {1}$$

Again for simplicity, we will assume that  $\tilde{e}_t$  is independent of  $\tilde{\eta}_t$  and  $\tilde{\mu}_t$ . Next, assuming that  $(x_t - x^*)$  and  $(z_t - z^*)$  are relatively small, we linearize the profit function around  $x^*, z^*$  so that

$$\pi_t = \pi(x^*, z^*, 0, 0) + \pi_x \tilde{\mu}_t (x_t - x^*) + \pi_z \tilde{\eta}_t (z_t - z^*)$$
 (2)

where  $\pi_x$  is the derivative of  $\pi$  with respect to  $\tilde{\mu}_t$   $(x_t - x^*)$  and  $\pi_z$  is the derivative of  $\pi$  with respect to  $\tilde{\eta}_t$   $(z_t - z^*)$ . By a suitable choice of units for  $(x_t - x^*)$  and  $(z_t - z^*)$ ,  $\pi_x$  and  $\pi_z$  can be set to unit vectors.<sup>3</sup> Then, after substitution from equation (1) into equation (2), equation (2) can be written as

$$\pi_{t} = \pi^{*} + \tilde{\mu}_{t}'(x_{t} - x^{*}) + \tilde{\eta}_{t}'(\bar{z} - z^{*}) + \tilde{u}_{t}$$
 (3)

where  $\pi^* \equiv \pi(x^*, z^*, 0, 0)$  and  $\tilde{u}_t = \tilde{\eta}_t' \tilde{e}_t$ . In this form, elements of  $\tilde{\mu}_t$  can be interpreted as the instrument uncertainties associated with use of the corresponding decision variables.

Finally, we assume a quadratic utility function in which case the firms objective function can be rewritten in terms of the mean and variance on  $\pi_t$  where

$$E(\pi_{t}) = \pi_{t} + \overline{\mu}_{t}'(x_{t} - x^{*}) + \overline{\eta}_{t}'(\overline{z_{t}} - z^{*}), \qquad (4)$$

since  $E\left[\tilde{\mu}_{t}\right]=E\left[\vec{e}_{t}\right]E\left[\bar{\eta}_{t}\right]=0,$  and

$$V(\pi_t) \equiv \sigma_{\pi}^2 = \sigma_u^2 + (\overline{z}_t - z^*)' V_{\eta} (\overline{z}_t - z^*) + (x_t - x^*)' V_{\mu} (x_t - x^*)$$

$$+ 2(\overline{z}_t - z^*)' C_{\mu,\eta} (x_t - x^*)$$
(5)

with  $V_{\eta}$  and  $V_{\mu}$  being the covariance matrices of  $\eta$  and  $\mu$  respectively and  $C_{\mu,\eta}$  being the  $\mu, \eta$  – covariance matrix.

Efficient combinations of  $(x_t - x^*)$ , the decision variables, are those which

min. 
$$\sigma_{\pi}^2$$
 subject to  $E(\pi) \ge \pi_0$ .

These take the form

$$\Delta x_t^* \equiv x_t^* - x^* = \frac{1}{2} \lambda V_u^{-1} \overline{\mu}_t - V_u^{-1} C_{u,\eta}' (\overline{z}_t - z^*). \tag{6}$$

where  $\lambda > 0$  is the Lagrange multiplier associated with the expected profit constraint.

In equation (6), the first term on the right hand side represents the the "active" component of a firm's response to changing external conditions. The multiplier  $\lambda$  is determined by the tangency of mean-variance efficient frontier with the firm's (management's) utility function. As firms become more risk averse,  $\lambda$  falls and "active" adjustments to changing conditions are curtailed. However, as this is done the mix of "active" adjustments, characterized by the vector  $V_{\mu}^{-1} \cdot \bar{\mu}_t$  remains unchanged. To see how this portfolio of optimal "active" adjustments varies across

instruments consider the case in which  $V_{\mu}$  is a diagonal and all the elements of  $\bar{\mu}_t$  are equal. Then

$$(V_{\mu}^{-1} \cdot \overline{\mu}_{t})_{i} \equiv i^{th}$$
 element of the optimal active adjustment portfolio =  $\frac{\overline{\mu}_{t}}{\sigma^{2}}$ 

where  $\mu_t$  is the common expected return to adjustments in the decision variables and  $\sigma_i^2$  is the variance of the instrument uncertainty concerning the impact of the  $i^{th}$  decision variable. Clearly the greater this instrument uncertainty the smaller will be the extent to which the decision variable is "actively" adjusted in response to changing external circumstances.

The second right hand side term in equation (6) represents a defensive, variance minimizing response to an expected exogenous change  $(\bar{z}_t - z^*)$ . To see why this is so, consider the  $i^{th}$  column of the matrix  $(V_{\mu}^{-1})$   $C'_{\mu,\eta}$  which is

$$\beta_i' \equiv [V_{u}^{-1} C_{u,u}']_i = V_{u}^{-1} [C_{u,n}']_i$$

where  $[C_{\eta,\mu}^{'}]_i$  is the  $i^{th}$  row of the matrix  $C_{\eta,\mu}$ . The elements of the  $i^{th}$  row of  $C_{\eta,\mu}$  are the n-covariances of  $\bar{\eta}_i$  with the various  $\bar{\mu}_j$ . Thus, the vector  $\beta_i^{'}$  represents the projection of  $\bar{\eta}_i$  on the  $\bar{\mu}$  uncertainties and the second right hand side term in equation (6), which is the sum of  $\beta_i^{'}(\bar{z}_{it}-z^*)$ , is the projection of the sum of  $\bar{\eta}_i^{'}(\bar{z}_{it}-z^*)$  on the  $\bar{\mu}$  instrument uncertainties. Changing the instruments by these amounts, thus, minimizes the expected residual uncertainty induced by the  $(\bar{z}_t-z^*)$  changes in the exogenous variables.

A simple example may illustrate how this works. Suppose that a firm's price is its only decision variable, that the overall price level is the only significant exogenous variable and profits (and demand) depend only on relative prices. Then even though, the impact of relative prices on profitability may not be known, the effect of a change in the price level,  $\tilde{\eta}_1$ , is equal and opposite to the effect of a change in the firm's own price,  $\tilde{\mu}_1$ . A regression of  $\tilde{\eta}_1$  observations on  $\tilde{\mu}_1$  observations would produce a slope of minus one in the limit. Thus, in this simple case,

$$V_{\mu}^{-1} C_{\mu,\eta} = -1$$

and

 $\Delta p^*$  = optimal change in the firm's own price =  $\Delta p^e + \frac{1}{2}\lambda V_{\mu}^{-1} \overline{\mu}_t$  where  $\Delta p^e$  is the expected change in the overall price level. The adjustment to exogenous change, therefore, consists of two components; a variance minimizing adjustment to neutralize the expected change in the overall price level and a portfolio of "active" adjustments from that point, whose extent depends on the degree to which the firm is risk averse. Rigidity is defined, in this simple example, in terms of real rather than nominal prices. Price versus quantity rigidity from that base depends on the relative uncertainties concerning the impact of price changes compared to quantity changes. This is the subject of the next section.

# Labor Market Adjustment with Efficiency Wages

To consider the application of the model described above, we examine a very simple model of the labor market built around standard efficiency wage considerations. A firm produces output using only labor as an input. The amount of labor

supplied by each worker is an increasing function of the worker's real wage (for simplicity we will consider real wages as the firm's decision variable). This may arise either for incentive, selection or turnover reasons. Let  $g(\tilde{\mu}(w-w^*),w^*)$  denote the amount of labor supplied as a function of the real wage w. Firms are assumed to be uncertain about the impact of changes in the real wages on this level of productivity per worker (hence the factor  $\tilde{\mu}$ ) and this uncertainty increases with the size of deviations from the existing wage level,  $w^*$ . Total output is just output per worker, g, times the number of workers hired, N. This output is sold in a competitive international market at a price, p. We assume that average output per work at the existing wage,  $w^*$ , is observed essentially without error (i.e. the number of workers is sufficiently large so that even if individual output cannot be observed, average output once it has settled down to an equilibrium level can be). Thus, the profits of the firm are

$$\pi = p N \cdot g(\tilde{\mu}(w - w^*), w^*) - (w - w^*) N - w^*N$$

Finally we assume the future prices and the number of individuals hired can be observed without error and that  $\tilde{\alpha}$  is independent of other variables.

Consider then the firms adjustment to a change in prices,  $\Delta p$ . Linearizing about  $w^*$ ,

$$\pi \stackrel{\sim}{=} N \cdot g \cdot \Delta p + [pN \cdot g' \cdot \bar{\mu} - N] (w - w^*) + (p \cdot g - w) \cdot N .$$

The expected returns to changing wages and employment are  $(p \cdot N \cdot g' \cdot \overline{\alpha} - N)$  and  $(p \cdot g - w)$  respectively. The changes in the variances of profits that result from changes in wages and employment respectively are  $p^2 N^2 (g')^2 \sigma_{\mu}^2$  and zero

respectively. Since the average productivity per worker is known with certainty there is no instrument uncertainty associated with employment changes. The optimal portfolio of changes in wages and employment, therefore, consists entirely of employment changes.<sup>4</sup> The initial adjustment in response to the price change falls entirely on employment rather than wages.

This is, of course, an extreme example, but the general principal involved applies more generally. Wage changes affect all workers in ways that are not easy to predict. Consequent and imperfectly predictable changes in turnover (how many people quit), in worker effort and in the quality of the retained labor force all generate uncertainty about profits. In contrast, when workers are separated there is much less uncertainty about the amount of labor they are likely to supply. As long as this is true, labor force adjustments by risk averse firms will tend initially to fall more heavily on prices than quantities. Moreover, actual employment changes during cyclical fluctuations seem to be structured in ways which minimize the resulting uncertainties. Thus, lay-offs appear to be much more common relative to work sharing than most implicit contract models would suggest. Since work sharing and hours reductions, which affect all workers, may generate uncertain changes in labor supply (through quits, etc.), a risk averse firm would tend to avoid such measure in making short-run adjustments.

# Price Rigidities and Product Markets

In product markets, prices will tend to be more rigid than output levels as long as uncertainties concerning the impact of prices on demand are greater than uncertainties concerning the impact of output costs. Consider an imperfectly competitive firm that adapts to variations in demand by accumulating or reducing inventory.

Assume its financial position at the end of any period consists solely of its level of liquid assets and its accumulated inventory. Let

 $m_t \equiv \text{end-of-period liquid asset position} = m_{t-1} + p_t d (\tilde{\mu}_t p_t) - C (q_t)$  where  $p_t$  is the price level in period t,  $\tilde{\mu}_t$  embodies uncertainty concerning the impact of price on demand,  $q_t$  is output and  $c(q_t)$  is cost of production which is assumed to be known with certainty. Terminal inventories are

$$i_t = i_{t-1} - d(\tilde{\mu}_t p_t) + q_t$$
.

If the firm maximizes utility which is a function of its terminal position, it should be immediately clear that uncertainties concerning the effect of prices on demand (whether directly or as a result of uncertain competitor price responses) will produce instrument uncertainty associated with price changes that is greater than the instrument uncertainty associated with output changes.<sup>5</sup> Consequently, short term response to changes in external conditions will be weighted toward output rather than price adjustments. It should be noted again, however, that if demand is known to depend on relative rather than absolute price, the resulting price rigidity will be real rather than nominal.<sup>6</sup>

## A Final Note: Rigidity versus Inertia

The theory developed so far is a theory of price inertia (relative to quantity changes) rather than a theory of price rigidity strictly construed. Prices do change in response to changes in the economic environment, although they do so relatively slowly. However, the existence of fixed costs of changing prices may create actual

rigidities (i.e. prices which do not change at all in the short term) even while the existence of comparable or greater fixed costs of output changes might not create quantity rigidities. Instrument uncertainty regarding the effect of price (or wage) changes inhibits immediate price adjustments and also the size of the expected return from such adjustments. The utility gain from a small (but optimal) price adjustment might thus not be sufficient to cover the fixed cost of making such a change. At the same time, the utility gain from a larger (but also optimal) quantity adjustment might exceed the perhaps larger fixed cost associated with quantity (employment) adjustments.

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#### **Footnotes**

- A further difficulty with the menu cost literature is that empirically price rigidities appear to exist to a greater extend regarding past rates of change (i.e. inflation inertia) rather than past levels (i.e. pure price level inertia). In the menu cost model, pure price level inertia is explained.
- Uncertainty associated with the consequences of actions is sometimes referred
  to as instrument uncertainty. The model we present here is similar to that
  developed in a somewhat different context by Brainard [1967].
- 3. If  $x^*$  represents an optimal level of the decision variables in response to a steady-state level of the exogenous variables,  $z^*$ , then  $\pi_x$  will be zero. However, in that case, the model can be relinearized about  $\bar{z}_t$  and with suitable variable redefinitions an equation analytically equivalent to, but more complicated than, equation (3) can be obtained. In such an alternative formulation  $\pi_x$  will no longer be zero since the shift from  $z^*$  to  $\bar{z}_t$  will disturb the initial first order conditions.
- 4. Also since there are no price uncertainties, the variance minimizing shift in real wages is zero. Therefore, in this case, the change in decision variables consists solely of "active" changes. It should further be noted that with zero uncertainty in the impact of employment changes and a linear profit function, the optimal level of employment is undefined. However, non-linearities in the profit function and/or slight uncertainties concerning the impact of employment changes would eliminate this problem.

- 5. To see that this is so consider an imperfect competitor who determines a quantity sold  $(s_t)$  and a quantity produced  $(q_t)$ . Assume prices are determined by the market reaction to the quantity sold. In this case there will be no uncertainty about inventories, but uncertainty about the impact of sales on prices will produce uncertainty about the impact on profits of sales changes as opposed to production changes. When firms set prices rather than sales levels further uncertainties are introduced concerning the level of inventories. As a rule, this will intensify uncertainties concerning the effects of price changes relative to quantity changes.
- 6. There are, in reality, opportunities for experimentation and learning about the slopes of demand curves but experimentation is imperfect which leaves significant residual price risks and itself entails risk. Thus, particularly in recessions when firms are likely to be highly risk averse, experimentation possibilities are unlikely to eliminate price inertia.