NBER WORKING PAPER SERIES

THE COMPETITIVE EXTERNALITIES AND THE OPTIMAL SEIGNIORAGE

Joshua Aizenman

Working Paper No. 2937

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 1989

The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the author not those of the National Bureau of Economic Research. This paper was written while the author was at the Institute for Advanced Studies at the Hebrew University, attending the program on International Economics and Market structure, organized by Elhanan Helpman. The author wishes to thank Jacob A. Frenkel, Leonardo Leiderman and Nissan Liviatan for useful discussions that motivated this paper. Any errors, however, are mine.

NBER Working Paper #2937
April 1989

COMPETITIVE EXTERNALITIES AND THE OPTIMAL SEIGNIORAGE

ABSTRACT

The purpose of this study is to analyze the behavior of the inflation tax in an economy where, due to coordination failure, the inflation rate is not determined by a unique policy maker but by several competing decision makers. Each decision maker can effectively print more paper money via the central bank, which operates only as the printing agency of nominal balances. This market structure generates a competitive externality. A key result is that the 'optimal' inflation rate depends positively on the competitive externality. We provide two examples of scenarios where these externalities are relevant. First, the case in which the central bank is a powerless agent whose only responsibility is to print money upon demand by the ministers. The second example is a common currency area, where several countries operate in a monetary union. Alternatively, this may be the case of a country composed of several states or provinces, where the centralized government system is weak and local governments can use seigniorage to their advantage. The effect of competitive externalities is to increase the inflation rate, to an extent that puts the economy on the wrong side of the inflation tax Laffer curve.

Joshua Aizenman
Economics Department
The Hebrew University
Jerusalem, 91905, Israel

1. INTRODUCTION AND SUMMARY

The inflation tax approach originated in Friedman (1969). It considers the case of a central bank with monopoly power in supplying nominal money balances, which may use this power to tax money holders by inflation. This approach generated a number of influential results. First, in the absence of fiscal revenue motives, welfare maximization requires following optimal deflation, so as to minimize the costs of using money balances.[1] Second, allowing for fiscal revenue motives requires an inflation rate that is derived as part of the public finance problem, determining the vector of optimal taxes (see Phelps (1973)).

The above results were derived for the case of a centralized decision maker, assuming away coordination problems. This assumption may be questionable in certain interesting economic situations. For example, if both the central bank and the treasury are weak, one could envision a situation in which various ministers may compete for seigniorage. Alternatively, a common currency area composed of several countries may generate a similar situation. The purpose of this study is to analyze the behavior of the inflation tax in an economy where, due to coordination failure, the assumption of a centralized decision maker determining the optimal inflation rate does not hold. This will allow us to examine how the implicit market structure influences the predictions of the traditional inflation tax argument. Specifically, we replace the monopolist decision maker (like the central bank) with several competing decision makers.

For simplicity we assume that the inflation tax is determined by decisions makers who are organized in a 'monopolistic competitive' structure. Each decision maker can effectively print more paper money via the central bank, which operates only as the printing agency of nominal balances. The decision makers thus compete among

---

1. See Friedman (1969). Further research on optimal inflation can by found in, for example, Phelps (1973), Frenkel (1976), Helpman and Sadka (1979), Jovanovic (1982), Fischer (1982) and Kimbrough (1986).

themselves for the inflation tax revenues. We start by assuming that their number is large enough for each one's marginal printing decision not to affecting that of the other decision makers. Latter we extend our analysis to the conjecture variations equilibrium. We show that the 'monopolistic competitive' market structure generates a competitive externality in which each decision maker overlooks the externalities he himself introduced.[2] Our analysis focuses on the role of these externalities in explaining the inflation tax. A key result is that the 'optimal' inflation rate depends positively on the competitive externality. This externality may generate an inflation rate that put the economy on the wrong side of the Laffer curve.

Two examples are given of economic scenarios where these externalities are relevant. First, we consider the case of the central bank being a powerless agent whose only responsibility is to print money upon demand by the ministers. If the ministers have equal power and can not collude into a binding cartel, the economy is characterized by the above competitive externality. We show that if each minister aims at maximizing his inflation tax revenue, the Nash equilibrium will yield an inflation that puts the economy on the wrong side of the inflation tax Laffer curve. The second example considers an environment where the ministers are replaced with small countries that operate as part of a common currency area, as may be the case in Europe in the future.[3] Alternatively, this may be the case of a country composed of several states or provinces, where the centralized government system is weak and the local governments can use seigniorage to their advantage (as may be the case in Brazil

---

2. These externalities are similar to those identified in the literature on international coordination (see Hamada (1976)).

3. A recent study of monetary policy in a common currency union (Casella and Feinstein (1988)) analyze the way monetary arrangement influences the optimum financing of a public good, in a two country world. A pioneering study of this topic is Mundell (1961).

and Argentina). We derive the optimal inflation rate for each country in a Nash equilibrium, where each country can print more money. We show that in the absence of a cartel agreement on the division of seigniorage, the 'optimal' inflation rate for each country will exceed the optimal inflation rate obtained in a cooperative solution, and may put the economy on the wrong side of the inflation tax Laffer curve. In both examples the same externality operates: each minister (or country) is aware that marginal printing of money for his own benefit will increase inflation and thereby erode the base of the inflation tax (and reduce welfare), but he overlooks the adverse welfare consequences of inflation on all other ministers (or countries).

The above results can solve a puzzle recently observed in the context of the inflation tax. The public finance approach to inflation tax generated predictions that cast doubt on the validity of the inflation tax argument as an explanation for inflation. Specifically, maximizing inflation tax revenues is similar to maximizing the revenue of a monopoly whose marginal costs of production are nil, yielding an equilibrium at an inflation rate that yields a version of the unitary elasticity rule. While such inflation exceeds the socially optimal rate, it sets an upper limit to the inflation rate supported by the inflation tax argument. Recently, however, several studies detected inflation rates that seem to exceed those that maximize the revenue from inflation tax,[4] putting the economy on the wrong side of the inflation tax Laffer curve. One explanation for this phenomenon is the possibility of multiple equilibria, where the same inflation tax can be raised by either a low or a high inflation rate (see Liviatan (1984) and Bruno

---

4. See Cukierman (1987), Bruno (1988), Edwards and Tabellini (1988), Kiguel and Liviatan (1988)). For other relevant empirical studies see Fischer (1982) and Eckstein and Leiderman (1988). Note that an alternative explanation for the above observations may refer to time-inconsistent behavior of the type analyzed in Calvo (1978).

and Fischer (1984))[5]. This approach explains the high inflation equilibrium by focusing on parameters like the speed of adjustment and learning. It has problems, however, in explaining the economic process that may lead the policy maker to such an inefficient equilibrium. Our analysis explains the process that leads decision makers to the inefficient equilibrium, without relying on multiple equilibria. We demonstrate that the competitive externality identified in our analysis may put the economy on the wrong side of the Laffer curve due to the lack of coordination among the decision makers. This may occur even if the conditions needed to generate multiple equilibria are not met. By focusing on the decision making process and lack of coordination we suggest that a reform fails to address the underlying coordination problem will not work, thereby shifting the emphasis to the political structure of the economy.

Section 2 describes the derivation of the revenue maximizing inflation tax for the case of competing ministers; Section 3 considers the case of the optimal inflation tax in a common currency area, where various states compete for seigniorage. Section 4 closes with concluding remarks. The Appendix derives several of the equations used in the paper.

2. COMPETITION AMONG MINISTERS

Consider an economy with zero growth and two assets: money and real bonds. For simplicity of exposition, let us assume that there is perfect foresight and no uncertainty. Real output (Y) is constant, equal to 1. The Appendix provides a detailed example of the maximization problem and the budget constraints that yield the following demand structure. The demand for real money balances is given by:

(1) $$m_t = m(\pi_t),$$

5. For a related study see Obstfeld (1984).

where $\pi_t = \frac{P^e_{t+1} - P_t}{P_t}$ is the expected inflation rate, P_t the price level at time t, and P^e_{t+1} is the price level expected in period t+1. Henceforth we will identify P^e_{t+1} with P_{t+1}. We assume a very simple fiscal structure: the only tax is seigniorage. Let us denote by G_t total nominal revenue from seigniorage at time t:

$$(2) \qquad G_t = M_t - M_{t-1}.$$

There are n ministers, with symmetric power. Each of them can print money $G_{t;j}$ such that

$$(3) \qquad G_t = \sum_{j=1}^{n} G_{t;j}\,.$$

Each minister prints money so as to maximize his revenue, taking the printing of other ministers as given. Thus, the condition determining the level of printing by minister h is given by

$$(4) \qquad \frac{\partial(G_{t;h}/P_t)}{\partial G_{t;h}} = 0.$$

We focus our attention on the properties of a symmetric equilibrium, where $G_{t;h} = G_t/n$. To simplify, we henceforth suppress the time index. Direct derivation of G_h/P yields

$$(5) \qquad \frac{\partial(G_h/P)}{\partial G_h} = \frac{1}{P}\left[1 - \frac{1}{n}\frac{G}{P}\frac{\partial P}{\partial G}\right].$$

The direct effect of a marginal dollar printed by minister h is to increase his revenue by $\frac{1}{P}$. The indirect effect is to tax his total revenue ($\frac{1}{n}\frac{G}{P}$) at a rate of the induced price effect, $\frac{1}{P}\frac{\partial P}{\partial G}$.

Note that the total tax revenue is given by[6]

$$(6) \qquad \frac{G}{P} = m\frac{\pi}{1+\pi}.$$

Taking derivatives of the two sides of (6) yields

$$(7) \qquad \frac{1}{P}[1 - \frac{G}{P}\frac{\partial P}{\partial G}] = m\,\frac{1-\varepsilon(1+\pi)}{(1+\pi)^2}\frac{\partial \pi}{\partial G},$$

where ε is the elasticity of demand for money with respect to inflation, defined to be positive ($\varepsilon = -\frac{\pi}{m}\frac{\partial m}{\partial \pi}$). From (6) we get that in the steady state equilibrium

$$(8) \qquad \frac{G}{M} = \frac{\pi}{1+\pi}.$$

Applying (8) we get that at an increase in nominal seigniorage (i.e., $\Delta G = \Delta M$), will affect inflation by[7]

---

6. This follows from the fact that in a steady state equilibrium
$\frac{G}{P} = \frac{M_t - M_{t-1}}{P_t} = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} = m\,(1 - \frac{P_{t-1}}{P_t}) = m\frac{\pi}{1+\pi}$.

7. Note that
$\frac{\partial(G/M)}{\partial G} = \frac{1}{M(1+\pi)}$; and $\frac{\partial(\pi/(1+\pi))}{\partial G} = \frac{\partial \pi}{\partial G}\frac{1}{(1+\pi)^2}$. Equating the two results we get (9).

(9) $$\frac{\partial\pi}{\partial G} = \frac{1+\pi}{M}.$$

Substituting (9) in (7), solving for $\frac{G}{P}\frac{\partial P}{\partial G}$, and applying the solution to (5) yields a key equation:

(10) $$\frac{\partial(G_h/P)}{\partial G_h} = \frac{1}{P}\left[\frac{1}{1+\pi} - \varepsilon + \frac{n-1}{n}\left(\varepsilon + \frac{\pi}{1+\pi}\right)\right],$$

from which we obtain that the revenue-maximizing inflation rate for the noncooperative, competitive equilibrium (denoted by $\tilde{\varepsilon}$) is given by the following elasticity condition

(11) $$\tilde{\varepsilon} = n - 1 + \varepsilon_c, \qquad \text{where} \qquad \varepsilon_c = \frac{1}{1+\pi}.$$

We denote by ε_c the elasticity that yields the cooperative solution, where a unique policy maker maximizes the revenue (n=1). He will set inflation at a rate that will put the economy at the peak of the inflation tax Laffer curve, where $\varepsilon = \varepsilon_c$.[8] At that rate a marginal increase in printing money will generate a direct revenue effect that is fully offset by the indirect tax revenue of higher inflation. In terms of (5), for n=1 at the revenue-maximizing inflation rate, the indirect tax effect of the higher inflation rate equals unity (i.e., $1 = \frac{G}{P}\frac{\partial P}{\partial G}$). Suppose now that we operate at such an inflation rate, with competing minsters. For each of them the tax effect of higher inflation is only 1/n of the total effect, as can seen from (5). Thus, at the cooperative

8. It can be shown that this rule is equivalent in continuous time to the unitary elasticity rule.

equilibrium ($\varepsilon_c = \frac{1}{1+\pi}$) each minister has the incentive to print more.[9] This printing will move us to a non-cooperative equilibrium, with a higher inflation rate, and will place the economy on the elastic part of demand for money (see (11))[10].

Suppose now that we allow for limited strategic interaction among the various minsters, by assuming a conjecture variation equilibrium, where each minister reacts according the the following rule:

$$\text{(12)} \qquad \frac{\partial G_i}{\partial G_h} = \tau \quad \text{for } i \neq h.$$

In a symmetric equilibrium we now get:

$$\text{(13)} \qquad \frac{\partial (G_h/P)}{\partial G_h} = \frac{1}{P}\left[1 - \frac{1+(n-1)\tau}{n} \frac{G}{P} \frac{\partial P}{\partial G}\right].$$

Following the same steps as in the previous discussion we get that:

$$\text{(10')} \qquad \frac{\partial (G_h/P)}{\partial G_h} = \frac{1}{P}\left[\varepsilon_c - \varepsilon + \frac{(1-\tau)(n-1)}{n}\left(\varepsilon + \frac{\pi}{1+\pi}\right)\right],$$

and the revenue-maximizing inflation rate is given by the condition:

---

9. In terms of (10), at $\varepsilon = \frac{1}{1+\pi}$ $\frac{\partial (G_h/P)}{\partial G_h} = \frac{1}{P} \frac{n-1}{n}$.

10. To gain further insight it is constructive to consider an example in which the demand for money has the Cagan form, $m = Y \exp(a - b\,\pi)$. Direct calculation reveals that the revenue-maximizing inflation is given by $\pi = \frac{n-1-b+\sqrt{(b+1-n)^2+4bn}}{2b}$. As the decision process becomes more diffuse (higher n), we observe higher inflation rates, moving to the elastic portion of the demand for money.

$$(14) \qquad \tilde{\varepsilon} = (1 - \tau)\frac{n-1}{1+(n-1)\tau} + \varepsilon_c .$$

Note that as long as we get a partial response ($\tau < 1$) all our previous results continue to hold, where the competitive externality is proportional to $1 - \tau$.[11]

3. COMPETITION AMONG COUNTRIES IN A COMMON CURRENCY AREA

We can apply our previous analysis to the case where the coordination problem arises in a common currency area composed by several countries, each having the capacity to print money, directly or indirectly. We will solve this problem for the case where the relevant criterion for each decision maker is to raise a given revenue in a way that will minimize the welfare loss of taxes. Specifically, suppose that there are two taxes: seigniorage and income (or endowment) tax (denoted by ρ). The income tax is associated with net collection costs (ξ) . Thus, each dollar paid by a consumer yields only $1 - \xi$ to the fiscal authorities.[12] We preserve the assumptions of the previous part, and for

---

11. This also suggests that one way of overcoming the competitive externality is by devising a system with a complete response (i.e., where $\frac{\partial G_i}{\partial G_h} = 1$ for $i \neq h$). Note that such a system will not solve the coordination problems in the presence of incomplete information on the true printing of each minister.

12. Alternatively, to collect one dollar the authorities must spend $\xi/(1-\xi)$ on enforcing the law. Consequently, the private sector pays $1/(1-\xi)$ for each dollar of net taxes. Throughout the analysis we assume a constant collection costs. We allow for collection costs because this issue plays a key role in explaining the use of inflation tax. As was shown by Kimbrough (1986), the optimal inflation tax is zero. According to Aizenman (1987) and Vega (1988), the inflation tax is positive if one allows for collection

simplicity assume a neutral rate of time preference (in terms of the Appendix, we assume that $\beta = 0$). For exposition simplicity we assume a symmetric environment, where all the members of the currency area are similar in size and in the underlying economic structure.[13] The periodic utility of the representative consumer in country h is given by

$$(15) \qquad u[C] + v[m_h] ,$$

where u and v stand for the utility from consumption and money balances, and m_h are the real balances. The Appendix demonstrates that

$$(16) \qquad C = Y(1 - \rho) - \frac{\pi}{1+\pi} m_h .$$

The periodic consumption equals the output net of income tax and net of seigniorage; which is given by the resources spent on preserving the real balances at their steady state level. The Appendix demonstrates that the first order condition determining optimal money balances is given by[14]

$$(17) \qquad \frac{\pi}{1+\pi} u' = v' .$$

---

costs. For a study on collection costs, political instability and seigniorage see Cukierman and Tabellini (1989).

13. For an analysis of seigniorage in Europe that focuses on the differences among countries see Grilli (1988).

14. See equation (A6), applied for the case where $\beta = 0$.

We denote by $m_h(\pi)$ the demand for real money balances derived form the above first order condition.

The revenue of the fiscal authorities of country h is given by

$$Y\rho(1-\xi)+\frac{G_h}{P} \quad . \tag{18}$$

The global money market equilibrium condition is

$$\sum_{j=1}^{n} G_j/P = \frac{\pi}{1+\pi} \sum_{j=1}^{n} m_j(\pi) \quad , \tag{19}$$

that is, the aggregate printing of real balances (the left hand side) should match the aggregate loss in real balances due to inflation.
Alternatively,

$$\bar{G} = \frac{\pi}{1+\pi}\bar{M} \tag{19'}$$

where $\bar{M} = \sum_{j=1}^{n} M_j$ and $\bar{G} = \sum_{j=1}^{n} G_j$.

In the Appendix we solve the problem of optimal taxes for the case where the welfare loss associated with the various taxes is given by a quadratic loss function. Here we take the direct approach. The problem facing the authorities in country h is to raise a given real revenue target, denoted by $\Re$, in a way that will maximize the welfare of the, representative consumer in country h. Applying (15), (16) and (18), this is equivalent to choosing G_h so as to maximize

$$(20) \qquad u[Y + \frac{G_h/P}{1-\xi} - \frac{R}{1-\xi} - \frac{\pi}{1+\pi} m_h] + v[m_h] ,$$

subject to the global money market clearing conditions, as given by (19). Direct derivation of (20) yields the following first order condition: [15]

$$(21) \qquad \frac{1}{1-\xi} \frac{\partial(G_h/P)}{\partial G_h} = \frac{m_h}{(1+\pi)^2} \frac{\partial \pi}{\partial \bar{G}} .$$

Equation (21) has a simple interpretation: the condition for optimal seigniorage is that extra printing will save as much income tax payment (the left hand side) as equals the induced capital loss on the domestic holder of money (the right hand side).

Taking derivatives of the two sides of (19) yields that in a symmetric equilibrium the following condition holds

$$(22) \qquad \frac{1}{P} [1 - \frac{\bar{G}}{P} \frac{\partial P}{\partial \bar{G}}] = n\,m \; \frac{1-\varepsilon(1+\pi)}{(1+\pi)^2} \frac{\partial \pi}{\partial \bar{G}} ,$$

where m stands for the real balances in each country.

Applying (19') we get that at in a symmetric equilibrium an increase in nominal seigniorage (i.e., $\Delta\ \bar{G} = \Delta\ \bar{M}$) will affect inflation by:

$$(23) \qquad \frac{\partial \pi}{\partial \bar{G}} = \frac{1+\pi}{n m P}$$

Note that $\frac{\partial(G_h/P)}{\partial G_h} = \frac{1}{P} [1 - \frac{1}{n} \frac{\bar{G}}{P} \frac{\partial P}{\partial \bar{G}}]$. Applying this result, (21), (22), and (23), we get that the condition determining optimal inflation in the non-cooperative solution is

---

15. In deriving (21) we make use the fact that the consumer optimizes his money balances, and thus (17) holds.

(24) $$\tilde{\varepsilon} = n - 1 + \frac{\xi}{1+\pi}$$

or, alternatively

(25) $$\pi = [n - 1 + \frac{\xi}{1+\pi}] \frac{m}{-\partial m/\partial \pi}.$$

Note that in the absence of collection costs ($\xi = 0$), the cooperative solution ($n = 1$) yields zero as the optimal inflation rate, in line with the standard result. With positive collections costs we get that the optimal inflation rate is positive, depending positively on the collection costs (formally, the cooperative solution yields $\varepsilon_c = \frac{\xi}{1+\pi}$). Alternatively, if we operate in a non-centralized system with competing decision makers ($n > 1$), we get positive inflation even in the absence of collection costs. This occurs because each country at the margin can export part of the inflation tax abroad.[16] If the competitive feature is powerful enough we may end on the backward portion of the Laffer curve, where $\varepsilon > \varepsilon_c$.

Similar to the analysis in Section 3, allowing for a conjecture variation equilibrium where $\frac{\partial G_i}{\partial G_h} = \tau$ for $i \neq h$ will yield

(26) $$\tilde{\varepsilon} = (1 - \tau) \frac{n-1}{1+(n-1)\tau} + \frac{\xi}{1+\pi}.$$

Consequently, we conclude that the competitive externality is proportional to $1 - \tau$, which measures the degree to which there is coordination failure.

16. The possibility of exporting the U.S. inflation tax due to elements of currency substitution and the presence of tradable nominal assets is the topic of Tabellini (1988).

4. CONCLUDING REMARKS

Our paper analyzed the determination of seigniorage when several decision makers determine the seigniorage and the resulting inflation. We focused on the monopolistic competitive equilibrium, and extended the discussion to the conjecture variation equilibrium. Our analysis did not attempt to model the political factors underlying a non-cooperative solution that prevent reaching a more efficient cooperative solution. Taking these factors as given, we focused on the consequences of the interaction among the various policy makers. We identify the presence of a competitive externality. The problem here is similar to the case of a fragile cartel, where each agent ignores the adverse consequences of his cheating on other cartel members. As in the cartel example, the welfare consequences are clear: it is advantageous to delegate, if possible, the printing power to one agent, who will generate the cooperative optimal solution and then distribute the revenue. In the case of the ministers, this will occur if there is either a powerful finance minister or a powerful central banker. In the case of the various countries, it will be useful to delegate the power regarding monetary policy to the central bank, or to one dominant country, subject to the appropriate revenue sharing rule.

Our analysis refers to the case where there is no uncertainty. Under this assumption one can solve the problems generated by the competitive externality by a sharing rule, where each marginal printing of an atomistic decision maker will trigger equal printing by all other decision maker. The consequence of such a rule is to internalize the competitive externality, but its practical application is limited by the availability of full information on the printing of each agent and costless monitoring. A useful extension of our analysis may consider the case where monitoring is costly and partial, increasing the importance of competitive externalities.

APPENDIX

The purpose of the Appendix is to provide a detailed description of the economy, yielding the demand structure applied throughout the paper.

A. The Consumer

We consider consumers whose preferences are given by

$$\text{(A1)} \qquad \sum_{t=1}^{\infty} [\, u(C_t) + v(\frac{M_t}{P_t})] \,/(1+\beta)^t$$

where u is the sub-utility from consumption, and v is the sub-utility from money. The periodic budget constraints for $t > 0$ are given by

$$\text{(A2)} \qquad P_tC_t + P_tB_t + M_t = P_t\, Y(1-\rho) + P_{t-1}B_{t-1}(1+\beta) + M_{t-1}$$

where ρ stands for an income tax, and we assume the existence of a real, one-period bond, yielding an interest rate β. To simplify, we assume that both output Y and the real interest rate are time invariant, and that $B_0 = 0$. We can combine the above periodic budget constraints into a unified intertemporal budget constraint:

$$\text{(A3)} \qquad \sum_{t=1}^{\infty} [\, C_t + \frac{\Delta M_t}{P_t}]/(1+\beta)^t = \qquad \sum_{t=1}^{\infty} [Y(1-\rho)] \,/(1+\beta)^t$$

where $\Delta M_t = M_t - M_{t-1}$. The consumer problem is to choose the vector of consumption ($\{C_t\}$ and $\{M_t\}$) and money balances that maximizes (A1) subject to (A3). The first order conditions are:

$$\text{(A4)} \qquad \frac{\partial u(C_t)}{\partial C_t} = \lambda$$

$$\text{(A5)} \qquad \frac{\partial v(m_t)}{\partial m_t} = \lambda \frac{(1+\beta)(1+\pi)-1}{(1+\beta)(1+\pi)}$$

where λ is multiplier associated with the budget constraint. Note that our assumptions imply that the solution of (A4) and (A5) are time invariant. The simultaneous solution of (A4) and (A5) yields the steady state values of C and m. Taking the ratios of the two first order conditions we get:

$$\text{(A6)} \qquad \frac{\partial v(m)}{\partial m} = \frac{\partial u(C)}{\partial C} \frac{(1+\beta)(1+\pi)-1}{(1+\beta)(1+\pi)} .$$

Note that from the budget constraint we get that

$$\text{(A7)} \qquad C + m \frac{\pi}{1+\pi} = Y(1 - \rho).$$

Solving (A6) and (A7) simultaneously yields the functional solution for m:

$$\text{(A8)} \qquad m = m(\pi) .$$

B. Optimal taxes with a quadratic loss function.

Our discussion in Section 3 was implemented in terms of the utility of the representative agent. We here describe the derivation of the optimal seigniorage for the case where the welfare loss associated with the various taxes is given by a loss function. Suppose that the that the loss associated with the taxes is the net present value of the periodic loss, given by

$$\text{(A9)} \qquad L = C_\rho \rho^2 + C_\pi \pi^2 ,$$

where ρ is the income tax and C_ρ and C_π are constants, measuring the size of the welfare triangles measuring the welfare losses due to the presence of distorted taxes.[17]

The aggregate demand for real balances, $\bar{m}$, is given by

$$\text{(A10)} \qquad \bar{m} = n\, k(\pi) Y \, ,$$

where n is the number of the countries in the common currency area, $k(\pi)$ summarizes the functional dependency of the demand for money balances on inflation, and Y is the fixed GNP in each country. For exposition simplicity we assume a symmetric environment, where all the members of the currency area are similar in size and in the underlying economic structure , yielding equal C's .

The problem facing the policy maker h is to raise a given real revenue target, $\mathfrak{R}$, so as to minimize the loss from taxes. Formally, $\mathfrak{R}$ is given by

$$\text{(A11)} \qquad \mathfrak{R} = \rho Y + \frac{G_h}{P}$$

where G_h is the seigniorage and P is the general price level.

The problem facing the policy maker is to choose (ρ, G_h) such as to minimize the welfare loss, L, subject to the given revenue target, $\mathfrak{R}$. Applying the (A9), (A11) we can summarize this problem as choosing G_h so as to minimize

---

17. This welfare loss can be derived as a second order approximation of the welfare losses around the undistorted equilibrium. It is a useful approximation as long as the various taxes are small. The size of the various C's are determined by the various elasticities and by the presence of collection costs associated with the various taxes.

$$\text{(A12)} \qquad C_\rho[\mathcal{R}' - \frac{G_h}{PY}]^2 + C_\pi \pi^2$$

where $\mathcal{R}' = \mathcal{R}/Y$ is the GNP share of the revenue target. Direct derivation of (A12) yields:

$$\text{(A13)} \qquad C_\rho[\mathcal{R}' - \frac{G_h}{PY}] \frac{\partial(G_h/P)}{\partial G_h} = C_\pi \pi \frac{\partial \pi}{\partial(G/P)} \frac{\partial(G/P)}{\partial G_h} .$$

In deriving (A13) we make use of the fact that the inflation rate is determined in the steady state equilibrium by the global seigniorage, denoted by G/P. Applying the fact that $G/P = \bar{m} \frac{\pi}{1+\pi}$ we get that in the symmetric equilibrium

$$\text{(A14)} \qquad C_\rho[\mathcal{R}' - \pi\, k(\pi)] = C_\pi \pi \frac{(1+\pi)^2}{\bar{m}(1-(1+\pi)\varepsilon)} \frac{\dfrac{\partial(G/P)}{\partial G_h}}{\dfrac{\partial(G_h/P)}{\partial G_h}} .$$

Applying the logic of Section 2 (equation (5)) we get a key equation determining the condition for the optimal non-cooperative inflation rate:

$$\text{(A15)} \qquad C_\rho[\mathcal{R}' - \pi\, k(\pi)] = C_\pi \pi \frac{(1+\pi)^2}{(1-(1+\pi)\varepsilon)\,\bar{m}} \frac{1-\dfrac{G}{P}\dfrac{\partial P}{\partial G}}{1-\dfrac{1}{n}\dfrac{G}{P}\dfrac{\partial P}{\partial G}} .$$

Equation (A15) can be interpreted as follows: a marginal increase in revenue collected as seigniorage by country h will generate two effects: a marginal benefit due to the reduction in the tax rate, ρ, needed to collect the given revenue target and a marginal cost due to the induced higher inflation rate. The first effect (the marginal benefit) is given by the left hand side of (A15), the second effect (the marginal cost) is given by the right hand side of (A15).

The presence of the competitive externality is reflected in the fact that each country view the effective inflation tax base as the aggregate supply of money balances, n m. As a result, each country internalizes only a portion 1/n of the total effect of a higher inflation. The cooperative solution is characterized by full internalization, which, in terms of (A15), will occur for $n = 1$. The effect of the competitive externality is to reduce the marginal cost of higher inflation, thereby reducing the right hand side of (A15), and increasing the inflation rate.

REFERENCES

Aizenman, Joshua "Inflation, Tariffs and Tax Enforcement Costs," Journal of International Economic Integration, 2 (2), 1987, pp. 12-28.

Bruno, Michael. "Econometrics and the Design of Economic Reform," NBER Working paper no. 2718, 1988.

Bruno, Michael and Stanley Fischer. "Expectations and the High Inflation Trap," manuscript, 1985.

Calvo Guillermo A. "On the Time Consistency of Optimal Policy in a Monetary Economy," Econometrica, November, Volume No. 46, 1978.

Casella Alessandra and Jonathan Feinstein. "Management of a Common Currency," NBER Working Paper No. 2740, 1988.

Cukierman, Alex. "Rapid Inflation: Deliberate Policy or Miscalculation," presented at the Carnegie-Rochester Conference on Public Policy, 29, pp. 11 - 76, 1988.

_________ and Guido Tabellini. "Seigniorage, Political Instability, Polarization and Taxation in Developing Countries," manuscript, 1989.

Edwards Sebastian and Guido Tabellini. "Fiscal Policies, Inflation and Stabilization in Developing Countries, a Political Economy Approach," manuscript, U.C.L.A., January 1989.

Eckstein, Zvi and Leonardo Leiderman. "Intertemporal Models of Consumption and Money Holding: Their Implications for Seigniorage and Inflation," manuscript, Tel-Aviv University, 1988.

Fischer, Stanley. "Seigniorage and the Case for a National Money," Journal of Political Economy, 90, April 1982.

Frenkel, Jacob A. "Some Dynamic Aspects of the Welfare Cost of Inflationary Finance." In Ronald I. Mckinnon (ed.), Money and Finance in Economic Growth and Development , Marcel Dekker, Inc., 1976.

Friedman, Milton. "The Optimum Quantity of Money," in the The Optimum Quantity of Money and Other Assays, Aldine, Chicago, IL. pp 1-50.

Grilli Vittorio. "Seigniorage in Europe," manuscript, Yale University, 1988.

Hamada, Koichi. "A Strategic Analysis of Monetary Interdependency,"Journal of Political Economy , Vol. 84, 1976.

Helpman, Elhanan and Sadka, Efraim. "Optimal Financing of Government's Budget: Taxes, Bonds or Money?" American Economic Review, 69, 1979, 152-160.

Jovanovic, Boyan. "Inflation and Welfare in the Steady State." Journal of Political Economy , 90, June 1982; 561-77.

Kiguel Miguel A. and Nissan Liviatan. "Inflationary Rigidities and Orthodox Stabilization Policies: Lessons From Latin America," The World Bank Economic Review, September, Volume no. 2, 1988.

Kimbrough, Knet P. "The Optimal Quantity of Money Rule in the Theory of Public Finance," Journal of Monetary Economics, November 1986, pp. 277 - 285.

Liviatan, Nissan. "Inflation and the Composition of Deficit Finance," in Adams, F.E. (ed.) Global Econometrics, Cambridge, Mass., MIT Press.

Mundell, Robert A. "A Theory of Optimum Currency Areas," American Economic Review, Vol. 51, 1961.

Obstfeld, Maurice. "Multiple Stable Equilibria in an Optimizing Perfect-Foresight Model," Econometrica , Vol. 52, 1984.

Phelps, Edmund S., "Inflation in the Theory of Public Finance," Swedish Journal of Economics, 75, 1973, 67- 82.

Tabellini Guido "Exporting the Inflation Tax," manuscript, U.C.L.A., January 1989.

Vega, Carlos. "Essays on Currency Substitution and Inflation Tax," Ph.D. thesis, University of Chicago, 1987.

</ref_section>