NBER WORKING PAPER SERIES

MARKET FREEZES

Chao Gu
Guido Menzio
Randall Wright
Yu Zhu

Working Paper 29210
http://www.nber.org/papers/w29210

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2021

We thank Luis Araujo, Daniela Puzzello and Shengxing Zhang for useful comments or conversations, and Philip Coyle for excellent research assistance. Gu acknowledges financial support from Hong Kong Institute for Monetary and Financial Research. Wright acknowledges support from the Ray Zemon Chair in Liquid Assets, and the Kenneth Burdett Chair in Search Theory and Applications, at the Wisconsin School of Business. The Federal Reserve Bank of Minneapolis also provided support. This paper represents the views of the authors, which are not necessarily the views of the Hong Kong Monetary Authority, Hong Kong Institute for Monetary and Financial Research, or its Board of Directors or Council of Advisers.The views expressed are those of the authors and not those of the Bank of Canada, the Federal Reserve Bank of Minneapolis, the Federal Reserve System, or the National Bureau of Economic Research.

Market Freezes
Chao Gu, Guido Menzio, Randall Wright, and Yu Zhu
NBER Working Paper No. 29210
September 2021
JEL No. D53,D83,E30,E44,E52,G14

ABSTRACT

During the financial crisis apparently centralized markets continued to function while trade in OTC markets froze. We use search-and-bargaining theory to ascertain conditions that allow trade to temporarily freeze in decentralized markets, focusing on the roles of liquidity and self-fulfilling prophecies. We show standard models can have recurrent, belief-driven hot and cold spells, but not freezes and thaws. A simple specification that has freezes assumes negative returns. A more realistic one incorporates information frictions (costly asset-quality verification). Another uses different frictions to get credit freezes. We also discuss policy implications, and go into detail on the nature of OTC markets.

Chao Gu
University of Missouri
guc@missouri.edu

Guido Menzio
Department of Economics
New York University
19 West 4th Street
New York, NY 10012
and NBER
gm1310@nyu.edu

Randall Wright
Department of Finance and Department of Economics
University of Wisconsin-Madison
Grainger Hall
975 University Ave
Madison, WI 53706
and NBER
rwright@bus.wisc.edu

Yu Zhu
Bank of Canada
zhuyuzlf57@gmail.com

1 Introduction

As Chiu and Koeppl (2016) describe the most recent financial crisis, "there was a stunning difference in how asset markets were affected according to their infrastructure. Markets with centralized trading functioned rather well. To the contrary, in over-the-counter markets... trading came to a halt." An over-the-counter (OTC) asset market is one where agents must find counterparties and negotiate the terms of trade (Duffie et al. 2005). While this is absent from traditional general equilibrium theory, it is captured by search-and-bargaining theory.[1] This paper uses that kind of theory to ascertain under what conditions trade can temporarily halt, called a *market freeze,* with particular interest in the role of *liquidity*, and in the possibility it could be a *self-fulfilling prophecy.*

Market freezes are commonly considered puzzling. Consider Leitner (2011): "In normal times, investors buy and sell financial assets because there are gains from trade. However, markets do not always function properly – they sometimes 'freeze.' An example is the collapse of trading in mortgage-backed securities during the recent financial crisis. Why does trade break down despite the potential gains from trade? Can the government intervene to restore the normal functioning of markets?" Highlighting these questions suggests the answers are not obvious.[2]

While the phenomenon seems interesting and important, there is no consensus model for analyzing market freezes. Our theory builds on a standard approach to

[1] The search-and-bargaining framework has a long tradition in modeling markets with frictions, including studies of unemployment (Mortensen and Pissarides 1994, Pissarides 2000); money (Shi 1995, Trejos and Wright 1995); middlemen (Rubinstein and Wolinsky 1987); financial markets (Duffie et al. 2005, Lagos and Rocheteau 2009); and the foundations of Walrasian equilibium (Rubinstein and Wolinsky 1985, Gale 1986).

[2] As additional motivation, consider Benmelech and Bergman (2012): "Financial market freezes – by which we mean large declines in the volume of transactions in both the primary and the secondary markets that occur over a non-trivial period of time – are typically observed during financial crises." After describing several episodes going back to the 1870's, they say "a market freeze took place during the financial crisis of 2008-09 with the collapse of the structured finance market... not only did the market for mortgage-backed securities such as RMBS and CDOs collapse, but also, other, non-housing segments of the structured finance markets – ranging from commercial loans securitizations to asset-backed securities – came to a halt." See also Bebchuk and Goldstein (2010) and Razin (2014).

the analysis of liquidity in different contexts, Lagos and Wright (2005) (see Lagos et al. 2017 and Rocheteau and Nosal 2017 for surveys of work using the framework). The specification nests several applications in the literature, including those where in decentralized markets households acquire goods from each other, households acquire goods from producers or retailers, firms acquire inputs or ideas from each other, firms acquire funding from financial institutions, or investors acquire assets from each other. It also takes an abstract approach to price formation, nesting various bargaining solutions, competitive pricing and other mechanisms. Moreover, liquidity can be interpreted as assets facilitating immediate settlement or serving as collateral for deferred settlement, and we consider different combinations of real assets, currency and credit with endogenous debt limits.

In a benchmark model, which is fairly general, as described above, freezes *cannot* occur under standard assumptions. To explain this, first, we interpret assets as in the literature following Lucas (1978) as equity shares in firms, where a firm is identified by its technology. A common metaphor refers to it as a "tree" – a rudimentary technology generating a dividend ρ, colorfully called "fruit," with no additional inputs. In financial economics, where assets are held for their returns, $\rho > 0$. In monetary economics, where they are held for liquidity, typically $\rho = 0$, capturing the "intrinsic uselessness" of fiat currency that Wallace (1980) takes as its identifying characteristic. In some commodity money models, such as Kiyotaki and Wright (1989), $\rho < 0$ is used and interpreted as a storage cost.

As is known, in this setting there are deterministic equilibria where prices and quantities fluctuate as a self-fulfilling prophecy, which we call hot and cold spells since the market goes up and down, but we show there do not exist deterministic equilibria with recurrent freezes, where the market completely stops and restarts. Hence we consider stochastic (sunspot) equilibria. There are stochastic equilibria with hot and cold spells, but with $\rho \geq 0$, which we consider a standard assumption, again there are no recurrent freezes and thaws. We then show $\rho < 0$ allows recurrent freezes, but $\rho < 0$ is nonstandard in financial economics and arguably irrelevant for modern financial markets. Still, the methods developed for this relatively simple case are building blocks for more realistic settings.

To pursue this, we consider $\rho \geq 0$ with a transaction cost emerging from information frictions related to the recognizability friction in Lester et al. (2012). Namely, assets come in high and low quality – as an extreme, genuine and counterfeit – and agents accepting them must pay a cost to verify their value (Appendix A reinterprets this as concern over paper payment instruments in a pandemic where there is a cost to verify their safety). Now stochastically recurrent freezes are possible with $\rho \geq 0$: in some (sunspot) states quality is vetted and assets are traded; in others exchange breaks down. More economic intuition is provided below, but for now we just mention that the self-referential nature of liquidity plays an important role, since whether agents are willing to trade assets depends on what they think others are doing.

In an extension with multiple assets, trade can freeze in some but not others. When one asset is currency, we discuss monetary policy, and show freezes are less likely at low inflation. We also discuss regulating assets' use as payment instruments. Another extension studies what we call genuine OTC markets, meaning this: in our other models assets are used to acquire objects with direct payoffs; in this one, those objects are acquired with cash, while cash is acquired by selling assets in an OTC market. Stochastically recurrent freezes are again possible. Another extension uses a fixed entry cost.[3] This model shows how the decentralized (bilateral) nature of OTC markets can matter: similar fixed costs in Walrasian markets do not lead to freezes. A final extension considers costs of using credit. We show how freezes can occur here, too, as is relevant to the extent that credit market and asset market freezes are related but distinct phenomena.

[3]Entry costs are featured in many prominent search models (e.g., Diamond 1982; Pissarides 1990), but usually they do not generate interesting dynamics without additional ingredients, like increasing returns in matching (Diamond and Fudenberg 1989) or production (Mortensen 1999). We get interesting dynamics without such ingredients due to liquidity considerations.

The rest of the paper is organized as follows. Section 2 analyzes the benchmark model. Section 3 builds on that in the environment with verification costs. Section 4 discusses genuine OTC markets. Section 5 takes up other extensions and Section 6 concludes. The Appendix contains details and proofs.

2 Baseline Model

2.1 Environment

As in Lagos and Wright (2005), in every period of discrete time, a continuum of infinite-lived agents interact in two ways: first they trade in a decentralized market, or DM; then they trade in a frictionless centralized market, or CM. This captures an asynchronicity of expenditures and receipts central to any analysis of money or credit – agents may want to buy something in the DM while their incomes accrue in the CM, so they must pay either using assets acquired in a previous CM or debt due in a future CM. It is also nice for our application since it easily accommodates various specifications for search, bargaining, information and asset characteristics.

A measure $n_b = 1$ of agents called buyers want something, denoted q, provided in the DM by a measure n_s of agents called sellers. In applications in the literature, the agents are interpreted as households, firms, retailers or financial institutions, and q can be a good, an input, an asset or an idea, all of which fit within our formalization (see Appendix A for details). In the DM buyers and sellers meet bilaterally, where $n = n_b/n_s$ is the buyer/seller ratio, α is the probability a buyer meets a seller, and α/n is the probability a seller meets a buyer. In the CM, all agents consume a numeraire good x, supply labor ℓ, adjust asset holdings and settle debts. Period (CM plus DM) payoffs for buyers and sellers are

$$U(x) - \ell + u(q) \text{ and } U(x) - \ell - c(q),$$

where $u(q)$ and $c(q)$ are the benefit and cost to trading q, with $U'(x), u'(q), c'(q) > 0$, $U''(x), u''(q) < 0 \leq c''(q)$, $u(0) = c(0) = 0$ and $u'(0) > c'(0)$.

For now there is only one storable object, an asset a that pays dividend ρ in numeraire in the CM. We want the environment to be stationary so that all dynamics are due to beliefs. When the asset is fiat currency, with $\rho = 0$, the supply can evolve according to $A_{t+1} = (1+\mu) A_t$, with changes engineered in the CM by lump sum transfers if $\mu > 0$ or taxes if $\mu < 0$, as that is is consistent with constant real money balances if the price level also increases at rate μ. When $\rho \neq 0$, the asset supply is fixed and normalized to $A = 1$.

We are mainly interested in stochastic, or sunspot, equilibria because market freezes cannot happen in deterministic equilibrium (Lemma 4); still, it is useful to start with deterministic outcomes. To proceed, let buyers' value functions in the CM and DM be $W(a)$ and $V(a)$, respectively. Then

$$W(a) = \max_{x,\ell,\hat{a}} \{U(x) - \ell + \beta V_{+1}(\hat{a})\} \quad (1)$$

$$\text{st } x = \ell + \rho a + \phi(a - \hat{a}) - T, \quad (2)$$

where ϕ is the price of a in terms of x, T is the tax, and the real wage is 1 since 1 unit of ℓ is assumed to produce 1 unit of x (this is easy to relax). Also, $\beta = 1/(1+r)$ is discounting between the CM and DM (without loss of generality discounting between the DM and CM is ignored). Given interior solutions, the FOC for $\hat{a}$ is $\phi = \beta V'_{+1}(\hat{a})$ and the envelope condition is $W'(a) = \rho + \phi$, observations that immediately yield a standard result in this class of models:[4]

Lemma 1: $\hat{a}$ is independent of a and $W(a)$ is linear.

To get assets valued for liquidity we must rule out perfect credit in the DM, where trade is financed by unsecured promises of payment in the CM. As is well known (Kocherlakota 1988), this requires a lack of commitment, plus imperfect monitoring or record-keeping to hinder credit without commitment supported, as

[4] This assumes an interior solution $\ell_t \in (0, \bar{\ell})\ \forall t$, where $\bar{\ell}$ is time endowment, but that can be relaxed to $\ell_t \in (0, \bar{\ell})$ for some t. Also, while here it follows directly from quasi-linear utility, Wong (2016) shows Lemma 1 also holds for any $U(x, \bar{\ell} - \ell)$ with $U_{11}U_{22} = U_{12}^2$, while Rocheteau et al. (2008) show it holds for any concave, monotone $U(x, \bar{\ell} - \ell)$ if labor is indivisible, $\ell \in \{0, \bar{\ell}\}$, and agents trade using lotteries as in Rogerson (1988).

in Kehoe and Levine (1993), by threats to punish renegers (we take up that kind of credit in Section 5). Then assets can facilitate transactions in two ways: a can be used as a medium of exchange for immediate settlement; or a can be pledged as collateral, with those reneging on debt getting their assets seized. For our purposes the story does not matter: the equations are the same whether a is called a medium of exchange or collateral.[5]

When a buyer holding a units of the asset meets a seller in the DM, they choose a quantity q and a payment $a' \leq a$ in units of the asset, where the constraint $a' \leq a$ simply says that one cannot hand over, or pledge, more than one has. Since $W'(a) = \rho + \phi$, this payment is worth $p = (\rho + \phi) a'$ in CM numeraire, or equivalently in utility, given the wage and marginal utility of leisure are both 1. It is convenient to define $z = (\rho + \phi) a$ and write the *liquidity constraint* as $p \leq z$.

The next step is to determine (p, q). Since there is no consensus on the "correct" approach in models of decentralized exchange, and since more generality is preferred when it does not detract from tractability, we use a generic mechanism (Gu and Wright 2016). This is a function $p = v(q)$ that says to get q a buyer must provide a payoff p to the seller. Naturally $v(0) = 0$ and, assuming almost-everywhere differentiability, $v'(q) > 0$. There only other requirement is that when trade is possible – i.e., when the buyer has $z > 0$ – the outcome depends on $p^* = v(q^*)$, where q^* is defined by $u'(q^*) = c'(q^*)$, as follows: if $z \geq p^*$ he pays p^* and gets q^*; and if $z < p^*$ he pays $p = z$ and gets $q = v^{-1}(z) < q^*$.

Many standard mechanisms are consistent with this specification, including as a simple example the Kalai bargaining solution, represented in our notation by $v(q) = \theta c(q) + (1 - \theta) u(q)$ where θ is buyer's share of the trade surplus. The

[5]This is not a new point (e.g., see Lagos 2010). A detail is that some papers using the collateral story, following Kiyotaki and Moore (1997), say only a fraction $\chi < 1$ of assets can be pledged as collateral, since one can always abscond with the rest. But when asset quality is private information, Li et al. (2012) show there is a χ limiting the amount of a one can use as a medium of exchange, so the equations are still the same. We emphasize this since some people seem to think collateral is important while medium of exchange considerations are not, even though they are typically equivalent (for exceptions, see Loberto 2018 or Madison 2018).

generalized Nash solution is the same if $p \leq z$ is slack, $\theta = 1$ or $\theta = 0$, but otherwise is different, and Aruoba et al. (2007) argue that Kalai has advantages in monetary models.[6] Since most results hold for any $v(\cdot)$ with the above properties, we need not take a stand on the trading protocol for now. One obvious result is:

Lemma 2: When a buyer and seller meet they trade iff $z > 0$.

The next step toward defining equilibrium is to derive the Euler equation. By Lemma 1, $W(a)$ is linear, and hence a buyer's DM value function satisfies

$$V(a) = W(a) + \alpha [u(q) - v(q)], \tag{3}$$

where again α is the probability a buyer has a trade opportunity, and q depends on his a, or equivalently his $z = (\rho + \phi) a$. On the one hand, $z \leq 0$ implies no trade and $z \geq p^*$ implies trade at $q = q^*$, so $z < 0$ or $z > p^*$ implies $\partial q / \partial a = 0$ and $V'(a) = W'(a) = \rho + \phi$. On the other hand, $0 < z < p^*$ implies trade at the solution to $v(q) = (\rho + \phi) a$, so $\partial q / \partial a = (\rho + \phi) / v'(q)$ and

$$V'(a) = \rho + \phi + \alpha (\rho + \phi) \lambda(q),$$

where $\lambda(q) \equiv u'(q) / v'(q) - 1$ is the Lagrange multiplier on $a' \leq a$, often called the *liquidity premium*.

It is convenient to define

$$L(z) \equiv \begin{cases} \lambda \circ v^{-1}(z) & \text{if } 0 < z < p^* \\ 0 & \text{if } z < 0 \text{ or } z > p^* \end{cases} \tag{4}$$

where $\lambda \circ v^{-1}(z) \equiv \lambda(v^{-1}(z))$ denotes a composite function. This is simply the multiplier expressed in terms of z rather than a, which is of course 0 when the constraint is slack. Then we can write $V'(a) = (\rho + \phi)[1 + \alpha L(z)]$, insert $V'(a)$ into the FOC for $\hat{a}$ from the CM, and, being careful with time subscripts, arrive at the Euler equation

$$\phi_t = \beta \left(\rho + \phi_{t+1}\right) [1 + \alpha L(z_{t+1})]. \tag{5}$$

[6] Other solution concepts consistent with our generic $v(\cdot)$ include strategic bargaining as in Zhu (2019), perfectly or imperfectly competitive pricing as in Gu and Wright (2016), and more exotic mechanisms designed to deliver efficiency as in Hu et al. (2009).

There is a technical issue about what to do at $z = 0$ and $z = p^*$, where $V(q)$ may not be differentiable, but, as explained below, it turns out not to matter. Another issue is that the global properties of $L(z)$ depend on details – e.g., for Kalai bargaining or Walrasian pricing $L'(z) < 0$ while with Nash bargaining it is ambiguous – but in any case Gu and Wright (2016) show $L'(z) < 0$ at steady state equilibrium (defined below) for any $v(q)$ satisfying the assumptions made above. These technicalities notwithstanding, given a time path for ϕ_t buyers' demand for z_t is described by (5). Sellers' have a similar problem but it is omitted since, with no need for DM liquidity, they demand $z_t = 0$.[7]

2.2 Deterministic Equilibrium

Market clearing entails $a_t = A_t$, and (5) can be written $z_t = F_{t+1}(z_{t+1})$ where

$$F_t(z_t) \equiv \rho A_t + \frac{\beta z_t}{1+\mu}\left[1 + \alpha L(z_t)\right]. \tag{6}$$

Here $F_t(z) = F(z)\ \forall t$ since, for stationary, as mentioned, when $\rho \neq 0$ we set $\mu = 0$ and $A_t = 1\,\forall t$. The domain of F is $[\rho A, \infty)$, since $z \geq \rho A$ is equivalent to $\phi \geq 0$. A deterministic equilibrium is a time path for z_t solving $z_t = F(z_{t+1})$ with $z_t \geq \rho A$ and $\lim_{t\to\infty} \beta^t \phi_t A_t = 0$, where the latter is the relevant transversality condition in this kind of model (Rocheteau and Wright 2013).[8]

At this point we can say why it does not matter if $L(z)$ is not differentiable at $z = 0$ or $z = p^*$. Our interest is in paths for z_t satisfying the difference equation (6), and for generic parameters such paths do not go through $z = 0$ or $z = p^*$.

[7] To verify this, notice (5) implies either $\phi_t = \beta(\rho + \phi_{t+1})$, which makes sellers indifferent to holding any z, or $\phi_t > \beta(\rho + \phi_{t+1})$, which makes them strictly prefer 0. So saying they demand $z = 0$ looks innocuous – but a detail must be mentioned. While for many $v(q)$ specifications (e.g., Kalai bargaining) buyers' surplus $u(q) - v(q)$ is increasing in q and hence in a, for some (e.g., Nash with $\theta < 1$) $u(q) - v(q)$ is decreasing near q^*. In that case, if ρA is big buyers do not want to hold it all, as their DM surplus is higher when they are more constrained. If buyers want $a < A$ one option is to let sellers hold the rest; instead, following Geromichalos et al. (2007) and Lagos and Rocheteau (2008), we assume buyers hold $a = A$ but only bring $\tilde{a} < A$ to the DM. This does not matter for much, but it simplifies market clearing.

[8] There are other endogenous variables, but given z_t those in the DM variables follow from $\phi_t = z_t/A_t - \rho$, $q_t = v^{-1}(z_t)$, etc., while those in the CM satisfy $U'(x_t) = 1$, etc.

A qualification is that $z_t = 0\,\forall t$ is a nonmonetary equilibrium that always exists if $\rho = 0$, which is the nongeneric but important case of fiat money; still, one can handle case that separately, and moreover it is not relevant for our purposes. Hence, F can have a kink at 0 or p^*, but that causes no problems.

Before starting on dynamics we describe steady states, given by solutions to $z_S = F(z_S)$. First consider $\rho > 0$ and $A = 1$, so the domain of F is $[\rho, \infty) \subset (0, \infty)$. In this case there exists a unique steady state z_S. To describe it, define a's fundamental price (its worth purely as a store of value) by $\phi^F \equiv \rho/r$ and let $z^F \equiv \rho(1+r)/r$. Then $z^F \geq p^*$ implies $\phi = \phi^F$ and $q = q^*$, in which case we say liquidity is abundant because it is sufficient to buy q^*, while $z^F < p^*$ implies $\phi > \phi^F$ and $q = v^{-1}(z) < q^*$, in which case we say liquidity is scarce. These standard results are shown in Fig. 1 for two examples, both with liquidity scarce, but one has F is monotone and the other nonmonotone, with functional forms and parameter values for all examples in Appendix B. Notice the graph delineates two sets (intervals), $\mathcal{Z}_1$ and $\mathcal{Z}_2$, where $z \in \mathcal{Z}_1$ is defined by $F(z) < z$ (F is below the 45^o line) and $z \in \mathcal{Z}_2$ by $F(z) > z$ (F is above the 45^o line).

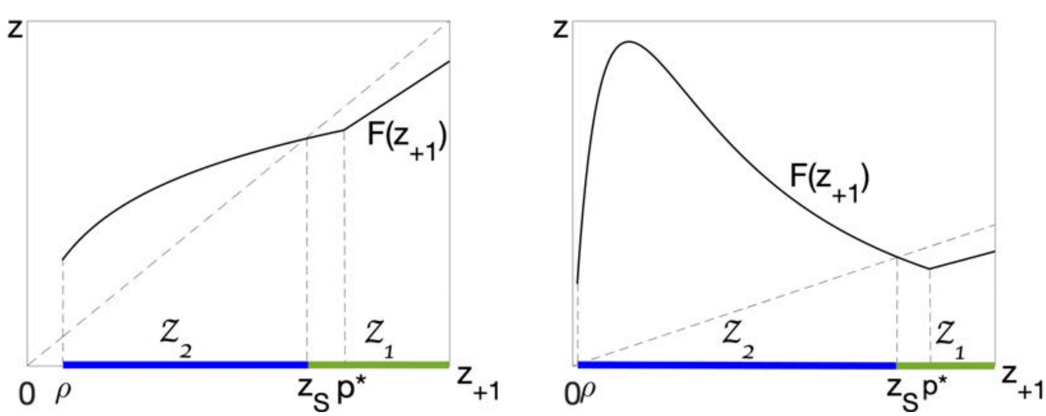


Fig. 1a: $\rho > 0$, monotone Fig. 1b: $\rho > 0$, nonmonotone

Now consider $\rho = 0$ and any $\mu > \beta - 1$, where the limit $\mu \to \beta - 1$ is the Friedman rule. To clarify this, define a nominal interest rate by letting $1 + i$ be the dollars agents require in the next CM to give up a dollar in this CM. Then

$1+i = (1+\mu)/\beta$, and $\mu \to \beta - 1$ is the same as $i \to 0$. Algebra reduces $z_S = F(z_S)$ to $\alpha L(z_S) = i$. A solution $z_S > 0$ to $\alpha L(z_S) = i$ exists iff $i < \bar{\imath}$, where $\bar{\imath} > 0$ except for extreme cases (e.g., bargaining when $\theta = 0$). When steady state $z_S > 0$ exists it is generically unique, with $q = q^*$ or $q < q^*$ depending on the specification. We do not show a graph for $\rho = 0$, because it looks similar to Fig. 1, except the domain is now $[0, \infty)$. In particular, $F(0) = 0$, so $z = 0$ is a steady state, the nonmonetary equilibrium mentioned above, where a is not traded in the DM, so agents may as well dispose of it.

Now consider $\rho < 0$. There is again a steady state where a is not traded in the DM, but now, with $\rho < 0$, agents definitely dispose of it. As long as $|\rho|$ is not too big there are also steady states, generically an even number, where a is accepted in the DM and hence is valued in the CM – i.e., $\phi > 0$, even though $\rho < 0$, due to the liquidity premium. Fig. 2 shows a case with two steady states, $z_L > 0$ and $z_H > z_L$, and in what follows we usually frame the discussion as if there are exactly two, but the results apply to any even number with the obvious changes in presentation. Notice the domain of F now includes some $z < 0$, and in addition to $\mathcal{Z}_1$ and $\mathcal{Z}_2$ Fig. 2 shows $\mathcal{Z}_0 = \{z | \rho A \leq z < 0\}$.

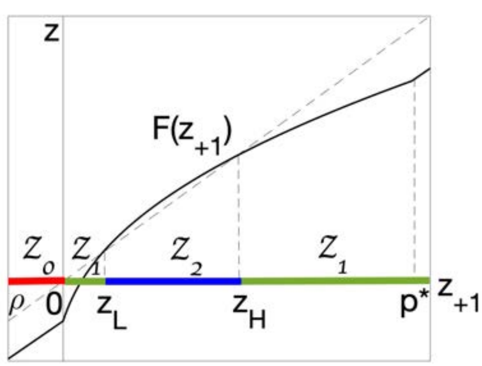


Figure 2a: $\rho < 0$, monotone

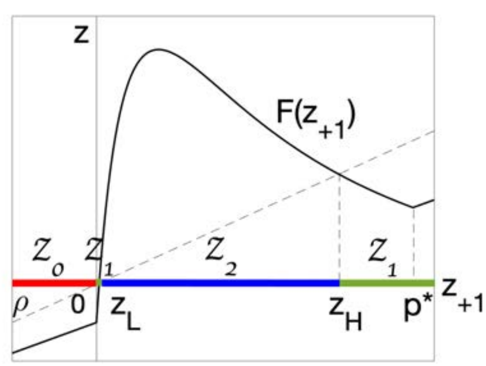


Figure 2b: $\rho < 0$, nonmonotone

The set $\mathcal{Z}_0$ is crucial to the analysis since $z < 0$ means a does not trade in the DM while $z > 0$ means it does. The following is also obvious:

Lemma 3: $\mathcal{Z}_0 \neq \varnothing$ iff $\rho < 0$.

For $z \geq 0$ steady state results are known, as are deterministic dynamics. For $z < 0$ the results are somewhat novel, but we do not dwell on them since our interest is mainly in recurrent freezes and they cannot occur in deterministic equilibrium. To verify this, notice there is DM trade at some dates but not others if $z > 0$ at some dates and $z \leq 0$ at others. For $\rho > 0$, this is impossible since $z \geq 0$ on the domain of F. For $\rho = 0$, $z = 0$ is possible, but $F(0) = 0$ and $F(z) > 0\,\forall z > 0$, so z cannot change sign. For $\rho < 0$, $z < 0$ is possible, but $F(z) < 0\,\forall z < 0$ so again z cannot change sign.

While deterministic equilibria cannot have recurrent episodes where the market shuts down, there can be cycles interpretable as hot and cold spells where market activity oscillates. As is standard, if $F'(z_S) < -1$, which can occur in Fig. 1 at z_S or in Fig. 2 at z_H, there are 2-period cycles.[9] This is summarized as follows:

Lemma 4: Deterministic equilibria can have hot and cold spells but not recurrent freezes and thaws.

2.3 Stochastic Equilibrium

Given Lemma 4 we look to sunspot equilibria. Consider a stochastic process s_t that is publicly observed at the start of period t, and has no impact on fundamentals (preferences, technologies and government policies), but could potentially affect endogenous variables. It suffices here to use a 2-state Markov process, with transition probabilities $\sigma_j = \Pr(s_{t+1} = s_{-j} | s_t = s_j)$, $j = 1, 2$. Let z_j be liquidity in state j. Proceeding as above, we get a two-state version of (6),

$$z_{1,t} = \rho + (1 - \sigma_1)\beta z_{1,t+1}[1 + \alpha L(z_{1,t+1})] + \sigma_1 \beta z_{2,t+1}[1 + \alpha L(z_{2,t+1})] \quad (7)$$

$$z_{2,t} = \rho + \sigma_2 \beta z_{1,t+1}[1 + \alpha L(z_{1,t+1})] + (1 - \sigma_2)\beta z_{2,t+1}[1 + \alpha L(z_{2,t+1})] \quad (8)$$

[9]See Azariadis (1993) for textbook results on cyclic dynamics. With the specifications in Appendix B, $F'(z_S) < -1$ occurs for moderately high values of the curvature parameter γ in $u(q)$. As γ gets higher other cycles emerge, including 3-cycles. When 3-cycles exist, N-cycles exist for all N, plus chaotic dynamics (again see Azariadis 1993).

or, more compactly,

$$z_{1,t} = (1-\sigma_1) F(z_{1,t+1}) + \sigma_1 F(z_{2,t+1}) \tag{9}$$

$$z_{2,t} = \sigma_2 F(z_{1,t+1}) + (1-\sigma_2) F(z_{2,t+1}). \tag{10}$$

Equilibrium is now a path $\{z_{1,t}, z_{2,t}\}$ solving (9)-(10), $z_{j,t} \geq \rho$, and the transversality condition. If agents ignore the sunspot there are equilibria with $z_{1,t} = z_{2,t}$, but we always mean a proper sunspot equilibrium where $z_{2,t} \neq z_{1,t}$. Further, we concentrate on stationary outcomes in the sense that $z_{j,t} = z_j \ \forall t$, where without loss of generality $z_2 > z_1$. We also distinguish between two cases: $z_2 > z_1 > 0$ is called an *intensive sunspot equilibria* or ISE, which is a stochastic version of the hot and cold spells discussed above; and $z_2 > 0 > z_1$ is called an *extensive sunspot equilibria* or ESE, which has stochastically recurrent freezes and thaws.[10]

Following methods going back to Azariadis (1981), we first solve (9)-(10) for (σ_1, σ_2) given (z_1, z_2), then find conditions on (z_1, z_2) to guarantee $(\sigma_1, \sigma_2) \in (0,1)^2$. The first step yields

$$\sigma_1 = \frac{z_1 - F(z_1)}{F(z_2) - F(z_1)} \tag{11}$$

$$\sigma_2 = \frac{F(z_2) - z_2}{F(z_2) - F(z_1)}. \tag{12}$$

Then it is easy to check $(\sigma_1, \sigma_2) \in (0,1)^2$ iff one of the following holds:

$$\text{Condition A, } F(z_2) < z_1 < z_2 < F(z_1)$$

$$\text{Condition B, } F(z_1) < z_1 < z_2 < F(z_2)$$

Condition A is relevant when F crosses the 45^o line from *above*, as in Fig. 1 at z_S, or in Fig. 2 at z_H. If $F'(z) < -1$ at steady state then there are $z_1 \in \mathcal{Z}_1$ and $z_2 \in \mathcal{Z}_2$ in the neighborhood of steady state such that Condition A holds. Hence,

[10] The ISE and ESE labels are from Trejos and Wright (2016), who discuss similar issues but only in environments where agents are restricted to hold $a \in \{0,1\}$, as was common in early monetary search theory. Obviously it is important to allow more general asset holdings when studying financial markets, even if some models with $a \in \{0,1\}$ are still useful.

$F'(z) < -1$ at steady state is sufficient for the existence of (z_1, z_2) that, together with the (σ_1, σ_2) in (11)-(12), constitute a sunspot equilibria. More important for our purposes, $z_1 \in \mathcal{Z}_1$ and $z_2 \in \mathcal{Z}_2$ are necessary for Condition A, which means $z_1, z_2 > 0$. Hence Condition A can be used to get ISE but not ESE.

Now consider Condition B, which is relevant when F crosses the 45^o line from *below*, as in Fig. 2 at z_L. It is easy to see from the graph that Condition B holds iff $z_1 \in \mathcal{Z}_0 \cup \mathcal{Z}_1$ and $z_2 \in \mathcal{Z}_2$, and then any $z_1 \in \mathcal{Z}_1$ and $z_2 \in \mathcal{Z}_2$ together with the implied (σ_1, σ_2) constitute an ISE. Moreover, Lemma 3 implies $\mathcal{Z}_0 \neq \varnothing$, and any $z_1 \in \mathcal{Z}_0$ and $z_2 \in \mathcal{Z}_2$ together with the implied (σ_1, σ_2) constitute an ESE. We summarize as follows:

Proposition 1: For $\rho \geq 0$, positive steady state z_S exists, is generically unique, and satisfies $F'(z_S) < 1$. If $F'(z_S) < -1$ there exist ISE around z_S for some (z_1, z_2) in the neighborhood of z_S. There do not exist ESE.

Proposition 2: For $\rho < 0$, positive steady states exist if $|\rho|$ is not too big, and generically come in pairs (z_L, z_H), with $z_H > z_L > 0$ and $F'(z_H) < 1 < F'(z_L)$. If $F'(z_H) < -1$ there exist ISE around z_H for some (z_1, z_2) in the neighborhood of z_H, and there exist ISE around z_L for any $z_1 \in \mathcal{Z}_1$ and $z_2 \in \mathcal{Z}_2$. There exist ESE around z_L for any $z_1 \in \mathcal{Z}_0$ and $z_2 \in \mathcal{Z}_2$.

These results deliver ESE, but only for $\rho < 0$, which one may find a poor description of assets in modern financial markets, even if it might (at best) apply to historical episodes with commodity monies. At one point we entertained the idea that $\rho < 0$ captures the notion of a *toxic asset*, something intimately connected with market freezes in convention wisdom, i.e., in Wikipedia:

> Toxic asset is a popular term for certain financial assets whose value has fallen significantly and for which there is no longer a functioning market, so that such assets cannot be sold at a price satisfactory to the holder... The term became common during the financial crisis of 2007-2008, in which [such assets] played a major role. When the market for toxic assets ceases to function, it is described as 'frozen'

Upon reflection, we were not able to convince ourselves (or others) that $\rho < 0$ is a good way to model toxic assets. Therefore, in what follows we maintain $\rho \geq 0$ and introduce other modeling ingredients, then use the tools developed above to derive results that better relate to modern financial markets, toxic assets and crises.

3 Information and Verification

An information friction is a reasonable modeling ingredient since an oft-mentioned feature of troubled financial assets is an opacity of quality. As reported in the *Liber8* (sic) *Economic Information Newsletter* (FRB St Louis, March 2009): "The TARP [troubled asset relief program] was originally conceived to purchase troubled assets directly from banks. However, as quickly became apparent, properly valuing these assets was extremely difficult." Following Lester et al. (2012), a tractable way to capture this is to let agents produce worthless assets at no cost and make others pay δ to verify quality (Appendix A suggests an alternative interpretation). In particular, they must pay δ to check, and hence to accept, a in the DM.[11]

3.1 One Asset

For now assume DM buyers make take-it-or-leave-it offers, consistent with Nash or Kalai bargaining at $\theta = 1$. This is not a big restriction: while a general $v(q)$ is preferable in Proposition 1 when the goal is to show ESE is *impossible* with $\rho \geq 0$ in that environment, $\theta = 1$ suffices when the goal is to show ESE is *possible* here. Now with a verification cost depending on the number of assets accepted, each unit of a that a DM buyer gives a seller is worth only $\phi+\rho-\delta$ to the latter, so bargaining

[11] We assume there is no such cost in the CM. One story is that CM trading goes through an exchange where experts certify asset quality so individuals can trade with impunity. Or, perhaps agents in the CM trade a to specialists who easily recognize quality while random DM counterparties do not. In fact, note different agents acquire a in the CM and DM: in the former it is buyers and in the latter it is sellers. All we need here is that buyers recognize a costlessly while sellers do not, although it gets more subtle in Section 4. Also, as in Lester et al. (2012), signaling or screening cannot avoid verifying quality in each trade because buyers can produce low quality assets on the spot; in contrast, in a related environment Li et al. (2012) have agents commit to quality before meeting counterparties, which is more complicated.

with $\theta = 1$ implies $(\phi + \rho - \delta)\, a' = c(q)$ (payments just covers production-plus-verification costs). Of course this is only valid if $\phi + \rho - \delta > 0$; otherwise there is no trade. Let $\tilde{q} \in (0, q^*)$ solve $u'(\tilde{q})(\phi + \rho - \delta) = c'(\tilde{q})(\phi + \rho)$, which exists if δ is not too big. Also, note that the lowest possible price of a is $\phi^F = \rho/r$, and assume $\delta > (1 + r)\rho/r$, so that $\phi = \phi^F$ implies there is no DM trade, which holds as long as ρ is not too big.

Letting $z = (\phi + \rho - \delta)\, a$, we have: no trade if $z \leq 0$; trade at $q = \tilde{q}$ if $z \geq c(\tilde{q})$; and trade at $q = c^{-1}(z)$ if $0 < z < c(\tilde{q})$. Thus $z < 0$ or $z > c(\tilde{q})$ implies $\phi_t = \beta\left(\phi_{t+1} + \rho\right)$, while $0 < z < c(\tilde{q})$ implies $\partial q / \partial a = (\phi + \rho - \delta) / c'(q)$ and

$$\phi_t = \beta\left[1 + \alpha\lambda\left(q_{t+1}\right)\right]\left(\phi_{t+1} + \rho - \delta\right) + \beta(1 - \alpha)\,\delta.$$

Emulating the methods developed above we arrive at

$$z_t = F\left(z_{t+1}\right) \equiv \beta z_{t+1}\left[1 + \alpha\tilde{L}\left(z_{t+1}\right)\right] + \beta(1 - \alpha) + \rho - \delta, \tag{13}$$

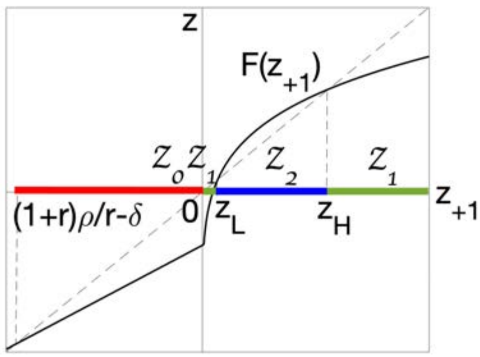
where after routine algebra the analog to L is given by $\tilde{L}$:[12]

$$\tilde{L}(z) \equiv \begin{cases} \lambda \circ c^{-1}(z) & \text{if } 0 < z < c \circ \tilde{q}(z) \\ \delta / z & \text{if } z < 0 \text{ or } z > c \circ \tilde{q}(z) \end{cases} \tag{14}$$

Figs 3 shows two cases, each with two positive steady states. Notice $F(0) < 0$, similar to Fig. 2, although the reason is different: in Section 2 there is a cost $-\rho$ to holding the asset; now there is a cost to transacting with it. Also notice the domain of F is $[\rho - \delta, \infty)$, but $z < 0$ does not imply $F(z)$ is below the 45^o line, as it crosses it at $(1 + r)\rho/r - \delta < 0$. Hence we keep $\mathcal{Z}_1$ and $\mathcal{Z}_2$ as above but redefine $\mathcal{Z}_0 = \{z | (1 + r)\rho/r - \delta \leq z < 0\}$. In particular, $z \in \mathcal{Z}_0$ not only guarantees $\phi \geq 0$, it also guarantees $\phi \geq \phi^F$. Also, using similar logic to Lemma 4, we have:

[12] Notice that in the lower branch of (14) $\tilde{L}(z) = \delta/z > 0$, different from Section 2 where in the lower branch $L(z) = 0$. The idea is that a marginal unit of a relaxes the liquidity constraint when it is binding, in the upper branch, at a cost δ, and does not relax the constraint when it is slack, in the lower branch, but it still costs δ to use it.

Lemma 5: With $\delta > 0$, deterministic equilibria can have hot and cold spells but still not recurrent freezes and thaws.



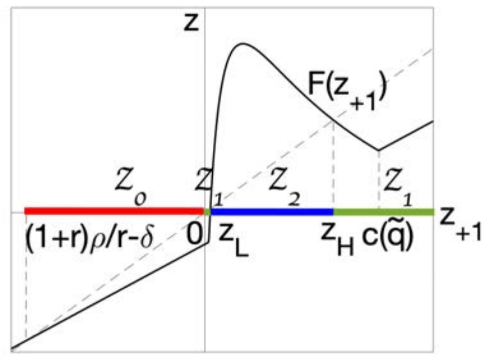


Fig. 3a: Verification cost, monotone   Fig. 3b: Verification cost, nonmonotone

Moving to sunspot equilibria, assuming δ is not too big, generically we have an even number of positive steady states, and hence some where F crosses the 45^o line from below, like z_L in Fig. 3. Condition B says any $z_1 \in \mathcal{Z}_0 \cup \mathcal{Z}_1$ and $z_2 \in \mathcal{Z}_2$ with $z_2 > z_1$ constitutes a sunspot equilibrium, where $z_1 \in \mathcal{Z}_1$ means it is an ISE and $z_1 \in \mathcal{Z}_0$ means it is an ESE.

Proposition 3: With $\rho \geq 0$ and $\delta > 0$, positive steady states exist if δ is not too big and generically come in pairs (z_L, z_H), with $z_H > z_L > 0$ and $F'(z_H) < 1 < F'(z_L)$. If $F'(z_H) < -1$ there exist ISE around z_H for some (z_1, z_2) in the neighborhood of z_H, and there exist ISE around z_L for any $z_1 \in \mathcal{Z}_1$ and $z_2 \in \mathcal{Z}_2$. There exist ESE around z_L for any $z_1 \in \mathcal{Z}_0$ and $z_2 \in \mathcal{Z}_2$.

The last part of Proposition 3 delivers the heretofore elusive ESE with $\rho \geq 0$. Intuitively, in the thaw state agents know there is always a probability of freezing, but as long as the probability is not too big a is traded in the DM, and the liquidity-enhanced demand for it keeps the CM price ϕ high enough to cover the cost δ of confirming asset quality. Then in the freeze state, as long as the probability of a thaw is not too big, a is not used in the DM, keeping ϕ low and preserving the frozen market.

3.2 Multiple Assets

Suppose there are two instruments that can facilitate DM exchange: a real asset a with dividend ρ and verification cost δ; and fiat money m with no such cost or dividend. This allows us to ask how m affects the possibility and properties of a freeze in a, and how that depends on monetary policy. For this exercise we assume both m and a are accepted by all DM sellers (but see Section 4), although of course they prefer m due to a's verification cost.[13]

With two assets the CM problem is

$$W(a,m) = \max_{x,\ell,\hat{a},\hat{m}} \{U(x) - \ell + \beta V_{+1}(\hat{a},\hat{m})\} \tag{15}$$

$$\text{st } x = \ell + (\phi^a + \rho)\, a + \phi^m m - \phi^a \hat{a} - \phi^m \hat{m} - T, \tag{16}$$

where ϕ^a and ϕ^m are the prices of a and m. The FOC's for an interior solution $(\hat{a},\hat{m})$ are $\phi^a = \beta\partial V_{+1}(\hat{a},\hat{m})/\partial\hat{a}$ and $\phi^m = \beta\partial V_{+1}(\hat{a},\hat{m})/\partial\hat{m}$. Generalization Lemma 1, $(\hat{a},\hat{m})$ is independent of (a,m) and W is linear.

With more payment options it is important to look into DM transactions in more detail. Buyers' take-it-or-leave-it offer can be described as choosing q plus payments m' and a' to maximize their surplus subject to covering sellers' costs:

$$\max_{a'\leq a, m'\leq m, q} [u(q) - (\phi^a + \rho)\, a' - \phi^m m'] \text{ st } c(q) = (\phi^a + \rho - \delta)\, a' + \phi^m m'.$$

There are four cases. Case (i) $\phi^a + \rho - \delta < 0$ implies the seller gets negative value from accepting a, so it is not used in the transaction. This means a is valued fundamentally, $\partial V/\partial a = \phi^a + \rho$, while the value of m is $\partial V/\partial m = [1 + \alpha L(\phi^m m)]\, \phi^m$. Case (ii) $\phi^a + \rho - \delta > 0$ and $\phi^m m > c(\tilde{q})$ implies the buyer can get more than $\tilde{q}$ using only m, which again means a is not used and m is used to purchase up to q^*, so $\partial V/\partial a$ and $\partial V/\partial m$ are the same as case (i).

[13] While it is assumed that m has no recognizability problem, that is not the only possibility, and it may be worth allowing counterfeit money, perhaps broadly interpreted to include bad checks. Relatedly, there are other issues with paper payment instruments not incorporated in the model, like the fact that they are hard to use for transactions at a distance, including internet trade. In principle some of these problems are solved by either private or public e-money, e.g. Bitcoin or Central Bank Digital Currency.

Case (iii) $\phi^a + \rho - \delta > 0$, $\phi^m m < c(\tilde{q})$ and $(\phi^a + \rho - \delta) a + \phi^m m > c(\tilde{q})$ implies the buyer spends all his m and enough a to get $\tilde{q}$, which means a is valued fundamentally and

$$\partial V / \partial m = \left(1 + \frac{\alpha\delta}{\phi^a + \rho - \delta}\right) \phi^m.$$

In this case notice the buyer does not acquire more than $\tilde{q}$ when he has higher m, but uses less a to pay for it and thus lowers transaction costs. Case (iv) $\phi^a + \rho - \delta > 0$ and $(\phi^a + \rho - \delta) a + \phi^m m < c(\tilde{q})$ implies the buyer uses all his m and a but is still is constrained, which means

$$\begin{aligned}
\partial V / \partial a &= \{1 + \alpha L [\phi^m m + (\phi^a + \rho - \delta) a]\} (\phi^a + \rho - \delta) + (1 - \alpha) \delta \\
\partial V / \partial m &= \{1 + \alpha L [\phi^m m + (\phi^a + \rho - \delta) a]\} \phi^m.
\end{aligned}$$

As always, the Euler equations follow from substituting $\partial V / \partial a$ and $\partial V / \partial m$ into the FOC's. To proceed, define $z^a = (\rho + \phi^a - \delta) a$, $z^m = \phi^m m$, and

$$\tilde{L}(z_t^m + z_t^a) = \begin{cases} \lambda \circ c^{-1}(z_t^m + z_t^a) & \text{if } 0 < z_t^m + z_t^a < c \circ \tilde{q}(z_t^a) \\ \delta / z_t^a & z_t^m + z_t^a < 0 \text{ or } z_t^m + z_t^a > c \circ \tilde{q}(z_t^a) \end{cases}$$

and write the dynamic system as

$$\begin{bmatrix} z_t^a \\ z_t^m \end{bmatrix} = \begin{bmatrix} F\left(z_{t+1}^a, z_{t+1}^m\right) \\ G\left(z_{t+1}^a, z_{t+1}^m\right) \end{bmatrix}, \tag{17}$$

where

$$F(z_t^a, z_t^m) = \beta z_t^a \left[1 + \alpha \tilde{L}(z_t^m + z_t^a)\right] + \beta (1 - \alpha) \delta + \rho - \delta$$

and

$$G(z_t^a, z_t^m) = \begin{cases} \dfrac{\beta z_t^m}{1 + \mu} [1 + \alpha L(z_t^m)] & \text{if } z_t^a > 0 \text{ and } z_t^m > c \circ \tilde{q}(z_t^a), \text{ or } z_t^a < 0 \\ \dfrac{\beta z_t^m}{1 + \mu} \left(1 + \dfrac{\alpha\delta}{z^a}\right) & \text{if } z_t^a > 0 \text{ and } z_t^m < c \circ \tilde{q}(z_t^a) < z_t^m + z_t^a \\ \dfrac{\beta z^m}{1 + \mu} [1 + \alpha L(z_t^m + z_t^a)] & \text{if } z_t^a > 0 \text{ and } z_t^m + z_t^a < c \circ \tilde{q}(z_t^a) \end{cases}.$$

Of course G has multiple branches because there are multiple cases as described above. The key economic point is that, since paying with a is costly, it is never used unless the buyer cashes out before getting $\tilde{q}$.

Here we focus on the case where both a and m are used for DM payments. To first examine steady state, note that (17) implies

$$L\left(z_S^m + z_S^a\right) = \frac{i}{\alpha} \text{ and } z_S^a = \frac{(\alpha + r)\,\delta - (1+r)\,\rho}{i - r}. \tag{18}$$

For a to be valued we need $i > r$ (inflation must be positive so m does not dominate a). For m to be valued we need $z_S^m = L^{-1}(i/\alpha) - z_S^a > 0$ (the liquidity provided by a does not render m useless). Together we need

$$\alpha L\left[\frac{(\alpha + r)\,\delta - (1+r)\,\rho}{i - r}\right] > i > r. \tag{19}$$

If $\alpha L(0) > r$ and δ as well as ρ are small, the set of i's satisfying (19) is nonempty.

An immediate but important result is that the liquidity provided by a does not affect total liquidity, which is pinned down by $\alpha L(z_S^a + z_S^m) = i$. Hence, there is *complete crowding out* of z_S^m when z_S^a increases. This makes it desirable to ban the use of a in the DM: if possible, regulation should discourage the use of opaque assets for payments when cash is an option, because cash saves on the cost δ and in principle can provide sufficient liquidity. Of course m is assumed here to not have its own set of issues, as discussed in fn. 13, but to the extent that central banks can provide safe, convenient payment instruments it is desire to use those as much as possible. Intuitively, the case for regulation can be understood in terms of a coordination failure: agents use a for DM trade because the value of m is low; and the value of m is low because agents use a for DM trade.

Next consider ESE, where the DM acceptance of a starts and stops, but when a freezes the DM still operates using m. With a Markov process as above, and a frozen in state 1 but not 2, we have:

$$z_1^a = (1-\sigma_1)\,F\left(z_1^a, z_1^m\right) + \sigma_1 F\left(z_2^a, z_2^m\right) \tag{20}$$

$$z_2^a = \sigma_2 F\left(z_1^a, z_1^m\right) + (1-\sigma_2)\,F\left(z_2^a, z_2^m\right) \tag{21}$$

$$z_1^m = (1-\sigma_1)\,G\left(z_1^a, z_1^m\right) + \sigma_1 G\left(z_2^a, z_2^m\right) \tag{22}$$

$$z_2^m = \sigma_2 G\left(z_1^a, z_1^m\right) + (1-\sigma_2)\,G\left(z_2^a, z_2^m\right) \tag{23}$$

Given $(\sigma_1, \sigma_2) \in (0,1)^2$ an ESE is a solution $(z_1^a, z_2^a, z_1^m, z_2^m)$ to (20)-(23) satisfying

$$(1+r)\,\rho/r - \delta < z_1^a < 0 < z_2^a,\ z_1^m > 0, \text{ and } z_2^m > 0.$$

To explain the conditions, first, having a traded in state 2 but not state 1 requires $z_1^a < 0 < z_2^a$. Then in any equilibrium the price of a is at least $\phi^F = (1+r)\,\rho/r$, which implies z_1^a is at least $(1+r)\,\rho/r - \delta$. Then having m valued and traded both states requires $z_1^m, z_2^m > 0$. These properties are obvious. After further investigating the equilibrium conditions we obtain the following additional properties, proved in Appendix C:

Lemma 6: In any ESE, $z_2^m < z_1^m < z_S^a + z_S^m < z_2^a + z_2^m < c \circ \tilde{q}\,(z_2^a)$, $z_2^a \in \mathcal{Z}_2$, and $z_2^a > z_S^a$.

This result gives properties of ESE assuming it exists. Intuitively, since a and m are both used in state 2, $z_2^a + z_2^m < c(\tilde{q})$. Total liquidity fluctuates around the steady state, with z^a above z_S^a in state 2 and below 0 in state 1. Money is less valuable in state 2 when the asset is used. Finally, as in the univariate case, we need $z_2^a \in \mathcal{Z}_2$. These properties provide guidance on how to construct ESE, and some of the properties are also sufficient for an ESE.

To proceed, we pick (z_1^a, z_2^a), solve for $(z_1^m, z_2^m, \sigma_1, \sigma_2)$ from (20)-(23), then check $(z_1^m, z_2^m) \in R_+^2$ and $(\sigma_1, \sigma_2) \in (0,1)^2$. Using the definitions of the $\mathcal{Z}$'s in Section 3.1, $(1+r)\,\rho/r - \delta < z_1^a < 0$ implies $z_1^a \in \mathcal{Z}_0$. By Lemma 6, we must pick $z_2^a \in \mathcal{Z}_2$ and $z_2^a > z_S^a$ because these are necessary for ESE. It turns out these conditions are also sufficient: for any (z_1^a, z_2^a) such that $z_1^a \in \mathcal{Z}_0$, $z_2^a \in \mathcal{Z}_2$ and $z_2^a > z_S^a$, we can find $(z_1^m, z_2^m) \in R_+^2$ and $(\sigma_1, \sigma_2) \in (0,1)^2$ consistent with ESE. This is provided in Appendix C:

Proposition 4: With a real asset a and money m, $\rho \geq 0$ and cost $\delta > 0$ only on a, if i satisfies (19) and $\delta > (1+r)\,\rho/r$, there exists a unique monetary steady state described by (18). There exist ESE where $z_1^m, z_2^m > 0$ for any (z_1^a, z_2^a) such that $z_1^a \in \mathcal{Z}_0$, $z_2^a \in \mathcal{Z}_2$ and $z_2^a > z_S^a$.

Although (17) a bivariate dynamic system, the ESE, or at least the part involving z^a, can be described using the univariate system from Section 3.1. The existence of ESE requires $\widetilde{\mathcal{Z}}_2 \equiv \{z_2^a | z_2^a \in \mathcal{Z}_2 \,\&\, z_2^a > z_S^a\} \neq \varnothing$, which can be reduced to (19). Therefore, the conditions for the existence of a steady state where m and a are both used are identical to the conditions for the existence of ESE. Based on that one might say the possibility of ESE seems fairly robust and empirically plausible.

Now a freeze in a does not shut down the DM, but can still be bad for output and welfare. Can monetary policy help? Consider the condition $z_S^a < z_H$ that is necessary for $\widetilde{\mathcal{Z}}_2 \neq \varnothing$. It can be written $i \geq \hat{\imath} \equiv r + [(\alpha + r)\,\delta - (1+r)\,\rho]\,/z_H^a$, implying freezes cannot occur if $i < \hat{\imath}$, i.e., if a central bank engineers a low nominal interest (equivalently inflation) rate. The intuition is straightforward: high i makes the value of m low, opening the door for a to be valued for liquidity and making freezes possible; low i drives a out of DM use, precluding freezes plus saving on verification cost. While this is a strong result, one should bear in mind something mentioned earlier: in reality m may have properties not in the model that make it less than perfect. That doesn't mean there isn't a grain of truth here.

For comparison, consider two real assets a and b, where a has a verification cost while b does not, and without loss of generality the supply of each is set to 1 by adjusting ρ^a and ρ^b. So that b does not dominate a, assume b depreciates, indeed assume it depreciates fully after one period. Thus, b is a one-period asset, or equivalently a one-period bond, issued in the CM at price ϕ^b with payout ρ^b in the next CM. To maintain stationarity, a new vintage is issued every period, and without affecting anything interesting suppose the government is endowed with the new assets or issues the new bonds. The key point is that one-period assets or bonds are convenient because we peg their DM value at $z^b = \rho^b$, putting us back in a univariate dynamic system in z^a, and thus avoiding the major complications in Proposition 4.

To make things interesting, assume b does not satiate liquidity needs. Then

$$z_t^a = F\left(z_{t+1}^a\right) \equiv \beta z_{t+1}^a \left[1 + \alpha \tilde{L}\left(z_{t+1}^a + z_{t+1}^b\right)\right] + \beta(1-\alpha)\delta + \rho^a - \delta,$$

which generically has an even number of steady states with $\phi^a > 0$ if δ is not too big. Notice $F(\cdot)$ includes z^b in $\tilde{L}\left(z^a + z^b\right)$, so z^b affects z^a even if z^a does not affect z^b. In particular, since b does not satiate liquidity needs, there is a benefit to having a used in the DM. This is in stark contrast to the case with a and m, and can be understood by noting that a real asset can be scarce while fiat currency cannot.[14] While a can contribute to total liquidity, it introduces the possibility of a freeze, so it may or may not be desirable on net to regulate its use. In any case, defining $\mathcal{Z}_j$ as in Section 3.1, we have this:

Proposition 5: With real assets a and b, $\rho^a \geq 0$, $\rho^b \geq 0$ and cost δ only on a, positive steady states exist if δ is not too big and generically come in pairs (z_L^a, z_H^a) with $z_H^a > z_L^a > 0$ and $F'(z_H^a) < 1 < F'(z_L^a)$. There exist ESE for any $z_1^a \in \mathcal{Z}_0$ and $z_2^a \in \mathcal{Z}_2$.

3.3 Discussion

We expect readers will find these results more satisfying than those in Section 2 because $\delta > 0$ is more palatable than $\rho < 0$, and because the notion of assets troubled due to opacity is compelling. As Razin (2014) puts it: "A major friction in the operation of financial markets is the presence of asymmetric information... If sellers have private information about the quality of the assets, buyers will be reluctant to buy the asset from them because they realize that the sale represents negative information about the asset. In extreme situations, when the only motivation to trade is based on information, this leads to a market freeze." This is standard lemon logic, but we prefer to let those acquiring assets get the requisite information at some cost. When we do, freezes can still occur.

[14] In case that is not clear, note that ϕ^m can adjust to make real balances big for any nominal supply M, while ϕ^b cannot be adjusted arbitrarily because the dividend/price ratio ρ^b/ϕ^b matters.

In terms of implications, in an economy with a and m, the message is that regulating the use of opaque assets for payment purposes is desirable, and, short of that, low i makes a freeze less likely. In an economy with a and b, the message is less clear since the liquidity provided by a is useful when the b is scarce. This may be the empirically relevant case, as people (e.g., Andolfatto and Williamson 2015; Caballero et al. 2017; Gorton and Ordonez 2021) suggest the supply of safe liquid assets may be tight, and in this context safe means transparent, not low risk. While the scarcity of such assets is an empirical issue, theory elucidates two problems arising from the use of opaque substitutes: resources are spent checking quality; and the likelihood of a freeze emerges.

ESE here is reminiscent of the description of the crisis in the Introduction (from Chiu and Koeppl 2016): "markets with centralized trading functioned rather well" but OTC trading "came to a halt." We say this because during a DM freeze it is business as usual in the CM in the sense that there is no disruption in trading x or ℓ.[15] Still, notice that during a DM freeze asset trading in the CM stops: when a is not being traded in the DM there is no need to reallocate it in the CM to get agents back to their preferred positions: buyers simply hold it and hold out for a thaw. This is relevant because we recall financial market freezes involve "large declines in the volume of transactions in both the primary and the secondary markets" (from Benmelech and Bergman 2012), and we think primary and secondary markets correspond well to our CM and DM.

In general, one might describe our freezes and thaws as crises and normal times. Yet one might instead describe freezes as normal times and dub thaws periods of "exuberance" since the liquidity premium means the asset price satisfies standard definitions of a bubble $\phi > \phi^F$. But this is not Greenspan's "irrational exuberance" because optimism is a self-fulfilling prophecy. Indeed, even more optimism can be

[15] It is worth mentioning versions of the model where q is capital that gets traded in the DM among firms realizing idiosyncratic productivity shocks (see Appendix A). Since capital is used in CM production, a DM freeze exacerbates misallocation, which affects consumption and employment even if x and ℓ are still easily traded.

consistent with equilibrium, as in the higher steady state z_H, and even beyond that in some periods in an ISE around z_H.[16]

4 Genuine OTC Markets

To model indirect asset liquidity, first assume m is needed for DM trade, say because the verification cost of a is too high for sellers while the verification cost for m is 0. Then add an OTC market between the CM and DM where buyers might trade a and m among themselves, for the following reason: after bringing (a, m) out of the CM they learn whether they will have in the next DM a trade opportunity – i.e., whether they will meet a seller, although it can also include whether they will have a use/need for q. This resembles actual OTC trade, with agents selling a to get m when they desire liquidity.[17]

Here we impose verification cost δ in the OTC market, where α is now the measure of buyers that learn they will have a DM trade opportunity, and $1 - \alpha$ the measure that learn they will not. Then the CM problem is

$$\begin{aligned} W(a,m) &= \max_{x,\ell,\hat{a},\hat{m}} \{U(x) - \ell + \beta [\alpha J_1(\hat{a},\hat{m}) + (1-\alpha) J_0(\hat{a},\hat{m})]\} \\ \text{st } x &= \ell + (\phi^a + \rho) a + \phi^m m - \phi^a \hat{a} - \phi^m \hat{m} - T \end{aligned}$$

where J_1 and J_0 are the OTC value functions of those that will have a DM trade opportunity and those that will not, respectively. The former want to trade a for m in this market, with the latter willing to take the other side.

Suppose those selling a for m in the OTC market always meet a counterparty, while those on the other side may or may not, assuming $\alpha < 1/2$. Also, suppose

[16] Since we just mentioned ISE, a little more can be said even if ESE is the main focus. The ISE results in Propositions 1 and 3 are interesting especially when they use Condition B. Recall Condition A concerns (z_1, z_2) in a (perhaps small) neighborhood of z_H, while Condition B applies to any $z_1 \in \mathcal{Z}_1$ and $z_2 \in \mathcal{Z}_2$. Relatedly, given two steady states $z_H > z_L > 0$, one must choose parameters carefully to get $F'(z_H) < -1$ and use Condition A, while $F'(z_L) > 0$ and Condition B hold for any parameters. Also, although in general having two assets complicates things, in the version with a and b ISE exist under the same conditions as Proposition 3.

[17] This setup is similar to He et al. (2015), Geromichalos and Herrenbrueck (2016, 2017), Mattesini and Nosal (2016) and Lagos and Zhang (2020), but they do not consider freezes.

those selling a for m, and later buying q for m, make take-it-or-leave-it offers in both markets. These assumptions are special, but again the goal is to show freezes can happen for some parameters not all, and, moreover, continuity implies similar results hold if the relevant meeting rates and bargaining powers are not too far from 1. With these assumptions,

$$J_1(a,m) = \max_{a',m'} \{u(q) - c(q) + W(a - a', m + m')\}$$

where a' is the asset sold, m' is the cash acquired, $c(q) = \phi^m(m + m')$, and $(\phi^a + \rho - \delta)a' = \phi^m m'$. Also, $J_0(a,m) = W(a,m)$ since agents receiving take-it-or-leave-it offers get no surplus.

OTC trade is constrained by $a' \leq a$ and $m' \leq \bar{m}$ where a is for the asset seller and $\bar{m}$ is for asset buyer. Hence, the maximum trade is $\min\{z^a, \bar{z}^m\}$, where $\bar{z}^m = \phi^m\bar{m}$. Letting $z^{m\prime} = \phi^m m'$ and $z^{a\prime} = (\phi^a + \rho - \delta)a'$, we have three cases:

(i) If $z^m > c(\tilde{q})$ then $z^{m\prime} = z^{a\prime} = 0$.

(ii) If $z^m < c(\tilde{q}) < z^m + \min\{z^a, \bar{z}^m\}$ then $z^{m\prime} = z^{a\prime} = c(\tilde{q}) - z^m$.

(iii) If $z^m + \min\{z^a, \bar{z}^m\} < c(\tilde{q})$ then $z^{m\prime} = z^{a\prime} = \min\{\bar{z}^m, z^a\}$.

As before, in the DM buyers do not purchase more than $\tilde{q}$. In case (i), their m coming out of the CM is sufficient to get $\tilde{q}$, so there is no need for OTC trade. In case (ii), their m coming out of the CM is not sufficient to get $\tilde{q}$, but after OTC trade it is. In case (iii), even after OTC trade they cannot get $\tilde{q}$, which implies that $a' \leq a$ or $m' \leq \bar{m}$ must bind.

If $z^a < \bar{z}^m$ the Euler equations for $\hat{m}$ and $\hat{a}$ are identical to the ones in Section 3.2. Hence results for the earlier model apply directly. Hence we have the following, with details supplied in Appendix C:

Proposition 6: With a genuine OTC market for a and m, $\rho \geq 0$ and an OTC verification cost δ on a, if i satisfies (19), $\delta > (1+r)\rho/r$, and ρ, δ are not too big, there exists a unique monetary steady state described by (18), and there exist ESE where $z_1^m, z_2^m > 0$ for any (z_1^a, z_2^a) such that $z_1^a \in \mathcal{Z}_0$, $z_2^a \in \mathcal{Z}_2 \cap (z_S^a, (z_S^a + z_S^m)/2)$.

4.1 More Discussion

ESE here is even more like the crisis where "markets with centralized trading functioned" but OTC trade "came to a halt." We say this because during an OTC freeze it is business as usual in the CM, in the sense used above, and almost business as usual in the DM, in the sense that trading m for q is not affected directly. Yet the DM can be affected indirectly by the OTC breakdown. Typically, the option to swap m for a before the DM convenes raises CM money demand, as agents know they can off-load idle cash when they learn they do not need it.[18] Thus OTC trade can increase z_m and DM trade. However, when agents trading a for m bargain with $\theta = 1$ this effect is inoperative; still, as we said above, similar results when $\theta < 1$, where it is operative, as long as θ is not too small.

We also suggest that the inability to liquidate a in a freeze is consistent with calling it a toxic asset "whose value has fallen significantly and for which there is no longer a functioning market." And at the root of the problem is opacity. Further, without working out the details, it is easy to imagine two OTC markets with agents selling a_1 for m in DM_1 and selling a_2 for m in DM_2. As should be apparent, one may freeze while the other does not, due entirely to luck.

5 Extensions

In what follows we revert to the case of one asset, with $\rho \geq 0$ and $\delta = 0$, and let it trade directly for q, then introduce a few new modeling ingredients.

5.1 Endogenous Entry

Suppose sellers must pay $\kappa > 0$ to enter the DM. As mentioned in the Introduction, participation costs are standard in search theory, where there is typically a CRS matching technology $\Upsilon(n_b, n_s)$ generating, in our context, the probability a buyer

[18] This insight is similar Berentsen et al. (2008), even if they use banking instead of OTC trade (see Appendix D), and do not consider stochastic equilibria. If one did consider such equilibria in their setting, ISE or ESE might resemble partial or total bank runs.

meets a seller in the DM, $\alpha(\tau) = \Upsilon(1, 1/\tau)$, and the probability a seller meets a buyer in the DM, $\alpha(\tau)\tau = \Upsilon(\tau, 1)$, with $\tau = n_b/n_s$ denoting market tightness.[19] Given this, except for $\alpha = \alpha(\tau)$ being endogenous, the analysis is the same as in the benchmark model and leads to

$$z_t = F(z_{t+1}) = \rho + \beta z_{t+1}\left[1 + \alpha(\tau_{t+1}) L(z_{t+1})\right]. \tag{24}$$

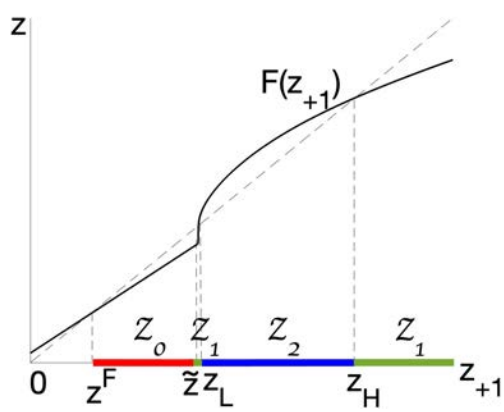


Fig. 4a: Fixed entry cost

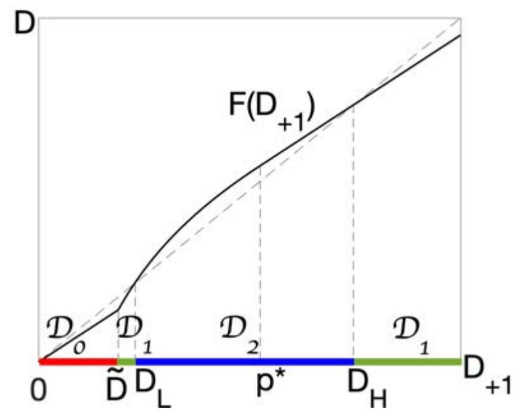


Fig. 4b: Fixed credit cost

A seller's DM trade surplus is $S(\bar{z}) = \min\{p^*, \bar{z}\} - c\left(\min\{q^*, v^{-1}(\bar{z})\}\right)$, where $\bar{z}$ is buyer liquidity. Sellers make entry decisions by comparing κ to the expected surplus, taking $\bar{z}$ and other sellers' actions as given. As usual, equilibria can entail entry by all, some or no sellers, depending on parameters. Given sellers' surplus is monotone in buyers' $\bar{z}$, we can write $\tau = \varsigma(\bar{z})$ and express (24) as

$$z_t = F(z_{t+1}) = \rho + \beta z_{t+1}\left[1 + \alpha \circ \varsigma(z_{t+1}) L(z_{t+1})\right].$$

See Fig. 4a for a case with one steady state at z^F, and two others, $z_H > z_L > z^F$.

With these results in hand, we can construct sunspot equilibrium using Condition B. Letting $\tilde{z}$ solve $S(\tilde{z}) = \kappa$ and redefining $\mathcal{Z}_0 = \left\{z | z^F \leq z < \tilde{z}\right\}$, $\mathcal{Z}_1 = \{z | z > F(z)\ \&\ z > \tilde{z}\}$ and $\mathcal{Z}_2 = \{z | F(z) > z\ \&\ z > \tilde{z}\}$, we summarize as follows:

[19] We adopt the usual assumptions in, e.g., Pissarides (2000): $\Upsilon(n_b, n_s)$ is strictly increasing and concave, while $\alpha(\tau)$ is strictly decreasing and $\alpha(\tau)\tau$ is strictly increasing in τ. Also $\lim_{\tau\to 0}\alpha(\tau) = \lim_{\tau\to\infty}\alpha(\tau)\tau = 1$ and $\lim_{\tau\to\infty}\alpha(\tau) = \lim_{\tau\to 0}\alpha(\tau)\tau = 0$.

Proposition 7: With $\rho \geq 0$ and entry cost κ, positive steady states above z^F exist if κ is not too big and generically come in pairs (z_L, z_H), with $z_H > z_L > 0$ and $F'(z_H) < 1 < F'(z_L)$. There exist ESE around z_L for any $z_1 \in \mathcal{Z}_0$ and $z_2 \in \mathcal{Z}_2$.

The proof is omitted as it is similar to other results. Here we emphasize the connection to the remarks on market infrastructure in the Introduction. A Proposition 7 freeze hinges on decentralized – i.e., bilateral – trade; it cannot occur if we replace the DM by a Walrasian market. In Walrasian markets with entry costs, if fewer sellers enter, those that do sell more by effectively serving more buyers, thus dissipating costs. In our DM a seller can serve at most one buyer per period, which is key to getting ESE. So, suppose there are two markets, one where a_1 trades for q_1 and one where a_2 trades for q_2, both with entry cost κ, but one is like our DM while the other is Walrasian. Only the former can freeze.

5.2 Costly Credit

The next extension concerns not freezes in asset markets, but credit markets, which are also said to be important (Bebchuk and Goldstein 2010; Benmelech and Bergman 2012). To apply our methods in this case, let all agents enter the DM for free, and assume the only payment instrument is unsecured debt, but to use it buyers must pay cost κ before leaving the CM.[20] Also, without loss of generality, any DM debt d is assumed to to be settled in the next CM. This leads to

$$W(d) = \max_{x,\ell} \{U(x) - \ell + \beta V_{+1}\} \text{ st } x + d = \ell. \tag{25}$$

For buyers not paying κ, $V = W(0)$. For those paying κ, they can get q on credit, but there is a debt limit $d \leq D$ that will be endogenized shortly. For these buyers

$$V = -\kappa + \alpha[u(q) + W(d)] + (1-\alpha)W(0) = \alpha S(D) + W(0),$$

[20]This is like having to pay credit card fees before shopping; paying κ after meeting a DM seller can also be considered. Also, with credit the only means of payment, a cost to using it is the same as a DM entery cost on buyers, but in Section 5.3 liquid assets are brought back in, and buyers can trade those for q without paying κ.

where

$$S(D) = \begin{cases} u \circ v^{-1}(D) - D & \text{if } D < p^* \\ u(q^*) - p^* & \text{otherwise.} \end{cases}$$

Buyers choose to pay κ iff $\alpha S(D) \geq \kappa$.

As in the literature following Kehoe and Levine (1983), if a debtor reneges he gets no credit in the future, which means no DM trade. Normalizing $U(x^*) - x^*$ to 0, where $U'(x^*) = 1$, the punishment payoff is 0, and we get the endogenous debt limit $d \leq D \equiv W(0)$. Now use that to rewrite (25) using D instead of $W(d)$:

$$D_t = F(D_{t+1}) \equiv \begin{cases} \beta D_{t+1} & \text{if } \alpha S(D_{t+1}) < \kappa \\ \beta[-\kappa + \alpha S(D_{t+1}) + D_{t+1}] & \text{otherwise.} \end{cases}$$

This gives the debt limit at t as a function of the limit at $t+1$, as in Alvarez and Jermann (2000) or Gu et al. (2013). Steady state solves $D_t = F(D_{t+1})$.

First observe $F' > 0$, so deterministic cycles are impossible here. Then for stochastic outcomes, let $\tilde{D}$ solve $\alpha S(\tilde{D}) = \kappa$, and note that if κ is small there are two positive steady states, D_L and D_H, as shown in Fig. 4b, where $\mathcal{D}_0 = \{D|0 \leq D < \tilde{D}\}$, $\mathcal{D}_1 = \{D|D > F(z) \,\&\, D > \tilde{D}\}$ and $\mathcal{D}_2 = \{D|F(D) > D \,\&\, D > \tilde{D}\}$.

Proposition 8 With a fixed cost κ of credit, positive steady states exist if κ is not too big and generically come in pairs (D_L, D_H), with $0 < F'(D_H) < 1 < F'(D_L)$. There exist ESE around D_L for any $D_1 \in \mathcal{D}_0$ and $D_2 \in \mathcal{D}_2$.

The omitted proof is similar to earlier results. The novelty concerns breakdowns in unsecured lending with the endogenous debt limit dropping to 0. Recall from fn. 2 "An important aspect of the economic crisis of 2008-2009 has been the 'freezing' of credit... Some observers have attributed the reluctance of financial firms to lend to irrational fear, while others have attributed it to a rational assessment of the fundamentals of the economy which can be expected to make it difficult for operating firms to repay extended loans." A Proposition 8 freeze is not due to fundamentals, since with those constant freezes may or may not occur. One can say it is due to fears, but they are by no means irrational: if a loan were extended during a freeze, off the equilibrium path, debtors' best response is default.

5.3 Money and Credit

In Section 5.2 buyers get credit directly from sellers, like they trade a directly for q in the baseline model. As in Section 4, where a trades for m then m buys q, we can have agents get m on credit then use m to buy q. That model is relegated to Appendix D, and can be described as having money and credit complements. Here we let them be substitutes: buyers can always use m to get q, and can use credit iff they pay κ. Appendix D shows the results are similar.

In this version the CM problem is

$$W(m,d) = \max_{\hat{m}} \{U(x) - \ell + \beta V_{+1}(\hat{m})\} \text{ st } x + d + \phi\hat{m} = \ell + \phi m - T.$$

where $V(m) = \alpha S(\phi m) + W(m,0)$ if buyers do not pay κ, and $V(m) = -\kappa + \alpha S(D + \phi m) + W(m,0)$ if they do. Now we assume debtors that renege are excluded from all future trade.[21] That makes the debt limit $d \leq D \equiv W(0,0)$. Emulating the above methods, and using the government budget $\phi(M_{+1} - M) + T = 0$, we get $D_t + z_t = F(D_{t+1}, z_{t+1})$, where

$$F(D_t, z_t) \equiv \begin{cases} \beta\alpha S(z_t) + \beta(D_t + z_t) & \text{if } \alpha[S(D_t + z_t) - S(z_t)] \leq \kappa \\ \beta[-\kappa + \alpha S(D_t + z_t)] + \beta(D_t + z_t) & \text{if } \alpha[S(D_t + z_t) - S(z_t)] > \kappa \end{cases}. \tag{26}$$

In addition, the Euler equation for z yields $z_t = G(D_{t+1}, z_{t+1})$, where

$$G(D_t, z_t) \equiv \begin{cases} \dfrac{\beta z_t}{1+\mu}[1 + \alpha L(z_t)] & \text{if } \alpha[S(D_t + z_t) - S(z_t)] \leq \kappa \\ \dfrac{\beta z_t}{1+\mu}[1 + \alpha L(D_t + z_t)] & \text{if } \alpha[S(D_t + z_t) - S(z_t)] > \kappa \end{cases}. \tag{27}$$

Equilibrium is characterized by (26)-(27).

Proposition 9 Assume $L(0) < \infty$. If $\beta[-\kappa + \alpha u(q^*) + (1-\alpha)p^*] > p^*$, there exist i and $(z_1, z_2) \in R_+^2$ such that any (D_1, D_2), $D_1 \in \mathcal{D}_0$, $D_2 \in \mathcal{D}_2$ and $D_2 > p^*$, constitute an ESE.

[21] This means CM and DM trade, which is effectively the same as Section 5.2, but we could proceed differently here – e.g., we can let defaulters participate in the CM which matters because that is where the monetary taxes/transfers occur, or let them trade in the DM using cash only. Those versions are similar but more complicated (see Gu et al. 2016 in a related context).

The proof is in Appendix C. Here we emphasize that the DM uses money and credit in a thaw and only money in a freeze, so credit freezes do not shut down trade for q but hinder it by forcing buyers to use cash. Also, related to earlier remarks, it is easy to imagine agents trade for q_1 in DM_1 and for q_2 in DM_2, and even if the markets have the same infrastructure there can be a freeze in one but not the other, which could be due fundamentals or pure luck. Moreover, DM_1 and DM_2 could be independent, or there could be contagion across credit markets, and modeling that explicitly may be interesting.

6 Conclusion

This paper studied models of decentralized exchange to see if recurrent market freezes can occur as self-fulfilling prophecies. The framework allowed combinations of real assets, currency, and credit, and nested applications where households trade with each other, households trade with firms, firms trade with each other, etc. It also allowed general mechanisms for price determination, and can be applied to assets serving as media of exchange or collateral. In a baseline model, with one asset having return ρ, there were freezes iff $\rho < 0$. In more relevant specifications, including one with information frictions, there were freezes with $\rho \geq 0$. Extensions to multiple assets generated novel implications for monetary policy and regulation. Another extension captured what we called genuine OTC markets. Another had entry costs, which showed how the bilateral nature of trade matters. Yet another featured credit market freezes. All of this taught us quite a lot about the possibility of a market freeze, and more generally about the nature of decentralized exchange.

Appendix A: Alternative Interpretations

Here we discuss ways $u(q)$ and $c(q)$ show up in the literature. Early papers (Lagos and Wright 2005 with fiat money; Geromichalos et al. 2007 with Lucas trees; Lagos and Rocheteau 2008 with capital) had households trading goods with each other. The gains from trade depend on who meets whom, given agents specialize in consumption and production. Then $u(q)$ is the utility of consuming q while $c(q)$ is the disutility of producing it. Later papers had agents called sellers that only produce in the DM and agents called buyers that only consume in the DM (Rocheteau and Wright 2005; Lagos and Rocheteau 2005). Some versions (Berentsen et al. 2011) interpret sellers as firms, or as retailers, but $c(q)$ is still a production cost.

Alternatively $c(q)$ can be an opportunity cost. Consider consumers endowed with Q units of the DM good. Before the DM opens there are preference shocks making utility either $\varepsilon_L \tilde{u}(\cdot)$ or $\varepsilon_H \tilde{u}(\cdot)$, with $\varepsilon_H > \varepsilon_L$ (this can be generalized). In a meeting between an agent with ε_L and one with ε_H, there are gains from trade where the former gives the latter q units of Q. This generates an opportunity cost $c(q) = \varepsilon_L[\tilde{u}(Q) - \tilde{u}(Q-q)]$ and benefit $u(q) = \varepsilon_H[\tilde{u}(Q+q) - \tilde{u}(Q)]$. Everything then is as in the baseline model, except there is a constraint $q \leq Q$, but it is slack if $\tilde{u}'(0) = \infty$. With, e.g., Kalai bargaining, we get the liquidity premium in terms of q explicitly as

$$\lambda(q) = \theta \frac{\varepsilon_H \tilde{u}'(Q+q) - \varepsilon_L \tilde{u}'(Q-q)}{\theta \tilde{u}'(Q-q) + (1-\theta)\tilde{u}'(Q+q)} = \theta \frac{u'(q) - c'(q)}{\theta c'(q) + (1-\theta) u'(q)}.$$

Other papers interpret agents as firms with capital K and a technology for producing x (e.g., Wright et al. 2020; Cui et al. 2021; Silveira and Wright 2010 is similar except firms trade ideas). The production function is $\varepsilon f(K)$, where ε is an idiosyncratic shock at the beginning of the DM. Then $c(q) = \varepsilon_L[f(K) - f(K-q)]$ and $u(q) = \varepsilon_H[f(K+q) - f(K)]$, and everything proceeds as usual, except for the constraint $q \leq K$, but it is slack if $f'(0) = \infty$. Firms can be owned by households with utility over (x, ℓ), but their payoff from profit is still measured by $c(q)$ and $u(q)$ since W is linear in numeraire. Rather than take K as an endowment, it can be an investment in the CM, where

$$x = \ell + (\rho + \phi)a - \phi\hat{a} - T + \varepsilon f(K) + (1-\delta)K - \hat{K}$$

and δ is depreciation. Now

$$\begin{aligned} u(q, K_H) &= \varepsilon_H [f(K_H + q) - f(K_H)] + (1-\delta) q \\ c(q, K_L) &= \varepsilon_L [f(K_L) - f(K_L - q)] + (1-\delta) q. \end{aligned}$$

In Rocheteau et al. (2018) firms in the DM seek funding from financial institutions. In other papers agents are investors in financial assets. Suppose there are two assets, (a_1, a_2). As in the baseline model a_1 has a fixed ρ_1, but a_2 has an individual-specific return, e.g., $\rho_2 = \varepsilon f(a_2)/a_2$ where ε is random. One interpretation is that ρ_2 is in terms of goods (the proverbial "fruit") and agents have taste shocks, as in many models of OTC markets (e.g., Lagos and Rocheteau's 2008 generalization of Duffie et al. 2005). Related papers use assets for settlement purposes where agents must use one asset to acquire another (Koeppl et al. 2008; Afonso and Lagos 2015; the papers in fn. 17). DM gains from trade emerge when agents with low and high valuation exchange q units of a_2 for p units of a_1. We now get (suppressing dependence of u and c on ϕ_2 to ease notation)

$$\begin{aligned} u(q, a_{H,2}) &= \varepsilon_H [f(a_{H,2} + q) - f(a_{H,2})] + \phi_2 q, \\ c(q, a_{L,2}) &= \varepsilon_L [f(a_{L.2}) - f(a_{L,2} - q)] + \phi_2 q, \end{aligned}$$

Finally, consider an alternative interpretation to the information story for δ: it is a cost to check a is not dangerous, as deemed relevant during the pandemic, which has seen a dramatic decline in the use of paper payment instruments due to fear over the virus. This is taken seriously by policy makers: "Some central banks are deploying measures to sterilize paper money with heat or UV light," while "The Federal Reserve began a seven to 10-day quarantine for United States dollars returning from Europe and Asia" (Washington Post 05/15/20). For a discussion of cash, bacteria and viruses, see https://www.scientificamerican.com/article/dirty-money/. On payments in the pandemic, see https://www.payments.ca/about-us/news/covid-19-pandemic-dramatically-shifts-canadians'-spending-habits. One interesting application may be to analyze this in detail.

Appendix B: Parameters for Examples

The functional form used in all numeric examples are $c(q) = q$ and

$$u(q) = B\frac{(q+\xi)^{1-\gamma} - \xi^{1-\gamma}}{(1-\gamma)}.$$

This DM utility function, which is common in related work, perturbs standard CRRA preferences by forcing $u(0) = 0$. In Fig. 4a we also need a matching technology, given by the urn-ball function $\Upsilon(n_b, n_s) = n_b\left(1 - e^{-n_s/n_b}\right)$. The parameters are given in Table 1.

Table 1: Parameters for the Examples

	B	ξ	γ	r	θ	ρ	α	δ	κ
Fig 1a	1	0.1	0.75	0.25	1	0.1	1		
Fig 1b	1	0.24	3	0.25	1	0.01	1		
Fig 2a	1	0.5	0.5	0.25	1	-0.15	1		
Fig 2b	1	0.25	2.7	0.25	1	-0.2	1		
Fig 3a	1.3	0	0.8	0.54	1	0.01	1	0.4	
Fig 3b	0.35	0.05	1.75	0.33	1	0.01	1	0.35	
Fig 4a	1.2	0	0.75	0.25	0.7	0.05			0.45
Fig 4b	5	0.5	2.5	0.35	1		1		6

Appendix C: Proofs

Lemma 6: In any ESE, $z_2^m < z_1^m < z_S^a + z_S^m < z_2^a + z_2^m < c \circ \tilde{q}(z_2^a)$, $z_2^a \in \mathcal{Z}_2$, and $z_2^a > z_S^a$.

Proof: First, to facilitate the arguments, we write (20)-(21) more explicitly as

$$\begin{aligned} (1+r) z_1^a &= (1+r)\rho - r\delta + (1-\sigma_1) z_1^a \\ &\quad +\sigma_1 \left\{ z_2^a \left[1 + \alpha \tilde{L}(z_2^m + z_2^a)\right] - \alpha\delta \right\} &(28)\\ (1+r) z_2^a &= (1+r)\rho - r\delta + \sigma_2 z_1^a \\ &\quad + (1-\sigma_2)\left\{ z_2^a \left[1 + \alpha \tilde{L}(z_2^m + z_2^a)\right] - \alpha\delta \right\} &(29) \end{aligned}$$

Then rewrite (28) as

$$\sigma_1 = \frac{(1+r)(z_1^a - \rho) + r\delta - z_1^a}{z_2^a\left[1 + \alpha\tilde{L}(z_2^a + z_2^m)\right] - \alpha\delta - z_1^a}, \tag{30}$$

and (29) as

$$\sigma_2 = \frac{z_2^a\left[1 + \alpha\tilde{L}(z_2^a + z_2^m)\right] - \alpha\delta - [(1+r)(z_2^a - \rho) + r\delta]}{z_2^a\left[1 + \alpha\tilde{L}(z_2^a + z_2^m)\right] - \alpha\delta - z_1^a}. \tag{31}$$

We now show $z_2^a + z_2^m < c \circ \tilde{q}(z_2^a)$. Suppose otherwise, then $\tilde{L}(z_2^a + z_2^m) = \delta / z_2^a$. The denominator of (30) and (31) is $z_2^a - z_1^a > 0$. Then $\sigma_1 > 0$ and $\sigma_2 > 0$ require

$$\begin{aligned}(1+r)(z_1^a - \rho) + r\delta - z_1^a &> 0 \\ (1+r)(z_2^a - \rho) + r\delta - z_2^a &< 0\end{aligned}$$

which cannot occur because $z_2^a > z_1^a$. This shows $z_2^a + z_2^m < c \circ \tilde{q}(z_2^a)$, which implies $\tilde{L} = L$. From this point on, we replace $\tilde{L}$ by L. From (30) and (31)

$$\sigma_1 + \sigma_2 = 1 + \frac{(1+r)(z_1^a - z_2^a)}{z_2^a [\alpha L(z_2^a + z_2^m) + 1] - \alpha\delta - z_1^a}. \tag{32}$$

As $z_2^a + z_2^m < c(\tilde{q})$ and L is decreasing, $z_2^a [\alpha L(z_2^a + z_2^m) + 1] > \alpha\delta + z_2^a$. The denominator is positive and the numerator is negative. So $\sigma_1 + \sigma_2 < 1$, and we only need to check $\sigma_1, \sigma_2 > 0$.

Since $z_2^a + z_2^m < c(\tilde{q})$, we can write (22)-(23) as

$$(1+i) z_1^m = (1-\sigma_1) z_1^m [1 + \alpha L(z_1^m)] + \sigma_1 z_2^m [1 + \alpha L(z_2^m + z_2^a)], \tag{33}$$

$$(1+i) z_2^m = \sigma_2 z_1^m [1 + \alpha L(z_1^m)] + (1-\sigma_2) z_2^m [1 + \alpha L(z_2^m + z_2^a)]. \tag{34}$$

From (33) and (34), we get

$$\sigma_1 = \frac{z_1^m [\alpha L(z_1^m) - i]}{z_1^m [1 + \alpha L(z_1^m)] - z_2^m [1 + \alpha L(z_2^a + z_2^m)]}, \tag{35}$$

$$\sigma_2 = \frac{z_2^m [i - \alpha L(z_2^a + z_2^m)]}{z_1^m [1 + \alpha L(z_1^m)] - z_2^m [1 + \alpha L(z_2^a + z_2^m)]}. \tag{36}$$

Suppose the denominator of (35) is negative. Then $\sigma_1, \sigma_2 > 0$ implies $\alpha L(z_1^m) < i < \alpha L(z_2^a + z_2^m)$. As L is decreasing, $z_1^m > z_2^a + z_2^m$. Combine (35) and (36) to get

$$\sigma_1 + \sigma_2 = 1 + \frac{(1+i)(z_2^m - z_1^m)}{z_1^m [1 + \alpha L(z_1^m)] - z_2^m [1 + \alpha L(z_2^a + z_2^m)]}.$$

For $\sigma_1 + \sigma_2 < 1$, the above implies $z_2^m > z_1^m$. As this is a contradiction, the denominator of (35) is positive. By (35) and (36), for $\sigma_1, \sigma_2 > 0$, it must be that $z_2^m < z_1^m$ and $\alpha L(z_1^m) > i > \alpha L(z_2^a + z_2^m)$, which implies $z_2^m < z_1^m < z_S^a + z_S^m < z_2^a + z_2^m$.

Next we prove $z_2^a \in \mathcal{Z}_2$. Suppose not. Then $[1 + \alpha L(z_2^a)] z_2^a - (\alpha + r)\delta + (1 + r)\rho < (1 + r) z_2^a$ by definition of $\mathcal{Z}_2$. By (31), $\sigma_2 > 0$ requires

$$z_2^a [1 + \alpha L(z_2^a + z_2^m)] - (\alpha + r)\delta + (1 + r)\rho > (1 + r) z_2^a.$$

As L is decreasing, $z_2^a [1 + \alpha L(z_2^a)] - (\alpha + r)\delta + (1 + r)\rho > (1 + r) z_2^a$, which is a contradiction.

Lastly, we prove $z_2^a > z_S^a$. Combine (29) and (34) to get rid of $L(z_2^a + z_2^m)$ to get

$$\sigma_2 = \frac{(i - r) z_2^a - (\alpha + r)\delta + (1 + r)\rho}{z_1^m z_2^a [1 + \alpha L(z_1^m)] / z_2^m - \alpha\delta - z_1^a} \tag{37}$$

As $z_1^m < z_2^a + z_2^m$ and L is decreasing, $L(z_1^m) > L(z_2^a + z_2^m)$. As $z_2^a + z_2^m < c(\tilde{q})$, $L(z_1^m) > \delta / z_2^a$. As $z_1^m > z_2^m$ proved above, the denominator is positive. For $\sigma_2 > 0$, the numerator is positive, so $z_2^a > [(\alpha + r)\delta - (1 + r)\rho] / (i - r) = z_S^a$. ■

Proposition 4: With a real asset a and money m, $\rho \geq 0$ and cost $\delta > 0$ only on a, if i satisfies (19) and $\delta > (1 + r)\rho / r$, there exists a unique monetary steady state described by (18). There exist ESE where $z_1^m, z_2^m > 0$ for any (z_1^a, z_2^a) such that $z_1^a \in \mathcal{Z}_0$, $z_2^a \in \mathcal{Z}_2$ and $z_2^a > z_S^a$.

Proof: First notice that the set $\{z_2^a | z_2^a \in \mathcal{Z}_2 \ \& \ z_2^a > z_S^a\}$ is not empty if (19) is satisfied. Combine (37) and (29) to obtain

$$\begin{aligned} & z_2^a [1 + \alpha L(z_2^m + z_2^a)] - (\alpha + r)\delta - (1 + r)(z_2^a - \rho) \\ = & \frac{(i - r) z_2^a - (\alpha + r)\delta + (1 + r)\rho}{z_1^m z_2^a [1 + \alpha L(z_1^m)] / z_2^m - \alpha\delta - z_1^a} \{z_2^a [1 + \alpha L(z_2^m + z_2^a)] - \alpha\delta - z_1^a\}. \end{aligned} \tag{38}$$

If $z_2^m \to 0$, the LHS converges to

$$z_2^a [1 + \alpha L(z_2^a)] - (\alpha + r)\delta - (1 + r)(z_2^a - \rho) > 0$$

because $z_2^a \in \mathcal{Z}_2$. The RHS converges to 0. At $z_2^m = c(\tilde{q}) - z_2^a$, the LHS is $-r z_2^a + (1 + r)\rho - r\delta < 0$. Notice

$$z_2^a [1 + \alpha L(z_2^m + z_2^a)] - \alpha\delta - z_1^a \geq z_2^a - \alpha\delta - z_1^a > 0.$$

The numerator in (38) is positive as $z_2^a > z_S^a$. There are two cases for the denominator.

Case 1: The denominator does not change sign on $(0, c(\tilde{q}) - z_2^a)$. This implies

$$z_1^m z_2^a [1 + \alpha L(z_1^m)] / z_2^m - \alpha\delta - z_1^a > 0$$

on $(0, c(\tilde{q}) - z_2^a)$. Hence, there exists $z_2^m \in (0, c(\tilde{q}) - z_2^a)$ that solves (38).

Case 2: The denominator changes sign on $(0, c(\tilde{q}) - z_2^a)$. Then there exists $\tilde{z}_2^m$ such that

$$\tilde{z}_2^m = \frac{z_1^m z_2^a [1 + \alpha L(z_1^m)]}{\alpha\delta + z_1^a}.$$

If $z_2^m < \tilde{z}_2^m$ but $z_2^m \to \tilde{z}_2^m$,

$$z_1^m z_2^a [1 + \alpha L(z_1^m)] / z_2^m - \alpha\delta - z_1^a \to 0.$$

Hence the RHS of (38) goes to ∞, and as a result there exists $z_2^m \in (0, \tilde{z}_2^m)$ that solves (38).

To summarize, there exists $z_2^m \in (0, c(\tilde{q}) - z_2^a)$ that solves (38) for every $z_1^m > 0$. Then (38) defines $z_2^m = g(z_1^m)$. Then use (30) and (33) to obtain

$$\begin{aligned} 1 + i &= \left\{1 - \frac{(1+r)(z_1^a - \rho) + r\delta - z_1^a}{z_2^a [1 + \alpha L(z_2^a + g(z_1^m))] - \alpha\delta - z_1^a}\right\} [1 + \alpha L(z_1^m)] \\ &\quad + \frac{(1+r)(z_1^a - \rho) + r\delta - z_1^a}{z_2^a [1 + \alpha L(z_2^a + g(z_1^m))] - \alpha\delta - z_1^a} \frac{g(z_1^m)}{z_1^m} [1 + \alpha L(g(z_1^m) + z_2^a)]. \end{aligned}$$

If $z_1^m \to 0$, the RHS goes to infinity, and if $z_1^m \to \infty$, it goes to

$$1 - \frac{(1+r)(z_1^a - \rho) + r\delta - z_1^a}{z_2^a [1 + \alpha L(z_2^a + g(z_1^m))] - \alpha\delta - z_1^a} < 1 + i.$$

Hence there exists $z_1^m > 0$ solving the equation. Denote the solution by z_1^{m*} and let $z_2^{m*} = g(z_1^{m*})$. Then we can back out σ_1^* and σ_2^*. At $z_2^m = g(z_1^m)$, $\sigma_2^* > 0$ by (37). Similarly, it is easy to check (30) to establish $\sigma_1^* > 0$ at (z_1^{m*}, z_2^{m*}), and to show $\sigma_1^* + \sigma_2^* < 1$. This completes the proof. ■

Proposition 6: With a genuine OTC market for a and m, $\rho \geq 0$ and an OTC verification cost δ on a, if i satisfies (19), $\delta > (1+r)\rho/r$, and ρ, δ are not too big, there exists a unique monetary steady state described by (18), and there exist ESE where $z_1^m, z_2^m > 0$ for any (z_1^a, z_2^a) such that $z_1^a \in \mathcal{Z}_0$, $z_2^a \in \mathcal{Z}_2 \cap (z_S^a, (z_S^a + z_S^m)/2)$.

Proof: Notice first that the equations for ESE with an OTC market are (28)-(34) with the assumption that $z_2^a < z_2^m$. By the proof of Proposition 4, we can construct ESE for any (z_1^a, z_2^a) such that $z_1^a \in \mathcal{Z}_0$, $z_2^a \in \mathcal{Z}_2$ and $z_2^a > z_S^a$. We need to show there exists z_2^a such that $z_2^a \in \mathcal{Z}_2$ and $z_2^a > z_S^a$, and the solution for z_2^{m*} satisfies $z_2^a < z_2^{m*}$. By Lemma 6, $z_2^{m*} + z_2^a > z_S^m + z_S^a$. So it is equivalent to show $z_2^a < (z_S^m + z_S^a)/2$. Altogether, we need $\mathcal{Z}_2 \cap \left(z_S^a, (z_S^m + z_S^a)/2\right) \neq \varnothing$. Notice that when δ and ρ are close to 0, z_S^a is close to 0. So the interval $(z_S^a, (z_S^m + z_S^a)/2)$ is not empty. Also notice $F(z_S^a) = [\alpha L(z_S^a) - i + 1 + r] z_S^a > (1+r) z_S^a$. So $z_S^a \in \mathcal{Z}_2$ and $\mathcal{Z}_2$ overlaps with $(z_S^a, (z_S^m + z_S^a)/2)$. This completes the proof. ■

Proposition 9 If $\beta[-\kappa + \alpha u(q^*) + (1-\alpha) p^*] > p^*$, there exist i such that any (D_1, D_2), $D_1 \in \mathcal{D}_0$, $D_2 \in \mathcal{D}_2$ and $D_2 > p^*$, constitute an ESE.

Proof: First, notice that by the condition that $\beta[-\kappa + \alpha u(q^*) + (1-\alpha) p^*] > p^*$, $F(p^*) > p^*$. So the set $\{D_2 | D_2 \in \mathcal{D}_2 \ \& \ D_2 > p^*\} \neq \varnothing$. The proposed ESE satisfies

$$D_1 + z_1 = \frac{(1-\sigma_1)[\alpha S(z_1) + D_1 + z_1] + \sigma_1(-\kappa + \alpha S^* + D_2 + z_2)}{1+r} \quad (39)$$

$$D_2 + z_2 = \frac{\sigma_2[\alpha S(z_1) + D_1 + z_1] + (1-\sigma_2)(-\kappa + \alpha S^* + D_2 + z_2)}{1+r} \quad (40)$$

$$z_1 = \frac{(1-\sigma_1) z_1 [1 + \alpha L(z_1)] + \sigma_1 z_2}{1+i} \quad (41)$$

$$z_2 = \frac{\sigma_2 z_1 [1 + \alpha L(z_1)] + (1-\sigma_2) z_2}{1+i}. \quad (42)$$

Notice that (41) and (42) are independent of (D_1, D_2). Combine to get

$$\alpha L(z_1) = \frac{i(i + \sigma_1 + \sigma_2)}{i(1-\sigma_1) + \sigma_2}.$$

Let $\alpha L(0) = \bar{\imath}$. For any (σ_1, σ_2), pick i such that $i(i + \sigma_1 + \sigma_2) = \bar{\imath}[i(1-\sigma_1) + \sigma_2]$, so that $z_1 = z_2 = 0$. It is obvious that (39)-(40) describe the credit-only case and any (D_1, D_2) with $D_1 \in \mathcal{D}_0$, $D_2 \in \mathcal{D}_2$ and $D_2 > p^*$ constitute an ESE. Pick i to solve $i(i + \sigma_1 + \sigma_2) = (\bar{\imath} - \varepsilon)[i(1-\sigma_1) + \sigma_2]$, where ε is positive but small. This implies z_1 that solves $\alpha L(z_1) = \bar{\imath} - \varepsilon$ is positive but small. Given z_1, pick $D_1 \in \mathcal{D}_0, D_2 \in \mathcal{D}_2$ and $D_2 > p^*$, and solve for $(z_2, \sigma_1, \sigma_2)$ from (39)-(41). By continuity, a solution exists. ■

Appendix D: Money and Credit as Complements

As in Section 4, after the CM closes, with probability α a buyer learns that he will have a trade opportunity in the DM, and with probability $1-\alpha$ he learns that he will not. Buyers with such an opportunity can enter a credit market where if they pay a fixed cost κ they can borrow money, and those with no such opportunity can (at no cost) lend it. As in Berentsen et al. (2007) we can interpret the credit market as banking. One way to do this is to assume buyers cannot commit or to repay loans and cannot be punished for reneging by other buyers, while there are third parties that act as bankers, intermediating between buyers that want to borrow and those that want to lend, because they can be trusted to honor their loans (deposits) and have a comparative advantage at punishing renegers (e.g., they have a good record-keeping technology). Another interpretation is that buyers are firms with investment opportunities. They can buy investment good using internal financing or pay the cost κ to issue corporate bond. Historically, corporate bond issuance experienced freezes and thaws (Benmelech and Bergman 2017). In any case there is a debt limit D similar to the one in Sections 5.2 and 5.3.

Buyers' CM problem is

$$\begin{aligned} W(m,d) &= \max_{x,\ell,\hat{m}} \{U(x) - \ell + \beta [\alpha J_1(\hat{m}) + (1-\alpha) J_0(\hat{m})]\} \\ \text{st } x &= \ell + \phi m - \phi \hat{m} - d - T. \end{aligned}$$

After the CM, those with a trading opportunity decide whether to pay κ to get more funds. For simplicity, assume credit is allocated as if there were a competitive (Walrasian) market. (We can obtain the same result if borrowers make take-it-or-leave-it offers.) Also assume cash is abundant, $(1-\alpha)\phi m > \alpha D$, which is satisfied if α is small, and implies the lending rate is 0. As in Section 5.3, buyer access credit iff $S(\phi m + D) - S(\phi m) > \kappa$. Supposing again that renegers are punished with exclusion from future DM as well as the transfer or tax in the CM, we have $d \leq W(0,0)$. This generates exactly the same equilibrium conditions as in Section 5.3 and therefore Proposition 9 applies.

References

G Afonso, R Lagos (2015) “Trade Dynamics in the Market for Federal Funds,” *Econometrica* 83, 263-313.

D Andolfatto, S Williamson (2015) “Scarcity of Safe Assets, Inflation, and the Policy Trap,” *JME* 73, 70-92.

S Aruoba, G Rocheteau, C Waller (2007) “Bargaining and the Value of Money,” *JME* 54, 2636–55.

F Alvarez, U Jermann (2000) “Efficiency, Equilibrium, and Asset Pricing with Risk of Default,” *Econometrica* 68, 775-97.

C Azariadis (1981) “Self-fulfilling Prophecies,” *JET* 25, 380-96.

C Azariadis (1993) *Intertemporal Macroeconomics*, Blackwell.

L Bebchuk, I Goldstein (2010) “Self-Fulfilling Credit Market Freezes,” *RFS* 24, 3519-55.

E Benmelech, N Bergman (2012) “Credit Market Freezes,” NBER WP 23512.

A Berentsen, G Camera, C Waller (2007) “Money, Credit and Banking.” *JET* 135, 171-95.

A Berentsen, G Menzio, R Wright (2011), “Inflation and Unemployment in the Long Run,” *AER* 101, 371-98.

R Caballero, E Farhi, P Gourinchas (2017) “The Safe Assets Shortage Conundrum,” *JEP* 31, 29-46.

J Chiu, T Koeppl (2016) “Trading Dynamics with Adverse Selection and Search: Market Freeze, Intervention and Recovery,” *RES* 83, 969-1000.”

W Cui, R Wright and Y Zhu (2021) “Cyclical Capital Reallocation and Liquidity,” mimeo.

P Diamond (1982) “Aggregate Demand Management in Search Equilibrium,” *JPE* 90, 881-94.

P Diamond, D Fudenberg (1989) “Rational Expectations Business Cycles in Search Equilibrium,” *JPE* 97, 606-19.

D Duffie, N Gârleanu, L Pederson (2005) “Over-the-Counter Markets,” *Econometrica* 73, 1815-47.

D Gale (1986) "Bargaining and Competition Part I: Characterization," *Econometrica* 54, 785-806.

A Geromichalos, J Licari, J Suárez-Lledó (2007) "Asset Prices and Monetary Policy," *RED* 10, 761-79.

A Geromichalos, L Herrenbrueck (2016) "Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets," *JMCB* 48, 35-79.

A Geromichalos, L Herrenbrueck (2017) "A Tractable Model of Indirect Asset Liquidity," *JET* 168, 252-60.

G Gorton, G Ordonez (2021) "The Supply and Demand for Safe Assets," mimeo.

C Gu, F Mattesini, C Monnet, R Wright (2013) "Endogenous Credit Cycles," *JPE* 121, 940-65.

C Gu, F Mattesini, R Wright (2016) "Money and Credit Redux," *Econometrica* 84, 1–32.

C Gu, R Wright (2016) "Monetary Mechanisms," *JET* 163, 644-57.

C He, R Wright, Y Zhu (2015) "Housing and Liquidity," *RED* 18, 435-55.

T Hu, J Kennan, N Wallace (2009) "Coalition-Proof Trade and the Friedman Rule in the Lagos-Wright Model," *JPE* 117, 116-37.

T Hu, G Rocheteau (2015) ""Monetary Policy and Asset Prices: A Mechanism Design Approach," *JMCB* 47, 39-76.

T Kehoe, D Levine (1993) "Debt-Constrained Asset Markets," *RES* 60, 865-88.

N Kiyotaki, J Moore (1997) "Credit Cycles," *JPE* 105, 211-48.

N Kiyotaki, R Wright (1989) "On Money as a Medium of Exchange," *JPE* 97, 927-54.

T Koeppl, C Monnet, T Temzilides (2008) "A Dynamic Model of Settlement," *JET* 142, 233-346.

N Kocherlakota (1988) "Money is Memory," *JET* 81, 232-51.

R Lagos (2010) "Asset Prices, Liquidity, and Monetary Policy in the Search Theory of Money," *FRB St. Louis Review* 92, 303-09.

R Lagos, G Rocheteau (2005) "Inflation, Output, and Welfare." *IER* 46, 495–522.

R Lagos, G Rocheteau (2008) "Money and Capital as Competing Media of Exchange," *JET* 142, 247-58.

R Lagos, G Rocheteau (2009) “Liquidity in Asset Markets with Search Frictions,” *Econometrica* 77, 403-26.

R Lagos, G Rocheteau, R Wright (2017) “Liquidity: A New Monetarist Perspective,” *JEL* 55, 1-70.

R Lagos, R Wright (2005) “A Unified Framework for Monetary Theory and Policy Analysis,” *JPE* 113, 463-44.

R Lagos, S Zhang (2018) “On Money as a Medium of Exchange in Near-Cashless Credit Economies,” mimeo.

R Lagos, S Zhang (2020) “Turnover Liquidity and the Transmission of Monetary Policy,” *AER* 110, 1635-72.

Y Leitner (2011) “Why Do Markets Freeze?” *Phila Fed Business Rev* 2011-Q2, 12-19.

B Lester, A Postlewaite, R Wright (2012) “Information, Liquidity, Asset Prices, and Monetary Policy,” *RES* 1209-38.

Y Li, G Rocheteau, P Weill (2012) “Liquidity and the Threat of Fraudulent Assets,” *JPE* 120, 815-46.

M Loberto (2018) “Liquidity and Information-Sensitive Assets,” mimeo.

R Lucas (1978) “Asset Prices in an Exchange Economy,” *Econometrica* 46, 1429-45.

F Madison (2018) “Asymmetric Information in Markets for Liquidity,” mimeo.

F Mattesini, E Nosal (2016) “Liquidity and Asset Prices in a Monetary Model with OTC Asset Markets,” *JET* 164, 187-217.

D Mortensen (1999) “Equilibrium Unemployment Dynamics,” *IER* 40, 889-914.

D Mortensen, C Pissarides (1994) “Job Creation and Job Destruction in the Theory of Unemployment,” *RES* 61, 397-415.

C Pissarides (2000) *Equilibrium Unemployment.* MIT Press.

A Razin (2014) *Understanding Global Crises: An Emerging Paradigm.* MIT Press.

G Rocheteau , E Nosal (2017) *Money, Payments, and Liquidity.* MIT Press.

G Rocheteau, P Rupert, K Shell, R Wright (2008) “General Equilibrium with Nonconvexities and Money,” *JET* 142, 294-317.

G Rocheteau, R Wright (2005) “Money in Search Equilibrium, in Competitive

Equilibrium, and in Competitive Search Equilibrium," *Econometrica* 73, 175-202.

G Rocheteau, R Wright (2013) "Liquidity and Asset Market Dynamics," *JME* 60, 275-94.

G Rocheteau, R Wright, Zhang (2018) "Corporate Finance and Monetary Policy," *AER* 108 (2018), 1147-86,

R Rogerson (1988) "Indivisible Labor, Lotteries and Equilibrium," *JME* 21, 3-16.

A Rubinstein, A Wolinsky (1987) "Middlemen," *QJE* 102, 581-93.

A Rubinstein, A Wolinsky (1985) "Equilibrium in a Market with Sequential Bargaining," *Econometrica* 53, 1133-50.

S Shi (1995) "Money and Prices: A Model of Search and Bargaining," *JET* 67, 467-96.

R Silveira, R Wright (2010) "Search and the Market for Ideas," *JET* 145, 1550-73.

A Trejos and R Wright (1995) "Search, Bargaining, Money, and Prices," *JPE* 103, 118-41.

A Trejos, R Wright (2016) "Search-Based Models of Money and Finance: An Integrated Approach," *JET* 164, 10-31.

S Williamson, R Wright (1994) "Barter and Monetary Exchange under Private Information," *AER* 84, 104-23.

T Wong (2016) "A Tractable Monetary Model under General Preferences," *RES* 83, 402-20.

R Wright, S Xiao, Y Zhu (2020) "Frictional Capital Reallocation with Ex Post Heterogeneity," *RED* 37, S227-53.

Y Zhu (2019) "A Note on Simple Strategic Bargaining for Models of Money or Credit," *JMCB* 51, 739-54.