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ABSTRACT

This paper revisits and proposes a resolution to an empirical and theoretical controversy between Keynes and the “classics” (or monetarists). The controversy dates to Keynes’s General Theory (1936)—most famously formalized in Hicks’s (1937) classic Econometrica article, in which the IS-LM model is first formally stated. We first replicate empirical tests formulated in the late 1960s and ’70s and show that more recent data have more statistical power and resolve the empirical debate in favor of the Keynesians, at least according to the criteria of the literature at that time. We then show, using a simple dynamic stochastic general equilibrium (DSGE) model, that the empirical tests suffer from the Lucas (1976) critique, as the conclusion fundamentally depends upon the assumed policy regime. Nevertheless, we argue, this new empirical result is useful: it provides evidence for the existence of a “Keynesian policy regime” according to which traditional monetary expansion loses its impact in the absence of a policy regime change, in the sense of Sargent (1982).

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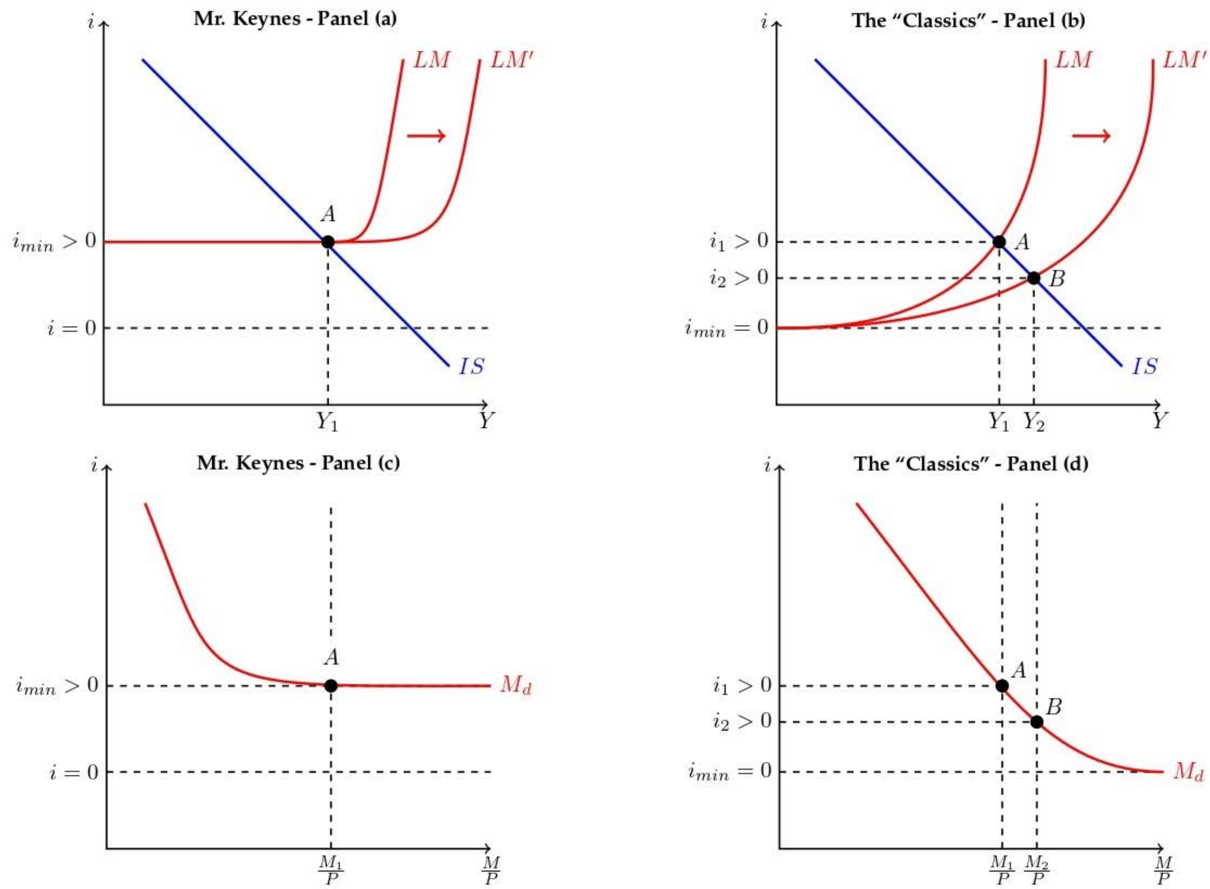

Figure 1. Hicks's suggested interpretation of Mr. Keynes and the "Classics" controversy

1. INTRODUCTION

Panels (a) and (b) of figure 1 are arguably the most influential diagrams in the history of macroeconomics. They were published in *Econometrica* in 1937 in a paper of the same title as this one, except we have replaced "Interpreted" with "Reinterpreted." Panel (a) is John Hicks's interpretation of John Maynard Keynes's *General Theory*. Panel (b) is his characterization of the view of the "classics" that Keynes set out to prove wrong. The figure, and the basic math underlying it, is the backbone of undergraduate macroeconomics teaching to this day. It represents what has become known as the IS-LM model.

The key difference between the Keynesian view, panel (a), and the view of the classics, panel (b), is that according to the former view, printing money, which shifts out the LM curve (from LM to LM'), has no effect on either output or prices at some positive long-term interest rate ($i > 0$). This is because at this interest rate (point A), the LM curve becomes flat. Meanwhile, according to the classics, the LM curve remains steep at point A, and thus monetary policy remains effective. The position that Hicks attributed to the classics is essentially the same as the one later associated with the monetarists.

Milton Friedman (1970), arguably the founding father of monetarism, casts the debate between Keynesians and monetarists in the same terms as Hicks, suggesting that "I regard the description of our position that 'money is all that matters for changes in nominal income and for short-run changes in real income' as an exaggeration but one that gives the right flavor of our conclusion." He then highlights that the assumed money-demand elasticity (the slope of the LM curve at point A) is *the* fundamental difference between the two schools of thought.

From figure 1 it might seem—as many economists, including Friedman, assumed at the time and later—that the debate between the Keynesians and the classics/monetarists was only empirical. This idea is portrayed in figure 1 in panel (c) and panel (d). The Keynesian view (panel [c]) predicts that at a certain level of money supply—again denoted by point A—increasing the money supply (on the x-axis) has no effect on *long-term* interest rates, even if at point A long-run interest rates are still positive. But according to the classics (or monetarists), there is no such point until *all interest rates*—long and short—reach zero, at which point money becomes a dominant asset.[1] The monetarists argued that such an environment had never existed in the United States and was unlikely to ever exist. Long-term interest rates on US corporate bonds, for example, have always been positive in US economic history, during both the Great Depression and the financial crisis of 2008. The same applies to long-term US government bonds.

That the controversy between Keynesians and the classics/monetarists could be resolved via a simple empirical test was not lost on the economics profession. Several articles in *Econometrica* attempted to resolve the controversy —for example, Bronfenbrenner and Mayer (1960), Eisner (1963), Pifer (1969), Eisner (1971), and White (1972). These papers propose formal econometric procedures to test for the existence of a liquidity trap using data from the United States from 1900 to 1958. The empirical test is that if the curves in figure 1, panels (c) and (d) asymptote at positive long-term interest rates, then as the money supply is increased further, long-term interest rates stop responding and the economy is in a liquidity trap. The conclusion of this debate was that there was no statistical evidence for a liquidity trap according to US data. The data, it would seem, handed a victory to the monetarists.

This paper makes three main contributions. First, in section 2, we survey and replicate the statistical evidence about the existence of a liquidity trap in the literature in *Econometrica* using the statistical procedure developed by Pifer (1969), Eisner (1971), and White (1972). Our contribution is to show that once we add data from the Great Inflation of the 1970s and the Financial Crisis of 2008, the evidence suggests that a liquidity trap exists in the sense defined by these authors. The estimation procedure by White (1972), which generalizes Pifer (1969), reveals a floor on long-term corporate-bond interest rates of about 2.23 percent, which is statistically different from zero at the 99 percent confidence level under White's suggested specification, while the floor on long-term government-bond interest rates is 1.72 percent.

[1] Consistent with this, leading monetarists often argued that for a liquidity trap to be a true trap for policy makers, the entire yield curve would need to be flat, which they suggested had never been observed. See Brunner and Meltzer (1968) for an exposition of this argument, reviewing both theory and empirics. See also Eisner (1963, pp. 532–33).

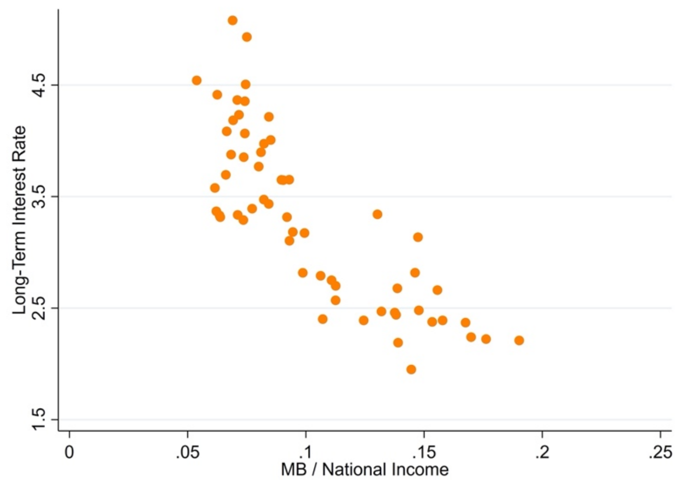

Figure 2. Long-Term Government-Bond Interest Rates versus Monetary Base over National Income from 1900-1958

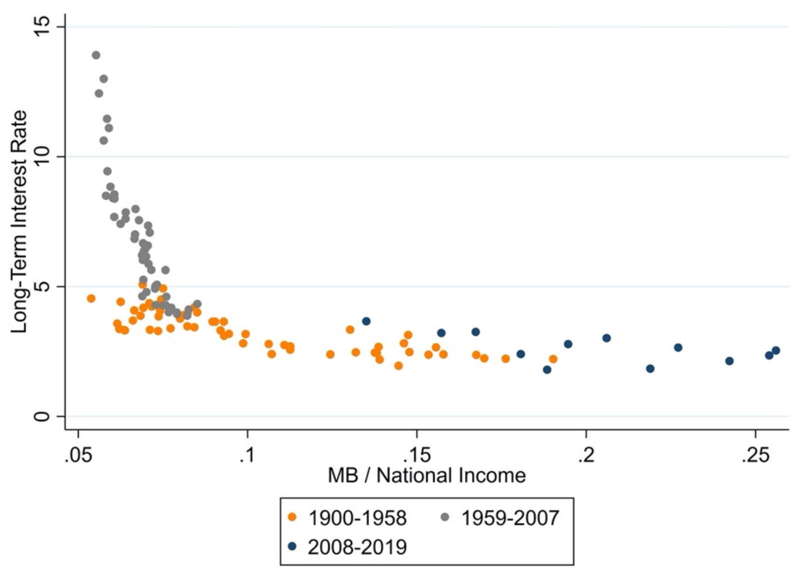

Figure 3. Long-Term Government-Bond Interest Rates versus Monetary Base over National Income from 1900-2019

The statistical power of the new data is most easily seen by considering figure 2 and 3. Figure 2 is a scatter diagram of long-term government-bond interest rates plotted against the ratio of the money supply to national income in the period 1900-1958, which is the sample period in the earlier literature. As the figure reveals, long-term interest rate fluctuated within a relatively narrow band between 1900-1958. Moreover the relationship appears close to linear and showing little tendency to asymptote at low long-term interest rate. In light of this figure, it is perhaps not surprising that the literature found no evidence for a bound on long-term interest rates, a conclusion we confirm statistically in the paper. Consider now figure 3. The orange dots represent the data points 1900–1958, the grey dots represent the data from 1959 to 2007, and the blue dots the data since the financial crisis, i.e. 2008-2019. With the addition of the two other time periods shown in figure 3, a curve emerges with obvious nonlinearities. Importantly, as the Fed increased the money supply following the financial crisis of 2008, long-term interest rates stopped declining and the curve asymptotes, as predicted by the Keynesians. Has the old empirical controversy been resolved in favor of the Keynesians?

The second contribution of the paper is to provide a structural reinterpretation of these econometric tests, building a stripped-down dynamic stochastic general equilibrium (DSGE) model. If we assume what we call a Keynesian policy regime the model implies the intersection of the IS and LM curves shown in panel (a) of figure 1. In contrast, if we assume what we term a monetarist policy regime, the model instead replicates the intersections of the IS and LM curves shown in panel (b) of figure 1. The key difference between the two policy regimes is that under the Keynesian policy regime, any increase in the money supply is expected to be temporary, while under the monetarist policy regime, then any increase in the money supply is expected to be permanent.

Critically, we show that the relationship between the supply of money and the long-term interest rate depends on the monetary policy regime. Under the Keynesian policy regime this relationship takes the form shown in panel (c) of figure 1, according to which any increase in the money supply has no effect on the long-term interest rate once the short-term interest rate is zero. In contrast, under the monetarist policy regime, this relationship takes the form shown in panel (d) of figure 1. In this case an increase in the supply of money always decreases long-term rates, even once the short rate is zero, provided the long-term rates are positive. It follows that the claims of either the Keynesians or the Monetarists can be supported by alternative assumptions about the underlying policy regime. The Keynesians are right in saying that increasing the money supply today is irrelevant once short-term rates are zero, if one implicitly assumes that the central bank cannot commit to future monetary expansions by raising the money supply today. Meanwhile, the monetarist are correct in saying that long-term rates can always be lowered (as long as they are positive) if one implicitly assumes that the central bank can send a signal of loose future policy by increasing the money supply today.

The theoretical analysis clarifies that the econometric test applied in the literature for the existence or nonexistence of a liquidity trap is subject to a classic Lucas critique: the estimated coefficients depend directly on the underlying policy regime. While the Lucas critique is well known as an argument against estimating structural relationships between inflation and output, we are not aware of its application to estimating money demand, where it appears even more striking, especially considering the historical importance of the controversy between the Keynesians and the monetarists. Furthermore, the theoretical analysis clarifies the conditions needed for the underlying shocks, and the policy regime, for the parameters of the model to be identified.[2]

We see as the third main contribution of the paper that it provides a stark and general illustration of the value of providing microfoundations in macro models. While this is obviously an old theme in the macro literature, most famously exemplified by the classic Lucas (1976), we think it is timely today, considering recent methodological controversies in macroeconomics following the crisis of 2008, when the microfounded approach to macroeconomics came under sustained criticism. Clearly stipulating micro-foundations is the key to resolving the controversy between the Keynesians and the classics/monetarists which animated the field of macroeconomics in the decades following the Keynes´ *General Theory*. This is a

[2] We thank Emi Nakamura for emphasizing the importance of the right specification of shocks for identification.

theme we return to at the end of the paper, where we put the contribution in the context of the current methodological debate in macroeconomics.

While the traditional literature on money demand focused on the existence of a liquidity trap, a more recent literature on the stability of money demand emerged following Lucas (1988). The latter literature is not subject to a Lucas critique, because it considers the short-term interest rate, not long-term rates, as the relevant opportunity cost of holding money. If there is a zero-lower bound on the nominal interest rate, then money demand defined in this way will always asymptote at the zero bound. Yet since this literature does not consider the effect of the money supply on long-term interest rates, or more generally the effect of money on aggregate demand, it does not directly address the controversy between the Keynesians and the classics/monetarists, which is the focus of this paper.

2. Empirical Estimates for the Existence of the Liquidity Trap

2.1 Background

The literature testing for the existence of a liquidity trap typically focuses on US data from the period 1900–1958. This literature originated in an *Econometrica* article written by Bronfenbrenner and Mayer (1960), who conclude that "neither the data nor theoretical considerations give any reason for expecting a liquidity trap." This article was followed by objections from Eisner (1963), a rebuttal by Bronfenbrenner and Mayer (1963), and an article by Meltzer (1963b), who concludes that "the evidence lends little or no support to the 'trap." Meltzer's (1963b) finding was interpreted as suggesting that central banks could always stimulate demand by increasing high-powered money.

The early literature estimated a demand function for money using the orange dotted data in figure 2 and 3 and asked whether money demand becomes more elastic at lower rates. The answer it found was no, even though no statistical tests were employed. It is not difficult to see why the literature came to this conclusion.

As the literature became more mature, starting with Pifer (1969), formal statistical tests were employed. The overall conclusion, however, remained the same: the data did not provide evidence in favor of a liquidity trap. Below we replicate this literature and show that newer data overturn the result and favor the Keynesian view.

2.2 Estimation Method and Data

The strategy the literature converged upon is best explained by considering the following demand function for money:

$$M_t = \frac{\gamma Y_t^{\omega}}{(i_t - i^{min})^{-\alpha}} e^{\xi_t} \qquad (1)$$

Here, M_t is money, Y_t is income, i_t is the long-term interest rate, i^{min} is the rate of interest below which long-term interest rates cannot fall and ξ_t is an exogenous disturbance. The empirical question is whether i^{min} is statistically different from zero.

There is of course the classic identification issues associated with estimating (1). What we observe is the intersection of demand and supply, and thus it is not obvious that the data will trace out equation (1). We leave discussion of this issue, curiously absent from the literature cited above, to Section 6 once it can be understood more clearly it the context of a structural model.

Leaving aside, for now, the classic identification problem, there are several other issues that need to be confronted in estimating (1), and the literature considered a variety of answers to each. First, what is the relevant measure of money? Two common measures are (1) M1 (currency in circulation outside of banks as well as bank deposits and close substitutes) and (2) the monetary base (all currency in circulation as well as nonborrowed bank reserves at the Federal Reserve). Second, what is the relevant security for measuring long-term interest rates: long-term corporate bonds, or long-term government bonds (which are arguably free of risk premia)? Third, what is the relevant income measure? Various authors used GDP or some measure of total assets or both.

Rather than taking a stance on these questions, which is difficult without an explicit model, we show that the results hold for *any combination of these choices*.[3] To streamline the discussion, we present the results using M1 as the measure of money, corporate and government bonds as measures of long-term interest rates, GDP as the measure of income, and an additional measure of total assets. We summarize the results using M1 in the body of the paper, whereas appendix C documents the same results using the monetary base as a proxy for the money supply.

We follow Pifer (1969) in formulating the baseline test for whether i^{min} is statistically different from zero. We run a nonlinear two-step maximum likelihood estimation. For each given i^{min}, we run the following regression, obtained by taking log of (1):

$$\log(M_t) = \log(\gamma) + \omega \log(Y_t) + \alpha \log(i_t - i^{min}) + \xi_t \tag{2}$$

We then choose the i^{min} that maximizes the likelihood function (see appendix B for details). We follow Eisner's (1971) method to correctly characterize the standard errors in the estimate of i^{min}.

We then follow White (1972) in extending this method by considering a more general functional form:

$$\frac{M_t^{\lambda} - 1}{\lambda} = \frac{\gamma^{\lambda} - 1}{\lambda} + \omega \frac{Y_t^{\lambda} - 1}{\lambda} + \alpha \, \frac{(i_t - i^{min})^{\lambda} - 1}{\lambda} + \xi_t \tag{3}$$

Equation (3) generalizes the specification of equation (2), where (2) is the special case in which $\lambda \to 0$.

[3] All the data sources are report in table III in appendix A.

2.3 Empirical Results

Table I

Estimates of λ and i^{min} Using M1 and Long-Term Corporate Interest Rates

Time Period	(1900-1958)			(1900-2019)		
Specification	Pifer (1969)	White (1972)		Pifer (1969)	White (1972)	
	i^{min}	λ	i^{min}	i^{min}	λ	i^{min}
i_t, Y_t	2.17	0.18	2.33	2.30	-0.10	2.25
	(0.01, 2.33)[1]	(-0.09, 0.35)[3]	(0.67, 2.34)[3]	(2.17, 2.33)[1]	(-0.16, -0.05)[3]	(2.02, 2.32)[3]
	(-1.50, 2.34)[2]	(-0.16, 0.39)[4]	(-1.50, 2.34)[4]	(2.09, 2.34)[2]	(-0.17, -0.03)[4]	(1.93, 2.33)[4]
i_t, Y_t, A_t	2.24	-0.36	1.70	2.30	-0.24	2.23
	(1.94, 2.32)[1]	(-0.60, -0.12)[3]	(-0.94, 2.21)[3]	(2.17, 2.34)[1]	(-0.28, -0.19)[3]	(2.10, 2.29)[3]
	(1.69, 2.33)[2]	(-0.66, -0.05)[4]	(-1.50, 2.27)[4]	(2.09, 2.34)[2]	(-0.30, -0.18)[4]	(2.05, 2.30)[4]

[1] 95%, and [2] 99% confidence interval, see Hayashi (2000, p. 52-53); [3] 95%, and [4] 99% confidence interval, see Greene (2018, p. 554).

The first column in table I shows the results for our replication of Pifer's (1969) two-step maximum likelihood estimation, both with and without assets as an additional explanatory variable, focusing on the sample 1900–1958. This results in an estimate of i^{min} as 2.24. While Pifer concludes that this number is not significantly different from zero, Eisner (1971) finds it is once one computes this error using the correct likelihood ratio test. As White (1972) stresses, however, if one considers the more general functional form of (3), the result is no longer statistically significant. White's result is replicated in the second and third columns of table I. As shown there, the point estimate of i^{min} is 1.70; however, the 95 percent confidence interval runs from −0.94 to 2.21. These results generated the consensus of the literature that there was no empirical evidence for a liquidity trap in the US data for the period 1900–58.

The last column of table I shows that later data overturn this result, as they greatly reduce the confidence bands. The point estimate is 2.23 with a 99 percent confidence interval from 2.05 to 2.30. Table I focuses on corporate-bond rates. In table II we show that the results are unchanged if government bonds are used to measure long-term interest rates. Table IV in appendix C presents the results using the monetary base instead of M1 as the measure of money. Thus, data from the second half of the twentieth century and the early part of the twenty-first seems to resolve the clash between Keynes and the classics/monetarists: the Keynesians won.

Table II

Estimates of λ and i^{min} Using M1

Specification	$\frac{M_t^\lambda - 1}{\lambda} = \frac{\gamma^\lambda - 1}{\lambda} + \omega \frac{Y_t^\lambda - 1}{\lambda} + \alpha \frac{(i_t - i^{min})^\lambda - 1}{\lambda} + \delta \frac{A_t^\lambda - 1}{\lambda} + u_t$	
	λ	i^{min}
Government Interest Rates	-0.19 (-0.27, -0.08)[1] (-0.29, -0.04)[2]	1.72 (1.36, 1.87)[1] (1.20, 1.89)[2]
Corporate Interest Rates	-0.24 (-0.28, -0.19)[1] (-0.30, -0.18)[2]	2.23 (2.10, 2.29)[1] (2.05, 2.30)[2]

[1] 95%, and [2] 99% confidence interval, see Greene (2018, p. 554).

Or did they? That is the question we address next from a theoretical perspective. For the second half of the twentieth century did not only bring new data to test theories. It also witnessed substantial theoretical advances, with the emergence of microfoundations for economic decisions and expectations. The explicit account of expectations is the crucial omission of the IS-LM model—one highlighted as possibly important by Hicks (1937). A more modern analysis shows that the LM curve of the form envisioned by either the Keynesians or the classics/monetarists can emerge from the data under different stipulations about the underlying monetary policy regime. Here we use the term "policy regime" following Sargent (1982). It refers to the rules by which monetary and fiscal authorities determine current *and future* monetary and fiscal policy.

3. A MINIMALISTIC NEW KEYNESIAN IS-LM MODEL

We now present a miniature New Keynesian dynamic general equilibrium (DSGE) IS-LM model that we use to generate the IS-LM diagram originated by Hicks under different assumptions about the underlying policy regime. Consider a standard New Keynesian model. The model can be summarized by three equations. All variables are expressed in terms of percentage (or log) deviation from steady state, with $\hat{Y}_t$ denoting output, $\hat{\imath}_t$ the short-term risk-free nominal interest rate, $\widehat{M}_t$ the money supply, π_t inflation, and $\hat{r}_t^e$ is an exogenous disturbance. The IS equation is

$$\text{IS} \qquad \hat{Y}_t = \delta E_t \hat{Y}_{t+1} - \sigma\delta(\hat{\imath}_t - E_t \pi_{t+1} - \hat{r}_t^e), \qquad (4)$$

where $0 < \delta \leq 1$ is a discounting term and $\sigma > 0$ is the inverse of the coefficient of relative risk aversion of the representative household. The IS equation is derived from the household optimization problem. The discounting term δ arises because of, for example, relative wealth in the utility function as in Michaillat and Saez (2018), an OLG structure as in Eggertsson, Mehrotra, and Robbins (2019), behavioral factors as in Gabaix (2020), or heterogeneity between borrowers and spenders as in Bilbiie (2020). We provide an explicit example of wealth in the utility function in appendix D, following Michaillat and Saez (2018).

This equation can be forwarded to yield

$$\hat{Y}_t = -\sigma\delta \sum_{j=0}^{\infty} \delta^j \left(\hat{\imath}_{t+j} - E_t\pi_{t+j+1} - \hat{r}^e_{t+j}\right), \tag{5}$$

illustrating that output depends not just on the current interest rate but also on the expected future path of the real interest rate. Introducing δ is useful because, as stressed by Eggertsson, Mehrotra, and Robbins (2019), it allows us to consider zero lower bound (ZLB) episodes of arbitrary duration.[4]

Inflation and output are related by a simple Phillips-curve equation:

AS $$\pi_t = \kappa\hat{Y}_t \tag{6}$$

Here, $\kappa > 0$. We adopt a slightly simpler version of the Phillips curve than is common in the modern literature, in which a forward-looking expectation term appears on the righthand side. This is only to simplify the exposition. This equation can be derived under the assumption that firms are monopolistically competitive with a fixed fraction of firms indexing their prices to the previous period's aggregate price level while the remaining fraction set their prices optimally (see appendix D).

To simplify the exposition, we now assume that $\kappa \to 0$ so that prices are completely fixed and $\pi_t = 0$. This is the assumption Hicks (1937) makes when he introduces the IS-LM model. Nothing substantive that follows depends on this. The more general case, however, is useful for the discussion in Section 5.

The LM equation is

LM/ELB $$\widehat{M}_t \geq \eta_y\hat{Y}_t - \eta_i\hat{\imath}_t + \varepsilon^d_t;\ \hat{\imath}_t \geq i^{elb}, \tag{7}$$

where $\eta_y > 0$, $\eta_i > 0$, and at least one of the inequalities holds with an equality at any given time. The shock ε^d_t represents a money demand shock. We set this shock at zero for now and return to it in section 6. Since the variables are expressed as deviations from steady state, i^{elb} is negative under the assumption that the interest rate cannot go below zero. This equation is derived by assuming that real money balances provide transaction services for the household and appear directly in the utility function. At some point, however, households have enough liquidity to satisfy all their transaction needs (they are satiated), at which point the first inequality is slack and $\hat{\imath}_t = i^{elb}$. The satiation point is denoted by M^*. Policy sets $\widehat{M}_t$ via open market operations in short-term risk-free government bonds, and, through this, the short-term nominal interest rate.

We consider the following assumption about the exogenous disturbance:

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[4] As stressed by Eggertsson, Mehrotra, and Robbins (2019), a permanent reduction in the natural rate of interest cannot be considered in the standard representative-agent New Keynesian model.

A1. At time 0, $\hat{r}_t^e = \hat{r}_S^e < 0$; it then reverts to steady state with a fixed probability μ in each of the following periods so that $\hat{r}_t^e = 0$. The stochastic period in which the shock reverts to steady state is denoted *T*. Once the shock reaches steady state, it stays there forever. We call the periods $t < T$ the short run, denoted *S*, and the periods $t \geq T$ the long run, denoted *L*.

We consider two types of monetary policy regimes:

A2. Under the *Keynesian policy regime*, then $\widehat{M}_t = \widehat{M}_L = 0$ for $t \geq T$ and $\widehat{M}_t = \widehat{M}_S$ for $t < T$ is a policy choice.

A3. Under the *monetarist policy regime*, then $\widehat{M}_t = \widehat{M}_S = \widehat{M}_L$ for $\forall\, t$ is a policy choice.

The key difference between the two regimes, thus, is that under the Keynesian policy regime, if there is a monetary expansion today the central bank is expected to reverse it as soon as economic conditions improve (when the shock $\hat{r}_S^e$ reverts to steady state). In contrast, in the case of the monetarist policy regime, any short-run monetary expansion is expected to be permanent.

Under assumptions A1–A3, the endogenous variables take on the same value in all periods $t < T$, which we denote by the subscript *S*, and the same value for $t \geq T$, which we denote with the subscript *L*. We can then write the expectation of future output as $E_t\hat{Y}_{t+1} = \mu\hat{Y}_L + (1-\mu)\hat{Y}_S$.

The short-run IS equation is then:

$$\hat{Y}_S = \frac{\mu\delta}{1-(1-\mu)\delta}\hat{Y}_L - \frac{\sigma\delta}{1-(1-\mu)\delta}\hat{\imath}_S + \frac{\sigma\delta}{1-(1-\mu)\delta}\hat{r}_S^e \tag{8}$$

while the LM equation is

$$\widehat{M}_S \geq \eta_y\hat{Y}_S - \eta_i\hat{\imath}_S;\ \hat{\imath}_S \geq i^{elb}. \tag{9}$$

Under the assumption that the LM equation holds with equality in the long run, we can solve for output in the long run to yield

$$\hat{Y}_L = \frac{\frac{\delta\sigma}{\eta_i}}{1-\delta+\sigma\delta\frac{\eta_y}{\eta_i}}\widehat{M}_L. \tag{10}$$

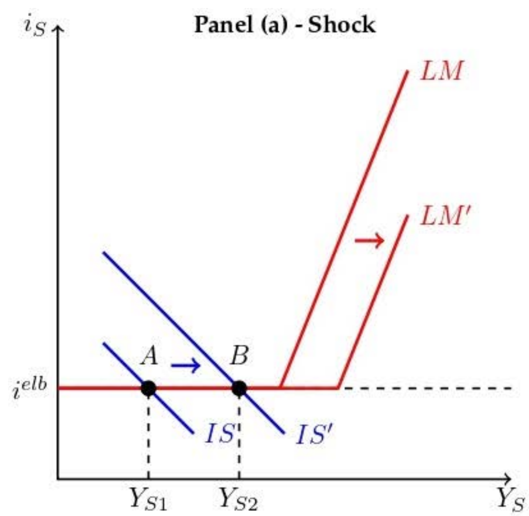

Figure 4. A Minimalistic Dynamic IS-LM Model

The IS and LM equations are portrayed in figure 4. Panel (a) shows the solution under the assumption that $\hat{r}_S^e < i^{elb}$ so that the effective lower bound (ELB) is binding, while panel (b) shows the intersection of the two curves when the ELB is not binding—that is, $\hat{r}_S^e = 0 > i^{elb}$. While a shift in the LM curve will increase output when there is no shock, it will do nothing once the shock brings the IS curve to the flat region of the LM curve, as in panel (a).

At a superficial level, these figures look exactly like the figures in the introduction, with Mr. Keynes corresponding to panel (a) in figure 1 and the classics to panel (b) in figure 1. It would thus seem the data give a straightforward way to discriminate between the two views. Panel (a) suggests that the Keynesian regime reigns when the short-term nominal interest rate is zero. Short-term nominal interest rates were close to zero from 1931 to 1947 and from 2008 until December 2015. According to this reading, the economy operated according to the Keynesian logic during the Great Depression and the Great Recession but according to the classical/monetarist logic in other periods.

Yet this clearly is not the correct interpretation of the controversy, and it would make our previous empirical test moot. Keynes and Hicks were well aware of that the short-term interest rate had been close to zero during the '30s in several countries and could not go below it. In Hicks's (1937) words, "If the costs of holding money can be neglected, it will always be profitable to hold money rather than lend it out, if the rate of interest is not greater than zero. Consequently the (short) rate of interest must always be positive."

The Keynesian theory of liquidity demand, however, depends on the *long-term* interest rate. Hicks (1937) argues that the ZLB on the short-term interest rate, in turn, implies that the long-term interest rate also has a bound and that this bound is greater than zero. To quote Hicks (1937):

> In an extreme case, the shortest short-term rate may perhaps be nearly zero. But if so, the long-term rate must lie above it, for the long rate has to allow for the risk that the short rate may rise during the currency of the loan, and it should be observed that the short rate can only rise, it cannot fall. This does not only mean that the long rate must be a sort of average of the probable short rates over its duration, and that this average must lie above the current short rate. There is also the more important risk to be considered, that the lender on long term may desire to have cash before the agreed date of repayment, and then, if the short rate has risen meanwhile, he may be involved in a

substantial capital loss. It is this last risk which provides Mr. Keynes' "speculative motive" and which ensures that the rate for loans of indefinite duration (which he always has in mind as the rate of interest) cannot fall very near zero.

Hence, to resolve the controversy, one needs to account for the behavior of the long-term interest rate and the extent to which it is insensitive to an expansion in the money supply. Before getting there, however, it is useful to understand a key difference between the model we have just derived, and the classic IS-LM model developed by Hicks.

As we can see in equation (5), the IS equation depends on the entire expected future path of the short-term nominal interest rate. In the context of our simple example, equation (8) reveals that this implies that output depends on expected future output, or $\hat{Y}_L$, which in turn is determined by the expectation about the long-term money supply as shown in equation (10).

Figure 4 shows the effect of a monetary expansion in the short run when there is a shock that puts the intersection of the IS and LM curves at the flat part of the LM curve. What is the effect of a monetary expansion—that is, $\widehat{M}_S \uparrow$—assuming either the Keynesian or the monetarist policy regime? Under both policy regimes, the LM curve shifts rightward from *LM* to *LM.'*

Under the Keynesian policy regime, this is the end of the story. Under the monetarist policy regime, however, the increase in $\widehat{M}_S$ also signals a future monetary expansion—that is, an increase in $\widehat{M}_L$—once the ZLB is no longer binding. This, however, has no effect on the LM curve. Instead, it shifts out the IS curve because it increases expectations about future income, thus stimulating spending today, as shown by the rightward shift of the IS curve from *IS* to *IS '*.

It is straightforward to show that the equilibrium is characterized by

$$\hat{Y}_S = \frac{1}{1-\delta(1-\mu)}\left[\frac{(\mu\delta)\frac{\sigma\delta}{\eta_i}}{1-\delta+\sigma\frac{\eta_Y}{\eta_i}}\widehat{M}_L + \sigma\delta(\hat{r}_S^e - i^{elb})\right], \tag{11}$$

with $\widehat{M}_L = 0$ under the Keynesian policy regime and $\widehat{M}_L = \widehat{M}_S$ under the monetarist policy regime. Thus, the monetary expansion is effective under the monetarist regime, while it is not under the Keynesian one.

The effectiveness of the monetary expansion under the monetarist regime is not coming from the short-run money supply increase per se. Instead, the increase in the money supply is effective because it signals a *permanent* monetary expansion.

The simple theory highlights that the effect of a monetary expansion depends critically on whether it is expected to be permanent, a point originally developed by Krugman (1998) in a classic article that helped launch the modern literature on the liquidity trap. As it turns out, this insight is also critical to understand why the slope of the estimated money demand depends on the policy regime.

4. Resolving the Controversy

A major advantage of building macroeconomic models from microfoundations is that it gives a straightforward way to add various new elements to a model. Thus, while the IS equation is a pricing equation for one-period debt, the underlying model can also be used to price any asset, such as a loan of infinite duration, whose price, according to Hicks, is the interest rate in Keynesian thought.

To capture the idea that the long-term interest rate is the price of loan of "infinite duration" define the long-term interest rate as the implied yield, denoted by i_t^{long}, of a perpetuity whose coupon payment declines geometrically at a rate ρ. The implied duration of such as bond is given by $(1-\rho\beta)^{-1}$ so by appropriate choice of ρ we can approximate a bond of arbitrary duration. The case in which $\rho = 0$ then correspond to one period risk-free bond, while $\rho = 1$ is a classic consol. This slight generalization of the classic consol allows us to simplify the analytics considerably, by an appropriate choice of ρ, as we will see below. Appendix D details the pricing of this bond and how its yield is defined.

The long-term interest rate defined by this consol, is up to a first order approximated by

$$\hat{\imath}_t^{long} = (1-\tfrac{\rho\beta}{\delta})E_t \textstyle\sum_{j=0}^{\infty} \left(\tfrac{\rho\beta}{\delta}\right)^j \hat{\imath}_{t+j}, \tag{12}$$

where β is the time-discount factor of the representative household. As suggested by this expression, the long-term rate is a weighted average of current and future short-term interest rates. To simplify the analytics we choose $\rho = \frac{\delta^2}{\beta}$ so that the long-rate correponds to yield on a bonds with duration $(1-\delta\,)^{-1}$. If we further assume that $\delta^2 = \beta$ then this bond is a classic consol (we do not, however, make this parameter restrictions below and thus keep the model in its more general form).

Under these assumptions the IS equation in the short run can be written as

$$\hat{Y}_S = -\tfrac{\sigma\delta}{1-\delta}(\hat{\imath}_S^{long} - \hat{r}_S^{e,l}), \tag{13}$$

where $\hat{r}_t^{e,l} \equiv (1-\delta)E_t \sum_{j=0}^{\infty} \delta^j \hat{r}_{t+j}^e$.

We now show how one can rewrite the LM equation in terms of the long-term interest rate. As we have documented Keynes, Hicks and the literature that followed assumed that money demand depended on the long rate. As we will show, however, the formulation of the LM equation in terms of the long-term interest rate takes different shape depending on the assumed policy regime. Accordingly, we will refer to the resulting relationships as "quasi-money demand" with one corresponding to the Keynesian policy regime while the other corresponds to the monetarist policy regime.

To derive a Keynesian LM curve—that is, a quasi-money demand function expressed in terms of the long-term interest rate—we first write the long-term interest rate in the short run as

$$\hat{\imath}_S^{long} = \tfrac{1-\delta}{1-\delta(1-\mu)}\hat{\imath}_S + \tfrac{\delta\mu}{1-\delta(1-\mu)}\hat{\imath}_L. \tag{14}$$

Under the Keynesian policy regime, the interest rate turns to its steady state in the long run so that $\hat{\imath}_L = 0$. Using (14), the bound on the long-term nominal interest rate is therefore

$$\hat{\imath}_S^{long} \geq i^{lelb,K} = \frac{1-\delta}{1-\delta(1-\mu)} i^{elb}. \tag{15}$$

Note that $i^{lelb,K} > i^{elb}$, so the lower bound on long-term interest rates is necessarily higher than the bound on the short-term rates for $\mu > 0$.

Intuitively, suppose the short-term interest rate cannot go below zero. Since the variables are expressed in deviation from steady-state values, this implies that i^{elb} is negative. Equation (15) then says that the long-term rate is bounded by people's expectations about *when* the short-term rate will rise since under the Keynesian policy regime people's expectations about the interest rate once the shock is over are fixed.

Substituting (14) into the LM equation allows us to derive a Keynesian quasi–LM curve,

LM^K $$\widehat{M}_S \geq \eta_y \widehat{Y}_S - \eta_i \alpha_i^K \hat{\imath}_S^{long};\ \hat{\imath}_S^{long} \geq i^{lelb,K}, \tag{16}$$

where $\alpha_i^K \equiv \frac{1-\delta(1-\mu)}{1-\delta}$. This is not a structural equation. Instead, it is a theoretical relationship derived conditional on the Keynesian policy regime.

Let us now consider a quasi-LM relationship in the case of a monetarist policy regime. The key difference is that in this case $\hat{\imath}_L$ is no longer fixed at its steady-state value because any increase in the money supply is expected to be permanent—that is, $\widehat{M}_L = \widehat{M}_S$. Combining the long run IS and LM equations, we obtain

$$\hat{\imath}_L = -\frac{1}{\eta_i + \frac{\sigma\delta}{1-\delta}\eta_y} \widehat{M}_S \ if\ \widehat{M}_S < M^* \text{ and } \hat{\imath}_L \geq i^{elb}, \tag{17}$$

where M^* is the money-satiation point in the long run and $\hat{\imath}_L = i^{elb}$ if $\widehat{M}_S \geq M^*$. This implies that, under the monetarist policy regime, the ELB is only binding on i_S^{long} when both i_S *and* i_L reach their lower bound. Accordingly, from equation (14) we see that the lower bound on the long-term interest rate under the monetarist regime is the same as that on the short-term interest rate—that is, i^{elb}. Using the expression for i_L in (17), and using (14) to substitute $\hat{\imath}_S^{long}$ for $\hat{\imath}_S$, in the LM equation we obtain a monetarist quasi–LM curve in terms of the long-term interest rate given by

LM^M $$\widehat{M}_S \geq \eta_y \alpha_y^M \widehat{Y}_S - \eta_i \alpha_i^M \hat{\imath}_S^{long};\ \hat{\imath}_S^{long} \geq i^{elb}, \tag{18}$$

where $\alpha_y^M = \frac{(1-\delta)\eta_i + \sigma\eta_y}{(1-\delta(1-\mu))\eta_i + \sigma\eta_y}$ and $\alpha_i^M = \frac{1-\delta(1-\mu)}{1-\delta}\alpha_y^M$.

We see that the key difference between the LM curves under the two policy regimes is that the slope of the LM curve is steeper under the monetarist regime. Moreover, the bound on the long-term interest rate is lower under the monetarist policy regime and coincides with the bound on the short-term interest rate.

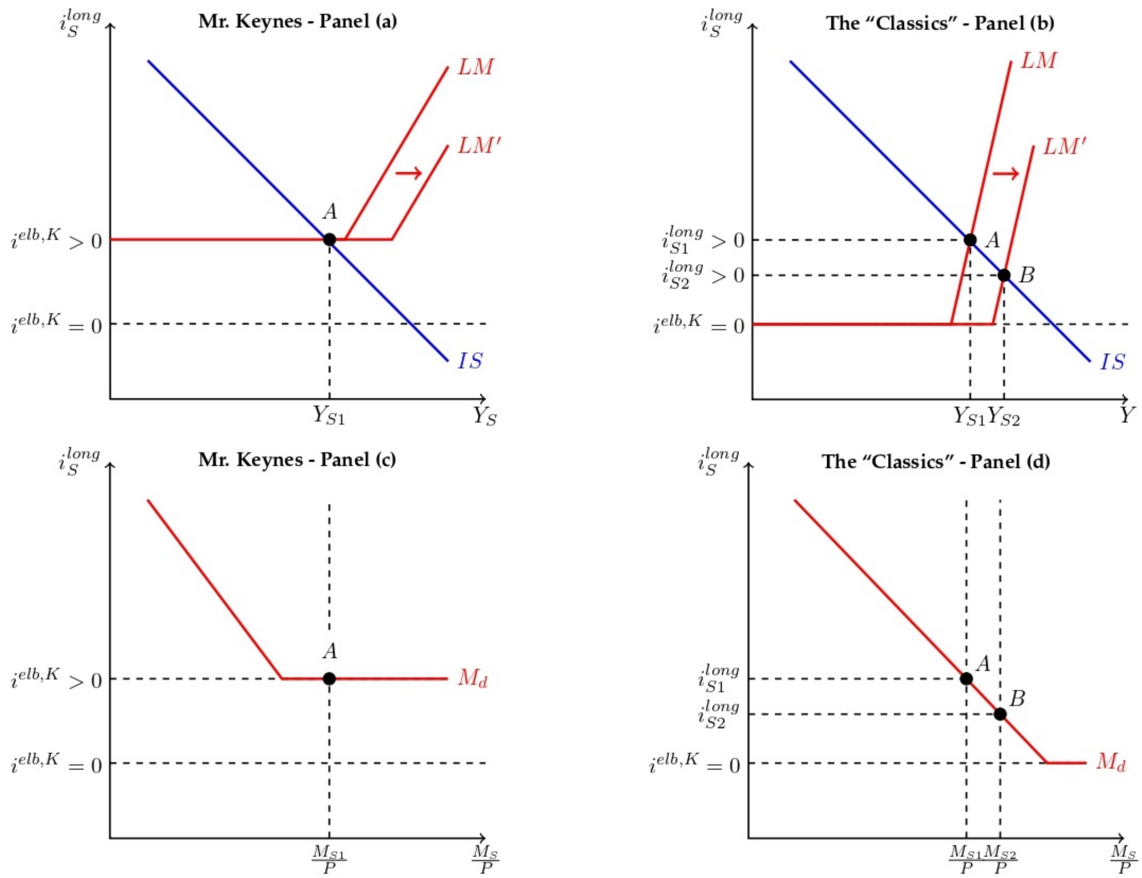

Figure 5. Suggested Reinterpretation

The IS and LM equations under the two policy regimes are shown in figure 5, which replicates the original figure from Hicks (1937). The analysis, however, makes clear that the money-demand equations written in terms of the long-term interest rate are not structural but instead depend upon the assumed policy regime.

Under what condition is the flat region of the LM curve reached under the two policy regimes? In the Keynesian policy regime, a sufficient condition is that the current short-term interest rate, i_S, reaches the lower bound. In this policy regime, the expectation of the future short-term interest rate once the shock is over, i_L, is fixed at the steady-state value. This implies that under the Keynesian policy regime, the term structure will always be upward sloping. Importantly, however, the lower bound on the long-term rate, $i^{elb,K}$, depends on the expected duration of the shock that gives rise to the ELB. Hence, if the shock is expected to last longer, then this will automatically reduce the lower bound on the long-term interest rate, $i^{elb,K}$, and make the term structure flatter.

In the monetarist regime, however, both i_S and i_L need to reach the ELB for the policy makers to find themselves at the flat region of the LM curve. This implies not only that the short-term interest rate needs to reach zero, but that the entire term structure is flat.

This analysis suggests the following reformulation of the Keynesian and monetarist perspectives: The Keynesians asserted that long-term interest rates could not be lowered further once the short-term interest rate reached zero. The analysis suggests that this assertion implicitly relies upon the assumption that the central bank cannot commit to a future monetary expansion. Meanwhile, the monetarists, in contrast asserted that long-term interest rates could always be lowered even when the short-term rate reached zero provided that long-term rates were still positive. To do so the central bank simply needed to increase the money supply. The analysis suggests that this assertion implicitly assumes that the central bank, by increasing the money supply today, is credibly signaling loose monetary policy *in the future*.

The relationships above were derived under the assumption that the measure of interest rate is a risk-free bond rate (typically measured by government bond rates). The same relationships can be derived using an interest rate that incorporates a risk of default which we denote by $\hat{\imath}_t^d$. Consider a loan contract according to which there is a ω_{t+1} probability of default in period t+1. As we shown in appendix E, this interest rate satisfies

$$\hat{\imath}_t^d = \hat{\imath}_t + E_t\widehat{\omega}_{t+1}$$

where $E_t\widehat{\omega}_{t+1}$ is the risk-premia in deviation from steady state. The same quasi-LM curve can now be derived by adjusting both the interest rates and the lower bound by the risk-premia. The model now predicts that the lower bound on the long-term risky interest rate is higher than for the risk-free rate. The exact same analysis, however, applies.

5. Interpretation of the Policy Regimes

There is a simple interpretation of the Keynesian policy regime. Consider the following social welfare criterion, which can be derived via a second-order approximation of the household utility function:

$$-E_t\sum_{t=0}^{\infty}\beta^t(\pi_t^2 + \lambda\hat{Y}_t^2) \qquad (19)$$

Consider a government that maximizes this objective, subject to the IS, AS, LM, and ELB constraints, taking expectations as given. Consider first this maximation problem, in the long run, once the ELB is no longer binding. The first-order condition of the government's problem can be summarized by

$$\pi_t + \frac{\lambda}{\kappa}\hat{Y}_t = 0, \qquad (20)$$

which, together with the AS equation, implies that $\pi_t = \hat{Y}_t = 0$ for $t \geq T$, which in turn implies that $\widehat{M}_t = 0$ for $t \geq T$. Hence a natural microfoundation for the Keynesian regime is that it corresponds to the optimal policy under discretion—that is, the case in which the government cannot commit to future policy as in Eggertsson (2006). Meanwhile, the monetarist regime is an example of a policy regime in which the government can make a credible commitment about future policies. A monetary expansion in the short run implies a permanent increase in the money supply and a commitment to future inflation. To be clear, however, the monetarist policy regime is only one example of a monetary policy regime where the policy maker signals a future monetary example, a theme we develop better in Section 7.

6. EMPIRICALLY IDENTIFYING QUASI-MONEY DEMAND

As we already mentioned in the empirical section, the estimation may suffer from a classic identification problem. Should we expect the data to trace out the quasi-demand for money, which does seem to pop up in Figure 1? What we see in the data, of course, represents the intersection of demand and supply, so whether the data traces out demand, supply, or neither, hinges on assumptions about the underlying shocks and the monetary policy regime. Fortunately, the structural model gives a clear guidance for how to interpret the evidence.

Consider the microfoundations for the Keynesian regime given by equation (20), assuming again full rigidities for simplicity so that $\pi_t = 0$. It implies that policy is set so that output (or more generally the output gap) is at steady state, i.e. $\hat{Y}_t = 0$, as long as the ELB is not binding. Then we have by equation (8) that $\hat{\imath}_S = \hat{r}_S^e$.

Consider again the two-state Markov process for the natural rate of interest, $\hat{r}_S^e$, but let us also incorporate the money demand shock ε_t^d which we assume follows the same two-state Markov process. Relative to our earlier exposition we allow the shocks, i.e. $\hat{r}_{t,S}^e$ and $\varepsilon_{t,S}^d$ to vary with time in the short run. Furthermore, we impose the assumption that they have a martingale property in the short-run, that is $E_t\hat{r}_{t+1,S}^e$=$E_t\hat{r}_{t,S}^e$ and $E_t\varepsilon_{t+1,S}^d$=$E_t\varepsilon_{t,S}^d$. It is easy to show, following the same steps as before, that we obtain the same expressions for the IS and the LM equations as in Section 4, but the short-run variables can now vary stochastically with time.[6]

Using (14) and (16), and that $\hat{Y}_t = 0$ if the ELB is not binding we obtain following expression for the long-term interest rate and the money supply

$$\hat{\imath}_{t,S}^{long} = \frac{1-\delta}{1-\delta(1-\mu)}\hat{r}_{t,S}^e \; if \; \hat{r}_{t,S}^e \geq i^{elb} \qquad (21)$$

$$\hat{M}_{t,S} \geq -\eta_i \alpha_i^K \frac{1-\delta}{1-\delta(1-\mu)}\hat{r}_{t,S}^e + \varepsilon_{t,S}^d; \;\; \hat{\imath}_{t,S}^{long} \geq i^{elb} \qquad (22)$$

Consider first random draws for $\hat{r}_{t,S}^e$ in the absence of money demand shocks. In this case the data traces the line shown in Figure 5 panel c). Moreover, for negative values of $\hat{r}_{t,S}^e$ the curve traced out asymptodes at i^{elb}. This suggests that under the assumption that there are only shocks to the natural rate of interest, the quasi-money demand is exactly identified.

[6] The reason is that by using the martingale property we once again have that $E_t\hat{Y}_{t+1} = \mu\hat{Y}_{t,L} + (1-\mu)\hat{Y}_{t,S} = (1-\mu)\hat{Y}_{t,S}$

The first key to identification is the assumption that money supply is set so the output gap is zero. To see the importance of this assumption, consider the alternative assumption that $\widehat{M}_t = 0$. In this case, assuming only shocks to the natural rate of interest, the data traces out a vertical line starting at i^{elb}.[7]

The other key identifying assumption is that there are only shocks to the natural rate of interest. Consider now money demand shocks and suppose there are no shock to the natural rate of interest. Then the data traces out a horizonal line at a constant natural rate of interest.

To sum up, the key identifying assumptions required to empirically recover quasi-money demand using the empirical strategy in the previous literature is that shocks to the natural rate of interest are the main shocks perturbing the model and that the money supply is determined according to optimal discretion (i.e. money supply is set to stabilize the output gap whenever possible). Under these two identifying assumptions then quasi-money demand is exactly identified. We do not attempt to empirically validate these identifying assumptions, even if we note that the Figure 3 seems well consistent with them. Instead, our main point is that *even if* we take the validity of these assumptions as granted, the identified relationship is *not structural*. It has fundamentally different economic interpretation than suggested by the previous literature.

Before proceeding let us comment further on the fact that the empirical identification for the quasi-money demand is conditional on the policy being set according to the Keynesian policy regime. Thus, if we can identify a positive floor on long-term interest rate, the models suggests that this is evidence for the existence of a Keynesian policy regime. The converse, however, is not true. If the empirical test does not find floor statistically different from zero, this should not necessarily be interpreted as evidence in favor of a monetarist policy regime, or even contradict a Keynesian policy regime. We discuss this point further in the next section that considers more broadly how the empirical test can be interpreted, assuming the conditions for a correct empirical identification are satisfied.

7. Interpretation of the Empirical Results

How should one interpret our empirical results in light of the theoretical analysis and assuming that the quasi-money demand is correctly identified empirically? The first major empirical implication of the theory concerns panels (c) and (d) in figure 5. According to the model, the LM curve estimated in section 2 is not structural but depends upon the assumed policy regime. If the policy regime is Keynesian, then the quasi-LM curve asymptotes at $i^{lelb,K}$; if it is monetarist, then the quasi-LM curve asymptotes at i^{elb}. For the United States, this number is approximately zero, in countries that have experimented by negative policy rates the effective lower bound is slightly below that.

The empirical analysis estimated the bound on long-term rates to be well above zero. The empirical results are therefore consistent with the existence of a Keynesian policy regime. The key aspect of the Keynesian policy regime is that an increase in the money supply today does not signal future monetary expansion. Hence increases in the money supply, beyond a certain point, leave long-term interest rates unchanged; more specifically, the point estimate is 1.72 percent using long-term government bonds and 2.2 percent using long-term corporate-bond rates.

[7] The key identifying assumption here is that the money supply is set to stabilize fluctuations in output (and more generally output and inflation). It is not important, however, that this is achieved exactly. Suppose, for example, that there is some control error, so that $\hat{Y}_t = \varepsilon_t^y$. The implication of this is that this error term would appear in both (21) and (22), thus further increasing the "cloud" around the line traced by the data in Figure 5 c).

Should this result be surprising? The consensus in the economics profession prior to the Great Recession was that the Federal Reserve had the power to prevent major recessions, such as the Great Depression. This was a key lesson many took from Friedman and Schwartz's (1963) classic book *A Monetary History of the United States*.

The monetarist thesis was that the money-demand equation is relatively stable. With a stable money demand, if the Federal Reserve increased the money supply, this would lead to a reduction in the long-term interest rate and an increase in demand. This thesis was summarized by Milton Friedman in discussing Bank of Japan policy options after a talk he delivered at the Bank of Canada in 2000: "Now, the Bank of Japan's argument is, 'Oh well, we've got the interest rate down to zero; what more can we do?' It's very simple. They can buy long-term government securities, and they can keep buying them and providing high-powered money until the high-powered money starts getting the economy in an expansion."

Our empirical result rejects this monetarist hypothesis. The data suggest that at a certain point, a monetary expansion does not lead to an expansion in aggregate demand, at least as measured by a reduction in long-term interest rates. The most natural interpretation of this finding is that static changes in the money supply do not necessarily translate into changes in expectations about future monetary policy.

It is worth stressing, however, that this interpretation does not contradict the central thrust of the monetarist argument—namely, that monetary policy can have an effect, even if the ZLB is binding. The result, instead, simply suggests that monetary policy does not operate via static changes in the supply of money.

But through what alternative mechanism can monetary policy operate. In the model we outlined, any policy that changes expectations about the future money supply (or more generally the future–interest rate reaction function of the central bank) will influence aggregate demand. This is indeed the major theme in the modern literature that emerged following the Japanese malaise (see Krugman (1998) and Eggertsson and Woodford (2003) for two early examples). It is via the effect on expectations that monetary policy operates, not via changes in the supply of the money supply today.

There are interesting examples from economic history of policy regimes where the policy maker manipulated expectations of future policy at the zero lower bound. Here we give two examples. As argued by Eggertsson (2008), Franklin Delano Roosevelt's (FDR's) abandonment of the gold standard and commitment to inflating the price level, together with an aggressive fiscal expansion, was arguably an effective way of signaling a monetary expansion in the spring of 1933. Considerable evidence suggests that inflation expectations shifted upward in the spring of 1933, which should have directly increased aggregate spending.

The type of monetary easing described by Eggertsson (2008) did not depend on any static changes in the money supply. The monetary base did not change much in 1933 when inflation expectations abruptly changed from people expecting rapid deflation to expecting modest inflation (in fact, the real money supply was lower in the fall of 1933 than the spring of that year, as documented by Eggertsson (2008)). In contrast to the traditional monetarist view, what was important was not static variations in the money supply but instead expectations about the future money supply. In 1933 the expectation of the future money supply, or more precisely the future monetary policy regime, abruptly changed with FDR's rise to power.

The expansionary policy regime we considered (what we called a monetarist regime) is only one example of a policy regime that can generate credible signals of future monetary easing. The FDR policy is another one (and it required no change in the current money supply). Yet another one is the policy the Federal Reserve implemented in the fall of 2012 when it stipulated it would not increase the interest rate until certain conditions were met.[8] By some accounts, this policy was successful in lowering the long-term interest rate, yet it required no change in the current money supply.

That the quasi-money-demand function is estimated to asymptote at a positive interest rate is suggestive of a Keynesian policy regime. What if we had found it to asymptote at zero? Should this be interpreted as contradicting that policy is set according to a Keynesian policy regime and perhaps evidence for a monetarist regime? The answer is no. The reason can be seen in the formula for $i^{lelb,K}$ in (15). As this formula suggests, it is possible that quasi-money-demand under the Keynesian policy regime also asymptotes at zero if people believe the natural rate of interest to be permanently negative. Indeed, we conjecture that if a similar statistical analysis is conducted as we do in section 2 for the case of Japan, we suspect that the bound on long term interest rates is not statistically different from zero. This, however, need not be a rejection of the existence of a Keynesian policy regime in Japan, but instead a reflection of that the public expects the ZLB to be binding for a very long time. It may also reflect that the zero lower bound is in fact not exactly zero in Japan, but instead modestly negative (at -0.1 since 2015), with the Bank of Japan experimenting with negative policy rates since 2015. The theory then suggests that the quasi-LM curve will asymptote at a rate above -0.1 if markets expect the current rate to correspond to the effective lower bound of policy rates.

8. Conclusions

A narrative emerged following the 2008 crisis that the standard IS-LM model was “good enough for government work” to use a popular saying. The argument was that despite the revolution in macroeconomics over the previous fifty-plus years, at the end of the day, the Hicksian IS-LM model provides a sufficient policy framework—one that is in some cases even superior to modern analysis, typically based upon microfounded DSGE models. This argument is, for example, forcefully put forth by Krugman (2018). Several other prominent economists voiced similar sentiments.[9]

Since Hicks’s original article was written in 1937, some observers may have the impression that little has been gained in the past eighty-plus years and some insights even lost. In other words, the rational-expectations revolution, and the quest for microfoundations for macroeconomics added little if anything to macroeconomic theory—not to mention practical policy.

There is some truth to this narrative. After all, those wedded to the old paradigm correctly anticipated, following the crisis of 2008, that budget deficits would not lead interest rates to soar, that the massive increase in monetary aggregates would not lead to inflation, that an increase in government spending would lead to more than a one-to-one increase in output, and that the turn of some governments during this period to fiscal austerity would be counterproductive. None of these predictions were obvious, and many economists armed with more modern modeling frameworks were led to take the opposite view, which arguably was less in tune with how things turned out.

Yet this narrative glosses over a fundamental issue: the IS-LM model, as proposed by Hicks, left unanswered the basic question of Keynes’s *General Theory*. Is monetary policy unable to stimulate demand in a recession, while fiscal policy retains its power (per Keynes)? Or is monetary policy effective at fighting

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[8] In a statement in December 2012, for example, the Federal Open Market Committee of the Federal Reserve announced that it would not raise interest rates until certain conditions for unemployment and inflation were met. Many empirical accounts suggest that this policy was successful in lowering long-term interest rates.

[9] See, for example, Martin Wolf’s discussion with Lawrence Summers on April 8, 2011, at a conference organized by INET, cited by Mark Toma at *Economist View*. Another example is Romer (2016).

recessions, while fiscal policy is not (per the classics/monetarists)? Hicks's analysis was consistent with *either* answer and suggested that which scenario applies depends on the elasticity of money demand. Moreover, several authors later argued in a series of articles in *Econometrica* that a relatively straightforward empirical test could be brought to bear on this question, using Hicks IS-LM framework. The literature converged on the view that there was no evidence of a liquidity trap based upon the IS-LM framework, using data that included the Great Depression, a period of extended low nominal interest rates.

At a broad level, one of the main contributions of this paper is to give a clear illustration of the general value of microfoundations for economic analysis. It is hard to think of a better example than Hicks's original formalization of Keynes's *General Theory* to make this point, and the proposed empirical test of the liquidity trap, the very topic that animated *The General Theory*.

Why are microfoundations helpful? In Hicks's analysis, there is no way of determining what, exactly, is the correct measure of interest rates, what is the most suitable measure of money, how short-term rates are related to long-term rates, how the risk-free interest rate is related to risky ones, and so on. We invite the reader to review some of the older literature we have cited in this paper. Reading these articles, the modern reader is immediately struck by the amount of space various authors have filled arguing over the correct measures of each variable, the correct specification of how one variable relates to another one, and so on. Should total assets enter the money-demand function? Should aggregate output? Should the short-term or long-term interest rate enter money demand? Should the measure of interest rate include risk premia? Authors take different positions in answering these questions, and each provides reasonable arguments for their positions. Ultimately, however, several areas of disagreement are left largely unresolved. Even more distressingly, it is not even obvious, given the IS-LM framework, how one can resolve these controversies *in principle*.

The key problem is that the older literature does not have clear criteria for how to answer these questions. In contrast, the modern literature offers a clear way forward. Since modern DSGE models are derived by fully specifying the environment agents live in and how they make decisions, the researcher can simply *ask* the agents that live inside the model. For example, in equation (13), we arrived at a formula relating long-term to short-term rates, derived from the agents' optimal asset holdings. Equation (7) relates money, aggregate output and the short-term interest rate, similarly defined by agents' optimal condition for holding cash. And perhaps more importantly, since all agents' problems are explicitly stated, along with a policy regime, it becomes clear how each relationship depends on the assumed policy regime, as illustrated by equations (17) and (18).

The fact that the model we sketched out here is derived from microfoundations does of course not necessarily make it correct. It does, however, allow one to conclusively answer how each variable is related to another and provides a systematic way of documenting how these relationships change depending on the assumptions. As it turns out, this provides fundamentally new insights into the literature that emerged about the liquidity trap following Hicks's statement of the IS-LM model and into the later work in the Keynesian and monetarist traditions.

The analysis provides a fundamentally different interpretation of the empirical tests for the existence of a liquidity trap found in the earlier literature. As we have documented, new data overturns existing empirical results. This would seem to allow the Keynesians to snatch victory from the monetarists, at least according to the old interpretation of this literature. Yet our theoretical analysis, based upon explicit microfoundations, shows that the money-demand equation this literature studied is not "structural," to use the language of modern general equilibrium theory.

This means that what we should expect to find using this statistical test depends on the underlying policy regime. The evidence, in other words, does not prove the existence of a Keynesian liquidity trap, but instead indicates that the data is consistent with a Keynesian monetary policy regime, in which static increases in the supply of money do not convey any useful information about future monetary policy.

Yet, despite this new empirical result, and in sharp contrast to the Keynesian proposition in Hicks's IS-LM model, there is indeed a fundamental role for monetary policy. However, contra Friedman and the monetarists, the key to successful monetary policy is not static variation in some monetary aggregate, which then feeds into the long-term interest rate and aggregate demand via a stable money-demand equation. Instead, monetary policy is successful only via the successful management of expectations about *futur*e monetary policy actions once interest rates are no longer constrained by the zero-lower bound. This type of expectation management, however, does not need to depend upon the supply of money at the ZLB.

This insight, of course, is the major theme of the modern literature on the liquidity trap, see for example Krugman (1998) and Eggertsson and Woodford (2003). It is thus the modern treatment of expectations—a critical omission of the IS-LM model readily admitted by Hicks[10] —that is the fundamental element needed to understand both the effect of monetary and fiscal policy on aggregate demand as well as giving a useful interpretation of existing empirical evidence. It seems difficult to imagine how these insights could have been obtained without the rational expectation revolution or in the absence of microfoundations for macroeconomic theory.

[10] Modigliani (1944, p. 56) also highlights the limits of the IS-LM model and the role of expectations: "In the diagram we have assumed that there is a single rate of interest r, instead of a whole system of rates for loans of different duration. While it may be assumed that in principle all the rates tend to move in the same direction, we must bear in mind that the extent to which a change in the supply of money changes the rates on loans of different maturities depends on the character of interest expectations."

APPENDIX A: DATA SOURCES

Table III

Data Sources

	(1900-1958)	(1959-2019)
M1 (M_t)	*Historical Statistics of the United States*[1]	Federal Reserve Economic Data[1]
Corporate Interest Rates (i_t)	*Historical Statistics of the United States*[2]	*Historical Statistics of the United States* and Federal Reserve Economic Data[2]
National Income (Y_t)	*Historical Statistics of the United States and Federal Reserve Economic Data*[3]	Federal Reserve Economic Data[3]
Assets (A_t)	*A Study of Saving in the United States* and Board of Governors of Federal Reserve System[4]	Board of Governors of Federal Reserve System[4]
Government Interest Rates (i_t)	Jordà-Schularick-Taylor Macrohistory Database and Federal Reserve Economic Data[5]	Federal Reserve Economic Data[5]
Monetary Base (M_t)	Jordà-Schularick-Taylor Macrohistory Database[6]	Jordà-Schularick-Taylor Macrohistory Database and Federal Reserve Economic Data[6]

[1] From 1899 to 1957, Series X 267 in U.S. Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1957*, Washington, D.C., 1960—original source of Pifer (1969) and White (1972); for 1958, Series Cj49 in *Historical Statistics of the United States, Millennial Edition Online*, Cambridge University Press, Cambridge (England), 2006; from 1959 to 2019, 'M1SL' Series in Federal Reserve Economic Data.

[2] From 1900 to 1957, Series X 346 (20-year maturity) in U.S. Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1957*, Washington, D.C., 1960—original source of Pifer (1969) and White (1972); from 1958 to 1975, Series Cj1241 (20-year maturity) in *Historical Statistics of the United States, Millennial Edition Online*, Cambridge University Press, Cambridge (England), 2006; from 1975 to 2019, 'AAA' Series (20-year-and-above maturity) in Federal Reserve Economic Data.

[3] From 1900 to 1957, Series F 7 in U.S. Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1957*, Washington, D.C., 1960; from 1958 to 2019, 'NICUR' Series in Federal Reserve Economic Data.

[4] From 1900 to 1949, the series has been constructed following the original source of Pifer (1969) and White (1972)—Meltzer (1963a):
"Total assets (A) were taken from Goldsmith, [R. W. Goldsmith, *A study of Savings in the United States*, Princeton University Press, Princeton N.J., 1956], Tables W-9 through W-16, and were available for eight benchmark dates from 1900-1949. The series used in the estimates for 1900-1945 was obtained by taking the ratio of total assets, A, to wealth, G (from Table W-1). Denote this ratio for any base year as b_0. Let b_n = the next base year ratio, and let $i = 0, 1, \ldots, n$ be an index of the years between base year dates inclusive of the end points. Then $A_i = [\frac{(n-i)}{n} b_0 + \frac{i}{n} b_n]$."; from 1950 to 2019, the series has been built summing the Series FL102000005.A, FL112000005.A, FL152000005.A, FL212000095.A, FL312000095.A, and FL792000095.A in the Financial Accounts of Board of Governors of Federal Reserve System.

[5] From 1900 to 1953, 'ltrate' Series (10-year maturity) in Jordà-Schularick-Taylor Macrohistory Database; from 1954 to 2019, 'GS20' Series (20-year maturity) in Federal Reserve Economic Data.

[6] From 1900 to 2016, 'narrowm' Series in Jordà-Schularick-Taylor Macrohistory Database; from 2017 to 2019, 'BOGMBASE' Series in Federal Reserve Economic Data.

APPENDIX B: ESTIMATION METHODS

B.1 PIFER (1969) ESTIMATION METHOD

Pifer (1969) proposes the following nonlinear two-step maximum likelihood method to estimate the i^{min}:

$$\max_{i^{min},\gamma,\omega,\alpha} L(y; i^{min}, \gamma, \omega, \alpha) = \max_{i^{min}}[\max_{\gamma,\omega,\alpha} L(y; \gamma, \omega, \alpha | i^{min})]$$

$$\text{subject to } L(\gamma, \omega, \alpha | i^{min}) = -e'e$$

$$y = \log(M_t) = \log(\gamma) + \omega \log(Y_t) + \alpha \log(i_t - i^{min}) + \xi_t$$

The first step is constructing a grid for i^{min} and for each possible value of i^{min} running the above regression to calculate the sum of squared residuals.[11] The second step is minimizing such sum, that is maximizing the likelihood function L, to identify the maximum likelihood estimate of i^{min}.

[11] We construct a grid for i^{min} from −1.5 to the minimum value of the series of long-term government-bond (corporate-bond) interest rates, in increments of 0.01.

B.2 WHITE (1972) ESTIMATION METHOD

White (1972) proposes the following generalization of Pifer 's estimation method to estimate the i^{min}:

$$\max_{\lambda, i^{min}, \gamma, \omega, \alpha} L(y; \lambda, i^{min}, \gamma, \omega, \alpha) = \max_{\lambda, i^{min}} [\max_{\gamma, \omega, \alpha} L(y; \gamma, \omega, \alpha | \lambda, i^{min})]$$

$$\text{subject to } L(\gamma, \omega, \alpha | \lambda, i^{min}) = -\frac{T}{2} \log(e'e) + (\lambda - 1) \sum_{t=1}^{T} \log(M_t)$$

$$y = \frac{M_t^{\lambda} - 1}{\lambda} = \frac{\gamma^{\lambda} - 1}{\lambda} + \omega \frac{Y_t^{\lambda} - 1}{\lambda} + \alpha \frac{(i_t - i^{min})^{\lambda} - 1}{\lambda} + \xi_t$$

The first step is constructing a two-dimensional grid for λ and i^{min} and for each possible point in this grid running the above regression to calculate the corresponding likelihood function L.[12] The second step is maximizing the latter to identify the maximum likelihood estimate of i^{min}.

[12] We construct a two-dimensional grid for λ and i^{min} from −1.5 to 1.5 (excluding 0) and from −1.5 to the minimum value of the series of long-term government-bond (corporate-bond) interest rates, in increments of 0.01.

APPENDIX C: RESULTS WITH THE MONETARY BASE

Table IV

Estimates of λ and i^{min} Using the Monetary Base

Specification	$\frac{M_t^\lambda - 1}{\lambda} = \frac{\gamma^\lambda - 1}{\lambda} + \omega \frac{Y_t^\lambda - 1}{\lambda} + \alpha \frac{(i_t - i^{min})^\lambda - 1}{\lambda} + \delta \frac{A_t^\lambda - 1}{\lambda} + u_t$	
	λ	i^{min}
Government Interest Rates	-0.07	1.69
	(-0.12, -0.03)[1]	(1.35, 1.84)[1]
	(-0.13, -0.02)[2]	(1.21, 1.86)[2]
Corporate Interest Rates	-0.13	2.13
	(-0.18, -0.08)[1]	(1.84, 2.26)[1]
	(-0.19, -0.07)[2]	(1.72, 2.28)[2]

[1] 95%, and [2] 99% confidence interval, see Greene (2018, p. 554).

APPENDIX D: MODEL DERIVATION

D1. HOUSEHOLDS

There is a continuum of identical households of measure 1. Household j solves the following maximization problem

$$\max_{\{C_t(j), M_t(j), B_t(j), B_t^r(j), B_t^l(j), N_t(j)\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t(j)) + \chi\left(\frac{M_t(j)}{P_t}\right)\xi_t^d + w\left(\frac{A_t(j) - A_t}{P_t}\right) - v(N_t(j)) \right] \xi_t$$

subject to
$$P_t C_t(j) + M_t(j) + B_t(j) + B_t^r(j) + S_t B_t^l(j)$$
$$= W_t N_t(j) + M_{t-1}(j) + (1 + i_{t-1}) B_{t-1}(j) + (1 - \omega_t)(1 + i_{t-1}^r) B_{t-1}^r(j) + (1 + \rho S_t) B_{t-1}^l(j)$$
$$+ \int_0^1 Z_t(i, j) di$$

$$A_t(j) = M_t(j) + B_t(j) + B_t^r(j) + S_t B_t^l(j)$$

$$i_t \geq 0$$

Here β is an intertemporal discount factor, $C_t(j) \equiv \left[\int_0^1 c_t(i,j)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$ is the aggregate consumption, $M_t(j)$ is the amount of dollars that the household holds at the end of period t, $B_t(j)$ is a risk-free nominal bond that pays i_t number of dollars at time $t+1$, $B_t^r(j)$ is a nominal bond that pays i_t^r number of dollars at time $t+1$ but with probability ω_{t+1} it will not be repaid, $B_t^l(j)$ is a perpetuity that pays out ρ^j dollars in period j+1 and S_t is its price, $N_t(j)$ is the labor supply that the household offers, ξ_t^d is a money demand shock, and ξ_t is a preference shock.

The function $u(.)$ is the period utility of consumption, it is increasing and concave in its argument and at least twice differentiable. The function $\chi\left(\frac{M_t(j)}{P_t}\right)$ denotes utility of holding real money balances. It is increasing and concave in its argument. Define the real money balance $m_t \equiv \frac{M_t}{P_t}$, we assume that there is satiation at m^*, so that, the partial derivative of the function χ, $\chi_m(m_t) = 0$ for $m_t \geq m^*$. The function w represents utility households have from its asset holding, $A_t(j)$, relative to the aggregate asset holding in the economy A_t as in Michaillat and Saez (2019). $P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$ denotes the aggregate price index, and W_t the nominal wage rate, $Z_t(i)$ is profit of firm i.

In equilibrium all household hold the same assets so that $A_t(j) = A_t = 0$. We substitute this equilibrium condition in the optimality conditions below, to simplify the notation, and also omit reference to j.

The solution of the maximation problem is standard and can be obtained by formulating a Lagrangian. Combining the first-order conditions of the Lagrangian problem results in the Euler equation, the money demand equation, the asset pricing equations, and the labor supply equation reported below

EE
$$u_C(C_t) = \beta(1+\ i_t)E_t[u_C(C_{t+1})\frac{\xi_{t+1}}{P_{t+1}}]\frac{P_t}{\xi_t} + w_A(0)$$

MD
$$\frac{\chi_m(m_t)}{u_C(C_t)}\xi_t^d \geq \frac{i_t}{(1+i_t)} - \frac{i_t}{(1+i_t)}\frac{w_A(0)}{u_C(C_t)}; \qquad i_t \geq 0$$

AP1
$$u_C(C_t) = \beta(1+\ i_t^r)E_t\left[(1-\omega_{t+1})u_C(C_{t+1})\frac{\xi_{t+1}}{P_{t+1}}\right]\frac{P_t}{\xi_t} + w_A(0)$$

AP2
$$u_C(C_t) = \beta E_t\left[(1+\rho S_{t+1})u_C(C_{t+1})\frac{\xi_{t+1}}{P_{t+1}}\right]\frac{P_t}{\xi_t\, S_t} + w_A(0)$$

LS
$$\frac{v_N(N_t)}{u_C(C_t)} = \frac{W_t}{P_t}$$

D2. ASSET PRICING

The duration of the perpetuity is defined as

$$D \equiv \frac{\beta \sum_{j=0}^{\infty}(j+1)(\beta\rho)^j}{\beta \sum_{j=0}^{\infty}(\beta\rho)^j}$$

while its yield at time t is the i_t^{long} that solves the equation:

$$S_t = \sum_{j=0}^{\infty} \frac{(\rho)^j}{\left(1 + i_t^{long}\right)^{j+1}}$$

which implies

AP3 $$1 + i_t^{long} = S_t^{-1} + \rho$$

D3. FIRMS

There is one firm for each good i which faces the demand function $y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t$ which is implied by the optimal spending decision of the household across good types.

We assume that a fixed fraction of firms γ set their prices flexibly, while the remaining fraction $1 - \gamma$ index their prices to the past price level.

The flexible-price firm i maximize period profits at time t

$$\max_{p_t(i),\, N_t(i)} Z_t(i) = p_t(i) y_t(i) - W_t N_t(i)$$

subject to $y_t(i) = N_t(i)$

$$y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t$$

We obtain the optimal pricing condition for the firm i from the first-order condition of the profit maximation, and assuming a symmetric equilibrium so that $p_t(i) = p_t^{flex}$

$$\frac{p_t^{flex}}{P_t} = \frac{\theta}{\theta - 1} \frac{W_t}{P_t}$$

The firms that index their prices set

$$p_t^{index} = P_{t-1}$$

The price index at time t can be now written as

$$P_t = \left[\gamma(p_t^{flex})^{1-\theta} + (1-\gamma)(p_t^{index})^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

Since production is linear in labor, aggregate hours are given by

$$N_t = \int_0^1 y_t(i)di = Y_t \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\theta} di = Y_t \mathit{\Delta}_t$$

where $\mathit{\Delta}_t \equiv \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\theta} di$.

The labor supply can be now expressed as

$$\frac{v_N(Y_t \mathit{\Delta}_t)}{u_C(C_t)} = \frac{W_t}{P_t}$$

Using this and the expressions for p_t^{flex} and p_t^{index} we obtain the price dispersion

PD $$\mathit{\Delta}_t = \gamma\left(\frac{\theta}{\theta-1}\frac{v_N(Y_t \mathit{\Delta}_t)}{u_C(C_t)}\right)^{-\theta} + (1-\gamma)(\Pi_t^{-1})^{-\theta}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$.

Similarly, following the same steps, the price index can be used to state a non-linear Phillips curve

PC $$1 = \gamma\left(\frac{\theta}{\theta-1}\frac{v_N(Y_t \mathit{\Delta}_t)}{u_C(C_t)}\right)^{1-\theta} + (1-\gamma)(\Pi_t^{-1})^{1-\theta}$$

D4. Market Clearing

Assume that all production is consumed

MC $$Y_t = C_t$$

D5. EQUILIBRIUM DEFINITION IN THE NON-LINEAR MODEL

An equilibrium is a collection of stochastic processes for $\{P_t, C_t, Y_t, i_t, i_t^r, S_t, i_t^l, \Delta_t\}$ that solve the Euler equation (EE), the money demand equation (MD), the asset pricing equations (AP1, AP2, AP3), the price dispersion equation (PD), the Phillips curve (PC), and the market clearing equation (MC) given the exogenous shocks $\{\xi_t^d, \xi_t, \omega_t,\}$ and policy specification for the sequence $\{M_t\}$.

D6. STEADY STATE

We consider a steady state in which inflation is zero, i.e. $\bar{\Pi}_t = 1$, and there is no price dispersion, i.e. $\Delta_t = 1$. Steady state output, denoted by $\bar{Y}$, then solves

$$\frac{v_N(\bar{Y})}{u_C(\bar{Y})} = \frac{\theta - 1}{\theta}$$

Define

$$\delta \equiv 1 - \frac{w_A(0)}{u_C(\bar{Y})}$$

We assume that $u_C(\bar{Y}) > w_A(0) \geq 0$, so that $0 < \delta \leq 1$.

The Euler equation in steady state implies that

$$1 + \bar{\imath} = \beta^{-1}\delta$$

We assume that the steady-state interest rate is positive, which implies the restriction that δ> β.

Let us denote by $\bar{\omega}$ the "steady state" value of ω_t; the steady-state risky interest rate is implied by the Euler equation and the first asset pricing equation

$$1 + \bar{\imath}^r = \frac{1 + \bar{\imath}}{1 - \bar{\omega}} = \frac{\beta^{-1}\delta}{1 - \bar{\omega}}$$

The Euler equation and the second asset pricing equation implies that

$$\bar{S} = \frac{1}{1 + \bar{\imath} - \rho} = \frac{1}{\beta^{-1}\delta - \rho}$$

The money demand equation can be used to solve for steady-state real money balance $\bar{m}$

$$\frac{\chi_m(\bar{m})}{u_C(\bar{Y})}\bar{\xi}^d \geq \frac{\bar{\imath}}{(1+\bar{\imath})} - \frac{\bar{\imath}}{(1+\bar{\imath})}\frac{w_A(0)}{u_C(\bar{Y})};\ \bar{\imath} > 0$$

Finally, from the second and the third asset pricing equation, we obtain

$$1 + \bar{\imath}^l = \frac{1 + \rho\bar{S}}{\bar{S}} = (1+\bar{\imath})$$

D7. LOG-LINEAR APPROXIMATION

We define the elasticity of intertemporal substitution $\sigma \equiv -\frac{u_C(\bar{Y})}{u_{CC}(\bar{Y})\bar{Y}} > 0$ and an approximation of the Euler equation yields the IS curve

is $$\hat{Y}_t = \delta E_t\hat{Y}_{t+1} - \sigma\delta(\hat{\imath}_t - E_t\pi_{t+1} - \hat{r}_t^n)$$

where $\hat{r}_t^n \equiv \hat{\xi}_t - E_t\hat{\xi}_{t+1}$, with $\hat{\xi}_t \equiv \log\xi_t - \log\bar{\xi}$, and with the other variables defined as $\hat{Y}_t \equiv \log Y_t - \log\bar{Y}$, $\pi_t \equiv \log P_t - \log P_{t-1}$, and $\hat{\imath}_t \equiv \log(1+i_t) - \log(1+\bar{\imath})$.

Approximating the Euler equation and the first asset pricing equation yields

as1 $$\hat{\imath}_t = \hat{\imath}_t^r - E_t\hat{\omega}_{t+1}$$

where $\hat{\imath}_t^r \equiv \log(1+i_t^r) - \log(1+\bar{\imath}^r)$, and $\hat{\omega}_t = \frac{\omega_t - \bar{\omega}}{1-\bar{\omega}}$.

Approximating the Euler equation and the second asset pricing equation yields

as2 $$\hat{S}_t = \frac{\beta\rho}{\delta}E_t\hat{S}_{t+1} - \hat{\imath}_t$$

where $\hat{S}_t \equiv \log S_t - \log\bar{S}$.

Approximating the second asset pricing equation yields

as3 $$\hat{S}_t = -\frac{\delta}{\delta - \beta\rho}\hat{\imath}_t^l$$

where $\hat{\imath}_t^l \equiv \log(1+i_t^l) - \log(1+\bar{\imath}^l)$.

Using the last two equations, we then obtain a relation between i_t^l and all the series of the short-term risk-free interest rates under the assumption that $\rho < \frac{\delta}{\beta}$

$$\hat{\imath}_t^l = (\frac{\delta-\beta\rho}{\delta})E_t \sum_{j=0}^{\infty}(\frac{\beta\rho}{\delta})^j \hat{\imath}_{t+j},$$

We define the interest elasticity of real money balances $\psi \equiv -\frac{\chi_m(\bar{m})}{\chi_{mm}(\bar{m})\bar{m}} > 0$ and an approximation of the money demand equation yields the LM curve

lm/elb $$\widehat{m}_t \geq \eta_y \hat{Y}_t - \eta_i \hat{\imath}_t + \varepsilon_t^d; \quad \hat{\imath}_t \geq i^{elb}$$

where $\eta_y \equiv \psi\sigma^{-1}\frac{\bar{\imath}}{\bar{\imath}+1-\delta} \geq 0$, $\eta_i \equiv \psi\frac{\beta}{1-\beta} \geq 0$, $\widehat{m}_t \equiv \log m_t - \log\bar{m}$, and $\varepsilon_t^d = \psi(\log\xi_t^d - \log\bar{\xi}^d)$

Linearizing the Phillips curve yields the AS curve

as $$\pi_t = \kappa\hat{Y}_t$$

where $\kappa \equiv \frac{\gamma}{1-\gamma}(\sigma^{-1} + \varphi) > 0$ and $\varphi \equiv \frac{v_{NN}(\bar{N})\bar{N}}{v_N(\bar{N})} > 0$ (inverse Frisch elasticity).

Let us define the nominal money growth by $\mu_t \equiv \frac{M_t}{M_{t-1}}$. This definition implies that $m_t \equiv m_{t-1}\mu_t\Pi_t^{-1}$ or, expressed with a log-linear approximation

mg $$\widehat{m}_t \equiv \widehat{m}_{t-1} + \hat{\mu}_t - \pi_t$$

D8. Approximated Equilibrium Definition

An approximated equilibrium is a collection of stochastic processes $\{\pi_t, \hat{Y}_t, \hat{\imath}_t, \hat{\imath}_t^r, \hat{S}_t, \hat{\imath}_t^l, \widehat{m}_t\}$ that solve the IS curve (is), the approximated asset pricing equations (as1, as2 and as3), the LM curve (lm/elb), the Phillips Curve (as), and the nominal money growth equation (mg) given the exogenous shocks $\{\varepsilon_t^d, \hat{r}_t^n, \widehat{\omega}_t\}$ and a policy specification for the sequence $\{\hat{\mu}_t\}$.

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