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ENDOGENOUS PRODUCT CYCLES

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ABSTRACT

We construct a model of the product cycle featuring endogenous innovation and endogenous technology transfer. Competitive entrepreneurs in the North expend resources to bring out new products whenever expected present discounted value of future oligopoly profits exceeds current product development costs. Each Northern oligopolist continuously faces the risk that its product will be copied by a Southern imitator, at which time its profit stream will come to an end. In the South, competitive entrepreneurs may devote resources to learning the production processes that have been developed in the North. There too, costs (of reverse engineering) must be covered by a stream of operating profits. We study the determinants of the long-run rate of growth of the world economy, and the long-run rate of technological diffusion. We also provide an analysis of the effects of exogenous events and of public policy on relative wage rates in the two regions.

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I. Introduction

The product cycle features prominently in trade between the Northern developed countries and the Southern newly-industrializing countries. In his seminal article on the subject, Vernon (1966) described the "life cycle" of a typical manufactured product. Development and initial manufacturing of new products takes place in the North, he argued, because R&D capabilities are well developed there and because proximity to large, high-income markets facilitates the innovation process. After a while, the production methods become more standardized. Then, technology transfer or imitation by Southern firms takes place, whereupon the bulk of production migrates to the South to capitalize on the relatively cheap labor there. Interregional trade in manufactured goods involves exchange of the latest, innovative goods, produced only in the North, for older, more established goods, produced predominantly or entirely in the South.

The first attempt at formal modeling of this phenomenon was carried out by Krugman (1979). He posited an exogenous rate, g (our notation), of introduction of new products in the North, and an exogenous rate, μ, of technology transfer to the South. By hypothesis, then, the total number of products known to the world evolves according to $\dot{n}/n = g$, while the number of products that the South is able to produce evolves according to $\dot{n}_S = \mu n_N$, where n_N is the number of products in which the North temporarily maintains exclusive productive capacity. These exogenous processes ensure the existence of a steady state in which the share of Northern products in the total number of products, $\sigma_N = n_N/n$, is equal to $g/(g+\mu)$. Adding some economic structure to the model, Krugman finds a positive relationship between the relative wage paid to Northern labor (w_N/w_S) and μ/g, and an inverse relationship between the relative wage and the relative size of the Northern labor force.

Krugman's work has since been extended by Dollar (1986) and Jensen and Thursby (1986, 1987). Dollar maintains Krugman's assumption of an exogenous rate of product innovation, but relates the rate of technology transfer to the North-South terms of trade, albeit in an entirely ad hoc manner. Jensen and Thursby (1986) attempt to capture the resource costs of product development and technology transfer and the decision processes that determine these expenditures, but they assume that all innovation is carried out by a single, monopolist entrepreneur in the North, and that the allocation of resources to reverse engineering in the South is made by a social planner. Their later (1987) paper does allow for a fixed number (perhaps greater than one) of innovators in the North, but reverts to the assumption of an exogenous rate of Southern imitation. Moreover, their analysis in both papers is partial equilibrium in nature, inasmuch as they take the interest rate as given.[1]

In this paper, we build upon our earlier work on product development and international trade (1988, 1989a) to construct a model of the product cycle featuring endogenous innovation and endogenous technology transfer. In our model, competitive entrepreneurs in the North expend resources to bring out new products whenever the expected present discounted value of future oligopoly profits exceeds current product development costs. Each Northern oligopolist continuously faces the risk that its product will be copied by a Southern imitator, at which time its profit stream will come to an end. Thus, the length of the initial phase in the life cycle for each product (i.e., when production occurs in the North) is a random variable. In the South,

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[1] A recent paper by Segerstrom et.al (1987) does provide, in a somewhat different framework, a more satisfactory depiction of the competitive process leading to the introduction of "improved" products, but they also assume that technology transfer is automatic and costless, and occurs after a fixed, exogenous period of time.

competitive entrepreneurs may devote resources to learning the production processes that have been developed in the North. There too, costs (of reverse engineering) must be covered by a future stream of operating profits. In all this, interest rates are determined endogenously so as to equate savings and investment.

Our approach enables us to discuss the determinants of the long-run rate of growth of the world economy and the long-run rate of technological diffusion. We find steady-state values for g and μ, and relate these to underlying structural characteristics of the world economy (the sizes of the two trading blocs, the productivities of resources in their various uses, and the nature of demand for the differentiated manufactured goods), and to the commercial and industrial policies enacted by the two governments. Also, we provide an analysis of the effects of exogenous events and of public policy on relative wage rates in the two regions, and find that Krugman's (1979) results derived for the case of g and μ exogenous may in fact be misleading. For example, when we allow for the changes in the steady-state rates of innovation and imitation that are induced by variations in the two labor forces, we find that the direction of movement in relative wages in the steady state is exactly the opposite of that predicted by Krugman.

The remainder of our paper is organized as follows. We develop our model of the product cycle in the next section. In Section III, we solve for the steady-state equilibrium and discuss its dependence on structural features of the world economy. We devote Section IV to policy analysis, considering there the long-run effects of subsidies to innovation in the North, of subsidies to reverse engineering (or learning) in the South, and of trade policies in both regions. The concluding section contains a summary of our findings.

II. A Model of the Product Cycle

We study a world economy comprising two countries or regions, denoted by "North" and "South". The regions differ only in their abilities to innovate. The North enjoys absolute (and comparative) advantage in developing new products and bringing them to market. Indeed, for much of the paper we shall assume that Southern productivity in product innovation is sufficiently low that the South performs none of this activity in the trading equilibrium.

We consider a world of symmetrically differentiated products. There exists a continuum of potential goods that are desirable to consumers, but only a subset of these (of finite measure) are produced at any point in time. Before any product can be manufactured and sold to consumers resources must be devoted to "developing" the product; that is, the good must be designed, the production techniques perfected, etc.

All consumers worldwide share identical preferences for the differentiated products. Each consumer seeks to maximize the time-separable intertemporal utility function

$$(1) \qquad U_t = \int_t^{\infty} e^{-\rho(\tau-t)} \log[u(\tau)]d\tau,$$

where ρ is the subjective discount rate and $u(\cdot)$ is the instantaneous sub-utility function given by

$$(2) \qquad u(\tau) = \left[\int_0^n x(\omega)^{\alpha} d\omega\right]^{1/\alpha}, \quad 0 < \alpha < 1 .$$

In (2), $x(\omega)$ is consumption of differentiated product ω and n (a function of τ) is the (measure of the) number of varieties available on the market.

The representative consumer maximizes (1) subject to an intertemporal budget constraint

$$(3) \qquad \int_t^\infty e^{-[R(\tau)-R(t)]}E(\tau)d\tau \le \int_t^\infty e^{-[R(\tau)-R(t)]}Y(\tau)d\tau + A(t) \ ,$$

where $R(t)$ is the cumulative interest factor from time 0 to t that the consumer faces on the local capital market, $E(\tau)$ and $Y(\tau)$ are his spending and factor income at time τ, respectively, and $A(t)$ represents the value of his asset holdings at t. Our results concerning the steady state do not hinge on whether capital is traded internationally or not; but for ease of exposition we shall assume in what follows that all agents face the same interest rate.

As is well known (see, for example, Grossman and Helpman (1988)), the solution to the intertemporal maximization problem requires

$$(4) \qquad \frac{\dot{E}}{E} = \dot{R} - \rho \ .$$

while (2) implies an instantaneous demand for variety ω given by

$$(5) \qquad x(\omega) = \frac{p(\omega)^{-\epsilon}}{\int_0^n p(\omega')^{1-\epsilon}d\omega'} E \ ,$$

where $p(\omega)$ is the price of variety ω and $\epsilon = 1/(1-\alpha) > 1$ is the (constant) elasticity of substitution between any two varieties.

Consumers use their savings to acquire riskless bonds and/or a portfolio of shares in profit-making firms. As we shall see, the profits of Northern firms are random variables. But all risks are firm specific, so that the consumer-investor can earn a sure return by holding a portfolio consisting of a continuum of such firms. Then arbitrage ensures that, in equilibrium, the return to such a diversified portfolio of Northern firms equals the riskless interest rate, which in turn equals the (certain) return on a share of any Southern firm.

The production sector comprises two distinct activities. Before a producer can begin to manufacture any variety, she must learn the production technique specific to that variety. If the product is a new one (i.e., not previously available in the marketplace), then this learning represents <u>innovation</u>. If, instead, the product already exists on the market, then the learning represents <u>imitation</u>. In either event, the learning activity requires an expenditure of resources by the entrepreneur (presumably more for innovation than for imitation), with productivity parameters that vary by region. After the production technique has been learned by the entrepreneur, she can manufacture the product according to a constant-returns-to-scale production function.

We suppose that there is a single, primary input, which we call labor. Consider first the manufacturing activity. Production of any variety of consumer good in either country requires a_X units of labor per unit of output. Hence, marginal cost for any good produced in country i is $w_i a_X$, where w_i is the wage there, for $i = S$ (South) and N (North). At any point in time, the set of available products and the number of entrepreneurs of each nationality able to produce every product is given. The producers behave as Bertrand competitors, taking the prices of other firms' products and the level of aggregate spending as fixed. They maximize profits by setting marginal revenue equal to marginal cost, as usual.

A Northern producer with proprietary know-how concerning the production technique for some particular variety faces, according to (5) and our assumptions about market structure, a demand curve with constant elasticity equal to $-\epsilon$. Such a firm maximizes instantaneous profits by charging a fixed mark-up over marginal costs, implying a price p_N that satisfies

$$(6) \qquad \alpha p_N = w_N a_X \ .$$

The resulting instantaneous profits are

$$(7) \qquad \pi_N = (1-\alpha) p_N x_N \ ,$$

where x_N is the equilibrium output level, calculated using (5). If two Northern firms were to be capable of producing the same variety (one having imitated the innovation of the other), then as Bertrand competitors with a homogeneous product they would each set a price equal to marginal cost and earn zero profits. It follows, therefore, that no Northern entrepreneur could recoup the fixed costs of imitation, and so none of this activity will take place in the North.

We suppose that the equilibrium wage rate in the South is below that of the North.[2] We rule out the possibility that a Southern firm will have exclusive production capability for some varieties by assuming that the productivity of Northern entrepreneurs as innovators far exceeds that of Southern entrepreneurs. This implies that all innovation occurs in the North, and that all knowledge acquisition in the South takes the form of imitation.

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[2] We will see that this assumption is consistent with the conditions for a steady-state equilibrium for a wide range of parameter values.

If two Southern firms have copied the same variety of consumer good, these two will set prices equal to their marginal costs and earn zero profits. So the second of the imitators could never justify bearing the cost of reverse engineering. It follows that the only case to consider for Southern manufacturers is one where a single Southern firm (imitator) competes with a single Northern firm (innovator) in the market for some particular variety.

Two outcomes may result in this competition, depending on the size of the gap between Northern and Southern wages. If the gap is large, the Southern firm can charge its monopoly price without paying any regard to competition from the Northern innovator. This price for Southern products prevails whenever $w_S a_X/\alpha$ (the monopoly price) falls short of the marginal cost of Northern production, $w_N a_X$, or when $w_S \leq \alpha w_N$. We shall refer to this as the <u>wide-gap case</u>. If relative wages in the South are somewhat higher, then a Southern firm charging its monopoly price would be undercut by its Northern rival. In this <u>narrow-gap case</u> the Southern firm prices just below the marginal cost of the Northern producer and thereby captures the entire market. Thus, we have

(8a) $$p_S = w_S a_X/\alpha \qquad \text{if } w_S \leq \alpha w_N ,$$

(8b) $$p_S = w_N a_X \qquad \text{if } w_S \geq \alpha w_N .$$

The instantaneous profits of a Southern firm are

(9a) $$\pi_S = (1-\alpha) p_S x_S \qquad \text{if } w_S \leq \alpha w_N ,$$

(9b) $$\pi_S = (1-w_S/w_N) p_S x_S \qquad \text{if } w_S \geq \alpha w_N ,$$

where x_S is the firm's level of sales in equilibrium (calculated from (5)). Notice that, in either case, the Southern firm uses its cost advantage to capture the entire sub-market and the Northern firm makes no further sales once its variety has been copied abroad. This feature of the model captures the migration of production from North to South, as first described by Vernon (1966).

We turn next to the learning activities. As in Romer (1988) and Grossman and Helpman (1989a,b), we assume that the resources devoted to industrial research generate two sorts of outputs. First, when an entrepreneur hires labor for purposes of innovation or imitation she derives an appropriable output in the form of a "blueprint" for production of a particular variety. This blueprint is the "entry ticket" into the oligopolistic competition in the final-goods sector and carries the reward of the associated stream of oligopoly profits. At the same time, the development activity (in the North) and the imitation activity (in the South) create non-appropriable, by-product benefits in the form of additions to general knowledge. Knowledge here includes scientific information, as well as some forms of engineering data with widespread applicability, that is generated in the course of developing or copying some particular product, and that contributes to the productivity of later learning efforts. We shall assume here that the stocks of industrial knowledge are specific to the countries in which the knowledge was created (but see footnote 3 below).

Consider now the imitation activity in the South. A Southern entrepreneur chooses at random one of the existing and not-previously-imitated Northern products to copy. In order to learn the production process, the entrepreneur must devote a_I/K_S units of labor to the task, where a_I is a fixed productivity parameter ("I" for imitation) and K_S is the stock of disembodied knowledge capital in the South. We take the stock of knowledge to be proportional to cumulative experience in the learning sector in the South, and choose units so that $K_S = n_S$, where n_S is the number of varieties that Southern firms have imitated in the past. Under this specification,[3]

$$(10) \qquad \dot{n}_S = n_S L_I / a_I ,$$

where L_I represents total labor employed in reverse engineering in the South.

Entry into imitation is assumed free, and can be financed either by a bond issue or an equity offering. If this activity were to offer a pure profit at any point in time, then incipient entry by entrepreneurs would generate excess demand for Southern labor. It follows therefore, that in an equilibrium with some labor devoted to imitation in the South, the present value of Southern profits from manufacturing must just equal the cost of entry into the final-goods sector via reverse engineering, or

$$\int_t^{\infty} e^{-[R(\tau)-R(t)]} \pi_S(\tau) d\tau = w_S(t) a_I / n_S(t) \ .$$

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[3] Some alternative specifications may be equally plausible. First, productivity in imitation might depend on both imitation experience and on knowledge accumulated in the North. This specification would apply if information disseminated internationally, and if the knowledge generated in the course of innovation were also helpful to imitators. Then the labor input coefficient in imitation would be $a_I/\phi(n_S,n)$. If, instead, productivity in imitation were enhanced by the existence of a greater number of products subject to copying, the input coefficient would be $a_I/\psi(n_S,n_N)$. Either of these specifications generates a steady-state equilibrium with properties similar to the one we describe, provided that $\phi(\cdot)$ and $\psi(\cdot)$ are assumed to be homogenous of degree one in their arguments. We will note instances where the alternative formulations yield different results in later footnotes.

Differentiating this break-even condition with respect to t, we find

$$(11) \qquad \frac{\pi_S}{w_S a_I/n_S} + \left(\frac{\dot{w}_S}{w_S} - \frac{\dot{n}_S}{n_S} \right) = \dot{R} \ .$$

Equation (11) expresses a no-arbitrage condition, equating the sum of the instantaneous profit rate (first term on the left-hand side) and the capital gain (second term on the left-hand side) to the instantaneous rate of interest. The capital-gain term reflects the fact that the value of a Southern firm equals the present cost of imitation, and so varies positively with the wage rate and negatively with productivity in the learning activity.

A potential Northern innovator faces a similar, though somewhat more complex decision problem. The development of a new product in the North requires a_D/K_N units of labor, where a_D is another productivity parameter ("D" for development) and K_N represents the level of scientific and engineering know-how in the North. We assume that each development project contributes a similar amount to the stock of knowledge in the North, so that K_N is proportional to cumulative experience in innovation, n. We choose units so that $K_N = n$. Then the measure of the set of available products grows according to

$$(12) \qquad \dot{n} = nL_D/a_D \ ,$$

where L_D is the aggregate amount of labor hired by entrepreneurs for purposes of innovation.

A Northern innovator who brings out a new product at time t faces thereafter a positive probability that her product will be selected by some Southern entrepreneur for imitation. If the product is copied at time T, the innovator's stream of monopoly profits ends then. In that event, the innovator earns, in total, a sum whose present discounted value at t is

$$\Pi(t,T) = \int_t^T e^{-[R(\tau)-R(t)]}\pi_N(\tau)d\tau \ .$$

At the time of development of some particular product, the date T at which imitation of that product will occur is a random variable. However, as we noted above, shareholders of the firm can diversify away this product-specific risk by holding a portfolio of Northern shares. The individual firm therefore maximizes its stock market value by maximizing the <u>expected</u> present discounted value of the stream of monopoly profits less innovation costs. We will assume that Northern agents have rational expectations. Letting F(t,T) denote the cumulative distribution function for T for a product developed at t (i.e., the probability that monopoly power will be lost to a Southern imitator before time T), we can write the expected present value of profits for a time-t innovator as

$$V(t) = \int_t^\infty \Pi(t,T)F_T(t,T)dT \ .$$

Since we allow free entry by Northern entrepreneurs into product development, a positive rate of innovation implies

$$V(t) = w_N(t)a_D/n(t) \ . \tag{13}$$

The evolution of imitation activity in the South after time t determines the distribution of the terminal date T for a product developed at t. Since

Southern entrepreneurs choose their target products at random, each existing Northern monopoly faces the same chance of being imitated. So the hazard rate of F(t,T), which is given by $F_T/(1-F)$, is equal to the instantaneous rate of imitation, $\mu(T) = \dot{n}_S(T)/n_N(T)$. This in turn implies

$$(14) \qquad F(t,T) = 1 - e^{-\int_t^T \mu(\tau)d\tau} .$$

Using (14), and the definitions of V(t) and $\Pi(t,T)$, we calculate

$$(15) \qquad \dot{V} = -\pi_N + (\dot{R} + \mu)V .$$

Now, differentiating (13) with respect to t, and using (15), we find

$$(16) \qquad \frac{\pi_N}{w_N a_D/n} + \left(\frac{\dot{w}_N}{w_N} - \frac{\dot{n}}{n}\right) = \dot{R} + \mu .$$

Equation (16) expresses a no-arbitrage relationship similar to (11). It equates the sum of the instantaneous profit rate of a Northern firm (first term on the left-hand side) and the capital gain (second term on the left-hand side) to the risk-adjusted interest rate. The capital gain here is the increase in the value of the firm, which equals the rate of increase in Northern wages minus the rate of productivity growth in the innovation activity. The risk premium is just equal to the rate of imitation, because this we have shown is the conditional density for the event that the firm suffers a total loss in earnings potential.

We complete our description of the equilibrium by appending the two labor-market clearing conditions. In each country, labor is employed in both manufacturing and learning activities. Letting $X_i = n_i x_i$ denote the aggregate output of final products in country i, i=N,S, and L_i denote the exogenous labor supply there, we equate labor supply and demand in each country in the following equations:

$$(17) \qquad (a_I/n_S)\dot{n}_S + a_X X_S = L_S ;$$

$$(18) \qquad (a_D/n)\dot{n} + a_X X_N = L_N .$$

The equations that we have derived in this section fully determine the evolution of the world economy from any initial conditions (i.e., numbers of goods produced in the North and South), provided that we choose an initial level of spending, E(0), consistent with long-run convergence to a steady state. We proceed now to examine the steady-state properties of our model.

III. Determinants of Imitation and Innovation in the Long Run

In the steady state, the number of products grows at constant rate g, and Southern firms imitate at constant rate μ. We are interested in the determinants of these long-run rates of innovation and imitation. We are also concerned with growth of log $u(\tau)$, since this measures instantaneous utility in our model. But it is easy to show that $d[\log u(\tau)]/dt = (1-\alpha)g/\alpha$, so the factors that affect the long-run rate of innovation similarly influence the steady-state growth in utility.[4]

[4] This claim can be verified using (2), once we recognize that the shares of Northern and Southern products in the total number of varieties are constant in the steady state, and that consumption of each variety falls at a rate equal to the rate of growth in the number of products (so that aggregate output in each country is constant).

We begin by normalizing nominal prices so that $w_N = n$. With this choice of numeraire, all prices and wages grow at the common rate g in the steady state, as does nominal spending E. Then (4) implies

$$(19) \qquad \dot{R} = g + \rho \ .$$

In the steady state, the share of Northern products in the total number of varieties, $\sigma_N = n_N/n$, is constant, and is equal to $g/(g+\mu)$. Using this fact, and substituting (6), (7) and (19) into the no-arbitrage condition (16), we find

$$(20) \qquad (1-\alpha)a_X X_N = \alpha a_D (g+\mu+\rho)\frac{g}{g+\mu} \ .$$

Now we combine (18) and (20) to derive

$$(21) \qquad \frac{(1-\alpha)(h_N-g)}{\alpha g/(g+\mu)} = g + \mu + \rho \ ,$$

where $h_N = L_N/a_D$ is the "effective" Northern labor force measured in terms of productivity in innovation.

Equation (21) expresses a steady-state relationship between g and μ. We depict this relationship by the curve NN in Figure 1.[5] The curve shows combinations of steady-state rates of innovation and imitation that are consistent with labor-market clearing in the North and a profit rate there equal to the risk-adjusted interest rate. Its positive slope can be understood as follows. A <u>ceterus paribus</u> increase in the rate of innovation

[5] This figure was suggested to us by Paul Krugman.

lowers the profit rate per variety for two reasons. First, an increase in g draws resources out of manufacturing into the learning sector, thereby decreasing aggregate output of final goods in the North, and hence the sales base over which mark-up profits are earned. Second, an increase in g raises the share of Northern products in the total number of varieties, and thus lowers the output per Northern firm for a given n and X_N. At the same time, an increase in the rate of innovation raises the interest rate, _ceterus paribus_. So an increase in g opens a positive gap between the risk-adjusted interest rate and the profit rate. An increase in the rate of imitation is needed, then, to restore equality between the two. The increase in μ raises the risk-premium, thereby exacerbating the disequilibrium, but it also raises the profit rate. As can be seen from (21), the effect on the profit rate (the left-hand side) dominates. The effect of μ on the Northern profit rate stems from the implied reduction in σ_N and thus the increase in sales for each Northern firm at given n and X_N.

To derive a second relationship between g and μ, we must bring in the equilibrium conditions for the South. The nature of this second relationship varies according to the size of the gap between Northern and Southern wages. We take up the wide-gap and narrow-gap cases in turn, discussing in each instance the determinants of (steady-state) g and μ.

A. _The Wide-Gap Case_

Recall that, when $w_S/w_N < \alpha$, the Southern imitator of a particular variety charges its monopoly price without regard to competition from the original Northern developer of that variety. In this case, we substitute (8a), (9a) and (19) into the no-arbitrage condition for Southern firms, (11), which gives

$$(22)\qquad (1-\alpha)a_X X_S = \alpha a_I(g + \rho) \ .$$

Now we use the labor-market clearing condition for the South, (17), to substitute for X_S in (22). Recognizing that the number of Southern products n_S grows at rate g in the steady state, we have

$$(23)\qquad g = (1-\alpha)h_S - \alpha\rho \ ,$$

where $h_S = L_S/a_I$ represents the effective labor force of the South measured in units of productivity at imitation.

We represent equation (23) by the horizontal line SS in Figure 1. The steady-state rates of innovation and imitation for the wide-gap case are given by the intersection of the curves SS and NN in the figure.[6] Of course, at this intersection point we must have $w_S \leq \alpha w_N$ for the wide-gap case to apply. (More on this point below.)

We note first the effects of international trade on steady-state growth in the two regions. The North's autarky rate of growth is found at the intersection of the NN curve and the horizontal axis, where $\mu=0$. Since the curve NN is everywhere upward sloping, the North grows faster in the steady state of a wide-gap trading equilibrium than it does in the absence of trade with the South. Southern imitation enables a release of resources from the

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[6] If the SS curve lies everywhere above the NN curve in the figure, then the narrow-gap case, rather than the wide-gap case, must apply. If the SS curve lies everywhere below NN, then there can be no steady-state with a positive rate of imitation in the South. Instead, the South will imitate for a while, then produce a fixed set of goods.

Northern manufacturing sector for redeployment in product development. This reallocation of resources is mediated by an increase in the profit rate, which results when, at given n, a smaller number of Northern firms share the total market for Northern products.

The growth rate for the South in autarky is determined by (23), after we replace a_I in that equation with the parameter reflecting Southern productivity in innovation, say a_{DS}. Since $a_{DS} > a_I$, it follows that the South too grows faster with trade than without. There are two sources of this faster growth. First, imitation of Northern goods saves resources relative to the development of new products from scratch. Second, the incentive to undertake the learning activity in the South is strengthened by the presence of Northern demand for Southern products, which raises the profit rate.

In the steady state of a wide-gap trading equilibrium, the growth rate is proximately determined by economic forces in the South. An increase in either the North's labor force or in its productivity at innovation shifts the NN curve upward. This suppresses the rate of Southern imitation and hence the steady-state share of products manufactured by the South, but has no effect on the steady-state growth rate.[7] The explanation for this lies in the determination of a Southern firm's profit rate, which in the steady state must equal $g+\rho$. Consider a shock in the North that alters the derived demand for Southern labor, hence the equilibrium relative wage w_S/w_N. Since Southern prices in the wide-gap case are a fixed mark-up over production costs there,

[7] If Southern productivity at imitation were influenced by either the number of products in the North, or by the stock of knowledge capital there (as described in footnote 3), then shocks in the North would have an effect on the steady-state growth rate. In these cases, the SS curve slopes downward (provided that $\phi(\cdot)$ and $\psi(\cdot)$ are homogenous of degree one), so that expansion of the North's effective labor force accelerates steady-state growth.

the change in w_S/w_N alters profits per variety and the cost of imitation equiproportionately. The shock in the North also may change the fraction of products manufactured in the South. But given aggregate Southern output of manufactures X_S, a change in n_S affects similarly the profits of a given variety and productivity in imitation. So, by either channel, the net effect on the profit rate in the South is nil. It follows that the initial values of g and X_S continue to satisfy the conditions for a steady-state equilibrium.

An improvement in productivity at manufacturing, a_X, has no effect on SS, hence no effect on the steady-state values of μ or g. But an expansion of effective labor in the South, precipitated either by an increase in L_S or a decline in a_I, causes the SS curve to shift upward and generates an increase in the steady-state rates of imitation and innovation. The impact effect entails a rise in the rate of imitation. In the North, this raises the risk premium for product development, but also boosts profits for each surviving monopoly, as output per Northern brand expands. As we mentioned before, the latter effect dominates, so innovation responds positively.

We close our discussion of the wide-gap case by considering the determination of relative wages. We evaluate (5) at the equilibrium prices given in (6) and (8a), and then take the ratio of outputs per variety in the North and South, to derive

$$\text{(24)} \qquad \left(\frac{w_S}{w_N}\right)^{\epsilon} = \frac{X_N(1-\sigma_N)}{X_S\sigma_N} \; .$$

Substituting for X_N/X_S using (20) and (22), and noting $\sigma_N = g/(g+\mu)$, we find

$$\text{(25a)} \qquad \left(\frac{w_S}{w_N}\right)^{\epsilon} = \frac{a_D}{a_I}\,\frac{\mu}{g+\mu}\left(1 + \frac{\mu}{g+\rho}\right) \; .$$

We note that the right-hand side of (24) is increasing in μ and declining in g and in g/μ.

An expansion of the labor force in the North alters the right-hand side of (25a) only via its effect on μ. We see, therefore, that this shock causes w_S/w_N to fall. A larger labor force in the South, on the other hand, implies acceleration of both innovation and imitation, but a fall in the ratio of g to μ (the slope of a ray from the origin to E falls as we move up along a given NN curve), hence a larger value of w_S/w_N. We conclude that the relative wage of the South moves inversely with the relative size of the North.

The response of relative wages to the productivity parameters is similar. An improvement in productivity in Northern innovation slows the steady-state rate of imitation, and also has a direct negative effect on the right-hand side of (25a), so the relative wage of the South falls in response. An improvement in Southern productivity in imitation raises μ, depresses g/μ, and directly increases the right-hand side of (25a), and so causes w_S/w_N to rise.

Our results concerning relative-wage effects stand in stark contrast to those of Krugman (1979), who took μ and g to be exogenous. The sources of the difference can be seen in the equation

$$\text{(25b)} \qquad \left(\frac{w_S}{w_N}\right)^{\epsilon} = \frac{(L_N-a_D g)}{(L_S-a_I g)}\,\frac{\mu}{g}\,,$$

derived by substituting (17) and (18) and $\sigma_N = g/(g+\mu)$ into (24). Here, when μ and g are taken as parameters, L_N and L_S have the effects on relative wages predicted by Krugman. These effects derive from the relative pricing of final goods, if we assume, as Krugman does, that all extra resources are devoted to manufacturing. But (25b) points to two ways in which the sizes of the labor forces affect relative wages that are missing from Krugman's analysis. First, when learning requires resources, changes in g alter the derived demand for labor in the learning sector, hence the residual available for manufacturing. Second, changes in outputs per firm caused by an expansion of either labor force necessitate a reallocation of resources between manufacturing and learning in each country in order to preserve the equality between the profit rate in the South and the risk-adjusted profit rate in the North.

B. The Narrow-Gap Case

In Figure 2 we have reproduced the NN curve (equation (21)), which continues to apply. We find a second relationship between the steady-state values of μ and g for the narrow-gap case by first substituting the equilibrium prices in (6) and (8b) into (5), then taking the ratio of aggregate outputs in the North and South, and finally using the result together with the market-clearing conditions, (17) and (18), to derive

$$\alpha^{\epsilon} = \frac{a_D}{a_I} \frac{(h_N - g)}{(h_S - g)} \frac{\mu}{g} . \tag{26}$$

We plot the combinations of g and μ that satisfy (26) as the curve XX in Figure 2. This curve describes combinations of the rates of growth and imitation that are consistent with simultaneous clearing of the labor markets and product markets in each country. It is easy to show that the XX curve slopes upward, once we recall that $h_S > h_N$ is required for the existence of a steady-state equilibrium with a positive rate of imitation in the South.[8] We

[8] See the argument in footnote 6 above. A necessary and sufficient condition for the SS curve to lie above the intersection of the NN curve with the vertical axis is $h_S > h_N$. This condition also is necessary for $w_S > \alpha w_N$

prove in the appendix that the XX curve must be steeper than the NN curve at any point of intersection, so the two curves can intersect at most once. Moreover, since the XX curve asymptotes to h_N as μ grows large, while the NN curve asymptotes to $(1-\alpha)h_N$, they must intersect exactly once. This intersection (at Q) represents the unique narrow-gap equilibrium, provided that the relative wage associated with that point satisfies $w_S/w_N \geq \alpha$.[9]

An increase in h_S, caused either by an increase in L_S or a decline in a_I, shifts the XX curve to the right (not shown), while leaving the NN curve unaffected. So the rates of growth and imitation are faster in the steady state of a narrow-gap equilibrium the larger is the effective labor force of the South. An increase in the effective labor force of the North, on the other hand, causes both curves to shift to the left, as depicted by the broken lines in the figure. It is easy to show that the leftward shift of the NN curve must be larger at the initial g, so the new steady state results at a point such as Q'.[10] Clearly the growth rate is larger at Q' than at Q, so the long-run rate of innovation now varies positively with the effective size of the North. Since the XX curve shifts upward by more than the NN curve at the

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(see (27) and note that $g > (1-\alpha)h_N-\alpha\rho$), as required for the narrow gap case.

[9] Using equation (27) below, we find that the condition on the relative wage corresponds to $g < (1-\alpha)h_S-\alpha\rho$ at point Q. But this implies that point Q must lie below the SS curve of Figure 1. We conclude that the wide-gap case applies whenever Q lies above the SS curve, and that the narrow-gap case applies when Q lies below this curve. Hence, the steady-state equilibrium with positive innovation and imitation, when it exists, must be unique.

[10] The XX shifts to the left by $\mu/(h_N-g)$. Using (21), which applies at the initial equilibrium, this distance equals $(1-\alpha)(g+\mu)\mu/\alpha g(g+\mu+\rho)$. The leftward shift of the NN curve is given by $(1-\alpha)(g+\mu)^2/\alpha g\rho$, which is larger.

initial value of μ, the rate of imitation must be smaller at Q′ than at Q.[11] Thus, the long-run rate of imitation is inversely related to the effective size of the North.

What are the effects of trade on the growth rate in each region? As before, the fact that the North must grow faster with trade than in autarky is immediate. The North's autarky rate of steady-state growth is once again given by the intersection of the NN curve and the vertical axis. Since the NN curve slopes upward, trade accelerates growth in the North, and for the same reasons as were discussed above.

If the South's rate of steady-state growth under autarky, given by $g_S = (1-\alpha)L_S/a_{DS} - \alpha\rho$, falls short of that in the North (surely the most plausible case), then trade clearly speeds growth for the South as well, since both regions grow at the same rate in the steady state of a trading equilibrium. But if the South grows faster under autarky than the North, then trade might slow growth in the South. Combining the South's labor-market clearing condition, (17), and its no-arbitrage condition, (11), after substituting for π_S and p_S in the latter from (8b) and (9b), we find for the narrow-gap trading equilibrium that

$$(27) \qquad g = (1-w_S/w_N)h_S - (w_S/w_N)\rho \ .$$

Since $w_S/w_N > \alpha$, we could have $g_S > g$ if the input requirements for product development in the South were only slightly larger than those for imitation. More plausibly, $a_I \ll a_{DS}$, in which case $g > g_S$.

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11 From (21) and (26) we compute that the XX curve shifts up at the initial value of μ by $[h_N/g - (h_N-g)/(h_S-g)]^{-1}$, while the NN curve shifts up by $[h_N/g + g/(1-\alpha)\mu]^{-1}$. Clearly, the shift in XX is larger.

We can use equation (27) to explore the effects of variations in economic structure on relative wages in a narrow-gap equilibrium. An expansion of the effective labor force in the North, for example, raises g, so by (27) it increases the relative wage of the North. And total differentiation of (27), together with (21) and (26), allows us to establish that the relative wage of the South is greater, the larger is the effective labor supply there. These results are, of course, qualitatively the same as those for the wide-gap case.

IV. Trade and Industrial Policies

Nations often contemplate the use of various policies in order to speed their growth, slow the rate of loss of markets to foreign competitor countries, etc. We can use our model of the product cycle to study the effects of these policies on long-run rates of growth and imitation, and on relative wages in the steady state. In this section, we shall consider subsidies to the learning activity and protective trade policies in each region. We limit our analysis here to positive issues; a complete welfare analysis for a small country with an economic structure similar to the one described here is carried out in Grossman and Helpman (1989b).

Let λ_i, $i=S,N$, be the fraction of learning costs borne by entrepreneurs in country i after subsidies have been applied, so that $1-\lambda_i$ is the subsidy rate. We assume that subsidies are financed by lump-sum taxes. Since Ricardian neutrality applies in our model, we need not specify the inter-temporal pattern of the tax collections, so long as the present value of the government's cash flow is zero.

The presence of subsidies to learning alters the no-arbitrage conditions for each country, hence the curves that determine the steady-state equilibrium. Specifically, we need to multiply the right-hand side of (21) by λ_N, while (23) becomes

$$(1-\alpha+\alpha\lambda_S)g = (1-\alpha)h_S - \alpha\lambda_S\rho \ .$$

The XX curve, (26), is not affected.

In a wide-gap equilibrium, a subsidy to innovation in the North shifts the NN curve to the left, while leaving the SS curve unchanged. This policy reduces the long-run rate of imitation and boosts the steady-state share of varieties produced in the North, but has no effect on the long-run growth rate. Using a modified version of (25a), it is easy to see that the Northern government's intervention raises the relative wage of its laborers in the steady state.

A subsidy to imitation or to technology adaptation in the South shifts the SS curve of the wide-gap case upward. The growth rate and the imitation rate rise, as does the share of varieties produced in the South. Like an improvement in productivity in imitation, this serves to raise the South's relative wage in the long run. The positive effect on the growth rate should be well understood by now. Although the speeding of the product cycle directly reduces the profitability of product development, this effect is more than offset by the expansion of sales for Northern products that survive. So the incentive to innovate is strengthened by faster imitation in the South.

The effects of these industrial policies in the narrow-gap case are easily derived. A subsidy to imitation shifts neither the NN nor the XX curve, and serves only to alter relative wages. As before, the relative wage of the South rises when its government subsidizes learning.[12] An R&D subsidy in the North causes the NN curve to shift up, and the equilibrium to move along a fixed XX curve. The growth rate and the rate of imitation (hence the average time to loss of competitiveness) both increase. The subsidy also serves to increase the relative wage of the North in the long run.

We turn now to trade policy. For the wide-gap case, the analysis is quite simple. An <u>ad valorem</u> tariff or export subsidy imposed by either country does not affect the elasticity of demand perceived by producers of any nationality. Therefore, it does not affect the prices charged by them. The profit rates do not change with trade policy, nor do the no-arbitrage conditions. Of course, the labor-market-clearing conditions, (17) and (18), continue to apply. It follows that trade policies in either country do not affect the NN or the SS of the wide-gap case, and therefore they do not alter the steady-state rates of innovation or imitation.

The conclusion for the narrow-gap case turns out the same, though the reasoning is more subtle. We must distinguish now between the prices charged by Southern manufacturers in the two different markets. Let T_i, $i=S,N$, be one plus the <u>ad valorem</u> tariff rate imposed by the government in country i, and let p_{Sj} be the price charged by a Southern firm for sales in country j.[13] In order to capture its home market, a Southern firm must undercut the <u>delivered</u> price of its Northern rival, so $p_{SS} = w_N a_X T_S$. Similarly, the Southern firm must set a price in the Northern market so that the tariff-inclusive consumer price falls below the unit cost of the Northern producer; i.e., $p_{SN} = w_N a_X / T_N$.

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[12] The equation for the relative wage $w=w_S/w_N$ that replaces (27) when subsidies to learning are present is: $(1-w)h_S - \lambda_S \rho w = (1-w+w\lambda_s)g$.

[13] We focus here on tariffs, though the conclusion for export subsidies is the same.

The Northern monopolists continue to price as before, since the trade barriers do not alter the demand elasticities perceived by them.

We substitute consumer prices (producer prices augmented by any applicable tariffs) into (5), and use E_i to represent aggregate spending by residents of country i, to find the sales by each firm in each market. Then we form the profit rate for a Northern firm and equate it to the steady-state risk-adjusted interest rate, whence

$$\frac{(1-\alpha)}{a_D}\left(\frac{g+\mu}{g+\mu\alpha^{1-\epsilon}}\right)\left(e_N+\frac{e_S}{T_S}\right) = g+\mu+\rho\ , \tag{28}$$

where $e_i = E_i/n$. We also substitute for aggregate Northern and Southern manufacturing output in (17) and (18), to derive

$$a_D g + \left(\frac{\alpha g}{g+\mu\alpha^{1-\epsilon}}\right)\left(e_N+\frac{e_S}{T_S}\right) = L_N\ ; \tag{29}$$

$$a_I g + \left(\frac{\mu\alpha^{1-\epsilon}}{g+\mu\alpha^{1-\epsilon}}\right)\left(e_N+\frac{e_S}{T_S}\right) = L_S\ . \tag{30}$$

Equations (28)-(30) determine the steady-state values of g, μ, and $z = e_N+e_S/T_S$. These steady-state values are invariant to the level of T_S or T_N. We conclude that trade policies cannot be used by either region to alter the steady-state rate of growth in a narrow-gap equilibrium. Nor can these policies be used to speed up or slow down the average length of the initial phase of the product cycle.

We should note that our finding that trade policy does not affect long-run growth would not survive in a modified version of our model. In particular, the introduction of a second production sector in each country would suffice to open a channel by which trade policy could influence steady-state growth. If, for example, we were to adopt an economic structure like that in Grossman and Helpman (1989a), where all differentiated products are intermediate goods and are combined with labor to produce a final output in each country, then trade policy would affect growth in the steady state. In such a three-activity economy, trade policy alters the allocation of resources between the joint activity of developing and producing differentiated products and that of producing final goods. However, the nature of this effect on resource allocation is rather complex and so lies beyond the scope of the present paper.

V. Conclusions

The product cycle describes an ever-evolving pattern of inter-regional trade. Goods are developed in the North and initially produced there. Later on in the life of an individual product the location of production migrates to the South, and the North comes to import the very same items that formerly it exported. In this paper, we have developed a model of this dynamic process in which the average length of the cycle and the speed with which new products are introduced to the market are both determined endogenously. We have used our model to study the determinants of the long-run rates of imitation and innovation, and the long-run distribution of labor income.

As in previous studies of technology-driven growth (e.g., Romer (1988), Grossman and Helpman (1989a)), we found that the size of the resource base and the productivity of resources in the learning activities are important determinants of the steady-state growth rate. Steady-state growth is faster the larger is the resource base of the South, and the more productive are its resources in learning the production processes for products originally developed in the North. This is perhaps surprising, because faster imitation by the South means on average a shorter period over which a Northern entrepreneur can earn monopoly profits. But profits during the monopoly phase are higher when a smaller number of Northern producers compete for resources in the manufacturing sector. We found the latter effect to dominate, so faster imitation by the South ultimately strengthens the incentive to innovate in the North.

Steady-state growth also is faster when the North is larger or its resources are more productive in product development, provided that the gap between wages in the North and South is not too large (what we have called the "narrow-gap case"). An increase in the size of the effective labor force in the North always slows the rate of imitation (hence the average length of the first stage of the product cycle) and reduces the steady-state share of varieties produced in the South. An increase in the effective labor force in the South has just the opposite effect on the imitation rate and on product shares in the long run.

Perhaps surprisingly, we find that the relative wage in the North rises when the effective size of the North expands in relation to the effective size of the South. This result, which is the opposite of that derived by Krugman (1979) in his product-cycle model with exogenous rates of innovation and imitation, stems ultimately from the increasing-returns nature of the technologies for production of goods and knowledge.

In comparing the product-cycle equilibrium to one with autarky in each region, we find that international trade always leads to faster growth in the North in the long run. The migration of some production to the South frees resources for use in the product development sector in the North. In the steady-state equilibrium with trade, the North firms have greater incentive to undertake R&D than in autarky, because each earns a higher profit rate, albeit for a shorter period of time. The South too grows faster with trade than without, except in the unlikely event that its autarky growth rate is faster than that of the North, and the resources required there for developing new products from scratch only slightly exceed the requirements for copying a product previously developed in the North.

We studied the long-run effects of two sorts of policy instruments. Subsidization of the learning activities (innovation in the North or imitation in the South) tends to boost the long-run rate of growth. The only exception to this occurs for subsidies to imitation when the North-South wage gap is small, in which case the growth effect is nil. Industrial policy of this sort always increases the relative wage of workers in the policy-active country.

Trade policies in either the North or the South have no effect on the long-run rates of growth and imitation in our model. These policies serve only to alter relative wages and the steady-state levels of real spending in the two regions. This finding, which is perhaps reminiscent of similar results that apply in neoclassical models of growth, relies strongly on the two-sector structure of our regional economies. If, instead, we were to allow a second manufacturing activity in each country in a manner that preserved the existence of a steady state, then trade policy would indeed play a role in determining the long-run growth rate.

APPENDIX

We prove in this appendix that, at any intersection of the NN curve defined by equation (21) and the XX curve defined by equation (26), the NN curve must be the steeper of the two. This proof is central to our demonstration that if there exists a steady-state equilibrium with positive innovation and imitation, it must be unique.

From (21), we solve for μ in terms of g along the NN curve, and write

$$\text{(A1)} \qquad \mu_N(g) = \frac{g(g-g_N)}{(1-\alpha)h_N-g} \ ,$$

where $g_N = (1-\alpha)h_N-\alpha\rho$. Similarly, from (26) we obtain

$$\text{(A2)} \qquad \mu_X(g) = \alpha^\epsilon \frac{a_I}{a_D} \frac{(h_S-g)}{(h_N-g)} g \ ,$$

which defines μ in terms of g along the XX curve. From these we compute the slopes

$$\text{(A3)} \qquad \mu_N' = \frac{\mu}{g} + \mu \frac{(1-\alpha)h_N-g_N}{(g-g_N)[(1-\alpha)h_N-g]} \ ;$$

$$\text{(A4)} \qquad \mu_X' = \frac{\mu}{g} + \mu \frac{(h_S-h_N)}{(h_S-g)(h_N-g)}$$

$$= \frac{\mu}{g} - \frac{\mu}{h_N-g} + \frac{\mu}{h_S-g} \ .$$

In these computations, (A1) has been used to substitute for μ in (A3) and (A2) has been used to substitute for μ in (A4). Now we define $\Delta_\mu = (\mu_N'-\mu_X')/\mu$, at a point of intersection where $\mu_N(g)=\mu_X(g)=\mu$. Then

$$\Delta_\mu = \frac{1}{h_S - g} + \frac{\Omega}{(g-g_N)(h_N-g)[(1-\alpha)h_N-g]} \quad ,$$

where

$$\Omega = [(1-\alpha)h_N - g_N](h_N-g) - [(1-\alpha)h_N-g](g-g_N) \; .$$

Labor-market clearing in the North (18) implies $h_N > g$, which in turn implies

$$\Omega > [(1-\alpha)h_N - g_N](h_N-g) - [(1-\alpha)h_N-g](h_N-g_N)$$

$$= \alpha h_N (g-g_N) > 0.$$

Since the NN curve asymptotes to $(1-\alpha)h_N$, we must have $g < (1-\alpha)h_N$ at any point of intersection of the XX and NN curves. Hence, $\Delta_\mu > 0$, as claimed.

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Figure 1

The Wide-Gap Case

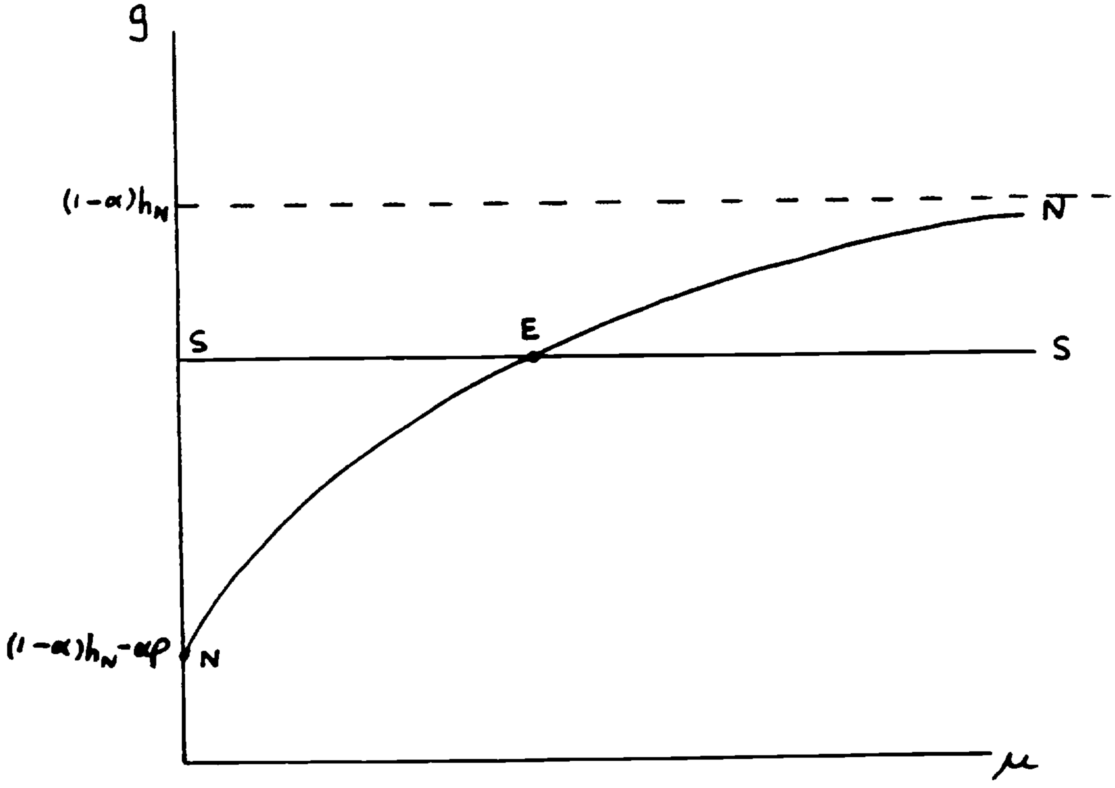

Figure 2

The Narrow-Gap Case