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ABSTRACT

The interdisciplinary circular economy literature recommends longer lasting products, to reduce pollution from repeated production and disposal. For any type of appliance, we assume consumers choose among variants with different durability. Firms are competitive. Standard Pigovian analysis shows that optimal taxes depend on pollution and not on product life. Here, we find conditions where consumers choose lives that are too short – a “durability gap”. First, we show that suboptimal existing output taxes imply suboptimal durability. An increase in uniform tax on all variants encourages purchase of a more durable variant and raises welfare. Second, welfare also is raised by a subsidy for choosing a more durable variant or by a marginally binding durability mandate. Third, we find that a social discount rate less than the private rate is the strongest case for policy to favor durability. Fourth, the consumer misperceptions we study have ambiguous implications for durability policy.

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Excess waste might be created by a “linear economy” that proceeds from resource extraction to production, use, and landfill disposal. In contrast, a “circular economy” (CE) could extract less, design products to last longer, design products to be recycled, and then encourage recycling. These ideas were introduced by architect Walter Stahel (1982) and design engineers (*e.g*., Hendrickson *et al*. 1998). Interdisciplinary CE literature reviewed in Stahel (2016) advocates for policies to encourage repair and reuse of products with longer lives – to reduce pollution from extraction through disposal. Product durability regulations are discussed in both the E.U. and U.S.[1] Ironically, circular economy has attracted little interest within economics. Here, we omit recycling because it is already well covered.[2] Instead, we focus on product durability.

With optimal corrections for production and disposal externalities and no other market failure, then those optimal policies do not depend on durability. After all, the choice of durability itself does not generate an externality. Here, in contrast, we ask what *uncorrected* market failures can justify calls in the interdisciplinary CE literature for policies that do depend on durability.

Economists since Pigou (1932) show that optimal corrective taxes apply to externality-generating activities at a rate equal to marginal external damages (MED), at least under what we call the “perfect assumptions”: perfect competition, full information, perfect enforcement, certainty, constant returns to scale (CRTS), and many identical, perfectly rational consumers (*e.g*., Baumol and Oates 1988). Producers pay the MED per ton of emissions, and consumers pay the MED per ton of disposal. Many subsequent papers relax each of those assumptions.[3]

Below, we review a large literature in which market power allows a producer to choose product durability. Consumers can only buy what firms choose to offer. That literature makes important contributions, but we find no economics literature that focusses on consumers’ unconstrained choices about durability and the market failures that could distort consumer choices. In other words, we do not try to extend further that existing literature. Instead, we focus on the opposite perspective. We revert to the simple assumption of perfect competition with free entry and exit, which means that producers are willing to offer whatever consumers want to buy.

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[1] The E.U. suggests requiring more durability to increase GDP and environmental benefits (Montalvo, *et al.* 2016, pp.10-12). Richter *et al*. (2019) calculate optimal durability of LED light bulbs and find that “longer lifetimes … in the E.U. could be appropriate” (p.107). They then discuss minimum durability standards and labelling requirements. The U.S. Federal Trade Commission (2021) also discusses durability regulations.

[2] The large economics literature on recycling includes early examples like Fullerton and Kinnaman (1996) and Palmer and Walls (1997), and it extends through the research and citations in Taylor (2020) or Berck *et al*. (2021).

[3] See reviews in Bovenberg and Goulder (2002) and Greenstone and Jack (2015).

Our questions about consumer choice of durability are analogous to those about the energy efficiency gap, defined as "a wedge between the cost-minimizing level of energy efficiency and the level actually realized" (Allcott and Greenstone 2012, p.4). This gap might be driven partly by externalities that prevent minimizing social costs and partly by "internalities" such as present bias or inattention (see Gerarden *et al.* 2017, or the review by Allcott 2016). Consumers may spend too little now on greater energy efficiency, even if it would save money on electricity in the long run. Analogously, we define a "durability gap" as a wedge between the cost-minimizing level of durability and the level actually realized. This gap also can be driven partly by externalities, internalities, or consumer mistakes. Consumers might not pay more now for a product that lasts longer, even if it saves money in the long run. If so, then policies can help consumers maximize their own welfare. In over-simplified-but-intuitive terms, making products that last twice as long can cut in half the repeated costs of production and disposal.[4]

These gaps are analogous but different. The energy efficiency gap focuses on policies to fix externalities from energy use during a product's fixed lifetime – ignoring the choice of product durability.[5] In contrast, we focus on policies to fix externalities from production and disposal when consumers can *choose* durability – ignoring energy efficiency and externalities from the use of the product.

We start with all the perfect market assumptions. Perfect competition means that firms cannot limit durability, plan obsolescence, or prevent repairs. Instead, selling at cost, firms will offer product varieties with any combination of characteristics desired by consumers. We ask whether one or more market failures creates a "durability gap" that makes chosen durability suboptimal. When we find a durability gap, we ask what policy could optimally address it.

As explained below, we allow for three types of market failure: (1) pollution externalities from extraction, production, or disposal; (2) consumers underweighting future costs, so the private rate of discount exceeds the social rate; and (3) consumer errors about product durability.

We solve two different models, for two different definitions of "durability". In both cases, we assume consumers can choose among variants of a particular type of product. One

[4] Future empirical work can address questions about the durability gap that are similar to those already addressed about the energy efficiency gap (Gillingham *et al.* 2009). Are the true long-run costs of durability known to the consumer and incorrectly measured by the analyst, or the other way around? Are consumers making rational decisions or not? Are external social cost being ignored? What is the size of each component of the gap?

[5] For example, Heutel (2015) studies policies to address an energy efficiency gap from present-bias internalities, using a model with a single durable good that has a fixed lifespan and that causes externalities from its use of energy. He shows that the optimum requires both a Pigovian tax and another policy to address the internality.

model assumes different variants are designed and built in ways that will require different maintenance or repair schedules (*e.g.*, cars, or laptops). Consumers can then decide when each variant would optimally be retired, and they use that information to choose the one to buy. The other model has no maintenance or repair, but variants are designed and built to last different lifetimes (*e.g.*, light bulbs). In both models, free entry and competition mean that some producer will build the desired variant and sell it at cost. We use the present value of all private costs over each variant's lifetime to derive annualized or "levelized" cost to consumers, and we show that the rational consumer would choose the variant with the lowest levelized cost.

Then we account for market failures and the present value of all social costs over a variant's life to derive levelized cost to society. We solve analytically for the optimal tax upon purchase of each variant that induces buyers to choose the variant that minimizes social costs. Our closed-form solution is called the "optimal" tax, because it encompasses either a first-best optimal (FBO) tax that corrects externalities without any other market failure, or a second-best optimal (SBO) tax in the presence of one or more uncorrected market failure (*e.g.*, social discount rate less than private discount rate, or if consumers make mistakes).

If that optimal tax falls with durability, then we say that a "durability gap" exists and can be addressed by a policy that explicitly favors long product lifetimes. We ask whether our results validate claims in the CE literature that optimal policy would need to address durability.

Using our models where consumers choose durability, the case where pollution externalities are the only type of market failure is solved easily to replicate the result in Bernard (2019) that the FBO purchase tax equals marginal environmental damage. This standard Pigovian tax does not favor products with longer lifetimes. We then prove five new results.

First, when that product tax is less than MED, even if it is a uniform tax for all durability variants of the relevant product, we show that a uniform increase in this suboptimal tax induces consumers to pay more for a variant with extra durability. The reason is that buying a longer-lasting product delays the time that the tax must be paid to buy its replacement.

Second, if the externality remains imperfectly corrected by this suboptimal per-unit tax, then a second-best policy can raise welfare in this model either by increasing that tax, or by introducing a durability subsidy, or by a marginally binding durability mandate. Then we add the other two categories of market failures, and we solve analytically for SBO purchase tax rates.

Third, when the social rate of discount is less than the private rate, and this market failure is not corrected directly, then the SBO product tax falls below MED for products that last longer (a durability gap, because optimal policy *does* encourage durability). The logic is that private decisions favor cheaper and less-durable products, failing to account for future excess social costs of disposal or of producing replacements. An exception is the case where producing a more durable good itself generates extra environmental damages that require a higher tax.

Fourth, we find that an error by consumers who underestimate (or overestimate) a product's life is an uncorrected market failure that effectively raises (reduces) the product's perceived annual cost of ownership. A higher perceived levelized cost is a disincentive that offsets part of any external cost from extraction of materials or production of the variant. In this case, the SBO tax is less than MED. A large underestimate can make the SBO tax negative (or an overestimate of durability can raise the SBO tax above MED). But any such mistake has ambiguous effects on how SBO taxes depend on product life. Thus, it conveys no clear message about a durability gap to be fixed by inducing changes in durability.

Fifth, for a numerical illustration of all these results with different combinations of market failures, we use data on 4,362 lightbulbs in the U.S. from 2015 to 2019, which we scraped from the website of the largest U.S. online lightbulb retailer. We regress sales price on the stated expectation of bulb lifetime and other attributes to estimate some parameters, and we calibrate other parameters. We insert all those parameters into our analytical formulas to calculate optimal tax rates, so that we can show graphically whether and how much these tax rates depend on durability – for alternative specifications about existing market failures.

1. Some Background on Particular Market Failures

Green designs of products that can be disassembled for recycling were discussed early by economists (*e.g.*, Fullerton and Wu 1998; Calcott and Walls 2000; Eichner and Pethig 2001), but the terminology of "circular economy" (CE) was not introduced until a decade ago by non-economists (see Stahel 2016 and Geissdoerfer *et al*. 2017). Interdisciplinary CE literature points to the importance of interactions around the entire circle, starting at mineral extraction, extending through product design for durability and recycling, and continuing through production processes, forward product supply chains, reverse supply chains for used materials, consumer recycling behavior, and methods of using waste back in production. The combination and interactions of all these activities is not well studied by economists.

This paper asks whether and when a market failure makes chosen durability suboptimal, but the list of possible market failures is far too long for one paper. One market failure omitted here is that asymmetric information can lead to adverse selection in markets for used durables (Akerlof 1970; Hendel and Lizzeri 1999a). If so, then consumers buying a new car know that they cannot sell it at a price that reflects its true value. Rather than get locked into the ownership of a long-lasting durable they cannot sell, consumers may choose less durability. Our model below assumes perfect information and many identical consumers (with no need to trade).

A second market failure omitted here is how imperfect competition can allow producer choice of durability. Early papers on this topic ignore environmental consequences and just discuss whether and when market power affects durability (*e.g.*, Levhari and Srinivasan 1969; Swan 1970). Durability is independent of market structure under strong assumptions, including the ability of firms to pre-commit to future prices, but a large subsequent literature finds reasons firms with market power underprovide durability: when allowing for maintenance (Schmalensee 1974); when firm are unable to lease the product to reduce time consistency problems (Bulow 1986); stochastic depreciation rates (Rust 1986); and preference heterogeneity (*e.g*., Kim 1989; Hendel and Lizzeri 1999b). These papers and others are reviewed by Waldman (2003).

Later, similar models were extended to add environmental externalities. For example, Boyce and Goering (1997) look at polluting durable-goods producers with market power, but they assume fixed durability. Our focus is endogenous durability. With imperfect competition that allows producer choice of durability, Goering and Boyce (1999) and Runkel (2003, 2004) look at FBO and SBO pollution policy. Overall, they show optimal emission taxes can be higher or lower than MED, depending on assumptions (*e.g.*, whether demand is linear, whether the cost function has increasing returns, or whether the firm can pre-commit or lease the durable). When firms can pre-commit, Runkel (2003) shows the emissions tax is less than MED. Without pre-commitment, Kinokuni *et al.* (2019) show a disposal fee greater than MED can help solve the time consistency problem and curb planned obsolescence, which raises durability and welfare.

Looking at other market failures, Eichner and Runkel (2003) consider missing markets for product attributes such as recyclability and durability. Finally, in Bernard (2019), a monopoly that makes products more durable may add production steps that raise emissions. In her other examples, emission reductions increase durability. When durability and pollution abatement are neutral or complementary attributes, an emission tax reduces the monopoly's pollution and increases durability; when these attributes are "competitive", effects are ambiguous.

Rather than try to extend this large producer choice literature even further, we choose a new perspective. Here, we focus on perfectly competitive firms that enter or exit the market with no fixed costs, so that only consumers choose what type of product will be built. If consumers want a more durable product, then that is what competitive producers will design and provide.

We limit consideration here to three categories of market failure that are prominent in the environmental economics literature and that could have important effects on consumer choice of durability. The first category we consider includes negative external effects from extraction, production, and disposal. These external social costs arise from mining and other virgin materials extraction (*e.g.*, water pollution, tailings, loss of biodiversity) and other stages of production (*e.g.*, greenhouse gases, local air pollutants, soil contamination). For a given type of product, these effects are summarized in our model below for each variant (j=1,…,N) by the difference between the private unit cost or competitive price P_j and the social cost P_j^*. This category also includes negative externalities from disposal (*e.g.*, litter and noise from garbage collection plus methane and leachate emissions from landfills). These effects appear in our model as the difference between the private cost of disposal D_j and the social cost D_j^*.

A second category of market failures is a possible divergence between the social rate of discount ρ and the private rate of discount r. The huge and complex literature on this topic is summarized here using just one prominent debate. Discussing climate policy, Stern (2007) argues for a social discount rate of 1.4%, partly on the grounds that the social welfare function should weight all generations equally and not underweight consumption of future generations. In response, Nordhaus (2007) argues for a higher social discount rate of 4.3% based on market interest rates, because future generations could gain more by investing additional capital at market interest rates. This debate is explained by Goulder and Williams (2012) as a debate about two different concepts, not about two values for a single "social rate of discount". The two concepts are blurred in models of Nordhaus and others that uses a representative, infinitely-lived consumer, since intertemporal choice then reflects both the consumer's utility maximization and social welfare maximization. This type of model has only one discount rate to be used both for private behavior and for social welfare. In contrast, an overlapping generations model might have individuals who underweight future generations. If so, then current generations can affect future generations without fully taking their welfare into consideration.[6]

[6] The debate about discounting also inevitably involves uncertainty, irrational behavior, and short-run versus long-run discounting. Goulder and Williams (2012) argue that uncertainty would affect both the private and social discount rates in the same direction, though not necessarily by the same amount. Here, we model consumer mistakes about perceived product lifetimes rather than mistakes about their own discount rate.

We focus on this market failure as an externality imposed by current consumers on future generations, but the difference between private and social rates of discount could reflect other market failures such as tax distortions, liquidity constraints, or consumer mistakes. For example, Heutel (2015) models “present bias” as quasi-hyperbolic discounting, where consumers’ short-run discount rate is too high relative to their *own* long-run rate. He assumes the social planner uses the “long-run criterion,” employing only the lower long-run discount rate. In fact, a model of consumers with infinite lives requires some internality if consumers put less weight on future utility. But the same effect appears with current generations of perfectly rational consumers that underweight costs on future generations. We adopt this latter externality interpretation. Our numerical illustrations simply use a market interest rate of 4% for private optimization, while we vary the social discount rate from 4% down to 3%, 2%, or 1%.

In a third category of market failure, consumers could be subject to inattention, irrational behavior, bias, simple errors, or other problems studied in behavioral economics. See reviews in Dellavigna (2009) and Gerarden, *et al*. (2017). These market failures again include too many possibilities to analyze here. Using our second model below, we investigate whether optimal policy favors durability in the simple case where consumers mis-estimate product lifetimes.

2. Consumer Choice of Durability

We consider the consumer’s choice from among a set of product variants, any one of which can provide the same stream of services, but where durability and other attributes can differ. As an example, consumers choose from a set of washing machines that can all provide the same washing services. Or, a set of phones can all provide the same calling services. Our numerical example below is a set of lightbulbs that provide the same stream of light. We assume that many identical consumers face enough varieties to be able to acquire any combination of other desired characteristics, allowing us to focus on the choice among variants that differ only by durability. We consider consumers with an arbitrarily long horizon who must choose one variant from the relevant set (ignoring the opt-out decision).[7]

Analysis requires a specific definition of durability. It could be based on economic depreciation, the annual fall in market value, but the resale price is not relevant for our many identical consumers who would have no interest in trading used appliances. In normal parlance, a

[7] Similarly, in Bernard’s (2019) monopoly model, “An infinitely lived representative household needs a given functionality or service supplied by the produced good. For instance, the household needs one washing machine, one toaster, or one refrigerator.” (p. 1187).

more durable product might be one that requires less maintenance and repair. Our first model here allows consumers to choose among variants with different repair cost schedules, where costs rise with variant age. A vehicle can be repaired, repeatedly, but eventually the repair cost becomes high enough that the owner chooses to dispose of it and buy a replacement. Sometimes, however, a more durable product is simply one that lasts longer. A light bulb or a toaster-oven is a "one-hoss-shay" investment with no repair costs at all, and a constant service flow, until the product fails entirely. In our second model below, we explore this case where each variant has no maintenance or repair but instead a fixed lifetime.

Specifically, consumers must choose one variant from a set indexed by $j = 1, \dots, N$, where all variants in this set provide the same constant flow of benefits B (such as washing services for a set of washing machines). Each consumer needs this stream of services to last indefinitely, but the N variants differ by their design for durability (built-in during construction), their purchase price P_j, and their final disposal cost D_j. All N variants are available to consumers because of full information, competition, and free entry or exit. With many identical consumers, only one variant is chosen, produced, and observed.[8]

In our first model, each variant has its own maintenance and repair cost schedule $\gamma m_j(t)$ that depends on time since purchase, t. A change in the scalar γ is used below to represent a proportional change in the maintenance schedule $\gamma m_j(t)$, where $m_j(t)$ is continuous, twice differentiable, and strictly convex $[m_j'(t) \equiv \partial m_j/\partial t > 0]$. Eventually, these repair costs rise enough that the consumer chooses to retire variant j at chosen time T_j, but differing repair cost schedules $m_j(t)$ imply that each variant can provide a different useful life.

To acquire this constant stream of benefits into the future, the consumer must make a plan about which variant to buy, when to retire it, and how to replace it. All characteristics of each variant are known in advance and fixed over time, so the consumer can plan from time zero and then just stick to the plan. Because the stream of benefits is fixed and necessary, the goal is simply to minimize the present value of the cost of buying this stream of services indefinitely.

We proceed first by describing the consumer's initial choice of variant, and then we prove that this same choice would be repeated. The initial choice requires two steps. First, fully

[8] In this partial equilibrium model, competition and CRTS imply that the supply curve is flat. Given consumer market demand for each variant, a unit tax on each variant would raise its flat supply curve and the equilibrium price by exactly the amount of the tax.

informed consumers plan the cost-minimizing retirement date ($0 < T_j < \infty$) for every variant j by balancing its rising maintenance cost against costs of disposal and replacement. Second, they then compare the N variants to choose the one with the lowest constant "levelized" flow cost. A longer-lasting variant may cost more initially but have a lower long-run annual cost.[9]

Consumers may also face various tax rates. Upon purchase, consumers must pay the price P_j, a purchase tax τ_j^P, plus an amount ε (which we use later to represent a change in the price or tax). Upon disposal, consumers must pay the private disposal charge D_j plus a disposal tax τ_j^D. Any tax rate can be positive or negative. A private disposal company or a municipality might collect a positive charge per unit of disposal, or they may charge only a fixed monthly fee to collect all garbage. In the latter case, the monthly fee is inframarginal, and the private marginal cost D_j is zero (ignoring scrap value).

Using continuous time, where r is the private discount rate, we define PV_j as the present value of all private costs of variant j (including taxes).[10] The consumer does not just minimize this present value, however, because useful lives differ. We want the lifetime T_j that minimizes the equivalent long-run annual (*i.e.*, "levelized") cost of ownership, c_j. To find c_j, we first set the present value of actual private costs equal to the present value of levelized costs c_j:[11]

$$PV_j(T_j) \equiv P_j + \tau_j^P + \varepsilon + \int_0^{T_j} \gamma m_j(t) e^{-rt} dt + (D_j + \tau_j^D) e^{-rT_j} \equiv \int_0^{T_j} c_j \, e^{-rt} dt \qquad (1)$$

This equation essentially defines c_j, so we can rearrange and solve for that levelized cost:

$$c_j(T_j) = \frac{r}{1 - e^{-rT_j}} PV_j(T_j) = \frac{r}{1 - e^{-rT_j}} \left[P_j + \tau_j^P + \varepsilon + \int_0^{T_j} \gamma m_j(t) e^{-rt} dt + (D_j + \tau_j^D) e^{-rT_j} \right] \qquad (2)$$

This levelized cost c_j is the "user cost of capital" for comparison with an asset's return, a concept that dates back at least to Hall and Jorgenson (1967). It is the flat "rental rate" paid by a renter to an owner who just breaks even when the owner pays for purchase, maintenance, and disposal.

The levelized cost $c_j(T_j)$ for each variant is U-shaped across the possible choices for

[9] For example, Miotti *et al*. (2016) calculate the present value of all ownership costs for 125 light-duty vehicles (and find that electric vehicles have higher initial costs but lower annualized costs than most fossil-fuel-powered cars).

[10] We ignore heterogeneity, including consumer discount rates (e.g., preferences, credit constraints, or tax rates). On heterogeneity, see Heutel (2015). If some consumers cannot borrow enough to pay the higher up-front cost of the variant with optimal durability, then then the taxes considered here cannot achieve first-best outcomes.

[11] Because of continuous time, this c_j is not a flat annual cost but an instantaneous flow cost rate.

retirement date ($0 < T_j < \infty$), first falling as purchase price is spread over more years of use, and then rising because we assume repair costs rise with variant age (enough for a minimum at finite life). Our numerical section below illustrates the U-shaped curve later in Figure 2.

Fully informed consumers with no liquidity constraints essentially differentiate (2) to find the age T_j at which $\partial c_j/\partial T_j$ is zero. They invest in the variant that has minimum levelized costs (whenever the flow of services is first required). Because the decision is made with perfect certainty, and because future technology does not change, they will not revise the retirement date. To see how this simple model relates to a more complete dynamic model, see Dixit and Pindyck (1994, pp. 136-52). They start with a fully dynamic investment problem with both a time trend and uncertainty, and they show how it reduces to our simpler problem as the trend and uncertainty go to zero (using either dynamic programing or option pricing methods).[12]

We use the Leibniz Rule to differentiate (2), and then we interpret the derivative:

$$\frac{\partial c_j}{\partial T_j} = 0 = -\frac{r^2 e^{-rT_j}}{(1-e^{-rT_j})^2}\left[P_j + \tau_j^P + \varepsilon + \int_0^{T_j} \gamma m_j(t) e^{-rt} dt + (D_j + \tau_j^D) e^{-rT_j}\right] + \frac{r e^{-rT_j}}{1-e^{-rT_j}}\left[\gamma m_j(T_j) - r(D_j + \tau_j^D)\right] \qquad (3)$$

To simplify, define the ratio $R_j \equiv \frac{re^{-rT_j}}{1-e^{-rT_j}}$ and divide everything in (3) by that ratio. We also use the definition of PV_j and move the first term to the left side:

$$\frac{r}{1-e^{-rT_j}} PV_j(T_j) \;\equiv\; c_j(T_j) \;=\; \left[\gamma m_j(T_j) - r(D_j + \tau_j^D)\right] \qquad (3')$$

Later propositions use these definitions of PV_j, R_j, and the fact that $c_j = \gamma m_j(T_j) - r(D_j + \tau_j^D)$ at this chosen point. To interpret marginal conditions, however, we re-arrange (3'):

$$\gamma m_j(T_j) \;=\; r\left[D_j + \tau_j^D + \frac{PV_j(T_j)}{1-e^{-rT_j}}\right] \qquad (3'')$$

[12] Dixit and Pindyck (1994, p.136) consider when to invest I in a project with value V that evolves by geometric Brownian motion, $dV = \alpha V dt + \sigma V dz$ (where dz is the increment of a Wiener process). In the general stochastic case, either a higher trend α or higher uncertainty σ leads to a greater value of waiting. The investment rule is characterized by a critical value V^* such that it is optimal to invest when $V > V^*$. They solve this problem first using dynamic programing (p.140), and then again using option pricing ("contingent claims analysis", p.147). The two solutions are equivalent. In the deterministic case ($\sigma = 0$), they show how optimizing investors might wait if $\alpha > 0$ (p.138). Our problem has no trend ($\alpha = 0$) and no uncertainty ($\sigma = 0$), so the equation above implies $dV = 0$. Thus, the investment is immediate, as long as $V \geq I$. Proposition 1 below shows that the same variant will be purchased repeatedly. Thus, we do not need the value function here, but future extensions can consider uncertainty, technical progress, or a trend in returns, especially if the rate of technical progress differs across goods of different durability.

To a consumer who contemplates using the variant slightly longer than T_j, the marginal cost of delay on the left is the additional maintenance cost at that point in time, $\gamma m_j(T_j)$. The marginal benefit on the right is the time value (at interest rate r) of delaying a capital cost that includes not only the disposal cost and tax, but also the present value cost of buying repeated replacements forever afterwards.[13] The marginal cost of delay $\gamma m_j(T_j)$ is rising, and it crosses the marginal benefit of delay at chosen age T_j (which is flat because the U-shaped c_j curve is flat at that T_j).

The N variants have different chosen lifetimes because of different costs of purchase, repair, and disposal. After choosing T_j to minimize the levelized cost for *each* variant j, the consumer then chooses the one with the lowest minimum c_j. We assume all variant attributes are constant over time, and that consumers live indefinitely, so we can now prove that consumers would always make the same choice again at the time of replacement.[14]

PROPOSITION 1: To minimize the present value cost of acquiring the needed stream of services forever, in this model, the variant with the lowest levelized c_j would be purchased repeatedly.

Proof: All variant characteristics remain constant into the future, so the levelized cost c_j could be incurred every period forever by a consumer who buys variant j again after each lifetime T_j. The present value of that stream of costs into the future is c_j/r. Suppose variant k is the one that minimizes levelized cost $[c_k = min(c_1, \ldots, c_N)]$. Then variant k must also minimize the cost of repeatedly buying any other variant $[c_k/r = min(c_1/r, \ldots, c_N/r)]$. Moreover, the consumer cannot reduce costs by switching from variant k, because that other variant cannot have a levelized cost lower than c_k. ■

While still looking at consumer behavior, we prove two more propositions to establish a logical baseline for use in later propositions. Each proof is short and intuitive, so each is included in the text. First, how does maintenance cost affect the choice of lifetime?

PROPOSITION 2: An increase in the scalar γ that multiplies all maintenance costs $m_j(t)$ in the model above induces consumers to choose an earlier retirement date.

Proof: Using the U-shaped curve for $\partial c_j/\partial T_j$ in equation (3), we allow each T_j in that equation

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[13] The last term within the brackets of (3") equals c_j/r, the present value of paying c_j forever after this delay. We are grateful to Armon Rezai for pointing out that each repeated purchase incurs the same PV_j, so the total cost with an infinite planning horizon is the sum of all discounted present values: $TC_j = \sum_{n=0}^{\infty} e^{-rnT} PV_j = \frac{1}{1-e^{-rT_j}} PV_j$.

[14] An extension might consider technical progress that reduces the future cost of a replacement purchase, but this extension would preclude the simple comparison of levelized costs used here. Even if all variants have the same rate of technical progress, a consumer might rationally choose a variant with higher levelized cost if its lifetime is short enough to take advantage of changes in technology that offer rapidly falling replacement cost, or expansion of services, or changes that enhance the durable's other characteristics.

to be an implicit function of γ. At the chosen point, we insert $T_j = T_j(\gamma)$ and take the derivative of (3) with respect to γ. The resulting long expression can be simplified using the definition of PV_j in equation (1), as well as $R_j \equiv re^{-rT_j}/(1-e^{-rT_j})$. We also replace $\gamma m_j(T_j) - r(D_j + \tau_j^D)$ by c_j, as shown in (3'). Then the derivative of (3) is:

$$-\frac{r}{1-e^{-rT_j}} R_j PV_j \frac{\partial T_j}{\partial \gamma} + \frac{r}{1-e^{-rT_j}} \int_0^{T_j} m_j(t) e^{-rt} dt + R_j c_j \frac{\partial T_j}{\partial \gamma} - m_j(T_j) - \gamma m_j'(T_j) \frac{\partial T_j}{\partial \gamma} = 0$$

Re-arranging terms, we have:

$$\frac{\partial T_j}{\partial \gamma} = \frac{\frac{r}{1-e^{-rT_j}} \int_0^{T_j} m_j(t) e^{-rt} dt - m_j(T_j)}{\left\{\gamma m_j'(T_j) + R_j \left[\frac{r}{1-e^{-rT_j}} \cdot PV_j - c_j\right]\right\}} < 0\,.$$

In the denominator, the first term is positive, and the second term is zero (equation 3'). The numerator is negative, because strict convexity ($m_j'(t) > 0$) means that the maintenance cost at the end of variant life, $m_j(T_j)$, is greater than the average levelized maintenance cost, $\frac{r}{1-e^{-rT_j}} \int_0^{T_j} m_j(t) e^{-rt} dt$. Thus, a proportional increase in maintenance cost γ induces consumers to dispose of the variant sooner. ■

Below, we use the converse of that proposition: reduced maintenance costs induce longer product lives – for any values of the other parameters. Next, how does initial purchase cost ($P_j + \tau_j^P + \varepsilon$) affect the choice of variant (*i.e.*, product lifetime)?

PROPOSITION 3: With all other parameters fixed, any increase in the initial purchase cost for variant j induces the consumer to choose a longer lifetime for that variant.

Proof: Using the U-shaped curve for $\partial c_j / \partial T_j$ in equation (3), we allow each T_j to be an implicit function of ($P_j + \tau_j^P + \varepsilon$). Any change in the purchase cost can be represented by a change in ε. We insert $T_j = T_j(\varepsilon)$ and take the derivative of (3) with respect to ε at the chosen point. The result can be simplified using (3') and the definitions for PV_j and R_j. Then the derivative of (3) is:

$$-\frac{r}{1-e^{-rT_j}} R_j PV_j \frac{\partial T_j}{\partial \varepsilon} + \frac{r}{1-e^{-rT_j}} + R_j c_j \frac{\partial T_j}{\partial \varepsilon} - \gamma m_j'(T_j) \frac{\partial T_j}{\partial \varepsilon} = 0$$

Re-arranging terms, we have:

$$\frac{\partial T_j}{\partial \varepsilon} = \frac{\frac{r}{1-e^{-rT_j}}}{\left\{\gamma m_j'(T_j) + R_j \left[\frac{r}{1-e^{-rT_j}} \cdot PV_j - c_j\right]\right\}} > 0$$

The numerator is positive. The denominator is positive because $m_j'(T_j)$ is positive and the term in brackets is zero (from 3'). For any values of the other parameters, an increase in the purchase price induces the consumer to spend more on maintenance to keep the product working longer. ■

We use this proposition to make two important points. First, intuitively, the consumer responds to any increase in purchase cost by choosing to delay the extra cost of replacement.

Second, the proposition also shows that the choice of lifetime T_j is a monotonic increasing function of ε (or more importantly, the tax τ_j^P). The next section shows how a social planner could use all social costs to choose the FBO or SBO lifetime, T_j^*. The fact that consumer choice of T_j increases monotonically with τ_j^P means that setting purchase tax τ_j^P uniquely determines the consumer's choice of lifetime. Thus, we can solve for an optimal tax such that the chosen T_j is the optimal T_j^* for each variant. Then we show that the choice of variant also is optimal.

3. The Social Planner's Choice of Durability

Of our three categories of market failure, the first category includes production and disposal externalities. The social cost of production P_j^* could exceed the sales price P_j if producers are not forced to cover all social costs during each phase from mineral extraction to final sale. And social costs of disposal D_j^* could exceed private disposal cost D_j if disposal companies cover only their own marginal costs (or if the per unit disposal charge is zero).[15] We show how all these externalities can be corrected by τ_j^P on purchase and τ_j^D upon disposal.[16]

The social planner's choice of optimal variant and optimal lifetime must involve a two-step process strictly analogous to the consumer's choice problem. First, the planner finds each variant's optimal lifetime T_j^* (for $j = 1, \dots, N$), and then the planner chooses the variant that minimizes all social costs of acquiring the necessary service flow.

Thus, the planner would use the social discount rate ρ to calculate the present value of all social costs for each variant, PV_j^*, and then set that PV_j^* equal to the present value of the equivalent levelized social cost flow c_j^*:

$$PV_j^*(T_j^*) \equiv P_j^* + \int_0^{T_j^*} \gamma m_j(t)\, e^{-\rho t} dt \ + D_j^* e^{-\rho T_j^*} = \int_0^{T_j^*} c_j^*\, e^{-\rho t} dt \tag{4}$$

Next, solve for c_j^* as:

[15] Social costs D_j^* increase with toxicity, especially with improper dumping. Assuming a known disposal method allows the tax to be collected at purchase, but we see below whether P_j^* and D_j^* are separate terms in the optimal tax.

[16] We omit both social cost $m_j^*(t)$ and tax $\tau_j^m(t)$ to correct negative externalities from maintenance. Time-specific taxes $\tau_j^m(t)$ are not very feasible and do not add much to this analysis. One could introduce $m_j^*(t)$ to capture the external cost of energy chosen to operate the product, but we omit that discussion here because energy costs in our model are fixed. The large energy efficiency literature already studies the endogenous choice of energy use. We also omit the possibility that product usage affects product life.

$$c_j^*(T_j^*) = \frac{\rho}{1 - e^{-\rho T_j^*}} PV_j^*(T_j^*) \tag{5}$$

Using (5), the planner chooses lifetime T_j^* that minimizes c_j^* for each variant and then chooses the variant with the minimum levelized social cost.[17]

Given consumer behaviors, we next find variant-specific tax rates that induce consumers to match the planner's choices.[18] For consumers to make the same choice of variant and lifetime as the planner, a sufficient condition is that private costs c_j in (2) exactly match social costs c_j^* in (5) for each variant.[19] If so, then $c_j = c_j^*$ (for all j), and the consumer's choice of lifetime T_j and variant with minimum c_j will match the planner's choices. We set $c_j = c_j^*$ and use the definitions of PV_j and PV_j^* to solve for the optimal purchase tax (in the case where $\varepsilon = 0$):[20]

$$\tau_j^P = \frac{\rho}{r} \cdot \frac{1 - e^{-rT_j}}{1 - e^{-\rho T_j^*}} \left(P_j^* + \int_0^{T_j^*} \gamma m_j(t) e^{-\rho t}\, dt + D_j^* e^{-\rho T_j^*} \right) - \left(P_j + \int_0^{T_j} \gamma m_j(t) e^{-rt} dt + \left[D_j + \tau_j^D \right] e^{-rT_j} \right) \tag{6}$$

This optimal tax may be FBO or SBO, depending on whether a market failure remains uncorrected. If abatement technology can alter pollution *per unit* of output, then only a tax on pollution can be FBO (any output tax can only be SBO). Here, we assume pollution is fixed per unit of output, so this output tax can be FBO in the absence of any other market failure.

This one equation cannot uniquely determine both tax rates (τ_j^P and τ_j^D), but this optimal tax rule can easily be confirmed. Start with *any* value for τ_j^D (*e.g.*, the simple case where $\tau_j^D = D_j^* - D_j$). Substitute that τ_j^D into (6), and then substitute that expression for τ_j^P into equation (2). Then all private costs within the formula for c_j in (2) match the social costs within the formula for c_j^* in (5). Thus, the consumer minimizes all the same costs as the social planner and must

[17] The consumer takes purchase price P_j and other private costs as fixed, but a welfare-maximizing planner would take account of the possible supply-side determination of costs. To simplify, our partial equilibrium model employs a flat supply curve (which would occur even in a general equilibrium model with perfect competition, constant returns to scale, and a single primary factor of production that also serves as numeraire).

[18] Since implementation would require many variant-specific parameters, these taxes are not feasible policy. Instead, the goal is to understand conceptually if optimal policy would address durability, which itself is not polluting.

[19] The choice of variant j is a discrete choice, so the minimum costs c_j and c_j^* need not match – as long as consumers make the socially optimal choice. Other tax rates may lead consumers to the socially optimal choice. Thus, our solution shows a set of τ_j that is sufficient but not necessary.

[20] We show below that this policy is FBO with no other market distortions, and it is SBO with market failures we model (*e.g.*, $\rho < r$), but it is not SBO in a world with tax distortions or other market failures not considered here.

make the same choice of lifetime ($T_j = T_j^*$ for each variant). Also, then the consumer chooses the same variant as the social planner. In other words, informed consumers minimizing their own c_j also minimize social cost c_j^*. Thus, we can replace all T_j^* with T_j in (6), to obtain:

$$\tau_j^P = \frac{\rho}{r} \cdot \frac{1 - e^{-rT_j}}{1 - e^{-\rho T_j}} \left(P_j^* + \int_0^{T_j} \gamma m_j(t) e^{-\rho t}\, dt + D_j^* e^{-\rho T_j} \right) - \left(P_j + \int_0^{T_j} \gamma m_j(t) e^{-rt} dt + \left[D_j + \tau_j^D \right] e^{-rT_j} \right) \tag{6'}$$

In any case, implementing tax rates in equation (6) would require a different tax on each variant j, and too much specific and detailed information about ρ, r, and every variable with a subscript. Instead, the point is to show generally how optimal policy depends on product lifetimes – to identify reasons why consumer choice of durability can be nonoptimal. Therefore, we next use private and social first order conditions to investigate welfare effects starting from a second-best world where tax rates are not the FBO tax rates in (6).

4. Results from the First Model: Maintenance Costs Affect Consumer Choice of Lifetime

We first look at a special case of the full equation (6), to prove some simple analytical results.

PROPOSITION 4: If the social discount rate ρ matches the private rate r in the model above, then the first-best social optimum can be achieved by a combinations of tax rates such as:
(A) $\tau_j^P = P_j^* - P_j$ *and* $\tau_j^D = D_j^* - D_j$*; or* (B) $\tau_j^P = (P_j^* - P_j) + (D_j^* - D_j)e^{-\rho T_j}$*and* $\tau_j^D = 0$.

Proof: Substitute tax rates from (A) or from (B) into equation (2). In either case, informed consumers minimizing their own c_j would also minimize social cost c_j^*. ■

In part (A) of this proposition, the product tax is the MED per unit of production, and the disposal tax is MED per unit of disposal. These standard Pigovian tax rates do not depend on the chosen variant lifetime T_j, but consumers nonetheless choose FBO durability T_j^*. In other words, this solution leaves no “durability gap” where policy needs to address suboptimal durability.

Similarly, tax τ_j^P in part (B) achieves the same optimum, even with no disposal tax. Instead, τ_j^P includes both the MED per unit of production and an “advance disposal fee” (ADF) equal to the present value of the external damage per until of disposal, $(D_j^* - D_j)e^{-\rho T_j}$. This solution (B) is FBO in our simple model only because we assumed that each variant’s characteristics are known and fixed (a known method of disposal with a known social cost at a known date). The consumer’s only choice is which variant to buy (*i.e*. which make and model of the product). Therefore, only one policy instrument is needed to get the consumer to choose the variant with the minimum levelized social costs.

In general, of course, consumers have multiple methods of disposal that may include recycling, landfill, or illegal dumping. In that case, consumer behavior can be re-directed toward the optimal method of disposal only at the time of disposal, and only by applying a different disposal tax to each method of disposal. We omit that problem here to focus on the choice of variant lifetime (durability), but economics literature shows an ADF is useful as a deposit, collected at the time of purchase, so that an optimal refund at the time of disposal can ensure that consumers are facing a net tax on each disposal method equal to its external cost.[21]

When *can* policy improve welfare by inducing or requiring longer product lives? We next study the case where policy is not optimal, that is, where a change in policy can improve welfare.

Proposition 3 for the general case showed that the consumer's choice of variant lifetime increases monotonically with the tax rate τ_j^P. Since the consumer's only choice for each variant is its lifetime T_j, then any conditions in the full model where the existing tax is less than the optimal tax in (6) must be a case with $T_j < T_j^*$. Thus, in such a case, an increase in τ_j^P would raise the chosen lifetime toward the optimal lifetime for that variant. If all variant tax rates were raised by the same uniform amount, then the chosen lifetime for every variant would rise, including the chosen lifetime of the chosen variant. It would presumably raise welfare, but a closed-form expression for welfare is not possible in the general case where ρ can differ from r. We conduct simulations with $\rho < r$ below, but here we can derive a simple closed-form expression for effects on economic welfare in the case where $\rho = r$.

First, define $EC_j \equiv P_j^* - P_j$ as the external cost per unit of variant j (at production, so it does not depend on choice of retirement date). In general, a levelized welfare cost is the extent to which levelized social cost exceeds private cost. Any levelized cost *does* depend on lifetime, however, so we define $WC_j(T_j) \equiv c_j^*(T_j) - c_j(T_j)$, where both c_j^* and c_j can be evaluated at any lifetime T_j (that is, c_j^* in equation (5) can be evaluated at $T_j \neq T_j^*$). Then we have:

PROPOSITION 5: Assume $\rho = r$ and $\tau_j^D = D_j^* - D_j$ as in Proposition 4A, but $\tau_j^P < P_j^* - P_j$ for all j. Then the chosen lifetime T_j is less than T_j^*, and any small increase in that chosen lifetime T_j for each variant would raise social welfare, because it reduces every welfare cost $WC_j(T_j)$.

[21] For early examples of an optimal deposit-refund system (DRS), see Fullerton and Kinnaman (1995), Palmer and Walls (1997), Fullerton and Wu (1998), Calcott and Walls (2000) or Eichner and Pethig (2001). More recently, Lemoine (2021) uses a dynamic model to derive an optimal DRS for a stock pollutant.

Proof: In these conditions, the existing suboptimal tax τ_j^P is less than the FBO tax (which Proposition 4 shows is $\tau_j^P = P_j^* - P_j$). Since Proposition 3 shows that the chosen lifetime rises monotonically with τ_j^P, the chosen T_j is less than FBO T_j^*. Using equation (2) for c_j, insert $\rho = r$ and $\tau_j^D = D_j^* - D_j$. Then use equation (5) for c_j^* but evaluated at T_j instead of T_j^*. The result is:

$$WC_j(T_j) \equiv c_j^*(T_j) - c_j(T_j) = \frac{\rho}{1 - e^{-\rho T_j}}\left[EC_j - \tau_j^P\right] \tag{7}$$

This welfare cost is zero if $\tau_j^P = EC_j$ (because then $T_j = T_j^*$). If $\tau_j^P < EC_j$, then $WC_j(T_j) > 0$. Next, differentiate (7) to get:

$$\partial WC_j/\partial T_j = \frac{\rho^2 e^{-\rho T_j}}{(1 - e^{-\rho T_j})^2}\left[\tau_j^P - EC_j\right] \tag{8}$$

The first ratio is unambiguously positive. If $\tau_j^P < EC_j$ for all j, then the derivatives in (8) are all negative, so an increase in lifetime T_j of the chosen variant reduces WC_j (raises social welfare).■

For each variant, lifetime T_j is the only choice variable. Thus, in this model with $\rho = r$ and $\tau_j < EC_j$, the welfare-raising increase in product lives can be induced or forced.

PROPOSITION 6: Under the same conditions ($\rho = r$, $\tau_j^D = D_j^* - D_j$ *and* $\tau_j^P < P_j^* - P_j$)*, then social welfare is raised by* (A) *a small uniform increase in every variant's purchase tax*, (B) *a small subsidy that proportionately reduces all maintenance costs, or* (C) *a marginally binding mandate on firms to design longer lasting products.*

Proof: (A) Proposition 3 shows that an increase in tax increases the chosen lifetime, and proposition 5 shows that the longer lifetime raises welfare. (B) A proportional subsidy is a reduction in γ and $\gamma m_j(t)$. Proposition 2 shows that an increase in γ reduces the chosen lifetime, so a subsidy that reduces γ must increase the chosen product lifetime and raise welfare. (C) In this simple model where T_j is the consumer's only choice for each variant, a forced increase in lifetimes of all variants has the same positive effect on welfare. ■

This proposition holds in the rarified environment of this model, but not more generally. For example, the subsidy to maintenance costs must be proportional. If it were a constant amount subtracted from $m_j(t)$ at any t, then the derivative $\partial c_j/\partial T_j$ would still be zero at the original T_j. Also, even the proportional subsidy might not raise welfare in a world with distorting taxes, as might be needed to pay for the subsidy, or with other market failures omitted here. Also, we assumed full information and perfect certainty. To identify just the perfect mandate for each variant (T_j^* for each choice $j = 1, \dots N$), information problems would likely be prohibitive. Still, the conceptual result is important: policy to encourage or require durability can raise welfare.

So far, Proposition 4 shows that $\rho = r$ in our model means that first-best-optimal (FBO) Pigovian taxes correct all externalities with no role for policy to favor long-lived products (no durability gap). But even with $\rho = r$, Propositions 5 and 6 find a durability gap in the second-best world where policy is not optimal (all $\tau_j^P < P_j^* - P_j$). In other words, the interdisciplinary CE literature is not necessarily wrong to call for durability regulations, but a justification in this case would require showing that existing externality policy is permanently suboptimal. If so, then welfare can be raised by raising product taxes, a subsidy for repairs, or a durability mandate.

Next, we investigate the durability gap in the presence of two market failures: production externalities and $\rho < r$. If this latter market failure cannot be corrected by policy that directly affects consumer discounting, then we find the SBO purchase tax for correcting production externalities. This SBO tax does not achieve the first-best optimal correction of all market failures; it cannot correct the externality between generations. But given that consumers put too little weight on the future, it maximizes welfare in the choice of variant. Does this tax depend on durability? In this first model, durability is defined by the maintenance cost schedule $\gamma m_j(t)$, which drives the consumer's choice of T_j. So, we take derivative of (6') with respect to γ:

$$\partial\tau_j^P/\partial\gamma = \frac{\rho(1-e^{-rT_j})}{r(1-e^{-\rho T_j})} \cdot \int_0^{T_j} m_j(t)\, e^{-\rho t} dt - \int_0^{T_j} m_j(t)\, e^{-rt} dt \tag{9}$$

The following lemma will be used in several subsequent proofs.

LEMMA 1: $\frac{re^{-rT_j}}{1-e^{-rT_j}} < \frac{\rho e^{-\rho T_j}}{1-e^{-\rho T_j}}$ *if and only if* $r > \rho$.

Proof: See Appendix A.

PROPOSITION 7: Where the social discount rate is less than the private rate ($\rho < r$), *then the SBO tax* τ_j^P *rises with actual maintenance costs* ($\partial\tau_j^P/\partial\gamma > 0$). *This durability gap means that the SBO tax is reduced for variants with more durability* (*lower* γ). *Moreover, that incentive for durability is larger at lower values of the social discount rate* (*that is,* $\partial^2\tau_j^P/\partial\gamma\partial\rho < 0$).

Proof: See Appendix B.

The tax reduction for durability is enlarged at lower social discount rates. Why? In our model, the consumer's choice of durability is repeated indefinitely, so longer-lasting products reduce the frequency that external costs are imposed – both from disposal and from production of a replacement. When $\rho = r$, private decisions with τ_j^P are first-best optimal, but $\rho < r$ means consumers overly discount those future costs and choose suboptimal durability.

To summarize the big picture for this section, we showed that having no market failures other than externalities means that the FBO taxes are at Pigovian rates ($P_j^* - P_j$ and $D_j^* - D_j$), which do not depend on durability. In a second-best world where $\tau_j^P < P_j^* - P_j$, *or* where $\rho < r$, then the SBO policy in this model does depend on durability.

5. Effects of Consumer Errors in the Second Model with Fixed Product Lifetimes

A more durable product might be one that requires less repair or one that simply lasts longer. We now turn to the latter model where maintenance and repair costs are zero, each variant's lifetime is fixed, and then it goes to disposal. Examples include a lightbulb or toaster oven. In this model, consumers can choose from a *continuum* of possible variants with different lifetimes, so lifetime T is a continuous variable.[22] Thus, this model has no index j for each of N variants. Durability is measured by lifetime, a continuous variable, so the consumer's cost minimization problem has only one step: the choice of lifetime T *is* the choice of variant.

We still assume that each price reflects the cost of production. If P were the same for all product lifetimes, then the longest lifetime would always minimize the levelized private cost $c(T)$. Whereas the prior model used rising maintenance costs to ensure a U-shaped levelized cost curve, this model must assume that a product's price $P(T)$ rises with its durability $[P'(T) > 0]$. After all, if firms minimize the cost of producing at durability T, then increasing that durability all else equal would likely require additional cost (more metals or stronger materials). Estimation for lightbulbs below confirms that $P'(T) > 0$.

We retain simplifying assumptions like perfect certainty, but this model can consider all three categories of market failure (externalities, consumer mistakes, and divergence between social and private discount rates).[23] Specifically, we assume that identical consumers face a tax upon purchase of each product variant with lifetime T while thinking that the lifetime is δT. We write the purchase tax as $\tau^P(T)$, and we find how τ^P depends on T (to identify a durability gap). For simplicity, we assume disposal costs do not depend on T. To proceed, consumers minimize levelized private cost by choosing the product variant with minimum $c(T)$ in:

$$(P(T) + \tau^P(T)) + (D + \tau^D)e^{-r\delta T} = \int_0^{\delta T} c(T)\, e^{-rt} dt = \frac{1 - e^{-r\delta T}}{r} c(T) \tag{10}$$

The socially optimal choice of durability is the one that minimizes social cost $c^*(T)$ in:

[22] Competitive firms in our model are willing to produce whatever consumers want, and CRTS means that these firms can enter or exit with no fixed costs. Thus, we only need to assume that firms *can* make this product with any durability. Identical consumers can choose any lifetime T, but their one choice is the only one produced.

[23] With behavioral effects, however, results may depend on whether the tax is paid by the buyer or the seller.

$$P^*(T) + D^* e^{-\rho T} = \int_0^T c^*(T)\, e^{-\rho t} dt = \frac{1 - e^{-\rho T}}{\rho} c^*(T) \tag{11}$$

We solve for $c(T)$ and $c^*(T)$, and we set them equal to each other. Then we solve for tax rates on consumers that ensure $c(T) = c^*(T)$ for each variant's durability (T):

$$\tau^P(T) = \frac{\rho}{r} \cdot \frac{1-e^{-r\delta T}}{1-e^{-\rho T}} [P^*(T) + D^* e^{-\rho T}] - [P(T) + (D + \tau^D) e^{-r\delta T}] \equiv \mathcal{R} \cdot PV^*(T) - PV(T) \tag{12}$$

where the initial ratio is $\mathcal{R} \equiv \frac{\rho}{r} \cdot \frac{1-e^{-r\delta T}}{1-e^{-\rho T}}$. This tax might be FBO (with no other market failure), or it is SBO with an uncorrected market failure (such as $\rho \neq r$ or $\delta \neq 1$). Given such a market failure, however, the tax in (12) gets consumers to make the SBO choice of product. Facing this tax, consumers who minimize $c(T)$ also minimize $c^*(T)$. The multiple terms in (12) enable taxes to minimize social costs of this purchase given all three types of market failures.

The optimal tax τ^P in equation (12) is still a function of τ^D, as it was before (just as any formula for the optimal τ^D would necessarily depend on τ^P). One equation does not determine both optimal tax rates. So, we can use a particular value of τ^D when we solve for τ^P using (12).

To see how τ^P depends on the lifetime of the product, T, we rewrite equation (12) as $\tau^P(T) = \mathcal{R}P^*(T) - P(T) + \mathcal{R}D^* e^{-\rho T} - (D + \tau^D) e^{-r\delta T}$, and differentiate:

$$\begin{aligned} \partial\tau^P/\partial T = {} & \frac{\rho}{1 - e^{-\rho T}} \left(\delta e^{-r\delta T} - \mathcal{R} e^{-\rho T}\right) P^*(T) + \left(\mathcal{R} P^{*\prime}(T) - P'(T)\right) \\ & + \frac{\rho e^{-\rho T}}{1 - e^{-\rho T}} \left(\delta e^{-r\delta T} - \mathcal{R} e^{-\rho T}\right) D^* - \mathcal{R}\rho e^{-\rho T} D^* + r\delta e^{-r\delta T}(D + \tau^D) \end{aligned} \tag{13}$$

In general, the sign of this derivative is ambiguous. To interpret equation (13), we consider each of the three categories of market failure one at a time and discuss corrective policies.

Interestingly, if perceived lifetimes are correct ($\delta = 1$), then this model is a special case of the first model above (where maintenance costs $m_j(t)$ are zero until time T when the product stops working). Therefore, propositions 4, 5, and 6 also hold in this second model when $\delta = 1$. Those propositions assume $\rho = r$, to find the FBO tax on externalities, and to find welfare effects of raising a suboptimal tax rate.

In this second model where $\rho < r$, however, we have:

PROPOSITION 8: Each variant in this model has no maintenance but a fixed lifetime ($T > 0$). *Assume* $\delta = 1$, $\rho < r$, *and* $\tau^D = \frac{e^{-\rho T}}{e^{-rT}} \mathcal{R} D^* - D$ (*where* $\mathcal{R} \equiv \frac{\rho(1-e^{-rT})}{r(1-e^{-\rho T})} < 1$). *Then:*

(A) *This SBO disposal tax is greater than the FBO Pigovian rate* ($\tau^D > D^* - D$)*, but the SBO product tax is* $\tau^P = \mathcal{R}P^*(T) - P(T)$*, which is less than the Pigouvian rate,* $\tau^P < P^*(T) - P(T)$.

(B) *The condition for* $\tau^P(T)$ *to fall for variants with more durability is complicated. A sufficient condition is* $P'(T) > \mathcal{R}P^{*\prime}(T)$*, where* $P^{*\prime}(T)$ *can be greater than* $P'(T)$*, but not by too much.*

(C) *The condition for* $\tau^P(T)$ *to rise with durability* T *is also complicated, but it requires that* $P^{*\prime}(T) \gg P'(T)$*. The exact condition is shown in the proof.*

Proof: See Appendix C.

We first explain why part (A) says the SBO rate τ^D is higher than $D^* - D$. Consumers buy this product knowing that disposal tax will be paid in T years, but the fact that their discount rate is too high ($r > \rho$) means they think the present value of a tax such as $D^* - D$ is less than the social present value of the social cost. They do pay private cost D, so the optimal cost ($\tau^D + D$) in part (A) has the same present value as D^*. In contrast, the product tax is paid at the time of purchase. In the limit as T approaches zero, then the difference in discount rates does not matter, so $\mathcal{R}$ approaches 1, and τ^P approaches the Pigovian rate $P^* - P$. If the lifetime T is above zero, however, then $\mathcal{R} < 1$, and the τ^P is always less than MED [*i.e.*, $\tau^P < P^*(T) - P(\mathrm{T})$].

The rest of Proposition 8 is about the durability gap, defined as the case where τ^P falls for variants with more durability ($\partial\tau^P/\partial T < 0$). Key to this determination is whether pollution externalities are larger from production of variants designed and produced to live longer. In part (B) where the MED is constant or falling with durability [$\mathcal{R}P^{*\prime}(T) < P'(T)$], then $\partial\tau^P/\partial T < 0$, and so second-best optimal policy indeed encourages durability.

While part (A) says that τ^P lies below MED at positive lifetimes, part (C) appears to contradict that result by saying that τ^P can rise with durability T. But in part (C), more durability T raises the size of the MED itself, because $P^{*\prime}(T) \gg P'(T)$. Thus, the SBO product tax can rise with T even though it is always less than MED. This case of part (C) where the SBO tax rises with durability does not imply a durability gap.

Which is it? In one case, durability may require design and construction with stronger materials that require new extraction or toxic production processes. Then the SBO tax rises with T. In other cases, if pollution does not rise with durability, then part (B) says the optimal tax is lower on products that last longer. Production technology depends on the product, and nobody has empirically investigated whether pollution per unit of production depends on durability.[24]

[24] As described in Bernard (2019, p. 1184): "Design choices influence material choices, production technologies,

Thus, a simple benchmark assumption might be in the middle, where all lifetimes T have the same MED. If so, then Proposition 8B says that the tax falls with T (a durability gap).

Both Propositions 7 and 8 study $\rho < r$ as a market failure where consumers overly discount future costs and so may choose suboptimal durability. Both show exactly when the SBO purchase tax falls with durability, but results differ slightly. When consumers choose durability defined by maintenance cost schedule $m_j(t)$ in the first model, their variant-specific tax τ_j^P already covers production externalities (which do not change if consumers extend the product life). This case provides the unambiguous result in Proposition 7 that the tax rate falls with durability if $\rho < r$. When consumers chose durability defined by product life T in the second model, however, the tax rate can rise with T if negative production externalities rise with T.

Next, we turn to the question of how mistaken lifetimes ($\delta \neq 1$) affect SBO tax rates. We first show how δ affects the SBO tax τ^P, but our main question is whether and how mistakes δ alter the way durability affects that SBO tax rate ($\partial\tau^P/\partial T$). We focus on the case where consumers underestimate product lives ($\delta < 1$), consistent with some labeling evidence.[25]

PROPOSITION 9: Regardless of whether $\rho < r$, the derivative of τ^P with respect to δ is strictly positive, so a lower perceived lifetime δT (given actual lifetime T) means a lower SBO product tax τ^P (and higher δ means higher τ^P). Next, for definitive results about the effect of δ on $\partial\tau^P/\partial T$ in this model with fixed lifetimes, assume $\rho = r$ and $\tau^D = \frac{e^{-\rho T}}{e^{-\rho\delta T}}\mathcal{R}D^* - D$ (where $\mathcal{R} \equiv \frac{1-e^{-\rho\delta T}}{1-e^{-\rho T}}$). Then the SBO product tax is $\tau^P = \mathcal{R}P^*(T) - P(T)$, and we have:

(A) *If $\delta > 1$, then $\mathcal{R} > 1$. The SBO τ^D exceeds the FBO rate ($D^* - D$), and the SBO τ^P also exceeds the FBO rate ($P^* - P$). The exact condition for $\tau^P(T)$ to **fall** for variants with longer lives is complicated. A sufficient condition is $\mathcal{R}P^{*\prime}(T) \leq P'(T)$. But $\mathcal{R} > 1$, so $\partial\tau^P/\partial T < 0$ would require $P^{*\prime}(T)$ to be substantially less than $P'(T)$. The general condition for $\tau^P(T)$ to **rise** with durability T is also complicated, but it has no simple sufficient condition. The exact condition shown in the proof is that $\mathcal{R}P^{*\prime}(T)$ is "enough" bigger than $P'(T)$.*

(B) *If $\delta < 1$, then $\mathcal{R} < 1$. The SBO τ^D is less than FBO rate ($D^* - D$), and SBO τ^P is less than the FBO rate, $P^*(T) - P(T)$. The complicated condition for $\tau^P(T)$ to **fall** with durability is shown in the proof, but it requires that $\mathcal{R}P^{*\prime}(T)$ is "enough" less than $P'(T)$. Since $\mathcal{R} < 1$, this*

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energy performance during use, recyclability, durability, and so on." Her contrasting examples are: "New composite materials in aircraft design reduce aircraft weight [but] these materials are almost completely nonrecyclable. … Conversely, the withdrawal of asbestos in the automotive industry has simultaneously reduced the emissions of hazardous particles during use and eased the repair and remanufacturing processes for longer life duration."

[25] The European Economic and Social Committee (2016, , p.12) states that "lifespan labeling has an influence on purchasing decisions in favor of products with longer lifespans" by an average of 13.8%. Labeling has a significant effect on chosen durability in eight of nine products tested, including *e.g.* suitcase (+23.7%), printer (+20.1%), trousers (+15.9%), or smartphone (+11.4%). Those examples explain our calculations with δ reduced by 20%.

*condition is more likely (but not sufficient) in the simple case where $P^{*\prime}(T) = P'(T)$. The condition for the SBO tax to **rise** with durability is also complicated, but a sufficient condition is $\mathcal{R}P^{*\prime}(T) \geq P'(T)$. Because $\mathcal{R} < 1$, this condition requires $P^{*\prime}(T) > P'(T)$.*

Proof: See Appendix D.

Proposition 9 first shows $\partial\tau^P/\partial\delta > 0$. A higher perceived product lifetime δT means a higher SBO product tax τ^P, but a lower perceived lifetime δT implies a lower SBO tax. It might become a subsidy. Why? A consumer who underestimates the product lifetime ($\delta T < T$) must think that the initial payment of $P(T)$ will yield services over a shorter life. Thus, a lower δ raises the perceived levelized cost $c(T)$. The consumer thinks this product's services cost more than their true cost. This mistake can be corrected by a policy that *reduces* perceived annual cost, using a subsidy. Policymakers can address production and disposal externalities with taxes, but they also can reduce the distortion caused by this consumer error by reducing τ^P. If the mistake is large enough, the externality tax is more than offset, making τ^P negative.[26]

On our main topic, the durability gap, Proposition 9A shows that *overestimation* of product lives ($\delta > 1$) implies that both τ^D and τ^P *exceed* MED. Also, finding a durability gap would be difficult, because $\partial\tau^P/\partial T < 0$ would require that the production externality *fall* with durability, more than a just little: $P^{*\prime}(T) \ll P'(T)$.

In Proposition 9B, the *underestimation* of product lives ($\delta < 1$) means that both taxes are less than their respective MED. It also says that finding a durability gap is a bit easier. The proof includes no simple sufficient condition, but it shows that production externalities might be flat or even rise with durability and still yield a "durability gap" where $\partial\tau^P/\partial T < 0$. Conversely, it shows that production externalities would have to rise rapidly with durability [$P^{*\prime}(T) \gg P'(T)$] to find a case where the SBO tax rate rises for products with longer lifetimes.

Specific results in Proposition 9 show that these mistakes can change the slope $\partial\tau^P/\partial T$ in either direction, but these specific results are not ready for policy implementation. First, this simple model omits too many other relevant variables. Second, despite our simplifications, the optimal tax in (12) has complicated components. Third, more evidence is needed to on whether consumers underestimate or overestimate product lives. Fourth, we have no proofs about how the SBO tax relates to durability with all three market failures simultaneously. The purpose here is not to determine and enact optimal tax rates, but to demonstrate conceptual results for intuition

[26] If the error $\delta < 1$ is the *only* market failure, then the SBO tax rate is unambiguously negative.

about how these market failures affect optimal policy regarding durability.

So what did we learn? It's certainly intuitive that τ^P can rise with durability if producing a good with more durability entails more damaging externalities. Also intuitive is how $\rho < r$ in Proposition 8 makes consumers underinvest (reducing the optimal tax on goods with longer lifetimes). But Propositions 9A and 9B assume $\rho = r$. So then how does the mistake δ alone affect the slope $\partial\tau^P/\partial T$? The intuition here is that the mistake δ enters $\mathcal{R} \equiv \frac{1-e^{-r\delta T}}{1-e^{-\rho T}}$ essentially by changing the private discount rate to $r\delta$. Consumers who underestimate the product life ($\delta < 1$) effectively discount at a lower rate. They buy a product that lasts longer than they want, so they effectively give more weight to the future than intended. If $\rho = r$, they weight the future by more than socially optimal. The policy corrects that mistake by discouraging durability.

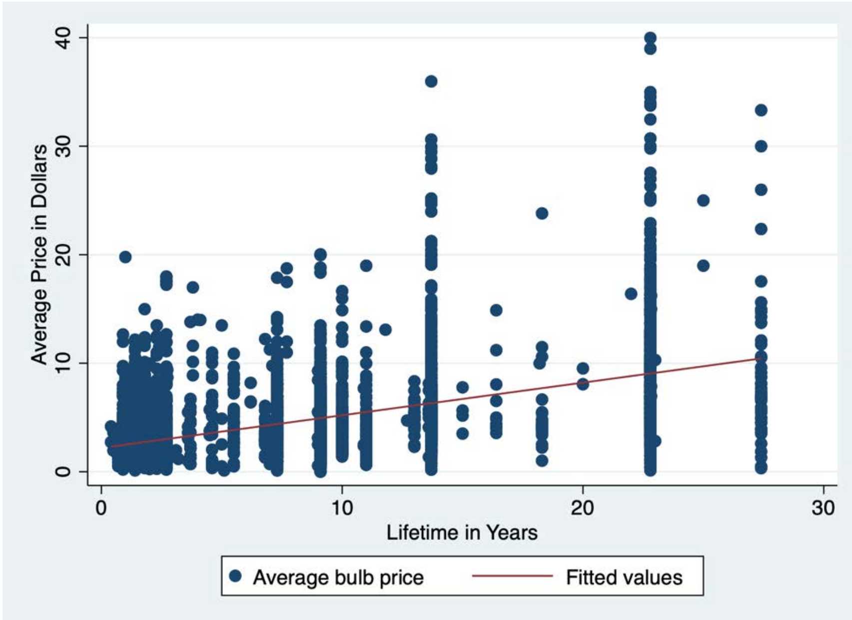
6. Simple Numerical Illustrations

Using this second model, we can show graphically how the SBO product tax changes with durability in a particular example for alternative parameter values regarding each market failure. For this "one-hoss-shay" technology, we consider a continuum of variants with different lifetimes ($0 < T < \infty$). They cannot be repaired, but together they offer a continuous choice of lifetime T. Lightbulbs are a set of products that provide the same stream of services and have differing lifetimes. We use data on 4,362 lightbulbs to estimate the relationship between price and bulb life, $P(T)$, and we choose values for other parameters in equations above. We insert parameter values into equation (12) and show how the SBO tax rate depends on durability.

Our data for lightbulbs in the U.S. market from 2015 to 2019 are scraped off the web from the largest online lightbulb retailer (1000Bulb.com). They include the most common types of bulbs purchased by households.[27] For each bulb in each U.S. state in each year, we have information on average sold price and on key bulb attributes (*e.g.*, bulb type, energy use, color temperature, and stated lifetime). Figure 1 displays a scatterplot showing each bulb's stated lifetime and price. In the figure, each bulb's price is averaged across five years of data, across fifty U.S. states, and across other bulb attributes. It shows generally that an increase in durability increases price, and that consumers are willing to pay extra for variants that last longer.

[27] These bulbs are candelabra (e12), intermediate (e17), and medium (e26). The bulb shapes are arbitrary (A), blunt tip (B), candle (C), reflector (BR, MR, R, PAR, RD), globe (G), straight (S, ST), tubular (T), and spiral. Among the 4,362 lightbulbs, 1649 are incandescent bulbs, 625 are CFL bulbs, and 2,088 are LED bulbs.

Figure 1: Scatterplot and Fitted Value of Lightbulb Price vs. Lifetime



To estimate the production cost $P(T)$ from these data, we use both nonparametric and parametric regressions. Because omitted variables might bias the estimated effects of durability T, we also control for other attributes available in the data (bulb type, shape, base, brightness, energy use, color temperature, and "Energy Star" status). We find that a linear functional form is sufficient to fit the data just as well as does the nonparametric regression. Thus, we estimate $P(T) = 2.981 + 0.328T$, using linear regression (controlling for observable bulb attributes).

Next, the social cost $P^*(T)$ includes both private costs $P(T)$ and all external costs from production of a lightbulb (but not from the use of it). Negative externalities may include water pollution from mining the materials in a lightbulb, local waste from drilling fossil fuels used for power during production, emissions of local air pollutants, and global greenhouse gas pollution. First, for climate damages from CO2, we use $50/ton as the estimate of the social cost of carbon (SCC) from the U.S. Interagency Working Group (2021, p.7). Then, from the life-cycle assessment study of Scholand and Dillon (2012), we obtain an estimate of CO2 emissions during the manufacturing of each bulb type.[28] Weighting by the quantity of each type, we find that 0.0029 tons of CO2 were emitted per unit of production of the average bulb.

But burning fossil fuels also creates emissions of local air pollutants such as PM2.5, SO2, and NOX. Estimates of local damages from these co-pollutants are sometimes found to be less

[28] To omit carbon emissions from consumer use, we sum CO2 produced from the following production stages of each lightbulb type: raw materials, manufacturing, transport, and disposal.

than the SCC per ton of carbon, but they are most often estimated to exceed the SCC. Roth *et al*. (2020) find that a tax equal to their estimate of SCC has co-benefits that are almost *twice* the carbon abatement benefits.[29] To be conservative, we assume damages from co-pollutants do not exceed our SCC = \$50/ton but instead just equal the same damages (another \$50/ton). Thus, we use \$100 as the overall MED per ton of CO2 emissions. This figure is also conservative because it omits externalities from mining, drilling, and spills. This \$100 per ton of CO2 is multiplied by 0.0029 tons of CO2 per unit production to get MED of \$0.29 per bulb. The corresponding social cost function is $P^*(T) = P(T) + 0.29 = 3.281 + 0.328T$.

Figure 2: How Annualized Costs Depend on Product Lifetime

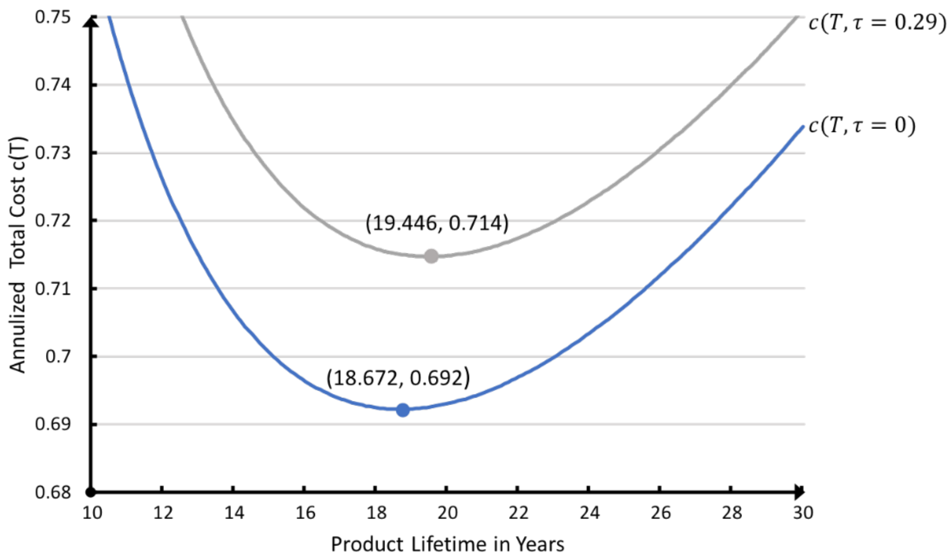


Figure 2 illustrates the consumer's choice of variant for the case with $\rho = r = 0.04$ and $\delta = 1$. It shows the U-shaped annualized cost curves across possible variant lifetimes up to thirty years.[30] Untaxed consumers choose a lifetime at the minimum of the private cost curve (labeled τ = 0). They choose the bulb that can be produced with a life of 18.7 years, where the annualized

[29] Looking at various CO2 reduction activities, Balbus *et al*. (2014) find that the health co-benefits from reduced PM2.5 would yield benefits between \$40 and \$198 per metric ton of CO2. Dedoussi *et al*. (2019) find co-benefits to be about 120% of the SCC measure of climate damages per ton of CO2. Karlsson *et al*. (2020) review 239 recent studies of co-benefits from climate policy and describe eight specific US studies that find local air quality co-benefits between \$8 and \$148 per ton of CO2, depending on the location and method of CO2 abatement. Most of this range exceeds the estimate of the direct benefits from carbon abatement. A caveat is that the degree to which $P^*(T)$ exceeds $P(T)$ also depends on existing policy. The U.S. has no carbon tax, but the use of fossil fuels faces various regulations that might raise $P(T)$. Fossil fuels also receive implicit subsidies through tax rules and under-priced leases of public land. We ignore these offsetting policies because their net effects are unknown.

[30] This illustration abstracts from many complications mentioned throughout this paper, including the possibility that durability is correlated with other attributes like product recyclability. For example, see Eichner and Runkel (2003).

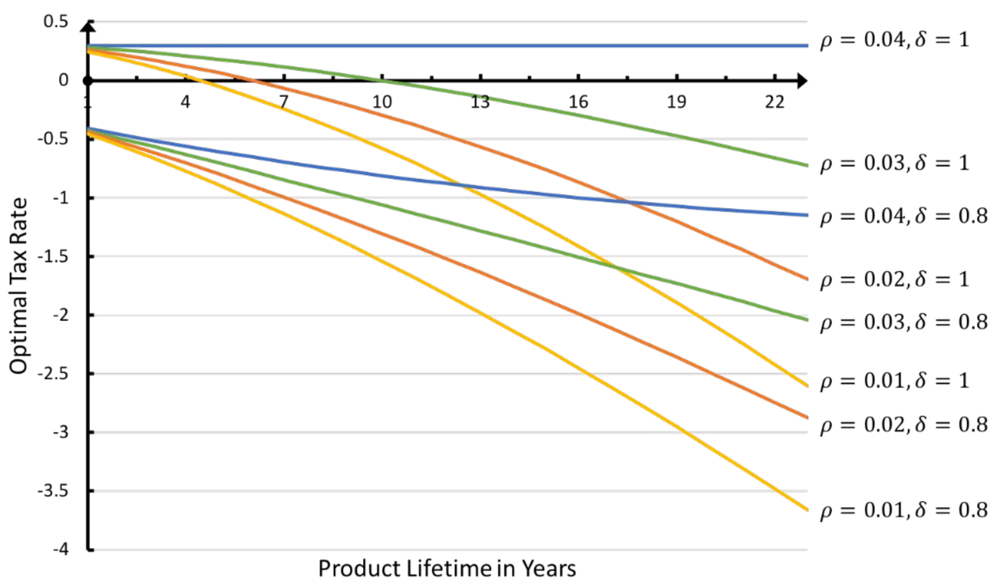
welfare cost from equation (7) is the difference in annualized costs: 0.715– 0.692 = \$0.023 per year (per bulb purchased). Facing a tax of \$0.29 per bulb, however, consumers choose the first-best optimum T of 19.45 years, where the welfare cost is zero.

Next, we illustrate how the optimal tax depends on durability – for alternative market failure parameters. Using each set of those parameters, Figure 3 looks at the calculated optimal purchase tax and shows how it depends on variant lifetime in this example where MED = $P^* - P$ = \$0.29 (regardless of durability). Initially, look at only the top four curves (omitting any errors, so $\delta = 1$). The top-most curve with $\rho = r = 0.04$ shows the special case where the FBO tax is a flat \$0.29 and does not depend on durability (Proposition 4). In fact, all curves with $\delta = 1$ and $r = 0.04$ start at \$0.29 at $T = 0$, but any $\rho < 0.04$ yields an optimal tax less than MED (Proposition 8A). This tax declines with durability T (Proposition 8B). Perhaps surprisingly, even though ρ = 0.03 is only slightly lower than r = 0.04, the tax then starts at \$0.29 at $T = 0$ and falls gradually to a net subsidy for all products with lifetimes over ten years.

For the case with all market failures simultaneously (equation 12), Proposition 9 says that an underestimated lifetime ($\delta < 1$) implies that the SBO tax is less than MED, and that the derivative of the tax with respect to lifetime can be positive when MED from production rises with durability. Our simple example here assumes that the MED does not depend on T, and so the slope is not positive for any curve in Figure 3.

Figure 3: How the Optimal Tax Rate on Purchase Depends on Product Lifetime

$(r = 0.04,\ P(T) = 2.981 + 0.328T,\ P^*(T) = 3.271 + 0.328T)$



Finally, in Figure 3, the four curves for δ = 0.8 are broadly similar to those for $\delta = 1$. For all values of δ, the other market failure ($\rho < r$) makes the SBO tax decline with durability.

While this example de-emphasizes the case where the SBO tax rises with durability, the important point from earlier sections is that internalities or mistakes can interact with discount rates and affect slope in either direction. The consumer mistake here is a fixed proportional error in perceived life, from T to δT. If instead δ were to *vary* with T, then the optimal tax could vary correspondingly with T. We cannot analyze all models, so we make no general claims about how internalities or errors affect the way optimal taxes relate to durability. Among others, Heutel (2015) discusses various models of present-bias internalities and policies to correct them.

With no mistakes or internalities ($\gamma = 1$), and with environmental damages that do not rise substantially when producing goods with more durability, then our key result is that a low social discount rate ($\rho < r$) leads to a tax that optimally falls with durability. This case provides the most support for those who think optimal policy would encourage durability.

7. Conclusion

The background for our analysis is that the choice of durability itself does not generate an externality that requires direct intervention. Consumers already make socially optimal choices about durability if they are informed and rational optimizers facing all social costs of production and disposal (e.g., facing Pigovian taxes). Given this background, we make five contributions.

First, if the tax is less than marginal external damage per unit output – as seems likely in many jurisdictions – then raising the tax increases chosen durability and increases welfare. Second, if the tax remains too low, then a durability subsidy or mandate can increase welfare. Third, a social rate of discount less than the private rate does cause a durability gap (if external costs of production do not increase too rapidly with chosen durability). If so, the optimal tax explicitly encourages durability. A lower social discount rate on a long-lived product can turn the second-best product tax into subsidy (despite negative externalities from production). Fourth, if consumers underestimate product lifetime, then they overstate the annual cost of its services. A purchase subsidy can correct that problem, offsetting the Pigovian tax, but that subsidy does not systematically relate to durability. Fifth, we estimate how the price of 4,362 types of lightbulbs depend on stated lifetime – controlling for other attributes – and we use damages from CO2 and local pollutants to calibrate the external cost per lightbulb. Inserting these parameters into our formulas allow graphical illustration of how second-best tax rates fall for longer-lived lightbulbs in cases where the private discount rate exceeds the social rate.

Yet much work remains. This paper introduces a "durability gap" and begins analysis of it, which helps with understanding the issues, but extensions could make the model more applicable. One extension could consider uncertainty about product lifetimes, with or without risk aversion of consumers. Another extension is to consider technological progress that reduces the expected price to be paid for product replacement. Other extensions might consider market power or other market imperfections, general equilibrium effects, or consumer heterogeneity.

The "Circular Economy" is a popular topic in the interdisciplinary literature, but economic analysis is lacking. Further analysis can extend not only to the durability issues just listed, but also to other circular economy issues about how to delay and reduce creation of waste, how durability relates to recyclability, and how to encourage the conversion of waste into valuable inputs that can re-enter production.

References

Akerlof, George A. 1970. "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." *Quarterly Journal of Economics* 84 (3): 488-500. https://doi.org/10.2307/1879431.

Allcott, Hunt. 2016. "Paternalism and Energy Efficiency: An Overview." *Annual Review of Economics* 8: 145-176. https://doi.org/10.1146/annurev-economics-080315-015255.

Allcott, Hunt, and Michael Greenstone. 2012. "Is There an Energy Efficiency Gap?" In *Journal of Economic Perspectives* 26 (1): 3–28. https://doi.org/10.1257/jep.26.1.3.

Balbus, John M., Jeffery B. Greenblatt, Ramya Chari, Dev Millstein, and Kristie L. Ebi. 2014. "A Wedge-Based Approach to Estimating Health Co-Benefits of Climate Change Mitigation Activities in the United States." *Climatic Change* 127 (2): 199–210. https://doi.org/10.1007/s10584-014-1262-5.

Baumol, William J., and Wallace E. Oates. 1988. *The Theory of Environmental Policy*. Cambridge: Cambridge University Press. https://doi.org/10.1017/cbo9781139173513.

Berck, Peter, Marshall Blundell, Gabriel Englander, Samantha Gold, Shelley He, Janet Horsager, Scott Kaplan, *et al*. 2021. "Recycling Policies, Behavior and Convenience: Survey Evidence from the CalRecycle Program." *Applied Economic Perspectives and Policy* 43 (2): 641–58. https://doi.org/10.1002/aepp.13117.

Bernard, Sophie. 2019. "Multidimensional Green Product Design." *Environmental and Resource Economics* 72 (4): 1183–1202. https://doi.org/10.1007/s10640-018-0243-y.

Bovenberg, A. Lans, and Lawrence H. Goulder. 2002. "Environmental Taxation and Regulation." Chapter 23 in A. Auerbach and M. Feldstein, eds., *Handbook of Public Economics* 3:1471–1545. https://doi.org/10.1016/S1573-4420(02)80027-1.

Boyce, John R., and Gregory E. Goering. 1997. "Optimal Taxation of a Polluting Durable Goods Monopolist." *Public Finance Review* 25 (5): 522–41. https://doi.org/10.1177/109114219702500505.

Bulow, Jeremy. 1986. "An Economic Theory of Planned Obsolescence." *Quarterly Journal of Economics* 101 (4): 729–49. https://doi.org/10.2307/1884176.

Calcott, Paul, and Margaret Walls. 2000. "The Economics of Waste: Can Downstream Waste Disposal Policies Encourage Upstream 'Design for Environment'?" *American Economic Review* 90 (2): 233–37. https://doi.org/10.1257/aer.90.2.233.

Dedoussi, Irene C., Florian Allroggen, Robert Flanagan, Tyler Hansen, Brandon Taylor, Steven R.H. Barrett, and James K. Boyce. 2019. "The Co-Pollutant Cost of Carbon Emissions: An Analysis of the US Electric Power Generation Sector." *Environmental Research Letters* 14 (9): 1-12. https://doi.org/10.1088/1748-9326/ab34e3.

Dellavigna, Stefano. 2009. "Psychology and Economics: Evidence from the Field." *Journal of Economic Literature* 47 (2): 315–72. https://doi.org/10.1257/jel.47.2.315.

Dixit, Avinash K., and Robert S. Pindyck. 1994. *Investment under Uncertainty*. Princeton: Princeton University Press. https://doi.org/10.2307/2329279.

Eichner, Thomas, and Rüdiger Pethig. 2001. "Product Design and Efficient Management of Recycling and Waste Treatment." *Journal of Environmental Economics and Management* 41 (1): 109–34. https://doi.org/10.1006/jeem.2000.1126.

Eichner, Thomas, and Marco Runkel. 2003. "Efficient Management of Product Durability and Recyclability under Utilitarian and Chichilnisky Preferences." *Journal of Economics/ Zeitschrift Fur Nationalokonomie* 80 (1): 43–75. https://doi.org/10.1007/s00712-002-0607-0.

European Economic and Social Committee. 2016. "The Influence of Lifespan Labelling on Consumers." *European Economic and Social Committee*. https://data.europa.eu/doi/10.2864/103864.

Fullerton, Don, and Thomas C. Kinnaman. 1995. "Garbage, Recycling, and Illicit Burning or Dumping." *Journal of Environmental Economics and Management* 29 (1): 78–91. https://doi.org/10.1006/jeem.1995.1032.

———. 1996. "Household Responses to Pricing Garbage by the Bag." *American Economic Review* 86 (4): 971–84. https://doi.org/10.2307/2118314.

Fullerton, Don, and Wenbo Wu. 1998. "Policies for Green Design." *Journal of Environmental Economics and Management* 36 (2): 131–48. https://doi.org/10.1006/jeem.1998.1044.

Geissdoerfer, Martin, Paulo Savaget, Nancy M.P. Bocken, and Erik Jan Hultink. 2017. "The Circular Economy – A New Sustainability Paradigm?" *Journal of Cleaner Production* 143: 757-68. https://doi.org/10.1016/j.jclepro.2016.12.048.

Gerarden, Todd D., Richard G. Newell, and Robert N. Stavins. 2017. "Assessing the Energy-Efficiency Gap." *Journal of Economic Literature* 55 (4): 1486–1525. https://doi.org/10.1257/jel.20161360.

Gillingham, Kenneth, Richard G. Newell, and Karen Palmer. 2009. "Energy Efficiency Economics and Policy." *Annual Review of Resource Economics* 1 (1): 597–620.

https://doi.org/10.1146/annurev.resource.102308.124234.

Goering, Gregory E., and John R. Boyce. 1999. "Emissions Taxation in Durable Goods Oligopoly." *Journal of Industrial Economics* 47 (1): 125–43. https://doi.org/10.1111/1467-6451.00093.

Goulder, Lawrence H., and Roberton C. Williams. 2012. "The Choice of Discount Rate for Climate Change Policy Evaluation." In *Climate Change Economics*. Vol. 3. https://doi.org/10.1142/S2010007812500248.

Greenstone, Michael, and B. Kelsey Jack. 2015. "Envirodevonomics: A Research Agenda for an Emerging Field." *Journal of Economic Literature* 53 (1): 5–42. https://doi.org/10.1257/jel.53.1.5.

Hall, Robert E, and Dale W Jorgenson. 1967. "Tax Policy and Investment Behavior." *The American Economic Review* 57 (3): 391–414. http://www.jstor.org/stable/1812110.

Hendel, Igal, and Alessandro Lizzeri. 1999a. "Adverse Selection in Durable Goods Markets." *American Economic Review* 89 (5): 1097–1115. https://doi.org/10.1257/aer.89.5.1097.

———. 1999b. "Interfering with Secondary Markets." *The RAND Journal of Economics* 30 (1): 1-21. https://doi.org/10.2307/2556043.

Hendrickson, Chris, Arpad Horvath, Satish Joshi, and Lester Lave. 1998. "Economic Input-Output Models for Environmental Life-Cycle Assessment." *Environmental Science and Technology* 32 (7). https://doi.org/10.1021/es983471i.

Heutel, Garth. 2015. "Optimal Policy Instruments for Externality-Producing Durable Goods under Present Bias." *Journal of Environmental Economics and Management* 72: 54–70. https://doi.org/10.1016/j.jeem.2015.04.002.

Karlsson, Mikael, Eva Alfredsson, and Nils Westling. 2020. "Climate Policy Co-Benefits: A Review." *Climate Policy* 20 (3): 292–316. https://doi.org/10.1080/14693062.2020.1724070.

Kim, Jae Cheol. 1989. "Trade in Used Goods and Durability Choice." *International Economic Journal* 3 (3): 53–63. https://doi.org/10.1080/10168738900080020.

Kinokuni, Hiroshi, Shuichi Ohori, and Yasunobu Tomoda. 2019. "Optimal Waste Disposal Fees When Product Durability Is Endogenous: Accounting for Planned Obsolescence." *Environmental and Resource Economics* 73 (1): 33–50. https://doi.org/10.1007/s10640-018-0248-6.

Lemoine, Derek. 2021. "Incentivizing Negative Emissions Through Carbon Shares." *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.3705090.

Levhari, David, and TN Srinivasan. 1969. "Durability of Consumption Goods: Competition Versus Monopoly." *The American Economic Review* 59 (1): 102–107. http://www.jstor.org/stable/1811096.

Miotti, Marco, Geoffrey J. Supran, Ella J. Kim, and Jessika E. Trancik. 2016. "Personal Vehicles Evaluated against Climate Change Mitigation Targets." *Environmental Science and Technology* 50 (20): 10795–804. https://doi.org/10.1021/acs.est.6b00177.

Montalvo, Carlos, David Peck, and Elmer Rietveld. 2016. "A Longer Lifetime for Products: Benefits for Consumers and Companies." *Study for the IMCO Committee (European*

Parliament). https://www.europarl.europa.eu/RegData/etudes/STUD/2016/579000/IPOL_STU(2016)579000_EN.pdf.

Nordhaus, William D. 2007. "A Review of the Stern Review on the Economics of Climate Change." *Journal of Economic Literature* 45 (3): 686-702. https://doi.org/10.1257/jel.45.3.686.

Palmer, Karen, and Margaret Walls. 1997. "Optimal Policies for Solid Waste Disposal Taxes, Subsidies, and Standards." *Journal of Public Economics* 65 (2): 193–205. https://doi.org/10.1016/S0047-2727(97)00028-5.

Pigou, Arthur C. 1932. *The Economics of Welfare*. 4th ed., London: Macmillan & Co. https://doi.org/10.4324/9781351304368.

Richter, Jessika Luth, Robert Van Buskirk, Carl Dalhammar, and Peter Bennich. 2019. "Optimal Durability in Least Life Cycle Cost Methods: The Case of LED Lamps." *Energy Efficiency* 12 (1): 107–21. https://doi.org/10.1007/s12053-018-9662-4.

Roth, Michael Buchdahl, Peter J. Adams, Paulina Jaramillo, and Nicholas Z. Muller. 2020. "Near Term Carbon Tax Policy in the US Economy: Limits to Deep Decarbonization." *Environmental Research Communications* 2 (5): 1-13. https://doi.org/10.1088/2515-7620/ab8616.

Runkel, Marco. 2003. "Product Durability and Extended Producer Responsibility in Solid Waste Management." *Environmental and Resource Economics* 24 (2): 161–82. https://doi.org/10.1023/A:1022800206337.

———. 2004. "Optimal Emissions Taxation under Imperfect Competition in a Durable Good Industry." *Bulletin of Economic Research* 56 (2): 115–32. https://doi.org/10.1111/j.1467-8586.2004.00192.x.

Rust, John. 1986. "When Is It Optimal to Kill off the Market for Used Durable Goods?" *Econometrica* 54 (1): 65-86. https://doi.org/10.2307/1914157.

Schmalensee, Richard. 1974. "Market Structure, Durability, and Maintenance Effort." *Review of Economic Studies* 41 (2): 277–87. https://doi.org/10.2307/2296716.

Scholand, Michael, and Dillon, Heather E. 2012. "Life-Cycle Assessment of Energy and Environmental Impacts of LED Lighting Products Part 2: LED Manufacturing and Performance". U.S. Department of Energy, Office of Scientific and Technical Information. https://doi.org/10.2172/1044508. https://www.osti.gov/servlets/purl/1044508.

Stahel, Walter R. 1982. "The Product-Life Factor." Geneva: Product-Life Institute. http://www.product-life.org/en/major-publications/the-product-life-factor.

———. 2016. "The Circular Economy." *Nature* 531(7595): 435-8. https://doi.org/10.1038/531435a.

Stern, Nicholas. 2007. *The Economics of Climate Change: The Stern Review*. Cambridge: Cambridge University Press. https://doi.org/10.1017/CBO9780511817434.

Swan, Peter L. 1970. "Durability of Consumption Goods." *American Economic Review* 60 (5): 884-94.

Taylor, Rebecca L.C. 2020. "A Mixed Bag: The Hidden Time Costs of Regulating Consumer Behavior." *Journal of the Association of Environmental and Resource Economists* 7 (2): 345–78. https://doi.org/10.1086/707039.

U.S. Federal Trade Commission (2021). *Nixing the Fix: An FTC Report to Congress on Repair Restrictions*, Washington DC. At: https://www.ftc.gov/system/files/documents/reports/nixing-fix-ftc-report-congress-repair-restrictions/nixing_the_fix_report_final_5521_630pm-508_002.pdf

U.S. Interagency Working Group on Social Cost of Greenhouse Gases (2021). "Technical Support Document: Social Cost of Carbon, Methane, and Nitrous Oxide – Interim Estimates under Executive Order 13990". At: https://perma.cc/5B4Q-3T5Q.

Waldman, Michael. 2003. "Durable Goods Theory for Real World Markets." *Journal of Economic Perspectives* 17 (1): 131–54. https://doi.org/10.1257/089533003321164985.

Appendices

Appendix A: Prove Lemma 1, that $\frac{re^{-rT_j}}{1-e^{-rT_j}} < \frac{\rho e^{-\rho T_j}}{1-e^{-\rho T_j}}$ if and only if $r > \rho$.

First, notice that both sides of this inequality have the form $y = \frac{xe^{-xT_j}}{1-e^{-xT_j}}$ (where x could be either r or ρ). Differentiate this expression with respect to x :

$$\partial y/\partial x = \frac{e^{-xT_j}\,(1 - xT_j - e^{-xT_j})}{(1 - e^{xT_j})^2}$$

The first term in the numerator (e^{-xT_j}) is always positive, but the bracketed term is zero if xT_j is zero. For all $xT_j > 0$, the derivative of that bracketed term with respect to xT_j is (e^{-xT_j}–1), which is negative. So, as xT_j rises from zero, the numerator starts at zero and then falls. Thus, for $xT_j > 0$, the numerator of $\partial y/\partial x$ is negative. The denominator is positive, so $\partial y/\partial x < 0$.

To look at the two sides of the key inequality above, we note that $\rho < r$. If the value of x were to rise from ρ toward r, then the ratio y falls below $\frac{\rho e^{-\rho T_j}}{1-e^{-\rho T_j}}$. Thus, $\frac{re^{-rT_j}}{1-e^{-rT_j}} < \frac{\rho e^{-\rho T_j}}{1-e^{-\rho T_j}}$. ■

Appendix B: Prove Proposition 7

A sufficient condition for $\partial\tau_j^P/\partial\gamma > 0$ in equation (9) is:

$$\int_0^{T_j} m_j(t)\,e^{-rt}dt \Big/ \int_0^{T_j} m_j(t)\,e^{-\rho t}dt < \frac{\rho(1 - e^{-rT_j})}{r(1 - e^{-\rho T_j})}$$

To show that the right-hand-side (RHS) of this inequality exceeds the left-hand-side (LHS), first note that $\rho < r$ means $0 < e^{-rT_j} < e^{-\rho T_j} < 1$. Because $m_j(t)$ is rising with t [so $m_j{}'(t) > 0$], the LHS is always less than $\frac{e^{-rT_j}}{e^{-\rho T_j}}$ at any T_j. Also, Appendix A shows that $\frac{re^{-rT_j}}{1-e^{-rT_j}} < \frac{\rho e^{-\rho T_j}}{1-e^{-\rho T_j}}$ (where $\rho < r$, and $\delta = 1$). Thus, $\frac{e^{-rT_j}}{e^{-\rho T_j}}$ is less than the RHS (proving $\partial\tau_j^P/\partial\gamma > 0$). Then, we further differentiate $\partial\tau_j^P/\partial\gamma$ in (9) with respect to ρ to get:

$$\partial^2\tau_j^P/\partial\gamma\partial\rho = \left[-\frac{\rho T_j e^{-\rho T_j}}{\left(1 - e^{-\rho T_j}\right)^2}\cdot\int_0^{T_j} m_j(t)\,e^{-\rho t}dt\right] + \left[-\frac{\rho\left(1 - e^{-rT_j}\right)}{r\left(1 - e^{-\rho T_j}\right)}\cdot\int_0^{T_j} tm_j(t)\,e^{-\rho t}dt\right]$$

With $\rho < r$, then $\frac{\rho T_j e^{-\rho T_j}}{\left(1-e^{-\rho T_j}\right)^2} > 0$ and $\frac{\rho\left(1-e^{-rT_j}\right)}{r\left(1-e^{-\rho T_j}\right)} > 0$. Thus, both the first and the second terms in brackets are negative, and $\partial^2\tau_j^P/\partial\gamma\partial\rho < 0$. ■

Appendix C: Prove Proposition 8

With $\delta = 1$, we first show that $\mathcal{R} < 1$. In the limit, as T approaches zero, the ratio $\mathcal{R} \equiv \frac{\rho(1-e^{-rT})}{r(1-e^{-\rho T})}$ approaches 1. For $T > 0$, differentiate $\mathcal{R}$ as:

$$\partial\mathcal{R}/\partial T = \frac{\rho}{1-e^{-\rho T}}\left(e^{-rT} - \frac{\rho}{r}\cdot\frac{1-e^{-rT}}{1-e^{-\rho T}}e^{-\rho T}\right) \equiv \frac{\rho}{1-e^{-\rho T}}(e^{-rT} - \mathcal{R}e^{-\rho T})$$

The first ratio is positive. If $\rho < r$, the term in brackets is negative, following the logic of Appendix A (with simple rearranging). Thus, $\rho < r$ implies $\partial\mathcal{R}/\partial T < 0$, and T rising from zero means that $\mathcal{R}$ falls below 1.

For part (A), because $\frac{e^{-\rho T}}{e^{-rT}}\mathcal{R} > 1$ from Appendix A, the SBO tax $\tau^D = \frac{e^{-\rho T}}{e^{-rT}}\mathcal{R}D^* - D$ exceeds the FBO Pigovian rate $D^* - D$. Next, substitute that expression for τ^D into the long expression for τ^P in equation (12), which then reduces to $\tau^P(T) = \mathcal{R}P^*(T) - P(T)$. Then, because $\mathcal{R} < 1$, the tax $\tau^P = \mathcal{R}P^*(T) - P(T)$ is less than the Pigouvian rate $P^*(T) - P(T)$.

For part (B), the derivative of $\tau^P = \mathcal{R}P^*(T) - P(T)$ with respect to T is:

$$\partial\tau^P/\partial T = \frac{\rho}{1-e^{-\rho T}}(e^{-rT} - \mathcal{R}e^{-\rho T})P^*(T) + \left(\mathcal{R}P^{*\prime}(T) - P'(T)\right)$$

We know that $\frac{re^{-rT}}{1-e^{-rT}} < \frac{\rho e^{-\rho T}}{1-e^{-\rho T}}$ (from Appendix A), which implies $e^{-rT} - \mathcal{R}e^{-\rho T} < 0$. So the long first term is negative. The second term can be either sign, so $\partial\tau^P/\partial T < 0$ if and only if the second term, $\mathcal{R}P^{*\prime}(T) - P'(T)$, is negative or not very positive (*i.e.*, does not exceed the absolute value of the negative first term). A simpler sufficient condition is $\mathcal{R}P^{*\prime}(T) \leq P'(T)$, so that the second term is not positive. Because $\mathcal{R} < 1$, then $P^{*\prime}(T)$ can be greater than $P'(T)$, but not by too much. Then the negative first term means $\partial\tau^P/\partial T < 0$.

For part (C), the condition for the tax to *rise* with durability is

$$\partial\tau^P/\partial T = \frac{\rho}{1-e^{-\rho T}}(e^{-rT} - \mathcal{R}e^{-\rho T})P^*(T) + \left(\mathcal{R}P^{*\prime}(T) - P'(T)\right) > 0.$$

The long first term is negative. Thus, the overall sign is positive if and only if the second term is positive and greater than the absolute value of the negative first term. But $\mathcal{R} < 1$, however, so $\partial\tau^P/\partial T > 0$ if and only if $P^{*\prime}(T)$ is much bigger than $P'(T)$. ■

Appendix D: Prove Proposition 9

Allowing $\rho \neq r$, differentiate (12) with respect to δ:

$$\partial\tau^P/\partial\delta = \frac{\rho}{r}\cdot\frac{1}{1-e^{-\rho T}}\cdot rTe^{-\rho\delta T}[P^*(T) + D^*e^{-\rho T}] + rTe^{-\rho\delta T}(D + \tau^D) > 0$$

Each term in this expression is positive, so the overall derivative is unambiguously positive. Thus, any perceived lifetime δT_j substantially below the actual lifetime T_j means that the optimal tax is lower. If consumers *overestimate* the product lifetime, then τ_j^P is above $P_j^* - P_j$.

Next assume $\rho = r$. With production and disposal externalities, then (12) reduces to:

$$\tau^P(T) = \mathcal{R}[P^*(T) + D^* e^{-\rho T}] - [P(T) + (D + \tau^D)e^{-\rho\delta T}]$$

Insert $\tau^D = \frac{e^{-\rho T}}{e^{-\rho\delta T}}\mathcal{R}D^* - D$ into that equation and rearrange. The result is $\tau^P = \mathcal{R}P^*(T) - P(T)$. In the limit, as T approaches zero, the ratio $\mathcal{R} \equiv \frac{1-e^{-\rho\delta T}}{1-e^{-\rho T}}$ approaches 1.

For part (A) where $\delta > 1$ and $T > 0$, then $\frac{e^{-\rho T}}{e^{-\rho\delta T}}\mathcal{R} > 1$, following logic of Appendix A. (Appendix A says "if and only if $r > \rho$", and here $r = \rho$, but r here is multiplied by $\delta > 1$, which makes $r\delta > \rho$.) Then the SBO tax $\tau^D = \frac{e^{-\rho T}}{e^{-\rho\delta T}}\mathcal{R}D^* - D$ exceeds the FBO rate ($D^* - D$). Also, $\delta > 1$ means that $\mathcal{R} \equiv \frac{1-e^{-\rho\delta T}}{1-e^{-\rho T}} > 1$, so the SBO tax $\tau^P = \mathcal{R}P^*(T) - P(T)$ is *greater* than the FBO product tax $P^*(T) - P(T)$.

Next, for $T > 0$, differentiate τ^P as:

$$\partial\tau^P/\partial T = \frac{\rho}{1 - e^{-\rho T}}\left(\delta e^{-\rho\delta T} - \mathcal{R}e^{-\rho T}\right)P^*(T) + \left(\mathcal{R}P^{*\prime}(T) - P'(T)\right)$$

where $\delta > 1$. Appendix A shows $\frac{re^{-rT_j}}{1-e^{-rT_j}} < \frac{\rho e^{-\rho T_j}}{1-e^{-\rho T_j}}$ if and only if $r > \rho$. Here, we have $r = \rho$, but $\delta > 1$, so $\delta r = \delta\rho > \rho$. Replace r in Appendix A with $\delta\rho$, to get $\frac{\rho\delta e^{-\rho\delta T}}{1-e^{-\rho\delta T}} < \frac{\rho e^{-\rho T}}{1-e^{-\rho T}}$. Cancel the ρ in both numerators, which implies $\delta e^{-\rho\delta T} - \mathcal{R}e^{-\rho T} < 0$. Thus, the long first term in $\partial\tau^P/\partial T$ is negative. The second term can be either sign, so the tax falls with durability ($\partial\tau^P/\partial T < 0$) if and only if the second term, $\mathcal{R}P^{*\prime}(T) - P'(T)$, is negative or not very positive (*i.e.*, does not exceed the absolute value of the negative first term). A simpler sufficient condition is $\mathcal{R}P^{*\prime}(T) \leq P'(T)$, so that the second term is not positive. Note that $\mathcal{R} > 1$ here, so this sufficient condition means that $P^{*\prime}(T)$ must be much smaller than $P'(T)$. If so, then the negative first term means $\partial\tau^P/\partial T < 0$.

Finally, within part (A), the condition for the tax to *rise* with durability is that $\partial\tau^P/\partial T$ (shown above) is positive. The first term is negative, so the overall sign is positive if and only if the second term, $\mathcal{R}P^{*\prime}(T) - P'(T)$, is positive *and* greater than the absolute value of the negative first term. Then, $\partial\tau^P/\partial T > 0$. But $\mathcal{R} > 1$, so we have no simple sufficient condition for $\mathcal{R}P^{*\prime}(T)$ to be "enough" bigger than $P'(T)$ to offset the negative first term of $\partial\tau^P/\partial T$.

For part (B), $\delta < 1$ implies $\frac{e^{-\rho T}}{e^{-\rho\delta T}}\mathcal{R} < 1$ (Appendix A). The SBO tax $\tau^D = \frac{e^{-\rho T}}{e^{-\rho\delta T}}\mathcal{R}D^*–D$ is less than the FBO Pigovian rate ($D^* - D$). Next, $\delta < 1$ means that $\mathcal{R} \equiv \frac{1-e^{-\rho\delta T}}{1-e^{-\rho T}} < 1$, so the SBO tax $\tau^P = \mathcal{R}P^*(T) - P(T)$ is also less than the FBO product rate $P^*(T) - P(T)$.

The condition for the tax to fall with durability is:

$$\partial\tau^P/\partial T = \frac{\rho}{1-e^{-\rho T}}\left(\delta e^{-\rho\delta T} - \mathcal{R}e^{-\rho T}\right)P^*(T) + \left(\mathcal{R}P^{*\prime}(T) - P'(T)\right) < 0.$$

In this case, $\delta < 1$ implies $\frac{\delta e^{-\rho\delta T}}{1-e^{-\rho\delta T}} > \frac{e^{-\rho T}}{1-e^{-\rho T}}$ (Appendix A), which implies $\delta e^{-\rho\delta T} - \mathcal{R}e^{-\rho T} > 0$. So the long first term of $\partial\tau^P/\partial T$ is positive. The sign of the second term is ambiguous. Thus, the sign of $\partial\tau^P/\partial T$ is negative if and only if the second term is negative *and* its absolute value exceeds the positive first term. The fact that $\mathcal{R} < 1$ makes the second term $\mathcal{R}P^{*\prime}(T) - P'(T)$ negative in the simple case where $P^{*\prime}(T) = P'(T)$, but that simple case is not sufficient for $\partial\tau^P/\partial T < 0$. We have no simple sufficient condition for $\partial\tau^P/\partial T < 0$.

Finally, the tax rises with durability ($\partial\tau^P/\partial T > 0$) if and only if the second term, $\mathcal{R}P^{*\prime}(T) - P'(T)$, is positive or not very negative (so that it does not completely offset the positive first term). A simpler sufficient condition is a nonnegative second term, $\mathcal{R}P^{*\prime}(T) \geq P'(T)$, because then the positive first term implies $\partial\tau^P/\partial T > 0$. Because $\mathcal{R} < 1$, this sufficient condition requires $P^{*\prime}(T)$ to be greater than $P'(T)$. ■