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ABSTRACT

In many recent location choice models, households randomly vary with respect to their utility of living in a location. We demonstrate that the distribution generating this randomness is fundamentally not identifiable from location choice data and as a result the optimal allocation as chosen by a social planner is not identified. We propose an algorithm for setting the distribution generating the random utility across locations that implies a planner will optimally choose no redistribution in the absence of externalities or equity motives between different groups of people. Our algorithm preserves a planner's motives to redistribute due to equity considerations between different types of people and efficiency in production, the focus of many recent studies.

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1. Introduction

For decades, federal, state and local governments have directly or indirectly redistributed income across locations. This redistribution can take many forms: It can be a subsidy for development of new low-income housing (Davis et al., 2019); a subsidy to local businesses operating in low-income areas such as Empowerment Zones (Busso et al., 2013); a large-scale government works projects (Kline and Moretti, 2014); or other forms. Thus, a central area of investigation in economics is to understand the context in which redistribution across locations improves welfare.

Recent papers by Fajgelbaum and Gaubert (2019), Rossi-Hansberg et al. (2020) and Gaubert et al. (2020) extend this tradition by studying optimal transfers of income across households and locations using sophisticated equilibrium location choice models. The models include well-documented externalities in production and multiple types of households, for example low- and high-skill. These papers quantify, using the filter of the calibrated model, transfers across people and locations that improve expected utility for reasons of both efficiency and equity.

We show that in location choice models a planner will have three motives to redistribute resources across locations and people relative to an environment in which households consume the income they generate and do not receive (or pay) transfers. The first, which we call "across-type equity," is to narrow inequality in consumption across different types of households, for example low-skill and high-skill. The second, which we call "efficiency," arises from externalities and spillovers across types in production; the planner will transfer resources to provide incentives for households to internalize the external impacts of their decisions. Understanding motives for redistribution arising from these first two reasons has been the focus of recent studies.

We show that a planner has a third reason to redistribute in these models: To equate the average marginal utility of consumption of otherwise identical households that make different location choices. A typical prediction is that a planner will redistribute resources from ex-ante identical households choosing to live and work in high-income locations to households choosing low-income locations. We call this third motive "within-type transfers."

To understand why a planner may wish to make within-type transfers in location choice models, we need to provide some background. For all locations to be occupied in models with ex-ante identical households, some households must choose to live in low-income locations. In an older literature that relies on the Rosen-Roback model (Roback, 1982) to describe the economic environment, utility in every location is assumed identical and each household is indifferent as to its location. The Rosen-Roback model implies that population elasticities are infinite with respect to a small change in location attributes such as consumption or amenities holding all else fixed.

This infinite elasticity is not realistic and many researchers now use a different framework where utility in every location is not assumed to be equal. Instead, households receive "location attachment" draws that affect the utility of living in each location. These draws randomly vary across locations and households, and with these draws included in the model households are not indifferent to where they live. Some households will not leave their location in response to marginal changes in utility, and researchers can calibrate the distribution of the draws to match empirical population elasticities with respect to changes in wages, amenities, or other location characteristics. The fact that some households are sticky with respect to location choice raises the possibility of welfare-improving place-based policies. The calibration of the distribution of the location attachment draws that generates this stickiness enables accurate predictions about behavioral responses to policy.

Unfortunately, we document that the exact distribution of these location attachment draws is fundamentally not identifiable from location choice data: A continuum of distributions predict exactly the same elasticities and probability distribution over all location choices. We show this lack of identification implies the size and direction of optimal within-type transfers are not identified, even when a model includes all three motives for redistribution. Different, untestable assumptions about the distribution of location attachment draws can lead to large swings in predicted optimal within-type transfers and the uncertainty this creates potentially swamps predicted redistribution arising from the motives of across-type equity or efficiency. When researchers compare policies across a number of scenarios we often do not know the role played by within-type transfers in generating changes to policy as compared to the roles of across-type equity or efficiency. We therefore propose an adjustment to the standard planning problem that eliminates within-type transfers, while preserving motives for the planner to redistribute for reasons of across-type equity or efficiency.

Our paper proceeds as follows. We start by studying a simple model with only one type of household and no externalities where the only reason for redistribution arises from within-type transfers. We assume the location attachment draws are iid from the Fréchet distribution, as is typical. We then derive the result that a planner will redistribute income from households choosing to live in high-income locations to households choosing to live in low-income locations.

Next, we use the intuition of optimal unemployment insurance from the work of Chetty (2006) to show why this result arises. Optimal transfers across locations equate across-location differences in the average marginal utility of consumption to the deadweight loss in tax collection arising from a marginal change in these transfers. The Fréchet distribution implies a particular outcome for differences in the average marginal utility of consumption across locations. The Fréchet distribution is just one possible distribution for the location attachment draws. If researchers are only interested in predictions as to how location choices respond to various changes in wages or amenities, then the choice of the Fréchet from the class of distributions we describe is harmless, as the other distributions imply exactly the same location choice elasticities. It is only when researchers wish to study optimal policy that the choice of the Fréchet has consequences. We show using both theory and simulations that we can choose from an infinite class of distributions of location attachment draws to have the following features: (i) the probability distribution over location choices is exactly the same as when the draws are from the Fréchet but (ii) differences in the average marginal utility of consumption across locations can vary by a wide margin, fundamentally changing predicted optimal within-type transfers. We show this uncertainty can overwhelm predicted transfers arising from across-type equity or efficiency considerations in more complicated models.

Since location choice data cannot be used to identify the distribution of the location attachment draws, and therefore optimal within-type transfers, we propose a method for researchers to set the distribution of these draws that does not change any aspect of the distribution over location choices or alter any location choice elasticity, but sets differences in the average marginal utility of consumption across locations equal to zero. In a simple model with one type of household and no externalities, after applying our adjustment a planner will not redistribute resources across locations. In a more complex model, with multiple types of households and an externality and spillovers across types in production, our adjustment removes motives for redistribution arising from within-type transfers, but preserves motives to redistribute arising from across-type equity and efficiency.

2. Proving Lack of Identification of Within-Type Transfers

2.1. A Common Model

We start by considering the predictions of a simple location choice model with no externalities and one type of household that is at the core of some more complicated models. The economy consists of a measure 1 of ex-ante identical households and each household must choose where to live from one of $n = 1, \ldots, N$ discrete locations. Households value consumption, which is produced and transferrable across locations. Each household living in location n produces z_n units of output. L_n is the measure of households living and working in n.

Denote c_n as consumption enjoyed by each household living in location n, not necessarily equal to z_n. The utility of household i choosing to live in location n is

$$u_{ni} \quad = \quad A_n c_n e_{ni}$$

A_n are amenities freely enjoyed by all households living in location n. e_{ni} is a level of attachment to location n by household i that varies across locations and households. Each household draws and observes e_{ni} for $n = 1, \ldots, N$ before making a location choice. Households differ only with respect to these draws. We assume, as is common, that the e_{ni} are drawn iid across locations for each household and iid across all households from the Fréchet distribution with shape parameter ν.

Consider a planner with the objective to maximize expected utility subject to satisfying aggregate feasibility, $\sum_n z_n L_n = \sum_n c_n L_n$, population feasibility, $1 = \sum_n L_n$, and respects that households choose the location offering the maximum value of u_{ni}, i.e. household i chooses n_i^* when $n_i^* = \text{argmax}\{u_{ni}\}_{n=1}^{N}$. We show in Appendix A that a planner that maximizes expected utility will relate the per-household consumption differential between any two locations n and n' to the per-household income differential of those locations as follows

$$
\begin{aligned}
c_n - c_{n'} &= (1-\tau)\,(z_n - z_{n'}) \\
\text{with} \quad \tau &= \frac{1}{1+\nu}
\end{aligned}
\tag{1}
$$

Equation (1) illustrates what we call within-type transfers, as the planner optimally redistributes consumption from households living in high income locations to those living in low income locations. Households choosing to work and live in low income locations receive subsidies that are funded by otherwise identical households choosing to work to work and live in high income locations. At a typical calibration of $\nu = 2$ (Rossi-Hansberg et al., 2020), $\tau = 1/3$.

2.2. Economics of Within-Type Transfers

So why does the planner wish to redistribute income in this model? After all, there are no externalities and all households have the ability to choose any location in which to live and earn the income of that location. We use intuition from the literature on optimal unemployment insurance show why a planner makes within-type transfers. Consider a simple setup with only 2 locations where locations differ in their income per household, denoted w_1 and w_2. Assume residents of location 2 pay a tax t to finance a subsidy b paid to residents of location 1. For example, when $z_1 = w_1$ and $z_2 = w_2$ it can be shown equation (1) implies

$$
b = \frac{(1-L_1)\,(z_2 - z_1)}{1+\nu} \quad \text{and} \quad t = \frac{L_1\,(z_2 - z_1)}{1+\nu}
$$

Each household draws idiosyncratic preferences for locations we label as ε_1 and ε_2 and chooses the location that provides the highest utility. Utility is derived from consumption bundled with each individual's draws of ε_1 and ε_2. Expected utility in this simple model is

$$
V = E_{\varepsilon_1,\varepsilon_2} \max\Big(u\big(w_1 + b, \varepsilon_1\big), u\big(w_2 - t(b), \varepsilon_2\big)\Big) \quad \text{with} \quad t(b) = \frac{L_1}{1-L_1} b
$$

The second expression is the government balanced budget condition assuming there is a measure 1 of households in the economy.

The optimal subsidy is the value of b at which $\frac{dV}{db} = 0$. To characterize this optimal subsidy, begin with the expression,

$$\frac{dV}{db} = \frac{\partial V}{\partial b} + \frac{dt}{db}\frac{\partial V}{\partial t} + \left(\frac{\partial L_1}{\partial b} + \frac{dt}{db}\frac{\partial L_1}{\partial t}\right)\underbrace{\frac{\partial V}{\partial L_1}}_{=0}$$

Importantly, the third term is equal to zero because individuals who switch locations in response to a marginal policy change receive the same utility in both locations.[1]

The partial effects of changing b and t depend only on the populations of the two locations and the average marginal utility of consumption of residents in each location, denoted μ_1 and μ_2:

$$\frac{\partial V}{\partial b} = L_1 \overbrace{E\left[\frac{\partial u(w_1+b,\varepsilon_1)}{\partial w_1}\middle| u(w_1+b,\varepsilon_1) > u(w_2-t,\varepsilon_2)\right]}^{\equiv \mu_1}$$

$$\frac{\partial V}{\partial t} = -(1-L_1)\underbrace{E\left[\frac{\partial u(w_2-t,\varepsilon_1)}{\partial w_2}\middle| u(w_1+b,\varepsilon_1) < u(w_2-t,\varepsilon_2)\right]}_{\equiv \mu_2}$$

The government balanced budget condition implies that

$$\frac{dt}{db} = \frac{L_1}{1-L_1}\left(1 + \frac{\frac{dL_1}{db}\frac{b}{L_1}}{1-L_1}\right)$$

The first term is the mechanical change in the tax is necessary to finance the change in transfers in the absence of any behavioral responses. The second term captures the fact that an additional tax increase is necessary to offset the population response to the change in

[1] dt/db is not a partial derivative both because people move as a result of the policy and because of the need to balance the budget.

taxes/transfers. We can now write

$$\begin{aligned}\frac{dV}{db} &= L_1\mu_1 - \frac{L_1}{1-L_1}\left(1+\frac{\frac{dL_1}{db}\frac{b}{L_1}}{1-L_1}\right)(1-L_1)\mu_2 \\ &= L_1\left[\mu_1 - \mu_2\left(1+\frac{\frac{dL_1}{db}\frac{b}{L_1}}{1-L_1}\right)\right]\end{aligned}$$

In the case when the current transfer is $b=0$, the planner wishes to transfer consumption to the location with the higher marginal utility of consumption. At the optimal transfer b (satisfying $dV/db=0$), we have

$$\frac{\mu_1-\mu_2}{\mu_2} = \frac{\frac{dL_1}{db}\frac{b}{L_1}}{1-L_1} \tag{2}$$

This result is exactly analogous to the Baily-Chetty formula of Chetty (2006) characterizing the optimal generosity of unemployment insurance. The left-hand side is the insurance benefit of moving 1 dollar (in total) from location 2 to location 1, which, for most commonly considered utility functions, is decreasing in b. The right-hand side describes the marginal cost of raising one dollar in total from location 2, which captures the fact that as benefits rise, the population of location 1 also increases which causes an excess burden of transferring one additional dollar.

2.3. Location Choice and Location Specific Preference Draws

In a two location model where e_{in} are iid drawn from the Fréchet, we can derive the left-hand side of equation (2). A household chooses location 1 whenever $e_1 > e_2 t$ where $t = A_2c_2/(A_1c_1)$. This implies the following expected values

$$\begin{aligned}E\left[e_1 \mid e_1 > e_2 t\right] &= (1+t^\nu)^{1/\nu}\,\Gamma\left(1-\frac{1}{\nu}\right) \\ E\left[e_2 \mid e_2 > e_1/t\right] &= (1+(1/t)^\nu)^{1/\nu}\,\Gamma\left(1-\frac{1}{\nu}\right)\end{aligned}$$

where Γ is the gamma function. The average marginal utility of consumption of households living in locations 1 and 2 is equal to the appropriate expression above multiplied by A_1 for location 1 and A_2 for location 2. After cancelling redundant terms, the left-hand side of equation (2) is equal to[2]

$$\left(\frac{A_1}{A_2}\right)\left[\frac{1+t^\nu}{1+(1/t)^\nu}\right]^{1/\nu} - 1 \;=\; \left(\frac{A_1}{A_2}\right)\left[\frac{\dfrac{(A_1c_1)^\nu+(A_2c_2)^\nu}{(A_1c_1)^\nu}}{\dfrac{(A_2c_2)^\nu+(A_1c_1)^\nu}{(A_2c_2)^\nu}}\right]^{1/\nu} - 1$$

$$= \frac{1/c_1 \; - \; 1/c_2}{1/c_2}$$

We now write down a transformation of the location attachment draws that yields exactly the same probability distribution over all location choices but different values for the left-hand side of equation (2) and therefore different optimal within-type transfers. Start by noting that the optimal location choice for household i, call it n_i^*, satisfies

$$n_i^* \;=\; \text{argmax}\left[A_1c_1e_{1i},\; A_2c_2e_{2i},\; \ldots,\; A_Nc_Ne_{Ni}\right]$$

Suppose a researcher had considered a different distribution for the location attachment draws, $\widetilde{e}_{ni}$, such that the optimal location choice for household i resulting from this distribution, call it $\widetilde{n}_i^*$, satisfies

$$\widetilde{n}_i^* \;=\; \text{argmax}\left[A_1c_1\widetilde{e}_{1i},\; A_2c_2\widetilde{e}_{2i},\; \ldots,\; A_Nc_N\widetilde{e}_{Ni}\right]$$

When $\widetilde{e}_{ni} = D_ie_{ni}$, with D_i random but taking on a single realized value for each household i, optimal location choices for every household are identical to those when household utility is $A_nc_ne_{ni}$:

$$\begin{aligned}\widetilde{n}_i^* &= \text{argmax}\left[A_1c_1\widetilde{e}_{1i},\; A_2c_2\widetilde{e}_{2i},\; \ldots,\; A_Nc_N\widetilde{e}_{Ni}\right]\\ &= \text{argmax}\left[A_1c_1D_ie_{1i},\; A_2c_2D_ie_{2i},\; \ldots,\; A_Nc_ND_ie_{Ni}\right]\\ &= \text{argmax}\left[A_1c_1e_{1i},\; A_2c_2e_{2i},\; \ldots,\; A_Nc_Ne_{Ni}\right]\\ &= n_i^*\end{aligned}$$

[2]Note that the Fréchet shape parameter ν only determines the marginal deadweight loss from increasing transfers, the right-hand side of equation (2). This expression shows ν does not determine any benefits, the left-hand side of equation (2).

D_i is fundamentally not identifiable from location choice data as optimal choices from $\widetilde{e}_{ni}$ are all exactly the same as with e_{ni}.[3]

2.4. Lack of Identification of Optimal Transfers

For predicting population responses to various changes in location attributes such as consumption and amenities, setting $D_i = 1$ for all households is harmless as location choice predictions do not depend on D_i. For the purposes of deriving optimal within-type transfers, setting $D_i = 1$ for all households is an arbitrary assumption with significant consequences as any correlation of D_i with one or more of the draws of e_{ni} for $n = 1, \ldots, N$ can change predicted optimal transfers.

To see this, define utility in location n for household i as

$$A_n c_n \widetilde{e}_{ni} \quad \text{with} \quad \widetilde{e}_{ni} \;=\; D_i e_{ni}$$

where e_{ni} are drawn iid from the Fréchet and D_i is defined as:

$$D_i \;=\; \left[\prod_{n=1}^{N} e_{ni}^{\phi_n}\right]^{-1}$$

The parameters $\phi_1, \phi_2, \ldots, \phi_N$ govern the correlation of D_i and each e_{ni}; for now, we assume these parameters are the same for all households. Throughout the text, we describe $\phi_1, \phi_2, \ldots, \phi_N$ as nuisance parameters since they are not identifiable from location choice data. Now consider three cases of (ϕ_1, ϕ_2) for the two location model with $A_1 = A_2 = 1$ and $z_1 = z_2$.

[3]We can use the results of Matzkin (1993) to formally state what is identified in this model. Define utility in location n for household i as $A_n c_n D_i e_{ni}$. For arbitrary location m, the probability a household chooses to live in m, call it ρ_m, is

$$\begin{aligned}
&\rho_m\left(\{A_n\}_{n=1}^N, \{c_n\}_{n=1}^N\right)\\
=\;& \text{Prob}\{\log A_m + \log c_m + \log D_i + \log e_{mi} > \log A_n + \log c_n + \log D_i + \log e_{ni}\} \quad \text{for } n \neq m\\
=\;& \text{Prob}\{\log e_{mi} - \log e_{ni} > \log A_n - \log A_m + \log c_n - \log c_m\} \quad \text{for } n \neq m
\end{aligned}$$

The joint CDF of the N−1 terms $(\log e_{mi} - \log e_{ni})$ is all that is nonparametrically identified, assuming sufficient continuous variation in consumption or amenities. Notice that the D_i terms do not appear and therefore they are not identifiable.

- *Case 1:* $\phi_1 = \phi_2 = 0$

$$\text{Utility in location 1} \quad = \quad c_1 e_{1i} \quad \text{and} \quad \text{Utility in location 2} \quad = \quad c_2 e_{2i}$$

Household i chooses location 1 as long as $e_{1i}/e_{2i} \geq c_2/c_1$. Given the e_{ni} are drawn iid from the Fréchet distribution, optimal transfers are characterized by equation (1), and since $z_1 = z_2$ the planner optimally sets $c_1 = c_2$ and no resources are transferred across locations.

- *Case 2:* $\phi_1 = 0$ and $\phi_2 = 1$

$$\text{Utility in location 1} \quad = \quad c_1 \left(\frac{e_{1i}}{e_{2i}} \right) \quad \text{and} \quad \text{Utility in location 2} \quad = \quad c_2$$

Household i chooses location 1 as long as $e_{1i}/e_{2i} \geq c_2/c_1$ implying that for any given values of e_{1i} and e_{2i}, households choose exactly the same locations as in case 1.

The planner will *not* optimally choose to set $c_1 = c_2$. Suppose that $c_1 = c_2$ and households choose location 1 whenever $e_{1i}/e_{2i} > 1$. The marginal utility of consumption for all households living in location 2 is always 1. The average marginal utility of consumption for all households choosing to live in location 1 at at this allocation is

$$E\left[\frac{e_{1i}}{e_{2i}} \mid \frac{e_{1i}}{e_{2i}} > 1\right] \quad > \quad 1$$

When $c_1 = c_2$, the average marginal utility of consumption of residents optimally choosing to live in location 1 is strictly larger than the average marginal utility of consumption of residents choosing to live in location 2. Therefore, the planner will transfer some consumption from location 2 to location 1 and $c_1 > c_2$.

- *Case 3:* $\phi_1 = 1$ and $\phi_2 = 0$ such that

$$\text{Utility in location 1} \quad = \quad c_1 \quad \text{and} \quad \text{Utility in location 2} \quad = \quad c_2 \left(\frac{e_{2i}}{e_{1i}} \right)$$

As with cases 1 and 2, household i chooses location 1 as long as $e_{1i}/e_{2i} \geq c_2/c_1$. For any given values of e_{1i} and e_{2i}, households optimally sort into exactly the same locations as in cases 1 and 2. Now consider the allocation $c_1 = c_2$, such that households choose to live in location 2 whenever $e_{2i}/e_{1i} > 1$. The marginal utility of consumption for all households living in location 1 is 1. The average marginal utility of consumption for all households choosing to live in location 2 is

$$E\left[\frac{e_{2i}}{e_{1i}} \mid \frac{e_{2i}}{e_{1i}} > 1\right] \quad > \quad 1$$

At the allocation $c_1 = c_2$, the average marginal utility of consumption of households optimally choosing to live in location 2 is strictly larger than the average marginal utility of consumption of households choosing to live in location 1. The planner will transfer some consumption from location 1 to location 2 and $c_2 > c_1$, exactly the opposite result as in case 2.

In each of cases 1-3, households choose to live in location 1 as long as $e_{1i}/e_{2i} \geq c_2/c_1$ and this choice is completely independent of the values of ϕ_1 and ϕ_2. Yet in case 1 the planner chooses no transfers, in case 2 the planner transfers consumption from location 2 to location 1, and in case 3 the planner transfers consumption from location 1 to location 2. Thus, the size and direction of the transfers is determined by the nuisance parameters ϕ_1 and ϕ_2.

This simple example is sufficient for the general point we wish to make: Since $\phi_1, \phi_2, \ldots, \phi_N$ are not identified from location choice data, optimal within-type transfers across locations are also not identified.

2.5. Numerical Examples

To illustrate the potential quantitative significance of this problem, we simulate the planning solution to a two location version of the model when utility for household i in location n is defined as

$$\begin{aligned} u_{ni} &= A_n c_n \widetilde{e}_{ni} \\ \text{with} \quad \widetilde{e}_{ni} &= D_i e_{ni} \quad \text{and} \quad D_i = \left[\prod_{n=1}^{N} e_{ni}^{\phi_n}\right]^{-1} \end{aligned}$$

where e_{ni} is drawn iid from the Fréchet distribution with shape parameter $\nu = 2$. In simulations we consider values of $\phi_1 \in \{0.0, 0.5, 1.0\}$ and $\phi_2 \in \{0.0, 0.5, 1.0\}$. For all combinations of ϕ_1 and ϕ_2, we consider the case of equally productive locations, $z_1 = z_2 = 1.0$, and location 1 more productive, $z_1 = 4/3$ and $z_2 = 2/3$. We set $A_1 = A_2 = 1.0$ in all simulations.

Given the draws of $\widetilde{e}_{ni}$, we assume each household chooses the location that provides the highest level of utility and then determine the allocation of consumption to residents of each location that maximizes overall average utility in the economy, subject to the resource constraint $\sum_n L_n (z_n - c_n) = 0$ and population constraint $\sum_n L_n = 1$. Note that when $\phi_1 = \phi_2 = 1$, $\widetilde{e}_{ni}$ is drawn iid from the Weibull distribution, the focus of an early draft of our paper.

The top panel of Figure 1 shows results for the case in which residents of both locations are equally productive and the bottom panel shows results when residents of location 1 are more productive. The y-axis of the top panel marks per-household consumption in location 1 less that of location 2, $c_1 - c_2$, and the y-axis of the bottom panel marks the ratio of the difference in per-household consumption to the difference in per-household income, $(c_1 - c_2) / (z_1 - z_2)$. The x-axis of both panels marks the value of ϕ_2 and the different lines show results for various values of ϕ_1. The dashed black lines mark allocations where consumption in each location equals production in that location and no within-type transfers occur.

The case of iid draws from the Fréchet is shown at $(\phi_1, \phi_2) = (0, 0)$, the dark blue circle in each panel. In this case, when the two locations are equally productive, there are no transfers (top panel); and when households in location 2 are more productive than in location 1, the planner redistributes $(c_1 - c_2) / (z_1 - z_2) = 2/3 = 1 - (1 + \nu)^{-1}$ of that difference, exactly as predicted by equation (1). Unfortunately, Figure 1 also makes clear that optimal transfers can vary quite a lot depending on the other values of the nuisance parameters (ϕ_1, ϕ_2). For any value of ϕ_1, increasing ϕ_2 – a movement from left to right along any given line – increases consumption allocated to households living in location 1 relative to those living in location 2. For any given value of ϕ_2, increasing ϕ_1 – moving down from a higher line to a lower line holding ϕ_2 fixed – increases allocations of consumption to households living in location 2 relative to those in location 1. These patterns are consistent with the intuition of the three cases discussed in section 2.4.

3. Method for Eliminating Motive for Within-Type Transfers

3.1. The Method

Since location choice data do not identify optimal within-type transfers, we advocate setting the distribution of the location attachment draws such that the planner optimally chooses no within-type transfers in a simple model with one type of household and no externalities. Below, we propose a 5-step algorithm to find a distribution of location attachment draws that accomplishes this objective and does not change any household's optimal location choice. Throughout the text, we describe this algorithm as implementing an adjustment to the planning problem:

1. Guess a variable called $\omega_i = 1.0$ for all households in the simulation
2. Multiply the utility function by ω_i. Find the allocation of c_n^τ for all $n = 1, \ldots, N$ and $\tau = 1, \ldots, T$ that is feasible and maximizes the planner's objectives at the current guess for ω_i.
3. Given each household's optimal choice at this allocation, $\widehat{n}_i$, compute $\widehat{\omega}_i$ as the inverse of the marginal utility of consumption for that household at the optimally chosen location. For example, using the framework described in section 2.5, if we define $\widehat{n}_i$ as the optimally chosen location for household i at the current guess of c_n^τ then we set $\widehat{\omega}_i = (A_{\widehat{n}_i}\ D_i\ e_{\widehat{n}_i})^{-1}$.
4. Compute $\omega_i' = \omega_i + d \cdot (\widehat{\omega}_i - \omega_i)$, where $d \in (0, 1]$ is a dampening factor.
5. Update $\omega_i = \omega_i'$ and repeat steps 2-5 until c_n^τ has converged.

This algorithm finds the solution the planner's problem that is consistent with the marginal utility of consumption equal to 1 for all households, thus setting the left-hand side of equation (2) to zero.

3.2. The Adjustment Applied to the Simple Model

Denote the planner's objective as $\mathcal{O}$. In the model we have analyzed so far, our adjustment normalizes the location attachment draws as follows

$$\mathcal{O} \;=\; E_i\left[\max_n \left\{A_n c_n \widehat{e}_{ni}\right\}_{n=1}^N\right] \quad \text{where} \;\; \widehat{e}_{ni} \;=\; \omega_i \widetilde{e}_{ni} \;\; \text{and} \;\; \widetilde{e}_{ni} \;=\; D_i e_{ni} \tag{3}$$

In the above, $\widehat{e}_{ni}$ are the normalized draws and ω_i is set to the inverse of the marginal utility of consumption for household i at the planner's optimal allocation. Explaining, if household i optimally chooses location n_i^* given the planner's allocation $c_1^*, c_2^*, \ldots, c_N^*$, then $\omega_i = \left(A_{n_i^*}\ D_i\ e_{n_i^* i}\right)^{-1}$. Since ω_i is fixed across locations for any given household it does not affect any location choices of households; additionally, since ω_i rescales the draws such that all households have the same marginal utility of consumption of 1, the planner has no motives for within-type transfers. Given any initial researcher chosen distribution of location attachment draws $\widetilde{e}_{ni}$, $\widehat{e}_{ni}$ implies exactly the same optimal location choices but removes motives for the planner to implement within-type transfers.

Referring to Figure 1, when we set ω_i in this way, the planner optimally chooses the dashed line at 0.0 in the top panel ($c_1 = c_2 = z_1 = z_2$) and the dashed line at 1.0 in the bottom panel for any combination of the nuisance parameters (ϕ_1, ϕ_2) determining D_i.

3.3. More Complicated Models

3.3.1. Theory

In our introduction, we describe three possible motives for a planner to redistribute resources across people and locations: Across-type equity, efficiency, and within-type transfers. So far, we have analyzed a simple model where a planner has no motives for redistribution due to across-type equity (as there is only one type of household) or efficiency (as there are no externalities). We now show that in a more complicated model where all three motives may be present, our adjustment removes motives for within-type redistribution but motives for redistribution due to across-type equity and efficiency remain.

Consider an environment in which there are are $n = 1, \ldots, N$ discrete locations, $\tau = 1, \ldots, T$ distinct types of people, and possible externalities and complementarities across types in production. We assume a planner can choose any level of consumption for any type of household in any location, as long as the overall allocation satisfies aggregate feasibility conditions and respects individual optimization, i.e. households are assumed to optimally choose locations given their location attachment draws and given the allocation of consumption across locations.[4]

[4] In this framework, the planner does not need to and will not want to implement across-location transfers

The planner chooses consumption for each type in each location to maximize the social welfare function

$$\sum_{\tau} \Pi^{\tau} L^{\tau} U\left(V^{\tau}\right) \tag{4}$$

where Π^{τ} is the planner's Pareto weight on type τ households in the economy, L^{τ} is the total population of type τ, V^{τ} is the expected utility associated with a type τ household and U is a concave function. The constraints on the problem are listed below, with Lagrange multipliers placed to the left of the brackets:

$$\begin{array}{lrcl}
\text{Expected Utility, by type: } \tau = 1, \ldots, T & \lambda^{\tau}\left[E_{e_{ni}}\left(\max_{n'} u^{\tau}_{n'i}\right) - V^{\tau}\right] & = & 0 \\
\text{Resource constraint:} & P\left[\sum_n \sum_{\tau} t^{\tau}_n L^{\tau}_n\right] & = & 0 \\
\text{Population, by type: } \tau = 1, \ldots, T & \gamma^{\tau}\left[L^{\tau} - \sum_n L^{\tau}_n\right] & = & 0 \\
\text{Optimization, by type and location: } \tau = 1, \ldots, T \text{ and } n = 1, \ldots, N & W^{\tau}_n\left[\rho^{\tau}_n L^{\tau} \; - \; L^{\tau}_n\right] & = & 0
\end{array}$$

L^{τ}_n is the population of type τ in location n, u^{τ}_{ni} for agent i of type τ is the function $u_n\left(c_n, D_i, e_{ni}\right)$ with $c^{\tau}_n = z^{\tau}_n - t^{\tau}_n$ where z^{τ}_n is income generated by one type τ worker in location n.[5] ρ^{τ}_n is the probability that $n = \text{argmax}_{n'} u^{\tau}_{n'i}$ for $n' = 1, \ldots, N$.

In Appendix B.1 we derive the solution to this problem; below we copy the equation from that Appendix that characterizes optimal location- and type-specific transfers for type τ in location n

$$\underbrace{\frac{\kappa \mathcal{U}^{\tau} \Pi^{\tau} \mu^{\tau}_n - \overline{\mathcal{U}\Pi}\bar{\mu}}{\overline{\mathcal{U}\Pi}\bar{\mu}}}_{(1)} \; - \; \underbrace{\frac{\kappa \epsilon^{\tau}_n}{\overline{\mathcal{U}\Pi}\bar{\mu}}}_{(2)} \; = \; \underbrace{\left(\frac{1}{L^{\tau}_n}\right) \sum_m t^{\tau}_m L^{\tau} \left(\frac{\partial \rho^{\tau}_m}{\partial t^{\tau}_n}\right)}_{(3)}$$

This solution is similar to the solution of the simple model, equation (2), but modified to

as a means to implement across-type transfers. We can modify the environment to restrict transfers across locations to be identical across types. If different types tend to occupy different locations, then across-location transfers accomplish some across-type redistribution (Gaubert et al., 2020). This changes details of the solution but does not affect our general conclusions.

[5] This can be a function of $L^{\tau'}_n$ for $\tau' = 1, \ldots, T$, for example $z^{\tau}_n = z\left(z_n, L^1_n, L^2_n, \ldots, L^T_n\right)$ where z_n is TFP for location N.

allow for multiple types of people in the economy and the possibility of complementarities across types and externalities in production. For a given type τ in location n, the first term on the left-hand side captures the difference in the Pareto-weighted ($\kappa\mathcal{U}^\tau\Pi^\tau$) marginal utility of consumption of that type in that location (μ_n^τ) from the economywide-average marginal utility of consumption ($\overline{\mathcal{U}\Pi}\bar{\mu}$) and the second term captures the economy-wide utility net benefit of production spillovers generated by that type in that location ($\kappa\epsilon_n^\tau$).[6] The difference of these two terms is equated to the marginal deadweight loss from increasing transfers, the third term.

In Appendix B.2, we derive the impact of our procedure on the solution for the optimal transfer to type τ in location n, which we copy below

$$\left(\kappa\mathcal{U}^\tau\Pi^\tau - \overline{\mathcal{U}\Pi}\right) \;-\; \kappa\epsilon_n^\tau \;=\; \left(\frac{1}{L_n^\tau}\right)\sum_m t_m^\tau L^\tau\left(\frac{\partial\rho_m^\tau}{\partial t_n^\tau}\right) \tag{5}$$

After the adjustment, the planner continues to have motives to transfer resources across types and locations. The right-hand side of equation (5), the marginal deadweight loss from increasing transfers, does not change. The term in parentheses on the left-hand side, $\kappa\mathcal{U}^\tau\Pi^\tau - \overline{\mathcal{U}\Pi}$, is constant across locations for any given type, but this term allows for transfers arising from motives of across-type equity based on differences in Pareto weights Π^τ and the slope of the concave function U evaluated at the optimal policy.[7] The term $\kappa\epsilon_n^\tau$ measures the impact of spillovers and externalities in production and within-type variation in this term determines across-location, within-type transfers.

3.3.2. Numerical Examples

We now demonstrate that optimal transfers across locations are unidentified in more complicated models by examining these transfers in an environment with two locations and

[6]The $\mathcal{U}^\tau$ term is the derivative of the U function in the planner's objective function for type τ. κ is a scalar related to economy-wide fiscal externalities, marginal utilities of consumption and average production externalities and spillovers.

[7]Our adjustment causes the marginal utility of consumption of all households of all types to be identical. This means that the planner will not redistribute from high-income types to low-income types unless the Pareto weights for low-income types are higher than for high-income types. Researchers can pick Pareto weights to replicate optimal across-type transfers that are the solution to the planner's problem prior to applying our adjustment, if desired.

two types of people. In what follows, the planner maximizes equation (4) where U is the natural logarithm function and $\Pi^1 = \Pi^2 = 0.5$ such that the planner cares about both types equally. Denote c_n^τ as the consumption the planner allocates to type τ agents living in location n. Utility of household i of type τ in location n is

$$\begin{aligned} & A_n c_n^\tau \widetilde{e}_{ni} \\ & \text{where } \widetilde{e}_{ni} = D_i^\tau e_{ni} \text{ and } D_i^\tau = \left[e_{1i}^{\phi_1^\tau} e_{2i}^{\phi_2^\tau}\right]^{-1} \end{aligned}$$

implying $V^\tau = E_{i\in\tau}\left[\max A_n c_n^\tau D_i^\tau e_{ni}\right]$. In all simulations, we draw e_{ni} iid from the Fréchet distribution with shape parameter $\nu = 2$. We always set $(\phi_1^\tau, \phi_2^\tau) = (0,0)$ for type $\tau = 2$ and consider $(\phi_1^\tau, \phi_2^\tau) = (0,0), (0,1), (1,0)$ and $(1,1)$ for type $\tau = 1$. As before, $A_1 = A_2 = 1$.

In each location, the two types of labor are aggregated as follows to form a composite labor input

$$\mathcal{L}_n = \left[\left(\lambda_n^1 \cdot L_n^1\right)^\rho + \left(L_n^2\right)^\rho\right]^{1/\rho}$$

L_n^1 and L_n^2 are the measures of type 1 and type 2 households in location n, respectively. Total output produced in location n is $z_n \mathcal{L}_n$ with $z_n = 4/3$ in location 1 and 2/3 in location 2. We consider two different values for ρ, $\rho = 1.0$ (perfect substitutes) and $\rho = 0.5$ (complementarities across types in production).

In all simulations, we set $\lambda_n^1 = 1$ for location 2 ($n = 2$). In simulations without any externalities, we set $\lambda_n^1 = 1$ for location 1. In simulations with an externality, we specify an increasing impact of the share of type 1 workers on type 1 productivity in location 1

$$\lambda_n^1 = \left(\frac{L_n^1}{L_n^1 + L_n^2}\right)^\delta$$

for $n = 1$ (only) with $\delta = 0.15$. In all simulations we specify population measures of 0.32 type 1 households and 0.68 type 2 households.

Figure 2 shows results for optimal within-type redistribution across locations for type 1 agents for the economy without an externality (top panel) and the economy with an externality (bottom panel). We measure this redistribution as $\left(c_1^1 - c_2^1\right) / \left(w_1^1 - w_2^1\right)$, where

w_n^1 is the marginal product of labor of type 1 agents living in location n, measured as

$$w_n^1 \;\;=\;\; z_n \mathcal{L}^{1-\rho} \left(\lambda_n^1\right)^{\rho} \left(L_n^1\right)^{\rho-1}$$

A value of 1.0 indicates that no within-type redistribution across locations occurs for type 1 agents; a value less than 1.0 indicates that consumption is redistributed from type 1 households living in location 1 to those living in location 2; and, and a value greater than 1.0 indicates consumption is redistributed from type 1 households living in location 2 to those living in location 1.

For now, ignore the black lines in Figure 2 and focus only on the blue and red lines. The solid blue and red lines in both panels correspond to $\rho = 1$, perfect substitutes, and the dashed blue and red lines correspond to $\rho = 0.5$. The x-axis of each graph shows the value of ϕ_2^τ for $\tau = 1$, either 0 or 1. The y-axis shows the measure of within-type redistribution for Type 1 households. Finally, the blue lines correspond to values for $\phi_1^\tau = 0$ for $\tau = 1$ and the red lines correspond to values for $\phi_1^\tau = 1$, also for $\tau = 1$.

A few points jump out from this figure. First, by comparing the results in the bottom panel to those in the top panel, we can see the impact of the externality on redistribution: For any given parameterization of ρ, ϕ_1^τ and ϕ_2^τ for $\tau = 1$, the planner wants less redistribution from type 1 residents in location 1 to location 2 when the externality is present. Second, by comparing the dashed to solid lines for either the blue or red lines, we can observe how the elasticity of substitution of the two types in production affects optimal redistribution from type 1 residents in location 1 to location 2. Finally, the figure shows how the nuisance parameters ϕ_1^τ and ϕ_2^τ for $\tau = 1$ affect optimal redistribution. When the value of ϕ_1^τ is large relative to ϕ_2^τ for $\tau = 1$, the marginal utility of consumption of households optimally choosing location 1 is relatively low, and the planner optimally redistributes consumption away from location 1 and towards location 2. When the value of ϕ_2^τ is large relative to ϕ_2^τ for $\tau = 1$, the opposite happens. As the panels make clear, the unidentifiable variation in ϕ_1^τ and ϕ_2^τ for $\tau = 1$ can cause optimal transfers across locations for type 1 agents to be almost anything, even in the presence of a large externality in production in one location.

To implement our adjustment, we redefine the location attachment draws for household

i of type τ in location n such that utility is

$$A_n c_n^\tau \widehat{e}_{ni} \tag{6}$$

$$\text{where } \widehat{e}_{ni} = \omega_i \widetilde{e}_{ni},\ \widetilde{e}_{ni} = D_i^\tau e_{ni},\ D_i^\tau = \left[e_{1i}^{\phi_1^\tau} e_{2i}^{\phi_2^\tau}\right]^{-1} \text{ and } \omega_i = \left[A_{n_i^* i}\ D_i^\tau\ e_{n_i^* i}\right]^{-1}$$

where n_i^* is the optimally chosen location for agent i given realized e_{ni} at the planner's optimal allocation of consumption for that type $c_n^{\tau *}$ for $n = 1, \ldots, N$. Including ω_i in utility in this way ensures that the average marginal utility of consumption in a location is constant across locations at the planner's chosen allocation, and thus the planner has no motive to redistribute to equate within-type marginal utilities of consumption.[8]

The impact of the adjustment on redistribution among Type 1 agents is shown by the black lines in Figure 2. In the top panel, the version of the model with no externalities, the planner chooses no redistribution. This result does not depend on the the value of ρ, ϕ_1^τ or ϕ_2^τ for $\tau = 1$. In the bottom panel, the planner chooses to redistribute from type 1 agents living in location 2 to type 1 agents living in location 1, to incentivize type 1 agents to live in location 1 due to the production externality. The amount of redistribution does not depend on ϕ_1 or ϕ_2 but does depend on the elasticity of substitution in production between the two types of agents. This example shows our adjustment allows for the direct study of optimal place-based transfers in response to production externalities and spillovers across types, without the size and the direction of those transfers influenced by differences in the marginal utility of consumption across locations that are not identifiable from location choice data.

3.4. Discussion of Uniqueness

In all simulations where we apply our adjustment, we use the 5-step procedure outlined in section 3.1 to find optimal allocations. Since we are embedding something that looks like a fixed point into the planning problem, readers might be concerned this procedure may not produce a unique candidate solution. The procedure finds an allocation with the following

[8] As discussed, the planner may still want to redistribute across locations and types for reasons of across-type equity or efficiency.

property: With weights ω_i set equal to the inverse of the marginal utility of consumption at the candidate allocation, the candidate allocation solves the adjusted planner's problem – adjusted to include ω_i. An allocation has this property if and only if it is a solution to equation (5). Therefore, if equation (5) has a unique solution our procedure also produces a unique solution. Uniqueness of equation (5) depends on researcher choices that determine elasticities, externalities, and Pareto weights.[9] As long as researchers can prove equation (5) has a unique solution, then our experience suggests our procedure will find that solution as long as the dampening factor d is sufficiently small.[10]

4. Additional Thoughts and Conclusions

The essence of results we document have been identified in at least one other area of economics. For example, we can relate the differences in predicted optimal transfers between our cases 2 and 3 from section 2.4 to intuition from Klevin et al. (2009) on optimal taxation of married couples. In that paper, the secondary earner can choose not to work for one of two reasons: he/she either receives a bad draw of market earnings or a good draw of home productivity. Klevin et al. (2009) show that optimal policy depends on which of the two explanations caused the secondary earner to not work in the market.[11]

One path for future research may be to use data to estimate differences across locations in the average marginal utility of consumption of otherwise identical households. Researchers in other fields of economics have attempted to estimate state dependence in the marginal utility of consumption. For example, health economists have tried to identify how the state of a person's health affects their marginal utility of consumption. Finkelstein et al. (2009) survey the various approaches and results in the literature and conclude, "Currently available estimates offer little in the way of a consensus on the sign or magnitude of health state dependence." The hurdle for estimation is high in location choice models, as researchers need to understand variation in the average marginal utility of consumption across locations and we believe this will be difficult to measure. Even if a policy experiment exogenously

[9] For example, multiple candidate solutions may exist depending on the properties of agglomeration externalities in the model.

[10] We set the dampening factor to 1 (no dampening) in all the numerical examples in this paper.

[11] We thank Patrick Kline for suggesting this connection.

shifts location choices of some marginal households, optimal policy depends on the marginal utility of consumption of *all* households including – perhaps most importantly – those least likely to move. Until we have direct evidence on differences in the average marginal utility of consumption across locations, we advocate imposing a solution to planning problems that removes a planner's incentives for within-type transfers across locations absent motives of productive efficiency or externalities.

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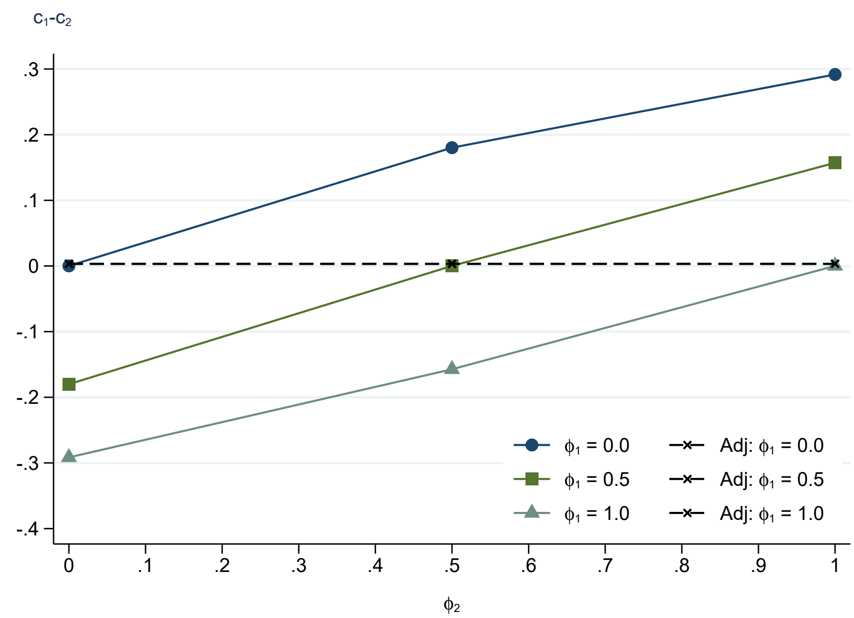
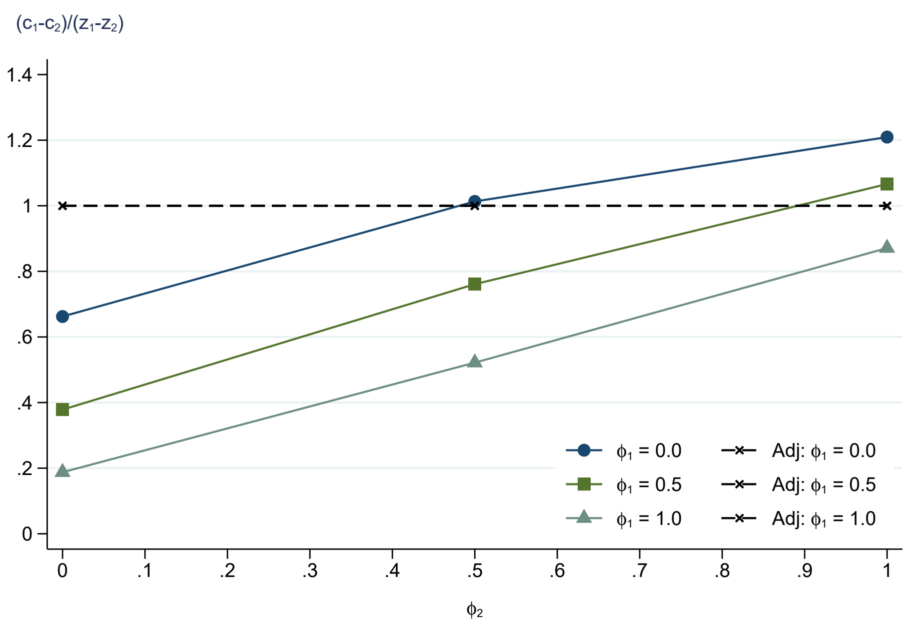
Figure 1: Redistribution from Location 2 to 1, Various Values of ϕ_1 and ϕ_2

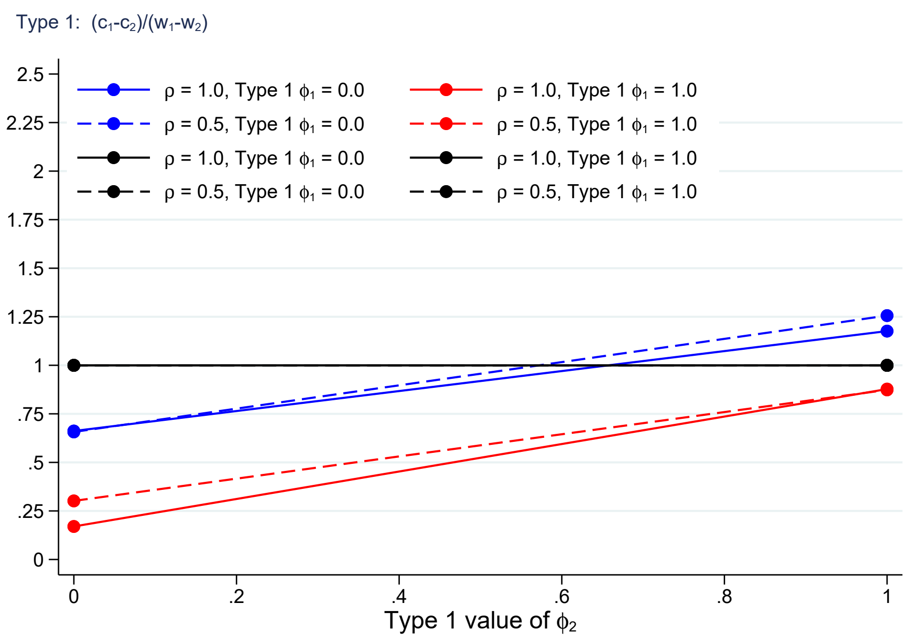


(a) $z_1 = z_2 = 1.0$

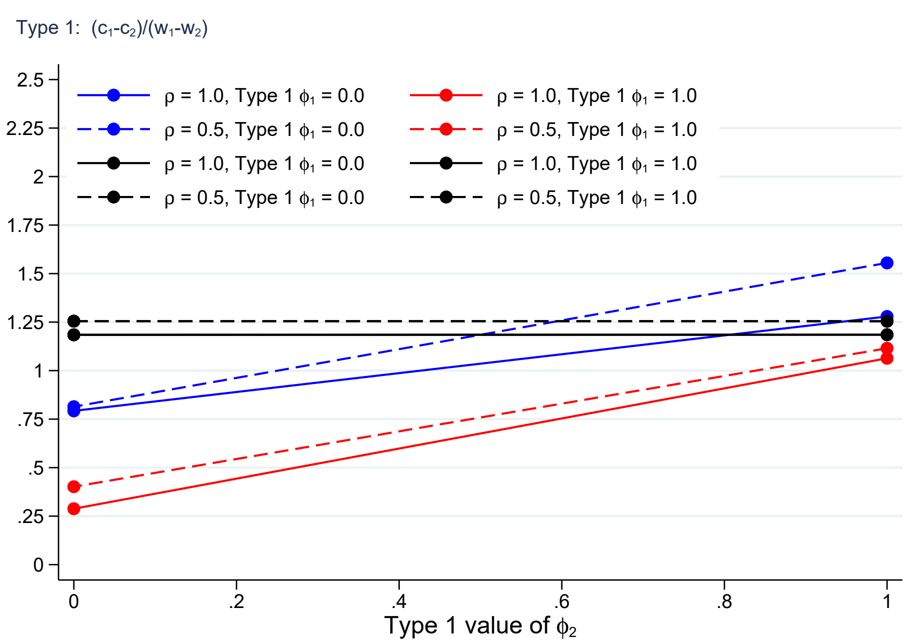


(b) $z_1 = 4/3$, $z_2 = 2/3$

Figure 2: Redistribution from Location 2 to 1, Type 1, Various Values of ϕ_1^τ and ϕ_2^τ



(a) No Externalities



(b) Externality in Location 1 from L_1^1

Appendix A. Planning Solution

Denote A_n as amenities in location n and c_n as consumption in location n such that the deterministic portion of utility in location n is $u_n = A_n c_n$ and utility for person i in location n is $u_{ni} = u_n e_{ni}$ where e_{ni} is drawn iid from the Fréchet distribution with parameter ν. Also denote L_n as the population in location n and let G denote the pre-determined amount of government expenditure that needs to be funded by taxation. The planner solves:

$$\max_{\{c_n, L_n\}_{n=1}^{N}} U$$

subject to the following constraints (Lagrange multipliers are to the left of the brackets)

$$\begin{array}{lrcl}
\text{Expected Utility} & \lambda\left[\left(\sum_n u_n^{\nu}\right)^{\frac{1}{\nu}} - U\right] & = & 0 \\
\text{Resource constraint} & P\left[\sum_n L_n z_n - \sum_n L_n c_n - G\right] & = & 0 \\
\text{Population:} & \mu\left[1 - \sum_n L_n\right] & = & 0 \\
\text{Utility n=1,...,N} & \theta_n\left[A_n c_n - u_n\right] & = & 0 \\
\text{Individual optimization n=1,...,N:} & W_n\left[\left(\frac{u_n}{U}\right)^{\nu} - L_n\right] & = & 0
\end{array}$$

First-order conditions are

$$\begin{array}{lrcl}
u_n: & 0 & = & \lambda L_n U - \theta_n u_n + \nu W_n L_n \\
c_n: & 0 & = & \theta_n u_n - P L_n c_n \\
L_n: & 0 & = & P\left(z_n - c_n\right) L_n - W_n L_n - \mu L_n \\
U: & 0 & = & 1 - \lambda - (\nu/U)\sum_n W_n L_n
\end{array}$$

From the FOC for U we have $(\nu/U)\left(\sum_n W_n L_n\right) = 1 - \lambda$. Add the Focs for u_n to get $1 = \sum_n \theta_n \left(u_n/U\right)$. Now add the FOCs for c_n to get $(U/P) = GDP - G$ where $GDP = \sum_n z_n L_n$.

Now start with the FOC for L_n

$$0 = P\left(z_n - c_n\right) L_n - W_n L_n - \mu L_n$$

Use FOC for u_n

$$W_n L_n \;=\; \frac{1}{\nu}\left(\theta_n u_n\right) \;-\; \frac{1}{\nu}\left(\lambda L_n U\right) \;=\; \frac{1}{\nu}\left(P L_n c_n\right) \;-\; \frac{1}{\nu}\left(\lambda L_n U\right)$$

Insert

$$\begin{aligned}
0 \;&=\; P L_n z_n \;-\; P L_n c_n \;-\; \frac{1}{\nu}\left(P L_n c_n\right) \;+\; \frac{1}{\nu}\left(\lambda L_n U\right) \;-\; \mu L_n \\
0 \;&=\; z_n \;-\; c_n \;-\; \frac{1}{\nu}\left(c_n\right) \;+\; \frac{1}{P\nu}\left(\lambda U\right) \;-\; \frac{\mu}{P} \\
&=\; z_n \;-\; \left[\frac{1+\nu}{\nu}\right] c_n \;+\; \left(\frac{U}{P}\right)\left(\frac{\lambda}{\nu} - \frac{\mu}{U}\right)
\end{aligned}$$

Rearrange terms and substitute for U/P to get

$$c_n \;=\; \left[\frac{\nu}{1+\nu}\right] z_n \;+\; \left(\frac{\lambda - \frac{\mu\nu}{U}}{1+\nu}\right)\left(GDP - G\right) \tag{A.1}$$

If we multiply the above equation by L_n and then sum over n, we get the expression

$$\begin{aligned}
\left(\lambda - \frac{\mu\nu}{U}\right)\left(GDP - G\right) \;&=\; \left(1+\nu\right)\left(GDP - G\right) \;-\; \nu GDP \\
&=\; GDP - \left(1+\nu\right) G
\end{aligned} \tag{A.2}$$

After inserting equation (A.2) into (A.1), we get the following expression for optimal consumption in location n

$$\begin{aligned}
c_n \;&=\; \left(1-\tau\right) z_n \;+\; T \\
\text{where} \quad \tau \;&=\; \frac{1}{1+\nu} \quad \text{and} \quad T \;=\; \tau \cdot GDP \;-\; G
\end{aligned}$$

Appendix B. Multiple Types of Households, Multiple Locations and Production Externalities

Appendix B.1. No Adjustment

We now consider an environment with $n = 1, \ldots, N$ discrete locations and $\tau = 1, \ldots, T$ types. We assume a planner can choose any level of consumption for any type in any location, as long as the allocation satisfies aggregate feasibility conditions and respects individual optimization, i.e. households optimally choose locations given their location attachment draws and given the allocation of consumption across locations.

The objective of the planner is as follows

$$\max_{\left\{\{t_n^\tau, L_n^\tau\}_{\tau=1}^T\right\}_{n=1}^N} \sum_\tau \Pi^\tau L^\tau U\left(V^\tau\right)$$

where U is a concave function, L_n^τ is the population of type τ in location n, L^τ is the total population of type τ and V^τ is the expected utility associated with type τ. Then planner maximizes this function subject to constraints listed below. Note that in the list of constraints the Lagrange multipliers are to the left of the brackets:

Expected Utility, by type: $\tau = 1, \ldots, T$	$\lambda^\tau \left[E_{e_{ni}}\left(\max_{n'} u_{n'i}^\tau\right) - V^\tau\right]$	$=$	0
Resource constraint:	$P\left[\sum_n \sum_\tau t_n^\tau L_n^\tau\right]$	$=$	0
Population, by type: $\tau = 1, \ldots, T$	$\gamma^\tau\left[L^\tau - \sum_n L_n^\tau\right]$	$=$	0
Optimization, by type and location: $\tau = 1, \ldots, T$ and $n = 1, \ldots, N$	$W_n^\tau\left[\rho_n^\tau L^\tau - L_n^\tau\right]$	$=$	0

u_{ni}^τ for agent i of type τ is the function $u_n\left(c_n, D_i, e_{ni}\right)$ with $c_n^\tau = z_n^\tau - t_n^\tau$ where z_n^τ is income generated by one type τ worker in location n which can be a function of $L_n^{\tau'}$ for $\tau' = 1, \ldots, T$, for example $z_n^\tau = z\left(z_n, L_n^1, L_n^2, \ldots, L_n^T\right)$ where z_n is TFP for location N. As specified, this framework allows for complementarities across types or externalities involving one or more types in production.[12] ρ_n^τ is the probability that $n = \text{argmax}_{n'} u_{n'i}^\tau$ for $n' = 1, \ldots, N$.

[12] As an example, in a decentralized economy firms may take as given in location n multifactor productivity of a_n where $a_n = z_n\left(L_n^{\tau*}\right)^\delta$, with $L_n^{\tau*}$ an externality in type $\tau*$ workforce. The planner explicitly takes into consideration the impact of allocations on the externality.

The first-order conditions are:

$$\begin{aligned}
V^\tau: \quad & 0 &=& \left(\frac{\partial U}{\partial V^\tau}\right)\Pi^\tau L^\tau - \lambda^\tau \\
L_n^\tau: \quad & 0 &=& \sum_{\tau'} \lambda^{\tau'}\left(\frac{\partial E_{e_{ni}}\left(\max_{n'} u_{n'i}^{\tau'}\right)}{\partial L_n^\tau}\right) + P t_n^\tau - \gamma^\tau - W_n^\tau \\
t_n^\tau: \quad & 0 &=& -\lambda^\tau\left(\frac{\partial E_{e_{ni}}\left(\max_{n'} u_{n'i}^{\tau}\right)}{\partial c_n^\tau}\right) + P L_n^\tau + \sum_m W_m^\tau L^\tau\left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau}\right)
\end{aligned}$$

where we have made use in the last equation that $\partial c_n^\tau / \partial t_n^\tau = -1$.

To reduce notation, for any given type τ' define the derivative of the expected value with respect to L_n^τ as

$$\frac{\partial E_{e_{ni}}\left(\max_{n'} u_{n'i}^{\tau'}\right)}{\partial L_n^\tau} = \frac{\partial E_{e_{ni}}\left(\max_{n'} u_{n'i}^{\tau'}\right)}{\partial c_n^{\tau'}} \cdot \frac{\partial c_n^{\tau'}}{\partial L_n^\tau} = \frac{\partial E_{e_{ni}}\left(\max_{n'} u_{n'i}^{\tau'}\right)}{\partial c_n^{\tau'}} \cdot \epsilon_n^{\tau\to\tau'}$$

Also define

$$\frac{\partial E_{e_{ni}}\left(\max_{n'} u_{n'i}^{\tau}\right)}{\partial c_n^{\tau}} = \left(\frac{L_n^\tau}{L^\tau}\right)\mu_n^\tau$$

where μ_n^τ is the average of the marginal utility of consumption of type τ agents that have chosen to live in location n:

$$\mu_n^\tau = E\left[\frac{\partial u_{ni}^\tau}{\partial c_n^\tau} \;\middle|\; n = \operatorname{argmax} u_{n'i}^\tau\right]$$

After substituting $\mathcal{U}^\tau = \partial U / \partial V^\tau$, this allows us to rewrite the FOCs as:

$$\begin{aligned}
V^\tau: \quad & 0 &=& \mathcal{U}^\tau \Pi^\tau L^\tau - \lambda^\tau \\
L_n^\tau: \quad & 0 &=& \sum_{\tau'} \lambda^{\tau'}\left(\frac{L_n^{\tau'}}{L^{\tau'}}\right)\mu_n^{\tau'}\epsilon_n^{\tau\to\tau'} + P t_n^\tau - \gamma^\tau - W_n^\tau \\
t_n^\tau: \quad & 0 &=& -\lambda^\tau\left(\frac{L_n^\tau}{L^\tau}\right)\mu_n^\tau + P L_n^\tau + \sum_m W_m^\tau L^\tau\left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau}\right)
\end{aligned}$$

Consider the FOC for L_n^τ after reducing for λ^τ:

$$0 \;=\; \sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_n^{\tau'} \mu_n^{\tau'} \epsilon_n^{\tau \to \tau'} + P t_n^\tau - \gamma^\tau - W_n^\tau$$

Multiply everything by L_n^τ and sum over n

$$0 \;=\; \sum_n L_n^\tau \left[\sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_n^{\tau'} \mu_n^{\tau'} \epsilon_n^{\tau \to \tau'} \right] + P \sum_n t_n^\tau L_n^\tau - \gamma^\tau \sum_n L_n^\tau - \sum_n W_n^\tau L_n^\tau$$

Define total tax revenues collected for type τ residents as $\mathcal{T}^\tau$. After rearranging terms, this reduces to

$$\gamma^\tau \;=\; \sum_n \left(\frac{L_n^\tau}{L^\tau} \right) \left[\sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_n^{\tau'} \mu_n^{\tau'} \epsilon_n^{\tau \to \tau'} \right] + P \left(\frac{\mathcal{T}^\tau}{L^\tau} \right) - \sum_n W_n^\tau \left(\frac{L_n^\tau}{L^\tau} \right)$$

Insert this expression for γ^τ into the FOC for L_n^τ, rearrange terms, and replace n with m everywhere:

$$W_m^\tau = $$
$$P t_m^\tau - P \left(\frac{\mathcal{T}^\tau}{L^\tau} \right) + \sum_{m'} W_{m'}^\tau \left(\frac{L_{m'}^\tau}{L^\tau} \right) + \sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_m^{\tau'} \mu_m^{\tau'} \epsilon_m^{\tau \to \tau'} - \sum_{m'} \left(\frac{L_{m'}^\tau}{L^\tau} \right) \left[\sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_{m'}^{\tau'} \mu_{m'}^{\tau'} \epsilon_{m'}^{\tau \to \tau'} \right]$$

Now return to the t_n equation and substitute for λ^τ

$$\mathcal{U}^\tau \Pi^\tau L_n^\tau \mu_n^\tau \;=\; P L_n^\tau + \sum_m W_m^\tau L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right)$$

Insert for W_m^τ

$$\mathcal{U}^\tau \Pi^\tau L_n^\tau \mu_n^\tau =$$
$$P L_n^\tau + \sum_m \left\{ P t_m^\tau - P \left(\frac{\mathcal{T}^\tau}{L^\tau} \right) + \sum_{m'} W_{m'}^\tau \left(\frac{L_{m'}^\tau}{L^\tau} \right) + \sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_m^{\tau'} \mu_m^{\tau'} \epsilon_m^{\tau \to \tau'} - \sum_{m'} \left(\frac{L_{m'}^\tau}{L^\tau} \right) \left[\sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_{m'}^{\tau'} \mu_{m'}^{\tau'} \epsilon_{m'}^{\tau \to \tau'} \right] \right\} L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right)$$

Note that since the overall population of type τ is fixed, this implies $\sum_m (\partial \rho_m^\tau / \partial t_n^\tau) = 0$.

Thus, the above can be reduced to:

$$\mathcal{U}^{\tau}\Pi^{\tau}L_{n}^{\tau}\mu_{n}^{\tau} \quad = \quad PL_{n}^{\tau} + P\sum_{m} t_{m}^{\tau}L^{\tau}\left(\frac{\partial\rho_{m}^{\tau}}{\partial t_{n}^{\tau}}\right) + \sum_{m}\sum_{\tau'}\mathcal{U}^{\tau'}\Pi^{\tau'}L_{m}^{\tau'}\mu_{m}^{\tau'}\epsilon_{m}^{\tau\to\tau'}L^{\tau}\left(\frac{\partial\rho_{m}^{\tau}}{\partial t_{n}^{\tau}}\right)$$

Add across τ and rearrange:

$$\sum_{\tau}\mathcal{U}^{\tau}\Pi^{\tau}L_{n}^{\tau}\mu_{n}^{\tau} - \sum_{\tau}\sum_{m}\sum_{\tau'}\mathcal{U}^{\tau'}\Pi^{\tau'}L_{m}^{\tau'}\mu_{m}^{\tau'}\epsilon_{m}^{\tau\to\tau'}L^{\tau}\left(\frac{\partial\rho_{m}^{\tau}}{\partial t_{n}^{\tau}}\right) \quad = \quad P\sum_{\tau}L_{n}^{\tau} + P\sum_{\tau}\sum_{m}t_{m}^{\tau}L^{\tau}\left(\frac{\partial\rho_{m}^{\tau}}{\partial t_{n}^{\tau}}\right)$$

Sum over n

$$\sum_{n}\sum_{\tau}\mathcal{U}^{\tau}\Pi^{\tau}L_{n}^{\tau}\mu_{n}^{\tau} - \sum_{n}\sum_{\tau}\sum_{m}\sum_{\tau'}\mathcal{U}^{\tau'}\Pi^{\tau'}L_{m}^{\tau'}\mu_{m}^{\tau'}\epsilon_{m}^{\tau\to\tau'}L^{\tau}\left(\frac{\partial\rho_{m}^{\tau}}{\partial t_{n}^{\tau}}\right) \quad = \quad P\sum_{n}\sum_{\tau}L_{n}^{\tau} + P\sum_{n}\sum_{\tau}\sum_{m}t_{m}^{\tau}L^{\tau}\left(\frac{\partial\rho_{m}^{\tau}}{\partial t_{n}^{\tau}}\right)$$

Define $\overline{\mathcal{U}\Pi}$, $\bar{\mu}$, ℓ and Δ as follows

$$\begin{aligned}
\overline{\mathcal{U}\Pi} &= \sum_{n}\sum_{\tau}\mathcal{U}^{\tau}\Pi^{\tau}L_{n}^{\tau} \\
\bar{\mu} &= \left(\overline{\mathcal{U}\Pi}\right)^{-1}\sum_{n}\sum_{\tau}\mathcal{U}^{\tau}\Pi^{\tau}L_{n}^{\tau}\mu_{n}^{\tau} \\
\ell &= \sum_{n}\sum_{\tau}\sum_{m}t_{m}^{\tau}L^{\tau}\left(\frac{\partial\rho_{m}^{\tau}}{\partial t_{n}^{\tau}}\right) \\
\Delta &= \sum_{n}\sum_{\tau}\sum_{m}\sum_{\tau'}\mathcal{U}^{\tau'}\Pi^{\tau'}L_{m}^{\tau'}\mu_{m}^{\tau'}\epsilon_{m}^{\tau\to\tau'}L^{\tau}\left(\frac{\partial\rho_{m}^{\tau}}{\partial t_{n}^{\tau}}\right)
\end{aligned}$$

For economic interpretation, $\bar{\mu}$ is the Pareto-weighted average marginal utility of consumption in the economy; ℓ measures the impact on the tax base generated by the location responses to a marginal increase in taxes that is uniformly applied across locations and types; Δ measures the Pareto-weighted sum of the marginal change in economy-wide utility arising from spillovers generated by the location responses to a marginal increase in taxes that is uniformly applied across locations and types.[13]

[13] In the special case in which utility is linear in consumption and location-specific preferences are additive to utility then $\sum_{n}\left(\partial\rho_{m}^{\tau}/\partial t_{n}^{\tau}\right) = 0$ giving $\ell = \Delta = 0$.

With this notation, we write

$$P \;=\; \frac{\overline{\mathcal{U}\Pi\bar{\mu}} \;-\; \Delta}{1+\ell} \;=\; \overline{\mathcal{U}\Pi\bar{\mu}} \left(\frac{1 \;-\; \dfrac{\Delta}{\overline{\mathcal{U}\Pi\bar{\mu}}}}{1+\ell} \right)$$

Insert this definition of P and return to the FOC for t_n

$$\mathcal{U}^{\tau}\Pi^{\tau}L_n^{\tau}\mu_n^{\tau} - \sum_m \sum_{\tau'} \mathcal{U}^{\tau'}\Pi^{\tau'}L_m^{\tau'}\mu_m^{\tau'}\epsilon_m^{\tau\to\tau'}L^{\tau}\left(\frac{\partial \rho_m^{\tau}}{\partial t_n^{\tau}}\right) \;=\; \overline{\mathcal{U}\Pi\bar{\mu}} \left(\frac{1 - \dfrac{\Delta}{\overline{\mathcal{U}\Pi\bar{\mu}}}}{1+\ell} \right) \left[L_n^{\tau} + \sum_m t_m^{\tau} L^{\tau} \left(\frac{\partial \rho_m^{\tau}}{\partial t_n^{\tau}}\right) \right]$$

Divide by L_n^{τ}

$$\mathcal{U}^{\tau}\Pi^{\tau}\mu_n^{\tau} - \left(\frac{1}{L_n^{\tau}}\right) \sum_m \sum_{\tau'} \mathcal{U}^{\tau'}\Pi^{\tau'}L_m^{\tau'}\mu_m^{\tau'}\epsilon_m^{\tau\to\tau'}L^{\tau}\left(\frac{\partial \rho_m^{\tau}}{\partial t_n^{\tau}}\right) \;=\; \overline{\mathcal{U}\Pi\bar{\mu}} \left(\frac{1 - \dfrac{\Delta}{\overline{\mathcal{U}\Pi\bar{\mu}}}}{1+\ell} \right) \left[1 + \left(\frac{1}{L_n^{\tau}}\right) \sum_m t_m^{\tau} L^{\tau} \left(\frac{\partial \rho_m^{\tau}}{\partial t_n^{\tau}}\right) \right]$$

Define $\kappa = \frac{1+\ell}{1-\frac{\Delta}{\overline{\mathcal{U}\Pi\bar{\mu}}}}$. Then

$$\kappa\mathcal{U}^{\tau}\Pi^{\tau}\mu_n^{\tau} - \kappa\left(\frac{1}{L_n^{\tau}}\right) \sum_m \sum_{\tau'} \mathcal{U}^{\tau'}\Pi^{\tau'}L_m^{\tau'}\mu_m^{\tau'}\epsilon_m^{\tau\to\tau'}L^{\tau}\left(\frac{\partial \rho_m^{\tau}}{\partial t_n^{\tau}}\right) \;=\; \overline{\mathcal{U}\Pi\bar{\mu}} \left[1 + \left(\frac{1}{L_n^{\tau}}\right) \sum_m t_m^{\tau} L^{\tau} \left(\frac{\partial \rho_m^{\tau}}{\partial t_n^{\tau}}\right) \right]$$

Subtract $\overline{\mathcal{U}\Pi\bar{\mu}}$ and then divide.

$$\frac{\kappa\mathcal{U}^{\tau}\Pi^{\tau}\mu_n^{\tau} - \overline{\mathcal{U}\Pi\bar{\mu}}}{\overline{\mathcal{U}\Pi\bar{\mu}}} - \frac{\kappa\left(\frac{1}{L_n^{\tau}}\right) \sum_m \sum_{\tau'} \mathcal{U}^{\tau'}\Pi^{\tau'}L_m^{\tau'}\mu_m^{\tau'}\epsilon_m^{\tau\to\tau'}L^{\tau}\left(\frac{\partial \rho_m^{\tau}}{\partial t_n^{\tau}}\right)}{\overline{\mathcal{U}\Pi\bar{\mu}}} \;=\; \left(\frac{1}{L_n^{\tau}}\right) \sum_m t_m^{\tau} L^{\tau} \left(\frac{\partial \rho_m^{\tau}}{\partial t_n^{\tau}}\right)$$

Define ϵ_n^{τ} as the Pareto-weighted sum of the marginal change in economy-wide utility arising from spillovers generated by the location responses to a marginal increase in taxes that is applied in location n to type τ:

$$\epsilon_n^{\tau} \;=\; \left(\frac{1}{L_n^{\tau}}\right) \sum_m \sum_{\tau'} \mathcal{U}^{\tau'}\Pi^{\tau'}L_m^{\tau'}\mu_m^{\tau'}\epsilon_m^{\tau\to\tau'}L^{\tau}\left(\frac{\partial \rho_m^{\tau}}{\partial t_n^{\tau}}\right)$$

Rewrite the above as

$$\underbrace{\frac{\kappa\mathcal{U}^{\tau}\Pi^{\tau}\mu_n^{\tau} - \overline{\mathcal{U}\Pi}\bar{\mu}}{\overline{\mathcal{U}\Pi}\bar{\mu}}}_{(1)} - \underbrace{\frac{\kappa\epsilon_n^{\tau}}{\overline{\mathcal{U}\Pi}\bar{\mu}}}_{(2)} = \underbrace{\left(\frac{1}{L_n^{\tau}}\right)\sum_m t_m^{\tau}L^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right)}_{(3)} \tag{B.1}$$

For a given type τ in location n, the first term on the left-hand side captures the difference in the Pareto-weighted marginal utility of consumption of that type in that location from the economywide-average and the second term captures the economy-wide utility (net) benefit of production spillovers generated by that type in that location. The difference of these two terms is equated to the the marginal deadweight loss from increasing transfers for that type in that location, the third term.

We can rewrite this third term to gain some intuition. To start, note the following

$$\frac{\partial\rho_n^{\tau}}{\partial t_n^{\tau}} = -\sum_{m\neq n}\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}$$

Then the third term becomes

$$\left(\frac{1}{L_n^{\tau}}\right)\frac{\partial\rho_n^{\tau}}{\partial t_n^{\tau}}L^{\tau}\left(t_n^{\tau} - \frac{\sum\limits_{m\neq n} t_m^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right)}{\sum\limits_{m\neq n}\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}}\right) \tag{B.2}$$

This "fiscal externality" is the amount by which the tax from type τ in location n exceeds the tax that the marginal leavers of type τ will be exposed to, on average, conditional on leaving location n.

Appendix B.2. With our Proposed Adjustment

It is convenient to rewrite equation (B.1) as follows

$$\frac{\kappa\mathcal{U}^{\tau}\Pi^{\tau}\left(\mu_n^{\tau} - \mu^{\tau}\right) + \kappa\mathcal{U}^{\tau}\Pi^{\tau}\mu^{\tau} - \overline{\mathcal{U}\Pi}\bar{\mu}}{\overline{\mathcal{U}\Pi}\bar{\mu}} - \frac{\kappa\epsilon_n^{\tau}}{\overline{\mathcal{U}\Pi}\bar{\mu}} = \left(\frac{1}{L_n^{\tau}}\right)\sum_m t_m^{\tau}L^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right)$$

where $\mu^{\tau} = \sum\limits_n \left(L_n^{\tau}/L^{\tau}\right)\mu_n^{\tau}$. We have shown earlier that location data do not pin down within-type transfers that are only based on differences in within-type marginal utility of consumption across locations, the term involving $\mu_n^\tau - \mu^\tau$. For this reason, we advocate setting $\mu_n^\tau = \mu^\tau$, thereby eliminating the desire for a planner to redistribute for this motive.

If researchers wish to eliminate these transfers, they simply need to ensure that the average marginal utility of consumption for a given type does not vary across locations. There are many possible ways to generate this outcome. We propose simply setting $\mu_n^\tau = \mu^\tau = 1$ for all households, which also implies $\bar{\mu} = 1$. After this adjustment, the optimal tax on type τ at location n satisfies:

$$\left(\kappa \mathcal{U}^\tau \Pi^\tau - \overline{\mathcal{U}\Pi}\right) \; - \; \kappa \epsilon_n^\tau \;\; = \;\; \left(\frac{1}{L_n^\tau}\right) \sum_m t_m^\tau L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau}\right) \tag{B.3}$$

With one type and no externalities in production, $\epsilon_n^\tau = 0$ and $\Delta = 0$. The condition for optimality can be written as

$$\ell \cdot \mathcal{U} \;\; = \;\; \left(\frac{1}{L_n}\right) \sum_m t_m \left(\frac{\partial \rho_m}{\partial t_n}\right)$$

Notice that the left-hand side does not vary across locations. The only solution that satisfies this equation for every location is $t_n = 0$ for all n.[14]

Returning to the multiple-type case of equation (B.3) , the framework has the capacity to deliver both transfers across and within types. The term $\kappa \mathcal{U}^\tau \Pi^\tau - \overline{\mathcal{U}\Pi}$ is constant across locations for any given type, but allows transfers of consumption across types based on differences in Pareto weights and the slope of the concave function U evaluated at the optimal policy. The term $\kappa \epsilon_n^\tau$ measures the impact of spillovers and externalities in production (which we believe the data can identify).[15] Within-type variation in this term determines across-location, within-type transfers.

[14] $\ell = 0$ when $t_n = 0$ at every n.

[15] Recall we have set $\mu_n^\tau = 1$, such that ϵ_n^τ contains Pareto weights, elasticities of location choices with respect to income, and production-function spillovers and externalities.