NBER WORKING PAPER SERIES

TESTABLE IMPLICATIONS OF INDETERMINACIES IN MODELS WITH RATIONAL EXPECTATIONS

Robert P. Flood

Robert J. Hodrick

Working Paper No. 2903

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 1989

This paper is part of NBER's research programs in International Studies and in Financial Markets and Monetary Economics. Any opinions expressed are those of the authors not those of the National Bureau of Economic Research.

NBER Working Paper #2903
March 1989

TESTABLE IMPLICATIONS OF INDETERMINACIES IN MODELS WITH RATIONAL EXPECTATIONS

ABSTRACT

The possibility that movements in market prices of assets or goods may be caused by self-fulfilling prophecies, called bubbles or sunspots, has long intrigued market observers. If bubbles or sunspots exist, market prices differ from their fundamental values, and markets do not necessarily allocate resources to their best possible uses. Some might argue then that public policies would be needed to alleviate such problems.

This paper surveys the current state of the empirically-oriented literature concerning rational dynamic indeterminacies by which we mean a situation of self-fulfilling prophecy within a rational expectations model. The empirical work in this area concentrates primarily on indeterminacies in price levels, exchange rates and equity prices. We first examine a particular type of explosive indeterminacy, usually called a rational bubble, in a familiar model of equity pricing. We then consider empirical work relating to price level and exchange rate indeterminacies, before examining empirical studies of indeterminacies in stock prices. Finally, we take up some interpretive issues. We find that existing bubbles tests do not establish that rational bubbles exist in asset prices.

Robert P. Flood
Research Department
International Monetary Fund
Washington, DC 20431

Robert J. Hodrick
Kellogg Graduate School
of Management
Northwestern University
Evanston, IL 60208

The possibility that movements in prices could be due to the self-fulfilling prophecies of market participants has long intrigued observers of free markets. Such self-fulfilling prophecies are often called bubbles or sunspots to denote their dependence on events that are extraneous to the market. The folklore of such episodes includes the Tulip Bubble (see Peter Garber (1987)), the South Sea Bubble and the Mississippi Bubble (see Charles Kindleberger (1978)), and the increase in equity prices during the "Roaring 20's" followed by the 1929 crash. More recently, the rise and crash of stock prices from 1982 to 1987, the appreciation of the dollar on foreign exchange markets that peaked in 1985, and sudden housing price increases from California to Massachusetts have been attributed to speculative bubbles. The idea that bubbles might exist is often traced to John Maynard Keynes's (1936) description of an equity market as an environment in which speculators anticipate "what average opinion expects average opinion to be," rather than focusing on things fundamental to the market.

Explanations of movements in asset prices in the popular press often mention self-fulfilling prophecies. For example, in discussing reasons for the fall in stock prices on October 19, 1987, Gary Becker (1987) argues, "Unfortunately, expectations based on extraneous factors and on guesses about the behavior of others can become self-fulfilling, generating large swings in prices for a while."

If bubbles exist in asset markets, market prices of assets will differ from their fundamental values. Markets would not necessarily be allocating savings of individuals to best possible uses. Public policies would need to be designed to rid the markets of bubbles. Although these problems have been known for a long time, until recently, academic economists conducted relatively little formal theoretical analysis of these issues or empirical examinations of actual markets, probably because economists' analytical tools were inadequate. Since economic theory placed essentially no restrictions on how agents formed expectations, empirical analysts had little direction for studying the possibility of self-fulfilling prophecy. The widespread adoption of the assumption of rational expectations provided the required discipline for theoretical and empirical study of the issues.[1]

Many rational expectations models have an indeterminate aspect.[2] Usually, it arises when agents' current decisions depend both on the current market price and on their expectations of future prices. For example, consider a model of an investor's demand for an equity that makes the amount demanded depend on the expected return. If there is a fixed amount of the equity outstanding, the current price is determined by equality of investors' demands with the existing supply. The equilibrium depends upon the current equity price and the agents' beliefs about equity prices in the future, since realized returns depend on the cost of the equity today, on its resale value in the future and on any intermediate dividends paid to holders of the stock. Armed just with this information, the theory can only describe an expected price path; it does not predict which of the many possible paths will be realized by the market.

In such circumstances economic models require additional information to make predictions about the current market price. Sometimes theory supplies additional information and suggests ways of testing whether these theoretical restrictions are correct. If the theoretical arguments apply to the real world, large numbers of price paths can be excluded as possible dynamic paths leaving the researcher with an unique path or only a few paths to study. It may be, though, that the researcher prefers not to include among his maintained hypotheses theoretical considerations that exclude all indeterminacies. The researcher's prior beliefs may be quite eclectic across a wide range of sensible background models, some of which exclude indeterminacies and some of which don't.

The researcher can look also to empirical work on actual markets to see what kinds of price paths were chosen in actual experience. He can then extrapolate from the previous empirical work to his current theoretical work and adopt the type of price path previous research indicates to be appropriate to his particular circumstance. Additionally, he can test the new model for bubbles and other indeterminacies.

The purpose of this paper is to survey the current state of the empirically-oriented literature concerning rational dynamic indeterminacies by which we mean a situation of self-fulfilling prophecy within a rational expectations model. The empirical work in this area concentrates primarily on indeterminacies in price levels, exchange rates and equity prices.[3] Our survey focuses on these market prices. After examining a particular type of explosive indeterminacy, usually called a rational bubble, in a familiar example of the market for equities, in order to provide a common ground for later analysis, we consider empirical work relating to price level and exchange rate indeterminacies. Then, we examine empirical studies of indeterminacies in stock prices. Finally, we take up some interpretive issues.

A Common Framework

Consider the equity pricing discussion above. If people in the economy are not averse to risk, and if they discount future utility at a constant rate, r, in equilibrium all assets would have the same constant expected real return. The price of the equity share, q_t, would be the expected present value of the dividend accruing to ownership of the equity share during the ownership period, d_{t+1}, plus the price at which the share can be sold at the end of the ownership period, q_{t+1}, or:

(1) $$q_t = \rho E_t(d_{t+1} + q_{t+1}),$$

where $\rho = 1/(1+r)$, and $E_t(\cdot)$ denotes the mathematical expectation operator conditional on time t information. In this setting, today's market price, q_t, depends on agents' expectations of price in the future, $E_t(q_{t+1})$. The typical asset pricing formula derived from equation (1) sets price equal to the expected present value of all future dividends:[4]

(2) $$q_t^f = \sum_{i=1}^{\infty} \rho^i E_t(d_{t+i}).$$

We attach a superscript "f" to this price because we define it to be the <u>market fundamentals price</u> for this model. Equation (2), however, does not give the only mathematical solution to equation (1). To characterize other solutions let the market price be the fundamentals price plus B_t. The fundamentals price continues to solve (1), and terms like B_t must satisfy:

(3) $$B_t = \rho E_t(B_{t+1}).$$

From equation (3) we see that anything, sensible or silly, can be added to equation (2) as long as it fulfills equation (3), which we rewrite as:

(4) $$B_{t+1} = \rho^{-1} B_t + b_{t+1},$$

where $b_{t+1} = B_{t+1} - E_t(B_{t+1})$. According to the terminology adopted by Olivier Blanchard (1979), Robert Flood and Peter Garber (1980b) and Blanchard and Mark Watson (1982), B_t is a bubble in the equity price, and b_{t+1} is the innovation in the bubble at time t+1.[5]

Theory is helpful in thinking about whether terms like B_t can exist in rational markets. For example, William Brock (1982) notes that if the agents in the model can be aggregated into a representative investor, there is a terminal condition (known as a transversality condition) that allows the analyst to deduce that rational bubbles are absent. Thus, if one is willing to maintain such a model, then a test for bubbles is a test of the underlying model; and a rejection of a no bubbles hypothesis is a rejection of the model inclusive of the transversality condition.

Intuition about the transversality condition can be gained by considering the consequences of various buy and hold strategies. Equation (1) balances the cost of the investment against the discounted expected value of the payoff on the investment from resale in the following period. But, the investor must also be indifferent between holding the asset for one period and holding it for several periods. If the investor's family is effectively infinitely lived, as in the representative agent framework, consideration must also be given to the buy and hold forever strategy. This produces a marginal gain equal to the right-hand side of (2). Hence, if the price of the asset were less than the fundamentals price, agents could increase their utility by buying the asset and planning to hold it forever. This increased demand would increase the market price. Similarly, no one would buy the asset if its price exceeded the market fundamental price, because the utility cost would exceed the utility gain from holding it forever. The decrease in demand would cause the market price to fall. Essentially, the transversality condition requires $\lim_{t\to\infty} E_0(\rho^{-t} q_t) = 0$, which rules out rational bubbles.[6]

Behzad Diba and Herschel Grossman (1987) also note that bubbles can never be negative because the bubble process (3) would imply that the stock price was expected to be negative within finite time. This violates limited liability and free disposal. Ruling out negative bubbles is important since it implies that if a bubble ever is zero it cannot start again because the innovation in the bubble would not be mean zero. Hence, any bubbles currently present would have had to start at the initiation of the market. Diba and Grossman (1988) extend this argument to rational inflationary bubbles in the price level.

Most of the empirical work on bubbles is concerned with the rational indeterminacy introduced above. In the next two sections we explore the empirical implications of this type of indeterminacy in two settings -- price level models and equity pricing models.

Price-Level Bubble Tests

Indeterminacies in the price level usually result when the demands for nominal assets depend on the expected rate of inflation. The first tests of such an indeterminacy were conducted by Robert Flood and Peter Garber (1980b) in a monetary model of the German hyperinflation first studied by Phillip Cagan (1956). The model consists of a money demand equation, a money supply rule, and money market equilibrium. The money market equilibrium that combines supply and demand is:

$$(5) \qquad m_t - p_t = \beta - \alpha\left[E_t(p_{t+1}) - p_t\right] + v_t \ , \ \alpha > 0,$$

where m_t is the logarithm of the money supply at time t, p_t is the logarithm of the price level at time t and v_t is a mean-zero random error. We obtain the market fundamentals solution to equation (5) by analogy with equation (1). Think of p_t as playing the role played previously by q_t; think of $k_t = (m_t - \beta - v_t)/(1+\alpha)$ as playing the role of d_t; and think of $\alpha/(1+\alpha)$ as the counterpart of ρ. By analogy with equation (2), the market fundamentals solution to equation (5) is:

$$(6) \qquad p_t^f = \sum_{i=0}^{\infty} E_t(k_{t+i})[\alpha/(1+\alpha)]^i.$$

Notice that, as before, equation (6) does not give all of the possible solutions to equation (5). To obtain additional solutions, add a variable B_t to equation (6). The augmented expression defines a price path that satisfies (5) as long as $E_t(B_{t+1}) = [(1+\alpha)/\alpha]B_t$. Although indeterminacies such as B_t had been discussed in the theoretical literature, Flood and Garber's (1980b) attempt to identify, estimate and test for a bubble process removed it from the realm of pure theory and inserted it into empirical economics.

Flood and Garber allow the data to suggest time series processes for the elements of k_t, and they estimate a bubble-augmented reduced form equation for p_t assuming a nonstochastic bubble process. Consequently, $B_t = B_0[(1+\alpha)/\alpha]^t$, and the no bubbles hypothesis is the restriction $B_0 = 0$. The empirical methodology, the particular markets studied and the interpretations of the study of bubbles have been extended in numerous directions. First, we consider methodological contributions, and second we examine additional market coverage and alternative interpretations.

Improvements in Methodology

Flood and Garber mention three important potential methodological weaknesses of their study. First, they assume that money is exogenous. Second, they allow only for a deterministic bubble process, and third, their statistical inference does not have solid foundations in asymptotic distribution theory.

Edwin Burmeister and Kent Wall (1982) use the Flood and Garber data and the Cagan model to address the first two issues. They allow money growth to depend on past money growth and past inflation, thus relaxing the exogeneity assumption, and they allow a constant nonzero variance for b_t, making the bubble stochastic.

Both of these studies develop consistent parameter estimates, but they lack convincing tests of the no bubbles hypothesis because of an exploding regressor problem. Roughly, because estimation is conducted under the alternative hypothesis that bubbles are present in the economy, the reduced form regression with price as the dependent variable has $[(1+\alpha)/\alpha]^t$ as the regressor associated with the parameter B_0, the initial value of the bubble. This regressor is exploding quite fast, indeed, so fast that the information content of its most recent observation never goes to zero as a fraction of the information content of all previous observations. This situation makes it easy to prove consistency of the estimator of B_0 since convergence is quick, but it presents serious problems for testing hypotheses concerning B_0. The information structure of the exploding regressor ensures that any time series sample no matter how large is always a small sample, and standard central limit theorems do not apply.

Two methods of circumventing the asymptotic problem have been implemented in the literature. The first method, due to Flood, Garber and Louis Scott (1984) approaches the large sample limit from a cross-sectional dimension. The second method, due to Kenneth West (1987a), does not estimate the bubble directly, but tests for bubbles indirectly.

Flood, Garber and Scott use the fact that several countries experienced simultaneous hyperinflations following World War I to test the no bubbles hypothesis in a time series-cross section framework. An asymptotic distribution for the bubble coefficient is obtained by approaching the hypothetical limit in the cross-sectional dimension. Unfortunately, Flood, Garber and Scott have only three simultaneous hyperinflations. Therefore, while they reject the hypothesis of no bubbles in the three simultaneous hyperinflations, their appeal to large sample distribution results is probably suspect.

More recently, the empirical literature regarding the no bubbles hypothesis has taken an indirect approach. West (1987a) develops the indirect approach in an application to the stock market, which is discussed below. Alessandra Casella (1986) applies the West-style test to the German hyperinflation data, and it is in that context that we introduce the test. The fundamental insight involves estimating the parameters of a reduced-form price equation by two different methods.

For Casella's hyperinflation application the bubble test requires two estimates of α, the semi-elasticity of money demand with respect to the expected rate of inflation in (5). The first estimation method delivers consistent (but inefficient) estimates of the parameter and its standard error regardless of the presence of bubbles, while the second delivers parameter estimates and standard errors that are consistent and efficient if bubbles are absent but that are inconsistent if bubbles are present.

Obtaining the first set of consistent estimates, regardless of bubbles, is accomplished through instrumental variable estimation of the money demand function. Obtaining the second set of estimates requires simultaneous estimation of a market-fundamentals forecasting process and a stationary transformation of the price-level reduced form equation subject to the rational expectations cross equation restrictions, as derived by Lars Hansen and Thomas Sargent (1981).

Since the two estimates of α and their standard errors will yield numerically different values, the natural question is why. Are the differences due to sampling error or are they due to a bias in the second estimates? Such a bias would be introduced by a bubble in the price-level solution. For example, suppose that market price contains a bubble that is correlated with some of the market fundamentals. In such a circumstance the second method, based on the reduced-form price-level regression, will yield biased coefficient estimates. A Hausman (1978) specification test is exactly what is required to distinguish between parameter differences due to sampling error and those due to specification error.[7] In this circumstance when all other elements of the model are inserted into the maintained hypothesis, the Hausman test becomes a bubble test. One of the strengths of this type of test is that the researcher does not have to specify an ad hoc restriction on the variance of b_t--the weaknesses of the test will be discussed below.

When Casella implements her version of the West bubble test on the German data, her results are consistent with price-level bubbles if the money supply is maintained to be exogenous to inflation, but they are consistent with the no bubbles hypothesis if the money supply is modeled as an endogenous process.

Exchange-Rate Bubble Tests

A bubble that appears in a model of the price level, which is the value of goods in terms of a particular currency, usually appears also in the foreign exchange value of that currency. Consequently, there is an equivalence in many models between price level bubbles and exchange rate bubbles as Kenneth Singleton (1987) notes.

Richard Meese (1986) applies the West (1987a) bubble test to the U.S. dollar values of the deutschemark and the pound sterling exchange rates.[8] The West test indicates very strong evidence of bubbles in these exchange rates during the period October 1973-November 1982. Meese supplemented these tests with some other indirect tests involving these same exchange rates and their assumed market fundamentals, the relative money supplies and relative incomes of the countries whose currencies are being priced. Meese's indirect tests are not supportive of a no bubbles hypothesis.

Kenneth West (1987b) conducts some additional bubble tests on the deutschemark-dollar exchange rate and the associated market fundamentals from January 1974 through May 1984. The test West examines here is different than the test we describe above. West uses a construction similar to the variance bounds tests of Stephen LeRoy and Richard Porter (1981) to conclude that exchange rate variability, in the absence of bubbles, is consistent with the standard monetary model augmented to include money demand errors and deviations from purchasing power parity. These additional features are not entertained by Meese and possibly explain the difference in inference regarding the presence or absence of bubbles. There are also additional criticisms of the Meese analysis that we discuss below.

George Evans (1986) conducts nonparametric tests on the risk-adjusted excess U.S. dollar return to holding the U.K. pound over the period January 1981 to December 1984. Although these tests are not obviously related to those mentioned above, Evans refers to the tests as "bubble tests." An interesting point about Evans's study is his interpretation of finding excess returns. He notes that one possible explanation of the excess returns is that the sample period contained a growing bubble that skewed returns during the period. Evans's argument is interesting because it demonstrates that a sample that might have been thought to be large enough for standard testing with bubbles absent can become a small sample in the presence of bubbles.

Bubbles and Stock Price Volatility

This section examines the issues of bubbles in stock prices and the relation of bubble tests to excess volatility tests. A simple model that forms the foundation of much of the asset price bubble and excess volatility literature is the constant expected real return model presented in equation (1).[9] Although many authors, including those of popular financial textbooks such as Richard Brealey and Stewart Myers (1981, pp. 42-45), often refer to this model as "a standard efficient markets model," it should be understood that it is quite restrictive. This is only a simple characterization of what one could mean by the concept of an efficient market.[10]

Although bubbles could make asset prices more volatile than their market fundamentals, certain kinds of asset price volatility tests are not well-designed to provide tests for bubbles. Gregory Mankiw, David Romer, and Matthew Shapiro (1985) note this point, and Robert Flood and Robert Hodrick (1986) elaborate upon it. The problem is that the specification of the null hypothesis underlying the tests includes bubbles, if they exist, into a composite null hypothesis. Consequently, rejection of the null hypothesis cannot be attributable to bubbles.

This point is easily understood by consideration of the construction of the volatility tests that have typically been conducted within the confines of the constant expected rate of return model. Robert Shiller (1981) proposes a comparison between the volatilities of actual prices and of ex post rational prices. He defines the ex post rational price to be the discounted present value of actual dividends:

$$(7) \qquad q_t^* = \sum_{i=1}^{\infty} \rho^i d_{t+i} .$$

The expected value of the right-hand side of equation (7) is the fundamental price of equation (2), and the validity of the constant expected return model can be tested by examination of the null hypothesis that $q_t = E_t(q_t^*)$. Since the realization of a variable can be decomposed into its expectation conditional on a given information set plus an innovation that is not correlated with the information set, the validity of the model also implies the variance bound inequality $V(q_t) \le V(q_t^*)$, if the unconditional variance is well defined.

Notice that since it is impossible to measure the right-hand side of equation (7), econometric analysis must infer measurements of the discounted future value of dividends. Sanford Grossman and Robert Shiller's (1981) measurable ex post rational price, $\hat{q}_t$, truncates the infinite discounted sum of dividends in the last period of the sample, say at time T, and substitutes the discounted market price at time T for the indefinite future. With this definition, the actual market price, conditional on the validity of the model, is the expected value of measurable ex post rational price,

$$(8) \qquad \hat{q}_t = \sum_{i=1}^{T-t} \rho^i d_{t+i} + \rho^{T-t} q_T, \qquad t = 0, 1, \ldots, T-1,$$

and the null hypothesis is $V(q_t) \le V(\hat{q}_t)$. To understand why bubbles are included in this null hypothesis, notice that inclusion of $q_T = q_T^f + B_T$ on the right-hand side of equation (8) implies that $q_t = E_t(\hat{q}_t)$ even if bubbles are present because $q_t = q_t^f + B_t$, and bubbles are expected to grow each period at the gross rate of ρ^{-1}. Hence, evidence of violation of the variance bounds inequality in these tests cannot be due to bubbles.

Shiller's (1981) first method of measuring ex post rational prices, on the other hand, substitutes the discounted average price during the sample as the forecast of the indefinite post-sample discounted sum of dividends. Unfortunately, there is no reason why the hypothesis that market price is equal to expected ex post rational price should continue to be satisfied by this construction. Furthermore, Terry Marsh and Robert Merton (1986) demonstrate that this construction could have misleading properties if dividends are smoothed by management to be an exact function of current and past prices, since, by construction, the variance bound inequality must be violated in this case.

Much of Shiller's (1981) and Grossman and Shiller's (1981) evidence against the constant discount rate model is due to simple plots of the time series of actual prices and of constructed ex post rational prices. The plots of the time series of constructed ex post rational prices are considerably smoother than the time series of actual prices. Allan Kleidon (1986) effectively criticizes these plots by demonstrating that simulated data, generated to satisfy the model, produce plots that look very much like the plots from actual data. Kleidon's dividend process is the lognormal random walk.

West's Specification Test

As noted above, Kenneth West (1987a) develops an ingenious test for bubbles. We now interpret the results of West's investigation of the Standard and Poor's Composite Price Index and the Dow-Jones data that were first used by Shiller (1981).

West uses the constant expected return model in the development of the null hypothesis of no bubbles. He estimates a dividend forecasting equation in which future dividends depend upon the past history of dividends. He conducts a battery of tests to check that the return equation and the dividend forecasting equation are consistent with the data. Since there is generally a substantial difference between the parameter estimates constructed to satisfy the Hansen-Sargent (1981) formulas and the parameter estimates of the projections of stock prices onto the information set used to forecast dividends, West (1987a, p. 554) concludes, "The data reject the null hypothesis of no bubbles. The rejection appears to result at least in part because the coefficients in the regression of price on dividends are biased upwards."

One aspect of West's test is criticized by Flood, Hodrick and Paul Kaplan (1987). They note that estimation of ρ in the specification of the return generating model (1) involves use only of the one-period relation between current price and expected next period dividend and price, while testing the constructed relation of the ρ and the parameters from the dividend forecasting model to the reduced form coefficients involves implicit iteration of the return generating model an infinite number of times. Although West does not find strong evidence against the specification of the constant expected return model, when using the levels of real variables, Flood, Hodrick and Kaplan find substantive evidence of misspecification of the model when they iterate the equation for a second period. The latter authors change the specification in two other ways. They formulate the model in returns, and they use dividend-price ratios as instruments.[11]

West (1987a) acknowledges this significant evidence against his model of equilibrium expected returns when these alternative instruments are used. He also attempts to allow for time variation in expected returns within a linearized model with mixed results. The support for finding bubbles in some of his specifications increases while it decreases for others.

A second area of criticism of the West (1987a) specification test for bubbles is that he assumes the dividend forecasting equations are stationary in either the levels of real dividends or their first differences. Since most macroeconomic time series appear to be stationary in first differences of natural logarithms of the real variables, both of these specifications are somewhat suspect. In addition, the likelihood that a constant dividend process characterizes over one hundred years of data seems somewhat small given what little is known about the dividend process.[12]

If we restrict attention to what West (1987a) actually estimates, for the

Standard and Poor's Composite dividend process from 1871 to 1980, the superior specification of the dividend process appears to require first differencing and a second order autoregression. When variability of returns is allowed, and the test statistics are recalculated, there is no evidence against the null hypothesis of no bubbles.

West (1987a) also notes that a popular model of the dividend process is the lognormal random walk. In this case a closed-form expression for the price of the stock is available in terms of ρ and the mean and variance of the growth rate of dividends. Since the asymptotic distribution theory necessary to provide a distribution for the coefficient in the projection of price onto dividends is inapplicable in this case, West does no formal tests. Although the point estimates of the model are inconsistent with the no bubbles hypothesis, the results are sensitive to the value of ρ, and the no bubbles hypothesis cannot be rejected for plausible values of ρ. Since there is sensitivity of the test to the estimated parameters, West interprets the results as mild evidence against the null of no bubbles. But, the evidence seems just as easily interpretable in the opposite way, especially in light of potential for misspecification of the model.

Recent Evidence on Stock Price Volatility

We conclude this section with a discussion of some of the current literature on the excess variability of stock prices relative to dividends. A number of authors including Mankiw, Romer and Shapiro (1985), West (1988a), and Campbell and Shiller (1988a,b) test implications of the variability of stock prices relative to dividends. All find that simple models such as the constant expected return model are inconsistent with the data.[13] The sensitivity of these tests to the assumed structure of the dividend process and the model of returns is an outstanding issue for research.[14]

Examples of recent findings that intrigue us include the Campbell and Shiller (1988a,b) studies and the West (1988a) volatility test. Campbell and Shiller (1988a,b) estimate a vector autoregression (VAR) of the logarithm of the dividend-price ratio, the logarithm of a long average of earnings relative to price, and the first difference of the logarithm of dividends. The hypothesis of a constant expected rate of return then implies a restriction across the coefficients of the VAR that is easily rejected by the data. A byproduct of the estimation is a ratio of the standard deviation of calculated returns that are constructed from the coefficients of the VAR assuming that the model is true relative to the standard deviation of actual returns. This value is 0.277 with a standard error of 0.069. Thus, the false model's predicted returns are much less variable than actual returns. When the expected return on the stock market is allowed to be variable but is postulated to be equal to a constant plus the expected real return on commercial paper, the model is still rejected by the data at very low levels of significance, but the ratio of the standard deviation of returns implied by the model to the standard deviation of actual returns increases to 0.478 with a standard error of 0.044.

West (1988a) develops a volatility test that is quite similar in its estimated equations to the specification test described above. The test involves a comparison of estimates, constructed using two different information sets, of the innovation variance in the expected infinite sum of current and future dividends discounted at a constant rate. One information set is taken to be current and past dividends, which is a proper subset of the market's information set. The other information set is taken to be the market price under the hypothesis that constant expected returns are correct. Forecasting with a smaller information set than the market's ought to result in a larger innovation variance, but West finds the opposite and attributes a large part of the volatility of prices to either bubbles or fads. He argues that time-varying expected returns are unlikely to overturn the results.

Whether the actual volatility of equity returns is due to time variation in the rational equity risk premium or to bubbles, fads, and market inefficiencies is an open issue.[15] Bubble tests require a well specified model of equilibrium expected returns that has yet to be developed, and this makes inference about bubbles quite tenuous.

Sunspot Models

An alternative type of indeterminacy in rational expectations models that is not explosive or divergent is often called a sunspot to reflect the dependence of the equilibrium on something extraneous to the model. Such theoretical models have been constructed by Azariadis (1981) and others. Theorists often construct sunspot models by linearizing a nonlinear rational expectations model and exploring parameter values that allow stationary nonuniqueness of equilibrium solutions. Blanchard and Kahn (1980) explain how to determine whether a linear rational expectations model will have an unique nonexplosive solution that depends only on market fundamentals. Since the theoretical literature is quite broad and the published empirical literature on these indeterminacies is small, we cannot devote much space to the development of the ideas. Michael Woodford (1986, 1988) has made some progress in developing the empirical implications of sunspot models for explaining business cycle.

Some Matters of Interpretation

Many researchers think that bubbles and sunspots are not present in economic data because they can be ruled out by economic theory. Should these researchers still be interested in empirical tests of bubbles? We answer yes, primarily because bubble tests are an interesting specification test of the model.

Since bubbles and sunspots arise in economic models that incorporate market fundamentals, tests for these indeterminacies require correct specification of market fundamentals. Bubble tests examine a composite null hypothesis of no bubbles and correctly specified market fundamentals, which must be construed broadly to be both the data series and the equations that constitute the economic model. Since bubble tests can only legitimately be done on models that are not rejected by the data, researchers must first conduct a battery of diagnostic tests. Bubble tests may be powerful at detecting misspecifications of the model, even if it has passed other specification tests.

Flood and Garber (1980b) note that an omitted variable problem can bias bubble tests toward rejection of the no bubbles hypothesis. Consider the possibility that agents may have been expecting some future event, which is relevant to the determination of the price level, that the unwary researcher does not include in the model's market fundamentals. For example, suppose agents had information during the sample that there would be an increase in the money supply at some future date, and suppose that this information is not imbedded in the historical money supply statistics used by the researcher to generate forecasts of future money supplies. In this circumstance the dynamics of the price level will rationally have anticipated the increase in the money supply in a manner that is indistinguishable from the dynamics induced by a bubble in the market. The structure of a rational expectations model of the price level forces the dynamics of the price level in response to all omitted expected future variables to be indistinguishable from dynamic paths caused by bubbles. Flood and Hodrick (1986) demonstrate the analogous point in an equity market example in which agent's are anticipating a change in taxation of dividend income.[16]

Consider the biases that could plague West's (1987a) stock price bubble test. Applications of the specification test for bubbles require a forecasting equation based on a subset of the agents' information set and an unrejected return generating process. West (1987a) uses ARIMA models of dividends and tests for changes in the structure of the dividend process with a Chow test. Since he cannot reject the hypothesis of no difference in the structure of the dividend process, he proceeds with the bubble test, but this does not mean that agents were not anticipating a change in the structure of dividends that did not materialize during the sample. Similarly, although West is unable to reject the hypothesis that his return generating process is correctly specified, we note above that extension of the model to longer horizons points strongly toward model misspecification. How this misspecification biases his tests is an open issue.

Similar problems plague the study of hyperinflations if agents think that a hyperinflation will not last indefinitely, since they consequently must be anticipating a reform of the monetary process.[17] In such an environment, the price level is changing with changes in the probability of monetary reform, and without modelling this issue, researchers may associate movements in the price level caused by changes in the probability of monetary reform with changes induced by a nonexistent bubble.

In the foreign exchange market, a large body of research initiated by Richard Meese and Kenneth Rogoff (1983) indicates that standard exchange rate models forecast quite badly. When bubble tests are conducted on these models, they find bubbles. According to much research, though, it is very unlikely that the models are correct. If the models are false, rejection of the null hypothesis of no bubbles cannot be attributed solely to bubbles since it could equally well be caused by the misspecification of the model.

The moral of this section is that research ought to find apparent evidence of bubbles when models work poorly or when agents expect the future to be somewhat different than history. We think this point presents a serious interpretive problem for all bubble tests. The current empirical tests for bubbles do not successfully establish the case that bubbles exist in asset prices. Nevertheless, bubble tests are interesting specification tests and should continue to be an important part of the econometrician's tool kit.

Footnotes

* This research was supported by a grant from The Lynde and Harry Bradley Foundation to whom the authors express their gratitude. The comments of John Cochrane and Mark Watson are also gratefully acknowledged.

1. Rational expectations is the requirement that the subjective expectations of the agents in an economic model be identical to the mathematical expectations of the model that are produced by the exogenous sources of uncertainty interacting with the behavior of the agents.

2. See William Brock (1974), John Taylor (1977), and Robert Shiller (1978) for discussions of this indeterminacy.

3. Olivier Blanchard and Mark Watson (1982) also study the market for gold.

4. Equation (2) is derived from (1) by recursively substituting for future prices and using the law of iterated expectations, $E_t(E_{t+1}(d_{t+2})) = E_t(d_{t+2})$.

5. The bubble process is the homogenous part of the solution to the difference equation. Edwin Burmeister, Robert Flood and Peter Garber (1983) explain several indeterminacies discussed in the literature in terms of the homogenous part of the solution. This type of indeterminacy is explosive since $\rho^{-1} > 1$.

6. Jean Tirole (1985) explores an overlapping generations model of real asset pricing that does not exclude explosive indeterminacies as equilibrium phenomena, but they occur only if the rate of growth of the economy is higher than the steady state rate of return on capital. Price level models that are consistent with many researcher's prior beliefs but that still fail to exclude explosive indeterminacies are discussed by Brock (1974) and subsequently by Maurice

Obstfeld and Kenneth Rogoff (1983, 1986). Interestingly, explosive price level indeterminacies are much harder to rule out with a priori theoretical arguments than are indeterminacies concerning real asset prices. Nonexplosive indeterminacies seem to be even harder to rule out with theoretical arguments than are explosive indeterminacies.

7. The bubble need not be correlated with market fundamentals to apply the Hausman test. While a bias may be created by correlation of the bubble innovations with innovations in any of the fundamental variables, it may also be created by the bubble's mean biasing the estimate of the constant in the reduced form or because the improperly excluded bubble has exploding variance. See Casella (1986).

8. The first exchange-rate bubble paper was by Wing Woo (1987) who uses a portfolio balance model to test for bubbles in the exchange rate of the U.S. dollar versus the currencies of Germany, France, Canada and Japan. An interesting aspect of Woo's investigation is that he takes a stand on the initiation mechanism for an exchange rate bubble by looking for bubbles just after major monetary disturbances. This method, probably more than most, runs the risk of confusing bubbles with expected changes in market fundamentals. We return to this possible confusion below. Additional bubble tests involving foreign exchange markets are by Kunio Okina (1985). Jeffrey Frankel (1985) and Paul Krugman (1986) develop empirical analyses that the value of the dollar relative to foreign currencies is not "sustainable." While they motivate their analyses by the strength of the dollar, which they attribute possibly to a bubble, they do not test formally for bubbles.

9. See Robert Shiller's (1989) article in this issue and the critical survey by Christian Gilles and Stephen LeRoy (1987) for additional viewpoints.

10. See Eugene Fama (1976) for a discussion of the fact that market efficiency is always a joint hypothesis that depends on a model of appropriate expected asset returns and on an information set of investors.

11. These findings are consistent with the predictability of returns at long horizons that is documented by Eugene Fama and Kenneth French (1988), James Poterba and Lawrence Summers (1988), and John Campbell and Robert Shiller (1987, 1988a,b).

12. Terry Marsh and Robert Merton (1987) investigate the aggregate dividend process and conclude that there is support for the idea that managers smooth dividends. They argue that the only constraint on the dividend process is that its present value be equal to the present value of earnings. Campbell and Shiller (1987) question whether the findings of Marsh and Merton actually reflect dividend smoothing or simply additional ability of the market to predict future dividends from information that is in addition to the past history of dividends. Campbell and Shiller (1988a) note that a long average of the earnings of the Standard and Poor's composite relative to current price is useful in predicting dividends.

13. Stephen LeRoy and William Parke (1988) examine the unconditional variance of the price-dividend ratio while assuming that the dividend process is a geometric random walk. They conclude that stock price variability is consistent with the constant expected rate of return model although they acknowledge that the test may have little power.

14. Joe Mattey and Richard Meese (1986) investigate Monte Carlo simulations, using six different data generating environments, one of which includes a stochastic bubble, of twenty-four test statistics that have been proposed as tests of asset pricing models. Their results indicate that some tests may have poor small sample properties.

15. See Colin Camerer (1987) and West (1988b) for further discussion of these issues.

16. James Hamilton and Charles Whiteman (1985) extend this omitted variable argument by formally demonstrating the observational equivalence of omitted state variables and stochastic bubbles.

17. Flood and Garber (1980a) derive probabilities of monetary reform during the German hyperinflation by identifying them with the probability that the process for the money supply is inconsistent with a finite price level. Inconsistency is defined to be a monetary growth rate that is too fast to be discounted at the discount rate implied by the model. Such a money supply process implies an infinite price level if agent's thought that it would last forever.

References

Azariadis, Costas, "Self-fulfilling Prophecies," Journal of Economic Theory, December 1981, 25, 380-96.

Becker, Gary S., "Why a Depression Isn't in the Cards," Business Week, November 9, 1987, 22.

Blanchard, Olivier J., "Speculative Bubbles, Crashes and Rational Expectations," Economics Letters, 1979, 3, 387-89.

Blanchard, Olivier J., and Charles Kahn, "The Solution of Linear Difference Models Under Rational Expectations," Econometrica 48, November 1980, 1305-1311.

Blanchard, Olivier J., and Mark W. Watson, "Bubbles, Rational Expectations and Financaial Markets." In Wachtel, Paul, ed., Crisis in the Economic and Financial System. Lexington, MA: Lexington Books, 1982, pp. 295-315.

Brealey, Richard, and Stewart Myers, Principles of Corporate Finance. New York: McGraw-Hill, 1981.

Brock, William A., "Money and Growth: The Case of Long-run Perfect Foresight," International Economic Review, October 1974, 15, 750-777.

Brock, William A., "Asset Prices in a Production Economy." In McCall, John, ed., The Economics of Information and Uncertainty. Chicago: University of Chicago Press, 1982, pp. 1-43.

Burmeister, Edwin, Robert P. Flood, and Peter M. Garber, "On the Equivalence of Solutions in Rational Expectations Models," Journal of Economic Dynamics and Control, May 1983, 5, 224-234.

Burmeister, Edwin, and Kent Wall, "Kalman Filtering Estimation of Unobserved Rational Expectations with an Application to the German Hyperinflation," Journal of Econometrics, November 1982, 20, 255-84.

Cagan, Phillip, "The Monetary Dynamics of Hyperinflation." In Friedman, Milton, ed., Studies in the Quantity Theory of Money. Chicago: University of Chicago Press, 1956.

Camerer, Colin, "Bubbles and Fads in Asset Prices: A Review of Theory and Evidence," unpublished paper, University of Pennsylvania, 1987.

Campbell, John Y., and Robert J. Shiller, "Cointegration and Tests of Present Value Models," Journal of Political Economy, October 1987, 95, 1062-1088.

Campbell, John Y., and Robert J. Shiller, "Stock Prices, Earnings and Expected Dividends," Journal of Finance, July 1988a, 43, 661-676.

Campbell, John Y., and Robert J. Shiller, "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," Review of Financial Studies, Fall 1988b, 1, 195-228.

Casella, Alessandra, "Testing for Price Level Bubbles: The German Hyperinflation Once More," unpublished paper, University of California, Berkeley, December 1986.

Diba, Behzad T. and Herschel I. Grossman, "On the Inception of Rational Bubbles," Quarterly Journal of Economics, August 1987, 102, 697-700.

Diba, Behzad T. and Herschel I. Grossman, "Rational Inflationary Bubbles," Journal of Monetary Economics, May 1988, 21, 35-46.

Evans, George W., " A Test for Speculative Bubbles and the Sterling-Dollar Exchange Rate: 1981-1984," American Economic Review, September 1986, 76, 621-636.

Fama, Eugene F., Foundations of Finance. New York: Basic Books, 1976.

Fama, Eugene F. and Kenneth R. French, "Permanent and Temporary Components of Stock Prices," Journal of Political Economy, April 1988, 96, 246-273.

Flood, Robert P. and Peter M. Garber, "An Economic Theory of Monetary Reform," Journal of Political Economy, February 1980a, 88, 24-58.

Flood, Robert P. and Peter M. Garber, "Market Fundamentals Versus Price Level Bubbles: The First Tests," Journal of Political Economy, August 1980b, 88, 745-70.

Flood, Robert P., Peter M. Garber and Louis Scott, "Multi-Country Tests for Price Level Bubbles," Journal of Economic Dynamics and Control, December 1984, 8, 329-40.

Flood, Robert P. and Robert J. Hodrick, "Asset Price Volatility, Bubbles and Process Switching," Journal of Finance, September 1986, 41, 831-842.

Flood, Robert P., Robert J. Hodrick and Paul Kaplan, "An Evaluation of Recent Evidence on Stock Market Bubbles," unpublished paper, Northwestern University, revised May 1987.

Frankel, Jeffrey A., "The Dazzling Dollar," Brookings Papers on Economic Activity, 1985, 1, 199-217.

Garber, Peter M., "The Tulip Mania Legend," unpublished paper, Brown University, August 1987.

Gilles, Christian, and Stephen F. LeRoy, "The Variance-Bounds Tests: A Critical Survey," unpublished paper, University of California, Santa Barbara, August 1987.

Grossman, Sanford, and Robert J. Shiller, "The Determinants of the Variability of Stock Market Prices," American Economic Review, May 1981, 71, 222-27.

Hamilton, James D., and Charles H. Whiteman, "The Observable Implications of Self-fulfilling Expectations," Journal of Monetary Economics, November 1985, 16, 353-74.

Hansen, Lars Peter, and Thomas J. Sargent, "Formulating and Estimating Dynamic Linear Rational Expectations Models." In Lucas, Robert E. Jr., and Thomas J. Sargent, eds., Rational Expectations and Econometric Practice. Minneapolis: University of Minnesota Press, 1981, pp. 91-126.

Hausman, Jerry A., "Specification Tests in Econometrics," Econometrica, November 1978, 46, 1251-71.

Keynes, John Maynard, The GEneral Theory of Employment, Interest and Money. London: Macmillan, 1936.

Kindleberger, Charles, Manias, Panics and Crashes. New York: Basic Books, 1978.

Kleidon, Allan W., "Variance Bounds Tests and Stock Price Valuation Models," Journal of Political Economy, October 1986, 94, 953-1001.

Krugman, Paul R., "Is the Srong Dollar Sustainable?" in The U.S. Dollar - Recent Developments, Outlook, and Policy Options. Fedreal Reserve Bank of Kansas City, 1986, pp. 103-132.

LeRoy, Stephen F., and Richard D. Porter, "The Present Value Relation: Tests Based on Implied Variance Bounds," Econometrica, May 1981, 49, 555-74.

LeRoy, Stephen F., and William R. Parke, "Stock Price Volatility: An Inequality Based on the Geometric Random Walk," unpublished paper, University of California, Santa Barbara, 1988.

Mankiw, N. Gregory, David Romer, and Matthew Shapiro, "An Unbiased Reexamination of Stock Market Volatility," The Journal of Finance, July 1985, 40, 677-89.

Marsh, Terry A., and Robert C. Merton, "Dividend Variability and Variance Bounds Tests for the Rationality of Stock Market Prices," American Economic Review, June 1986, 76, 483-498.

Marsh, Terry A., and Robert C. Merton, "Dividend Behavior for the Aggregate Stock Market," Journal of Business, January 1987, 60, 1-40.

Mattey, Joe, and Richard A. Meese, "Empirical Assessment of Present Value Relations," Econometric Reviews, 1986, 5, 171-234.

Meese, Richard A., "Testing for Bubbles in Exchange Markets: The Case of Sparkling Rates," Journal of Political Economy, April 1986, 94, 345-73.

Meese, Richard A., and Kenneth Rogoff, "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?" Journal of International Economics, February 1983, 14, 3-24.

Obstfeld, Maurice, and Kenneth Rogoff, "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" Journal of Political Economy, August 1983, 91, 675-87.

Obstfeld, Maurice, and Kenneth Rogoff, "Ruling Out Divergent Speculative Bubbles," Journal of Monetary Economics, May 1986, 17, 349-62.

Okina, Kunio, "Empirical Tests of 'Bubbles' in the Foreign Exchange Market," Bank of Japan Monetary and Economic Studies, May 1985, 3, 1-45.

Poterba, James M., and Lawrence H. Summers, "Mean Reversion in Stock Prices: Evidence and Implications," Journal of Financial Economics, October 1988, 22, 27-59.

Shiller, Robert J., "Rational Expectations and the Dynamic Structure of Macroeconomic Models: A Critical Review," Journal of Monetary Economics, January 1978, 4, 1-44.

Shiller, Robert J., "Do Stock Prices Move By Too Much to be Justified by Subsequent Changes in Dividends?," American Economic Review, June 1981, 71, 421-36.

Singleton, Kenneth J., "Speculation and the Volatility of Foreign Currency Exchange Rates." In Brunner, Karl and Allan H. Meltzer, eds., Bubbles and Other Essays, Carnegie-Rochester Conference on Public Policy, Spring 1987, 26, 9-56.

Taylor, John, "Conditions for Unique Solutions in Stochastic Macroeconomic Models with Rational Expectations," Econometrica, November 1977, 45, 1377-1385.

Tirole, Jean, "Asset Bubbles and Overlapping Generations," Econometrica, November 1985, 53, 1071-1100.

West, Kenneth D., "A Specification Test for Speculative Bubbles," Quarterly Journal of Economics, August 1987a, 102, 553-580.

West, Kenneth D., "A Standard Monetary Model and the Variability of the Deutschemark-Dollar Exchange Rate," Journal of International Economics, August 1987b, 23, 57-76.

West, Kenneth D., "Dividend Innovations and Stock Price Volatility," Econometrica, January 1988a, 56, 37-61.

West, Kenneth D., "Bubbles, Fads and Stock Price Volatility Tests: A Partial Evaluation," Journal of Finance, July 1988b, 43, 639-656.

Woo, Wing T., "Some Evidence of Speculative Bubbles in the Foreign Exchange Markets," Journal of Money, Credit and Banking, November 1987, 19, 499-514.

Woodford, Michael, "Stationary Sunspot Equilibria in a Finance Constrained Economy," Journal of Economic Theory, 1986, 40, 128-137.

Woodford, Michael, "Expectations, Finance, and Aggregate Instability." In Kohn, Meir and S. C. Tsiang, eds., Finance Constraints, Expectations and Macroeconomics. Oxford: Oxford University Press, 1988.