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ABSTRACT

We develop a simple (incumbent versus entrant) strategic deterrence model to study the economic and geopolitical interactions underlying a strategic international activity, such as trade-related infrastructure projects like the Panama Canal. We study the incentives for global geopolitical players to support allied satellite countries where the strategic activity takes place or could potentially be initiated. We show that even if no effective competitor emerges, the appearance of a geopolitical challenger capable of credibly supporting the entrant has a pro-competition economic effect which benefits consumers all over the world. Thus, we provide a mechanism through which geopolitical rivalry between global powers leads to better economic outcomes for the global economy (i.e., less market power in the provision of international trade-related infrastructure). This contrasts with previous research on politics and market power which emphasizes the negative effects of political interference as well as research on international relations which often highlights the negative global effects of rising geopolitical tensions between an established power and a emerging challenger.

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1 Introduction

There is a growing and fascinating body of empirical literature on the effects of large-scale infrastructure projects (e.g., the expansion of the Panama Canal) on the volume and pattern of international and regional trade (Maurer and Yu, 2008; Feyrer, 2009; Hugot and Dajud, 2016). These studies use disruptions in the operation of trade-related infrastructures or new infrastructure projects as invaluable exogenous shocks that affect trade costs across locations and products. This is definitely a well founded empirical approach to estimate the causal effect of trade costs on the volume and pattern of international and regional trade. In this paper, we adopt a completely different but complementary approach. Our goal is to explore the strategic economic and political forces that underlie some of these infrastructure projects. Strategic considerations are relevant for at least two reasons. First, the construction of large-scale trade-related infrastructure, such as ports and canals, tends to be undertaken on a non-competitive basis, as such projects are often carried out under monopolistic or oligopolistic conditions or are conducted by government-owned firms. Thus, the scope for strategic economic decisions is simply larger than it is, say, for standard shipping and transportation services. Second, since major infrastructure projects have the potential to redirect trade flows and foreign direct investment and, in the event of open conflict, to influence military operations, they are often considered to be of key importance for geopolitical reasons.

As the Panama Canal provides such a strategic link between the Atlantic and Pacific Oceans, it is an excellent example of a trade-related infrastructure project that is subject to substantial economic and geopolitical strategic considerations. Ever since its construction, the Panama Canal has been an almost uncontested monopoly. Initially, it was owned by the United States and, although in 1999 it was transferred to the Republic of Panama, it is still considered to be within the orbit of influence of the United States (Sabonge and Sánchez, 2014). During the twentieth century, several projects to build alternative routes between the Atlantic and Pacific Oceans were envisioned, but it was not until the economic and geopolitical rise of China that a more serious challenge emerged. That challenge took the form of China's inclusion of a proposal for an alternative transoceanic canal running through Nicaragua as part of its Belt and Road Initiative. However, the project has since been postponed and the initial construction works have been suspended. We argue that the expansion of the Panama Canal played an important role in China's decision to suspend the project, but we also contend that the threat of a Chinese-financed rival canal through Nicaragua was a factor in Panama's decision to expand its canal and in the United States' decision to support that plan.

To formally capture the strategic interactions illustrated by the Panama Canal, we extend the standard game-theoretic model of strategic entry deterrence to include a geopolitical component. In this model, there is one incumbent (e.g., Panama) and a potential entrant (e.g., Nicaragua) that play an entry game and two global powers (e.g., United States and China) that try to influence the outcome of this entry game for economic and geopolitical reasons. To do so, each global power subsidizes its geopolitical ally.

When the global power allied with the incumbent wins the subsidy race, in equilibrium, there is deterrence (e.g., no canal is built in Nicaragua). This does not imply that geopolitics does not matter. Under deterrence, the incumbent, supported by its global ally, overinvests in capacity to deter the entrant that has received a credible promise of support from the other global power. In equilibrium, this credible promise is not acted upon, but it plays an important role in prompting the incumbent and its global ally to further expand capacity/ support the expansion of that capacity. Thus, even when no effective competitor emerges, the rise of a geopolitical challenger has a pro-competition economic effect which benefits consumers all over the world.

When the global power that is allied with the entrant wins the subsidy race, in equilibrium, there is accommodated entry. In other words, there is entry (e.g., a canal is built in Nicaragua with Chinese support) when the global ally of the entrant is willing to provide significant support and the global ally of the incumbent (e.g., the United States) is not willing to provide the substantial funds required to deter entry. In this case, the rise of a geopolitical challenger has economic as well as geopolitical effects. From an economic perspective, the market structure changes from a monopoly to a duopoly, which in turn leads to a reduction in the equilibrium price. Once again, consumers of all regions benefit from this change. From a geopolitical perspective, in equilibrium, there is effective entry by a new global power, which breaks up the geopolitical monopoly of the incumbent's global ally.

Regardless of which global power wins the geopolitical subsidy race, the rise of a geopolitical challenger makes consumers better off. Do consumers prefer any specific outcome? More precisely, is it better for consumers that the global power allied with the incumbent wins the subsidy race and, hence, there is deterrence, or that the global power that is allied with the entrant wins the subsidy race and, hence, there is accommodated entry? We show that, from a consumer welfare perspective, in equilibrium entry deterrence is always preferred to accommodation. Intuitively, this occurs because as the rising global power increases its support to the potential entrant, the entry deterrence capacity level increases, eventually, surpassing the aggregate capacity under accommodation. Moreover, provided that the rising global power can commit enough funds to support the potential entrant, which is the case in our baseline setting, the equilibrium of the geopolitical subsidy race always induces a capacity level under deterrence above the aggregate capacity under accommodation. Therefore, consumers are better off when the global power allied with the incumbent wins the subsidy race, i.e., when the geopolitical status quo is not altered.

Additionally, we also consider six extensions to our baseline model:

First, in the baseline model, we implicitly assume that the subsidies promised by both global powers are contingent but binding decisions. This implies that the mere threat of subsidizing the entrant can allow the rising global power to make the incumbent and its ally increase capacity. In an extension, we explore what happens when we limit this threatening capability. In particular, we assume that the rising global power faces a budget constraint to subsidize the potential entrant. This scenario gives rise to three novel results. When the rising global power only faces a mild budget constraint, there is no serious change in the equilibrium. Entry is less likely to occur, and deterrence becomes easier to sustain, as the established global power cannot be bullied with non-credible promises of large subsidies. When the rising global power faces a more severe budget constraint, it stops presenting a geopolitical threat for the established global power because the incumbent is willing to deter entry even with no support. Then, at the margin, both global powers are better off if the rising global power's budget constraint is relaxed, which prompts the incumbent to further expand its capacity to deter entry. When the rising global power has a very limited budget, in equilibrium, entry is blocked and, once again, both global powers will be better off if rising global power's budget constraint is relaxed to the point that the incumbent is forced to expand its capacity to deter entry. Finally, it is worth mentioning that a stricter budget constraint for the rising global power might have a pro-competitive economic effect only if the rising global power would have outbid the incumbent global power in an unconstrained environment.

Second, in the first extension we show that even when the incumbent global power could economically benefit from a capacity expansion of its ally, if the rising global power does not pose an effective geopolitical threat, it decides not to support this potentially beneficial expansion. The reason is that we implicitly assume that the global power allied with the incumbent only offers a subsidy to deter entry. In other words, its support to the incumbent is triggered by a geopolitical logic.[1] In an extension we allow the global power allied with the incumbent to support capacity expansions motivated purely by economic reasons. Since this is particularly interesting when there is no geopolitical threat, we consider a scenario in which the rising global power faces a severe budget constraint and, hence, using the subsidy schedule in the baseline model, the resulting equilibrium would be blocked entry and no capacity expansion. On the contrary, in this extension we show that the global power allied with the incumbent subsidizes some capacity expansion because consumers are willing to compensate the incumbent for the loss in profits associated with a capacity expansion beyond the monopoly level. Nevertheless, we show that geopolitical competition still has a pro-competition economic effect in the sense that in any equilibrium of the baseline model (no matter if there is deterrence of accommodated entry), the aggregate capacity will be even higher.

Third, to distinguish between geopolitical rivalry and the pure effect of a rising global power, we study a scenario in which the global power allied with the incumbent is entirely incapable of supporting its ally. We show that in such circumstances, the equilibrium is accommodated entry, which is better for consumers than when there is no geopolitical threat (i.e., the equilibrium for the first extension when the rising global power has a very limited budget), but weakly worse than the equilibrium in the baseline model, i.e., when the global power allied with the incumbent can support its ally to deter entry. Thus, to fully obtain the economic benefits of geopolitical competition, consumers do not only need a rising global power capable of effectively supporting the potential entrant, but also a global power allied with the incumbent that can effectively react to the geopolitical challenge subsidizing the expansion of incumbent's capacity.

Fourth, we explore the possibility that the global power allied with the incumbent does not value its geopolitical monopoly in the strategic transportation service. In other words, in this extension, we study a scenario with no geopolitical rivalry. When this occurs, the global power allied with the incumbent enjoys no geopolitical benefit from deterring the potential entrant and its geopolitical ally, the rising global power. Therefore, compared with the baseline model, in equilibrium, it is more likely that the rising global power will win the subsidy race, which implies that deterrence will be less likely and accommodated entry more likely. As entry deterrence implies a higher consumer surplus in comparison to accommodation, no geopolitical rivalry leads to weakly worse outcomes for consumers.

Fifth, in the baseline model global powers chose their subsidies simultaneously. Although reasonable, an odd feature of this arrangement is that the incumbent global power commits to subsidize the incumbent for an action of the entrant, which is not directly under its control. In an extension, we study a scenario in which the rising global power moves first, which serves two purposes. It allows us to simplify the contract offered by the global power allied with the incumbent, making the payment contingent only on the capacity level built by the incumbent rather than on the entry decision of the potential entrant. It also helps eliminate equilibrium multiplicity. Crucially, it confirms that key results do not depend on global powers choosing simultaneously or an odd artifact in the contract offered by the global power

[1]This should not be misinterpreted as implying that economic gains are disregarded. It only states that if there is no entry threat, then the global power allied with the incumbent does not consider subsidizing the incumbent for pure economic reasons.

allied with the incumbent.

Finally, we explore the possibility of luring the entrant into the incumbent global power's sphere of influence. To do so, we allow the incumbent global power to preempt the geopolitical subsidy race and lure the potential entrant offering a lump-sum payment to break its geopolitical ties with the rising global power and become an ally of the incumbent global power. Moreover, we assume that this geopolitical realignment is not costless for the potential entrant, for example, because it will suffer sanctions from its former ally. Naturally, if the switching cost for the potential entrant is high enough, the equilibrium in the baseline setting is not affected. More interesting, if the switching cost is low enough, the incumbent global power will find attractive to lure the potential entrant and, in equilibrium, entry will be deterred. Indeed, we show that in such circumstances, the incumbent global power can induce a very convenient equilibrium, in which it does not pay any additional subsidy and obtains an expansion of the incumbent's capacity for free, i.e., just threatening to further subsidize the potential entrant.

1.1 Related Literature

There are four areas of the literature related to this paper. First, in industrial organization there is an extensive body of literature on strategic entry deterrence. Second, in the area of international relationships, there is also an extensive body of literature on geopolitics and, in particular, on the interactions between an established global or regional power and a rising challenger. Third, the paper is related to economics of conflict and the rising literature on the connections between geopolitics and economics as well as foreign influence. Finally, in industrial organization, there is a recent literature on the relationship between political decisions and market power.

The classical literature on strategic entry deterrence has highlighted several mechanisms that an incumbent can use to deter entry. We focus on a group of papers that consider that an incumbent can use strategic investments to deter entry.[2,3] Our model closely follows Tirole (1988), who drew on the results of Kreps and Scheinkman (1983) and Fudenberg and Tirole (1984) to study a two-stage entry game where firms select their capacities in the first stage and then compete on prices in the second stage. We augment this model by introducing two new players (the global powers) with the ability and willingness to influence the incumbent and entrant, respectively. Models of entry deterrence have been extended in several directions.[4] However, to the best of our knowledge, there is no extension that has studied how geopolitical considerations affect the equilibrium. At a pure theoretical level, our model suggests that once we introduce a player with the ability and willingness to expand the equilibrium quantity (e.g., the rising global power in our model), blocked entry will never be an equilibrium of the deterrence model.

[2] An alternative group of models focuses on pricing decisions which can be used to build up the reputation of an incumbent (Kreps et al., 1982) or to signal the existence of a low cost to the potential entrant (Milgrom and Roberts, 1982).

[3] The foundational work in entry deterrence are Spence (1977) and Dixit (1979, 1980). Spence (1977) formalizes the idea that investments in capacity are a credible commitment capable of deterring country, while Dixit (1979) expands this model to allow the incumbent to choose between deterring and accommodating entry. Dixit (1980) goes on to explore different post-entry scenarios, including those involving a quantity leadership role for the entrant and price competition.

[4] For example, Maggi (1996) introduced uncertainty regarding conditions in the contested market, while Bagwell and Ramey (1996) explored the role of avoidable costs, and Eaton and Ware (1987) looked at how the market structure might vary with technology. Additionally, several theoretical implications of these models have been tested in a variety of markets. For example, Thomas (1999) focused on cereals, Lieberman (1987) on chemical industries, Conlin and Kadiyali (2006) on lodging properties, and Ellison and Ellison (2011) on pharmaceuticals.

The reason being that such a player can always induce deterrence without actually incurring any cost. The only remaining question is whether this player is interested in escalating its support to induce entry.

There is a vast body of literature within the field of international relations on the interactions between an established power and a rising challenger (e.g., Nye Jr (1991); Ikenberry (2011)). Our paper emphasizes the dilemma between economic gains and geopolitical threats. Overall, a rising economic power opens up excellent new economic opportunities for the established power via specialization, international trade and foreign direct investment. The cost for the established power is the sharing of political influence with the rising power. We make three contributions to this literature. First, we formally model one possible way in which an established power and a rising challenger can interact and explore under what conditions and why a dilemma between economic gains and geopolitical threats emerges. Second, our model also allows us to explore what the consequences are for the countries being influenced by the global powers as well as third countries. Finally, we identify a mechanism through which geopolitical competition and considerations shape a strategic international activity (trade infrastructure).

In the literature on economics of conflict, several papers have studied the connections between international integration and conflict. Theoretical papers include Skaperdas and Syropoulos (2001), Syropoulos (2006), Garfinkel et al. (2012), Garfinkel et al. (2015), Jackson and Nei (2015), Lopez Cruz and Torrens (2019, 2022). Empirical papers include Polachek and Seiglie (2007), Polachek et al. (2007) and Kamin (2022). None of these papers, however, considers how global powers compete for key trade-related infrastructures with strategic geopolitical importance. Closer to our work are Camboni and Porcellacchia (2021), Ambrocio and Hasan (2021), Gelpern et al. (2021), and Aidt et al. (2021). Camboni and Porcellacchia (2021) study how countries compete for a geopolitical sphere of influence. Ambrocio and Hasan (2021) and Gelpern et al. (2021) show that countries that align with a global power can obtain economic benefits, such as improvements in borrowing conditions. Aidt et al. (2021) provide a theoretical framework and a survey of the political economy literature on foreign influence. Our results suggest that geopolitical rivalry might have a pro-competition global economic effect and, hence, some of its economic benefits might extend to third countries.[5] The paper is also connected with the dynamic games approach to conflict, in particular, Hendrickson and Salter (2016), who study participation (i.e., entry) in a revolt. Analogously to Hendrickson and Salter (2016), in the current model, the rising global power must obtain a markup over the participation cost (i.e., its subsidy) before it enters the geopolitical arena. Otherwise, it is deterred by the subsidies offered by the existing global power to the incumbent country. In contrast to Hendrickson and Salter (2016), in the current model, the rising global power can still influence economic outcomes (inducing a capacity expansion by the incumbent) even when no entry happens in equilibrium. The reason is that capacity expansion to assure deterrence works as an economic concession extracted by the rising global power.

Finally, in industrial organization, there is a renewed interest in how politics influences market power. With evidence of firms' market power increasing over time (De Loecker and Eeckhout, 2018; De Loecker et al., 2020), recent research has explored politics as an additional source of market power (e.g., Cowgill et al. (2021) and Lancieri et al. (2022)). There is growing evidence that policy decisions such as regulations (Trebbi and Zhang, 2022), competition policy (Ha et al., 2021) and entrant exclusion (Callander et al. (2022) and Kang and Xiao (2023)) can significantly influence market configuration and market power. These policy decisions, however, have been either assumed as given or selected by a single maximizing

[5]In a similar fashion, Thompson and Hickson (2012) argue that geopolitical rivalry might intensify the competition for scarce labor, which increases wages.

political agent (usually a politician or bureaucrat). A consistent conclusion of this research is that political interferences increase market power and, hence, create inefficiencies.[6] Our paper shares with Callander et al. (2022) and Kang and Xiao (2023) the idea of connecting politics with market entry and deterrence. There are, however, substantial differences. First, none of these papers consider the role of political competition. Second, both papers argue that political influence is negative for consumers; by disincentivizing investment of leading firms (Callander et al., 2022) or by enhancing the commitment power of the leading firm to crowd-out other firms (Kang and Xiao, 2023). In our model this is not necessarily the case. The rise of geopolitical competition has a pro-competition economic effect that benefits consumers. Finally, both papers assume that the politician has full credibility, while we also explore a scenario in which one of the political players (the rising power) has limited commitment. Moreover, this leads to an interesting result. Both political players (the incumbent and the rising power) benefit from intermediate levels of credibility for the rising power.

The rest of the paper is organized as follows. Section 2 presents a standard model of strategic economic deterrence, augmented with the subsidies offered by the global powers. Section 3 introduces the geopolitical dimension by looking at the equilibrium interactions between the two global powers. Section 4 develops six extensions of the model. Section 5 applies the model to the case of the Panama Canal. Section 6 presents the conclusions.

2 A Simple Model of Economic Deterrence

Consider two countries that, by virtue of their locations, could provide a strategic transportation service such as a connection between the Atlantic and Pacific Oceans (e.g., Panama and Nicaragua). The demand for this service comes from three countries and/or regions that we interpret as two global powers denoted G_1 and G_2 (e.g., the United States and China) and the rest of the world denoted RW, respectively. To simplify things, suppose that the strategic transportation service is an homogeneous product for which the demand in country j is a linear function of the price: $Q_j = A_j\,(a - P)$ for $j \in J = \{G_1, G_2, RW\}$, where $P \geq 0$ is the price of the service, $a > 0$, and $A_j > 0$ for all j. Therefore, the inverse demand of the service is $P = a - bQ$, where $b = \left(\sum_{j\in J} A_j\right)^{-1}$ and $Q = \sum_{j\in J} Q_j$.

The countries that are strategically located to provide this service are not symmetric. One country, denoted by I, is the market incumbent (e.g., Panama) and the other country, denoted by E, is a potential entrant (e.g., Nicaragua). I and E play a deterrence game. Countries first make a capacity decision (e.g., build or expand the canal) and later compete on prices. Specifically:

1. I selects capacity $k_I \geq 0$.

[6] Multiple mechanisms have been proposed as possible political sources of market power: mergers simplify industry lobbying helping firms overcome collective action problems (Cowgill et al., 2021; Moshary and Slattery, 2023); use of political connections through US Congress committee members (Fan and Zhou, 2023); and political exclusion to induce preemption (Callander et al., 2022; Kang and Xiao, 2023). Our paper is more closely related to the political exclusion mechanism. Callander et al. (2022) considers a single politician with the capacity to impose minimum quality standards, which might result in the exclusion of a potential entrant. The problem for the politician is that a leading firm capturing a larger market share makes political protection less attractive. Thus, to avoid losing political rents, the politician must keep some level of competition. Kang and Xiao (2023) argue that a leading firm can preempt pro-competitive government policies. They consider a single politician who can enact costly policies that increase consumers' welfare, but it is less willing to do so if a leading firm has previously committed to a larger capacity (and production).

2. E observes k_I and selects capacity $k_E \geq 0$.

3. Given (k_I, k_E), there is price competition. I and E simultaneously and independently select prices (p_I, p_E) and the demand of each country is determined according to the efficient-rationing rule.[7]

For both countries building capacity has a unit cost $c > 0$ but E must also incur an entry cost $F > 0$. Additionally, global powers provide subsidies. In particular, G_1 offers a subsidy $S_1 \geq 0$ to the incumbent if it manages to avoid entry (i.e., when $k_E = 0$), while G_2 offers an entry subsidy of $S_2 \in [0, F]$ to the potential entrant (i.e., when $k_E > 0$). There are several ways to justify these subsidies. For example, assume that for geopolitical reasons an existing global power (i.e., G_1) is interested in keeping the strategic transportation service under the exclusive control of a close ally or satellite state (i.e., I), while a rising global power (i.e., G_2) is interested in opening an alternative route that it is not under the control and/or heavy influence of G_1.[8]

2.1 Economic Equilibrium

The subgame perfect Nash equilibrium of this game can be easily solved through backward induction. Moreover, we impose restrictions on capacity choices and the set of parameters which ensure that, in equilibrium, under a duopoly, both countries set the same price and use all their installed capacity. In particular, we assume that:

Assumption 1 *i.* $k_I \in \left[0, \frac{a-c}{b}\right]$, *ii.* $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$, *and iii.* $a \leq 2c$.

The first two restrictions state that I can select any k_I up to the competitive capacity level (i.e., $k_I \leq \bar{k}^c = \frac{a-c}{b}$) and E can build k_E up to the difference between the competitive capacity and the capacity selected by the incumbent (i.e., $k_E \leq \bar{k}^c - k_I$). If I and E expect that installed capacity will be fully used and the same price will be set, these restrictions are inconsequential, given that E has no incentive to enter and select $k_E > \bar{k}^c - k_I$ (because this will always induce negative profits), and I has no incentive to select $k_I > \bar{k}^c$ (because $k_I = \bar{k}^c$ is enough to deter entry). The third restriction (i.e., $a \leq 2c$) implies that $k_E \leq (a - bk_I)/2b$ and $k_I \leq (a - bk_E)/2b$ hold for all $k_I \in \left[0, \bar{k}^c\right]$ and $k_E \in \left[0, \bar{k}^c - k_I\right]$, which ensures that both countries will use all their installed capacity and set the same price. More precisely, given capacity choices (k_I, k_E) and that the demand of each country is determined according to the efficient-rationing rule, price competition between I and E will lead to Nash equilibrium prices $p_I = p_E = a - b(k_I + k_E)$, provided that capacity choices satisfy $k_E \leq (a - bk_I)/2b$ and $k_I \leq (a - bk_E)/2b$ (see Appendix A.1 for more details).[9]

[7] The efficient-rationing rule indicates that consumers with the highest willingness to pay will be served first. This rule has the advantage of maximizing the consumer surplus. For more details see Tirole (1988).

[8] In Section 3 we introduce and discuss the payoff functions of the global powers, including geopolitical payoffs. In Section 4 we further discuss the contractual arrangements supporting subsidy offers and consider an alternative subsidy schemes for global power G_1 that only depends on actions taken by the incumbent rather than the potential entrant.

[9] Alternatively, we can consider that I and E compete a la Cournot, in which case, no further restriction is required to ensure that $P = a - b(k_I + k_E)$ for all $k_I \in \left[0, \frac{a-c}{b}\right]$ and $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$.

Thus, under Assumption 1, the equilibrium market price as a function of (k_I, k_E) is $P = a - b(k_I + k_E)$ and, hence, profit functions are given by:

$$\pi_I(k_I, k_E) = [a - b(k_I + k_E) - c]\, k_I + (1 - \chi_{k_E>0})\, S_1$$
$$\pi_E(k_I, k_E) = [a - b(k_I + k_E) - c]\, k_E - \chi_{k_E>0}\,(F - S_2)$$

where $\chi_{k_E>0} = 1$ if $k_E > 0$ and $\chi_{k_E>0} = 0$ if $k_E = 0$.

Next we characterize capacity choices:

Potential entrant: Assume that I has selected $k_I \in \left[0, \frac{a-c}{b}\right]$. Then, the profit maximizing capacity level for E is given by:

$$k_E(k_I) = \left(1 - \chi_{k_I \geq \bar{k}^d}\right)\left(\frac{a - bk_I - c}{2b}\right)$$

where $\chi_{k_I \geq \bar{k}^d} = 1$ if $k_I \geq \bar{k}^d$, $\chi_{k_I \geq \bar{k}^d} = 0$ if $k_I < \bar{k}^d$, and

$$\bar{k}^d = \frac{a - c - 2\sqrt{b(F - S_2)}}{b}$$

is the deterrence capacity level when G_2 offers an entry subsidy of S_2.[10] The intuition behind $k_E(k_I)$ is as follows. If E decides to enter, its best response is given by $k_E = \frac{a-bk_I-c}{2b} \in \left[0, \frac{a-c}{b} - k_I\right]$, which induces profits $\pi_E = \frac{(a-bk_I-c)^2}{4b} - F + S_2$. On the contrary, if E does not enter, it obtains zero profits. We assume that E enters whenever $\pi_E > 0$, which holds if and only if $k_I < \bar{k}^d$. Three remarks apply. First, the greater the entry subsidy offered by G_2, the greater the expansion in capacity that I needs to incur in order to deter entry. Formally, $\bar{k}^d$ is strictly increasing in S_2. Second, for $S_2 = F$, we have $\bar{k}^d = \bar{k}^c = \frac{a-c}{b}$, and, hence, E will always enter unless I builds capacity to the competitive level. Finally, for $S_2 = 0$, we have $\bar{k}^d = \frac{a-c-2\sqrt{bF}}{b}$ and there are two possibilities. If the entry cost is above monopoly profits (formally, $F \geq \frac{(a-c)^2}{4b}$), we have $\bar{k}^d \leq 0$ and, hence E will never enter regardless of I's choice. On the contrary, if the entry cost is below monopoly profits ($F < \frac{(a-c)^2}{4b}$), we have $\bar{k}^d > 0$ and, hence, E's entry choice depends on k_I. Both situations will be considered.

Incumbent: Given $k_E(k_I)$, the problem of I is:

$$\max_{k_I \in \left[0, \frac{a-c}{b}\right]} \left\{ \pi_I(k_I, k_E(k_I)) = \chi_{k_I \geq \bar{k}^d}\left[(a - bk_I - c)\, k_I + S_1\right] + \left(1 - \chi_{k_I \geq \bar{k}^d}\right)\left[\frac{(a - bk_I - c)\, k_I}{2}\right] \right\}$$

The solution of this problem is often expressed as a function of $\bar{k}^d$ and $\bar{k}^m$, deterrence and monopoly capacity levels, respectively. However, for our purposes, it will be more convenient to express the equilibrium outcome as a function of the subsidies offered by G_1 and G_2.

[10] Given that $k_E(k_I) = 0$ if and only if $k_I \geq \bar{k}^d$, the subsidy offered to the incumbent can be alternatively specified as $\chi_{k_I \geq \bar{k}^d} S_1$, where $\chi_{k_I \geq \bar{k}^d} = 1$ if $k_I \geq \bar{k}^d$ and $\chi_{k_I \geq \bar{k}^d} = 0$ if $k_I < \bar{k}^d$ (a payment contingent on the incumbent's capacity choice) rather than $(1 - \chi_{k_E>0})\, S_1$, where $\chi_{k_E>0} = 1$ if $k_E > 0$ and $\chi_{k_E>0} = 0$ if $k_E = 0$ (a payment contingent on the potential entrant's capacity choice). Note, however, that $\bar{k}^d = \frac{a-c-2\sqrt{b(F-S_2)}}{b}$ depends on S_2. Thus, it is still the case that $\chi_{k_I \geq \bar{k}^d} S_1$ is contingent on actions taken by a third party, in this case the subsidy offered by the other global power. We will return to this issue in Sections 4.2 and 4.4.

Blocked entry: Suppose that $\bar{k}^d \leq \bar{k}^m = \frac{a-c}{2b}$, where $\bar{k}^m$ is the monopoly capacity level. Thus, even if I selects the monopoly capacity, E will not enter. Then, the unique subgame perfect Nash equilibrium outcome is $k_I = \bar{k}^m$, $k_E = 0$, and the equilibrium price is $P = a - b\bar{k}^m$. Note that $\bar{k}^d \leq \bar{k}^m$ if and only if

$$S_2 \leq \bar{S}^b = F - \frac{(a-c)^2}{16b}$$

Intuitively, if I selects $\bar{k}^m$ and E chooses to enter, its best response will be $k_E = \frac{a-c}{4b}$, which is just the Stackelberg equilibrium. In such circumstances, E's profits (including the entry cost and G_2's entry subsidy) will be $\pi_E\left(\bar{k}^m, \frac{a-c}{4b}\right) = \frac{(a-c)^2}{16b} - F + S_2$. Thus, $\bar{S}^b$ is the minimum subsidy that G_2 must offer to induce $\pi_E\left(\bar{k}^m, \frac{a-c}{4b}\right) \geq 0$, i.e., before E considers entering when I behaves as an unchallenged monopoly. We assume that

Assumption 2 *The entry cost satisfies* $F > \frac{(a-c)^2}{16b}$.

Note that $\bar{S}^b > 0$ if and only if Assumption 2 holds, which means that a positive but small entry subsidy (formally, any $S_2 \leq \bar{S}^b$) will not induce entry even when I behaves as an unchallenged monopoly. In other words, Assumption 2 implies that E (e.g., Nicaragua) always requires the support of G_2 (e.g., China) to enter.

Deterrence or accommodation?: Suppose that $\bar{k}^d > \bar{k}^m$ or, which is equivalent, $S_2 > \bar{S}^b$. Thus, I needs to expands capacity beyond the monopoly level in order to deter entry because now G_2 is offering a subsidy that will induce entry if I insists behaving as an unchallenged monopoly. In such circumstances, I must choose between $\bar{k}^m$ and $\bar{k}^d$ (any other capacity choice will be either dominated by $\bar{k}^m$ or $\bar{k}^d$). Therefore, there are two possible cases:

Deterred entry: Suppose that $\pi_I\left(\bar{k}^d, k_E\left(\bar{k}^d\right)\right) \geq \pi_I\left(\bar{k}^m, k_E\left(\bar{k}^m\right)\right)$. Then, the unique subgame perfect Nash equilibrium outcome is $(k_I, k_E) = \left(\bar{k}^d, 0\right)$, and the equilibrium price is $P = a - b\bar{k}^d$. Intuitively, to deter entry, I must select capacity level $k_I = \bar{k}^d$, which induces $k_E\left(\bar{k}^d\right) = 0$ and the following profits (including G_1's subsidy) for I:

$$\pi_I\left(\bar{k}^d, 0\right) = \left(a - b\bar{k}^d - c\right)\bar{k}^d + S_1 = 2(a-c)\sqrt{\frac{F - S_2}{b}} - 4(F - S_2) + S_1$$

On the contrary, if I insists on choosing $k_I = \bar{k}^m$, E enters and, hence, profits for I will be given by:

$$\pi_I\left(\bar{k}^m, \frac{a-c}{4b}\right) = \left(\frac{a - b\bar{k}^m - c}{2}\right)\bar{k}^m = \frac{(a-c)^2}{8b}$$

Thus, I prefers to deter entry when $\pi_I\left(\bar{k}^d, 0\right) \geq \pi_I\left(\bar{k}^m, \frac{a-c}{4b}\right)$ or, which is equivalent,

$$S_1 \geq \bar{S}^d(S_2) = \frac{(a-c)^2}{8b} - 2(a-c)\sqrt{\frac{F - S_2}{b}} + 4(F - S_2)$$

The idea is that the greater the entry subsidy offered by G_2, the greater the expansion in capacity that I needs to do in order to deter entry and, hence, the lower the profits that I obtains (formally, $\bar{k}^d$ is increasing in S_2 and $\pi_I\left(\bar{k}^d, 0\right)$ is decreasing in S_2). Eventually, deterring entry becomes unprofitable for I, unless G_1 offers a subsidy that covers the difference between profits under no expansion (i.e., $\pi_I = \frac{(a-c)^2}{8b}$) and profits (excluding any subsidy) under the required expansion to deter entry (i.e., $\pi_I = 2(a-c)\sqrt{\frac{F-S_2}{b}} - 4(F-S_2)$). In other words, $\bar{S}^d(S_2)$ is the minimum subsidy that G_1 must offer to I in order to deter entry when G_2 offers a subsidy of S_2 to E.

Accommodated entry: Alternatively, suppose that $\pi_I\left(\bar{k}^d, k_E\left(\bar{k}^d\right)\right) \leq \pi_I\left(\bar{k}^m, k_E\left(\bar{k}^m\right)\right)$ or, which is equivalent, $S_1 \leq \bar{S}^d(S_2)$. Then, the unique subgame perfect Nash equilibrium outcome is $(k_I, k_E) = \left(\bar{k}^m, \frac{a-c}{4b}\right)$, and the equilibrium price is $P = \frac{a+3c}{4}$. Intuitively, when $S_1 \leq \bar{S}^d(S_2)$, the subsidy offered by G_1 is not enough to compensate I for the difference between profits under no expansion (i.e., $\pi_I = \frac{(a-c)^2}{8b}$) and profits (excluding any subsidy) under the required expansion to deter entry (i.e.,

Equilibrium multiplicity: Note that for $S_1 = \bar{S}^d(S_2)$ we have $\pi_I\left(\bar{k}^d, k_E\left(\bar{k}^d\right)\right) = \pi_I\left(\bar{k}^m, k_E\left(\bar{k}^m\right)\right)$ and, hence, there are two subgame perfect Nash equilibrium outcomes: the equilibrium described under deterred entry and the equilibrium described under accommodated entry.

The following proposition summarizes the economic equilibrium for any pair of subsidies (S_1, S_2).

Proposition 1 ***Economic equilibrium.*** *Suppose that Assumptions 1 and 2 hold.*

1. *Suppose that* $0 \leq S_2 \leq \bar{S}^b$. *Then, the entry of* E *is* ***blocked***. *Specifically, in equilibrium,* $(k_I, k_E) = \left(\bar{k}^m, 0\right) = \left(\frac{a-c}{2b}, 0\right)$ *and* $P = \frac{a+c}{2}$.
2. *Suppose that* $\bar{S}^b < S_2 \leq F$.
   (a) *If* $S_1 > \bar{S}^d(S_2)$, *then the entry of* E *is* ***deterred***. *Specifically, in equilibrium,* $(k_I, k_E) = \left(\bar{k}^d, 0\right) = \left(\frac{a-c-2\sqrt{b(F-S_2)}}{b}, 0\right)$ *and* $P = a - b\bar{k}^d = c + 2\sqrt{b(F-S_2)}$.
   (b) *If* $S_1 = \bar{S}^d(S_2)$, *then there are two equilibria: in one equilibrium the entry of* E *is* ***deterred****, while in the other* I ***accommodates*** *the entry of* E. *Under deterrence (accommodation),* (k_I, k_E, P) *is as in part a (c).*
   (c) *If* $S_1 < \bar{S}^d(S_2)$, *then* I ***accommodates*** *the entry of* E. *Specifically, in equilibrium,* $(k_I, k_E) = \left(\bar{k}^m, \frac{\bar{k}^m}{2}\right) = \left(\frac{a-c}{2b}, \frac{a-c}{4b}\right)$ *and* $P = a - \frac{3}{4}\bar{k}^m = \frac{a+3c}{4}$.

Proof: *See Appendix A.1.* ■

Figure 1 illustrates Proposition 1. When the subsidy provided to E by global power G_2 is below a certain threshold (formally, $0 \leq S_2 \leq \bar{S}^b$), then E will not enter even if I keeps capacity at the monopoly level. Under such circumstances, I does not need to invest in extra capacity to deter E. Then, the equilibrium outcome coincides with the standard equilibrium under a monopoly. For the case of the Panama Canal, this can be interpreted as a situation in which China is not seriously committed to subsidizing Nicaragua and, lacking China's backing, Nicaragua finds it too costly to build a new canal even when Panama does not expand its capacity.

When the subsidy provided to E by global power G_2 is above a certain threshold (formally, $S_2 > \bar{S}^b$) and if I keeps capacity at the monopoly level, then E will have incentives to enter. Under such circumstances, I's only choice is between accommodating and overinvesting in capacity to deter the entry of E. Indeed, when the subsidy provided by global power G_1 is generous enough (formally, $S_1 > \bar{S}^d(S_2)$), it is profitable for I to install extra capacity to deter E's entry. The market then becomes a monopoly. For the case of the Panama Canal, this can be interpreted as a situation in which the United States helps Panama to build extra capacity in order to deter Nicaragua from building a new canal with the support of China. It is worth mentioning that, although the market becomes a monopoly under both deterred and blocked entry, equilibrium quantities and prices are not the same. The reason for this is that when $S_2 > \bar{S}_2^b$, I must overinvest in capacity to deter E.

When the subsidy provided by G_1 is not generous enough (formally, $S_1 < \bar{S}^d(S_2)$), I prefers to accommodate entry and the equilibrium outcome coincides with the equilibrium of the Stackelberg's model. For the case of the Panama Canal, this can be interpreted as a situation in which the United States does not provide enough support to Panama to deter Nicaragua from building a new canal with the support of China. Finally, $S_1 = \bar{S}^d(S_2)$ is a knife edge situation in which the subsidies are such that I is indifferent to the choice between deterrence and accommodation. This knife edge situation will prove to be important in Section 3, where we endogenize the subsidies provided by the global powers.

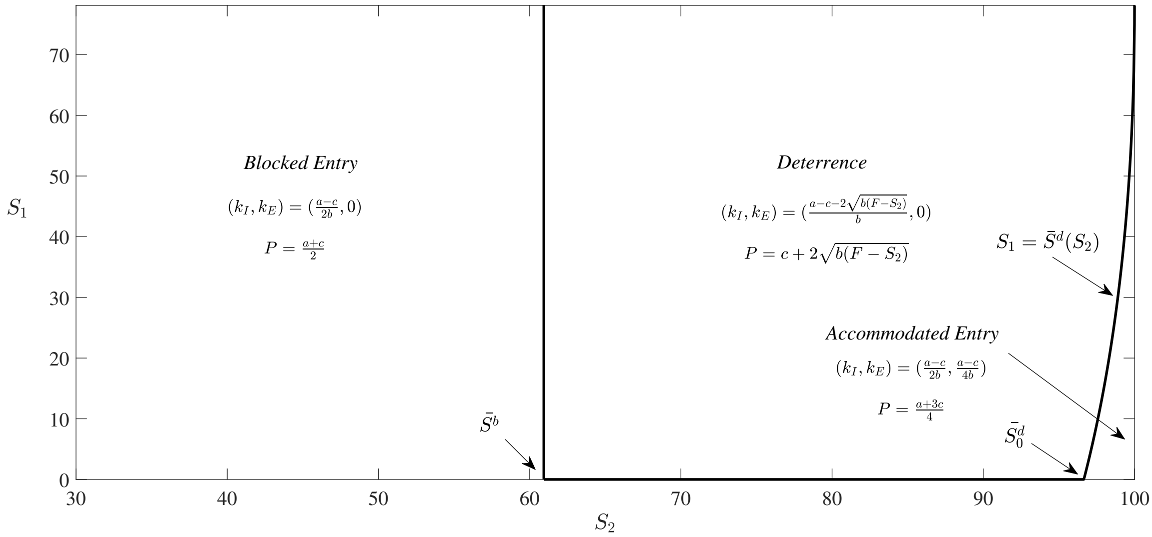

Figure 1. Economic equilibrium given (S_1, S_2). Note: The figure has been plotted assuming $a = 3.75$, $b = 1/400$, $c = 2.5$, and $F = 100$.

Further characterization of $\bar{S}^d(S_2)$: In Appendix A.1, we prove that, when Assumptions 1 and 2 hold, $\bar{S}^d(S_2)$ is strictly increasing and strictly convex in S_2 for all $S_2 \in [\bar{S}^b, F]$, and that there exists a unique $\bar{S}_0^d \in (\bar{S}^b, F)$ such that[11]

$$\bar{S}^d(S_2) < 0 \text{ for all } S_2 \in \left[\bar{S}^b, \bar{S}_0^d\right), \ \bar{S}^d\left(\bar{S}_0^d\right) = 0 \text{ and } \bar{S}^d(S_2) > 0 \text{ for all } S_2 \in \left(\bar{S}_0^d, F\right]$$

Using these properties, Proposition 1.2 can be written as follows. Let $\left(\bar{S}^d\right)^{-1}$ denote the inverse of $\bar{S}^d$ for all $S_1 \geq 0$. Then:

- If $\bar{S}^b < S_2 < \left(\bar{S}^d\right)^{-1}(S_1)$, then the entry of E is deterred. Moreover, for all $\bar{S}^b < S_2 < \bar{S}_0^d$, the

[11]Indeed, solving $\bar{S}^d\left(\bar{S}_0^d\right) = 0$, we have that $\bar{S}_0^d = F - \left(1 - \frac{\sqrt{2}}{2}\right)^2 \frac{(a-c)^2}{16b}$.

entry of E is deterred even for $S_1 = 0$.

- If $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$, then there are two equilibria: in one equilibrium the entry of E is deterred, while in the other I accommodates the entry of E.

- If $\left(\bar{S}^d\right)^{-1}(S_1) < S_2 \leq F$, then I accommodates the entry of E.

Intuitively, $\bar{S}_0^d$ is the minimum subsidy that G_2 must offer to E in order for E to consider entering when I is willing to expand its capacity, but it does not receive any support from G_1. Thus, if G_2 offers $S_2 \in \left(\bar{S}^b, \bar{S}_0^d\right)$, I can deter entry without the support of G_1. On the contrary, if G_2 offers $S_2 > \bar{S}_0^d$, entry deterrence requires the support of G_1; specifically, G_1 must offer $S_1 > \bar{S}^d(S_2) > 0$.

Equilibrium prices and quantities: It is useful to compare equilibrium prices under blocked entry, deterrence and accommodation.

Corollary 1 ***Economic equilibrium.*** *Under the assumptions in Proposition 1.*

1. *The equilibrium prices (aggregate quantity) under deterrence and accommodation are lower (higher) than under blocked entry.*

2. *The equilibrium price (aggregate quantity) under deterrence is lower (higher) than under accommodation if and only if*

$$S_2 > F - \frac{(a-c)^2}{64b} \in \left(\bar{S}^b, \bar{S}_0^d\right)$$

Thus, whenever $S_2 \geq \bar{S}_0^d$, the equilibrium price (aggregate quantity) under deterrence is lower (higher) than under accommodation.

Proof*: See Appendix A.1.* ■

The intuition behind Corollary 1.1 is clear. Under deterrence, I expands its capacity beyond the monopoly level in order to deter entry, which reduces the equilibrium price. Under accommodation, I does not expand its capacity, but the entry of E rises aggregate capacity, which also reduces the equilibrium price. Corollary 1.2 is more subtle. Suppose that $S_2 \in \left(\bar{S}^b, \bar{S}_0^d\right)$. Then, I can deter entry without the support of G_1. In such a case, the capacity expansion required to deter entry might not be enough to surpass the equilibrium quantity under accommodated entry. As S_2 increases, the deterrence capacity level rises and, eventually, it surpasses the aggregate capacity level under accommodation. Crucially, this occurs before I requires the support of G_1 to deter entry. Thus, whenever $S_2 > \bar{S}_0^d$, the deterrence capacity level is always greater than the aggregate capacity level under accommodation. Intuitively, for I is better to expand capacity up to the aggregate capacity level under accommodation if this is what it takes to deter entry, rather than accept entry and only supply a share of the aggregate capacity level under accommodation. In other words, even without the support of G_1, I is willing to expand capacity beyond the aggregate capacity level under accommodation in order to deter the entry of E.

3 Geopolitics and Political Deterrence

This section introduces geopolitical conflict between the global powers that use the strategic transportation service and characterize the equilibrium subsidies and corresponding capacity choices. In particular, suppose that before I and E play the economic deterrence game, global powers play an international influence game in which they simultaneously and independently select (S_1, S_2) and the payoff function of global power G_j is given by:

$$W_j(k_I, k_E) = CS_j(k_I, k_E) + B_j(k_I, k_E)$$

$CS_j(k_I, k_E) = \frac{A_j b^2 (k_I + k_E)^2}{2}$ is the consumer surplus enjoyed by country G_j, and $B_j(k_I, k_E)$ is the net geopolitical benefits for G_j (i.e., geopolitical benefits minus subsidies). Following the literature on the economics of conflict (e.g., Garfinkel and Skaperdas (2007)), geopolitical benefits are determined by a contest success function:

$$B_1(k_I, k_E) = \theta(k_I, k_E) B_1^M - (1 - \chi_{k_E > 0}) S_1 = \frac{(k_I)^m}{(k_I)^m + (k_E)^m} B_1^M - (1 - \chi_{k_E > 0}) S_1$$

$$B_2(k_E, k_I) = [1 - \theta(k_I, k_E)] B_2^M - \chi_{k_E > 0} S_2 = \frac{(k_E)^m}{(k_I)^m + (k_E)^m} B_2^M - \chi_{k_E > 0} S_2$$

where $m \in (0, 1]$ and $B_j^M > 0$ is the geopolitical benefit for global power G_j when its ally has an exclusive (i.e., monopolistic) control of the strategic transportation service. For the case of the Panama Canal, B_1^M (B_2^M) would be the geopolitical benefits for the United States (China) when Panama (Nicaragua) is the only available transoceanic canal in Central America. The contest success function $\theta(k_I, k_E)$ captures the geopolitical rivalry between the global powers. The greater (lower) the ratio of k_I/k_E the greater the geopolitical benefit of G_1 (G_2) the global power allied with the incumbent (entrant). Next, we discuss and motive the global powers' payoff functions.

Liberalism or Realism?: The payoff functions of the global powers encompass in a stylized fashion the perspectives on the goal of states supported by the two most influential schools of thought in international relations: liberalism and realism. While liberals often emphasize the importance of economic gains through international cooperation, realists focus on security dilemmas, relative positions and zero-sum games (e.g., Shiraev and Zubok (2015)). Since we assume that each global power values economic as well as geopolitical payoffs, our specification can handle both schools. Indeed, as geopolitical benefits rise (decrease), our payoff functions become more realists (liberal).

War, chocking points and international negotiations: Besides a realist perspective of international relations, there are several ways to motivate that geopolitical benefits depend on relative capacity levels installed by the incumbent and the potential entrant. Suppose that in case of war or escalation of geopolitical tensions, each global power must rely only on its ally to move military assets through this strategic transportation point. That is, global power G_1 can only use the incumbent (which has capacity k_I) and global power G_2 can only use the potential entrant (which has capacity k_E). Then, the ratio of capacity choices affects the ratio of military assets that each global power can deploy. Moreover, even during peacetime, the relative control of key chocking points might influence global powers' bargaining power in international negotiations, with the ratio k_I/k_E being a measure of G_1's control of the strategic transportation point relative to G_2.

Geopolitical rivalry? The specification of geopolitical benefits assumes a rival geopolitical relation between global powers G_1 and G_2, with G_1 interested in keeping its exclusive control of the strategic transportation service and G_2 interested in breaking this geopolitical monopoly. It is not difficult, however, to keep this specification while at the same time we relax how effective geopolitical rivalry is. Indeed, in two extensions (Sections 4.1 and 4.2) we explore environments in which the rising global power (i.e., G_2) poses limited or no geopolitical threat (for example, because it faces a budget constraint to subsidize E's entry). In another pair of extensions (Sections 4.3 and 4.4) we consider a rising global challenger (i.e., G_2) that faces no active reaction by the incumbent global power (i.e., G_1), and a situation strategically equivalent to a no-rivalry environment in which each global power is only interested in controlling a satellite state with the strategic transportation service, not whether the other global power controls one.

Timing of events for subsidy offers: We treat global powers symmetrically and assume that they simultaneously and independently select subsidies. In one of the extensions (Section 4.5), we relax this assumption and explore an environment in which the rising global power (i.e., G_2) moves first, which gives G_2 a first mover advantage. Note, however, that even if both global powers move simultaneously, the geopolitical payoffs favor the existing global power (i.e., G_1) given that its initial position is exclusive geopolitical control (i.e., $(k_I, k_E) = \left(\frac{a-c}{2b}, 0\right)$, which implies $\theta(k_I, k_E) = 1$) and its worst possible geopolitical outcome would be under accommodated entry (i.e., $(k_I, k_E) = \left(\frac{a-c}{2b}, \frac{a-c}{4b}\right)$, which implies $\theta(k_I, k_E) = \frac{(2)^m}{1+(2)^m} > \frac{1}{2}$). Thus, the geopolitical influence game resembles a defender-attacker interaction with a defense strategic advantage, G_1 playing the role of the defender and G_2 playing the role of the attacker. The model also admits a second potential asymmetry between global powers in the specification of the geopolitical payoffs, namely, that the value of having exclusive control of the strategic transportation service might be different for G_1 and G_2.

Geopolitical alliances and subsidies: The setting assumes that one global power is allied with the incumbent and the other with the potential entrant and each global power only provides subsidies to its ally. It is reasonable to assume that due to a long relationship between the existing global power and the incumbent, the rising global power cannot easily change the geopolitical alignment of the incumbent (e.g., China cannot convince Panama to become its satellite). This justifies the assumption that the rising global power only considers subsidizing the potential entrant. Still, it is not clear why the existing global power cannot approach the potential entrant (e.g., U.S. might consider subsidizing Nicaragua to build a canal). In one extension we explore this possibility (Section 4.6).

Geopolitical competition and market power: Finally, it is worth emphasizing how global powers compete in this model and differentiate it from other forms of geopolitical competition that often exacerbates market power. Consider, for example, colonial wars among European nations with the goal of imposing exclusive trading rights and other mercantilist restrictions to the colonized territory (Lopez Cruz and Torrens, 2022). The equivalent in the context of strategic transportation services would be that global powers fight to conquer a unique available location with access to the strategic service. On the contrary, the current setting implicitly assumes that alternative locations can be developed if properly subsidized, which opens the door to a pro-competitive effect of geopolitical competition. Alternatively, consider the US support to some international commodity cartels during the Cold War period. As shown by Galiani et al. (2022) this can be rationalized as a strategy to share the burden of containing the spread of communism with other commodity importers and escaping the free riding problem associated with international contributions to finance foreign aid. The equivalent in the context of strategic transportation services would be an existing global power that chooses to be lenient with an ally that monopolizes such a service to avoid its geopolitical realignment. Once again, the key difference is that in the current model, the rising power is subsidizing a new competitor in a different location rather than funding a regime change followed by a geopolitical realignment in the incumbent location.

3.1 Geopolitical Equilibrium

Equilibrium selection. To characterize equilibrium subsidies, it is useful to employ a selection criterion to deal with multiple economic equilibria for the knife edge situation in Proposition 1. Recall that when $S_1 = \bar{S}^d(S_2)$, deterrence and accommodation are both subgame perfect Nash equilibria (see Proposition 1.2.b). A convenient criterion is to assume that if $S_1 = \bar{S}^d(S_2)$, then the economic equilibrium will be accommodation when accommodation strictly dominates deterrence for G_2. Otherwise, the economic equilibrium will be deterrence. One advantage of this criterion is that G_2 always has a best response for any S_1.

The following lemma employs the results in Proposition 1 to rewrite geopolitical payoffs as a function of the profile of subsidies offered by the global powers.

Lemma 1 ***Geopolitical benefits***. *Suppose that Assumptions 1 and 2 hold. Then, geopolitical benefits are given by:*

$$B_1(S_1,S_2) = \begin{cases} B_1^M - S_1 & \text{if } \left[0 \le S_2 \le \bar{S}^b\right] \text{ or } \left[\bar{S}^b < S_2 \le F \text{ and } S_1 \ge \bar{S}^d(S_2)\right] \\ B_1^D & \text{if } \left[\bar{S}^b < S_2 \le F \text{ and } S_1 \le \bar{S}^d(S_2)\right] \end{cases}$$

$$B_2(S_1,S_2) = \begin{cases} 0 & \text{if } \left[0 \le S_2 \le \bar{S}^b\right] \text{ or } \left[\bar{S}^b < S_2 \le F \text{ and } S_1 \ge \bar{S}^d(S_2)\right] \\ B_2^D - S_2 & \text{if } \left[\bar{S}^b < S_2 \le F \text{ and } S_1 \le \bar{S}^d(S_2)\right] \end{cases}$$

where $B_1^D = \frac{2^m}{1+2^m}B_1^M$ *and* $B_2^D = \frac{1}{1+2^m}B_2^M$.[12] ***Proof***: *See Appendix A.2.* ■

B_1^M is the geopolitical benefits enjoyed by G_1 when there is no entry, i.e., under a monopoly, while B_1^D is the geopolitical benefits enjoyed by G_1 when there is entry, i.e., under a duopoly. Thus, $B_1^M - B_1^D > 0$ is the geopolitical benefits for G_1 of avoiding entry. Similarly, $B_2^D > 0$ is the geopolitical benefits enjoyed by G_2 when there is entry, while, under no entry, G_2 has no geopolitical benefits.

To characterize the equilibrium subsidies chosen by the global powers it is useful to define:

$$\Delta(S) = \frac{\left[a - c - 2\sqrt{b(F-S)}\right]^2}{2} - \frac{9(a-c)^2}{32} \tag{1}$$

$$\Delta W_1(S_1) = B_1^M - B_1^D + A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) - S_1 \tag{2}$$

$$\Delta W_2(S_2) = B_2^D - A_2\Delta(S_2) - S_2 \tag{3}$$

$A_j\Delta(S)$ is the change in the consumer surplus experienced by consumers of country j when the economic equilibrium moves from accommodation to deterrence (with G_2 offering entry subsidy S). Note that, due

[12] For $S_1 = \bar{S}^d(S_2)$ we employ the selection criterion. In particular, if accommodation strictly dominates deterrence for G_2, then $k_E > 0$ and, hence, $\theta(k_I, k_E) = \frac{2^m}{1+2^m}$. On the contrary, if deterrence strictly dominates accommodation for G_2, then $k_E = 0$ and, hence, $\theta(k_I, k_E) = 1$.

to Corollary 1, $\Delta(S) > 0$ for all $S \geq \bar{S}_0^d$. $\Delta W_1(S_1)$ is the net benefit of deterrence for G_1 when G_1 offers S_1 to deter entry and G_2 offers an entry subsidy $\left(\bar{S}^d\right)^{-1}(S_1)$. $\Delta W_2(S_2)$ is the net benefit of entry for G_2, when G_2 offers an entry subsidy S_2. In Appendix A.2 we prove that:

- $\Delta W_1(S_1)$ is strictly decreasing in S_1 for all $S_1 \in \left[0, \bar{S}^d(F)\right]$ if and only $A_1 b \leq 2\left(\sqrt{2}-1\right)$.
- $A_1\Delta(F) - \bar{S}^d(F) < 0$ if and only if $A_1 b < 4/7$.
- $\Delta W_2(S_2)$ is strictly decreasing in S_2 for all $S_2 \in \left[\bar{S}_0^d, F\right]$.

The first result states that the higher the subsidy that G_1 offers to deter entry, the lower the marginal benefit of deterrence. In other words, any extra dollar spent on subsidizing I generates a lower increase in the consumer surplus of G_1. Note that this result requires that $A_1 b \leq 2\left(\sqrt{2}-1\right) \approx 0.828$, where $A_1 b = Q_1/\sum_{j\in J} Q_j = A_1/\sum_{j\in J} A_j$ is the fraction of the strategic service demanded by G_1. The second result ensures that it is not necessary the case that $\Delta W_1(S_1) > 0$ for all $S_1 \in \left[0, \bar{S}^d(F)\right]$. For this result, the more stringent condition $A_1 b \leq 4/7 \approx 0.57$ is required. Finally, the third result states that the higher the entry subsidy that G_2 offers, the lower the marginal benefit of accommodation. The intuition is simple because the higher S_2, the lower the equilibrium price under deterrence and, hence, the higher the loss in consumer surplus when the economic equilibrium moves from deterrence to accommodation. To ensure that these results hold, we impose:

Assumption 3 *The market share of global power G_1 satisfies $A_1 b = A_1/\sum_{j\in J} A_j < 4/7$.*

The intuition behind this assumption is as follows. Besides the geopolitical benefits of deterring G_2, G_1 also obtains an economic benefit from the associated expansion in I's capacity. The reason is that consumers prefer to pay a lump sum to I in exchange for a reduction in the price of the strategic transportation service that would induce the same reduction in I's profits and the greater the fraction of the strategic service demanded by G_1, the greater this economic benefit. In particular, for $A_1 b > 4/7$, G_1 is willing to provide a subsidy to I that outbids $S_2 = F$ just on economic grounds.[13]

The following two propositions characterize the equilibrium subsidies. Proposition 2 states necessary and sufficient conditions for a profile of equilibrium subsidies to induce accommodation and deterrence, respectively. Proposition 3 further characterizes these conditions.

Proposition 2 ***Geopolitical equilibrium (conditions)***. *Suppose that Assumptions 1, 2, and 3 hold, $B_1^M - B_1^D \leq \bar{S}^d(F) - A_1\Delta(F)$ and $B_2^D \leq F + A_2\Delta(F)$. Then, the equilibrium subsidy profiles are those that satisfy:*

$$S_1 = \bar{S}^d(S_2), \; S_2 \in \left[\bar{S}_0^d, F\right] \tag{4}$$

[13] In one extension (see Section 4.2) we further elaborate the possibility that G_1 offers to subsidize I only on economic grounds.

and

$$[B_2^D > A_2\Delta(S_2) + S_2 \text{ and } B_1^M - B_1^D \leq S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)] \tag{5}$$

or

$$[B_2^D \leq A_2\Delta(S_2) + S_2 \text{ and } B_1^M - B_1^D \geq S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)] \tag{6}$$

Moreover, if (5) holds there is accommodated entry, while if (6) holds, entry is deterred. ***Proof****: See Appendix A.2.* ■

To see the intuition behind Proposition 2, we must understand the logic behind equations (4), (5) and (6).

Best response function of G_2: In equilibrium, it is always the case that $S_1 = \bar{S}^d(S_2)$ and $S_2 \in \left[\bar{S}_0^d, F\right]$. This is because $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ is the best response function of G_2. Figure 2 illustrates why this is the case. Panel a shows the payoff of G_2 as a function of S_2 for any $S_2 \in [0, F]$, while Panel b zooms in to the key range $S_2 \in \left[\bar{S}_0^d, F\right]$. The intuition is as follows. Given S_1, from Proposition 1, we know that if G_2 offers $S_2 \in \left[0, \left(\bar{S}^d\right)^{-1}(S_1)\right)$, then entry will be deterred, while if G_2 offers $S_2 \in \left(\left(\bar{S}^d\right)^{-1}(S_1), F\right]$, then there will be accommodation. Offering $S_2 \in \left[0, \left(\bar{S}^d\right)^{-1}(S_1)\right)$ is not a best response to S_1. With this offer E will not enter and, hence, G_2 will not need to pay any subsidy. However, the higher the subsidy offered by G_2, the greater the amount of capacity that I will need to install to deter E and, hence, the lower the equilibrium price. Formally, $W_2(S_1, S_2)$ is strictly increasing in S_2 for all $S_2 \in \left[0, \left(\bar{S}^d\right)^{-1}(S_1)\right)$. Offering $S_2 \in \left(\left(\bar{S}^d\right)^{-1}(S_1), F\right]$ is not a best response to S_1 either. With this offer, E will enter and, hence, G_2 will need to pay the subsidy; however, the equilibrium price under accommodation does not depend on S_2. Formally, $W_2(S_1, S_2)$ is strictly decreasing in S_2 for all $S_2 \in \left(\left(\bar{S}^d\right)^{-1}(S_1), F\right]$. Thus, the only remaining possibility is $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$. But are we sure that $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ is the best response function of G_2? In particular, note that $W_2(S_1, S_2)$ is not continuous at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ (see Figure 2.b). Our selection criterion, however, implies that $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ leads to the economic equilibrium that maximizes $W_2(S_1, S_2)$, which ensures that $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ is indeed the best response function of G_2.

Deterrence or accommodation? Does $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ lead to deterrence or accommodation? There are two possible situations to consider. Suppose that G_1 offers a relatively low subsidy (formally, S_1 such that $B_2^D > \left(\bar{S}^d\right)^{-1}(S_1) + A_2\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$). Then, $W_2(S_1, S_2)$ adopts its maximum at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ when there is accommodation (see Figure 2.b). On the other hand, suppose that G_1 offers a relatively high subsidy (formally, S_1 such that $B_2^D \leq \left(\bar{S}^d\right)^{-1}(S_1) + A_2\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$). Then, $W_2(S_1, S_2)$ adopts its maximum at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ when there is deterrence (see Figure 2.b). Intuitively, when G_1 offers a relatively low (high) subsidy, it is (not) worth it for G_2 to pay $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ in order to enjoy the geopolitical gains associated with E's entry. Summing up, in order for accommodation (deterrence) to be an equilibrium it must be the case that $B_2^D > S_2 + A_2\Delta(S_2)$ ($B_2^D \leq S_2 + A_2\Delta(S_2)$).

Best response of G_1: What about G_1's incentives? Considering the best response function of G_2, there are two types of candidates for equilibrium subsidy profiles. Any profile in which $S_1 = \bar{S}^d(S_2)$, $S_2 \in \left[\bar{S}_0^d, F\right)$ and $B_2^D > S_2 + A_2\Delta(S_2)$ hold, leads to accommodation. For those profiles, $B_1^M - B_1^D \leq S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$ ensures that G_1 does not have an incentive to unilaterally deviate to $S_1 < \bar{S}^d(S_2)$, which would lead to deterrence. Any profile in which $S_1 = \bar{S}^d(S_2)$, $S_2 \in \left[\bar{S}_0^d, F\right)$ and $B_2^D \leq S_2 + A_2\Delta(S_2)$ hold, leads to deterrence. For those profiles, $B_1^M - B_1^D \geq S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$ ensures that G_1 does not have an incentive to unilaterally deviate to $S_1 > \bar{S}^d(S_2)$, which would lead to accommodation. The intuition behind these inequalities is as follows. $B_1^M - B_1^D > 0$ is the geopolitical gain for G_1 associated with maintaining its geopolitical monopoly. To enjoy those benefits, G_1 must pay a subsidy of S_1 to the incumbent. Moreover, switching the economic equilibrium from accommodated entry to deterrence is valuable for G_1. Specifically, it induces a change in the consumer surplus of $A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$

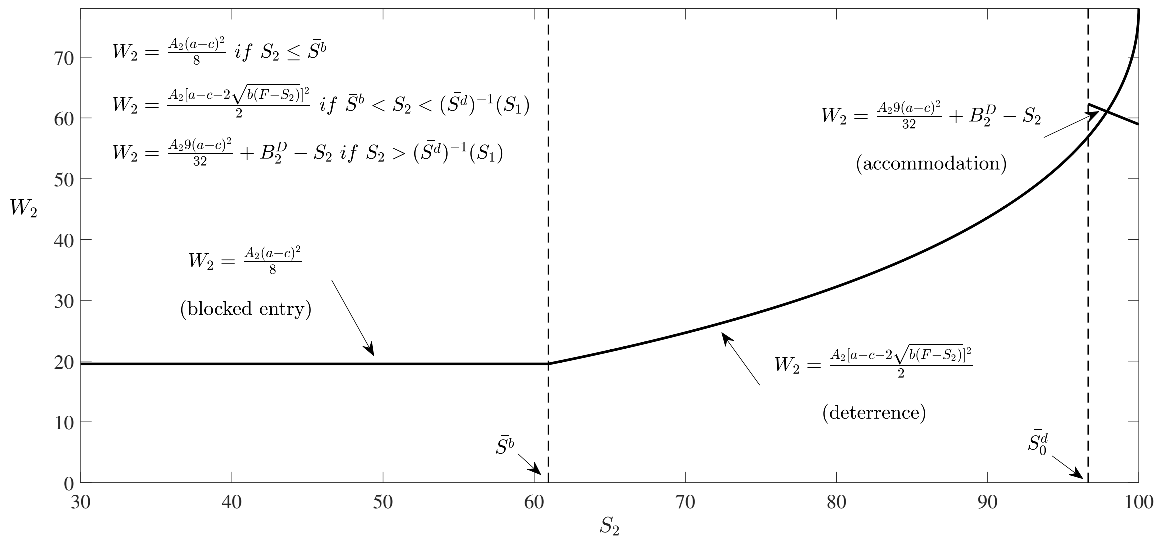

Figure 2.a. Geopolitical equilibrium. Notes: The figure has been plotted assuming $a = 3.75$, $b = 1/400$, $c = 2.5$, $F = 100$, $A_2 = 100$, and $B_2^D = 115$.

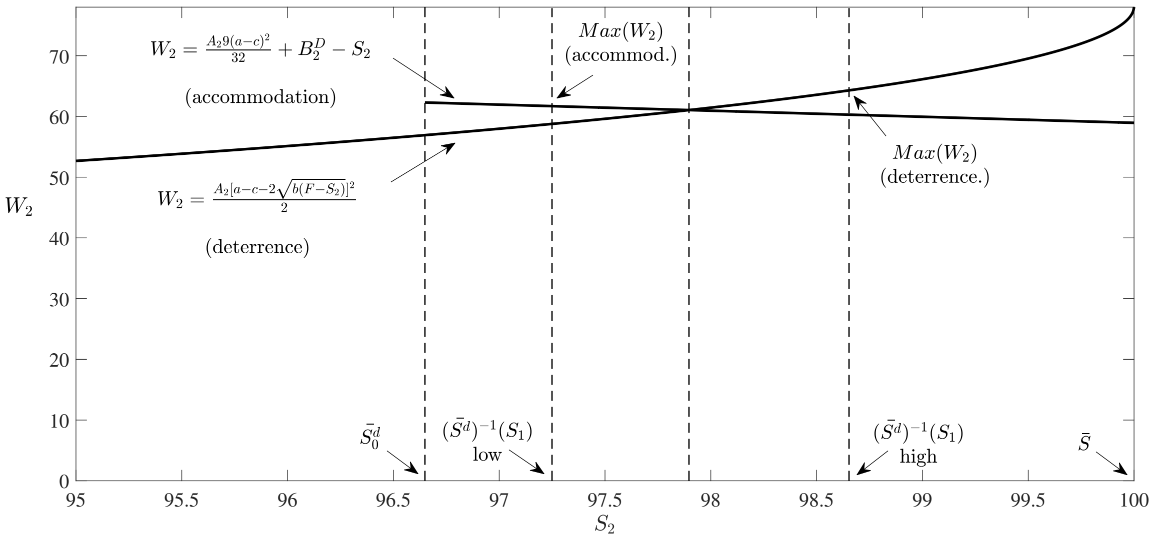

Figure 2.b. Geopolitical equilibrium. Notes: The figure has been plotted assuming $a = 3.75$, $b = 1/400$, $c = 2.5$, $F = 100$, $A_2 = 100$, and $B_2^D = 115$.

Proposition 3 ***Geopolitical equilibrium (subsidy race).*** *Suppose that Assumptions 1, 2, and 3 hold,* $B_1^M - B_1^D \leq \bar{S}^d(F) - A_1\Delta(F)$, *and* $B_2^D \leq F + A_2\Delta(F)$. *Let* $\tilde{S}_1 \in \left(0, \bar{S}^d(F)\right]$ *be the unique solution to:*

$$B_1^M - B_1^D = \tilde{S}_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) \tag{7}$$

For $B_2^D \leq \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$, *let* $\tilde{S}_2 = \bar{S}_0^d$, *while for* $B_2^D > \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$, *let* $\tilde{S}_2 \in \left(\bar{S}_0^d, F\right]$ *be the unique solution to:*

$$B_2^D = \tilde{S}_2 + A_2\Delta\left(\tilde{S}_2\right) \tag{8}$$

1. *If* $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, *then the equilibrium subsidy profiles are those that satisfy:* $S_1 = \bar{S}^d(S_2)$ *and* $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$. *Moreover, in all these equilibria entry is deterred.*

2. *If* $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, *then the equilibrium subsidy profiles are those that satisfy:* $S_1 = \bar{S}^d(S_2)$ *and* $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$. *Moreover, in all these equilibria there is accommodated entry.*

Proof: *See Appendix A.2.* ■

The intuition behind Proposition 3 is as follows: $\tilde{S}_1$ is the maximum subsidy that G_1 is willing to pay in order to deter entry. Indeed, (7) simply equates the geopolitical and economic benefits derived from deterrence (i.e., $B_1^M - B_1^D + A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$) with its economic costs (i.e., $\tilde{S}_1$). Since $A_1\Delta\left(\bar{S}_0^d\right) > 0$, we have $B_1^M - B_1^D > -A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(0)\right) = -A_1\Delta\left(\bar{S}_0^d\right)$ and, hence, it is always the case that $\tilde{S}_1 > 0$. In other words, G_1 is always willing to offer some subsidy to induce deterrence because for a small S_1, the increase in the consumer surplus of G_1 when the equilibrium changes from accommodation to deterrence is greater than S_1.

Similarly, $\tilde{S}_2$ is the maximum subsidy that G_2 is willing to pay in order to induce entry, with (8) equating the geopolitical benefits of entry (i.e., B_2^D) with its economic costs (i.e., $\tilde{S}_2 + A_2\Delta\left(\tilde{S}_2\right)$). Note, however, that $A_2\Delta\left(\bar{S}_0^d\right) > 0$ implies that that if $B_2^D \leq \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$, the maximum that G_2 is willing to offer is $\tilde{S}_2 = \bar{S}_0^d$. Intuitively, the geopolitical benefits for G_2 are not high enough to compensate for the minimum economic costs required to induce accommodated entry. On the contrary, if $B_2^D > \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$, G_2 is willing to offer up to $\tilde{S}_2 > \bar{S}_0^d$ in order to induce entry.

Finally, there are two possible equilibrium outcomes for this subsidy race. When $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, G_1 is willing to offer a subsidy higher than or equal to $\bar{S}^d\left(\tilde{S}_2\right)$ in order to deter entry, while G_2 is not willing to pay more than $\tilde{S}_2$ to induce entry. Then, G_1 outbids G_2 in the subsidy race and, in equilibrium, entry is always deterred. On the other hand, when $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, G_1 is not willing to offer more than $\bar{S}^d\left(\tilde{S}_2\right)$ in order to deter entry, while G_2 is willing to pay up to $\tilde{S}_2$ to induce entry. Then, G_2 outbids G_1 in the subsidy race and, in equilibrium, there is accommodated entry.

Several remarks regarding Propositions 2 and 3 are called for here.

Panama Canal: Propositions 2 and 3 suggest a simple but coherent explanation for the expansion of the Panama Canal. (See Section 5 for further details.) China threatened to support Nicaragua's effort to build a new canal, and Panama reacted by expanding its canal to deter entry. Does the United States need to subsidize the expansion of the Panama Canal in order for this to be an equilibrium? According to Propositions 2 and 3, not necessarily. Depending on the parameters of the model, $(S_1, S_2) = \left(0, \bar{S}_0^d\right)$ could be a Nash equilibrium that leads to deterrence. In particular, if $B_2^D \leq \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$, then, the maximum that G_2 (i.e., China) is willing to offer is $S_2 = \bar{S}_0^d$ and, hence, in equilibrium, $(S_1, S_2) = \left(0, \bar{S}_0^d\right)$. Thus, Panama deters Nicaragua's entry without the support of the United States. On the contrary, if $B_2^D > \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$, the maximum that G_2 is willing to offer to Nicaragua surpasses $\bar{S}_0^d$ and, therefore, Panama is not anymore willing to deter Nicaragua's entry unless it receives a subsidy from the United States.

Subsidies as credible promises: Consider the equilibria that induce deterrence. In those equilibria, G_2 does not actually pay any subsidy to E. G_2 just offers a subsidy, which triggers a response from G_1 and I, which move to overinvest in capacity to deter E's entry. Of course, this raises the question as to how credible G_2's offer to subsidize the entrance of E actually is. The model implicitly assumes that S_2 is fully credible, but it is not difficult to envision situations in which G_2 must at least incur some cost in order to signal its commitment. Similarly, in the equilibria that induce accommodated entry, G_1 does not actually pay any subsidy to I, but the subsidy promised by G_1 is not completely irrelevant either. Indeed, the higher S_1, the more generous S_2 needs to be in order to induce E's entrance. In Section 4 we consider several extensions in which global powers face a budget constraint and, hence, they cannot credibly promise any subsidy.

Pro-competitive economic effect of geopolitical competition (no blocked entry in equilibrium): Note that if we impose that $S_2 \leq \bar{S}^b$, then the entry of E is blocked (Proposition 1.1). However, this never occurs when global powers are allowed to endogenously select their subsidies. Moreover, from Corollary 1, we know that the equilibrium prices (aggregate quantity) under deterrence and accommodation are lower (higher) than under blocked entry. Thus, G_2's geopolitical challenge (i.e., its willingness and commitment to support E's entry) has a pro-competition economic effect (i.e., lower equilibrium price), which benefits consumers all over the world (including consumers who are not associated with any global power), an example of good economic outcomes resulting from political competition. That geopolitical competition averts blocked entry does not necessarily imply that more geopolitical competition always leads to lower equilibrium prices, something that we explore in the following comparative statics exercises.

3.2 Comparative Statics Analysis

We explore how geopolitical factors affect the equilibrium. The following proposition summarizes the results.

Proposition 4 ***Comparative statics.*** *Suppose that Assumptions 1, 2, and 3 hold,* $B_1^M - B_1^D < \bar{S}^d(F) - A_1\Delta(F)$, *and* $B_2^D \in \left(\bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right), F + A_2\Delta(F)\right]$. *Then,* $\tilde{S}_1 \in \left(0, \bar{S}^d(F)\right)$ *and* $\tilde{S}_2 \in \left(\bar{S}_0^d, F\right)$. *Moreover,* $\tilde{S}_1$ $(\tilde{S}_2)$ *is strictly increasing in* $B_1^M - B_1^D$ (B_2^D); *and* $\tilde{S}_1$ *and* $\tilde{S}_2$ *are both strictly increasing in* F. ***Proof****: See Appendix A.2.* ■

How do geopolitical benefits affect equilibrium subsidies and, ultimately, the entry decision?

Change in $B_1^M - B_1^D$: An increase in $B_1^M - B_1^D$ makes G_1 more willing to pay a higher subsidy in order to deter entry. Formally, $\tilde{S}_1$ is strictly increasing in $B_1^M - B_1^D$. If it was initially the case that $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, then a rise in $B_1^M - B_1^D$ does not affect the nature of the equilibrium, i.e., before as well as after the increase in $B_1^M - B_1^D$ entry is deterred. However, the rise in $B_1^M - B_1^D$, increases the maximum equilibrium subsidy offered by G_2, which decreases the lowest possible equilibrium price. Thus, the rise in $B_1^M - B_1^D$ opens the way for improving the situation for consumers all over the world.[14] On the other hand, if it was initially the case that $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, then a marginal rise in $B_1^M - B_1^D$ does not affect the nature of the equilibrium. Before, as well as after, the increase in $B_1^M - B_1^D$, there is accommodated entry. Moreover, since, under accommodated entry, neither capacity choices nor the equilibrium price depend on the subsidies, a marginal rise in $B_1^M - B_1^D$ has no effect on the well-being of consumers. Starting from $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, a sufficiently large rise in $B_1^M - B_1^D$ reverses this inequality and, hence, the equilibrium changes from accommodated entry to deterrence. Since the equilibrium price under deterrence is always lower than under accommodated entry (recall Corollary 1), this large rise in $B_1^M - B_1^D$ makes consumers all over the world better off. Summing up, a rise in the geopolitical benefits of G_1 has a positive effect on the well-being of consumers all over the world. The following corollary summarizes the results:

Corollary 2 ***Change in*** $B_1^M - B_1^D$. *Under the assumptions in Proposition 4.*

1. *Suppose that before and after an increase in* $B_1^M - B_1^D$ *we have* $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$. *Then, deterrence persists and a new range of equilibrium subsidy profiles is added in which consumers are better off.*

[14] Since there are multiple equilibrium subsidy profiles, we cannot state that consumers will be better off after the increase in $B_1^M - B_1^D$. More formally, every equilibrium subsidy profile before the rise in $B_1^M - B_1^D$ will also be an equilibrium subsidy profile after the rise in $B_1^M - B_1^D$. In addition, after the rise in $B_1^M - B_1^D$, there will be a new range of equilibrium subsidy profiles with higher S_2 than in the equilibrium subsidy profiles before the rise in $B_1^M - B_1^D$.

2. *Suppose that before and after an increase in* $B_1^M - B_1^D$ *we have* $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$. *Then, accommodated entry persists and there is no effect on consumers.*

3. *Suppose that before an increase in* $B_1^M - B_1^D$ *we have* $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$ *and after we have* $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$. *Then, the equilibrium changes from accommodated entry to deterrence and consumers are better off.*

Proof: *Immediate from Propositions 3, 4 and Corollary 1.* ■

Change in B_2^D: An increase in B_2^D makes G_2 more willing to pay a higher subsidy in order to induce entry. Formally, $\tilde{S}_2$ is strictly increasing in B_2^D. If it was initially the case that $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, a rise in B_2^D does not affect the nature of the equilibrium. Before, as well as after, the increase in B_2^D, there is accommodated entry. Moreover, since, under accommodated entry, neither capacity choices nor the equilibrium price depend on the subsidies, a rise in B_2^D has no effect on the well-being of consumers. On the other hand, if it was initially the case that $\tilde{S}_1 > \bar{S}^d\left(\tilde{S}_2\right)$, then a marginal a rise in B_2^D does not affect the nature of the equilibrium. Before, as well as after, the increase in B_2^D, there is deterrence. However, this marginal rise in B_2^D increases the minimum equilibrium subsidy offered by G_2, which reduces the highest possible equilibrium price. Thus, the rise in B_2^D opens the way for improving the situation for consumers.[15] Starting from $\tilde{S}_1 > \bar{S}^d\left(\tilde{S}_2\right)$, a sufficiently large rise in B_2^D makes $\tilde{S}_2$ greater than or equal to $\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ and, hence, the equilibrium changes from deterrence to accommodated entry. Since the equilibrium price under accommodated entry is always higher than under deterrence (recall Corollary 1), this change unambiguously makes consumers worse off. Summing up, a rise in the geopolitical benefits of G_2 has an ambiguous effect on he well-being of consumers. The following corollary summarizes the results:

Corollary 3 ***Change in*** B_2^D. *Under the assumptions in Proposition 4.*

1. *Suppose that before and after an increase in* B_2^D *we have* $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$. *Then, accommodated entry persists and there is no effect on consumers.*

2. *Suppose that before and after an increase in* B_2^D *we have* $\tilde{S}_1 > \bar{S}^d\left(\tilde{S}_2\right)$. *Then, deterrence persists and a range of equilibrium subsidy profiles is added in which consumers are better off.*

3. *Suppose that before an increase in* B_2^D *we have* $\tilde{S}_1 > \bar{S}^d\left(\tilde{S}_2\right)$ *and after we have* $\tilde{S}_1 \leq \bar{S}^d\left(\tilde{S}_2\right)$. *Then, the equilibrium changes from deterrence to accommodated entry and consumers are worse off.*

Proof: *Immediate from Propositions 3, 4 and Corollary 1.* ■

[15] Since there are multiple equilibrium subsidy profiles, we cannot state that consumers will be better off after the increase in B_2^D. More formally, the rise in B_2^D eliminates a range of equilibrium subsidy profiles with the lowest S_2 and, hence, the highest equilibrium prices.

Change in F: A rise in F makes both global powers more willing to pay a higher subsidy. Formally, $\tilde{S}_1$ and $\tilde{S}_2$ are both strictly increasing in F. The intuition behind this result is as follows. Consider the economic and geopolitical calculus of G_1. The geopolitical benefits derived from deterrence (i.e., $B_1^M - B_1^D$) are not affected by a change in F, while its economic costs (i.e., $\tilde{S}_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$) decrease with a rise in F. This might seem counterintuitive given that $A_1\Delta = A_1\left\{\frac{\left[a-c-2\sqrt{b(F-S)}\right]^2}{2} - \frac{9(a-c)^2}{32}\right\}$ is decreasing in F (as the consumer surplus obtained by G_1 under deterrence decreases with F, while the consumer surplus obtained by G_1 under accommodated entry is not affected by F). However, F also influences $\left(\bar{S}^d\right)^{-1}$. Indeed, an increase in F leads to a decrease in $\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ and this 'indirect' change dominates the direct effect of F on Δ. Thus, a higher F leads to a higher $\tilde{S}_1$.

For G_2, neither the geopolitical benefits of entry (i.e., B_2^D) nor its cost (i.e., $\tilde{S}_2$) are affected by F, while the economic costs of entry (i.e., $A_2\Delta\left(\tilde{S}_2\right)$) increase with F as the consumer surplus obtained by G_2 under accommodated entry is not affected by F while the consumer surplus obtained by G_2 under deterrence decreases with F. Thus, a higher F leads to a higher $\tilde{S}_2$.

What about the nature of the equilibrium? It is easy to verify that if it was initially the case that $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, then this inequality will also hold after a rise in F. Thus, if before the rise in F there was deterrence, there will also be deterrence after the rise in F. Moreover, it is also possible to prove that a change in F has an ambiguous impact on the well-being of consumers. (See Appendix A.2 for details). If, on the contrary, it was initially the case that $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, we must distinguish two possible situations. First, if after the rise in F it is still the case that $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, then there is accommodated entry before as well as after the change in F. Since under accommodated entry, neither capacity choices nor the equilibrium price depend on the subsidies or the entry cost, a rise in F has no effect on the well-being of consumers. Second, if after the rise in F we have $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, then the equilibrium changes from accommodated entry to deterrence, making consumers better off.

Corollary 4 ***Change in*** F. *Under the assumptions in Proposition 4.*

1. *Suppose that before the increase in* F *we have* $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$. *Then, the increase in* F *has an ambiguous effect on consumers*[16] *and no effect on geopolitical outcomes.*

2. *Suppose that before and after the increase in* F *we have* $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$. *Then, the increase in* F *has no effect on consumers or geopolitical outcomes.*

3. *Suppose that before the increase in* F *we have* $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$ *and after we have* $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$. *Then, the equilibrium changes from accommodated entry to deterrence, making consumers better off.*

[16] More precisely, the increase in F induce a new range of equilibria with higher equilibrium price than the highest equilibrium price before the increase in F, but also induces a new range of equilibria with lower equilibrium price than the lowest equilibrium price before the increase in F.

Proof: *See Appendix A.2.* ■

4 Extensions

This section explores six extensions. First, we consider a scenario in which G_2 faces a budget constraint to subsidize E and, hence, it cannot fully commit to support the entrant. That is, we study an environment with limited geopolitical threat from the global ally of the entrant. Second, we explore a scenario in which G_2 poses no geopolitical threat at all but G_1 can offer a subsidy to expand I's capacity due to economic concerns even when there is no possibility that E enters. These extensions allow us to build reasonable counterfactuals to compare with the equilibria in Proposition 3 and, hence, better appreciate the role played by geopolitics. Third, we study a scenario in which G_1 cannot subsidize the incumbent and, hence, G_2 can rise unchallenged. This extension is interesting because it allows us to isolate the pure effect of a rising global power willing to subsidize entry and compare it with a situation in which there is effective geopolitical rivalry between G_1 and G_2. Fourth, we explore a scenario in which G_1 has no geopolitical interest in subsidizing I, which we interpret as a situation of no geopolitical rivalry in the sense that each global power is merely interested in controlling a satellite state with the strategic transportation service, not whether the other global power controls one or not. Fifth, we explore a variation in the sequence of play in which the rising global power moves first, which eliminates an odd feature of the contract offered by G_1, namely, a payment conditional on an action taken by the potential entrant rather than the incumbent itself. Finally, we consider an environment in which the incumbent global power can also approach the entrant.

4.1 Limited Geopolitical Threat from G_2

An important assumption in Section 3 is that global powers do not face any relevant financial constraint to support their respective allies. Formally, we have assumed that $S_1 \geq 0$ and $S_2 \in [0, F]$. This is critical even when, in equilibrium, one of the global powers does not actually pay any subsidy. The reason is that only genuine subsidy offers can influence economic decisions. Next, we explore a scenario in which the rising global power faces a financial constraint that affects its ability to compete in the subsidy race.

Suppose that G_2 has a limited budget to subsidize E's entry $S_2 \leq \rho F$ with $\rho \in [0, 1]$. That is, for $\rho = 0$, G_2 is totally incapable of supporting E, while for $\rho = 1$, we return to the scenario in Section 3 in which G_2 can fully afford E's entry cost. It is not difficult to see how this budget constraint affects Proposition 1. (See Appendix A.3 for details). When $\rho \in \left[\bar{\rho}_0^d, 1\right]$, where $\bar{\rho}_0^d = \bar{S}_0^d / F$, it is possible for entry to be blocked, deterred or accommodated depending on S_1 and S_2, as it is the case in Proposition 1. Thus, for $\rho \in \left[\bar{\rho}_0^d, 1\right)$, G_2's budget constraint has no major impact on Proposition 1. When $\rho \in \left(\bar{\rho}^b, \bar{\rho}_0^d\right)$, where $\bar{\rho}^b = \bar{S}^b / F$, G_2 can only promise to pay an amount lower than $\bar{S}_0^d$, which could be enough to induce I to increase its capacity to deter entry, but it will never be enough to induce accommodation. In other words, for intermediate values of ρ, entry will be either blocked or deterred. Finally, when $\rho \in [0, \bar{\rho}]$, entry is always blocked for all values of S_2. Intuitively, with a low enough ρ, G_2 can only offer to pay an amount lower than $\bar{S}^b$, which is never enough to induce the incumbent to deter entry or to induce an accommodated entry.

The following proposition characterizes the Nash equilibrium subsidies chosen by the global powers for different values of ρ.

Proposition 5 ***Limited geopolitical threat from*** G_2. *Suppose that Assumptions 1, 2, and 3 hold and* $S_2 \leq \rho F$. *Let* $\bar{\rho}^b = \frac{\bar{S}^b}{F}$ *and* $\bar{\rho}_0^d = \frac{\bar{S}_0^d}{F}$.

1. *Suppose that* $\bar{\rho}_0^d \leq \rho < 1$, $B_1^M - B_1^D \leq \bar{S}^d(F) - A_1\Delta(F)$ *and* $B_2^D \leq F + A_2\Delta(F)$.
   (a) *If* $\tilde{S}_1 \geq \bar{S}^d(\rho F)$, *then equilibrium subsidies are* $S_1 = \bar{S}^d(\rho F)$ *and* $S_2 = \rho F$. *Moreover, in equilibrium entry is deterred.*
   (b) *If* $\tilde{S}_1 < \bar{S}^d(\rho F)$ *and* $\tilde{S}_2 \leq \rho F$, *then Proposition 3 holds.*
   (c) *If* $\tilde{S}_1 < \bar{S}^d(\rho F)$ *and* $\tilde{S}_2 > \rho F$, *then equilibrium subsidies are* $S_1 = \bar{S}^d(S_2)$ *and* $S_2 \in \left[(\bar{S}^d)^{-1}\left(\tilde{S}_1\right), \rho F\right)$. *Moreover, in all these equilibria there is accommodated entry.*
2. *Suppose that* $\bar{\rho}^b < \rho < \bar{\rho}_0^d$. *Then, equilibrium subsidies are* $S_1 = 0$ *and* $S_2 = \rho F$. *Moreover, in equilibrium, entry is deterred.*
3. *Suppose that* $0 \leq \rho \leq \bar{\rho}^b$. *Then, equilibrium subsidies are* $S_1 = 0$ *and* $S_2 \in [0, \rho F]$. *Moreover, in equilibrium, entry is blocked.*

Proof: *See Appendix A.3.* ■

Proposition 5.1 is similar to Proposition 3. In equilibrium, entry is deterred when global power G_1 wins the subsidy race, and there is accommodated entry when global power G_2 wins the subsidy race. The difference is that while in Proposition 3 the winner is the global power that is willing to go further in the subsidy race, now G_2 faces a budget constraint that restricts how much it can offer to E. As a consequence, if G_1 is willing to offer $S_1 \geq \bar{S}^d(\rho F)$ (formally, if $\tilde{S}_1 \geq \bar{S}^d(\rho F)$), then there is nothing that G_2 can do to induce entry. In equilibrium, entry is deterred, even when G_2 would be willing to outbid G_1, (formally, even if $\tilde{S}_2 > (\bar{S}^d)^{-1}\left(\tilde{S}_1\right)$). The problem is that G_2 cannot offer its willingness to pay to induce entry. When G_1 is not willing to offer $S_1 \geq \bar{S}^d(\rho F)$ (formally, when $\tilde{S}_1 < \bar{S}^d(\rho F)$), then there are two possible situations. If G_2's budget constraint is not binding (formally, if $\tilde{S}_2 \leq \rho F$), then Proposition 3 still holds. All that matters is the global players' willingness to pay to deter or to induce entry. If G_2's budget constraint is binding (formally, if $\tilde{S}_2 > \rho F$), then it must be the case that G_2 is willing to and capable of outbidding G_1. Then, in equilibrium, there is accommodated entry. The only difference with Proposition 3 is that now $S_2 \in \left[(\bar{S}^d)^{-1}\left(\tilde{S}_1\right), \rho F\right)$ instead of $S_2 \in \left[(\bar{S}^d)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$.

Table 1 summarizes the differences between Proposition 5.1 and Proposition 3. First, suppose that $\bar{S}^d(\rho F) \leq \tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$. Then, when the maximum subsidy that G_2 can offer is F, there is accommodated entry, while when the maximum subsidy that G_2 can offer is ρF, there is deterrence. In other words, G_2's budget constraint changes the nature of the equilibrium outcome (from accommodation to deterrence). This induces a drop in the equilibrium price, which makes consumers better off. Second, suppose that $\bar{S}^d(\rho F) \leq \bar{S}^d\left(\tilde{S}_2\right) \leq \tilde{S}_1$. Then, G_2's budget constraint does not change the nature of the equilibrium outcome (i.e., with or without it there is deterrence). However, the equilibrium price is higher when G_2 faces a budget constraint because it is forced to bid a subsidy lower than its willingness to pay to induce entry (formally, $\bar{S}^d(\rho F) \leq \bar{S}^d\left(\tilde{S}_2\right)$). In this case, G_2's budget constraint negatively affects consumers. Finally, suppose that $\tilde{S}_1 < \bar{S}^d(\rho F) < \bar{S}^d\left(\tilde{S}_2\right)$. Then, G_2's budget constraint does not change the nature of the equilibrium outcome (i.e., with or without it, there is accommodated entry). However, when G_2 faces a budget constraint, there is a lower maximum subsidy that G_2 pays to support entry. Since, under accommodation, subsidies do not change the equilibrium price, consumers are not affected.

	Maximum S_2		
Situation	F (Proposition 3) $S_1 = \bar{S}^d(S_2)$ and $S_2 \in$	ρF (Proposition 5) $S_1 = \bar{S}^d(\min\{S_2, \rho F\})$ and $S_2 \in$	Main effects of G_2's budget constraint
$\bar{S}^d(\rho F) \leq \tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$	$\left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$ (accommodation)	ρF (deterrence)	- From accommodation to deterrence - Lower price
$\bar{S}^d(\rho F) \leq \bar{S}^d\left(\tilde{S}_2\right) \leq \tilde{S}_1$	$\left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$ (deterrence)	ρF (deterrence)	- Lower S_1 - Higher price
$\bar{S}^d\left(\tilde{S}_2\right) \leq \tilde{S}_1 < \bar{S}^d(\rho F)$	$\left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$ (accommodation)	$\left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$ (accommodation)	- No effect
$\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right) \leq \bar{S}^d(\rho F)$	$\left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$ (deterrence)	$\left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$ (deterrence)	- No effect
$\tilde{S}_1 < \bar{S}^d(\rho F) < \bar{S}^d\left(\tilde{S}_2\right)$	$\left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$ (accommodation)	$\left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \rho F\right)$ (accommodation)	- Lower S_2

Table 1: Effect of G_2's budget constraint when $\bar{\rho}_0^d \leq \rho < 1$.

Proposition 5.2 brings about new results. For $\bar{\rho}^b < \rho < \bar{\rho}_0^d$, G_2 can only offer an amount lower than $\bar{S}_0^d$, which implies that E will not enter, even when $S_1 = 0$. This does not imply that there is no room for strategic subsidies, however. In particular, to induce the incumbent to expand its capacity, G_2 has an incentive to offer the highest possible subsidy to E (i.e., $S_2 = \rho F$). On the other hand, G_1 does not need to offer any subsidy to induce deterrence. Thus, in equilibrium, $S_1 = 0$, $S_2 = \rho F$ and entry is deterred. In the context of the Panama Canal, this would be a scenario where China, by promising to support Nicaragua, forces Panama to expand its capacity without the need for any subsidy from the United States. Compared with Proposition 3, now G_2' budget constraint has a more radical impact on the equilibrium outcome. For $\bar{\rho}^b < \rho < \bar{\rho}_0^d$, G_2 does not pose any geopolitical threat for G_1. This is because there is no promise that G_2 can make that will induce E to enter. Moreover, in economic terms, G_1 benefits from G_2's support to E because it forces I to increase its capacity, which reduces the equilibrium price of the transportation service. Indeed, it is easy to verify that when $\bar{\rho}^b < \rho < \bar{\rho}_0^d$, the payoffs for both global powers are increasing in ρ. Thus, this is a situation in which the United States would prefer that China's budget constraint is relaxed up to $\rho < \bar{\rho}_0^d$.

Proposition 5.3 also brings about novel results. For $0 \leq \rho \leq \bar{\rho}^b$, regardless of the subsidy offered by G_2, entry will be blocked. Then, G_1 does not have any incentives to offer a positive subsidy. G_2, on the other hand, is indifferent to any subsidy because, given its limited budget, G_2's offer will not affect capacity decisions. In the context of the Panama Canal, this would be a scenario in which China lacks the resources to push Panama to expand its canal and, hence, entry remains blocked. Once again, this is not a good outcome for the global powers. Both would be better off if China's budget constraint is relaxed and I were forced to increase its capacity in order to deter entry.

Summing up, except when $\bar{S}^d(\rho F) \leq \tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, consumers are weakly better off when G_2 does not face a budget constraint that limits its ability to bid in the subsidy race. In other words, the only possible situation in which restricting G_2's budget constraint has a pro-competition economic effect is when G_2 would outbid G_1 without any budget constraint (formally, $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$) but the opposite is true when G_2 can only offer up to ρF (formally, $\bar{S}^d(\rho F) \leq \tilde{S}_1$). The reason is that in such circumstances, G_2's budget constraint will induce a switch from accommodation to deterrence and, as we have already seen (Corollary 1), the equilibrium price (aggregate quantity) under deterrence is always lower (higher) than under accommodation.

4.2 No Geopolitical Threat from G_2 and G_1 Subsidizes I's Capacity Expansion

Propositions 5.2 and 5.3 suggests that even when G_2 is not capable of supporting E's entry, G_1 might benefit from an expansion in the capacity of I beyond the monopoly level. However, lacking any geopolitical threat, in equilibrium, G_1 offers $S_1 = 0$, which induces I to select $k_I = \bar{k}^m$. The reason is that G_1's subsidy is a geopolitically motivated payment to avoid E's entry (i.e., when $k_E = 0$). Thus, when there is no entry threat, G_1 has no incentive to offer any subsidy to avoid E's entry. Next, we allow G_1 to offer a subsidy to expand I's capacity even when there is no chance that E enters.

As we have assumed in the previous subsection, suppose that G_2 has a limited budget ρF to subsidize E's entry. Moreover, assume that $0 \leq \rho \leq \bar{\rho}^b$. But, now suppose that G_1 can make any offer $\left(S_1, \bar{k}_I\right)$ of the form:

$$S_1(k_I) = \chi_{\bar{k}_I} S_1, \text{ where } \chi_{\bar{k}_I} = 1 \text{ if } k_I \geq \bar{k}_I \geq \bar{k}^m \text{ and } \chi_{\bar{k}_I} = 0 \text{ if } k_I < \bar{k}_I \tag{9}$$

For $\bar{k}_I = \bar{k}^d$, (9) is identical to the offer in Section 3, that is, G_1 offers S_1 if entry is deterred and 0, otherwise. However, (9) is more general than the subsidy schedule employed in Section 3; in particular, it allows G_1 to offer a subsidy to expand I's capacity even if $k_I = \bar{k}^m$ is enough to avert E's entry.

Suppose that G_1 makes an offer $\left(S_1, \bar{k}_I\right)$. If I accepts this offer and selects capacity $k_I = \bar{k}_I$, its profits will be $\pi_I\left(\bar{k}_I\right) = \left(a - b\bar{k}_I - c\right)\bar{k}_I + S_1$. On the contrary, if I rejects this offer and, provided that Assumption 2 holds, I will select the monopoly capacity level and, hence, its profits will be $\pi_I\left(\bar{k}^m\right) = \left(a - b\bar{k}^m - c\right)\bar{k}^m = \frac{(a-c)^2}{4b}$. Since for all $\bar{k}_I \geq \bar{k}^m$, $\pi_I\left(\bar{k}_I\right)$ is strictly decreasing in $\bar{k}_I$, if G_1 wants to induce capacity level $\bar{k}_I$ it must offer at least $S_1 = \frac{(a-c)^2}{4b} - \left(a - b\bar{k}_I - c\right)\bar{k}_I$. Thus, G_1's problem becomes:

$$\max_{S_1 \geq 0, \bar{k}_I \geq \bar{k}^m} \left\{W_1 = CS_1\left(\bar{k}_I\right) - S_1\right\}$$
$$s.t.\text{: } S_1 = \frac{(a-c)^2}{4b} - \left(a - b\bar{k}_I - c\right)\bar{k}_I$$

where $CS_1\left(\bar{k}_I\right) = \frac{A_1 b^2 \left(\bar{k}_I\right)^2}{2}$. Solving, we obtain:

$$S_1 = \left[\frac{(2 - A_1 b)^2 - 4(1 - A_1 b)}{4(2 - A_1 b)^2 b}\right](a-c)^2 > 0,\ \bar{k}_I = \frac{a-c}{(2 - A_1 b)b} > \bar{k}^m$$

The reason G_1 is interested in subsidizing the expansion of I's capacity level is that consumers prefer to pay a lump sum to I in exchange for a reduction in the price of the strategic transportation service that would induce the same drop in I's profits. Put in another way, if a lump sum tax is employed, consumers are willing to compensate I for the loss in profits associated with a capacity expansion beyond the monopoly level. How far G_1 is willing to go depends on the fraction of the strategic service it demands. The higher $A_1 b$, the greater G_1's consumers benefit from a reduction in the price and, hence, the greater the subsidy that they are willing to pay and capacity expansion they are willing to finance. Formally, S_1 and $\bar{k}_I$ are both strictly increasing in $A_1 b$.

The following proposition summarizes the results and compares $\bar{k}_I$ with the aggregate quantity under accommodation and deterrence.

Proposition 6 ***No geopolitical threat from G_2 and G_1 subsidizes I's capacity expansion.*** *Suppose that Assumptions 1 and 2 hold, $S_2 \leq \rho F$ with $\rho \in \left[0, \bar{\rho}^b\right]$, and G_1 can offer any contract of the form (9). Then, G_1 offers $S_1 = \left[\frac{(2-A_1 b)^2 - 4(1-A_1 b)}{4(2-A_1 b)^2 b}\right](a-c)^2$ and $\bar{k}_I = \frac{a-c}{(2-A_1 b)b}$; I accepts it and selects $k_I = \bar{k}_I$; and E selects $k_E = 0$. Moreover:*

1. *$\bar{k}_I < \frac{3(a-c)}{4b}$ (i.e., the aggregate quantity under accommodation) if and only if $A_1 b < 2/3$.*
2. *$\bar{k}_I = \frac{a-c}{(2-A_1 b)b} < \frac{(a-c)}{2b}\left(1 + \frac{\sqrt{2}}{2}\right)$ (i.e., the minimum aggregate quantity under deterrence in Proposition 3) if and only if $A_1 b < 2\left(\sqrt{2} - 1\right)$.*

Proof: *See Appendix A.3.* ■

In Sections 2 and 3, our implicit counterfactual has been that under no geopolitical threat from G_2, entry will be blocked; more precisely, in equilibrium, $k_I = \frac{a-c}{2b}$ and $k_E = 0$. When compared against this counterfactual, entry deterrence and accommodation (the equilibrium under geopolitical competition in Proposition 3) always induces higher aggregate quantities and lower equilibrium prices. Thus, compared to a situation with no geopolitical threat, geopolitics has a pro-competition effect. Proposition 6 allows us to consider an alternative counterfactual under no geopolitical threat from G_2 in which G_1 subsidizes I to expand capacity beyond the monopoly level for economic rather than geopolitical reasons. Clearly, this alternative counterfactual is considered, the economic benefits of geopolitical competition are lower, given that $\bar{k}_I > \frac{a-c}{2b}$. Note, however, that it is still the case that the equilibrium in Proposition 3 induces higher aggregate quantities and, hence, lower equilibrium prices than the equilibrium in Proposition 6. More formally, for the aggregate quantity under accommodation to be greater than $\bar{k}_I$ we require $A_1 b < 2/3$, while for the minimum aggregate quantity under deterrence (i.e., k^d for $S_2 = \bar{S}_0^d$) to be greater than $\bar{k}_I$ we only need to impose that $A_1 b < 2\left(\sqrt{2} - 1\right) \approx 0.828$. Moreover, both conditions are weaker than Assumption 3.

Summing up, if the benchmark we employ to compute the impact of geopolitical competition is a counterfactual in which the established global power is already subsidizing the expansion of k_I beyond the monopoly level for economic reasons, it is still the case that geopolitical competition has a pro-competition economic effect.

4.3 Unchallenged Rising Power

As we have already shown (see the discussion immediately after Proposition 3), the incumbent global power (i.e., G_1) is always willing to offer some subsidy to induce deterrence. Formally, it is always the case that $\tilde{S}_1 > 0$. This implies that in order to induce entry, G_2 must outbid G_1 in the subsidy race. On the contrary, in this subsection we explore a scenario in which G_2 does not face any effective reaction by the incumbent global power. In other words, when G_2 can rise unchallenged. To do so, assume that $S_1 = 0$. That is, G_1 is not allowed to participate and/or capable of participating in the subsidy race. This scenario is interesting because it isolates the pure effect of a rising power willing to subsidize entry.

Proposition 7 ***Unchallenged rising power***. *Suppose that Assumptions 1 and 2 hold, $S_1 = 0$ and $B_2^D \in \left(\bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right), F + A_2\Delta\left(F\right)\right]$. Then, the equilibrium subsidy offer by G_2 is $S_2 = \bar{S}_0^d$. Moreover, in this equilibrium there is accommodated entry.* ***Proof:*** *See Appendix A.3.* ■

Comparing Proposition 7 with Proposition 3, we conclude that consumers are always weakly worse off when global power G_1 is not allowed to participate in the subsidy race because the equilibrium price under accommodated entry is always higher than the equilibrium price under deterred entry. Comparing Proposition 7 with Propositions 5, we conclude that if $0 \leq \rho \leq \bar{\rho}^b$, then consumers are better off when G_2 rises unchallenged than when G_2 does not pose a geopolitical threat because the equilibrium price under accommodated entry is lower than under blocked entry. This formalizes the idea that, for consumers, geopolitical rivalry (i.e., a rising global power facing an active incumbent global power capable subsidizing deterrence) is the best possible scenario. Geopolitical competition is better than unchallenged rising power, which is better than no geopolitical threat.

4.4 No Geopolitical Rivalry

Rather than completely eliminating G_1 from the geopolitical subsidy race, an interesting scenario to explore is to assume that G_1 has no geopolitical interest in deterring entry, which requires $B_1^M = B_1^D$. Since, in equilibrium, $B_1^D = \frac{2^m}{1+2^m} B_1^M$ (see Lemma 1), this can only occur if $B_1^D = B_1^M = 0$, that is, when G_1 has no geopolitical interest at all. More generally, note that Propositions 2 and 3 hold for any B_1^M and B_1^D. Thus, we can always set $B_1^D = B_1^M = B_1 \geq 0$ and interpret B_1 as the geopolitical benefit that G_1 obtains from controlling a satellite state with the strategic transportation service. Similarly, Propositions 2 and 3 hold for any B_2^D (and not only when $B_2^D = \frac{1}{1+2^m} B_2^M$). Thus, B_2^D can be interpreted as the geopolitical benefit that G_2 obtains from controlling a satellite state with the strategic transportation service. In other words, setting $B_1^M = B_1^D$, we have a situation of no geopolitical rivalry, in which each global power is merely interested in controlling a satellite state with a trans-oceanic canal, not whether the other global power controls one or not.

Proposition 8 ***No geopolitical rivalry.*** *Suppose that Assumptions 1, 2, and 3 hold, $B_1^M = B_1^D$, and $B_2^D \leq F + A_2\Delta(F)$. Let $\tilde{S}_1^{no-rival} \in (0, \bar{S}^d(F)]$ be the unique solution to:*

$$A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1^{no-rival}\right)\right) = \tilde{S}_1^{no-rival}$$

Then, Proposition 3 holds with $\tilde{S}_1^{no-rival}$ replacing $\tilde{S}_1$. Moreover, $\tilde{S}_1^{no-rival} < \tilde{S}_1$. ***Proof****: See Appendix A.3.* ■

Comparing Propositions 8 and 3, we conclude that consumers are always weakly better off when there is geopolitical rivalry.

4.5 Sequential Geopolitical Subsidy Race

The setup in Section 3 assumes that global power G_2 offers an entry subsidy to the potential entrant and that global power G_1 offers a subsidy to the incumbent to avoid entry. The entry subsidy provided by global power G_2 can be interpreted as a bonus for completing the project or a grant in exchange for a commitment by the entrant to build the project (i.e., if the project is not built funds must be returned). In any case, the payment offered by global power G_2 only depends on actions taken by the potential entrant and, hence, the contractual arrangement does not need to include contingencies on the actions taken by the incumbent or any other actor beyond the signatories of the agreement. On the contrary, the subsidy offered by global power G_1 constitutes a payment conditional on an action taken by the potential entrant rather than the incumbent itself. Once again, this can be interpreted as a bonus paid by global power G_1 if the incumbent manages to deter entry, but the odd legal issue persists: the bonus depends on what the potential entrant does, which is only indirectly affected by the incumbent.

To deal with this issue in this subsection we explore an alternative timing for the geopolitical game. In particular, we assume that the rising global power moves first (i.e., G_2 makes an offer S_2 to the entrant), which is observed by G_1, who thereafter makes its offer to the incumbent. Now it is still possible to interpret the offered made by G_1 as a bonus for deterring entrance (which of course depends on an action taken by E rather than I) but we can also consider that for any given S_2, G_1 computes the incumbent's capacity required to deter entry and it offers a bonus for reaching at least such capacity level. Under such interpretation, no contractual arrangement contingent on third party choices is employed. More formally, we modify the timing of the geopolitical game as follows[17]:

1. G_2 offers $S_2(k_E) = \chi_{k_E>0}S_2$, where $\chi_{k_E>0} = 1$ if $k_E > 0$ and $\chi_{k_E>0} = 0$ if $k_E = 0$.

2. G_1 observes S_2 and then offers $S_1(k_I) = \chi_{k_I \geq \bar{k}^d}S_1$, where $\chi_{k_I \geq \bar{k}^d} = 1$ if $k_I \geq \bar{k}^d = \frac{a-c-2\sqrt{b(F-S_2)}}{b}$ and $\chi_{k_I \geq \bar{k}^d} = 0$ if $k_I < \bar{k}^d$.

[17]The timing of the economic game between the entrant and the incumbent is not altered.

Note that $S_1(k_I)$ might depend on S_2, but since S_2 has already been selected and observed, the contract that G_1 offers to I does not need to specify S_2. It simply states that a bonus of S_1 dollars will be paid if the incumbent builds capacity $\bar{k}^d$ or greater; otherwise, no payment. Since $\bar{k}^d$ is the deterrence capacity level, firms face exactly the same incentives as in the setup in Section 2. Formally, since $k_E = 0$ if and only if $k_I \geq \bar{k}^d$, it is still the case that S_1 is a subsidy from global power G_1 to the incumbent to stop entry and S_2 is an entry subsidy from global power G_2 to the potential entrant. Thus, the economic equilibrium is not affected. Formally, Proposition 1 still holds. Making the geopolitical game sequential, however, might significantly affect the subsidy race between the global powers. Fortunately, as the following proposition formally shows, this is not the case.

Proposition 9 ***Sequential geopolitical subsidy race****.* *Suppose that Assumption 1, 2, and 3 hold,* $B_1^M - B_1^D \leq \bar{S}^d(F) - A_1\Delta(F)$, *and* $B_2^D \in \left(\bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right), F + A_2\Delta(F)\right]$, *but assume that* G_2 *moves first in the subsidy race.*

1. *Suppose that* $\tilde{S}_1 > \bar{S}^d\left(\tilde{S}_2\right)$*. Then, the unique subgame perfect Nash equilibrium outcome is* $S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ *and* $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \tilde{S}_1$*. Moreover, in this equilibrium, entry is deterred.*

2. *Suppose that* $\tilde{S}_1 = \bar{S}^d\left(\tilde{S}_2\right)$*. Then, the set of subgame perfect Nash equilibrium outcomes is* $S_2 = \tilde{S}_2$ *and* $S_1\left(\tilde{S}_2\right) \in \left[0, \tilde{S}_1\right]$*. For* $S_1\left(\tilde{S}_2\right) \in \left[0, \tilde{S}_1\right)$*, there is accommodated entry, while for* $S_1\left(\tilde{S}_2\right) = \tilde{S}_1$*, entry is deterred.*

3. *Suppose that* $\bar{S}^d\left(\tilde{S}_2\right) > \tilde{S}_1$*. Then, the set of subgame perfect Nash equilibrium outcomes is* $S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ *and* $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) \in \left[0, \tilde{S}_1\right]$*. Moreover, in equilibrium, there is accommodated entry.*

Proof*: See Appendix A.3.* ■

Comparing Propositions 9 and 3, we observe that key results in Proposition 3 are not significantly altered. When the incumbent global power is willing to offer a higher subsidy than the rising global power (formally, $\tilde{S}_1 > \bar{S}^d\left(\tilde{S}_2\right)$), entry is deterred, while the opposite happens when the rising global power is willing to outbid the incumbent global power (formally, $\bar{S}^d\left(\tilde{S}_2\right) > \tilde{S}_1$). Not surprisingly, since in Proposition 9, the rising global power moves first, it uses this first-mover advantage to induce a more favorable equilibrium outcome. When $\tilde{S}_1 > \bar{S}^d\left(\tilde{S}_2\right)$, G_2 pushes to the best possible deterrence equilibrium in which G_1 must offer its maximum willingness to pay to deter entry, i.e., $\tilde{S}_1$. When $\bar{S}^d\left(\tilde{S}_2\right) > \tilde{S}_1$, G_2 is capable of inducing entry, offering the minimum subsidy that outbids G_1, i.e., $S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$.

The multiplicity of equilibrium outcomes does not fully disappear in Proposition 9 but only comes from multiple equilibrium reactions by G_1, which can be easily adjusted assuming that G_1 always reacts with the maximum subsidy among its best response set, provided that such a maximum exists. Introducing this simple criteria, we obtain the following corollary.

Corollary 5 *Under the assumptions in Proposition 9. Suppose that global power* 1 *always reacts with the highest possible best response subsidy. Then, the unique subgame perfect Nash equilibrium outcome is:* $S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ *and* $S_1 = \tilde{S}_1$*. Moreover, if* $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$*, then entry is deterred, while if* $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$*, there is accommodated entry.* ***Proof****: See Appendix A.3.* ■

4.6 Geopolitical Realignment of the Potential Entrant

The setup in Section 3 assumes that geopolitical alliances are given, namely, G_1 is allied with I and G_2 with E and each global power only subsidizes its ally. Next, we relax this assumption and explore a scenario in which the established global power (i.e., G_1) has the opportunity to seduce E offering a payment to break its geopolitical ties with the rising global power G_2. Thus, we consider that G_1 can deal with the geopolitical threat posed by G_2 inducing a geopolitical realignment of the potential entrant, denying to G_2 a location where it can build an alternative strategic infrastructure.

To do so we augment the game in Section 3 with an initial stage in which G_1 can approach E, preempting the subsidy race. In particular, suppose that before the geopolitical subsidy race between the global powers, G_1 can approach E and make the following offer. In exchange for E's commitment to reject any offer made by G_2, E receives a lump sum payment $F_1^E \geq 0$. In addition, if this offer is accepted, then G_1 has the chance of offering an extra payment $S_1^E \geq 0$ to E, provided that $k_E > 0$. In other words, G_1 offers E a payment of $F_1^E \geq 0$, if it breaks its geopolitical alliance with G_2 and forms an alliance with G_1. Moreover, assume that E must pay a cost $C^E > 0$ for breaking its geopolitical ties with G_2, which captures, for example, a sanction that G_2 will impose on E. The following proposition characterizes the equilibrium.

Proposition 10 ***Geopolitical realignment of*** E.[18] *Suppose that Assumptions 1, 2, and 3 hold,* $B_1^M - B_1^D \leq \bar{S}^d(F) - A_1\Delta(F)$*, and* $B_2^D \leq F + A_2\Delta(F)$*, but assume that before the geopolitical subsidy race,* G_1 *can offer* E *a deal to break its geopolitical ties with* G_2*.*

1. *Suppose that* $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$*. Then, if the following condition holds,*

$$C^E > \bar{C}^E = A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)\right] + \tilde{S}_1 \tag{10}$$

   G_1 *is never interesting in making an offer that* E *is willing to accept and, hence, the geopolitical alliance between* E *and* G_2 *and the equilibrium in Proposition 3.1 persists. On the contrary, if* $C^E \leq \bar{C}^E$*, there are equilibria in which* E *accepts to realign with* G_1*. In those equilibria,* G_1 *always offers* $S_1^E = \bar{S}_0^d$ *to* E*,* $S_1^I = 0$ *to* I *entry is deterred.*

2. *Suppose that* $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$*, Then, if the following condition holds*

$$C^E > \bar{C}^E - \left[\frac{(a-c)^2}{16b} - F + \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right] \tag{11}$$

[18] In Appendix A.3 we prove a more detailed proposition that fully characterizes the equilibrium for any set of parameters.

G_1 is never interesting in making an offer that E is willing to accept and, hence, the geopolitical alliance between E and G_2 and the equilibrium in Proposition 3.2 persists. On the contrary, if $C^E \leq \bar{C}^E - \left[\frac{(a-c)}{16b}^2 - F + \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$, there are equilibria in which E accepts to realign with G_1. In those equilibria, G_1 always offers $S_1^E = \bar{S}_0^d$ to E, $S_1^I = 0$ to I entry is deterred.

Proof: *See Appendix A.3.* ■

As expected, if the cost that E would incur for breaking its geopolitical ties with G_2 is high enough, then G_1 will never be interested in offering a subsidy that E is willing to accept. The reason is that G_1 needs to compensate E for this cost. In such circumstances, the equilibrium in Proposition 3 is not affected. Note that the required threshold for Proposition 3.2 to be unaltered is lower than for Proposition 3.1. The intuition is that when $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, if E rejects to break its geopolitical ties with G_2, then the equilibrium will be accommodated entry and, hence, E will collect positive profits. Therefore, G_1 must not only compensate E for the cost of breaking its geopolitical ties with G_2 but also for the lost profits. On the contrary, when $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, if E rejects to break its geopolitical ties with G_2, then entry will be deterred and, hence, E will obtain zero profits. Thus, all that G_1 must offer E to break its ties with G_2 is C^E.

More novel results emerge if the cost of realigning E is low enough; formally, when $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$ and (10) does not hold, or $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$ and (11) does not hold. In those circumstances, there are equilibria in which E accepts G_1's offer to break its geopolitical alliance with G_2. Moreover, in those equilibria it is always the case that entry is deterred, but the expansion of I's capacity required to deter entry is not subsidize by G_1. The intuition is as follows. Once the alliance between E and G_2 is severed, G_1 must decide which set of subsidies it will offer to E and I. Moreover, G_1 can completely disregard geopolitical issues in its decision because now E and I are both G_1's allies. It turns out that the optimal decision for G_1 is to offer $S_1^E = \bar{S}_0^d$ to E and $S_1^I = 0$ to I, which leads to deterred entry. Thus, in equilibrium, G_1 does not pay any subsidy and obtains an expansion of I's capacity for free, i.e., just threatening to subsidize E.

5 The Case of the Panama Canal

The Panama Canal's monopoly on passage between the Atlantic and Pacific Oceans has periodically been threatened by the possibility of a project to build a new canal through Nicaragua. In the last decade, this threat became more credible because such a project was part of China's Belt and Road worldwide infrastructure initiative aimed at developing logistical infrastructure to facilitate Chinese engagement in foreign markets and military actions (Cai, 2017).

Panama' existence as a state and an economy backed by American interests in transoceanic travel has been defined by the Panama Canal ever since its construction. Sigler (2014) shows just how much the Panama Canal has shaped Panama's national economy and its internal politics and goes on to show how disruptive a rival, such as a canal in Nicaragua, could be for that country.

The geopolitical implications of the possibility of constructing canals to span Central America are closely linked to the inception of the state of Panama itself. The Panamanian isthmus was part of the sovereign territory of Colombia and became a place of interest to the French government in the late nineteenth century when France started dredging a trans-American canal through the swamps and jungles of that territory to create a sea lane to connect the Atlantic and Pacific Oceans. The French eventually failed when malaria and yellow fever decimated their workers. This opened up an opportunity for the United States, under President Theodore Roosevelt, to take over the project. As Panama was part of Colombia at the time, the negotiations concerning the building of the canal took place between the United States and Colombia. Those talks led to the signing of the Hay-Herrán Treaty, which, however, ended up being rejected by the Colombian Senate. This set the stage for the separation of Panama from Colombia and resulted in the Hay-Bunau-Varilla Treaty, which was signed by the French plenipotentiary ambassador of Panama to Washington. The United States then bought the French interest in Panama for US$40 million (Sabonge and Sánchez, 2014). When the United States purchased the rights to the canal project, the population of the isthmus rebelled against Colombia and declared independence in 1903. Colombia tried to retake the isthmus, but the new state of Panama was shielded by a fleet of US Navy ships (Sánchez, 2019).

The Hay-Bunau-Varilla Treaty gave the United States the rights, in perpetuity, to a strip of land (the Canal Zone) where the laws of the United States would apply. The arrangement for operating the canal did not allow for Panama to share in the revenue or other financial benefits derived from it. All that Panama received was a modest lease payment (Sabonge and Sánchez, 2014). All this changed, however, with the signing of the Torrijos-Carter Treaty in 1977, which provided for the Canal Zone to be abolished and for the Panama Canal to be handed over to the Republic of Panama at the end of 1999 (Sabonge and Sánchez, 2014).

Since the construction of the Panama Canal in 1914, the value of that route has changed over time. In the beginning, the Canal was primarily of strategic value from a military standpoint. In the years following the Second World War, it gained increasing economic and commercial value. And since its handover to the Republic of Panama, it has become a significant generator of wealth for Panama, whose monopoly position has essentially been uncontested until fairly recently, when a robust push for a canal through Nicaragua began to emerge.

The Nicaragua Interoceanic Grand Canal Master Plan was aimed at creating a faster route through the Americas while also industrializing the adjacent corridor. As it would be located to the north of the Panama Canal, the Nicaraguan canal would provide a faster route for ships bound for the Northern Hemisphere and would be able to accommodate ships that are too large to fit through the Panama Canal. The project was to be organized by the Hong Kong Nicaragua Canal Development Investment Company (HKND). In 2013, a 100-year concession contract for the management of the Nicaraguan Canal Authority was signed between HKND and the Government of Nicaragua. The first stages of the canal's construction began the following year. It has often been speculated that the HKND receives funds directly from the Chinese government (Sabonge and Sánchez, 2014). Arturo Cruz, the former Ambassador of Nicaragua to the United States, has said that "if the canal goes ahead… it will be because the Chinese government wants it to, and the financing will come from China's various state firms" (Sánchez, 2019).

However, although the Nicaraguan canal project nominally still forms part of China's Belt and Road Initiative, China has distanced itself from the project, and construction has been suspended. At the same time, Panama has effectively doubled the capacity of the Panama Canal by adding a new lane of traffic so that a larger number of ships can transit the canal at the same time and increasing the width and depth of the lanes and locks in order to accommodate larger container ships. The new ships, called New Panamax, are about one and a half times the previous Panamax size and can carry over twice as much cargo. The expansion was approved by a national referendum in 2006, but because of the 2008 financial crisis, construction did not actually begin until later, and the expanded facilities were finally completed in 2016.

In 2006, the Panama Canal Authority (PCA) estimated the cost of the third set of locks at US$5.25 billion. The PCA also estimated that the investment could be recouped thanks to the increased revenues that the project would yield. Opponents of the project contend that these estimates are based on uncertain projections of maritime trade and world economic trends. Indeed, Former President Jorge Illueca, former Assistant Administrator of the Panama Canal Commission Fernando Manfredo, shipping consultant Julio Manduley, and industrial entrepreneur George Richa M. have said that the expansion was not necessary and claimed that the construction of a mega-port on the Pacific side would be sufficient to meet probable future demand. At the moment, the projections presented to support the financial viability of the project appear to be grounds for optimism; the delay in the construction works has also substantially altered the initial financial estimates. External finance for the project was provided by several international financial institutions in which the United States Government has a great deal of influence, such as the Inter-American Development Bank (IDB) and the International Finance Corporation (IFC), as well as by the Japan Bank of International Cooperation (JBIC) and the European Investment Bank (EIB).

Although it is often argued that China has stepped back from the Nicaragua canal project in response to Panama's decision to cut diplomatic ties with Taiwan and to recognize the People's Republic of China as the only sovereign Chinese republic (Cheng and Lohman, 2017), Propositions 2-4 offer a more plausible explanation. Indeed, these propositions suggest several mechanisms that explain the observed behavior of the parties involved. First, most likely the Panama Canal has very high geopolitical value for the United States and much more limited geopolitical value for China. For example, some works in International Relations indicate that powerful countries put special interest in keeping other powerful countries out of their areas of influence (e.g., Mearsheimer (2001)). In terms of our model, this translates into $B_1^M - B_1^D$ relatively higher than B_2^D, which makes deterrence more likely. Second, the entry cost for Nicaragua-China was probably very high. Some initial estimates for the Nicaragua Canal were US$ 50 billion (almost 10 times the cost of the Panama Canal expansion). As we discussed before Corollary 4, a rise in F makes deterrence more likely. This, however, does not imply that China should have not considered doing the project. As Proposition 5 shows, even when China knew that, in equilibrium, entry will be deterred, it was rational to include the Nicaragua Canal in the Belt and Road Initiative, start serious conversations with the Nicaraguan government about the project, and sign a contract for the concession of the Nicaraguan Canal Authority to HKND. We interpret these decisions as strategic moves to establish the credibility of China's intentions. Ultimately, China did not finance the Nicaragua Canal, but creating a credible threat was probably useful to influence the expansion of the Panama Canal, a non-negligible improvement as China is the second most important user of the canal.[19]

6 Concluding Remarks

We have developed a simple model of strategic deterrence between an incumbent country in which strategic trade-related infrastructure is located and a potential entrant. An established global power allied with the incumbent and a rising global power aligned with the entrant strategically influence the

[19]Measure by tonnage either as country of origin or destiny. See Panama Canal Authority (2022).

game by making funding available in order to advance their economic and geopolitical interests. Our main finding is that, even if the entrant is deterred, a geopolitical challenger that credibly commits to supporting the entrant has a pro-competition economic effect on the market for this type of strategic transportation service. This effect makes consumers of the transportation service in all regions better off, reduces the profits of the incumbent, and has no effect on the entrant. The established global power might be forced to pay out more generous subsidies in order to support the incumbent's deterrence effort, but it will not suffer a geopolitical loss. The rising global power will enjoy a larger consumer surplus at no cost, but it will not secure any geopolitical advantage.

We have applied our theoretical results to the case of the expansion of the Panama Canal and China's negotiations with Nicaragua to build an alternative transoceanic passage and argued that the model provides a reasonable rationalization for this important case. Nevertheless, more rigorous empirical tests should be performed to confirm the mechanism in our model. Next, we briefly discuss some possibilities.[20] Consider China's public announcements of infrastructure projects within its Belt and Road Initiative and find those that can be seen as supporting entry challenging some existing infrastructure controlled by the US and its closed allies. Consider the US and G7's response to the Belt and Road initiative, the Build Back Better World, and find projects that can be viewed as supporting the expansion of existing infrastructures potentially challenged by the Belt and Road Initiative. Finally, verify if there is evidence of deterrence, i.e., if some projects initially supported by China are cancelled and/or postponed after Build Back Better World's announcements. Another possibility would be to use historical data on projects proposed and developed/abandoned during infrastructural booms that took place in periods where a new great power rose or when geopolitical tensions among existing powers intensified.

Beyond finding empirical support for the mechanism proposed in this paper, the model and the resulting findings are just the tip of the iceberg for a more ambitious research agenda focusing on the international political economy of strategic trade-related infrastructure, in particular, and geopolitics and international trade, more generally. That research should address questions such as the following: When does rivalry between global powers lead to market restrictions that distort international trade flows (e.g., colonial powers and mercantilist policies), and when does it generate pro-competition economic effects by breaking up monopoly positions or forcing agents to engage in more competitive behavior?

Our model also highlights the importance of political competition in understanding the political economy of market power. Although in some cases, models with a single political agent are reasonable (for example, when a public utility company faces a unique regulatory agency), in other cases, accounting for political competition is crucial. This is particularly relevant in international politics, where powerful states usually compete for control and influence over international activities that they consider geopolitically important. Thus, in this paper we advance our understanding of how politics affects market power by considering competition among political actors. Crucially, modeling geopolitical competition does more than simply add realism to the single political actor model. Indeed, conclusions from previous research can be completely reversed: Political interference might reduce market power and improve market outcomes.

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Online Appendix to "Geopolitics and International Trade Infrastructure Deterrence"

This appendix presents the proofs of all lemmas and propositions.

A.1 Economic Equilibrium

In this section we prove Proposition 1, further characterize $\bar{S}^d(S_2)$ (which leads to a more precise version of Proposition 1.2), and prove Corollary 1.

Proposition 1 ***Economic equilibrium***. *Suppose that Assumptions 1 (i.e., $k_I \in \left[0, \frac{a-c}{b}\right]$, $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$, and $a \leq 2c$) and 2 (i.e., $F > \frac{(a-c)^2}{16b}$) hold.*

1. *Suppose that $0 \leq S_2 \leq \bar{S}^b$. Then the entry of E is* ***blocked***. *Specifically, in equilibrium $(k_I, k_E) = \left(\frac{a-c}{2b}, 0\right)$ and $P = \frac{a+c}{2}$.*
2. *Suppose that $\bar{S}^b < S_2 \leq F$.*
   (a) *If $S_1 > \bar{S}^d(S_2)$, then the entry of E is* ***deterred***. *Specifically, in equilibrium, $(k_I, k_E) = \left(\frac{a-c-2\sqrt{b(F-S_2)}}{b}, 0\right)$ and $P = c + 2\sqrt{b(F-S_2)}$.*
   (b) *If $S_1 = \bar{S}^d(S_2)$, then there are two equilibria: in one equilibrium the entry of E is deterred, while in the other I accommodates the entry of E. Under deterrence (accommodation), (k_I, k_E, P) is as in part a (c).*
   (c) *If $S_1 < \bar{S}^d(S_2)$, then I* ***accommodates*** *the entry of E. Specifically, in equilibrium, $(k_I, k_E) = \left(\frac{a-c}{2b}, \frac{a-c}{4b}\right)$ and $P = \frac{a+3c}{4}$.*

Proof. We proceed through backward induction.

Efficient-rationing rule and price competition: According to the efficient-rationing rule, demands are given by:

$$Q_I(p_E, p_I) = \begin{cases} \min\left\{\max\left\{\frac{a-p_I}{b} - k_E, 0\right\}, k_I\right\} & \text{if } p_I > p_E \\ \min\left\{\max\left\{\frac{a-p}{2b}, \frac{a-p}{b} - k_E\right\}, k_I\right\} & \text{if } p_E = p_I = p \\ \min\left\{\frac{a-p_I}{b}, k_I\right\} & \text{if } p_I < p_E \end{cases}$$

$$Q_E(p_E, p_I) = \begin{cases} \min\left\{\max\left\{\frac{a-p_E}{b} - k_I, 0\right\}, k_E\right\} & \text{if } p_E > p_I \\ \min\left\{\max\left\{\frac{a-p}{2b}, \frac{a-p}{b} - k_I\right\}, k_E\right\} & \text{if } p_E = p_I = p \\ \min\left\{\frac{a-p_E}{b}, k_E\right\} & \text{if } p_E < p_I \end{cases}$$

To see the logic behind the efficient-rationing rule, assume that $p_E = p_I$ and focus on I (analogous logic applies to E). Then, demand will be split evenly between both countries at $(a-p)/2b$, unless E is capacity-constrained. If so, I will be the only service provider over the excess of demand $(a-p)/b - k_E$. Since I also needs to consider its own capacity constraint, we have $Q_I(p_E, p_I) = \min\left\{\max\left\{\frac{a-p}{2b}, \frac{a-p}{b} - k_E\right\}, k_I\right\}$. Next, assume that $p_E > p_I$. Then, as consumers try to buy from the low-priced firm first, I's demand is $(a-p_I)/b$, provided that its capacity constraint (k_I) does not bind. Therefore, $Q_I(p_E, p_I) =$

$\min\left\{\frac{a-p_I}{b}, k_I\right\}$. E obtains the residual demand $\max\left\{\frac{a-p_E}{b} - k_I, 0\right\}$ (if any) after taking into account its own capacity constraint (k_E). Then, $Q_E(p_E, p_I) = \min\left\{\max\left\{\frac{a-p_E}{b} - k_I, 0\right\}, k_E\right\}$. A similar reasoning follows for $p_E < p_I$.

Suppose that I and E have selected capacity levels $k_I \geq 0$ and $k_E \geq 0$, respectively. We will prove that, under proper conditions, it is a Nash equilibrium for I and E to set $p_I = p_E = p^* = a - b(k_I + k_E)$. To do so, suppose that I sets $p_I = p^* = a - b(k_I + k_E)$ and recall that E's demand is given by:

- If $p_E > p_I$, then $x_E = \min\left\{\max\left\{\frac{a-p_E}{b} - k_I, 0\right\}, k_E\right\}$.
- If $p_E < p_I$, then $x_E = \min\left\{\frac{a-p_E}{b}, k_E\right\}$.
- If $p_E = p_I = p$, then $x_E = \min\left\{\max\left\{\frac{a-p}{2b}, \frac{a-p}{b} - k_I\right\}, k_E\right\}$.

Then, E has three possible choices to consider:

1. If E also sets $p_E = p^*$, then E's demand is given by $x_E = \min\{\max\{(k_I + k_E)/2, k_E\}, k_E\} = k_E$ and, therefore, E's revenue is $R_E(p^*) = p^* k_E = [a - b(k_I + k_E)] k_E$.

2. If E sets $p_E < p^*$, then E's demand is given by $x_E = \min\{(a - p_E)/b, k_E\}$ and, therefore, E's revenue is $R_E = p_E \min\{(a - p_E)/b, k_E\}$. Since $p_E < p^*$, it must be the case that $(a - p_E)/b > (k_I + k_E)$ and, hence, $R_E = p_E k_E < p^* k_E$. Thus, E obtains higher revenues if it sets $p_E = p^*$.

3. If E sets $p_E > p^*$, then E's demand is given by $x_E = \min\{\max\{(a - p_E)/b - k_I, 0\}, k_E\}$ and, therefore, E's revenue is $R_E = p_E \min\{\max\{[(a - p_E)/b] - k_I, 0\}, k_E\}$. Since $p_E > p^*$, it must be the case that $[(a - p_E)/b - k_I] < k_E$ and, hence, $R_E = p_E\{[(a - p_E)/b] - k_I\}$. This implies that E's maximum revenue is attained at $p_E = \hat{p} = (a - bk_I)/2$. In order for $p_E = \hat{p}$ not to be a possible deviation, we need that $\hat{p} \leq p^*$, which holds if and only if $k_E \leq (a - bk_I)/2b$.

Summing up, E's best response to $p_I = p^* = a - b(k_I + k_E)$ is to set $p_E = p^*$ if and only if $k_E \leq (a - bk_I)/2b$. Following the same steps it is easy to prove that I's best response to $p_E = p^*$ is to set $p_I = p^*$ if and only if $k_I \leq (a - bk_E)/2b$. We want these conditions to hold for every profile of capacity choices such that $k_I \in \left[0, \frac{a-c}{b}\right]$ and $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$. Note that $k_E \leq (a - bk_I)/2b$ and $k_I \leq (a - bk_E)/2b$ hold for all $k_E \in [0, (a - c)/b - k_I]$ if and only if they hold for $k_E = (a - c)/b - k_I$, i.e., if and only if $(a - 2c)/b \leq k_I \leq \frac{c}{b}$. These inequalities hold for all $k_I \in [0, (a - c)/b]$ if and only if $a \leq 2c$. Thus, Assumption 1 ensures that these inequalities always hold.

Capacity choices: Assume that I has selected $k_I \in \left[0, \frac{a-c}{b}\right]$. Then, the problem of E is given by:

$$\max_{k_E \in \left[0, \frac{a-c}{b} - k_I\right]} \left\{ \pi_E = [a - b(k_I + k_E) - c] k_E - \left\{ \begin{array}{ll} F - S_2 & if\ k_E > 0 \\ 0 & if\ k_E = 0 \end{array} \right\} \right.$$

If E selects $k_E > 0$, its best response is $k_E = (a - bk_I - c)/2b < (a - c - bk_I)/b$. Thus, E's profits are $\pi_E = \left[(a - bk_I - c)^2/4b\right] - (F - S_2)$. On the contrary, if E selects $k_E = 0$, E's profits are $\pi_E = 0$. Thus, E's best response is given by:

$$k_E(k_I) = \left\{ \begin{array}{ll} 0 & if\ \bar{k}^d \leq k_I \leq \frac{a-c}{b} \\ \frac{a-bk_I-c}{2b} & if\ 0 \leq k_I < \bar{k}^d \end{array} \right. ,\ where\ \bar{k}^d = \frac{a - c - 2\sqrt{b(F - S_2)}}{b}$$

Given the reaction function of E, the problem of I is:

$$\max_{k_I \in \left[0, \frac{a-c}{b}\right]} \left\{ \pi_I = \left\{ \begin{array}{ll} \pi_I^m = (a - bk_I - c)\, k_I + S_1 & \textit{if } k_I \geq \bar{k}^d \\ \pi_I^s = \left(\frac{a-bk_I-c}{2}\right) k_I & \textit{if } k_I < \bar{k}^d \end{array} \right. \right\}$$

Let $\bar{k}^m = (a-c)/2b$ be the monopoly capacity level. It is easy to verify that π_I^s is increasing in k_I for all $k_I \in \left[0, \bar{k}^m\right)$, decreasing in k_I for all $k_I \in \left(\bar{k}^m, (a-c)/b\right]$ and it has a maximum at $k_I = \bar{k}^m$. Similarly, π_I^m is increasing in k_I for all $k_I \in \left[0, \bar{k}^m\right)$, decreasing in k_I for all $k_I \in \left(\bar{k}^m, (a-c)/b\right]$ and it has a maximum at $k_I = \bar{k}^m$. Thus, to solve this problem we must consider two possible cases.

Case 1 (blocked entry): Suppose that $\bar{k}^d \leq \bar{k}^m$, which holds if and only if

$$S_2 \leq \bar{S}^b = F - \frac{(a-c)^2}{16b}$$

Note that Assumption 2 ensures that $\bar{S}^b > 0$. Then π_I^d is increasing in k_I for all $k_I < \bar{k}^d$ and π_I^m has a global maximum at $k_I = \bar{k}^m$. Since $\pi_I^m\left(\bar{k}^m\right) \geq \pi_I^m\left(\bar{k}^d\right) > \pi_I^s\left(\bar{k}^d\right)$, π_I has a global maximum at $k_I = \bar{k}^m$. Summing up, when $\bar{k}^d \leq \bar{k}^m$, the unique subgame perfect Nash equilibrium outcome is $k_I = \bar{k}^m$, $k_E = 0$, the equilibrium price is $P = a - b\bar{k}^m$, and the equilibrium profits of I and E are $\pi_I = \left[(a-c)^2/4b\right] + S_1$ and $\pi_E = 0$, respectively.

Case 2 (deterred or accommodated entry): Suppose that $\bar{k}^m < \bar{k}^d$, which holds if and only if

$$S_2 > \bar{S}^b = F - \frac{(a-c)^2}{16b}$$

Then, π_I^s has a global maximum at $k_I = \bar{k}^m$ and π_I^m is decreasing in k_I for all $k_I \geq \bar{k}^d$, which means that π_I^m has a global maximum at $k_I = \bar{k}^d$. If I selects $k_I = \bar{k}^m$, then it gets $\pi_I^s\left(\bar{k}^m\right) = \left(a - b\bar{k}^m - c\right)\bar{k}^m/2$. If I selects $k_I = \bar{k}^d$, then it gets $\pi_I^m\left(\bar{k}^d\right) = \left(a - b\bar{k}^d - c\right)\bar{k}^d + S_1$. $\pi_I^m\left(\bar{k}^d\right) > \pi_I^s\left(\bar{k}^m\right)$ if and only if $S_1 > \left[(a-c)^2/8b\right] - 2(a-c)\sqrt{(F-S_2)/b} + 4(F-S_2)$, $\pi_I^m\left(\bar{k}^d\right) = \pi_I^s\left(\bar{k}^m\right)$ when $S_1 = \left[(a-c)^2/8b\right] - 2(a-c)\sqrt{(F-S_2)/b} + 4(F-S_2)$, and $\pi_I^m\left(\bar{k}^d\right) < \pi_I^s\left(\bar{k}^m\right)$ if and only if $S_1 < \left[(a-c)^2/8b\right] - 2(a-c)\sqrt{(F-S_2)/b} + 4(F-S_2)$. Therefore, we have the following cases:

Case 2.a (deterred entry). Suppose that

$$S_1 > \bar{S}^d(S_2) = \frac{(a-c)^2}{8b} - 2(a-c)\sqrt{\frac{F-S_2}{b}} + 4(F-S_2)$$

Then, the unique subgame perfect Nash equilibrium outcome is $k_I = \bar{k}^d$, $k_E = 0$, the equilibrium price is $P = a - b\bar{k}^d$, and the equilibrium profits of I and E are $\pi_I = \left\{\left[2\sqrt{b(F-S_2)}\right]\left[a - c - 2\sqrt{b(F-S_2)}\right]/b\right\} + S_1$ and $\pi_E = 0$, respectively.

Case 2.b (deterred or accommodated entry). Suppose that

$$S_1 = \bar{S}^d(S_2) = \frac{(a-c)^2}{8b} - 2(a-c)\sqrt{\frac{F-S_2}{b}} + 4(F-S_2)$$

Then, there are two subgame perfect Nash equilibrium outcomes: the equilibrium described in case 2.a and the equilibrium described in case 2.c.

Case 2.c (accommodated entry): Suppose that

$$S_1 < \bar{S}^d(S_2) = \frac{(a-c)^2}{8b} - 2(a-c)\sqrt{\frac{F-S_2}{b}} + 4(F-S_2)$$

Then, the unique subgame perfect Nash equilibrium outcome $k_I = \bar{k}^m$, $k_E = (a-c)/4b$, the equilibrium price is $P = (a+3c)/4$, and the equilibrium profits of I and E are $\pi_I = (a-c)^2/8b$ and $\pi_E = \left[(a-c)^2/16b\right] - (F-S_2)$, respectively.

This completes the proof of Proposition 1. ■

Further characterization of $\bar{S}^d(S_2)$: It is possible to further characterize the equilibrium for $\bar{S}^b < S_2 \leq \bar{S}$. In particular, note that:

- $\bar{S}^d$ is a continuous function of S_2 for all $\bar{S}^b \leq S_2 \leq F$.
- $\bar{S}^d\left(\bar{S}^b\right) = -(a-c)^2/8b < 0$.
- $d\bar{S}^d(S_2)/dS_2 = \left[(a-c)/\sqrt{(F-S_2)b}\right] - 4 > 0$ if and only if $S_2 > \bar{S}^b$. Thus, $\bar{S}^d(S_2)$ is strictly increasing in S_2 for all $\bar{S}^b \leq S_2 \leq F$.
- $d^2\bar{S}^d(S_2)/(dS_2)^2 = (a-c)/2b^{1/2}(F-S_2)^{3/2} > 0$. Thus, $\bar{S}^d(S_2)$ is strictly convex in S_2 for all $\bar{S}^b \leq S_2 \leq F$.
- $\bar{S}^d(F) = \frac{(a-c)^2}{8b} > 0$.

Therefore, there exists $\bar{S}^d_0 \in \left(\bar{S}^b, F\right)$ such that $\bar{S}^d(S_2) < 0$ for all $S_2 \in \left[\bar{S}^b, \bar{S}^d_0\right)$, $\bar{S}^d\left(\bar{S}^d_0\right) = 0$, and $\bar{S}^d(S_2) > 0$ for all $S_2 \in \left(\bar{S}^d_0, F\right]$. Moreover, $\bar{S}^d(S_2)$ has a continuous inverse and, hence, $S_1 = \bar{S}^d(S_2)$ if and only if $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$. Hence, we obtain the following more precise version of Proposition 1.2.

2.a *If* $\bar{S}^b < S_2 < \left(\bar{S}^d\right)^{-1}(S_1)$, *then the entry of* E *is* ***deterred****. Specifically, in equilibrium,* $(k_I, k_E) = \left(\frac{a-c-2\sqrt{b(F-S_2)}}{b}, 0\right)$ *and* $P = c + 2\sqrt{b(F-S_2)}$.

2.b *If* $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$, *then there are two equilibria: in one equilibrium the entry of* E *is deterred, while in the other* I *accommodates the entry of* E*. Under deterrence (accommodation),* (k_I, k_E, P) *is as in part a (c).*

2.c *If* $\left(\bar{S}^d\right)^{-1}(S_1) < S_2 \leq F$ *, then* I ***accommodates*** *the entry of* E*. Specifically, in equilibrium,* $(k_I, k_E) = \left(\frac{a-c}{2b}, \frac{a-c}{4b}\right)$ *and* $P = \frac{a+3c}{4}$.

Corollary 1. ***Economic equilibrium.*** *Under the assumptions in Proposition 1.*

1. *The equilibrium prices (aggregate quantity) under deterrence and accommodation are lower (higher) than under blocked entry.*

2. *The equilibrium price (aggregate quantity) under deterrence is lower (higher) than under accommodation if and only if*

$$S_2 > F - \frac{(a-c)^2}{64b} \in \left(\bar{S}^b, \bar{S}_0^d\right)$$

Thus, whenever $S_2 \geq \bar{S}_0^d$, the equilibrium price (quantity) under deterrence is lower (higher) than under accommodation.

Proof: From Proposition 1, the equilibrium capacity levels and price under blocked entry, deterrence and accommodation are given by: $(k_I, k_E) = \left(\bar{k}^m, 0\right) = \left(\frac{a-c}{2b}, 0\right)$, $P = \frac{a+c}{2}$; $(k_I, k_E) = \left(\bar{k}^d, 0\right) = \left(\frac{a-c-2\sqrt{b(F-S_2)}}{b}, 0\right)$, $P = a - b\bar{k}^d = c + 2\sqrt{b\left(F - S_2\right)}$; and $(k_I, k_E) = \left(\bar{k}^m, \frac{\bar{k}^m}{2}\right) = \left(\frac{a-c}{2b}, \frac{a-c}{4b}\right)$, $P = a - \frac{3}{4}\bar{k}^m = \frac{a+3c}{4}$, respectively. Clearly, $\bar{k}^m + \frac{\bar{k}^m}{2} = \frac{3(a-c)}{4b} > \bar{k}^m = \frac{a-c}{2b}$. Thus, The equilibrium quantity under accommodation is lower than under blocked entry. Since, $P = a - b\left(k_I + k_E\right)$, the equilibrium price under accommodation is lower than under blocked entry. $\bar{k}^d > \bar{k}^m$ if and only if $S_2 > \bar{S}^b = F - \frac{(a-c)^2}{16b}$. From Proposition 1.2, deterrence can only be an equilibrium when $S_2 > \bar{S}^b$. Therefore, it is always the case that the equilibrium quantity under deterrence is higher than under blocked entry. Since, the equilibrium price is given by $P = a - b\left(k_I + k_E\right)$, the equilibrium price under deterrence is always lower than under blocked entry. Finally, $\bar{k}^d > \bar{k}^m + \frac{\bar{k}^m}{2}$ if and only if $S_2 > F - \frac{(a-c)^2}{64b}$. Thus, the equilibrium quantity under deterrence is higher than under accommodation if and only if $S_2 > F - \frac{(a-c)^2}{64b}$. Since, $P = a - b\left(k_I + k_E\right)$, the equilibrium price under deterrence is lower than under accommodation if and only if $S_2 > F - \frac{(a-c)^2}{64b}$. Moreover, note that $F - \frac{(a-c)^2}{64b} > F - \frac{(a-c)^2}{16b} = \bar{S}^b$ and $\bar{S}^d\left(F - \frac{(a-c)^2}{64b}\right) = -\frac{(a-c)^2}{16b} < 0$, which implies that $F - \frac{(a-c)^2}{64b} < \bar{S}_0^d$. Thus, $F - \frac{(a-c)^2}{64b} \in \left(\bar{S}^b, \bar{S}_0^d\right)$, which implies that whenever $S_2 \geq \bar{S}_0^d$, the equilibrium price (quantity) under deterrence is lower (higher) than under accommodation. ■

A.2 Geopolitical Equilibrium

In this section we prove three lemmas (including Lemma 1) that help us characterize the geopolitical trade-off faced by each global power. Then, we prove Propositions 2, 3 and 4, and Corollary 4.

Lemma 1 ***Geopolitical benefits***. *Suppose that Assumptions 1 (i.e., $k_I \in \left[0, \frac{a-c}{b}\right]$, $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$, and $a \leq 2c$) and 2 (i.e., $F > \frac{(a-c)^2}{16b}$) hold. Then:*

$$B_1 = \begin{cases} B_1^M - S_1 & \text{if } \left[0 \leq S_2 \leq \bar{S}^b\right] \text{ or } \left[\bar{S}^b < S_2 \leq F \text{ and } S_1 \geq \bar{S}^d\left(S_2\right)\right] \\ B_1^D & \text{if } \left[\bar{S}^b < S_2 \leq F \text{ and } S_1 \leq \bar{S}^d\left(S_2\right)\right] \end{cases}$$

$$B_2 = \begin{cases} 0 & \text{if } \left[0 \leq S_2 \leq \bar{S}^b\right] \text{ or } \left[\bar{S}^b < S_2 \leq F \text{ and } S_1 \geq \bar{S}^d\left(S_2\right)\right] \\ B_2^D - S_2 & \text{if } \left[\bar{S}^b < S_2 \leq F \text{ and } S_1 \leq \bar{S}^d\left(S_2\right)\right] \end{cases}$$

where $B_1^D = \frac{2^m}{1+2^m}B_1^M$ and $B_2^D = \frac{1}{1+2^m}B_2^M$.

Proof: Geopolitical payoff functions are given by:

$$B_1(k_I,k_E)=\theta(k_I,k_E)B_1^M-S_1(k_E)=\frac{(k_I)^m}{(k_I)^m+(k_E)^m}B_1^M-S_1(k_E)$$
$$B_2(k_E,k_I)=[1-\theta(k_I,k_E)]B_2^M-S_2(k_E)=\frac{(k_E)^m}{(k_I)^m+(k_E)^m}B_2^M-S_2(k_E)$$

From we Proposition 1 we have:

If $0\le S_2\le \bar{S}^b$, then entry is blocked. Hence, $k_E=0$, which implies $\theta(k_I,k_E)=1$. Thus, $B_1=B_1^M-S_1$ and $B_2=0$.

If $\bar{S}^b<S_2<(\bar{S}^d)^{-1}(S_1)$, then entry is deterred. Hence, $k_E=0$, which implies $\theta(k_I,k_E)=1$. Thus, $B_1=B_1^M-S_1$ and $B_2=0$.

If $S_2=(\bar{S}^d)^{-1}(S_1)$, then entry is either deterred or accommodated. If entry is deterred, then $k_E=0$, which implies $\theta(k_I,k_E)=1$. Thus, $B_1=B_1^M-S_1$ and $B_2=0$. If entry is accommodated, then $k_E=(a-c)/4b$ and $k_I=\bar{k}^m=(a-c)/2b$, which implies that $\theta(k_I,k_E)=2^m/(1+2^m)$. Thus, $B_1=[2^m/(1+2^m)]B_1^M$ and $B_2=[1/(1+2^m)]B_2^M-S_2$.

If $(\bar{S}^d)^{-1}(S_1)<S_2\le F$ entry is accommodated. Moreover, $k_E=(a-c)/4b$ and $k_I=\bar{k}^m=(a-c)/2b$, which implies that $\theta(k_I,k_E)=2^m/(1+2^m)$. Thus, $B_1=[2^m/(1+2^m)]B_1^M$ and $B_2=[1/(1+2^m)]B_2^M-S_2$. ■

Lemma 2 ***Geopolitical trade-off for*** G_1**.** *Suppose that Assumptions 1 (i.e.,* $k_I\in\left[0,\frac{a-c}{b}\right]$*,* $k_E\in\left[0,\frac{a-c}{b}-k_I\right]$*, and* $a\le 2c$*), 2 (i.e.,* $F>\frac{(a-c)^2}{16b}$*), and 3 (i.e.,* $A_1b<4/7$*) hold. Let* $\Delta(S)=\frac{\left[a-c-2\sqrt{b(F-S)}\right]^2}{2}-\frac{9(a-c)^2}{32}$*. Then:*

1. *If* $B_1^M-B_1^D\le\bar{S}^d(F)-A_1\Delta(F)$*, then there exists a unique* $\tilde{S}_1\in\left(0,\bar{S}^d(F)\right]$ *such that* $B_1^M-B_1^D>S_1-A_1\Delta\left((\bar{S}^d)^{-1}(S_1)\right)$ *for all* $S_1\in\left[0,\tilde{S}_1\right)$*,* $B_1^M-B_1^D=\tilde{S}_1-A_1\Delta\left((\bar{S}^d)^{-1}\left(\tilde{S}_1\right)\right)$*, and* $B_1^M-B_1^D<S_1-A_1\Delta\left((\bar{S}^d)^{-1}(S_1)\right)$ *for all* $S_1\in\left(\tilde{S}_1,\bar{S}^d(F)\right]$*.*

2. *If* $B_1^M-B_1^D>\bar{S}^d(F)-A_1\Delta(F)$*, then* $B_1^M-B_1^D>S_1-A_1\Delta\left((\bar{S}^d)^{-1}(S_1)\right)$ *for all* $S_1\in\left[0,\bar{S}^d(F)\right]$*.*

3. *Moreover,* $\bar{S}^d(F)-A_1\Delta(F)>0$ *if and only if* $A_1b<4/7$*.*

Proof: Define

$$\Delta W_1(S_1)=B_1^M-B_1^D+A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)-S_1,$$

where $\Delta(S)=\frac{\left[a-c-2\sqrt{b(F-S)}\right]^2}{2}-\frac{9(a-c)^2}{32}$ and $(\bar{S}^d)^{-1}$ is the inverse of $\bar{S}^d(S)=\frac{(a-c)^2}{8b}-2(a-c)\sqrt{\frac{F-S}{b}}+4(F-S)$. $\Delta W_1(S_1)$ is continuously differentiable for all $S_1\in\left[0,\bar{S}^d(F)\right]$. Take the derivative of $\Delta W_1(S_1)$ with respect to S_1:

$$\frac{\partial\Delta W_1(S_1)}{\partial S_1}=A^1\left[\frac{\partial\Delta\left((\bar{S}^d)^{-1}(S_1)\right)}{\partial(\bar{S}^d)^{-1}(S_1)}\right]\left[\frac{\partial(\bar{S}^d)^{-1}(S_1)}{\partial S_1}\right]-1$$

where

$$\frac{\partial\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)}{\partial\left(\bar{S}^d\right)^{-1}(S_1)} = \frac{\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]\sqrt{b}}{\sqrt{F-\left(\bar{S}^d\right)^{-1}(S_1)}}$$

Due to the implicit function theorem,

$$\frac{d\left(\bar{S}^d\right)^{-1}(S_1)}{dS_1} = \left[\frac{d\bar{S}^d\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)}{dS_1}\right]^{-1} = \left[\frac{a-c}{\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}}-4\right]^{-1}$$

It is easy to verify that $\partial\Delta W_1(S_1)/\partial S_1 < 0$ if and only if $\left(\bar{S}^d\right)^{-1}(S_1) > S_2' = F - \left[\left(1-A^1b\right)^2(a-c)^2/4\left(2-A^1b\right)^2b\right]$. Thus, $\partial\Delta W_1(S_1)/\partial S_1 < 0$ if and only if $S_1 > \bar{S}^d(S_2')$. Note that $\bar{S}^d(S_2') < 0$ if and only if $A^1b < 2\left(\sqrt{2}-1\right) \approx 0.828$, which holds due to Assumption 3. Therefore, $\partial\Delta W_1(S_1)/\partial S_1 < 0$ for all $S_1 \geq 0$, which implies that $\Delta W_1(S_1)$ is an strictly decreasing function of S_1 for all $S_1 \in \left[0, \bar{S}^d(F)\right]$. Since $\Delta W_1(S_1)$ is a continuous and strictly decreasing function of S_1 for all $S_1 \in \left[0, \bar{S}^d(F)\right]$ and $\Delta W_1(0) = B_1^M - B_1^D + A_1\Delta\left(\bar{S}_0^d\right) > 0$, there are two possible cases to consider:

Case 1: Suppose that $\Delta W_1\left(\bar{S}^d(F)\right) \leq 0$ or, which is equivalent, $B_1^M - B_1^D \leq \bar{S}^d(F) - A^1\Delta(F)$. Then, there exists a unique $\tilde{S}_1 \in \left(0, \bar{S}^d(F)\right]$ such that $B_1^M - B_1^D > S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$ for all $S_1 \in \left[0, \tilde{S}_1\right)$, $B_1^M - B_1^D = \tilde{S}_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$, and $B_1^M - B_1^D < S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$ for all $S_1 \in \left(\tilde{S}_1, \bar{S}^d(F)\right]$.

Case 2: Suppose that $\Delta W_1\left(\bar{S}^d(F)\right) > 0$ or, which is equivalent, $B_1^M - B_1^D > \bar{S}^d(F) - A_1\Delta(F)$. Then, $B_1^M - B_1^D > S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$ for all $S_1 \in \left[0, \bar{S}^d(F)\right]$.

Finally, note that $B_1^M - B_1^D \leq \bar{S}^d(F) - A_1\Delta(F)$ can hold only if $\bar{S}^d(F) - A_1\Delta(F) > 0$. Since $\bar{S}^d(F) - A_1\Delta(F) = \frac{(a-c)^2}{8b} - A_1\frac{7(a-c)^2}{32}$, $\bar{S}^d(F) - A_1\Delta(F) > 0$ if and only if $A_1b < 4/7$, which holds due to Assumption 3. ■

Lemma 3 ***Geopolitical trade-off for*** G_2**.** *Suppose that Assumptions 1 (i.e.,* $k_I \in \left[0, \frac{a-c}{b}\right]$*,* $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$*, and* $a \leq 2c$*) and 2 (i.e.,* $F > \frac{(a-c)^2}{16b}$*) hold. Let* $\Delta(S) = \frac{\left[a-c-2\sqrt{b(F-S)}\right]^2}{2} - \frac{9(a-c)^2}{32}$*. Then:*

1. *If* $B_2^D \leq \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$*, then* $B_2^D < S_2 + A_2\Delta(S_2)$ *for all* $S_2 \in \left(\bar{S}_0^d, F\right]$*.*

2. *If* $\bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq F + A_2\Delta(F)$*, then, there exists a unique* $\tilde{S}_2 \in \left(\bar{S}_0^d, F\right]$ *such that* $B_2^D > S_2 + A_2\Delta(S_2)$ *for all* $S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$*,* $B_2^D = \tilde{S}_2 + A_2\Delta\left(\tilde{S}_2\right)$*, and* $B_2^D < S_2 + A_2\Delta(S_2)$ *for all* $S_2 \in \left(\tilde{S}_2, F\right]$*.*

3. *If* $B_2^D > F + A_2\Delta(F)$*, then* $B_2^D > S_2 + A_2\Delta(S_2)$ *for all* $S_2 \in \left[\bar{S}_0^d, F\right]$*.*

Proof: Define

$$\Delta W_2(S_2) = B_2^D - A_2\Delta(S_2) - S_2,$$

where $\Delta(S_2) = \frac{\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} - \frac{9(a-c)^2}{32}$. Note that $\Delta W_2(S_2)$ is a continuous and strictly decreasing function of S_2 for all $S_2 \in \left[\bar{S}_0^d, F\right]$. Thus, that there are three possible cases to consider.

Case 1: Suppose that $\Delta W_2\left(\bar{S}_0^d\right) \leq 0$ or, which is equivalent, $B_2^D \leq \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$. Then, $B_2^D < S_2 + A_2\Delta(S_2)$ for all $S_2 \in \left(\bar{S}_0^d, F\right]$.

Case 2: Suppose that $\Delta W_2(F) \leq 0 < \Delta W_2\left(\bar{S}_0^d\right)$ or, which is equivalent, $\bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq F + A_2\Delta(F)$. Then, there exists a unique $\tilde{S}_2 \in \left(\bar{S}_0^d, F\right]$ such that $B_2^D > S_2 + A_2\Delta(S_2)$ for all $S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$, $B_2^D = \tilde{S}_2 + A_2\Delta\left(\tilde{S}_2\right)$, and $B_2^D < S_2 + A_2\Delta(S_2)$ for all $S_2 \in \left(\tilde{S}_2, F\right]$.

Case 3: Suppose that $\Delta W_2(F) > 0$ or, which is equivalent, $B_2^D > F + A_2\Delta(F)$. Then, $B_2^D > S_2 + A_2\Delta(S_2)$ for all $S_2 \in \left[\bar{S}_0^d, F\right]$. ■

Propositions 2 and 3. *Suppose that Assumptions 1 (i.e.,* $k_I \in \left[0, \frac{a-c}{b}\right]$*,* $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$*, and* $a \leq 2c$*), 2 (i.e.,* $F > \frac{(a-c)^2}{16b}$*), and 3 (i.e.,* $A_1 b < 4/7$*) hold,* $B_1^M - B_1^D \leq \bar{S}^d(F) - A_1\Delta(F)$ *and* $B_2^D \leq F + A_2\Delta(F)$*. Let* $\Delta(S) = \frac{\left[a-c-2\sqrt{b(F-S)}\right]^2}{2} - \frac{9(a-c)^2}{32}$*. Then:*

1. *Suppose that* $B_2^D \leq \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$*. Then, the set of equilibrium subsidies is given by* $S_1 = \bar{S}^d(S_2)$ *with* $S_2 \in \left[\bar{S}_0^d, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$*, where* $\tilde{S}_1 \in \left(0, \bar{S}^d(F)\right]$ *is the unique solution to* $B_1^M - B_1^D = \tilde{S}_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$*. Moreover, in all these equilibria entry is deterred.*

2. *Suppose that* $\bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq F + A_2\Delta(F)$*. Then, the set of equilibrium subsidies is given by* $S_1 = \bar{S}^d(S_2)$ *with:*
$$\begin{array}{ll} S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right) & \text{if } \tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right) \\ S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right] & \text{if } \tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right) \end{array}$$
where $\tilde{S}_1 \in \left(0, \bar{S}^d(F)\right]$ *is the unique solution to* $B_1^M - B_1^D = \tilde{S}_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$ *and* $\tilde{S}_2 \in \left(\bar{S}_0^d, F\right]$ *is the unique solution to* $B_2^D = \tilde{S}_2 + A_2\Delta\left(\tilde{S}_2\right)$*. Moreover, in all the equilibria in which* $S_2 \in \left[\tilde{S}_2, F\right]$ *entry is deterred, while in all the equilibria in which* $S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$ *there is accommodated entry.*

Proof:

Payoff functions: The consumer surplus of country j as a function of the price is $CS_j(P) = A_j(a-P)^2/2$. Thus, employing Proposition 1, the consumer surplus of each country as a function of (S_1, S_2) is given by:

$$CS_j(S_1, S_2) = A_j \begin{cases} \frac{(a-c)^2}{8} & \text{if } 0 \leq S_2 \leq \bar{S}^b \\ \frac{\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ \frac{\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}_0^d \leq S_2 < \left(\bar{S}^d\right)^{-1}(S_1) \\ \frac{\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} \text{ or } \frac{9(a-c)^2}{32} & \text{if } \bar{S}_0^d \leq S_2 = \left(\bar{S}^d\right)^{-1}(S_1) \\ \frac{9(a-c)^2}{32} & \text{if } \bar{S}_0^d \leq \left(\bar{S}^d\right)^{-1}(S_1) < S_2 \leq F \end{cases}$$

Employing Proposition 1 and Lemma 1, the geopolitical payoff of each global power as a function of (S_1, S_2) is given by:

$$B_1(S_1,S_2)=\begin{cases} B_1^M - S_1 & \textit{if } 0 \le S_2 \le \bar{S}^b \\ B_1^M - S_1 & \textit{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ B_1^M - S_1 & \textit{if } \bar{S}_0^d \le S_2 < \left(\bar{S}^d\right)^{-1}(S_1) \\ B_1^M - S_1 \textit{ or } B_1^D & \textit{if } \bar{S}_0^d \le S_2 = \left(\bar{S}^d\right)^{-1}(S_1) \\ B_1^D & \textit{if } \bar{S}_0^d \le \left(\bar{S}^d\right)^{-1}(S_1) < S_2 \le F \end{cases}$$

$$B_2(S_1,S_2)=\begin{cases} 0 & \textit{if } 0 \le S_2 \le \bar{S}^b \\ 0 & \textit{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ 0 & \textit{if } \bar{S}_0^d \le S_2 < \left(\bar{S}^d\right)^{-1}(S_1) \\ B_2^D - S_2 \textit{ or } 0 & \textit{if } \bar{S}_0^d \le S_2 = \left(\bar{S}^d\right)^{-1}(S_1) \\ B_2^D - S_2 & \textit{if } \bar{S}_0^d \le \left(\bar{S}^d\right)^{-1}(S_1) < S_2 \le F \end{cases}$$

Finally, the payoff function of each global power as a function of (S_1, S_2) is given by:

$$W_j(S_1,S_2) = CS_j(S_1,S_2) + B_j(S_1,S_2)$$

Selection criterion: From Proposition 1.2.b, if $S_1 = \bar{S}^d(S_2)$, deterrence and accommodation are both subgame perfect Nash equilibria. In such a case, the equilibrium with accommodation is selected when it strictly dominates the equilibrium with deterrence for G_2. Otherwise, the economic equilibrium with deterrence is selected. Thus,

$$W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right) = \max\left\{ \begin{array}{l} \frac{A_2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]^2}{2}, \\ \frac{A_2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}(S_1) \end{array} \right\}$$

Best response correspondence of G_2: Employing the above selection criterion, the payoff function of G_2 as a function of (S_1, S_2) is given by:

$$W_2(S_1,S_2)=\begin{cases} \frac{A_2(a-c)^2}{8} & \textit{if } 0 \le S_2 \le \bar{S}^b \\ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \textit{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \textit{if } \bar{S}_0^d \le S_2 < \left(\bar{S}^d\right)^{-1}(S_1) \\ \max\left\{ \begin{array}{l} \frac{A_2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]^2}{2}, \\ \frac{A_2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}(S_1) \end{array} \right\} & \textit{if } S_2 = \left(\bar{S}^d\right)^{-1}(S_1) \\ \frac{A_2 9(a-c)^2}{32} + B_2^D - S_2 & \textit{if } \left(\bar{S}^d\right)^{-1}(S_1) < S_2 \le F \end{cases}$$

$W_2(S_1,S_2)$ is a constant for all $S_2 \in \left[0, \bar{S}^b\right]$, it is strictly increasing in S_2 for all $S_2 \in \left[\bar{S}^b, \left(\bar{S}^d\right)^{-1}(S_1)\right)$, and it is strictly decreasing in S_2 for all $S_2 \in \left(\left(\bar{S}^d\right)^{-1}(S_1), F\right]$. This does not immediately imply that

$W_2(S_1, S_2)$ has its unique global maximum at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$. The reason is that $W_2(S_1, S_2)$ might not be continuous at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$.[21] However, note that $W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right)$ adopts the maximum between the left and right limits of the function at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ and both of these limits exist. Therefore, it is always the case that $W_2(S_1, S_2)$ adopts its unique global maximum at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$. Thus, the best response correspondence of G_2 is given by:

$$S_2 = \left(\bar{S}^d\right)^{-1}(S_1) \text{ for all } S_1 \in \left[0, \bar{S}^d(F)\right]$$

Economic equilibrium selection under $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$: To determine if $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ leads to deterrence or accommodated entry, we must study $W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right)$. Note that

$$W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right) = \begin{cases} \frac{A_2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]^2}{2} & \text{if } \Delta W_2\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) \leq 0 \\ \frac{A_2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}(S_1) & \text{if } \Delta W_2\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) > 0 \end{cases}$$

where $\Delta W_2(S_2) = B_2^D - A_2\Delta(S_2) - S_2$.

Since $B_2^D \leq F + A_2\Delta(F)$, Lemma 3 implies that there are two possible cases to consider.

Case 1: Suppose that $B_2^D \leq \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$. Then, $B_2^D < S_2 + A_2\Delta(S_2)$ for all $S_2 \in \left(\bar{S}_0^d, F\right]$. Therefore, $W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right) = A_2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]^2/2$ for all $S_1 \in \left[0, \bar{S}^d(F)\right]$. That is, $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ leads to deterrence.

Case 2: Suppose that $B_2^D > \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$. Then, there exists a unique $\tilde{S}_2 \in \left(\bar{S}_0^d, F\right]$ such that $B_2^D > S_2 + A_2\Delta(S_2)$ for all $S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$, $B_2^D = \tilde{S}_2 + A_2\Delta\left(\tilde{S}_2\right)$, and $B_2^D < S_2 + A_2\Delta(S_2)$ for all $S_2 \in \left(\tilde{S}_2, F\right]$. Therefore,

$$W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right) = \begin{cases} \frac{A_2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}(S_1) & \text{if } 0 \leq S_1 < \bar{S}^d\left(\tilde{S}_2\right) \\ \frac{A_2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]^2}{2} & \text{if } \bar{S}^d\left(\tilde{S}_2\right) \leq S_1 \leq \bar{S}^d(F) \end{cases}$$

That is, $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ leads to accommodated entry when $S_1 < \bar{S}^d\left(\tilde{S}_2\right)$ and to deterrence when $S_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$.

Best response correspondence of G_1. The payoff function of G_1 as a function of (S_1, S_2) is given

[21] $W_2(S_1, S_2)$ is always a continuous function of S_2 for all $S_2 \in \left[0, \left(\bar{S}^d\right)^{-1}(S_1)\right)$ and $S_2 \in \left(\left(\bar{S}^d\right)^{-1}(S_1), F\right]$. In particular, it is continuous at $S_2 = \bar{S}^b$.

by:

$$W_1(S_1,S_2)=\begin{cases}\frac{A_1(a-c)^2}{8}+B_1^M-S_1 & if\ 0\le S_2\le \bar{S}^b\\ \frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}+B_1^M-S_1 & if\ \bar{S}^b<S_2<\bar{S}_0^d\\ \frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}+B_1^M-S_1 & if\ \bar{S}_0^d\le S_2\le F\ and\ S_1>\bar{S}^d(S_2)\\ \frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}+B_1^M-S_1\ or\ \frac{A^19(a-c)^2}{32}+B_1^D & if\ \bar{S}_0^d\le S_2\le F\ and\ S_1=\bar{S}^d(S_2)\\ \frac{A_19(a-c)^2}{32}+B_1^D & if\ \bar{S}_0^d\le S_2\le F\ and\ S_1<\bar{S}^d(S_2)\end{cases}$$

If $0\le S_2\le \bar{S}^b$, then, $W_1(S_1,S_2)=\left[A_1(a-c)^2/8\right]+B_1^M-S_1$, which is strictly decreasing in S_1. Thus, the best response to $0\le S_2\le\bar{S}^b$ is always $S_1=0$. Similarly, if $\bar{S}^b<S_2<\bar{S}_0^d$, then $W_1(S_1,S_2)=A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2/2+B_1^M-S_1$, which is strictly decreasing in S_1. Thus, the best response to $\bar{S}^b<S_2<\bar{S}_0^d$ is always $S_1=0$.

If $\bar{S}_0^d\le S_2\le F$, there are two possible cases to consider:.

Case 1: Suppose that $B_2^D\le\bar{S}_0^d+A_2\Delta\left(\bar{S}_0^d\right)$. Then:

$$W_1(S_1,S_2)=\begin{cases}\frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}+B_1^M-S_1 & if\ S_1>\bar{S}^d(S_2)\\ \frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}+B_1^M-\bar{S}^d(S_2) & if\ S_1=\bar{S}^d(S_2)\\ \frac{A_19(a-c)^2}{32}+B_1^D & if\ S_1<\bar{S}^d(S_2)\end{cases}$$

Therefore, if $B_1^M-B_1^D>\bar{S}^d(S_2)-A_1\Delta(S_2)$, then $W_1(S_1,S_2)$ adopts its maximum at $S_1=\bar{S}^d(S_2)$; if $B_1^M-B_1^D=\bar{S}^d(S_2)-A_1\Delta(S_2)$, then $W_1(S_1,S_2)$ adopts its maximum at $S_1\in\left[0,\bar{S}^d(S_2)\right]$; finally if $B_1^M-B_1^D<\bar{S}^d(S_2)-A_1\Delta(S_2)$, then $W_1(S_1,S_2)$ adopts its maximum at $S_1\in\left[0,\bar{S}^d(S_2)\right)$. Since $B_1^M-B_1^D\le\bar{S}^d(F)-A_1\Delta(F)$, Lemma 2 implies that there exists a unique $\tilde{S}_1\in\left(0,\bar{S}^d(F)\right]$ such that $B_1^M-B_1^D>S_1-A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$ for all $S_1\in\left[0,\tilde{S}_1\right)$, $B_1^M-B_1^D=\tilde{S}_1-A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$, and $B_1^M-B_1^D<S_1-A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$ for all $S_1\in\left(\tilde{S}_1,\bar{S}^d(F)\right]$. Therefore, the best response correspondence of G_1 is given by:

$$S_1=\begin{cases}0 & if\ 0\le S_2\le\bar{S}_0^d\\ \bar{S}^d(S_2) & if\ \bar{S}_0^d<S_2<\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\\ \left[0,\bar{S}^d(S_2)\right] & if\ S_2=\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\\ \left[0,\bar{S}^d(S_2)\right) & if\ \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)<S_2\le F\end{cases}$$

Case 2: Suppose that $B_2^D > \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$. Then:

$$W_1\left(S_1, S_2\right) = \begin{cases} \frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - S_1 & \textit{if } S_1 > \bar{S}^d\left(S_2\right) \\ \frac{A_1 9(a-c)^2}{32} + B_1^D & \textit{if } S_1 = \bar{S}^d\left(S_2\right) \textit{ and } S_2 < \tilde{S}_2 \\ \frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - \bar{S}^d\left(S_2\right) & \textit{if } S_1 = \bar{S}^d\left(S_2\right) \textit{ and } S_2 \geq \tilde{S}_2 \\ \frac{A_1 9(a-c)^2}{32} + B_1^D & \textit{if } S_1 < \bar{S}^d\left(S_2\right) \end{cases}$$

$$W_1\left(S_1, S_2\right) = \begin{cases} \frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - S_1 & \textit{if } S_1 > \bar{S}^d\left(S_2\right) \\ \frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - \bar{S}^d\left(S_2\right) & \textit{if } S_1 = \bar{S}^d\left(S_2\right) \\ \frac{A_1 9(a-c)^2}{32} + B_1^D & \textit{if } S_1 < \bar{S}^d\left(S_2\right) \end{cases}$$

If $S_2 < \tilde{S}_2$, $W_1\left(S_1, S_2\right)$ adopts its maximum at $S_1 \in \left[0, \bar{S}^d\left(S_2\right)\right]$ if and only if $B_1^M - B_1^D \leq \bar{S}^d\left(S_2\right) - A_1\Delta\left(S_2\right)$. Otherwise, there is no $S_1 \in \left[0, \bar{S}^d\left(F\right)\right]$ that maximizes $W_1\left(S_1, S_2\right)$. If $S_2 \geq \tilde{S}_2$, $W_1\left(S_1, S_2\right)$ adopts its maximum at $S_1 = \bar{S}^d\left(S_2\right)$ when $B_1^M - B_1^D > \bar{S}^d\left(S_2\right) - A_1\Delta\left(S_2\right)$; it adopts its maximum at $S_1 \in \left[0, \bar{S}^d\left(S_2\right)\right]$ when $B_1^M - B_1^D = \bar{S}^d\left(S_2\right) - A_1\Delta\left(S_2\right)$; and it adopts its maximum at $S_1 \in \left[0, \bar{S}^d\left(S_2\right)\right)$ when $B_1^M - B_1^D < \bar{S}^d\left(S_2\right) - A_1\Delta\left(S_2\right)$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d\left(S_2\right)\right] & \textit{if } \bar{S}_0^d \leq S_2 < \tilde{S}_2 \textit{ and } B_1^M - B_1^D \leq \bar{S}^d\left(S_2\right) - A_1\Delta\left(S_2\right) \\ \bar{S}^d\left(S_2\right) & \textit{if } \tilde{S}_2 \leq S_2 \leq F \textit{ and } B_1^M - B_1^D > \bar{S}^d\left(S_2\right) - A_1\Delta\left(S_2\right) \\ \left[0, \bar{S}^d\left(S_2\right)\right] & \textit{if } \tilde{S}_2 \leq S_2 \leq F \textit{ and } B_1^M - B_1^D = \bar{S}^d\left(S_2\right) - A_1\Delta\left(S_2\right) \\ \left[0, \bar{S}^d\left(S_2\right)\right) & \textit{if } \tilde{S}_2 \leq S_2 \leq F \textit{ and } B_1^M - B_1^D < \bar{S}^d\left(S_2\right) - A_1\Delta\left(S_2\right) \end{cases}$$

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d\left(S_2\right)\right] & \textit{if } \bar{S}_0^d \leq S_2 < \tilde{S}_2 \textit{ and } B_1^M - B_1^D \leq \bar{S}^d\left(S_2\right) - A_1\Delta\left(S_2\right) \\ \bar{S}^d\left(S_2\right) & \textit{if } \tilde{S}_2 \leq S_2 \leq F \textit{ and } B_1^M - B_1^D \geq \bar{S}^d\left(S_2\right) - A_1\Delta\left(S_2\right) \\ \left[0, \bar{S}^d\left(S_2\right)\right) & \textit{if } \tilde{S}_2 \leq S_2 \leq F \textit{ and } B_1^M - B_1^D \leq \bar{S}^d\left(S_2\right) - A_1\Delta\left(S_2\right) \end{cases}$$

Since $B_1^M - B_1^D \leq \bar{S}^d\left(F\right) - A_1\Delta\left(F\right)$, Lemma 2 implies that there exists a unique $\tilde{S}_1 \in \left(0, \bar{S}^d\left(F\right)\right]$ such that $B_1^M - B_1^D > S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(S_1\right)\right)$ for all $S_1 \in \left[0, \tilde{S}_1\right)$, $B_1^M - B_1^D = \tilde{S}_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$, and $B_1^M - B_1^D < S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(S_1\right)\right)$ for all $S_1 \in \left(\tilde{S}_1, \bar{S}^d\left(F\right)\right]$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \bar{S}^d(S_2) & \textit{if } \tilde{S}_2 \leq S_2 < (\bar{S}^d)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } S_2 = (\bar{S}^d)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } (\bar{S}^d)^{-1}\left(\tilde{S}_1\right) < S_2 \leq F \end{cases} \quad \textit{when } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \geq \tilde{S}_2$$

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } (\bar{S}^d)^{-1}\left(\tilde{S}_1\right) \leq S_2 < \tilde{S}_2 \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } \tilde{S}_2 \leq S_2 \leq F \end{cases} \quad \textit{when } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) < \tilde{S}_2$$

Nash equilibrium: We must consider two possible cases.

Case 1: Suppose that $B_2^D \leq \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 \leq \bar{S}_0^d \\ \bar{S}^d(S_2) & \textit{if } \bar{S}_0^d < S_2 \leq (\bar{S}^d)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } (\bar{S}^d)^{-1}\left(\tilde{S}_1\right) \leq S_2 \leq F \end{cases} \quad \textit{and } S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d(S_2) \; \textit{and } S_2 \in \left[\bar{S}_0^d, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$$

Moreover, in all these equilibria entry is deterred. From Lemma 2, we have that $S_1 \in \left[0, \tilde{S}_1\right]$ if and only $B_1^M - B_1^D \geq S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$. Thus, in all these equilibria we have that $B_1^M - B_1^D \geq S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$. From Lemma 3, we have that $B_2^D \leq S_2 + A_2\Delta(S_2)$ for all $S_2 \in \left[\bar{S}_0^d, F\right]$. Thus, in all these equilibria we have that $B_2^D \leq S_2 + A_2\Delta(S_2)$.

Case 2: Suppose $B_2^D > \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \bar{S}^d(S_2) & \textit{if } \tilde{S}_2 \leq S_2 < (\bar{S}^d)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } S_2 = (\bar{S}^d)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } (\bar{S}^d)^{-1}\left(\tilde{S}_1\right) < S_2 \leq F \end{cases} \quad \textit{when } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \geq \tilde{S}_2$$

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } (\bar{S}^d)^{-1}\left(\tilde{S}_1\right) \leq S_2 < \tilde{S}_2 \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } \tilde{S}_2 \leq S_2 \leq F \end{cases} \quad \textit{when } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) < \tilde{S}_2$$

$$\textit{and } S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$[S_1 = \bar{S}^d(S_2) \ \ and \ S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)] \ when \ \tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$$
$$[S_1 = \bar{S}^d(S_2) \ \ and \ S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]] \ when \ \tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$$

Moreover, in all the equilibria in which $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$, entry is accommodated, while in the equilibria in which $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$ entry is deterred.

Suppose that $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, where $\tilde{S}_1 \in \left(0, \bar{S}^d(F)\right]$ is the unique solution to $B_1^M - B_1^D = \tilde{S}_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$ and $\tilde{S}_2 \in \left(\bar{S}_0^d, F\right]$ is the unique solution to $B_2^D = \tilde{S}_2 + A_2\Delta\left(\tilde{S}_2\right)$. Then, the set of equilibrium subsidies is given by $S_1 = \bar{S}^d(S_2)$ with $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$. From Lemma 2, $S_1 \in \left(\tilde{S}_1, \bar{S}^d(F)\right]$ if and only if $B_1^M - B_1^D < S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$. From Lemma 3, $S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$ if and only if $B_2^D > S_2 + A_2\Delta(S_2)$. Thus, in all these equilibria, $B_1^M - B_1^D < S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$ and $B_2^D > S_2 + A_2\Delta(S_2)$.

Suppose that $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, where $\tilde{S}_1 \in \left(0, \bar{S}^d(F)\right]$ is the unique solution to $B_1^M - B_1^D = \tilde{S}_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$ and $\tilde{S}_2 \in \left(\bar{S}_0^d, F\right]$ is the unique solution to $B_2^D = \tilde{S}_2 + A_2\Delta\left(\tilde{S}_2\right)$. Then, the set of equilibrium subsidies is given by $S_1 = \bar{S}^d(S_2)$ with $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$. From Lemma 2, $S_1 \in \left[0, \tilde{S}_1\right]$ if and only if $B_1^M - B_1^D \geq S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$. From Lemma 3, $S_2 \in \left[\tilde{S}_2, F\right]$ if and only if $B_2^D \leq S_2 + A_2\Delta(S_2)$. Thus, in all these equilibria, $B_1^M - B_1^D \geq S_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$ and $B_2^D \leq S_2 + A_2\Delta(S_2)$.

This completes the proofs of Propositions 2 and 3. ■

Proposition 4 ***Comparative statics.*** *Suppose that Assumptions 1 (i.e.,* $k_I \in \left[0, \frac{a-c}{b}\right]$*,* $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$*, and* $a \leq 2c$*), 2 (i.e.,* $F > \frac{(a-c)^2}{16b}$*), and 3 (i.e.,* $A^1 b < 4/7$*) hold,* $B_1^M - B_1^D < \bar{S}^d(F) - A_1\Delta(F)$*, and* $\bar{S}_0^d + A^2\Delta\left(\bar{S}_0^d\right) < B_2^D < F + A_2\Delta(F)$*. Then,* $\tilde{S}_1 \in \left(0, \bar{S}^d(F)\right)$ *and* $\tilde{S}_2 \in \left(\bar{S}_0^d, F\right)$*. Moreover,* $\tilde{S}_1$ *(*$\tilde{S}_2$*) is strictly increasing in* $B_1^M - B_1^D$ *(*B_2^D*); and* $\tilde{S}_1$ *and* $\tilde{S}_2$ *are both strictly increasing in* F*.*

Proof:

Nash equilibrium: $\tilde{S}_1 \in \left(0, \bar{S}^d(F)\right)$ *and* $\tilde{S}_2 \in \left(\bar{S}_0^d, F\right)$ are immediate from Propositions 2 and 3.

Comparative statics with respect to $B_1^M - B_1^D$ ***and*** B_2^D: $\tilde{S}_1$ is given by $B_1^M - B_1^D = \tilde{S}_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$. Employing the implicit function theorem we have:

$$\frac{\partial \tilde{S}_1}{\partial\left(B_1^M - B_1^D\right)} = \left\{1 + - \left[A_1 \frac{\partial \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}{\partial\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}\right]\left[\frac{\partial\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}{\partial \tilde{S}_1}\right]\right\}^{-1}$$

We have already proved that $1 - A_1 \left[\frac{\partial \Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)}{\partial \left(\bar{S}^d\right)^{-1}(S_1)} \right] \left[\frac{\partial \left(\bar{S}^d\right)^{-1}(S_1)}{\partial S_1} \right] < 0$ for all $S_1 \in \left[0, \bar{S}^d(F)\right]$ (see the proof of Lemma 2). Therefore, $\frac{\partial \tilde{S}_1}{\partial \left(B_1^M - B_1^D\right)} > 0$.

$\tilde{S}_2$ is given by $B_2^D = \tilde{S}_2 + A_2 \Delta\left(\tilde{S}_2\right)$. Employing the implicit function theorem we have:

$$\frac{\partial \tilde{S}_2}{\partial B_2^D} = \frac{1}{1 + A_2 \frac{\partial \Delta\left(\tilde{S}_2\right)}{\partial \tilde{S}_2}}$$

We have already proved that $1 + A_2 \frac{\partial \Delta\left(\tilde{S}_2\right)}{\partial \tilde{S}_2} > 0$ for all $S_2 \in \left[\bar{S}_0^d, F\right]$ (see the proof of Lemma 3). Therefore, $\frac{\partial \tilde{S}_2}{\partial B_2^D} > 0$.

Comparative statics with respect to F: $\tilde{S}_1$ is given by $B_1^M - B_1^D = \tilde{S}_1 - A_1 \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$. Employing the implicit function theorem we have:

$$\frac{\partial \tilde{S}_1}{\partial F} = \frac{A_1 \left[\frac{\partial \Delta\left(\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)\right)}{\partial F} \right] + A_1 \left[\frac{\partial \Delta\left(\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)\right)}{\partial \left(\bar{S}^d\right)^{-1}(\tilde{S}_1)} \right] \left[\frac{\partial \left(\bar{S}^d\right)^{-1}(\tilde{S}_1)}{\partial F} \right]}{1 - A_1 \left[\frac{\partial \Delta\left(\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)\right)}{\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)} \right] \left[\frac{\partial \left(\bar{S}^d\right)^{-1}(\tilde{S}_1)}{\partial \tilde{S}_1} \right]}$$

where

$$\frac{\partial \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}{\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)} = \frac{\left[a - c - 2\sqrt{b\left(F - \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}\right] b}{\sqrt{b\left(F - \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}}$$

$$\frac{\partial \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}{\partial \tilde{S}_1} = \left[\frac{(a-c)}{\sqrt{b\left(F - \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}} - 4 \right]^{-1}$$

$$\frac{\partial \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}{\partial F} = -\frac{\partial \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}{\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}$$

$$\frac{\partial \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}{\partial F} = -\frac{\partial \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}{\partial \tilde{S}_1}$$

Introducing these expressions we obtain:

$$\frac{\partial \tilde{S}_1}{\partial F} = \frac{-A_1\left[\frac{\partial \Delta\left(\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)\right)}{\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)}\right]\left[1+\frac{\partial\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)}{\partial \tilde{S}_1}\right]}{1-A_1\left[\frac{\partial \Delta\left(\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)\right)}{\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)}\right]\left[\frac{\partial\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)}{\partial \tilde{S}_1}\right]}$$

We have already proved that $1 - A_1\left[\frac{\partial \Delta\left(\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)\right)}{\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)}\right]\left[\frac{\partial\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)}{\partial \tilde{S}_1}\right] < 0$ (see the proof of Lemma 2), $\frac{\partial \Delta\left(\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)\right)}{\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)} > 0$, and $\frac{\partial\left(\bar{S}^d\right)^{-1}(\tilde{S}_1)}{\partial \tilde{S}_1} > 0$. Therefore, $\frac{\partial \tilde{S}_1}{\partial F} > 0$.

$\tilde{S}_2$ is given by $B_2^D = \tilde{S}_2 + A_2\Delta\left(\tilde{S}_2\right)$. Employing the implicit function theorem we have:

$$\frac{\partial \tilde{S}_2}{\partial F} = \frac{-A_2\frac{\partial \Delta(\tilde{S}_2)}{\partial F}}{1 + A_2\frac{\partial \Delta(\tilde{S}_2)}{\partial \tilde{S}_2}}$$

where

$$\frac{\partial \Delta\left(\tilde{S}_2\right)}{\partial \tilde{S}_2} = \frac{-\left[a-c-2\sqrt{b\left(F-\tilde{S}_2\right)}\right]b}{\sqrt{b\left(F-\check{S}_2\right)}}, \quad \frac{\partial \Delta\left(\tilde{S}_2\right)}{\partial F} = -\frac{\partial \Delta\left(\tilde{S}_2\right)}{\partial \tilde{S}_2}$$

Introducing these expressions we obtain:

$$\frac{\partial \tilde{S}_2}{\partial F} = \frac{A_2\frac{\partial \Delta(\tilde{S}_2)}{\partial \tilde{S}_2}}{1 + A_2\frac{\partial \Delta(\tilde{S}_2)}{\partial \tilde{S}_2}}$$

Note that $\frac{\partial \Delta(\tilde{S}_2)}{\partial \tilde{S}_2} > 0$, which implies that $\frac{\partial \tilde{S}_2}{\partial F} > 0$. This completes the proof of Proposition 4. ■

Corollary 4 ***Change in*** *F. Under the assumptions in Proposition 4.*

1. *Suppose that before the increase in F we have $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$. Then, the increase in F has an ambiguous effect on consumers and no effect on geopolitical outcomes.*

2. *Suppose that before and after the increase in F we have $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$. Then, the increase in F has no effect on consumers or geopolitical outcomes.*

3. *Suppose that before the increase in F we have $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$ and after we have $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$. Then, the equilibrium changes from accommodated entry to deterrence, making consumers better off.*

Proof of Part 1: We first prove that whenever $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, a rise in F does not reverse this inequality and, hence, before and after the change in F, in equilibrium, entry is deterred. Then, we study the effect of F on the equilibrium prices under deterrence.

Effect of F ***on*** $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$: Take the derivative of $\tilde{S}_1 - \bar{S}^d\left(\tilde{S}_2\right)$ with respect to F:

$$\frac{\partial\left[\tilde{S}_1 - \bar{S}^d\left(\tilde{S}_2\right)\right]}{\partial F} = \frac{\partial \tilde{S}_1}{\partial F} - \frac{\partial \bar{S}^d\left(\tilde{S}_2\right)}{\partial F} - \frac{\partial \bar{S}^d\left(\tilde{S}_2\right)}{\partial \tilde{S}_2}\frac{\partial \tilde{S}_2}{\partial F}$$

where

$$\frac{\partial \bar{S}^d\left(\tilde{S}_2\right)}{\partial \tilde{S}_2} = \frac{a-c}{\sqrt{\left(F-\tilde{S}_2\right)b}} - 4, \quad \frac{\partial \bar{S}^d\left(\tilde{S}_2\right)}{\partial F} = -\frac{\partial \bar{S}^d\left(\tilde{S}_2\right)}{\partial \tilde{S}_2}$$

Therefore

$$\begin{aligned}\frac{\partial\left[\tilde{S}_1 - \bar{S}^d\left(\tilde{S}_2\right)\right]}{\partial F} &= \frac{\partial \tilde{S}_1}{\partial F} + \frac{\partial \bar{S}^d\left(\tilde{S}_2\right)}{\partial \tilde{S}_2}\left(1 - \frac{\partial \tilde{S}_2}{\partial F}\right) \\ &= \frac{\partial \tilde{S}_1}{\partial F} + \frac{\partial \bar{S}^d\left(\tilde{S}_2\right)}{\partial \tilde{S}_2}\frac{1}{1 + A_2\frac{\partial \Delta\left(\tilde{S}_2\right)}{\partial \tilde{S}_2}}\end{aligned}$$

We have already proved that $\frac{\partial \tilde{S}_1}{\partial F} > 0$. Also note that $\frac{\partial \bar{S}^d\left(\tilde{S}_2\right)}{\partial \tilde{S}_2} > 0$ and $\frac{\partial \Delta\left(\tilde{S}_2\right)}{\partial \tilde{S}_2} > 0$. Therefore, $\frac{\partial\left[\tilde{S}_1 - \bar{S}^d\left(\tilde{S}_2\right)\right]}{\partial F} > 0$. Thus, if $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$ holds for $F = F_0$, then $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$ must also hold for $F = F_1 > F_0$.

Effect of F ***on*** $F - \tilde{S}_2$: Take the derivative of $F - \tilde{S}_2$ with respect to F:

$$\frac{\partial\left(F - \tilde{S}_2\right)}{\partial F} = 1 - \frac{\partial \tilde{S}_2}{\partial F} = \frac{1}{1 + A_2\frac{\partial \Delta\left(\tilde{S}_2\right)}{\partial \tilde{S}_2}} > 0$$

Thus, a rise in F increases $F - \tilde{S}_2$.

Effect of F ***on*** $F-\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$: Take the derivative of $F-\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ with respect to F:

$$\begin{aligned}\frac{\partial\left(F-\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}{\partial F} &= 1-\frac{\partial\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}{\partial F}-\frac{\partial\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}{\partial\tilde{S}_1}\frac{\partial\tilde{S}_1}{\partial F}\\ &= 1+\frac{\partial\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}{\partial\tilde{S}_1}\left[1-\frac{\partial\tilde{S}_1}{\partial F}\right]\\ &= 1+\frac{\partial\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}{\partial\tilde{S}_1}\frac{1+A_1\left[\frac{\partial\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}{\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}\right]}{1-A_1\left[\frac{\partial\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}{\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}\right]\left[\frac{\partial\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}{\partial\tilde{S}_1}\right]}\\ &= \frac{1+\frac{\partial\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}{\partial\tilde{S}_1}}{1-A_1\left[\frac{\partial\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}{\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}\right]\left[\frac{\partial\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}{\partial\tilde{S}_1}\right]}<0\end{aligned}$$

Thus, a rise in F decreases $F-\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$.

Let $F=F_0$ and suppose that $\tilde{S}_1\left(F_0\right)\geq\bar{S}^d\left(\tilde{S}_2\left(F_0\right),F_0\right)$. Then, in equilibrium, entry is deterred and the equilibrium price is given by $P\left(S_2,F_0\right)=a-b\bar{k}^d=c+2\sqrt{b\left(F_0-S_2\right)}$ with $S_2\in\left[\tilde{S}_2\left(F_0\right),\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\left(F_0\right),F_0\right)\right]$. Now, suppose that $F=F_1>F_0$. We have proved that $\tilde{S}_1\left(F_1\right)\geq\bar{S}^d\left(\tilde{S}_2\left(F_1\right),F_1\right)$. Then, in equilibrium, entry is deterred and the equilibrium price is given by $P\left(S_2,F_1\right)=a-b\bar{k}^d=c+2\sqrt{b\left(F_1-S_2\right)}$ for $S_2\in\left[\tilde{S}_2\left(F_1\right),\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\left(F_1\right),F_1\right)\right]$. Comparing $P\left(S_2,F_0\right)$ with $P\left(S_2,F_1\right)$, note that for a given S_2 it is always the case that $P\left(S_2,F_1\right)>P\left(S_2,F_0\right)$. Moreover, we have proved that $F-\tilde{S}_2$ is increasing in $\tilde{S}_2$, which implies that $P\left(\tilde{S}_2\left(F_1\right),F_1\right)>P\left(\tilde{S}_2\left(F_0\right),F_0\right)$. However, we have also proved that $F-\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ is decreasing in F, which implies that $P\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\left(F_1\right),F_1\right),F_1\right)<P\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\left(F_0\right),F_0\right),F_0\right)$. In other words, the increase in F brings a new range of equilibria with higher equilibrium price than the highest equilibrium price before the rise in F, but also a brings a new rage of equilibria with lower equilibrium price than the lowest equilibrium price before the rise in F. Thus, the increase in F has an ambiguous effect on consumers.

Proofs of part 2 and 3: Immediate from Propositions 3, 4 and Corollary 1. ■

A.3 Extensions

In this section we present the proofs of Propositions 5 to 10.

0.1 A.3.1 Limited Geopolitical Threat from G_2 (Proposition 5)

We begin reconsidering Proposition 1 when the maximum subsidy that G_2 can offer is ρF. Then, we prove Proposition 5.

Proposition 1bis *Suppose that Assumptions 1 (i.e., $k_I \in \left[0, \frac{a-c}{b}\right]$, $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$, and $a \leq 2c$) and 2 (i.e., $F > \frac{(a-c)^2}{16b}$) hold, and $S_2 \leq \rho F$ with $\rho \in [0,1]$. Let $\bar{\rho}_0^d = \frac{\bar{S}_0^d}{F}$ and $\bar{\rho}^b = \frac{\bar{S}^b}{F}$.*

1. *Suppose that $\bar{\rho}_0^d \leq \rho \leq 1$. Then:*
   (a) *If $0 \leq S_2 \leq \bar{S}^b$ entry is blocked, $(k_I, k_E) = \left(\frac{a-c}{2b}, 0\right)$ and $P = \frac{a+c}{2}$.*
   (b) *If $\bar{S}^b < S_2 < \bar{S}_0^d$ entry is deterred, $(k_I, k_E) = \left(\frac{a-c-2\sqrt{b(F-S_2)}}{b}, 0\right)$ and $P = c + 2\sqrt{b(F-S_2)}$.*
   (c) *If $\bar{S}_0^d \leq S_2 \leq \rho F$ and $S_1 > \bar{S}^d(S_2)$ entry is deterred, $(k_I, k_E) = \left(\frac{a-c-2\sqrt{b(F-S_2)}}{b}, 0\right)$ and $P = c + 2\sqrt{b(F-S_2)}$.*
   (d) *If $\bar{S}_0^d \leq S_2 \leq \rho F$ and $S_1 = \bar{S}^d(S_2)$, then there are two equilibria: in one equilibrium entry is deterred, while in the other entry is accommodated. Under deterrence (accommodation), (k_I, k_E, P) is as in part c (e).*
   (e) *If $\bar{S}_0^d \leq S_2 \leq \rho F$ and $S_1 < \bar{S}^d(S_2)$ entry is accommodated, $(k_I, k_E) = \left(\frac{a-c}{2b}, \frac{a-c}{4b}\right)$ and $P = \frac{a+3c}{4}$.*

2. *Suppose that $\bar{\rho}^b < \rho < \bar{\rho}_0^d$. Then,*
   (a) *If $0 \leq S_2 \leq \bar{S}^b$ entry is blocked, $(k_I, k_E) = \left(\frac{a-c}{2b}, 0\right)$ and $P = \frac{a+c}{2}$.*
   (b) *If $\bar{S}^b < S_2 \leq \rho F$ entry is deterred, $(k_I, k_E) = \left(\frac{a-c-2\sqrt{b(F-S_2)}}{b}, 0\right)$ and $P = c + 2\sqrt{b(F-S_2)}$.*

3. *Suppose that $0 \leq \rho \leq \bar{\rho}^b$. Then, entry is blocked, $(k_I, k_E) = \left(\frac{a-c}{2b}, 0\right)$ and $P = \frac{a+c}{2}$ for all $0 \leq S_2 \leq F$.*

Proof. From Proposition 1 we have: If $0 \leq S_2 \leq \bar{S}^b$, then entry is blocked; if $\bar{S}^b < S_2 < \bar{S}_0^d$, then entry is deterred; if $\bar{S}_0^d \leq S_2 < \left(\bar{S}^d\right)^{-1}(S_1)$ or, which is equivalent, $[\bar{S}_0^d \leq S_2 \leq F$ and $S_1 > \bar{S}^d(S_2)]$ entry is deterred; if $\bar{S}_0^d < S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ or, which is equivalent, $[\bar{S}_0^d \leq S_2 \leq F$ and $S_1 = \bar{S}^d(S_2^c)]$, then entry is either deterred or accommodated; and, finally, if $\left(\bar{S}^d\right)^{-1}(S_1) < S_2 \leq F$ or, which is equivalent, $[\bar{S}_0^d \leq S_2 \leq F$ and $S_1 < \bar{S}^d(S_2)]$ entry is accommodated. Let $\bar{\rho}^b = \bar{S}^b/F$, $\bar{\rho}^d = \bar{S}_0^d/F$. Then, Proposition 1bis follows by imposing $S_2 \leq \rho F$. ■

Proposition 5 *Suppose that Assumptions 1 (i.e., $k_I \in \left[0, \frac{a-c}{b}\right]$, $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$, and $a \leq 2c$), 2 (i.e., $F > \frac{(a-c)^2}{16b}$), and 3 (i.e., $A_1 b < 4/7$) hold, $B_1^M - B_1^D \leq \bar{S}^d(F) - A_1\Delta(F)$, $B_2^D \leq F + A_2\Delta(F)$, and $S_2 \leq \rho F$ with $\rho \in [0,1]$. Let $\bar{\rho}_0^d = \frac{\bar{S}_0^d}{F}$ and $\bar{\rho}^b = \frac{\bar{S}^b}{F}$.*

1. *Suppose that $\bar{\rho}_0^d \leq \rho < 1$, $B_1^M - B_1^D \leq \bar{S}^d(F) - A_1\Delta(F)$* and *$B_2^D \leq F + A_2\Delta(F)$.*

(a) *If* $\tilde{S}_1 \geq \bar{S}^d(\rho F)$*, then equilibrium subsidies are* $S_1 = \bar{S}^d(\rho F)$ *and* $S_2 = \rho F$*. Moreover, in equilibrium entry is deterred.*

(b) *If* $\tilde{S}_1 < \bar{S}^d(\rho F)$ *and* $\tilde{S}_2 \leq \rho F$*, then Proposition 3 holds.*

(c) *If* $\tilde{S}_1 < \bar{S}^d(\rho F)$ *and* $\tilde{S}_2 > \rho F$*, then equilibrium subsidies are* $S_1 = \bar{S}^d(S_2)$ *and* $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \rho F\right)$*. Moreover, in all these equilibria there is accommodated entry.*

2. *Suppose that* $\bar{\rho}^b < \rho \leq \bar{\rho}_0^d$*. Then, equilibrium subsidies are* $S_1 = 0$ *and* $S_2 = \rho F$*. Moreover, in equilibrium, entry is deterred.*

3. *Suppose that* $0 < \rho \leq \bar{\rho}^b$*. Then, the set of equilibrium subsidies is given by* $S_1 = 0$ *and* $S_2 \in [0, \rho F]$*. Moreover, in equilibrium, entry is blocked.*

Proof of Part 1: Suppose that $\bar{\rho}_0^d \leq \rho < 1$.

Selection criterion: From Proposition 1bis, if $S_1 = \bar{S}^d(S_2)$, deterrence and accommodation are both subgame perfect Nash equilibria. In such a case, the equilibrium with accommodation is selected when it strictly dominates the equilibrium with deterrence for G_2, provided that $\bar{S}^d(S_2) < \rho F$. Otherwise, the economic equilibrium with deterrence is selected. Thus,

$$W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right) = \begin{cases} \max\left\{ \begin{array}{l} \frac{A^2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]^2}{2}, \\ \frac{A^2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}(S_1) \end{array} \right\} & \text{if } \left(\bar{S}^d\right)^{-1}(S_1) < \rho F \\ \frac{A^2\left[a-c-2\sqrt{b\left(F-\rho\bar{S}\right)}\right]^2}{2} & \text{if } \left(\bar{S}^d\right)^{-1}(S_1) \geq \rho F \end{cases}$$

Best response correspondence of G_2. Suppose that $\left(\bar{S}^d\right)^{-1}(S_1) \geq \rho F$ (equivalently, $S_1 \geq \bar{S}^d(\rho F)$). Then, employing the above selection criteria, the payoff function of G_2 as a function of (S_1, S_2) is given by:

$$W_2(S_1, S_2) = \begin{cases} \frac{A^2(a-c)^2}{8} & \text{if } 0 \leq S_2 \leq \bar{S}^b \\ \frac{A^2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ \frac{A^2\left[a-c-2\sqrt{b(F-\min\{S_2,\rho F\})}\right]^2}{2} & \text{if } \bar{S}_0^d \leq \min\{S_2, \rho F\} \leq \left(\bar{S}^d\right)^{-1}(S_1) \end{cases}$$

which adopts a maximum at $S_2 = \rho F$. Suppose that $\left(\bar{S}^d\right)^{-1}(S_1) < \rho F$ (equivalently, $S_1 < \bar{S}^d(\rho F)$). Then, employing the above selection criterion, the payoff function of G_2 as a function of (S_1, S_2) is given by:

$$W_2(S_1,S_2)=\begin{cases}\frac{A_2(a-c)^2}{8} & \text{if } 0\le S_2\le \bar{S}^b\\ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}^b< S_2<\bar{S}_0^d\\ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}_0^d\le S_2<\left(\bar{S}^d\right)^{-1}(S_1)\\ \max\left\{\begin{array}{c}\frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2},\\ \frac{A_2 9(a-c)^2}{32}+B_2^D-S_2\end{array}\right\} & \text{if } S_2=\left(\bar{S}^d\right)^{-1}(S_1)\\ \frac{A_2 9(a-c)^2}{32}+B_2^D-S_2 & \text{if } \left(\bar{S}^d\right)^{-1}(S_1)<S_2\le\rho F\end{cases}$$

which adopts a maximum at $S_2=\left(\bar{S}^d\right)^{-1}(S_1)$. Thus, the best response correspondence of G_2 is given by:

$$S_2=\begin{cases}\left(\bar{S}^d\right)^{-1}(S_1) & \text{if } 0\le S_1<\bar{S}^d(\rho F)\\ \rho F & \text{if } S_1\ge \bar{S}^d(\rho F)\end{cases}$$

Economic equilibrium selection under the best response correspondence of G_2. We must consider three possible cases:

Case 1: Suppose that $S_1\ge\bar{S}^d(\rho F)$. Then, using Proposition 1.bis and the economic selection criterion, $S_2=\rho F$ leads to deterrence.

Case 2: Suppose that $0\le S_1<\bar{S}^d(\rho F)$. To determine if $S_2=\left(\bar{S}^d\right)^{-1}(S_1)$ leads to deterrence or accommodated entry, we use Lemma 3. There are two possible cases to consider:

Case 2.a: Suppose that $\tilde{S}_2\le\rho F$ or, which is equivalent, $B_2^D\le\rho F+A_2\Delta(\rho F)$. Then, $S_2=\left(\bar{S}^d\right)^{-1}(S_1)$ leads to accommodated entry when $S_1<\bar{S}^d\left(\tilde{S}_2\right)$ and to deterrence when $S_1\ge\bar{S}^d\left(\tilde{S}_2\right)$.

Case 2.b: Suppose that $\rho F<\tilde{S}_2\le F$ or, which is equivalent, $\rho F+A_2\Delta(\rho F)<B_2^D\le F+A_2\Delta(F)$. Then, $S_2=\left(\bar{S}^d\right)^{-1}(S_1)$ leads to accommodated entry.

Best response correspondence of G_1. If $0\le S_2\le\bar{S}^b$, then, $W_1(S_1,S_2)=\left[A^1(a-c)^2/8\right]+B_1^M-S_1$, which is strictly decreasing in S_1. Thus, the best response to $0\le S_2\le\bar{S}^b$ is always $S_1=0$. Similarly, if $\bar{S}^b<S_2<\bar{S}_0^d$, then $W_1(S_1,S_2)=A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2/2+B_1^M-S_1$, which is strictly decreasing in S_1. Thus, the best response to $\bar{S}^b<S_2<\bar{S}_0^d$ is always $S_1=0$.

If $\bar{S}_0^d\le S_2\le\rho F$, there are two possible cases to consider:

Case 1: Suppose that $B_2^D\le\rho F+A_2\Delta(\rho F)$. Then, $\tilde{S}_2\le\rho F$ and, hence,

$$W_1(S_1,S_2)=\begin{cases}\frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}+B_1^M-S_1 & \text{if } \bar{S}_0^d\le S_2<\tilde{S}_2 \text{ and } S_1>\bar{S}^d(S_2)\\ \frac{A_1 9(a-c)^2}{32}+B_1^D & \text{if } \bar{S}_0^d\le S_2<\tilde{S}_2 \text{ and } S_1\le\bar{S}^d(S_2)\\ \frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}+B_1^M-S_1 & \text{if } \tilde{S}_2\le S_2\le\rho F \text{ and } S_1\ge\bar{S}^d(S_2)\\ \frac{A_1 9(a-c)^2}{32}+B_1^D & \text{if } \tilde{S}_2\le S_2\le\rho F \text{ and } S_1<\bar{S}^d(S_2)\end{cases}$$

For $\bar{S}_0^d\le S_2<\tilde{S}_2$, $W_1(S_1,S_2)$ adopts its maximum at $S_1\in\left[0,\bar{S}^d(S_2)\right]$ if and only if $B_1^M-B_1^D\le\bar{S}^d(S_2)-A_1\Delta(S_2)$. Otherwise, there is no $S_1\in\left[0,\bar{S}^d(F)\right]$ that maximizes $W_1(S_1,S_2)$. For $\tilde{S}_2\le S_2\le$

ρF, adopts its maximum at $S_1 = \bar{S}^d(S_2)$ if and only if $B_1^M - B_1^D \geq \bar{S}^d(S_2) - A_1\Delta(S_2)$, while it adopts its maximum at $S_1 \in \left[0, \bar{S}^d(S_2)\right)$ if and only if $B_1^M - B_1^D \leq A^1\Delta(S_2) + \bar{S}^d(S_2)$. For $\rho F < S_2 \leq F$, adopts its maximum at $S_1 = \bar{S}^d(\rho F)$ if and only if $B_1^M - B_1^D \geq \bar{S}^d(\rho F) - A_1\Delta(\rho F)$, while it adopts its maximum at $S_1 \in \left[0, \bar{S}^d(\rho F)\right)$ if and only if $B_1^M - B_1^D \leq \bar{S}^d(\rho F) - A_1\Delta(\rho F)$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } \bar{S}_0^d \leq S_2 < \tilde{S}_2 \textit{ and } B_1^M - B_1^D \leq \bar{S}^d(S_2) - A_1\Delta(S_2) \\ \bar{S}^d(S_2) & \textit{if } \tilde{S}_2 \leq S_2 \leq \rho F \textit{ and } B_1^M - B_1^D \geq \bar{S}^d(S_2) - A_1\Delta(S_2) \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } \tilde{S}_2 \leq S_2 \leq \rho F \textit{ and } B_1^M - B_1^D \leq \bar{S}^d(S_2) - A_1\Delta(S_2) \end{cases}$$

Thus, employing Lemma 2, we must consider two possible subcases:

Case 1.a: Suppose that $B_1^M - B_1^D \leq \bar{S}^d(\rho F) - A_1\Delta(\rho F)$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \leq S_2 < \tilde{S}_2 \\ \bar{S}^d(S_2) & \textit{if } \tilde{S}_2 \leq S_2 \leq \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } \max\left\{\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right\} \leq S_2 \leq \rho F \end{cases}$$

Case 1.b: Suppose that $B_1^M - B_1^D > \bar{S}^d(\rho F) - A_1\Delta(\rho F)$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \bar{S}^d(S_2) & \textit{if } \tilde{S}_2 \leq S_2 \leq \rho F \end{cases}$$

Case 2: Suppose that $\rho F + A_2\Delta(\rho F) < B_2^D \leq F + A_2\Delta(F)$. Then, that $\tilde{S}_2 > \rho F$ and, hence,

$$W_1(S_1, S_2) = \begin{cases} \frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - S_1 & \textit{if } \bar{S}_0^d \leq S_2 < \rho F \textit{ and } S_1 > \bar{S}^d(S_2) \\ \frac{A_1 9(a-c)^2}{32} + B_1^D & \textit{if } \bar{S}_0^d \leq S_2 < \rho F \textit{ and } S_1 \leq \bar{S}^d(S_2) \\ \frac{A_1\left[a-c-2\sqrt{b(F-\rho F)}\right]^2}{2} + B_1^M - S_1 & \textit{if } S_2 = \rho F \textit{ and } S_1 \geq \bar{S}^d(\rho F) \\ \frac{A_1 9(a-c)^2}{32} + B_1^D & \textit{if } S_2 = \rho F \textit{ and } S_1 < \bar{S}^d(\rho F) \end{cases}$$

Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } \bar{S}_0^d \leq S_2 < \rho F \textit{ and } B_1^M - B_1^D \leq \bar{S}^d(S_2) - A_1\Delta(S_2) \\ \left[0, \bar{S}^d(\rho F)\right) & \textit{if } S_2 = \rho F \textit{ and } B_1^M - B_1^D \leq \bar{S}^d(\rho F) - A_1\Delta(\rho F) \\ \bar{S}^d(\rho F) & \textit{if } S_2 = \rho F \textit{ and } B_1^M - B_1^D \geq \bar{S}^d(\rho F) - A_1\Delta(\rho F) \end{cases}$$

Thus, employing Lemma 2, we must consider three possible subcases:

Case 2.a: Suppose that $B_1^M - B_1^D \leq \bar{S}^d(\rho F) - A_1\Delta(\rho F)$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \le S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \le S_2 < \rho F \\ \left[0, \bar{S}^d(\rho F)\right) & \textit{if } S_2 = \rho F \textit{ and } \tilde{S}_1 < \bar{S}^d(\rho F) \\ \bar{S}^d(\rho F) & \textit{if } S_2 = \rho F \textit{ and } \tilde{S}_1 = \bar{S}^d(\rho F) \end{cases}$$

Case 2.b: Suppose that $B_1^M - B_1^D > \bar{S}^d(\rho F) - A_1\Delta(\rho F)$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \le S_2 < \bar{S}_0^d \\ \bar{S}^d(\rho F) & \textit{if } S_2 = \rho F \end{cases}$$

Nash equilibrium: We must consider two possible cases:

Case 1: Suppose that $B_2^D \le \rho F + A_2\Delta(\rho F)$ (i.e., $\tilde{S}_2 \le \rho F$).

Case 1.a: Suppose that $B_1^M - B_1^D \le \bar{S}^d(\rho F) - A_1\Delta(\rho F)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \le S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \le S_2 < \tilde{S}_2 \\ \bar{S}^d(S_2) & \textit{if } \tilde{S}_2 \le S_2 \le \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } \max\left\{\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right\} \le S_2 \le \rho F \end{cases}$$

$$S_2 = \begin{cases} \left(\bar{S}^d\right)^{-1}(S_1) & \textit{if } 0 \le S_1 < \bar{S}^d(\rho F) \\ \rho F & \textit{if } S_1 \ge \bar{S}^d(\rho F) \end{cases}$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d(S_2) \textit{ and } S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$$

$$S_1 = \bar{S}^d(S_2) \textit{ and } S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$$

$$S_1 = \bar{S}^d(\rho F) = \tilde{S}_1 \textit{ and } S_2 = \rho F$$

Moreover, in all the equilibria in which $\tilde{S}_1 \ge \bar{S}^d\left(\tilde{S}_2\right)$ entry is deterred, while in all the equilibria in which $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, entry is accommodated. In the equilibrium in which $S_2 = \rho F$ and $S_1 = \bar{S}^d(\rho F) = \tilde{S}_1$, entry is deterred.

Case 1.b: Suppose that $B_1^M - B_1^D > \bar{S}^d(\rho F) - A_1\Delta(\rho F)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \le S_2 < \bar{S}_0^d \\ \bar{S}^d(S_2) & \textit{if } \tilde{S}_2 \le S_2 \le \rho F \end{cases} \quad \textit{and } S_2 = \begin{cases} \left(\bar{S}^d\right)^{-1}(S_1) & \textit{if } 0 \le S_1 < \bar{S}^d(\rho F) \\ \rho F & \textit{if } S_1 \ge \bar{S}^d(\rho F) \end{cases}$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d(S_2) \textit{ and } S_2 \in \left[\tilde{S}_2, \rho F\right)$$
$$S_1 = \bar{S}^d(\rho F) \textit{ and } S_2 = \rho F$$

Moreover, in all these equilibria entry is deterred.

Summing up, for $B_2^D \leq \rho F + A_2\Delta(\rho F)$ we have:

- If $\tilde{S}_1 \geq \bar{S}^d(\rho F)$, then the equilibrium subsidy profiles are those that satisfy $S_1 = \bar{S}^d(\rho F)$ and $S_2 = \rho F$. Moreover, in this equilibrium entry is deterred.
- If $\bar{S}^d\left(\tilde{S}_2\right) \leq \tilde{S}_1 < \bar{S}^d(\rho F)$, then the equilibrium subsidy profiles are those that satisfy $S_1 = \bar{S}^d(S_2)$ and $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$. Moreover, in all these equilibria entry is deterred. That is, Proposition 3.1 holds.
- If $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, then the equilibrium subsidy profiles are those that satisfy $S_1 = \bar{S}^d(S_2)$ and $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$. Moreover, in all these equilibria there is accommodated entry. That is, Proposition 3.2 holds.

Case 2: Suppose that $\rho F + A_2\Delta(\rho F) < B_2^D \leq F + A_2\Delta(F)$.

Case 2.a: Suppose that $B_1^M - B_1^D \leq \bar{S}^d(\rho F) - A_1\Delta(\rho F)$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \leq S_2 < \rho F \\ \left[0, \bar{S}^d(\rho F)\right) & \textit{if } S_2 = \rho F \\ \bar{S}^d(\rho F) & \textit{if } S_2 = \rho F \textit{ and } \tilde{S}_1 = \bar{S}^d(\rho F) \end{cases}$$
$$S_2 = \begin{cases} \left(\bar{S}^d\right)^{-1}(S_1) & \textit{if } 0 \leq S_1 < \bar{S}^d(\rho F) \\ \rho F & \textit{if } S_1 \geq \bar{S}^d(\rho F) \end{cases}$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d(S_2) \textit{ and } S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \rho F\right)$$
$$S_1 = \bar{S}^d(\rho F) = \tilde{S}_1 \textit{ and } S_2 = \rho F$$

Moreover, in all the equilibria in which $\tilde{S}_1 < \bar{S}^d(\rho F)$, entry is accommodated, while in the equilibrium in which $S_2 = \rho F$ and $S_1 = \bar{S}^d(\rho F) = \tilde{S}_1$, entry is deterred.

Case 2.b: Suppose that $B_1^M - B_1^D > \bar{S}^d(\rho F) - A_1\Delta(\rho F)$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \bar{S}^d(\rho F) & \textit{if } S_2 = \rho F \end{cases} \quad \textit{and } S_2 = \begin{cases} \left(\bar{S}^d\right)^{-1}(S_1) & \textit{if } 0 \leq S_1 < \bar{S}^d(\rho F) \\ \rho F & \textit{if } S_1 \geq \bar{S}^d(\rho F) \end{cases}$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d(\rho F) \;\; and \;\; S_2 = \rho F$$

Moreover, in all these equilibria entry is deterred.

Summing up, for $\rho F + A_2\Delta(\rho F) < B_2^D \leq F + A_2\Delta(F)$ (i.e., $\tilde{S}_2 > \rho F$) we have:

- If $\tilde{S}_1 \geq \bar{S}^d(\rho F)$, then the equilibrium subsidy profile is $S_1 = \bar{S}^d(\rho F)$ and $S_2 = \rho F$. Moreover, in this equilibrium entry is deterred.
- If $\tilde{S}_1 < \bar{S}^d(\rho F)$, then the equilibrium subsidy profiles are those that satisfy $S_1 = \bar{S}^d(S_2)$ and $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \rho F\right)$. Moreover, in all these equilibria there is accommodated entry.

This completes the proof of Proposition 5.1.

Proof of Part 2: Suppose that $\bar{\rho}^b < \rho < \bar{\rho}_0^d$. Then, employing Proposition 1bis (Part 2), the consumer surplus of each country as a function of (S_1, S_2) is given by:

$$CS_j(S_1, S_2) = A_1 \begin{cases} \frac{(a-c)^2}{8} & if \; 0 \leq S_2 \leq \bar{S}^b \\ \frac{\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & if \; \bar{S}^b < S_2 \leq \rho F \end{cases}$$

while the geopolitical payoff of each global power as a function of (S_1, S_2) is given by:

$$B_1(S_1, S_2) = \begin{cases} B_1^M - S_1 & if \; 0 \leq S_2 \leq \bar{S}^b \\ B_1^M - S_1 & if \; \bar{S}^b < S_2 \leq \rho F \end{cases}, \; B_2(S_1, S_2) = \begin{cases} 0 & if \; 0 \leq S_2 \leq \bar{S}^b \\ 0 & if \; \bar{S}^b < S_2 \leq \rho F \end{cases}$$

Therefore, the payoff function of each global power is given by:

$$W_1(S_1, S_2) = \begin{cases} \frac{A_1(a-c)^2}{8} + B_1^M - S_1 & if \; 0 \leq S_2 \leq \bar{S}^b \\ \frac{A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - S_1 & if \; \bar{S}^b < S_2 \leq \rho F \end{cases}$$

$$W_2(S_1, S_2) = \begin{cases} \frac{A_2(a-c)^2}{8} & if \; 0 \leq S_2 \leq \bar{S}^b \\ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & if \; \bar{S}^b < S_2 \leq \rho F \end{cases}$$

Best response correspondence of G_1. Fix $S_2 \geq 0$. Suppose that $0 \leq S_2 \leq \bar{S}^b$. Then, $W_1(S_1, S_2) = \left[A_1(a-c)^2/8\right] + B_1^M - S_1$, which is strictly decreasing in S_1. Thus, the best response to $0 \leq S_2 \leq \bar{S}^b$ is $S_1 = 0$. Suppose that $\bar{S}^b < S_2 \leq \rho F$. Then, $W_1(S_1, S_2) = A_1\left[a-c-2\sqrt{b(F-S_2)}\right]^2/2 + B_1^M - S_1$, which is strictly decreasing in S_1. Thus, the best response to $\bar{S}^b < S_2 \leq \rho F$ is $S_1 = 0$.

Best response correspondence of G_2. Fix $S_1 \geq 0$. $W_2(S_1, S_2)$ is a continuous function of S_2 for all $S_2 \in [0, \rho F]$ (in particular, $W_2(S_1, S_2)$ is continuous for $S_2 = \bar{S}^b$); $W_2(S_1, S_2)$ is a constant for all $S_2 \in \left[0, \bar{S}^b\right]$; $W_2(S_1, S_2)$ is strictly increasing in S_2 for all $S_2 \in \left[\bar{S}^b, \rho F\right]$. Thus, the best response to $S_1 \geq 0$ is $S_2 = \rho F$.

Nash equilibrium. The Nash equilibrium profile is given by $S_1 = 0$ and $S_2 = \rho F$.

This completes the proof of Proposition 5.2.

Proof of Part 3: Suppose that $0 < \rho \leq \bar{\rho}^b$. Then, employing Proposition 1bis (Part 3), the consumer surplus of each country as a function of (S_1, S_2) is given by:

$$CS_j(S_1, S_2) = A_j \frac{(a-c)^2}{8}$$

while the geopolitical payoff of each global power as a function of (S_1, S_2) is given by:

$$B_1(S_1, S_2) = B_1^M - S_1,\ B_2(S_1, S_2) = 0$$

Therefore, the payoff function of each global power is given by:

$$W_1(S_1, S_2) = \frac{A_1(a-c)^2}{8} + B_1^M - S_1,\ W_2(S_1, S_2) = \frac{A_2(a-c)^2}{8}$$

Best response correspondence of G_1. Fix $S_2 \geq 0$. Then, $W_1(S_1, S_2) = \left[A_1(a-c)^2/8\right] + B_1^M - S_1$, which is strictly decreasing in S_1. Thus, the best response to $S_2 \geq 0$ is $S_1 = 0$.

Best response correspondence of G_2. Fix $S_1 \geq 0$. Then, $W_2(S_1, S_2) = A_2(a-c)^2/8$, which does not depend on S_2. Thus, the best response to $S_1 \geq 0$ is $S_2 \in [0, \rho F]$.

Nash equilibrium. The set of Nash equilibrium profiles is given by $S_1 = 0$ and $S_2 \in [0, \rho F]$.

Most preferred equilibrium for each global power: In any Nash equilibrium it must be the case that $S_1 = 0$, which implies that the payoffs of the global powers as a function of the equilibrium profile of subsidies are given by:

$$W_1(0, S_2) = \frac{A_1(a-c)^2}{8} + B_1^M,\ W_2(0, S_2) = \frac{A_2(a-c)^2}{8}$$

Thus, G_1 and G_2 are indifferent among the Nash equilibrium profiles (S_1, S_2). This completes the proof of Proposition 5.3. ■

A.3.2 No Geopolitical Threat from G_2 and G_1 Subsidizes I's Capacity Expansion (Proposition 6)

In this subsection we prove Proposition 6.

Proposition 6 ***No geopolitical threat from*** G_2 ***and*** G_1 ***subsidizes*** I***'s capacity expansion***. *Suppose that Assumptions 1 (i.e.,* $k_I \in \left[0, \frac{a-c}{b}\right]$, $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$, *and* $a \leq 2c$*) and 2 (i.e.,* $F > \frac{(a-c)^2}{16b}$*) hold,* $S_2 \leq \rho F$ *with* $\rho \in \left[0, \bar{\rho}^b\right]$, *and* G_1 *can offer any contract of the form*

$$S_1(k_I) = \chi_{\bar{k}_I} S_1,\ \text{where}\ \chi_{\bar{k}_I} = 1\ \text{if}\ k_I \geq \bar{k}_I \geq \bar{k}^m\ \text{and}\ \chi_{\bar{k}_I} = 0\ \text{if}\ k_I < \bar{k}_I$$

Then, G_1 *offers* $S_1 = \left[\frac{(2-A_1 b)^2 - 4(1-A_1 b)}{4(2-A_1 b)^2 b}\right](a-c)^2$ *and* $\bar{k}_I = \frac{a-c}{(2-A_1 b)b}$; I *accepts it and selects* $k_I = \bar{k}_I$; *and* E *selects* $k_E = 0$. *Moreover:*

1. $\bar{k}_I < \frac{3(a-c)}{4b}$ *(i.e., the aggregate quantity under accommodation) if and only if* $A_1 b < 2/3$.

2. $\bar{k}_I = \frac{a-c}{(2-A_1 b)b} < \frac{(a-c)}{2b}\left(1+\frac{\sqrt{2}}{2}\right)$ *(i.e., the minimum aggregate quantity under deterrence in Proposition 3) if and only if* $A_1 b < 2\left(\sqrt{2}-1\right)$.

Proof: Since $S_2 \leq \bar{\rho}^b F = \bar{S}^b$, following the same steps employed to prove Proposition 1.1, we have that $k_E = 0$ for all $k_I \geq \bar{k}^m$. Now, suppose that G_1 offers $\left(S_1, \bar{k}_I\right)$. If I accepts this offer, its profits will be given by:

$$\pi_I\left(\bar{k}_I, S_1\right) = \left(a - b\bar{k}_I - c\right)\bar{k}_I + S_1$$

On the contrary, if I rejects this offer, following the same steps employed to prove Proposition 1.1, we have that I will select $k_I = \bar{k}^m$ and, hence, its profits will be given by:

$$\pi_I\left(\bar{k}^m\right) = \left(a - b\bar{k}^m - c\right)\bar{k}^m = \frac{(a-c)^2}{4b}$$

Since for all $\bar{k}_I \geq \bar{k}^m$, $\pi_I\left(\bar{k}_I, S_1\right)$ is strictly decreasing in $\bar{k}_I$. Thus, $\pi_I\left(\bar{k}_I, 0\right) < \pi_I\left(\bar{k}^m\right)$. Therefore, for I to accept this offer we need that

$$S_1 \geq \pi_I\left(\bar{k}^m\right) - \pi_I\left(\bar{k}_I, 0\right) = \frac{(a-c)^2}{4b} - \left(a - b\bar{k}_I - c\right)\bar{k}_I$$

Thus, G_1's problem becomes:

$$\max_{S_1 \geq 0, \bar{k}_I \geq \bar{k}^m} \left\{W_1\left(\bar{k}_I, S_1\right) = CS_1\left(\bar{k}_I\right) + B_1^M - S_1\right\}$$

$$s.t.\text{: } S_1 \geq \frac{(a-c)^2}{4b} - \left(a - b\bar{k}_I - c\right)\bar{k}_I$$

where $CS_1\left(\bar{k}_I\right) = \frac{A_1 b^2 \left(\bar{k}_I\right)^2}{2}$. Clearly, for any solution it must be the case that $S_1 = \frac{(a-c)^2}{4b} - \left(a - b\bar{k}_I - c\right)\bar{k}_I$. Hence, the problem becomes

$$\max_{\bar{k}_I \geq \bar{k}^m} \left\{W_1\left(\bar{k}_I\right) = \frac{A_1 b^2 \left(\bar{k}_I\right)^2}{2} + B_1^M - \frac{(a-c)^2}{4b} + \left(a - b\bar{k}_I - c\right)\bar{k}_I\right\}$$

Note that $\frac{\partial^2 W_1}{\left(\partial \bar{k}_I\right)^2} = b\left(A_1 b - 2\right) < 0$, which implies that $W_1\left(\bar{k}_I\right)$ is strictly concave. Therefore, the unique solution to this problem is

$$\bar{k}_I = \frac{a-c}{\left(2 - A_1 b\right) b}$$

Note that $\bar{k}_I > \bar{k}^m = \frac{a-c}{2b}$ and $\bar{k}_I < \frac{a-c}{b}$. Thus, $\bar{k}_I$ is strictly greater than the monopoly capacity level (as required) but also strictly lower than the competitive capacity. Solving for S_1 we have $S_1 = \left[\frac{(2-A_1 b)^2 - 4(1-A_1 b)}{4(2-A_1 b)^2 b}\right](a-c)^2$. Thus, in equilibrium, G_1 offers $S_1 = \left[\frac{(2-A_1 b)^2 - 4(1-A_1 b)}{4(2-A_1 b)^2 b}\right](a-c)^2$ and $\bar{k}_I = \frac{a-c}{(2-A_1 b)b}$; I accepts it and selects $k_I = \bar{k}_I$; and E selects $k_E = 0$.

From Proposition 1.2, the aggregate quantity under accommodation is $k_I + k_E = \frac{3(a-c)}{4b}$. Thus, $\bar{k}_I < \frac{3(a-c)}{4b}$ if and only if $A_1 b < 2/3$, which is always satisfied if Assumption 3 (i.e., $A_1 b < 4/7$) holds.

From Proposition 1.2, the aggregate quantity under deterrence is $k_I + k_E = \bar{k}^d = \frac{a-c-2\sqrt{b(F-S_2)}}{b}$. From Propositions 2 and 3, the lowest possible $\bar{k}^d$ in an equilibrium with entry deterrence occurs when $S_2 = \bar{S}_0^d$, which is implicitly given by:

$$\bar{S}^d\left(\bar{S}_0^d\right) = \frac{(a-c)^2}{8b} - 2(a-c)\sqrt{\frac{F-\bar{S}_0^d}{b}} + 4\left(F - \bar{S}_0^d\right) = 0$$

Solving and employing that $\bar{S}_0^d > \bar{S}^b = F - \frac{(a-c)^2}{16b}$, we obtain

$$\bar{S}_0^d = F - \left(1 - \frac{\sqrt{2}}{2}\right)^2 \frac{(a-c)^2}{16b}$$

Then, the lowest possible $\bar{k}^d$ in an equilibrium with entry deterrence is $\bar{k}^d = \frac{(a-c)}{2b}\left(1 + \frac{\sqrt{2}}{2}\right)$. Thus, $\bar{k}_I < \frac{(a-c)}{2b}\left(1 + \frac{\sqrt{2}}{2}\right)$ if and only if $A_1 b < 2\left(\sqrt{2} - 1\right)$, which is always satisfied if Assumption 3 (i.e., $A_1 b < 4/7$) holds. This completes the proof of Proposition 6. ■

A.3.3 Unchallenged Rising Power (Proposition 7)

In this subsection we prove Proposition 7.

Proposition 7 ***Unchallenged rising power***. *Suppose that Assumptions 1 (i.e., $k_I \in \left[0, \frac{a-c}{b}\right]$, $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$, and $a \le 2c$) and 2 (i.e., $F > \frac{(a-c)^2}{16b}$) hold, $S_1 = 0$ and $\bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right) < B_2^D \le F + A_2\Delta(F)$. Then, the equilibrium subsidy offer by G_2 is $S_2 = \bar{S}_0^d$. Moreover, in this equilibrium there is accommodated entry.*

Proof: Using Proposition 1 and Lemma 1 we have:

$$W_2(0, S_2) = \begin{cases} \frac{A_2(a-c)^2}{8} & \text{if } 0 \le S_2 \le \bar{S}^b \\ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ \max\left\{ \begin{array}{c} \frac{A_2\left[a-c-2\sqrt{b\left(F-\bar{S}_0^d\right)}\right]^2}{2}, \\ \frac{A_2 9(a-c)^2}{32} + B_2^D - \bar{S}_0^d \end{array} \right\} & \text{if } S_2 = \bar{S}_0^d \\ \frac{A_2 9(a-c)^2}{32} + B_2^D - S_2 & \text{if } \bar{S}_0^d < S_2 \le F \end{cases}$$

$W_2(0, S_2)$ is a constant for all $S_2 \in \left[0, \bar{S}^b\right]$, it is strictly increasing in S_2 for all $S_2 \in \left[\bar{S}^b, \bar{S}_0^d\right)$, and it is strictly decreasing in S_2 for all $S_2 \in \left(\bar{S}_0^d, F\right]$. This does not immediately imply that $W_2(0, S_2)$ has its unique global maximum at $S_2 = \bar{S}_0^d$. The reason is that $W_2(0, S_2)$ might not be continuous at $S_2 = \bar{S}_0^d$. However, note that $W_2(0, S_2)$ adopts the maximum between the left and right limits of the function at $S_2 = \bar{S}_0^d$ and both of these limits exist. Therefore, it is always the case that $W_2(0, S_2)$ adopts its unique global maximum at $S_2 = \bar{S}_0^d$. Thus, in equilibrium, $S_2 = \bar{S}_0^d$. Finally, since, $B_2^D > \bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right)$, our equilibrium selection criteria implies that G_2 prefers accommodated entry. This completes the proof of Proposition 7. ■

A.3.4 No Geopolitical Rivalry (Proposition 8)

In this subsection we prove Proposition 8.

Proposition 8 ***No geopolitical rivalry.*** *Suppose that Assumptions 1 (i.e., $k_I \in \left[0, \frac{a-c}{b}\right]$, $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$, and $a \leq 2c$), 2 (i.e., $F > \frac{(a-c)^2}{16b}$) , and 3 (i.e., $A_1 b < 4/7$) hold, $B_1^M = B_1^D$, and $B_2^D \leq F + A_2\Delta(F)$. Let $\tilde{S}_1^{no-rival} \in \left(0, \bar{S}^d(F)\right]$ be the unique solution to:*

$$A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1^{no-rival}\right)\right) = \tilde{S}_1^{no-rival}$$

Then, Proposition 3 holds with $\tilde{S}_1^{no-rival}$ replacing $\tilde{S}_1$. Moreover, $\tilde{S}_1^{no-rival} < \tilde{S}_1$.

Proof: The proof is immediate because Lemmas 2 and 3 and Propositions 2 and 3 still hold for $B_1^M - B_1^D = 0$. Finally, Corollary 2 implies that $\tilde{S}_1$ is decreasing in $B_1^M - B_1^D$, which implies that $\tilde{S}_1^{no-rival} < \tilde{S}_1$, where is the unique solution to $B_1^M - B_1^D + A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \tilde{S}_1$. ■

A.3.5 Sequential Geopolitical Subsidy Race (Proposition 9)

In this subsection we proof Proposition 9.

Proposition 9 ***Sequential geopolitical subsidy race.*** *Suppose that Assumptions 1 (i.e., $k_I \in \left[0, \frac{a-c}{b}\right]$, $k_E \in \left[0, \frac{a-c}{b} - k_I\right]$, and $a \leq 2c$), 2 (i.e., $F > \frac{(a-c)^2}{16b}$) , and 3 (i.e., $A_1 b < 4/7$) hold, $B_1^M - B_1^D \leq \bar{S}^d(F) - A_1\Delta(F)$, and $B_2^D \in \left(\bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right), F + A_2\Delta(F)\right]$.*

1. *Suppose that $\tilde{S}_1 > \bar{S}^d\left(\tilde{S}_2\right)$. Then, the unique subgame perfect Nash equilibrium outcome is $S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ and $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \tilde{S}_1$. Moreover, in this equilibrium, entry is deterred.*

2. *Suppose that $\tilde{S}_1 = \bar{S}^d\left(\tilde{S}_2\right)$. Then, the set of subgame perfect Nash equilibrium outcomes is $S_2 = \tilde{S}_2$ and $S_1\left(\tilde{S}_2\right) \in \left[0, \tilde{S}_1\right]$. For $S_1\left(\tilde{S}_2\right) \in \left[0, \tilde{S}_1\right)$, there is accommodated entry, while for $S_1\left(\tilde{S}_2\right) = \tilde{S}_1$, entry is deterred.*

3. *Suppose that $\tilde{S}_2 > \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$. Then, the set of subgame perfect Nash equilibrium outcomes is $S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ and $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) \in \left[0, \tilde{S}_1\right]$. Moreover, in equilibrium, there is accommodated entry.*

Proof: We have already seen that Proposition 1 still holds. Then, we can employ the best response correspondence of G_1 that we have computed in Propositions 2 and 3. There are two cases to consider, when $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$ and when $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$.

Case 1: Suppose that $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$ [equivalently, $\tilde{S}_2 \leq \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$]. Then, the best response correspondence of G_1 is given by:

$$S_1(S_2) = \begin{cases} 0 & \text{if } 0 \leq S_2 < \bar{S}_0^d \\ \bar{S}^d(S_2) & \text{if } \tilde{S}_2 \leq S_2 < \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right] & \text{if } S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right) & \text{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) < S_2 \leq F \end{cases}$$

The payoff function of G_2 as a function of (S_1, S_2) is given by:

$$W_2(S_1, S_2) = \begin{cases} \frac{A_2(a-c)^2}{8} & \text{if } 0 \leq S_2 \leq \bar{S}^b \\ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}_0^d \leq S_2 < \left(\bar{S}^d\right)^{-1}(S_1) \\ \max\left\{ \begin{array}{c} \frac{A_2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]^2}{2}, \\ \frac{A_2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}(S_1) \end{array} \right\} & \text{if } S_2 = \left(\bar{S}^d\right)^{-1}(S_1) \\ \frac{A_2 9(a-c)^2}{32} + B_2^D - S_2 & \text{if } \left(\bar{S}^d\right)^{-1}(S_1) < S_2 \leq F \end{cases}$$

We must introduce $S_1(S_2)$ into $W_2(S_1, S_2)$ in order to obtain $W_2(S_1(S_2), S_2)$. There are five cases to consider:

Case 1.a: Suppose that $S_2 \in \left[0, \bar{S}_0^d\right)$. Then, $S_1(S_2) = 0$. Therefore, we have

$$W_2(S_1(S_2), S_2) = \begin{cases} \frac{A_2(a-c)^2}{8} & \text{if } 0 \leq S_2 \leq \bar{S}^b \\ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}^b < S_2 < \bar{S}_0^d \end{cases}$$

Case 1.b: Suppose that $S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$. Then, G_1 does not have a best response to S_2. However, suppose that G_2 expects that $S_1(S_2) > \bar{S}^d(S_2)$. Then, we have:

$$W_2(S_1(S_2), S_2) = \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}$$

Case 1.c: Suppose that $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$. Then, $S_1(S_2) = \bar{S}^d(S_2)$. Therefore, we have:

$$W_2(S_1(S_2), S_2) = \max\left\{ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}, \frac{A_2 9(a-c)^2}{32} + B_2^D - S_2 \right\}$$

Since $\bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq F + A_2\Delta(F)$ and $S_2 \geq \tilde{S}_2$ we have:

$$W_2\left(S_1\left(S_2\right), S_2\right) = \frac{A_2\left[a - c - 2\sqrt{b\left(F - S_2\right)}\right]^2}{2}$$

Case 1.d: Suppose that $S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$. Then, $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \left[0, \tilde{S}_1\right]$. If $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) < \tilde{S}_1$, then

$$W_2\left(S_1\left(S_2\right), S_2\right) = \frac{A_2 9\left(a - c\right)^2}{32} + B_2^D - S_2$$

If $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \tilde{S}_1$, then the analysis in case 1.c applies and, hence,

$$W_2\left(S_1\left(S_2\right), S_2\right) = \frac{A_2\left[a - c - 2\sqrt{b\left(F - S_2\right)}\right]^2}{2}$$

Case 1.e: Suppose that $S_2 \in \left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), F\right]$. Then, $S_1\left(S_2\right) = \left[0, \bar{S}^d\left(S_2\right)\right)$. Therefore, we have

$$W_2\left(S_1\left(S_2\right), S_2\right) = \frac{A_2 9\left(a - c\right)^2}{32} + B_2^D - S_2$$

Combining all the cases, we obtain:

$$W_2\left(S_1\left(S_2\right), S_2\right) = \begin{cases} \frac{A_2(a-c)^2}{8} & \text{if } 0 \leq S_2 \leq \bar{S}^b \\ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}^b \leq S_2 < \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \\ \frac{A_2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) & \text{if } S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \text{ and } S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) \in \left[0, \tilde{S}_1\right) \\ \frac{A_2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}\right]^2}{2} & \text{if } S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \text{ and } S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \tilde{S}_1 \\ \frac{A_2 9(a-c)^2}{32} + B_2^D - S_2 & \text{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) < S_2 \leq F \end{cases}$$

Suppose that G_1 plays $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) \in \left[0, \tilde{S}_1\right)$. Then, $W_2\left(S_1\left(S_2\right), S_2\right)$ is a constant for all $S_2 \in \left[0, \bar{S}^b\right]$, it is strictly increasing in S_2 for all $S_2 \in \left[\bar{S}^b, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$, and it is strictly decreasing in

S_2 for all $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), F\right]$. Since $\tilde{S}_2 \leq \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$, it must be the case that

$$\lim_{S_2 \to \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]^-} W_2\left(S_1\left(S_2\right), S_2\right) = \frac{A_2\left[a - c - 2\sqrt{b\left(F - \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}\right]^2}{2}$$

$$\geq$$

$$W_2\left(S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right), \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \frac{A_2 9\left(a-c\right)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$$

with strict inequality if $\tilde{S}_2 < \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$. Thus, if $\tilde{S}_2 < \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$, there is no S_2 that maximizes $W_2\left(S_1\left(S_2\right), S_2\right)$. On the contrary, if $\tilde{S}_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$, $W_2\left(S_1\left(S_2\right), S_2\right)$ has its unique global maximum at $S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$. The best response of G_1 is $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \left[0, \tilde{S}_1\right)$.

Suppose that G_1 plays $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \tilde{S}_1$. Then, $W_2\left(S_1\left(S_2\right), S_2\right)$ is a constant for all $S_2 \in \left[0, \bar{S}^b\right]$, it is strictly increasing in S_2 for all $S_2 \in \left[\bar{S}^b, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$, and it is strictly decreasing in S_2 for all $S_2 \in \left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), F\right]$. Since $\tilde{S}_2 \leq \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$, it must be the case that

$$W_2\left(S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right), \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \frac{A_2\left[a - c - 2\sqrt{b\left(F - \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}\right]^2}{2}$$

$$\geq$$

$$\lim_{S_2 \to \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]^+} W_2\left(S_1\left(S_2\right), S_2\right) = \frac{A_2 9\left(a-c\right)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$$

Thus, $W_2\left(S_1\left(S_2\right), S_2\right)$ has its unique global maximum at $S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$. The best response of G_1 is $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \tilde{S}_1$.

Summing up, if $\tilde{S}_2 < \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$, the unique subgame perfect Nash equilibrium outcome is $S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ and $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \tilde{S}_1$. In this equilibrium, entry is deterred. If $\tilde{S}_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$, the set of subgame perfect Nash equilibrium outcomes is $S_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ and $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) \in \left[0, \tilde{S}_1\right]$. For $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) \in \left[0, \tilde{S}_1\right)$, there is accommodated entry, while for $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) = \tilde{S}_1$, entry is deterred.

Case 2: Suppose that $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$ [equivalently, $\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) < \tilde{S}_2$]. Then, the best response correspondence of G_1 becomes:

$$S_1(S_2) = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \leq S_2 < \tilde{S}_2 \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } \tilde{S}_2 \leq S_2 \leq F \end{cases}$$

We must introduce $S_1(S_2)$ into $W_2(S_1, S_2)$ in order to obtain $W_2(S_1(S_2), S_2)$. There are five cases to consider:

Case 2.a: Suppose that $S_2 \in \left[0, \bar{S}_0^d\right)$. Then, $S_1(S_2) = 0$. Then, therefore was have:

$$W_2(S_1(S_2), S_2) = \begin{cases} \frac{A_2(a-c)^2}{8} & \textit{if } 0 \leq S_2 \leq \bar{S}^b \\ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \textit{if } \bar{S}^b < S_2 < \bar{S}_0^d \end{cases}$$

Case 2.b: Suppose that $S_2 \in \left[\bar{S}_0^d, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$. Then, G_1 does not have a best response. Provided that G_2 expects that G_1 will select $S_1 \geq \tilde{S}_1$ we have:

$$W_2(S_1(S_2), S_2) = \frac{A_2\left[a - c - 2\sqrt{b(F - S_2)}\right]^2}{2}$$

Case 2.c: Suppose that $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$. Then, $S_1(S_2) \in \left[0, \bar{S}^d(S_2)\right]$. If $S_1(S_2) \in \left[0, \bar{S}^d(S_2)\right)$, then we have $W_2(S_1(S_2), S_2) = \frac{A_2 9(a-c)^2}{32} + B_2^D - S_2$. If $S_1(S_2) = \bar{S}^d(S_2)$, then

$$W_2(S_1(S_2), S_2) = \max\left\{\frac{A_2\left[a - c - 2\sqrt{b(F - S_2)}\right]^2}{2}, \frac{A_2 9(a-c)^2}{32} + B_2^D - S_2\right\}$$

Since $\bar{S}_0^d + A_2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq F + A_2\Delta(F)$ and $S_2 < \tilde{S}_2$, we have

$$W_2(S_1(S_2), S_2) = \frac{A_2 9(a-c)^2}{32} + B_2^D - S_2$$

Case 2.d: Suppose that $S_2 \in \left[\tilde{S}_2, F\right]$. Then, $S_1(S_2) \in \left[0, \bar{S}^d(S_2)\right)$. Therefore, we have

$$W_2(S_1(S_2), S_2) = \frac{A_2 9(a-c)^2}{32} + B_2^D - S_2$$

Combining all the cases, we obtain:

$$W_2\left(S_1\left(S_2\right),S_2\right)=\begin{cases}\frac{A_2(a-c)^2}{8} & \text{if } 0\le S_2\le \bar{S}^b\\ \frac{A_2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}^b\le S_2<\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\\ \frac{A_2 9(a-c)^2}{32}+B_2^D-S_2 & \text{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\le S_2\le F\end{cases}$$

$W_2\left(S_1\left(S_2\right),S_2\right)$ is a constant for all $S_2\in\left[0,\bar{S}^b\right]$, it is strictly increasing in S_2 for all $S_2\in\left[\bar{S}^b,\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$, and it is strictly decreasing in S_2 for all $S_2\in\left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right),F\right]$. Since $\tilde{S}_2>\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$, it must be the case that

$$\lim_{S_2\to\left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]^-}W_2\left(S_1\left(S_2\right),S_2\right)=\frac{A_2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}\right]^2}{2}$$

$$<$$

$$W_2\left(S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right),\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)=\frac{A_2 9\left(a-c\right)^2}{32}+B_2^D-\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$$

Thus, $W_2\left(S_1\left(S_2\right),S_2\right)$ has its unique global maximum at $S_2=\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$. The best response of G_1 is $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)=\left[0,\tilde{S}_1\right]$.

Summing up, if $\tilde{S}_2>\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$, the set of subgame perfect Nash equilibrium outcomes is $S_2=\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ and $S_1\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)\in\left[0,\tilde{S}_1\right]$. Moreover, in all these equilibria, there is accommodated entry. This completes the proof of Proposition 9. ■

Corollary 6 *Under the assumptions in Proposition 9. Suppose that global power* 1 *always reacts with the highest possible best response subsidy. Then, the unique subgame perfect Nash equilibrium outcome is:* $S_2=\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ *and* $S_1=\tilde{S}_1$. *Moreover, if* $\tilde{S}_1\ge\bar{S}^d\left(\tilde{S}_2\right)$, *then entry is deterred, while if* $\tilde{S}_1<\bar{S}^d\left(\tilde{S}_2\right)$, *there is accommodated entry.*

Proof: . ■

A.3.6 Geopolitical Realignment of the Potential Entrant (Proposition 10)

Proposition 10 ***Geopolitical realignment of*** E. *Suppose that Assumptions 1, 2, and 3 hold,* $B_1^M-B_1^D\le\bar{S}^d\left(F\right)-A_1\Delta\left(F\right)$, *and* $B_2^D\le F+A_2\Delta\left(F\right)$, *but assume that before the geopolitical subsidy race,* G_1 *can offer* E *a deal to break its geopolitical ties with* G_2.

1. *Suppose that* $\tilde{S}_1\ge\bar{S}^d\left(\tilde{S}_2\right)$.

(a) *If* $C^E > A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)\right] + \tilde{S}_1$*, then there is no geopolitical realignment of* E *and Proposition 3.1 holds, i.e., in equilibrium,* G_1 *offers* $F_1^E < C^E$*, which is rejected by* E*, then* G_1 *and* G_2 *offer* $S_1 = \bar{S}^d(S_2)$ *and* $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$*, respectively. Moreover, in all these equilibria entry is deterred.*

(b) *If* $A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\tilde{S}_2\right)\right] + \bar{S}^d\left(\tilde{S}_2\right) < C^E \leq A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)\right] + \tilde{S}_1$*, there is geopolitical realignment of* E *if and only if* G_1 *expects* $S_1 \in \left[\bar{S}^d\left(\tilde{S}_2\right), \tilde{S}_1\right]$ *such that*

$$C^E \leq A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)\right] + S_1$$

If there is geopolitical realignment, the equilibrium is as in Proposition 10.1.c. Otherwise, in equilibrium, G_1 *offers* $F_1^E < C^E$*, which is rejected by* E*, then* G_1 *and* G_2 *offer* S_1 *and* $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$*, respectively. Moreover, entry is deterred.*

(c) *If* $C^E \leq A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\tilde{S}_2\right)\right] + \bar{S}^d\left(\tilde{S}_2\right)$*, then there is geopolitical realignment of* E*. In equilibrium,* G_1 *offers* $F_1^E = C^E$*, which is accepted by* E*, then* G_1 *offers* $S_1^E = \bar{S}_0^d$ *and* $S_1^I = 0$*. Moreover, in equilibrium entry is deterred.*

2. *Suppose that* $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$*.*

(a) *If* $C^E > A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)\right] + \tilde{S}_1 - \left[\frac{(a-c)^2}{16b} - F + \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$*, then there is no geopolitical realignment of* E *and Proposition 3.2 holds, i.e., in equilibrium,* G_1 *offers* $F_1^E < C^E + \frac{(a-c)^2}{16b} - F + \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$*, which is rejected by* E*, then* G_1 *and* G_2 *offer* $S_1 = \bar{S}^d(S_2)$ *and* $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$*, respectively. Moreover, in all these equilibria there is accommodated entry.*

(b) *If* $A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)\right] + \tilde{S}_1 - \left[\frac{(a-c)^2}{16b} - F + \tilde{S}_2\right] < C^E \leq A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)\right] + \tilde{S}_1 - \left[\frac{(a-c)^2}{16b} - F + \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$*, there is geopolitical realignment of* E *if and only if* G_1 *and* E *expect* $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$ *such that*

$$C^E \leq A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)\right] + \tilde{S}_1 - \left[\frac{(a-c)^2}{16b} - F + S_2\right]$$

If there is geopolitical realignment, the equilibrium is as in Proposition 10.2.c. Otherwise, in equilibrium, G_1 *offers* $F_1^E < C^E + \frac{(a-c)^2}{16b} - F + \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$*, which is rejected by* E*, then* G_1 *and* G_2 *offer* $S_1 = \bar{S}^d(S_2)$ *and* S_2*, respectively. Moreover, in equilibrium, there is accommodated entry.*

(c) *If* $C^E \leq A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)\right] + \tilde{S}_1 - \left[\frac{(a-c)^2}{16b} - F + \tilde{S}_2\right]$*, then there is geopolitical realignment of* E*. In equilibrium,* G_1 *offers* $F_1^E = C^E + \frac{(a-c)^2}{16b} - F + S_2$*, where*

$S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$ *is the equilibrium* S_2 *that* G_1 *and* E *expect to occur if* E *rejects this offer, then* G_1 *offers* $S_1^E = \bar{S}_0^d$ *and* $S_1^I = 0$*. Moreover, in equilibrium entry is deterred.*

Proof: We proceed through backward induction.

Subgame in which G_1***'s offer is accepted***. Suppose that G_1 offers F_1^E and E accepts this offer. Assume that then G_1 selects $\left(S_1^E, S_1^I\right)$. Then, from Proposition 1, we have:

- If $0 \leq S_1^E \leq \bar{S}^b$, then the entry of E is blocked.
- If $\bar{S}^b < S_1^E < \left(\bar{S}^d\right)^{-1}\left(S_1^I\right)$, then the entry of E is deterred. Moreover, for all $\bar{S}^b < S_1^E < \bar{S}_0^d$, the entry of E is deterred even for $S_1^I = 0$.
- If $S_1^E = \left(\bar{S}^d\right)^{-1}\left(S_1^I\right)$, then there are two equilibria: in one equilibrium the entry of E is deterred, while in the other I accommodates the entry of E.
- If $\left(\bar{S}^d\right)^{-1}\left(S_1^I\right) < S_1^E \leq F$, then I accommodates the entry of E.

Thus, the payoff of G_1 as a function of $\left(S_1^E, S_1^I\right)$ will be given by:

$$W_1\left(S_1^E, S_1^I\right) = B_1^M + \begin{cases} \frac{A_1(a-c)^2}{8} - S_1^I & \text{if } 0 \leq S_1^E \leq \bar{S}^b \\ \frac{A_1\left[a-c-2\sqrt{b\left(F-S_1^E\right)}\right]^2}{2} - S_1^I & \text{if } \bar{S}^b < S_1^E < \left(\bar{S}^d\right)^{-1}\left(S_1^I\right) \\ \max\left\{\frac{A_1\left[a-c-2\sqrt{b\left(F-S_1^E\right)}\right]^2}{2} - S_1^I, \frac{A_1 9(a-c)^2}{32} - S_1^E\right\} & \text{if } S_1^E = \left(\bar{S}^d\right)^{-1}\left(S_1^I\right) \\ \frac{A_1 9(a-c)^2}{32} - S_1^E & \text{if } \left(\bar{S}^d\right)^{-1}\left(S_1^I\right) < S_1^E \leq F \end{cases}$$

Fix any $S_1^I = \bar{S}_1^I$. Then, $W_1\left(S_1^E, \bar{S}_1^I\right)$ is a constant for all $S_1^E \in \left[0, \bar{S}^b\right]$, it is strictly increasing in S_1^E for all $S_1^E \in \left[\bar{S}^b, \left(\bar{S}^d\right)^{-1}\left(\bar{S}_1^I\right)\right)$, and it is strictly decreasing in S_1^E for all $S_1^E \in \left(\left(\bar{S}^d\right)^{-1}\left(\bar{S}_1^I\right), F\right]$. Moreover, $W_1\left(S_1^E, \bar{S}_1^I\right)$ adopts the maximum between the left and right limits of the function at $S_1^E = \left(\bar{S}^d\right)^{-1}\left(\bar{S}_1^I\right)$ and both of these limits exist. Therefore, it is always the case that $W_1\left(S_1^E, \bar{S}_1^I\right)$ adopts its unique maximum at $S_1^E = \left(\bar{S}^d\right)^{-1}\left(\bar{S}_1^I\right)$. Thus, it only remains to determine the value of $\bar{S}_1^I$ that maximizes $W_1\left(\left(\bar{S}^d\right)^{-1}\left(\bar{S}_1^I\right), \bar{S}_1^I\right)$, where

$$W_1\left(\left(\bar{S}^d\right)^{-1}\left(\bar{S}_1^I\right), \bar{S}_1^I\right) = B_1^M + \max\left\{\frac{A_1\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}\left(\bar{S}_1^I\right)\right)}\right]^2}{2} - \bar{S}_1^I, \frac{A_1 9(a-c)^2}{32} - \left(\bar{S}^d\right)^{-1}\left(\bar{S}_1^I\right)\right\}$$

From the proof of Lemma 2, provided that $A^1 b < 2\left(\sqrt{2}-1\right) \approx 0.828$, $\frac{A_1\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}\left(\bar{S}_1^I\right)\right)}\right]^2}{2} - \bar{S}_1^I$ is strictly decreasing in $\bar{S}_1^I$. Thus, $\frac{A_1\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}\left(\bar{S}_1^I\right)\right)}\right]^2}{2} - \bar{S}_1^I$ adopts a maximum for $\bar{S}_1^I =$

0. Similarly, $\frac{A_1 9(a-c)^2}{32} - \left(\bar{S}^d\right)^{-1}\left(\bar{S}_1^I\right)$ is strictly decreasing in $\bar{S}_1^I$, which implies that it also adopts a maximum for $\bar{S}_1^I = 0$. Therefore, the value of $\bar{S}_1^I$ that maximizes $W_1\left(\left(\bar{S}^d\right)^{-1}\left(\bar{S}_1^I\right), \bar{S}_1^I\right)$ is always $\bar{S}_1^I = 0$, which implies $S_1^E = \bar{S}_0^d = \left(\bar{S}^d\right)^{-1}(0)$. Finally, to determine if $\left(S_1^E, S_1^I\right) = \left(\bar{S}_0^d, 0\right)$ leads to deterrence or accommodated entry, we must compare $\frac{A_1\left[a-c-2\sqrt{b\left(F-\bar{S}_0^d\right)}\right]^2}{2}$ with $\frac{A_1 9(a-c)^2}{32} - \bar{S}_0^d$. Note that $\bar{S}_0^d > F - \frac{(a-c)^2}{64b}$, which implies $\frac{A_1\left[a-c-2\sqrt{b\left(F-\bar{S}_0^d\right)}\right]^2}{2} > \frac{A_1 9(a-c)^2}{32}$. Therefore, $\left(S_1^E, S_1^I\right) = \left(\bar{S}_0^d, 0\right)$, always leads to deterrence.

Summing up, if E accepts G_1's offer, then G_1 will select $\left(S_1^E, S_1^I\right) = \left(\bar{S}_0^d, 0\right)$, entry will be deterred, E will obtain

$$\pi_E = F_1^E - C^E$$

and G_1 will obtain

$$W_1\left(\bar{S}_0^d, 0\right) = B_1^M + \frac{A_1\left[a-c-2\sqrt{b\left(F-\bar{S}_0^d\right)}\right]^2}{2} - F_1^E$$

Subgame in which G_1's offer is rejected. Suppose that G_1 offers F_1^E and E rejects this offer. Then, Propositions 2 and 3 apply, which implies that we have two possible cases to consider:

Case 1: Suppose that $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$. Then, the equilibrium subsidy profiles are those that satisfy: $S_1 = \bar{S}^d(S_2)$ and $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$. Moreover, in all these equilibria entry is deterred. Hence, E will obtain

$$\pi_E = 0$$

and G_1 will obtain

$$W_1 = B_1^M + \frac{A_1\left[a-c-2\sqrt{b\left(F-S_2\right)}\right]^2}{2} - \bar{S}^d(S_2)$$

Case 2: Suppose that $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$. Then, the equilibrium subsidy profiles are those that satisfy: $S_1 = \bar{S}^d(S_2)$ and $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$. Moreover, in all these equilibria there is accommodated entry. Hence, E will obtain

$$\pi_E = \frac{(a-c)^2}{16b} - F + S_2$$

and G_1 will obtain:

$$W_1 = B_1^D + \frac{A_1 9(a-c)^2}{32}$$

G_1's initial offer to E: To determine the optimal F_1^E selected by G_1, we must consider two possible situations:

Case 1: Suppose that $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$. Then, for E to accept F_1^E, G_1 must offer $F_1^E \geq C^E$. This is a good deal for G_1 if and only if

$$\frac{A_1\left[a-c-2\sqrt{b\left(F-\bar{S}_0^d\right)}\right]^2}{2}-C^E \geq \frac{A_1\left[a-c-2\sqrt{b\left(F-S_2\right)}\right]^2}{2}-\bar{S}^d\left(S_2\right)$$

where $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$. Using $S_1 = \bar{S}^d\left(S_2\right)$, subtracting $\frac{A_1 9(a-c)^2}{32}$ on both sides of the inequality and rearranging terms we have

$$C^E \leq A_1\Delta\left(\bar{S}_0^d\right) - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(S_1\right)\right) + S_1$$

where $S_1 \in \left[\bar{S}^d\left(\tilde{S}_2\right), \tilde{S}_1\right]$. In the proof of Lemma 2 we have shown that provided that $A^1 b < 2\left(\sqrt{2}-1\right) \approx 0.828$, $A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(S_1\right)\right) - S_1$ is strictly decreasing in S_1. Thus, there are three cases to consider:

Case 1.a: Suppose that $C^E > A_1\Delta\left(\bar{S}_0^d\right) - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$. Then, G_1 will never offer $F_1^E \geq C^E$. Thus, Proposition 3.1 holds. More precisely, G_1 offers $F_1^E < C^E$, which is rejected by E, then G_1 and G_2 offer $S_1 = \bar{S}^d\left(S_2\right)$ and $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$, respectively. Moreover, in all these equilibria entry is deterred.

Case 1.b: Suppose that $A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\tilde{S}_2\right)\right] + \bar{S}^d\left(\tilde{S}_2\right) < C^E \leq A_1\Delta\left(\bar{S}_0^d\right) - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$. Then, depending on the equilibrium that G_1 expects to occur if it offers $F_1^E < C^E$, G_1 will prefer to offer $F_1^E = C^E$ (which will be accepted by E) or $F_1^E < C^E$ (which will be rejected by E). In particular, G_1 will offer $F_1^E = C^E$ if and only if it expects $S_1 \in \left[\bar{S}^d\left(\tilde{S}_2\right), \tilde{S}_1\right]$ such that

$$C^E \leq A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\left(\bar{S}^d\right)^{-1}\left(S_1\right)\right)\right] + S_1$$

Thus, if the above condition holds, the equilibrium is as in case 1.c. Otherwise, in equilibrium, G_1 offers $F_1^E < C^E$, which is rejected by E, then G_1 and G_2 offer S_1 and $S_2 = \left(\bar{S}^d\right)^{-1}\left(S_1\right)$, respectively. Moreover, entry is deterred.

Case 1.c: Suppose that $C^E \leq A_1\left[\Delta\left(\bar{S}_0^d\right) - \Delta\left(\tilde{S}_2\right)\right] + \bar{S}^d\left(\tilde{S}_2\right)$. Then, G_1 will offer $F_1^E = C^E$ and E will accept this offer. Therefore, in equilibrium, G_1 will select $\left(S_1^E, S_1^I\right) = \left(\bar{S}_0^d, 0\right)$ and entry will be deterred.

Case 2: Suppose that $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$. Then, for E to accept F_1^E, G_1 must offer $F_1^E \geq C^E + \frac{(a-c)^2}{16b} - F + S_2$, where $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$. This is a good deal for G_1 if and only if

$$B_1^M + \frac{A_1\left[a-c-2\sqrt{b\left(F-\bar{S}_0^d\right)}\right]^2}{2} - \frac{(a-c)^2}{16b} + F - C^E - S_2 \geq B_1^D + \frac{A_1 9\left(a-c\right)^2}{32}$$

where $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$. Rearranging terms, we have:

$$C^E \leq B_1^M - B_1^D + A_1\Delta\left(\bar{S}_0^d\right) - \left[\frac{(a-c)^2}{16b} - F + S_2\right]$$

where $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$. There are three cases to consider:

Case 2.a: Suppose that $C^E > B_1^M - B_1^D + A_1\Delta\left(\bar{S}_0^d\right) - \left[\frac{(a-c)^2}{16b} - F + \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$. Then, G_1 will never offer $F_1^E \geq C^E + \frac{(a-c)^2}{16b} - F + S_2$ for any $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$. Thus, Proposition 3.2 holds. More precisely, G_1 offers $F_1^E < C^E + \frac{(a-c)^2}{16b} - F + \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$, which is rejected by E, then G_1 and G_2 offer $S_1 = \bar{S}^d\left(S_2\right)$ and $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$, respectively. Moreover, in all these equilibria there is accommodated entry.

Case 2.b: Suppose that $B_1^M - B_1^D + A_1\Delta\left(\bar{S}_0^d\right) - \left[\frac{(a-c)^2}{16b} - F + \tilde{S}_2\right] < C^E \leq B_1^M - B_1^D + A_1\Delta\left(\bar{S}_0^d\right) - \left[\frac{(a-c)^2}{16b} - F + \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$. Then, depending on the equilibrium that G_1 and E expect to occur if E rejects the offer, G_1 will be willing to offer $F_1^E = C^E + \frac{(a-c)^2}{16b} - F + S_2$ or $F_1^E < C^E + \frac{(a-c)^2}{16b} - F + S_2$. In particular, suppose that G_1 and E expect $S_1 = \bar{S}^d\left(S_2\right)$ and $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$. Then, G_1 will offer $F_1^E = C^E + \frac{(a-c)^2}{16b} - F + S_2$ if and only if

$$C^E \leq B_1^M - B_1^D + A_1\Delta\left(\bar{S}_0^d\right) - \left[\frac{(a-c)^2}{16b} - F + S_2\right]$$

Thus, if the above condition holds, the equilibrium is as in case 2.c. Otherwise, in equilibrium, G_1 offers $F_1^E < C^E + \frac{(a-c)^2}{16b} - F + \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$, which is rejected by E, then G_1 and G_2 offer $S_1 = \bar{S}^d\left(S_2\right)$ and S_2, respectively. Moreover, in equilibrium, there is accommodated entry.

Case 2.c: Suppose that $C^E \leq B_1^M - B_1^D + A_1\Delta\left(\bar{S}_0^d\right) - \left[\frac{(a-c)^2}{16b} - F + \tilde{S}_2\right]$. Then, G_1 will offer $F_1^E = C^E + \frac{(a-c)^2}{16b} - F + S_2$, where $S_1 = \bar{S}^d\left(S_2\right)$ and $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$ is the equilibrium that G_1 and E expect to occur if E rejects this offer. Therefore, in equilibrium, G_1 will select $\left(S_1^E, S_1^I\right) = \left(\bar{S}_0^d, 0\right)$ and entry will be deterred. This completes the characterization of the equilibrium.

Finally, using $B_1^M - B_1^D = \tilde{S}_1 - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)$, the condition in case 2 for G_1 to be willing to offer $F_1^E = C^E + \frac{(a-c)^2}{16b} - F + S_2$ becomes:

$$C^E \leq A_1\Delta\left(\bar{S}_0^d\right) - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1 - \left[\frac{(a-c)^2}{16b} - F + S_2\right]$$

Thus, for $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$ (i.e., case 1), the equilibrium in Proposition 3.1 holds if and only if

$$C^E > A_1\Delta\left(\bar{S}_0^d\right) - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$$

while for $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$ (i.e., case 2), the equilibrium in Proposition 3.2 holds if and only if

$$C^E > A_1\Delta\left(\bar{S}_0^d\right) - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1 - \left[\frac{(a-c)^2}{16b} - F + \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$$

Note that $\frac{(a-c)^2}{16b} - F + \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) > 0$ if and only if $\tilde{S}_1 > \bar{S}^d\left(F - \frac{(a-c)^2}{16b}\right) = -\frac{(a-c)^2}{8}$, which always holds because $\tilde{S}_1 > 0$. Thus, $C^E > A_1\Delta\left(\bar{S}_0^d\right) - A_1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$ is a sufficient condition for Proposition 3 to hold even when G_1 can can offer E a deal to break its geopolitical ties with G_2. ■