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THE TERM STRUCTURE OF INTEREST RATES AND THE EFFECTS OF MACROECONOMIC POLICY

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ABSTRACT

This paper analyzes the effects of monetary and fiscal policy shocks on the term structure of interest rates. The effects of temporary versus permanent, unanticipated versus anticipated, policy disturbances and the responses of long versus short, and real versus nominal, rates are contrasted. The main results are summarized in a series of propositions. Among them, the finding that an unanticipated permanent fiscal expansion impacts more on long-term rates, may help explain their observed excessive volatility. The effects of structural changes on the relative variances are also discussed, with the effect which operates through the impact on private speculative behavior being emphasized.

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1. INTRODUCTION

The term structure of interest rates is an important mechanism for the transmission of macroeconomic policy. Monetary policy conducted through the transaction of short-term assets has effects, via the term structure, on long-term interest rates, which in turn influence the rate of investment and the growth rate of the economy. Yet to date, there has been relatively little attention devoted to analyzing the effects of macroeconomic policy on the term structure. Exceptions to this include Blanchard (1981), Mascaro and Meltzer (1983), Turnovsky and Miller (1984) and McCafferty (1986), although these models are restrictive in various ways.[1]

The past 30 years or so has seen substantial twists in the yield curve. For example, during the latter half of the 1950's the yield spread on 10 year U.S. Treasury bonds over 3 month Bills averaged around .7 percentage points. It increased during the first half of the 1960's to around 1.2, dropping off during the second half of the decade to under .3. During the middle and latter parts of the 1970's it increased to 1.6 and after becoming negative at the end of the decade, it has averaged around 2.5 during the economic expansion of the 1980's.

There is an extensive literature investigating the empirical relationships between short-term and long-term interest rates.[2] The starting point for these studies is typically some version of the capital asset pricing—efficient markets relationship, which expresses the current long-term rate as a discounted sum of the expected future short-term rates, over the time to maturity. Certain aspects of this relationship have been investigated in detail and shown to be somewhat at variance with the underlying theory. In a seminal paper, Shiller (1979) has shown how empirical estimates of the volatility of the long-term interest rate, as measured by the variance of its short-term holding yields, vastly exceeds that implied by the underlying theory. This finding has generated considerable interest and stimulated further work by Singleton (1980, Flavin (1983), Kleidon (1986) and others, on the variance bound test used by Shiller. These results have been viewed as casting serious doubt on the expectations approach to the term structure.

By focusing on only a single structural relationship between the short-term and long-term rates, the framework adopted by the empirical literature is a strictly partial equilibrium one. In fact, both short-term and long-term rates are jointly determined as part of a complete macroeconomic system. As such, their stochastic properties reflect the stochastic processes determining policy and other exogenous disturbances impinging on the economy. The purpose of this paper is to examine the behavior of the term structure within such a macroeconomic framework, under the assumption that agents hold rational expectations. Using this approach, the solutions for the current long-term and short-term interest rates are obtained in terms of current and expected future government policy instruments, which we take to be monetary and fiscal policies. We then analyze the effects on the term structure of various policy changes. In particular, we contrast the effects of (i) temporary versus permanent policy changes; and (ii) unanticipated versus anticipated changes; and the effects on (iii) long versus short rates; and (iv) real versus nominal rates.

This macroeconomic general equilibrium approach to the term structure offers several important insights. First, it is clear that the term structure is sensitive to different types of policy (and other disturbances). Indeed, these are presumably what are being reflected in the types of twists in the yield curve which we noted. This equilibrium approach enables us to address this issue in a rather general way. Secondly, it also provides a useful framework which may aid in our understanding of the empirical results obtained. In this respect, we will show below that the small variance of the long rate relative to the short rate, which has formed the basis for much criticism of the expectations approach to the term structure, holds for only some, but not all, disturbances. For example, if the underlying disturbances in the economy are transitory shocks in the money supply, then indeed the ratio of the variances of the long-term to the short-term interest rates, implied by the model, will be unrealistically small. On the other hand, if the underlying shocks are permanent disturbances in fiscal policy, then the variance of the long rate in fact typically exceeds that of the short rate. To the extent that stochastic disturbances in fiscal policy have been the dominant sources of interest rate fluctuations and can be approximated by such a process, this can indeed help explain the apparent excess volatility of long-term rates characteristic of the empirical literature.

A further important consequence of a complete macroeconomic approach is that we are able to establish how, with risk averse speculators, an increase in the variance of government policy gives rise to two effects on the variances of the interest rates. First there is a direct effect; given the parameters of the model, a larger variance in policy will translate to a larger variance in the rates. Secondly, by influencing private speculative behavior, it also has another indirect effect. This may either reinforce or counteract the direct effect, depending upon the nature of the disturbance.

The remainder of the paper proceeds as follows. The next two sections outline the model and its solution. Sections 4 and 5 then analyze the effects of various kinds of monetary and fiscal disturbances, respectively, on the term structure, while Section 6 discusses the implication of these for the observed relative variances of short and long rates. Section 7 then briefly analyzes the impact of structural changes on the term structure, while the final section highlights the main conclusions.

2. COMPLETE STOCHASTIC MACRO MODEL WITH TERM STRUCTURE

The model we shall use is a conventional stochastic new classical macro model. It consists of the following relationships, all expressed for convenience in deviation form

$$Y_t = -dR_t + G_t \qquad d > 0 \tag{1a}$$

$$M_t - P_t = \alpha_1 Y_t - \alpha_2 i_t \qquad \alpha_1 > 0, \alpha_2 > 0 \tag{1b}$$

$$Y_t = \gamma(P_t - P^*_{t,t-1}) \qquad \gamma > 0 \tag{1c}$$

$$r_t = i_t - (P^*_{t+1,t} - P_t) \tag{1d}$$

$$r_t = R_t - \frac{1}{\eta}(R^*_{t+1,t} - R_t). \tag{1e}$$

where

Y_t = real output, measured in logarithms,

R_t = long-term real interest rate, measured in natural units,

i_t = short-term nominal interest rate, measured in natural units,

r_t = short-term real interest rate, measured in natural units,

G_t = real government expenditure, measured in logarithms,

P_t = price of output, measured in logarithms,

$P^*_{t+i,t}$ = expectation of P for time $t+i$, formed at time $t, i = 1, \ldots$; all t,

M_t = nominal supply of money, measured in logarithms.

Equation (1a) is the *IS* curve, where the relevant interest rate is taken to be the long-term real rate. This reflects the fact that real expenditures include investment, which given adjustment costs depend upon the long-term real rate.[3] Money market equilibrium is described by (1b), where the demand for money depends upon the short-term nominal rate. The third equation describes the supply of output in terms of a Lucas supply function; i.e., output (as a deviation about its natural rate level) depends upon the unanticipated change in the current price level. This formulation abstracts from persistence in output, but since this issue is not the focus of our analysis, the simple supply function (1c) suffices for our purposes.

The critical part of the model involves the relationships between the interest rates. The first of these, (1d) is the standard relationship between the short-term real and nominal interest rates.[4] The relationship involving the term structure is specified by (1e), where the long-term real rate of interest is defined to be the yield on a consol paying a constant (real) coupon flow of unity. If we denote such a yield by R, the price of the consol is $\frac{1}{R}$. The relationship (1e) can be obtained as a linear approximation to a simple capital asset pricing relationship.

An important aspect of this relationship is the parameter η. On the assumption of risk neutrality, $\eta = \bar{R}$, where $\bar{R}$ is the mean long-term real rate. This is the form adopted in the empirical literature. On the other hand, a relationship such as (1e) with η constant, can also be obtained in the more general framework of a two-period mean-variance utility maximizing framework with risk averse speculators. In this case, η can be shown to be of the form

$$\eta = \bar{R} + k\sigma_R^2(1)$$

where the constant k reflects the degree of risk aversion and $\sigma_R^2(1)$ is the one period variance of the long-term rate.[5] In this case, as long as investors are risk averse, $k > 0$, and η increases with the variance $\sigma_R^2(1)$. It can therefore be interpreted as being the "risk adjusted discount rate" and is endogenously determined along with $\sigma_R^2(1)$. As long as the underlying stochastic economic structure remains unchanged, so that $\sigma_R^2(1)$ remains constant, then η can also be treated as constant. This will be the assumption maintained throughout Sections 3-5. In Section 6, however, we will consider the effects of changes in the structure, which involve considering their impact on $\sigma_R^2(1)$ and hence on η.

One further point worth noting is that the portfolio choice has been decomposed into a money demand function and the asset pricing relationship, with the latter depending upon the one-period variance of the long term real rate and k through η. In general, the coefficients of the money demand function, derived as part of this optimization will also depend upon these same parameters. The decomposition we have adopted may be shown to arise in the important case where the underlying two-period utility function is separable in consumption and current real money balances, on the one hand, and future wealth, on the other. The money demand function (1b), which is independent of $\sigma_R^2(1)$ can be derived as an approximation to the optimality conditions derived from this form of utility function; see Eaton and Turnovsky (1981).

The complete macro model consisting of equations (1a) – (1e) jointly determines the five variables Y_t, R_t, r_t, i_t and P_t. In addition, there is a relationship between the short-term and long-term nominal rates. While the latter does not appear explicitly in any of the behavioral relationships, it is nevertheless of some interest and is discussed below. To preserve simplicity, we assume that the only long-term bond is real. Under this condition we can then show that to a linear approximation the short-term and long-term nominal rates are given by

$$i_t = I_t - \frac{1}{I}(I^*_{t+1,t} - I_t) \tag{2}$$

where $\bar{I}$ = equilibrium of the long-term nominal rate, and $\bar{I} = \bar{R} = \bar{r}$. The assumption being made that the long-term bonds are real, while appearing restrictive, is not unreasonable. They can simply be viewed as being equities issued by firms to finance their investment.

This model possesses the usual long-run neutrality properties. The steady state (denoted by bars) attained when all expectations are realized and unchanging is described by the set of relationships

$$\overline{Y} = 0 \tag{3a}$$

$$\overline{R} = \overline{r} = \overline{i} = \overline{I} = \frac{G}{d} \tag{3b}$$

$$\overline{P} = M + \alpha_2 \frac{G}{d}. \tag{3c}$$

In the long run, all interest rates are equal and depend only on the level of government expenditure. The price level, in addition to depending upon the level of government expenditure, is proportional to the stock of money.

3. SOLUTION TO THE MODEL

To solve the model, we proceed by first determining expectations and then substituting the resulting expressions back into the system. Taking expectations of the supply and aggregate demand functions at time t, for time $t+j$, yields:

$$Y^*_{t+j,t} = 0 \qquad j = 1, 2, \ldots \tag{4a}$$

$$R^*_{t+j,t} = G^*_{t+j,t}/d. \tag{4b}$$

Given the Lucas supply function, output for any period in the future is expected to be zero, while the expected long-term real rate for time $t+j$ depends upon only the expected level of real government expenditure for that period. Next, the expectations of the term structure relationship (1e) implies

$$r^*_{t+j,t} = R^*_{t+j,t} - \frac{1}{\eta}(R^*_{t+j+1,t} - R^*_{t+j,t})$$

which, using (4b) becomes

$$r^*_{t+j,t} = \frac{1}{d}(1 + \frac{1}{\eta})G^*_{t+j,t} - \frac{1}{d\eta}G^*_{t+j+1,t}. \tag{4c}$$

An expected transitory increase in government expenditure for time $t+j$ is seen to have a larger effect on the expected short-term rate for that period, than it does on the long. The reason is that it lowers the expected price of the long- term asset for that period, thereby increasing the expected capital gain on holding that asset, and raising the expected short-term rate above the long rate. By contrast, an expected increase in government expenditure for the following period $t+j+1$, lowers $r^{*}_{t+j,t}$. The reason is that it lowers the expected price of the long-term asset for the following period, thereby lowering the expected capital gain on holding that asset, and reducing the expected short-term rate of return. Combining these two effects, an increase in government expenditure which is expected to be sustained for (at least) two period raises the expected price of the long term bond equally for the two periods. The expected capital gain is eliminated and the expected response of the short-term rate equals that of the long rate.

Substituting (4a), (4c) into the money market equilibrium condition (1b) and taking expected values, yields the following difference equation in the expected price level:

$$\begin{aligned} \alpha_2 P^{*}_{t+j+1,t} - (1+\alpha_2)P^{*}_{t+j,t} &= -M^{*}_{t+j,t} - \alpha_2 r^{*}_{t+j,t} \\ &= -M^{*}_{t+j,t} - \frac{\alpha_2}{d}[(1+\frac{1}{\eta})G^{*}_{t+j,t} - \frac{1}{\eta}G^{*}_{t+j+1,t}] \end{aligned} \tag{5}$$

the stable solution to which is[7]

$$\begin{aligned} P^{*}_{t+j,t} &= \frac{1}{1+\alpha_2}[\sum_{k=0}^{\infty}[M^{*}_{t+j+k,t} + \alpha_2 r^{*}_{t+j+k,t}](\frac{\alpha_2}{1+\alpha_2})^{k}] \\ &= \frac{1}{1+\alpha_2}[\sum_{k=0}^{\infty} M^{*}_{t+j+k,t}(\frac{\alpha_2}{1+\alpha_2})^{k} + \frac{\alpha_2}{d}(1+\frac{1}{\eta})G^{*}_{t+j,t} \\ &\quad + \frac{(\alpha_2\eta - 1)}{d\eta}\sum_{k=1}^{\infty} G^{*}_{t+j+k,t}(\frac{\alpha_2}{1+\alpha_2})^{k}]. \end{aligned} \tag{6}$$

Setting $j = 1$ in this equation, the expected price level is the discounted sum of all expected future money supplies and expected future short-term real interest rates. An increase in expected government expenditure for just one period ahead, $G^{*}_{t+1,t}$, raises the expected short-term real interest rate for period one and this increases the expected price for that period. Expected increases in government expenditure for subsequent periods have two effects. For example, an increase in $G^{*}_{t+2,t}$ say, raises $r^{*}_{t+2,t}$, while lowering $r^{*}_{t+1,t}$; see (4c). The net effect on $P^{*}_{t+1,t}$ depends upon the quantity $(\alpha_2\eta - 1)$. Recognizing that α_2 is the semi-elasticity of the demand for money, $\alpha_2\eta$ is the interest elasticity of the demand for money, when the interest rate equals the risk adjusted discount rate η. Assuming $\alpha_2\eta < 1$ as being the plausible case, we see that expected increases in government expenditure for subsequent periods will lower the first period's expected price level.

To obtain the solutions of or the current variables of the model, we take the one period expectations of equations (1a)- -(1e), (4) and (5) and subtract from the original equations. Eliminating the nominal interest rate i_t, this system, written in terms of unanticipated current changes, can be expressed in terms of the following matrix equation:

$$\begin{pmatrix} 1 & d & 0 & 0 \\ \alpha_1 & 0 & -\alpha_2 & 1+\alpha_2 \\ 0 & 1+\frac{1}{\eta} & -1 & 0 \\ -1 & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} Y_1 \\ R_t - R^*_{t,t-1} \\ r_t - r^*_{t,t-1} \\ P_t - P^*_{t,t-1} \end{pmatrix}$$

$$= \begin{pmatrix} G_t - G^*_{t,t-1} \\ M_t - M^*_{t,t-1} + \alpha_2(P^*_{t+1,t} - P^*_{t+1,t-1}) \\ \frac{1}{\eta}(R^*_{t+1,t} - R^*_{t+1,t-1}) \\ 0 \end{pmatrix} \tag{7}$$

Given the expressions for expectations in (4b), (4c), and (6), this system may be solved for the current equilibrium of the economy, in terms of the policy variables G_t, M_t and their expectations. But the solutions are also obtained in terms of η, which, if speculators are risk averse, is positively related to the one-period variance of the real long rate, $\sigma^2_R(1)$, and therefore is itself endogenous to the system. As long as the structure of the economy is fixed, this can be treated as a given parameter. However, any structural change, resulting from the change in some parameter, will lead to a change in $\sigma^2_R(1)$ and in η, and this needs to be taken into account. Examples of changes in the degree of risk aversion and in the variances of exogenous policy changes are discussed in Section 7 below. We shall restrict our comments to the solutions for the interest rates, and discuss both the real and nominal in turn.

A. Real Rates

Solving (7) for the unanticipated change in the current long-term real rate, yields

$$\begin{aligned} R_t - R^*_{t,t-1} = \frac{1}{D}[&(1+\alpha_2+\alpha_1\gamma)(G_t - G^*_{t,t-1}) - \gamma(M_t - M^*_{t,t-1}) \\ &- \gamma\alpha_2(P^*_{t+1,t} - P^*_{t+1,t-1}) + \frac{\alpha_2\gamma}{\eta}(R^*_{t+1,t} - R^*_{t+1,t-1})] \end{aligned} \tag{8}$$

where

$$D \equiv d[1+\alpha_2+\alpha_1\gamma] + \gamma\alpha_2(1+\frac{1}{\eta}) > 0.$$

According to (8), the unanticipated change in the long-term rate depends upon: (i) the unanticipated change in current government expenditure; (ii) the unanticipated change in the current money stock; (iii) the revision to the forecast of the price level for time $t+1$, updated between time $t-1$ and t; (iv) the revision to the forecast of the next period's long-term real rate, updated between time $t-1$ and t.

From (6), the revision to the forecast of the price level is

$$
\begin{aligned}
P^*_{t+1,t} - P^*_{t+1,t-1} = \frac{1}{1+\alpha_2}\Bigg\{ & \sum_{k=0}^{\infty}(M^*_{t+k+1,t} - M^*_{t+k+1,t-1})(\frac{\alpha_2}{1+\alpha_2})^k \\
& + \frac{\alpha_2}{d}(1+\frac{1}{\eta})(G^*_{t+1,t} - G^*_{t+1,t-1}) + (\frac{\alpha_2\eta - 1}{d\eta})\sum_{k=1}^{\infty}(G^*_{t+k+1,t} - G^*_{t+k+1,t-1})(\frac{\alpha_2}{1+\alpha_2})^k \Bigg\}
\end{aligned} \tag{9}
$$

while from (4b)

$$
R^*_{t+1,t} - R^*_{t+1,t-1} = \frac{1}{d}[G^*_{t+1,t} - G^*_{t+1,t-1}]
$$

Substituting expressions into (8) yields the solution for the long term real rate. The resulting expression is reported in equations (ia) of Table 1.

The unanticipated change in the long-term real rate depends positively upon the current unanticipated change in government expenditure and negatively upon the unanticipated change in the current money supply. These first two effects are standard. In addition, it depends inversely upon revisions to forecasts of all future money stocks. This comes from the fact that an upward revision to the forecast of the money stock for time $t+j$ say leads to an upward revision to the forecast price level for time $t+1$, $(P^*_{t+1,t} - P^*_{t+1,t-1})$. This leads to a partial upward adjustment in the short-term nominal rate and a corresponding partial reduction in the corresponding real rate. The long rate, being an average of future short-term rates, is therefore also reduced. Finally, the adjustment in the long-term real rate depends positively or negatively upon revisions to all future forecasts of government expenditure, depending upon whether the elasticity of the demand for money $\alpha_2\eta$ 1. The mechanism for this again is through the revision to the price forecast. We may note that these (and all other solutions) are of the general form considered by Plosser (1982).

The response of the short-term real rate is obtained from the term structure relationship, which we may write as

$$
r_t - r^*_{t,t-1} = (1+\frac{1}{\eta})(R_t - R_{t,t-1^*}) - \frac{1}{\eta}(R^*_{t+1,t} - R^*_{t+1,t-1}). \tag{10}
$$

The first component of this is a proportional magnification of the unanticipated component of the long-term real rate. But in addition, the upward revision to the prediction $R^*_{t+1,t}$ implies a reduction in the expected price of the long-term bond. This lowers the real rate of return, thereby lowering the short-term real interest rate. The resulting expression for the adjustment of the short-term real rate, expressed in terms of the policy variables, is given in equations (ii) of Table 1.

B. Nominal Rates

From the short-term real rate, the short-term nominal rate can be easily determined. Taking one-period expectations of (1d) and subtracting yields

$$i_t - i^*_{t,t-1} = (r_t - r^*_{t,t-1}) + (P^*_{t+1,t} - P^*_{t+1,t-1}) - (P_t - P^*_{t,t-1}). \tag{11}$$

The expressions for $(r_t - r^*_{t,t-1}), (P^*_{t+1,t} - P^*_{t+1,t-1})$ are obtained from (iia) of Table 1, and equation (9), respectively. The remaining quantity, the unanticipated change in the current price level, $P_t - P^*_{t,t-1}$, obtained by solving (7), is given by the expression

$$\begin{aligned} P_t - P^*_{t,t-1} = \frac{1}{D}[\alpha_2(1+\frac{1}{\eta})(G_t - G^*_{t,t-1}) + d(M_t - M^*_{t,t-1}) + \alpha_2 d(P^*_{t+1,t} - P^*_{t+1,t-1}) \\ - \frac{\alpha_2 d}{\eta}(R^*_{t+1,t} - R^*_{t+1,t-1})]. \end{aligned} \tag{12}$$

Combining these components yields the solution for the unanticipated change in the current short-term nominal rate, $i_t - i^*_{t,t-1}$, in terms of the actual and anticipated future policy variables. This is reported in (iiia) of Table 1.

The expected short-term nominal rate is obtained by taking conditional expectations of the money market equilibrium condition (1b). Noting (4a), this is given by

$$i^*_{t+j,t} = \frac{1}{\alpha_2}[P^*_{t+j,t} - M^*_{t+j,t}] \qquad j = 1, 2, \ldots \tag{13}$$

so that the expected future short-term nominal rate is inversely related to the expected future real money stock for that period. The solution for the one-period ahead expected future nominal rate is obtained by substituting (6) into (13), and setting $j = 1$. This is reported in (iiib). In contrast to the expected future real rates, it depends upon the discounted sum of all expected future policy variables. This occurs because of their impact on the expected inflation rate.

Finally, the long-term nominal interest rate is obtained from the term structure relationship (2), the solution to which (with $\bar{I} = \bar{R}$) is

$$I_t = \frac{1}{1+1/\bar{R}}[i_t + \sum_{j=1}^{\infty} i^*_{t+j,t}(\frac{1}{1+\bar{R}})^j]$$

$$I^*_{t,t-1} = \frac{1}{1+1/\bar{R}}\sum_{j=0}^{\infty} i^*_{t+j,t-1}(\frac{1}{1+\bar{R}})^j.$$

The relevant expressions are obtained from (13), (6), with solutions being reported in (iva), (ivb) of Table 1.

4. IMPACT OF MONETARY DISTURBANCES ON TERM STRUCTURE

Table 2 summarizes the effects of monetary and fiscal policy disturbances on the various interest rates. These effects are determined on the assumption that the parameter η remains fixed, so that they are to be interpreted as pertaining to a given structure of the economy. The shock being analyzed must therefore be viewed as coming from a given probability distribution, rather than reflecting any change in the variance. Both transitory and permanent disturbances are considered. The policy variable Z_t say $(Z_t = M_t, G_t)$, is described by the stochastic process

$$Z_t = Z_{t-1} + \epsilon_t^P + \epsilon_t^T - \epsilon_{t-1}^T \tag{14}$$

The random variables $\epsilon_t^P, \epsilon_t^T$ have zero means, finite variances σ_p^2, σ_T^2 respectively, and are independently distributed over time. Under these conditions ϵ_t^P represents a permanent shift while ϵ_t^T represents a transitory shock. To ensure that η remains constant across disturbances, we shall assume that $\sigma_P^2 + \sigma_T^2$ remains constant.

Table 2 draws the following distinctions between the responses of the various interest rates:

(i) real vs. nominal rates;

(ii) long vs. short rates.

The disturbances are characterized as follows:

(i) transitory vs. permanent disturbances;[8]

(ii) unanticipated vs. anticipated current disturbances; i.e., whether or not the stochastic components $\epsilon_t^T, \epsilon_t^P$, are anticipated in advance.

To discuss the effects of the policy shocks it is convenient to decompose the various interest rates x_t into their unanticipated component $(x_t - x_{t,t-1}^*)$ and their anticipated component $x_{t,t-1}^*$. Any unanticipated policy shock has no effect on $x_{t,t-1}^*$ and so the response of the actual interest rate is simply that of $(x_t - x_{t,t-1}^*)$. Analogously, any anticipated policy shock has no effect on $(x_t - x_{t,t-1}^*)$ and so the complete response of the interest rate is fully reflected in $x_{t,t-1}^*$. In describing the behavior of the interest rates in response to the various shocks we will therefore in fact be referring to the appropriate components reported in Table 2.

The effects of various monetary disturbances, which are summarized in Part A of Table 2, are discussed in the remainder of this section. Fiscal disturbances are reported in Part B and are considered in Section 5 below. In all cases, the disturbances are taken to be of unit magnitude. One expression which occurs both explicitly in Part A of Table 2 and plays a role in subsequent comparisons we shall make is $(1 + 1/\eta - d\alpha_1)$. We shall assume that

$$1 + \frac{1}{\eta} - d\alpha_1 > 0 \tag{15}$$

a condition which is surely met for all plausible parameter values.[9]

A. Unanticipated Monetary Disturbances

We begin with a consideration of unanticipated monetary disturbances and from Part A of Table 2, draw the following observations:

1. An *unanticipated temporary* or *permanent* increase in the nominal money supply will *lower* both the *short-term* and *long-term real* interest rates, with the effects on the former being proportionately greater by a factor $(1 + 1/\eta)$ in the two cases.
2. An *unanticipated temporary* or *permanent* increase in the nominal money supply will *lower* both the *short-term* and *long-term nominal* interest rates, with the effects on the former being proportionately greater by a factor $(1 + 1/\bar{R})$ in the two cases.
3. An *unanticipated temporary* monetary expansion leads to a *greater* reduction in *short-term nominal* than in *short-term real* rates. The same is true with respect to *long-term* rates, when speculators are risk neutral, but is not necessarily so when they are risk averse.[10] An *unanticipated permanent* monetary expansion causes a *greater* reduction in *real* than it does in *nominal* interest rates (both long-term and short-term).
4. An *unanticipated temporary* monetary expansion leads to a *greater* reduction in *nominal* interest rates (both long-term and short-term), but a *smaller* reduction in *real* interest rates (both long-term and short-term), than does an equivalent *permanent* monetary expansion.

Many of these effects are straightforward. The proportionality in the effects of monetary expansions on the short-term and long-term rates follows directly from the term structure relationship. The fact that a temporary monetary expansion has a greater effect on short-term nominal rates than it does on short-term real rates follows from the fact that being temporary, such an expansion raises the current price level P_t, while leaving all future price expectations unchanged. This reduces the current expected rate of inflation $(P^*_{t+1,t} - P_t)$, thereby lowering the short-term nominal rate relative to the short-term real rate. The fall in the former is therefore greater. If speculators are risk neutral, these effects on the short-term rates translate proportionately to corresponding effects on the long-term rates. But with risk averse behavior, the response of the long-term nominal rate involves a greater dampening of the response of the short-term nominal rate, than is the case with the corresponding real rate. It is therefore possible for the fall in the long-term real rate to exceed that of the long-term nominal rate, as noted.[11] An unanticipated permanent monetary expansion, on the other hand, has a greater effect on the short-term real rate, than it does on the short-term nominal rate. This is due to the fact that being permanent, it raises the expected future price more than it does the current price level, thereby increasing the expected rate of inflation $(P^{*}_{t+1,t} - P_t)$.[12] This reduces the fall in the nominal rate, relative to that of the real rate. These effects are transmitted, via the term structure relationship, to the long-term rates, this being true irrespective of the degree of risk aversion.

The comparison between the effects of temporary and permanent monetary expansions on the real and nominal interest rates is also of interest. Hardly surprisingly, a temporary monetary expansion lowers the short-term nominal interest rate. While one effect of a permanent monetary expansion is to do the same, at the same time, it will raise expected future inflation rates, thereby putting upward pressure on long- term nominal rates. From the term structure relationship the fall in the short-term rate is mitigated; i.e., the short- term nominal rate falls less than if the monetary expansion were only transitory. Consider now real rates. A temporary monetary expansion reduces the short-term real rate, although by a lesser amount than the nominal rate, as we have seen. In addition to this effect, an unexpected permanent monetary increase leads to an upward revision in the prediction of future prices, which causes the current price level to increase unexpectedly, leading to an increase in the current level of output through the supply function. This in turn adds to the fall in the long term real rate, required to maintain product market equilibrium, and by arbitrage, the fall in the short-term real rate is increased.

B. Anticipated Monetary Disturbances

The effects of anticipated current monetary changes operate through their impacts on the anticipated components of the interest rates. Since anticipated real interest rates have been shown to depend only upon fiscal policy, $R^{*}_{t,t-1}, r^{*}_{t,t-1}$ are independent of any monetary disturbances. It therefore follows that the actual real rates R_t, r_t, respond equally to anticipated or unanticipated monetary disturbances of a given type. Also, since an expected permanent increase in the money supply leads to a proportionate expected increase in the price level, the expected real money stock for each future period is unchanged, so that the effect on all future expected nominal interest rates, $i^{*}_{t+j,t}$ are zero. Furthermore, since the current expected future long-term nominal rate is just a discounted sum of all expected future short-term rates, all of which are unaffected by an expected permanent monetary expansion, it too is unchanged. On the other hand, an expected temporary increase in the money supply, lowers the expected short-term rate $i^*_{t,t-1}$ so that the current actual short-term rate falls by more than if the expansion were unanticipated. The same pattern applies to the long- term nominal rate, although the responses are dampened in accordance with the term structure relationship.

5. IMPACT OF FISCAL DISTURBANCES ON TERM STRUCTURE

We turn now to Part B of Table 2 and consider the effects of unit increases in real government expenditure on the term structure.

A. *Unanticipated Fiscal Disturbances*

From Part B of Table 2, the following propositions can be derived:

1. An *unanticipated temporary* increase in government expenditure will *raise* the *short-term* and *long-term real* interest rates, with the effects on the former being proportionately greater by the factor $(1 + 1/\eta)$.
2. An unanticipated temporary increase in government expenditure will *raise* both the *short-term* and *long-term nominal* interest rates, with the effects on the former being proportionately greater by the factor $(1 + 1/\bar{R})$.
3. Assuming the interest elasticity of the demand for money $\alpha_2\eta < 1$, an *unanticipated permanent* increase in government expenditure *raises* the *long-term real* rate, doing so by an amount which *exceeds* its effects on the *short-term real* rate. In fact, the response of the short-term rate is ambiguous, and under quite plausible conditions it may well fall.
4. An *unanticipated permanent* increase in government expenditure *raises* both the *long-term* and *short-term nominal* rates, with its effect on the former being greater.
5. An *unanticipated temporary* increase in government expenditure has a *greater* effect on *real* than it does on nominal rates (both long-term and short-term). An *unanticipated permanent* increase in government expenditure has a *greater* effect on *nominal* rates than it does on real rates (both long-term and short-term).
6. Provided $\alpha_2\eta < 1$, an *unanticipated temporary* increase in government expenditure has a *greater* effect on *short-term* rates (both real and nominal) than it does a *permanent* increase. An *unanticipated permanent* increase has a *greater* effect on *long-term* rates (both real and nominal) than does a *temporary* increase.

The proportionality of the short-term and long-term rates described in 1 and 2 follows immediately from the term structure relationship and the fact that transitory fiscal disturbances have no expectational effects beyond the current period. The results contained in 3 and 4, namely that permanent increases in government expenditure will have *greater* effects on long-term rates than on short-term rates, we consider to be the most significant propositions. In the first place, the notion that government expenditure is set as a stochastic adjustment from the previous period's level, does not seem to be unreasonable. The finding that this leads to a larger response in long-term rates, relative to short-term rates, means that to the extent that fluctuations in interest rates are in fact generated by fiscal disturbances of this kind, the variation of long-term rates relative to that of short-term rates will in fact be much larger than the simple expectations theory suggests and also much more consistent with the empirical evidence. And this is true even under the assumption of risk neutrality.

The basic reason for this finding can be seen by comparing the response of the current short-term r_t and the expected future short-term rates r^*_{t+i}. From equation (iia) in Table 1, a permanent increase in government expenditure raises the current short-term real rate by an amount[13]

$$dr_t = \frac{1}{D}[(1 + \alpha_2 + \alpha_1\gamma) - \frac{\alpha_2^2\gamma}{d}(1 + \frac{1}{\eta})]$$

while it raises all expected future short-term real rates by

$$dr^*_{t+i,t} = \frac{1}{d} > dr_t.$$

Comparing these two expressions, a permanent fiscal expansion is seen to have a greater effect on expected future short-term rates than on the present rate, thereby causing a greater adjustment in the current long-term rate. The same general argument applies to nominal rates.

A more intuitive explanation is the following. A temporary increase in government expenditure say, shifts out the IS curve, thereby raising the current long term rate. If it is expected to be permanent, in addition to having this effect, it will also raise the expected long-term real rates for all subsequent periods. Since in the short run, some of the variation originating with G_t is borne by output, it follows from the IS curve (1a) that the rise in the current long-term rate R_t is less than $1/d$. On the other hand, since output in all future periods is expected to remain fixed, the effects of expected future increases in government expenditure are expected to be borne fully by corresponding rises in expected future long-term rates, the amount of the expected adjustment being $1/d$. Since future long-term rates are therefore expected to rise more than the current, the present price of long-term bonds is expected to fall, and by arbitrage, the short-term real interest rate rises less than the long rate. In fact, if the expected fall in the price of long-term real bonds is sufficient, it is possible for the current short-term real rate to actually decline. Much the same type of argument can be given with respect to nominal rates.

Turning to the comparison of nominal and real rates given in 5, as noted, an unanticipated transitory fiscal expansion shifts the IS curve out, thereby increasing both the long-term and short-term real rates. At the same time, the current price level is raised, although since the disturbance is transitory, there is no effect on the expected price level in any future period. The current expected inflation rate therefore falls and the short-term nominal rate therefore rises less than the real. By contrast, an unanticipated permanent fiscal expansion will raise expected future prices more than it does the present. The current expected inflation rate therefore rises and the nominal rate rises more than the real.

Finally, we turn to the comparison of the effects of temporary and permanent fiscal expansions. In addition to raising the current long-term real rate, a permanent fiscal expansion will raise both expected future long-term and short-term real rates by amounts which exceed the adjustment in the current long-term real rate. This causes the current long-term real rate to adjust more than if the expansion were merely temporary. On the other hand, a temporary fiscal expansion leaves next period's forecast of the long-term real rate unchanged; $R^*_{t+1,t} - R_t$ therefore falls. This fall, as compared to the rise when the increase is permanent, means that the short-term real rate is more responsive to temporary than to permanent fiscal expansions.

B. Anticipated Disturbances

An anticipated current fiscal expansion, which is expected to be permanent, raises all current expected rates (real and nominal, long-term and short-term) by the same amounts, $1/d$. Since the long-term expected real rate depends upon government expenditure for only that period (see (4b)), its response is the same whether the disturbance is in fact temporary or permanent. The short-term expected real rate in response to an anticipated transitory shock rises by an amount augmented by the factor $(1 + 1/\eta)$. The expected increase in the current short-term nominal rate depends upon the response in the expected price level (see (13)). The response of the latter to a temporary fiscal expansion exceeds that in response to an expected permanent expansion if and only if $\alpha_2\eta < 1$. Finally, the effects of anticipated current disturbances on actual current variables can be obtained by combining these effects with those discussed in 7.

6. RELATIVE VARIANCES OF SHORT- AND LONG-TERM RATES

The results summarized in Table 2 and discussed in Sections 4 and 5 offer important insights into the observed relative variances of short-term and long-term rates. From these results we see that if the underlying stochastic disturbances consist of some linear combination of: (i) temporary monetary, (ii) permanent monetary, and (iii) temporary fiscal shocks, then the relative variances of the long-term and short-term real rates are

$$\frac{\sigma_R^2}{\sigma_r^2} = \frac{1}{(1+1/\eta)^2} \tag{16a}$$

while the corresponding relative variances of the nominal rates are

$$\frac{\sigma_I^2}{\sigma_i^2} = \frac{1}{(1+1/\overline{R})^2}. \tag{16b}$$

In the case where speculators are risk neutral, $\eta = \overline{R}$ and these two expressions coincide. Taking $\overline{R} = .05$ as being a reasonable value for the long-term equilibrium interest rate, then these ratios imply $\frac{\sigma_R^2}{\sigma_r^2} = \frac{\sigma_I^2}{\sigma_i^2} = .0023$; the variance of the long rate is a negligible fraction of that of the short rate. Empirically, the ratio is much larger than .0023 and this is precisely the excess volatility of long rates. We may note that risk averse speculators raise the ratio $\frac{\sigma_R^2}{\sigma_r^2}$ for real rates, and in the limiting case when $k \to \infty$, this ratio approaches unity.

By contrast, if the only disturbances are permanent fiscal disturbances, the relationship bewtween the variances of the short-term and long-term rates is more complicated. In the case of real rates it becomes

$$\frac{\sigma_R^2}{\sigma_r^2} = [\frac{1+\alpha_2+\alpha_1\gamma+\frac{\gamma\alpha_2(1-\alpha_2\eta)}{d\eta}}{1+\alpha_2+\alpha_1\gamma-\frac{\alpha_2^2}{d}[1+\frac{1}{\eta}]}]^2. \tag{17a}$$

In general, we can show $\sigma_R^2 > \sigma_r^2$ if and only if

$$2[1+\alpha_2+\alpha_1\gamma-\frac{\gamma\alpha_2}{d}]+\frac{\gamma\alpha_2}{d\eta}[1-\alpha_2\gamma] > 0 \tag{18}$$

an inequality which under plausible conditions will be met. For example, taking the set of parameter values noted in footnote 13, (18) is true for all d. But this parameter set requires assigning a value to η, which itself is endogenous, unless $k = 0$. Sufficient conditions, expressed in terms of basic parameters which ensure that (18) is met include that each parenthesis be positive, which too is likely to be met. Otherwise, to obtain a necessary and sufficient condition for (18) to hold in terms of only basic parameters requires us to solve for η. This involves a nonlinear equation and in general is intractable.[14]

The relative variances of the nominal rates are

$$\frac{\sigma_I^2}{\sigma_i^2} = \frac{[\frac{(1+\alpha_1\gamma)(1+\alpha_2)}{D}+\frac{1}{dR}]^2(\frac{1}{1+1/R})^2}{[\frac{(1+\alpha_1\gamma)(1+\alpha_2)}{D}]^2} \tag{17b}$$

which can be shown to imply $\sigma_I^2 > \sigma_i^2$ if and only if (15) holds, as we have assumed. This latter condition may be written as

$$\eta[1-d\alpha_1]+1 \equiv [\bar{R}+k\sigma_R^2(1)][1-d\alpha_1]+1>0 \tag{19}$$

which will clearly be met for all degrees of risk aversion, and hence for all values of η, if $1 > d\alpha_1$. If on the other hand, $1 < d\alpha_1$, (19) imposes an upper bound on $\sigma_R^2(1)$. Again to express (19) in terms of the underlying parameters requires the elimination of η and again the nonlinearity involved makes this intractable.

But even if (18) and (19) are not met, it is clear that the relative variances of long to short rates (both real and nominal) are much larger when the underlying disturbances are permanent fiscal shocks. To the extent that elements of all such disturbances are occurring simultaneously, our framework is capable of generating relative variances of long to short rates which are perfectly consistent with empirical evidence.

7. STRUCTURAL CHANGES AND THE TERM STRUCTURE

In the policy changes considered so far, the underlying structure of the economy is assumed to remain fixed. The consequence of this is that the risk-adjusted discount rate η does not change. We now consider the effects of changes in: (i) the degree of risk aversion as reflected by k; (ii) an exogenous variance, on the variances of the short-term and long-term interest rates. We shall restrict our analysis to real rates, but nominal rates can be analyzed in a similar way.

Let us assume that the fluctuations in the economy are due to temporary monetary disturbances, u_t^m say, having variance σ_u^2. From Table 2A, the short-run fluctuations in the real rates are

$$R_t - R^*_{t,t-1} = -\frac{\gamma}{D}u_t^m, \quad r_t - r^*_{t,t-1} = \frac{-\gamma(1+1/\eta)}{D}u_t^m$$

with the corresponding one-period variances being

$$\sigma_R^2(1) = \frac{\gamma^2\sigma_u^2}{[d(1+\alpha_2+\alpha_1\gamma)+\gamma\alpha_2(1+1/\eta)]^2} \tag{20a}$$

$$\sigma_r^2(1) = \frac{\gamma^2(1+1/\eta)^2\sigma_u^2}{[d(1+\alpha_2+\alpha_1\gamma)+\gamma\alpha_2(1+1/\eta)]^2}. \tag{20b}$$

But as noted previously

$$\eta = \bar{R} + k\sigma_R^2(1) \tag{20c}$$

so that the short-run variance of the long real rate, $\sigma_R^2(1)$, and the risk adjusted discount rate η, are jointly determined by (20a), (20c).[15] The non-linearity of the former equation raises the possibility of non-existence and non-uniqueness of equilibrium, an issue which has been discussed in related contexts by others; see e.g., Driskill and McCafferty (1980), Turnovsky (1983). However, as evident from Fig. 1, such problems do not arise here.

Fig. 1 illustrates the joint determination of $\sigma_R^2(1), \eta$, and $\sigma_r^2(1)$. The locus XX in the right-hand side panel is a straight line relating η to $\sigma_R^2(1)$ in accordance with (20c). The locus YY is the relationship between $\sigma_R^2(1)$ and η described by the solution (20a). This is an upward sloping curve, passing through the origin, having the indicated curvature and asymptote. The unique point of intersection A is the equilibrium solution for η and the short-run real variance $\sigma_R^2(1)$. In the left-hand panel, the curve ZZ relates the one-period variance of the short-run real rate $\sigma_r^2(1)$ to η, and this is negatively sloped as indicated by (20b). Corresponding to the equilibrium value of η obtained from the point of intersection A, is the point B in the ZZ locus, which in turn yields the equilibrium value for the short-run variance of the short-term real rate $\sigma_r^2(1)$.[16]

Consider now an increase in the degree of risk aversion ω as reflected by an increase in k. This is illustrated by a rotation of the XX line to XX'. The equilibria shift from A to A', B to B' respectively, resulting in an increase in the one-period variance of the long-term rate $\sigma_R^2(1)$, and a decrease in the one-period variance of the short-term rate $\sigma_r^2(1)$. This suggests that the assumption of risk neutrality (when the line XX becomes horizontal) *overstates* the variance of the short-term rate, relative to that of the long-term rate.

The movement from A to A' can be decomposed into several components. First, the direct effect of the increase in ω, for *given* variance $\sigma_R^2(1)$, is to raise η. This is represented by the move from A to C. This rise in η in turn leads to an increase in $\sigma_R^2(1)$ [see (20a)] and this is illustrated by the move from C to D. This induced rise in $\sigma_R^2(1)$ then increases η further, which in turn increases $\sigma_R^2(1)$, etc., and this is illustrated by the move along the locus DA'. The mirror images of these adjustments in the left-hand panel are given by a jump from B to E, followed by a gradual adjustment along EB'.

Fig. 1.B illustrates an increase in the variance of the transitory money shock, σ_u^2. In this case XX remains unchanged; YY rotates outwards, while ZZ shifts outwards. The result is that the equilibrium points A, B shift to A' and B' respectively. The variance of the long-term rate definitely increases, while the variance of the short-term rate may either increase as illustrated, or decrease. The move can be broken down into two effects. The first are the direct increases due to the outward shifts in the XX and ZZ curves and are given by the movements AC, BD. These effects assume that η remains constant. But as the variance of the long-term rate increases, $\sigma_R^2(1)$ the discount rate η increases. This tends to raise the long-term real rate, while lowering the short-term rate. These moves, illustrated by the movements CA', DB', reinforce the direct effect in the former case, but counteract it in the latter.

The analysis for permanent monetary shocks and temporary fiscal disturbances is essentially identical to that given. The case of permanent fiscal disturbances is however, different, and is illustrated in Figure 2. Space limitations permit only a brief discussion of this case.

The lines XX, YY, and ZZ are analogous to those in Fig. 1. The relationship between $\sigma_R^2(1)$ and η is now negatively sloped, while that between $\sigma_r^2(1)$ and η changes sign with η. As before, there is a unique equilibrium. Since in the limit as $\eta \to \infty, \sigma_R^2(1) = \sigma_r^2(1)$, we see from the figure that $\sigma_R^2(1) > \sigma_r^2(1)$ at least as long as η is sufficiently large so that $\sigma_r^2(1)$ lies to the right of the vertical through Q (note $QO = OR$). An increase in the degree of risk aversion now lowers the variance of the long rate and may or may not increase the short variance, depending upon η. The direct effect of an increase in the variance of the permanent fiscal shock is to raise the variances of both the long-term and short-term rates by amounts AC, BD respectively. But the accompanying increase in η leads to a partially offsetting reduction in $\sigma_R^2(1)$, illustrated by CA'. The induced effect on $\sigma_r^2(1)$ is measured by DB', which as illustrated represents a further rise, but which may be a decline if η is sufficiently small.

8. CONCLUSIONS

This paper has analyzed the term structure of interest rates within a complete stochastic macroeconomic model. Two main aspects have been discussed

First, we have analyzed in some detail the effects of various monetary and fiscal policies on the term structure. In so doing, we have contrasted the effects of temporary vs. permanent, and unanticipated vs. anticipated, disturbances on the one hand, and the responses of long vs. short, and real vs. nominal rates, on the other. The main results are summarized in a series of propositions outlined in Sections 4 and 5. These underscore the general conclusion that the response of the term structure is highly sensitive to the nature of the underlying shocks impinging on the economy. Although little purpose would be served by reviewing these propositions, it is worth highlighting some of the sharp contrasts we have obtained.

For example, whereas an unanticipated temporary monetary expansion tends to have a greater effect on nominal rates, an unanticipated permanent monetary expansion has a greater effect on real rates. These relative responses are reversed for fiscal policy. While an unanticipated temporary fiscal expansion has greater effects on real rates, an unanticipated permanent expansion has a greater impact on nominal rates. Thirdly, and most importantly, although a temporary fiscal expansion has a greater effect on short rates, an unanticipated permanent fiscal expansion has greater impact on long term rates. To the extent that such shifts represent a significant source of the stochastic fluctuations in interest rates, this result may help significantly in explaining the observed volatility of long-term rates.

The second major aspect we have discussed is to show how with risk averse behavior, any structural change has two effects on the variances of the interest rates. In addition to a direct effect, there is an indirect effect which operates through the impact of the change on private speculative behavior. The latter may reinforce or counteract the former and will have an impact on the relative variances of the short and long rates.

TABLE 1

Solutions for Interest Rates

Long-term Real Rates

$$R_t - R^*_{t,t-1} = \frac{1}{D}\Big[-\gamma(M_t - M^*_{t,t-1}) + (1 + \alpha_2 + \alpha_1\gamma)(G_t - G^*_{t,t-1})$$

$$- \gamma\sum_{k=1}^{\infty}(M^*_{t+k,t} - M^*_{t+k,t-1})\left[\frac{\alpha_2}{1+\alpha_2}\right]^k + \frac{\gamma(1-\alpha_2\eta)}{d\eta}\sum_{k=1}^{\infty}(G^*_{t+k,t} - G^*_{t+k,t-1})\left[\frac{\alpha_2}{1+\alpha_2}\right]^k\Big] \qquad \text{(ia)}$$

$$R^*_{t,t-1} = G^*_{t,t-1}/d \qquad \text{(ib)}$$

Short-Term Real Rate

$$r_t - r^*_{t,t-1} = \frac{(1+\frac{1}{\eta})}{D}\Big[-\gamma(M_t - M^*_{t,t-1}) + (1 + \alpha_2 + \alpha_1\gamma)(G_t - G^*_{t,t-1})$$

$$- \gamma\sum_{k=1}^{\infty}(M^*_{t+k,t} - M^*_{t+k,t-1})\left[\frac{\alpha_2}{1+\alpha_2}\right]^k + \frac{\gamma(1-\alpha_2\eta)}{d\eta}\sum_{k=1}^{\infty}(G^*_{t+k,t} - G^*_{t+k,t-1})\left[\frac{\alpha_2}{1+\alpha_2}\right]^k\Big] \qquad \text{(iia)}$$

$$- \frac{1}{d\eta}(G^*_{t+1,t} - G^*_{t+1,t-1})$$

$$r^*_{t,t-1} = \frac{1}{d}(1 + \frac{1}{\eta})G^*_{t,t-1} - \frac{1}{d\eta}G^*_{t+1,t-1} \qquad \text{(iib)}$$

Short-Term Nominal Rate

$$i_t - i^*_{t,t-1} = \frac{1}{D}\left[-[\gamma(1+\frac{1}{\eta}) + d](M_t - M^*_{t,t-1}) + (1+\frac{1}{\eta})(1+\alpha_1\gamma)(G_t - G^*_{t,t-1}) + \frac{d}{\alpha_2}(1+\alpha_1\gamma)\sum_{h=1}^{\infty}(M^*_{t+h,t} - M^*_{t+h,t-1})\left(\frac{\alpha_2}{1+\alpha_2}\right)^h + \frac{(1+\alpha_1\gamma)(\alpha_2\eta - 1)}{\alpha_2\eta}\sum_{h=1}^{\infty}(G^*_{t+h,t} - G^*_{t+h,t-1})\left(\frac{\alpha_2}{1+\alpha_2}\right)^h\right] \tag{iiia}$$

$$i^*_{t,t-1} = -(\frac{1}{1+\alpha_2})M^*_{t,t-1} + \frac{1}{d(1+\alpha_2)}(1+\frac{1}{\eta})G^*_{t,t-1} + \frac{1}{\alpha_2(1+\alpha_2)}\left[\sum_{h=1}^{\infty}M^*_{t+h,t-1}\left(\frac{\alpha_2}{1+\alpha_2}\right)^h + \left(\frac{\alpha_2\eta - 1}{d\eta}\right)\sum_{h=1}^{\infty}G^*_{t+h,t-1}\left(\frac{\alpha_2}{1+\alpha_2}\right)^h\right] \tag{iiib}$$

Long-Term Nominal Rate

$$I_t - I^*_{t,t-1} = \frac{\bar{R}}{1+\bar{R}}\left[(i_t - i^*_{t,t-1}) + \sum_{j=1}^{\infty}(i^*_{t+j,t} - i^*_{t+j,t-1})\left(\frac{1}{1+\bar{R}}\right)^j\right] \tag{iva}$$

where

$$I^*_{t,t-1} = \frac{\bar{R}}{1+\bar{R}}\sum_{j=0}^{\infty} i^*_{t+j,t-1}\left(\frac{1}{1+\bar{R}}\right)^j \tag{ivb}$$

$$i^*_{t+j,t} = -\frac{1}{1+\alpha_2}M^*_{t+j,t} + \frac{1}{d(1+\alpha_2)}(1+\frac{1}{\eta})G^*_{t+j,t} + \frac{1}{\alpha_2(1+\alpha_2)}\left\{\sum_{h=1}^{\infty}\left[M^*_{t+j+h,t} + \left(\frac{\alpha_2\eta - 1}{d\eta}\right)G^*_{t+j+h,t}\right]\left(\frac{\alpha_2}{1+\alpha_2}\right)^h\right\}$$

$$D \equiv d[1 + \alpha_2 + \alpha_1\gamma] + \gamma\alpha_2(1+\frac{1}{\eta}) > 0$$

TABLE 2

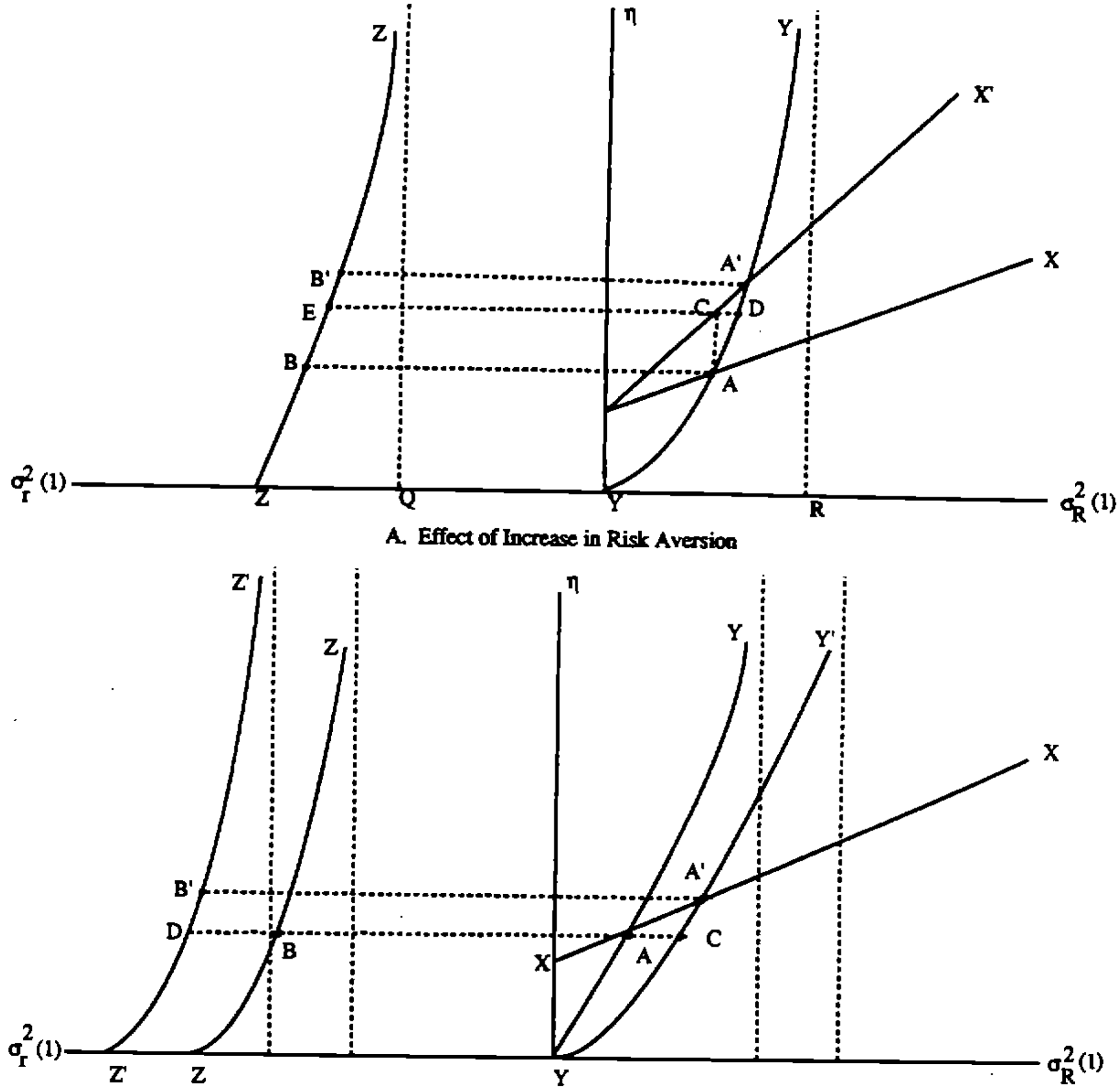
A. *Current Monetary Disturbances*

		Temporary	Permanent
unanticipated:	$R_t - R^*_{t,t-1}$	$-\frac{\gamma}{D}$	$-\frac{\gamma(1+\alpha_2)}{D}$
anticipated:	$R^*_{t,t-1}$	0	0
unanticipated:	$r_t - r^*_{t,t-1}$	$-\frac{\gamma(1+1/\eta)}{D}$	$-\frac{\gamma(1+\alpha_2)(1+1/\eta)}{D}$
anticipated:	$r^*_{t,t-1}$	0	0
unanticipated:	$I_t - I^*_{t,t-1}$	$-\frac{[\gamma(1+1/\eta)+d]}{(1+1/\bar{R})D}$	$-\frac{\gamma[1+1/\eta-d\alpha_1]}{(1+1/\bar{R})D}$
anticipated:	$I^*_{t,t-1}$	$-\frac{(1+\alpha_2)}{(1+1/\bar{R})}$	0
unanticipated:	$i_t - i^*_{t,t-1}$	$-\frac{[\gamma(1+1/\eta)+d]}{D}$	$-\frac{\gamma[(1+1/\eta)-d\alpha_1]}{D}$
anticipated:	$i^*_{t,t-1}$	$-\frac{1}{1+\alpha_2}$	0

TABLE 2 (continued)

B. *Current Fiscal Disturbances*

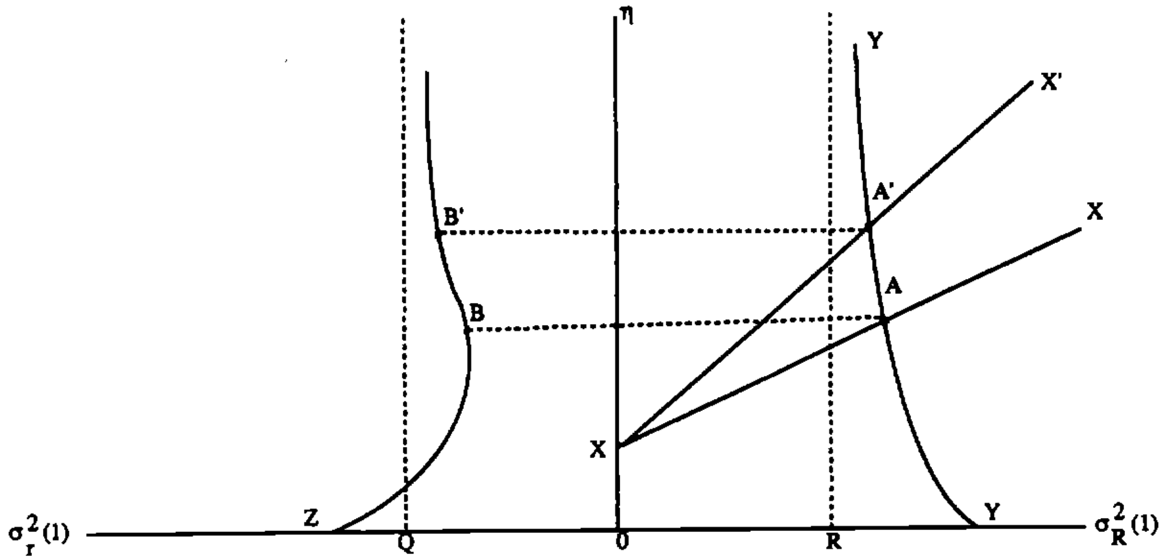
		Temporary	Permanent
unanticipated:	$R_t - R^*_{t,t-1}$	$\frac{1+\alpha_2+\alpha_1\gamma}{D}$	$\frac{1}{D}\left[(1+\alpha_2+\alpha_1\gamma)+\frac{\gamma\alpha_2(1-\alpha_2\eta)}{d\eta}\right]$
anticipated:	$R^*_{t,t-1}$	$\frac{1}{d}$	$\frac{1}{d}$
unanticipated:	$r_t - r^*_{t,t-1}$	$\frac{(1+\alpha_2+\alpha_1\gamma)(1+1/\eta)}{D}$	$\frac{(1+\alpha_2+\alpha_1\gamma)-\frac{\alpha_2^2\gamma}{d}(1+1/\eta)}{D}$
anticipated:	$r^*_{t,t-1}$	$(1+\frac{1}{\eta})/d$	$\frac{1}{d}$
unanticipated:	$I_t - I^*_{t,t-1}$	$\frac{(1+\alpha_1\gamma)(1+1/\eta)}{D(1+1/\bar{R})}$	$\left[\frac{1}{1+1/\bar{R}}\right]\left[\frac{(1+\alpha_1\gamma)(1+\alpha_2)}{D}+\frac{1}{d\bar{R}}\right]$,
anticipated:	$I^*_{t,t-1}$	$\frac{(1+1/\eta)}{d(1+\alpha_2)(1+1/\bar{R})}$	$\frac{1}{d}$
unanticipated:	$i_t - i^*_{t,t-1}$	$\frac{(1+\alpha_1\gamma)(1+1/\eta)}{D}$	$\frac{(1+\alpha_2)(1+\alpha_1\gamma)}{D}$,
anticipated:	$i^*_{t,t-1}$	$\frac{(1+1/\eta)}{d(1+\alpha_2)}$	$\frac{1}{d}$



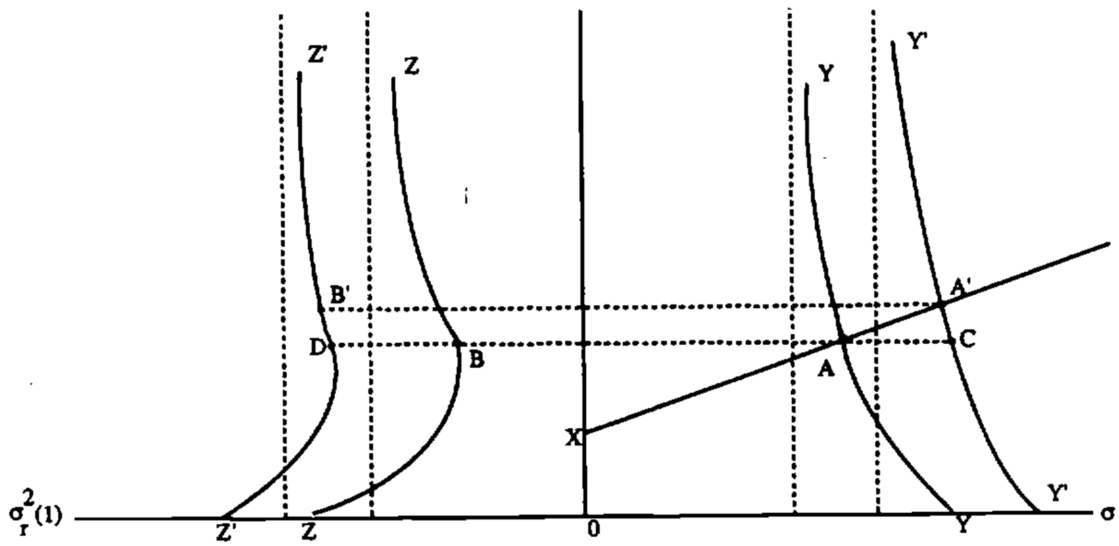
A. Effect of Increase in Risk Aversion

B. Effect of Increase in Variance of Money Supply

FIGURE 1



A. Effect of Increase in Risk Aversion



B. Effect of Increase in Variance of Permanent Fiscal Disturbance

FIGURE 2

FOOTNOTES

*The constructive comments of two referees are gratefully acknowledged.

[1]For example, the Blanchard and Turnovsky–Miller models are both non–stochastic in which prices remain fixed or at best are introduced in a restrictive way. Both analyses lead to saddlepoint–type dynamics, with the forward–looking behavior of the long rate, being combined with the backward–looking behavior arising from some sluggishness in the system. In the Blanchard model, this is introduced through gradual adjustments in output, while in the Turnovsky–Miller model, it arises through the process of wealth accumulation. The Mascaro and Meltzer paper introduces risk but as a given exogenous parameter. The analysis is therefore essentially deterministic, as well. The McCafferty paper is closest to the present. It derives the term structure from underlying mean–variance utility maximization of risk–averse speculators. In his model, however, prices remain fixed, while the range of exogenous disturbances considered is much less extensive. Finally, Cox, Ingersoll and Ross (1985) present the most general stochastic utility maximizing approach to the term structure. However, their analysis does not address macroeconomic policy issues, which are the prime focus of the present analysis.

[2]Some of the well–known early work includes Modigliani and Sutch (1966), Modigliani and Shiller (1972), while among the more recent contributions are Shiller (1979), Mankiw and Miron (1985), Mankiw (1986).

[3]Other justifications for the inclusion of the long–term rate is that to the extent that real private expenditure include investment goods, it depends upon Tobin's q, which in turn is inversely related to the long–term real rate. Also, the long–term real rate reflects asset values and their impact through wealth on current consumption. But, it is also possible for consumption to depend upon the short–term real rate as well. The inclusion of this variable, in addition to R, does not alter the substance of our analysis in any way.

[4]This specification assumes risk neutrality. If more generally, one assumes a mean–variance utility maximizing framework, then under somewhat restrictive conditions this relationship can be amended to simply include a risk premium, which is a function of the underlying one–period variance of the price level. While this constant may be treated as fixed for any given probability distribution, it does of course change as the price variance changes across distributions. This aspect is discussed further in a expanded version of this paper.

[5]Suppose, for example, that speculators maximize as a mean–variance utility function

$$E_t(\pi_{t+1}) - \frac{1}{2}\omega E_t[\pi_{t+1} - E_t(\pi_{t+1})]^2$$

where π_{t+1} denotes profits, and ω measures the degree of risk aversion. Then one can show that this will give rise to the following linearized speculative demand function for long bonds, L_t

$$L_t^s = \frac{\bar{R}^3}{\omega\sigma_R^2(1)}[R_t - \frac{(R_{t+1,t}^* - R_t)}{\bar{R}} - r_t].$$

Suppose, in addition there are non–speculators, whose demand function for long bonds depends upon the interest differential

$$L_t^{ns} = \beta(R_t - r_t) \qquad \beta > 0.$$

Equilibrium in the bond market, $L_t^s + L_t^{ns} = 0$ implies

$$[\beta + \frac{\bar{R}^3}{\omega\sigma_R^2(1)}](R_t - r_t) - \frac{\bar{R}^2}{\omega\sigma_R^2(1)}(R_{t+1,t}^* - R_t) = 0$$

and this may be solved for r_t in the form

$$r_t = R_t - \frac{1}{\eta}(R_{t+1,t}^* - R_t)$$

where

$$\eta \equiv \bar{R} + \beta\omega\sigma_R^2(1)/\bar{R}^2 \equiv \bar{R} + k\sigma_R^2(1).$$

This procedure is essentially the stock analogue to the flow formulation adopted by McCafferty (1986). It is discussed further in an expanded version of this paper.

[6]Throughout we shall adopt the following notation. For any variable X say, $X_{t+j,t}^*$ denotes the prediction of X for time $t+j$, formed at time t.

[7]By considering the stable solution, we are ruling out speculative bubbles.

[8]The analysis is based on the assumption that private agents can distinguish between permanent and transitory shocks. This is clearly restrictive. More generally, agents will be faced with trying to infer the permanent and transitory components from observations on the current and past policy variables themselves. For an example of this using the optimal linear forecasting methods of Muth (1960) see Brunner, Cukierman and Meltzer (1980). Our separate treatment of permanent and transitory disturbances corresponds to two polar cases of this more general approach.

[9]For example, taking the risk adjusted discount rate η to be say .10, the income elasticity of the demand for money α_1 to be 1, (15) is met as long as $d < 11$, which is surely the case for any plausibly sloped IS curve.

[10]The formal condition for the effect on the long–term nominal rate to be greater is $d > \gamma[(1/\eta) - 1/\bar{R}]$. This is certainly met under risk neutrality (when $\eta = \bar{R}$), but need not hold otherwise.

[11]For example, taking values of $\eta = .10, \bar{R} = .05, \gamma = 1$, the fall in the long–term real rate exceeds that of the long–term nominal rate provided $d < 10$, which seems reasonable.

[12]The response of the expected inflation rate can be determined by combining equations (6′), (9) and (12).

[13]Taking values of $\alpha_1 = 1, \alpha_2 = .5, \gamma = 1, \eta = .10$ as being plausible the short–term real rate will rise or fall depending upon whether d 1.1. While a rise would seem more likely, a fall definitely cannot be ruled out.

[14]The nonlinearity involved is illustrated for temporary monetary shocks by equations (20a), (20c) below.

[15]The role of the endogeneity of speculative behavior in the determination of the term structure is also discussed by McCafferty (1986).

[16]Note that since both $\sigma_R^2(1), \sigma_r^2(1)$ converge to $1/d$ as $\eta \to \infty$, the distance $QY = QR$. Figure 1 illustrates how for these disturbances $\sigma_r^2(1) > \sigma_R^2(1)$.

[17]This is based on the plausible assumption made earlier that R responds positively to a permanent fiscal shock.

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