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Voting Rights, Agenda Control and Information Aggregation
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ABSTRACT

This paper examines the comparative properties of voting rules based on the richness of their ballot spaces, assuming a given distribution of voting rights. We focus on how well voting rules aggregate the information dispersed among voters. We consider how different voting rules affect both voters' decisions at the voting stage and the incentives of the agenda-setter who decides whether to put the proposal to a vote. Without agenda-setter, the voting efficiency of rules is higher when their ballot space is richer. Moreover, full-information efficiency requires full divisibility of the votes. In the presence of an agenda-setter, we uncover a novel trade-off: in some cases, rules with high voting efficiency provide worse incentives to the agenda-setter to select good proposals. This negative effect can be large enough to wash out the higher voting efficiency of even the most efficient rules.

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1 Introduction

Voting is ubiquitous in our society, serving as a fundamental means for decision-making in various contexts. Whether citizens select representatives, shareholders approve management proposals, politicians determine governmental policies, jurors decide the fate of a defendant, or creditors approve debt restructuring plans, voting plays a crucial role. Despite the wide array of theoretically available voting rules, practical constraints often limit the feasible options, particularly when exogenous factors influence the distribution of voting rights. Fairness and equity concerns frequently call for symmetrically distributed voting rights among voters, adhering to the one-person-one-vote principle, a foundational tenet of democratic elections worldwide. However, in scenarios where voters have varying stakes in the decisions, the same fairness principles may necessitate asymmetric treatment (Brighouse and Fleurbaey, 2010). This gives rise to situations where voters possess different numbers of ballots or their ballots weigh differently on the final outcome.[1]

Even when the distribution of voting rights is given, the set of possible voting rules remains vast. A crucial factor in distinguishing among voting rules is then the extent to which voters can express the intensity of their support for or against the proposal via their vote. Some voting rules provide voters with a wide variety of ballots, which they can use to reveal the strength of their support. Other rules, however, offer only a limited set of ballots, which do not allow voters to convey more than the direction of their support.

In this paper, we study the comparative properties of voting rules based on the richness of their ballot spaces, assuming a given distribution of voting rights. We focus on how well voting rules aggregate the information dispersed among voters, in the spirit of the Condorcet Jury Theorem literature. We consider how different voting rules affect both voters' decisions at the voting stage and the incentives of the agenda-setter who shapes the proposal before the vote. The main insight of our analysis is that, for a given distribution of voting rights, richer ballot spaces are desirable when there is no (strategic) agenda-setter, but not necessarily when there is one.

[1] Such asymmetric voting rights emerge, for instance, in shareholders meetings due to heterogeneity in shareholdings, in debt restructuring votes with heterogeneous creditor claims, and in international organizations like the European Union, the International Monetary Fund, and the World Bank, where countries' contributions or population sizes result in distinct voting powers. Bouton et al. (2018) discuss how asymmetric voting rights may be necessary to guarantee participation in international institutions. See also Maggi and Morelli (2006) and Macé and Treibich (2021).

We consider a general framework that includes the following components. First, we focus on binary group decisions, where voters must decide whether to accept or reject a proposal. Such decisions can arise in various contexts, such as a retention election for a governor or a judge, a vote by shareholders on a management proposal, a vote in the national assembly on a particular piece of legislation, a jury vote to convict or acquit a defendant, or a creditors vote to approve or reject a debt restructuring plan. Second, we consider any distribution of voting rights among voters, where each voter is assigned a specific number of ballots to cast. Voting rules then differ based on the range of ballot options available to voters. Third, in line with the literature on information aggregation, we consider cases where (most) voters have the same state-contingent preferences. That is, they agree that the proposal should be approved in some situations, and rejected in others. For example, most citizens prefer a representative who is honest, skilled, and hard-working; most shareholders aim for high profits; most politicians prefer projects that can be completed successfully; most jurors want to convict the guilty and acquit the innocent; and most creditors seek to minimize debt reduction while ensuring the solvency of the debtor firm or country. The issue for voters is that, at the time of the vote, they are only imperfectly informed about the desirability of the proposal. Moreover, some voters may be more precisely informed than others, potentially creating or exacerbating an imbalance between voting rights and information precision.

For the sake of concreteness, we present our analysis and results for the case of shareholders' meetings. As we discuss in details in the Appendix, this case fits particularly well the key components of our framework. Shareholders frequently vote on proposal by the management–i.e. binary choices.[2] The distribution of voting rights among shareholders depends on the distribution of shares, with corporations mostly following the *one-share-one-vote principle*,[3] and can be either symmetric or (most frequently) asymmetric, as different shareholders may hold different number of shares.[4] Finally, it is natural that (most) shareholders share the common goal of maximizing the value of the firm.

[2] As discussed in Christoffersen et al. (2007) and Yermack (2010), management's proposals include, e.g., possible mergers and acquisitions, the issuance of new shares, the sale of the firm, amendments to governance procedures, changes in voting rights of directors, and new compensation package for directors.

[3] Yet, recent trends are towards more progressive systems in which some shareholders are awarded an oversized share of the voting rights, e.g., dual-class stock (see, e.g., Adams and Ferreira, 2008, and Hayden and Bodie, 2008).

[4] For instance, 96% of the firms in a representative sample of US firms have at least one *blockholder*, i.e., a shareholder who own more than 5% of the outstanding shares (Holderness, 2009). By contrast, retail shareholders hold much fewer shares of a given firm. Moreover, the dispersion of shareholdings among retail shareholders is substantial (Brav et al., 2022).

In the first part of the paper, we focus on the strategic behavior of shareholders at the meeting, taking the management proposal as given. We establish two main results for this case with an exogenous proposal. First, we compare rules with a finite ballot space in terms of their information aggregation efficiency (i.e., the likelihood of selection of the best alternative). We show that, for any given distribution of voting rights and any informational asymmetry among shareholders, a voting rule with a richer ballot space dominates a voting rule with a poorer one. This is because strategic shareholders can use the richer ballot space to adjust their impact on the voting outcome to the accuracy of their information. This normative result has direct implications for the comparison of commonly used voting rules from an information aggregation standpoint. For instance, contrarily to what is usually feasible in practice, shareholders who own multiple shares should be allowed to vote them differently (or partially abstain).

Second, we show that efficiency requires votes to be fully divisible. This is so because, under any voting rule with fully divisible votes (the *generalized flexible rules*), all shareholders have the ability to reveal their information fully, independently of the number of shares they own. This second normative result highlights that, from an information aggregation standpoint, the distribution of voting rights across shareholders becomes irrelevant when votes are fully divisible (and all shareholders have at least one vote). Thus, as long as votes are made fully divisible, decisions about how to distribute voting rights across shareholders can focus on other facets of the issue (e.g., effects on takeovers or on the firms' choice of ownership and financing).

We further show that the desirability of a richer ballot space is robust to the presence of (i) partisan shareholders, (ii) expressive/non-strategic shareholders, (iii) super-majority thresholds, (iv) ambiguity about the information technology of other shareholders, and (v) endogenous acquisition of information by shareholders.

In the second part of the paper, we explore how voting rules affect the management's incentives before the meeting. We allow the management to decide whether to put a proposal to the vote after observing a signal about its quality. If the management blocks the proposal, then the status quo remains. The properties of a voting rule then depend both on (i) *selection* (i.e., the incentives it provides the management to select good proposals and block bad ones), and (ii) *voting efficiency* (i.e., the quality of information aggregation at the meeting).

Following the corporate governance and finance literature, we allow for conflict between shareholders and the management along two dimensions (see Tirole 2010 and references therein). First, the manager may be *misaligned*, in the sense that she wants the proposal to be adopted even when it is undesirable for the shareholders. Second, the manager may incur a (reputation) cost if the proposal is rejected at the shareholder meeting.

The existence of an agenda-setter whose preferences are not necessarily aligned with those of the voters is not confined to shareholders' meetings. For instance, activists triggering a retention election may not care much about the valence of the governor or judge; committee chairs may put more weight on the party line than other legislators; prosecutors might care more than jurors about the probability of winning a case than doing the right thing (i.e. convict the guilty and acquit the innocent) when choosing which charges to press; negotiators of bankruptcy cases may be more lenient on debtors than creditors (e.g., due to moral hazard or career concerns) when designing the debt restructuring proposal.

We uncover a trade-off between selection and voting efficiency underlying the comparison of generalized flexible rules and other, less flexible, voting rules, with lower voting efficiency. The key is the impact of the voting rule on the behavior of the misaligned manager. When a rejection of the proposal at the meeting is costly for the manager, she may have incentives to block the proposal even when misaligned. We show that, in equilibrium, the misaligned manager blocks bad proposals with some probability; a selection of proposals that is beneficial to shareholders. We then find that the incentives to block bad proposals can be stronger under lower voting efficiency rules, if shareholders are more likely to reject the proposal if it reaches the meeting. In some cases, the difference in selection of proposals between generalized flexible rules and lower voting efficiency rules can be sufficiently strong to compensate for the higher voting efficiency of the former rules. Then, shareholders are better off under lower voting efficiency rules than under generalized flexible ones. This occurs when it is sufficiently likely that the manager is misaligned and the cost of rejection of the proposal at the meeting is in an intermediate range. Indeed, for low costs of rejection, misaligned managers do not block any proposal in equilibrium under any voting rule, and hence voting efficiency is all that matters for the quality of outcomes; while when rejection costs are very high, blocking proposals is very frequent under any voting rule—and therefore the outcomes are quite similar.

1.1 Related Literature

The exogenous proposal part of our paper contributes to the literature on information aggregation in binary elections with exogenous alternatives (see, e.g., Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1996, 1997, 1998; Myerson, 1998; Bhattacharya, 2013; Bouton et al., 2018; Barelli et al., 2022). That literature focuses on a specific symmetric distribution of voting rights (one-person-one-vote) and voting rules with finite ballot spaces (voters can vote in favor the proposal, against it, or abstain).[5] By contrast, we consider any asymmetric distribution of voting rights and allow for richer ballot spaces. That allows us to show the desirability of richer ballot spaces in terms of information aggregation for a broad class of situations.[6] In particular, we show that (i) among rules with finite ballot spaces, a rule with richer ballot spaces dominates one with poorer ballot spaces, and that (ii) generalized flexible rules are the only efficient rules.

The second result highlights that vote divisibility is a key feature to aggregate information efficiently. While the literature has studied how to adjust specific features of the voting rule to voters' environment in order to improve information aggregation, such as the super-majority threshold (Feddersen and Pesendorfer, 1998; Maug and Rydqvist, 2009) or the allocation of voting rights (Nitzan and Paroush, 1982; Azrieli, 2018), we show that full divisibility of the votes is sufficient to achieve efficient information aggregation, independently of the environment. This observation is important for institutional design as it implies that the other features of the voting rule (than vote divisibility) can then be chosen to comply with other desiderata without compromising on efficiency.

Our analysis also complements the typical results on strategic abstention, i.e., that less informed

[5] One notable exception is Bar-Isaac and Shapiro (2020). They consider a setup with one blockholder who owns many shares, and many retail shareholders who own one share each. We generalize their model, and their result of the desirability of partial abstention, by considering (i) any distribution of shares among shareholders, including but not limited to multiple blockholders, (ii) any correlation between shareholdings and information precision, and (iii) imperfect information about shareholders' information precision. Beyond this generalization, a key difference is that we study the information aggregation properties of voting rules taking into account of their effects on the management's incentives. That allows us to identify situations in which partial abstention, through its effects on the management's incentives, is detrimental to information aggregation.

[6] This result is reminiscent of the finding of Ahn and Oliveros (2016) that Approval Voting dominates other voting rules in multi-candidate elections because of its richer ballot space. Beyond our focus on two-alternative elections for any distribution of voting rights, another difference is that we show that extending the ballot space allows for better equilibria but does not generate worse ones.

voters abstain from voting in equilibrium (Feddersen and Pesendorfer, 1996; McMurray, 2013; Herrera et al., 2019). We show that, when allowed, as under the flexible rule, voters partially abstain, with the extent of abstention diminishing with the informativeness of their signal. As a result, shareholders endogenously determine the impact of their vote on the final decision as a (smooth) function of their signal's informativeness.

The endogenous proposal part of our paper contributes to the literature on information aggregation with endogenous alternatives. We are only aware of two papers exploring that question: Henry (2008), and Bond and Eraslan (2010). The latter is the closest to our paper. It stresses the importance of endogenizing the proposal when one studies the information properties of a voting rule. But our model and insights are quite different than those in Bond and Eraslan (2010). Crucially, we consider any distribution of voting rights and compare rules which differ along two dimensions: the divisibility of the votes, and the super-majority threshold. By contrast, Bond and Eraslan (2010) focus on the super-majority threshold comparing one-person-one-vote rules such as majority and unanimity. Moreover, by comparing rules which admit an unambiguous ranking in terms of voting efficiency (richer vs. poorer ballot space), we uncover a general tradeoff between voting efficiency and selection incentives that is not possible to detect when limiting attention to classes of one-person-one-vote rules.[7] Also, we focus on the extensive margin of action for the manager, instead of the intensive margin in Bond and Eraslan (2010). That allows us to show that selection incentives can upset the welfare ordering of rules even if the manager only controls the agenda and has limited power to adjust the details of the proposal.

Finally, our paper also contributes to the literature on our main application: voting at shareholder meetings. There is a large empirical literature studying the effect of shareholder voting on firms performance and management's behavior. Overall, this literature finds that control by shareholders (i) affects positively firms' performance, and (ii) is key to provide proper incentives to the management.[8] The specifics of corporate governance rules and procedures appear to play

[7] This is because super-majority rules–a prominent class of one-person-one-vote rules–do not admit a clear ranking in terms of voting efficiency: each super-majority rule is optimal under certain informational assumptions (see, e.g., Maug and Rydqvist, 2009).

[8] For instance, Appel et al. (2016) find that even passive investors affect positively firms' longer-term performance through voting; Fos et al. (2018) find that the proximity to an election affects the behavior of directors; Li et al. (2018) finds that, in the US, shareholder voting help mitigate agency problems that plague corporate acquisitions; Becht et al. (2016) find that shareholder voting has a substantial positive effect on the quality of acquisitions by firms in the UK; Richardson (2000) finds a positive relationship between information asymmetry between managers and

an important role. As summarized by Yermack (2010, p. 106): research that studies the "[...] general effects of voting restrictions on firm value and performance, often [finds] that firms perform worse when the shareholder franchise is curtailed [...]. Notable recent papers in this large literature include Gompers et al. (2003), which examines a range of takeover defenses and voting restrictions; Bebchuk and Cohen (2005) and Faleye (2007), both of which focus on staggered boards; and Gompers et al. (2010), which studies dual-class voting structures."[9] More directly connected to us, Burkart and Lee (2008) review the theoretical literature on the one-share-one-vote principle and highlight three classes of effects of the security-voting structure (aka the distribution of voting rights): effects on takeovers, effects on incentives of blockholders, and effects on the firms' choice of ownership and financing. We complement that literature by highlighting the effects of voting rules on the quality of decisions at shareholders meetings, and how they shape managers' incentives.

2 A Simple Example

Three shareholders must decide through voting whether to approve a proposal, A, or maintain the status quo, B. The shareholders' payoffs are contingent on the outcome: they obtain a payoff of one if the outcome is A in state α or B in state β; otherwise, if the outcome is A in state β (type-I error) or B in state α (type-II error), they receive a payoff of zero. The issue is that, at the time of the vote, shareholders do not know whether the proposal is desirable (state α) or not (state β). These two states are considered equiprobable. Prior to voting, each shareholder receives a conditionally independent signal either in favor of the proposal, s_a, or against it, s_b. The information structure is asymmetric in the sense that, in both states, shareholders are more likely to receive the signal s_a in favor of the proposal. In particular, we assume that $Pr\left(s_a|\alpha\right) = 0.75$ and $Pr\left(s_b|\beta\right) = 0.45$. Despite the asymmetric information structure, the efficient outcome for shareholders is to approve the proposal if and only if the group receives a majority of s_a signals.

Shareholders have different voting rights: there is one "blockholder" who holds three shares,

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shareholders, which impedes the ability of the latter to control efficiently the former, and earnings management (i.e., funky accounting by managers); Cai et al. (2009) and Aggarwal et al. (2019) focus on directors elections in US firms and find that even uncontested elections affect the firms and directors in various dimensions; Conyon and Sadler (2010), Ferri and Maber (2013), and Alissa (2015) study the effects of say-on-pay votes in the UK and find that they constrain the size and the structure of top managers' pay.

[9] Note however that Frankenreiter et al. (2021) have recently cast some serious doubts on the reliability of the datasets used in this literature.

and two small shareholders who hold one share each, with all shares carrying one vote. Taking that distribution of voting rights as given, we compare the performance of two voting rules with different ballot spaces: the *rigid rule*, under which shareholders have to cast all their votes in favor of the proposal, or all their votes against it, and the *flexible rule*, which allows shareholders to fully adjust the intensity of their support in favor or against the proposal as each share carries one fully divisible vote, which can be voted independently.

We first consider the exogenous proposal case: the manager always puts the proposal to the vote. Under the flexible rule, there exists an equilibrium that achieves the optimal outcome: the small shareholders vote in line with their respective signals (i.e., cast their only vote in favor of the proposal if receiving signal s_a, and against it if receiving signal s_b), while the blockholder casts one of her votes in the same way as small shareholders, but abstains with the other two. The key factor behind the effectiveness of the flexible rule lies in the ability of the blockholder not to use all her votes in the same way. By doing so, she can strategically adjust the intensity of her support to the information structure, leading to an efficient outcome. Note that this strategic adjustment does not always take the form of abstention. For other information structures, the blockholder would indeed cast all her votes in the efficient equilibrium.[10]

Under the rigid rule, by contrast, the group faces a fundamental limitation in implementing the optimal decision. The blockholder, holding a majority of the votes, exercises unilateral control over the outcome. The best she can do is to follow her own signal. This inherent imbalance significantly undermines the efficiency of the rigid rule, as it essentially discards the minority shareholders' valuable information. Under the rigid rule, the expected utility of shareholders is only 0.6, compared to 0.6345 under the flexible rule.

We now consider the endogenous proposal case in which a manager holds the authority to decide whether to put the proposal to the vote or to block it. The manager knows the state of the world at the time of that decision but is misaligned with shareholders in the sense that she obtains a payoff of one if the proposal is adopted, and zero otherwise, irrespective of the state of the world. She may nonetheless block the proposal as she incurs a rejection cost, $c > 0$, if the proposal is turned

[10]For instance, for $Pr\left(s_a|\alpha\right) = 0.4$ and $Pr\left(s_b|\beta\right) = 0.75$, the blockholder casts two of her votes in favor of the proposal independently of her signal, and then casts her last vote based on her signal (small shareholders also follow their signal). This guarantees that the proposal is adopted when it is the efficient outcome (i.e. when shareholders receive at least one signal s_a).

down by shareholders.

The manager blocks the proposal if the expected cost of rejection is excessively high. Since shareholders vote informatively, it is more likely that the proposal is rejected when undesirable (in state β). Hence, the manager has stronger incentives to block the proposal when it is not desirable (in state β) than when it is (in state α). The key determinant of the manager's behavior is thus the probability that shareholders approve an undesirable proposal (i.e., the likelihood of a type-I error). If that probability is sufficiently high compared to the rejection cost c, the manager must be blocking the proposal with positive probability in equilibrium, and weakly more when the proposal is undesirable, which benefits shareholders.[11]

We have shown above that, when the manager always puts the proposal to the vote, the flexible rule produces less errors overall than the rigid rule. Yet, this does not mean that each type of error is less likely under the flexible rule. And indeed, for the example under consideration, the flexible rule produces more type-I errors than the rigid rule: 0.57475 vs. 0.55. This implies that there are values of the rejection cost c such that the manager wants to block the proposal under the rigid rule but not under the flexible one. For instance, when the cost of rejection is $c = 1.25$, in the equilibrium under the rigid rule, the manager blocks the undesirable proposal with probability of $\frac{4}{9}$, while she does not block it under the flexible rule. This greater selectivity of proposals by the manager benefits shareholders, effectively compensating for the lower efficiency of the rigid rule at the voting stage: shareholders' expected utility under the rigid rule sees a substantial increase to 0.7222 (compared to the utility of 0.6345 under the flexible rule). For the endogenous proposal case, the rigid rule thus dominates the flexible rule.

3 Model

In this section we present our baseline model. In Appendix D we provide a discussion of some of the key assumptions, including restrictions on communication, share trading, and vote trading.

Consider a firm with a set $N = \{1, 2, ..., n\}$ of shareholders, with $n > 2$. Each shareholder $i \in N$ holds d_i shares, where d_i is a positive integer. We denote the share distribution by $d = (d_i)_{i \in N}$.

[11] We can show that, for appropriate values of the rejection cost, c, the equilibrium is such that the manager puts the proposal to the vote with probability one when it is desirable for shareholders (in state α), and blocks it with positive probability when it is not desirable (in state β).

At the shareholders meeting, shareholders have to choose, through voting, whether to approve a proposal by the management, A, or keep the status quo, B. We denote the set of alternatives by $O = \{A, B\}$. For now, we assume that the proposal is exogenously given. In Section 6 we allow the management to block the proposal.

Shareholders are uncertain about the quality of the proposal. There are two states of the world, $\omega \in \Omega = \{\alpha, \beta\}$ that are unobserved at the time of the vote. For the sake of simplicity, we assume that the states are equiprobable.

Preferences. All shareholders agree that the proposal is good in state α but bad in state β:[12]

$$\begin{aligned} &u_i\left(A|\alpha\right) = 1,\ u_i\left(A|\beta\right) = -1, \\ &u_i\left(B|\alpha\right) = u_i\left(B|\beta\right) = 0. \end{aligned} \tag{1}$$

Information. Before the meeting, each shareholder i receives a signal $s_i \in S := [0, 1]$ distributed according to a shareholder-specific distribution function, $F_i\left(\cdot|\omega\right)$, with density $f_i\left(\cdot|\omega\right)$. Conditional on the state, signals are drawn independently. The type of shareholder i after the draw of the signal is $t_i = \frac{f_i(s_i|\alpha)}{f_i(s_i|\beta)} \in T_i = [\delta_i, 1/\delta_i]$.[13] We make the following assumption about the signal technology:

Assumption 1 (Strong MLRP and Bounded Support)**.** *For every shareholder $i \in N$, t_i is strictly increasing in s_i and there exists $\delta_i \in (0, 1)$ such that $\frac{f_i(0|\alpha)}{f_i(0|\beta)} = \delta_i$ and $\frac{f_i(1|\alpha)}{f_i(1|\beta)} = 1/\delta_i$.*

The first part of Assumption 1 means that shareholders who receive higher signals attach a larger probability to the state of the world being state α, while the second part means that there is no shareholder with arbitrarily precise information about the state of the world. This assumption allows for various structures of shareholders' information technology. For instance, it allows for arbitrarily large differences in the (expected) information quality of two shareholders, and for any type of correlation between the (expected) information quality of a shareholder and the number of shares she owns.

[12]Note that shareholders' stakes may well vary in practice (e.g. they can be proportional to their shareholdings) but this does not affect the structure of the game. The utility definition is thus, in that sense, without any loss of generality.

[13]The type t_i of a shareholder summarizes the strategically relevant information contained in her signal s_i, which corresponds to the relative likelihood of state α once the signal has been received.

Voting Rules. We consider a broad class of voting rules. A voting rule is described by its ballot spaces $X = (X_1, X_2, ..., X_n)$, such that for each $i \in N$, $X_i \subseteq \mathbb{R}$ is a closed and bounded set with at least two elements. Each shareholder $i \in N$ chooses $x_i \in X_i$ and the outcome is:

$$G(x) = \begin{cases} A & \text{if } \sum_{i \in N} x_i > 0 \\ AB & \text{if } \sum_{i \in N} x_i = 0 \\ B & \text{if } \sum_{i \in N} x_i < 0, \end{cases}$$

where AB denotes the fair lottery between A and B.[14] For each shareholder $i \in N$, we denote by $w_i = \frac{1}{2}(\max X_i - \min X_i)$ her voting right under the voting rule X (which boils down to the number of votes she can cast in favor or against the proposal if $\max X_i = -\min X_i$).

This class of voting rules nests most popular rules. For instance, the majority rule, or *one-person-one-vote* rule, $X_i^{1P1V} = \{-1, 1\}$, is such that the allocation of voting rights is independent of the share distribution d, i.e., $w_i = 1$ for all i. Each shareholder has to choose among two ballots, voting for A (action 1) or voting for B (action -1). When voting rights are allocated according to the *one-share-one-vote principle*, i.e. $w_i = d_i$ for all i, different degrees of vote divisibility can be considered. Under the *one-share-one-vote* rule, $X_i^{1S1V} = \{-d_i, -d_i + 1, \ldots, d_i\}$ and each shareholder i can choose from a rich set of ballots: either fully supporting A (action d_i), fully supporting B (action $-d_i$), or intermediate intensities of support (integers between $-d_i$ and d_i). An alternative, coarser, version of the one-share-one-vote rule is such that $X_i^{1S1V-r} = \{-d_i, 0, d_i\}$. Under that coarse version, shareholders cannot partially abstain, i.e., cast only some of their shares in favor of one alternative. They have to cast all their votes in favor or against the proposal, or fully abstain. As discussed below, this is more in line with what is done in practice as partial abstention is often not feasible (or very costly) for most shareholders.

For any given distribution of voting rights $w = (w_i)_{i \in N}$, two rules stand out in terms of the divisibility of the votes. At one extreme, under the *flexible* rule, $X_i^f = [-w_i, w_i]$, votes are fully divisible and shareholders can thus fully adjust the intensity of their support in favor or against the proposal. By contrast, under the *rigid* rule, $X_i^r = \{-w_i, w_i\}$, votes are fully non-divisible and

[14] Note that this definition allows for super-majority threshold requirements. Indeed, the rule where A passes whenever $\sum_{i \in N} x_i > m$ is equivalent to the rule where A passes whenever $\sum_{i \in N}(x_i - m/n) > 0$. Hence, for instance, the 2/3-supermajority rule is such that A passes whenever $\sum_{i \in N} \frac{x_i + 1}{2} > \frac{2n}{3}$, or $\sum_{i \in N} x_i > \frac{n}{3}$, and can be written as $X_i = \{-4/3, 2/3\}$ for all $i \in N$.

shareholders can thus only indicate the direction of their support.

It will be useful to distinguish between cases in which there is a shareholder who can always affect the outcome independently of the choices of the other shareholders, and cases in which such a player is not present. We will say that a shareholder i is *decisive* if $w_i > \sum_{j \neq i} w_j$, and that there is *no decisive shareholder* if no such shareholder exists.

Strategies and Equilibrium Concept. For each voting rule X and each shareholder $i \in N$, a strategy is a function $\sigma_i : T_i \to \Delta(X_i)$. As is standard in the literature, $\Delta(X_i)$ is the set of all probability distributions on X_i. When σ_i is a pure strategy, we sometimes abuse notation and denote by $\sigma_i(t_i)$ the action x that the shareholder picks with probability 1. Since σ_i can be a mixed strategy, it is useful to distinguish the random variable, σ_i, from a potential realization, $\widehat{\sigma}_i$: we say that $\widehat{\sigma}_i$ is a potential realization of σ_i if and only if $\widehat{\sigma}_i$ belongs to the support of σ_i. Consequently, when σ_i is a pure strategy we have $\sigma_i(t_i) = \widehat{\sigma}_i(t_i)$ for every $t_i \in T_i$.

We focus on (interim) *Bayesian Nash Equilibria* (BNE) such that equilibrium strategies are best responses at the interim stage (when each shareholder knows her own type).

4 Welfare Benchmarks

We consider two different welfare benchmarks, which serve different purposes: one is to assess the efficiency of a voting rule, and the other to compare voting rules which are not efficient.

Our efficiency benchmark corresponds to the preferred outcome of shareholders when they have access to all the information dispersed in the electorate (i.e., if they were able to observe the signal profile). In that case, shareholders would prefer the alternative that is most likely to match the state of the world, conditional on the available information:

Definition 1. *Given a vector of signals $s = (s_1, s_2, ..., s_n)$, the efficient outcome, E, is equal to A if $\Pr(\alpha|s) > 1/2$, B if $\Pr(\beta|s) > 1/2$, and any outcome is efficient if $\Pr(\alpha|s) = \Pr(\beta|s) = 1/2$.*

This leads to a natural implementation notion:[15]

[15]In the sequel, we abuse notation and refer to a voting rule when in fact mean the game that is defined by that voting rule. Hence, we will talk about an equilibrium of a voting rule, and say that a voting rule implements an outcome.

Definition 2. *Given a voting rule X, a BNE $\sigma = (\sigma_1, \ldots, \sigma_n)$ is efficient if $G(\sum_{i \in N} \widehat{\sigma}_i(t_i)) = E$ for every $t \in \Pi_{i \in N} T_i$ and every potential realization $\widehat{\sigma}(t)$ of the random variables $\{\sigma_1(t_1), \ldots, \sigma_n(t_n)\}$. A voting rule X implements the efficient outcome in equilibrium if it admits an efficient BNE.*

Our second welfare benchmark corresponds to the preferred outcome of shareholders when they know the state of the world: A in state α, and B in state β.

The two welfare benchmarks are compatible: when the state of the world is unobservable, then the alternative that is most likely the correct outcome given the available information coincides with the efficient outcome. Moreover, in our common value environment, both these welfare benchmarks are aligned with utilitarian principles. The correct outcome is the utilitarian outcome (i.e. the alternative that maximizes the sum of ex-post utilities), and the efficient outcome is the outcome most likely to be the utilitarian one given all the shareholders' information.[16]

We then compare the performance of voting rules focusing on the ex-ante (i.e. before the state of the world and types are drawn) probability with which they implement the correct outcome, considering both the best equilibria (in terms of selecting the correct outcome), and the worst ones:[17]

Definition 3. *Voting rule X dominates voting rule X' if (i) for every BNE under X', there is a BNE under X such that the ex-ante probability of implementing the correct outcome is higher under X than X'; and (ii) for every BNE under X, there is a BNE under X' such that the ex-ante probability of implementing the correct outcome is lower under X' than X. If, moreover, either (i) or (ii) (or both) hold strictly, we say that X strictly dominates X'.*

Voting rules typically admit multiple equilibria. Therefore, assessing the potential performance of a rule considering only the best (worst) equilibrium, in terms of the ex-ante probability of selecting the correct outcome, might be overly optimistic (pessimistic). For this reason, we opt for a comparative criterion that combines both the best and the worst possible equilibrium outcomes.

[16] Instead of definition 2, one might prefer a seemingly weaker efficiency notion. That is, one could only require that, given any profile of types, the probability that the equilibrium outcome is correct is equal to the probability that E is correct. These two efficiency notions are, in fact, equivalent (proof available upon request).

[17] Given our setup with binary states and outcomes, and common values, comparing two equilibria of two voting rules with respect to the probability that each implements the correct outcome, is equivalent to comparing them with respect to the corresponding sums of expected utilities across agents.

We say that a rule dominates another if and only if the former is superior to the latter in both dimensions.[18] This is particularly important in our case because, as we will illustrate below, rules with richer ballot spaces allow for both better and worse outcomes compared to rules with poorer ballot spaces. Hence, focusing only on the best (worst) equilibrium of each rule might not be very informative with respect to the range of equilibrium performances.[19]

5 Equilibrium Analysis: Exogenous Proposal

In this section, we analyze the equilibrium performances of various voting rules when the manager is passive, i.e., the proposal is exogenously given. We split the section in two parts. First, we compare the information aggregation properties of finite voting rules, i.e., rules with finite ballot spaces such as the *1S1V* rule or the *rigid* rule. Second, we focus on the informational efficiency of all types of voting rules. In Appendix C, we discuss the robustness of our results to various extensions of the model: (i) allowing for the presence of partisan and/or expressive (non-strategic) shareholders, and (ii) endogenizing the acquisition of information.

We first introduce the following Lemma, which is not only useful to prove some of our key results, but also to understand the benefits and pitfalls of a voting rule with a richer ballot space.

Lemma 1. $\Pr(\alpha|s) > \frac{1}{2} \Leftrightarrow \sum_{i \in N} \ln(t_i) > 0$.

Proof. See Appendix A. □

This lemma, which is an application of Bayes' rule, is reminiscent of the results in Nitzan and Paroush (1982), which characterizes the weights that a rule should attach to votes in situations where agents have heterogeneous information precision in a canonical jury setting.

[18] Since we work with a wide set of voting rules, an explicit characterization of the equilibrium set of each rule is not plausible (even if one applies standard refinement tools). There is a trade-off between focusing on specific pairs of voting rules, where the scope of the analysis is more targeted and characterizing equilibrium sets is tractable (see, e.g., Bouton and Castanheira, 2012), and conducting comparisons across a wide set of alternative rules, where the scope of the exercise is wider, but a detailed description of the equilibria for each rule is not tractable (see, e.g., Goertz and Maniquet, 2011). We can nevertheless compare rules based on the comparison of their best and worst equilibria.

[19] Note that we do not limit attention to equilibria in undominated strategies. Importantly, we will show next that both the best and the worst equilibrium are typically not in dominated strategies. Hence, in most cases, the comparison between voting rules cannot become any sharper by focusing on this oft-used refinement in voting games (see, e.g., Bouton and Castanheira, 2012).

This result suggests that if a voting rule allows shareholders with different information precision to cast votes in proportion to the logarithm of their type, then efficiency can be reached. For instance, let us consider a group of shareholders with the same type space $T = \{e^{-10}, e^{-1}, e, e^{10}\}$. The voting rule $X = \times_{i \in N}\{-10, -1, 1, 10\}$ allows them to secure the efficient outcome if they each cast $\ln(t_i)$ votes in favor of A. Of course, such a rule also makes the "inefficient" outcome (i.e., A when $\Pr(\alpha|s) < \frac{1}{2}$ and B when $\Pr(\alpha|s) > \frac{1}{2}$) attainable, since a shareholder of type t_i could cast $-\ln(t_i)$ votes in favor of A. By contrast, the rigid rule $X' = \times_{i \in N}\{-10, 10\}$ does not allow the shareholders to reach the efficient outcome, nor the inefficient one. Hence, it is not obvious how to rank voting rules X and X' in terms of potential outcomes. As we prove below, when we focus on equilibrium outcomes, this indeterminacy is resolved.

5.1 Comparison of Finite Voting Rules

We start this section by comparing two arbitrary finite voting rules, with one having ballot spaces that are subsets of the other's ballot spaces. To state our results in a compact manner it is useful to pin down this possible relationship between two voting rules.

Definition 4. *Consider two voting rules, X and X'. If $X'_i \subseteq X_i$ for every shareholder i, then X is said to be richer than X'.*

Thus, we have that, e.g., the flexible rule is richer than the *1S1V* rule, which itself is richer than the *rigid* rule. Note that the flexible rule is richer than any other voting rule.

Richer ballot spaces differ from poorer ones in one important respect. When a shareholder is allowed to cast, for instance, either 10 votes in favor of the proposal, or 10 votes against it, then that shareholder is pivotal if the election is tied, or up to 10 votes short of being tied. However, when a shareholder is allowed to choose both whether to support the proposal and how much to support it, she can adjust strategically the set of cases in which her action matters: e.g. by casting a unique vote in favor of the proposal, she is pivotal only when there is a tie or the election is one vote away from a tie. Hence, by enriching the ballot space we allow shareholders to more accurately align the influence they exert on the outcome with their information precision. The following proposition shows that giving shareholders such flexibility is efficiency enhancing:[20]

[20]We prove the result under Assumption 1 for clarity of exposition. The result would still hold under the milder

Proposition 1. *Under Assumption 1 and when there is no decisive shareholder, if a finite voting rule X is richer than voting rule X', then X dominates X'.*

Proof. See Appendix A. □

Here is a sketch of the proof. First, let us consider the comparison of the best equilibria under those two rules. Recall from McLennan (1998) that in a pure common value environment, a strategy profile producing the maximal ex-ante utility must be an equilibrium.[21] Given that voting rule X is richer, any outcome under rule X' can be reproduced under X by simply replicating the strategies. Hence, there always exists an equilibrium under X (the utility-maximizing strategy profile) that produces an ex-ante utility at least as high as in the best equilibrium under X'.[22] Notice that for this argument to hold it is not necessary that the best equilibrium under X is efficient, nor that a decisive player is absent. As long as X provides more ballot options to the shareholders than X', then the best equilibrium under X leads to the correct outcome with at least as high a probability as the best equilibrium under X'.

Second, let us consider the comparison of the worst equilibria under those two voting rules. The proof has two main steps. In the first step, we prove the intuitive result that under any voting rule, a profile of monotone strategies is at least as good as a profile where all voters vote in favor of the same outcome independently of their signal. This last profile is in fact an equilibrium in undominated strategies under any voting rule, provided that there is no decisive shareholder.[23] The second step is more involved and consists in proving that any equilibrium is welfare-equivalent to a profile of monotone strategies. We show that if an equilibrium exhibits strategies that are not monotonic, with a shareholder i casting x_i votes when of type t_i but a larger amount $x'_i > x_i$ when of lower type $t'_i < t_i$, then it must be that the difference between x'_i and x_i is small enough that it

assumption of Weak Monotone Likelihood Property. The proof can indeed be directly adapted to that case, by defining shareholders' strategies as functions defined on the (compact) signal space rather than on the type space.

[21] McLennan (1998) assumes that types are finite to guarantee the existence of such a utility-maximizing profile. We prove that profiles that maximize ex-ante utility under any finite rule also exist in settings with infinite types, like ours.

[22] Ahn and Oliveros (2016) use a similar argument to show the superiority of approval voting over plurality in multi-alternative elections.

[23] When the shares are fragmented across several shareholders, and all are expected to employ a sufficiently biased strategy towards acceptance of the proposal, then one has incentives to vote against the proposal with all one's votes for any possible signal. Hence, casting all your votes against the proposal is the unique best response to certain beliefs, and thus an undominated strategy.

does not affect the outcome. By following this line of reasoning, the equilibrium can be shown to be equivalent to a profile of monotone strategies.

Proposition 1 has various implications for the comparison of finite voting rules, holding the allocation of voting rights w constant. Consider for instance variations of the *one-share-one-vote* rule, i.e. rules such that $w_i = d_i$. First, more flexible versions dominate less flexible ones: e.g., *1S1V* dominates both *1S1V-r* which itself dominates the *rigid* rule (since it reduces the ballots even further by preventing any type of abstention). This provides an argument in favor of allowing shareholders to partially abstain (as we discuss below, this is often not possible or very costly in practice). Second, the *one-share-one-vote* rule becomes more efficient after a stock split since $X_i^{1S1V} \subset X_i^{1S1V \otimes k} = \{-d_i, -d_i + \frac{1}{k}, \ldots, d_i\}$, for any integer $k > 1$. This result shows that, through their effect on the ballot space of shareholders, stock splits indeed increase shareholders' ability to reveal their information about the quality of management proposals through voting.

While the focus of this paper is on the comparison of voting rules for a given distribution of voting rights, it is worth noting that Proposition 1 also allows for the comparison of voting rules with different distribution of voting rights. For instance, since the *one-share-one-vote* rule has a richer ballot space than the *one-person-one-vote* rule, Proposition 1 implies that the former rule dominates the latter.[24] Proposition 1 also has implications for dual class capital structures in which some shares do not carry voting rights. Allowing for shares with no voting rights reduces the effective ballot space of some shareholders, limiting their ability to convey information through voting. This affects negatively information aggregation. Therefore, the *one-share-one-vote* rule where all shares have voting rights dominates dual class rules.

5.2 Efficient Voting Rules

In this section, we consider all voting rules and focus on their efficiency. We can achieve a full characterization, both of the voting rules that lead to efficiency in equilibrium, and of the complete set of efficient equilibria corresponding to each such rule. The overall message is two-fold: (i) to be efficient, a voting rule must have a ballot space at least as rich as the type space, and (ii) when votes are fully divisible, the distribution of voting rights across shareholders is irrelevant from an

[24] While this result may be intuitive when information accuracy and shareholdings are correlated, it also holds when they are not. See Appendix C in a previous version of the paper (Bouton et al., 2021) for an illustration.

information aggregation standpoint.

To prove the results in this section, we need one additional assumption:

Assumption 2. *For every shareholder $i \in N$, $T_i = T$ (i.e. $\delta_i = \delta \in (0,1)$) and t_i is continuous in s_i.*

This assumption requires that all types that are possible for one shareholder are possible–but not necessarily equally likely–for any other. Even with this assumption, our model allows for arbitrarily large differences in the expected information quality of two shareholders.[25]

We first characterize the unique efficient equilibrium under the *flexible* rule:

Proposition 2. *Under Assumptions 1 and 2, the* flexible *rule, $X_i^f = [-w_i, w_i]$, admits a unique (up to admissible multiplicative and additive constants) efficient BNE, such that $\sigma_i^f(t_i) = c \ln t_i + \kappa_i$ with $\sum_{i \in N} \kappa_i = 0$ and $c \in (0, \min\{\frac{-w_i - \kappa_i}{\ln \delta}, \frac{-w_i + \kappa_i}{\ln \delta}\}]$ for every $i \in N$.*

Proof. See Appendix A. □

The equilibrium strategy implies that A wins if and only if $\sum_{i \in N} c \ln(t_i) > 0$, which guarantees that the outcome is efficient. As mentioned above, we know from McLennan (1998) that a strategy that maximizes ex ante welfare must be an equilibrium.[26] Uniqueness (up to a multiplicative and an additive constant) follows from the fact that any efficient BNE σ must guarantee that $sgn\left(\sum_{i \in N} \sigma_i(t_i)\right) = sgn\left(\sum_{i \in N} \ln(t_i)\right)$. Only strategies such that $\sigma_i(t_i) = c \ln t_i + \kappa_i$ with $\sum_{i \in N} \kappa_i = 0$ and $c \in (0, \min\{\frac{-w_i - \kappa_i}{\ln \delta}, \frac{-w_i + \kappa_i}{\ln \delta}\}]$ for every $i \in N$ satisfy that condition.[27] The maximum value of c simply guarantees that the ballot of any given type of voter fits in the ballot space.

Next, we characterize the set of efficient voting rules. For that purpose, we introduce the following class of rules, which includes the flexible rule.

[25] The continuity of the mapping from signals to types amounts to having a connected type space. This helps us to fully focus on the essential nature of the strategies in an efficient equilibrium, without being distracted by the possibility of multiple best responses. Indeed, when gaps in the type space are allowed all the equilibria that we identify still exist, but additional ones emerge due to indifferences of types close to points of discontinuity.

[26] We can even show that σ^f is an ex-post equilibrium: no shareholder has incentives to deviate ex post, when all types are known.

[27] This result highlights a challenge for shareholders: they need to coordinate on the correct multiplicative and additive constants. It seems that a focal equilibrium is the one in which shareholders use the highest possible vote intensity when they receive the most informative signal and zero votes when completely indifferent.

Definition 5. *A voting rule X belongs to the class of* ***generalized flexible rules*** *if there exists $(\psi_i)_{i\in N} \in \times_{i\in N} int(X_i)$, such that $\sum_{i\in N} \psi_i = 0$.*

The class of generalized flexible rules is such that (i) all shareholders have access to a continuous ballot space, and (ii) the ballot space allows for a tie in which each voter can increase and decrease the net vote total by any arbitrarily small degree.

Proposition 3. *Under Assumptions 1 and 2, a voting rule implements the efficient outcome in equilibrium if and only if it belongs to the class of generalized flexible rules.*

Proof. See Appendix A. □

The "if" part of the proposition follows from the fact that the efficient equilibrium under the *flexible* rule can be properly re-scaled to fit the ballot space of any rule that belongs to the class of generalized flexible rule (i.e. to fit within any open set around any vector $(\psi_i)_{i\in N}$ such that $\sum_{i\in N} \psi_i = 0$). The "only if" part of the proposition follows from the fact that any efficient equilibrium must be strictly increasing in the shareholder's type, and only rules that belong to the class of generalized flexible rules satisfy that requirement. To understand why it must be strictly increasing, just consider two type profiles t and t' such that (i) $t_{-i} = t'_{-i}$, (ii) $\Pr(\alpha|t) > \Pr(\beta|t)$, and (iii) $\Pr(\alpha|t') < \Pr(\beta|t')$. It must then be that $t_i > t'_i$. But, if $\sigma_i(t_i) \leq \sigma_i(t'_i)$, then either the outcome for t or for t' is not efficient (or both).

Proposition 3 shows that all rules that belong to the class of generalized flexible rules are efficient. This is a broad class of voting rules, which encompasses vastly different distributions of voting rights across shareholders. The equivalence of those different rules from an information aggregation standpoint highlights that vote divisibility is a crucial feature of voting rules, which makes imbalances in voting rights across shareholders irrelevant. This means that decisions about how to distribute voting rights across shareholders can be oblivious to information aggregation issues as long as votes are made fully divisible.

Generalized flexible rules address specific weaknesses inherent to finite voting rules, such as the *one-person-one-vote* and *one-share-one-vote* rules. For instance, the *one-person-one-vote* (with abstention) rule is not efficient when voters receive signals of sufficiently different precision (as in our setup). Indeed, in such cases, the equilibrium (and optimal) strategy under that rule is such that some voters abstain (see, e.g., Feddersen and Pesendorfer, 1996 and McMurray 2013). Some of the information dispersed among voters is thus lost in the voting process, which prevents efficiency. Note that, while the *one-person-one-vote* (with abstention) rule is not efficient independently of the group size, it still features nice asymptotic properties. Indeed, the literature has shown that, in various scenarios, the *one-person-one-vote* (with abstention) rule guarantees full information equivalence (A wins in state α and B in state β) with a probability that tends to one when the size of the groups tends to infinity (see, e.g., Feddersen and Pesendorfer, 1997; Myerson, 1998; Bhattacharya, 2013 and Barelli et al., 2022). This highlights that the advantage of the generalized flexible rules is particularly pronounced when the group of voters is relatively small.

Another limitation of finite voting rules is that shareholders need to accurately know the information technology of all other shareholders, i.e., $F_i(\cdot|\omega)$, to determine their optimal (typically mixed) strategy. If they are mistaken or have ambiguous beliefs about the information technology of others, then the *one-person-one-vote* rule may even fail to aggregate information asymptotically. By contrast, it is clear from Proposition 2 that the equilibrium strategies under the *flexible* rule are independent of other shareholders' information technology. As a result, the *flexible* rule, along with other generalized flexible rules, remains efficient even in the presence of such mistakes or ambiguities.

Both the superiority of voting rules with richer ballot spaces, and the efficiency of generalized flexible rules rely on some shareholders partially abstaining, i.e. not casting all their votes in favor or against the proposal. In practice, it is often not feasible (or very costly) for shareholders to vote their shares differently.[28] Hence, the fact that such a behavior is not widely observed is not evidence that shareholders would not partially abstain if given the opportunity. And indeed, shareholders

[28]There are different reasons why partial abstention is often not possible (or very costly) in practice. First, most shareholders simply are not offered the choice to do so. As discussed in Brav et al. (2022), for shareholders who do not attend meetings in-person, which represents a vast majority of shareholders, voting occurs through proxies. As illustrated on the SEC website (https://www.sec.gov/spotlight/proxymatters/proxy_materials.shtml), proxy cards typically only give shareholders the options to vote all their shares in the same way (i.e., all votes in favor or against any given proposal, or abstention on all votes). Voting information forms, which are used by beneficial owners to instruct their brokers on how to cast their votes, feature the same limitations as paper and internet proxy cards (see, e.g., Nicks, 2014). Note that according to Racanelli (2018), 75% to 80% of all public issuers' shares are held as beneficial shares. Voting information forms are thus ubiquitous. Second, as discussed in Bar-Isaac and Shapiro (2020), for investment advisers, any type of abstention (or voting shares in opposite directions) may be viewed as a failure of their fiduciary duty to their clients. And indeed, Bolton et al. (2020) find that abstentions occurred in only 0.1% of all the proposal-institution pairs in their dataset of proxy votes by mutual and pension funds.

use extensively another form of partial abstention, i.e., across proposals in a given meeting.[29] In the dataset of US retail shareholders assembled by Brav et al. (2022), out of 51.7 million ballots cast, 6.9 millions ballots include abstentions on some (but not all) proposals.[30]

6 Equilibrium Analysis: Endogenous Proposal

The first part of our analysis abstracted from the issue of managers' incentives with respect to the selection of proposals, to instead focus solely on the voting stage. In this section, we explore the information aggregation properties of voting rules taking into account of their effects on managers' incentives. To do so, we introduce in our model a manager that has to decide whether to put a proposal (of exogenously given quality) to the vote. The management thus has veto power on the proposal. Our setup allows for the manager's preferences to differ from shareholders' preferences over two dimensions. First, the manager may prefer the proposal to be adopted in both states. We then say that the manager is *misaligned*. Second, the manager may incur a (reputation) cost if a proposal is rejected at the shareholder meeting. We discuss these two dimensions of conflict between shareholders and the manager in Appendix D. We explore the empirical implications of this section's findings in Appendix E.

6.1 A General Model

We introduce a general model that encompasses all the cases discussed in this section. The shareholders are modeled as in our baseline model. We focus on the comparison between generalized flexible rules, that are efficient at the voting stage, and finite voting rules, that are not efficient at the voting stage. Our objective is to show that, when the proposal is endogenous, the richness of the ballot space is no longer sufficient to rank the efficiency of voting rules. To do so, we fix the distribution of voting rights w and we focus on two polar cases in terms of ballot space richness: the *rigid* rule and the *flexible* rule.

[29]This form of partial abstention would be predicted by our model if we were to consider that shareholders have multiple proposals to vote on. In our model, as in the classical swing voter's curse (Feddersen and Pesendorfer, 1996), shareholders would abstain on proposals for which they are relatively less informed than other shareholders, and vote in favor or against proposals for which they are relatively better informed.

[30]We are very grateful to Alon Brav, Matthew Cain, and Jonathon Zytnick for generously providing us these statistics.

We introduce a new player: the manager, denoted by M. She does not belong to the set of shareholders.[31] After receiving a signal (more details below), the manager decides to either put the proposal to a vote ($x_M = P$) or veto it ($x_M = V$). If the manager vetoes, the proposal is not considered by the shareholders, and the outcome is B. If the manager calls for a vote, shareholders decide whether to accept it (outcome A) or reject it (outcome B).

Before making her decision, the manager receives a signal $s_M \in [0, 1]$. In any state ω, the signal s_M is drawn from a distribution $F_M(\cdot|\omega)$, with density $f_M(\cdot|\omega)$, independently from the signals of the shareholders. We assume that the manager's type t_M is weakly increasing in s_M. In what follows, we consider two cases: either the manager's type space T_M is compact, i.e. $T_M = [\delta_M, 1/\delta_M]$ (as for the shareholders), or the manager knows the state of nature, i.e., $T_M = \{0, \infty\}$.

The utility of the manager u_M can be decomposed into two parts: an outcome-utility u^o_M and a reputation cost c. The outcome-utility depends on whether the manager is *aligned* ($a = 1$) with the shareholders or *misaligned* ($a = 0$). When aligned, the manager has the same outcome-utility as the shareholders. In particular, for any decision $O \in \{A, B\}$ and state $\omega \in \{\alpha, \beta\}$, we have:

$$u^o_M(O|\omega, a = 1) = \mathbf{1}_{\{O=A,\omega=\alpha\}} - \mathbf{1}_{\{O=A,\omega=\beta\}}.$$

When misaligned, the manager wants the proposal to pass in both states:

$$u^o_M(O|\omega, a = 0) = \mathbf{1}_{\{O=A\}}.$$

We assume that the manager has an ex-ante probability $\mu \in [0, 1]$ to be misaligned, and that the draw of a is independent of both the state and the signals. Only the manager knows whether she is aligned.

The manager also incurs a cost $c \geq 0$ if the proposal is turned down by shareholders at the meeting. Shareholders know the value of c. The utility of the manager can be written as:

$$u_M(O, x_M|\omega, a) = u^o_M(O|\omega, a) - c \times \mathbf{1}_{\{x_M=P,O=B\}}.$$

[31] This assumption is not crucial for our results except for the case of a fully aligned manager under the *flexible* rule. Without that assumption, the manager never vetoes in that case, making it less interesting.

While the game has now an additional player, we still use the notion of dominance defined for the exogenous proposal case: a voting rule dominates another if the best and worst equilibria under the former rule (in terms of informational efficiency, taking both shareholders' and the manager's signals into account) outperform the best and worst equilibria under the latter rule.

In order to analyze the endogenous proposal case, we need to allow for asymmetric priors. Indeed, as the manager may veto the proposal at different rates in the two states, shareholders may attach different probabilities to each state when a proposal is put to a vote. Crucial to the analysis that follows, our result that any rule that belongs to the class of generalized flexible rules dominates any finite voting rule is robust to asymmetric priors. In fact, generalized flexible rules remain efficient in that case.[32]

To accommodate the sequential nature of the game and the fact that it involves incomplete and imperfect information, we rely on the concept of Perfect Bayesian Equilibrium (PBE).

6.2 Costless Rejection

We start by investigating the case for which the manager does not incur a cost if her proposal is turned down at the shareholders meeting ($c = 0$). The only potential source of conflict with shareholders is then whether the proposal should pass only in state $\omega = \alpha$, or in both states. This case is useful to explore the effect of voting rules on the behavior of the manager when aligned. Indeed, when rejection is costless for the manager, the misaligned manager always calls for a vote, independently of the voting rule. We show that the full divisibility of the votes remains desirable in this case (i.e., the *flexible* rule dominates the *rigid* rule).

6.2.1 Comparison of Voting Rules

The following proposition shows that, when rejection of the proposal at the meeting is costless for the manager, the *flexible* rule continues to perform better than the *rigid* rule:

[32] A simple modification of Lemma 1 to allow for asymmetric priors shows that $\Pr(\alpha|s) > \frac{1}{2}$ if and only if $\sum_{i \in N} \ln(t_i) > \ln\left(\frac{1-\Pr(\alpha)}{\Pr(\alpha)}\right)$. Thus, in order to implement the efficient decision at the voting stage, shareholders need to compensate for the different likelihood across states. Under the *flexible* rule, shareholders can still implement the efficient decision with the equilibrium $\sigma_i^f(t_i) = c \ln t_i + \kappa_i$, with $\sum \kappa_i = c \ln\left(\frac{\Pr(\alpha)}{1-\Pr(\alpha)}\right)$.

Proposition 4. *Under Assumptions 1 and 2, for every probability $\mu \in [0,1]$ that the manager is misaligned, the* flexible *rule X^f dominates the* rigid *rule X^r. For some parameter values, the dominance is strict.*

Proof. See Appendix B. □

To understand this result, let us first suppose that the manager is aligned with shareholders with probability one (i.e., $\mu = 0$). When an aligned manager takes the behavior of shareholders as given, she calls for a vote only if she believes that the proposal is sufficiently likely to be good (i.e., that the state is α). The definition of "sufficiently likely" depends on the magnitude of type-I errors (A in state β) and type-II errors (B in state α) at the voting stage. The higher the overall probability of error, the higher the manager's incentives to veto. As we have seen in the previous section, for any given information structure, the overall probability of error at the voting stage is higher under the *rigid* rule than the *flexible* rule. Hence, the aligned manager has stronger incentives to veto the proposal under the *rigid* rule than the *flexible* rule.

Given that the manager is (partially) informed about the state of the world, her decision of whether to veto influences the beliefs of shareholders about the quality of the proposal. The higher propensity of the manager to veto under the *rigid* rule implies that the decision to call for a vote is a stronger signal that the proposal is good under the *rigid* rule than the *flexible* rule. Hence, shareholders start their meeting with more precise information (and more favorable to A) under the *rigid* rule than the *flexible* rule, which increases the probability that they make a correct decision.

There is thus a trade-off between the informational efficiency of the *flexible* rule in the voting phase and the weaker selection incentives it gives to the manager before the meeting. In Proposition 4, we prove that, despite this trade-off, the *flexible* rule continues to dominate the *rigid* rule. For the case of a perfectly aligned manager, the intuition is similar to that of previous results: conditional on the state, all players prefer the same outcome. Since the *flexible* rule gives more flexibility to transmit information than the *rigid* rule, it dominates. When the manager is misaligned with positive probability, i.e., $\mu > 0$, the trade-off between selection and voting efficiency is attenuated. Under both the *flexible* rule and the *rigid* rule, when the manager is misaligned ($a = 0$), she always puts the proposal to a vote. This is true even if she receives a very precise signal that the proposal is undesirable. Hence, even if shareholders take into account that the ratio of bad proposals over good ones proposed by a misaligned manager is higher than the same ratio for an aligned manager, the effect is the same under the two rules. We thus have that the dominance of the *flexible* rule over the *rigid* rule is stronger the higher the probability μ that the manager is misaligned. When $\mu = 1$, the comparative properties of the *flexible* rule and the *rigid* rule are the same as in the previous section (with exogenous proposals).

6.3 Costly Rejection

We now analyze the case in which the manager incurs a cost when her proposal is rejected at the meeting ($c > 0$). Costly rejection moderates the incentives of the manager to call for a vote. In contrast with the costless rejection case, the misaligned manager may then choose to veto the proposal. This moderating effect is stronger (i) the higher the cost of rejection c, and (ii) the higher the probability of rejection by shareholders. Given that the probability of rejection is generically different under the *flexible* rule and the *rigid* rule, the moderating effect affects manager's incentives differently under the two rules.

Key to the overall information aggregation performance of a voting rule is how it incentivizes the misaligned manager to veto the proposal when it is bad. To center the analysis on this dimension of the problem, we consider the special case of our model in which the manager is perfectly informed about the quality of the proposal (i.e., she knows the state of the world), all shareholders draw their signals from a common distribution (i.e. $F_i(\cdot|\omega) = F(\cdot|\omega)$ for every i and every ω), and they have equal voting rights (i.e. w_i is the same for all shareholders). These assumptions also help with tractability since they allow us to compare voting rules by focusing on symmetric shareholder behavior and monotone strategies on behalf of the manager.[33]

Our main finding is that there are situations in which the moderating effect is sufficiently stronger under the *rigid* rule so that the better selection of proposals by the misaligned manager under that rule more than compensates for the higher voting efficiency of the *flexible* rule. In such cases, the *rigid* rule dominates the *flexible* rule.

[33] That is, from now on, whenever we refer to an equilibrium of the game, we assume that it is a PBE that involves a monotone strategy on behalf of each type of manager (i.e. the probability of vetoing is weakly higher when the proposal is bad), and the best symmetric equilibrium (with respect to shareholders' welfare) at the voting stage, given correct beliefs about the manager's behavior.

6.3.1 Equilibrium Behavior

The following Lemma stems directly from the assumption that the manager is perfectly informed, the fact that in the good state ($\omega = \alpha$) both types of managers have the same preferences, and the fact that, under both the *flexible* rule and the *rigid* rule, the probability of rejection is higher in the bad state ($\omega = \beta$).

Lemma 2. *In equilibrium, (i) in state α, the manager makes the same decision whether aligned or misaligned, (ii) in state β, the aligned manager always vetoes, and (iii) the misaligned manager vetoes with a (weakly) higher probability in state β than in state α.*

Lemma 2 narrows down the potential equilibrium strategies to three types: (i) the manager only vetoes when aligned and the state is bad; (ii) when the state is good, the manager never vetoes, and when the state is bad, the manager vetoes with positive probability (< 1) if misaligned and with probability 1 if aligned; (iii) the manager vetoes in both states. That latter type of equilibria, which requires specific out-of-equilibrium beliefs for shareholders or very large c, always exists (under both the *flexible* and the *rigid* rules), and is necessarily the worst in terms of shareholders' welfare. To determine which rule dominates, we can thus focus on the other types of potential equilibria, which necessarily include the best equilibrium of each rule.

In types of equlbria (i) and (ii), the manager never vetoes in the good state (regardless of her type), she always vetoes a bad proposal when aligned, and vetoes in the bad state with probability $\gamma_V \in [0, 1)$ when misaligned.[34] Together, μ (the probability the manager is misaligned) and γ_V determine the shareholders' prior π that the proposal is good conditional on a vote (i.e., $\pi := \Pr(\alpha|\gamma_V) = (1 + \mu(1 - \gamma_V))^{-1}$), which in turn determines the shareholders' optimal behavior at the meeting. In equilibrium, it must be that the probability of vetoing in the bad state is optimal given the best response of shareholders.

A key driver of the manager's behavior is the probability that the proposal is rejected by shareholders. The following Lemma highlights that for the misaligned manager, the decision whether to veto the proposal in the bad state relies only on the probability that the proposal is rejected in

[34] Note that if $\gamma_V = 1$, the shareholders can make the inference that conditional on voting, the state must be good. In that case, the incentives of shareholders is to always approve the proposal. But this would give the manager incentives to deviate from $\gamma_V = 1$. Hence, this cannot be an equilibrium.

that state, i.e., 1 minus the probability of type-I error, $p_I(\gamma_V)$.

Lemma 3. *In equilibrium, the probability that the misaligned manager vetoes in the bad state, γ_V^*, must be such that $p_I(\gamma_V^*) \geq \frac{c}{1+c}$. When $p_I(0) > \frac{c}{1+c}$, the equilibrium is such that the manager never vetoes, i.e., $\gamma_V^* = 0$. When $p_I(0) < \frac{c}{1+c}$, the equilibrium is such that the misaligned manager vetoes in the bad state with positive probability, i.e. $\gamma_V^* \in (0,1)$, with $p_I(\gamma_V^*) = \frac{c}{1+c}$.*

Proof. See Appendix B. □

To understand the behavior of the manager under the different voting rules, we thus need to understand how the probability of type-I error, p_I, varies across rules. The misaligned manager has stronger incentives to veto the proposal if the probability of type-I error is low. The difficulty is that the *flexible* rule and the *rigid* rule cannot be neatly ranked based on that probability of error: depending on the situation, it may be higher or lower under the *flexible* rule than the *rigid* rule. This is illustrated in Figure 1, which shows the probability of type-I errors as a function of the prior for the case of an exogenous proposal. This prior is relevant because, as explained above, it increases with γ_V, the probability that the manager vetoes the proposal in the bad state. The figure thus implicitly shows that for some values of γ_V, the probability of type-I error is higher under the *flexible* rule and for other values it is higher under the *rigid* rule. Whether the misaligned manager ends up vetoing more under one rule or the other in equilibrium thus depends on the specifics of the situations.[35]

6.3.2 Comparison of Voting Rules

We are now in position to compare the flexible and rigid rules. First, we can show that if either the probability of misalignment μ or the cost of rejection c is small enough (or both are), then the *flexible* rule continues to dominate the *rigid* rule.

Proposition 5. *Under Assumptions 1 and 2, when the manager is perfectly informed, there exist thresholds $\bar{c} > 0$ and $\bar{\mu} > 0$ such that, if $c < \bar{c}$ or $\mu < \bar{\mu}$, then the flexible rule X^f dominates the* rigid *rule X^r.*

[35] In the case of binary signals, the optimal decision is simply implemented with a threshold: the proposal is only accepted given a number of signals s_a. All priors that have the same optimal threshold generate the same probability of type-I error. This, together with the fact that the optimal threshold decreases with the prior explains the step function under the *flexible* rule in Figure 1.

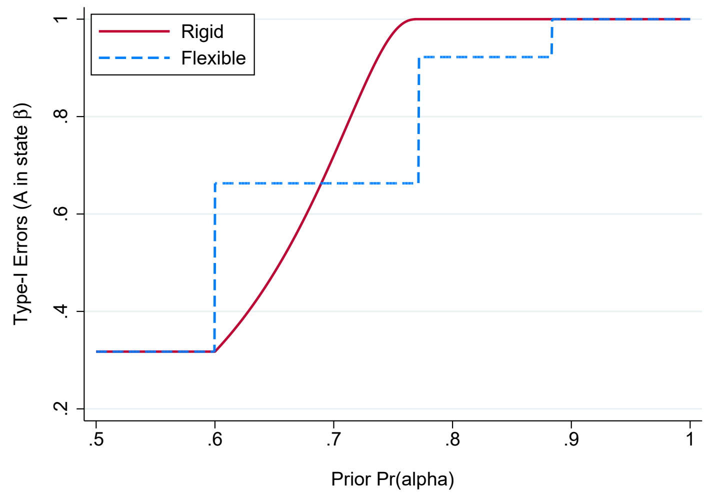

Figure 1: Probability of a type-I error under the rigid rule and the flexible rule as a function of the prior on state α (with an exogenous agenda). Parameters assumed: $n = 5$, and binary signals with $\Pr(s_\alpha|\alpha) = \Pr(s_\beta|\beta) = 0.6$.

Proof. See Appendix B. □

The intuition is as follows. As we have seen above, the incentives to veto of the misaligned manager depend on the cost of rejection c and the probability of type-I error. When c is sufficiently small, the incentives to veto of the misaligned manager are very weak under both the *flexible* rule and the *rigid* rule and she never vetoes. The comparison of the two rules then depends exclusively on their voting efficiency, and we know that the flexible rule dominates in that case.

When μ is sufficiently small, managers are most often aligned, in which case only desirable proposals are put to a vote. The prior $\pi \approx 1$ becomes more informative than shareholders' signals, and they are better off always voting for the proposal, which can be achieved under any rule. The flexible and rigid rules are then equivalent.

For larger cost of rejection c and probability of misalignement μ, the behavior of the misaligned manager in the bad state generally differs under the two rules. However, the effect can either reinforce or compensate the higher voting efficiency of the *flexible* rule (which is defined in terms of both types of errors). When the probability of type-I error is lower under the *flexible* rule in equilibrium, the manager vetoes more often in the bad state under that rule. The better selection of proposals by the manager then reinforces the voting efficiency advantage of the *flexible* rule. By contrast, when the probability of type-I error is lower under the *rigid* rule in equilibrium, the manager vetoes more often in the bad state under that rule. The better selection of proposals by the manager under the *rigid* rule then compensates for its lower voting efficiency. As we show in the following proposition, in equilibrium, the selection effect can be strong enough to overturn the higher voting efficiency of the *flexible* rule.[36]

Proposition 6. *When n is odd and signals are binary, with symmetric informativeness across states, there exist $\underline{\mu} < 1$ and $0 < c_1 < c_2$ such that for any $\mu > \underline{\mu}$ and any $c \in (c_1, c_2)$, the rigid rule X^r dominates the flexible rule X^f.*

To understand this result, it is useful to consider a numerical example: there are 5 shareholders ($n = 5$), the prior of the good state is $\Pr(\alpha) = 0.5$, and there are binary signals with $\Pr(s_a|\alpha) = \Pr(s_b|\beta) = 0.6$. Suppose, moreover, that the manager is misaligned with probability $\mu = 0.6$, and that the cost of rejection $c \in [0, 2.5]$. Figure 2 (top panel) illustrates how the utility of shareholders varies with c. For sufficiently low values of c (below 0.63), $p_I^f(0)$, $p_I^r(0) > \frac{c}{1+c}$. Hence, it is a best response for the manager not to veto, i.e. $\gamma_V^* = 0$, under both the *flexible* rule and the *rigid* rule (as illustrated on the bottom panel of Figure 2). Since there is no differential selection, the only difference across rules comes from their ability to aggregate the information dispersed among shareholders. Consistent with what we have seen in the case of an exogenous proposal, the *flexible* rule then dominates the *rigid* rule. This illustrates the result in Proposition 5. By contrast, when c becomes higher than 0.63, the same strategy by the manager implies $p_I^f(0) > \frac{c}{1+c} > p_I^r(0)$. It is thus not a best response for the manager to always call for a vote under the *rigid* rule (but it is under the *flexible* rule). In equilibrium, she vetoes the proposal in the bad state with probability $\gamma_V^* \in (0, 1)$ under the *rigid* rule, with γ_V^* increasing in c. This selection of proposal by the manager is beneficial to shareholders. For c sufficiently large (above 0.85), the *rigid* rule dominates the *flexible* rule. However, as indicated in the figure, for very large c (above 1.97), then the *rigid* rule no longer dominates. This is because, for c large enough, the misaligned manager also vetoes the proposal in the bad state with positive probability under the *flexible* rule. This eliminates (and even flips) the advantage of the *rigid* rule in terms of selection of proposal and hence prevents it to dominate the *flexible* rule. This illustrates the result in Proposition 6.

Expanding on the previous example, we can compare the utility of shareholders under the

[36]Note that the result is set in a simplified benchmark with binary signals, where Assumptions 1 and 2 are not satisfied. Yet, the (weakly) higher voting efficiency of X^f over X^r remains valid in this environment.

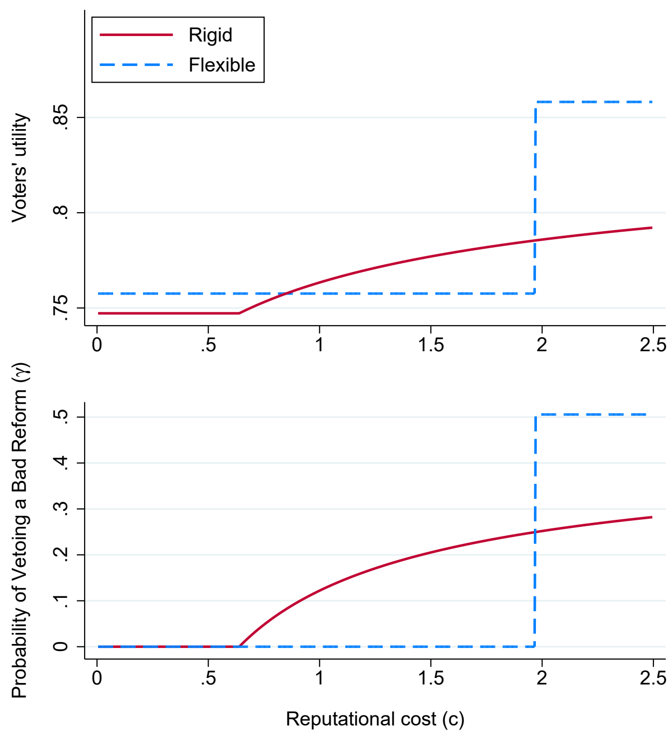

Figure 2: Welfare comparison between the flexible and the rigid rule. The upper panel displays the voters' utility under each rule, and the lower panel the probability of the misaligned manager to veto a bad proposal. The parameters assumed for this comparison are $n = 5$, $\Pr(\alpha) = 0.5$, and binary signals with $\Pr(s_a|\alpha) = \Pr(s_b|\beta) = 0.6$.

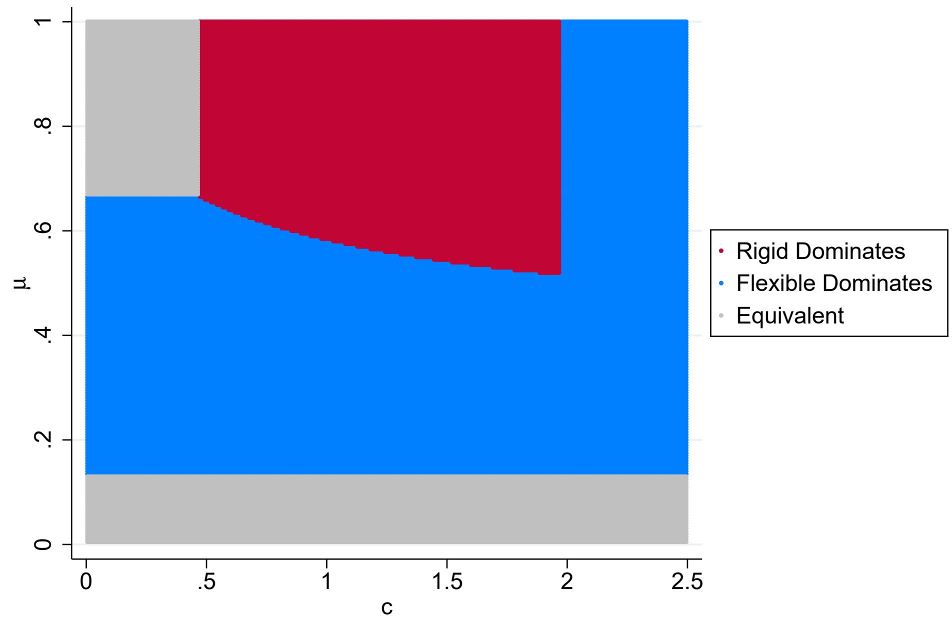

Figure 3: Welfare comparison across rules for different combinations of c (horizontal axis) and μ (vertical axis). The parameters assumed for this comparison are $n = 5$, $\Pr(\alpha) = 0.5$, and binary signals with $\Pr(s_a|\alpha) = \Pr(s_b|\beta) = 0.6$.

flexible rule and the *rigid* rule for various values of c and μ. In line with Proposition 6, Figure 3 shows that the *rigid* rule dominates the *flexible* rule when (i) μ is sufficiently large, and (ii) c is in the right intermediary range. The values of c guarantee that the manager vetoes in some situations under the *rigid* rule but not under the *flexible* rule, and hence that the stronger selection advantage of the *rigid* rule is present (as explained above). The high value of μ guarantees that the manager is often misaligned, and hence that the selection advantage of the *rigid* rule is large enough.[37]

7 Conclusions

In this paper, we studied the comparative properties of voting rules based on the richness of their ballot spaces, assuming a given distribution of voting rights. Our focus was on the informational efficiency of voting rules and we considered how different voting rules affect both voters' decisions

[37] When aligned, the manager vetoes for sure when the state is bad. She does so under both the *flexible* rule and the *rigid* rule. When the state is good she has similar incentives as the misaligned manager (that is, she trades off the risk of rejection with the gain in case the proposal is adopted). Thus, the decision to put the proposal to a vote has a different effect on shareholders' beliefs than when the manager is always misaligned. In particular, due to the veto by the aligned manager in the bad state, the expected quality of the proposal conditional on a vote being called increases. This is beneficial for shareholders. Given that this positive selection effect when the manager is aligned occurs both under the *flexible* rule and the *rigid* rule, it results in a decrease of the overall advantage of the *rigid* rule in terms of selection (this advantage only materializes when the manager is misaligned).

at the voting stage and the incentives of the agenda-setter who decides whether to put the proposal to a vote. For the sake of concreteness, we presented our analysis and results for the case of shareholders' meetings as it fits particularly well the key components of our framework. Yet, here we summarize our result for the general case.

We first considered the case in which the agenda-setter is passive and does not select the proposal being voted on. We proved two main results. First, for any distribution of voting rights, a finite rule with richer ballot spaces dominates a rule with poorer ones, independently of whether information accuracies and voting rights are correlated. Second, efficiency requires votes to be fully divisible.

We then considered the case in which the agenda-setter decides whether to put the proposal to a vote. We uncovered a trade-off between selection and voting efficiency underlying the comparison of the flexible rule (which is efficient at the voting stage) and the rigid rule (which is not efficient at the voting stage): in some cases, the higher voting efficiency of the flexible rule implies worse selection incentives for the agenda-setter. We found that the negative effect of worse selection incentives on voters' welfare can be large enough to wash out the higher voting efficiency of the flexible rule. Then, the rigid rule is better for voters.

The key insight underlying this welfare-reversal result is that although the overall probability of error is lower under a flexible rule compared to a rigid one, the specific information structure can lead to a reduced likelihood of a certain type of error under the rigid rule. Consequently, stakeholders who strongly favor this type of error over other ones may be more motivated to enhance proposal quality under the rigid rule. This dynamic—which is present, but is not limited to the specific agenda-setting model that we explore in the paper—could significantly impact voters' payoffs, and upset the welfare consequences of enriching the ballot space.

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Appendix A: Proofs for Section 5

Proof of Lemma 1. First, note that, by Bayes' rule:

$$\mathbb{P}(\alpha|\mathbf{s}) = \frac{\prod_{i\in N} f_i(s_i|\alpha)\,\mathbb{P}(\alpha)}{\prod_{i\in N} f_i(s_i|\alpha)\,\mathbb{P}(\alpha) + \prod_{i\in N} f_i(s_i|\beta)\,\mathbb{P}(\beta)}.$$

Thus, $\mathbb{P}(\alpha|\mathbf{s}) > 1/2$ requires $\prod_{i\in N} f_i(s_i|\alpha) > \prod_{i\in N} f_i(s_i|\beta)$, or equivalently $\sum_{i\in N} \ln\left(\frac{f_i(s_i|\alpha)}{f_i(s_i|\beta)}\right) > 0$. □

Proof of Proposition 1 . In the sequel, we denote the conditional density of shareholder i's type t_i in state ω by $g_i(t_i \mid \omega)$, it is defined by $g_i(t_i \mid \omega) = f_i\left((s_i)^{-1}(t_i) \mid \omega\right)$. We denote by $G_i(\cdot \mid \omega)$ the corresponding cumulative distribution function. Moreover, we denote the (unconditional) density associated to shareholder i's type t_i by $g_i(t_i)$, i.e. defined by $g_i(t_i) = \mathbb{P}(\alpha)g_i(t_i \mid \alpha) + \mathbb{P}(\beta)g_i(t_i \mid \beta)$.

We know from Lemma 4 (in this Appendix) that the best BNE under X is weakly better than the best BNE under X'. Applying Lemma 5 (in this Appendix), we obtain that the worst BNE under X, σ, yields at least an expected utility of 0 to each shareholder. As there exists a BNE under X', σ', which exactly yields an expected utility of 0 to each shareholder (as there is no decisive shareholder by assumption, any profile yielding a sure outcome with no decisive shareholder is an equilibrium), we also obtain that the worst BNE under X (that is, σ) is no worse than the worst BNE under X' (that is, σ'). □

Lemma 4. *Let X and X' be finite voting rules with $X'_i \subseteq X_i$ for all i. Then, for any BNE σ' under X', there exists a BNE σ under X such that the ex-ante probability of implementing the correct outcome is (weakly) higher at σ than at σ'.*

Proof. As the utility of every shareholder is linearly increasing with the probability that the correct outcome is implemented, we employ in this proof as in the later proofs the terms "welfare" and "more efficient" to refer to this common utility. The main task of the proof is to show that a welfare-maximizing (and thus a BNE) exists for any finite voting rule X. This is shown in two steps.

Claim 1: for any profile σ, there is a profile σ' such that $u_i(\sigma') \geq u_i(\sigma)$ and where for all $i \in N$, $\sigma'_i : T_i \to X_i$ is a pure, weakly increasing strategy.

For any $i \in N$, we may write:

$$\begin{aligned}\mathbb{E}[u_i(x,\sigma_{-i}) \mid t_i] = \mathbb{P}(\alpha \mid t_i) &\left(\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j > -x \mid \alpha) + \frac{1}{2}\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j = -x \mid \alpha)\right) \\ &- (1-\mathbb{P}(\alpha \mid t_i))\left(\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j > -x \mid \beta) + \frac{1}{2}\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j = -x \mid \beta)\right).\end{aligned}$$

Hence,

$$\frac{\partial \mathbb{E}[u_i(x,\sigma_{-i}) \mid t_i]}{\partial t_i} = \frac{\partial \mathbb{P}(\alpha \mid t_i)}{\partial t_i} \Bigg(\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j > -x \mid \alpha) + \mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j > -x \mid \beta) + \frac{1}{2}\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j = -x \mid \alpha) + \frac{1}{2}\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j = -x \mid \beta) \Bigg).$$

We have $\mathbb{P}(\alpha \mid t_i) = \frac{g_i(t_i|\alpha)}{g_i(t_i|\alpha)+g_i(t_i|\beta)} = \frac{t_i}{1+t_i} = 1 - \frac{1}{1+t_i}$, and we get $\frac{\partial \mathbb{P}(\alpha|t_i)}{\partial t_i} = \frac{1}{(1+t_i)^2} > 0$. This implies that $\frac{\partial \mathbb{E}[u_i(x,\sigma_{-i})|t_i]}{\partial t_i}$ is weakly increasing in x (*increasing differences*). As X_i is finite, let us define the best reply σ_i' to σ_{-i} as the smallest value of $x \in X_i$ maximizing the expected utility (we abuse notation as σ_i' is a pure strategy) :

$$\forall t_i \in T_i, \quad \sigma_i'(t_i) = \min\{x \in X_i \mid \mathbb{E}[u_i(x,\sigma_{-i}) \mid t_i] \geq \mathbb{E}[u_i(y,\sigma_{-i}) \mid t_i], \quad \forall y \in X_i\}.$$

The strategy σ_i' must be weakly increasing. Assume by contradiction that $x = \sigma_i'(t_i) > y = \sigma_i'(t_i')$ for $t_i < t_i'$. Then, by definition of $\sigma_i'(t_i)$ as a minimum, we have $\mathbb{E}[u_i(x,\sigma_{-i})|t_i] > \mathbb{E}[u_i(y,\sigma_{-i})|t_i]$ and thus, using the property of *increasing differences*, we obtain :

$$\begin{aligned}\mathbb{E}[u_i(x,\sigma_{-i})|t_i'] &= \mathbb{E}[u_i(x,\sigma_{-i})|t_i] + \int_{t_i}^{t_i'} dt \frac{\partial \mathbb{E}[u_i(x,\sigma_{-i})|t]}{\partial t} \\ &> \mathbb{E}[u_i(y,\sigma_{-i})|t_i] + \int_{t_i}^{t_i'} dt \frac{\partial \mathbb{E}[u_i(y,\sigma_{-i})|t]}{\partial t} = \mathbb{E}[u_i(y,\sigma_{-i})|t_i'],\end{aligned}$$

a contradiction with y being a best reply at t_i'. Hence, the (pure) strategy σ_i' is weakly increasing in t_i'. By applying the same reasoning iteratively for $i = 1, \ldots, n$, we obtain the profile σ', as desired.

Claim 2: the rule X admits a welfare-maximizing strategy profile, which is thus a BNE.

Let us consider the family of profiles consisting in pure, weakly increasing strategies. Let us write $X_i = \{x^1, \ldots, x^k\}$ with $x^1 < \ldots < x^k$. A pure, weakly increasing strategy σ_i is thus described by a series of cutoffs $(t_i^j)_{0 \leq j \leq k} \in (T_i)^{k+1}$, with $t_i^0 = \delta_i$ and $t_i^k = \frac{1}{\delta_i}$, and such that $\forall j$, $t_i^j \leq t_i^{j+1}$ and $t_i \in (t_i^j, t_i^{j+1}) \Rightarrow \sigma_i(t_i) = x^{j+1}$. A profile of such strategies is thus described by a series of cutoffs for each shareholder $i \in N$. Now, as each distribution $G_i(\cdot \mid \omega)$ does not admit any atom, the expected utility attached to such profile is a continuous function of its cutoffs. As cutoffs are taken in a compact set, there is a profile σ^* maximizing the expected utility among all profiles in the family. By application of Claim 1, σ^* maximizes the expected utility among all profiles. As the game is of common interest, the profile σ^* must be a BNE, and hence a welfare-maximizing BNE (this is the original argument of Mc Lennan, 1998).

Finally, to conclude, whenever two rules X and X' are such that $\forall i \in N$, $X_i' \subseteq X_i$, then each profile under rule X' can be reproduced under rule X. It follows that the welfare-maximizing profile (BNE) σ under X achieves at least as much expected utility as the welfare-maximizing profile (BNE) σ' under X'. This concludes the proof. □

Lemma 5. *For any BNE σ under a finite rule X, we have $\forall i \in N$, $u_i(\sigma) \geq 0$.*

Proof. We introduce a couple of notations for the proof. For a strategy profile σ and a type vector $\mathbf{t} = (t_i)_{i \in N}$, we denote by $p^A(\sigma(\mathbf{t}))$ the probability that A is implemented given the votes $\sigma(t)$:

$$p^A(\sigma(\mathbf{t})) = \mathbb{P}\left(\sum_{i \in N} \widehat{\sigma}_i(t_i) > 0\right) + \frac{1}{2}\mathbb{P}\left(\sum_{i \in N} \widehat{\sigma}_i(t_i) = 0\right).$$

Note that we have $\int p^A(\sigma(\mathbf{t})) \prod_{i=1}^n g_i(t_i) dt_i = \mathbb{P}(O = A|\sigma)$.

Claim 1: For any profile of pure, weakly increasing strategies σ, we have $u_i(\sigma) \geq 0$ for all i.

Let σ be a profile of pure, weakly increasing strategies. Let us denote by $U(\sigma) = u_i(\sigma)$ the common utility. We may then write:

$$\begin{aligned}
U(\sigma) &= \int (\mathbb{P}(\alpha|\mathbf{t}) - \mathbb{P}(\beta|\mathbf{t}))\, p^A(\sigma(\mathbf{t})) \prod_{i=1}^n g_i(t_i) dt_i \\
&= \int (2\mathbb{P}(\alpha|\mathbf{t}) - 1)\, p^A(\sigma(\mathbf{t})) \prod_{i=1}^n g_i(t_i) dt_i \\
&= 2 \underbrace{\int \mathbb{P}(\alpha|\mathbf{t}) p^A(\sigma(\mathbf{t})) \prod_{i=1}^n g_i(t_i) dt_i}_{\widetilde{U}(\sigma)} - \mathbb{P}(O = A|\sigma).
\end{aligned}$$

To prove the claim that $U(\sigma) \geq 0$, it thus suffices to show that $\widetilde{U}(\sigma) \geq \frac{1}{2}\mathbb{P}(O = A|\sigma)$. We first observe that, for any $k \in N$, the function $g^k : t_k \mapsto \int \mathbb{P}(\alpha|\mathbf{t}) \prod_{i=1}^{k-1} g_i(t_i) dt_i$ is weakly increasing (for each t_{-k}). Moreover, for any $k \in N$, the function $h^k : t_k \mapsto \int p^A(\sigma(\mathbf{t})) \prod_{i=1}^{k-1} g_i(t_i) dt_i$ is weakly increasing (for each t_{-k}) since σ_k is weakly increasing. By repeated application of Lemma 6 (in this Appendix), we thus obtain:

$$\begin{aligned}
\widetilde{U}(\sigma) &= \int \mathbb{P}(\alpha|\mathbf{t}) p^A(\sigma(\mathbf{t})) \prod_{i=1}^n g_i(t_i) dt_i \\
&\geq \int \left(\int \mathbb{P}(\alpha|\mathbf{t}) g_1(t_1) dt_1\right) \times \left(\int p^A(\sigma(\mathbf{t})) g_1(t_1) dt_1\right) \prod_{i=2}^n g_i(t_i) dt_i \\
&\geq \dots \\
&\geq \int \left(\int \mathbb{P}(\alpha|\mathbf{t}) \prod_{i=1}^k g_i(t_i) dt_i\right) \times \left(\int p^A(\sigma(\mathbf{t})) \prod_{i=1}^k g_i(t_i) dt_i\right) \prod_{i=k+1}^n g_i(t_i) dt_i \\
&\geq \dots \\
&\geq \left(\int \mathbb{P}(\alpha|\mathbf{t}) \prod_{i=1}^n g_i(t_i) dt_i\right) \times \left(\int p^A(\sigma(\mathbf{t})) \prod_{i=1}^n g_i(t_i) dt_i\right) = \frac{1}{2}\mathbb{P}(O = A|\sigma).
\end{aligned}$$

This concludes the proof of Claim 1.

Claim 2: For any BNE σ, there exists a profile σ^+ in pure, weakly increasing strategies such that $\forall i \in N,\ u_i(\sigma) = u_i(\sigma^+)$.

Let σ be a BNE. For any strategy σ_i of shareholder i, we consider a re-ordering σ_i^+, i.e. a strategy such that:

- σ_i^+ is pure and weakly increasing
- for any ballot $x_i \in X_i$, we have $\mathbb{P}(\widehat{\sigma}_i \leq x_i) = \mathbb{P}(\sigma_i^+ \leq x_i)$.

To construct such a re-ordering, we define σ_i^+ by (abusing notation as σ_i^+ is pure):

$$\forall t_i \in T_i, \qquad \sigma_i^+(t_i) = \min\left\{ x_i \in X_i \mid \sum_{x \in X_i, x \leq x_i} \int_{\delta_i}^{\frac{1}{\delta_i}} \sigma_i(t_i')(x) g_i(t_i') dt_i' > \int_{\delta_i}^{t_i} g_i(t_i') dt_i' \right\}.$$

The strategy σ_i^+ is pure, weakly increasing and continuous (and even flat) everywhere but on a finite number of points. We shall prove that for almost any type vector $\mathbf{t} = (t_i)_{i \in N}$, the sign of $\sum_{i \in N} \widehat{\sigma}_i(t_i)$ is the same as that of $\sum_{i \in N} \sigma_i^+(t_i)$. In the sequel, we refer to the sign of a number x as positive if $x > 0$, negative if $x < 0$, and null (neither positive nor negative) if $x = 0$.

Let $t_i \in T_i$ be a type such that σ_i^+ is continuous at t_i and assume that there exists $x_i \in X_i$ for which $\sigma_i(t_i)(x_i) > 0$ and $x_i \neq \sigma_i^+(t_i)$. We focus on the case for which $\sigma_i^+(t_i) > x_i$ (the other case can be treated analogously) and we further assume that $x_i = \min\{x \in X_i \mid \sigma_i(t_i)(x) > 0\} := \min \widehat{\sigma}_i(t_i)$. Observe that there must exist $t_i' < t_i$ such that $\sigma_i(t_i')(y_i) > 0$ with $y_i \geq \sigma_i^+(t_i) > x_i$. Indeed, if this type t_i' didn't exist, we would have $\forall t_i' < t_i, \widehat{\sigma}_i(t_i') < \sigma_i^+(t_i)$ for any realization of $\sigma_i(t_i')$, which would imply:

$$\sum_{x < \sigma_i^+(t_i)} \int_{\delta_i}^{\frac{1}{\delta_i}} \sigma_i(t"_i)(x) g_i(t"_i) dt"_i \geq \int_{\delta_i}^{t_i} \left(\sum_{x < \sigma_i^+(t_i)} \sigma_i(t"_i)(x) \right) g_i(t"_i) dt"_i = \int_{\delta_i}^{t_i} g_i(t"_i) dt"_i.$$

We would then have for any $t < t_i$ (since g_i is positive on T_i):

$$\sum_{x < \sigma_i^+(t_i)} \int_{\delta_i}^{\frac{1}{\delta_i}} \sigma_i(t"_i)(x) g_i(t"_i) dt"_i > \int_{\delta_i}^{t} g_i(t"_i) dt"_i.$$

By definition of σ_i^+, we would have $\forall t < t_i, \sigma_i^+(t) < \sigma_i^+(t_i)$. This contradicts the fact that σ_i^+ is continuous at t_i. We thus obtained the existence of $y_i \geq \sigma_i^+(t_i) > x_i$ such that $\sigma_i(t_i')(y_i) > 0$ for some $t_i' < t_i$.

As σ is a BNE, x_i must be optimal for i at t_i and y_i must be optimal for i at t_i':

$$\Delta_i := u_i(x_i, \sigma_{-i} | t_i) - u_i(y_i, \sigma_{-i} | t_i) \geq 0$$
$$\Delta_i' := u_i(y_i, \sigma_{-i} | t_i') - u_i(x_i, \sigma_{-i} | t_i') \geq 0.$$

By summation, we obtain that $\Delta_i + \Delta_i' \geq 0$. Now, we may write:

$$\Delta_i = \int \left(2\mathbb{P}(\alpha | t_i, t_{-i}) - 1\right) \left(p^A\left(x_i, \sigma_{-i}(t_{-i})\right) - p^A\left(y_i, \sigma_{-i}(t_{-i})\right)\right) \prod_{j \neq i} g_j(t_j) dt_j.$$

Similarly,

$$\Delta_i' = \int \left(2\mathbb{P}(\alpha|t_i', t_{-i}) - 1\right) \left(p^A\left(y_i, \sigma_{-i}(t_{-i})\right) - p^A\left(x_i, \sigma_{-i}(t_{-i})\right)\right) \prod_{j \neq i} g_j(t_j) dt_j.$$

We thus have:

$$\begin{aligned}\Delta_i + \Delta_i' &= 2 \int \left(\mathbb{P}(\alpha|t_i', t_{-i}) - \mathbb{P}(\alpha|t_i, t_{-i})\right) \left(p^A\left(y_i, \sigma_{-i}(t_{-i})\right) - p^A\left(x_i, \sigma_{-i}(t_{-i})\right)\right) \prod_{j \neq i} g_j(t_j) dt_j \\ &\geq 0.\end{aligned}$$

As $t_i' < t_i$, we have that for all t_{-i}, $\mathbb{P}(\alpha|t_i', t_{-i}) - \mathbb{P}(\alpha|t_i, t_{-i}) < 0$. Moreover, as $y_i > x_i$, we have by definition of p^A that for all t_{-i}, $p^A\left(y_i, \sigma_{-i}(t_{-i})\right) \geq p^A\left(x_i, \sigma_{-i}(t_{-i})\right)$. To reconcile the three inequalities, it must be that $\Delta_i + \Delta_i' = 0$ and that for almost all t_{-i}, $p^A\left(y_i, \sigma_{-i}(t_{-i})\right) = p^A\left(x_i, \sigma_{-i}(t_{-i})\right)$. This last equality implies, by definition of p^A, that $\mathbb{P}\left(x_i \leq -\sum_{j \neq i} \widehat{\sigma}_j \leq y_i\right) = 0$. As $x_i = \min \widehat{\sigma}_i(t_i)$ and $\sigma_i^+(t_i) \leq y_i$, we obtain

$$\mathbb{P}\left(\min \widehat{\sigma}_i(t_i) \leq -\sum_{j \neq i} \widehat{\sigma}_j \leq \sigma_i^+(t_i)\right) = 0.$$

Following a symmetrical argument, we also obtain

$$\mathbb{P}\left(\sigma_i^+(t_i) \leq -\sum_{j \neq i} \widehat{\sigma}_j \leq \max \widehat{\sigma}_i(t_i)\right) = 0.$$

It follows that $\mathbb{P}\left(sgn\left(\widehat{\sigma}_i(t_i) + \sum_{j \neq i} \widehat{\sigma}_j\right) \neq sgn\left(\sigma_i^+(t_i) + \sum_{j \neq i} \widehat{\sigma}_j\right)\right) = 0$. Moreover, by construction of the strategies $(\sigma_j^+)_{j \neq i}$, the probability of the previous event remains null if some strategy realizations $\widehat{\sigma}_j$ are transformed into σ_j^+ (the transformation from σ_j to σ_j^+ is measure-preserving by design). This can be written: for all $S \subseteq N \backslash \{i\}$,

$$\mathbb{P}\left(sgn\left(\widehat{\sigma}_i(t_i) + \sum_{j \in S} \widehat{\sigma}_j + \sum_{j \in (N \backslash S) \backslash \{i\}} \sigma_j^+\right) \neq sgn\left(\sigma_i^+(t_i) + \sum_{j \in S} \widehat{\sigma}_j + \sum_{j \in (N \backslash S) \backslash \{i\}} \sigma_j^+\right)\right) = 0. \quad (2)$$

We have just shown that (2) holds whenever σ_i^+ is continuous at t_i. As σ_i^+ is continuous almost everywhere, we have: for all $S \subseteq N \backslash \{i\}$,

$$\mathbb{P}\left(sgn\left(\widehat{\sigma}_i + \sum_{j \in S} \widehat{\sigma}_j + \sum_{j \in (N \backslash S) \backslash \{i\}} \sigma_j^+\right) \neq sgn\left(\sigma_i^+ + \sum_{j \in S} \widehat{\sigma}_j + \sum_{j \in (N \backslash S) \backslash \{i\}} \sigma_j^+\right)\right) = 0. \quad (3)$$

To conclude, we observe that $sgn(\sum_{j \in N} \widehat{\sigma}_j) \neq sgn(\sum_{j \in N} \sigma_j^+)$ can be satisfied only if there exists some index k for which $sgn(\sum_{j=1}^{k-1} \widehat{\sigma}_j + \sum_{j=k}^{n} \sigma_j^+) \neq sgn(\sum_{j=1}^{k} \widehat{\sigma}_j + \sum_{j=k+1}^{n} \sigma_j^+)$. Hence, we may write, applying (3):

$$\mathbb{P}\left(sgn(\sum_{j\in N}\widehat{\sigma}_j) \neq sgn(\sum_{j\in N}\sigma_j^+)\right) \leq \sum_{k=1}^{n}\mathbb{P}\left(sgn(\sum_{j=1}^{k-1}\widehat{\sigma}_j + \sum_{j=k}^{n}\sigma_j^+) \neq sgn(\sum_{j=1}^{k}\widehat{\sigma}_j + \sum_{j=k+1}^{n}\sigma_j^+)\right) = 0.$$

Therefore, σ and σ^+ lead to the same outcome with probability one, i.e. for almost any type vector t. It follows that $\forall i \in N$, $u_i(\sigma) = u_i(\sigma^+)$. This concludes the proof of Claim 2.

To conclude the proof, note that any BNE σ yields the same utilities as a profile σ^+ of pure, weakly increasing strategies (Claim 2), under which all expected utilities are positive (Claim 1). Thus for all $i \in N$, $u_i(\sigma) \geq 0$. □

Lemma 6. *Let f be a density function on a real interval T, let $g, h : T \to \mathbb{R}$ be two weakly increasing functions. Then:*

$$\int g(t)h(t)f(t)dt \geq \left(\int g(t)f(t)dt\right) \times \left(\int h(t)f(t)dt\right).$$

Proof. The statement is a direct consequence of the property that, for any real random variable Y (drawn with density f) and for any pair of weakly increasing functions (g, h), we have:

$$Cov[g(Y), h(Y)] = \mathbb{E}[g(Y)h(Y)] - \mathbb{E}[g(Y)]\mathbb{E}[h(Y)] \geq 0.$$

□

Proof of Proposition 2. The fact that each $i \in N$ employing $\sigma_i^f(t_i) = c\ln t_i$ with $c = \min_{i\in N}\{\frac{-w_i}{\ln(\delta)}\}$ is an efficient BNE of the flexible rule X^f is straightforward, by application of Lemma 1.

Claim: If σ is an efficient BNE, then for any $\mathbf{t}$ such that $\sum_{i=1}^n \log(t_i) = 0$, we must have $\sum_{i=1}^n \sigma_i(t_i) = 0$ with probability one.

By contradiction, suppose there exists a type profile $\mathbf{t}^*$ such that $\sum_{i=1}^n \log(t_i^*) = 0$ and assume w.l.o.g that for some realizations $(\hat{\sigma}_i(t_i^*))_{i=1\dots n}$ we have $\Delta := \sum_{i=1}^n \hat{\sigma}_i(t_i^*) < 0$.

As $\sum_{i=1}^n \log(t_i^*) = 0$, we may assume without loss of generality that $\log(t_1^*) \leq 0$, $\log(t_2^*) \leq 0$ and $\log(t_3^*) \geq 0$ (if only one of the $\log(t_i^*)$ was negative, there would be at least two weakly positive one, so that a symmetric argument applies). It follows that $t_1^* < \max T$, $t_2^* < \max T$ and $t_3^* > \min T$. In the sequel, for each type t_i for which $\sigma_i(t_i)$ is mixed, we denote by $\underline{\sigma}_i(t_i)$ the infimum of realizations of the random variable $\sigma_i(t_i)$ and by $\overline{\sigma}_i(t_i)$ the supremum of realizations of the random variable $\sigma_i(t_i)$, in other words these are the lower and upper bounds of the support of $\sigma_i(t_i)$. We define:

$$\Delta_1 = \liminf_{\substack{t_1\to t_1^*\\ t_1>t_1^*}} \underline{\sigma}_1(t_1) - \hat{\sigma}_1(t_1^*), \quad \Delta_2 = \liminf_{\substack{t_2\to t_2^*\\ t_2>t_2^*}} \underline{\sigma}_2(t_2) - \hat{\sigma}_2(t_2^*) \quad \text{and} \quad \Delta_3 = \hat{\sigma}_3(t_3^*) - \limsup_{\substack{t_3\to t_3^*\\ t_3<t_3^*}} \overline{\sigma}_3(t_3).$$

Let $\varepsilon > 0$. There exists $t_1 > t_1^*$ such that $\underline{\sigma}_1(t_1) - \hat{\sigma}_1(t_1^*) \leq \Delta_1 + \varepsilon$. It follows that there is a realization $\tilde{\sigma}_1(t_1)$ such that $\tilde{\sigma}_1(t_1) - \hat{\sigma}_1(t_1^*) \leq \Delta_1 + 2\varepsilon$. Now, for any $t_3 < t_3^*$ and any realization $\tilde{\sigma}_3(t_3)$, we have $\hat{\sigma}_3(t_3^*) - \tilde{\sigma}_3(t_3) \geq \Delta_3$. If we take $t_3 < t_3^*$ close enough to t_3^*, we have that $\sum_{i\neq 1,3} \log(t_i^*) + \log(t_1) + \log(t_3) > 0$. As σ is efficient, we must have $\sum_{i\neq 1,3} \hat{\sigma}_i(t_i^*) + \tilde{\sigma}_1(t_1) + \tilde{\sigma}_3(t_3) > 0$, which is equivalent to

$$\Delta + (\tilde{\sigma}_1(t_1) - \hat{\sigma}_1(t_1^*)) + (\tilde{\sigma}_3(t_3) - \hat{\sigma}_3(t_3^*)) > 0.$$

We thus obtain $\Delta + \Delta_1 + 2\varepsilon - \Delta_3 > 0$. At the limit ($\varepsilon \to 0$), we obtain $\Delta + \Delta_1 \geq \Delta_3$. Following a similar argument, by fixing first $t_3 > t_3^*$ close to t_3^*, and then $t_1 < t_1^*$ close enough to t_1^*, i.e. such that $\sum_{i \neq 1,3} \log(t_i^*) + \log(t_1) + \log(t_3) < 0$, we obtain $\Delta + \Delta_1 \leq \Delta_3$. We thus have $\Delta_1 = \Delta_3 - \Delta$.

Now, we may iterate the argument by fixing both 1 and 2's types simultaneously. First, we fix $t_1 > t_1^*$ close to t_1^* and $t_2 > t_2^*$ close to t_2^*, and then $t_3 < t_3^*$ close enough to t_3^*. Efficiency then entails $\Delta + \Delta_1 + \Delta_2 \geq \Delta_3$. Second, we fix first t_3 and then t_1 and t_2 to get $\Delta + \Delta_1 + \Delta_2 \leq \Delta_3$. We thus have $\Delta_1 + \Delta_2 = \Delta_3 - \Delta = \Delta_1$, so that $\Delta_2 = 0$.

To conclude, $\Delta_2 = 0$ means that there exists $t_2 > t_2^*$ arbitrarily close to t_2^* with a realization $\tilde{\sigma_2}(t_2)$ arbitrarily close to $\hat{\sigma_2}(t_2^*)$. Thus, there exists t_2 with $\sum_{i \neq 2} \log(t_i^*) + \log(t_2) > 0$ and $\sum_{i \neq 2} \hat{\sigma_i}(t_i^*) + \tilde{\sigma_2}(t_2) < 0$. This contradicts the premise that σ is efficient and thus proves the claim.

We can easily rule out the existence of a non-degenerate mixed efficient BNE. If σ is a non-degenerate mixed BNE of the game, there exists at least one $i \in N$ and at least one $y \in T$ such that the random variable $\sigma_i(y)$ admits at least two distinct potential realizations. Consider without loss of generality that this player is the first shareholder, and let $\mathbf{t} = (t_1, t_2, t_3, t_4, ...) = (y, \frac{1}{y}, 1, 1, ...)$. If σ is efficient, then by application of the previous claim, given any two vectors of potential realizations $\widehat{\sigma} = (\widehat{\sigma}_1(y), \widehat{\sigma}_2(\frac{1}{y}), \widehat{\sigma}_3(1), \widehat{\sigma}_4(1), ...)$ and $\widetilde{\sigma} = (\widetilde{\sigma}_1(y), \widetilde{\sigma}_2(\frac{1}{y}), \widetilde{\sigma}_3(1), \widetilde{\sigma}_4(1), ...)$ with $\widehat{\sigma}_i(t_i) = \widetilde{\sigma}_i(t_i)$ for every $i > 1$, we must have that $\sum_{i \in N} \widehat{\sigma}(t_i) = 0$ and $\sum_{i \in N} \widetilde{\sigma}(t_i) = 0$. But this means that $\widehat{\sigma}_1(y)$ must be identical to $\widetilde{\sigma}_1(y)$ and hence $\sigma_1(y)$ cannot admit at least two distinct potential realizations, which contradicts the assumption above. Hence, if σ is an efficient BNE, it must be pure.

We now turn attention to pure equilibria. First we argue that an efficient pure BNE σ must be symmetric across shareholders up to an additive constant (i.e. there exist $\phi_{i,j}$ such that $\sigma_i(y) = \sigma_j(y) + \phi_{i,j}$, for every $i, j \in N$ and every $y \in T$). If σ is efficient then for every $y \in T$, for $\mathbf{t} = (t_1, t_2, t_3, t_4, ...)$ with $t_i = y$, $t_j = \frac{1}{y}$ and $t_k = 1$ for all $k \notin \{i, j\}$, we need to have $\sum_{k \in N} \sigma_k(t_k) = 0$ and thus $\sigma_i(y) = -\sigma_j(\frac{1}{y}) - \sum_{k \in N - \{i.j\}} \sigma_k(1)$. By keeping j fixed and varying i we get that all players, except possibly j, employ the same strategy up to an additive constant. By varying j as well, we get that all players use the same strategy up to an additive constant. That is, in an efficient pure BNE σ a player i uses the strategy $\sigma_i(y) = \theta(y) + \kappa_i$, with $\sum_{i \in N} \kappa_i = 0$ and $\theta(1) = 0$.

Notice that if σ is an efficient equilibrium of the flexible rule X^f characterized by some θ and $\kappa = (\kappa_1, \kappa_2, .., \kappa_n)$ such that $\sum_{i \in N} \kappa_i = 0$ and $\kappa_j \neq 0$ for at least one $j \in N$, it follows that σ', characterized by the same θ and $\kappa = (0, 0, ..., 0)$, is an efficient equilibrium of the rule $X' = (\mathbb{R}, \mathbb{R}, ..., \mathbb{R})$. Hence, to characterize all efficient equilibria of the flexible rule X^f, it suffices to characterize all admissible θs that are feasible under X^f and that lead to full information equivalence when $\kappa = (0, 0, ..., 0)$ under rule X'. In the remaining part of the proof, we slightly abuse terminology, and instead of saying "a pure-strategy efficient equilibrium of X' characterized by θ and $\kappa = (0, 0, ..., 0)$" we simply say "an efficient equilibrium θ".

In an efficient equilibrium θ, it must be the case that for every $\mathbf{t} \in T^n$ we have $sgn(\sum_{i \in N} \theta(t_i)) = sgn(\mathbb{P}(\alpha|\mathbf{t}) - \mathbb{P}(\beta|\mathbf{t}))$. But we know from Lemma 1 that $\mathbb{P}(\alpha|\mathbf{t}) - \mathbb{P}(\beta|\mathbf{t}) > 0 \Leftrightarrow \sum_{i \in N} \ln(t_i) > 0$, $\mathbb{P}(\alpha|\mathbf{t}) - \mathbb{P}(\beta|\mathbf{t}) < 0 \Leftrightarrow \sum_{i \in N} \ln(t_i) < 0$, and $\mathbb{P}(\alpha|\mathbf{t}) - \mathbb{P}(\beta|\mathbf{t}) = 0 \Leftrightarrow \sum_{i \in N} \ln(t_i) = 0$. In other words, for every $\mathbf{t} \in T^n$ it must hold that $sgn(\sum_{i \in N} \theta(t_i)) = sgn(\sum_{i \in N} \ln(t_i))$.

First, we prove that every efficient equilibrium θ is monotone (increasing, in particular) and symmetric (i.e., $\theta(y) = -\theta(\frac{1}{y})$ for every $y \in T$), then that it is differentiable on $int(T) = (\delta, \frac{1}{\delta})$ and continuous on $T = [\delta, \frac{1}{\delta}]$, and, finally, we provide a full characterization by showing that each efficient equilibrium θ is equal to the natural logarithm multiplied by some positive constant.

Monotonicity and symmetry of equilibria: For every $t_i < \frac{1}{\delta}$ there exists a $t_{-i} \in T^{n-1}$ such that $\mathbb{P}(\alpha|\mathbf{t}) = \mathbb{P}(\beta|\mathbf{t})$, so that $\sum_{i \in N} \ln(t_i) = 0$. Hence, for such a $\mathbf{t} = (t_i, t_{-i})$ and every

$\varepsilon \in (0, \frac{1}{\delta} - t_i]$, it is true that, $\mathbb{P}(\alpha|(t_i + \varepsilon, t_{-i})) > \mathbb{P}(\beta|(t_i + \varepsilon, t_{-i}))$, so that $\sum_{j \in N - \{i\}} \ln(t_j) + \ln(t_i + \varepsilon) > 0$. Since every efficient equilibrium θ delivers the efficient outcome, it follows that for every $y < \frac{1}{\delta}$ and $\varepsilon \in (0, \frac{1}{\delta} - y]$ there exists a $t_{-i} \in T^{n-1}$ such that $\sum_{j \in N - \{i\}} \theta(t_j) + \theta(y) = 0$ and $\sum_{j \in N - \{i\}} \theta(t_j) + \theta(y + \varepsilon) > 0$. In other words, $\theta(y + \varepsilon) > \theta(y)$ for every $y < \frac{1}{\delta}$ and $\varepsilon \in (0, \frac{1}{\delta} - y]$; θ is strictly increasing in the player's type. To establish symmetry, consider that $\mathbf{t} \in T^n$ is such that $t_i = 1$ for every $i \in N$. By application of the above Claim, we must have $\sum_{i \in N} \theta(t_i) = n\theta(1) = 0$, which implies $\theta(1) = 0$. Now consider a $\mathbf{t} \in T^n$ such that $t_1 = y \in T$, $t_2 = \frac{1}{y} \in T$ and $t_i = 1$ for every $i > 2$. We have $\sum_{i \in N - \{1,2\}} \ln(1) + \ln(y) + \ln(\frac{1}{y}) = 0$ and hence $\sum_{i \in N - \{1,2\}} \theta(1) + \theta(y) + \theta(\frac{1}{y}) = 0$, which implies $(n - 2) \times 0 + \theta(y) + \theta(\frac{1}{y}) = 0$, for every $y \in T$. In other words, $\theta(y) = -\theta(\frac{1}{y})$, for every $y \in T$.

Differentiability: By Lebesgue's theorem for the differentiability of monotone functions defined over open intervals we have that every equilibrium $\theta : T \to \mathbb{R}$ is differentiable at almost every $y \in int(T) = (\delta, \frac{1}{\delta})$. We will now establish that θ is actually differentiable at every $y \in (\delta, \frac{1}{\delta})$. Notice that in all profiles with $t_1 = y \in (\delta, 1]$, $t_2 = y' \in (1, \frac{1}{\delta})$, $t_3 = \frac{1}{yy'} \in (\delta, \frac{1}{\delta})$, and $t_i = 1$ for every $i > 3$, it must hold that $\theta(y) + \theta(y') + \theta(\frac{1}{yy'}) = 0 \Leftrightarrow \theta(y) = -\theta(y') - \theta(\frac{1}{yy'})$ by the fact $\theta(1) = 0$ and that $\sum_{i \in N} \ln(t_i) = 0$. Assume that θ is not differentiable at a particular $\tilde{y} \in (\delta, 1]$. Then it follows that $-\theta(y') - \theta(\frac{1}{yy'})$ is not differentiable with respect to y at $\tilde{y}$, for every $y' \in (1, \frac{1}{\delta})$. But due to the fact that θ is differentiable at almost every $y \in int(T)$, it follows that for every $y \in (\delta, 1]$, there exists $y' \in (1, \frac{1}{\delta})$ such that θ is differentiable at $\frac{1}{yy'}$. This contradicts the claim that there exists $\tilde{y} \in (\delta, 1]$ at which θ is not differentiable, and, by symmetry it follows that θ is differentiable at every $y \in int(T)$.

Continuity at the boundary: We know that θ is differentiable, and thus continuous, on $int(T) = (\delta, \frac{1}{\delta})$. Let us show that it is continuous at $y = \frac{1}{\delta}$. Suppose by contradiction, that there is a discontinuity. As θ is increasing, it must be of the form: $\theta(y) - \theta(y - \varepsilon) > \tilde{\varepsilon}$ for every $\varepsilon \in (0, \bar{\varepsilon}]$, where $\bar{\varepsilon}$ and $\tilde{\varepsilon}$ are positive constants. Then there exists $\lambda \in (0, 1 - \delta)$ such that $(n - 2)\theta(1 - \lambda) + \theta(y) + \theta(\frac{1}{y - \varepsilon}) > 0$ for every $\varepsilon \in (0, \bar{\varepsilon}]$. But for every $\lambda \in (0, 1 - \delta)$ one can find $\varepsilon > 0$ small enough such that $(n - 2)\ln(1 - \lambda) + \ln(y) + \ln(\frac{1}{y - \varepsilon}) < 0$. This contradicts the fact that θ leads to the efficient outcome for every possible realization of types. Thus θ is continuous at $\frac{1}{\delta}$, and for the same reason, it must be continuous at δ. We conclude that θ is continuous on $T = [\delta, \frac{1}{\delta}]$.

Characterization: We fix an efficient equilibrium θ and an arbitrary pair of values $(y', \tilde{y}) \in (\delta, \frac{1}{\delta})^2$, such that $y' < 1$ and $\tilde{y} > 1$. Consider now a $\mathbf{t} \in (\delta, \frac{1}{\delta})^n$ such that $t_1 = y'$, $t_2 = \tilde{y}$, $t_3 = \frac{1}{y'\tilde{y}} \in (\frac{1}{\tilde{y}}, \frac{1}{y'}) \subset (\delta, \frac{1}{\delta})$, and $t_i = 1$ for every $i > 3$. If we define $r = y' \times \tilde{y}$ we get $\sum_{i \in N - \{1,2,3\}} \theta(1) + \theta(y') + \theta(\frac{r}{y'}) + \theta(\frac{1}{r}) = 0$. Since, $\ln(y) + \ln(\frac{r}{y}) + \ln(\frac{1}{r}) = 0$ for every y in an open ball around y', and since θ is differentiable at y', it follows that we can take the derivative of $\sum_{i \in N - \{1,2,3\}} \theta(1) + \theta(y) + \theta(\frac{r}{y}) + \theta(\frac{1}{r}) = 0$ with respect to y and evaluate it at y'. By doing that, we get, $\theta'(y') + \theta'(\frac{r}{y'})(-\frac{r}{y'^2}) = 0$. This can be written as $y' \times \theta'(y') = \tilde{y} \times \theta'(\tilde{y})$. But since this holds for any pair of values $(y', \tilde{y}) \in int(T)^2$, such that $y' < 1$ and $\tilde{y} > 1$, it is true that, for any fixed $\tilde{y} \in (1, \frac{1}{\delta})$, we have $y \times \theta'(y) = \tilde{y} \times \theta'(\tilde{y})$ for every $y \in (\delta, 1)$. In other words, for every $y \in (\delta, 1)$ we have $y \times \theta'(y) = c \Longrightarrow \theta'(y) = \frac{c}{y} \Longrightarrow \theta(y) = c \ln y + \hat{c}$, for some $c > 0$ and $\hat{c} \in \mathbb{R}$. By the fact that $\theta(1) = 0$, it follows that $\hat{c} = 0$ and, hence, $\theta(y) = c \ln y$ for every $y \in (\delta, 1)$, with $c > 0$. By symmetry of θ it follows that for every $y \in (1, \frac{1}{\delta})$, we have $\theta(y) = -\theta(\frac{1}{y}) = -c \ln \frac{1}{y} = c \ln y$. That is, $\theta(y) = c \ln y$ for every $y \in (\delta, \frac{1}{\delta})$, with $c > 0$, and by continuity at the boundary, the formula must hold for every $y \in [\delta, \frac{1}{\delta}]$.

By the fact that in every equilibrium σ of the flexible rule X^f, we must have $\sigma_i(\frac{1}{\delta}) \leq w_i$ and $\sigma_i(\delta) \geq -w_i$, and by the above analysis, it follows that in an efficient equilibrium it should hold that $\sigma_i(t_i) = c \ln t_i + \kappa_i$ with $\sum_{i \in N} \kappa_i = 0$ and $c \in (0, \min_{i \in N}\{\frac{\min(w_i - \kappa_i, w_i + \kappa_i)}{-\ln(\delta)}\}]$. □

Proof of Proposition 3. Let X be a generalized flexible rule: there exists $(\psi_i)_{i \in N} \in \times_{i \in N} int(X_i)$ such that $\sum_{i \in N} \psi_i = 0$. To see why X admits an efficient equilibrium, notice that any efficient equilibrium σ of the flexible rule X^f, as characterized in Proposition 2, can be properly re-scaled so that, for each $i \in N$, σ_i fits within any open set around ψ_i.

To understand why only generalized flexible rules admit an efficient equilibrium, let X be a voting rule with an efficient equilibrium σ. Since every strategy that is feasible according to this rule is also feasible under the rule $X' = (\mathbb{R}, \mathbb{R}, ..., \mathbb{R})$, it must be the case that σ is an efficient equilibrium of X' too. We replicate the reasoning in the proof of Proposition 2, and get the following result: as σ is an efficient equilibrium of X', it should hold that $\sum_{i \in N} \sigma_i(1) = 0$ and, for every $i \in N$, $\sigma_i(y)$ should be continuous and strictly increasing in an open ball around $y = 1$. Hence, noting $\psi_i = \sigma_i(1)$, we have that $\sum_{i \in N} \psi_i = 0$ and $\psi_i \in int(X_i)$ for every $i \in N$, that is X is a generalized flexible rule. □

Appendix B: Proofs of Section 6

Proof of Proposition 4. The result is straightforward when the manager is perfectly informed, i.e. $T_M = \{0, \infty\}$. In that case, the aligned manager vetoes in the bad state, while all other manager's types never veto. The setting is thus equivalent to a voting game with fixed prior $\Pr(\alpha) = \frac{1}{1+\mu}$, for which the dominance of X^f over X^r has already been established. In the rest of the proof, we thus focus on the case where T_M is compact

As for Proposition 1, we divide the dominance statement in two lemmas (Lemma 7 and 8 in this Appendix), focusing respectively on the best and the worst equilibria under each voting rule. A further lemma (Lemma 9) shows that dominance can be strict. Throughout the proof, we treat the game between the manager and the shareholders as a simultaneous game and we continue to apply the equilibrium notion of a BNE, as in Section 5. We note however that the same results hold for the equilibrium concept of weak perfect Bayesian equilibrium in the sequential version of the game when there is no decisive shareholder. □

Lemma 7. *For any $\mu \in [0, 1]$ and any voting rights distribution w, for any BNE σ under X^r, there exists a BNE σ' under X^f that makes all shareholders (weakly) better off in expectation.*

Proof. The strategy of the proof is to construct a two-player common value game between an aligned manager and an aggregate shareholder (holding all the shareholders' signals). When voting under X^f, shareholders can implement (i.e. decentralize) the aggregate shareholder's strategy of the most efficient equilibrium of the two-player game. The corresponding profile is then the most efficient equilibrium of the original game under X^f. Using the argument of McLennan (1998), shareholders' welfare at equilibrium cannot improve with X^r.

Two-player game. We consider a game with two players: a manager M and an aggregate shareholder AS. The manager receives the signal s_M, while the aggregate shareholder receives the signals $s_1, \ldots, s_n$. After receiving their (private) signals, players simultaneously choose to pass ($x_j = P$) or to veto ($x_j = V$) the proposal. The proposal is accepted with probability 1 if both players choose P, and it is accepted with probability μ if the manager vetoes while the aggregate shareholder passes.[38] Both players share the same utility: for all $j \in \{M, AS\}$,

$$u_j(A|\alpha) = 1,\ u_j(A|\beta) = -1,$$
$$u_j(B|\alpha) = u_j(B|\beta) = 0.$$

Claim 1: the two-player game admits a most efficient strategy profile, which is also an equilibrium of that game. The expected utility of each player is at least 0 at this equilibrium.

We first show the existence of a most efficient equilibrium, and we start by computing players' best replies. Given a strategy σ_{AS} for the aggregate shareholder, the expected utility difference between actions P and V for the manager, $\Delta u_M := \mathbb{E}[u_M(x_M = P, \sigma_{AS}) \mid s_M] - \mathbb{E}[u_M(x_M = V, \sigma_{AS}) \mid s_M]$, can be written as:

$$\Delta u_M = (1-\mu)\mathbb{P}(\sigma_{AS} = P)\left(2\mathbb{P}(\omega = \alpha \mid \sigma_{AS} = P, s_M) - 1\right).$$

Indeed, the only difference between the actions $x_M = P$ and $x_M = V$ arises when the aggregate shareholder chooses to pass ($\sigma_{AS} = P$) and the manager has control over her own veto (with probability $1-\mu$). Hence, playing $x_M = P$ is a best reply for the manager whenever $\mathbb{P}(\omega = \alpha \mid \sigma_{AS} = P, s_M) \geq 1/2$, or equivalently $\frac{\mathbb{P}(\omega=\alpha|\sigma_{AS}=P,s_M)}{\mathbb{P}(\omega=\beta|\sigma_{AS}=P,s_M)} \geq 1$. Noting $t_M = \frac{f_M(s_M|\alpha)}{f_M(s_M|\beta)}$ and $t_{\sigma_{AS}} = \frac{\mathbb{P}(\sigma_{AS}=P|\alpha)}{\mathbb{P}(\sigma_{AS}=P|\beta)}$, we may write $\frac{\mathbb{P}(\omega=\alpha|\sigma_{AS}=P,s_M)}{\mathbb{P}(\omega=\beta|\sigma_{AS}=P,s_M)} = t_M t_{\sigma_{AS}}$. We obtain that there is a cutoff $\bar{t}_M := \frac{1}{t_{\sigma_{AS}}}$ such that $x_M = P$ is a best reply whenever $t_M \geq \bar{t}_M$, and that $x_M = V$ is a best reply otherwise.

Similarly, we denote the aggregate shareholder's type by $t_{AS} = \frac{\prod_{i=1}^n f_i(s_i|\alpha)}{\prod_{i=1}^n f_i(s_i|\beta)} = \prod_{i=1}^n t_i$. As for the manager, we obtain that $\Delta u_{AS} := \mathbb{E}[u_{AS}(\sigma_M, x_{AS} = P) \mid s_1, \ldots, s_n] - \mathbb{E}[u_{AS}(\sigma_M, x_{AS} = V) \mid s_1, \ldots, s_n]$, can be written as:

$$\Delta u_{AS} = (\mathbb{P}(\sigma_M = P) + \mu\mathbb{P}(\sigma_M = V)) \times$$
$$\left(2\mathbb{P}\left(\omega = \alpha | \tilde{\mathbb{P}}(\sigma_M = P) = \frac{\mathbb{P}(\sigma_M = P)}{\mathbb{P}(\sigma_M = P) + \mu\mathbb{P}(\sigma_M = V)}, s_1, \ldots, s_n\right) - 1\right),$$

where $\tilde{\mathbb{P}}$ denotes the posterior probability once one knows that the manager has passed the proposal (either because she chose to do so, $x_M = P$, or because she tried to veto, $x_M = V$, but the veto was not registered, which arises with probability μ). Hence, as for the manager, there is a cutoff $\bar{t}_{AS}$, function of the manager's strategy σ_M, such that $x_{AS} = P$ is a best reply to σ_M whenever $t_{AS} \geq \bar{t}_{AS}$, and that $x_{AS} = V$ is a best reply otherwise.

We have shown that the best reply of each player $j \in \{M, AS\}$ is characterized by a cutoff $\bar{t}_j$ above which j plays P, and below which j plays V. Now, observe that the players' type spaces $T_M = [\delta_M, \frac{1}{\delta_M}]$ and $T_{AS} = [\delta^n, \frac{1}{\delta^n}]$ are compact. Moreover, if one denotes by $g_j(t_j \mid \omega)$ the density according to which player j is of type t_j in state ω, we obtain the expected utility of both players given the cutoffs $(\bar{t}_M, \bar{t}_{AS})$ as:

$$\mathbb{E}[u_j \mid \bar{t}_M, \bar{t}_{AS}] = \int_{\bar{t}_{AS}}^{\frac{1}{\delta^n}} dt_{AS} \int_{\delta_M}^{\frac{1}{\delta_M}} dt_M \left(\mu + (1-\mu)\mathbf{1}_{\{t_M \geq \bar{t}_M\}}\right) \times$$
$$\left(\mathbb{P}(\omega = \alpha) g_{AS}(t_{AS}|\alpha) g_M(t_M|\alpha) - \mathbb{P}(\omega = \beta) g_{AS}(t_{AS}|\beta) g_M(t_M|\beta)\right).$$

As in each state, the conditional distributions of t_M and t_{AS} are continuous, the expected utility

[38] In other words, the manager has incomplete control over her own veto: with probability μ, she is "transformed" into a misaligned manager, who automatically passes the proposal.

is continuous in both players' cutoffs. It follows that there exists an optimal couple of cutoffs $(\bar{t}^*_M, \bar{t}^*_{AS})$ which maximizes the common utility. The corresponding strategy profile must thus be an equilibrium, the most efficient equilibrium, and also the most efficient strategy profile. As the strategy profile $(x_M = V, x_{AS} = V)$ yields an expected utility of 0, the expected utility of the most efficient profile must be at least 0.

Claim 2: The most efficient profile of the two-player game is replicable in the original game under X^f. In that game, the corresponding profile is both the most efficient profile such that the misaligned manager always proposes and the most efficient equilibrium.

In the original game under X^f, consider the strategy profile σ^* where: the aligned manager behaves as the manager of the two-player game; the misaligned manager always proposes; the shareholders decentralize the aggregate shareholder's strategy of the two-player game by playing the log-strategy identified in Proposition 2.

First, observe that σ^* is an equilibrium. Indeed, we know from Claim 1 that the aligned manager and the shareholders are playing optimally (if one individual shareholder could improve her utility, then the aggregate shareholder could do it as well in the two-player game, a contradiction). Moreover, passing is always a best reply for the misaligned manager.

Second, σ^* is the most efficient profile among those for which the misaligned manager always proposes. Indeed, if there was a (strictly) more efficient such profile, then there would be a (strictly) more efficient profile than the one identified in the two-player game, a contradiction with Claim 1.

Third, assume by contradiction that there is a (strictly) more efficient equilibrium. By virtue of the previous assertion, it must be a strategy profile such that the misaligned manager proposes with probability strictly less than one. For such a strategy to be a best reply, it must be that shareholders always turn the proposal down. Such a profile yields an expected utility of 0 for the shareholders (and the aligned manager), and thus cannot be a strict improvement over σ^*, since σ^* yields at least 0 (applying Claim 1).

Claim 3: The original game under X^r admits equilibria, but none of them is more efficient than the most efficient profile under X^f.

First, observe that the profile in which every manager's type vetoes and all shareholders vote against the proposal is an equilibrium, thus an equilibrium exists.

Second, any profile under X^r for which the misaligned manager always proposes is replicable in the game under X^f, and thus cannot be (strictly) more efficient than σ^* (applying Claim 2). Thus, no equilibrium under X^r for which the misaligned manager always proposes can improve upon σ^*.

Finally, any equilibrium under X^r such that the misaligned manager proposes with a probability strictly less than one must yield an expected utility of 0 for the shareholders and the aligned manager (same argument as under X^f). Therefore, no equilibrium under X^r can improve upon σ^*. □

Lemma 8. *For any $\mu \in [0,1]$ and any voting rights distribution w, for any BNE σ under X^f, there exists a BNE σ' under X^r that makes all shareholders (weakly) worse off in expectation.*

Proof. First, observe that the profile for which any manager vetoes the proposal and all shareholders vote against it is an equilibrium under X^r, with an expected utility of 0 for each shareholder.

Second, let $\sigma = (\sigma_M, \sigma_1, \ldots, \sigma_n)$ be a BNE under X^f. We will show that this equilibrium yields an expected utility of at least 0 to all shareholders. If $\mathbb{P}(\sum_{i=1}^n \widehat{\sigma}_i(t_i) \geq 0) = 0$, the profile yields an expected utility of 0 (the proposal is never accepted), and the previous statement holds. We may thus focus on the case for which $\mathbb{P}(\sum_{i=1}^n \widehat{\sigma}_i(t_i) \geq 0) > 0$. As σ is an equilibrium, it must be that the misaligned manager always proposes. Then, by applying the same argument as in the proof of Lemma 7 (Claim 1), we obtain that the aligned manager's strategy must be weakly increasing.

Now, given the (aligned and misaligned) managers' strategies, the game among shareholders can be seen as a game with an exogenous proposal (as in Section 4), albeit with a possibly biased prior $\mathbb{P}(\omega = \alpha) \geq 1/2$. In that game, we can apply the same reasoning as in the proof of Lemma 4 (Claim 1), and we obtain that there exists a strategy profile σ' for the shareholders, such that each σ'_i is pure, weakly increasing and: $\forall i \in N, u_i(\sigma_M, \sigma_1, \ldots, \sigma_n) = u_i(\sigma_M, \sigma'_1, \ldots, \sigma'_n)$. As each player's strategy under the profile $(\sigma_M, \sigma'_1, \ldots, \sigma'_n)$ is weakly increasing, a similar argument as the one used in Lemma 5 (Claim 1) shows that this profile yields an expected utility of at least 0 to all shareholders. Therefore, $\forall i \in N, u_i(\sigma) \geq 0$. This concludes the proof. □

Lemma 9. *For some parameter values, the dominance of X^f over X^r is strict.*

Proof. To establish strict dominance, consider an instance for which, when the proposal is exogenous, the best BNE under X^f, σ^f , implements the correct outcome with a probability p^f strictly higher than p^r, attained at the best BNE under X^r, σ^r. We may further assume $p^f > p^r > \frac{1}{2}$. We will establish the strict dominance of X^f over X^r when the proposal is endogenous by taking δ_M sufficiently close to 1.

First, observe that for δ_M close enough to 1, the profile where the manager (either aligned or misaligned) always proposes and the shareholders play a BNE σ of the exogenous proposal game, such that $p^\sigma > 1/2$, must be an equilibrium. Indeed, on the shareholders' side, the game is the same as the one with an exogenous proposal since the manager always proposes. On the aligned manager's side, the utility of vetoing is 0, while the utility of proposing can be made arbitrarily close to $2p^\sigma - 1 > 0$ (for δ_M close to 1), in which case proposing is indeed a best reply. Finally, as the probability of accepting the proposal is always positive (since $p^\sigma > 1/2$), the misaligned manager's best reply is also to propose.

Second, assume that there is a BNE σ under X^r which implements the correct outcome with a probability $p^\sigma > p^r$. As $p^\sigma > 1/2$, the misaligned manager always proposes (for δ_M close enough to 1) . As p^r is the highest probability to implement the correct outcome under X^r when the proposal is exogenous, it must be that the aligned manager sometimes vetoes the proposal, for some type $t_M \in T_M$. It must thus be that $\mathbb{P}(\alpha|t_M)\mathbb{P}(O = A|\alpha) + \mathbb{P}(\beta|t_M)\mathbb{P}(O = B|\beta) \leq \frac{1}{2}$, where $\mathbb{P}(O|\omega)$ denotes the probability of alternative O passing in state ω when the proposal is passed to the shareholders. By choosing δ_M sufficiently close to 1, we can make $\mathbb{P}(\alpha|t_M)$ and $\mathbb{P}(\beta|t_M)$ arbitrarily close to 1/2 for all $t_M \in [\delta_M, \frac{1}{\delta_M}]$, and, as $p^r > \frac{1}{2}$, we can thus make sure that

$$\left(\exists t_M \in T_M, \quad \mathbb{P}(\alpha|t_M)\mathbb{P}(O = A|\alpha) + \mathbb{P}(\beta|t_M)\mathbb{P}(O = B|\beta) \leq \frac{1}{2}\right)$$
$$\Rightarrow \quad (\forall t_M \in T_M, \quad \mathbb{P}(\alpha|t_M)\mathbb{P}(O = A|\alpha) + \mathbb{P}(\beta|t_M)\mathbb{P}(O = B|\beta) < p^r).$$

If σ_M denotes the aligned manager's strategy, we obtain:

$$p^\sigma = \int (\mu + (1-\mu)\sigma_M(t_M)(P)) \times$$
$$(\mathbb{P}(\alpha|t_M)\mathbb{P}(O = A|\alpha) + \mathbb{P}(\beta|t_M)\mathbb{P}(O = B|\beta))\, g_M(t_M)dt_M < p^r.$$

Hence a contradiction. We have shown that for δ_M close enough to 1, the dominance of X^f over X^r can be strict. □

Proof of Lemma 3. The expected payoff of the misaligned manager when she calls for a vote in the bad state is $p_I(\gamma_V) - c(1 - p_I(\gamma_V))$. This has to be compared to a payoff of 0 if she vetoes the proposal. As a result, the manager strictly prefers to call for a vote if $p_I(\gamma_V) > \frac{c}{1+c}$, and is indifferent between vetoing and putting the proposal to a vote if $p_I(\gamma_V) = \frac{c}{1+c}$. Given that the misaligned manager cannot veto with probability 1 in equilibrium, it must be that $p_I(\gamma_V^*) \geq \frac{c}{1+c}$.

Given that $p_I(\gamma_V)$ is (weakly) increasing in γ_V, we have that $p_I(0) > \frac{c}{1+c}$ guarantees that $p_I(\gamma_V) > \frac{c}{1+c}$ $\forall \gamma_V \in (0,1)$. Hence, the unique equilibrium must be such that the misaligned manager does not veto in the bad state, i.e., $\gamma_V^* = 0$. It follows that the manager neither vetoes in the good state. By contrast, when $p_I(0) < \frac{c}{1+c}$, $\gamma_V^* = 0$ is not an equilibrium. Given that $p_I(\gamma_V) \to 1$ when $\gamma_V \to 1$, we have that there must be a value of $\gamma_V^* \in (0,1)$ such that $p_I(\gamma_V^*) = \frac{c}{1+c}$. This pins down the equilibrium behavior of the misaligned manager in the bad state. □

Proof of Proposition 5. **Small cost.** First remember that, when perfectly informed, (i) the aligned manager always vetoes the proposal in the bad state, and (ii) in the good state, the manager behaves in the same way whether aligned or misaligned. We also know that the equilibrium must be such that the manager, whether aligned or misaligned, never vetoes in the good state. So, we need to prove that there is a c sufficiently small such that the misaligned manager never vetoes in the bad state.

When the misaligned manager never vetoes in the bad state, the probability of a type-I error in that state is positive, i.e., $p_I(0) > 0$. This is because shareholders are then uncertain about the state of nature at the time of the meeting and hence make mistakes of both types. Thus, there must be a c sufficiently small such that $p_I(0) > \frac{c}{1+c}$. Given that this later condition guarantees that not vetoing is a best response for the misaligned manager in the bad state, we have that there must be a c sufficiently small such that the misaligned manager vetoes with probability 0 in the bad state. Given that the manager behaves in the same way under X^f and X^r, the comparison of these two voting rules depends exclusively on their voting efficiency. We know from Proposition 1 that X^f has a higher voting efficiency than X^r. Thus, X^f dominates X^f when c is sufficiently small.

Small probability of misalignment. First, observe that when μ tends to 0, then the prior π tends to 1, for any value of the vetoing probability γ_V. When $\pi = (1 + \mu(1 - \gamma_V))^{-1}$ is close enough to 1, at the voting stage, the optimal mapping from signals to decision for the shareholders consists in always approving the proposal. This mapping can be achieved under both X^f and X^r (any shareholder always voting in favor of the proposal independent of her signal), and since the mapping is optimal for any value of γ_V, it must be achieved at the optimal symmetric equilibrium under both X^f and X^r. Under both rules, a misaligned manager never vetoes the proposal, i.e. $\gamma_V = 0$. Thus both rules achieve the same expected utility, it follows that X^f weakly dominates X^r. □

Proof of Proposition 6. The first step of the proof consists in computing the optimal symmetric equilibrium and the associated type-I and type-II error probabilities under each voting rule under a relevant range of priors. Throughout, $\pi = (1+\mu(1-\gamma_V))^{-1}$ is shareholders' prior that the state is α (that the proposal is good). We will assume $\mu > \frac{1-p}{p}$, so that $\frac{1}{1+\mu} < p$. Thus, when the manager does not veto, the prior is $\pi = \frac{1}{1+\mu} < p$. The set of signals is binary, $S = \{s_a, s_b\}$, and p is the probability of receiving a correct signal, i.e. $\Pr(s_a \mid \alpha) = \Pr(s_b \mid \beta) = p$. Finally, we denote by n_a and n_b the realized numbers of signals s_a and s_b (respectively).

Flexible rule. When the prior is $\pi = 1/2$, the optimal mapping from signals to decision consists in rejecting the proposal whenever $n_b \geq n_a + 1$. The optimal symmetric equilibrium under X^f implements this optimal mapping by letting every shareholder vote according to her signal. When the prior is close enough to 1/2, the same equilibrium remains optimal. Under this equilibrium, error probabilities are given by:

$$p_I^s = p_{II}^s = \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} (1-p)^k p^{n-k}.$$

Consider now a larger prior $\pi > 1/2$. When the prior is larger, shareholders' expected utility put more weight on type-II errors (rejecting in the good state). As a result, the optimal mapping from signal profiles to decision (and thus the optimal symmetric equilibrium under X^f) must be biased towards approving the proposal. In the lowest prior range such that the optimal mapping is strictly biased in favor of the proposal, a proposal is rejected whenever $n_b \geq n_a + 3$. Under this mapping, error probabilities are given by:

$$p'_I = p_I^s + \binom{n}{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2}} p^{\frac{n+1}{2}} \qquad \text{and} \qquad p'_{II} = p_{II}^s - \binom{n}{\frac{n+1}{2}} (1-p)^{\frac{n+1}{2}} p^{\frac{n-1}{2}}.$$

The prior cutoff π at which the two mappings are welfare-equivalent is given by:

$$(1-\pi)p'_I + \pi p'_{II} = (1-\pi)p_I^s + \pi p_{II}^s \quad \Leftrightarrow \quad (1-\pi)\binom{n}{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2}} p^{\frac{n+1}{2}} = \pi \binom{n}{\frac{n+1}{2}} (1-p)^{\frac{n+1}{2}} p^{\frac{n-1}{2}}$$
$$\Leftrightarrow \quad \pi = p.$$

Observe that this is not surprising: a rational decision maker wants to change the mapping precisely when the prior becomes more informative than one private signal.

To conclude, there exists $\varepsilon > 0$ such that: in an optimal symmetric equilibrium under X^f, the type-I error probability is p_I^s for $\pi \in [1/2, p)$, any value in $[p_I^s, p'_I]$ for $\pi = p$ and $p'_I > p_I^s$ for $\pi \in (p, p+\varepsilon)$.[39]

Rigid rule. When the prior is $\pi \in [1/2, p)$, the optimal mapping (reject whenever $n_b \geq n_a + 1$, as shown above) is achievable under the rigid rule. This is thus the best symmetric equilibrium.

For a larger prior $\pi \geq p$, it becomes optimal to bias the mapping from signal profiles to decision towards proposal approval. The most efficient way to approach this optimal benchmark at a symmetric profile under X^r is to let shareholders with a bad signal s_b approve the proposal with a fixed probability $q \geq 0$ (while shareholders with a good signal s_a keep approving the proposal). Under this symmetric profile, error probabilities are given by:

$$p_I(q) = \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} (1-p)^k p^{n-k} + \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} (1-p)^k p^{n-k} \sum_{l=\frac{n+1}{2}-k}^{n-k} \binom{n-k}{l} q^l (1-q)^{n-k-l}$$

[39] When $\pi = p$, for any candidate value $p^\lambda = p_I^s + \lambda(p'_I - p_I^s)$ with $\lambda \in [0,1]$, there is an optimal symmetric equilibrium where (i) the manager vetoes with probability $\gamma_V = 1 - \frac{1-p}{\mu p}$, so that $\pi = p$, (ii) after a positive signal, each shareholder votes for the proposal with a weight of 1 , (iii) after a negative signal, each voter is indifferent and mixes between a vote (against the proposal) with a weight of 1 and a vote with a weight $v < 1$. The weight v is chosen so as to implement the rule $n_b \geq n_a + 3$, while the mixing probability m is chosen so as to reach the desired type-I error probability p^λ. Formally, we can pick $v = \frac{n-2}{n+2}$ and m such that $\lambda = H(\frac{n+2}{4})$ where H is the cumulative distribution function of the binomial distribution with parameters $\frac{n+1}{2}$ and m.

and

$$p_{II}(q)=\sum_{k=0}^{\frac{n-1}{2}}\binom{n}{k}(p)^k(1-p)^{n-k}\sum_{l=0}^{\frac{n-1}{2}-k}\binom{n-k}{l}q^l(1-q)^{n-k-l}.$$

We thus have[40]

$$\frac{\partial p_I}{\partial q}=\sum_{k=0}^{\frac{n-1}{2}}\binom{n}{k}(1-p)^kp^{n-k}\sum_{l=\frac{n+1}{2}-k}^{n-k}\binom{n-k}{l}\left(lq^{l-1}(1-q)^{n-k-l}-(n-k-l)q^l(1-q)^{n-k-l-1}\right)$$

so that

$$\left.\frac{\partial p_I}{\partial q}\right|_{q=0}=\binom{n}{\frac{n-1}{2}}(1-p)^{\frac{n-1}{2}}p^{\frac{n+1}{2}}\left(\frac{n+1}{2}\right).$$

We also have

$$\frac{\partial p_{II}}{\partial q}=\sum_{k=0}^{\frac{n-1}{2}}\binom{n}{k}p^k(1-p)^{n-k}\sum_{l=0}^{\frac{n-1}{2}-k}\binom{n-k}{l}\left(lq^{l-1}(1-q)^{n-k-l}-(n-k-l)q^l(1-q)^{n-k-l-1}\right)$$

so that

$$\begin{aligned}\left.\frac{\partial p_{II}}{\partial q}\right|_{q=0}&=\sum_{k=0}^{\frac{n-3}{2}}\binom{n}{k}p^k(1-p)^{n-k}\left(n-k-(n-k)\right)-\binom{n}{\frac{n-1}{2}}p^{\frac{n-1}{2}}(1-p)^{\frac{n+1}{2}}\left(\frac{n+1}{2}\right)\\&=-\binom{n}{\frac{n-1}{2}}p^{\frac{n-1}{2}}(1-p)^{\frac{n+1}{2}}\left(\frac{n+1}{2}\right).\end{aligned}$$

The cutoff prior π such that the symmetric profile with a $q>0$ becomes optimal is such that:

$$\begin{aligned}\pi\left.\frac{\partial p_{II}}{\partial q}\right|_{q=0}+(1-\pi)\left.\frac{\partial p_I}{\partial q}\right|_{q=0}=0\quad&\Leftrightarrow\quad-\pi(1-p)+(1-\pi)p=0\\&\Leftrightarrow\quad\pi=p.\end{aligned}$$

We have shown that the optimal symmetric profile, and thus the optimal symmetric equilibrium by application of Theorem 2 in McLennan (1998), is such that $q=0$ for $\pi\in[1/2,p]$ and $q>0$ for $\pi>p$ (since, by the previous computation, $\left.\frac{\partial W}{\partial q}\right|_{q=0}>0$ for $\pi>p$). Next, we show that the optimal mixing probability q^* is continuous on the prior range $[p,p+\eta]$ for some $\eta>0$.

Note that, for a prior $\pi\geq p$, the optimal mixing probability $q^*(\pi)$ is determined by the equation

$$\pi\left.\frac{\partial p_{II}}{\partial q}\right|_{q=q^*}+(1-\pi)\left.\frac{\partial p_I}{\partial q}\right|_{q=q^*}=0\quad\Leftrightarrow\quad\frac{-\pi}{1-\pi}=\left.\frac{\frac{\partial p_I}{\partial q}}{\frac{\partial p_{II}}{\partial q}}\right|_{q=q^*}:=G(q^*).$$

To show that $q^*(\pi)$ is continuous in a neighborhood of p, it is sufficient to show that the derivative of the function $G(q):=\frac{\frac{\partial p_I}{\partial q}}{\frac{\partial p_{II}}{\partial q}}$ is not null at $q=0$.

[40] Note that we use here the usual convention that $0^0=1$ (for instance $q^{l-1}=1$ for $l=1$ and $q=0$).

We may write:

$$\frac{\partial^2 p_I}{\partial q^2} = \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} (1-p)^k p^{n-k} \sum_{l=\frac{n+1}{2}-k}^{n-k} \binom{n-k}{l} \times$$
$$\Big(l(l-1)q^{l-2}(1-q)^{n-k-l} - 2l(n-k-l)q^{l-1}(1-q)^{n-k-l-1} + (n-k-l)(n-k-l-1)q^l(1-q)^{n-k-l-2} \Big)$$

so that

$$\left.\frac{\partial^2 p_I}{\partial q^2}\right|_{q=0} = \binom{n}{\frac{n-3}{2}} (1-p)^{\frac{n-3}{2}} p^{\frac{n+3}{2}} \binom{\frac{n+3}{2}}{2} \times 2 + \binom{n}{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2}} p^{\frac{n+1}{2}} \left(\binom{\frac{n+1}{2}}{2} \times 2 - 2\binom{\frac{n+1}{2}}{1} \left(\frac{n+1}{2} - 1\right) \right)$$
$$= \frac{n!}{(\frac{n-1}{2})!(\frac{n-3}{2})!} (1-p)^{\frac{n-3}{2}} p^{\frac{n+1}{2}} (2p-1) > 0.$$

We may also write:

$$\frac{\partial^2 p_{II}}{\partial q^2} = \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} p^k (1-p)^{n-k} \sum_{l=0}^{\frac{n-1}{2}-k} \binom{n-k}{l} \times$$
$$\Big(l(l-1)q^{l-2}(1-q)^{n-k-l} - 2l(n-k-l)q^{l-1}(1-q)^{n-k-l-1} + (n-k-l)(n-k-l-1)q^l(1-q)^{n-k-l-2} \Big)$$

so that

$$\left.\frac{\partial^2 p_{II}}{\partial q^2}\right|_{q=0} = \sum_{k=0}^{\frac{n-5}{2}} \binom{n}{k} p^k (1-p)^{n-k} \Big((n-k)(n-k-1) - 2(n-k)(n-k-1) + (n-k)(n-k-1) \Big)$$
$$+ \binom{n}{\frac{n-3}{2}} p^{\frac{n-3}{2}} (1-p)^{\frac{n+3}{2}} \Big(\frac{n+3}{2} \times (-2(\frac{n+3}{2} - 1)) + \frac{n+3}{2}(\frac{n+3}{2} - 1) \Big)$$
$$+ \binom{n}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1-p)^{\frac{n+1}{2}} \frac{n+1}{2}(\frac{n+1}{2} - 1)$$
$$= -\binom{n}{\frac{n-3}{2}} p^{\frac{n-3}{2}} (1-p)^{\frac{n+3}{2}} (\frac{n+3}{2})(\frac{n+1}{2}) + \binom{n}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1-p)^{\frac{n+1}{2}} (\frac{n+1}{2})(\frac{n-1}{2})$$
$$= \frac{n!}{(\frac{n-1}{2})!(\frac{n-3}{2})!} p^{\frac{n-3}{2}} (1-p)^{\frac{n+1}{2}} (2p-1) > 0.$$

Finally we obtain

$$\left.\frac{\partial^2 p_I}{\partial q^2}\right|_{q=0} \times \left.\frac{\partial p_{II}}{\partial q}\right|_{q=0} - \left.\frac{\partial^2 p_{II}}{\partial q^2}\right|_{q=0} \times \left.\frac{\partial p_I}{\partial q}\right|_{q=0} < 0$$

and thus $G'(0) < 0$.

To conclude, there exists $\eta > 0$ such that: in the optimal symmetric equilibrium under X^r, the type-I error probability is p_I^s for $\pi \in [1/2, p)$, and a continuous function $p_I(\pi)$ on $[p, p+\eta)$, such that $p_I(\pi) > p_I(p) = p_I^s$ whenever $\pi > p$ (this derives from the fact that $p_I(\cdot)$ is a continuous function of q and that q^* is continuous as a function of π).

Comparison of voting rules. We consider a cost c such that $\frac{c}{1+c} \in (p_I^s, p_I')$. We have $p_I(\gamma_V = 0) = p_I^s < \frac{c}{1+c}$ for both voting rules. By application of Lemma 3, we must have $p_I^r(\gamma_V^r) = p_I^f(\gamma_V^f) = \frac{c}{1+c}$.

As $\frac{c}{1+c} \in (p_I^s, p_I')$, for the flexible rule, the probability of vetoing must make the prior equal to p. That is $\pi^f := \frac{1}{1+\mu(1-\gamma_V^f)} = p$, or equivalently $\gamma^f = \frac{p(1+\mu)-1}{p\mu}$.

As $\frac{c}{1+c} > p_I^s$, for the rigid rule, the probability of vetoing must make the prior strictly above p. That is $\pi^r := \frac{1}{1+\mu(1-\gamma_V^r)} > p$, or equivalently $\gamma_V^r > \gamma_V^f = \frac{p(1+\mu)-1}{p\mu}$.

We now focus on the average error probability (conditional on a vote), defined by $\xi = \pi p_{II} + (1-\pi)p_I$.

Under the flexible rule X^f, the prior $\pi^f = p$ is such that the optimal symmetric equilibrium is welfare-equivalent (at the voting stage) to the sincere voting profile, we thus have

$$\xi^f = \pi^f p_{II}^s + (1-\pi^f)p_I^s = p_I^s.$$

Under the rigid rule X^r, the prior $\pi^r > p$ is such that the optimal symmetric equilibrium is (strictly) welfare-superior (at the voting stage) to the sincere voting profile (since, with the above notations $q^* > 0$), we thus have

$$\xi^r = \pi^r p_{II}^r + (1-\pi^r)p_I^r < \pi^r p_{II}^s + (1-\pi^r)p_I^s = p_I^s = \xi^f.$$

To conclude, note that the expected common utility W attached to any strategy profile can be written as:

$$\begin{aligned} W := \Pr(0 = A, \omega = \alpha) - \Pr(0 = A, \omega = \beta) &= \frac{(1-p_{II}) - \mu(1-\gamma_V)p_I}{2} \\ &= \frac{\pi(1-p_{II}) - (1-\pi)p_I}{2\pi} = \frac{1}{2} - \frac{\xi}{\pi}. \end{aligned}$$

As $\pi^r > \pi^f$ and $\xi^r < \xi^f$, we obtain that $W^r > W^f$. This conclusion holds true whenever $\mu > \frac{1-p}{p}$ and $\frac{c}{1+c} \in (p_I^s, p_I')$, which is equivalent to $c \in (\frac{p_I^s}{1-p_I^s}, \frac{p_I'}{1-p_I'})$. □

Appendix C: Extensions

In this Appendix, we explore the robustness of the results in Section 5 when allowing for partisan or expressive shareholders, and when endogenizing the acquisition of information by shareholders. To lighten the notations, we consider throughout that Appendix only voting rules X such that $\max X_i = -\min X_i = w_i$ for all $i \in N$. All the arguments can be adapted to voting rules that do not satisfy this condition.

Behavioral (partisan and expressive) shareholders

We consider a new class of behavioral shareholders, whose actions are a fixed function of their signals, independent of their environment. We focus on three relevant behavioral types:

- A-partisans (behavioral type A) are such that $x_i = w_i$ for any $t_i \in T_i$.
- B-partisans (behavioral type B) are such that $x_i = -w_i$ for any $t_i \in T_i$.

- Expressive (or naïve) shareholders (behavioral type E) are such that :

$$x_i(t_i) = \begin{cases} w_i & \text{if } t_i \geq 1 \\ -w_i & \text{if } t_i < 1. \end{cases}$$

Partisans behavioral types can be thought of as strategic shareholders who: (i) prefer one particular action (A or B) independent of the state of the world ω, and (ii) use undominated strategies. This way of modeling partisan shareholders is in line with Bar-Isaac and Shapiro (2020). In practice, there is evidence that disagreement among shareholders may not only stem from information asymmetries, but also from differences in preferences (Bolton et al., 2020). As mentioned in Cvijanovic et al. (2020, p.3), preferences may vary due to differences in, e.g., portfolio allocation (Cohen and Schmidt, 2009), business ties (Davis and Kim, 2007, and Cvijanovic et al., 2020), reputational concerns (Chevalier and Ellison, 1999), and political and social goals (Woidtke, 2002).

Expressive (or naïve) voters can be thought of as shareholders with the same preferences as regular shareholders, but who do not use their information optimally. When they receive a signal indicating that a state (say α) is more likely than the other, they use all their voting rights to vote in favor of the the state-matching decision (A in that case), without taking into account their environment (possibly, because they do not understand that the information held by their fellow shareholders affect their pivotality).

Finally, we refer to regular (common-value, strategic) shareholders as of type C. While the game now includes non-common value shareholders, we still use the notion of dominance and efficiency defined in Section 4. The main reason to do so is that our focus is on the informational efficiency of voting rules. Another reason is that these welfare benchmarks, even if they are better suited for the analysis of common-value elections, also admit a utilitarian interpretation in certain contexts with heterogeneous preferences.[41]

Only partisan shareholders.

When all behavioral voters are partisans and the overall voting rights of each behavioral type is common knowledge, all our main results mostly hold. In particular, we can prove that (i) richer rules dominate poorer ones if these rules do not differ in their voting rights distribution, and (ii) the flexible rule X^f is efficient and its superiority over other (finite) rules can be reinforced.

We denote the total voting rights held by A (resp. B)-partisans by w_A (resp. w_B). Common-value shareholders own the remaining voting rights w_C.

Proposition 7. *Consider that (w_A, w_B, w_C) is common knowledge. Under Assumption 1: (i) if two rules X and X' have the voting rights distribution w with no decisive shareholder, and X is richer than X', then X dominates X'; and (ii) if $w_C > |w_A - w_B|$, then X^f admits an efficient BNE.*

[41] When there are partisan shareholders, the correct outcome (i.e., A in state α, and B in state β) does not necessarily coincide with the utilitarian one. Under both the rigid and the flexible rules, when partisans have so many votes that common-value shareholders cannot affect the final outcome, then in one state the correct outcome coincides with the one that maximizes the sum of utilities; and in the other state they differ. Importantly, those situations are the same for both rules. In the remaining situations, when the behavior of the common-value shareholders matters for the final outcome, the correct outcome coincides with the utilitarian one. Hence, if one of the two voting rules is found to admit a BNE that implements the correct outcome with higher probability than any BNE of the other rules, then it follows that it also implements the utilitarian outcome with higher probability.

Proof. In this proof, we denote by N_A (resp. N_B) the set of A-partisans (resp. B-partisans) and by N_C the set of common-value shareholders.

(i) Consider two rules X and X' with for all $i \in N$, $X'_i \subseteq X_i$. The proof's strategy consists in re-writing the voting games associated to these rules as voting games taking place among common-value shareholders, and to apply Proposition 1.

For a C-shareholders' vote profile $\mathbf{x} = (x_i)_{i \in N_C}$, a proposal passes (for sure) if and only if:

$$w_A - w_B + \sum_{i \in N_C} x_i > 0 \;\Leftrightarrow\; \sum_{i \in N_C} \left(x_i + \frac{w_A - w_B}{\# N_C} \right) > 0.$$

For $i \in C$, we let $Y_i = \{x_i + \frac{w_A - w_B}{\# N_C} \mid x_i \in X_i\}$ and $Y'_i = \{x_i + \frac{w_A - w_B}{\# N_C} \mid x_i \in X'_i\}$. By application of Proposition 1, the rule Y dominates the rule Y', it follows from the previous equivalence that we also have that X dominates X'.

(ii) when $w_C > |w_A - w_B|$, an efficient equilibrium can be constructed under X^f as follows. Assume without loss of generality that $w_B \geq w_A$. To implement the efficient outcome, C-shareholders must then find a way to compensate for that bias. An easy way to do so is for common-value shareholders to make two modifications to the efficient equilibrium strategy under X^f as characterized in Proposition 2. First, common-value shareholders have to re-scale their strategy by a factor of $(1 - \frac{w_B - w_A}{w_C})$ to leave room for compensation. Second, each common-value shareholder i includes $w_i \times \frac{w_B - w_A}{w_C}$ points in favor of A on her ballot. Formally, consider

$$\sigma_i(t_i) = \left(1 - \frac{w_B - w_A}{w_C}\right) \times \frac{c \ln(t_i)}{\ln(1/\delta)} + w_i \times \frac{w_B - w_A}{w_C}.$$

It is straightforward that $w_A - w_B + \sum_{i \in N_C} \sigma_i(t_i) \geq 0 \;\Leftrightarrow\; \sum_{i \in N_C} \ln(t_i) \geq 0$. Hence, σ is an efficient BNE. □

To illustrate the second part of the proposition, consider a case with 10 common-value shareholders holding 10 voting rights each, and partisan shareholders, all of them of type B, holding cumulatively 20 voting rights. Assume that, without the partisans, each common-value shareholder would give 10 points to A when receiving the most informative signal in favor of A, 2 points to A when receiving another, less informative signal in favor of A, and -5 points to A (i.e. 5 points to B) when receiving a signal in favor of B. With partisan shareholders, each common-value shareholder would re-scale her strategy by a factor of 0.8 (i.e., giving 0.8 times the number of points she would have given without partisan shareholders), and add 2 points for A. Thus, she would still give 10 ($= 10 \times 0.8 + 2$) points to A when receiving the most informative signal in favor of A, but she would now give 3.6 ($= 2 \times 0.8 + 2$) points to A when receiving the less informative signal in favor of A, and -2 ($= -5 \times 0.8 + 2$) points to A (i.e. 2 points to B) when receiving a signal in favor of B.

Partisan and expressive shareholders.

To explore this case, we assume that the distribution of voting rights $w = (w_i)_{i \in N}$ is common knowledge among shareholders, but the behavioral type of each shareholder is unknown. Specifically, the behavioral type of each shareholder $i \in N$ is drawn from a distribution $p_i \in \Delta(\{A, B, C, E\})$. We let $p = (p_i)_{i \in N}$ denote the (shareholder-specific) behavioral-type distribution.

In this extended model, we show that richer rules still dominate poorer ones (provided that voting rights are distributed similarly) when we restrict our attention on their best equilibria.

Proposition 8. *Consider two finite voting rules X and X', with the same distribution of voting rights w, and a behavioral-type distribution p. Under Assumption 1, if X is richer than X', then for any BNE under X' there exists a BNE under X such that the probability of making the correct decision is higher.*

Proof. Let X be a finite voting rule. The voting game can then be re-written as one of common values. In this game, each shareholder $i \in N$ has the (common) utility u_i as defined in (1). When she is of type t_i and chooses an action $x_i \in X_i$, the action is recorded with probability p_i^C, while it is transformed into $+w_i$ with probability p_i^A, into $-w_i$ with probability p_i^B and into $(2 \times 1_{\{t_i \geq 1\}} - 1)w_i$ with probability p_i^E.

The result of Proposition 8 is obtained by the same argument as in the proof of Lemma 4. The only difference arises with the expressions (in the proof of Claim 1) $\mathbb{P}_{\sigma_{-i}}(\sum_{j \neq i} x_j > -x \mid \omega) + \frac{1}{2}\mathbb{P}_{\sigma_{-i}}(\sum_{j \neq i} x_j = -x \mid \omega)$ which should be replaced by $\mathbb{P}_{\tilde{\sigma}_{-i}}(\sum_{j \neq i} x_j > -x \mid \omega) + \frac{1}{2}\mathbb{P}_{\tilde{\sigma}_{-i}}(\sum_{j \neq i} x_j = -x \mid \omega)$, where the strategy $\tilde{\sigma}_j$ is defined for each shareholder j by:

$$\tilde{\sigma}_j(t_j) = \begin{cases} \sigma_j(t_j) & \text{with probability } p_j^C \\ w_j & \text{with probability } p_j^A \\ -w_j & \text{with probability } p_j^B \\ (2 \times 1_{\{t_j \geq 1\}} - 1)w_j & \text{with probability } p_j^E. \end{cases}$$

□

To understand this result, first note that behavioral shareholders behave in the same way under X and X': partisan shareholders give as many points as possible to their favorite alternative, while expressive ones simply follow their signal. Thus, behavioral shareholders create the same noise under the two voting rules. To implement their desired outcomes, common-value shareholders have to correct for that noise while still finding a way to reveal their information. The relatively richer ballot space under X than X' gives more leeway to common-value shareholders to achieve this.

While a richer rule improves information aggregation, the noise arising from the presence of behavioral shareholders (either because the number of partisans is unknown or because there are expressive shareholders) leads to inefficiencies, even under the flexible rule. Figure 4 illustrates that point: the probability of reaching the correct decision given the collective information decreases with the frequency of B-partisans (we assume for simplicity that shareholders are either of type C or B) under X^f. Note that this is also true under X^r. This suggests an explanation for the empirical finding that firms with more heterogeneous shareholder base under-perform (see, e.g., Kandel et al., 2011, and Schwartz-Ziv and Volkova, 2021).

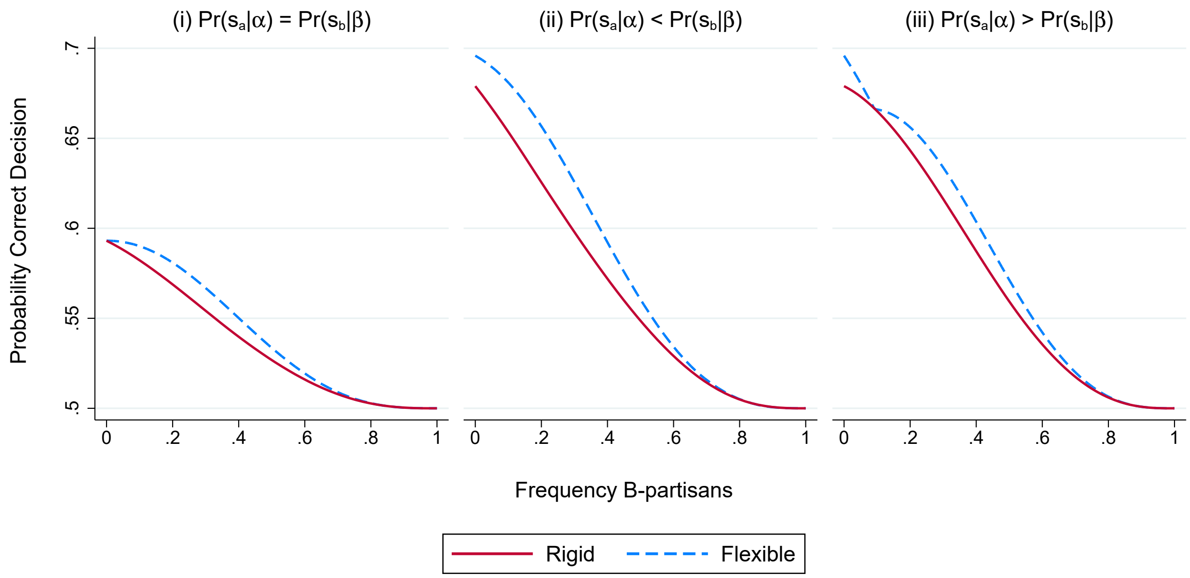

Figure 4: Probability of correct decision under X^f and X^r with binary signals, a random number of B-partisans, $n = 6$, even voting rights and an even prior. The conditional probabilities of receiving the correct signals in each state are $Pr(s_a|\alpha) = Pr(s_b|\beta) = 55\%$ in (i), $Pr(s_a|\alpha) = 55\%$ and $Pr(s_b|\beta) = 65\%$ in (ii), and $Pr(s_a|\alpha) = 65\%$ and $Pr(s_b|\beta) = 55\%$ in (iii).

Endogenous Information

Consider now that the signal is not free. Specifically, we introduce a pre-stage to the game in which shareholders independently and simultaneously decide whether to acquire an informative signal at cost $l > 0$, or to stay only with the prior. In the next stage of the game, the shareholders observe who acquired a signal, and then vote. For tractability, we focus on equilibria in which shareholders use pure strategies in the first stage, and the BNE that maximizes the probability of making the correct decision in each subgame.

This is the model considered in Persico (2004), with two differences: (i) we allow for more general signal structures and a more general class of voting rules, and (ii) he allows for imperfect information (i.e. shareholders do not necessarily observe who acquired a signal). This simplifying assumption helps us deal more easily with the large variety of voting rules and signals that we consider here, but we argue after the statement of the result why it is robust to having unobservable information acquisition at least as far as the comparison of best equilibria of different rules is concerned.

In this setup, we can show that the flexible rule X^f still dominates all the other rules but with a different welfare criterion in mind: the probability of making the correct decision net of total information acquisition costs. It seems reasonable that shareholders should care both about the accuracy of their choice and the cost of information acquisition. Now, this does not exclude the possibility that another rule is superior in terms of maximizing the probability of making the correct decision independent of the information acquisition costs.

Proposition 9. *Under Assumptions 1 and 2, the probability of making the correct decision net of total information costs is higher under the flexible rule compared to any other voting rule.*

Proof. A subgame is essentially defined by the set of shareholders that acquire a signal. Let us denote the set of informed shareholders by I. By Proposition 1 and by the argument of McLennan

(1998) we have that, from an ex-ante point of view, the probability of the firm making the correct decision in subgame I under voting rule X, denoted by $\mathbb{P}(I, X)$, must satisfy $\mathbb{P}(I, X) \geq \mathbb{P}(I', X)$ when $\#I > \#I'$ for any X, and $\mathbb{P}(I, X^f) \geq \mathbb{P}(I, X)$ for any I and any X. We also notice that since we have fixed a certain BNE in each subgame, the whole game may be viewed as a single stage game in which the shareholders only decide whether to draw an informative signal or not. This simplified version of the game is a potential game with potential function $\mathbb{P}(I(z), X) - \#I(z) \times l$, where z is the vector of information acquisition decisions with $z_i = 1$ when shareholder i acquires a signal and $z_i = 0$ otherwise; and $\#I(z) = \sum_{i \in N} z_i$ is the number of informed shareholders. By the fact that there are finitely many alternative vectors z, the potential function obtains a maximum value for (at least) one of these vectors, which is also a pure strategy equilibrium of this simplified game (see for instance Monderer and Shapley, 1996). Moreover, every pure strategy equilibrium of this simplified game must be a maximizer of this potential function. Assume now, that a pure strategy equilibrium, z_f^*, exists under X^f such that $\mathbb{P}(I(z_f^*), X^f) - \#I(z_f^*) \times l < \mathbb{P}(I(z_X^*), X) - \#I(z_X^*) \times l$, where z_X^* is an equilibrium of some other rule X. Since $\mathbb{P}(I, X^f) \geq \mathbb{P}(I, X)$ for any I and any X, it follows that $\mathbb{P}(I(z_X^*), X^f) - \#I(z_X^*) \times l \geq \mathbb{P}(I(z_X^*), X) - \#I(z_X^*) \times l$, which contradicts the fact that z_f^* is an equilibrium – and, thus, a maximizer of the corresponding potential function – under X^f. Therefore, there is no rule X that admits a better equilibrium than X^f. □

The intuition behind this result relies on the following fact. Consider a given profile of information acquisition decisions of the other shareholders. Then, an increase in the expected utility of a shareholder if she acquires information corresponds to the increase in the probability of making the correct decision net of the increase in total information acquisition costs. Hence, the information acquisition game is a potential game. Its potential is the probability of making the correct decision net of the total information acquisition costs.

Now, let us consider shareholders using the same profile of information acquisition decisions under the flexible rule as in the equilibrium of another voting rule. The value of the potential corresponding to the flexible rule must be at least as high as that of the other rule. This follows from the superior information aggregation properties of the flexible rule. For that profile of information acquisition decisions, the information costs are the same under the two rules, but the probability of making the correct decision is higher under the flexible rule. Hence, in every equilibrium under the flexible rule, the value of the potential must be larger compared to the value of the potential in any equilibrium of any other rule.

The above result is robust to information acquisition decisions being unobservable. To see this, notice that the best equilibrium of the game with observable information acquisition decisions remains an equilibrium of the game with unobservable information decisions under any voting rule. Indeed, under observable information acquisition, in the best equilibrium of a voting rule when a shareholder who is expected to acquire information does not acquire information, she decreases her expected utility (otherwise, we would not be in equilibrium), but less so compared to the case of unobservable information acquisition decisions: in the first case, all shareholders adjust and use the welfare maximizing BNE of the voting subgame, but in the latter they cannot do so and the probability of making the correct decision decreases even more. Hence, the flexible rule delivers a higher probability of making the correct choice net of total information acquisition costs, even when information acquisition decisions are unobservable.

Appendix D: Discussion of Modeling Assumptions

State-Contingent Preferences

A central assumption of our baseline model is that shareholders have state-contingent preferences: conditional on the state of the world, they all agree whether the management's proposal should be approved or rejected. This is a standard assumption in the literature on shareholders voting (see, e.g., Maug and Yilmaz, 2002; Marquez and Yılmaz, 2008; Levit and Malenko, 2011; Esö et al., 2014; Malenko and Malenko, 2019; Bar-Isaac and Shapiro, 2020; Meirowitz and Pi, 2022; Ma and Xiong, 2021). It indeed seems natural to assume that (most) shareholders share the common goal of maximizing the value of the firm.

There are various pieces of empirical evidence that are coherent with the state-contingent preferences assumption. More precisely, the literature uncovers facts that are in line with models of strategic voting making that assumption, similar to the one developed above. For instance, Maug and Rydqvist (2009) structurally estimate such a model of strategic voting using data about U.S. shareholders meetings between 1994 and 2003. They find that the voting behavior of shareholders at those meetings is in line with their model. As predicted: (i) shareholders vote more in favor of proposals when the supermajority threshold increases, and (ii) there is essentially no effect of supermajority thresholds on the acceptance rate.

Christoffersen et al. (2007) study vote trading in the US and the UK and find patterns that are in line with information aggregation theory of voting (see, e.g., Esö et al., 2014). They indeed uncover an active market for votes, both in the US and the UK, where the average vote sells for a price of *zero*. Moreover, as predicted by the theory, vote trading increases (i) with asymmetric information among shareholders, (ii) the importance of the proposal at stake (proxied by poor performance of firm), and (iii) if the pivot probability is high. Finally, warnings of votes that violate corporate governance standards (which they interpret as a negative public signal about the proposal that reduces information asymmetry among shareholders) reduce vote trading.

Calluzzo and Dudley (2019) study the influence of proxy advisors on firm voting outcomes, policies and values. They find that, as predicted by Malenko and Malenko (2019) based on a model including shareholders with state-contingent preferences, proxy advisors have a large influence when shareholders have weak incentives to acquire information.

There is also evidence that, at first sight, appears to contradict the predictions of a model of strategic voting including shareholders with state-contingent preferences: Li et al. (2022) find that there is substantial trading by mutual funds after shareholder meetings. Yet, Meirowitz and Pi (2022) and Bouton et al. (2022) show that this is actually consistent with such a model. For instance, Meirowitz and Pi (2022) takes into account that shareholders who vote in the meetings are also traders after the meeting. In such a setting, shareholders do not fully reveal information through their vote, which prevents information aggregation. This creates opportunities to trade after shareholder meetings. In our model, under the *one-person-one-vote* or *one-share-one-vote* rules, there would be a different reason for trading after the vote. Depending on the precision of their signal, shareholders have different beliefs about the probability that the decision at the meeting was correct. Hence, shareholders with sufficiently precise signals against the decision made at the meeting would be willing to sell their shares, and those with sufficiently precise signals aligned with the decision made at the meeting would be willing to buy more shares. These trading patterns are in line with the findings of Li et al. (2022) about the behavior of mutual funds after the meetings.[42]

[42] Our model also predicts that the trading patterns would be systematically different under the flexible rule

Last but not least, it is important to stress that we are not trying to argue that every single shareholder has state-contingent preferences. Indeed, as we discuss in the previous Appendix, there is evidence suggesting that disagreement among shareholders may not only stem from information asymmetries. This is exactly the reason why, in that Appendix, we consider an extension of our model that allows for the presence of partisan shareholders. And we show that, in the presence of such shareholders, the fexible rule still outperforms other voting rules in terms of information aggregation.

Information Asymmetry

Another key assumption of our model is that some shareholders are better informed than others. We view this assumption as uncontroversial. First, as explained in Knyazeva et al. (2018, p. 681): "the precision of a trader's [...] private information may be a function of the trader's overall or company specific investment experience, local knowledge, or the extent of resources that the trader can allocate to information gathering." And, indeed, Kim and Verrecchia (1991) show that traders react differently to release of public information about a given firm, i.e., less informed traders, who revise the beliefs more, react more. Also, Iliev and Lowry (2015) and Iliev et al. (2021) find that mutual funds vary greatly in their reliance on proxy advisory recommendations, with the more informed voting less in line with the recommendations.

Second, the literature provides evidence that shareholders have different incentives to invest in acquisition of information (see, e.g., Chen et al., 2007, and Fich et al., 2015).

Third, information asymmetries among shareholders help explain phenomena that are difficult to explain without such asymmetries (see, e.g., Glosten and Milgrom, 1985's discussion of the bid-ask spread).

Finally, there is an empirical literature studying information asymmetry among shareholders, using different measures (see, e.g., Brown and Han, 1992; Healy et al., 1995; Welker, 1995; Iliev and Lowry, 2015; Knyazeva et al., 2018). It points toward substantial information asymmetries among shareholders/investors. This is true both across types of shareholders (see, e.g., Sias et al. (2006) for evidence of the informational advantage of institutional investors over other types of investors), and within a given type (see, e.g., Knyazeva et al. (2018) for evidence of heterogeneity among institutional investors).

No Communication

In our baseline model, we assume that shareholders cannot communicate before the vote. This is not an innocuous assumption. If costless, communication can indeed improve information aggregation (see, e.g., Coughlan, 2000), and mute differences between voting rules (see, e.g., Gerardi and Yariv, 2007). The idea is simple: when shareholders have state-contingent preferences, they have incentives to truthfully reveal their private information to one another, and then vote unanimously for the efficient outcome.

In practice, it is not infrequent for shareholders to declare their intention to vote, especially large blockholders and institutional investors. But it is far from clear that such communication is truthful and that it leads to the aggregation of all information before the vote. Indeed, there are several hurdles to the truthful revelation of information through pre-vote communication among

(compared to finite rules). Under that rule, when voting fully aggregates information, shareholders' posteriors are identical. There is then no room for trade after the meeting.

shareholders.

First, in the presence of partisan shareholders, communication is impeded (Coughlan, 2000). The problem is that those shareholders have incentives to pretend that they have state-contingent preferences but that they have received a signal in favor of their preferred alternatives. And, as we show in the previous Appendix, the flexible rule still dominates other voting rules in the presence of such shareholders (without communication).

Second, even if there are no partisan shareholders, communication among shareholders is far from costless. In the case of most public firms, shares are distributed among many, scattered, individuals and institutions. It is thus logistically challenging to organize communication. Moreover, as explained in Malenko and Malenko (2019, p.2470), "[...] investors fear that communication with others can be considered "forming a group" [...]," which would trigger costly administrative filling requirements and, in some cases, a poison pill. There could also be a cost of publicly disclosing your information: "[...] investors are often reluctant to publicly disclose their intention to vote against management, fearing that doing so would be viewed as an activist campaign and lead to managerial retaliation."

Vote Trading

Our baseline model does not allow shareholders to trade votes before the meeting. Yet, we know that there is an active market for votes (Christoffersen et al., 2007) and that vote trading can be beneficial for information aggregation. Esö et al. (2014) study vote trading and, assuming one share per shareholder, prove the existence of an efficient equilibrium under the rigid rule (*one-person-one-vote*) in which vote trades at a price of zero. In that equilibrium, uninformed shareholders sell their votes to informed shareholders. As in the case of communication, allowing for vote trading could then mute differences between the flexible rule and other (finite) voting rules. But, there are reasons to believe this is not the case.

First, note that vote trading is irrelevant under the flexible rule: there is no gain from trade because the equilibrium is efficient. Second, there are various hurdles to vote trading under other voting rules. For instance, the efficient equilibrium in Esö et al. (2014) is not robust to the presence of sufficiently many partisan shareholders. Moreover, we conjecture that differences in signal precision would also prevent efficient aggregation of information. The problem in that case is that shareholders need to know how precise their information is compared to that of other shareholders in order to decide optimally whether to "buy" or "sell" votes. There is no clear way for shareholders to do so. This issue becomes even worse if there is ambiguity about the information technology of other shareholders.

Share Trading

Our baseline model does not allow shareholders to trade votes before or after the meeting. A general treatment of this question is out of the scope of this paper (for recent contributions on the topic of share trading and voting, see, e.g., Levit et al., 2019; Bar-Isaac and Shapiro, 2020; Meirowitz and Pi, 2022; Levit et al., 2021; Bouton et al., 2022). However, it would be erroneous to believe that our results are not robust to some forms of share trading before or after the meeting. For instance, we could consider a pre-meeting trading model similar to Bar-Isaac and Shapiro (2020)'s. They assume that a shareholder "trades so as to maximize the overall expected value of the firm." Due to efficiency of the flexible rule, the overall expected value of the firm is maximized without trading. Shareholders would thus not have any incentive to trade shares among them before the meeting.

For voting rules that are not efficient, shareholders might have incentives to trade. But even if *shares* are fully divisible, it would not be always possible to reach the efficient outcome through share trading. What prevents efficiency is private information about the precision of the signal of the blockholder.

To see this, let us consider a case in which there is a unique blockholder holding multiple shares and all other shareholders hold one share each. Before the meeting, the blockholder can post a price and sell some of her shares to individuals who are currently not owning any fraction of the firm and are completely uninformed about the issue at hand, or to other shareholders. Other shareholders are characterized by an identical precision of information. We focus on the properties of a (rather coarse) version of *one-share-one-vote* that compels shareholders either to use all their votes or to fully abstain. We also focus on a case in which the signal precision of the blockholder is not too high so that she would never want to buy shares in equilibrium.

If the blockholder's signal precision is publicly known, then trade (i.e., the blockholder sells some of her shares) would result in an optimal distribution of votes and hence in an efficient outcome. In that case the price of a share simply reflects the probability that the firm makes the correct choice. Note, however, that, as discussed in Bar-Isaac and Shapiro (2020), there may be hurdles to share trading that could prevent the efficient outcome to materialize even when signal precision is publicly known.

If the blockholder's signal precision is her private information (consider, for instance, that her precision is identical to that of the other shareholders with some probability and otherwise higher), then there cannot be a separating equilibrium with prices corresponding to the two potential eventualities (i.e. a higher price when the blockholder's precision is high and a lower price when her information precision is low). If it were the case, then the low precision blockholder would pretend to have high precision signal in order to sell her shares at a higher price. The equilibrium price is then too low: the blockholder does not sell enough shares, and hence casts too many votes from an information aggregation standpoint. This leads to an inefficient outcome under that (rather coarse) version of *one-share-one-vote*.[43] This implies that divisibility of votes may remain a useful tool for information aggregation when trading of shares is allowed before the meeting.

Conflict Between Shareholders and the Manager

In our model with endogenous proposal we allow for management's preferences to differ from shareholders' preferences over two dimensions. First, the manager may be misaligned with shareholders and prefer the proposal to be adopted in both states. As discussed in Becht et al. (2016), there is empirical evidence showing that a substantial share of corporate acquisitions are associated with negative returns for acquirer shareholders. One explanation is that, in the case of M&A, managerial wealth and shareholder wealth are decoupled (Grinstein and Hribar, 2004; Harford and Li, 2007; Fu et al., 2013). Another area of conflict is Say-on-Pay proposals (Cuñat et al., 2016). Finally, Bach and Metzger (2016) and Babenko et al. (2019) find evidence that managers manipulate the voting process to increase the success rate of their proposals. Those manipulations appear to be value-destroying. For a more thorough discussion of conflict between managers and shareholders see, e.g., chapter 1 in Tirole (2010).

Second, the manager may incur a cost if a proposal is rejected at the shareholder meeting. Becht et al. (2016) and Gantchev and Giannetti (2021) mention the existence of such reputation costs but

[43] The formal arguments backing this claim are similar to the ones employed by Bar-Isaac and Shapiro (2020) when they consider trade with endogenous information acquisition, and are therefore omitted.

we are not aware of direct evidence. However, there is suggestive evidence. For instance, Cai et al. (2009) and Aggarwal et al. (2019) find that dissent votes in uncontested director elections have negative consequences for both directors and executives. The rejection of a management proposal at the meeting could have similar implications for the management. Also, Li et al. (2018) find that managers trying to acquire another corporation implement various strategies to avoid shareholder voting on the acquisition, and hence the risk of an embarrassing rebuke at the shareholder meeting.

Appendix E: Empirical Implications

The analysis in Section 6 sheds a new light on the very high approval rate of management's proposals by shareholders in practice (see, e.g., Maug and Rydqvist, 2009; Babenko et al., 2019; Bach and Metzger, 2019). It has been argued (informally) that such a high rate can be explained by the selection of proposals by managers (see, e.g., Becht et al., 2016). The idea is that the fear of having a proposal turned down, which has negative consequences for the managers, gives managers incentives to withhold low quality proposals. This means that only high quality proposals are put to a vote, and are approved at a very high rate.

In our model, shareholders approve the proposal more frequently when the manager has the power to veto it than when she does not. Yet, the mechanism at play may be different. As we explained above, it does not necessarily rely on the management incurring a cost when its proposal is rejected by shareholders, as this effect arises even when $c = 0$. In that case, it instead relies on the manager's willingness to make decisions that are beneficial for the firm and shareholders. This is another source of selection for proposals at shareholders meetings that has implications for the empirical literature studying the effects of shareholders voting on firms' performance.

Our analysis also produces a testable prediction: the approval rate of proposals at the shareholder meeting is decreasing in μ, the probability that the manager is misaligned. Different measures of alignment between shareholders and management could be used to test this prediction, such as the extent of the CEO equity-based compensation as in Datta et al. (2001), bonus-compensation as in Grinstein and Hribar (2004), and the sensitivity of the CEO compensation to stock performance post acquisition as in Harford and Li (2007).

Finally, in equilibrium, the quality of the proposal conditional on a vote being called is increasing in c. Thus, the probability of approval of the proposal is also increasing in c. This result suggests an explanation for the much higher approval rate of management's proposals than shareholders' proposals in practice (Bach and Metzger, 2019): shareholders do not suffer (as high) reputation cost when their proposals are turned down at the meeting. Differences in this cost among shareholders could potentially help explain why the approval rate of shareholder proposals is strongly associated with the identity of the sponsor (Gillan and Starks, 2000).