NBER WORKING PAPER SERIES

THE VARIABILITY OF VELOCITY IN CASH-IN-ADVANCE MODELS

Robert J. Hodrick

Narayana Kocherlakota

Deborah Lucas

Working Paper No. 2891

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 1989

This paper is part of NBER's research program in Financial Markets and Monetary Economics. Any opinions expressed are those of the authors not those of the National Bureau of Economic Research.

NBER Working Paper #2891
March 1989

THE VARIABILITY OF VELOCITY IN CASH-IN-ADVANCE MODELS

ABSTRACT

Early cash-in-advance models have the feature that the cash-in-advance constraint always binds, implying that the velocity of money is constant. Lucas (1984) and Svensson (1985) propose a change in information structure that potentially allows velocity to vary. By calibrating a version of these models using a new solution algorithm, and using U.S. time series data on consumption growth and money growth, we find that in practice the cash-in-advance constraint almost always binds. This result is robust to changes in the forcing process, the inclusion of credit goods along with cash goods, various preference specifications, and changes in the precision of the agents' information. We conclude that there is little practical gain in using these more complicated informational specifications in future applications of a cash-in-advance technology.

Robert J. Hodrick
Kellogg Graduate School
of Management
Northwestern University
Evanston, IL 60208

Narayana Kocherlakota
Kellogg Graduate School
of Management
Northwestern University
Evanston, IL 60208

Deborah Lucas
Kellogg Graduate School
of Management
Northwestern University
Evanston, IL 60208

I. Introduction

It has become quite common to introduce a demand for money in general equilibrium models through a cash-in-advance (CIA) constraint. Simple CIA models require that the money held in a given period be at least sufficient to cover perfectly anticipated expenditures. Agents facing positive nominal interest rates never hold idle cash balances in these economies, so the entire money supply turns over each period. Consequently, these models incorrectly predict that the consumption velocity of money is always unity.

In response to this difficulty, Lucas (1984) and Svensson (1985) modify the information structure in the basic CIA setup. We focus upon Svensson's formulation of a single agent exchange economy in which output and the rate of growth of the nominal money stock follow a stationary Markov chain. Cash balances must be chosen before the quantity of output is known. Therefore, agents may choose to carry unspent cash across periods, and velocity can in principle vary. Lucas and Stokey (1987) further weaken the tie between money and expenditures by allowing substitution between "cash" goods and "credit" goods (ones that are not subject to the CIA constraint).

These models make predictions about the interactions of inflation, real and nominal interest rates, real balances, velocity, money growth and consumption growth. Yet the precise nature of these predictions is known for only a few special cases. Svensson's analytical results, for example, depend on the clearly unrealistic assumption that the state of the world is independently and identically distributed (i.i.d.). On the other hand, the work of Giovannini (1987) and Hodrick (1989) relaxes the i.i.d. assumption but assumes parameters are such that the CIA constraint always binds.

This paper examines two CIA models that retain Svensson's original timing of information flows and transactions in markets. In the first, agents must buy all goods with cash (the "cash" model), while in the latter, they can substitute between goods bought with cash and those bought using credit (a "cash-credit" model). It sets forth new algorithms to solve these models without restricting the CIA constraint to be always binding or assuming the state of the world is i.i.d.. We consider a representative agent exchange economy in which the joint stochastic process governing money growth and consumption growth is a Markov chain calibrated to accord with United States time series data. This exercise is similar to that of Mehra and Prescott (1985), but we do not use their calibration procedure. Instead, we estimate a first order bivariate vector autoregression (VAR) using quarterly and annual consumption growth and money growth data. We then approximate the VAR by a Markov chain using the quadrature method of Tauchen (1987). Using this forcing process, we calculate the joint distribution of endogenous and exogenous variables for several homothetic utility functions.

We use this method to address two related issues. First, we wish to understand the models' qualitative predictions about the joint distribution of endogenous and exogenous variables. To do this, we graph a variety of unconditional population moments as a function of the preference parameters.

The second issue is whether either model can generate predictions about contemporaneous first and second unconditional moments that are consistent with sample estimates. Svensson (1985) and Lucas and Stokey (1987) allow for possible slackness in the CIA constraint, which accounts for much of the analytical difficulty in using these models. For this reason, it is natural to focus attention upon the models' predictions of the volatility of velocity and its correlations with other variables. We find that the cash model typically fails to generate any variability of velocity for almost all parameter specifications; the CIA constraint almost always binds. On the other hand, the cash-credit model can generate large fluctuations in velocity, especially when the two goods are close substitutes, even though the CIA constraint generally binds for the cash good. Unfortunately, whenever the cash-credit model predicts a realistic variance in velocity, it also predicts implausibly large values of expected velocity. Moreover, the cash-credit model is unable to generate realistic predictions about the sample moments of many other variables.

Finally, we do extensive sensitivity analyses by altering the timing of information, the nature of the driving processes, and the specification of the utility function. We examine the sensitivity of the models to changes in the information structure by providing agents with signals about the future. In Svensson's model, agents know next period's money supply and this period's output when they trade in the asset market. We modify this framework by giving them noisy signals of next period's output and money. In general, such changes have only small effects. Similarly, the models' predictions are highly invariant to changes in the forcing processes. However, changing preferences to reflect "habit formation" (Ryder and Heal (1973)) generates substantially different predictions.

The algorithm used to find stationary equilibria for the models is new, and has several nice properties. Because it does not rely on a contraction principle, the speed of convergence does not depend explicitly upon the discount factor, is generally quite fast, and accommodates discount factors greater than one. In a growth model, a discount factor greater than one cannot be dismissed *a priori* (Kocherlakota (1988)); however, we find that the models perform no better in this region.

(5b) $$Q_t = q(\gamma_t,\omega_t)P_t y_{t-1}.$$

Define the multipliers on the constraints (2) and (3), expressed in real terms, to be functions of the state that do not depend upon the money supply and that can be written as $\mu(\gamma_t,\omega_t)y_{t-1}^{-\alpha}$ and $\lambda(\gamma_t,\omega_t)y_{t-1}^{-\alpha}$ respectively, and define $m(\gamma_t,\omega_t) = 1/p(\gamma_t,\omega_t)$. Then a <u>stationary equilibrium</u> is a set of functions $\{m(\gamma_t,\omega_t),\ q(\gamma_t,\omega_t),\ \mu(\gamma_t,\omega_t),\ \lambda(\gamma_t,\omega_t)\}$ such that:

(6) $m(\gamma_t,\omega_t) \geq \gamma_t, \qquad \{\mu(\gamma_t,\omega_t) \geq 0 \text{ and } \mu(\gamma_t,\omega_t)[m(\gamma_t,\omega_t) - \gamma_t] = 0\},$

(7) $\mu(\gamma_t,\omega_t) + \lambda(\gamma_t,\omega_t) = \gamma_t^{-\alpha},$

(8) $\lambda(\gamma_t,\omega_t)m(\gamma_t,\omega_t) = \beta E[(\gamma_{t+1}^{-\alpha}m(\gamma_{t+1},\omega_{t+1})|(\gamma_t,\omega_t)]\gamma_t^{1-\alpha}/\omega_t,$

(9) $\lambda(\gamma_t,\omega_t)q(\gamma_t,\omega_t) = \beta E[\lambda(\gamma_{t+1},\omega_{t+1})(q(\gamma_{t+1},\omega_{t+1}) + \gamma_{t+1})|(\gamma_t,\omega_t)]\gamma_t^{1-\alpha},$

where (6) is the liquidity constraint, with the Kuhn-Tucker multiplier in braces, (7) is the first order condition associated with c_t, and (8) and (9) ar modifications of the first order conditions associated with M_{t+1} and z_{t+1}. $E[x|y]$ denotes the expectation of x conditional on y. An equilibrium must also satisfy the necessary conditions that expected utility and expected nominal wealth are finite, which is equivalent to requiring that the eigenvalues of the matrix A, with typical element $A_{ij} = \beta\Pi_{ij}(\gamma_j)^{1-\alpha}$, lie within the unit circle.[1]

In general, equations (6) - (9) do not admit an analytical solution. Nevertheless, the algorithm in the next section finds the stationary equilibriu (if one exists), which allows numerical exploration of predictions of the model

B. The Cash-Credit Model

Modifying the above model to allow cash goods and credit goods as in Lucas and Stokey (1987) is straightforward. We specialize their model by assuming a particular preference specification, by choosing an information structure identical to that of Svensson, and by adding growth to the endowment.[2]

The cash-credit model allows the agent to consume two goods. Good 1 can be purchased only with cash, while good 2 is bought on credit. Equations (2) and (3) are replaced with the following:

(2') $P_t c_{1t} \leq M_t$,

(3') $M_{t+1} + Q_t z_{t+1} = z_t P_t y_t + Q_t z_t + (\omega_t - 1)X_t + M_t - P_t(c_{1t} + c_{2t})$.

The goods sell for the same price because sellers receive payments for both goods in time to make purchases the following period, and because the goods are perfect substitutes in production. The technology is linear:

(4a') $c_{1t} + c_{2t} = y_t$.

Finally, we generalize (1) by using the period utility function:

$$(10) \qquad u(c_{1t}, c_{2t}) = \frac{\left[c_{1t}^{\psi} c_{2t}^{(1-\psi)}\right]^{(1-\alpha)} - 1}{(1-\alpha)}.$$

As above, a stationary equilibrium is a set of functions, independent of time and the level of the money supply, that support the agent's first order conditions and the market clearing conditions. We again assume that the equilibrium functions are separable in y_{t-1}, and that consumption of both goods is linear in y_t. Then $\{c_1(\gamma_t,\omega_t), m(\gamma_t,\omega_t), \mu(\gamma_t,\omega_t), \lambda(\gamma_t,\omega_t), q(\gamma_t,\omega_t)\}$ is a stationary equilibrium if it satisfies the following:

(11) $c_1(\gamma_t,\omega_t) \leq m(\gamma_t,\omega_t)$, $\{\mu(\gamma_t,\omega_t) \geq 0$, and $\mu(\gamma_t,\omega_t)[m(\gamma_t,\omega_t) - c_1(\gamma_t,\omega_t)] = 0\}$

$$(12) \qquad u_1(\gamma_t,\omega_t) = u_2(\gamma_t,\omega_t) + \mu(\gamma_t,\omega_t)$$

$$(13) \qquad \lambda(\gamma_t,\omega_t) = u_2(\gamma_t,\omega_t)$$

$$(14) \qquad \mu(\gamma_t,\omega_t) = u_1(\gamma_t,\omega_t) - \frac{\beta E[u_1(\gamma_{t+1},\omega_{t+1}) m(\gamma_{t+1},\omega_{t+1}) \mid \gamma_t,\omega_t]\gamma_t^{1-\alpha}}{\omega_t m(\gamma_t,\omega_t)}$$

(15) $\lambda(\gamma_t,\omega_t)q(\gamma_t,\omega_t) = \beta E[\lambda(\gamma_{t+1},\omega_{t+1})(q(\gamma_{t+1},\omega_{t+1}) + \gamma_{t+1})|(\gamma_t,\omega_t)]\gamma_t^{1-\alpha}$,

where the marginal utilities of consumption of the two goods are

(16a) $$u_1(\gamma_t,\omega_t) = \psi\gamma_t^{-\alpha}[1-c_1(\gamma_t,\omega_t)]^{(1-\psi)(1-\alpha)}c_1(\gamma_t,\omega_t)^{[\psi(1-\alpha)-1]}$$

(16b) $$u_2(\gamma_t,\omega_t) = (1-\psi)\gamma_t^{-\alpha}[1-c_1(\gamma_t,\omega_t)]^{[(1-\psi)(1-\alpha)-1]}c_1(\gamma_t,\omega_t)^{\psi(1-\alpha)}.$$

Note that if the CIA constraint does not bind in state (γ,ω), equations (12) and (16) imply that $c_1(\gamma,\omega) = \psi$. Also, we can solve for (c_1, μ, m, λ) using only equations (11)-(14), and then use equation (15) to obtain q.

C. Formulas for the Endogenous Variables

Endogenous stochastic processes for velocity, realized real and nominal interest rates, inflation, and the growth of real balances can be calculated from the above equilibrium functions. Since these expressions are straightforward to derive (see Svensson (1985)), we present them in Table 1 without derivation. Although these quantities are algebraically identical in the cash good and cash-credit models, their predicted values may vary significantly because the functions are based on different equilibrium conditions. Although the only assets outstanding in the economy are money and the endowment stock, we price other assets using the market clearing condition that they be in zero net supply. When calculating returns, we assume that all potential assets are traded in the securities market after the goods market closes. Hence, the payoff received at time t from a one period bond purchased at time t-1 would not be available for consumption purchases until time t+1.

III. A Solution Algorithm

Both models are solved by a similar method, but the algorithm is most straightforward for the cash model. With the equilibrium of the form described above, the system (6), (7) and (8) reduces to two functional equations in two unknown functions μ and m:

(17) $\gamma_t \leq m(\gamma_t,\omega_t) \qquad \{\mu(\gamma_t,\omega_t) \geq 0, \;\; \mu(\gamma_t,\omega_t)[\gamma_t - m(\gamma_t,\omega_t)] = 0\}$

(18) $\mu(\gamma_t,\omega_t) = \gamma_t^{-\alpha} - \dfrac{\beta E[\gamma_{t+1}^{-\alpha}\, m(\gamma_{t+1},\omega_{t+1}) \mid \gamma_t,\omega_t]\gamma_t^{1-\alpha}}{\omega_t m(\gamma_t,\omega_t)}$

The algorithm takes the following steps:

<u>Step 1</u>: Set $m_0(\gamma,\omega) = \gamma$ for all (γ,ω). (This is equivalent to assuming that the CIA constraint binds in all states.)

<u>Step 2</u>: Use (18) to solve for $\mu_0(\gamma,\omega)$. If $\mu_0(\gamma,\omega) \geq 0$ for all (γ,ω), this is an equilibrium. If not, go to Step 3.

<u>Step 3</u>: If $\mu_0(\gamma,\omega) < 0$ in all states, stop since no equilibrium exists. For any state in which $\mu_0(\gamma,\omega) \geq 0$, set $m_1(\gamma,\omega) = m_0(\gamma,\omega)$. If $\mu_0(\gamma_j,\omega_j) < 0$ in some state (γ_j,ω_j), set $m_1(\gamma_j,\omega_j)$ in the denominator of the right-hand side of (18) so that $\mu(\gamma_j,\omega_j) = 0$ when (18) is solved using the vector $m_0(\gamma,\omega)$ in the numerator of the right-hand side.

<u>Step 4</u>: Use (18) to solve for $\mu_1(\gamma,\omega)$ using $m_1(\gamma,\omega)$ on the right-hand-side. If $\mu_1(\gamma,\omega) \geq 0$ in all states, this is an equilibrium. If not, set $\mu_0(\gamma,\omega) = \mu_1(\gamma,\omega)$ and $m_0(\gamma,\omega) = m_1(\gamma,\omega)$ for all (γ,ω); and repeat Step 3.

<u>Theorem 1</u>: The algorithm converges to a stationary equilibrium if one exists.

The proof of Theorem 1 is in Appendix A.

A similar algorithm solves the cash-credit model (see Appendix B).[3] The programs also check that the conditions for existence are satisfied.[4]

IV. <u>Data and Estimation of the Vector Autoregressions</u>

This section describes the calibration of the Markov forcing processes. We first estimate a bivariate VAR using U.S. consumption growth and money growth data, and approximate this VAR by a Markov chain as in Tauchen (1987).

We estimate the VARs from quarterly (1959:I-1987:IV) and annual (1950-1986) data. Quarterly per capita money and consumption growths are constructed from data on the Citibase data tape (see Appendix C for a more complete description). Real consumption per capita is the sum of quarterly consumption in 1982 dollars of nondurables and services divided by total population. The corresponding price level is the sum of the current dollar series divided by consumption measured in 1982 dollars. Monthly observations on the money stock, measured by M2, are converted into per capita quarterly series by averaging the money supply for the quarter and dividing by total population.

While M2 may not be the monetary aggregate that most closely corresponds to the money supply of the theories, we use M2 instead of M1 for the following reason. A first order Markov process in the growth rates of money and endowments implies a stationary velocity in the model. Since M1 velocity appears to be nonstationary during the sample, the model would be rejected immediately.[5] M2 velocity, on the other hand, appears to be stationary. Thus, in this sense, M2 is the more appropriate monetary aggregate for calibrating these simple CIA models. The sensitivity analyses (reported below) also suggest robustness of the results to some misspecifications in the money growth process.

One problem with using M2 as the monetary aggregate is that its expected velocity is in general smaller than that predicted by the models. To adjust for this, Eckstein and Leiderman (1988) assume that real balances exceed the amount spent on the cash good by a constant fraction. We follow this approach by focussing on the coefficient of variation of velocity as a measure of volatility. The coefficient of variation is unaffected by scaling factors, while the standard deviation would be.

In the theory above, the exogenous processes are a first order Markov process, but our method can accommodate higher order processes. Hence, in each case, the appropriate order of the VAR of money growth and consumption growth was assessed informally by examination of the autocorrelations of the raw data and formally by examination of the Schwartz (1978) criterion and by likelihood ratio tests. A first-order VAR is adequate for both series (see Table 2). The marginal levels of significance of the test statistics indicate that the restricted models are not rejected at standard levels of significance.

The estimated quarterly VAR is the following:

(19a) $$\omega_t = \underset{(0.134)}{0.501} + \underset{(0.074)}{0.623}\,\omega_{t-1} - \underset{(0.118)}{0.116}\,\gamma_{t-1} + \varepsilon_{\omega t}$$

(19b) $$\gamma_t = \underset{(0.103)}{0.655} + \underset{(0.057)}{0.109}\,\omega_{t-1} + \underset{(0.091)}{0.238}\,\gamma_{t-1} + \varepsilon_{\gamma t}$$

The estimated standard deviations of the residuals are $\sigma_\omega = 0.00636$ and $\sigma_\gamma = 0.00489$, and their contemporaneous covariance is $\sigma_{\omega\gamma} = 0.282\text{E-}06$ implying a correlation coefficient of 0.009. The R^2 for (19a) is .375, and the R^2 for (19b) is .079. Standard errors are in parentheses. The unconditional mean values implied by the VAR are 1.0196 for ω and 1.0054 for γ.

Tauchen (1987) describes a quadrature procedure that constructs approximating Markov chains for VARs. Application of his method to the VAR in (19) with sixteen states provides a good approximation. We check this by estimating the following VAR using data generated from the Markov chain:

(20a) $$\omega_t = 0.508 + 0.613\,\omega_{t-1} - 0.113\,\gamma_{t-1} + \varepsilon_{\omega t}$$

(20b) $$\gamma_t = 0.655 + 0.109\,\omega_{t-1} + 0.237\,\gamma_{t-1} + \varepsilon_{\gamma t}$$

where $\sigma_\omega = 0.00628$, $\sigma_\gamma = 0.00489$, and $\sigma_{\omega\gamma} = 0.287\text{E-}06$. This corresponds very closely to (19a-b).

For the annual data, the consumption series is also nondurables plus services, with prices constructed as above. The money stock is also M2, and per capita series are obtained by dividing by total population.

The estimated VAR for the annual data set is the following:

$$\text{(21a)} \qquad \omega_t = \underset{(0.350)}{0.742} + \underset{(0.116)}{0.685}\, \omega_{t-1} - \underset{(0.321)}{0.400}\, \gamma_{t-1} + \varepsilon_{\omega t}$$

$$\text{(21b)} \qquad \gamma_t = \underset{(0.163)}{0.680} + \underset{(0.054)}{0.091}\, \omega_{t-1} + \underset{(0.150)}{0.239}\, \gamma_{t-1} + \varepsilon_{\gamma t}$$

The estimated residual variances are $\sigma_\omega = 0.02243$ and $\sigma_\gamma = 0.01072$; their contemporaneous covariance is $\sigma_{\omega\gamma} = -0.00011$, a correlation coefficient of -0.443; the R^2 for (21a) is .489, while for (21b) it is .084. The unconditional mean values implied by the annual VAR are 1.0599 for ω and 1.0203 for γ.

Applying Tauchen's (1987) method to this VAR requires a 16 state Markov chain. Again, we find a close fit between the actual VAR (21a-b) and the following VAR estimated using data generated from the Markov chain:

$$\text{(22a)} \qquad \omega_t = 0.756 + 0.635\, \omega_{t-1} - 0.362\, \gamma_{t-1} + \varepsilon_{\omega t}$$

$$\text{(22b)} \qquad \gamma_t = 0.680 + 0.091\, \omega_{t-1} + 0.239\, \gamma_{t-1} + \varepsilon_{\gamma t}$$

where $\sigma_\omega = 0.02220$, $\sigma_\gamma = 0.01072$, and $\sigma_{\omega\gamma} = -0.00011$.

The next section compares simulation results to sample values calculated from quarterly and annual data on velocity, real and nominal interest rates, inflation, and real balances. Velocity is calculated as the ratio of nominal consumption to nominal money balances. Ex post real interest rates are calculated by subtracting one from one plus the nominal interest rate divided by one plus the inflation rate, which is the ratio of the price level at time t+1 to the price level at time t. Real balances are the nominal money supply divided by the price level.

V. Simulation Results

Given the specification of the exogenous processes of money growth and consumption growth, our numerical algorithms find equilibria for a range of preference parameters. We focus on two related questions:

1) How do changes in the preference parameters α, β, and ψ, affect the joint distribution of the endogenous and exogenous variables?

2) To what extent does the joint distribution of endogenous and exogenous variables accord with the behavior of U.S. time series data?

For a partial answer to the first question, consider Figures 1-6. They graph six unconditional moments, generated using the Markov chain approximation to the annual VAR, of the joint distribution of money growth, consumption growth, inflation, velocity, and interest rates as functions of α, β, and four values of ψ (.3, .6, .9, 1).[6] Note that the cash-credit model nests the cash model since the latter is equivalent to the former at $\psi = 1$. For small values of α and large values of β no equilibrium exists. This is denoted by the low flat regions on the Figures.

A. Effects of Preference Parameters on Predictions of the Model

Figures 1-3 present the expected value of velocity, its standard deviation, and its coefficient of variation. In the cash model, there is virtually no variation in velocity. This can be explained as follows. Consider the agent's marginal choice in the time t asset market. He can either hold additional cash or invest in an interest bearing bond that returns $(1+i_t)$ in the next asset market. The benefit of the former is that money provides liquidity services, while the bond cannot be converted into consumption in period t+1. As long as the interest rate is sufficiently high and the variation in the marginal utility of consumption across future states is sufficiently small, agents economize on cash balances and hold just enough to cover time t+1 purchases in all states.

In the cash-credit model, there are two distinct mechanisms that can generate variation in velocity. The first is the same as in the cash model; agents may carry unspent cash across periods. The second is through a substitution effect between cash and credit goods. Once again we find that agents rarely carry any cash across periods: the CIA constraint is almost always binding. However, the substitution effect allows substantial variation in velocity. Because the CIA constraint generally binds, velocity is approximately one plus the ratio of the credit good to the cash good, and we can write:

$$(23)\qquad cv[v_t] \approx \sigma[c_{2t}/c_{1t}]/(1 + E[c_{2t}/c_{1t}])$$

where cv[•] is the coefficient of variation, and $\sigma[\cdot]$ is the standard deviation. As ψ decreases, the agent substitutes toward the credit good, which for these particular forcing processes multiplies c_{2t}/c_{1t} by a (nearly) time and state invariant constant that is larger than one. Hence, decreasing ψ increases the expectation, standard deviation, and coefficient of variation of velocity. The coefficient of variation of velocity is fairly insensitive to α and β.

Figure 4 illustrates the correlation of velocity and nominal interest rates. The two are highly correlated when $\psi < 1$.[7] When interest rates are high, real balances are expensive, and individuals economize by demanding less of the cash good and more of the credit good. This analysis also helps to explain why velocity and money growth rates are typically highly positively correlated (Figure 5). Money growth is positively serially correlated; thus, when ω_{t-1} is high, the conditional expectation of ω_t is high, which increases expected inflation and nominal interest rates. Hence, agents hold fewer real balances when money growth is high. Figure 6 shows that consumption growth and velocity are usually positively correlated.

In Figure 7, the unconditional expected real interest rate is essentially linear in α and β and is insensitive to ψ. Its most important determinant is β;

a higher subjective discount factor leads to a lower real interest rate. Because the models' predictions of unconditional expected inflation are insensitive to parameter values, the unconditional expected nominal interest rate behaves similarly to the expected real interest rate.

B. Consistency with U.S. Time Series Data

Our second question is whether the models' predictions are consistent with U.S. data. Since we have weak a priori beliefs about the preference parameters, and to allow the models the greatest chance of success, we calculate the unconditional moments of equilibrium variables over a large parameter range. Tables 3 and 4 show maximum and minimum predictions for each of 17 statistics considered individually and the corresponding sample statistics from the annual data with an approximate large sample standard error.[8] The parameters generating the maximum and minimum prediction for each quantity are in parentheses next to the estimated value.

Tables 3 and 4 illustrate the poor performance of both models. The sample value falls outside the range of the cash model (Table 3) for 14 out of 17 statistics, for β between .9 and 1.000, and α between 0 and 9.5. When allowance is made for asymptotic standard errors around the point estimates of the sample statistics, the model is still unable to produce 5 of the sample values. The model predicts virtually no variation in velocity in this range. The cash-credit model also performs poorly for the same α-β range, and ψ between .2 and .8 (Table 4). It fails to reproduce 11 of the 17 point estimates of the statistics and cannot reproduce 3 of the sample statistics after allowance for standard errors. Tables 5 and 6 present the same information for the quarterly implementation, while Tables 7 through 10 consider $\beta > 1$, both annually and quarterly. None of these changes improves the performance of the models.

It is important to note that these tests are extremely weak because we only consider one moment at a time; an accurate model should match all sample moments for a fixed set of preference parameters. Thus, although the cash-credit model generates plausible values of the coefficient of variation of velocity, it is not a successful model. In particular, matching the variation in velocity (with the restriction $\beta < 1$) requires β close to one, ψ close to zero, and α very large. For these preference parameters, the expected real interest rate is on the order of 20% per year. When the expected real interest rate is more realistic, the highest coefficient of variation of velocity that the model generates is approximately .03 (the sample average is .0456). Thus it appears impossible to reconcile the low value of average interest rates with the volatility of velocity in this framework.

Relation to Interest Elasticity of Money Demand

Above we examine the correlation of velocity and nominal interest rates, which is very similar to the estimation of traditional money demand functions. Here we consider whether a money demand function estimated from model-generated time series data resembles the results of its sample counterpart.[9]

A standard functional form used to estimate money demand relates real balances, M/P, to real consumption, y, and the nominal interest rate, i:

$$(24)\qquad \ln(M/P) = \delta_0 + \delta_1 \ln(y) + \delta_2 i + \varepsilon,$$

Our assumption that equilibria are linear in y implies that real balances are linear in y and a function of γ and ω. This is equivalent to restricting δ_1 to be equal to 1 in (24). Thus, we examine the regression:

$$(25)\qquad \ln(M/(Py)) = \delta_0 + \delta_2 i + \varepsilon, \text{ or } \ln(1/v) = \delta_0 + \delta_2 i + \varepsilon$$

The annual estimate is of δ_2 is -.995, with a standard error of .358. The corresponding model estimates are found by calculating $\text{cov}[\ln(1/v), i]/\sigma^2[i]$. When $\psi \leq .3$, δ_2 is within two standard deviations of the sample estimate for a large range of α and β, which is consistent with the generally better performance of the cash-credit model for low ψ.

VI. Sensitivity Analysis

A. Varying the Agent's Information

An important contribution of Lucas and Stokey (1987) was their generalization of the information available to the agent in the asset market. In this section, we expand the state space of the cash-credit model to include a noisy signal about next period's money supply or output. This allows us to examine the sensitivity of the model's predictions to changes in the precision of the signal.

In the above models, agents received information about the time t output and the time t+1 money stock at the beginning of period t, and the actual transfer of money occurred in the asset market at time t. To consider the effects of a signal about the monetary injection, we must assume instead that the monetary transfer occurs after the close of the time t asset market since otherwise the agent would learn from his own transfer. This timing is similar to that in Lucas and Stokey.

First, assume agents in the asset market at time t receive a signal about the growth of the money supply between the current and next period equal to the true money growth rate plus i.i.d. noise: $S_t = \omega_t + \eta_t$ with $\eta_t \sim N(0,\sigma_\eta^2)$ and uncorrelated with the time t information set. Time t information includes S_t, ω_{t-1}, and γ_t, and the evolution of the stationary component of the state can be estimated with the following VAR:

$$S_{t+1} = a_0 + a_1 S_t + a_2 \omega_{t-1} + a_3 \gamma_t + \epsilon_{St}$$

(26) $\omega_t = b_0 + b_1 S_t + b_2 \omega_{t-1} + b_3 \gamma_t + \varepsilon_{\omega t}$

$\gamma_{t+1} = c_0 + c_1 S_t + c_2 \omega_{t-1} + c_3 \gamma_t + \varepsilon_{\gamma t}.$

We want to estimate this VAR and discretize the distribution as before, varying σ_η to reflect different degrees of uncertainty about next period's money supply. It is straightforward to calculate the resulting coefficients of the VAR and the resulting covariance matrix as a function of σ_η and the matrix of independent variables (see Appendix D).

To test the sensitivity of the predictions, we set σ_η equal to 0.01, 0.5, 1, and 2 times the unconditional standard deviation of money growth in the data (σ_ω). The equilibrium conditions are unchanged except that ω_t in (14) must be inside the expectation operator. The results in Table 11 indicate that the unconditional expectations of velocity, inflation, and the real interest rate are relatively insensitive to the agent's information about future money growth, as is the coefficient of variation of velocity.

Similarly, we allow agents at time t to observe a signal $S_t = \gamma_{t+1} + \eta_t$ of next period's output growth, where η_t is distributed as above. In this case, σ_η is set to 0.01, 0.5, 1, and 2 times the unconditional standard deviation of output growth. Again, the precision of the signal has little affect on the predictions of the model, as reported in Table 12.

B. CES and Habit Formation Preferences

It is possible that the cash-credit model generates a larger range of predictions for an alternative utility specification. The functional form in (10) is restrictive because it implies a unitary intratemporal elasticity of substitution between the cash and credit goods for all ψ. If agents regard cash and credit goods as closer substitutes than is implied by this utility function, velocity may vary more as they substitute more freely between the two goods. To assess this conjecture, we consider a CES alternative:

$$(27) \qquad u(c_{1t}, c_{2t}) = [(c_{1t}^{\eta} + c_{2t}^{\eta})^{(1-\alpha)/\eta} - 1]/(1-\alpha)$$

in which the intratemporal elasticity of substitution is $1/(1-\eta)$. We examine the model's predictions for $\eta < 1$ in Table 13. A comparison with Tables 4 indicates no great improvement in the predictions of the model.

If utility this period depends on the increase in consumption over that in the previous period, preferences in the cash good model can be written as

$$(28) \qquad u(c_t) = [(c_t - b(\min[\gamma])c_{t-1})^{1-\alpha} - 1]/(1-\alpha)$$

where b can be varied between zero and one to reflect the degree of habit formation.[10] In (28) we scale b by the minimum growth rate of consumption to ensure that preferences are well-defined. The effect of these preferences on the range of predicted values is dramatic. Table 14 reports ranges for α and β as in Table 3, and $b \in \{0, .2, .4, .6, .8\}$ such that the marginal utility of consumption is positive. The model can generate predictions that lie within one standard deviation of all 17 statistics; eleven of the sample statistics lie within the range of possible moments generated by the model. Intuitively, these preferences induce extremely high risk aversion as b approaches 1 even when α is small because the point of infinite marginal utility in the current period utility function is based on last period's consumption. Such preferences generate a large precautionary demand for money in some states, which reduces the velocity of money and increases its volatility. This interpretation is consistent with the fact that using the standard preferences, the cash good model can generate large fluctuations in velocity and inflation (e.g., for values of $\alpha > 400$ and $\beta > 1.15$).

Again, though, the model is not successful when we require that it simultaneously fit the data on several dimensions. Parameters that generate realistic variability of velocity also generate unrealistically high variability of inflation and real interest rates. Any parameter specification that generates a coefficient of variation of velocity larger than 0.04 (the sample estimate is 0.0456) also produces a standard deviation of inflation larger than 0.05 and a standard deviation of real interest rates larger than 0.06. The sample estimates of the latter quantities are 0.03 (standard deviation .008) and 0.02 (standard deviation .005).

C. Sensitivity to Forcing Process and Sample Period

We test the robustness of the models' predictions using the quarterly data by (a) varying the parameters of the forcing processes, and (b) truncating the data at 1979:II. Neither variation substantially changes the ranges reported in Tables 3 through 6. In particular, the basic inability of the cash model to generate the observed variability of velocity remains.

In order to consider the robustness of the results to potential misspecification of the driving process, we consider 18 variations of the forcing processes (listed in Table 15), always adjusting the constants to preserve the unconditional means of the forcing processes. The effect of changing constants is examined in three other experiments. Mean preserving changes have little effect on the expected values of inflation, real balance growth, and interest rates. Although there are some changes in the second moments of the variables, the effects are small in terms of changing the ranges in Tables 3 and 4. We also look at the effects of changing the intercepts of the VARs. Importantly, while the second moments are not greatly affected, the predictions of the model for the expectations of inflation and real balance growth are quite sensitive to these parameters.

A strong assumption underlying our results is that quarterly consumption and money growths follow a stationary process from 1959 to 1987. Since many researchers argue that there was a fundamental change in the monetary regime in October 1979, we reconstruct Tables 3-6 using data from the period 1959:1-1979:2. We find that the model performs worse over this period in the sense that for all four tables, a larger number of sample estimates fall outside the ranges consistent with the model.

VII. Conclusions

Lucas (1984) and Svensson (1985) show that adding uncertainty about future cash needs can in principle allow velocity to vary in CIA models. Whether this change in the information structure has practical implications for the models' predictions is an empirical question. Using driving processes estimated from U.S. time series, we simulate the cash model over a wide range of preference parameters to generate predictions on the unconditional moments of several economic time series, including velocity. A striking result is that the model predicts essentially constant velocity, using the annual data process for any subjective discount factor between 0.9 and 1.06 and any coefficient of relative risk aversion between 0 and 9.5. Velocity is always constant with the quarterly driving process. Furthermore, the result that the CIA constraint always binds is robust to substitutability between cash and credit goods, and changes in the assumed information structure. This strongly suggests that in future applications of CIA models, assuming an information structure for which the CIA constraint always binds will simplify the analysis without significantly changing the results.

Including credit goods, as in Lucas and Stokey (1987), generates variability in velocity in CIA models without complicated information structures. Our simulations examine the implications of adding credit goods to the model. Although the variability of velocity increases, average velocity also increases well above empirically plausible levels. The model is also unable to generate realistic predictions about the first and second moments of other variables, even if considered individually. We emphasize, though, that we have not tested CIA models as a class against any of the popular alternatives, which might perform just as poorly when imbedded in similarly stylized models.

Our estimation/calibration procedure is fairly easy to implement, and quickly reveals the properties of these models under a variety of scenarios. Although this procedure provides no formal test statistic with which to reject a model, it is a relatively inexpensive way to assess the probable value of a theoretical model in explaining data[11]. Pretesting of this sort is valuable for understanding new theories that purport to explain the data.

If our calibrations had produced more promising results, we had planned to estimate the models with a technique such as Hansen's (1982) Generalized Method of Moments (GMM), which allows overidentified models to be tested formally. A true model would predict unconditional moments that are the same as estimated unconditional moments after allowance for sampling error. Therefore, orthogonality conditions could be constructed by taking deviations of sample moments from model moments. A serious methodological problem in estimating these models with GMM is that the models' moments depend on the estimated parameters of the VAR and on the discretization of the state space. The first problem might be overcome by simultaneous estimation of the VAR and the parameters of the model. It is not known how the discrete state approximation affects the GMM parameter estimates and standard errors. One can avoid these problems by examining Euler equation restrictions, as Finn, Hoffman and

Schlagenhauf (1988) and Eckstein and Leiderman (1988) do.[12]

Although we emphasize macroeconomic predictions, these models also generate asset pricing predictions. Labadie (1988) and Backus, Gregory and Zin (1988) investigate the empirical predictions of the cash model for asset prices, assuming that the CIA constraint always binds. Our findings provide support for their results. The asset pricing implications of the cash-credit model have not been explored; this is a topic of current research.

Appendix A: Proof of Theorem 1

Lemma 1: A sequence of $m(\gamma,\omega)$'s generated by the procedure described above, $\{m_t(\omega,\gamma)\}$, $t = 0,1,2,\ldots$, is non-decreasing.

Proof: On any iteration, if $\mu_{t-1}(\gamma,\omega) \geq 0$, $m_t(\gamma,\omega) = m_{t-1}(\gamma,\omega)$ in Step 3. If $\mu_{t-1}(\gamma,\omega) < 0$, then $m(\gamma,\omega)$ must be increased in the denominator of the RHS of (18) until $\mu(\gamma,\omega) = 0$.//

Lemma 2: The equilibrium $m(\gamma,\omega)$ is bounded for all γ and ω.

Proof: By examination of (18), if any $m(\gamma,\omega)$ is unbounded, $m(\gamma,\omega)$ is unbounded for all γ and ω. Say that all $m(\gamma,\omega)$ are unbounded. Since γ is bounded, the CIA constraint is slack in all states, and $\mu(\gamma,\omega) = 0$ for all γ and ω. This implies a zero nominal interest rate in all states, contradicting the definition of an equilibrium.//

Lemma 3: If in the sequence of μ's generated by the algorithm, $\{\mu_t(\omega,\gamma)\}$, $t = 0,1,2,\ldots$, it ever occurs that $\mu_t(\gamma,\omega) < 0$ for all γ and ω, the μ's will always be negative, and the algorithm does not converge.

Proof: Say that $\mu_t(\gamma,\omega) < 0$ for all γ and ω. Then by Lemma 1, $m_{t+1}(\gamma,\omega) > m_t(\gamma,\omega)$ for all γ and ω. Therefore on the next iteration, the numerator of the expectation in (18) increases. Using $m_{t+1}(\gamma,\omega)$ in the denominator of the expectation and solving for $\mu_{t+1}(\gamma,\omega)$ in (18) implies that $\mu_{t+1}(\gamma,\omega)$ is again less than zero, since μ was equal to zero when the numerator of the expectation was lower and the denominator the same. Thus $m_{t+2}(\gamma,\omega) > m_{t+1}(\gamma,\omega)$, and the stopping condition never is satisfied.//

Proof of Theorem 1: (by contradiction) Let $\{m^*(\gamma,\omega), \mu^*(\gamma,\omega)\}$ be the equilibrium defined by (17) and (18). By Lemma 1 the m's are a non-decreasing sequence, so that if the algorithm fails to converge to the equilibrium, the sequence must jump over $m^*(\gamma,\omega)$. Assume that $m_{t+1}(\gamma_j,\omega_j) > m^*(\gamma_j,\omega_j)$ for a subset of states $J = \{(\gamma_j,\omega_j)\}$, but that $m_t(\gamma,\omega) \le m^*(\gamma,\omega)$ for all γ and ω. For all (γ,ω) such that $\mu_t(\gamma,\omega) \ge 0$, $m_{t+1}(\gamma,\omega) = m_t(\gamma,\omega)$, so the equilibrium is not jumped in these states. Let $(\gamma_j,\omega_j) \in J$. Then $\mu_t(\gamma_j,\omega_j) < 0$, and $m_{t+1}(\gamma_j,\omega_j) > m^*(\gamma_j,\omega_j) > m_t(\gamma_j,\omega_j)$. Solving for μ in (18) with $m^*(\gamma_j,\omega_j)$ in place of $m_{t+1}(\gamma_j,\omega_j)$ but all else the same implies $\mu < 0$. Consider the effect of increasing $m(\gamma,\omega)$ to $m^*(\gamma,\omega)$ for all γ and ω in the numerator of the expectation in (18), and solve for μ again with $m^*(\gamma_j,\omega_j)$ in the denominator. Algebra establishes that the resulting μ is < 0. This contradicts $m^*(\gamma,\omega)$ being an equilibrium.

Appendix B: Solution Algorithm for Cash-Credit Model

First, the system (11)-(14) reduces to the three equation system:

(B1) $c_1(\gamma_t,\omega_t) \le m(\gamma_t,\omega_t)$

$\{\mu(\gamma_t,\omega_t) \ge 0, \text{ and } \mu(\gamma_t,\omega_t)[c_1(\gamma_t,\omega_t) - m(\gamma_t,\omega_t)] = 0\}$

(B2) $u_1(\gamma_t,\omega_t) = u_2(\gamma_t,\omega_t) + \mu(\gamma_t,\omega_t)$

(B3) $$\mu(\gamma_t,\omega_t) = u_1(\gamma_t,\omega_t) - \frac{\beta E_t[u_1(\gamma_{t+1},\omega_{t+1})m(\gamma_{t+1},\omega_{t+1}) \mid \gamma_t,\omega_t]\gamma_t^{1-\alpha}}{\omega_t m(\gamma_t,\omega_t)}$$

The algorithm solves for $c_1(\gamma,\omega)$, $m(\gamma,\omega)$ and $\mu(\gamma,\omega)$. We use the notation that c_{10} refers to a value of c_1 on the an initial iteration, and c_{11} refers to c_1 on the subsequent iteration. m_0 and m_1 are defined similarly.

Step 1: Set $m_0(\gamma,\omega) = c_{10}(\gamma,\omega) = \psi$ for all (γ,ω).

Step 2: Use (B3) to solve for $\mu_0(\gamma,\omega)$. If $\mu_0(\gamma,\omega) \ge 0$ for all (γ,ω), this is an equilibrium. If not, go to Step 3.

Step 3: If $\mu_0(\gamma,\omega) < 0$ in all states, stop since the algorithm will never converge. For any state in which $\mu_0(\gamma,\omega) \le 0$, use (B2) to define $c_{11}(\omega,\gamma)$ with $\mu = 0$, and solve for $ml(\gamma,\omega)$ in (B3) with $\mu = 0$. If $c_{11}(\omega,\gamma)$ is more than $ml(\gamma,\omega)$, reduce $c_{11}(\omega,\gamma)$ to $m_1(\gamma,\omega)$. On all future iterations, this procedure will determine c_1 and m in this state. For any state in which $\mu_0(\gamma,\omega) > 0$, substitute for μ in (B3) using (B2), and solve (B3) for $m_1(\gamma,\omega) = c_{11}(\gamma,\omega)$, using the vector of $m_0(\gamma,\omega)$ and $c_{10}(\gamma,\omega)$ from the previous iteration in the expectation on the right-hand side of (B3).

Step 4: Use (B3) to solve for $\mu_1(\gamma,\omega)$ using $m_1(\gamma,\omega)$ and $c_{11}(\gamma,\omega)$ on the right-hand-side. If $\mu_1(\gamma,\omega) \geq 0$ in all states, and $m_1(\gamma,\omega) = m_0(\gamma,\omega)$ and $c_{11}(\gamma,\omega) = c_{10}(\gamma,\omega)$ this is an equilibrium. If not, set $c_{10}(\gamma,\omega) = c_{11}(\gamma,\omega)$ and $m_0(\gamma,\omega) = m_1(\gamma,\omega)$ and repeat Step 3.

Appendix C: Data Sources

Quarterly data are from the Citibase Data tape of Northwestern University. The series and their corresponding Citibase acronyms are listed below. National Income and Product Accounts is denoted NIPA.

Money stock - FM2: The sum of currency, travelers checks, demand deposits, and other checkable deposits (M1) plus overnight repurchase agreements and overnight Eurodollars, money market mutual fund balances, money market deposit accounts, and savings and small time deposits.

Population - POP: 1st of the month estimate of population, including armed forces overseas.

Consumption of nondurables in 1982 (current) dollars - GCN82 (GCN): NIPA.

Consumption of services in 1982 (current) dollars - GCS82 (GCS): NIPA.

Nominal interest rates - FGYM3: Three month treasury bill yield in the secondary market, monthly average of daily rates.

Annual data is from the 1987 Economic Report of the President unless otherwise indicated.

Consumption of nondurables and services in 1982 (current) dollars: Table B-2, (B-1). 1987 observations from the 1988 Report.

Population: Table B-31, 1986-1987 observations from the 1988 Report.

Money stock: M2, 1948-83 from Balke and Gordon (1986), 1984-87 from 1988 Economic Report of the President.

Nominal Interest Rate: Commercial paper interest rates for 4-6 month maturity prior to 1979, 6 month thereafter. The rates are quoted on a bank discount basis. 1950-1987 from 1988 Economic Report of the President, Table B-71.

Appendix D: Derivation of Modified VARs for Information Experiments

To represent varying degrees of precision in the agent's knowledge of ω_t at time t, suppose there exist (T+2) observations of output growth and money growth. Define X to be a (T x 4) matrix such that its four columns are: Col. 1, Vector of ones; Col. 2, observations of ω beginning with the second period; Col. 3, observations of ω beginning with the first period; Col. 4, observations of γ beginning with the second period. Let Y be a vector containing T observations of ω beginning with the third period. Let $\sum$ be a (4 X 4) matrix with a single nonzero element, $\sum^{22} = \sigma_\eta^2$. Then a consistent estimator of the four elements of the vector c is:

$$c = (X'X + T\sum)^{-1}X'Y.$$

See Chow (pp. 105-6, 1983). We can similarly estimate the vectors a and b.

Decomposing S_t in (26), the error term can be written as the vector $(a_1\eta_t+\epsilon_{St}, b_1\eta_t+\epsilon_{\omega t}, c_1\eta_t+\epsilon_{\gamma t})$. Since η_t is uncorrelated with the time t information set, the covariance matrix is:

$$\begin{bmatrix} a_1^2\sigma_\eta^2+\sigma_S^2 & a_1b_1\sigma_\eta^2+\sigma_{S\omega} & a_1c_1\sigma_\eta^2+\sigma_{S\gamma} \\ a_1b_1\sigma_\eta^2+\sigma_{S\omega} & b_1^2\sigma_\eta^2+\sigma_\omega^2 & b_1c_1\sigma_\eta^2+\sigma_{\omega\gamma} \\ a_1c_1\sigma_\eta^2+\sigma_{S\gamma} & b_1c_1\sigma_\eta^2+\sigma_{\omega\gamma} & c_1^2\sigma_\eta^2+\sigma_\gamma^2 \end{bmatrix}$$

References

Backus, David K., Allan W. Gregory, and Stanley E. Zin, 1988, "Risk Premiums in the Term Structure: Evidence From Artificial Economies," manuscript, Federal Reserve Bank of Minneapolis.

Balke, Nathan S. and Robert J. Gordon, 1986, "Appendix B Historical Data," in: Robert J. Gordon, ed., The American Business Cycle: Continuity and Change. Chicago: University of Chicago Press, 781-850.

Chow, Gregory C., 1983, Econometrics. New York: McGraw Hill.

Constantinides, George, 1988, "Habit Formation: A Resolution of the Equity Premium Puzzle," manuscript, The University of Chicago.

Eckstein, Zvi, and Leonardo Leiderman, 1988, "Estimating Intertemporal Models of Consumption and Money Holdings and Their Implications for Seigniorage and Inflation," manuscript, Tel-Aviv University.

Finn, Mary G., Dennis L. Hoffman and Don E. Schlagenhauf, 1988, "Intertemporal Asset-Pricing Relationships in Barter and Monetary Economies: An Empirical Analysis," manuscript, Arizona State University.

Giovannini, Alberto, 1987, "Uncertainty and Liquidity," manuscript, Columbia University.

Hansen, Lars Peter, 1982, "Large Sample Properties of Generalized Method of Moments Estimators," Econometrica 50, 1029-1054.

Hansen, Lars Peter and Ravi Jagannathan, 1988, "Restrictions on Intertemporal Marginal Rates of Substitution Implied by Asset Returns," Northwestern University, Department of Finance Working Paper No. 12.

Hartley, Peter R., 1988, "The Liquidity Services of Money," International Economic Review 29, 1-24.

Hodrick, Robert J., 1989, "Risk, Uncertainty and Exchange Rates." N.B.E.R. Working Paper No. 2429, forthcoming Journal of Monetary Economics.

Judge, George G., W. E. Griffiths, R. Carter Hill, Helmut Lutkepohl, Tsoung-Chao Lee, 1985, The Theory and Practice of Econometrics. 2nd Edition. New York: John Wiley and Sons.

Kocherlakota, Narayana, 1988, "A Note on the 'Discount' Factor in Growth Economies," Northwestern University, Department of Finance Working Paper No. 62.

Labadie, Pamela A., 1988, "The Effects of Stochastic Inflation on Asset Prices," manuscript, Columbia University.

Lucas, Robert E. Jr., 1984, "Money in a Theory of Finance," in: Karl Brunner and Allan H. Meltzer, eds., Essays on Macroeconomic Implications of Financial and Labor Markets and Political Processes, Carnegie-Rochester Conference Series on Public Policy, Volume 21, 9-45.

Lucas, Robert E. Jr., 1987, "Money Demand in the United States: A Quantitative Review," manuscript, The University of Chicago.

Lucas, Robert E. Jr., and Nancy L. Stokey, 1987, "Money and Interest in a Cash-in-Advance Economy," Econometrica 55, 491-514.

Marshall, David A., 1988, "Inflation and Asset Returns in a Monetary Economy, Empirical Results," manuscript, Northwestern University Department of Finance.

Mehra, Rajnish and Edward C. Prescott, 1985, "The Equity Premium: A Puzzle," Journal of Monetary Economics 15, 145-62.

Ryder, H. E., Jr., and G. M. Heal, 1973, "Optimal Growth with Intertemporally Dependent Preferences," Review of Economic Studies 40, 1-31.

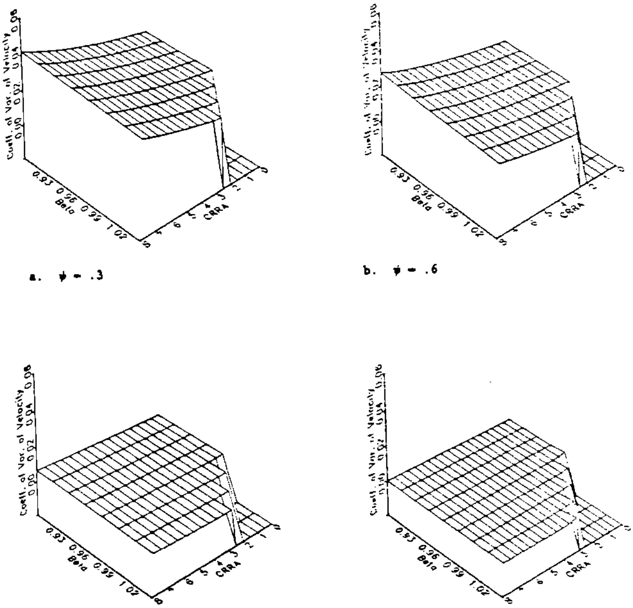
Schwartz, G., 1978, "Estimating the Dimension of a Model," The Annals of Statistics 6, 461-464.

Sims, Christopher A., 1980, "Macroeconomics and Reality," Econometrica 48, 1-49.

Svensson, Lars E. O., 1985, "Money and Asset Prices in a Cash-in-Advance Economy," Journal of Political Economy 93, 919-944.

Tauchen, George, 1987, "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," manuscript, Duke University.

Figure 1: Coefficient of Variation of Velocity (Annual Data



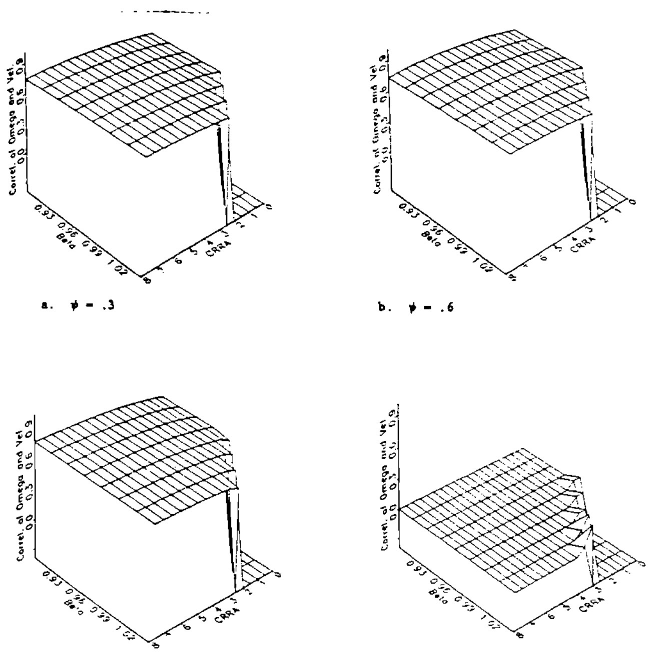
a. ψ = .3

b. ψ = .6

c. ψ = .9

d. ψ = 1. (Svensson)

Figure 2: Correlation of Velocity and Money Growth (Annual Data)



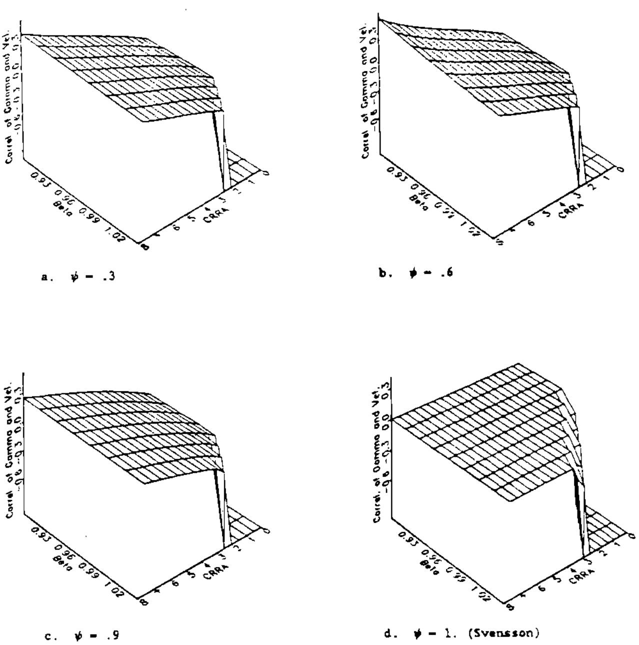
a. ψ = .3

b. ψ = .6

c. ψ = .9

d. ψ = 1 (Svensson

Figure 3. Correlation of Consumption Growth and Velocity (Annual Data)



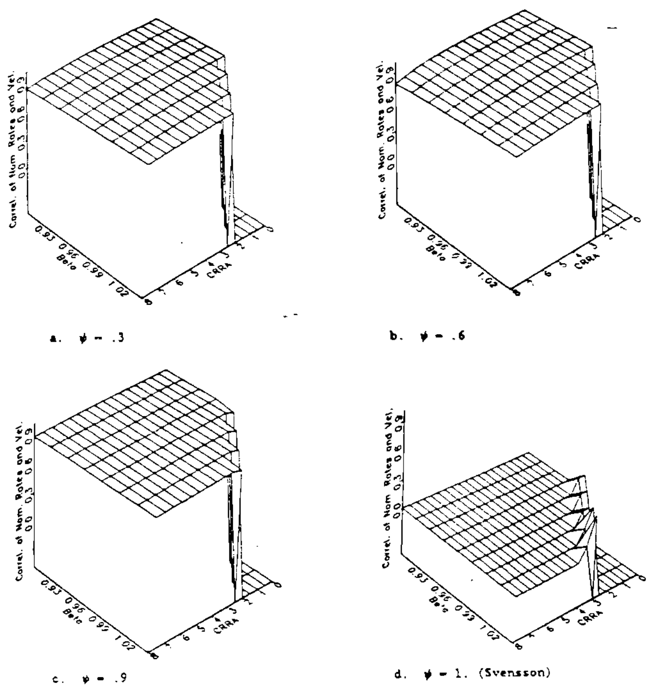
a. ψ = .3

b. ψ = .6

c. ψ = .9

d. ψ = 1. (Svensson)

Figure 4: Correlation of Nominal Interest Rates and Velocity (Annual Data)



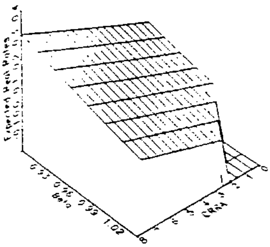
a. ψ = .3

b. ψ = .6

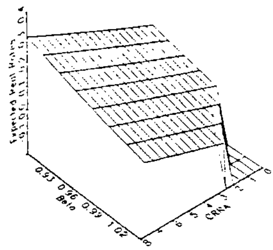
c. ψ = .9

d. ψ = 1. (Svensson)

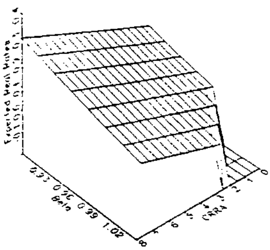
Figure 5: Expected Real Interest Rates (Annual Data)



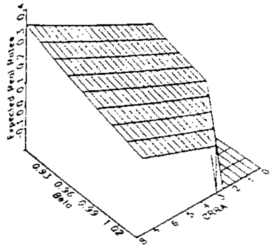
a. ψ = .3



b. ψ = .6

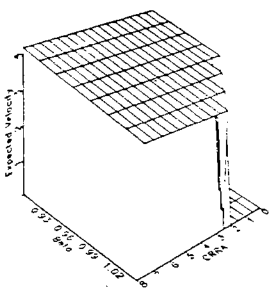


c. ψ = .9

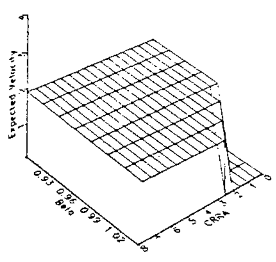


d. ψ = 1. (Svensson)

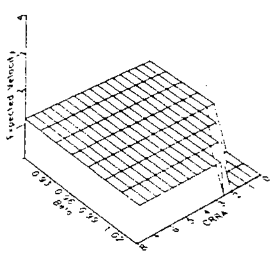
Figure 6: Expected Velocity (Annual Data)



a. ψ = .3



b. ψ = .6



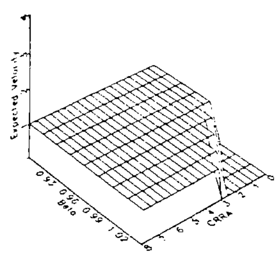
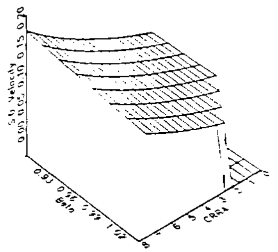
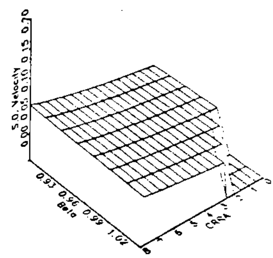


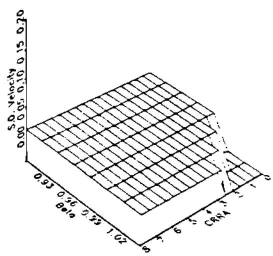
Figure 7: Standard Deviation of Velocity



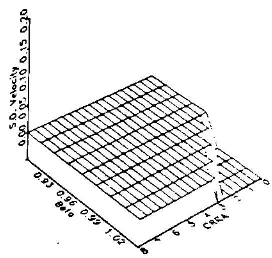
a. ψ = .3



b. ψ = .6



c. ψ = .9



d. ψ = 1. (Svensson)

Table 1: Expressions for Endogenous Variables

Consumption Velocity	$v = \dfrac{\gamma}{m(\gamma,\omega)}$
Inflation Rate (plus one)	$\pi(\gamma,\omega\|\gamma',\omega') = \dfrac{m(\gamma,\omega)\omega}{m(\gamma',\omega')\gamma}$
Nominal Interest Rate	$i(\gamma,\omega) = \dfrac{E[\mu(\gamma',\omega')(1/\pi(\gamma,\omega\|\gamma',\omega'))\|(\gamma,\omega)]}{E[\lambda(\gamma',\omega')(1/\pi(\gamma,\omega\|\gamma',\omega'))\|(\gamma,\omega)]}$
Realized Real Interest Rate	$r(\gamma,\omega\|\gamma',\omega') = \dfrac{1+i(\gamma,\omega)}{\pi(\gamma,\omega\|\gamma',\omega')} - 1$
Growth Rate of Real Balances	$m_g(\gamma,\omega\|\gamma',\omega') = \dfrac{m(\gamma',\omega')\gamma}{m(\gamma,\omega)} - 1$

Note: (γ',ω') denotes the state of the Markov chain in the next period. $(\gamma,\omega|\gamma',\omega')$ denotes a transition from the state (γ,ω) to the state (γ',ω') in the next period.

Table 2: Diagnostics for Choice of Lag Length in the Vector Autoregressions

Sample	Quarterly	Annual
SC(1)	- 20.401	- 16.171
SC(2)	- 20.038	- 15.561
SC(3)	- 19.740	- 15.038
LR 1 vs. 2	1.576	1.495
MLS	.813	.827
LR 2 vs. 3	9.575	4.063
MLS	.048	.398

Note: The value of the Schwartz (1978) criterion for lag length j is SC(j). The statistic is calculated as equation (16.6.7) of Judge, et.al. (1985, p. 687). The likelihood ratio test of lag length j versus length j+1 is denoted LR j vs. j+1. The marginal level of significance of the likelihood ratio test is denoted MLS. The likelihood ratio statistics incorporate the degrees of freedom correction recommended by Sims (1980).

Table 3

Cash Model Simulation Results vs. Sample Values
Annual Data: 1950 to 1987

β = .9,.92, ...,.98,1.0; α = 0,.5, ...,9.5

	min	min (α,β)	max	max (α,β)	sample value	std. dev. sample value
E[v]	0.9999	(1.,1.0)	1.0000	(9.5,1.0)	0.8826	0.0105 **
σ[v]	0.0000	(9.5,1.0)	0.0009	(1.0,1.0)	0.0405	0.0086 **
cv[v]	0.0000	(9.5,1.0)	0.0009	(1.0,1.0)	0.0456	0.0097 **
corr[v,γ]	-0.1585	(1.5,1.0)	0.0000	(9.5,1.0)	-0.5000	0.1447 **
corr[v,ω]	0.0000	(9.5,1.0)	0.0711	(0.,.98)	-0.0668	0.2263 *
corr[v,i]	0.0000	(2.5,1.0)	0.1555	(0.,.98)	0.5348	0.2245 *
E[π]	0.0389	all	0.0389	all	0.0434	0.0079 *
σ[π]	0.0297	all	0.0297	all	0.0283	0.0061 *
E[i]	0.0594	(0.,.98)	0.3901	(9.5,.90)	0.0587	0.0094 *
σ[i]	0.0182	(0.,.98)	0.0537	(9.5,.90)	0.0323	0.0076
E[ρ]	0.0201	(1.,1.0)	0.3377	(9.5,.90)	0.0148	0.0053 *
σ[ρ]	0.0116	(4.,1.0)	0.0218	(9.5,.90)	0.0200	0.0046
E[m_g]	0.0203	all	0.0203	all	0.0164	0.0063 *
σ[m_g]	0.0450	(9.5,1.0)	0.0474	(1.0,1.0)	0.0336	0.0061 *
corr[π,ω]	0.9227	(1.0,1.0)	0.9254	(9.5,1.0)	0.3421	0.1191 **
corr[π,i]	0.9165	(0.,.98)	0.9274	(5.0,.98)	0.7689	0.0805 *
corr[π,ρ]	-0.8812	(0.,.90)	0.4445	(9.5,.98)	-0.1808	0.1904

Note: A * indicates that the sample value falls outside the possible range predicted by model. A ** indicates that the sample value falls outside the possible range by more than two standard deviations. π = inflation; i = nominal interest rate; ρ = real interest rate; ω = money growth rate; γ = consumption growth rate; m_g = real balances growth rate. Correlation of any variable and money growth is contemporaneous (e.g.corr[v,ω] = corr[v_t,ω_{t-1}]). No equilibrium exists for β = 1, $\alpha \leq$ .5.

Table 4

Cash-Credit Model Simulation Results vs. Sample Values
Annual Data: 1950 to 1987

β = .9,.92,...,.98,1.; α = 0,.5,...,9.5; ψ = .2,.4,.6,.8

	min	min (α,β,ψ)	max	max (α,β,ψ)	sample value	std. dev. sample value	
E[v]	1.2649	(1.,1.0,.8)	6.5738	(9.5,.9,.2)	0.8826	0.0105	**
σ[v]	0.0071	(0.,.96,.8)	0.3351	(9.5,.9,.2)	0.0405	0.0086	
cv[v]	0.0056	(0.,.96,.8)	0.0510	(9.5,.9,.2)	0.0456	0.0097	
corr[v,γ]	-0.3319	(0,.98,.8)	0.5157	(9.,.92,.4)	-0.5000	0.1447	*
corr[v,ω]	0.6190	(0.,.98,.8)	0.7520	(7.5,.92,.8)	-0.0668	0.2263	**
corr[v,i]	0.7342	(9.5,.96,.4)	0.9879	(0.,.9,.6)	0.5348	0.2245	*
E[π]	0.0389	(1.,.9,.8)	0.0398	(9.5,.9,.2)	0.0434	0.0079	*
σ[π]	0.0290	(1.,.9,.8)	0.0529	(9.5,.9,.2)	0.0283	0.0061	*
E[i]	0.0594	(0.,.98,.6)	0.3902	(9.5,.9,.8)	0.0587	0.0094	*
σ[i]	0.0181	(0.,.98,.6)	0.0638	(9.5,.9,.8)	0.0323	0.0076	
E[ρ]	0.0201	(1.,1.0,.8)	0.3390	(9.5,.9,.2)	0.0148	0.0053	*
σ[ρ]	0.0138	(3.,1.0,.8)	0.0675	(9.5,.9,.2)	0.0200	0.0046	
E[m_g]	0.0203	(2.,1.0,.8)	0.0213	(9.5,.9,.2)	0.0157	0.0064	*
σ[m_g]	0.0648	(0.,.96,.8)	0.3232	(9.5,.9,.2)	0.0334	0.0060	**
corr[π,ω]	0.4665	(9.5,.9,.2)	0.8746	(2.,1.0,.8)	0.3421	0.1191	*
corr[π,i]	0.4518	(9.5,.9,.2)	0.8867	(3.5,1.0,.8)	0.7689	0.0805	
corr[π,ρ]	-0.8393	(0.,.9,.8)	0.3004	(9.5,1.0,.8)	-0.1808	0.1904	

Note: See also Table 3. The algorithm failed to converge for 13 out of the 400 possible parameter specifications.

Table 5

Cash Model Simulation Results vs. Sample Values
Quarterly Data, 1959:2 - 1988:1

β = .975,.98,...,.995,1.000; α = 0,.5,1,...,9.5

	min	min (α,β)	max	max (α,β)	sample value	std. dev. sample value
$E[v]$	1.0000	all	1.0000	all	0.2261	0.0018 **
$\sigma[v]$	0.0000	all	0.0000	all	0.0090	0.0014 **
$cv[v]$	0.0000	all	0.0000	all	0.0398	0.0061 **
$corr[v,\gamma]$	0.0000	all	0.0000	all	-0.3420	0.1112 **
$corr[v,\omega]$	0.0000	all	0.0000	all	-0.1634	0.1243 *
$corr[v,i]$	0.0000	all	0.0000	all	0.6208	0.1526 *
$E[\pi]$	0.0141	all	0.0141	all	0.0122	0.0014 *
$\sigma[\pi]$	0.0085	all	0.0085	all	0.0074	0.0012 *
$E[i]$	0.0195	(1.0,1.00)	0.0933	(9.5,.975)	0.0151	0.0014 **
$\sigma[i]$	0.0041	(0.0,.990)	0.0122	(9.5,.975)	0.0069	0.0013
$E[\rho]$	0.0054	(1.0,1.00)	0.0781	(9.5,.975)	0.0030	0.0011 **
$\sigma[\rho]$	0.0051	(3.0,1.00)	0.0075	(9.5,.975)	0.0062	0.0010
$E[m_g]$	0.0054	all	0.0054	all	0.0052	0.0016 *
$\sigma[m_g]$	0.0205	all	0.0205	all	0.0098	0.0014 **
$corr[\pi,\omega]$	0.8042	all	0.8042	all	0.1844	0.1012 **
$corr[\pi,i]$	0.7811	(9.5,.995)	0.8167	(0.5,.995)	0.6192	0.0942 *
$corr[\pi,\rho]$	-0.9113	(0.0,.985)	0.4570	(9.5,.990)	-0.5047	0.1392

Note: See also Table 3. Equilibrium does not exist for (α,β) = (0,.995), (0,1), (.5,1).

Table 6

Cash-Credit Model Simulation Results vs. Sample Values
Quarterly Data, 1959:2 - 1988:1

β = .975,.98,...,.995,1.; α = 0,.5,...,9.5; ψ = .2,.4,.6,.8

	min	min (α,β,ψ)	max	max (α,β,ψ)	sample value	std. dev. sample value	
E[v]	1.2549	(1.,1.00,.8)	5.3763	(9.5,.975,.2)	0.2261	0.0018	**
σ[v]	0.0018	(0.,.99,.8)	0.0890	(9.5,.975,.2)	0.0090	0.0014	
cv[v]	0.0014	(0.,.99,.8)	0.0166	(9.5,.975,.2)	0.0398	0.0061	**
corr[v,γ]	-0.0692	(0.,.975,.6)	0.6639	(9.5,.975,.6)	-0.3420	0.1112	**
corr[v,ω]	0.5667	(9.5,.975,.6)	0.6189	(2.5,.975,.2)	-0.1634	0.1243	**
corr[v,i]	0.8894	(9.5,.975,.6)	0.9997	(0.,.975,.6)	0.6208	0.1526	*
E[π]	0.0141	all	0.0141	all	0.0122	0.0014	*
σ[π]	0.0076	(8.5,.985,.8)	0.0147	(9.5,.975,.2)	0.0074	0.0012	*
E[i]	0.0195	(1.,1.00,.8)	0.0933	(9.5,.975,.6)	0.0151	0.0014	**
σ[i]	0.0041	(0.,.99,.6)	0.0132	(9.5,.975,.6)	0.0069	0.0013	
E[ρ]	0.0054	(1.,1.00,.8)	0.0783	(9.5,.975,.2)	0.0030	0.0011	**
σ[ρ]	0.0050	(3.,1.00,.8)	0.0192	(9.5,.975,.2)	0.0062	0.0010	
E[m_g]	0.0054	(6.,1.00,.8)	0.0055	(9.5,.975,.2)	0.0052	0.0016	*
σ[m_g]	0.0224	(2.5,1.00,.8)	0.1099	(9.5,.975,.2)	0.0098	0.0014	**
corr[π,ω]	0.1791	(9.5,.975,.2)	0.7784	(5.5,1.00,.8)	0.1844	0.1012	
corr[π,i]	0.0913	(9.5,.975,.2)	0.7808	(2.,1.00,.8)	0.6192	0.0942	
corr[π,ρ]	-0.8930	(0.,.975,.8)	0.0695	(9.5,1.00,.8)	-0.5047	0.1392	

Note: See also Table 3. Equilibrium does not exist for (α,β) = (0,.995), (0,1), (.5,1).

Table 7

Cash Model Simulation Results vs. Sample Values
Annual Data: 1950 to 1987

β = 1.00,1.02, ...,1.06; α = 0,.5, ...,9.5

	min	min (α,β)	max	max (α,β)	sample value	std. dev. sample value
$E[v]$	0.9996	(4.0,1.06)	1.0000	(9.5,1.06)	0.8883	0.0104 **
$\sigma[v]$	0.0000	(9.5,1.06)	0.0016	(4.0,1.06)	0.0405	0.0086 **
$cv[v]$	0.0000	(9.5,1.06)	0.0016	(4.0,1.06)	0.0456	0.0097 **
$corr[v,\gamma]$	-0.2390	(4.0,1.06)	0.0000	(9.5,1.06)	-0.5000	0.1447 *
$corr[v,\omega]$	0.0000	(9.5,1.06)	0.2651	(4.0,1.06)	-0.0668	0.2263 *
$corr[v,i]$	0.0000	(7.0,1.06)	0.4283	(4.0,1.06)	0.5348	0.2245 *
$E[\pi]$	0.0389	all	0.0389	all	0.0434	0.0079 *
$\sigma[\pi]$	0.0296	(4.0,1.06)	0.0297	(9.5,1.06)	0.0283	0.0061 *
$E[i]$	0.0595	(1.0,1.00)	0.2511	(9.5,1.)	0.0587	0.0094 *
$\sigma[i]$	0.0205	(1.0,1.00)	0.0483	(9.5,1.)	0.0323	0.0076
$E[\rho]$	0.0201	(1.0,1.00)	0.2040	(9.5,1.)	0.0148	0.0053 *
$\sigma[\rho]$	0.0111	(4.5,1.06)	0.0196	(9.5,1.00)	0.0200	0.0046 *
$E[m_g]$	0.0203	all	0.0203	all	0.0164	0.0063 *
$\sigma[m_g]$	0.0450	(9.5,1.06)	0.0496	(4.0,1.06)	0.0336	0.0061 *
$corr[\pi,\omega]$	0.9168	(4.0,1.06)	0.9254	(9.5,1.06)	0.3421	0.1191 **
$corr[\pi,i]$	0.9207	(4.0,1.06)	0.9274	(5.0,1.06)	0.7689	0.0805 *
$corr[\pi,\rho]$	-0.8162	(1.0,1.00)	0.4445	(9.5,1.06)	-0.1808	0.1904

Note: See also Table 3. There is no equilibrium for β = 1, $\alpha \le$ .5; β = 1.02, $\alpha \le$ 1.5; β = 1.04, $\alpha \le$ 2.5; β = 1.06, $\alpha \le$ 3.5;

Table 8

Cash-Credit Model Simulation Results vs. Sample Values
Annual Data: 1950 to 1987

β = 1.00, 1.02, 1.04; α = 0,.5,...,9.5; ψ = .2,.4,.6,.8

	min	min (α,β,ψ)	max	max (α,β,ψ)	sample value	std. dev. sample value
E[v]	1.2649	(1.,1.,.8)	6.0144	(9.5,1.,.2)	0.8826	0.0105 **
σ[v]	0.0078	(1.,1.,.8)	0.2860	(9.5,1.,.2)	0.0405	0.0086
cv[v]	0.0061	(1.,1.,.8)	0.0475	(9.5,1.,.2)	0.0456	0.0097
corr[v,γ]	-0.2411	(1.,1.,.8)	0.4690	(9.5,1.,.6)	-0.5000	0.1447 *
corr[v,ω]	0.6564	(1.,1.,.8)	0.7519	(8.,1.,.2)	-0.0668	0.2263 **
corr[v,i]	0.7628	(9.5,1.,.6)	0.9788	(1.,1.,.2)	0.5348	0.2245 *
E[π]	0.0389	(1.5,1.,.8)	0.0396	(9.5,1.,.2)	0.0434	0.0079 *
σ[π]	0.0290	(1.5,1.,.8)	0.0486	(9.5,1.,.2)	0.0283	0.0061 *
E[i]	0.0595	(1.,1.,.6)	0.2511	(9.5,1.,.6)	0.0587	0.0094 *
σ[i]	0.0205	(1.,1.,.8)	0.0570	(9.5,1.,.6)	0.0323	0.0076
E[ρ]	0.0201	(1.,1.,.8)	0.2049	(9.5,1.,.2)	0.0148	0.0053 *
σ[ρ]	0.0133	(3.5,1.04,.8)	0.0555	(9.5,1.,.2)	0.0200	0.0046
E[m_g]	0.0203	(4.,1.04,.8)	0.0211	(9.5,1.,.2)	0.0164	0.0063 *
σ[m_g]	0.0653	(2.,1.,.8)	0.2998	(9.5,1.,.2)	0.0334	0.0061 **
corr[π,ω]	0.4736	(9.5,1.,.2)	0.8758	(4.,1.04,.8)	0.3728	0.1043 *
corr[π,i]	0.4750	(9.5,1.,.2)	0.8885	(4.,1.04,.8)	0.7689	0.0805
corr[π,ρ]	-0.7762	(1.,1.,.2)	0.3099	(9.5,1.04,.8)	-0.1808	0.1904

Note: See also Table 3. There is no equilibrium for β = 1, $\alpha \le$ .5; β = 1.02, $\alpha \le$ 1.5; β = 1.04, $\alpha \le$ 2.5; β = 1.06, $\alpha \le$ 3.5;

Table 9

Cash Model Simulation Results vs. Sample Values
Quarterly Data, 1959:2 - 1988:1

β = 1.,1.005,...,1.015; α = 0,.5,...,9.5

	min	min (α,β)	max	max (α,β)	sample value	std. dev. sample value
E[v]	0.9998	(4.0,1.015)	1.0000	(9.5,1.010)	0.2261	0.0018 **
σ[v]	0.0000	(9.5,1.010)	0.0010	(4.0,1.015)	0.0090	0.0014 **
cv[v]	0.0000	(9.5,1.010)	0.0010	(4.0,1.015)	0.0398	0.0061 **
corr[v,γ]	0.0000	(9.5,1.015)	0.2643	(4.0,1.015)	-0.3420	0.1112 **
corr[v,ω]	0.0000	(9.5,1.015)	0.2865	(4.0,1.015)	-0.1634	0.1243 *
corr[v,i]	0.0000	(9.5,1.015)	0.3968	(4.0,1.015)	0.6208	0.1526 *
E[π]	0.0141	all	0.0141	all	0.0122	0.0014 *
σ[π]	0.0084	(4.0,1.015)	0.0085	(9.5,1.010)	0.0074	0.0012 *
E[i]	0.0195	(1.0,1.000)	0.0659	(9.5,1.000)	-0.0151	0.0014 **
σ[i]	0.0048	(1.0,1.000)	0.0119	(9.5,1.000)	0.0069	0.0013
E[ρ]	0.0054	(1.0,1.000)	0.0511	(9.5,1.000)	0.0030	0.0011 **
σ[ρ]	0.0050	(3.0,1.010)	0.0074	(9.5,1.000)	0.0062	0.0010
E[m_g]	0.0054	all	0.0054	all	0.0052	0.0016 *
σ[m_g]	0.0205	(5.0,1.010)	0.0218	(4.0,1.015)	0.0098	0.0014 **
corr[π,ω]	0.7992	(4.0,1.015)	0.8051	(3.5,1.010)	0.1844	0.1012 **
corr[π,i]	0.7811	(9.5,1.010)	0.8156	(1.0,1.000)	0.6192	0.0942 *
corr[π,ρ]	-0.8574	(1.0,1.000)	0.0483	(9.5,1.015)	-0.5047	0.1392

Note: See also Table 3. Equilibrium does not exist for $\beta = 1$, $\alpha \leq .5$; $\beta = 1.005$, $\alpha \leq 1.5$; $\beta = 1.01$, $\alpha \leq 2.5$; $\beta = 1.015$, $\alpha \leq 3.5$.

Table 10

Cash-Credit Model Simulation Results vs. Sample Values
Quarterly Data, 1959:2 - 1988:1

β = 1.00,1.005,...,1.015; α = 0,.5,...,9.5; ψ = .2,.4,.6,.8

	min	min (α,β,ψ)	max	max (α,β,ψ)	sample value	std. dev. sample value
E[v]	1.2549	(1.,1.00,.8)	5.2668	(9.5,1.,.2)	0.2261	0.0018 **
σ[v]	0.0020	(1.,1.00,.8)	0.0853	(9.5,1.,.2)	0.0090	0.0014
cv[v]	0.0016	(1.,1.00,.8)	0.0162	(9.5,1.,.2)	0.0398	0.0061 **
corr[v,γ]	0.1070	(1.,1.00,.8)	0.6515	(9.5,1.,.6)	-0.3420	0.1112 **
corr[v,ω]	0.5708	(9.5,1.0,.6)	0.6189	(2.5,1.0,.2)	-0.1634	0.1242 **
corr[v,i]	0.8934	(9.5,1.00,.6)	0.9928	(1.,1.0,.2)	0.6208	0.1526 *
E[π]	0.0141	all	0.0141	all	0.0122	0.0014 *
σ[π]	0.0076	(8.5,1.,.8)	0.0144	(9.5,1.,.2)	0.0074	0.0012 *
E[i]	0.0195	(1.,1.00,.8)	0.0660	(9.5,1.,.6)	0.0151	0.0014 **
σ[i]	0.0048	(1.,1.00,.8)	0.0126	(9.5,1.,.6)	0.0069	0.0013
E[ρ]	0.0054	(1.,1.00,.8)	0.0513	(9.5,1.,.2)	0.0030	0.0011 **
σ[ρ]	0.0049	(3.,1.01,.8)	0.0183	(9.5,1.,.2)	0.0062	0.0010
E[m_g]	0.0054	(6.,1.01,.8)	0.0055	(9.5,1.,.2)	0.0052	0.0016 *
σ[m_g]	0.0224	(2.5,1.005,.8)	0.1074	(9.5,1.,.2)	0.0098	0.0014 **
corr[π,ω]	0.1861	(9.5,1.00,.2)	0.7787	(5.5,1.01,.8)	0.0989	0.0917 *
corr[π,i]	0.1015	(9.5,1.,.2)	0.7810	(2.,1.005,.8)	0.6192	0.0942
corr[π,ρ]	-0.8264	(1.,1.00,.8)	0.0696	(9.5,1.01,.8)	-0.5047	0.1392

Note: See also Table 3. Equilibrium does not exist for β = 1, $\alpha \leq$ .5; β = 1.005, $\alpha \leq$ 1.5; β = 1.01, $\alpha \leq$ 2.5; β = 1.015, $\alpha \leq$ 3.5.

Table 11

The Effect of a Noisy Signal About Money Growth
in the Cash-Credit Model (β = .99)
Annual Data: 1958 to 1987

		$.01\sigma_\omega$	$.5\sigma_\omega$	$1\sigma_\omega$	$2\sigma_\omega$
(α=1, ψ=.1)					
	E[v]	10.648	10.705	10.686	10.681
	cov[v]	0.014	0.016	0.016	0.016
	E[π]	0.042	0.048	0.045	0.045
	E[ρ]	0.030	0.029	0.030	0.030
(α=1, ψ=.5)					
	E[v]	2.071	2.078	2.076	2.076
	cov[v]	0.008	0.009	0.009	0.009
	E[π]	0.042	0.048	0.045	0.045
	E[ρ]	0.029	0.029	0.030	0.030
(α=1, ψ=.9)					
	E[v]	1.119	1.120	1.120	1.120
	cov[v]	0.002	0.002	0.002	0.002
	E[π]	0.042	0.048	0.045	0.044
	E[ρ]	0.029	0.029	0.030	0.030
(α=4, ψ=.1)					
	E[v]	11.195	11.258	11.273	11.265
	cov[v]	0.017	0.019	0.019	0.019
	E[π]	0.042	0.048	0.045	0.045
	E[ρ]	0.087	0.088	0.093	0.092
(α=4, ψ=.5)					
	E[v]	2.132	2.140	2.141	2.141
	cov[v]	0.009	0.011	0.012	0.012
	E[π]	0.042	0.048	0.045	0.045
	E[ρ]	0.087	0.088	0.093	0.092
(α=4, ψ=.9)					
	E[v]	1.126	1.127	1.127	1.127
	cov[v]	0.002	0.002	0.002	0.002
	E[π]	0.042	0.048	0.045	0.044
	E[ρ]	0.087	0.088	0.092	0.092

Table 12

The Effect of a Noisy Signal About Output Growth
in the Cash-Credit Model (β = .99)
Annual Data: 1958 to 1987

		$.01\sigma_\omega$	$.5\sigma_\omega$	$1\sigma_\omega$	$2\sigma_\omega$
(α=1, ψ=.1)					
	E[v]	10.780	10.712	10.674	10.749
	cov[v]	0.015	0.016	0.016	0.016
	E[π]	0.058	0.049	0.044	0.053
	E[ρ]	0.028	0.029	0.030	0.030
(α=1, ψ=.5)					
	E[v]	2.087	2.079	2.075	2.083
	cov[v]	0.009	0.009	0.009	0.009
	E[π]	0.058	0.049	0.044	0.053
	E[ρ]	0.028	0.029	0.030	0.029
(α=1, ψ=.9)					
	E[v]	1.121	1.120	1.120	1.120
	cov[v]	0.002	0.002	0.002	0.002
	E[π]	0.058	0.049	0.044	0.053
	E[ρ]	0.028	0.029	0.030	0.029
(α=4, ψ=.1)					
	E[v]	11.301	11.269	11.253	11.311
	cov[v]	0.029	0.020	0.020	0.019
	E[π]	0.059	0.049	0.044	0.053
	E[ρ]	0.083	0.088	0.092	0.089
(α=4, ψ=.5)					
	E[v]	2.145	2.141	2.139	2.146
	cov[v]	0.018	0.012	0.012	0.012
	E[π]	0.058	0.049	0.044	0.053
	E[ρ]	0.082	0.088	0.092	0.089
(α=4, ψ=.9)					
	E[v]	1.127	1.127	1.127	1.127
	cov[v]	0.004	0.002	0.002	0.002
	E[π]	0.058	0.049	0.044	0.053
	E[ρ]	0.082	0.088	0.091	0.088

Table 13

Cash-Credit Model Simulation Results vs. Sample Values
Annual Data: 1950 to 1987
CES Utility

β = .92,.94,...,.98,1.; α = 0,.5,...,9.5; η = .5,.75,.8,.9,.95,.97

	min	min (α,β,η)	max	max (α,β,η)	sample value	std. dev. sample value
E[v]	2.1236	(0.,.98,.5)	27314.	(9.5,.92,.97)	0.8826	0.0105 **
σ[v]	0.0496	(0.,.98,.5)	3726.6	(9.5,.92,.97)	0.0405	0.0086 *
cv[v]	0.0234	(0.,.98,.5)	0.1572	(9.5,.92,.8)	0.0456	0.0097
corr[v,γ]	-0.4235	(0.,98,.8)	0.3955	(9.,.98,.5)	-0.5000	0.1447 *
corr[v,ω]	0.5777	(0.,.92,.97)	0.7481	(7,.92,.5)	-0.0668	0.2263 **
corr[v,i]	0.7715	(9.,.98,.5)	0.9960	(0.,.92,.97)	0.5348	0.2245 *
E[π]	0.0390	(0.,.98,.5)	0.0461	(9.5,.92,.8)	0.0434	0.0079
σ[π]	0.0325	(0.,.98,.5)	0.1273	(9.5,.92,.8)	0.0283	0.0061 *
E[i]	0.0579	(1.,1.,.97)	0.3599	(9.5,.92,.75)	0.0587	0.0094
σ[i]	0.0013	(0.,.94,.97)	0.0547	(8.5,.94,.5)	0.0323	0.0076
E[ρ]	0.0202	(0.,.98,.5)	0.3193	(9.5,.92,.8)	0.0148	0.0053 *
σ[ρ]	0.0265	(0.,.98,.5)	0.1665	(9.5,.92,.8)	0.0200	0.0046 *
E[m_g]	0.0206	(0.,.98,.5)	0.0285	(9.5,.92,.8)	0.0157	0.0064 *
σ[m_g]	0.1759	(0.,.98,.5)	1.0380	(9.5,.92,.8)	0.0334	0.0060 **
corr[π,ω]	-0.1591	(9.5,.98,.97)	0.5627	(0.,.98,.5)	0.3421	0.1191
corr[π,i]	-0.1153	(9.5,.98,.97)	0.5705	(0.,.98,.5)	0.7689	0.0805 **
corr[π,ρ]	-0.9968	(0.,.92,.97)	-0.7336	(7.,1.,.5)	-0.1808	0.1904 **

Note: See also Table 3. No equilibrium exists for β = 1, $\alpha \leq$ .5. The algorithm failed to converge for 61 out of 600 possible parameter specifications.

Table 14

Cash good model Simulation Results vs. Sample Values
Annual Data: 1950 to 1987
Habit Formation Preferences

β = .92,.94,...,.98,1.; α = 0,.5,...,9.5; b = 0,.2,.4,.6,.8

	min	min (α,β,b)	max	max (α,β,b)	sample value	std. dev. sample value	
E[v]	0.7881	(3.,1.,.8)	1.0000	(9.5,1.,.8)	0.8826	0.0105	
σ[v]	0.0000	(9.5,1.,.8)	0.2382	(3.5,.98,.8)	0.0405	0.0086	
cv[v]	0.0000	(9.5,1.,.8)	0.3001	(3.5,.98,.8)	0.0456	0.0097	
corr[v,γ]	-0.8422	(3.,1.,.8)	0.0000	(9.5,1.,.8)	-0.5000	0.1447	
corr[v,ω]	0.0000	(9.5,1.,.8)	0.2480	(1.,1.,.6)	-0.0668	0.2263	*
corr[v,i]	0.0000	(4.5,1.,.2)	0.8342	(3.,1.,.8)	0.5348	0.2245	
E[π]	0.0389	(6.5,.96,.4)	0.4304	(3.5,.98,.8)	0.0434	0.0079	
σ[π]	0.0297	(6.5,.96.4)	3.9709	(3.5,.98,.8)	0.0283	0.0061	*
E[i]	0.0592	(1.,1.,.8)	0.3599	(9.5,.92,0.)	0.0587	0.0094	*
σ[i]	0.0182	(0.,.98,.6)	0.0954	(9.5,.92,.6)	0.0323	0.0076	
E[ρ]	0.0201	(1.,1.,0.)	0.4773	(3.5,.98,.8)	0.0148	0.0053	*
σ[ρ]	0.0116	(4.,1.,0.)	3.6304	(9.5,1.,.8)	0.0200	0.0046	
E[m_g]	0.0204	(9.5,1.,.8)	0.3767	(3.5,.98,.8)	0.0157	0.0064	*
σ[m_g]	0.0450	(9.5,1.,.8)	19.000	(3.5,.98,.8)	0.0334	0.0060	*
corr[π,ω]	-0.3386	(2.5,1.,.8)	0.9254	(9.5,1.,.8)	0.3421	0.1191	
corr[π,i]	-0.4345	(2.5,1.,.8)	0.9274	(5.,.98.,0)	0.7689	0.0805	
corr[π,ρ]	-0.9667	(1.,1.,.8)	0.5510	(9.5,.94,.4)	-0.1808	0.1904	

Note: See also Table 3. No equilibrium exists for β = 1, $\alpha \leq$ .5.

Table 15

Sensitivity Tests on VAR Process

$$\gamma_t = A_{10} + A_{11}\ \gamma_{t-1} + A_{12}\ \omega_{t-1} + \varepsilon_{\gamma t}$$

$$\omega_t = A_{20} + A_{21}\ \gamma_{t-1} + A_{22}\ \omega_{t-1} + \varepsilon_{\omega t}$$

The basis for the parameters and standard errors (SE) are in equations (19a) and (19b).

1. double σ^2_γ	7. A_{11} - SE(A_{11})	13. A_{21} - SE(A_{21})
2. triple σ^2_γ	8. A_{11} + SE(A_{11})	14. A_{21} + SE(A_{21})
3. $\sigma_{\gamma\omega} = 0$	9. A_{11} + 2SE(A_{11})	15. A_{21} + 2SE(A_{21})
4. $\sigma_{\gamma\omega} = -\sigma_{\gamma\omega}$	10. A_{12} - SE(A_{12})	16. A_{22} - SE(A_{22})
5. double σ^2_ω	11. A_{12} + SE(A_{12})	17. A_{22} + SE(A_{22})
6. triple σ^2_ω	12. A_{12} + 2SE(A_{12})	18. A_{22} + 2SE(A_{22})

Footnotes

We thank Anthony Braun, John Cochrane, Martin Eichenbaum, Lars Peter Hansen, John Heaton, Robert Korajczyk, Pamela Labadie, Robert Lucas, Mark Watson and the participants of seminars at the University of Chicago, Northwestern University and the Federal Reserve Bank of Minneapolis for useful conversations during the course of writing the paper.

1. It can be demonstrated that if the state space is finite, there is a unique equilibrium of this form. Equilibrium may not be unique for a countably infinite state space, although we have been unable to construct examples of multiple equilibria. A stationary equilibrium may not exist if ω is sufficiently small or β too large. When ω is very low in many states, money has a high real rate of return. If the return is high enough, the agent tries to postpone consumption perpetually, and markets cannot clear.

2. Lucas and Stokey (1987) have a more general informational structure that allows a noisy signal about the time t+1 money supply in the time t information set, rather than the perfectly revealing signal of the Svensson model. We explore this possibility in Section 6. Lucas and Stokey also assume a different timing convention: the asset market convenes before the goods market.

3. The computer programs that solve the models are written in Gauss and are available from the authors.

4. Notice from (17) and (18) that if the CIA constraint is always binding,

$$(*) \qquad \mu_t = \gamma_t^{-\alpha}\left[1 - \beta E_t(\gamma_{t+1}^{1-\alpha}/\omega_t)\right] > 0.$$

In early CIA models the order of events in a period is information flow, asset market, and goods market; and positive nominal interest rates imply that the CIA constraint always binds. The expression in square brackets in (*) is the nominal interest rate divided by one plus the nominal interest rate in those models. The expression for nominal interest rates with the alternative timing of this paper is different. Hence, one cannot simply check that interest rates are always positive and conclude that the CIA constraint is always binding. Checking that (*) is always positive is equivalent to following the first two steps of our algorithm. We thank Andrew Atkeson for drawing our attention to the fact that changing the timing within a period does not change the states of the world in which the CIA constraint binds since (*) depends only on the preferences of agents and the time series properties of the forcing processes.

5. Marshall (1988) notes that M1 growth may be nonstationary and therefore works with a monetary transactions technology that incorporates technological change.

6. In this section, we focus on the results of the annual data because the models performed slightly better using these processes. The qualitative behavior using quarterly driving processes is similar.

7. At first glance, it may appear troubling that the correlations between velocity and other variables are discontinuous at $\psi = 1$. However, note that the function $f(a) = \mathrm{Corr}(ax_t, x_t)$ is discontinuous at $a = 0$. Similarly, changes in ψ leave the correlations in Figures 2-4 relatively unaffected until the credit good sector entirely disappears when $\psi = 1$. In other words, the discontinuity exhibited in Figures 2-4 is a property of the correlation function and not of the model.

8. The approximate asymptotic standard errors of the unconditional moments are calculated following the suggestions in Hansen and Jagannathan (1988). They are calculated as Taylor's series approximations of arbitrarily serially correlated time series.

9. Lucas (1987) notes that traditional money demand functions can be generated from the first order conditions of the agent's maximization problem in a simplified version of the Lucas and Stokey (1987) model. In that framework, the function fits exactly, which is not true with the timing of our markets and information flows. For example, with $\beta=.99$, $\alpha=7$, and any ψ, the R^2 statistic for the regression (25) is approximately .69.

10. These preferences were first suggested by Ryder and Heal (1973), and are used by Constantinides (1988) as a means of resolving the equity premium puzzle.

11. For example, it would be possible to examine the properties of models such as those of Hartley (1988) and Marshall (1988) in a similar fashion.

12. Finn, Hoffman, and Schlagenhauf (1988) use GMM to test an equity pricing Euler equation restrictions for the Svensson model and do not reject the model using monthly data over the period 1959:02-1985:12. Using Israeli data, Eckstein and Leiderman (1988) find that money in the utility function outperforms the cash-credit model.