NBER WORKING PAPER SERIES

DYNAMIC SEIGNIORAGE THEORY: AN EXPLORATION

Maurice Obstfeld

Working Paper No. 2869

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 1989

The initial work on this paper was done during a visit to the Institute of International Economic Studies, University of Stockholm, in May 1988. I am grateful for conversations with the research staff and visitors at the Institute, particularly Guillermo Calvo, Torsten Persson, and Lars Svensson. Thanks for helpful comments also go to Alberto Giovannini, Vittorio Grilli, Dale Henderson, Eric Maskin, and Jeffrey Miron. Research support has been provided by the NBER's Olin Visiting Scholar program and by the National Science Foundation. This paper is part of NBER's research programs in Economic Fluctuations and International Studies. Any opinions expressed are those of the author not those of the National Bureau of Economic Research.

NBER Working Paper No. 2869
February 1989

DYNAMIC SEIGNIORAGE THEORY: AN EXPLORATION

ABSTRACT

This paper shows that the optimal extraction of seigniorage implies a strong tendency for inflation to fall over time toward its socially optimal level. The point is made using a multi-period model in which (i) the government can finance deficits through bond issue or money creation, (ii) private-sector expectations are rational, and (iii) the government sets the inflation rate each period in a discretionary manner. One way to view the model is as a synthesis of the "tax-smoothing" theory of government deficits, which predicts that the inflation tax follows approximately a martingale, and of models of discretionary policy making, which predict (absent reputation effects) that inflation is likely to exceed its socially optimal level. Both predictions are modified when the two approaches to explaining inflation are merged. Reputation effects play no role in the analysis.

Maurice Obstfeld
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297

Introduction

High inflation can arise when the seigniorage derived from money creation plays a prominent role in the public finances. Economic research of the 1950s and 1960s elucidated the allocative effects of steady-state, fully anticipated inflation--the resource transfer to the government and the associated welfare loss of consumers. More recent work has aimed to explain observed inflation rates, and has proceeded along two main lines. The first of these focuses on the temptation to effect the resource transfer more efficiently through surprise inflation. The second adds a dynamic dimension to the problem, exploring official incentives to issue debt so as to spread over time the distortionary burden of the inflation tax. It is convenient to call the first approach to explaining inflation the *discretionary-policy* approach, the second the *tax-smoothing* approach.

Calvo (1978a,b), Barro (1983), and others have observed that when policy is made on a discretionary, period-by-period basis, the lure of surprise price-level movements may, in equilibrium, lead to higher inflation and lower revenue than would result if the government could bind itself to a prior choice of the inflation rate. The incentive to violate such a commitment in later periods is an example of the general problem of time inconsistency: optimal government plans that affect private choices may no longer appear optimal after the private sector has made those choices.[1]

[1]See Kydland and Prescott (1977) and Calvo (1978a,b). Seigniorage is not, of course, the only mechanism through which government

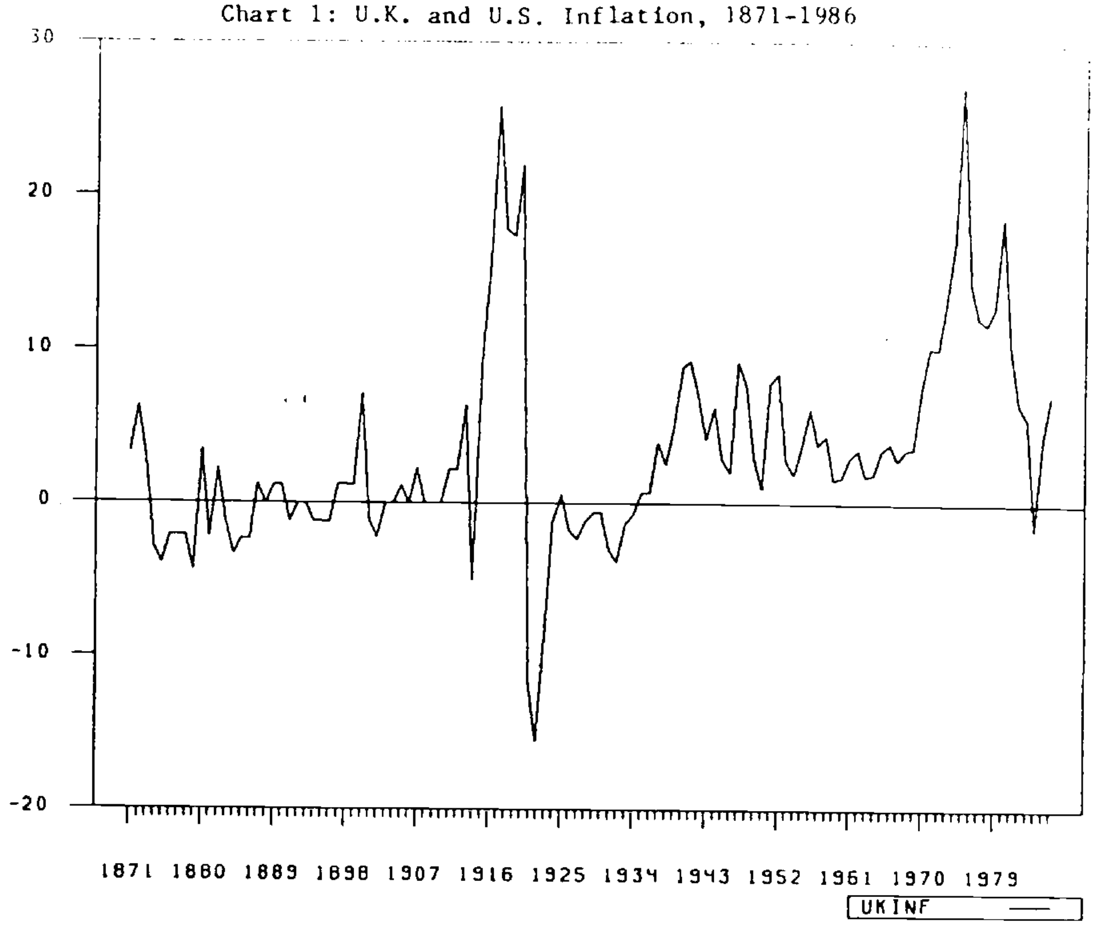
While the discretionary-policy approach predicts that inflation will be higher than is socially optimal (absent possible reputation effects), the tax-smoothing approach implies that inflation, whatever its level today, will be persistent. Barro (1979) suggested that an intertemporally maximizing government with exogenous spending needs would find it optimal to formulate contingency plans for taxes that equate over all future periods the expected marginal loss due to tax distortions. This "permanent-income" view of tax-setting behavior has been extended to the inflation tax by Mankiw (1987), Grilli (1988a,b), and others, who argue that, at least as an approximation, a country's inflation rate should follow a martingale if seigniorage is the main force driving inflation.

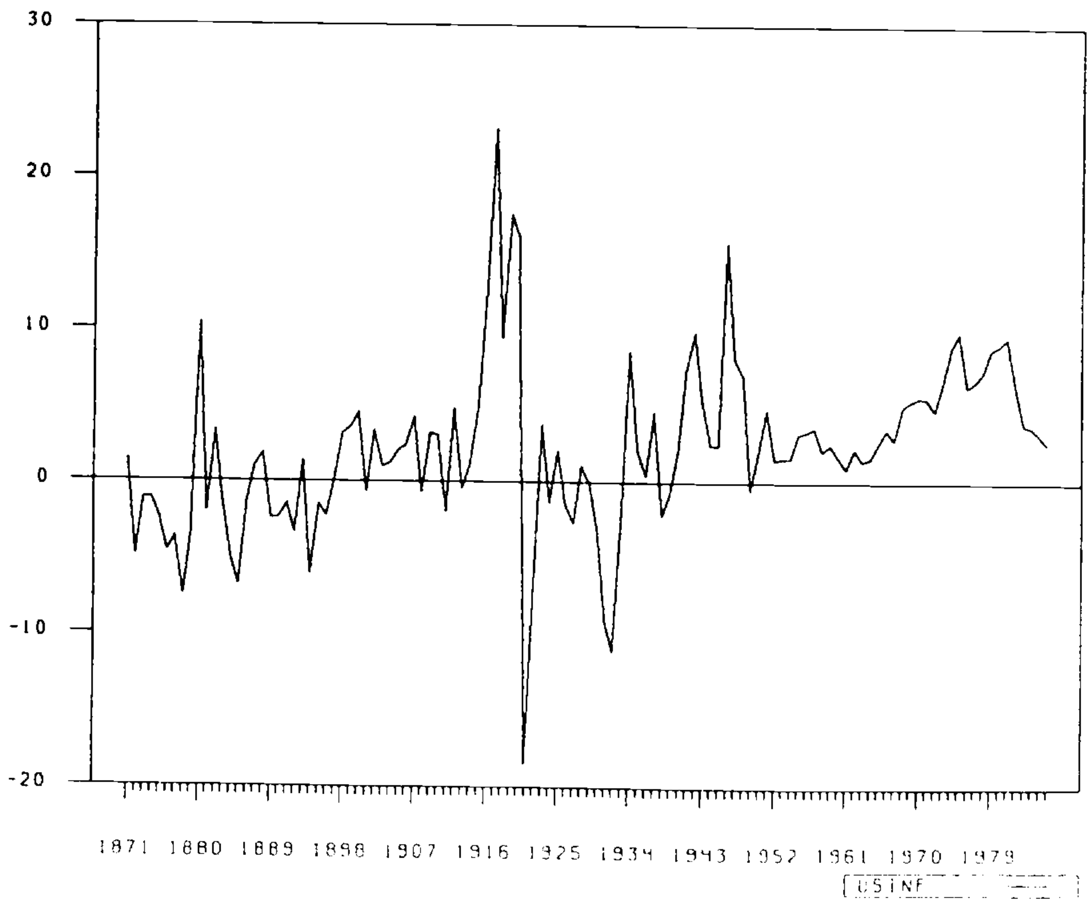
A look at some data gives cause for skepticism about both views of inflation--at least in the simple forms paraphrased above. Chart 1 shows the time series of annual inflation rates in the United Kingdom and the United States between 1871 and 1986.[2] In both countries, inflation shows occasional sharp upward spikes (for example, during the two world wars), but there is no clear

credibility problems may cause suboptimally high inflation (although it has been a key mechanism at some time in many countries). Some studies, for example, Kydland and Prescott (1977) and Barro and Gordon (1983), focus instead on attempts to exploit short-run tradeoffs between unanticipated inflation and employment.

[2] A number of authors have made the point that fairly long time series might be needed to detect the mean-reverting behavior tested for below. Given this need, it seems best to use data from countries, like the U.K. and the U.S., which have maintained stable political institutions over a long sample period. Vittorio Grilli generously provided the data plotted in chart 1.

Chart 1: U.K. and U.S. Inflation, 1871-1986





evidence of persistently high inflation. Indeed, when examined over a long time period, inflation apparently has a tendency to revert to a fairly low mean level.

These visual impressions are confirmed by the more formal econometric tests reported in table 1. For both countries, the favored univariate statistical model for inflation is a first-order autoregression with an autoregressive coefficent α that is positive, but well below unity. [The Dickey-Fuller criterion rejects decisively the hypothesis of a random walk (α = 1) in favor of a stationary alternative.] The implied steady-state annual inflation rate for the U.K. is 3.5 percent; that for the U.S. is 2.2 percent.[3]

Does a tendency for inflation to revert to a low level imply that discretionary policy-making and tax smoothing should be rejected as elements in a positive theory of inflation taxation? In this paper, I argue, to the contrary, that both concepts are central to understanding the dynamic behavior of seigniorage revenue and inflation. Existing models stressing the potential

[3]Mankiw (1987) finds that 1952-85 U.S. data lend some support to a version of the tax-smoothing theory of inflation. This finding may be due to the relatively low power of a short sample period, or it may reflect sub-sample instability, which Barsky (1987) suggests is present in long time series of U.S. inflation. Using a broader country (and time) sample, Grilli (1988b) and Poterba and Rotemberg (1988) find evidence against the tax-smoothing theory's general validity. The evidence in chart 1 suports Fischer's (1986, p. 14) observation that "Going further back in history, it is clear that inflationary bias is only a sometime thing. At the ends of the Napoleonic and Civil Wars, and World War I, Britain and the United States deflated to get back to fixed gold parities. These episodes too deserve attention in the dynamic inconsistency literature." Indeed, the generalized industrial-country disinflation of the early 1980s provides a more recent example, albeit less dramatic, of the type of event Fischer describes.

Table 1

Univariate Inflation Autoregressions Using Annual Data, 1871-1986

United Kingdom

$$\text{Inflation}_t = \underset{(0.0050)}{0.0102} + \underset{(0.0667)}{0.7067}\cdot\text{Inflation}_{t-1} + \text{Error}_t$$

$h = 0.39$ $\qquad d = 1.95$ $\qquad \bar{R}^2 = 0.49$

United States

$$\text{Inflation}_t = \underset{(0.0049)}{0.0113} + \underset{(0.0817)}{0.4953}\cdot\text{Inflation}_{t-1} + \text{Error}_t$$

$h = -0.04$ $\qquad d = 2.00$ $\qquad \bar{R}^2 = 0.24$

Notes: Standard errors are given in parentheses; each regression is based on 115 observations. h is Durbin's test statistic for first-order serial correlation, which is distributed N(0,1) under the null hypothesis of serially uncorrelated errors. d is the Durbin-Watson statistic (which is biased toward 2.00 in the above regressions). The AR(1) specification reported above was reached in tests against alternatives involving up to ten lags of inflation. Inflation data were constructed as $(P_t - P_{t-1})/P_{t-1}$, where P_t is the price level. For the U.K., the price level over 1870-1965 is the GNP deflator reported in table III.12 of Forrest Capie and Allan Webber, *A Monetary History of the United Kingdom, 1870-1982*, vol. 1 (London: George Allen and Unwin, 1985); over 1966-1986, it is the ratio of nominal to real GNP from the *Annual Abstract of Statistics*, various issues. (See Grilli 1988a for details.) For the U.S., the price level over 1870-1975 is the ratio of nominal to real income as reported in table 4.8 of Milton Friedman and Anna J. Schwartz, *Monetary Trends in the United States and the United Kingdom* (Chicago: University of Chicago Press, 1982); over 1976-86, it is the net national product deflator from the *Survey of Current Business*. Price indexes are based at 1929 = 100.

role of time inconsistency in inflation taxation do not offer predictions about the equilibrium dynamics of government debt and of tax rates; in contrast, monetary tax-smoothing models do offer an account of these dynamics, but one that fails as an equilibrium theory because of its failure to confront and resolve the time-inconsistency issue. By *merging* the two approaches below using a time-consistent equilibrium construct, however, I derive a theory consistent with the story told by chart 1. The theory predicts that at *each point in time* at which inflation is positive, it is higher than it would be if the government could precommit itself to future tax policies. But the theory also predicts that, given rational private expectations, government tax-smoothing behavior results in an inflation rate with a tendency to fall *over time* toward the government's preferred long-run inflation rate (zero in my model). Reputation effects play no role in this equilibrium.[4]

In technical terms, the investigation pursued here is an application of dynamic game theory to interactions between the

[4]See Lucas (1986) for accessible accounts of both the tax-smoothing idea and the obstacles to implementation implied by dynamic inconsistency. Lucas describes examples in which government debt management policies can be used to ensure the time consistency of optimal plans. Were such schemes generally feasible, the naive tax-smoothing models might be resuscitated as viable positive theories. Particularly in monetary economies, however, it appears that the scope for such fine-tuning is very limited. The nature of these limitations is discussed in section V below. Poterba and Rotemberg (1988) consider the role of time inconsistency in tax-smoothing inflation models, but they do not focus on the dynamics of tax rates and government debt. Bohn (1988) uses a framework similar to the one developed below to study government motives for nominal debt issue in a stochastic environment.

public and private sectors. The endogenous state variable responsible for the economy's dynamics is the stock of government debt. In the time-consistent intertemporal (Nash) equilibrium constructed below, the government's inflation policy reaction function is optimal, given the private rule for forecasting inflation; at the same time, private forecasts are rational, given the government's objectives and constraints. Phelps and Pollak (1968), in a model of capital accumulation by successive generations, presented an early economic application of the equilibrium concept applied here.[5]

The structure of the paper is as follows. Section I sets up a model economy in which the government maximizes an intertemporal welfare criterion that is decreasing in current inflation but increasing in private real money balances. Section II describes the government's optimal plan in a Stackelberg game, and shows why the plan generally is time inconsistent. Section III contains the central result that in a time-consistent Nash equilibrium, inflation has a tendency to fall over time. Section IV provides a closed-form linear example in which the earlier theoretical results can be checked numerically. Section V examines some special issues connected with domestic-currency public debt, particularly (i) multiple equilibriums and (ii) the possible elimination of the time-consistency problem through sophisticated

[5] More recent applications include (aside from those mentioned in the last footnote) Kydland and Prescott (1977), Calvo (1978a,b), Fischer (1980), Miller and Salmon (1985), Oudiz and Sachs (1985), Chari and Kehoe (1987), Cohen and Michel (1988), and Persson and Svensson (1988).

government open-market operations in real and nominal securities. Concluding remarks are contained in section VI.

I. The Model

The analysis is developed in a setting due to Brock (1974). A single representative private agent consumes a single perishable, nonproduced consumption good. At the start of period t, the representative agent maximizes

$$\sum_{s=t}^{\infty} \beta^{s-t}[c_s + g_s + v(m_s)],$$

where c is private consumption; g is government consumption, which enters linearly into private utility; m stands for real money balances (nominal balances M divided by the price level P); $\beta < 1$ is the subjective discount factor; and v(m) is twice continuously differentiable and strictly concave.

Conditions necessary for individual optimality are well known. The linearity of utility in consumption fixes the equilibrium real interest rate between periods s and s+1, ρ_s, at $\rho = (1 - \beta)/\beta$.[6] Real money demand at time s, m_s, satisfies

$$(1) \quad v'(m_s) = \frac{\rho + \pi^e_{s+1}}{1 + \rho} = \frac{i_s}{1 + i_s},$$

[6]By assuming a constant real interest rate, I am pushing to the background questions of the term structure of government debt. Implications of this modeling strategy are mentioned at various points in section V.

where superscript "e" denotes a private expectation,

$$\pi^e_{s+1} = (P^e_{s+1} - P_s)/P^e_{s+1},$$

and i_s is the nominal interest rate between dates s and s+1, defined as $(1+\rho)P^e_{s+1}/P_s$. The variable π_{s+1} will be referred to as the *inflation rate* between periods s and s+1. On this definition, π_{s+1} is positively correlated with, but differs from, the standard measure of the inflation rate, $(P_{s+1} - P_s)/P_s$; the former is, however, the measure relevant to the present setup. Notice that π approaches an upper bound of 1 as the conventionally-defined inflation rate becomes arbitrarily high. Thus, π has a natural intepretation as a tax rate, with $\pi = 1$ the confiscatory rate.[7]

The economy is endowed with a fixed "natural" output level y, but the amount of output available to be consumed falls when the actual inflation rate rises. Specifically, equilibrium private consumption is

$$(2) \quad c_s = y - g_s - \kappa(\pi_s),$$

where $\kappa(0) = 0$, $\kappa'(0) = 0$, $\kappa''(\pi) > 0$, and $\kappa'(\pi)$ has the same sign as π. The inflation-cost function in (2) is meant to capture

[7]The above conditions on private money demand and the real interest rate follow from the fact that the individual maximizes subject to a sequence of finance constraints of the form $w_s = (1+\rho_{s-1})w_{s-1} - c_s - (\rho_{s-1}+\pi_s)m_{s-1}$, where w_s is the sum (at the end of period s) of real money balances and the real present value of all other forms of wealth. See Brock (1974), Obstfeld and Rogoff (1983), or Persson, Persson, and Svensson (1987).

costs distinct from the inflation-tax distortion of money demand, for example, the reduction in production efficiency often said to accompany higher inflation. Alternatively, $\kappa(\pi)$ allows inflation "surprises" to inflict social costs distinct from those of perfectly anticipated inflation.

The government must undertake a fixed level of consumption each period, so $g_s = g$, a constant.[8] In the model, however, only two sources of public finance are available, the proceeds of money creation and the proceeds of borrowing from the private sector. The presence of nonmonetary taxes in the formal model would not change any main predictions about inflation; but a model extended along the lines of Phelps (1973) would have implications for tax rates that might be useful in empirical evaluation.[9] As this extension is not pursued here, the revenues from money creation and from public debt issue are now described in turn.

The seigniorage obtained from the public through money creation over period s, σ_s, is given by

$$\sigma_s = \frac{M_s - M_{s-1}}{P_s} .$$

Assuming money-market equilibrium leads to a useful formula for

[8]This assumption is easily relaxed, as shown at the end of section III. The model as specified does not account for an endogenous government spending level, however.

[9]Phelps argues that the inflation tax should covary positively with other taxes. Poterba and Rotemberg (1988) and Grilli (1988b) present empirical evidence on the covariation of inflation with other forms of taxation. Their evidence is not entirely consistent with Phelps's prescription.

σ_s. Let $\mu(\pi^e_{s+1})$ be the money demand function implied by (1). [Since $v''(m_s) < 0$, $\mu'(\pi^e_{s+1}) < 0$.] Then seigniorage can be written

$$(3) \qquad \sigma_s = \pi_s m_{s-1} + (m_s - m_{s-1}) = \pi_s \mu(\pi^e_s) + [\mu(\pi^e_{s+1}) - \mu(\pi^e_s)].$$

This equation states that seigniorage is derived from two sources, (i) the public's desire to maintain real balances in the face of inflation and (ii) any desired increase in real balances.

The second means by which the government can finance budget deficits (surpluses) is the sale (purchase) of *price-level indexed* bonds that are not subject to default. Many of the dynamic results derived below may continue to hold when government debt obligations are denominated entirely in domestic currency, rather than indexed. The introduction of nominal debt complicates the technical analysis considerably, however, and also creates a clear potential for multiple equilibriums. Consideration of these difficulties is postponed until section V.

A bond has a fixed real face value (of unity) and pays interest ρ after a period. The stock of such instruments held by the government at the end of period s, a_s, evolves according to

$$(4) \qquad a_s = (1 + \rho)a_{s-1} + \sigma_s - g.$$

By integrating this expression forward, applying the long-run solvency condition $\lim_{s\to\infty}(1+\rho)^{-s}a_s \geq 0$, and using (3), one obtains the government's intertemporal budget constraint,

$$(1+\rho)a_{t-1} + \sum_{s=t}^{\infty}(1+\rho)^{s-t}[\pi_s m_{s-1} + (m_s - m_{s-1})] \geq (1+\rho)g/\rho.$$

It is assumed that the government's budgetary obligations always lead to nonnegative inflation rates.

The government maximizes an objective that is increasing in private consumption (of private and public goods) and real balances,

$$(5)\quad W_t = \sum_{s=t}^{\infty}\beta^{s-t}[u(c_s + g) + z(m_s)] = \sum_{s=t}^{\infty}\beta^{s-t}\{u[y - \kappa(\pi_s)] + z(m_s)\}.$$

The period utility function $u(c + g) + z(m)$ appearing in (5) is concave and twice continuously differentiable. W_t is general enough (given a change in the inflation-cost function $\kappa(\pi)$) to admit government-specific socially preferred inflation rates. To ensure (somewhat arbitrarily) that the inflation rate most appealing to the government is zero, I assume that at the real-balance level $\bar{m}$ associated with a zero inflation rate [that is, where $v'(\bar{m}) = \rho/(1+\rho)$], $z'(\bar{m}) = 0$. Thus, my choice for W_t does not really allow the possibility that the government aims directly to maximize the welfare of the representative agent.[10] Notice that

[10] In the model with $\kappa(0) = 0$, equilibrium appears to be generically nonexistent if the government's objective function is the same as the representative agent's. One way to allow the two functions to coincide is to assume that output is lost when inflation deviates, not from zero, but from Friedman's full liquidity rate, $-\rho$. If it is assumed in addition that full liquidity occurs at a finite level of real balances m_f, with increments to real balances beyond this point yielding zero utility, then the economy converges, not to zero inflation as it does below, but to deflation at rate ρ, with a steady-state government asset stock a_f satisfying $\rho a_f - g =$

the formulation above assumes that the private sector and government have the same discount rate, an assumption made in most of the tax-smoothing literature. Consequences of relaxing this assumption are discussed briefly at the end of section III.

II. Precommitment Equilibrium

If the government can precommit to follow chosen policies, the resulting equilibrium is the outcome of a Stackelberg game in which the government moves first, and moves only once. If this game is played at $t = 1$, say, the government announces its choice of a policy path for the entire future, taking into account that each private individual's best response under precommitment is to believe the announcement. The equilibrium policy path is the one that maximizes W_1 of (5) when it is correctly anticipated by the public. This section examines the precommitment solution and describes how it breaks down as an equilibrium concept when policies are not pre-set, but are chosen instead on a discretionary basis in the period of their implementation.

In the model with precommitment, a government moving on date 1 picks an inflation path $\{\pi_s\}_{s=1}^{\infty}$ that maximizes W_1 subject to the public-sector intertemporal budget constraint, the private optimality conditions

ρm_f. (In this long-run equilibrium, government interest earnings in excess of g are simply used to retire circulating money.) The problem with this formulation is its lack of realism: the kinds of inflation costs summarized in the function $\kappa(\pi)$ are difficult to ascribe empirically to deviations of inflation from $-\rho$.

(6) $m_s = \mu(\pi_{s+1})$ $(s \geq 1)$,

and *given* inherited levels of aggregate real balances, m_0, and government assets, a_0. Constraint (6) embodies the government's rational belief that if all future policies are immutably set on date 1, it will be in each private individual's interest to use those known policy settings in forecasting inflation, and thus in deciding how much money to hold.[11]

Strictly speaking, the government does not choose the path of prices. That path is instead the outcome of a market equilibration process that the government controls indirectly through the amount of money it issues. Given the announced inflation path, the government must engineer a path for nominal money consistent with market clearing on each date. The constraints on the government's plan ensure that such a money-supply path exists and is unique; and if speculative price-level bubbles are ruled out, that unique money-supply path necessarily produces, in equilibrium, the announced path of prices.[12]

To derive necessary first-order conditions (for an interior maximum) from the standpoint of date 1, let $\{\lambda_s\}_{s=1}^{\infty}$ be a sequence of multipliers associated with the finance constraints (4), and consider the Lagrangian

[11]Notice that the price level in the period prior to government choice, P_0, is an historical datum. Therefore, choice of the price level's path from $t = 1$ on is equivalent to choice of the inflation rate from $t = 1$ on.

[12]A formal proof could be constructed along the lines of Calvo (1978b, appendix 1). On mechanisms for ruling out price-level bubbles, see Obstfeld and Rogoff (1983, 1986).

$$L_1 = \sum_{s=1}^{\infty} \beta^{s-1} \{u[y - \kappa(\pi_s)] + z[\mu(\pi_{s+1})]\}$$

$$- \sum_{s=2}^{\infty} \beta^{s-1} \lambda_s [a_s - (1+\rho)a_{s-1} + (1-\pi_s)\mu(\pi_s) - \mu(\pi_{s+1}) + g]$$

$$- \lambda_1 [a_1 - (1+\rho)a_0 + (1-\pi_1)m_0 - \mu(\pi_2) + g].$$

Define $\upsilon(\pi_s) = u[y - \kappa(\pi_s)]$ as the utility from consumption when the inflation rate is π_s; $\upsilon'(\pi_s) = -u'[y - \kappa(\pi_s)]\kappa'(\pi_s) \leq 0$ for $\pi_s \geq 0$. Differentiation of L_1 with respect to a_s $(s \geq 1)$ shows that λ_s is constant at level λ_1 [since $\beta(1+\rho) = 1$]. Differentiation with respect to π_1 and π_s $(s > 1)$ leads to

(7) $\quad -\upsilon'(\pi_1) = \lambda_1 m_0$.

and, for $s > 1$,

(8) $\quad -\upsilon'(\pi_s) - (1+\rho)z'[\mu(\pi_s)]\mu'(\pi_s) = \lambda_1 \mu(\pi_s)[1 - \eta(\pi_s)]$,

where $\eta(\pi_{s+1})$ is (minus) the elasticity of real money demand with respect to the period-s opportunity cost of money, $i_s/(1+i_s)$.[13] (η will be called, somewhat inaccurately, the *interest elasticity* of money demand.) The resulting optimal-control inflation path is

[13]Recall, from (1), the definition of $i_s/(1+i_s)$ as $(\rho+\pi_{s+1})/(1+\rho)$ (under perfect foresight). Equation (1) also implies that $\eta(\pi_{s+1}) = -(\rho + \pi_{s+1})/(1 + \rho)m_s v''(m_s) = -(\rho + \pi_{s+1})\mu'(\pi_{s+1})/\mu(\pi_{s+1})$, which is the relationship used to derive (8).

denoted by $\{\pi_s^*\}_{s=1}^{\infty}$; it corresponds to an optimal shadow value of public funds λ_1^*.[14]

Three aspects of this precommitment equilibrium are important:

1. Equation (8) implies that inflation is constant at a level $\pi_s^* = \pi^*$ (say) for $s > 1$. (π_1^* need not equal π^*, though.) Given that g is constant, government assets remain constant (at a_1) after date 1. The precommitment solution thus entails perfect tax smoothing after the initial period. At inflation level π^*, the marginal value of the additional seigniorage from a perfectly foreseen one-period rise in inflation equals the marginal cost of that rise. The additional revenue from raising inflation for a period s, leaving other periods' inflation rates unchanged, is (in terms of period-s goods) $\mu(\pi_s) + \pi_s\mu'(\pi_s) + (1+\rho)\mu'(\pi_s) - \mu'(\pi_s) = \mu(\pi_s)[1 - \eta(\pi_s)]$;[15] this sum multiplied by λ_1 is the marginal

[14]In what follows, it is assumed that the precommitment optimum exists, is unique, and is characterized by (7) and (8). (Section IV describes a linear problem, closely related to this one, in which existence and uniqueness can be verified.) As (8) makes clear, a necessary condition for the validity of these assumptions is that η not exceed unity everywhere. The important role of the interest elasticity of money demand is discussed further in a moment.

[15]The term $(1+\rho)\mu'(\pi_s) - \mu'(\pi_s) = \rho\mu'(\pi_s)$ comes from the effect of the increase in π_s on money-demand growth in periods s-1 and s: given π_{s-1} and π_{s+1}, money demand growth falls by $\mu'(\pi_s)$ in period s-1 and rises by that amount in period s. The net effect on the period-s value of government revenues is negative and proportional to ρ. This "stock-shift" effect of a change in inflation on money demand was emphasized by Auernheimer (1974) in his discussion of the transition between inflationary steady states; its presence explains why the interest elasticity of money demand, not the expected-inflation elasticity, is relevant to the discussion here.

benefit from the foreseen one-period rise in inflation. The marginal cost of the foreseen increase in π_s is the sum of the consumption loss, $-\upsilon'(\pi_s)$, and the period-s welfare value of the reduction in real balances held during period s-1, $-(1+\rho)z'[\mu(\pi_s)]\mu'(\pi_s)$.

2. At an interior optimum, the interest elasticity of money demand, η, is always less than 1. To see this, imagine that the government has formulated a plan in which, in some period s, $\eta(\pi_s) \geq 1$. The argument in the preceding paragraph shows that a foreseen *decrease* in π_s leads to an *increase* in seigniorage over period s. There is thus a feasible plan in which inflation is no higher on all dates and lower on at least one. It follows that the original plan could not have been optimal.

3. The precommitment solution would be time inconsistent if the government were, in fact, unable to precommit to the plan chosen on date 1. To see why, suppose the public believes the date-1 plan when it is announced, but that on date 2, the authorities suddenly find themselves able to reoptimize. They can thus choose a new optimal initial inflation level π_2^{**} and the inflation rate π^{**} optimal for dates 3, 4, etc. Time consistency requires that $\pi_2^{**} = \pi^* = \pi^{**}$. Since (by assumption) $m_1 = \mu(\pi^*)$, the date-2 analogues of (7) and (8) imply that π_2^{**} and π^{**} satisfy

$$(9) \qquad \upsilon'(\pi^{**}) - \upsilon'(\pi_2^{**}) = \lambda_2^*\{\mu(\pi^*) - \mu(\pi^{**})[1 - \eta(\pi^{**})]\} - (1+\rho)z'[\mu(\pi^{**})]\mu'(\pi^{**}).$$

Because $\eta > 0$ (at positive inflation rates), and because the last term on the right-hand side of (9) is nonnegative, (9) cannot hold if the three inflation rates appearing in it are the same, as required for time consistency. Thus, the optimal policy under precommitment is time inconsistent: on date 2, the planner, if free to do so, would choose an initial inflation rate π_2^{**} different from the rate π^* previously planned on date 1.

The time-inconsistency problem is perhaps most striking when $v'(\pi) = 0$; this is the case studied by Calvo (1978b), in which price-level "surpises" are not costly and inflation inflicts harm exclusively through its role as a tax on real balances. Provided $\lambda_1^* > 0$, it is optimal to set π_1 at its maximal value of 1 in this case.[16] This policy amounts, in effect, to an official declaration on date 1 that the existing currency is valueless, an action that allows the government sell a "new" currency to the public for use in subsequent periods. Of course, when the next period arrives it becomes optimal to have another currency "reform" if $\lambda_2^* > 0$.[17]

Even when the government puts no weight on money-demand distortions $[z(m) = 0]$, but instead cares only about the current-output costs of inflation, equation (9) shows that the

[16]Since $\lambda_1^* m_0 > 0$ and, from the perspective of date 1, no social costs depend on π_1, it is optimal on that date to maximize current inflation. Obviously, the solution for inflation is not interior in this special case. If some $\pi_1 < 1$ left the government with enough interest income to cover its expenditure forever without recourse to inflation, λ_1^* would be zero.

[17]I owe this interpretation to Guillermo Calvo. (Notice that the date-1 price level, measured in the new currency, is indeterminate.)

precommitment solution is time inconsistent. In planning the future (constant) inflation rate π^* on date 1, the government, as shown above, balances marginal revenue--equal to $\mu(\pi^*)[1 - \eta(\pi^*)]$--against marginal cost each period. Once date 2 arrives, however, the marginal revenue from raising π_2 from the planned value of π^* is simply $\mu(\pi^*)$, while the marginal revenue from raising subsequent π's from π^* is still $\mu(\pi^*)[1 - \eta(\pi^*)]$. Marginal costs don't vary as time passes in the present case. The optimum, as seen from date 2, will thus call for an initial inflation rate higher than that in later periods, in contrast to the flat inflation rate from date 2 onward called for by the date-1 plan. In other words, on date 2 the government need no longer take account of the effect its choice of π_2 has on date-1 money demand.

III. Time-Consistent Equilibrium

The time inconsistency of the precommitment solution disqualifies it as an equilibrium of a dynamic economy in which all agents continually reoptimize. Private agents who understand the government's objectives and constraints will attempt to forecast the government's actions on the basis of its budgetary conditions, and perhaps other variables. The government, in turn, will consider the effects of its policies on private forecasts; but it is likely to view itself as influencing private forecasts only through current and past policies, and not through the mere announcement of future policies. In such a setting, equilibrium is most naturally analyzed in terms of policy and forecasting *rules* that relate actions to the variables defining the economy's current state. A time-consistent equilibrium is decribed by a policy rule that maximizes the government's objective function each period, given that private agents forecast on the basis of the rule and that the government takes the private forecasting rule to be beyond its control.

In this section, the foregoing notion of time-consistent equilibrium is explored. The Nash game considered here posits decision rules for private individuals that are parametric for the policymaker and that depend, through the system's state variables, on past and current policies, but not on announced future policies. These features of the setup ensure time consistency.

In the present context, the state of the economy on any date t is assumed to be fully described by the predetermined variables entering the government's budget constraint for that date, m_{t-1} and a_{t-1}. This assumption means that only policy and forecasting rules without memory are considered. The state of the economy could conceivably include (in addition to m_{t-1} and a_{t-1}) policy actions taken in the past or even extrinsic variables, but this study does not adopt so broad a view. In particular, possible reputational equilibriums of the type Rogoff (1989) critically reviews are ignored.[18]

[18]Barro and Gordon (1983) and Grossman and Van Huyck (1986) are among the papers examining reputational equilibriums. The reputational equilibrium idea is generalized by Chari and Kehoe's (1987) notion of a "sustainable plan," which allows players' reaction functions to depend on state vectors that may explicitly include past government actions along with the more conventional state variables (for example, asset stocks) usually considered. (Chari and Kehoe refer to these vectors as "histories.") The equilibrium singled out below corresponds to the Markov perfect equilibrium studied, in a quite different setting, by Maskin and

Given the foregoing exclusion of strategies with memory, it is natural to assume that individuals base forecasts of price-level change between any two periods s and s+1 on *aggregate* real balances m_s and on a_s, the same variables that determine the government's choice of π_{s+1}. If each individual j calculates his expected inflation by the rule $\pi^e_{s+1} = \hat{\epsilon}(m_s, a_s)$, and if the number of agents is, for convenience, normalized to 1, then aggregate money demand is

$$m_s = \Sigma_j m^j_s = \Sigma_j \ \mu(\pi^e_{s+1}) = \mu[\hat{\epsilon}(m_s, a_s)].$$

Under reasonable conditions (for example, that $\partial\hat{\epsilon}/\partial m > 0$), the equation in m_s above has a unique solution, $m_s = m(a_s;\hat{\epsilon})$, which is illustrated in figure 1.[19] Without loss of generality, therefore, one can assume that any government views current and future aggregate money demand as given by the "reduced-form" relationship

$$m_s = \mu\{\hat{\epsilon}[m(a_s;\hat{\epsilon}), a_s]\} \equiv \mu[\epsilon(a_s)],$$

so that $\pi^e_{s+1} = \epsilon(a_s)$ for all periods s in which government

Tirole (1988). Barro's (1983) essentially static analysis implicitly uses the latter equilibrium concept, but his model contains no dynamic state variables when strategies without memory are excluded.

[19]One would expect $\partial\hat{\epsilon}/\partial m$ to be positive for two reasons. First, a rise in last period's real balances raises the base for the inflation tax, thereby raising the marginal revenue from inflation. Second, a rise in last period's real balances, all else the same, lowers the increase in real money demand over the current period, thereby lowering government wealth and raising λ, the marginal value of government revenue.

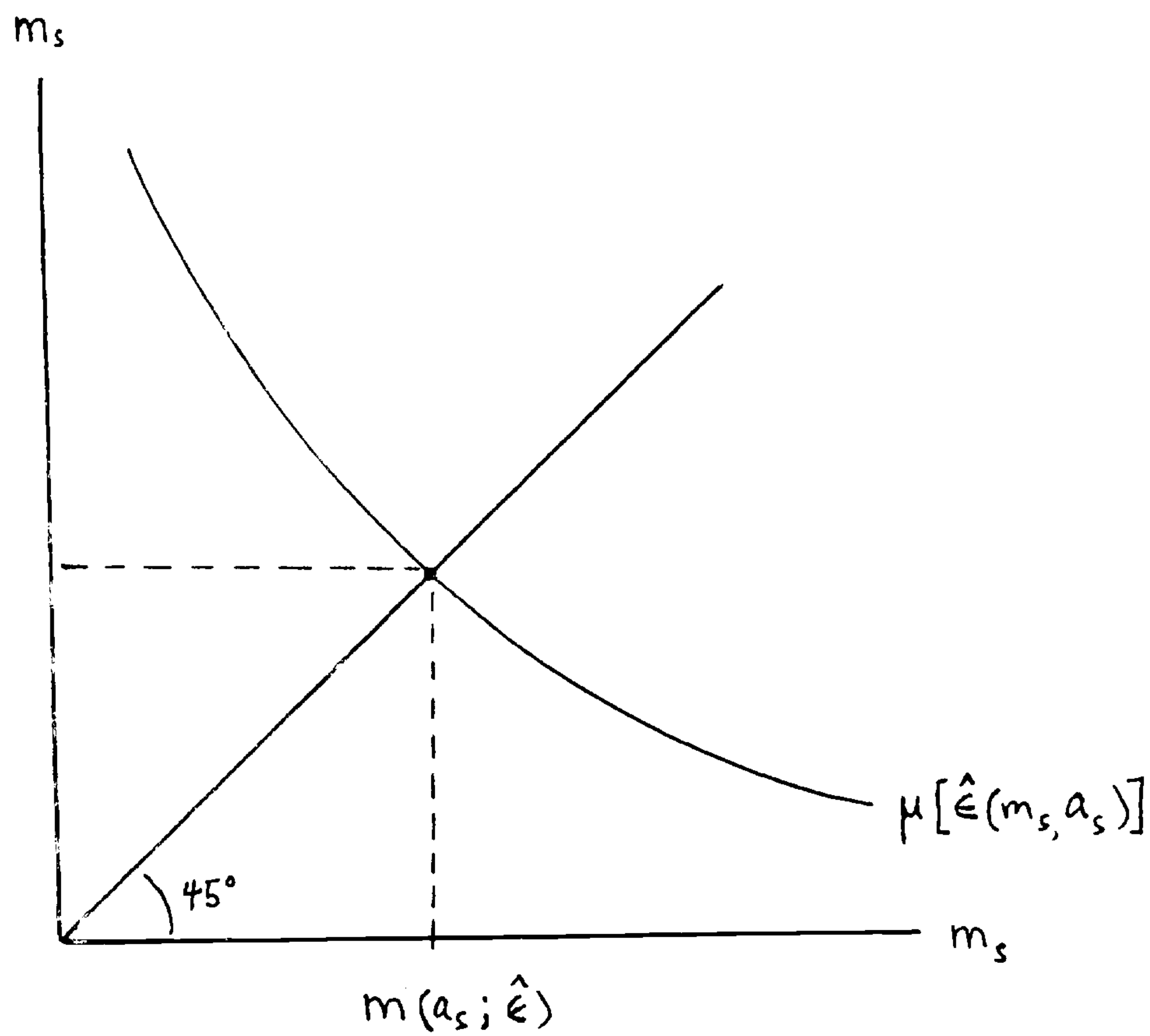


FIGURE 1: Determination of equilibrium aggregate real money balances

decisions still have to be made. Only the case $\epsilon'(a_s) < 0$ is relevant: higher public assets lead to lower expected inflation.[20]

A government planning on date 1, say, maximizes the function $\Sigma_{s=1}^{\infty}\beta^{s-1}[\upsilon(\pi_s) + z(m_s)]$ subject to $m_s = \mu[\epsilon(a_s)]$ (which it takes as a *given* constraint for $s \geq 1$) and the sequence (4) of finance constraints. If $\{\lambda_s\}_{s=1}^{\infty}$ is a sequence of multipliers associated with the finance constraints, the resulting Lagrangian is

$$L_1 = \upsilon(\pi_1) + z\{\mu[\epsilon(a_1)]\} + \sum_{s=2}^{\infty} \beta^{s-1}\Big(\upsilon(\pi_s) + z\{\mu[\epsilon(a_s)]\}$$

$$- \lambda_s\{a_s - (1+\rho)a_{s-1} + (1-\pi_s)\mu[\epsilon(a_{s-1})] - \mu[\epsilon(a_s)] + g\}\Big)$$

$$- \lambda_1\{a_1 - (1+\rho)a_0 + (1-\pi_1)m_0 - \mu[\epsilon(a_1)] + g\};$$

as discussed above, a_0 and m_0 are predetermined as of date 1. Differentiation with respect to π_1, π_s ($s > 1$), and a_s ($s \geq 1$) leads to the first-order conditions[21]

$$(10) \quad -\upsilon'(\pi_s) = \lambda_s m_{s-1} \qquad \text{(for } all\ s \geq 1\text{)},$$

$$(11) \quad \lambda_s = \beta(1+\rho)\lambda_{s+1} + \frac{z'\{\mu[\epsilon(a_s)]\}\mu'[\epsilon(a_s)]\epsilon'(a_s)}{1 - \mu'[\epsilon(a_s)]\epsilon'(a_s)}$$

$$+ \frac{\beta(\rho + \pi_{s+1})\mu'[\epsilon(a_s)]\epsilon'(a_s)\lambda_{s+1}}{1 - \mu'[\epsilon(a_s)]\epsilon'(a_s)},$$

[20]The reasoning in the previous footnote suggests that $\partial\hat{\epsilon}/\partial a < 0$. Since $\partial\hat{\epsilon}/\partial m > 0$ has been assumed, it follows that $\epsilon'(a) < 0$.

[21]For convenience, $\epsilon(a)$ is assumed to be differentiable.

which necessarily hold at an interior maximum. Of course, a feasible plan must also satisfy, for $s \geq 1$,

$$(12) \quad a_s = (1+\rho)a_{s-1} - (1-\pi_s)m_{s-1} + \mu[\epsilon(a_s)] - g.$$

Necessary condition (10) equates the marginal output cost of inflation to the shadow value of marginal revenue, which is proportional to last period's real balances. The second, intertemporal condition, (11), equates the current marginal value of public resources, λ_s, to the payoff from raising government saving by a unit in period s and then returning government assets to their initial path in period s+1.

The latter payoff can be viewed as the sum of three components. The first summand on the right-hand side of (11) is the *direct* marginal value product of an additional unit of saving, that is, the discounted gross rate of return on saving, $\beta(1+\rho)$, valued at next period's resource shadow price, λ_{s+1}. If this term described fully the marginal return to government saving, then since $\beta(1+\rho) = 1$, λ would be constant over time, as in the precommitment solution. The two remaining summands on the right-hand side of (11), however, capture additional, *indirect* effects of government saving on the government's objective function. Both indirect effects are a consequence of the assumption $\epsilon'(a_s) < 0$, which implies that higher government wealth, by lowering expected inflation, increases money demand.

Consider first the expression

$$\frac{z'(\mu[\epsilon(a_s)])\mu'[\epsilon(a_s)]\epsilon'(a_s)}{1 - \mu'[\epsilon(a_s)]\epsilon'(a_s)}$$

in (11). This term reflects the effect of an additional unit of government saving on the social utility derived from private real-balance holdings. Notice that the total indirect effect of additional government saving on government assets is not just $\mu'[\epsilon(a_s)]\epsilon'(a_s)$: that term measures only the first-round impact of higher period-s government saving on real money-demand growth over period s. There are further feedback effects that magnify this first round effect by a "multiplier" $1 - \mu'[\epsilon(a_s)]\epsilon'(a_s)$. To make the government's problem meaningful, the multipler process through which successive increments to a_s raise money-demand growth, raising a_s further, is assumed to converge, not explode. Convergence requires that

$$(13) \quad 1 > \mu'[\epsilon(a_s)]\epsilon'(a_s).$$

The last term in (11),

$$\frac{\beta(\rho + \pi_{s+1})\mu'[\epsilon(a_s)]\epsilon'(a_s)\lambda_{s+1}}{1 - \mu'[\epsilon(a_s)]\epsilon'(a_s)},$$

reflects the fact that the additional period-s money-demand growth induced by higher government saving in period s affects government revenue in periods s and s+1. All else equal, higher money-demand growth in period s implies lower growth in period s+1, but there is a net positive effect, proportional to $\beta\rho$, on the period-s present value of revenues. In addition, the inflation-tax base for period s+1 is higher, with a revenue effect proportional to π_{s+1}.

While the additional terms in (11) measure complicated effects, the basic message of the equation is simple: the marginal physical return to government saving exceeds the simple gross return on bonds, $1 + \rho$. As a result the government will be motivated to accumulate assets over time when $\beta = 1/(1+\rho)$. This is the main economic factor driving the declining time path of inflation described below. The rest of this section is devoted to analyzing the model's dynamic equilibrium.

To start, consider the implications of conditions (10)-(12) for the government's policy rule. Because $m_s = \mu[\epsilon(a_s)]$ for $s \geq 1$, equations (10)-(12) can be reduced to a (generally nonlinear) difference equation system in the costate variable λ and government assets a. Given m_0 and a_0, an initial costate value λ_1 determines π_1 [via (10) for s = 1] and thus a_1 [via (12) for s = 1]; the subsequent paths of λ and a are then determined by the first-order conditions, with inflation rates coming from (10). An optimal plan entails an initial choice λ_1^* such that the resulting path of the economy maximizes the government's objective function.

Let $\hat{V}(m_{s-1}, a_{s-1})$ be the value function for the government's problem at the start of period s, when the previous period's real balances and government assets are m_{s-1} and a_{s-1}, respectively. As shown in the appendix , the costate λ_s^* can be interpreted as

$\beta\partial\hat{V}(m_{s-1},a_{s-1})/\partial a_{s-1}$ (that is, λ_s^* is the shadow value of government wealth at the start of period s, and an extra unit of government wealth at the end of period s-1 implies, other things equal, $1/\beta$ extra units of wealth at the start of period s). In particular, $\lambda_1^* = \beta\partial\hat{V}(m_0,a_0)/\partial a_0$. Equation (10) therefore implies that inflation is chosen to satisfy the equation

$$-\upsilon'(\pi_s) = [\beta\partial\hat{V}(m_{s-1},a_{s-1})/\partial a_{s-1}]m_{s-1},$$

which implies, in turn, a government inflation rule of the form

$$\pi_{s+1} = \gamma(m_s,a_s;\hat{\epsilon}).$$

The reason for this rule's dependence on the private forecasting mechanism denoted by $\hat{\epsilon}$ is clear.[22]

A time-consistent *equilibrium* can now be defined. It is a function $\pi^e_{s+1} = \hat{\gamma}^*(m_s,a_s)$ such that $\gamma(m_s,a_s;\hat{\gamma}^*) = \hat{\gamma}^*(m_s,a_s)$. Kydland and Prescott (1977) explained how a time-consistent equilibrium can be viewed as the limit of a sequence of iterations in which, at each step, private forecasting rules are updated to reflect fully the policy rules that were optimal in the previous iteration, given the private forecasting rule then being used. In this equilibrium, it is optimal each period for the government to

[22]Recall that $\epsilon(a_s)$ depends on $\hat{\epsilon}$ through the equality $\hat{\epsilon}[\hat{m}(a_s;\epsilon),a_s] = \epsilon(a_s)$, where $m(a_s;\hat{\epsilon})$ is the solution to $m_s = \mu[\hat{\epsilon}(m_s,a_s)]$ (see figure 1).

use the rule $\pi_{s+1} = \hat{\gamma}^*(m_s, a_s)$, given that the private sector forms its expectations using the same rule, $\pi^e_{s+1} = \hat{\gamma}^*(m_s, a_s)$. No general results concerning the existence *or uniqueness* of this type of equilibrium seem to exist, except in the linear case (see Cohen and Michel). Section IV, below, contains a linear example that can be viewed as an approximation to the present model.[23]

Having defined a time-consistent equilibrium, the next step is to characterize it. Suppose that the public bases its forecasts on the rule $\pi^e_{s+1} = \hat{\gamma}^*(m_s, a_s)$, which, at the same time, describes the government's best inflation response given this rule. As shown earlier, the forecasting rule can be expressed, in equilibrium, in the alternative form $\pi^e_{s+1} = \gamma^*(a_s)$, where $\gamma^*(a_s)$ solves the equation in π^e_{s+1}, $\pi^e_{s+1} = \hat{\gamma}^*[\mu(\pi^e_{s+1}), a_s]$. Along the equilibrium path, the actual inflation rate π_s is given by $\hat{\gamma}^*(m_{s-1}, a_{s-1}) = \hat{\gamma}^*\{\mu[\gamma^*(a_{s-1})], a_{s-1}\} = \gamma^*(a_{s-1})$.

Return now to the intertemporal necessary condition (11). Use (10) to eliminate λ_s and λ_{s+1}, substitute $\gamma^*(a_s)$ for the arbitrary expectation rule $\epsilon(a_s)$, and rearrange to obtain:

[23]If there are multiple time-consistent equilibriums, it is at least conceivable (but far from obvious) that the government will be a dominant player, in the sense that the public will follow it if it announces a time-consistent plan. Such a dominant-player role could allow the government to coordinate expectations on the time-consistent equilibrium that leads to the highest value of its objective function. Barro and Gordon (1983, p. 112) seem to have this idea in mind when they assume, in their setting, that the government can choose the reputational inflation equilibrium it prefers.

$$
-\frac{\upsilon'[\gamma^*(a_{s-1})]}{\mu[\gamma^*(a_{s-1})]} = -\frac{\upsilon'[\gamma^*(a_s)]}{\mu[\gamma^*(a_s)]}\left\{1+\frac{\beta[\rho+\gamma^*(a_s)]\mu'[\gamma^*(a_s)]\gamma^{*\prime}(a_s)}{1-\mu'[\gamma^*(a_s)]\gamma^{*\prime}(a_s)}\right\}
$$

$$
(14) \qquad + \frac{z'(\mu[\gamma^*(a_s)])\mu'[\gamma^*(a_s)]\gamma^{*\prime}(a_s)}{1-\mu'[\gamma^*(a_s)]\gamma^{*\prime}(a_s)} .
$$

To interpret this condition further, note that if expectations are unresponsive to government assets $[\gamma^{*\prime}(a_s) = 0]$, the result is the condition $\upsilon'(\pi_s)/m_{s-1} = \upsilon'(\pi_{s+1})/m_s$ appearing in Mankiw (1987) and Grilli (1988a, 1988b): it is optimal for the government to stabilize the ratio of the marginal output cost of inflation to the inflation-tax base, which, with constant money demand, would require a flat inflation path and an unchanging government asset level. Since the government is taking money demand as given in this special case, inflation costs caused by reductions in the utility from money use [and represented by the function $z(m)$] do not appear in the intertemporal smoothing condition.

When $\gamma^{*\prime}(a_s) < 0$, however, (14) shows that the ratio $\upsilon'(\pi_s)/m_{s-1}$, and thus inflation itself, declines over time. As observed earlier, additional government saving now yields a gross physical return $1+\rho$ *plus* the extra benefits from the induced increase in money demand [see (11)]. Since the government's discount factor is just $1/(1+\rho)$, the government will accumulate bonds, which it can do by always setting inflation higher than the level consistent with an unchanging stock of government assets.

Let $A(a_s)$ denote the right-hand side of equation (14) and $B(a_{s-1})$ the left-hand side. Then on the assumption that $A(a_s)$ is invertible, the difference equation

$$A(a_s) = B(a_{s-1})$$

describes the evolution of government assets along a time-consistent equilibrium path for $s \geq 2$.

It is simple to find the stationary state of this system, $\bar{a}$. If $\bar{a} = g/\rho$, the government can finance its consumption without recourse to the inflation tax. It has also been assumed that at the level of real balances $\mu(0)$ corresponding to an inflation rate of zero, $z'[\mu(0)] = 0$. Thus, $\gamma^*(\bar{a}) = 0$ in any time-consistent equilibrium. Equation (14) confirms that $\bar{a} = g/\rho$ is the long-run equilibrium stock of government assets.

Figure 2 illustrates the dynamics of convergence (in the relevant region where $a < \bar{a}$) under the assumption that $\gamma^{*\prime}(a) < 0$. Equation (14) shows that for any asset level $a < \bar{a}$, $A(a) > B(a)$. Since $A(a)$ has been assumed invertible, there is a unique path converging to budget balance along which government assets rise--and inflation falls--monotonically. Notice that the path described by figure 2 is shown as beginning with a_1 rather than with a_0. The asset stock a_1 depends on a_0 (and m_0), however, through the government's finance constraint and its initial inflation choice.

It was mentioned in section I that the above model is special

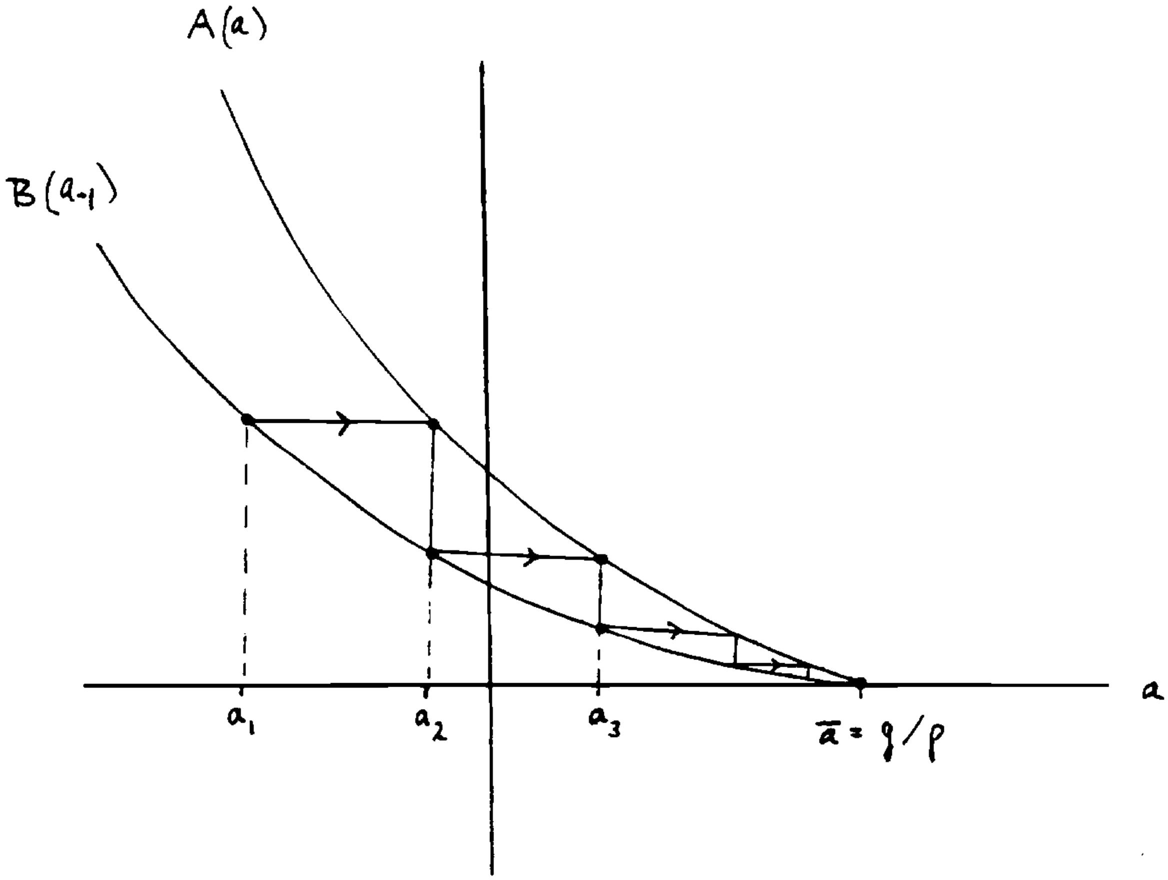


FIGURE 2: Convergence to the long-run equilibrium $\bar{a}$ follows the difference equation $A(a_s) = B(a_{s-1})$

in assuming that the government and private sector share a common discount rate. Under plausible assumptions, however, the government discount rate will exceed the private rate. For example, policymakers may discount the future more heavily than private agents because they face some probability of being removed from office. In formal terms, a positive difference between the government and private discount rates amounts to lowering β in (14) while holding ρ constant. Clearly, $\bar{a} = g/\rho$, with $\gamma^{*}(\bar{a}) = 0$, remains a stationary state of the model; however, the $A(a_s)$ schedule in figure 2 may now cross the $B(a_{s-1})$ schedule to the left of the origin, a possibility that would give rise to multiple stationary states, alternately stable and unstable.

Notice that the model does *not* imply that governments always run surpluses. As in any model with some tax smoothing, the time pattern of deficits depends on the time pattern of government spending which, for simplicity, was assumed above to be flat. The model is easily extended to account for an exogenously varying and perfectly foreseen path of government spending. Just respecify the equations in terms of a state variable equal to government assets *less* the present discounted value of future government spending:

$$(15) \quad a_s - \sum_{h=0}^{\infty} (1+\rho)^{-h+1} g_{s+h+1}.$$

Rewriting the model in terms of the state variable (15) leads to some predictions about the relation between expenditure changes and deficits. Sufficiently large, sufficiently temporary innovations in public spending, for example, will induce the government to engage in partial tax smoothing by raising inflation and running a larger deficit. (Inflation will be more sensitive to expenditure innovations than in a model with *complete* tax smoothing, however.) Once spending has returned to normal, however, inflation will not remain at a permanently higher level, but will instead tend to come down over time.

The steady-state of the economy is now one in which the deficit is zero *on average*: absent unexpected shocks, asset income just covers the annuity value of the government expenditure path, and there is no need ever to use the inflation tax.

IV. A Linear Example

A closed-form linear example illustrates the characteristics of the equilibrium described in the last section. For the purpose of the example, I assume that $z(m) = 0$, so that inflation reduces welfare only through its negative effect on current output.

The key trick in obtaining a linear example is to redefine the model, not in terms of inflation, but in terms of the inflation tax levied on money holders, τ_s, where

$$\tau_s = \pi_s m_{s-1}.$$

Major assumptions are that the aggregate demand for real balances is a linear decreasing function of the expected inflation tax,

$$(16) \quad m_s = \delta_0 - \delta_1 \tau^e_{s+1},$$

and that the output cost of inflation is given by the function

$$\kappa(\tau_s) = (1/2)\tau_s^2.$$

The government is assumed to maximize

$$(17) \quad -(1/2) \sum_{s=t}^{\infty} \beta^{s-t} \tau_s^2.$$

Formulation (16) is sensible if the interest elasticity of money demand is low enough that inflation-tax proceeeds and inflation do move together. As an example, suppose that individual j's money demand is

$$m_s^j = \left(\frac{\rho + \pi^e_{s+1}}{1 + \rho} \right)^{-1/\theta} = (1 - \beta + \beta\pi^e_{s+1})^{-1/\theta} \qquad (\theta > 1),$$

as it is when the private utility-of-money function is $v(m) = (1-\theta)^{-1} m^{1-\theta}$ and the interest elasticity of money demand is less than 1. Since $\tau^e_{s+1} = \pi^e_{s+1} m_s$, this money demand function can be expressed as $m_s^j = m_s^{1/\theta}[(1-\beta)m_s + \beta\tau^e_{s+1}]^{-1/\theta}$. Linearizing in a neighborhood of $\pi = 0$ leads to the approximate equality

$$m_s^j = (1-\beta)^{-1/\theta} - \frac{\beta\tau^e_{s+1}}{\theta(1-\beta)},$$

which, when summed over all individuals, is of form (16).

One assumption that will be needed to obtain a solution is that

(18) $\delta_1 < 1/\rho$;

this inequality certainly holds in the example just given [since $\rho = (1-\beta)/\beta$] when $\theta > 1$. Inequality (18) plays the same role here as did the assumption $\eta < 1$ in previous sections: it ensures that a fall in inflation-tax revenue does not raise total seigniorage σ.

The solution method is to solve first for the time-consistent inflation policy *along an equilibrium path*, that is, a path on which past as well as current expectations of future inflation taxation coincide with the policy. Once a solution along equilibrium paths is in hand, a general solution, valid for arbitrary values of the economy's state variables, can be inferred. Define the variable d_s as $\Sigma_{h=0}^{\infty}(1+\rho)^{-h+1}g_{h+s+1} - a_s$ [as in (15)]; d_s can be thought of as the government's debt inclusive of future expenditure commitments. The rule being sought has the general form $\tau_s = \hat{\gamma}(m_{s-1},d_{s-1})$; but along an equilibrium path $m_{s-1} = \delta_0 - \delta_1\hat{\gamma}(m_{s-1},d_{s-1})$, so the reduced-form policy rule makes τ_s a function of the beginning-of-period government debt alone, $\tau_s = \gamma^*(d_{s-1})$, with $\gamma^{*\prime}(d) > 0$ on the way to the steady state.

It is natural to conjecture a linear reaction function for the government,

(19) $\tau_s = \gamma d_{s-1}$;

in equilibrium, private expectations must obey

$$(20) \quad \tau^e_{s+1} = \gamma d_s .$$

These two equations lack intercept terms because the government will always choose $\tau_s = 0$ when $d_{s-1} = 0$, that is, when asset income fully finances public spending needs.

To determine γ, one can use a necessary condition for a time-consistent optimal plan: if the public follows (20), and if (19) is to be followed by the government in the future, (19) must be optimal today. This requirement will be shown to determine a unique coefficient in (19) and (20), $\gamma = \gamma^*$.[24]

Step one is to find the value function $V(d_t)$ for a government that starts out with inherited debt d_t at the beginning of period t+1, given that (19) is followed for s > t and that expectations are determined by (20) for s ≥ t. [V(d) is therefore the value function along an equilibrium path.] Under (17),

$$V(d_t) = -(1/2) \sum_{s=t+1}^{\infty} \beta^{s-t-1} (\gamma d_{s-1})^2 .$$

By using the government finance constraint,

$$d_s = (1 + \rho) d_{s-1} - (m_s - m_{s-1}) - \tau_s$$

[24]An alternative procedure, which leads to the same solution, is to calculate directly the government's optimal policy when it takes the public's forecasting rule as given.

$$= \gamma\delta_1 d_s + [(1 + \rho) - \gamma(1 + \delta_1)]d_{s-1}$$

$$= \frac{(1 + \rho) - \gamma(1 + \delta_1)}{1 - \gamma\delta_1} d_{s-1} ,$$

to solve for future levels of d in the preceding expression, one obtains

$$(21) \quad V(d_t) = -(1/2) \sum_{s=t+1}^{\infty} \beta^{s-t-1}(\gamma\psi^{s-t-1}d_t)^2$$

$$= -(1/2) \frac{\gamma^2 d_t^2}{1 - \beta\psi^2} ,$$

where

$$(22) \quad \psi = \frac{(1 + \rho) - \gamma(1 + \delta_1)}{1 - \gamma\delta_1} .$$

Notice that the private sector's forecasting rule (20) has been used, along with (19), in calculating $V(d_t)$. It will be verified at the end of the exercise that $\gamma^*\delta_1 < 1$ and that $0 < \psi < 1$, so that the debt process is directly convergent along the equilibrium path. [An inequality corresponding to $\gamma^*\delta_1 < 1$, inequality (13), was assumed in the preceding section.][25]

At the start of period t, the government's problem is to

[25]Here, $\gamma^*\delta_1$ is the marginal effect of higher government assets on real money demand. The derivative $\mu'[\epsilon(a_s)]\epsilon'(a_s)$ in (13) also measures this effect.

choose τ_t so as to maximize

$$-(1/2)\tau_t^2 + \beta V(d_t)$$

subject to

$$d_t = \frac{[(1 + \rho) - \gamma\delta_1]d_{t-1}}{1 - \gamma\delta_1} - \frac{\tau_t}{1 - \gamma\delta_1} .$$

At this point that the assumption of a continuing equilibrium path is used again, since the above expression for d_t is based on substituting $\delta_0 - \gamma\delta_1 d_{t-1}$ for the time-t state variable m_{t-1}. By (21), the first-order condition for an optimum is

$$-\tau_t + \frac{\beta\gamma^2}{(1 - \beta\psi^2)} \left\{ \frac{[(1 + \rho) - \gamma\delta_1]d_{t-1}}{(1 - \gamma\delta_1)^2} - \frac{\tau_t}{(1 - \gamma\delta_1)^2} \right\} = 0,$$

which can be solved to obtain

$$(23) \qquad \tau_t = \frac{\beta\gamma^2[(1 + \rho) - \gamma\delta_1]}{\beta\gamma^2 + (1 - \beta\psi^2)(1 - \gamma\delta_1)^2} d_{t-1}.$$

Equation (23) has the same form as the assumed rule (19); time consistency requires that the parameter γ in (19) equal the coefficient of d_{t-1} in (23), that is, that it satisfy the quadratic equation

$$(24) \qquad \phi(\gamma) = \delta_1(1 - \rho\delta_1)\gamma^2 - (1 + \rho)\gamma + \rho(1 + \rho) = 0.$$

[Equation (24) is derived by using (22) to eliminate ψ in (23).]

Equation (24) has two roots, given by

$$\gamma = \frac{(1+\rho) \pm \sqrt{(1+\rho)^2 - 4\rho\delta_1(1 - \rho\delta_1)(1+\rho)}}{2\delta_1(1 - \rho\delta_1)} .$$

Both roots are positive and real,[26] but it is the smaller of the two that is the optimal policy coefficient γ^*. To rule out the larger root, consider the graph of equation (24), which can be drawn with the help of the derivative

$$\phi'(\gamma) = 2\delta_1(1 - \rho\delta_1)\gamma - (1+\rho).$$

Clearly, $\phi(\rho) > 0$, $\phi'(\rho) < 0$ (see the last footnote), and $\phi'(\gamma) < 0$ for all $\gamma < \rho$. At $\gamma = 1/\delta_1$, however, $\phi(\gamma)$ is negative: $\phi(1/\delta_1) = -(\rho/\delta_1)(1 - \rho\delta_1) < 0$. Figure 3 therefore describes how the roots of (24) are determined. The fact that the larger root necessarily exceeds $1/\delta_1$ means that it can be ruled out as a meaningful solution. Clearly, the smaller root of (24), γ^*, is below $1/\delta_1$.

Because γ^* is above ρ in the time-consistent equilibrium, it is optimal for the authorities to accumulate assets when $d > 0$ by extracting seigniorage at a rate greater than the rate

[26]The condition for real roots is that $(1+\rho) > 4\rho\delta_1(1 - \rho\delta_1)$; however, $\rho\delta_1(1 - \rho\delta_1)$ reaches its maximum at $\rho\delta_1 = 1/2$, so the inequality is always satisfied for positive ρ.

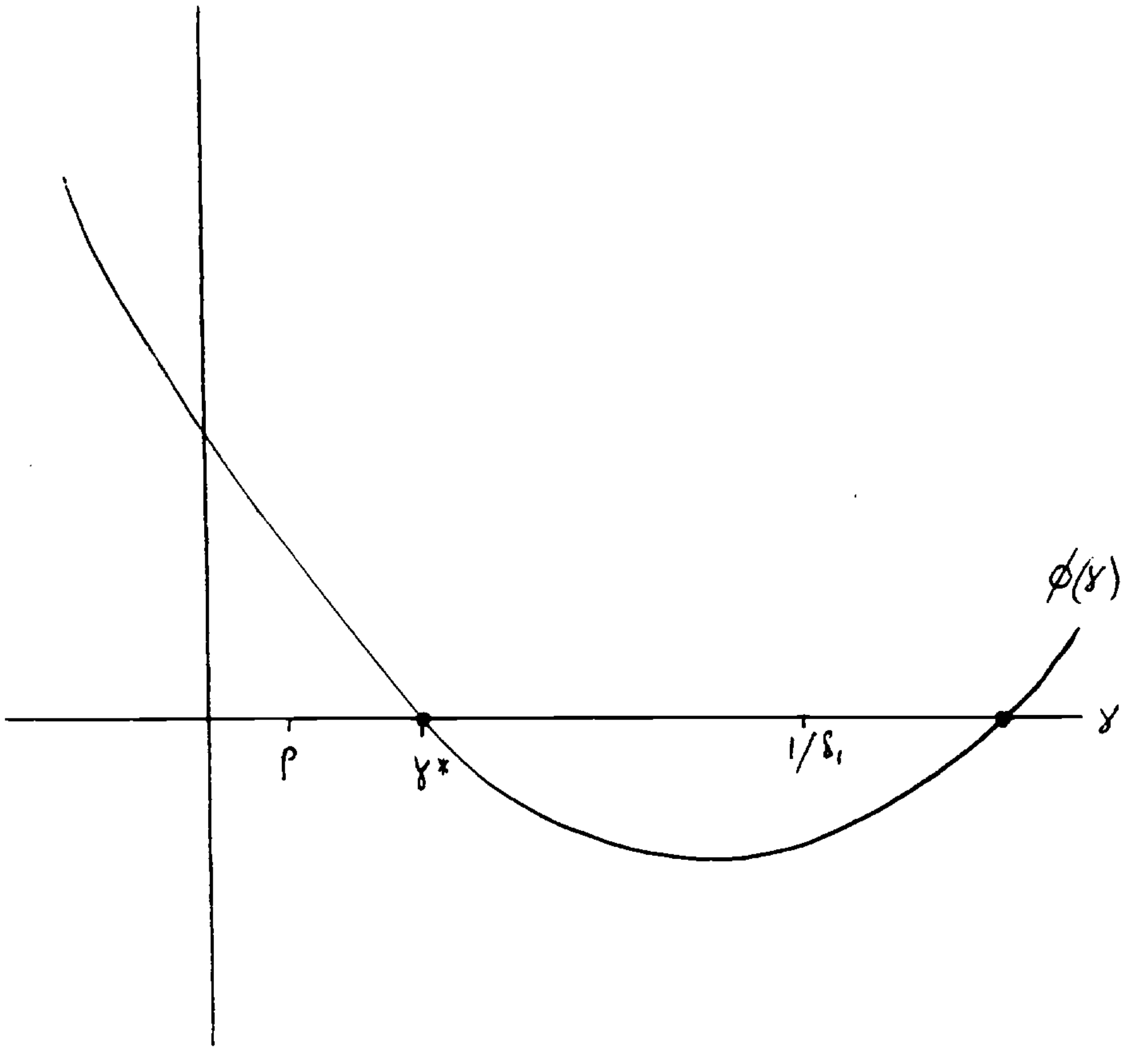


FIGURE 3: Zeroes of the quadratic function $\phi(\gamma) =$

$$\delta_1(1 - \rho\delta_1)\gamma^2 - (1 + \rho)\gamma + \rho(1 + \rho)$$

that would hold d constant, ρd. It is straightforward to check that when $\gamma = \gamma^*$ in (22), $0 < \psi < 1$. Debt evolves according to $d_s = \psi d_{s-1}$ along the equilibrium path, so for the relevant value of ψ, the debt variable converges monotonically to zero. The government thus reduces its (average) deficit to zero over time, as indicated by figure 2.[27] The inflation tax also converges to zero according to the equation $\tau_s = \psi \tau_{s-1}$.

As noted earlier, γ^* describes optimal policy, and optimal private expectations, along the economy's equilibrium path. In general, however, the economy could begin out of equilibrium, with the previous period's real balances m_{s-1} *not* given by $\delta_0 - \delta_1 \gamma^* d_{s-1}$. It can be verified that in this case the optimal policy rule for the government is

$$\tau_s = \hat{\gamma}^*(m_{s-1}, a_{s-1}) = \zeta[\gamma^* m_{s-1} + \gamma^*(1+\rho)d_{s-1} - \gamma^* \delta_0],$$

where $\zeta = (1 + \rho - \gamma^* \delta_1)^{-1}$.

A numerical example illustrates the properties of the time-consistent solution. For the purpose of this example, β is taken to be 0.96 (corresponding to a real interest rate, ρ, of 0.0417 per period) and δ_1 is taken to be 12 (corresponding to a money-demand interest elasticity of 1/2 when the utility of real

[27]Equation (22) shows that $\psi \in (0,1)$ when $\gamma^* < (1+\rho)/(1+\delta_1)$ and $\rho < \gamma^*$. The second inequality has already been established; the first turns out (after tedious algebra) to be equivalent to $\rho\delta_1(2 - \rho\delta_1) < 1$. The function $\rho\delta_1(2 - \rho\delta_1)$ reaches a maximum of 1 when $\rho\delta_1 = 1$, however, and the model assumes that $\rho\delta_1 < 1$.

money balances is isoelastic). It is assumed further that the economy starts out on the equilibrium path on date 1 (that is, with $m_0 = \gamma^* d_0$).

With the above parametrization, $\gamma^* = 0.0695$ and ψ [given by (22)] = 0.8319. In the time-consistent equilibrium, government debt therefore follows the difference equation $d_s = 0.8319\ d_{s-1}$; by the linearity of the example, inflation-tax revenue follows this same difference equation, starting from the level $0.0695\ d_0$.

V. Nominal Government Debt

The model has assumed until now that all public debt takes the form of promises to deliver specified amounts of real output on future dates. In many countries, however, much (or most) public debt is denominated in local currency, not output. This section briefly sketches some ways in which the presence of home-currency government debt could change conclusions reached above.

There are two main differences compared with the previous analysis. First, nominal debt may lead to multiple equilibriums through a channel absent above, the effect of inflation expectations on nominal interest rates.[28] If all public debt is nominal, however, there still appears to be a time-consistent equilibrium in which inflation falls over time.

The second main difference, which I regard as a theoretical curiosity because of its knife-edge nature, arises when the government can issue indexed and nominal debt side by side. Once

[28]Calvo (1988a) analyzes a two-period model in which such multiplicities can occur.

the menu of financial instruments is expanded in this way, the government may be able to attain the precommitment allocation, *even under discretion*, through sophisticated open-market operations in real and nominal bonds.

Multiple equilibriums. Suppose first that the government can buy and issue nominal debt only. A nominal bond issued in period s is a promise to pay $1+i_s$ currency units in period s+1, where i_s is the period-s nominal interest rate. If a^n_s denotes the real value of net nominal government claims on the public at the end of period s, the government's finance constraint takes the form

$$(25) \quad a^n_s = (1 - \pi_s)[(1 + i_{s-1})a^n_{s-1} - m_{s-1}] + m_s - g$$

for a constant g. [It is clear from (25) that the real inflation-tax base in period s now includes nominal public debt obligations, equal to $-(1+i_{s-1})a^n_{s-1}$, along with lagged real balances.] Given a nominal interest rate i_{s-1} that is predetermined from the perspective of date s, a government that is a net debtor in nominal terms ($a^n_{s-1} < 0$) has an additional incentive to engineer surprise inflation in period s (an unexpectedly low value of $1-\pi_s$). Of course, it will be true in equilibrium that

$$(26) \quad 1 + \rho = (1 - \pi^e_s)(1 + i_{s-1})$$

(the Fisher equation), where π^e_s is the rational forecast of the government's preferred inflation rate.

To adapt the earlier discussion to the case of nominal debt, notice the implication of (25) that the sole state variable relevant to government decisions in period s+1 is the real stock of nominal government assets (net of monetary liabilities) at the start of period s+1, $(1+i_s)a^n_s - m_s$. Individuals will therefore forecast π_{s+1} according to some rule $\pi^e_{s+1} = \hat{\epsilon}[(1+i_s)a^n_s - m_s]$; and by analogy with the tack taken before, it is natural to seek a reduced-form rule $\pi^e_{s+1} = \epsilon(a^n_s)$ governing expectations formation along an equilibrium path. Use of Fisher equation (26) shows that such a rule is the solution to the equation in π^e_{s+1},

$$(27) \quad \pi^e_{s+1} = \hat{\epsilon}[(1+\rho)a^n_s/(1-\pi^e_{s+1}) - \mu(\pi^e_{s+1})] \equiv \zeta(\pi^e_{s+1},a^n_s).$$

If the government is indebted to the public ($a^n_s < 0$), equation (27) may have multiple solutions, an outcome that could lead to multiple equilibriums. Figure 4 (which assumes that $\hat{\epsilon}' < 0$) illustrates this possibility. Notice that the slope of $\zeta(\pi^e_{s+1},a^n_s)$ is ambiguous when $a^n_s < 0$ because a rise in π^e_{s+1} both raises next period's public debt-service obligations, raising the incentive to inflate, and lowers real balances, lowering the incentive to inflate. In the case shown, $\zeta(\pi^e_{s+1},a^n_s)$ has two intersections with the 45^o line; but more generally, it could have more than two intersections or none.[29]

[29] I have pushed aside issues of term structure in assuming that all nominal debt matures after one period. Calvo (1988b) has shown that even under a constant real interest rate, as here, the government may have incentives to issue longer-term obligations when all public debt is currency-denominated. The presence of such

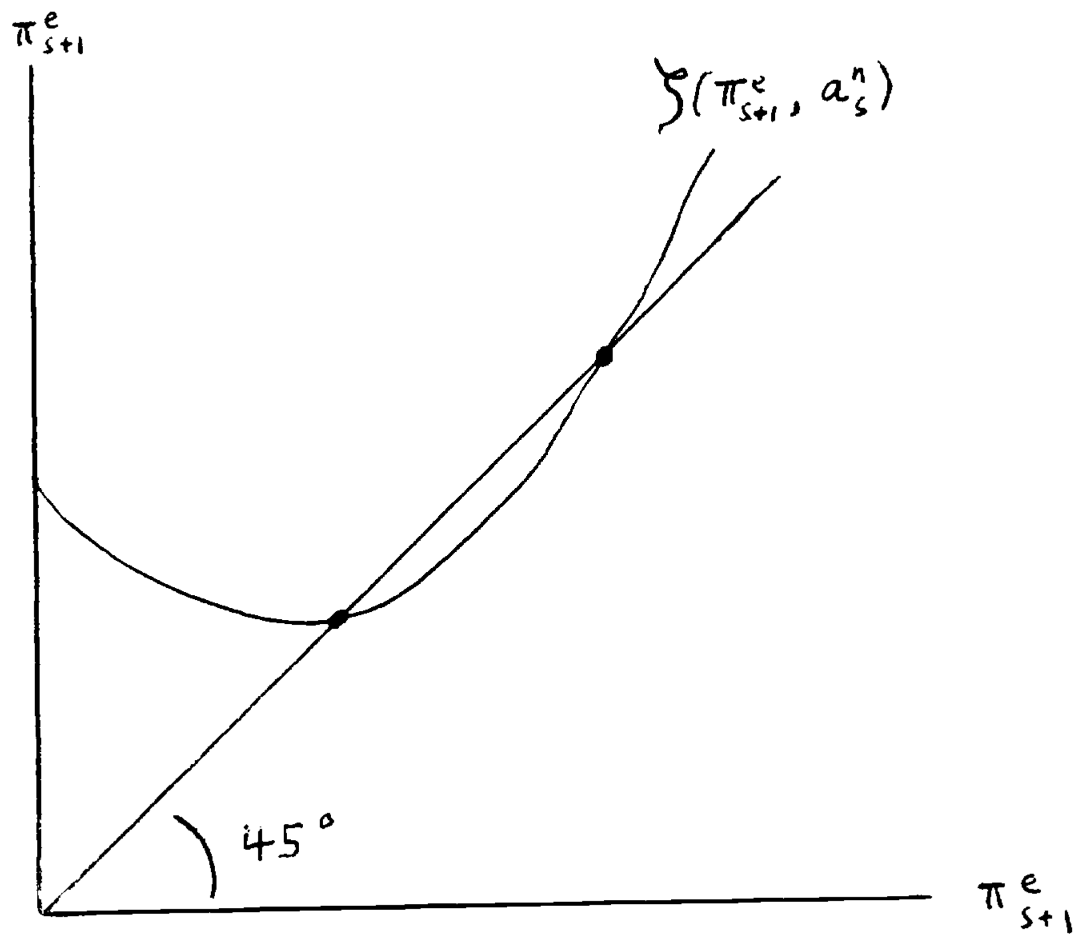


FIGURE 4: When government debt is nominal, it may be impossible to express inflation expectations uniquely as a function of real government assets

Only at the low-inflation intersection in figure 4 is it the case that higher government assets a_s^n (which shift the ς-function downward) lower equilibrium inflation expectations; at the high-inflation intersection, public asset accumulation appears to worsen the inflation outlook. Given a favorable outcome in figure 4, however, the government has the additional motives for asset accumulation that drive the results of the last two sections. If a time-consistent equilibrium exists in this case, one would therefore expect the earlier analysis to go through much as before, despite the currency denomination of the public debt.

Equilibrium dynamics in an economy where the unfavorable forecasting rule is adopted remain mysterious. It seems doubtful to me that a time-consistent equilibrium (in which the intertemporal government budget constraint is met) can exist in a setting where increasing public deficits are accompanied by falling inflation.[30]

Attaining the precommitment path. When the government can deal in both nominal and real securities, it gains the ability to

obligations could affect the number of equilibriums. For example, given a nominal public debt in the form of *consols*, the analogue of equation (29) has a unique solution because $\partial\varsigma/\partial\pi^e_{s+1} < 0$ everywhere. (Higher inflation expectations would lower the real present value of future public debt-service obligations if debt took the form of nominal consols.) In Calvo's (1988b) model, the presence of real as well as nominal government debt, as in the next case considered below, removes the motivation for a richer nominal term structure, given the fixed real interest rate.

[30] Of course, there could be a third, higher-inflation intersection in figure 4, at which the ς-function crossed the 45^o line from above. There, just as at the low-inflation intersection, higher government assets would lower equilibrium expected inflation.

rearrange its bond portfolio so as to influence the incentives it will face in future periods. Open-market switches between real and nominal debt, at a given price level, alter the inflation-tax base faced in the following period, and may allow the government to sustain the precommitment solution as an equilibrium.

If the government's price-level indexed bond holdings are a_s and its currency-denominated bond holdings are a^n_s, its finance constraint is

$$(28) \quad a_s + a^n_s = (1+\rho)a_{s-1} + (1-\pi_s)[(1+i_{s-1})a^n_{s-1} - m_{s-1}] + m_s - g.$$

As in section II, first-order conditions for the precommitment solution are

$$(29) \quad -v'(\pi_1) = \lambda_1[-(1+i_0)a^n_0 + m_0],$$

and, for s > 1, equation (8), which is repeated below:

$$-v'(\pi_s) - (1+\rho)z'[\mu(\pi_s)]\mu'(\pi_s) = \lambda_1\mu(\pi_s)[1 - \eta(\pi_s)].$$

(The government takes (26) into account for s > 1 when it maximizes on date 1, since it cannot surprise the public in future periods under precommitment.)

Suppose that on date 1 the government rearranges its asset portfolio so that a^n_1 satisfies

$$\lambda_1^*[-(1+i^*)a_1^n + m^*] = (1+\rho)z'[\mu(\pi^*)]\mu'(\pi^*) + \lambda_1^*[1 - \eta(\pi^*)],$$

where starred values are the (time-invariant) values associated with the government's date-1 optimization. Then (8) and (29) imply that when the government reoptimizes on date 2, its preferred inflation rate remains π^*; and by maintaining its nominal assets at a_1^n forever, the government can ensure that the date-1 precommitment plan remains optimal in all future periods.

Intuitively, the government buys enough nominal assets on date 1 to eliminate the incentive to inflate beyond the precommitment level on dates 2, 3, etc. Notice that the government retains some net home-currency liability under the foregoing plan: since $\pi^* > 0$, (29) shows that $m^* - (1+i^*)a_1^n > 0$.

In a nonmonetary setting, Lucas and Stokey (1983) suggested a method of managing the term structure of government debt that sustains a precommitment optimum under discretion. Persson, Persson, and Svensson (1987) argued that the natural extension of the Lucas-Stokey idea to a monetary economy involves an exact balancing of the government's net nominal bond holdings against its monetary liabilities, that is, a zero net position in home currency. Because Persson, Persson, and Svensson assumed away any explicit cost of inflation surprises, a zero net position in home currency appears necessary, in their framework, to ensure time consistency; but as Calvo and Obstfeld (1988) have shown, that prescription is also sufficient only under quite unrealistic conditions. Appropriate debt management avoids time inconsistency above because the real interest rate is fixed (so that the term-structure issues raised by Lucas and Stokey don't arise), because all nominal debt matures after a single period, and because unanticipated as well as anticipated inflation is socially costly.

Real interest rates vary in practice and governments do deal in long-term instruments; so the above scheme cannot be taken literally as describing official behavior in actual economies.[31] The example nonetheless highlights incentives that may infuence governments to some degree. Unfortunately, the problem of modeling equilibrium with both real and nominal securities appears to be quite difficult once a setting of any generality is assumed. Success in this area would yield, not only positive predictions about government deficits and inflation, but insight into the mix of real and nominal government debt issue actually observed.

It is worth noting that the paper's central conclusions about the dynamics of time-consistent equilibriums would probably survive the introduction of a variable real interest rate. With a flexible marginal utility of consumption, a declining inflation path would imply rising output and a *falling* real interest rate. A tendency for the real interest rate to fall would, in turn, reinforce the tendency for public-sector deficits and inflation to decline over time. Such an extension of the present model could help explain the "stylized" fact that real interest rates are high at the beginning of hyperinflation stabilization programs.

[31]This is probably fortunate, since few (if any) governments have positive net holdings of domestic-currency bonds.

VI. Conclusion

This paper has explored the dynamic behavior of seigniorage in a time-consistent equilibrium with government borrowing and lending. A major positive implication of the paper is that in the absence of unforeseen shocks, governments will shrink their budget deficits over time so as to reduce the need for inflationary finance (or, for that matter, for other distorting taxes).

Before matching theory to data, stochastic disturbances (to government spending requirements, say) would have to be worked into the model. At least as an approximation, such a model should replicate the stationary behavior of inflation evident in chart 1 of section I. The underlying cause of this behavior--that in a time-consistent equilibrium, there are incentives for debt reduction beyond the physical return on assets--should still operate in a stochastic setting.

Although the model yields predictions consistent with the long-term behavior of U.K. and U.S. prices, it is less clear that it captures well the apparently chronic inflation in many developing countries, where seigniorage is typically more important than in the industrialized world. The model helps explain, however, why governments in budgetary crisis often sharply devalue their currencies in the foreign exchange market, thereby spurring domestic inflation but (hopefully) promoting increases in official foreign reserves.[32] A partial rationale for devaluing at the outset of stabilization is to lower *future*

[32]See Kiguel and Liviatan (1988), who discuss some recent episodes.

inflation by objectively improving the budgetary situation and shifting expectations--just as in the account given above.

Much government-caused inflation is not linked to seigniorage, official preferences change over time, and inflation is subject to serially correlated shocks beyond government control. So at best, the theory set out above explains one of the underlying tendencies driving inflation, not all of inflation. Employment and distribution goals, two factors absent from the paper's model, appear particularly important.[33] Nonetheless, the paper seems useful as a first attempt at explaining one factor--the key one in some countries--driving the dynamics of inflation under discretionary policy formulation.

[33]The results of table 1 show long-run inflation rates for the U.K. and U.S. that are positive rather than zero. This result is inconsistent with a literal interpretation of the paper's model, but could be explained by a government target inflation rate that (for some reason) exceeds zero, or by policy considerations independent of the public finances that influence inflation choices.

Appendix. Interpreting the Government's Value Function

This appendix derives the link between the costate variable λ_s^* associated with a solution to the maximization problem of section III and the partial derivatives of the value function $\hat{V}(m_{s-1}, a_{s-1})$ for the problem.

After substituting $\mu[\epsilon(a_s)]$ for m_s in government finance constraint (12), one can write that equation in the form

$$\text{(A1)} \qquad a_s = \phi(m_{s-1}, a_{s-1}, \pi_s).$$

[Inequality (13) is still assumed.] With the aid of this notation, the value function can be defined as

$$\text{(A2)} \qquad \hat{V}(m_{s-1}, a_{s-1}) = \max_{\pi_s} \Big[\upsilon(\pi_s) + z(\mu\{\epsilon[\phi(m_{s-1}, a_{s-1}, \pi_s)]\}) + \beta\hat{V}(\mu\{\epsilon[\phi(m_{s-1}, a_{s-1}, \pi_s)]\}, \phi(m_{s-1}, a_{s-1}, \pi_s)) \Big].$$

The partial derivatives of $\phi(m_{s-1}, a_{s-1}, \pi_s)$ are easily computed. For example, the partial derivative $\partial\phi/\partial\pi_s$ is

$$\partial\phi/\partial\pi_s = m_{s-1}/\{1 - \mu'[\epsilon(a_s)]\epsilon'(a_s)\} > 0.$$

Thus, the first-order condition for a maximum can be written

$$\text{(A3)} \quad \upsilon'(\pi_s) + z'(m_s)\mu'[\epsilon(a_s)]\epsilon'(a_s)(\partial\phi/\partial\pi_s)$$
$$+ \beta[\partial\hat{V}(m_s, a_s)/\partial m_s]\mu'[\epsilon(a_s)]\epsilon'(a_s)(\partial\phi/\partial\pi_s)$$
$$+ \beta[\partial\hat{V}(m_s, a_s)/\partial a_s](\partial\phi/\partial\pi_s) = 0.$$

In view of (A1) and the constraint $m_s = \mu[\epsilon(a_s)]$, (A3) gives the optimal choice of π_s as a function $\gamma(m_{s-1}, a_{s-1}; \epsilon)$ of initial asset stocks.

Definition (A2) now leads to the envelope condition

$$\text{(A4)} \quad \partial\hat{V}(m_{s-1}, a_{s-1})/\partial a_{s-1} = z'(m_s)\mu'[\epsilon(a_s)]\epsilon'(a_s)(\partial\phi/\partial a_{s-1})$$
$$+ \beta[\partial\hat{V}(m_s, a_s)/\partial m_s]\mu'[\epsilon(a_s)]\epsilon'(a_s)(\partial\phi/\partial a_{s-1})$$
$$+ \beta[\partial\hat{V}(m_s, a_s)/\partial a_s](\partial\phi/a_{s-1}).$$

Since $\partial\phi/\partial a_{s-1} = [(1+\rho)/m_{s-1}]\partial\phi/\partial\pi_s$, (A3) and (A4) together imply

$$\text{(A5)} \quad -\upsilon'(\pi_s) = \beta[\partial\hat{V}(m_{s-1}, a_{s-1})/\partial a_{s-1}]m_{s-1}.$$

[Recall that $\beta = 1/(1 + \rho)$]. Compare (A3) and (A5) with equations (10) and (11) in the main text to derive the interpretations

$$\partial\hat{V}(m_{s-1}, a_{s-1})/\partial a_{s-1} = (1 + \rho)\lambda_s^*,$$

$$\partial\hat{V}(m_{s-1}, a_{s-1})/\partial m_{s-1} = (\pi_s - 1)\lambda_s^*.$$

References

Auernheimer, Leonardo. "The Honest Government's Guide to the Revenue from the Creation of Money." *Journal of Political Economy* 82 (May/June 1974): 598-606.

Barro, Robert J. "On the Determination of the Public Debt." *Journal of Political Economy* 87 (October 1979): 940-971.

_____. "Inflationary Finance under Discretion and Rules." *Canadian Journal of Economics* 16 (February 1983): 1-16.

_____ and David B. Gordon. "Rules, Discretion, and Reputation in a Model of Monetary Policy." *Journal of Monetary Economics* 12 (July 1983): 101-121.

Barsky, Robert. "The Fisher Hypothesis and the Forecastability and Persistence of Inflation." *Journal of Monetary Economics* 19 (January 1987): 3-24.

Bohn, Henning. "Why Do We Have Nominal Government Debt?" *Journal of Monetary Economics* 21 (January 1988): 127-140.

Brock, William A. "Money and Growth: The Case of Long-Run Perfect Foresight." *International Economic Review* 15 (October 1974): 750-777.

Calvo, Guillermo A. "Optimal Seigniorage from Money Creation." *Journal of Monetary Economics* 4 (August 1978a): 503-517.

_____. "On the Time Consistency of Optimal Policy in a Monetary Economy." *Econometrica* 46 (November 1978b): 1411-1428.

_____. "Servicing the Public Debt: The Role of Expectations." *American Economic Review* 78 (September 1988a): 647-661.

_____. "Optimal Maturity of Nominal Government Debt." Mimeo, 1988b.

_____ and Maurice Obstfeld. "Time Consistency of Fiscal and Monetary Policy: A Comment." Mimeo, 1988.

Chari, V.V. and Patrick J. Kehoe. "Sustainable Plans." Mimeo, 1987.

Cohen, Daniel and Philippe Michel. "How Should Control Theory Be Used to Calculate a Time-Consistent Government Policy?" *Review of Economic Studies* 55 (April 1988): 263-274.

Fischer, Stanley. "Dynamic Inconsistency, Cooperation, and the Benevolent Dissembling Government." *Journal of Economic Dynamics and Control* 2 (1980): 93-107.

_____. "Time Consistent Monetary and Fiscal Policies: A Survey."
Mimeo, 1986.

Grilli, Vittorio U. "Fiscal Policies and the Dollar/Pound Exchange Rate: 1870-1984." Mimeo, 1988a.

_____. "Seigniorage in Europe." Mimeo, 1988b.

Grossman, Herschel I. and John B. Van Huyck. "Seigniorage, Inflation, and Reputation." *Journal of Monetary Economics* 18 (July 1986): 21-31.

Kiguel, Miguel A. and Nissan Liviatan. "Inflationary Rigidities and Orthodox Stabilization Policies: Lessons from Latin America." *World Bank Economic Review* 2 (September 1988): 273-298.

Kydland, Finn E. and Edward C. Prescott. "Rules Rather Than Discretion: The Inconsistency of Optimal Plans." *Journal of Political Economy* 85 (1977): 473-492.

Lucas, Robert E., Jr. "Principles of Fiscal and Monetary Policy." *Journal of Monetary Economics* 17 (January 1986): 117-134.

_____ and Nancy L. Stokey. "Optimal Fiscal and Monetary Policy in an Economy without Capital." *Journal of Monetary Economics* 12 (July 1983): 55-93.

Mankiw, N. Gregory. "The Optimal Collection of Seigniorage: Theory and Evidence." *Journal of Monetary Economics* 20 (September 1987): 327-341.

Maskin, Eric and Jean Tirole. "A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs." *Econometrica* 56 (May 1988): 549-569.

Miller, Marcus and Mark Salmon. "Policy Coordination and Dynamic Games." In *International Economic Policy Coordination*, edited by Willem H. Buiter and Richard C. Marston. Cambridge: Cambridge University Press, 1985.

Obstfeld, Maurice and Kenneth Rogoff. "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" *Journal of Political Economy* 91 (August 1983): 675-687.

_____. "Ruling Out Divergent Speculative Bubbles." *Journal of Monetary Economics* 17 (May 1986): 349-362.

Oudiz, Gilles and Jeffrey Sachs. "International Policy Co-ordination in Dynamic Macroeconomic Models." In *International Economic Policy Coordination*, edited by Willem H. Buiter and Richard C. Marston. Cambridge: Cambridge University Press, 1985

Persson, Mats, Torsten Persson, and Lars E.O. Svensson. "Time Consistency of Fiscal and Monetary Policy." *Econometrica* 55 (November 1987): 1419-1431.

Persson, Torsten and Lars E.O. Svensson. "Checks and Balances on the Government Budget." In *Economic Effects of the Government Budget*, edited by Elhanan Helpman, Assaf Razin, and Efraim Sadka. Cambridge, Massachusetts: MIT Press, 1988.

Phelps, Edmund S. "Inflation in the Theory of Public Finance." *Swedish Journal of Economics* 75 (January/March 1973): 67-82.

_____ and Robert A. Pollak. "On Second-Best National Saving and Game-Equilibrium Growth." *Review of Economic Studies* 35 (April 1968): 185-199.

Poterba, James M. and Julio J. Rotemberg. "Inflation and Taxation with Optimizing Governments." Mimeo, 1988.

Rogoff, Kenneth S. "Reputation, Coordination, and Monetary Policy." In *Handbook of Modern Business Cycle Theory*, edited by Robert J. Barro. Cambridge, Massachusetts: Harvard University Press, 1989.

</ref_section>