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A HETEROGENOUS-AGENT PERSPECTIVE

Tobias Broer
Per Krusell
Erik Öberg

Working Paper 28366
<http://www.nber.org/papers/w28366>

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
January 2021

We thank Valerie Ramey for the key impetus to this paper. We also thank seminar participants and Søren Ravn for useful comments. All errors are our own. Financial support from Handelsbanken's research foundations is gratefully acknowledged. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 28366
January 2021
JEL No. E0

ABSTRACT

We use an analytically tractable heterogeneous-agent (HANK) version of the standard New Keynesian model to show how the size of fiscal multipliers depends on i) the distribution of factor incomes, and ii) the source of nominal rigidities. With sticky prices but flexible wages, the standard representative-agent (RANK) model predicts large multipliers because profits fall after a fiscal stimulus and the resulting negative income effect makes the representative worker work harder. Our HANK model, where workers do not own stock and thus do not receive profit income, predicts smaller fiscal multipliers. In fact, they are smaller with sticky prices than with flexible prices. When wages are the source of nominal rigidity, in contrast, fiscal multipliers are close to one, independently of income heterogeneity and price stickiness.

Tobias Broer
IIES, Stockholm University
10691 Stockholm
Sweden
and CEPR
Tobias.Broer@iies.su.se

Erik Öberg
Department of Economics
Uppsala University
P.O. Box 513
SE-751 20 Uppsala, Sweden
erik.oberg@nek.uu.se

Per Krusell
Institute for International Economic Studies
Stockholm University
106 91 STOCKHOLM
SWEDEN
and CEPR
and also NBER
per.krusell@iies.su.se

1 Introduction

As monetary policy interest rates have reached their lower bound of zero, a growing literature has looked at the ability of fiscal policy to stimulate aggregate activity. Particular attention has been paid to the *fiscal multiplier* (the ratio of the equilibrium output response to the increase in government spending) in New Keynesian economies where price rigidities make output partly demand determined. Because standard representative-agent New Keynesian (RANK) models are inconsistent with key empirical features of household consumption-savings behavior, this literature is moving fast towards heterogeneous-agent New Keynesian (HANK) models as its new benchmark.¹ This paper presents a very simple HANK environment to study the role of two central determinants of the fiscal multiplier in New Keynesian economies: i) the distribution of factor incomes (profit vs wage income), and ii) the source of nominal rigidities (sticky prices vs wages). In particular, our model allows us to clarify the role of income heterogeneity and wage stickiness for fiscal multipliers in richer, quantitative HANK economies compared to simpler RANK economies.²

Our HANK model extends the one we studied in an earlier paper ([Broer, Hansen, Krusell, and Öberg, 2020](#)) to include government spending shocks. In particular, relative to the standard RANK environment, the model has a more realistic specification of worker income that allows for idiosyncratic risk and accounts for the fact that the vast majority of workers do not own stock and thus do not receive firm profits in the form of dividends. This is in contrast to the representative worker in RANK who receives all factor incomes.³ A particularly attractive feature of our model is that the government spending multiplier has a closed-form solution in the benchmark cases of fully flexible and fully rigid wages.

The analysis of our simplified HANK environment shows, first, that in the RANK

¹For an introduction to the study of fiscal multipliers, see [Farhi and Werning \(2016\)](#). For a survey of the HANK literature, see [Kaplan and Violante \(2018\)](#).

²The importance of wage rigidity for the transmission mechanism that we document is, in our view, an appealing feature as it captures the importance of wage setting institutions for monetary transmission in data for advanced economies [Olivei and Tenreyro \(2007, 2010\)](#), [Björklund et al. \(2018\)](#).

³The distinction between workers who earn labor income and capitalists who only earn dividends captures the strong concentration of equity holdings in the data. It also makes our model different from the two-agent New Keynesian (“TANK”) model of [Bilbiie \(2008\)](#), which has a fraction of households that only earn labor income and consumes hand-to-mouth, and another fraction of households that earn both labor and profit income and are unconstrained. See also [Colciago \(2011\)](#).

model without wage rigidity the sticky-price amplification of fiscal shocks results entirely because workers receive profit income. The key intuitive insight here is that the representative worker adjusts her labor supply due to the combined income effects caused by the changes in taxes, wages and profits relative to the substitution effect caused by changes in wages. When a fiscal expansion increases wages, the output and labor supply response is thus stronger, the less positively total worker income responds relative to wages. By providing households with an extra source of income that is less procyclical than wages (in fact countercyclical in the typical version of the model), firm profits thus boost the output and labor supply response to a fiscal shock. This mechanism whereby sticky prices raise the fiscal multiplier through an income effect is implausible, not only because profits are procyclical in the data but also because workers barely hold stock. As any reasonably calibrated HANK model would respect this fact—and our simple HANK model makes this point by assuming workers own no stock at all and thus do not receive profits through dividends—we conclude that such models cannot offer large fiscal multipliers (at least not without adding other features). Moreover, while the countercyclical response of profits does not affect labor supply in our simple HANK model it implies more procyclical wages than with flexible prices. Workers thus reduce their labor supply, and the spending multiplier is therefore lower, compared to the flexible-price equilibrium.

A second feature of fiscal transmission in New Keynesian models that our simplified HANK model highlights is that, with rigid wages, the multipliers are larger relative to the flexible-price equilibrium in both the RANK model and our HANK model. In fact, with fully rigid wages, the multiplier is exactly equal to one in both models. More generally, when wages are rigid, households are constrained to supply whatever hours are needed to satisfy consumption demand, ignoring any labor-supply considerations. Therefore, it is sufficient to analyze the “demand side” of the economy to understand the fiscal multiplier in this case. The fiscal multiplier is larger than in the flexible-price benchmark because rigid wages dampen the response of inflation to fiscal shocks, which limits the monetary tightening that otherwise crowds out the fiscal stimulus. And because different assumptions regarding the distribution of factor incomes leave consumption demand unaffected, the dynamics of the HANK and RANK economies are isomorphic. A quantitative analysis shows that this result, of amplified and equal multipliers in RANK and HANK, also holds with a more realistic, partial

degree of wage rigidity.

We believe these results are useful for clarifying the interaction of household heterogeneity and nominal rigidities in determining the stabilizing effect of fiscal policy. In particular, models with flexible wages may obscure the role of heterogeneity in affecting aggregate dynamics, through their implausible implications for the dynamics of profit income. Based on our analysis, nominal wage rigidity makes a more plausible starting point, in line with the quantitative HANK models developed in [Auclert, Rognlie, and Straub \(2018\)](#) and [Hagedorn, Manovskii, and Mitman \(2019\)](#) to study fiscal multipliers.⁴

Our analysis also highlights the fact that the transmission of fiscal shocks in the NK setting is rather different from that of monetary shocks (which we analyzed in our previous paper), for at least two reasons. First, since a fiscal shock directly affects households' budgets, its effect directly depends on other sources of income and their endogenous dynamic responses over time. Assumptions about the distribution of factor incomes thus have a first-order effect on the propagation of fiscal shocks. Second, it is well known that the effect of fiscal shocks depends on the response of real interest rates ([Woodford, 2011](#)). Here, we note that accounting for wage rigidity dampens the inflation response to fiscal shocks, and thus the endogenous reaction of monetary policy that, typically, counteracts the demand-effect of fiscal shocks. This raises the fiscal multiplier relative to the standard version of the model with only price rigidities, but also makes it less sensitive to the current stance of monetary policy. In particular, previous analysis based on representative-agent models has shown that fiscal multipliers can be particularly potent in situations where the monetary authority is constrained by a zero lower bound (ZLB) on the nominal interest rate.⁵ In our concluding section we discuss the ZLB constraint and argue that the ZLB is actually not critical from the perspective of the simple models we consider here. The RANK model's high multipliers in such situations only materialize due to the implausible transmission mechanism,

⁴Two recent and complementary papers support this view: [Auclert, Bardóczy, and Rognlie \(2020\)](#) show that it is impossible for New Keynesian models with flexible labor markets to simultaneously match empirical estimates for marginal propensities to earn, marginal propensities to consume and fiscal multipliers. [Cantore and Freund \(2020\)](#) show that the implausible income effect of profit on labor supply can be reduced by introducing portfolio adjustment costs in a two-agent New Keynesian (TANK) model, but that this also implies smaller fiscal multipliers.

⁵See, e.g, [Woodford \(2011\)](#), [Christiano, Eichenbaum, and Rebelo \(2011\)](#), and [Eggertsson \(2011\)](#).

and the sticky-wage fiscal multiplier, whether in a RANK or HANK model, is close to one and largely unaffected by the presence of the ZLB.

2 Fiscal multipliers in RANK and HANK

The goal here is to compare the responses to fiscal shocks across two models: the standard RANK model and our simple HANK model. We describe the two models in turn. Like the “textbook” representative-agent New Keynesian (RANK) model we abstract from physical capital and government bonds, which allows us to solve the models analytically.

2.1 RANK

The representative household has “KPR preferences” over consumption and leisure as in [King, Plosser, and Rebelo \(1988\)](#), to be consistent with balanced growth; for convenience, we use the additively separable version from [MaCurdy \(1981\)](#), where φ regulates the (constant) Frisch elasticity of labor supply. Each of a continuum of the household’s members provides a differentiated labor service in a monopsonistic fashion and pays a [Rotemberg \(1982\)](#)-type adjustment cost when changing the wage.⁶ There is a continuum of monopolistically competitive firms operating a production function that is linear in the Dixit-Stiglitz composite of the differentiated labor inputs. Firms set their output prices subject to the [Calvo \(1983\)](#) friction.

The monetary authority sets interest rates according to a Taylor rule that only reacts to inflation and the fiscal authority taxes the household lump-sum to fully finance its spending.⁷

The log-linear approximation of the equilibrium around a zero-inflation steady

⁶An isomorphic model would also be obtained if assuming a Calvo-type friction in the wage setting problem, as in [Erceg, Henderson, and Levin \(2000\)](#). We opt for the Rotemberg adjustment cost as it is easier to solve when we move to our heterogeneous-agent version of the NK model, given that the equilibrium wage distribution becomes degenerate.

⁷In other words, we abstract from the distortionary effects of taxes and government debt.

state is described by the following equations.⁸

$$\text{IS : } \hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1}^p) \quad (1)$$

$$\text{Price Phillips : } \pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \quad (2)$$

$$\text{Wage Phillips : } \pi_t^w = \beta E_t \pi_{t+1}^w - \lambda_w (\hat{\omega}_t - (\hat{c}_t + \varphi \hat{n}_t)) \quad (3)$$

$$\text{Wage accounting : } \hat{\omega}_t = \hat{\omega}_{t-1} + \pi_t^w - \pi_t^p \quad (4)$$

$$\text{Resources : } (1 - \bar{\tau}) \hat{c}_t + \bar{\tau} \hat{\tau}_t = \hat{n}_t \quad (5)$$

$$\text{Taylor rule : } \hat{i}_t = \phi_\pi \pi_t^p \quad (6)$$

$$\text{Tax policy : } \hat{\tau}_t = \rho \hat{\tau}_{t-1} + \nu_t. \quad (7)$$

Here, \hat{x}_t denotes the log-deviation of x at t from its steady state value. Let us now briefly describe the equations one by one.

Equation (1) in the New Keynesian IS curve, an Euler condition that links current consumption \hat{c} to expected future consumption and the expected real interest rate. Equation (2) is the New Keynesian Phillips curve, relating current price inflation π^p to expected inflation and the current real marginal cost; the latter, with linear production, equals the real wage ω . λ_p is a combination of structural preference parameters and the price-resetting probability, which together govern the price response to changes in marginal costs.⁹ Equation (3) is the Phillips curve for wages implied by the household's wage-setting problem. It describes how nominal wage inflation π^w responds to changes in the difference between the real wage and the marginal rate of substitution between consumption and hours, where λ_w is another combination of structural parameters, including the adjustment-cost parameter for wages.¹⁰ Equation (4) is an accounting identity, describing the evolution of the real wage in terms of wage and price inflation. Equation (5) is the economy's resource constraint, with $\bar{\tau} = g$ representing the share of government expenditures, g , in output, $c + g$. It thus sets the share-weighted average of consumption and taxes equal to output, which absent productivity shocks and under linear production equals hours worked \hat{n} .

For given expectations, the system (1)–(5) determines the five unknown endoge-

⁸The derivation of our results here are standard; for details, see [Broer, Hansen, Krusell, and Öberg \(2020\)](#).

⁹Specifically, $\lambda_p = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$, where β is the discount factor and θ_p the per-period Calvo probability that a firm cannot reset its price.

¹⁰Specifically, $\lambda_w = -\frac{\epsilon_w - 1}{\chi}$ where ϵ_w is the elasticity of substitution between labor inputs and χ is the Rotemberg adjustment-cost parameter.

nous variables— \hat{c} , π^p , π^w , $\hat{\omega}$, and \hat{n} —uniquely as a function of the policy variables \hat{i} and $\hat{\tau}$. Equations (6) and (7) then provide the policy rules (a Taylor rule and an AR(1) for the fiscal shock) and for our calibrated model the implied full dynamic system has a unique stable solution around its zero-inflation steady state, which is also uniquely determined.

Income in this economy is the sum of labor earnings, $\hat{\omega} + \hat{n}$, and dividends from the monopolistic firms, d ; time- t dividends can therefore be solved residually from

$$\text{Household income : } \hat{n}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t. \quad (8)$$

Here, the weights \bar{S} and $1 - \bar{S}$ are the steady-state shares in output of earnings and dividends, respectively. The role of dividends is key for the intuitive comparison to the HANK model.

In the flexible limit of wage setting, $\lambda_w \rightarrow \infty$ in the wage Phillips curve (3), implying that the real wage equals the marginal rate of substitution:

$$\hat{\omega}_t = \hat{c}_t + \varphi \hat{n}_t. \quad (9)$$

In this case, the model equations can be collapsed into the familiar 3-equation representation, augmented with the tax policy equation.

2.2 HANK

The HANK model is the natural extension of the RANK model to the kind of incomplete-markets model studied in [Huggett \(1993\)](#): households can only imperfectly insure themselves against idiosyncratic labor productivity shocks by trading a risk-free bond subject to a borrowing constraint. We consider a particularly simple HANK model here. In particular, the household sector consists of *workers* and a (small) mass of *capitalists*. These are ex-ante identical in all aspects (including their tax share of income) except that the capitalists own the firms and derive income from firm dividends, whereas workers only receive wage income. This assumption captures the fact that equity ownership is extremely concentrated (see, e.g., [Kuhn and Rios-Rull \(2016\)](#)). As we shall see, the fact that workers do not receive dividends makes a crucial difference for the workings of the model. We also follow [Krusell, Mukoyama, and Smith \(2011\)](#), [Werning \(2015\)](#), [McKay and Reis \(2017\)](#), and [Ravn and Sterk \(2018\)](#) and assume a zero borrowing constraint such that households cannot borrow at all, and have no saving in

equilibrium. This assumption allows closed-form expressions for our key aggregates but is not essential for the key insights. We moreover posit that aggregate shocks are small relative to idiosyncratic shocks to worker productivity (and thus income), together with a small fixed cost of employment, implying that the agent with the highest propensity to save is always a worker in this economy and that capitalists do not work. For details, see [Broer, Hansen, Krusell, and Öberg \(2020\)](#).

In terms of the implied equations, the HANK model is different to the RANK model in two ways. One is that consumption, \hat{c} , now refers to worker (and not aggregate) consumption. The second, and implied, difference is that the resource constraint (5) is now replaced by the worker’s budget constraint, evaluated at equilibrium (where bond holdings are zero):

$$\text{Worker income : } \quad (1 - \bar{\tau})\hat{c}_t + \bar{\tau}\hat{\tau}_t = \hat{\omega}_t + \hat{n}_t. \quad (10)$$

The key here is that income, on the right-hand side, is not aggregate income \hat{n} (which, expressed in terms of factor payments as in (8), would include dividends) but rather worker earnings: $\hat{\omega} + \hat{n}$.

Before moving on to the analysis of fiscal multipliers let us quickly note that our HANK model, which is really like a two-agent model with workers and capitalists, is different than other so-called TANK models (see, e.g., [Galí, Lopez-Salido, and Valles \(2004\)](#) and [Bilbiie \(2008\)](#)), where the typical assumption is that a fraction of the consumers is hand-to-mouth, with the remainder being standard permanent-income consumers. The difference—that our workers do not receive dividend income whereas the hand-to-mouth consumers do—is crucial, and in all essential respects that we discuss below, the typical TANK model really functions like the RANK model.¹¹

2.3 Fiscal multipliers

We now consider the implications of an innovation in government spending in the RANK and HANK models. In particular, we ask how the models’ output multipliers are affected by the source of the nominal rigidity: sticky prices or sticky wages. In all results we assume that $\phi_\pi > 1$ and that all other parameters are restricted to their respective standard domain ($\beta \in (0, 1)$, $\varphi \in (0, \infty)$, etc.).

¹¹See [Galí and Debortoli \(2017\)](#) for a discussion of the implications of different profit-distribution schemes in a TANK model.

Flexible wages Consider first the case with flexible wages ($\lambda_w \rightarrow \infty$). Define the cumulative fiscal multiplier M as the ratio of the cumulative output (and therefore hours) response to the cumulative increase in fiscal spending following a positive innovation in period 0:

$$M = \frac{\sum_{t=0}^{\infty} \hat{n}_t}{\bar{\tau} \sum_{t=0}^{\infty} \hat{\tau}_t} \quad (11)$$

Furthermore, define M_{nat} as the cumulative fiscal multiplier in “natural” equilibrium, where both prices and wages are flexible. The multiplier gap, $M - M_{nat}$, is a measure of the contribution of sticky prices to the equilibrium response of output. Proposition 1 establishes that multiplier gap is positive in RANK but negative in HANK. In other words, sticky prices amplify the response to fiscal shocks in RANK only because the representative agent receives profit income.

Proposition 1 *Suppose $\lambda_w \rightarrow \infty$ and $\phi_\pi > 1$. Then in response to a positive fiscal innovation in period 0, $\nu_0 > 0$, we have that*

$$M - M_{nat} > 0 \quad \text{in RANK}$$

$$M - M_{nat} < 0 \quad \text{in HANK}$$

Proof: See the Appendix.

What is the mechanism that induces households to work more when prices are sticky in RANK but less in HANK? To see this, we combine the optimality condition for labor supply (9) with the worker’s equilibrium budget—(5) and (8) for the RANK model and (10) for the HANK model—to obtain the following conditions for the determination of hours worked:

$$\text{RANK: } [\varphi(1 - \bar{\tau}) + \bar{S}] \hat{n}_t = \bar{\tau}(\hat{\tau}_t - \hat{\omega}_t) + (1 - \bar{S})(\hat{\omega}_t - \hat{d}_t) \quad (12)$$

$$\text{HANK: } [\varphi(1 - \bar{\tau}) + 1] \hat{n}_t = \bar{\tau}(\hat{\tau}_t - \hat{\omega}_t) \quad (13)$$

Note that with flexible prices, the real wage is constant at its steady state level in both models (because production is linear in labor), implying $\hat{\omega}_t = 0$. In this case, the profit share of income is constant, and hours rise in both models solely because of the negative income effect of the increased taxes. With rigid prices, inflation and real wages rise in both models in response to a fiscal expansion. In HANK, this *depresses* hours worked because post-tax labor income is smaller than wage earnings (as the the share of government expenditures $\bar{\tau}$ is positive), such that the income effect of wage rises

dominates the substitution effect in (13) with standard KPR preferences. In RANK, the wage rise stimulates labor supply, for two reasons: First, workers also receive profit income ($\bar{S} < 1$ in 12), which reduces the relative income effect of the wage rise. Second, wage rises imply smaller profits, which in itself has a negative income effect on hours worked. In the realistic case where the profit share is smaller than the tax share of total income ($1 - \bar{S} < \tau$), profit income is not high enough to make households work more in response to higher wages. In this case, sticky prices deliver a higher spending multiplier solely because they imply a countercyclical response of profits \hat{d}_t .

Rigid wages With rigid wages hours worked are not determined by income and substitution effects on labor supply. Instead, households supply whatever hours are needed to satisfy consumption demand. Proposition 2 establishes that with fully rigid wages, the fiscal multiplier equals one in both models, irrespective of the degree of price stickiness.¹² In other words, there is amplification relative to the flexible-price equilibrium, but its strength does not depend on the distribution of factor incomes.

Proposition 2 *Suppose $\lambda_w \rightarrow 0$. Then in response to a positive fiscal innovation in period 0, $\nu_0 > 0$, we have that the cumulative fiscal multiplier $M = 1 > M_{nat}$ in both RANK and HANK.*

Proof: See the Appendix.

To understand the intuition behind Proposition 2, the key insight is that constant nominal wages imply constant prices set by firms, and thus constant real wages. With constant real wages, the profit share of income is again constant, so labor income and firm profits respond in exactly the same way to the fiscal expansion. This implies that in HANK worker and capitalist consumption aggregate to an IS equation for aggregate consumption that is identical to that in RANK (see equation (1)). Moreover, this consumption path is the same as in the flexible-price equilibrium, since prices, and therefore the real interest rate are affected by the fiscal shock. And with unchanged consumption, output increases exactly by the same amount as government spending,

¹²Proposition 2 focuses on the natural equilibrium where the output deviation is a function of the fiscal shock, the only state variable in the model. With fully rigid wages, there is an obvious multiplicity in the model (as wages never adjust to any constant deviation of output from its steady state) that disappears with a more general Taylor rule (that puts an arbitrarily small weight on the output gap), or when considering only equilibria that converge as $\lambda_w \rightarrow 0$.

implying a multiplier of unity.

Quantitative illustration We illustrate these results with a numerical example, in which the degree of price and wage rigidity are set to standard values. We look at three different cases: no rigidities, price rigidities, and wage rigidities.¹³ We set the tax/output share of 30 percent and study the quarterly response to a 1 percent shock to government spending with autocorrelation $\rho = 0.5$. The other parameter values are taken from Galí (2008, Ch. 3 and 6). The discount factor equals 0.99, the Frisch elasticity $1/\varphi$ is set at 1, the elasticity of substitution between goods as well as that between labor inputs equals 6, the price reset probability is $1/3$ and the Rotemberg adjustment cost for wage setting is set so as to replicate a corresponding Calvo wage reset probability of $1/4$, and the Taylor coefficient on inflation is 1.5.

The resulting IRFs are displayed in Figure 1, where the blue solid lines are the responses of the RANK model and the red dashed lines are the responses of the HANK model. All responses are expressed in terms of percent or percentage-point deviations from the steady state. The cumulative multiplier is displayed in Table 1.

Model	No rigidity		Price rigidity		Wage rigidity	
	RANK	HANK	RANK	HANK	RANK	HANK
Cumulative multiplier	0.59	0.59	0.78	0.53	0.98	0.98

Table 1: Cumulative Fiscal Multipliers

Using these standard parameter values, two things are worth noting. First, the price elasticity is $\epsilon = 6$, which implies $1 - \bar{S} = \frac{1}{6}$, which is lower than our $\bar{\tau} = 0.3$, so the net effect of the real wage increase on hours is negative. In RANK, sticky prices increase the output multiplier in RANK solely because price-stickiness implies a negative profit response. Second, even with a degree of wage rigidity that is far from the limit of full rigidity, the response of the RANK and the HANK model are still close to being equivalent.

¹³In the case of wage rigidities, we assume no price rigidities; the behavior of this version of the model does not depend on whether or not there are price rigidities.

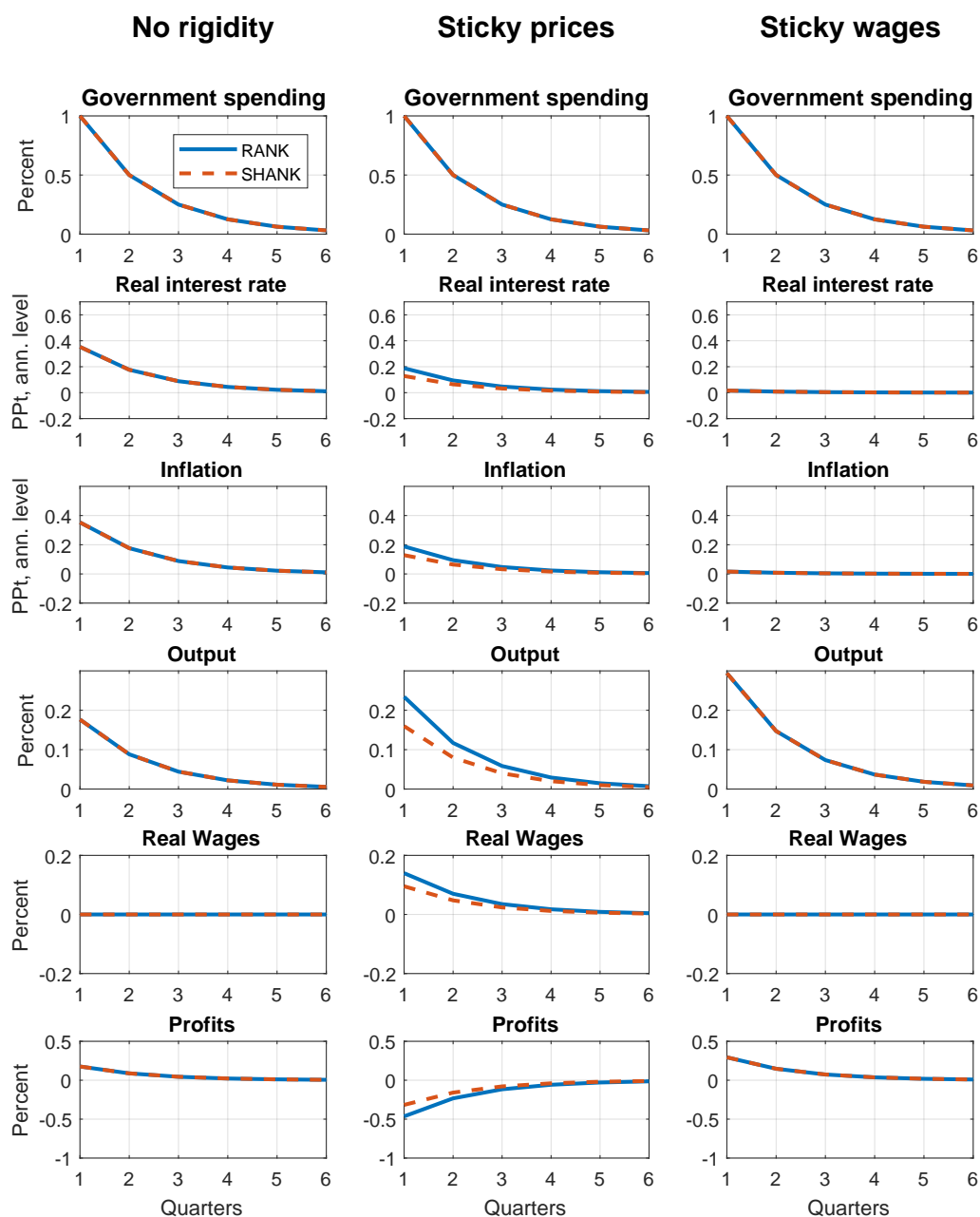


Figure 1: Responses to 1 percent rise in government spending

3 Concluding remarks

For robustness of the quantitative illustration, we also examined models with significantly less wage rigidity (by doubling the probability that firms can reset prices from 1/4 to 1/2) and found very small differences between the HANK and RANK models and only marginally lower multipliers. We also found that the assumption of linear production is not material for our results: with a labor input elasticity of output equal to 0.7, the responses in the sticky-wage versions of the RANK and HANK models are

still close to identical; with sticky prices, the difference between RANK and HANK increases, since profits respond even more countercyclically in this case.

A more interesting other case is that when the monetary authority is constrained by a zero lower bound (ZLB) on the interest rate. In particular, representative-agent models with nominal rigidities have been found to predict multipliers that can be significantly larger than one in those circumstances, providing a rationale for using discretionary fiscal policy as a stabilization tool (see, e.g. [Woodford \(2011\)](#), [Christiano, Eichenbaum, and Rebelo \(2011\)](#), and [Eggertsson \(2011\)](#)). The intuitive reason for their prediction is simple. With active monetary policy, real interest rates rise in response to higher inflation and partly crowd out the positive effect of a fiscal shock on output. When monetary policy is constrained and nominal interest rates unaffected by shocks, in contrast, the rise in inflation in response to a fiscal stimulus implies a *fall* in real interest rates, thus further stimulating consumption and increasing both the output response and the multiplier. What can we learn about this mechanism from our analysis? First, under sticky prices, the insight that under reasonable parametric restrictions, countercyclical profits are a necessary condition for amplification does not depend on the state of monetary policy, but follows directly from the household's labor supply decision together with the market clearing condition. Thus, the amplification of the multiplier is only consistent with the representative agent's optimal choice of labor supply because profits respond countercyclically. That is, for multipliers to be larger in the ZLB case, it must be that profits respond more strongly (negatively) to the fiscal stimulus. I.e., to the extent one agrees with our term "implausible" to describe the sticky-price model's amplification mechanism, this mechanism becomes even more implausible in the case of the ZLB. How does the behavior of the rigid-wage model change when monetary policy is constrained? Virtually not at all: the multiplier remains one. As is clear from the impulse responses with rigid wages in [Figure 1](#), there is virtually no response of inflation and the interest rate to the fiscal shock and, hence, the lower bound on the nominal interest rate has virtually no effect. More broadly, the rigid-wage setting dampens the fluctuations in marginal costs, and therefore restrains the amplification of the multiplier at the zero lower bound.¹⁴

¹⁴The ZLB can clearly play a role in a HANK setting with sticky prices and sticky wages, not only since the degree of wage rigidity is ultimately an (open) question for empirical research, see, e.g., [Beraja, Hurst, and Ospina \(2019\)](#). However, for modest departures from the benchmark here, including to cases where the production has some curvature, we found that the ZLB plays a very limited role.

The main take-away from our present note is not a critique of the three-equation representative-agent New Keynesian model. We do think that the representative-agent focus of this model is problematic and that the supply side is inadequate, but this model has been very important in providing a clear mechanism for the dynamics of demand (through its dynamic IS equation and monetary policy). Instead, the main goal here is to provide some guidance in the construction of quantitative HANK models. In particular, models with flexible wages may obscure the role of heterogeneity in affecting aggregate dynamics, through its implausible implications for the dynamics of profit income. Based on our analysis, nominal wage rigidity is a more plausible benchmark, which is also in line with recent developments of the HANK literature concerning fiscal multipliers (Auclert, Rognlie, and Straub, 2018, Hagedorn, Manovskii, and Mitman, 2019). We also think that there is a need to develop such models of rigid wages—in particular their foundations—further; they have received much less attention than models with price rigidities. Such work is at least on our own agenda.

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Appendix

In this appendix, we prove Propositions 1 and 2. Before proving each proposition separately, note that the log-linear equilibrium of the RANK model, described by Equations (1)-(7), and the HANK model, described by Equations (1)-(4), (6), (7) and (10), may both

be collapsed to the following five-equation system:

$$\bar{\tau} E_t \Delta \hat{\tau}_{t+1} = E_t \Delta \hat{n}_{t+1} + \zeta_i^1 E_t \Delta \hat{\omega}_{t+1} - (1 - \bar{\tau})(\phi_\pi \pi_t^p - E_t \pi_{t+1}^p) \quad (14)$$

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \quad (15)$$

$$\pi_t^w = \beta E_t \pi_{t+1}^w - \lambda_w \left(\zeta_i^2 \hat{\omega}_t - \left(\frac{1}{1 - \bar{\tau}} + \varphi \right) \hat{n}_t + \frac{\bar{\tau}}{1 - \bar{\tau}} \hat{\tau}_t \right) \quad (16)$$

$$\Delta \hat{\omega}_t = \pi_t^w - \pi_t^p \quad (17)$$

$$\hat{\tau}_t = \rho \hat{\tau}_{t-1} + \nu_t. \quad (18)$$

where $i \in RANK, HANK$ and

$$\begin{aligned} \zeta_{RANK}^1 &= 0 \\ \zeta_{RANK}^2 &= 1 \\ \zeta_{HANK}^1 &= 1 \\ \zeta_{HANK}^2 &= -\frac{\bar{\tau}}{1 - \bar{\tau}} \end{aligned}$$

Proof to Proposition 1

Suppose $\lambda_w \rightarrow \infty$ and $\phi_\pi > 1$. It can then easily be verified that the model has a unique bounded equilibrium. We compute this equilibrium by guess and verify.

Consider a positive fiscal innovation in period 0, $\nu_0 > 0$. With $\lambda_w \rightarrow \infty$, Equation (16) implies that the solution must satisfy

$$\zeta_i^2 \hat{\omega}_t = \left(\frac{1}{1 - \bar{\tau}} + \varphi \right) \hat{n}_t - \frac{\bar{\tau}}{1 - \bar{\tau}} \hat{\tau}_t.$$

The system may therefore be reduced to

$$\bar{\tau} E_t \Delta \hat{\tau}_{t+1} = E_t \Delta \hat{n}_{t+1} + \zeta_i^1 E_t \Delta \hat{\omega}_{t+1} - (1 - \bar{\tau})(\phi_\pi \pi_t^p - E_t \pi_{t+1}^p) \quad (19)$$

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \quad (20)$$

$$\zeta_i^2 \hat{\omega}_t = \left(\frac{1}{1 - \bar{\tau}} + \varphi \right) \hat{n}_t - \frac{\bar{\tau}}{1 - \bar{\tau}} \hat{\tau}_t \quad (21)$$

$$\hat{\tau}_t = \rho \hat{\tau}_{t-1}. \quad (22)$$

for $t > 0$ and $\tau_0 = \nu_0$. Given a solution to this system of four unknowns $\{\hat{n}_t, \hat{\omega}_t, \pi_t^p, \hat{\tau}_t\}$, the path for π_t^w can be solved residually from Equation (17).

We reduce the system further. Substituting Equation (21) into Equation (20) gives us

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \frac{\lambda_p}{\zeta_i^2} \left(\frac{1}{1 - \bar{\tau}} + \varphi \right) \hat{n}_t - \frac{\lambda_p \bar{\tau}}{\zeta_i^2 (1 - \bar{\tau})} \hat{\tau}_t \quad (23)$$

First differencing Equation (21) and substituting this into Equation (19) gives us

$$\left(\bar{\tau} + \frac{\zeta_i^1 \bar{\tau}}{\zeta_i^2 (1 - \bar{\tau})}\right) E_t \Delta \hat{\tau}_{t+1} = \left(1 + \frac{\zeta_i^1}{\zeta_i^2} \left(\frac{1}{1 - \bar{\tau}} + \varphi\right)\right) E_t \Delta \hat{n}_{t+1} - (1 - \bar{\tau})(\phi_\pi \pi_t^p - E_t \pi_{t+1}^p) \quad (24)$$

Furthermore, substituting Equation (22) into Equations (23) and (24), the system may be reduced to

$$\left(\bar{\tau} + \frac{\zeta_i^1 \bar{\tau}}{\zeta_i^2 (1 - \bar{\tau})}\right) (\rho - 1) \hat{\tau}_t = \left(1 + \frac{\zeta_i^1}{\zeta_i^2} \left(\frac{1}{1 - \bar{\tau}} + \varphi\right)\right) E_t \Delta \hat{n}_{t+1} - (1 - \bar{\tau})(\phi_\pi \pi_t^p - E_t \pi_{t+1}^p) \quad (25)$$

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \frac{\lambda_p}{\zeta_i^2} \left(\frac{1}{1 - \bar{\tau}} + \varphi\right) \hat{n}_t - \frac{\lambda_p \bar{\tau}}{\zeta_i^2 (1 - \bar{\tau})} \hat{\tau}_t \quad (26)$$

Now, we guess that the solution is linear in $\hat{\tau}_t$, i.e., we guess that $\hat{n}_t = \mu_n^i \hat{\tau}_t$ and $\pi_t^p = \mu_\pi^i \hat{\tau}_t$. With this guess the system can be written

$$\left(\bar{\tau} + \frac{\zeta_i^1 \bar{\tau}}{\zeta_i^2 (1 - \bar{\tau})}\right) (\rho - 1) \hat{\tau}_t = \left(1 + \frac{\zeta_i^1}{\zeta_i^2} \left(\frac{1}{1 - \bar{\tau}} + \varphi\right)\right) \mu_n^i (\rho - 1) \hat{\tau}_t - (1 - \bar{\tau})(\phi_\pi \mu_\pi^i \hat{\tau}_t - \mu_\pi^i \rho \hat{\tau}_t) \quad (27)$$

$$\mu_\pi^i \hat{\tau}_t = \beta \mu_\pi^i \rho \hat{\tau}_t + \frac{\lambda_p}{\zeta_i^2} \left(\frac{1}{1 - \bar{\tau}} + \varphi\right) \mu_n^i \hat{\tau}_t - \frac{\lambda_p \bar{\tau}}{\zeta_i^2 (1 - \bar{\tau})} \hat{\tau}_t \quad (28)$$

or

$$\left(\bar{\tau} + \frac{\zeta_i^1 \bar{\tau}}{\zeta_i^2 (1 - \bar{\tau})}\right) (\rho - 1) = \left(1 + \frac{\zeta_i^1}{\zeta_i^2} \left(\frac{1}{1 - \bar{\tau}} + \varphi\right)\right) \mu_n^i (\rho - 1) - (1 - \bar{\tau})(\phi_\pi \mu_\pi^i - \mu_\pi^i \rho) \quad (29)$$

$$\mu_\pi^i = \beta \mu_\pi^i \rho + \frac{\lambda_p}{\zeta_i^2} \left(\frac{1}{1 - \bar{\tau}} + \varphi\right) \mu_n^i - \frac{\lambda_p \bar{\tau}}{\zeta_i^2 (1 - \bar{\tau})} \quad (30)$$

which constitute a solution if and only if Equations (29) and (30) solve for μ_n^i and μ_π^i .

Rearranging, we have

$$\mu_n^i = \kappa_1^i + \kappa_2^i \mu_\pi^i \quad (31)$$

$$\mu_\pi^i = \kappa_3^i + \kappa_4^i \mu_n^i \quad (32)$$

where

$$\begin{aligned} \kappa_1^i &= \frac{\zeta_i^2 (1 - \bar{\tau}) \bar{\tau} + \zeta_i^1 \bar{\tau}}{\zeta_i^2 (1 - \bar{\tau}) + \zeta_i^1 (1 + \varphi (1 - \bar{\tau}))} \\ \kappa_2^i &= -\frac{(1 - \bar{\tau})(\phi_\pi - \rho) \zeta_i^2 (1 - \bar{\tau})}{(1 - \rho)(\zeta_i^2 (1 - \bar{\tau}) + \zeta_i^1 (1 + \varphi (1 - \bar{\tau})))} \\ \kappa_3^i &= -\frac{\lambda_p \bar{\tau}}{\zeta_i^2 (1 - \bar{\tau}) (1 - \beta \rho)} \\ \kappa_4^i &= \frac{\lambda_p (1 + (1 - \bar{\tau}) \varphi)}{\zeta_i^2 (1 - \bar{\tau}) (1 - \beta \rho)} \end{aligned}$$

The solution to the system (31)-(32) is

$$\mu_\pi = \frac{\kappa_3^i + \kappa_1^i \kappa_4^i}{1 - \kappa_2^i \kappa_4^i} \quad (33)$$

$$\mu_n = \frac{\kappa_1^i + \kappa_2^i \kappa_3^i}{1 - \kappa_2^i \kappa_4^i} \quad (34)$$

which confirms that our guess constitutes a solution. We now compute the cumulative fiscal multiplier using this solution for each model separately.

RANK model: In the RANK model $\zeta^1 = 0$ and $\zeta^2 = 1$. In this case,

$$\begin{aligned} \kappa_1 &= \bar{\tau} \\ \kappa_2 &= -\frac{(1 - \bar{\tau})(\phi_\pi - \rho)}{(1 - \rho)} \\ \kappa_3 &= -\frac{\lambda_p \bar{\tau}}{(1 - \bar{\tau})(1 - \beta\rho)} \\ \kappa_4 &= \frac{\lambda_p(1 + (1 - \bar{\tau})\varphi)}{(1 - \bar{\tau})(1 - \beta\rho)} \end{aligned}$$

and therefore

$$\begin{aligned} \mu_n &= \frac{\bar{\tau}}{(1 + (1 - \bar{\tau})\varphi)} \frac{1 + \frac{\lambda_p(\phi_\pi - \rho)}{(1 - \rho)(1 - \beta\rho)}}{\frac{1}{(1 + (1 - \bar{\tau})\varphi)} + \frac{\lambda_p(\phi_\pi - \rho)}{(1 - \rho)(1 - \beta\rho)}} \\ &> \frac{\bar{\tau}}{(1 + (1 - \bar{\tau})\varphi)}. \end{aligned} \quad (35)$$

In the case of flexible prices, $\lambda_p \rightarrow \infty$, and using l'Hôpital's rule, we have that

$$\mu_{n,nat} = \frac{\bar{\tau}}{(1 + (1 - \bar{\tau})\varphi)} \quad (36)$$

Therefore, the difference between the cumulative fiscal multiplier in the baseline and the flexible-price model is given by:

$$\begin{aligned} M - M_{nat} &= \frac{\sum_{t=0}^{\infty} \mu_n \hat{\tau}_t}{\bar{\tau} \sum_{t=0}^{\infty} \hat{\tau}_t} - \frac{\sum_{t=0}^{\infty} \mu_{n,nat} \hat{\tau}_t}{\bar{\tau} \sum_{t=0}^{\infty} \hat{\tau}_t} \\ &= \frac{1}{\bar{\tau}} (\mu_n - \mu_{n,nat}) \\ &> 0. \end{aligned}$$

HANK model: In the HANK model $\zeta^1 = 1$ and $\zeta^2 = -\frac{\bar{\tau}}{1 - \bar{\tau}}$. In this case,

$$\begin{aligned} \kappa_1 &= \frac{\bar{\tau}}{(1 + \varphi)} \\ \kappa_2 &= \frac{(\phi_\pi - \rho)\bar{\tau}}{(1 - \rho)(1 + \varphi)} \\ \kappa_3 &= \frac{\lambda_p}{(1 - \beta\rho)} \\ \kappa_4 &= -\frac{\lambda_p(1 + (1 - \bar{\tau})\varphi)}{\bar{\tau}(1 - \beta\rho)} \end{aligned}$$

and therefore

$$\begin{aligned}\mu_n &= \frac{\bar{\tau}}{(1 + (1 - \bar{\tau})\varphi)} \frac{1 + \frac{\lambda_p(\phi_\pi - \rho)}{(1-\rho)(1-\beta\rho)}}{\frac{(1+\varphi)}{(1+(1-\bar{\tau})\varphi)} + \frac{\lambda_p(\phi_\pi - \rho)}{(1-\rho)(1-\beta\rho)}} \\ &< \frac{\bar{\tau}}{(1 + (1 - \bar{\tau})\varphi)}.\end{aligned}\quad (37)$$

In the case of flexible prices, $\lambda_p \rightarrow \infty$, and using l'Hôpital's rule, we have that

$$\mu_n = \frac{\bar{\tau}}{(1 + (1 - \bar{\tau})\varphi)} \quad (38)$$

Therefore, the difference between the cumulative fiscal multiplier in the baseline and the flexible-price model is given by:

$$\begin{aligned}M - M_{nat} &= \frac{\sum_{t=0}^{\infty} \mu_n \hat{\tau}_t}{\bar{\tau} \sum_{t=0}^{\infty} \hat{\tau}_t} - \frac{\sum_{t=0}^{\infty} \mu_{n,nat} \hat{\tau}_t}{\bar{\tau} \sum_{t=0}^{\infty} \hat{\tau}_t} \\ &= \frac{1}{\bar{\tau}} (\mu_n - \mu_{n,nat}) \\ &< 0.\end{aligned}$$

This completes the proof.

Proof to Proposition 2

Suppose $\lambda_w \rightarrow 0$. We compute the resulting equilibrium by guess and verify. Consider a positive fiscal innovation in period 0, $\nu_0 > 0$. With $\lambda_w \rightarrow 0$, Equation (16) implies that

$$\pi_t^w = 0.$$

The system may therefore be reduced to

$$\bar{\tau} E_t \Delta \hat{\tau}_{t+1} = E_t \Delta \hat{n}_{t+1} + \zeta_i^1 E_t \Delta \hat{\omega}_{t+1} - (1 - \bar{\tau})(\phi_\pi \pi_t^p - E_t \pi_{t+1}^p) \quad (39)$$

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \quad (40)$$

$$\Delta \hat{\omega}_t = -\pi_t^p \quad (41)$$

$$\hat{\tau}_t = \rho \hat{\tau}_{t-1}. \quad (42)$$

for $t > 0$ and $\tau_0 = \nu_0$. This is a system with four unknowns $\{\hat{n}_t, \hat{\omega}_t, \pi_t^p, \hat{\tau}_t\}$. To solve it, we guess that the solution has $\hat{\omega}_t = 0$. With this guess, it follows directly that $\pi_t^p = 0$ from Equation (40). From Equation (39), we then have that

$$E_t \Delta \hat{n}_{t+1} = \bar{\tau} E_t \Delta \hat{\tau}_{t+1} \quad (43)$$

Further guess that the solution is linear in τ_t : $\hat{n}_t = \mu_n \hat{\tau}_t$, we have that

$$\mu_n E_t \Delta \hat{\tau}_{t+1} = \bar{\tau} E_t \Delta \hat{\tau}_{t+1} \quad (44)$$

which confirms that our guess constitutes a solution if

$$\mu_n = \bar{\tau} \quad (45)$$

This is true irrespective of the values of ζ_1^i and ζ_2^i . With fully rigid wages, the solution to both the RANK and the HANK model is given by

$$\hat{n}_t = \bar{\tau} \hat{\tau}_t, \quad (46)$$

$$\hat{\tau}_t = \rho \hat{\tau}_{t-1}, \quad (47)$$

for $t > 0$ and $\hat{\tau}_0 = \nu_0$. The cumulative fiscal multiplier in both models is given by

$$\begin{aligned} M &= \frac{\sum_{t=0}^{\infty} \mu_n \hat{\tau}_t}{\bar{\tau} \sum_{t=0}^{\infty} \hat{\tau}_t} \\ &= \frac{\bar{\tau} \sum_{t=0}^{\infty} \hat{\tau}_t}{\bar{\tau} \sum_{t=0}^{\infty} \hat{\tau}_t} \\ &= 1. \end{aligned}$$

Furthermore, by Equations (36) and (38), the multiplier in the natural equilibrium of the HANK and the RANK model is identical and given by

$$\begin{aligned} M_{nat} &= \frac{\sum_{t=0}^{\infty} \mu_{n,nat} \hat{\tau}_t}{\bar{\tau} \sum_{t=0}^{\infty} \hat{\tau}_t} \\ &= \frac{\bar{\tau}}{\bar{\tau}(1 + (1 + \bar{\tau})\varphi)} \frac{\sum_{t=0}^{\infty} \hat{\tau}_t}{\sum_{t=0}^{\infty} \hat{\tau}_t} \\ &= \frac{1}{1 + (1 + \bar{\tau})\varphi} \\ &< 1. \end{aligned}$$

This completes the proof.