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ABSTRACT

We derive the effect of plausible deniability on asset risk premia in a dynamic setting with correlated firm values, systematic risk, and risk-averse investors. Firms optimally exercise American disclosure options, which are more valuable due to the possibility that other correlated firms may disclose high values, lifting investors' perceptions of the values of nondisclosing firms. Risk premia rise (and average prices fall) prior to disclosures, because investors make inferences about aggregate risks from failures to disclose, resulting in higher state prices for bad states.

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The seminal work by Dye (1985) challenged the unraveling arguments of Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) and showed that plausible deniability could induce firms with news below an equilibrium threshold (boundary) to credibly withhold information from the market. Following this, uncertainty about whether asymmetric information exists has generally been shown to adversely affect the quality of public knowledge.[1]

But how strategic disclosure affects risk and asset prices has received little attention. This is a more challenging problem. In the classic paradigm, agents typically receive signals about their values that are independent draws from a defined distribution. But, idiosyncratic risk does not command a risk premium in securities markets, so the standard paradigm needs to be extended to a setting in which investors are risk-averse and firm values are correlated (i.e., systematic risk exists).

Additionally, to characterize the time-dependent risk premia that investors require over any horizon, it is necessary to consider a dynamic asset pricing model with Bayesian learning.[2] As Savor and Wilson (2016) point out, earnings announcement premia routinely arise as correlated information from issuing firms conveys signals about related firms and the general economy. Under this view, Bayesian investors learn from news and solve a signal extraction problem to determine how much of the

[1]See Matthews and Postlewaite (1985); Jung and Kwon (1988); Shin (2003); Acharya, DeMarzo, and Kremer (2011); Carlin, Davies, and Iannaccone (2012).

[2]Acharya, DeMarzo, and Kremer (2011) is the closest paper to ours and includes some of the elements our our model. Their model is dynamic with a single firm whose value is correlated with a public announcement. They assume the firm and investors are risk neutral. They find that a firm with good (bad) news discloses early (late), and there is clustering after bad news in the sense that the firm discloses more quickly after a bad public announcement than after a good announcement. They also characterize the effect of plausible deniability on volatility and skewness.

announcing firm's information is systematic in nature (Ben-Rephael et al., 2020).

In this paper, we analyze a dynamic game in continuous-time where competitive firms stochastically receive a signal about the value of their terminal cash flow. At each instant in time, if a firm is informed, it may choose plausible deniability, whereby its presence in a pool of other firms commands a rational price by risk averse investors. Otherwise, the firm discloses its value, separating from others.

When the managers of a firm are informed, they have an American *disclosure option*. They choose the optimal time to exercise this option, concealing the firm's value prior to exercise. If the pool price drops below the known value, they incur a cost at each instant of time they do not disclose. However, the possible benefit of remaining in the pool is that another firm may become informed and disclose a higher value, thereby raising the pool price and prolonging the value of plausible deniability.

Given this, the Nash equilibria that arise are fundamentally different from previous studies because they are history-dependent and strategic. The disclosure of one firm endogenously affects the equilibrium boundary of other firrms. Not only is the market price of firms in the pool a function of the probability of being informed (presence of plausible deniability), but it depends on previous disclosures (correlated values) *and* the time at which they disclosed. We obtain the equilibrium disclosure threshold by solving a fixed point condition at each point in time, conditioned on prior disclosures.

One main result is that risk premia rise over time prior to disclosures. Consequently, prices are lower on average than they would be if disclosure were mandatory.

Prior to disclosures, investors regard firm values as having a mixture distribution, with the mixing variable being whether a firm has learned its value and hence could feasibly disclose. The distribution conditional on having learned the value is truncated from above at the disclosure boundary, so the mixture is a combination of the unconditional and a truncated distribution. Investors make inferences about aggregate risks from firms' failures to disclose conditioning on the mixture distribution. These inferences combined with risk aversion cause state prices for bad states to rise faster than probabilities, producing rising risk premia for the firms.

A novel feature of our model is that more than two distributions are mixed after some firm has disclosed, because there can be different truncation boundaries depending on whether a firm learned its value before the other firm disclosed or after it disclosed. We study a setting with only two firms and hence at most two truncation boundaries. Some of our analysis can be extended to more than two firms, but dealing with multiple truncation boundaries becomes difficult as the number of firms and hence the number of such boundaries increases.

We document various properties of the equilibrium, some of which relate to prior literature. For example, we analyze the excess of the disclosure boundary over the price, which arises due to the real option effect that another firm may disclose and lift the pool price, prolonging the value of plausible deniability as mentioned above. Acharya, DeMarzo, and Kremer (2011) do not solve a multi-firm model, but they do point out that this real option effect will not exist for uncorrelated or for perfectly correlated firms. We confirm this and show that the excess of the disclosure boundary over the price is in fact hump-shaped in the correlation between the firms and is zero for a correlation of zero or one. The emphasis of Acharya, DeMarzo, and Kremer (2011) is on clustering of announcements in response to a negative exogenous announcement. A similar clustering happens in our model: the time between disclosures is smaller on average when the disclosing firm announces relatively bad news.

We separate disclosure returns into scheduling returns (the returns firms would experience if it were announced that a firm has decided to disclose, without conditioning on the content of the disclosure) and announcement returns (total returns less scheduling returns). Both scheduling returns and announcement returns are positive on average for the disclosing firm and also for the nondisclosing firm, though naturally smaller for the nondisclosing firm. The scheduling return is positive on average under both the risk-neutral probability and the physical probability, because it is good news that a firm is separating from the pool. The average announcement return is a pure risk premium. Both the disclosing firm and the nondisclosing firm earn a risk premium in our model because some risk is resolved for both. Because risk premia rise and prices fall prior to disclosures, average announcement returns are higher than unconditional risk premia, consistent with the high average announcement returns that have been documented empirically (e.g., Beaver, 1968; Ball and Kothari, 1991; Cohen et al., 2007).

Positive scheduling returns are natural in a model of plausible deniability but can be puzzling in other settings. For example, Ross (1989) shows, using only no-arbitrage assumptions, that a change in the rate of information flow should not affect asset prices. In Ross's model, the change in information flow is exogenous, whereas in our model the change is endogenous (a disclosure) and signals the firm's value.

Section 1 presents the model and some facts about the risk-neutral and conditional distributions. Section 2 solves the model with a single firm and explains how the change in the mixture distribution produces a risk premium that rises as time passes prior to the firm's disclosure. Section 3 solves the model with two firms. Unlike the single-firm model, the two-firm model contains the real option effect that delaying disclosure can be valuable because a firm that delays has the potential to benefit from the other firm disclosing. Section 4 discusses the implications for asset prices. Section 5 concludes. All proofs are in the appendices.

1. Model Set-Up

Assume firms learn their date–T values $\tilde{x}_i$ at random times $\tilde{\theta}_i \in [0,T]$. We make assumptions about firm values and market pricing that are consistent with the Capital Asset Pricing Model (CAPM). There is a representative investor with constant absolute risk aversion γ who maximizes the expected utility of her wealth $\tilde{w}$ at date T. The values $\tilde{x}_i$ are symmetrically distributed and joint normally distributed with $\tilde{w}$. Therefore,

$$\tilde{x}_i = \alpha + \beta\tilde{w} + \tilde{\varepsilon}_i$$

for some α and β, where the $\tilde{\varepsilon}_i$ are normally distributed mean-zero variables that are independent of $\tilde{w}$ and independent of each other. Denote the mean and standard deviation of $\tilde{w}$ by μ_w and σ_w, respectively. Let $\mu = \alpha + \beta\mu_w$ denote the mean of $\tilde{x}_i$, and let σ_ε denote the standard deviation of $\tilde{\varepsilon}_i$. The variance of $\tilde{x}_i$ is $\sigma^2 = \beta^2\sigma_w^2 + \sigma_\varepsilon^2$. The correlation of $\tilde{x}_i$ with $\tilde{x}_j$ is $\rho = \beta^2\sigma_w^2/\sigma^2$.

Assume the interest rate r is constant. The stochastic discount factor (SDF) at date 0 for pricing payoffs at date T is proportional to the representative investor's marginal utility, which is proportional to $e^{-\gamma\tilde{w}}$. Because the interest rate is r, the expected SDF must be e^{-rT}. Hence, the SDF is

$$\tilde{m} \;=\; e^{-rT}\frac{e^{-\gamma\tilde{w}}}{\mathsf{E}[e^{-\gamma\tilde{w}}]}\,.$$

We will use risk-neutral pricing. The risk-neutral expectation of any random variable $\tilde{y}$ is $\mathsf{E}^*[\tilde{y}] = e^{rT}\mathsf{E}[\tilde{m}\tilde{x}]$. Lemma 1.1 below shows that the risk-neutral and physical distributions differ only with regard to the mean of the $\tilde{x}_i$. Let μ^* denote the risk-neutral mean. The risk premium $\gamma\beta\sigma_w^2$ in Lemma 1.1 equals risk aversion multiplied by the covariance of the firm value $\tilde{x}_i$ with the representative investor's wealth $\tilde{w}$, as in the CAPM (the CAPM holds in our model at date 0 but not at subsequent dates, due to the non-normalities induced by investors' inferences in the presence of plausible deniability).

Lemma 1.1. *Under the risk-neutral probability, firm values $\tilde{x}_i$ are joint normally distributed with means $\mu^* = \mu - \gamma\beta\sigma_w^2$ and with the same standard deviations and correlation as under the physical probability. Conditional on $\tilde{x}_j$, the value $\tilde{x}_i$ of firm $i \neq j$ is normally distributed under the physical probability with mean $\rho\tilde{x}_j + (1-\rho)\mu$ and with standard deviation $\sigma\sqrt{1-\rho^2}$. The conditional distribution is the same under the risk-neutral probability except that the conditional mean is $\rho\tilde{x}_j + (1-\rho)\mu^*$.*

Assume the random times $\tilde{\theta}_i$ are uniformly distributed on $[0, T]$ and are independent of each other and independent of the $\tilde{x}_i$ and $\tilde{w}$. This implies that the $\tilde{\theta}_i$ are independent of each other and independent of the $\tilde{x}_i$ under the risk-neutral distribution also. Assume no information arrives to the market between 0 and T other than through the disclosures of firms (or the absence of disclosures). Let P_t denote the price at date t of all firms that have not disclosed (the pool price). This price evolves deterministically between disclosures and jumps up or down when there is a disclosure. After a firm discloses, its price is $\mathrm{e}^{-r(T-t)}\tilde{x}_i$.

Assume the firm's objective is to choose a disclosure date $\tau \geq \tilde{\theta}_i$ to maximize

$$\mathsf{E}^*\left[\int_0^\tau \mathrm{e}^{-rt}P_t\,\mathrm{d}t + \int_\tau^T \mathrm{e}^{-rt}\mathrm{e}^{-r(T-t)}\tilde{x}_i\,\mathrm{d}t\right] = \mathsf{E}^*\left[\int_0^\tau \mathrm{e}^{-rt}P_t\,\mathrm{d}t + (T-\tau)\mathrm{e}^{-rT}\tilde{x}_i\right]. \quad (1.1)$$

For any time t prior to $\tilde{\theta}_i$, the firm has no choice but to remain silent. However, for $t \geq \tilde{\theta}_i$, it optimally chooses τ to maximize its payoff from t onward. The disclosure date τ is chosen based on all information prior to that date, including the firm's own value and any disclosures made by other firms. Our choice of the objective function (1.1) is motivated by the assumption that the firm or its managers benefit from having a higher share price over the course of time until T. For simplicity, we assume the benefit is additive in time and that the benefit is valued according to the market's SDF, producing the objective (1.1).

2. A Single Firm

We first solve the model with only a single firm. This model is analyzed in Section IIIA of Acharya, DeMarzo, and Kremer (2011), but here we are assuming a risk-averse representative investor rather than risk neutrality. This enables us to calculate the dynamics of the risk premium.

The equilibrium price P_t before the firm discloses is the discounted risk-neutral expectation of $\tilde{x}$, conditional on the firm not having disclosed. We conjecture that P_t decreases as time passes, because the likelihood that the firm has learned its value and is exercising plausible deniability increases as time passes. In this circumstance, the firm will choose to disclose its value as soon as its discounted value exceeds the price; consequently, the equilibrium price will indeed decrease as time passes as conjectured. So, the firm discloses whenever $\tilde{x} \geq \mathrm{e}^{r(T-t)} P_t$. Thus, $B_t = \mathrm{e}^{r(T-t)} P_t$ is the disclosure threshold (boundary).

Let ϕ and Φ denote the standard normal density and distribution functions. At any date t, there are two possible events. With probability $(T-t)/T$, the firm has not received a signal, in which case its expected value is μ^*. With probability $(t/T)\Phi((B_t - \mu^*)/\sigma)$, it received a signal and $\tilde{x}_1 \leq B_t$, in which case (from the standard formula for the mean of a truncated normal) its expected value is $\mu^* - \sigma\phi((B_t - \mu^*)/\sigma)/\Phi((B_t - \mu^*)/\sigma)$. These facts lead immediately to the following formula for P_t.

Lemma 2.1. *The market price P_t of the firm prior to disclosure is*

$$P_t \;=\; \mathrm{e}^{-r(T-t)} \left[\mu^* - \sigma \frac{t\phi((B_t - \mu^*)/\sigma)}{T - t + t\Phi((B_t - \mu^*)/\sigma)} \right] . \tag{2.1}$$

When we combine (2.1) with the condition $B_t = \mathrm{e}^{r(T-t)} P_t$, we obtain a fixed-point problem. It turns out that we can solve this fixed-point problem for a standardized model ($\mu^* = 0$ and $\sigma = 1$) and then just scale and translate to solve it for general (μ^*, σ). We denote the negative of the solution of the standardized model as $z(t)$

and compute it as follows.

Lemma 2.2. *The equation*

$$z = \frac{t\phi(z)}{T - t\Phi(z)} \tag{2.2}$$

has a unique solution $z(t)$. The solution is positive and is increasing in t.

With Lemmas 2.1 and 2.2, we have

Proposition 2.1. *The unique equilibrium price is $P_t = \mathrm{e}^{-r(T-t)}B_t$, where*

$$B_t = \mu^* - \sigma z(t)\,. \tag{2.3}$$

We can use Proposition 2.1 to calculate the risk premium prior to disclosure. The risk premium is $(\mathrm{e}^{-r(T-t)}\mathsf{E}_t[\tilde{x}] - P_t)/P_t$, where E_t is the expectation conditional on date–t information. In Lemma 2.1, we compute the risk-neutral conditional expectation. The same reasoning can be applied to calculate the physical conditional expectation, which is

$$\mu - \sigma\frac{t\phi\left(\frac{\mu - B_t}{\sigma}\right)}{T - t\Phi\left(\frac{\mu - B_t}{\sigma}\right)}\,. \tag{2.4}$$

Figure 2.1 plots the risk premium and shows that it rises over time prior to disclosure.

The fundamental reason that the risk premium rises prior to disclosure is that investors make inferences about the aggregate state from the failure to disclose, producing rising state prices for bad states. A state price is the price of an Arrow security, meaning a security that pays \$1 in a particular state of the world and 0 otherwise. The price of an Arrow security (relative to Lebesgue measure) is the

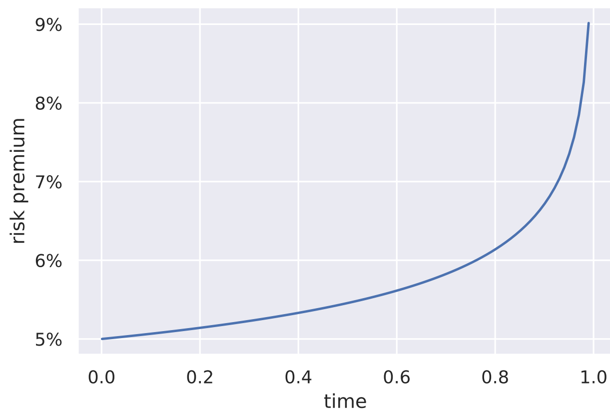

Figure 2.1: This figure shows the risk premium $(\mathrm{e}^{-r(T-t)}\mathsf{E}_t[\tilde{x}] - P_t)/P_t$, conditioning on the firm not having disclosed by t. The parameter values are $T = 1$, $\mu = 105$, $\mu^* = 100$, $\sigma = 15$, and $r = 0$.

SDF multiplied by the physical density.[3] State prices equal the discounted risk-neutral density. Figure 2.2 shows the risk-neutral and physical densities of $\tilde{x}$ at different dates, conditioning on the firm not having disclosed by that date.[4] In a risk-neutral world, state prices equal the discounted physical density. Even with risk neutrality, state prices for bad states rise prior to disclosure as Figure 2.2 shows,

[3] In our model, the SDF at date t for pricing payoffs that depend on $\tilde{x}$ at date T is

$$\mathrm{e}^{-r(T-t)}\mathrm{e}^{-\lambda(\tilde{x}-\mu)/\sigma} \Big/ \mathsf{E}_t\left[\mathrm{e}^{-\lambda(\tilde{x}-\mu)/\sigma}\right] , \tag{2.5}$$

where $\lambda = (\mu - \mu^*)/\sigma$ and where E_t denotes the physical expectation conditioned on date–t information.

[4] The event that the firm does not disclose by t is, as discussed preceding Lemma 2.1, the joint event $\{\tilde{\theta} > t\} \cup \{\tilde{\theta} \le t, \tilde{x} < B_t\}$. Under either the risk-neutral or physical probability, the probability that $\tilde{x} \le a$ for any real a conditioned on this event is

$$\frac{t\,\mathrm{prob}(\tilde{x} \le a \wedge B_t) + (T-t)\,\mathrm{prob}(\tilde{x} \le a)}{t\,\mathrm{prob}(\tilde{x} \le B_t) + T - t} .$$

Differentiating produces the densities shown in Figure 2.2.

because such states become more probable. However, the combination of risk aversion with systematic risk causes state prices for bad states to rise faster than the physical density rises (and state prices for good states to fall faster than the physical density falls), producing the rising risk premium.

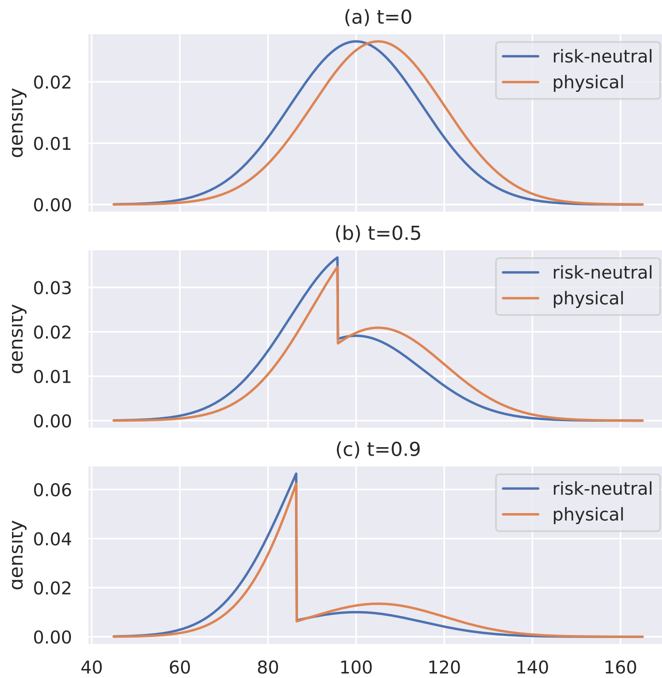

Figure 2.2: The risk-neutral and physical densities of the firm value $\tilde{x}$ are shown at different dates, conditional on the firm not having disclosed by that date. The parameter values are $T = 1$, $\mu = 105$, $\mu^* = 100$, $\sigma = 15$, and $r = 0$.

Because the firm can exercise plausible deniability, its announcement is frequently delayed relative to the time at which it acquires information. The probability that the firm discloses before any date t is $(t/T)\,\text{prob}(\tilde{x} > B_t)$. Differentiating through the fixed-point condition in Lemma 2.2 to calculate $z'(t)$, it is straightforward to calculate the density of the disclosure time. Under the physical probability, the

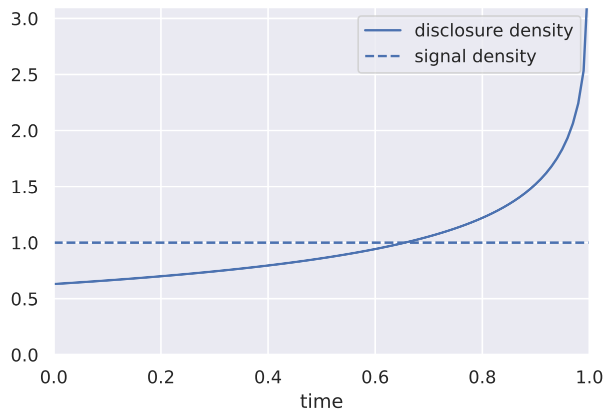

Figure 2.3: The physical density of the firm's disclosure date is shown, compared to the uniform density of the arrival time of information to the firm. The parameter values are $T = 1$, $\mu = 105$, $\mu^* = 100$, $\sigma = 15$, and $r = 0$.

density is

$$\frac{1}{T}\Phi\left(\frac{\mu - B_t}{\sigma}\right) + \frac{t}{T}\phi\left(\frac{\mu - B_t}{\sigma}\right)\frac{\phi(z(t)) + z(t)\Phi(z(t))}{T - t\Phi(z(t))}. \tag{2.6}$$

Figure 2.3 provides an illustration.

3. Multiple Firms

When multiple firms have correlated values and some have not yet disclosed, it may be optimal for a firm to continue to exercise plausible deniability even after the pool price drops below its discounted value. This is due to the possibility that another firm may become informed, disclose a high value, and cause the pool price to jump upwards, which prolongs the value of plausible deniability. In other words, like typical American options, the disclosure option must be sufficiently far in the money before it is optimal to exercise it.[5]

We analyze this disclosure option like any other American option. The optimal disclosure threshold is determined by a differential equation in the inaction region, in conjunction with value matching and smooth pasting conditions at the boundary of the region. As in Section 2, let P_t denote the price of any firm that has not disclosed, and let B_t denote the optimal disclosure boundary. At each date t after a firm learns its value $\tilde{x}_i$, define the value function

$$J_t = \sup_{\tau} \mathsf{E}_t^* \left[\int_t^{\tau} \mathrm{e}^{-r(u-t)} P_u \, \mathrm{d}u + \int_{\tau}^{T} \mathrm{e}^{-r(u-t)} \mathrm{e}^{-r(T-u)} \tilde{x}_i \, \mathrm{d}u \right] . \tag{3.1}$$

The supremum in (3.1) is taken over disclosure times that can depend on all prior information, and the value (3.1) depends on all prior disclosures and on the firm's value $\tilde{x}_i$.

We can rewrite (3.1) as

$$\mathsf{E}_t^* \left[\int_t^{T} \mathrm{e}^{-r(u-t)} P_u \, \mathrm{d}u \right] + \sup_{\tau} \mathsf{E}^* \left[\int_{\tau}^{T} \mathrm{e}^{-r(u-t)} \left\{ \mathrm{e}^{-r(T-u)} \tilde{x}_i - P_u \right\} \mathrm{d}u \right] . \tag{3.2}$$

The second term in (3.2) is the value of an American exchange option in which the firm exchanges the reward process P_u for the process $\mathrm{e}^{-r(T-u)} \tilde{x}_i$. The formulation (3.2) is natural from an option-pricing point of view, but another formulation is also

[5] Acharya, DeMarzo, and Kremer (2011) describe this same real option effect relative to the exogenous announcement that they study. The difference in our model is that all announcements are endogenous. Each firm takes into account the option values created by others, and, in turn, each firm's optimal reaction to the option value affects the options values of all other correlated firms.

interesting. We can write (3.1) as:

$$(T-t)\mathrm{e}^{-r(T-t)}\tilde{x}_i + \sup_{\tau} \; \mathsf{E}^* \left[\int_t^{\tau} \mathrm{e}^{-r(u-t)} \left\{ P_u - \mathrm{e}^{-r(T-u)}\tilde{x}_i \right\} \, \mathrm{d}u \right] . \tag{3.3}$$

The first term in (3.3) is the value the firm would achieve if disclosure were mandatory, and the second term is the value of plausible deniability.

Bellman's Principle of Optimality implies that the stochastic process

$$\int_0^t \mathrm{e}^{-ru} P_u + \mathrm{e}^{-rt} J_t \tag{3.4}$$

is a martingale under the risk-neutral probability. Taking the differential of (3.4), we see that the martingale property of (3.4) can be stated as

$$(P - rJ)\,\mathrm{d}t + \mathsf{E}^*[\mathrm{d}J] \;=\; 0 \,. \tag{3.5}$$

The value J_t is determined by equation (3.5) in conjunction with value matching and smooth pasting. The value matching condition is that, at the optimal disclosure time $t = \tau$,

$$J_t \;=\; \int_t^T \mathrm{e}^{-r(u-t)}\mathrm{e}^{-r(T-u)}\tilde{x}_i \,\mathrm{d}u \;=\; (T-t)\mathrm{e}^{-r(T-t)}\tilde{x}_i \,. \tag{3.6}$$

The smooth pasting condition at the boundary is that J paste together smoothly (in x) with the value on the right-hand side of (3.6).

We show in Appendix A that equation (3.5), value matching, and smooth pasting are equivalent to a simple marginal condition for the exercise boundary. Consider a firm with value $\tilde{x}_i$ such that $\mathrm{e}^{-r(T-t)}\tilde{x}_i > P_t$. In other words, the firm would trade at a higher price if it disclosed. The cost of delaying disclosure for an instant $\mathrm{d}t$

is $[e^{-r(T-t)}\tilde{x}_i - P_t]\,dt$. The benefit of delaying disclosure is that another firm may disclose a high value during the instant dt, lifting the pool price and producing a positive jump ΔJ_t in J_t.[6] The marginal condition is that the cost of delaying disclosure must equal the expected benefit for a firm that is at the optimal exercise boundary. Thus, the optimal exercise boundary at date t is the number B_t such that

$$\left[e^{-r(T-t)}B_t - P_t\right]\,dt \;=\; \mathsf{E}_t^*[\Delta J_t \mid \tilde{x}_i = B_t]\,. \tag{3.7}$$

The reasoning leading to (3.7) is valid for any number of firms, and it is also valid in a single-firm model with an exogenous disclosure as in Acharya, DeMarzo, and Kremer (2011). The complication that we address in this paper is that in our model (3.7) is an equilibrium condition rather than just an optimality condition, because the jumps ΔJ_t in our model depend on the disclosure policies of other firms, which in a symmetric equilibrium must also conform to the time-varying B_t defined by (3.7).

We are able to solve this equilibrium condition explicitly when there are only two firms, which we assume henceforth. Let us define Stage 1 of the game to be the (endogenous) period of time during which no firm has made a disclosure. Likewise, let Stage 2 be the time period when one firm is left, as in Section 2. Let P_{it} and B_{it} denote the price of any firm that has not disclosed and the optimal disclosure boundary, respectively, in stage i of the model. The price P and boundary B jump

[6] For a firm at the optimal exercise boundary, downward jumps in J_t are not possible, because a downward jump in the boundary does not affect the optimal policy of a firm that is already at the boundary – the firm should still disclose, so the value of J is still the right-hand side of (3.6) after such a jump. On the other hand, an upward jump in the boundary means that it is optimal to continue to exercise plausible deniability a while longer, so the value of J becomes larger than the right-hand side of (3.6), that is, there is a positive jump ΔJ_t.

from $P_{1,t}$ and $B_{1,t}$ to $P_{2,t}$ and $B_{2,t}$, respectively, at the time that the first firm discloses. We solve the model by working backwards.

3.1. Stage 2

After one firm has disclosed, the model is similar to the single-firm model studied in Section 2. The only difference is that the market's inference problem can be more complicated in Stage 2 of a two-firm model. Without loss of generality, denote the firm that discloses first by firm 1. Let τ denote the disclosure time. Consider a date $t \geq \tau$ prior to firm 2's disclosure. We know that firm 2 will disclose as soon as the price drops below its discounted value, that is, when $\tilde{x}_2 \geq \mathrm{e}^{r(T-t)} P_{2t}$. So, B_2 and P_2 are related as in Section 2, namely, $B_{2t} = \mathrm{e}^{r(T-t)} P_{2t}$.

The market computes P_{2t} by conditioning on $\tilde{x}_1$ and on the event

$$\left\{\tilde{\theta}_2 > t\right\} \cup \left\{\tilde{\theta}_2 \leq \tau, \tilde{x} < B_{1\tau} \wedge B_{2t}\right\} \cup \left\{\tau < \tilde{\theta}_2 \leq t, \tilde{x} < B_{2t}\right\}.$$

The three events in this union are (i) firm 2 learns its value after t, (ii) firm 2 learns its value before τ and the value is less than both $B_{1\tau}$ and B_{2t}, and (iii) firm 2 learns its value between τ and t and the value is less than B_{2t}. These are the three possibilities given that firm 2 has not disclosed by t. To understand event (ii), note that if firm 2 received its value before τ, then it would have disclosed before τ if $\tilde{x}_2 > B_{1\tau}$ and would have disclosed before t if $\tilde{x}_2 > B_{2t}$, so if it has not disclosed before t then it must be that both $\tilde{x}_2 < B_{1\tau}$ and $\tilde{x}_2 < B_{2t}$. On the other hand, in event (iii), if the firm learned its value between τ and t and has not disclosed by t, then we can be sure that its value is below B_{2t}, but we cannot be sure that it is below $B_{1\tau}$. In this case, both boundaries affect the pool price. However, if it happens to be true that

$B_{2t} \leq B_{1\tau}$, then in both (ii) and (iii), the discounted value is known to be lower than B_{2t}, and we can combine events (ii) and (iii) as: (ii′) firm 2 learns its value before t and the discounted value is less than B_{2t}. The case (ii′) is the same as the conditioning in Lemma 2.1. As time passes, B_{2t} will drop, and we will eventually be in case (ii′).

We are now conditioning on $\tilde{x}_1$ also. As stated in Lemma 1.1, the risk-neutral distribution of $\tilde{x}_2$ conditonal on $\tilde{x}_1$ is normal with mean $\tilde{\mu}_2^* \stackrel{\text{def}}{=} \rho\tilde{x}_1 + (1-\rho)\mu^*$ and standard deviation $\sigma_2 \stackrel{\text{def}}{=} \sigma\sqrt{1-\rho^2}$. As in Section 2, we can solve the model for a standardized distribution ($\tilde{\mu}_2^* = 0$ and $\sigma_2 = 1$) and then scale and translate to produce the solution for general ($\tilde{\mu}_2^*$, σ_2). For any real number y and any date $t \geq \tau$, consider the fixed point condition

$$z \; = \; \frac{\tau\phi(y) + (t-\tau)\phi(z)}{T - \tau\Phi(y) - (t-\tau)\Phi(z)} \, . \tag{3.8}$$

Lemma 3.1. *Equation* (3.8) *has a unique solution $z_2(\tau, t, y)$. It is positive and is an increasing function of t. Furthermore, $z_2(\tau, t, y) < y$ if and only if $z(t) < y$, where $z(t)$ is defined in Lemma 2.2.*

We are now able to describe the equilibrium price and boundary in Stage 2. The standardized boundary switches from the fixed point in Lemma 3.1 to the fixed point in Lemma 2.2 when we move to case (ii′) as described above. This appears in (3.10) below as the switch from z_2 to z.

Proposition 3.1. *Suppose firm 1 discloses $\tilde{x}_1$ at date τ and consider $t \geq \tau$. The unique equilibrium price of firm 2 after the disclosure of firm 1 and before the dis-*

closure of firm 2 is $P_{2t} = \mathrm{e}^{-r(T-t)} B_{2t}$*, where*

$$B_{2t} \;=\; \tilde{\mu}_2^* - \sigma_2 Z_{2t} \tag{3.9}$$

and where

$$Z_{2t} \;=\; \begin{cases} z_2(\tau, t, \tilde{y}) & \textit{if } z(t) < \tilde{y} \\ z(t) & \textit{otherwise}\,, \end{cases} \tag{3.10}$$

with $\tilde{y} = (\tilde{\mu}_2^* - B_{1\tau})/\sigma_2$*.*

3.2. Stage 1

Now, we back up and find the equilibrium price and exercise boundary before either firm discloses. We begin by finding the equilibrium price conditional on the boundary. Then, we will use the marginal condition (3.7) to find the boundary. Consider any date t and suppose neither firm disclosed prior to t. The equilibrium price is

$$P_{1t} \;=\; \mathrm{e}^{-r(T-t)} \mathsf{E}^*[\tilde{x}_i \mid \text{neither firm disclosed prior to } t]\,. \tag{3.11}$$

Unlike the single-firm model, in which we condition on a firm not disclosing, here we also have to condition on the lack of disclosure by another correlated firm that may also be exercising plausible deniability. This depresses the equilibrium price further. The event on which we are conditioning in (3.11) is the union of the following four disjoint events:

(i) neither firm learned its value prior to t;

(ii) firm 1 learned its value $\tilde{x}_1 < B_{1t}$ prior to t and firm 2 did not learn its value prior to t;

(iii) firm 2 learned its value $\tilde{x}_2 < B_{1t}$ prior to t and firm 1 did not learn its value prior to t;

(iv) both firms learned their values $\tilde{x}_i < B_{1t}$ at dates prior to t.

As before, we can work with standardized distributions and then scale and translate for general (μ^*, σ). We will obtain $B_{1t} = \mu^* - \sigma z_1(t)$, where z_1 is a deterministic function that does not depend on the parameters μ^* and σ. Let $\Gamma(\cdot \mid \rho)$ denote the bivariate distribution function for normal random variables with zero means, unit standard deviations, and correlation equal to ρ. By combining the risk-neutral expectations of the four events described above, we obtain the following description of the equilibrium price conditional on the boundary, that is, conditional on $z_1(t)$.

Lemma 3.2. *Suppose there has been no disclosure prior to date t. Then,*

$$\frac{\mathrm{e}^{r(T-t)} P_{1t} - \mu^*}{\sigma} = -\frac{t(1+\rho)\phi\left(z_1(t)\right)\left[T - t\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}} z_1(t)\right)\right]}{\left[T - t\Phi\left(z_1(t)\right)\right]^2 + t^2\left[\Gamma\left(-z_1(t), -z_1(t) \mid \rho\right) - \Phi(-z_1(t))^2\right]}, \quad (3.12)$$

where $z_1(t) \overset{def}{=} (\mu^* - B_{1t})/\sigma$.

From Lemma 3.2, the left-hand side of the marginal condition (3.7), that is, the cost of waiting to disclose for a firm at the boundary, can be computed in terms of the unknown $z_1(t)$ as

$$\mathrm{e}^{-r(T-t)}B_{1t} - P_{1t} \;=\; \sigma\mathrm{e}^{-r(T-t)} \times$$
$$\left[\frac{t(1+\rho)\phi\left(z_1(t)\right)\left[T - t\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}z_1(t)\right)\right]}{\left[T - t\Phi\left(z_1(t)\right)\right]^2 + t^2\left[\Gamma\left(-z_1(t), -z_1(t) \mid \rho\right) - \Phi(-z_1(t))^2\right]} - z_1(t)\right] . \quad (3.13)$$

We can derive a useful bound on the equilibrium disclosure boundary from the fact that (3.13) must be nonnegative. Nonnegativity of (3.13) follows from the fact that a firm will never disclose when the pool price is above its discounted value. The bound relates the boundary in Stage 1 of the two-firm model to the boundary in the single-firm model in Section 2. On the one hand, the boundary (and price) should be lower in the two-firm model because it is bad news that another correlated firm has also not disclosed; on the other hand, the boundary should be higher in the two-firm model because of the real option effect represented in the marginal condition (3.7). We can most conveniently express the relationship between the boundaries in terms of the negatives of the standardized boundaries (z_1 and z) as follows.

Lemma 3.3. *In equilibrium prior to either firm's disclosure, it must be true that*

$$z_1(t) \;\leq\; \sqrt{\frac{1+\rho}{1-\rho}}z(t)\,. \quad (3.14)$$

Now, consider the right-hand side of the marginal condition (3.7). To compute the value of waiting to disclose when at the boundary, suppose firm 2 is at the exercise boundary at t, that is, $\tilde{x}_2 = B_{1t}$. Suppose firm 1 discloses at t. As discussed previously, there will be a nonzero jump in firm 2's value function if and only if the boundary rises following the disclosure—that is, if and only if $B_{2t} > B_{1t}$.

So, which disclosures by firm 1 have the property that $B_{2t} > B_{1t}$? By combining the description of B_{2t} in Proposition 3.1 with the bound in Lemma 3.3, we obtain the following. We use the same definition of $\tilde{y}$ as in Proposition 3.1.

Lemma 3.4. *Suppose firm 1 discloses $\tilde{x}_1$ at date t. Then $B_{2t} > B_{1t}$ if and only if*

$$\tilde{y} \stackrel{def}{=} \frac{\rho\tilde{x}_1 + (1-\rho)\mu^* - B_{1t}}{\sigma_2} > z(t)\,. \tag{3.15}$$

A necessary condition for (3.15) *is that $\tilde{x}_1 > B_{1t}$.*

Lemma 3.4 shows that a jump in the value of firm 2 occurs at date t if and only if firm 1 learns its value at t and has a sufficiently high value $\tilde{x}_1 > B_{1t}$.[7] Now we turn to computing the jump in firm 2's value when $B_{2t} > B_{1t}$. Given our working assumption that firm 2 is at the boundary at t, its value at t is

$$J_t = \int_t^T \mathrm{e}^{-r(u-t)}\mathrm{e}^{-r(T-u)} B_{1t}\,\mathrm{d}u\,. \tag{3.16}$$

When firm 1 discloses at t and we have a jump in the boundary $B_{2t} > B_{1t}$, then firm 2 will continue to exercise plausible deniability and earn the reward P_{2u} until P_{2u} drops down to $\mathrm{e}^{-r(T-u)}B_{1t}$, equivalently, B_{2u} drops down to B_{1t}. The jump in

[7]This result is quite important for rendering the model tractable. It means that, for computing the marginal condition (3.7), we can ignore the case in which firm 1 learns its value before t and discloses at t (which happens only when $\tilde{x}_1 = B_{1t}$). The arrival rate of such disclosures depends on the slope of the boundary B_1 at t (more disclosures happen in a given time interval if the boundary is steeper) . Thus, absent a result like Lemma 3.4, we would have to solve the marginal condition (3.7) as a dynamical system, perhaps as a differential equation. Because of Lemma 3.4, we can instead solve for the boundary pointwise in t, by solving the fixed-point problem in Proposition 3.2 at each t.

firm 2's value function is the difference between the value of this and the value (3.16), which is

$$\int_t^s \mathrm{e}^{-r(u-t)}\left[P_{2u} - \mathrm{e}^{-r(T-u)}B_{1t}\right]\,\mathrm{d}u\,, \tag{3.17}$$

where s denotes the date after t at which $B_{2s} = B_{1t}$. From Proposition 3.1, the date s is the solution of $z(s) = \tilde{y}$, that is, it is $s = z^{-1}(\tilde{y})$. Furthermore, Proposition 3.1 shows that, for $u \in [t, z^{-1}(\tilde{y})]$,

$$\mathrm{e}^{-r(u-t)}\left[P_{2u} - \mathrm{e}^{-r(T-u)}B_{1t}\right] \;=\; \sigma_2\,\mathrm{e}^{-r(T-t)}[\tilde{y} - z_2(t,u,\tilde{y})]\,. \tag{3.18}$$

Combining these facts, we conclude that the risk-neutral expected jump in firm 2's value conditional on a jump occurring and conditional on firm 2 being at the boundary at t is

$$\sigma_2\,\mathrm{e}^{-r(T-t)}\mathsf{E}^*\left[\int_t^{z^{-1}(\tilde{y})}[\tilde{y} - z_2(t,u,\tilde{y})]\,\mathrm{d}u\;\middle|\;\tilde{x}_2 = B_{1t}, \text{firm 1 discloses}, \tilde{y} > z(t)\right]. \tag{3.19}$$

We need to derive the density of $\tilde{y}$ given the information on which we are conditioning in (3.19). Maintaining our assumption that $\tilde{x}_2 = B_{1t}$, the distribution of $\tilde{x}_1$ conditional on $\tilde{x}_2$ is normal with mean $\rho B_{1t} + (1-\rho)\mu^*$ and standard deviation σ_2. An easy calculation then shows that the distribution of $\tilde{y}$ conditional on $\tilde{x}_2$ is normal with mean $\sqrt{1-\rho^2}z_1(t)$ and standard deviation ρ. The distribution of $\tilde{y}$ conditional on $\tilde{x}_2$ and conditional on $\tilde{y} > z(t)$ is truncated normal with density

$$\frac{1}{\rho}\,\phi\left(\frac{\sqrt{1-\rho^2}z_1(t) - y}{\rho}\right)\Bigg/\;\Phi\left(\frac{\sqrt{1-\rho^2}z_1(t) - z(t)}{\rho}\right). \tag{3.20}$$

The last piece we need is the arrival rate of disclosures by firm 1 having $\tilde{y} > z(t)$.

For any $\epsilon > 0$, the probability that firm 1 learns its value at some $u \in (t, t+\epsilon)$ and $\tilde{y} > z(t)$, conditional on $\tilde{x}_2 = B_{1t}$, is

$$\frac{1}{T}\int_t^{t+\epsilon} \Phi\left(\frac{\sqrt{1-\rho^2}z_1(u) - z(t)}{\rho}\right) \mathrm{d}u\,.$$

The probability conditional on $\tilde{x}_2 = B_{1t}$ and conditional on firm 1 not having disclosed prior to t is

$$\int_t^{t+\epsilon} \Phi\left(\frac{\sqrt{1-\rho^2}z_1(u) - z(t)}{\rho}\right) \mathrm{d}u \bigg/ \left[T - t\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}z_1(t)\right)\right]\,.$$

The arrival rate (hazard rate) is the derivative of this ratio with respect to ϵ, evaluated at $\epsilon = 0$, so it is

$$\Phi\left(\frac{\sqrt{1-\rho^2}z_1(t) - z(t)}{\rho}\right) \bigg/ \left[T - t\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}z_1(t)\right)\right]\,. \tag{3.21}$$

Putting these things together, we arrive at the following formula for the right-hand side of the marginal condition (3.7).

Lemma 3.5.

$$\begin{aligned}\mathsf{E}_t^*[\Delta J_{2t} \mid \tilde{x}_2 = B_{1t}] \;=\; & \left\{\frac{\sigma\sqrt{1-\rho^2}}{\rho}\mathrm{e}^{-r(T-t)} \bigg/ \left[T - t\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}z_1(t)\right)\right]\right\} \\ & \times \int_{z(t)}^{\infty}\int_t^{z^{-1}(y)} [y - z_2(t,u,y)]\,\phi\left(\frac{\sqrt{1-\rho^2}z_1(t) - y}{\rho}\right) \mathrm{d}u\,\mathrm{d}y\,. \end{aligned} \tag{3.22}$$

We can now express the marginal condition (3.7) as a numerically tractable fixed-point condition. Note that the only model parameter that appears in (3.24) is the correlation ρ. The correlation determines the standardized boundary and price, and then we obtain the general boundary and price by scaling and translating as discussed before.

Proposition 3.2. *Prior to any disclosure, the equilibrium disclosure boundary at t is*

$$B_{1t} \;=\; \mu^* - \sigma z_1(t)\,, \tag{3.23}$$

where $z_1(t)$ satisfies

$$\frac{t(1+\rho)\phi\left(z_1(t)\right)\left[T - t\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}z_1(t)\right)\right]}{\left[T - t\Phi\left(z_1(t)\right)\right]^2 + t^2\left[\Gamma\left(-z_1(t), -z_1(t)\mid\rho\right) - \Phi(-z_1(t))^2\right]} - z_1(t) \;=$$
$$\frac{\sqrt{1-\rho^2}}{\rho} \times \frac{\int_{z(t)}^{\infty}\int_t^{z^{-1}(y)}\left[y - z_2(t,u,y)\right]\phi\left(\frac{\sqrt{1-\rho^2}z_1(t)-y}{\rho}\right)\mathrm{d}u\,\mathrm{d}y}{T - t\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}z_1(t)\right)}\,. \tag{3.24}$$

Figure 3.1 shows the equilibrium pool price and disclosure boundary described in Proposition 3.2 and also the pool price and disclosure boundary in the single-firm model. In all versions of the model, the price begins at μ^* at date 0 and decreases to $-\infty$ as $t \to T$. The pool price is uniformly decreasing in the correlation at each date, because the failure of another correlated firm to disclose is more meaningful, and hence the price is lower, when the correlation is higher. In the single-firm model, the boundary and price are the same (with $r = 0$); however, in the multi-firm model, the boundary exceeds the price due to the real option effect. Therefore, the boundary in the multi-firm model is initially higher than in the single-firm model.

Figure 3.2 shows the equilibrium timing of disclosures. The first announcement in the two-firm model generally occurs before the announcement in the single-firm

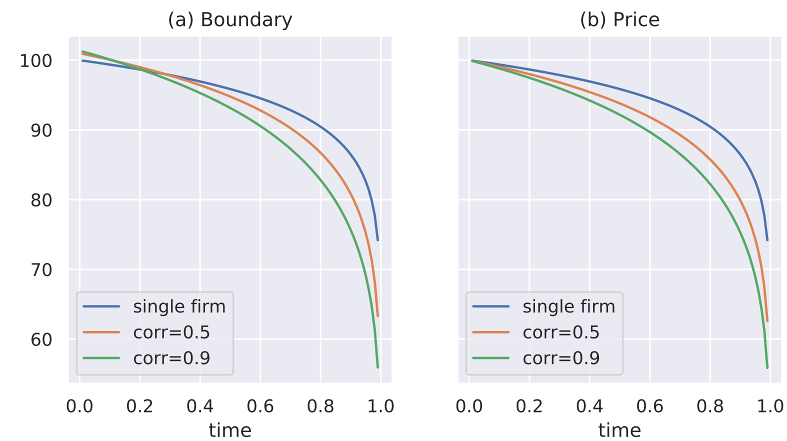

Figure 3.1: Panel (a) presents the equilibrium disclosure boundary, and Panel (b) presents the equilibrium pool price for the single-firm model and for Stage 1 of the two-firm model with different correlations ρ. The parameter values are $T = 1$, $\mu = 105$, $\mu^* = 100$, $\sigma = 15$, and $r = 0$.

model, both because the minimium of the two information arrival times is generally smaller than the arrival time in the single-firm model and because the boundary in the multi-firm model is generally lower than the boundary in the single-firm model. The second of the two announcements generally occurs after the announcement in the single-firm model. Recall that the boundary that determines the second of the two announcements is the Stage 2 boundary described in Proposition 3.1.

4. Implications for Asset Prices

We first present our main results regarding risk premia and average prices. Then, we discuss some issues considered by Acharya et al. (2011). Finally, we describe the scheduling and announcement returns for the disclosing and nondisclosing firms.

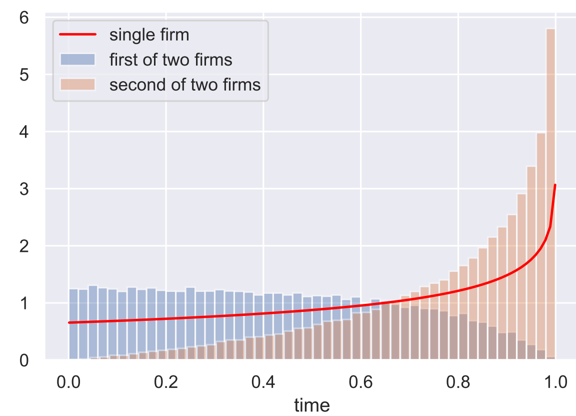

Figure 3.2: The red curve is the density function of the disclosure time in the single-firm model, as shown in Figure 2.3. The histograms are the distributions of the first and second disclosure times in the two-firm model, generated from 100,000 simulations of the model. The parameter values are $\rho = 0.5$, $T = 1$, $\mu = 105$, $\mu^* = 100$, $\sigma = 15$, and $r = 0$.

4.1. Risk Premia and Average Prices

As in the single-firm model, the risk premium rises over time prior to disclosures. Figure 4.1 shows that it rises faster with multiple firms than with a single firm and rises faster when the firms are more highly correlated. The risk premium rises for the same reason that it rises in the single-firm model: rising state prices for bad states. It rises faster with multiple firms and with higher correlation, because the market makes a stronger inference from the absence of disclosures by correlated firms.

Figure 4.2 shows the effect of plausible deniability on average prices. We compute the expectation under the physical probability of the average of the two firms' prices at each date, given the equilibrium disclosure times and the market pricing described in Section 3. We compare this to the expectation of the average price when disclosure

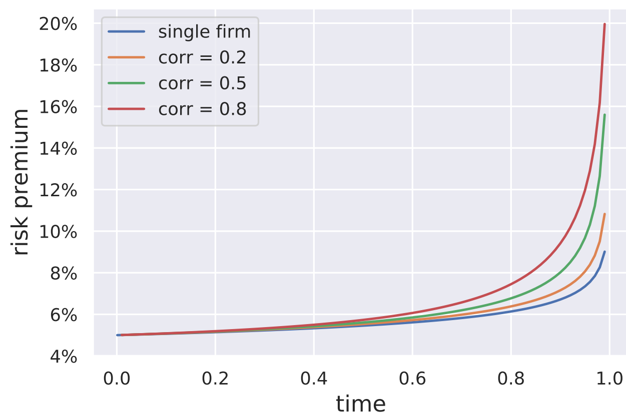

Figure 4.1: The Stage 1 risk premium is computed as the difference between the market's discounted conditional expectation of $\tilde{x}_i$ and the price divided by the price. The parameter values are $r = 0$, $\mu = 105$, $\mu^* = 100$, and $\sigma = 15$.

is mandatory.[8]

Figure 4.2 shows that the possibility of plausible deniability depresses average prices.[9] This is due to the higher risk premium that plausible deniability induces. Of course, the risk is eventually resolved as firms disclose, so the effect of plausible deniability vanishes as $t \to T$, as Figure 4.2 shows. The patterns for different correlations are due to (i) the risk premium rises faster in Stage 1 when the correlation is higher, as shown in Figure 4.1, so the average price initially falls faster when the correlation is higher, and (ii) when the correlation is higher, the risk reduction for the

[8]When disclosure is mandatory, the pool price is μ^* until one firm discloses, and then the price for the nondisclosing firm (firm 2) is $\rho\tilde{x}_1 + (1-\rho)\mu^*$. Integrating over the information arrival times, we can easily compute that the expected average price is $t^2\mu + t(1-t)[(1+\rho)\mu + (1-\rho)\mu^*] + (1-t)^2\mu^*$.

[9]Thus, firms might prefer, if possible, to commit to disclosing as soon as they learn their values. A preference for commitment occurs in many contexts. We should note however that the discounted prices are martingales under the risk-neutral probability, so it is only under the physical probability that the average price is depressed, and we are assuming firms maximize risk-neutral expectations.

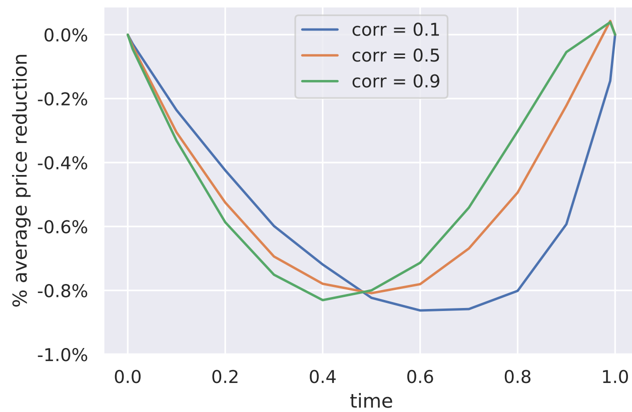

Figure 4.2: The figure presents the difference between the expected average price in the two-firm model (computed from 100,000 simulations for each correlation) and the expected average price with mandatory disclosure, shown as a percent of the expected average price with mandatory disclosure. The parameter values are $r = 0$, $\mu = 105$, $\mu^* = 100$, and $\sigma = 15$.

second firm to disclose is greater when the first firm discloses, so the risk premium and price reduction are generally smaller with higher correlations at later dates.

4.2. Real Option Effect and Clustering

Here we examine two issues addressed by Acharya et al. (2011). In their model, the firm is willing to allow the pool price to drop some distance below its value before disclosing, because of the possibility that the public announcement will be good news, which will lift the pool price and prolong the value of plausible deniability. This real option effect (relative to the other firm's disclosure rather than an exogenous announcement) is what underlies our marginal condition (3.7). Figure 4.3 plots the excess of the Stage 1 boundary over the price in our model, in relation to the price, as a function of the firms' correlation. In their discussion of a multi-firm model, Acharya et al. (2011) point out that this excess will be zero at correlations of 0 and

1 when there are multiple firms and no exogenous announcement. This is confirmed in Figure 4.3, which shows moreover that the pattern is in general hump-shaped.

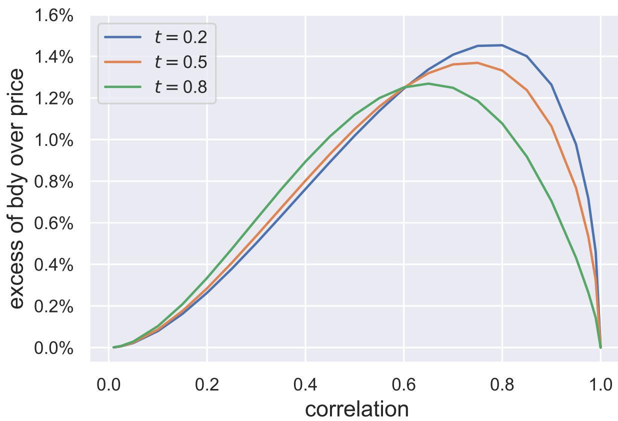

Figure 4.3: The figure presents the difference between the Stage 1 boundary and the Stage 1 price in relation to the price: $(B_{1t} - P_{1t})/P_{1t}$. The parameter values are $r = 0$, $\mu = 105$, $\mu^* = 100$, and $\sigma = 15$.

The marginal condition (3.7) demonstrates that the spread between the Stage 1 boundary and price shown in Figure 4.3 depends on the probability that the other firm discloses, the distribution of the other firm's potential disclosure, and the effect of the other firm's disclosure on the market's inference regarding the nondisclosing firm. Correlation of firm values has two opposing effects on the spread. First, the higher the correlation, the greater the inference the market will make regarding the nondisclosing firm. So, for any given disclosure by the other firm, the benefit for the nondisclosing firm is greater when the correlation is higher. Second, the higher the correlation, the less likely it is that the other firm will disclose a value markedly different from the nondisclosing firm's value. This channel reduces the potential benefit of disclosure by the other firm, and hence reduces the spread between boundary and price, when the correlation is increased. Figure 4.3 shows that the first effect dominates at low correlations—the spread rises with correlation—and the second effect dominates at high correlations—the spread falls with correlation.

The passage of time affects both channels described in the previous paragraph. The distributions of value from both the market's perspective and from the perspective of the nondisclosing firm become more skewed as time passes, as explained in Section 2. Section 2 discusses how the relation between risk-neutral and physical distributions changes with the truncation-induced skewness. The distributions that are relevant for Figure 4.3 are the nondisclosing firm's perception of the other firm's value and the nondisclosing firm's perception how the market will perceive its value after disclosure of the other firm's value. The first distribution (the distribution of $\tilde{x}_1$ given $\tilde{x}_2$) is a truncation of the normal distribution with mean $\rho\tilde{x}_2+(1-\rho)\mu^*$ and standard deviation $\sigma\sqrt{1-\rho^2}$. The second distribution (the distribution of $\rho\tilde{x}_1+(1-\rho)\mu^*$ conditional on $\tilde{x}_2$) is a truncation of the normal distribution with mean $\rho^2\tilde{x}_2+(1-\rho^2)\mu^*$ and standard deviation $\rho\sigma\sqrt{1-\rho^2}$. The difference between the distributions obviously depends on the correlation ρ. Thus, how the passage of time affects the spread between the boundary and the price depends on the correlation. The upshot is that the passage of time increases the spread between the boundary and price for low correlations (on the left in Figure 4.3, the green curve is highest) and decreases the spread for low correlations (on the right in Figure 4.3, the green curve is lowest).

Acharya et al. (2011) show that disclosures happen faster after a negative public announcement than a positive public announcement. The same phenomenon occurs

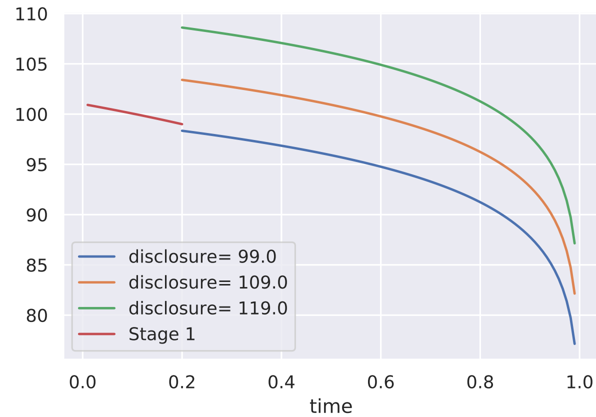

Figure 4.4: The figure presents Stage 1 and Stage 2 boundaries. The red curve is the Stage 1 boundary prior to $t = 0.2$. The other curves are Stage 2 boundaries resulting from different disclosures at $t = 0.2$. The parameter values are $\rho = 0.5$, $r = 0$, $\mu = 105$, $\mu^* = 100$, and $\sigma = 15$. At $t = 2$, the Stage 1 boundary is 99.0, so a disclosure at 99.0 causes the Stage 2 boundary to drop relative to the Stage 1 boundary, as shown in Lemma 3.4.

in our model relative to the endogenous announcements by the firms: the second firm delays less after the first firm's announcement when the first firm's announcement is relatively bad news. The second firm announces when its value is above the Stage 2 boundary, so this clustering effect is equivalent to the Stage 2 boundary being lower when the first firm's announcement is worse news. The clustering effect is an implication of Proposition 3.2. Figure 4.4 illustrates the phenomenon. It plots equilibrium Stage 2 boundaries conditional on various possible disclosures. The Stage 2 boundary is higher—implying that the second disclosure will be delayed longer on average—when the disclosure is better news.

When the first firm's disclosure is at or near the Stage 1 boundary, the Stage 2 boundary is lower at the disclosure date than the Stage 1 boundary, as shown in

Lemma 3.4. This is illustrated in Figure 4.4. If the second firm has learned its value and its value lies in the interval between the two boundaries, then it will disclose at the same time as the first firm. Thus, there is a nonzero probability that both firms will disclose at the same time, even though their information arrival times are independently and uniformly distributed. This is the extreme case of clustering.

4.3. Scheduling and Announcement Returns

Now, we look at announcement returns. Announcements in our model occur at unpredictable times. In practice, announcements are usually scheduled ahead of time. In a world where plausible deniability is a possibility, the mere act of scheduling an announcement conveys good news, because it demonstrates that the firm wants to separate from the pool. So, there should be a positive market reaction when the announcement is scheduled.[10] On the other hand, the market reaction to the actual news that is released should be zero on average under the risk-neutral probability and positive on average under the physical probability only if it is positively correlated with the pricing kernel. To separate the reaction to the scheduling and the reaction to the actual announcement in our model, we compute the risk-neutral expectation of the announcement at the announcement date, conditioning on an announcement occurring but not conditioning on the content of the announcement. This is the price at which the firm would trade an instant before the announcement if it were known at that time that an announcement was forthcoming. Call this price the interim

[10] As mentioned in the introduction, plausible deniability is one answer to a question asked by Ross (1989): "how could the mere announcement of the acceleration of the release of some information concerning its future payoff possibly influence the price of an asset?" In Ross's model (as apparently in the example of New York City bonds that he cites) the acceleration is not a signal, but it is a signal in a model with plausible deniability.

price. The total announcement return is the pre-announcement-to-interim return compounded with the interim-to-post-announcement return. We call the former the scheduling return and the latter the announcement return.

Figure 4.5 shows that the scheduling return for the disclosing firm is substantially larger than the expected announcement return (the announcement risk premium) in our model. The same is true for the non-disclosing firm, which also experiences a positive scheduling return and earns a positive risk premium upon announcement as shown in Panel (b) of Figure 4.5. Both the scheduling return and the announcement risk premium are smaller for the non-disclosing firm than for the disclosing firm, consistent with Savor and Wilson (2016). All of the returns are larger if the disclosure happens later, consistent with the rising risk premium shown in Figure 4.1.

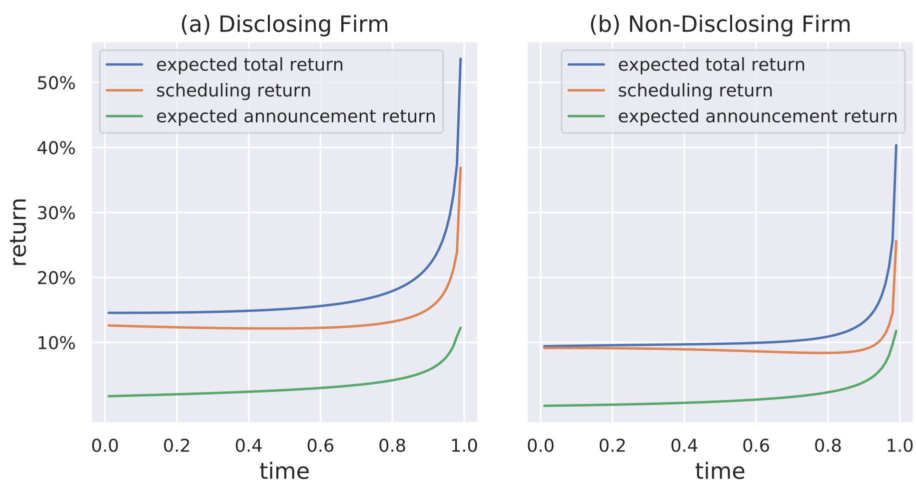

Figure 4.5: Average returns are calculated as a function of the first disclosure time. The definitions of the returns are given in the text. The parameter values are $\rho = 0.5$, $T = 1$, $\mu = 105$, $\mu^* = 100$, $\sigma = 15$, and $r = 0$.

5. Conclusion

When plausible deniability is possible, risk premia rise prior to disclosures and consequently prices decline on average. This is a consequence of investors making inferences about the aggregate risks from firms' failure to disclose. The rising risk premia and falling prices produce announcement returns that are higher than unconditional risk premia, consistent with the empirical evidence of Savor and Wilson (2016). These effects are stronger when firm values are more highly correlated.

We demonstrate these results in a model that advances the literature in two respects: risk-averse investors and competing firms. We solve the model by solving a fixed-point condition at each date. One might expect that the equilibrium disclosure boundary at any date would depend on the boundary at other dates and hence could be obtained at best as the solution of a differential equation. However, we show that disclosures at the boundary cause the boundary for the nondisclosing firm to fall, which simplifies the problem substantially, making it possible to solve for the boundary pointwise in time. This result also implies that there is a nonzero probability of simultaneous disclosure by the two firms, an extreme version of the clustering studied by Acharya et al. (2011). Whether this simplification can be attained with more than two firms is an open question. However, many of our results (in particular, the rising risk premia and the effect of correlation on the real option effect) should be robust to the number of firms.

Appendix A. Marginal condition for Optimal Stopping

We want to describe the equilibrium boundary in Stage 1 of the two-firm model. Consider a firm that has learned its value x. If it has disclosed at t, then its remaining value is equal to

$$\int_t^\infty \mathrm{e}^{-r(u-t)}\mathrm{e}^{-r(T-u)}x \;=\; (T-t)\mathrm{e}^{-r(T-t)}x \qquad \text{(A.1)}$$

Prior to disclosing, its value function depends on whether the other firm has disclosed. If not, then we are in Stage 1 and we write the value function as $J_1(t,x)$. If the other firm has disclosed, then we are in Stage 2 and the firm's value function depends on (t,x) and on the timing and content of the other firm's disclosure. Write it as $J_2(\tau,d,t,x)$ where τ denotes the date of the other firm's disclosure and d denotes the content of the other firm's disclosure. Set

$$J = \begin{cases} J_1(t,x) & \text{in Stage 1} \\ J_2(\tau,d,t,x) & \text{in Stage 2} \\ (T-t)\mathrm{e}^{-r(T-t)}x & \text{after disclosure} \end{cases}$$

The value matching condition is that J equal (A.1) when x is at the optimal disclosure boundary. The smooth pasting condition is that

$$\frac{\partial J}{\partial x} \;=\; (T-t)\mathrm{e}^{-r(T-t)} \qquad \text{(A.2)}$$

when x is at the optimal disclosure boundary.

For concreteness, label the firm whose value we are studying as firm 2, so x denotes the realization of $\tilde{x}_2$, and d denotes the realization of $\tilde{x}_1$. We need to compute $\mathsf{E}^*[\mathrm{d}J]$

in Stage 1. We have

$$\mathsf{E}^*[\mathrm{d}J] \;=\; \frac{\partial J_1(t,x)}{\partial t}\,\mathrm{d}t + \mathsf{E}^*[\Delta J] \tag{A.3}$$

where the possible jump ΔJ would be a result of a disclosure by firm 1. The jump size and arrival rate of disclosures is discussed in the text. Let $\lambda(t,x)$ denote the arrival rate of disclosures that create jumps, so

$$\mathsf{E}^*[\Delta J] \;=\; \big(\mathsf{E}^*[J_2 \mid \text{jump}] - J_1\big)\lambda(t,x)\,\mathrm{d}t\,.$$

Denote

$$\mathsf{E}^*[J_2 \mid \text{jump}]\lambda(t,x)$$

by $\omega(t,x)$. Substituting into the martingale condition (3.5), we have

$$P_{1t} - [r+\lambda(t,x)]J_1(t,x) + \frac{\partial J_1(t,x)}{\partial t} + \omega(t,x) \;=\; 0\,. \tag{A.4}$$

For each x, this is an ordinary differential equation (ODE) for J_1 as a function of t. The value matching condition is a terminal condition for the ODE. Let $\tau(x)$ denote the date t such that $x = B_{1t}$. Solving the ODE subject to the value matching condition gives, for $t \le \tau(x)$,

$$\begin{aligned} J_1(t,x) \;=\; & x[T-\tau(x)]\mathrm{e}^{-r(T-t)}\exp\left(-\int_t^{\tau(x)} \lambda(u,x)\,\mathrm{d}u\right) \\ & + \int_t^{\tau(x)} \mathrm{e}^{-r(u-t)}\exp\left(-\int_t^{u} \lambda(s,x)\,\mathrm{d}s\right)[P_{1u}+\omega(u,x)]\,\mathrm{d}u\,. \end{aligned} \tag{A.5}$$

To apply the smooth pasting condition, we differentiate with respect to x and evaluate at $t = \tau(x)$. Equating the derivative to $(T-t)\mathrm{e}^{-r(T-t)}$ gives

$$\mathrm{e}^{-r(T-t)}x - P_{1t} \;=\; \omega(t,x) - (T-t)\mathrm{e}^{-r(T-t)}x\lambda(t,x)\,. \tag{A.6}$$

This is the same as equation (3.7).

Appendix B. Proofs Excluding Lemma 3.3

Proof of Lemma 1.1. Project $\tilde{w}$ on $\tilde{x}_1 + \tilde{x}_2$ as

$$\tilde{w} - \mu_w = \lambda(\tilde{x}_1 + \tilde{x}_2 - 2\mu) + \tilde{u}, \tag{B.1}$$

where $\tilde{u}$ is the residual and is independent of $\tilde{x}_1$ and $\tilde{x}_2$, and where

$$\lambda \;=\; \frac{\mathrm{cov}(\tilde{w}, \tilde{x}_1 + \tilde{x}_2)}{\mathrm{var}(\tilde{x}_1 + \tilde{x}_2)} \;=\; \frac{\beta\sigma_w^2}{2\beta^2\sigma_w^2 + \sigma_\varepsilon^2}\,.$$

Thus,

$$\mathsf{E}[\tilde{w} \mid \tilde{x}_1, \tilde{x}_2] = \lambda(\tilde{x}_1 + \tilde{x}_2) + \mu_w - 2\lambda\mu\,.$$

The correlation of $\tilde{x}_1$ and $\tilde{x}_2$ is $\rho = \beta^2\sigma_w^2/\sigma^2$, and we have

$$\lambda = \frac{\rho}{(1+\rho)\beta}\,.$$

The conditional variance of $\tilde{w}$ given $\tilde{x}_1$ and $\tilde{x}_2$ is the variance of $\tilde{u}$ in (B.1), and we have

$$\mathrm{var}(\tilde{u}) \;=\; \sigma_w^2 - \lambda^2\left(4\beta^2\sigma_w^2 + 2\sigma_\varepsilon^2\right) \;=\; \left(\frac{1-\rho}{1+\rho}\right)\sigma_w^2\,.$$

We have

$$\mathsf{E}[\tilde{m} \mid \tilde{x}_1, \tilde{x}_2] \;=\; \frac{\mathrm{e}^{-rT}}{\mathsf{E}[\mathrm{e}^{-\gamma\tilde{w}}]}\mathsf{E}[\mathrm{e}^{-\gamma\tilde{w}} \mid \tilde{x}_1, \tilde{x}_2]\,.$$

From Step 1 and the usual formula for the mean of an exponential of a normal random variable,

$$\mathsf{E}[\mathrm{e}^{-\gamma\tilde{w}}] \;=\; \exp\left(-\gamma\mu_w + \frac{\gamma^2\sigma_w^2}{2}\right)$$

and

$$\mathsf{E}[\mathrm{e}^{-\gamma\tilde{w}} \mid \tilde{x}_1, \tilde{x}_2] = \;\exp\left(-\gamma\lambda(\tilde{x}_1+\tilde{x}_2) - \gamma\mu_w + 2\gamma\lambda\mu + \left(\frac{1-\rho}{1+\rho}\right)\frac{\gamma^2\sigma_w^2}{2}\right)\,.$$

Thus,

$$\mathsf{E}[\tilde{m} \mid \tilde{x}_1, \tilde{x}_2] \;=\; \exp\left(-rT - \gamma\lambda(\tilde{x}_1+\tilde{x}_2) + 2\gamma\lambda\mu - \left(\frac{\rho}{1+\rho}\right)\gamma^2\sigma_w^2\right)\,.$$

Consider any numbers a_1 and a_2 and let A denote the event $\{\tilde{x}_1 \le a_1, \tilde{x}_2 \le a_2\}$. The risk-neutral probability of the event is

$$\mathbb{Q}(A) \;=\; \mathrm{e}^{rT}\mathsf{E}[1_A\tilde{m}] \;=\; \mathrm{e}^{rT}\mathsf{E}\Big[1_A\mathsf{E}[\tilde{m} \mid \tilde{x}_1, \tilde{x}_2]\Big]\,.$$

Therefore, from Step 2,

$$\begin{aligned}\mathbb{Q}(A) \;&=\; \mathrm{e}^{-\rho\gamma^2\sigma_w^2/(1+\rho)}\mathsf{E}[1_A\mathrm{e}^{-\gamma\lambda(\tilde{x}_1+\tilde{x}_2-2\mu)}]\\ &=\; \mathrm{e}^{-\rho\gamma^2\sigma_w^2/(1+\rho)}\int_{-\infty}^{a_2}\int_{-\infty}^{a_1}\mathrm{e}^{-\gamma\lambda(x_1+x_2-2\mu)}g(x_1,x_2)\,\mathrm{d}x_1\,\mathrm{d}x_2\,, \qquad \text{(B.2)}\end{aligned}$$

where g denotes the density function of $(\tilde{x}_1, \tilde{x}_2)$.

Set $\mathbf{x} = (x_1\ x_2)'$, $\boldsymbol{\mu} = (\mu\ \mu)'$, and $\boldsymbol{\iota} = (1\ 1)'$. Let $\boldsymbol{\Sigma}$ denote the covariance matrix of $\tilde{x}$, which is

$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \sigma^2 .$$

We use the standard fact that for any vector $\mathbf{b}$,

$$\mathrm{e}^{-\mathbf{b}'(\mathbf{x}-\boldsymbol{\mu})} g(\mathbf{x}) \;=\; \mathrm{e}^{\mathbf{b}'\boldsymbol{\Sigma}\mathbf{b}/2} g(\mathbf{x} + \boldsymbol{\Sigma}\mathbf{b}) . \tag{B.3}$$

Set $\mathbf{b} = \gamma\lambda\boldsymbol{\iota}$. Direct calculations show that

$$\frac{\rho\gamma^2\sigma_2^2}{1+\rho} \;=\; \frac{1}{2}\mathbf{b}'\boldsymbol{\Sigma}\mathbf{b} ,$$

and $\boldsymbol{\Sigma}\mathbf{b} = \gamma\beta\sigma_w^2\iota$. Substituting these and (B.3) into (B.2), we obtain

$$\mathbb{Q}(A) \;=\; \int_{-\infty}^{a_2}\int_{-\infty}^{a_1} g(x_1 + \gamma\beta\sigma_w^2, x_2 + \gamma\beta\sigma_w^2)\, \mathrm{d}x_1\, \mathrm{d}x_2 .$$

This shows that $\tilde{x}_1$ and $\tilde{x}_2$ have the same distribution under the risk-neutral probability as under the physical probability, but with means shifted by $\gamma\beta\sigma_w^2$.

The remaining task is to compute the distribution of $\tilde{x}_i$ given $\tilde{x}_j$. Project $\tilde{x}_i$ on $\tilde{x}_j$ under the physical probability as

$$\tilde{x}_i - \mu = b(\tilde{x}_j - \mu) + \tilde{e} \tag{B.4}$$

where $\tilde{e}$ is the residual and is independent of $\tilde{x}_j$. Thus,

$$\mathsf{E}[\tilde{x}_i \mid \tilde{x}_j] = b\tilde{x}_j + (1-b)\mu\,,$$

and we have

$$b \;=\; \frac{\text{cov}(\tilde{x}_i, \tilde{x}_j)}{\text{var}(\tilde{x}_j)} \;=\; \rho\,.$$

The conditional variance of $\tilde{x}_i$ given $\tilde{x}_j$ is the variance of $\tilde{e}$, and (B.4) implies that this conditional variance is

$$\text{var}(\tilde{e}) = \sigma^2(1-\rho^2)\,.$$

The calculation is the same under the risk-neutral probability with μ^* replacing μ. □

Proof of Lemma 2.1. This follows from a direct calculation of the expected value. □

Proof of Lemma 2.2. Fix t and T such that $0 \le t \le T$. Equation (2.2) can be rewritten as

$$F(t,z) \;\stackrel{\text{def}}{=}\; z\,[T - t\Phi(z)] - t\phi(z) \;=\; 0\,. \tag{B.5}$$

We have $\partial F/\partial z = T - t\Phi(z) > 0$, $F(0) < 0$, and $\lim_{z\to\infty} F(t,z) = \infty$. Therefore, $F(t,z) = 0$ admits a unique solution for every t, and the solution is positive. Because F is continuously differentiable and $\partial F/\partial z \neq 0$, the implicit function theorem states that $z(t)$ is also continuously differentiable and

$$\frac{\mathrm{d}z}{\mathrm{d}t} \;=\; -\,\frac{\partial F}{\partial t}\bigg/\frac{\partial F}{\partial z} \;=\; \frac{z\Phi(z) + \phi(z)}{T - t\Phi(z)} \;>\; 0\,.$$

□

Proof of Proposition 2.1. Use Lemmas 2.1 and 2.2 and the substitutions $\phi(-z) = \phi(z)$

$$T - t + t\Phi(-z) \;=\; T - t + t[1 - \Phi(z)] \;=\; T - t\Phi(z)\,.$$

$\square$

Proof of Lemma 3.1. Equation (3.8) is equivalent to

$$z\big[T - \tau\Phi(y) - (t-\tau)\Phi(z)\big] - \tau\phi(y) - (t-\tau)\phi(z) \;=\; 0\,. \tag{B.6}$$

The left-hand side of (B.6) is negative at $z = 0$ and infinite at $z = \infty$. Furthermore, its derivative with respect to z is

$$T - \tau\Phi(y) - (t-\tau)\Phi(z) \;>\; 0\,,$$

so there is a unique solution z of (B.6), and the solution is positive. Denote the solution by $z_2(\tau, t, y)$. The implicit function theorem gives us

$$\frac{\partial z_2(\tau,t,y)}{\partial t} \;=\; \frac{z\Phi(z) + \phi(z)}{T - \tau\Phi(y) - (t-\tau)\Phi(z)} \;>\; 0\,.$$

Because (B.6) is strictly monotone in z, the condition $y < z_2(\tau, t, y)$ is equivalent to

$$y\big[T - \tau\Phi(y) - (t-\tau)\Phi(y)\big] - \tau\phi(y) - (t-\tau)\phi(y) \;<\; 0\,,$$

which simplifies to

$$y\big[T - t\Phi(y)\big] - t\phi(y) \;<\; 0\,. \tag{B.7}$$

The definition of $z(t)$ is that $z(t) = z$ where

$$z\big[T - t\Phi(z)\big] - t\phi(z) \;=\; 0\,. \tag{B.8}$$

The left-hand side of (B.8) is strictly monotone in z, as noted in the proof of Lemma 2.2, so (B.7) is equivalent to $y < z(t)$. $\square$

Proof of Proposition 3.1. The equilibrium boundary is the solution B_{2t} of the fixed-point condition

$$\begin{aligned} B_{2t} \;&=\; \\ &\mathsf{E}^*\left[\tilde{x}_2 \,\middle|\, \tilde{x}_1, \big\{\tilde{\theta}_2 > t\big\} \cup \big\{\tilde{\theta}_2 \le \tau, \tilde{x} < B_{1\tau} \wedge B_{2t}\big\} \cup \big\{\tau < \tilde{\theta}_2 \le t, \tilde{x} < B_{2t}\big\}\right]. \end{aligned} \tag{B.9}$$

There are two possible cases: (i) $B_{2t} \le B_{1\tau}$, and (ii) $B_{2t} > B_{1\tau}$. In case (i), the fixed point problem in (B.9) translates to a single-firm problem, whose solution is given in Proposition 2.1. Hence, in case (i),

$$B_{2t} \;=\; \tilde{\mu}_2^* - \sigma_2 z(t)\,.$$

The condition $B_{2t} \le B_{1\tau}$ translates to $z(t) \ge \tilde{y}$. In case (ii), the fixed point problem in (B.9) becomes

$$B_{2t} \;=\; \mathsf{E}^*\left[\tilde{x}_2 \,\middle|\, \tilde{x}_1, \big\{\tilde{\theta}_2 > t\big\} \cup \big\{\tilde{\theta}_2 \le \tau, \tilde{x} < B_{1\tau}\big\} \cup \big\{\tau < \tilde{\theta}_2 \le t, \tilde{x} < B_{2t}\big\}\right]. \tag{B.10}$$

Define $Z_{2t} = (\tilde{\mu}_2^* - B_{2t})/\sigma_2$. We can calculate the probabilities of the events on the right-hand side of (B.10) as

$$\begin{aligned} Pr\{\tilde{\theta}_2 > t\} &= \frac{T-t}{T}, \\ Pr\{\tilde{\theta}_2 \le \tau, \tilde{x} < B_{1\tau}\} &= \frac{\tau}{T}\big(1 - \Phi(\tilde{y})\big), \\ Pr\{\tau < \tilde{\theta}_2 \le t, \tilde{x} < B_{2t}\} &= \frac{t-\tau}{T}\big(1 - \Phi(Z_{2t})\big). \end{aligned}$$

Using these probabilities and the fact that $\mathsf{E}^*[\tilde{x}_2|\tilde{x}_1, \tilde{x}_2 \le a] = \tilde{\mu}_2^* - \sigma_2\phi(a)/\Phi(a)$, we can rewrite (B.10) as

$$\begin{aligned} B_{2t} = \; & \frac{T-t}{T - \tau\Phi(\tilde{y}) - (t-\tau)\Phi(Z_{2t})} \times \tilde{\mu}_2^* \\ & + \frac{\tau\big(1-\Phi(\tilde{y})\big)}{T - \tau\Phi(\tilde{y}) - (t-\tau)\Phi(Z_{2t})} \times \big(\tilde{\mu}_2^* - \sigma_2 \frac{\phi(\tilde{y})}{1-\Phi(\tilde{y})}\big) \\ & + \frac{\big(t-\tau\big)\big(1-\Phi(Z_{2t})\big)}{T - \tau\Phi(\tilde{y}) - (t-\tau)\Phi(Z_{2t})} \times \big(\tilde{\mu}_2^* - \sigma_2 \frac{\phi(Z_{2t})}{1-\Phi(Z_{2t})}\big) \,. \end{aligned}$$

This simplifies to

$$B_{2t} = \tilde{\mu}_2^* - \sigma_1 \frac{\tau\phi(\tilde{y}) + (t-\tau)\phi(Z_{2t})}{T - \tau\Phi(\tilde{y}) - (t-\tau)\Phi(Z_{2t})}, ,$$

which is equivalent to

$$Z_{2t} \;=\; \frac{\tau\phi(\tilde{y}) + (t-\tau)\phi(Z_{2t})}{T - \tau\Phi(\tilde{y}) - (t-\tau)\Phi(Z_{2t})} \,.$$

This implies $Z_{2t} = z_2(\tau, t, \tilde{y})$. □

Proof of Lemma 3.2. Set $k = z_1(t)$. The risk-neutral probabilities of the events (i)–

(iv) are, respectively,

$$
\begin{aligned}
p_1 &= \frac{t(T-t)}{T^2}\Phi\left(-k\right) \\
p_2 &= \frac{t(T-t)}{T^2}\Phi\left(-k\right) \\
p_3 &= \frac{t^2}{T^2}\Gamma\left(-k,-k,\rho\right) \\
p_4 &= \frac{(T-t)^2}{T^2}\,.
\end{aligned}
$$

The risk-neutral expectation of $\tilde{x}_1$ conditional on each of the four events is

$$
\begin{aligned}
a_1 &\stackrel{\text{def}}{=} \mathsf{E}^*[\tilde{x}_1 \mid \tilde{x}_1 < B_{1t}] \\
a_2 &\stackrel{\text{def}}{=} \mathsf{E}^*[\tilde{x}_1 \mid \tilde{x}_2 < B_{1t}] \\
a_3 &\stackrel{\text{def}}{=} \mathsf{E}^*[\tilde{x}_1 \mid \tilde{x}_1 < B_{1t}, \tilde{x}_1 < B_{1t}] \\
a_4 &\stackrel{\text{def}}{=} \mathsf{E}^*[\tilde{x}_1]\,.
\end{aligned}
$$

To compute a_1, a_2, and a_3, write $\tilde{x}_i = \mu^* + \sigma\xi_i$, where the ξ_i are correlated standard normals under the risk-neutral probability. We have

$$
\begin{aligned}
a_1 &= \mu^* + \sigma\mathsf{E}\left[\xi_1 \mid \xi_1 < -k\right] \\
&= \mu^* - \sigma\phi(-k)/\Phi(-k)\,, \\
a_2 &= (1-\rho)\mu^* + \rho\mathsf{E}[\tilde{x}_2 \mid \tilde{x}_2 < B_{1t}] \\
&= \mu^* + \rho\sigma\mathsf{E}\left[\xi_2 \mid \xi_2 < -k\right] \\
&= \mu^* - \rho\sigma\phi(-k)/\Phi(-k)\,.
\end{aligned}
$$

Also,

$$\begin{aligned} a_3 &= \mu^* + \sigma \mathsf{E}\left[\xi_1 \mid \xi_1 < -k, \xi_2 < -k\right] \\ &= \mu^* - \sigma \mathsf{E}\left[-\xi_1 \mid -\xi_1 > k, -\xi_2 > k\right] \\ &= \mu^* - \sigma \frac{(1+\rho)\phi(-k)\Phi(-\delta)}{\Gamma(-k,-k,\rho)} \,. \end{aligned}$$

We use equation (1) in Rosenbaum (1961) for the last line. The equilibrium price is

$$\mathrm{e}^{-r(T-t)} \cdot \frac{\sum_{i=1}^4 p_i a_i}{\sum_{i=1}^4 p_i} \,,$$

which simplifies to the formula stated in the lemma. □

Proof of Lemma 3.4. Notice that $\tilde{\mu}_2^* - \sigma_2 \tilde{y} = B_{1t}$. Furthermore, $B_{2t} = \tilde{\mu}_2^* - \sigma_2 Z_{2t}$. Therefore, $B_{2t} > B_{1t}$ if and only if $Z_{2t} < \tilde{y}$. It follows from the definition (3.10) of Z_{2t} and Lemma 3.1 that this is equivalent to $z(t) < \tilde{y}$. Now, if $\tilde{x}_1 = B_{1t}$, then

$$\tilde{y} \;=\; \frac{(1-\rho)(\mu^* - B_{1t})}{\sigma_2} \;=\; \frac{1-\rho}{\sqrt{1-\rho^2}} \cdot \frac{\mu^* - B_{1t}}{\sigma} \;=\; \sqrt{\frac{1-\rho}{1+\rho}} z_1(t) \,.$$

Combining this with Lemma 3.3, we see that if $\tilde{x}_1 = B_{1t}$, then $\tilde{y} \leq z(t)$. Hence, $\tilde{x}_1 > B_{1t}$ is a necessary condition for $z(t) < \tilde{y}$. □

Proof of Lemma 3.5. The calculation is explained in the text. □

Proof of Proposition 3.2. This follows directly from Proposition 3.1 and Lemmas 3.2–3.5.

□

Appendix C. Proof of Lemma 3.3

Set $k = z_1(t)$ and

$$d \;=\; \sqrt{\frac{1-\rho}{1+\rho}}k\,.$$

We need to establish that $d \le z(t)$. Because $z(t) > 0$, this is clearly true if $d < 0$, so we can assume $d \ge 0$. The definition of $z(t)$ in Lemma 2.2 is that $z(t) = z$ where

$$z[T - t\Phi(z)] - t\phi(z) \;=\; 0\,.$$

The derivative of the left-hand side of this is $T - t\Phi(z) > 0$, so the left-hand side is a monotone function of z. Hence, the condition $d \le z(t)$ is equivalent to

$$d[T - t\Phi(d)] - t\phi(d) \;\le\; 0\,.$$

So, what we need to show is that

$$d \;\le\; \frac{t\phi(d)}{T - t\Phi(d)}\,. \tag{C.1}$$

What we know from the nonnegativity of (3.13) is that

$$k \;\le\; \frac{t(1+\rho)\phi\left(k\right)\left[T - t\Phi\left(d\right)\right]}{\left[T - t\Phi\left(k\right)\right]^2 + t^2\left[\Gamma\left(-k,-k \mid \rho\right) - \Phi(-k)^2\right]}\,.$$

Multiplying by $\sqrt{(1-\rho)/(1+\rho)}$, we obtain

$$d \;\le\; \frac{t\sqrt{1-\rho^2}\phi\left(k\right)\left[T - t\Phi\left(d\right)\right]}{\left[T - t\Phi\left(k\right)\right]^2 + t^2\left[\Gamma\left(-k,-k \mid \rho\right) - \Phi(-k)^2\right]}\,.$$

Therefore, to establish (C.1), it suffices to show that

$$\frac{\sqrt{1-\rho^2}\phi\left(k\right)}{\left[T - t\Phi\left(k\right)\right]^2 + t^2\left[\Gamma\left(-k,-k \mid \rho\right) - \Phi(-k)^2\right]} \le \frac{\phi(d)}{\left[T - t\Phi(d)\right]^2}\,. \tag{C.2}$$

Given d, we can regard the left-hand side of (C.2) as a function of either k or ρ , because

$$k = d\sqrt{\frac{1+\rho}{1-\rho}} \quad \Leftrightarrow \quad \rho = \frac{k^2 - d^2}{k^2 + d^2}\,. \tag{C.3}$$

The right-hand sideof (C.2) is the same function evaluated at $\rho = 0 \Leftrightarrow k = d$. It suffices therefore to show that (C.2) is decreasing in k when we set $\rho = \rho(k)$ as defined in (C.3).

Taking the log of (C.2) and differentiating, it suffices to show that

$$\begin{aligned}\Big\{\big[T - t\Phi\,(k)\,\big]^2 + t^2\big[\Gamma\,(-k, -k \mid \rho(k)) - \Phi(-k)^2\big]\Big\}\,\frac{\mathrm{d}\log\big[\sqrt{1-\rho(k)^2}\phi\,(k)\,\big]}{\mathrm{d}k} \\ \leq \frac{\mathrm{d}}{\mathrm{d}k}\Big\{\big[T - t\Phi\,(k)\,\big]^2 + t^2\big[\Gamma\,(-k, -k \mid \rho(k)) - \Phi(-k)^2\big]\Big\}\,.\end{aligned} \tag{C.4}$$

We have

$$\sqrt{1-\rho(k)^2} = \frac{2dk}{d^2 + k^2}\,,$$

so

$$\frac{\mathrm{d}\log\big[\sqrt{1-\rho(k)^2}\phi\,(k)\,\big]}{\mathrm{d}k} = \frac{1}{k} - \frac{2k}{d^2+k^2} - k \;= -\frac{\rho}{k} - k\,.$$

Therefore, the left-hand side of (C.4) is

$$-\Big\{\big[T - t\Phi\,(k)\,\big]^2 + t^2\big[\Gamma\,(-k, -k \mid \rho(k)) - \Phi(-k)^2\big]\Big\}\left(\frac{\rho}{k} + k\right)\,,$$

Also,

$$\frac{\mathrm{d}}{\mathrm{d}k}\big[T - t\Phi\,(k)\,\big]^2 = -2t\big[T - t\Phi\,(k)\,\big]\phi(k) \tag{C.5}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}k}\left[-\Phi(-k)^2\right] = 2\Phi(-k)\phi(k)\,. \tag{C.6}$$

We prove the following at the end of this appendix.

Lemma C.1.

$$\frac{\mathrm{d}}{\mathrm{d}k}\Gamma(-k,-k) \mid \rho(k)) \;=\; 2\phi(k)\left[\Phi(-d)+\frac{d\phi(d)}{d^2+k^2}\right] \tag{C.7}$$

Equations (C.5)–(C.7) imply that the right-hand side of (C.4) is

$$-2t\phi(k)\big[T-t\Phi\left(k\right)\big]+2t^2\phi(k)\left[\Phi(-d)+\frac{d\phi(d)}{d^2+k^2}+\Phi(-k)\right].$$

Substituting $\Phi(-k)=1-\Phi(k)$, this simplifies to

$$-2tT\phi(k)+2t^2\phi(k)\left[1+\Phi(-d)+\frac{d\phi(d)}{d^2+k^2}\right]$$

The inequality (C.4) is therefore equivalent to

$$\begin{aligned}\left\{\big[T-t\Phi\left(k\right)\big]^2+t^2\big[\Gamma\left(-k,-k\mid\rho(k)\right)-\Phi(-k)^2\big]\right\}\left(\frac{\rho}{k}+k\right)&\\ \geq 2tT\phi(k)-2t^2\phi(k)\left[1+\Phi(-d)+\frac{d\phi(d)}{d^2+k^2}\right]&\end{aligned} \tag{C.8}$$

A sufficient condition for (C.8) is

$$\big[T-t\Phi\left(k\right)\big]^2\left(\frac{\rho}{k}+k\right)\;\geq\;2tT\phi(k)-2t^2\phi(k)\left[1+\Phi(-d)+\frac{d\phi(d)}{d^2+k^2}\right].$$

We can rewrite this as

$$\begin{aligned}2t^2\phi(k)\left[1+\Phi(-d)+\frac{d\phi(d)}{d^2+k^2}\right]+t^2\Phi(k)^2\left(\frac{\rho}{k}+k\right)&\\ -2tT\Phi\left(k\right)\left(\frac{\rho}{k}+k\right)-2tT\phi(k)+T^2\left(\frac{\rho}{k}+k\right)\geq 0&\end{aligned} \tag{C.9}$$

The left-hand side is quadratic in t with a unique minimum at

$$\frac{T\Phi\left(k\right)\left(\frac{\rho}{k}+k\right)+T\phi(k)}{2\phi(k)\left[1+\Phi(-d)+\frac{d\phi(d)}{d^2+k^2}\right]+\Phi(k)^2\left(\frac{\rho}{k}+k\right)}$$

Substituting this into (C.9) and cancelling a T^2 factor, we see that (C.9) holds for all t if and only if

$$\left\{2\phi(k)\left[1+\Phi(-d)+\frac{d\phi(d)}{d^2+k^2}\right]+\Phi(k)^2\left(\frac{\rho}{k}+k\right)\right\}\left(\frac{\rho}{k}+k\right) \geq \left[\Phi\left(k\right)\left(\frac{\rho}{k}+k\right)+\phi(k)\right]^2$$

This simplifies to

$$2\left[\Phi(-k)+\Phi(-d)+\frac{d\phi(d)}{d^2+k^2}\right]\left(\frac{\rho}{k}+k\right)-\phi(k)\geq 0\,. \tag{C.10}$$

Set $x=d/k$ so $d=xk$. Then, (C.10) is equivalent to

$$2\left[k\Phi(-k)+k\Phi(-xk)+\frac{x\phi(xk)}{1+x^2}\right]\left(\frac{1-x^2}{1+x^2}+k^2\right)-k^2\phi(k)\geq 0\,. \tag{C.11}$$

It will suffice to establish (C.11) for all $k>0$ and all $x\in(0,1)$. A sufficient condition for (C.11) is

$$2k\Phi(-k)\left(\frac{1-x^2}{1+x^2}+k^2\right)+\frac{2xk^2\phi(k)}{1+x^2}-k^2\phi(k)\geq 0\,. \tag{C.12}$$

The left-hand side of (C.12) is larger than $2k^3\Phi(-k)-k^2\phi(k)$, so (C.12) holds if $2k\Phi(-k)-\phi(k)\geq 0$.

Set $h(k)=2k\Phi(-k)-\phi(k)$. Then $h'(k)=2\Phi(-k)-k\phi(k)$ and $h''(k)=(k^2-3)\phi(k)$. Thus, h' is a decreasing function for $k>\sqrt{3}$. It is negative at $k=\sqrt{3}$ and hence negative for all $k > \sqrt{3}$. This means that h is decreasing for $k > \sqrt{3}$. The function h is positive at $k = \sqrt{3}$ and is equal to 0 at $k = \infty$, so it is positive for all $k > \sqrt{3}$. We have $h'' < 0$ for $k < \sqrt{3}$, so h is concave on the region $(0, \sqrt{3})$; hence, it is positive for all $k > k^* \stackrel{\text{def}}{=} \inf\{x \mid h(x) > 0\} \approx 0.62$ and negative for $k < k^*$.

We have shown that (C.12) holds for $k \geq k^*$. Consider $k < k^*$. Multiply by $(1 + x^2)/k$ to write (C.12) as

$$2\Phi(-k)\left[1 - x^2 + k^2(1 + x^2)\right] - (1 - 2x + x^2)k\phi(k) \geq 0\,. \tag{C.13}$$

This can be rearranged as

$$\Bigg[kh(k) - 2\Phi(-k)\Bigg]x^2 + 2k\phi(k)x + 2\Phi(-k) + kh(k) \geq 0\,. \tag{C.14}$$

The left-hand side of (C.14) is concave in x when $k < k^*$. It is equal to $4k^2\Phi(-k) > 0$ at $x = 1$, and it is equal to $j(k) \stackrel{\text{def}}{=} 2\Phi(-k) + kh(k)$ at $x = 0$. A concave function on $(0, 1)$ that is positive at 0 and at 1 must be positive everywhere on $(0, 1)$, so it suffices now to show that $j(k) > 0$ when $k < k^*$. The function j is decreasing on $(0, k^*)$, because its derivative is $2h(k) - (1 + k^2)\phi(k)$. Furthermore, $h(k^*) = 0$, so $j(k^*) = 2\Phi(-k^*) > 0$. Hence, j must be positive on $(0, k^*)$.

Proof of Lemma C.1. Consider two standard normals with correlation ρ defined as $\tilde{x}_1 = \tilde{\varepsilon}_1$ and $\tilde{x}_2 = \rho\tilde{\varepsilon}_1 + \sqrt{1 - \rho^2}\tilde{\varepsilon}_2$ where $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are independent standard normals.

We have

$$\begin{aligned}\Gamma(a,b\mid\rho) &= \text{prob}\left(\tilde{\varepsilon}_1 \le a, \tilde{\varepsilon}_2 \le \frac{b-\rho\tilde{\varepsilon}_1}{\sqrt{1-\rho^2}}\right) \\ &= \int_{-\infty}^{a}\int_{-\infty}^{(b-\rho\varepsilon_1)/\sqrt{1-\rho^2}} \phi(\varepsilon_2)\,\mathrm{d}\varepsilon_2\,\phi(\varepsilon_1)\,\mathrm{d}\varepsilon_1 \\ &= \int_{-\infty}^{a}\Phi\left(\frac{b-\rho\varepsilon_1}{\sqrt{1-\rho^2}}\right)\phi(\varepsilon_1)\,\mathrm{d}\varepsilon_1\end{aligned}$$

So, using the fact that

$$\frac{\partial}{\partial\rho}\frac{b-\rho\varepsilon_1}{\sqrt{1-\rho^2}} = (1-\rho^2)^{-3/2}(\rho b-\varepsilon_1)\,,$$

we obtain

$$\frac{\partial\Gamma(a,b\mid\rho)}{\partial\rho} = (1-\rho^2)^{-3/2}\int_{-\infty}^{a}\phi\left(\frac{b-\rho\varepsilon_1}{\sqrt{1-\rho^2}}\right)(\rho b-\varepsilon_1)\phi(\varepsilon_1)\,\mathrm{d}\varepsilon_1$$

Also,

$$\phi\left(\frac{b-\rho\varepsilon_1}{\sqrt{1-\rho^2}}\right)\phi(\varepsilon_1) = \frac{1}{2\pi}\exp\left(-\frac{1}{2}\left[\frac{(b-\rho\varepsilon_1)^2}{1-\rho^2}+\varepsilon_1^2\right]\right)$$

and

$$\frac{(b-\rho\varepsilon_1)^2}{1-\rho^2}+\varepsilon_1^2 = b^2+\frac{(\varepsilon_1-b\rho)^2}{1-\rho^2}$$

so

$$\phi\left(\frac{b-\rho\varepsilon_1}{\sqrt{1-\rho^2}}\right)\phi(\varepsilon_1) = \phi(b)\phi\left(\frac{\varepsilon_1-b\rho}{\sqrt{1-\rho^2}}\right)\,.$$

Hence,

$$\frac{\partial \Gamma(a, b \mid \rho)}{\partial \rho} = (1-\rho^2)^{-3/2} \phi(b) \int_{-\infty}^{a} (\rho b - \varepsilon_1) \phi\left(\frac{\varepsilon_1 - b\rho}{\sqrt{1-\rho^2}}\right) \, \mathrm{d}\varepsilon_1$$

Finally, making the change of variables $u = (\varepsilon_1 - b\rho)/\sqrt{1-\rho^2}$, we obtain

$$\begin{aligned} \int_{-\infty}^{a} (\rho b - \varepsilon_1) \phi\left(\frac{\varepsilon_1 - b\rho}{\sqrt{1-\rho^2}}\right) \, \mathrm{d}\varepsilon_1 &= (1-\rho^2) \int_{-\infty}^{(a-b\rho)/\sqrt{1-\rho^2}} -u\phi(u) \, \mathrm{d}u \\ &= (1-\rho^2) \phi\left(\frac{a - b\rho}{\sqrt{1-\rho^2}}\right) . \end{aligned}$$

Therefore,

$$\frac{\partial \Gamma(a, b \mid \rho)}{\partial \rho} = \frac{1}{\sqrt{1-\rho^2}} \cdot \phi(b) \phi\left(\frac{a - b\rho}{\sqrt{1-\rho^2}}\right) .$$

Also, we have

$$\frac{\partial}{\partial a} \Gamma(a, b \mid \rho) = \Phi\left(\frac{b - \rho a}{\sqrt{1-\rho^2}}\right) \phi(a)$$

and

$$\frac{\partial}{\partial b} \Gamma(a, b \mid \rho) = \Phi\left(\frac{a - \rho b}{\sqrt{1-\rho^2}}\right) \phi(b) \, .$$

Substituting $a = -k$ and $b = -k$, we obtain

$$\frac{b - \rho a}{\sqrt{1-\rho^2}} = \frac{a - \rho b}{\sqrt{1-\rho^2}} = -d \, .$$

Furthermore,

$$\sqrt{1 - \rho(k)^2} = \frac{2dk}{d^2 + k^2} \, ,$$

and

$$\rho'(k) = \frac{4d^2k}{(d^2+k^2)^2}\,,$$

so

$$\frac{\rho'(k)}{\sqrt{1-\rho(k)^2}} = \frac{2d}{d^2+k^2}\,.$$

Therefore,

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}k}\Gamma(-k,-k) \mid \rho(k)) &= 2\Phi(-d)\phi(-k) + \frac{1}{\sqrt{1-\rho^2}}\cdot\phi(-k)\phi\left(-d\right)\rho'(k) \\
&= 2\Phi(-d)\phi(k) + \frac{2d}{d^2+k^2}\phi(d)\phi(k) \\
&= 2\phi(k)\left[\Phi(-d) + \frac{d\phi(d)}{d^2+k^2}\right]\,.
\end{aligned}$$

$\square$

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