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ABSTRACT

When tastes are represented by a class of generalized isoelastic preferences which—unlike traditional Von-Neumann preferences—do not confuse behavior towards risk with attitudes towards intertemporal substitution, the true *beta* of an asset is, in general, an average of its consumption and market *betas*. We show that the two parameters measuring risk aversion and intertemporal substitution affect consumption and portfolio allocation decisions in symmetrical ways. A unit elasticity of intertemporal substitution gives rise to myopia in consumption-savings decisions (the future does not affect the optimal consumption plan), while a unit coefficient of relative risk aversion gives rise to myopia in portfolio allocation (the future does not affect optimal portfolio allocation). The empirical evidence is consistent with the behavior of intertemporal maximizers who have a unit coefficient of relative risk aversion and an elasticity of intertemporal substitution different from 1.

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The two most popular asset pricing models, the intertemporal—or dynamic, or "consumption based"—asset pricing model of Lucas [1978] and Breeden [1979], and the static—or "market based"—asset pricing model of Sharpe [1966] and Lintner [1965], are often viewed as mutually inconsistent. The alleged mutual inconsistency of these models stems from the differences in their so-called "*beta* representations", which express the equilibrium excess rate of return (over the riskfree rate) on an asset. According to the consumption asset pricing model (CCAPM), the equilibrium excess return on an asset is determined by its covariation with the marginal rate of substitution in consumption. When preferences are of the standard Von Neumann-Morgenstern (VNM), time-separable, isoelastic form, the marginal rate of substitution is an isoelastic function of consumption growth. By contrast, the static capital asset pricing model (SCAPM) predicts that the excess return on an asset is determined by its covariance with the return on the market portfolio.[1] As a consequence, the two models also imply different measures of systematic risk.[2]

Aside from their generally disappointing empirical performance,[3] these two models are questionable theoretically. The SCAPM is based on the strong assumption that the utility of end-of-period wealth is independent of returns beyond the current period. The CCAPM, by contrast, relies on a specification of preferences which characterizes the two distinct concepts of intertemporal substitution and risk aversion with a single parameter.[4] Thus, in the CCAPM, risk neutral agents also have an infinite elasticity of intertemporal substitution: clearly an *a priori* unacceptable restriction.

This paper studies a dynamic asset pricing model derived assuming a general family of generalized isoelastic preferences, which subsumes isoelastic VNM expected utility functions as a special case. The main advantage of our specification of preferences—derived from recent work by Epstein-Zin [1987a], Farmer [1987],

[1] See Hansen, Richard and Singleton [1981].

[2] See, for example, Mankiw and Shapiro [1986].

[3] See the original studies of Black, Jensen and Scholes [1972] and Hansen and Singleton [1982, 1983].

[4] Of course, these models have been criticized for many other assumptions: see, for example, Roll [1977] for a comprehensive survey.

and one of us (Weil [1987, 1988])—is our ability to parametrize independently investors' attitudes towards risk (the parameter that enters the SCAPM), and investors' willingness to substitute future for present consumption (the parameter that characterizes the CCAPM). Hence our model is free from the theoretical shortcoming that we mentioned above.

We characterize the optimal consumption and portfolio allocation programs which arise from our assumptions on preferences. As Epstein and Zin [1987b] first hinted, the model of asset prices which emerges is—in general—a hybrid of the SCAPM and the CCAPM. We compare the equilibrium conditions from our model with those that characterize the SCAPM and the CCAPM. We show that, under suitable restrictions on preferences and technology, our more general dynamic setup may give rise to equilibrium conditions which are equivalent to those of the SCAPM. On the other hand, the CCAPM arises only when preferences are VNM. We thus demonstrate that the static and the dynamic capital asset pricing models—far from being mutually inconsistent—are just special cases of a more general, but as tractable, specification of preferences.

Our study is related to earlier attempts by Hakansson [1971] (who studied a discrete-time model) and Merton [1973] (who used a continuous-time model) to establish which restrictions on preferences or asset returns would give rise to the SCAPM even in a fully specified, dynamic model. The restrictions uncovered by these authors, however, were difficult to interpret, since they involved magnitudes which could not be interpreted, in a VNM setting, as unambiguously referring to risk or intertemporal substitution. Our analysis suggests that myopia in consumption-saving—the case of a unit elasticity of intertemporal substitution—does not in general give rise to the static CAPM. Instead, we show that unit elasticity of relative risk aversion gives rise to myopia in portfolio allocation, and that myopia in portfolio allocation makes the SCAPM hold true even in the general dynamic setup that we consider.

Our paper is also indirectly related to recent work by Bergman [1985], Constantinides [1988] and Sundaresan [1989], who explore the implications for equilibrium asset pricing of time non-separable preferences. We, on our part, allow for preferences which are both time and state non-separable.

Section 1, which describes the model, outlines the axiomatic foundations of Kreps-Porteus non-expected utility preferences, and derives the Euler equations that characterize our model. Section 2 studies various restrictions on taste parameters and the parameters of the distribution of returns. Section 3 explores the theoretical and empirical implications of our results for the *beta* representation of excess returns. The conclusion summarizes our study, and indicates directions for future research.

1 Optimal consumption and portfolio allocation with Kreps-Porteus preferences

1.1 An introduction to Kreps-Porteus preferences

Researchers in finance and economics have long used the very convenient assumption of time and state separability to represent preferences over random consumption prospects. Unfortunately, specifying that utility can be written as

$$V_t = E_t \sum_{i=0}^{\infty} \delta^i \, u(c_{t+i}) \tag{1}$$

implicitly imposes, as is well known, severe restrictions on the consumers' attitudes toward risk and intertemporal substitution. For the case of isoelastic utility, for instance, if $u(c) = c^{1-\rho}/(1-\rho)$, the parameter ρ represents both the Arrow-Pratt coefficient of relative risk aversion for timeless gambles or lotteries on permanent consumption, and the inverse of the elasticity of intertemporal substitution.[5] This behavioral restriction is obviously devoid of any behavioral rationale—as it implies, for instance, that risk neutral agents necessarily have an infinite elasticity of substitution. Not only has it led to much semantic confusion, but it has also prevented any delineation of the respective role of attitudes toward risk and intertemporal substitution in portfolio choice and consumption-

[5]For the exponential sub-utility function $u(c) = \exp(-\zeta c)/(-\zeta)$, $\zeta > 0$, ζ is the coefficient of absolute risk aversion and ζc the elasticity of intertemporal substitution.

savings decisions.[6]

For that reason, a way has long been sought to relax the implicit restriction imposed by the combined assumptions of state and time separability, while maintaining the properties, deemed desirable, of stationarity of preferences and temporal consistency of optimal plans. As independent work by Epstein and Zin [1987a], Farmer [1988] and one of us (Weil [1987]) has established, and as Selden's [1978] early results on the two-period "ordinal certainty equivalence" preferences had hinted, that a class of preferences axiomatized in a series of seminal papers by Kreps and Porteus [1978, 1979a, 1979b], provide a simple and elegant way of separating risk aversion from intertemporal substitution while complying with all the above *desiderata.* Kreps-Porteus preferences generalize time-additive expected utility by relaxing one of the fundamental axioms of VNM theory, the originally static "axiom of reduction of compound lotteries",[7] which, when imposed on temporal gambles, implies that agents are indifferent to the way uncertainty on consumption lotteries resolves over time.[8]

Monotonicity and regularity conditions, together with axiom of independence of irrelevant alternatives and the requirement that preferences be temporally consistent and stationary, suffice, as Kreps and Porteus show, to obtain the following recursive representation

[6] See, for instance, Hall [1988] for the empirical difficulties raised in consumption theory by the time additive, expected utility restriction.

[7] Loosely speaking, this axiom imposes that when one is offered to participate in a lottery whose prizes consist in tickets to other lotteries, one is concerned only in the compound probability of each final prize, and does not care about which particular sequence of lotteries led to that prize.

[8] Consider, for instance, two consumption lotteries each offering prizes with the same compound probabilities, but which differ in terms of the dates at which uncertainty resolves. In lottery A, the agent consumes $c > 0$ with certainty at t. A fair coin is then flipped at $t+1$: if heads, the agent consumes c forever; if tails, he consumes c at $t+1$ and $c' \neq c$ thereafter. In lottery B, the agent consumes c with certainty at t and $t+1$, and the fair coined is tossed instead at $t+2$. If tails, consumption is c thereafter; if heads, it is c'. A VNM expected utility maximizer, who only looks at compound probabilities, is indifferent between the two lotteries, despite the fact that uncertainty on future consumption is resolved one period earlier in lottery A than in lottery B. A Kreps-Porteus agent, by contrast, may have a preference for late or early resolution of uncertainty. Note, in addition, that the foregoing remarks apply to consumption lotteries: even VNM consumers in general prefer early resolution of uncertainty on income lotteries, as it improves planning.

of preferences:

$$V_t = U[c_t, E_t V_{t+1}], \quad (2)$$

where V_t denotes utility at time t, E_t is the expected value operator conditioned by the time t information set, and $U[.,.]$ is, using Koopman's [1960] terminology, the "aggregator" function—which aggregates current consumption with future (expected) value.[9]

When the aggregator function $U[.,.]$ is linear in its second argument (a case which would result from the imposition of the "reduction" axiom on temporal consumption lotteries), one can easily see, by forward substitutions and the law of iterated projections, that utility V_t is the expected sum of discounted future "felicities," with the discount factor being a constant, the derivative, U_2, of the aggregator function with respect to its second argument. The standard time- and state-separable expected utility representation therefore obtains as a special case of Kreps-Porteus preferences.

More generally, the tastes represented in (2) are not part of the class of expected utility functionals,[10] and they allow, as Kreps and Porteus show, for preference for late or early resolution of uncertainty over consumption lotteries depending on the concavity or convexity of the aggregator function $U[.,.]$ in its second argument.

To understand why Kreps-Porteus preferences provide a way to separate attitudes towards risk from behavior towards intertemporal substitution, it is necessary to explain the link between the three concepts of preference for early or late resolution, risk aversion, and intertemporal substitution. In utility terms,[11] one can show lotteries in which uncertainty resolves early[12] are less risky than late resolution lotteries with the same distribution of prizes,[13] but that they present, at the same time, more sure fluctuations of utility over time. There is therefore, in general, a trade-off between safety and stability of utility. Agents who dislike risk "more" than intertemporal fluctuations prefer, *ceteris paribus*, early resolution; but consumers who have a stronger distaste for intertertemporal fluctuations than for

[9] In general, the aggregator function could be allowed to be time dependent.

[10] Equation (2) is not linear in probabilities when the aggregator function is not linear in its second argument.

[11] This, and not consumption, is in general the appropriate "metric."

[12] Lottery A in footnote 8.

[13] Lottery B.

risk prefer late resolution. For VNM consumers with time-additive utility, the cost and benefit of each lottery in terms of safety and stability of utility balance out, because the very same factor which leads them to dislike risk (a high coefficient of relative risk aversion) is necessarily conducive to a strong distaste for intertemporal fluctuations in utility (a low elasticity of intertemporal substitution). VNM consumers with time additive utility are, therefore, indifferent to the timing of resolution of uncertainty over consumption lotteries.

The particular parametric form of Kreps-Porteus preferences upon which this paper concentrates is based on the generalization of isoelastic utility independently proposed, in slightly different forms, by Epstein-Zin [1987a] and Weil [1987], and characterized by the following aggregator function:

$$U[c_t, E_t V_{t+1}] = \left\{(1-\delta)c_t^{1-\rho} + \delta\left(E_t V_{t+1}\right)^{\frac{1-\rho}{1-\gamma}}\right\}^{\frac{1-\gamma}{1-\rho}}, \qquad (3)$$

where $\gamma \geq 0$, $\gamma \neq 1$ can be interpreted as the Arrow-Pratt coefficient of relative risk aversion (CRRA), $1/\rho \geq 0$, $\rho \neq 1$ represents the elasticity of intertemporal substitution (EIS), and $\delta \in (0,1)$ is the subjective discount factor.[14] Under certainty, it is easy to see that the parameter γ is irrelevant, as it can be eliminated by an increasing monotone transformation: risk aversion does not matter under certainty. In the presence of uncertainty, the standard time additive, isoelastic, expected utility representation obtains as the special case in which γ is constrained to be equal to ρ, as the aggregator function is then linear in its second argument. In general,

[14]To justify this terminology and understand the connection of this parametrization with Selden's two-period isoelastic OCE preferences, write $W = U^{1/(1-\gamma)}$, and define two functions, $\mathcal{V}(x) = x^{1-\gamma}$ and $\mathcal{U}(c,x) = [(1-\delta)c^{1-\rho} + \delta x^{1-\rho}]^{1/(1-\rho)}$. Then the definition of preferences in (2) and (3) is equivalent to $W_t = \mathcal{U}(c_t, \mathcal{V}^{-1}[E_t\mathcal{V}(W_{t+1})])$ — a formulation used by Epstein [1988]. Current utility is hence the aggregate, through the function $\mathcal{U}$ which depends only on δ and ρ, of current consumption and the certainty equivalent of future utility, computed through the function $\mathcal{V}$ which depends only on γ. Attitudes toward intertemporal substitution are thus captured by $\mathcal{U}$, and behavior towards risk by $\mathcal{V}$. In the two-period case, the certainty equivalent of future utility is just the certainty equivalent of future consumption—and our preferences reduce to Selden's. Also note that the configurations $\gamma = 1$ and $\rho = 1$ can be dealt with as limiting cases, using de L'Hopital's rule, of the aggregator function in (3) after using the affine transformation $V_t = 1 + (1-\delta)(1-\gamma)v_t$. We follow that procedure in section 2 and its attendant appendices.

however, a consumer has a preference for late (resp. early) resolution of uncertainty if ρ is larger (resp. smaller) than γ.[15] The preferences defined by (2) and (3) are isoelastic in two dimensions: the coefficient of relative risk aversion, γ, is a constant, and so is the elasticity of intertemporal substitution, $1/\rho$. The Kreps-Porteus preferences defined in (3) achieve the desired separation between risk aversion and intertemporal substitution: for they do not impose any cross-restriction on the taste parameters γ and ρ.

1.2 Optimal consumption and portfolio allocation

We now turn to the characterization of the optimal consumption and portfolio decision of an infinitely-lived, representative[16] consumer whose tastes are defined by equations (2) and (3), and who can invest his wealth in N financial assets, with asset i, $i = 1, \ldots, N$, offering the random gross rate of return $R_{i,t+1}$ between periods t and $t+1$. Let

$$R_{M,t+1} \equiv \sum_{i=1}^{N} \alpha_{i,t} R_{i,t+1} \tag{4}$$

denote the rate of return on the optimally weighted portfolio (the "market" portfolio), where $\alpha_{i,t}$ denotes the share, determined at time t, of asset i in the optimal portfolio, and

$$\sum_{i=1}^{N} \alpha_{i,t} = 1. \tag{5}$$

The consumer's objective is to maximize utility, defined by (2) and (3), by choosing a sequence of consumption plans and portfolio shares, subject to (4), (5), and the following sequence of budget constraints:

$$w_{t+1} = R_{M,t+1}(w_t - c_t), \tag{6}$$

for all $t \geq 0$, where w_t and c_t denote, respectively, wealth and consumption at time t.

[15] This is because the aggregator function is then concave (resp., convex) in its second argument. See Kreps and Porteus [1978].

[16] Issues relative to the aggregation of heterogeneous agents are outside the scope of this paper.

Let S_t denote the history of asset returns until, and including, time t, which we assume is known to the agent at time t. The optimal program is then characterized, most simply, in terms of a value function, $V(w, S)$, which solves the following functional equation, reminiscent of standard Bellman equations:

$$V(w_t, S_t) \;=\; \max_{c_t, \{\alpha_{i,t}\}_{i=1}^{N}} U[c_t, E_t V(w_{t+1}, S_{t+1})] \tag{7}$$

subject to (4), (5) and (6)—with the aggregator function given in (3).

Given the homotheticity of the preferences defined in (3), the characterization of the optimal consumption program can be divorced, as in standard VNM settings, from that of the optimal portfolio composition. It is easy to show, performing the maximizations called for by (7) and using the envelope theorem, that in a consumption optimum, the marginal value of wealth must be equalized to the marginal utility of consumption, so that the optimal consumption decision is characterized by the following first-order condition:

$$U_{1,t} \;=\; E_t \left\{ R_{M,t+1}\; U_{2,t}\; U_{1,t+1} \right\}, \tag{8}$$

where $U_{i,t}$ denotes the derivative of the aggregator function with respect to its i-th argument, evaluated at $(c_t, E_t V_{t+1})$. With time additive, expected utility, the discount factor $U_{2,t}$ is constant and equal to δ—so that this Euler equation assumes its "familiar" form.

As for portfolio composition, it must be the case that, for any two distinct assets i and j which are held by our consumer,

$$E_t \left\{ U_{1,t+1}\; R_{i,t+1} \right\} \;=\; E_t \left\{ U_{1,t+1}\; R_{j,t+1} \right\}. \tag{9}$$

Equation (9) must hold, in particular, for the market portfolio (because the market must be held in equilibrium in this representative agent economy). Hence, for any asset i which is held (which includes the market)

$$E_t \left\{ U_{2,t} \frac{U_{1,t+1}}{U_{1,t}} \;\; R_{i,t+1} \right\} \;=\; 1, \tag{10}$$

which is the fundamental Euler equation exploited in this paper.[17]

[17]Giving up one unit of consumption today costs $U_{1,t}$ time-t utils. An additional

Let

$$MRS_{t+1} \equiv U_{2,t}\frac{U_{1,t+1}}{U_{1,t}} \tag{11}$$

denote the marginal rate of substitution between periods t and $t+1$. Then, as in all optimizing intertemporal models, it is the covariance of an asset with the marginal rate of substitution which determines, in equilibrium, its excess return.

But what are the determinants of this marginal rate of substitution, and thus of excess returns? Is it consumption growth—as in traditional dynamic models with time-additive expected utility? Is it the rate of return on the market—as in static models? In the next section we answer those questions by showing that, in general, the marginal rate of substitution is a geometric weighted average of the consumption growth rate and of the return on the market. It is only in a few special cases, which we catalogue, that the marginal rate of substitution is driven solely by the rate return on the market. We postpone discussion of the implications of those results for the *beta* representation of excess returns and the interpretation of empirical evidence to the last section of the paper.

2 Determinants of the marginal rate of substitution

In this section, we examine the determinants of the marginal rate of substitution, given in equation (11), for the parametrization of isoelastic Kreps-Porteus preferences introduced in (3). We proceed in two steps, first characterizing those determinants for general parameter values, then examining the interesting special cases which emerge for remarkable values of ρ and γ, and for specific distributional assumptions on S_t.

2.1 General case

The method followed here draws both on Epstein-Zin [1987a] and Weil [1987, 1988]. To compute the marginal rate of substitution

unit of consumption tomorrow provides $U_{1,t+1}$ time-$(t+1)$ utils, which are worth $U_{2,t}U_{1,t+1}$ time-t utils. In an optimum, investing the one unit of consumption given up at time t in any asset i, and eating the proceeds at $t+1$ should not, on average, increase utility.

along an optimal consumption path, it is necessary to characterize the value function which solves the functional equation in (7). It is easy to verify[18] that the value function is homogeneous of degree $1-\gamma$ in wealth, and that it can be written as

$$V(w_t, S_t) \;=\; \Phi(S_t)\, w_t^{1-\gamma}. \tag{12}$$

Therefore, the Arrow-Pratt coefficient of relative risk aversion for timeless gambles (the elasticity of the indirect marfinal utility of wealth) is simply γ—which confirms the interpretation of γ given *supra.*

The associated consumption function is

$$c_t \;= \mu(S_t)\, w_t. \tag{13}$$

Our generalized isoelastic preferences thus yield, as does their VNM time additive counterpart, consumption functions which are linear in wealth.[19] Using the properties of the functions $\Phi(.)$ and $\mu(.)$ spelled out in Appendix A together with the definition of the aggregator function given in (3), one can show after tedious but straightforward manipulations that the marginal rate of substitution can be expressed as :

$$MRS_{t+1} \;=\; \left[\delta\left(\frac{c_{t+1}}{c_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}} \left[\frac{1}{R_{M,t+1}}\right]^{1-\frac{1-\gamma}{1-\rho}}. \tag{14}$$

Equation (14), which was first derived by Epstein and Zin [1987a], shows that with Kreps-Porteus isoelastic preferences, the marginal rate of substitution is, in general, a geometric weighted average of the rate of growth of consumption and of the rate of return on the market portfolio.

What in general determines the equilibrium rate of return on asset i is thus the covariance with both consumption growth and the rate of return on the market. The SCAPM and CCAPM should hence not be viewed as contradictory approaches, but rather as constitutive elements of a more general theory.

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[18]See Appendix A for proofs of the following statements.

[19]This property is shared by the Kreps-Porteus generalization of exponential utility, which is easy to parametrize. We conjecture that it is true more generally for the *meta*-HARA class of utility functions which can in principle be derived from Kreps-Porteus preferences.

2.2 *Unit elasticity of intertemporal substitution*

We now establish that one of the remarkable feature of time-additive logarithmic utility—the constancy of the marginal propensity to consume—obtains more generally for preferences characterized by a unit elasticity of intertemporal substitution, independently of attitudes towards risk.

As we show in Appendix B, when $\rho = 1$ optimal consumption is proportional to wealth, independently of the value of the coefficient of relative risk aversion:

$$c_t = (1 - \delta) w_t. \tag{15}$$

However, this insensitivity to the current realization of the state does not carry over to portfolio allocation, for the optimal portfolio decision is truly dynamic—as shown by the Euler equation derived in Appendix B.

Therefore, *a unit elasticity of intertemporal substitution implies a form of (rational) myopia in consumption and savings decisions, but not in portfolio allocation.*

Under the additional assumption that the market rate of return at $t+1$ follows, conditionally on information available at t, a log-normal process:

$$\ln R_{M,t+1} = \pi \ln R_{M,t} + \epsilon_{t+1} \tag{16}$$

with $\epsilon \sim \mathcal{N}(0, \sigma^2)$, and supposing that there is no other uncertainty than rate of return uncertainty, the functional equation in (7) can be solved explicitly. The (indirect) utility of wealth is simply:

$$V(w_t, R_{M,t}) = \Phi R_{M,t}^{\eta} \, w_t^{1-\gamma}, \tag{17}$$

where $\eta \equiv (1-\gamma)\pi\delta/(1-\pi\delta)$, and Φ is a constant whose value is irrelevant for the present discussion. As equation (17) indicates, the process followed by $\ln R_M$ need not be stationary for the value function to be well defined. Instead, one can show that it is only required that the process for R_M be of exponential order less than $1/\delta$, i.e., that $\mid \pi \mid < 1/\delta$.[20] The elasticity of the value function with

[20] A similar condition was noted by Hansen and Sargent [1980] in their analysis of solutions of linear rational expectations models.

respect to a realization of R_M, η, thus reflects, up to a constant of proportionality equal to 1 minus the coefficient of relative risk aversion, the present value of future increments in $\ln R_M$, discounted at rate δ.

Notice that if R_M is i.i.d., then $\eta = \pi = 0$ and the marginal value of one unit of wealth is, of course, independent of the current realization of the interest rate process. Of course, the same (familiar) result arises when $\gamma = \rho = 1$ (the expected logarithmic utility case analyzed *infra*).

The Euler equation associated with (17) is shown, in Appendix B.2 D, to be:

$$E_t \left\{ R_{i,t+1} \; R_{M,t+1}^{-(\gamma-\eta)} \right\} \; = \; E_t \left\{ R_{i,t+1} \; R_{M,t+1}^{1-(\gamma-\eta)} \right\} \tag{18}$$

As equation (18) shows, with a unit elasticity of intertemporal substitution and conditionally lognormal returns, we obtain an Euler equation that is observationally equivalent to the first-order condition in the SCPAM (or the CCAPM). The difference between the standard SCPAM equation and that obtained from our (restricted) dynamic model is in the interpretation of the "risk aversion" parameter. The relative risk aversion coefficient γ of the SCPAM is here replaced by $\gamma - \eta$—which would lead, if (18) were used to estimate the coefficient of relative risk aversion, to a downward bias of size η.

2.3 Unit coefficient of relative risk aversion

In the case of logarithmic risk preferences ($\gamma = 1, \rho \neq 1$), and for any stochastic environment, the consumption function which characterizes an optimal program is given by

$$c_t = \mu(S_t) \; w_t, \tag{19}$$

where the function $\mu(.)$ is implicitly defined in Appendix C. Consumption is linear in wealth. The marginal propensity to consume is, however, in general state-dependent: logarithmic risk preferences do not, therefore, yield any myopia in consumption.

The marginal rate of substitution between current and future consumption reduces to [see (14) and Appendix C]:

$$MRS_{t+1} = \frac{1}{R_{M,t+1}}, \tag{20}$$

so that the Euler equation characterizing the optimal program becomes

$$E_t \{R_{i,t+1}/R_{M,t+1}\} \;=\; 1, \tag{21}$$

which is the first-order condition of the SCPAM.

We thus conclude that with logarithmic risk preferences, and without any distributional restrictions on asset returns, excess returns should conform, in equilibrium, to the SCPAM—whichever the magnitude of the elasticity of intertemporal substitution. In other terms, *a unit coefficient of relative risk aversion implies a form of (rational) myopia in portfolio allocation, but not in consumption decisions.*

2.4 Logarithmic expected utility

The logarithmic expected utility case combines, not surprisingly, the remarkable features of the unit EIS and unit CRRA cases examined *supra*. It is easy to show[21] that, when $\gamma = \rho = 1$, optimal consumption is a constant fraction of wealth:

$$c_t = (1-\delta)w_t, \tag{22}$$

so that, using the budget constraint, the Euler equation can be written in two algebraically identical forms:

$$E_t\{R_{i,t+1}/R_{M,t+1}\} \;=\; 1, \tag{23}$$

or

$$E_t\left\{\delta\left[\frac{c_{t+1}}{c_t}\right]^{-1} R_{i,t+1}\right\} \;=\; 1. \tag{24}$$

Excess returns are then governed, in equilibrium, by the SCAPM according to the first equation, by the CCAPM according to the second. These two formulations are not contradictory: the SCAPM and CCAPM must be equivalent under logarithmic expected utility, because consumption growth is perfectly correlated with the rate of return on the market when the marginal propensity to consume is constant.

Logarithmic expected utility thus features two forms of rational myopia: *myopia in consumption* (a constant marginal propensity to

[21] See Appendix B.1 for a short proof.

consume), and *myopia in portfolio allocation* (excess returned governed by the SCAPM). Our framework enables us to unambiguously attribute the former to a unit elasticity of intertemporal substitution, and the latter to a unit coefficient of relative risk aversion.

Our results on specific preference restrictions are collected in Table 1. The Table highlights the symmetric effects of $\rho = 1$ and $\gamma = 1$ on consumption and portfolio allocation, respectively; it shows that myopia in consumption is not equivalent to myopia in portfolio allocation unless, as shown *supra*, when $\gamma = \rho = 1$.

	MPC	True CAPM
$\rho = 1$	constant	Neither SCAPM nor CCAPM
$\gamma = 1$	non-constant	SCAPM
$\rho = \gamma = 1$	constant	SCAPM=CCAPM

Table 1: Implications of unit risk aversion and unit intertemporal substitution

2.5 *I.i.d. uncertainty*

Suppose that the state vector S_t is identically and independently distributed, so that current realizations convey no information about future states. It is obvious that the marginal propensity to consume must then be constant: it depends on the state- and time-invariant expected return on the market portfolio. Consumption growth is, therefore [see (6)], proportional to the rate of return on the market. Straightforward computations (see Appendix D) prove that the marginal rate of substitution, given in (14), then reduces to $MRS_{t+1} = R_{M,t+1}^{-\gamma} = [(c_{t+1}/c_t)/(1-\mu))]^{-\gamma}$, so that the Euler equation (10) becomes

$$E_t \left\{ R_{M,t+1}^{-\gamma} R_{i,t+1} \right\} = 1, \tag{25}$$

or, equivalently,

$$E_t \left\{ (1-\mu)^{\gamma} \left[\frac{c_{t+1}}{c_t} \right]^{-\gamma} R_{i,t+1} \right\} = 1, \tag{26}$$

where μ denotes the marginal propensity to consume, computed in Appendix D.

The first of these two algebraically identical Euler equations is simply the Euler equation characterizing the optimal portfolio allocation of an agent with coefficient of relative risk aversion γ who maximizes, period by period, the expected utility of his wealth;[22] the second equation tells us that the optimal program is observationally equivalent to that of a VNM maximizer with a CRRA of γ (or an EIS of $1/\gamma$), and a subjective discount factor of $(1-\mu)^\gamma$.

Therefore, with i.i.d. uncertainty, attitudes toward intertemporal substitution, as measured by ρ, are irrelevant for portfolio allocation;[23] only attitudes toward risk, as parametrized by γ, matter. Further, the SCAPM and CCAPM should provide, with i.i.d. uncertainty, identical descriptions of excess returns.[24] As we shall argue below, the apparent inconsistency of this result with the data can be interpreted as *prima facie* evidence that the i.i.d. assumption is unwarranted in reality. Our results on specific distributional assumptions are collected in Table 2.

	MPC	True CAPM
I.i.d. returns	constant	SCAPM=CCAPM
$\rho = 1$, lognormal returns	constant	SCAPM- and CCAPM-like

Table 2: Implications of distributional assumptions

[22] This is most easily seen by noting that equation (25) holds, in particular, for the market portfolio, so that $E_t\left\{R_{M,t+1}^{-\gamma}R_{i,t+1}\right\} = E_t\left\{R_{M,t+1}^{1-\gamma}\right\}$—the traditional formulation of the SCAPM.

[23] But obviously not for the consumption/savings program.

[24] The equivalence of the SCAPM and CCPAM under i.i.d. uncertainty is well known with VNM time-additive preferences (see, for instance, Huang and Litzenberger [1988] for a proof). It has also been noticed, for Kreps-Porteus (KP) preferences, by Kocherlakota [1987], who interprets it as implying the observational equivalence of VNM and KP preferences. This observational equivalence is however trivial, as it is confined to a case—i.i.d. uncertainty—in which time is irrelevant both for VNM and KP optimal portfolio allocation. We drive a more interesting observational equivalence result in section 2.4 below.

3 Interpreting the data

We have suggested above that our model, based on isoelastic Kreps-Porteus preferences, predicts that excess returns on any asset are, in general, governed by the covariation with both consumption growth and the rate of return on the market portfolio. This result—which unifies two theories, the static and consumption capital asset pricing models, often viewed as largely contradictory—is now analyzed in greater detail. We provide a theoretical rationalization of the existing empirical evidence on excess returns, and identify some testable implications of our theory.

3.1 Decomposition of the true beta

To derive an analytically tractable expression of an asset true *beta*, it suffices to assume that the logarithm of the return on any asset i, of the rate of the return on the market, and of consumption growth are conditionally jointly normal, with mean

$$\xi_i = (\bar{r}_i, \bar{r}_M, \bar{c})' \tag{27}$$

and variance-covariance matrix

$$\Sigma_i = \begin{pmatrix} \sigma_i^2 & \sigma_{iM} & \sigma_{ic} \\ \sigma_{iM} & \sigma_M^2 & \sigma_{Mc} \\ \sigma_{ic} & \sigma_{Mc} & \sigma_c^2 \end{pmatrix}, \tag{28}$$

where c is shorthand for the logarithm of the rate of growth of consumption.[25]

Using standard results on the multivariate lognormal distribution, and the property that $EX^a = \exp(a\xi + a^2\sigma^2/2)$ if $\ln X \sim$

[25] The assumption that individual asset returns and the market return are jointly lognormal is, of course, only an approximation, since the market return is a linear combination of individual returns and the lognormal distribution is not stable under addition; the results below however hold exactly over infinitesimal time periods when asset returns follow a diffusion process. As to the assumption that asset returns and consumption growth are jointly lognormal, it might, in discrete time, be inconsistent with utility maximization. This occurs not only in our model but also, for instance, in the Hansen-Singleton [1983] paper for the special case of logarithmic expected utility. This difficulty, when it arises, is also resolved by considering continuous time.

$\mathcal{N}(\xi, \sigma^2)$, the Euler equation, given by (10) and (14), can be rewritten as:

$$-\rho\theta\bar{c} + (\theta - 1)\bar{r}_M + \bar{r}_i + \frac{1}{2}[\rho^2\theta^2\sigma_c^2 + (\theta-1)^2\sigma_M^2 + \sigma_i^2] \\ - \rho\theta\sigma_{ic} - \rho\theta(\theta-1)\sigma_{Mc} + (\theta-1)\sigma_{iM} = -\ln\delta, \qquad (29)$$

where $\theta \equiv (1-\gamma)/(1-\rho)$.

This equation also applies to the riskfree rate, so that:

$$-\rho\theta\bar{c} + (\theta - 1)\bar{r}_M + r_F + \frac{1}{2}[\rho^2\theta^2\sigma_c^2 + (\theta-1)^2\sigma_M^2] \\ - \rho\theta(\theta-1)\sigma_{Mc} = -\ln\delta, \qquad (30)$$

where r_F denotes the logarithm of the riskfree rate, R_F.[26]

Subtracting (30) from (29), we find that

$$\bar{r}_i + \sigma_i^2/2 - r_F = \rho\frac{1-\gamma}{1-\rho}\sigma_{ic} + \frac{\gamma-\rho}{1-\rho}\sigma_{iM}, \qquad (31)$$

or, equivalently, that

$$\ln E_t R_{i,t+1} - r_F = \gamma\sigma_{ic} + \frac{\gamma-\rho}{1-\rho}(\sigma_{iM} - \sigma_{ic}). \qquad (32)$$

Equation (32) confirms our earlier results. If preferences are VNM, so that $\gamma = \rho$, the covariance between an asset's return and the market should play no role in the determination of that asset's excess return—except, as noted above, in the special logarithmic case $\gamma = \rho = 1$ for which the static asset pricing model obtains ($\ln E_t R_{i,t+1} - r_F = \sigma_{iM}$). On the other hand, if preferences are of the Kreps-Porteus variety ($\gamma \neq \rho$), the covariance with *both* consumption and the market should in general have explanatory power for excess returns—one exception being the case $\gamma = 1$ for which the SCAPM obtains independently of the value of ρ, since $\gamma = (\gamma - \rho)/(1-\rho)$ when $\gamma = 1$.

A further, and perhaps more attractive, implication of the transformed Euler equation (31) can be derived by noting that it applies,

[26] Because of our distributional assumptions, and as (29) and (30) indicate, conditional expected returns and the riskfree rate are constant in this economy.

in particular, to the market portfolio, so that

$$\bar{r}_M + \sigma_M^2/2 - r_F = \rho\frac{1-\gamma}{1-\rho}\sigma_{Mc} + \frac{\gamma-\rho}{1-\rho}\sigma_M^2. \tag{33}$$

Dividing (31) by (33), and defining the i-th asset's consumption and market *betas* as being, respectively,[27]

$$\beta_i^c = \sigma_{ic}/\sigma_{Mc}, \tag{34}$$

$$\beta_i^M = \sigma_{iM}/\sigma_M^2, \tag{35}$$

we immediately find that

$$\beta_i = \alpha\beta_i^c + (1-\alpha)\beta_i^M, \tag{36}$$

where

$$\beta_i = \frac{\ln E_t(R_{i,t+1}/R_F)}{\ln E_t(R_{M,t+1}/R_F)} \tag{37}$$

denotes the "true" *beta* of asset i (defined as the excess return on asset i over the riskfree rate, relative to the excess return on the market portfolio),[28] and where

$$\alpha = \frac{\rho(1-\gamma)(\sigma_{Mc}/\sigma_M^2)}{\rho(1-\gamma)(\sigma_{Mc}/\sigma_M^2) + (\gamma-\rho)}. \tag{38}$$

Hence, according to equation (36), the true *beta* of asset i is a linear combination of that asset's consumption and market *betas*. The weight in that linear combination, α, is, from equation (38), the same for every asset. It depends only on preferences and on the correlation between the rate of return on the market and the the rate of growth of consumption.

Two interesting parameter configurations confirm, once again, the results of section 2. When consumers have a unit coefficient of relative risk aversion ($\gamma = 1$), $\alpha = 0$ and the true *beta* is the market *beta* independently of the value of the elasticity of intertemporal substitution. When preferences are VNM ($\rho = \gamma \neq 1$), $\alpha = 1$ and the true *beta* is the consumption *beta*.

[27]The market consumption and market *betas* are equal to 1 by normalization.

[28]Again, the market true *beta* is 1 by normalization.

In what follows, we show that this theoretical *beta* decomposition provides an explanation to the empirical evidence: excess returns, we shall claim, are best explained by the market *beta* because the consumers' coefficient of relative risk aversion is not significantly different from 1 and the VNM restriction does not hold.

3.2 An empirical exploration

In this section we report some illustrative analyses of the data, by reinterpreting the cross-sectional regressions run by Mankiw and Shapiro [1986].[29]

Mankiw and Shapiro [1986] presented a test on the relative predictive ability of the market *beta* and the consumption *beta* in a linear regression to explain a cross section of stock returns. As we have shown above, equations (36) and (37)—formally similar to the one estimated by Mankiw and Shapiro—are the implication of a fully specified dynamic model of asset pricing: hence we can interpret the estimates of the coefficients of the consumption *beta* and the market *beta* in terms of structural parameters. In addition, equation (36) and suggests that the coefficients of the market *beta* and the consumption *beta* in such a cross section should sum to 1, and—being a function of risk aversion, intertemporal substitution, and the covariance of consumption and the market portfolio—should be constant across assets. We test these propositions by reconstructing and updating the sample of Mankiw and Shapiro. Our sample includes all the companies listed in the New York Stock Exchange (NYSE) continuously from January 1959 to May 1987, contained in the tape of the Center for Research in Security Prices (CRSP) at the University of Chicago. The total number of firms is 379.

The consumption measure is real *per capita* consumer expenditure in nondurables and services (measured in terms of nondurables) from the US National Income Accounts. Individual firms' returns include dividends and capital gains, while the market return is the value-weighted CRSP NYSE index. The nondurables price deflator is used to measure real returns. We construct quarterly series of

[29]The approach of Mankiw and Shapiro [1986] was pioneered by Douglas [1969], Miller and Scholes [1982], Blume and Friend [1973], and Fama and MacBeth [1973]. See Huang and Litzenberger [1988] for a valuable survey of tests of the CAPM.

real returns and consumption growth, measured at the first month of each quarter.[30] From these quarterly series, we compute—for each firm in our sample—the moments in (27) and (28) over the period from January 1959 to May 1987.[31]

The dependent variables in the regression is the ratio on the right-hand side of equation (37). Since the riskfree rate is not observable, we follow the findings of Fama [1985] for the period from 1953 to 1971, and assume it to equal 1 percent per annum. The right-hand side variables in the regression are a constant term,[32] and the consumption and market *betas* defined in equations (34) and (35).

OLS estimates of the relation between betas and stock returns are consistent under relatively weak assumptions, but the covariance matrix of the disturbances may not be spherical, since stock returns are correlated. Hence we compute both OLS and GLS estimates. Following Mankiw and Shapiro we compute GLS estimates assuming that the residuals v_i $(i = 1, \ldots, 379)$ have the form:

$$v_i = k_i \nu + \epsilon_i \tag{39}$$

where ν is a common stochastic factor, and $\mathrm{Cov}(\epsilon_i, \epsilon_j) = 0 \; \forall i \neq j$, and $\mathrm{Cov}(\nu, \epsilon_i) = 0 \; \forall i$. These assumptions imply that excess returns are driven by a common factor, which is the implication of the *beta* representation of our model. Under these conditions, it is easy to show that the off-diagonal elements of the covariance matrix of disturbances are proportional to $\beta_i \beta_j$, while the diagonal elements are proportional to σ_i^2/σ_M^2.

Tables 3 and 4 contain the results of the OLS and GLS regressions and tests of the relation between betas and stock returns.

The first implication of the model is that the coefficients on the market *beta* and the consumption *beta* should sum to 1. Table 5 reports the marginal significance levels for the null hypothesis that the coefficients sum to 1.[33] For both the OLS and GLS estimates the

[30]The use of average consumption the first month of every quarter allows us to minimize the time-aggregation problem.

[31]The data and all the Fortran and Gauss programs used in this paper are available from us on request.

[32]A constant is included to allow for measurement error in our riskfree rate assumption and in the estimation of the expected rate of return in the market.

[33]I.e., the probability that under the null hypothesis the test statistic exceeds the

	Constant	β_i^c	β_i^M	R^2
OLS	0.48	0.07	0.89	0.17
Unrestricted	(0.11)	(0.06)	(0.12)	
OLS	0.44	0.07	0.93	0.17
Restricted	(0.03)	(0.06)	(0.06)	
GLS	0.39	0.06	0.96	0.17
Unrestricted	(0.92)	(0.06)	(0.39)	
GLS	0.38	0.06	0.94	0.17
Restricted	(0.91)	(0.06)	(0.06)	
Standard errors are given in parentheses				

Table 3: The relation between stock returns and *betas*

	Test of the restriction sum of coefficients = 1	Stability test
OLS	0.701	0.146
GLS	0.961	0.045

Table 4: Stock returns and *betas*: test statistics

restriction is not rejected. Furthermore, the OLS and GLS estimates are very similar to each other, and similar in the constrained and unconstrained versions of the regression equation.

The second implication is that the coefficients should not vary across firms. We test this constraint using a Chow test in the OLS case and a Wald test in the GLS case.[34] In both cases the null hypothesis of no change in the coefficients across subsamples is not rejected at the 1 percent level, although it is rejected at the 5 percent level when we use GLS estimates. We also produce 50 arbitrarily rearranged replicas of the original sample, and perform Chow and Wald tests for each one of them. We cannot reject the hypothesis

computed value. The statistic is distributed as an F in the OLS case, and as a χ^2 in the GLS case.

[34] We stack together the equations in the subsamples.

of constant coefficients at the 1 percent level in all 50 cases when we use the Chow test, and in about 80 percent of the cases when we use the Wald test.

The estimates of Table 3 indicate that the consumption *beta* coefficient is insignificantly different from zero, thus confirming the results first reported by Mankiw and Shapiro [1986]. Our model, however, suggests an explanation for this finding—which was, up to now, viewed as puzzling. As we argued above, equations (36) and (37) imply that, when $\gamma \rightarrow 1$, the weight of the consumption *beta* tends to zero. Hence our estimates imply that γ is insignificantly different from 1.

Is the evidence consistent with logarithmic VNM preferences? As we showed above, we should observe, if it were the case that $\gamma = \rho = 1$, the equivalence of the SCAPM and CCAPM; in other terms, we should have $\beta_i^c = \beta_i^M$ for every asset i. The estimated standard errors clearly suggest that the collinearity between β^c and β^M is low.[35] Thus, the data do not seem to support the hypothesis of logarithmic VNM preferences, and suggest that we have $\gamma = 1$, $\rho \neq 1$.

Is the evidence consistent with i.i.d. uncertainty? Again, the lack of substantial collinearity between the market and consumption *betas*— a collinearity which, according to the analysis of section 2, should be present under i.i.d. uncertainty—rules out this interpretation of the evidence.

Hence we tend to conclude that the evidence both leans against the VNM restriction on preferences, and is consistent with the hypothesis that the coefficient of relative risk aversion is unity. These results are consistent with the time-series evidence of Epstein and Zin [1987b].

One drawback of the *beta* regressions of Table 3 is that they are conditional on assumed values for the riskfree rate. Perhaps a cleaner test of our model relies on equation (32). In a cross-sectional regression of asset returns on a constant, σ_{ic} and $\sigma_{iM} - \sigma_{ic}$, the constant should provide an estimate of the riskfree rate, the coefficient

[35] A regression of β^M on a constant and β^c produces an estimate of the constant term of 0.77 (standard error 0.03) and of the slope coefficient of 0.23 (standard error 0.02).

on σ_{ic} an estimate of the coefficient of relative risk aversion, and the coefficient on $\sigma_{iM} - \sigma_{ic}$ should be non-zero if preferences are not VNM. In Table 5 and 6 we provide the OLS and GLS estimates and test statistics of that equation.

	constant	σ_{ic}	$\sigma_{iM} - \sigma_{ic}$	R^2
OLS	0.98	5.85	1.84	0.17
	(0.17)	(3.43)	(0.25)	
GLS	0.83	5.60	2.00	0.17
	(0.14)	(4.22)	(0.87)	
Standard errors are given in parentheses				

Table 5: Covariance regressions

	Test of the restriction: $\gamma = 1$	Stability test
OLS	0.996	0.326
GLS	0.282	0.487

Table 6: Covariance regressions: test statistics

The results are consistent with those obtained in the *beta* regressions. Although the coefficient of relative risk aversion is very unprecisely estimated, we cannot reject the hypothesis that it is different from one, as shown by the test of the restriction that both the coefficients on σ_{ic} and $\sigma_{iM} - \sigma_{ic}$ are equal to 1.[36] Furthermore, the assumption that preferences are VNM is decisively rejected. However the estimates of the riskfree rate, which is expressed in percent per quarter, appear to be too high: the lower bound of a 95 confidence interval for both the OLS and the GLS estimates is about 2 percent per annum, a value that is twice the size of the one estimated by Fama [1975] on a partly overlapping sample period. Finally, the stability tests appear less strongly supportive of the hypothesis that

[36] For $\rho \neq 1$, $\gamma = 1$ implies that $\gamma = (\gamma - \rho)/(1 - \rho) = 1$—whence our test [see equation (32)].

coefficients are constant across equations. While in the OLS regression we found that the null hypothesis of no difference in the coefficients is never rejected in the 50 sample permutations we perform, in the GLS regression we reject stability in about 30 percent of the experiments.

The high estimates of the riskfree rate and the few rejections of the coefficient-stability tests suggest the presence of some specification errors. These errors might be associated with our method of estimation, that assumes the model holds on average during a specified time interval, with sampling errors in the covariance matrix, and with errors in the measurement of the market rate of return (which might plausibly also include human wealth). We plan to address these questions directly in future empirical research, in order to obtain full-information estimates of these structural parameters which exploit the time-variation of conditional first and second moments.

Concluding remarks

This paper has studied the Capital Asset Pricing Model under the assumption that investors' preferences display a coefficient of constant relative risk aversion which differs from the constant elasticity of intertemporal substitution. Our analysis was motivated both by the theoretical shortcomings of VNM preferences which confuse these two parameters, and by various theoretical and empirical papers which aimed at establishing the relative importance of intertemporal effects in asset pricing equations.

We have shown that the particular family of preferences employed in this paper allows the construction of asset-pricing equations that are both tractable and intuitive. In particular, the respective roles of intertemporal substitution and risk aversion are greatly clarified in this setup. We find that a unit elasticity of intertemporal substitution gives rise to mypia in consumption-savings decisions (the future does not affect optimal consumption) while unit relative risk aversion gives rise to myopia in portfolio allocation decisions (the future does not affect optimal portfolio allocations). Equilibrium asset returns are determined both by the covariance with the market portfolio and the covariance with consumption growth: the relative importance of these two effects depends on the relative magnitude of the risk aversion and intertemporal substitution parameters.

As Epstein and Zin [1987b] first suggested, our paper implies that regressions similar to those run by Mankiw and Shapiro [1986] can be reconciled with a fully specified dynamic model of asset pricing. Parameter estimates in our regressions are known functions of the coefficient of relative risk aversion and the elasticity of intertemporal substitution, so that these regressions can be used to test for various hypotheses on behavioral parameters and for the restriction of VNM preferences. We find that portfolio choice seems to conform mainly to static considerations (whence the apparent empirical support for the SCAPM), and that market and consumption *betas* are largely unrelated. This empirical evidence is consistent with the behavior of intertemporal maximizers who have a unit coefficient of relative risk aversion and an elasticity of intertemporal substitution different from 1.

Although the exploratory regressions reported here are broadly consistent with our theoretical framework, we feel that a full-fledged, maximum likelihood estimation and testing of the model would be of interest. We propose to undertake that task, which is beyond the scope of this paper, in future research.

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Appendix A Characterization of the optimal program

This appendix studies the optimal consumption program and portfolio allocation. We proceed in two steps, first characterizing the solution to the optimality equation (7) in the text, then turning to the computation of the marginal rate of substitution along an optimal consumption path.

A.1 Value function and consumption function

Because of the homogeneity properties of the aggregator function, of the interpretation of γ as the CRRA, of the fact that preferences are isoelastic and of the separation between the consumption-savings and portfolio allocation problems, guess that the value function can be written in the form

$$V(w_t, S_t) = \Phi(S_t) w_t^{1-\gamma}, \tag{A.1}$$

where $\Phi(.)$ is an unknown function, and that the consumption function is linear in wealth:

$$c_t = \mu(S_t) w_t, \tag{A.2}$$

where $\mu(S_t)$ is the state-dependent marginal propensity to consume.

It is easy to show, performing the maximization called for by (7), that the functions $\Phi(.)$ and $\mu(.)$ are related by the following two conditions:

$$(1-\delta)[\mu(S_t)]^{-\rho} = \delta \varphi_t [1-\mu(S_t)]^{-\rho} \tag{A.3}$$

and

$$\Phi(S_t) = (1-\delta)^{\frac{1-\gamma}{1-\rho}} [\mu(S_t)]^{-\rho\frac{1-\gamma}{1-\rho}}, \tag{A.4}$$

where $\varphi_t = E_t\{\Phi(S_{t+1})[R_{M,t+1}]^{1-\gamma}\}^{\frac{1-\rho}{1-\gamma}}$. Those two expressions can be combined to yield a functional equation in $\Phi(.)$. Although that equation has an explicit solution in only a few special cases, the analysis which follows avoids the need for closed forms.

Using the budget constraint (6) along with (A.2), (A.3) and (A.4) imply that, along an optimal program,

$$\frac{\varphi_t^{\frac{1-\gamma}{1-\rho}}}{\Phi(S_{t+1}) R_{M,t+1}^{1-\gamma}} = \delta^{\frac{1-\gamma}{1-\rho}} \left[\frac{c_{t+1}}{c_t}\right]^{\rho\frac{1-\gamma}{1-\rho}} R_{M,t+1}^{\frac{1-\gamma}{1-\rho}}, \tag{A.5}$$

an expression which will be used *infra*.

A.2 Computation of the marginal rate of substitution

From (3), it is straightforward to show that the marginal rate of substitution defined in (11) is, along an optimal path,

$$MRS_{t+1} = \delta \left[\frac{c_{t+1}}{c_t}\right]^{-\rho} \left[\frac{E_t V_{t+1}}{V_{t+1}}\right]^{\frac{1-\rho}{1-\gamma}-1}, \tag{A.6}$$

where V_{t+1} denotes the value function evaluated at (w_{t+1}, S_{t+1}). Using the budget constraint (6) along with (A.1) and (A.2), one finds that

$$\frac{E_t V_{t+1}}{V_{t+1}} = \frac{\varphi_t^{\frac{1-\gamma}{1-\rho}}}{\Phi(S_{t+1}) R_{M,t+1}^{1-\gamma}}, \tag{A.7}$$

so that, substituting (A.5) and (A.7) into (A.6), we find that

$$MRS_{t+1} = \left[\delta \left(\frac{c_{t+1}}{c_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}} R_{M,t+1}^{\frac{1-\gamma}{1-\rho}-1} \tag{A.8}$$

an expression which, when inserted in (11), yields the Euler equation (14) in the text.

Appendix B Unit elasticity of intertemporal substitution

As in the previous appendix, we start by computing the limit of a monotone transformation of the aggregator function in (3):

$$\begin{aligned} v_t &= \lim_{\rho \to 1} \frac{U[c_t, 1 + (1-\gamma)(1-\delta) E_t v_{t+1}] - 1}{(1-\gamma)(1-\delta)} \\ &= c_t^{(1-\delta)(1-\gamma)} (E_t v_{t+1})^{\delta}, \end{aligned} \tag{B.1}$$

and by guessing a functional form for the value function: $v(w_t, S_t) = \Phi(S_t) w_t^{1-\gamma}$. Hence

$$E_t v_{t+1} = \varphi_t (w_t - c_t)^{1-\gamma}, \tag{B.2}$$

with $\varphi_t = E_t(\Phi_{t+1} R_{M,t+1}^{1-\gamma})$. Following the same procedure as *supra*, we obtain the first order condition for the consumer-investor optimization problem,

$$(1-\delta)\, \frac{1}{c_t} = \delta\, \frac{1}{w_t - c_t} \tag{B.3}$$

which implies:

$$c_t = (1-\delta) w_t. \tag{B.4}$$

Substituting this result in the functional equation, we obtain the following functional equation for $\Phi(.)$:

$$\Phi(S_t) = B[E_t(\Phi(S_{t+1}) R_{M,t+1}^{1-\gamma})]^{\delta} \tag{B.5}$$

where $B = (1-\delta)^{(1-\delta)(1-\gamma)} \delta^{(1-\gamma)\delta}$. This equation cannot, in general, be solved explicitly.

The marginal rate of substitution between periods t and $t+1$ is, from (11) and (B.1),

$$U_{2,t} \frac{U_{1,t+1}}{U_{1,t}} = \frac{v_{t+1}}{E_t v_{t+1}}, \tag{B.6}$$

so that, using the functional form of the value function and (B.5) and (10), the Euler equation can be written as:

$$E_t \left\{ \Phi(S_{t+1}) R_{M,t+1}^{-\gamma} R_{i,t+1} \right\} = E_t \left\{ \Phi(S_{t+1}) R_{M,t+1}^{1-\gamma} \right\} = 1. \tag{B.7}$$

This Euler equation corresponds, in general, to neither the SCAPM nor the CCAPM.

B.1 *Logarithmic utility*[37]

In the particular case in which $\gamma = \rho = 1$ (logarithmic expected utility), the second equality in (B.7) implies that that $E_t\Phi(S_{t+1}) = 1$. But then, (B.5) implies that $\Phi(S_t) = E_t\Phi(S_{t+1}) = 1 \ \forall t$, since $B = 1$ when $\gamma = 1$. In addition, the constancy of the marginal propensity to consume, established in (B.4), implies that the rate of growth of consumption is proportional to the rate of return on the market ($c_{t+1}/c_t = \delta R_{M,t+1}$), so that the Euler equation becomes

$$E_t\{R_{i,t+1}/R_{M,t+1}\} = E_t\{\delta \left[\frac{c_{t+1}}{c_t}\right]^{-1} R_{i,t+1}\} = 1. \tag{B.8}$$

Hence, with logarithmic expected utility, excess returns are explained equally well by the SCAPM and CCAPM—since the constancy of the marginal propensity to consume makes them identical models by implying a perfect correlation between consumption growth and the rate of return on the market.

B.2 *Lognormal asset returns*

Suppose that the only uncertainty is Markovian rate of return uncertainty, with the conditional distribution of returns given in equation (16).

Under this distributional assumption, the functional equation can be solved explicitly (the solution is reported in (17)), and (B.7) becomes, after a few manipulations,

$$E_t\left\{R_{i,t+1}\ R_{M,t+1}^{-(\gamma-\eta)}\right\} \ = \ E_t\left\{R_{i,t+1}\ R_{M,t+1}^{1-(\gamma-\eta)}\right\}, \tag{B.9}$$

where

$$\eta = (1-\gamma)\frac{\pi\delta}{1-\pi\delta}. \tag{B.10}$$

Notice that, again, (B.9) is observationally equivalent to both the SCAPM and CCAPM: consumption is a fixed fraction of wealth, and we can replace $R_{M,t+1}$ by $\delta^{-1}(c_{t+1}/c_t)$ in (B.9).

Appendix C Unit coefficient of relative risk aversion

We first compute the limit of a monotone transformation of the aggregator function in (3):

$$\begin{aligned} v_t &= \lim_{\gamma\to 1}\frac{U[c_t, 1+(1-\gamma)(1-\delta)E_t v_{t+1}]-1}{(1-\gamma)(1-\delta)} \\ &= \frac{\ln[(1-\delta)c_t^{1-\rho}+\delta e^{(1-\rho)(1-\delta)E_t v_{t+1}}]}{(1-\rho)(1-\delta)}, \end{aligned} \tag{C.1}$$

[37]The results in this subsection can, of course, also be derived by considering the limiting case $\rho = 1$ in the previous appendix.

and guess the value function to be of the form $v(w_t, S_t) = \frac{\ln w_t}{1-\delta} + \Phi(S_t)$. Hence $\exp[(1-\rho)(1-\delta)E_t v_{t+1}] = \varphi_t(w_t - c_t)^{1-\rho}$, with $\varphi_t = \exp[E_t \ln[R_{M,t+1}^{1-\rho}\phi_{t+1}^{(1-\rho)(1-\delta)}]$, and $\phi = e^{\Phi}$. The consumer's optimization problem is then:

$$\max_{c_t} \left[(1-\delta)c_t^{1-\rho} + \delta\varphi_t(w_t - c_t)^{1-\rho}\right] \tag{C.2}$$

We guess the consumption function to be of the form $c_t = \mu_t w_t$ and obtain the first order conditions:

$$(1-\delta)\mu_t^{-\rho} = \delta\varphi_t(1-\mu_t)^{-\rho}, \tag{C.3}$$

Substituting (D.3) into the Bellman equation (7), we obtain following relation: $\Phi_t = \mu_t^{-\rho}/(1-\rho)$. Using (C.1) and (C.3) to rewrite (14) in the text, we get:

$$E_t[R_{i,t+1}/R_{M,t+1}] = 1. \tag{C.4}$$

Appendix D I.i.d. uncertainty

We guess the value function to be of the form

$$V(w_t, S_t) = \Phi(S_t)w_t^{1-\gamma} \tag{D.1}$$

Hence, $E_t(V_{t+1}) = (w_t - c_t)^{1-\gamma}E_t(\Phi_{t+1}R_{M,t+1}^{1-\gamma})$.[38] Under the assumption of i.i.d. uncertainty, we can write $E_t(\Phi_{t+1}R_{M,t+1}^{1-\gamma}) = \varphi^{\frac{1-\gamma}{1-\rho}}$, so that the optimization problem of the consumer-investor becomes:

$$\max_{c_t} \left[(1-\delta)c_t^{1-\rho} + \delta\varphi(w_t - c_t)^{1-\rho}\right]^{\frac{1-\gamma}{1-\rho}} \tag{D.2}$$

We guess the consumption function to be of the form $c_t = \mu(S_t)w_t$: the first order condition in (D.2) is:

$$(1-\delta)\mu_t^{-\rho} = \delta\varphi(1-\mu_t)^{-\rho}, \tag{D.3}$$

which implies $\mu(S_t) = \mu$. Substituting the consumption function into the Bellmann equation (7), it is easy to verify that $\Phi(S_t) = \Phi$—a constant whose value is not relevant to this analysis. Substituting (D.3) into (7), after some manipulations we obtain:

$$\mu = 1 - \delta^{\frac{1}{\rho}}(E_t R_{M,t+1}^{1-\gamma})^{\frac{1-\rho}{\rho}\frac{1}{1-\gamma}}. \tag{D.4}$$

Using this expression in the Euler equation (11) and (14) in the text, we obtain

$$E_t[R_{i,t+1}R_{M,t+1}^{-\gamma}] = E_t(R_{M,t+1}^{1-\gamma}) = 1. \tag{D.5}$$

[38] Henceforth we use the shorthand notation $\Phi_t = \Phi(S_t)$ and $\mu_t = \mu(S_t)$.