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ABSTRACT

We present a new approach for estimating private equity returns using secondary market prices for entrepreneurial business credit card securitizations. We show that the market requires a significantly higher premium for entrepreneurial credit risk than for household credit risk. Entrepreneurial risk is systematic in nature and has much in common with risks in corporate bond and real-estate-backed lending markets. The expected return on private equity is on the order of 14 percent and the volatility of private equity returns is comparable to that of the smallest quintile of publicly traded firms.

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1. INTRODUCTION

What is the expected return from entrepreneurial investment? This issue is of fundamental importance given that the total value of private entrepreneurial equity in the U.S. is more than $12 trillion and represents a substantial fraction of aggregate household wealth.[1] Despite the importance of this issue, however, there appears to be little consensus about the risk and return of private equity—estimates of the average return on private equity reported in the literature range from about 5 to 60 percent. A major reason for this may simply be that since private equity does not generally trade in the capital markets, researchers have had to turn to alternative methodologies to estimate returns. Although innovative, these alternative approaches face the challenge of having to rely on incomplete, noisy, and potentially biased data on limited partnership cash flows, NAVs provided by venture capital funds, or other self-reported measures to estimate private equity returns (see Korteweg and Sorensen (2010) and Korteweg (2019)). Without using actual market prices, it is difficult to estimate the returns on entrepreneurial investment directly. Furthermore, these alternative approaches are generally limited to the small subset of entrepreneurial firms receiving venture capital funding or buyout financing.[2]

This paper introduces a novel approach for estimating the investment returns on small private entrepreneurial firms. The key to our approach is the use of secondary market prices for business credit card securitizations, where the credit cards are targeted specifically to entrepreneurs with firms having fewer than ten

[1]For example, see Federal Reserve Statistical Release Z.1 Financial Accounts of the United States, First Quarter 2020, Table B.101.

[2]Venture capital financing of private firms accounts for only a small fraction (less than one percent) of the entire private equity market (Moskowitz, and Vissing-Jorgensen (2002)). Furthermore, firms receiving venture capital funding or buyout financing may not be fully representative of the broader class of entrepreneurial firms. For example, Chemmanur, Krishnan, and Nandy (2011) report that private firms that received venture capital funding between the years 1972 and 2000 had average total assets of $8.1 million, sales of $32.7 million and more than 250 employees. Axelson, Jenkinson, Strömberg, and Weisbach (2013) report that the average enterprise value of private companies receiving buyout financing was more than $600 million.

employees and less than three million dollars in revenues. From these secondary market prices we can directly identify the credit spreads and excess returns for a broad class of securities that are representative of the debt claims on the assets of these entrepreneurial firms. We can then invert a standard Merton (1974) structural credit model to solve for implied risk and return measures for equity investments in these firms.

Credit card borrowing represents an important source of debt capital for many small entrepreneurial firms. The pool of assets underlying the business credit card securitizations in our sample consists of receivables from over one million cardholders. These credit cards can only be used for business purposes. The entrepreneur that owns the firm is jointly and severally liable with the business for all transactions on the account. Thus, the credit risk inherent in these receivables is a direct reflection of the credit risk of entrepreneurship. Because of this, the receivables represent general debt claims on the underlying assets and associated cash flows of these small entrepreneurial firms, and their pricing can be used to shed light on the returns of private equity claims on those assets.

Several important results emerge from our analysis. First, we find that the market prices entrepreneurial credit risk very differently from conventional household credit risk. In particular, we find that even after controlling for the underlying credit risk and structural features of the securitizations, the credit spread for entrepreneurial credit card securitizations is more than 150 basis points higher than for consumer credit card securitizations.

Second, motivated by the theoretical literature suggesting that entrepreneurial risk may be very systematic or procyclical in nature, we contrast the credit losses for the business credit card securitizations in our sample with those for consumer credit cards and other types of personal and commercial loans. We find that charge-off rates for entrepreneurial credit cards are much more related to the credit risk of asset-based loans than is the case for consumer credit cards. We also show that the credit spread for entrepreneurial credit card securitizations is much more related to general macroeconomic and financial market factors. Furthermore, we show that entrepreneurial credit spreads are more related to corporate bond spreads than they are to consumer credit spreads. Together, these results suggest that entrepreneurial risk is strongly systematic in nature which, in turn, may explain the large risk premium incorporated into the credit spread of the securitizations.

Turning to the estimation of private equity returns, we adopt a three-step approach. First, we compute the returns on the securitized debt claims on the assets of the underlying entrepreneurial firms. Second, we use historical data to estimate a range of potential leverage ratios for these entrepreneurial firms. Third, we use these calibrated inputs in a standard Merton (1974) structural

credit framework to solve for the implied moments of returns on private equity claims on these entrepreneurial firms.

The results indicate that the expected return on private equity is about 14 percent during the 2000–2010 sample period, which implies a private equity premium of about 11 percent. This contrasts with the realized excess return for publicly traded stock which was essentially zero over the same time period. Our estimate of the expected return on private equity investments is towards the lower end of the broad range of estimates that appear in the private equity literature.

The results also demonstrate that private equity returns are very volatile. We find that the annualized volatility of private equity returns is on the order of 90 percent during the sample period. This estimate is comparable to the volatility of returns for the smallest quintile of CRSP/Compustat firms during the same time period. Furthermore, this estimate is comparable to the volatility of deal-level private equity returns reported by Chen, Baierl, and Kaplan (2002), Cochrane (2005), and others.

Finally, our results imply that the systematic risk of entrepreneurship is comparable to that of smaller publicly-traded firms. In particular, our results suggest that the market beta of private equity is on the order of 1.21.

In summary, this paper makes several contributions to the private equity literature. First, the results indicate that the market views entrepreneurs as being very different from ordinary consumers or households. In particular, entrepreneurial credit risk has much more in common with corporate credit risk than with household credit risk. Second, because of the systematic nature of entrepreneurial risk, the cost of capital for private equity is relatively high compared to more traditional types of investments. Thus, entrepreneurship faces a high hurdle in the competition for scarce investment capital. This result is particularly important given the fundamental role that entrepreneurship plays in the macroeconomy. Finally, our results demonstrate that using prices from securitized debt markets can open new windows in measuring risk and return for asset classes that have traditionally been difficult to study. In particular, our approach could potentially be applied to a broad set of securitized entrepreneurial and consumer/household debt claims such as Small Business Administration loans, business loan syndications, student loans, auto loans, home equity lines of credit, and personal lines of credit.

2. RELATED LITERATURE

This paper is related to a number of important literatures. Foremost among these is the extensive literature on the estimation of private equity returns. There are two primary empirical approaches within this literature. The first uses self-reported data such as the Survey of Consumer Finances (SCF) to estimate the returns on private entrepreneurial investment. Important examples include Moskowitz and Vissing-Jorgensen (2002) who find that returns on private equity are similar to those on public equity, a result they describe as the private equity premium puzzle. Their results also highlight the important point that the vast majority of firms in the private equity asset class are smaller ventures operated as sole proprietorships and/or family businesses by self-employed entrepreneurs. Kartashova (2014) extends the original data set and finds that private equity returns are higher than public equity returns during the subsequent decade. A number of recent papers, however, raise questions about the reliability of self-reported entrepreneurial income data (see Tedds (2010), Hurst, Li, and Pugsley (2014), and Astebro and Chen (2014)).

The second approach is to use limited partner cash flows and NAV values reported by venture capital funds to estimate the returns on private equity at either the deal level or the fund level. Kaplan and Sensoy (2015), Korteweg (2019), and Brown, Harris, Hu, Jenkinson, Kaplan, and Robinson (2020) provide excellent surveys of this extensive literature. Important examples include Gompers and Lerner (1997), Peng (2001), Ljungqvist and Richardson (2003), Kaplan and Schoar (2005), Cochrane (2005), Phalippou and Gottschalg (2009), Korteweg and Sorensen (2010, 2017), Franzoni, Nowak, and Phalippou (2012), Driessen, Lin, and Phalippou (2012), Ewens, Jones, and Rhodes-Kropf (2013), Harris, Jenkinson, and Kaplan (2014), Korteweg and Nagel (2016), Robinson and Sensoy (2016), Ang, Chen, Goetzmann, and Phalippou (2018), Harris, Jenkinson, Kaplan, and Stucke (2018), Gornall and Strebulaev (2019), and many others. Because of the nature of the cash flow and NAV data, however, researchers in this area face many challenges in estimating private equity returns. Some of the problems with the data identified and discussed in the literature include selection and survivorship biases, stale or incorrectly reported NAVs, incomplete or missing cash flow information, and the inconsistent treatment of fees. As examples, see Emery (2003), Jenkinson, Sousa, and Stucke (2013), Sorensen, Wang, and Yang (2014), Jegadeesh, Kräussl, and Pollet (2015), Barber and Yasuda (2017), Brown, Gredil, and Kaplan (2019), and Korteweg (2019).

The data issues inherent in both of the primary empirical approaches used in this literature may be a contributing factor to the lack of consensus about the risk and return of private equity. Korteweg (2019) states: "[There is] a

substantial degree of heterogeneity in risk-adjusted return estimates, depending on the time period, empirical method, and data source used. There is currently no consensus as to what the main empirical approach should be toward estimating risk and return in private equity or how relevant benchmark returns should be constructed."

This paper advances the literature on the risk and return of private equity by introducing a new empirical approach based on the secondary market prices of securitized entrepreneurial debt. An important advantage of our approach is that it allows us to estimate private equity returns using objective market prices rather than relying on self-reported estimates or the cash flow and NAV measures reported by private equity funds. We note that several other recent papers use secondary market prices for listed private equity funds and funds-of-funds or limited partnership transactions. For example, see Jegadeesh, Kräussl, and Pollet (2015), McCourt (2018), Boyer, Nadauld, Vorkink, and Weisbach (2019), and Nadauld, Sensoy, Vorkink, and Weisbach (2018). A key distinction of our approach, however, is that it allows us to estimate private equity returns for a much broader class of small entrepreneurial firms than just the very limited set that have access to the venture capital markets.[3]

This paper is also related to the literature on the nature and implications of entrepreneurial risk. One segment of this literature focuses on what motivates individuals to become entrepreneurs and how entrepreneurs differ from non-entrepreneurs. Examples include Lazear and Moore (1984), Evans and Leighton (1989), Hamilton (2000), Moskowitz and Vissing-Jorgensen (2002), Fairlie and Robb (2007), Vereshchagina and Hopenhayn (2009), Hyytinen, Ilmakunnas, and Toivanen (2013), Hvide and Panos (2014), and Levine and Rubinstein (2017). Another segment highlights the important roles that liquidity constraints and an individual's financial/housing wealth play in decisions to pursue entrepreneurship. Key examples of this literature include Evans and Jovanovic (1989), Evans and Leighton (1989), Holtz-Eakin, Joulfaian, and Rosen (1994a, 1994b), Black, de Meza, and Jeffreys (1996), Kiyotaki and Moore (1997), Blanchflower and Oswald (1998), Quadrini (2000), Hurst and Lusardi (2004), Cagetti and De Nardi (2009), Hall and Woodward (2010), Fairlie and Krashinsky (2012), Franzoni, Novak, and Phalippou (2012), Wang, Wang, and Yang (2012), Corradin and Popov (2015), Adelino, Schoar, and Severino (2016), Schmalz, Sraer, and Thesmar (2017), and Levine and Rubinstein (2019). A third segment focuses on entrepreneurial risk and the systematic/procyclical nature of entrepreneurial activity in the economy. Important examples include Kihlstrom and Laffont (1979), Shleifer (1986), Bernanke and Gertler (1989), Gertler and

[3]The Kauffmann Foundation (2019) reports that only 0.50 percent of small firms are funded by venture capital.

Gilchrist (1994), Rampini (2004), Caggese (2012), Koellinger and Thurik (2012), Parker (2012), Manso (2016), Levine and Rubinstein (2019), Brown, Harris, Hu, Jenkinson, Kaplan, and Robinson (2020), and Jovanovic, Ma, and Rousseau (2020). This paper extends the literature by providing new evidence that entrepreneurial risk is strongly systematic in nature and opens a new window on the relation between household financial/housing wealth and entrepreneurship by showing that entrepreneurial credit risk is related to credit risk in the housing and real estate markets.

3. CREDIT CARD SECURITIZATIONS

This section provides a brief introduction to the basic structure and characteristics of credit card asset-backed securities (ABS). The Internet Appendix provides full details about these securities and the credit card ABS market.[4]

Credit cards represent an important source of financing for entrepreneurs. For example, total transaction volume for small business credit cards was $245 billion in 2017.[5] Table 4 of Robb and Robinson (2014) indicates that credit card debt represents 46.3 percent of total outsider debt for the nearly 5,000 startup firms in the Kauffman Firm Survey data set. The Small Business Credit Survey published by the Federal Reserve Banks reports that applying for bank lines of credit/loans and credit cards are the two most common ways in which small entrepreneurial firms seek financing.[6]

Entrepreneurs, of course, make use of other types of financing besides credit card debt. Accordingly, it is important to emphasize that our approach does not rely on the assumption that credit card debt is the primary source of entrepreneurial credit. Rather, all our approach requires is that the pricing of credit card debt in the market be representative of how other forms of entrepreneurial debt would be priced. In other words, we simply use the secondary market prices of credit card securitizations as proxies for how the market would price other types of entrepreneurial credit. Given the general nature of credit card debt and its comparability to other types of unsecured credit, however, this assumption appears very plausible.

[4]This discussion parallels that in Fleckenstein and Longstaff (2020).

[5]Statista (www.statista.com) at https://www.statista.com/statistics/936159/leading-small-business-credit-cards-usa-by-purchase-volume/.

[6]See https://www.newyorkfed.org/medialibrary/media/smallbusiness/2016/SBCS-Report-EmployerFirms-2016.pdf

The underlying collateral for credit card ABS consists of a pool of receivables generated when cardholders make charges on their credit cards to purchase goods and services. From the credit card issuer's perspective, credit card receivables are simply short-term unsecured loans.[7] The issuer pools the receivables and transfers them to a separate entity (master trust), which issues series of notes to investors. The master trust receivables are not segregated by series. Instead, the pool of receivables supports all outstanding series.

To illustrate, imagine that an issuer transfers $100 of card receivables to a master trust. There are at least two ways in which these receivables can be securitized. In the first, the master trust issues a single class of notes with a total notional value of $100, and all of the cash flows from the receivables flow through directly to the noteholders. Since the cash flows received by the notes exactly mirror those from the receivables, however, it is clear that investing in the notes is economically equivalent to owning the receivables—owning the notes is simply an alternative legal form of holding the underlying assets. One example of this type of securitization is a mortgage-backed pass-through security in which an investor receives all of the principal and interest payments from an underlying pool of mortgages exactly as if the investor directly owned the mortgages.

The second and more common way of securitizing credit card receivables is for the master trust to issue a series of tranched notes. For example, the receivables could be securitized by issuing a series of A, B, and C tranches with notional amounts of $70, $20, and $10, respectively. The C tranche absorbs the first $10 of credit losses, the B tranche absorbs the next $20 of credit losses, while the A tranche absorbs the remaining credit losses. Alternatively, the C, B, and A tranches can be described as attaching and detaching at zero and 10 percent, 10 and 30 percent, and 30 and 100 percent, respectively. The attachment point represents the percentage of the receivables pool balance that can default before the tranche experiences first losses. The detachment point represents the level of credit card defaults that leads to a total loss of the tranche. Finally, it is important to recognize that if an investor were to purchase all of the A, B, and C tranches issued in the securitization, the resulting portfolio would be equivalent to owning the single class of notes issued in the first type of securitization described above. Intuitively, this follows since the collective cash flows that would be received by the portfolio would be identical to those that would be received by a single class of notes which, in turn, would be identical to the total cash flows generated by the underlying receivables. Thus, holding the portfolio of all of the tranches issued in a securitization can also be viewed as economically equivalent

[7]Cardholders pay back either the full principal on this unsecured loan or make partial payment. In the latter case, the issuer finances the remaining balance and earns interest (finance charges).

to owning the underlying pool of receivables.

The process by which cash flows are allocated to investors has two distinct periods: revolving and controlled amortization (in some cases, controlled accumulation). If there are no losses, the two-period structure mimics a traditional bond in the sense that interest is distributed every month and principal is paid in a single "bullet" cash flow on the maturity date.

Specifically, upon issuance, a credit card ABS begins the "revolving period," during which investor coupon cash flows are paid from finance charge collections on the credit card accounts, and principal collections are used to purchase new receivables. Any residual cash flows after paying investor interest and charge-offs is referred to as "excess spread" and, depending on the master trust, serves as credit enhancement or is released to the issuer.

The revolving period continues for a predetermined length of time, and then the controlled amortization (accumulation) period begins when principal collections are distributed to investors. For instance, a credit card ABS with a five-year expected maturity might revolve for 48 months and then enter amortization for the final 12 months. In the case of controlled amortization, principal cash flows are distributed in equal installments, for instance 1/12 of the invested amount every month for 12 months. In the case of controlled accumulation, principal cash flows are deposited into a collection account (principal funding account) every month and then paid out as a single cash flow at the end of the accumulation period.

Sustained defaults or charge-offs on the pool of credit card accounts trigger early amortization, independent of whether the credit card ABS is in the revolving period or in controlled amortization (accumulation). Typical early amortization trigger events include collateral performance deterioration (e.g. the three-month average excess spread falls below zero, or the collateral balance falls below the investor invested amount), seller/servicer problems (e.g. seller interest falls below the required minimum level, or the seller fails to transfer new receivables into the trust when necessary), but also legal issues (e.g. breach of representation or warranties by the issuer, or default, bankruptcy, and insolvency of the seller or servicer). Early amortization is a reflection of financial stress within the collateral pool, and the credit card ABS immediately starts to amortize with principal balances being paid to investors according to their seniority.

4. THE ADVANTA BUSINESS CREDIT CARD

The key to our ability to identify private equity returns is the use of secondary market prices for financial claims on the underlying assets of private entrepreneurial firms. Data for this asset class can be difficult to find given its highly specialized nature. Using a combination of hand-collected data and proprietary market sources, however, we were fortunate to be able to construct an extensive data set for the securitizations issued by the Advanta Business Card Master Trust. This unique data set provides us with a "natural experiment" from which we can identify the risk and return characteristics of entrepreneurial investment.

Advanta Corp. (Advanta) was a monoline credit card bank and one of the largest issuers of credit cards to small business entrepreneurs in the U.S. Advanta's business credit cards were targeted towards small firms with fewer than ten employees and less than three million dollars in annual revenues. Accounts were restricted to business owners, and applicants needed proof of business ownership or involvement and were asked to provide their Federal tax I.D. number and business phone and address before finalization of the card acceptance.[8] Among Advanta's core customers were small independent brick-and-mortar and online retailers, small business start-ups, business professionals such as consultants, lawyers, physicians, contractors, television writers and producers, and online content developers. By signing the account agreement, the entrepreneur acknowledged that the credit card was to be used for commercial and business purposes only. Advanta could monitor business credit card usage and cancel the card if patterns of personal use were detected.[9] Furthermore, small entrepreneurs signed as personal guarantors of the business credit card. Specifically, under the cardholder agreement, the entrepreneur and the business were jointly and severally liable for all transactions on the business card account.[10] Since the business credit cards are personally guaranteed, our estimates can be interpreted as a combination of the entrepreneurial firm and the personal assets of the entrepreneur.

Advanta's line of small business credit cards were tailored to the needs of small entrepreneurial firms and business professionals and thus had features not offered by regular consumer credit cards, including higher credit limits, longer

[8]See, https://www.financeglobe.com/credit-cards/card-205/.

[9]Advanta stipulates that "Under the terms of our cardholder agreements, our business cards may be used for business purposes only." See, Advanta Corp., 10-K, 2001.

[10]Many business credit card issuers such as Advanta report only to business credit bureaus, so the entrepreneur's personal credit score is not affected by usage patterns on its business credit card.

billing cycles, additional cards for employees, personalized business checks, business purchase reward incentives, and business purchase protections. In addition, through Advanta's website, cardholders were able to use an array of tools and services to set up and build their businesses. For instance, Advanta offered payroll management, employee expense tracking, and online tools for credit card accounting and bookkeeping, business and health insurance, discounts on business travel, tools to create web sites, tutorials on developing business plans, marketing, tax and legal advice on business and personal finances, and many other small business topics.[11]

Table 1 reports summmary statistics about Advanta cardholders from Advanta's 2006 10-K filing. As shown, Advanta was one of the largest issuers of business credit cards in the U.S. with more than one million accounts at the end of 2006. Credit limits for Advanta accounts ranged from about $5,000 to $25,000. Account balances typically ranged from zero to $10,000, with an average balance of about $6,000 (for non-zero balance accounts). Furthermore, Advanta cardholders had relatively high credit scores. In particular, Table 1 shows that while FICO scores for some Advanta cardholders were below 600, the majority were in excess of 720. As discussed in the Internet Appendix, these FICO scores are comparable to those for cardholders in typical consumer credit card securitizations. Finally, Table 1 also shows that the largest concentrations of Advanta cardholders were in California, Florida, Texas, and New York. The Internet Appendix provides additional details about Advanta's cardholders as well as an in-depth discussion of the history, business model, and regulatory environment of Advanta.

5. THE DATA

This section summarizes some of primary data used in the paper. This data is obtained from a combination of hand-collected sources and proprietary industry market data. The Internet Appendix provides a full description of all variables used in the study along with the sources of the data.

We collect secondary market price data for all A, B, and C tranches from the securitizations of the Advanta Master Trust business credit card receivables portfolio from August 2000 to December 2010. We also hand collect data on attachment/detachment points and the floating coupons paid by each tranche from 424(b) (5) filings with the SEC and servicer reports from 10-D filings with the SEC. Each of the tranches in the sample pays a floating coupon that equals

[11]See, www.advanta.com, accessed via https://archive.org/web/.

one-month Libor plus a fixed spread. The attachment and detachment points for the tranches are the same throughout the entire sample period.[12] Table 2 provides summary statistics for the secondary market prices of the tranches.

As discussed earlier, the portfolio consisting of all of the tranches issued in a securitization is economically equivalent to investing in an alternative single-class pass-through securitization which, in turn, is equivalent from a cash flow perspective to owning the underlying receivables directly. Since we are interested in the entrepreneurial credit risk associated with the Advanta business credit card receivables, we make use of this equivalence by focusing specifically on the risk and return characteristics of the portfolio of all tranches issued in a securitization (rather than on individual tranches). For expositional clarity, we refer to this portfolio simply as a "bond."

We also collect data on characteristics of the Advanta Master Trust's collateral pool of receivables. Specifically, we collect the monthly payment rate, portfolio yield, excess spread, and the charge-off rate for the collateral pool. The monthly payment rate (MPR) measures the speed at which cardholders pay down the amount owed on their credit card balances and it is computed as the ratio of total cash flows into the trust each month divided by the portfolio receivables balance, expressed as a percentage. The portfolio yield is the annualized percentage gross return on the receivables portfolio and it is calculated as the total monthly (gross) cash flows into the master trust divided by the outstanding principal balance at the beginning of the month.[13] Excess spread is the annualized percentage net return on the portfolio and it is calculated as the annualized rate of (gross) portfolio yield less servicing fees, coupon cash flows to noteholders, charge-offs, and any other trust expenses. Excess spread represents a source of credit enhancement for the notes. Intuitively, as long as the excess spread is positive, the securitization generates enough cash inflows to cover cash outflows. The charge-off rate measures the rate of default on the credit card receivables and it is calculated as the (one-month) annualized percentage rate of charge-offs on the portfolio. Credit card receivables are typically charged off after the cardholder has been delinquent in paying the revolving balance for more than 180 days.

[12]Specifically, the B tranche attaches at the 8.9918 percent detachment point of the C tranche, and the A tranche attaches at the 21.5805 percent detachment point of the B tranche.

[13]Gross cash inflows consist of interest on the revolving principal balances (finance charges) plus income from fees on the accounts such as late charges, card annual fees, cash advance fees, overdraft charges, and card interchange. Cash inflows include recoveries on defaulted receivables, but exclude charge-offs from the current month.

To provide additional perspective, it is useful to compare the results from the Advanta business credit card securitizations to those from a broader cross-section of more-conventional consumer credit card securitizations. Accordingly, we also collect the information described above for nine of the largest U.S. consumer credit card issuers over the same time period. Specifically, we extend the sample to include the credit card securitizations for the master trusts set up by American Express, Bank of America, Bank One, Citibank (Citi), J.P. Morgan Chase (Chase), Capital One, First National Bank (First National), and World Financial Network/Alliance Data System (World Financial). Table 3 reports summary statistics for the monthly payment rates, portfolio yields, excess spreads, and charge-off rates for Advanta and the other issuers.

6. ENTREPRENEURIAL CREDIT RISK

As a preliminary step in our analysis, it is useful to first provide some broad perspective about the nature of entrepreneurial credit risk. Accordingly, we begin by calculating credit spreads for the Advanta business credit card securitizations. We then contrast their properties with those of more conventional consumer credit card securitizations.

Recall that our approach is to calculate credit spreads at the bond level (rather than at an individual tranche level). An important advantage of this approach is that it allow us to interpret the credit spread for Advanta in exactly the same way that we would interpret, for example, a corporate bond spread. Furthermore, this approach has the advantage of making it easier to compare credit spreads across issuers (since individual tranches typically vary in terms of their subordination levels across issuers).

In calculating credit spreads, we follow Fleckenstein and Longstaff (2020) by first swapping the Libor-based floating coupons on the individual securities into fixed rate coupons using standard basis and interest rate swaps. We obtain the swap rate data from the Bloomberg system. We then solve for the credit spread for a bond by taking the difference between its yield and the yield on a Treasury bond with the identical fixed coupon rate and maturity. The price of the matched-maturity matched-coupon Treasury bond is calculated from zero-coupon bond prices obtained by bootstrapping the Treasury constant maturity rates provided by the Federal Reserve's Selected Interest Rates H.15 release using a standard spline methodology.

Table 4 provides summary statistics for the credit spreads for Advanta's business credit card securitizations as well as the consumer credit card securitizations of the other issuers. Figure 1 plots the credit spread for the Advanta bonds along

with the average credit spread for the consumer credit card bonds. As shown in Figure 1, the credit spreads for Advanta and the other issuers were generally similar during the pre-financial-crisis period. Beginning in 2008, however, the credit spreads began to diverge significantly with the Advanta credit spread reaching a maximum of more than 1750 basis points, substantially higher than the nearly 900 basis point maximum credit spread for the consumer credit card securitizations during the financial crisis. This is also reflected in the summary statistics reported in Table 4 showing that the average credit spreads over the entire sample period for the Advanta and consumer credit card bonds are 283.95 and 123.52 basis points, respectively. Figure 2 plots the time series of charge-off rates for the Advanta securitizations and those for the consumer credit card securitizations of the other issuers in the sample. These results appear consistent with the evidence in Table 3 and Figure 2 that charge-off rates for Advanta business credit cards are higher on average than the charge-off rates for consumer credit cards during the sample period.

These results, however, raise several intriguing research issues. Are the larger credit spreads for the Advanta bonds simply due to their higher realized credit losses? Or are the larger credit spreads at least partially due to differences in how the market perceives and prices entrepreneurial credit risk relative to household or consumer credit risk?

To address these issues, we use a simple panel regression approach. Specifically, we regress the monthly credit spreads for the individual securitizations on a number of structural variables controlling for the potential credit risk of the underlying collateral pool of receivables, and test whether there is a residual spread that is unique to Advanta. This approach parallels that of Collin-Dufresne, Goldstein, and Martin (2001) who regress corporate credit spreads on structural variables that serve as controls for the underlying credit risk of the bonds. As controls for the credit risk of the collateral pools underlying the securitizations, we use the maturity of the securitizations as well as the four variables summarized in Table 3—the monthly payment rate, the portfolio yield, the excess spread, and the charge-off rate. Recall that the monthly payment rate reflects the speed at which credit card balances are paid off by cardholders. Thus, we would expect lower credit losses to occur when borrowers pay off their balances more rapidly. Both the portfolio yield and the excess spread are measures of the cash flows available to the master trust to service securitized debt. We would expect higher portfolio yields and excess spreads to be associated with lower credit losses. The charge-off rate is a direct measure of realized credit losses on the underlying receivables portfolio. We would expect that higher charge-off rates should be associated with higher credit spreads. To test for the possibility that entrepreneurial credit risk is priced differently by the market, we include an indicator function that takes value one for Advanta securitizations, and zero

otherwise. Finally, we include annual fixed effects in the panel regression as additional controls.

Table 5 reports the results from the panel regression. As shown, credit spreads are generally related to the structural variables in the way we would expect in the specifications that include the control variables individually. In particular, the coefficient for the monthly payment rate is negative and significant, indicating that credit spreads are lower when credit card balances tend to be paid off more rapidly. Similarly, the coefficient for the excess spread is negative and significant, consistent with the interpretation that investors in the securitizations face less credit risk when the cash flows available to service coupon payments and principal payments increase. The coefficient for the monthly charge-off rate is positive and significant, suggesting that credit spreads increase in response to higher realized credit losses in the underlying credit card receivables.

Table 5 also shows that there is a significant credit premium for the Advanta business credit card securitizations even after controlling for the structural credit risks of the underlying collateral pool. In particular, the indicator for Advanta is positive and highly significant in each of the regression specifications shown in Table 5. Focusing on the final specification with all of the control variables included, the point estimate of the Advanta credit premium is 165.70 basis points and has a t-statistic of 4.50.

7. HOW SYSTEMATIC IS ENTREPRENEURIAL RISK?

The results from the panel regression in Table 5 strongly suggest that the market views entrepreneurial credit risk as fundamentally different in nature from that of conventional household or consumer credit risk and prices it accordingly. One possibility implied by the literature is that entrepreneurship may be procyclical in nature. For example, Levin and Rubinstein (2019) present a model in which entrepreneurs are selected on collateral, and entrepreneurship is procyclical (also see Koellinger and Thurik (2012)). If entrepreneurship is more sensitive to macroeconomic fluctuations, then the large credit premium incorporated into the prices of Advanta's securitizations may be a reflection of their systematic market risk. We examine these possibilities in this section.

As a first step, we explore the relation between ex post entrepreneurial credit risk and a number of broader types of consumer and business credit risk. In particular, we regress changes in the charge-off rates experienced by the Advanta Business Card Master Trust portfolio on changes in the charge-off rates reported by commercial banks for their consumer loan, single-family mortgage, commercial real estate, and business loan portfolios. For comparison, we also estimate a

similar regression of changes in the charge-off rates for consumer credit card debt on the same variables. The quarterly commercial bank charge-off rate data are obtained from the Federal Reserve.

Table 6 reports the results from the regressions. Focusing first on the results for the changes in the charge-off rate for consumer credit cards, the table shows that there is little relation between changes in these charge-off rates and those for other categories of loans. The one exception is that the coefficient for the change in the charge-off rate for business loans is positive and significant (at the ten-percent level).

In contrast, the regression of changes in the Advanta charge-off rate shows that there is a significant relation between these charge-off rates and those for other types of loans. In particular, the coefficients for changes in both the single-family and commercial real estate charge-off rates are positive and significant. Furthermore, the coefficient for the change in the charge-off rate for business loans is positive and significant (at the ten-percent level).

Taken together, these results suggest that entrepreneurial risk differs in important ways from conventional household or consumer risk. As shown earlier in Figure 2, entrepreneurial and consumer risk share some common drivers given the similarities in their time series patterns. The results indicate, however, that entrepreneurial risk is also driven by additional factors affecting the risk of other types of asset classes. In particular, the significant relations between changes in Advanta charge-off rates and those for single-family mortgages and commercial real estate suggest that entrepreneurial risk may have an important collateral-value dimension. This is consistent with the extensive literature about the impact of liquidity constraints and household housing wealth on entrepreneurial activity. For example, Evans and Jovanovic (1989) and Levine and Rubinstein (2019) present models in which individuals' wealth or collateral directly impacts their ability to pursue entrepreneurship. Black, de Meza, and Jeffreys (1996), Corradin and Popov (2015), and Schmalz, Sraer, and Thesmar (2017) provide empirical evidence of the link between housing values and entrepreneurship. Our results open a new window on this literature by showing that entrepreneurial credit risk has important commonalities with the credit risk experienced in other collateral-value-dependent lending markets. Furthermore, our results suggest that time variation in macroeconomic conditions and asset valuations may be important drivers of systematic entrepreneurial risk.

To explore this issue in more depth, we regress changes in Advanta credit spreads on a number of macroeconomic and asset pricing factors. As macroeconomic variables, we include the percentage change in industrial production, the percentage change in the Consumer Price Index, initial jobless claims, and changes in a number of measures of consumer and business sentiment and con-

fidence. As asset pricing measures, we include stock market returns (CRSP value-weight index), Treasury bond returns (Bloomberg Treasury total return index), and housing returns (percentage changes in the Case-Shiller index). We also include changes in the VIX index of stock market volatility as well as changes in the MOVE index of Treasury rate volatility. To provide additional perspective, we also estimate the regression using changes in the credit spread for the consumer credit card securitization. Table 7 reports the regression results.

As shown, both changes in consumer credit card and Advanta credit spreads are significantly related to changes in macroeconomic conditions. What is interesting, however, is that changes in credit spreads for the consumer credit card securitizations appear to be more closely tied to changes in macroeconomic variables that directly impact households such as inflation and employment. In contrast, changes in credit spreads for Advanta securitizations also appear to be related to a much broader set of macroeconomic factors such as industrial production and measures of general consumer, business, and economic confidence.

Table 7 also shows that changes in the Advanta credit spread are significantly positively related to changes in housing values. In addition, changes in the Advanta credit spread are significantly related to changes in the VIX and MOVE measures of volatility in the stock and Treasury bond markets. These results provide direct support for the hypothesis that entrepreneurial risk is linked to collateral values and the risks inherent in individuals' wealth. In contrast, none of the financial market variables is significant in the regression for changes in consumer credit card securitization credit spreads.

As a third way of exploring the properties of entrepreneurial risk, we next regress changes in Advanta credit spreads on changes in a number of other credit spreads observable in the secondary capital markets. In particular, we regress monthly changes in Advanta credit spreads on changes in the average credit spreads for the consumer credit card securitizations of the other issuers in the sample and on changes in AAA, BAA, and high-yield corporate bond spreads. Table 8 reports the results from the regression.

The regression results again indicate that the market views entrepreneurial risk as very different in character from household or consumer risk. As shown, changes in the Advanta credit spread are not significantly related to changes in the credit spreads for the consumer credit card securitizations. Instead, changes in the Advanta credit spreads are significantly related to corporate credit spreads. In particular, the coefficients for changes in the BBB and high yield corporate credit spread are both positive and significant. These results suggest that entrepreneurial credit risk is viewed by the market as having much more to do with the macroeconomic factors driving risk in the broader corporate business sector than it has with general consumer risk. Again, these results provide support

for models such as Levin and Rubinstein (2019) in which entrepreneurial risk behaves differently from other types of household or consumer risk because of its dependence on asset/collateral values.

8. IMPLEMENTING THE MERTON MODEL

In this section, we begin with a brief review of the Merton (1974) structural credit modeling framework we will use in estimating private equity returns. We next discuss how we apply this framework to Advanta and describe the calibration of the model.

8.1 The Merton Model

Using the Merton (1974) structural credit framework to model the relation between equity returns and credit spreads has become standard in the literature. Key examples include Campbell and Taksler (2003), Vassalou and Xing (2004), Campbell, Hilscher, and Szilagyi (2008), Bharath and Shumway (2008), Chen, Collin-Dufresne, and Goldstein (2009), Schaefer and Strebulaev (2008), Coval, Jurek, and Stafford (2009), Chava and Purnanandam (2010), Galappi and Yan (2011), He and Xiong (2012), and Friewald, Wagner, and Zechner (2014). Our approach follows this extensive literature in using the familiar Merton (1974) framework in studying the returns from entrepreneurial investment.

In its simplest form, the Merton (1974) framework is typically used to value a debt claim for an issuer that may default. The capital structure of the issuer is assumed to consist of equity with a value denoted as S, and a zero-coupon bond with a notional amount F, maturity T, and a value denoted by B. The underlying assets of the issuer are denoted V and are assumed to follow the dynamics,

$$dV = \mu\ V\ dt + \sigma\ V\ dZ, \tag{1}$$

$$dV = r\ V\ dt + \sigma\ V\ dZ, \tag{2}$$

under the objective and risk-neutral measures, respectively. In these dynamics, μ and σ denote constants, r is the riskless rate, and Z is a standard Brownian motion.

Merton (1974) shows that the issuer's equity can be represented as a call option on the underlying assets of the issuer, allowing the Black and Scholes (1973) model to be used to solve for S,

$$S \;=\; V\; N(d) \;-\; F\; e^{-rT}\; N(d \;-\; \sqrt{\sigma^2 T}) \tag{3}$$

$$d \;=\; \frac{\ln(V/F) + (r \;+\; \sigma^2/2)T}{\sqrt{\sigma^2 T}}. \tag{4}$$

where $N(\cdot)$ is the cumulative standard normal distribution function. By definition, $B = V - S$.

Given this closed-form expression for S, it is straightforward to solve for the moments of returns for both the debt and equity components of the issuer's capital structure under the objective measure. The Internet Appendix shows that the expected return μ_B and the standard deviation of returns σ_B for the bond can be expressed as

$$\mu_B = \frac{\mu - r(1-X) - (\mu - r)N(d)}{X}, \tag{5}$$

$$\sigma_B \;=\; \frac{\sigma(1 - N(d))}{X}, \tag{6}$$

where $X = B/V$ is the leverage ratio. Similarly, the expected return μ_S and standard deviation of returns σ_S for the equity can be expressed as

$$\mu_S \;=\; r + \frac{(\mu - r)N(d)}{1 - X}, \tag{7}$$

$$\sigma_S \;=\; \frac{\sigma N(d)}{1 - X}. \tag{8}$$

8.2 Applying the Merton Model to Advanta

Our approach will be to apply the Merton (1974) framework to the Advanta securitizations. In doing this, however, it is important to acknowledge that care must be taken in the mapping from a stylized theoretical model such as Merton (1974) into the structure of actual debt claims traded in the financial markets.

First, the Merton (1974) model assumes that the debt in the issuer's capital structure consists of a single zero-coupon bond. In reality, actual capital structures for debt issuers are far more complex. Following standard practice in the

financial literature, we will take some liberties by interpreting the debt claim in the Merton (1974) more broadly. Specifically, the portfolio of business credit card receivables underlying an Advanta securitization—consisting of a portfolio of debt claims on the assets of the entrepreneurs—can be viewed as playing the same economic role as the zero-coupon bond in the Merton (1974) model. Note that this also allows us to interpret the collective assets of the entrepreneurs carrying Advanta credit card balances as the underlying assets V in the Merton (1974) model.

Second, actual Advanta securitizations make monthly coupon payments to investors throughout the life of the security. In contrast, the zero-coupon bond in the Merton (1974) framework does not. We observe, however, that the coupon rates for securitizations are set in a way that allows them to be issued at or close to par. Because of this feature, a very natural way of adapting the Merton model to the Advanta securitizations is simply to assume that the debt claims are issued at par or, equivalently, that the notional amount F of the debt equals the initial value of the debt claim B. This interpretation is supported by the results in Table 2 showing that the average prices of the Advanta tranches over the sample period are all close to par.

8.3. Model Calibration

The final step is the calibration of the model. Table 9 reports the average value of the riskless rate during the sample period which we use as the calibrated value for r. Similarly, we use a calibrated value for T of three years which corresponds closely to the average maturity of the Advanta securitizations during the sample period.

Given these values, the remaining task is to identify values for the entrepreneurial leverage ratio X and the parameters μ and σ. In doing this, our approach will be to use a range of values for X that are consistent with the historical record, and then solve for the implied values of μ and σ from the realized returns of Advanta bonds over the sample period.

Although there is probably no direct way of measuring the leverage of Advanta cardholders, we can at least provide some realistic bounds using aggregate statistics on household and business leverage. In particular, we collect data on total household, nonfinancial noncorporate business, and nonfinancial corporate business assets and liabilities from the Federal Reserve's Financial Accounts of the United States Z.1 Release during the 2000–2010 sample period. Table 9 reports the average ratio of total liabilities to total assets for each of these categories. As shown, the average leverage ratios range from about 35 to 53 percent. This range is consistent with those discussed in Heaton and Lucas (2000), Robb and Robinson (2014), and Kartashova (2014). In light of this, we will simply

use values of X ranging from 25 to 60 percent in the calibration rather than selecting a specific point estimate. As it turns out, estimates of the private equity premium are very robust to the values of X used in the calibration.

Given a value of X (as well as values of r and T), the expected bond return and standard deviation of the bond return shown in Equations (5) and (6) above are explicit functions of the two parameters μ and σ. Thus, given estimates of these bond return moments, Equations (5) and (6) become a system of two equations in two unknowns which can be solved directly for the parameters μ and σ. We estimate the expected excess return and standard deviation of returns for the Advanta bonds over the sample period using the observed monthly returns. The Internet Appendix provides the details about how these monthly returns are calculated. Table 9 reports the estimated values for these two return moments.

9. PRIVATE EQUITY RISK AND RETURN

With the Merton (1974) model calibrated to the Advanta securitizations, we can now solve for the key moments of private equity returns. Table 10 reports the estimated values of the expected return, standard deviation, private equity premium, Sharpe ratio, and market beta of private equity returns across the range of leverage ratios.

9.1 The Expected Return

As shown, the expected return on private equity is on the order of about 14 percent for all of the leverage ratios in the calibrated range. The estimated expected return is slightly lower for leverage ratios that are closer to 50 percent, but all the values are very similar. The average value of the expected return across the range is 13.82 percent which we can use as a point estimate. These results indicate that the estimated expected return is very robust to the assumptions about the leverage ratio used in the calibration.[14]

To calculate the private equity premium implied by the model, we simply subtract the average riskless rate over the sample period from the estimated expected return on private equity. Table 10 shows that the private equity premium range from about 10.40 percent to 11.70 percent. The point estimate of the

[14]It is important to observe, however, that this robustness result does not imply that the equity returns are unaffected by leverage (which would be contrary to what we would expect). Rather, this result simply means that since we apply the same leverage assumption to both the bond and equity, the effect of the assumption largely "washes out" in the estimation methodology.

private equity premium across the range of leverage ratios is 10.72 percent.

This estimate of the private equity premium far exceeds the realized excess return of publicly-traded stocks during the sample period. For example, the average annualized value of the Fama and French (2015) market factor (stock market return minus the riskless rate) is −0.10 percent over the sample period.

We can also compare these estimates of expected returns or the private equity premium to those reported in the previous literature. One challenge in doing this, however, is that there is no clear consensus in the literature on what the average returns on private equity are. As discussed by Kaplan and Sensoy (2015), Korteweg (2019), and others, estimates of the average premium on private equity vary widely across studies based on the data source, time period, and empirical methodology used. Estimates of the expected return on private implied by various studies range from roughly 5 percent (Fang, Ivashina, and Lerner (2015) and Gupta and van Nieuwerburgh (2019)) to about 60 percent (Peng (2001), Cochrane (2005), and Korteweg and Nagel (2016)). Our point estimate of the expected return on private equity places our results towards the lower end of the estimates reported in the literature.[15]

9.2 Private Equity Return Volatility

The calibrated Merton (1974) model also allows us to estimate the standard deviation of private equity returns. These estimates are obtained directly from the closed form solution for the standard deviation of equity returns in Equation (8).

Table 10 shows that the estimated standard deviations are on the order of 90 percent across the range of leverage ratios. The overall average is 94.55 percent which we can again take as the point estimate. While these standard deviations may appear large relative to the volatility of broader stock market indexes, they are actually fairly comparable to the average volatilities of individual stocks. For example, Figure 1 of Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) shows that the average annualized volatility of stock returns for CRSP/Compustat firms in the smallest size quintile ranged from about 60 to 120 percent during the 2000–2010 sample period. Also recall that the sample period includes the 2008 financial crisis which was associated with historically high levels of return volatility. Our results confirm that entrepreneurial investment is a highly risky venture, particularly given the lack of diversification in their personal portfolios that entrepreneurs typically face (see the discussion in Moskowitz and Vissing-Jorgensen (2002) and Vereshchagina and Hopenhayn (2009)).

[15] See the Internet Appendix for a discussion of the estimated average returns on private equity reported in the literature.

There is again no clear consensus in the previous literature about the volatility of private equity returns. Estimates of the annualized volatility of fund-level private equity returns range from about 5 percent (Gompers and Lerner (1997)) to about 72 percent (McKenzie and Janeway (2011)). Estimates of the volatility of deal-level private equity returns tend to be substantially higher with values in the range of 107 percent to 116 percent (Cochrane (2005) and Chen, Baierl, and Kaplan (2002)).

9.3 Sharpe Ratios

We can also calculate the Sharpe ratio for private equity returns implied by our estimates of the private equity premium and private equity return volatility. As shown in Table 10, the implied Sharpe ratios for private equity returns are 0.113 across the range of leverage ratios.

9.4 Private Equity Betas

We can also use the calibrated model to estimate the systematic risk of private equity. To do this, we regress Advanta bond returns on the five Fama and French (2015) factors and take the coefficient of 0.127 for the market factor. We then multiply this bond beta by the ratio of equity volatility to bond volatility (the ratio of the expressions in Equations (8) and (6)). Table 10 shows that the estimated beta coefficients vary from about 1.17 to 1.32 across the range of leverage ratios. The average value is 1.21 which we will again interpret as a point estimate.

We can use this estimate to provide a simple back-of-the-envelope estimate of the implied expected excess return for private equity. Assuming a market equity premium of 7 percent, the single-factor CAPM implies that the private equity premium should be about $1.21 \times 7 = 8.47$ percent. This back-of-the-envelope calculation is comparable to the private equity premium estimates shown in Table 10.

In a comprehensive survey, Korteweg (2019) provides an extensive list of private equity beta estimates reported in literature. These estimates range from about 0.70 (Ewens, Jones, and Rhodes-Kropf (2013), Jegadeesh, Kräussl, and Pollet (2015)) to 3.20 (Korteweg and Sorensen (2010) and Buchner and Stucke (2014)). The specific values vary, however, depending on whether the estimates are for venture capital funds or buyout funds. These results suggest that our estimates of private equity betas are consistent with the general range of previously reported measures.

10. CONCLUSION

We introduce a new approach for estimating private equity returns. This approach uses secondary market prices for debt claims on entrepreneurial assets—in conjunction with the Merton (1974) structural credit modeling framework—to infer the moments of private equity returns. An important advantage of this market-price-based approach is that it avoids the well-known problems of having to rely on limited partnership cash flow information, NAV estimates provided by venture capital funds, or other types of self-reported data in estimating private equity returns.

We apply our approach to an extensive data set of prices for Advanta's business credit card securitizations. Advanta was a major issuer of credit cards to small entrepreneurial firms with fewer than ten employees and less than three million dollars in annual revenues. The underlying collateral for the securitizations consisted of credit card receivables from a set of over one million cardholders. Thus, our approach allows us to shed light on private equity returns for a very broad set of small entrepreneurial firms, rather than for just the limited number of firms that actually receive venture capital funding and for which cash flow and NAV data can be sourced from private equity funds.

We find that the market prices entrepreneurial credit risk very differently from conventional household credit risk. After controlling for default risk and structural credit features, we find that Advanta credit card securitizations incorporate an additional credit premium of more than 150 basis points relative to consumer credit card securitizations. We provide evidence that this additional premium may be a reflection of the much-more systematic nature of entrepreneurial risk.

We obtain estimates of the expected return on private equity that are on the order of 13 to 15 percent during the 2000–2010 sample period. These values are towards the lower end of the range of estimates reported in the private equity literature. We also find that the volatility of private equity returns is comparable to that of the smallest decile of publicly traded firms. Finally, we estimate the market beta of private equity to be around 1.21. These results have many important implications for private equity and entrepreneurship.

It is important to acknowledge that our results are limited to the set of small entrepreneurial firms that were Advanta cardholders. Clearly, additional research would be needed to determine whether our results are applicable to broader classes of private equity. One advantage of our approach, however, is that it is scalable and could be applied to other types of entrepreneurial and consumer/household securitization markets such as Small Business Administration

loans, business loan syndications, student loans, auto loans, and home equity lines of credit.

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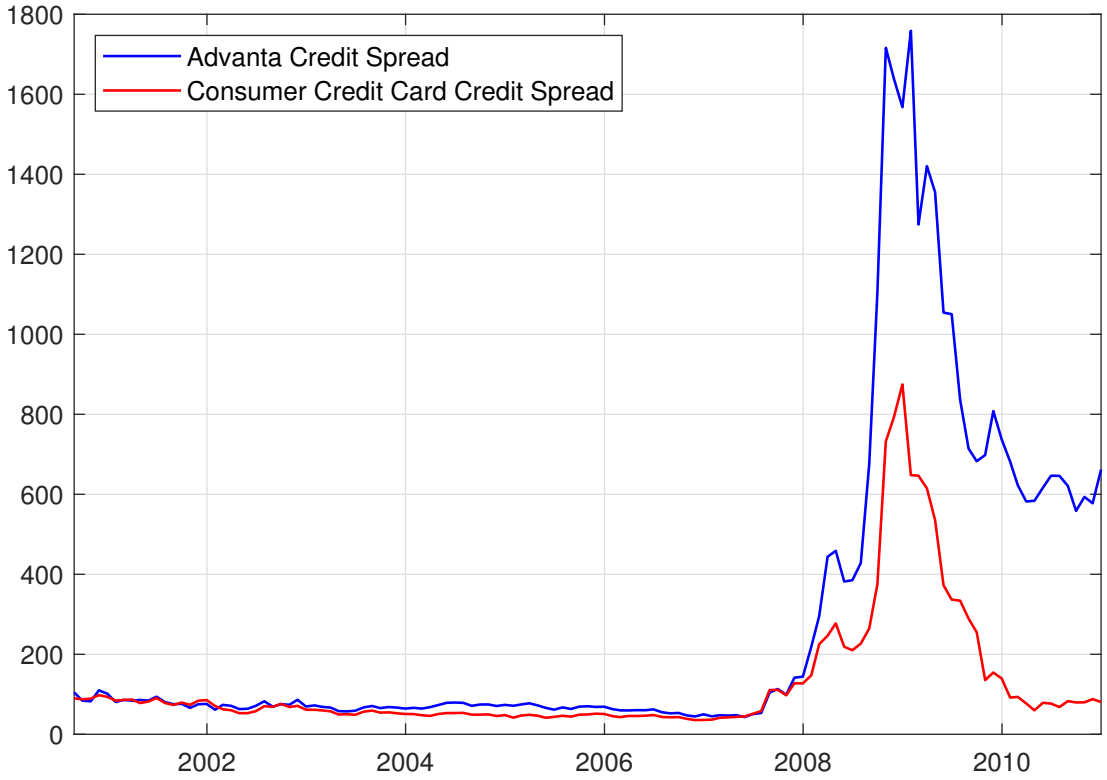

Figure 1. Credit Spreads for Credit Card Securitizations. This figure shows the credit spread for Advanta business credit card securitizations as well as the average credit spread for the consumer credit card securitizations of the other issuers in the sample. Credit spreads are expressed in basis points.

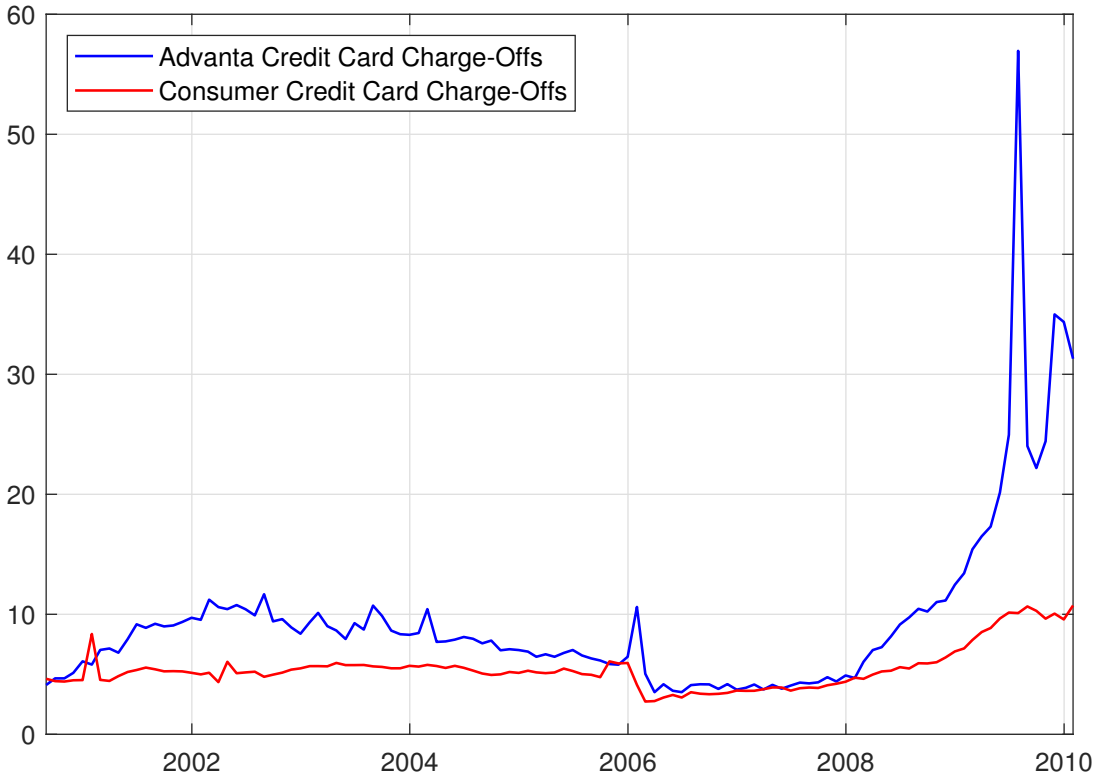

Figure 2. Charge-off Rates for Credit Card Securitizations. This figure shows the annualized percentage charge-off rate for Advanta business credit card securitizations as well as the average charge-off rate for the consumer credit card securitizations of the other issuers in the sample.

Table 1

Summary Statistics for Advanta Business Credit Cardholders. This table presents summary statistics about Advanta cardholders from Advantas 2006 10-K filing. Panel A presents summary statistics for card limits on the business credit cards. Panel B presents account balances on the credit card accounts. Panel C presents FICO scores of credit card account holders. Panel D presents the geographical location of account holders within the U.S. Number of accounts denotes the total number of business credit card accounts in the indicated ranges or categories. Accounts receivable denotes the total dollar amount of credit card receivables of the accounts in the indicated ranges or categories. Credit limits, account balances, and accounts receivable are expressed in dollars.

Panel A — Credit Limits

Credit Limit	Number of Accounts	Percentage of Total	Accounts Receivable	Percentage of Total
0.00	225,746	21.42	−767,267	−0.02
0.01 to 1,500	11,988	1.14	5,517,888	0.15
1,500 to 5,000	87,416	8.29	151,891,685	4.03
5,000 to 10,000	184,453	17.50	540,906,479	14.34
10,000 to 15,000	214,035	20.30	761,581,567	20.20
15,000 to 25,000	243,029	23.05	1,410,163,394	37.40
25,000 to 35,000	73,875	7.01	672,446,726	17.83
Over 35,000	13,555	1.29	228,735,760	6.07
Total	1,054,097	100.00	3,770,476,232	100.00

Panel B — Account Balances

Account Balance	Number of Accounts	Percentage of Total	Accounts Receivable	Percentage of Total
Less than 0.00	15,558	1.48	−5,184,772	−0.14
0.00	420,330	39.87	0	0.00
0.01 to 5,000	352,054	33.40	578,537,683	15.35
5,000 to 10,000	131,568	12.48	973,572,334	25.82
10,000 to 20,000	105,240	9.98	1,467,134,186	38.91
20,000 to 25,000	16,941	1.61	377,152,481	10.00
Over 25,000	12,406	1.18	379,264,320	10.06
Total	1,054,097	100.00	3,770,476,232	100.00

Table 1 Continued

Panel C — Most Recent FICO Score

FICO Score	Number of Accounts	Percentage of Total	Accounts Receivable	Percentage of Total
No FICO Score	197	0.02	18,123	0.00
Less than 600	42,425	4.02	216,116,946	5.73
600 to 660	114,589	10.87	466,005,333	12.36
661 to 719	328,621	31.18	1,404,664,820	37.26
720 and Higher	568,265	53.91	1,683,671,010	44.65
Total	1,054,097	100.00	3,770,476,232	100.00

Panel D — Geographical Location

Location	Number of Accounts	Percentage of Total	Accounts Receivable	Percentage of Total
California	145,700	13.82	498,881,414	13.23
Florida	79,158	7.51	272,669,419	7.23
Texas	76,208	7.23	269,426,356	7.15
New York	70,514	6.69	239,530,062	6.35
Illinois	42,698	4.05	155,931,372	4.14
Pennsylvania	42,169	4.00	144,917,210	3.84
Michigan	35,218	3.34	139,608,903	3.70
Ohio	33,868	3.21	139,524,963	3.70
New Jersey	37,767	3.58	122,940,182	3.26
Georgia	26,814	2.55	105,785,695	2.81
North Carolina	24,462	2.32	94,685,759	2.51
Colorado	25,229	2.39	94,468,323	2.51
Massachusetts	27,476	2.61	93,906,422	2.49
Washington	21,499	2.04	84,030,705	2.23
Virginia	22,696	2.15	83,042,113	2.20
All Others	342,621	32.51	1,231,127,334	32.65
Total	1,054,097	100.00	3,770,476,232	100.00

Table 2

Summary Statistics for Advanta Credit Card Tranches. This table presents summary statistics for the A, B, and C tranches of the Advanta credit card securitizations. Detach denotes the detachment point of the indicated tranches. Average maturity denotes the average number of months until maturity of the tranche. Average spread denotes the average spread above Libor paid by the tranche and is measured in basis points. Average price, Min price, Median price and Max price denote the average, minimum, median and maximum price of the tranche and is expressed in dollars per $100 notional value. N denotes the number of observations. The sample period is monthly from August 2000 to December 2010.

Tranche	Detach	Average Maturity	Average Spread	Average Price	Min Price	Median Price	Max Price	N
A Tranche	1.0000	27.05	15.51	96.86	43.52	99.98	100.93	893
B Tranche	0.2158	28.54	76.36	93.49	17.26	100.36	103.53	513
C Tranche	0.0899	26.60	174.10	91.80	5.01	101.06	108.99	503

Table 3

Summary Statistics for Credit Card Receivables Portfolio Characteristics. This table presents summary statistics for the indicated characteristics of the portfolios of credit card receivables underlying the securitizations. The statistics for the individual card issuers are computed by taking averages across all securitizations for each month, and then averaging the monthly averages. The monthly payment rate is the ratio of total cash flows collected each month divided by the portfolio balance, expressed as a percentage. Portfolio yield is the annualized percentage gross return on the portfolio. Excess spread is the annualized percentage net return on the portfolio. The Charge-off Rate is the one-month annualized percentage rate of charge-offs on the portfolio. N denotes the number of months. Average denotes summary statistics taken over the averages for the individual consumer credit card issuers. The sample period is monthly from August 2000 to December 2010.

Card Issuer	Monthly Payment Rate	Portfolio Yield	Excess Spread	Charge-off Rate	N
Advanta	19.321	22.242	7.101	9.423	114
American Express	22.269	21.370	10.261	5.477	125
Bank of America	15.379	18.946	7.164	6.866	125
Bank One	17.848	17.118	6.879	5.777	125
Citibank	18.682	16.405	6.638	6.069	123
Chase	17.497	16.339	6.061	5.636	125
Capital One	17.234	19.926	9.235	4.879	125
First National	13.601	17.704	5.768	7.382	94
MBNA	15.561	19.223	7.643	6.432	125
World Financial	17.684	28.887	12.066	7.236	125
Average	17.409	19.604	8.033	6.161	1092

Table 4

Summary Statistics for the Credit Spreads of Credit Card Securitizations. This table presents summary statistics for the credit spreads of the indicated categories of credit card asset-backed securities. The monthly credit spreads for each issuer are averages taken over all securitizations for that issuer for that that month. All credit spreads are expressed in basis points. Mean and Std. Dev. denote the means and standard deviations of credit spreads for the indicated issuer. The columns denoted 5%, 25%, 50%, 75%, and 95% represent the respective percentiles of the distribution of credit spreads. N denotes the number of monthly observations. Average denotes summary statistics taken over the averages for the individual consumer credit card issuers. The sample period is monthly from August 2000 to December 2010.

Card Issuer	Mean	Std. Dev.	5%	25%	50%	75%	95%	N
Advanta	283.95	400.66	47.83	66.02	75.27	432.03	1294.72	125
American Express	142.42	183.73	40.68	48.77	56.75	138.22	641.14	82
Bank of America	144.92	194.91	31.19	45.50	66.35	122.88	719.00	105
Bank One	122.20	161.54	34.14	43.18	51.38	107.85	508.74	88
Citibank	118.22	144.29	41.16	46.45	64.39	100.90	504.73	123
Chase	111.36	139.83	39.41	46.30	53.41	96.95	458.30	125
Capital One	150.72	202.66	46.35	55.72	77.79	108.38	733.00	125
First National	100.65	110.62	29.12	37.51	44.54	154.62	267.67	56
MBNA	122.14	153.98	38.61	49.36	61.27	97.50	536.09	125
World Financial	61.97	19.29	38.26	47.31	60.42	71.14	100.73	61
Average	123.52	157.74	41.54	49.11	68.09	93.52	555.27	890

Table 5

Results from Panel Regression of Credit Spreads on the Structural Credit Control Variables and the Indicator Variable for Advanta. This table reports the results from the panel regression of credit spreads for the Advanta and other credit card securizations on the structural credit control variables and an indicator variable that takes value one for Advanta securitizations, and zero otherwise. Credit spreads are expressed in basis points. Maturity is expressed in months. The monthly payment rate, the portfolio yield, and the charge-off rate are expressed as percentages. The time fixed effects represent indicator variables for the individual years during the sample period. Robust standard errors are clustered at the issuer level. The superscript * denotes significance at the ten-percent level; the superscript ** denotes significance at the five-percent level. The sample period is from August 2000 to December 2010.

	Coeff	t-Stat	Coeff	t-Stat	Coeff	t-Stat	Coeff	t-Stat	Coeff	t-Stat
Maturity	0.370	4.44**	0.341	3.38**	0.337	3.14**	0.350	3.43**	0.387	4.06**
Monthly Payment Rate	−9.866	−3.13**	–	–	–	–	–	–	−6.625	−1.49
Portfolio Yield	–	–	−2.512	−0.99	–	–	–	–	8.109	1.96*
Excess Spread	–	–	–	–	−9.300	−4.51**	–	–	−14.066	−3.18**
Charge-off Rate	–	–	–	–	–	–	11.036	2.39**	−5.291	−0.78
Indicator for Advanta	201.224	20.23**	201.253	18.30**	170.065	15.47**	136.661	6.27**	165.703	4.50**
Time Fixed Effects		Yes		Yes		Yes		Yes		Yes
Adj. R^2		0.583		0.571		0.583		0.580		0.592
N		16,614		16,614		16,614		16,614		16,614

Table 6

Results from Regressions of Changes in Advanta and Consumer Credit Card Charge-off Rates on Changes in Charge-off Rates for Other Types of Loans. This table reports the results from the regressions of changes in the charge-off rates for Advanta credit cards and consumer credit cards on changes in the charge-off rates for the indicated types of loans. The superscript * denotes significance at the ten-percent level; the superscript ** denotes significance at the five-percent level. The sample period is quarterly from 2000 Q3 to 2009 Q4.

	Consumer Credit Cards		Advanta Credit Card	
	Coeff	t-Stat	Coeff	t-Stat
Intercept	0.0181	0.12	0.2452	0.59
Lagged Dependent Variable	−0.2556	−1.37	−0.6496	−2.72**
Δ Charge-off Rate Consumer Loans	0.1478	0.29	−1.5403	−1.09
Δ Charge-off Rate Single-Family Mortgages	0.6232	0.78	4.7465	2.14**
Δ Charge-off Rate Commercial Real Estate	0.5949	0.81	5.8807	2.90**
Δ Charge-off Rate Business Loans	0.9727	1.77*	2.9962	1.92*
Adj. R^2		0.036		0.306
N		37		37

Table 7

Results from Regressions of Changes in the Credit Spreads for Consumer and Advanta Credit Card Securitizations on Macroeconomic and Financial Market Variables. This table reports the results from the regressions of changes in credit spreads on the indicated macroeconomic and financial market variables. Credit spreads are expressed in basis points. Industrial Production is the percentage change in the Industrial Production Index. Inflation denotes the percentage change in the Consumer Price Index. Jobless Claims denotes the monthly average of the Conference Board's weekly initial jobless claims index and is measured in thousands. Consumer Sentiment denotes changes in the Michigan Consumer Sentiment Index. Consumer, Business, and Economic Confidence denote changes in the Conference Board's Consumer, Business, and Economic Confidence Indexes, respectively. VIX and MOVE denote changes in the VIX Stock Market Volatility and Treasury Volatility Indexes, respectively. Stock Market Return denotes the percentage returns on the CRSP Value Weighted Index. Treasury Market Return denotes the percentage returns on the Bloomberg Treasury Total Return Index. Housing Market Return denotes the percentage returns on the Case-Shiller Housing Price Index. The superscript * denotes significance at the ten-percent level; the superscript ** denotes significance at the five-percent level. The sample period is quarterly from 2000 Q3 to 2009 Q4.

	Consumer Credit Cards		Advanta Credit Card	
	Coeff	t-Stat	Coeff	t-Stat
Intercept	79.2109	4.62**	158.4212	2.56**
Industrial Production	2.1839	0.33	−32.9000	−2.73**
Inflation	−61.3719	−2.37**	−68.2025	−1.81*
Jobless Claims	−0.1689	−4.32**	−0.3303	−2.18**
Consumer Sentiment	−0.5656	−0.36	6.0269	1.95*
Consumer Confidence	−2.3202	−1.59	−3.6869	−1.91*
Business Confidence	−2.2600	−1.14	−7.8153	−1.73*
Economic Confidence	−0.4141	−0.26	−5.2157	−1.77*
VIX	2.2108	1.02	13.2952	2.32**
MOVE	0.2395	0.70	−0.8731	−1.77*
Stock Market Return	−0.0497	−0.01	−11.3327	−1.51
Treasury Market Return	0.3192	0.25	4.0686	0.89
Housing Market Return	−7.5353	−1.66	−20.9703	−2.63**
Adj. R^2		0.324		0.367
N		124		124

Table 8

Results from Regression of Changes in Advanta Credit Spreads on Changes in Consumer Credit Card Credit Spreads and Corporate Bond Credit Spreads. This table reports the results from the regression of changes in the credit spreads of Advanta bonds on changes in the credit spreads of consumer credit card bonds and AAA, BBB, and high yield corporate bonds. Credit spreads are expressed in basis points. Robust standard errors are based on Newey and West (1987). The superscript * denotes significance at the ten-percent level; the superscript ** denotes significance at the five-percent level. The sample period is monthly from August 2000 to December 2010.

	Coeff	Stat
Intercept	4.5017	0.85
Δ Consumer Credit Card Spread	0.3776	0.67
Δ AAA Corporate Credit Spread	0.4575	0.65
Δ BBB Corporate Credit Spread	2.0424	2.21**
Δ High Yield Corporate Credit Spread	−0.4893	−2.24**
Adj. R^2		0.378
N		124

Table 9

Calibration of the Credit Model. This table reports the calibrated values for the credit model used to infer the moments of the returns on the underlying entrepreneurial assets from those for the Advanta credit card securitizations. The indicated values for the riskless rate, maturity, and excess returns are based on monthly values over the sample period. Non-corporate leverage denotes the average annual ratio of total liabilities to total assets for non-financial non-corporate businesses. Corporate leverage denotes the average annual ratio of total liabilities to total assets for non-financial corporate businesses. Household leverage denotes the average annual ratio of total liabilities to total non-financial assets for households and non-profit organizations. Total asset and liability values are taken from the Federal Reserve Z.1 Release for the Financial Accounts of the United States. Maturity is expressed in years. All other values are expressed as percentages. The sample period is August 2000 to December 2010.

Credit Model Inputs	Calibrated Value
Credit Model Parameters	
Average Riskless Rate	3.103
Average Maturity	3.000
Advanta Bond Returns	
Average Annual Excess Return	1.121
Annualized Standard Deviation	9.891
Market Beta	0.127
Leverage Ratio Proxies	
Non-Corporate Leverage	35.315
Corporate Leverage	52.593
Household Leverage	42.680

Table 10

Summary Statistics for Implied Private Equity Returns. This table reports summary statistics for the implied returns for equity investments in the assets underlying Advanta credit card securizations. The leverage ratio is the ratio of debt to total assets. Expected return and standard deviation denote the value of the implied expected return on equity investments and the standard deviation of the return on equity investments for the indicated leverage ratio values, respectively. Sharpe ratio denotes the Sharpe Ratio, and market beta denotes the loading on the Fama and French (2015) market portfolio. Returns are expressed as annualized percentages.

Leverage Ratio	Expected Return	Standard Deviation	Private Equity Premium	Sharpe Ratio	Market Beta
0.25	13.50	91.71	10.39	0.113	1.17
0.30	13.42	91.00	10.31	0.113	1.17
0.35	13.42	91.05	10.32	0.113	1.17
0.40	13.51	91.80	10.40	0.113	1.18
0.45	13.67	93.27	10.57	0.113	1.19
0.50	13.93	95.54	10.83	0.113	1.22
0.55	14.30	98.78	11.19	0.113	1.27
0.60	14.80	103.25	11.70	0.113	1.32

INTERNET APPENDIX FOR

PRIVATE EQUITY RETURNS:

EMPIRICAL EVIDENCE FROM THE BUSINESS CREDIT CARD SECURITIZATION MARKET

Matthias Fleckenstein
Francis A. Longstaff

INTERNET APPENDIX

A.1 Data Sources

Table A1 provides a description of all the data and variables used in the study along with their definitions and corresponding sources. The key to our ability to identify private equity returns is the use of secondary market prices and cash flows for financial claims on the underlying assets of private entrepreneurial firms. Since data for this asset class is not readily available given its highly specialized nature, we combine hand-collected data and proprietary market sources to construct an extensive data set for the securitizations issued by the Advanta Business Card Master Trust.

Specifically, we collect information on secondary market prices for all securitizations of the Advanta master trust over the period from August 2000 to December 2010 from the Bloomberg system by specifying ICAP/Tullett Prebon (TP ICAP) as the pricing sources. We supplement this pricing data with secondary market prices for the credit card securitizations of nine of the largest U.S. consumer credit card issuers over the same time period. These are the master trusts set up by American Express, Bank of America, Bank One, Citigroup (Citi), J.P. Morgan Chase (Chase), Capital One, First National Bank (First National), and World Financial Network/Alliance Data System (World Financial).

We also collect monthly cash flow information for all securitizations of the Advanta master trust and the consumer credit card master trusts over the same period from the Bloomberg system and supplement this data with hand-collected information from monthly servicer reports (10-D filings) with the SEC. We obtain the 10-D filings from the SEC's Electronic Data Gathering, Analysis, and Retrieval system (EDGAR) website. Specifically, we collect monthly payment rates, portfolio yields, charge-offs, and excess spreads for all securitizations of the Advanta master trust and the consumer credit card master trusts.

The monthly payment rate (MPR) is the monthly rate of total principal cash flows during the month divided by the total principal receivables balance at the beginning of the month. The MPR is a measure of the speed at which cardholders pay down the amount owed on their credit cards.

The charge-off rate is the amount charged off each month divided by the total outstanding principal balance at the beginning of the month. Credit card receivables are typically charged off after the cardholder has been delinquent in paying the revolving balance for more than 180 days. Since the outstanding principal is reduced by the amount of principal charge-offs at the start of the next month, charge-offs diminish the collateral value backing the issued notes

and the level of receivables backing the notes declines. If charge-offs reach a threshold specified in the prospectus supplement, an early amortization of the notes is triggered.

The excess spread is the annualized rate of (gross) portfolio yield less servicing fees, coupon cash flows to noteholders, charge-offs, and any other trust expenses.[1] Excess spread represents a source of credit enhancement for the notes. Intuitively, as long as the excess spread is positive, the securitization generates enough cash inflows to cover cash outflows. When the average excess spread is negative (typically calculated over a period of three months), many master trusts go into early amortization.[2]

Portfolio yield is the annualized rate of total monthly cash flows into the credit card master trust divided by the outstanding principal balance at the beginning of the month. Gross cash inflows consist of interest on the revolving principal balances (finance charges) plus income from fees on the accounts such as late charges, card annual fees, cash advance fees, overdraft charges, card interchange, and discounted receivables.[3] Cash inflows include recoveries on defaulted receivables, but exclude charge-offs from the current month. The portfolio yield is a measure of the income generated by the credit card receivables.

We also hand collect data on attachment and detachment points for all securitizations of the Advanta Business Card Master trusts and the consumer credit card master trusts from the prospectus supplements for each individual securitization. The primary source for obtaining the prospectus supplements are 424-B5 filings with the SEC which we accessed via the SEC's EDGAR website. For some securitizations in the early part of the sample, we were able to download

[1]The servicing fee is an annual fee expressed in percent and paid to the servicer of the credit card portfolio for servicing the portfolio on behalf of the master trust. It is often set as a fixed percentage of the total receivables in the credit card master trust.

[2]To illustrate how excess spread represents a source of credit enhancement, suppose a master trust generates a portfolio yield of 14.80% and experiences chargeoffs of 5.50%. The trust has issued notes paying LIBOR floating each month. Assuming that the floating coupon rate equals 2.05% and that the master trust is paying a servicing fee of 2%, the excess spread is $14.80\% - 2.05\% - 2.00\% - 5.50\% = 5.25\%$. Intuitively, this means that the master trust generates about five cents for each dollar invested each month above what is required to pay investor coupon interest, servicing fees, and other trust expenses.

[3]Some credit card master trusts can add receivables at a discount, typically ranging between 1% and 5%. The difference between the face value of receivables and their discount is added to finance charge collections.

prospectus supplements available in the Bloomberg system.

A.2 The Credit Card Asset-Backed Securities Market

The market for securities backed by business and consumer credit is a very large and mature market. Credit card receivables have been securitized since the late 1980s and by the late 1990s securitization had become the dominant funding source for credit card receivables.[4]

Annual issuance of asset-backed securities (ABS) increased from just under $40 billion in 1990 to around $200 billion in 2001. Between 2002 and 2007, annual issuance was consistently in the range of $230 billion to $275 billion. The volume of ABSs backed by revolving consumer credit grew at a compound rate of about 19.79 percent between 1989 and 2009. As of year-end 2019, there was more than $127 billion in credit card ABS outstanding, and $18 billion in credit card ABS was issued in 2019. Total ABS issuance in 2019 was $306 billion across consumer credit, automobile, equipment (floorplans, leases and transportation), student loans, CDO/CLOs and other ABS (franchises, insurance, timeshares). Consumer credit ABS made up about 7.06% of total $1.8 trillion ABS outstanding as of year-end 2019.[5]

Credit cards represent an important source of financing for entrepreneurs. For example, total transaction volume for small business credit cards was $245 billion in 2017.[6] Table 4 of Robb and Robinson (2014) indicates that credit card debt represents 46.3 percent of total outsider debt for the nearly 5,000 startup firms in the Kauffman Firm Survey data set. The Small Business Credit Survey published by the Federal Reserve Banks reports that applying for bank lines of credit/loans and credit cards are the two most common ways in which small entrepreneurial firms seek financing.[7]

The underlying collateral of a business credit card ABS consists of account receivables that are generated when businesses make charges on their credit cards

[4]See, FDIC Credit Card Securitization Manual 2007, https://www.fdic.gov/regulations/examinations/credit_card_securitization/.

[5]https://www.sifma.org/resources/research/us-abs-issuance-and-outstanding.

[6]Source: Statista (www.statista.com) at https://www.statista.com/statistics/936159/ leading-small-business-credit-cards-usa-by-purchase-volume/.

[7]Loans and lines of credit are the first choice for small businesses seeking capital—86% percent of credit applicants sought a loan or line of credit for their business, and 31% applied for a credit card. New companies (two years old or less) were more likely to seek credit cards, at 44 percent.

to purchase goods and services. From credit card issuer's perspective, credit card receivables are in effect a short-term unsecured loan. Cardholders pay back either the full principal of this unsecured loan or make partial payment. In the latter case, the issuer finances the remaining balance and earns interest (finance charges). Intuitively, the idea for pooling credit card receivables is to turn the volatile cash flows from individual cardholders paying off their credit card debts, into a stable cash flow that in aggregate resembles a bond which can then be tranched and sold to investors.

Credit card issuers typically use "master trusts" for the process of securitizing credit card receivables and creating notes that are subsequently sold to investors. This is because the master trust structure allows an issuer to sell multiple securities from the same trust, all of which rely on the same pool of receivables as collateral. To illustrate, an issuer could transfer say $1 billion of card receivables from one million accounts to a to a master trust, then issue multiple notes in various denominations and sizes.

Even though the receivables are transferred to the master trust, the seller must maintain an ownership interest in the trust. On the one hand, this "seller's interest" ensures that the issuer has some skin-in-the-game to maintain the credit quality of the pool, but it also absorbs seasonal fluctuations in credit card receivable balances so that the certificateholders' invested amount is always fully invested in credit card receivables. However, the seller's interest does not provide credit enhancement for the investors. In other words, the seller has a pari-passu claim on the master trust cash flows. The size of the seller's participation must remain at or above a minimum percentage of the trust receivables balance. The minimum required seller's interest for most master trusts tends to be in the 4% to 7% range of outstanding receivables. If the seller's interest falls below this threshold, the seller must add receivables or an early amortization event is triggered.

As cardholders pay back their card balances, the issuer transfers receivables from more accounts to the master trust. Moreover, the issuer can add receivables to the trust and create additional securities which are referred to as a new "series." However, even though master trusts issue different series over time, the receivables in the master trust are not segregated to indicate which series of securities they support. Instead, all the accounts support all the securities. Master trusts offer different "classes" of securities to investors which have some parallels with a CDO structure. Specifically, in a typical transaction, the master trust issues A, B, and C classes. The class C notes are similar to a CDO equity tranche in that class C investors bear the first losses in early amortization. Class B notes are junior to class A notes and are allocated the next layer of losses after class C notes are wiped out. Finally, class A notes are senior to other classes and

because class A securities have credit support from junior classes, they typically receive AAA credit ratings.

Each class can be thought of as having attachment and detachment points like a typical CDO. For example, a class C note could have an attachment points of zero and, say, a detachment point of 15% which means that the tranche thickness of the C class absorbs the first 15 cents of losses for each dollar of credit card receivables. Similarly, class A notes could have an attachment point, say, 25%, which means that class A investors would not experience losses unless total losses in the credit card portfolio exceeded 25 cents for each dollar of credit card receivables.[8]

The process by which cash flows are allocated to different investor classes has parallels to the CDO waterfall structure. Specifically, the typical setup has two different cash flow periods—revolving and controlled amortization (in some cases, controlled accumulation). A third, referred to as "early amortization" can be triggered when the master trust incurs sustained losses. If there are no losses, the two-period structure mimics a traditional bond in these sense that interest is distributed every month and principal is paid in a single "bullet" cash flow on the maturity date.

After issuance, a credit card ABS enters the "revolving period." In this phase, all cash flows on receivables are divided into finance charge collections and principal payments. Monthly finance charges are used to pay the investor coupon and servicing fees, as well as to cover any receivables that have been charged off in the month. Any residual cash flows are referred to as "excess spread" and, depending on the master trust, are used as credit enhancement or released to the seller. Cash flows from principal collections, by contrast, are not distributed to investors, but are used to purchase new receivables or to purchase a portion of the seller's interest if there are no new receivables. The revolving period continues for a predetermined length of time, typically from two to several years, and then the controlled amortization (accumulation) period begins where principal collections are distributed to investors. For instance, a credit card ABS with a five-year expected maturity might revolve for 48 months and then enter amortization for the final 12 months.

There are two primary mechanisms through which amortization is achieved. These are designated controlled amortization and controlled accumulation. In the case of controlled amortization, principal cash flows are distributed in equal installments, for instance 1/12 of the invested amount every month for 12 months.

[8]This example abstracts away other credit enhancements built into master trusts, such as cash collateral accounts, collateral invested amounts, and excess spread accumulation accounts.

During this period, interest cash flows are based on declining principal balances. In the case of controlled accumulation, by contrast, principal cash flows are deposited into a collection account—referred to as "principal funding account" (PFA)—every month and then paid out as a single "bullet" cash flows at the end of the accumulation period.

Independent of whether the credit card ABS is in the revolving period or controlled amortization (accumulation), early amortization can be triggered if there are sustained defaults or write-offs on the pool of credit card accounts. Common early amortization trigger events include collateral performance deterioration (e.g. the three-month average excess spread falls below zero, or the collateral balance falls below the investor invested amount), seller/servicer problems (e.g. seller interest falls below the required minimum level, the seller fails to transfer new receivables into the trust when necessary), but also legal issues (e.g. breach of representation or warranties by the issuer, or default, bankruptcy, and insolvency of the seller or servicer). Basically, once an early amortization even occurs, then a credit card ABS immediately enters starts to amortize and ABS investors start to receive principal cash flows. The principal received is distributed to the Class A certificate holders first until the Class A certificate holders are repaid in full, then to the Class B certificate holders until the Class B certificate holders are paid in full, and so on for any other classes issued in a particular series. Cash flow shortfalls owed to the Class A certificate holders on a specific distribution date reduce the principal amount of the subordinated classes.

Credit card ABS have different forms of credit enhancement because as unsecured revolving debt obligations, credit card receivables offer no collateral in the event of cardholder default. Common forms of credit enhancement are excess spread, a cash collateral account (CCA), a collateral invested amount (CIA), and subordination. Excess spread is the key form of internal credit enhancement. Intuitively, excess spread is simply the residual cash flow after investor interest, servicing fees, and charge-offs are allocated to each series. Excess spread is calculated as the gross portfolio yield less charge-offs less investor coupon less servicing fees, and is expressed as an annualized percentage of the outstanding principal balance. Typically, a negative three-month moving average excess spread for a particular series triggers early amortization. Available excess spread is often shared with other series, deposited into a spread as credit enhancement or released to the seller. A cash collateral account (CCA) is a segregated cash reserve account, funded at closing and held by the trust. Amounts deposited in the CCA are used to cover shortfalls in interest, principal, or servicing expense for a particular series if excess spread is reduced to zero. The cash to fund the CCA is usually lent by a third party and invested in high-grade short-term securities, all of which will mature on or before the next distribution date. Draws

on the CCA may be reimbursed from future excess spread. A collateral invested amount (CIA) is a privately-placed tranche of a series that is subordinate in payment rights to all investor certificates. It basically acts as another layer of subordination that is used to cover deficiencies if excess spread is reduced to zero. In some master trusts, the CIA has credit enhancement via a spread account and is allocated monthly excess spread, if available. Moreover, if the CIA is drawn on, it can be reimbursed from future excess spread.

The perhaps most straight-forward form of credit enhancement is subordination via senior/subordinate investor certificates. Typically, senior participation is in the form of class A certificates and subordinate participation is in the form of class B and class C certificates. For example, the receivables could be securitized by issuing a series of A, B, and C tranches with notional amounts of $70, $20, and $10, respectively. The C tranche absorbs the first $10 of credit losses (unless those losses can be covered by excess spread, CCA or CIA), the B tranche absorbs the next $20 of credit losses, while the A tranche absorbs the remaining credit losses. Principal collections are allocated to subordinate classes only after the senior certificates are fully repaid.

A.3 The Advanta Business Credit Card

Advanta was a one of the largest issuers of credit cards to small business entrepreneurs in the U.S. with more than one million small business credit card customer accounts. In June 2000, the Advanta Business Card Master Trust had more than $1.6 billion in credit card receivables and by year-end 2006, receivables had grown to more than $3.7 billion.[9] Advanta's business credit cards targeted small firms with fewer than ten employees and less than three million dollars in annual revenues. Among Advanta's core customers were small independent brick-and-mortar and online retailers, small business start-ups, business professionals, such as consultants, lawyers, physicians, contractors, television writers and producers and online content developers.

Accounts were restricted to business owners, and applicants needed proof of business ownership or involvement and were asked to provide their Federal tax I.D. number and business phone and address before finalization of the card acceptance.[10] By signing the account agreement, the entrepreneur acknowledged that the credit card was to be used for commercial and business purposes only.

[9]See, Advanta Business Card Master Trust, Prospectus for Series 2001-A Notes, dated March 2001; Prospectus for Series 2006-A Notes, dated March 2006; Advanta Corp., 10-K, 2001 and 2006.

[10]See, https://www.financeglobe.com/credit-cards/card-205/.

Advanta could monitor business credit card usage and cancel the card if patterns of personal use were detected.[11] Furthermore, small entrepreneurs signed as personal guarantors of the business credit card. Specifically, under the cardholder agreement, the entrepreneur and the business were jointly and severally liable for all transactions on the business card account.[12] There are also legal ramifications from commingling business and personal expenses if the entrepreneur were to file for bankruptcy. Specifically, entrepreneurs using limited liability companies (LLCs) and corporations to structure their enterprises, risk being held personally liable for all business debts when they "pierce the corporate veil."

Small entrepreneurs could apply for business credit cards from Advanta via its webpage, telephone, or direct-mail. In the initial credit inquiry, Advanta used information from personal credit reports such as the entrepreneur's FICO score, as well as the entrepreneur's prior business history.[13] Small entrepreneurs were declined credit if their "underwriting risk score" was below a certain threshold.[14]

Advanta's line of small business credit cards was tailored to the needs of small businesses and entrepreneurs, and thus had features not offered by regular consumer credit cards, including higher credit limits, longer billing cycles, additional cards for employees, personalized business checks, business purchase reward incentives, and business purchase protections. In addition, through Advanta's website, cardholders were able to use an array of tools and services to set up and build their businesses. For instance, Advanta offered payroll management, employee expense tracking, and online tools for credit card accounting and bookkeeping, business and health insurance, discounts on business travel, tools to create web sites, tutorials on developing business plans, marketing, tax and legal advice on business and personal finances, and many other small business

[11] Advanta stipulates that "Under the terms of our cardholder agreements, our business cards may be used for business purposes only." See, Advanta Corp., 10-K, 2001.

[12] Many business credit card issuers such as Advanta report only to business credit bureaus, so the entrepreneur's personal credit score is not affected by usage patterns on its business credit card.

[13] Advanta "scores potential applicants based on their current and historical business performance via information purchased from an external-reporting agency... The interest rate and credit line size offered varies and is ultimately determined based upon credit history and creditworthiness of the applicant." See, Advanta Corp., 10-K, 2000.

[14] Advanta's underwriting risk score "ranks applicants based on their expected creditworthiness" and is a "measure of credit risk or potential default [of the entrepreneur]." See, Advanta Corp., 10-K, 2000.

topics.[15]

Advanta had a line of different business credit cards.[16] For instance, the Advanta Platinum Unlimited Rewards Business Card had zero percent APR for 15 Months on balance transfers, and 7.99% fixed APR thereafter (7.99% Variable APR based on prime rate). The card had no annual fees and credit lines up to $50,000. The grace period on Advanta's Platinum Business Card was twenty days, so interest charges would not be applied for new purchases during this time if the entrepreneur paid the balance in full by the due date.[17] The card had rewards points on travel, merchandise and more (one point for every dollar in purchases). In addition, it had a tiered cash-back rewards program offering a quarter of a percent for the first $10,000 spent, one percent back at $80,000 and two percent at $100,000, with an additional one percent cash back in one area to be selected from a list of items bought most often, from gasoline to office supplies. The card had no blackout dates on air travel, no limit on the points the entrepreneur could earn, and free employee cards with no fee and no per-card monthly spending limits. The card had purchase protections and $0 fraud liability. Entrepreneurs could also custom-design their business credit card and order personalized business checks.[18] Finally, via online account access entrepreneurs had a set of tools for account monitoring, itemized expense reporting, and for setting up personalized billing dates, purchase protections and extended warranty coverage on purchases.[19] Other business credit cards offered by Advanta included the Advanta Business World Master Card and the Advanta Net 90 Platinum Business Card which had a 90 day grace period.

Table A2 reports summary statistics of Advanta business credit cardholders and consumer credit cardholders. Consumer credit cardholder data are from American Express Credit Account Master Trust, Chase Issuance Trust and Chase Master Trust, and First National Master Note Trust. As shown, Advanta's business credit card accounts had average limits between roughly $10,000 and $15,000, with limits ranging from about $2,000 to $50,000 throughout 2000 to 2010.[20] Account balances typically ranged from zero to $25,000. Furthermore

[15]See, www.advanta.com, accessed via https://archive.org/web/.

[16]See, http://www.moneybluebook.com/reviews-of-the-best-advanta-business-credit-cards-and-offers/.

[17]See, https://www.financeglobe.com/credit-cards/card-205/.

[18]https://www.nytimes.com/2004/09/21/business/businessspecial/credit-card.html, http://www.selectcreditcard.com/advanta.htm

[19]See, https://www.streetdirectory.com/travel_guide/164461/credit_cards/benefit_from_the_advanta_cash_back_credit_card_rewards.html

[20]See, Advanta, Conference call, Q4 2007.

Advanta cardholders had relatively high credit scores. In particular, Table A2 shows that while FICO scores for some Advanta cardholders were below 600, almost half the accounts had FICO scores of 720 or above. More specifically, the average entrepreneur had a FICO score of 660 or better, carried a card balance of $1,630 and had a credit card limit of $10,257.[21] Through June 2000, customers' FICO scores at origination averaged 716. The average increased to 730 by 2003, and ranged between 725 and 742 through the end of 2008.[22] Moreover, since June 2000, entrepreneurs were declined credit if their FICO score was lower than 660 or if their FICO score was unavailable at the time of underwriting.[23] Table A2 also shows that account balances and FICO scores of Advanta business cardholders are comparable to those for cardholders in typical consumer credit card securitizations.

A.4 Advanta Corporation

Advanta Corp. (Advanta) was a monoline credit card bank and one of the largest issuers of credit cards to small business entrepreneurs in the United States.[24] Small business credit cards became Advanta's sole focus when it exited its mortgage and leasing services businesses in early 2001, and Advanta's business cards segment was its only reportable business segment as of year-end 2007.[25] More-

[21] See, Advanta Business Card Master Trust, Prospectus for Series 2001-A Notes, dated March 2001.

[22] "We're continuing to attract a large number of high credit quality customers who are using the Advanta Business Card to help finance their businesses. By focusing only on the specific needs of the small business market, we understand what these business owners really want. From July through September, we added 85,000 new customers to our portfolio, with an average FICO score of 728." See, Advanta, Conference Call, October 31, 2006.

[23] See, Advanta Business Card Master Trust, Prospectus for Series 2001-A Notes, dated March 2001.

[24] Advanta did not have any other significant banking operations. It owned two insurance companies, Advanta Life Insurance Company and Advanta Insurance Company, which offered credit-related insurance to business card customers. Prior to 2001, Advanta also operated Advanta Mortgage, Advanta Leasing Services, and it had a consumer credit card business. However, Advanta exited the mortgage and leasing business in the first quarter of 2001 and sold its consumer credit card portfolio to Fleet Financial Group in 1997.

[25] Since Advanta was a monoline credit card bank for small businesses, the FDIC did not assign a functional peer group for regulatory comparative capital analysis to Advanta. See, FDIC (2010).

over, business cards accounted for over 90% of Advanta's revenues by fiscal-year end 2006.[26]

Throughout the sample period, Advanta was considered well-capitalized by the FDIC, as it maintained a Tier 1 risk-based capital ratio in excess of 20%, significantly higher than the regulatory requirements during the period from the early 2000s leading up to the 2008 financial crisis.[27] Advanta's cost of borrowing over the period from March 2001 to September 2009 averaged 4.987%.[28] Moreover, Advanta's securitizations were off-balance sheet and thus protected by the FDIC safe-harbor rule.[29] This means that Advanta's business credit card securitizations were bankruptcy remote and could receive higher credit ratings than Advanta as the originator of the business credit card receivables. All of Advanta's Class A securitizations were initially rated A by Moody's.[30]

Advanta was among the many financial services providers impacted by the 2008 financial crisis. Specifically, Advanta began to experience a rapid and significant increase in credit card delinquencies and charge-offs in 2008 and into 2009 as borrowers were impacted by rapidly deteriorating economic conditions. In the first quarter of 2009, Advanta reported a $76 million first-quarter loss as delinquencies rose to 11.5%, roughly twice the number from the previous year. Rising levels of charge-offs decreased the excess spread of the securitized business credit card receivables portfolio.[31] During the months of February, March, April

[26]See, Advanta Corp., 10-K Annual Report, 2007.

[27]See, Advanta Corp., Form 8-K (Oct.30, 2008). In the Press release reporting 3Q 2008 results, Advanta reported a Tier 1 capital ratio of 22.3%. The FDIC requires banks to maintain a Tier 1 risk-based capital ratio of 6% or greater. See 12 C.F.R. § 325.103(b)(1)(ii).

[28]See, S&P Capital IQ, Industry Specific Metrics for Advanta Corp.

[29]The Federal Deposit Insurance Corporation (FDIC) safe harbor rule, enacted in 2000, provided bankruptcy remoteness for assets transferred into securitizations, which meant that the FDIC, as receiver or conservator of a failed bank, could not repudiate contracts, or recover or reclaim financial assets transferred in connection with securitization transactions when these assets were off-balance-sheet for accounting purposes (Securitization Safe Harbor Rule, 12 C.F.R. § 360.6).

[30]Credit rating agencies assign credit ratings based on the risk of the underlying pool of unsecured small business credit card receivables. See, Moody's Approach to Rating Credit Card Receivables Backed Securities, available at www.moodys.com/researchdocumentcontentpage.aspx?doci d=PBS_1065701.

[31]Excess spread is the difference between the gross yield on the pool of securitized receivables less the cost of financing those receivables (weighted average

and May, analysts weighed the prospects of an early amortization event. In June 2009, Advantas Business Card Master Trust entered early amortization when its three-month average excess spread fell below zero.[32]

Despite the turmoil in the first quarter of 2009, however, Advanta's business card securitizations continued to pay interest to investors. Moreover, in May 2009, Advanta offered to buy back Class A notes which had a stabilizing effect on the market prices of Advanta's securitizations, and the FDIC considered Advanta adequately capitalized at the time. Over the summer of 2009, Advanta's securitization continued to recover and in September, Advanta announced that charge-offs on the business credit card receivables had declined significantly. Shortly after, in November 2009, Moody's confirmed the ratings of Advantas Class A notes.[33] Advantas securitization continued to stabilize and Class A notes were paid in full by 2011. Moreover, Moodys eventually upgraded Advantas Class B notes in September 2011.[34]

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coupon paid on the investor certificates), charge-offs, servicing costs, and any other trust expenses. Excess spread is typically a source of credit enhancement for the certificates since it is commonly available to absorb losses on the assets. Advanta's excess spread decreased from 9.5% in March 2006 to 4.9% in June 2008. During roughly the same time period, the industrys average excess spread ranged from 7% to 9%. See, FDIC (2010).

[32] Advanta is not the only master trust that had an early amortization event. The other seven public transactions that have triggered early amortization were issued by Chevy Chase FSB, Conseco Private LabelMaster Note Trust, First Consumers Credit Card Master Note Trust, Next Card Master Note Trust, Spiegel Master Trust, Southeast Bank, Republic Bank (Delaware). See Poon and Kohl (2016).

[33] "Driven primarily by a greater level of certainty with respect to collateral performance. The performance of the trust immediately following the beginning of the early amortization in May was initially very volatile, but not altogether unexpected. In recent months, however, there have been signs of performance stabilization. We believe the Class A notes will be paid in full even if performance deteriorates marginally from its present level." See, https://www.moodys.com/research/Moodys-confirms-Advanta-credit-card-trust-senior-notes-rating–PR_192101.

[34] "The decision to upgrade the Class B notes is driven primarily by a material improvement in the delinquency rate, a harbinger of future losses, while other performance metrics remain stable... This improvement in the delinquency rate is expected to cause charge-offs to fall in upcoming months." See, https://www.moodys.com/research/Moodys-upgrades-Advantas-card-backed-Class-B-notes–PR_225353.

A.5 The Merton Model

We use the familiar Merton (1974) framework to study the returns from entrepreneurial investment. This approach follows the extensive literature using the Merton (1974) structural credit framework to model the relation between equity returns and credit spreads.

In its basic form, the Merton (1974) framework is typically used to value a debt claim for an issuer that may default. The capital structure of the issuer is assumed to consist of equity denoted by S, and a zero-coupon bond with a notional amount F, maturity T, and value denoted by B. The standard accounting identify $V = B + S_t$ holds, and the riskfree rate, denoted by r, is assumed constant. The value of the underlying assets of the issuer are denoted V and are assumed to follow the dynamics

$$dV = \mu\ V\ dt\ +\ \sigma\ V\ dZ \tag{A1}$$
$$dV = r\ V\ dt + \sigma\ V\ dZ, \tag{A2}$$

under the objective and risk-neutral measures, respectively. In these dynamics, μ and σ denote constants, and Z is a standard Brownian motion. At maturity T, the payoffs to equity S and debt B, respectively, are

$$S = \max(V - F,\ 0) \tag{A3}$$
$$B = \min(V,\ F) = F\ -\ \max(F - V,\ 0) \tag{A4}$$

In this set up, Equation ($A3$) shows that the issuers equity can be represented as a call option on the underlying assets of the issuer, allowing the Black and Scholes (1973) model to be used to solve for S,

$$S = V\ N(d)\ -\ F\ e^{-rT}\ N(d\ -\ \sqrt{\sigma^2 T}) \tag{A5}$$

$$d = \frac{\ln(V/F) + (r\ +\ \sigma^2/2)T}{\sqrt{\sigma^2 T}}. \tag{A6}$$

where $N(\cdot)$ is the cumulative standard normal distribution function.

Since the sum of debt and equity equals the value of the underlying assets, the value of debt is simply $B = V - S$.

In the Merton (1974) model, the underlying assets have constant volatility σ. However, the volatility of the debt claim is stochastic and lower leverage is associated with lower volatility of debt. Specifically, let σ_B denote the volatility of the debt claim. Merton (1974) implies that

$$\sigma_B \;=\; (1 - N(d))\,\frac{V_t}{B_t}. \tag{A7}$$

Defining the leverage ratio as $X = B/V$, Equation $(A7)$ can be written as

$$\sigma_B \;=\; \frac{\sigma\,(1 - N(d))}{X}, \tag{A8}$$

which is Equation (6). Note, that as leverage goes to zero, the volatility of the debt claim approaches zero and debt becomes risk-free.

The volatility of equity is also stochastic and increases with leverage. Let σ_S denote the volatility of equity. Then,

$$\sigma_S = N\,(d)\,\frac{V}{S}\sigma. \tag{A9}$$

Using the definition for the leverage ratio $X = B/V$, Equation $(A9)$ can be written as

$$\sigma_S = \frac{\sigma N(d)}{1 - X}, \tag{A10}$$

which is Equation (8). This implies that for an unlevered issuer, equity volatility equals the volatility of the issuer's assets. Since there is a single source of risk in the model, deriving expected returns on equity and debt is straightforward. This is because the Sharpe ratios of assets, equity, and debt are equal (see Merton (1974), Equation (6)).

Let the expected returns on equity and debt be denoted by $\mu_S]$ and μ_B. Then,

$$\frac{\mu - r}{\sigma} = \frac{\mu_S - r}{\sigma_S}. \tag{A11}$$

Using Equations $(A10)$ and $(A11)$, the expected return on equity μ_S is given by

$$\mu_S = r + \frac{(\mu - r)N(d)}{1 - X}, \tag{A13}$$

which is Equation (7). Similarly, using Equations $(A8)$ and $A(9)$, the expected return on the debt claim can be written as

$$\mu_B = \frac{\mu - r(1 - X) - (\mu - r)N(d)}{X}, \tag{A14}$$

which is Equation (5).

A.6 Credit Spreads and Returns on Advanta Securitizations

Our approach to calculate credit spreads is based on the portfolio of all tranches issued in a securitization (rather than on individual tranches). An important advantage of this approach is that holding a portfolio of all tranches is economically equivalent to holding a single claim on the pool of business credit card receivables. In turn, this allows us to interpret the credit spread for Advanta in exactly the same way that we would interpret, for example, a corporate bond spread. For expositional clarity, we refer to this portfolio as an Advanta "bond."

As a preliminary step, we swap the Libo–based floating coupons on the individual tranches into fixed rate coupons using standard basis and interest rate swaps. To illustrate, consider the Class A notes of the Series 2000-B Advanta Business Card Master Trust securitization. This A tranche pays a floating coupon rate of one-month Libor plus 17 basis points every month until maturity which is expected to be July 21, 2003. Suppose the date is August 31, 2000. The market swap rate on that date for swapping a floating cash flows based on one-month Libor into fixed is 6.91 percent. Thus, by entering into the floating to fixed interest rate swap, we convert the floating rate Class A note into a fixed rate note with an annual coupon rate of 7.08 percent (6.91 percent + 17 basis points). We obtain all swap rate data used in this step from the Bloomberg system.

Turning next to how we create a "bond" by forming a portfolio of all the tranches. As discussed earlier, the portfolio consisting of all of the tranches issued in a securitization is economically equivalent to investing in an alternative single-class pass-through securitization which, in turn, is equivalent from a cash flow perspective to owning the underlying receivables directly. We first collect the attachment and detachment points for all the tranches issued by the Advanta Credit Card Master trust by reading the prospectus supplement of each series issued by the master trust. Recall that the attachment point represents

the percentage of the receivables pool balance that can default before the tranche experiences first losses, and the detachment point represents the level of credit card defaults that leads to a total loss of the tranche. In all series issued by the Advanta Master trust, the A tranches attach at the 21.5805 percent detachment point of the B tranches, and the B tranches attach at the 8.9918 detachment point of the C tranches. We note that Advanta issued some D tranches as equity tranches during our sample period which means that the C tranche technically has a non-zero attachment point. However, since the tranche thickness of D tranches was small relative to those of the C tranches, we make the mild assumption that the equity tranche has the same price and coupon as the C tranche, and we set the attachment point of the C tranche to zero percent.

First, at the end of each month during the sample period from August 2000 to December 2010, we match each A tranche with a B and a C tranche that are closest in expected maturity to the A tranche. To determine which B and C tranches have the closest maturity, we calculate the number of months to expected maturity for each tranche. For the small number of cases where the A tranche matures after the expected maturity date, we use the final maturity date from the prospectus supplement instead. We then take the minimum absolute difference in the number of months to maturity across all pairs of A/B tranches and A/C tranches to form triplets of A, B, and C tranches at the end of each month. In general, this results in a set of between two and 20 triplets each month.

Next, we create a single "bond" from each triplet by calculating the weighted average price and the weighted average coupon rate of the individual A, B, and C tranches in each triplet where the weights are the respective tranche thicknesses. Tranche thickness is just the difference between the detachment and the attachment point. For the Advanta A, B, and C tranches, these thicknesses are 0.784195, 0.125887, and 0.089918, respectively. To illustrate, at the end of August 2000, the triplet consists of the Series 2000-B A, B, and C tranches with prices (including accrued interest) of 100.198, 100.1396, and 100.0221, and swapped coupon rates of 7.0855, 7.4708, and 8.1805 percent, respectively. The A, B, and C tranches all have 35 months until maturity. Thus, using the attachment and detachment points of the A, B, and C tranches discussed previously, the price of the Advanta bond is $0.784195 \times 100.198 + 0.125887 \times 100.1396 + 0.089918 \times 100.0221 = 100.1748$, the fixed coupon rate is $0.784195 \times 7.0855 + 0.125887 \times 7.4708 + 0.089918 \times 8.1805 = 7.2325$ percent, and the maturity date is July 21, 2003. We calculate the yield to maturity of the Advanta bond using standard market conventions which is 7.145 percent in this example.

To calculate credit spreads, we use the following intuitive approach. We first calculate the price and the yield of the Advanta bond as if it were riskfree, and then we take the difference in yields between the actual Advanta bond and the

riskfree Advanta bond to obtain the credit spread. To get the riskfree price, we discount the fixed monthly coupon cash flows using the Treasury discount curve which we obtain by bootstrapping the Treasury constant maturity curve provided by the Federal Reserve. Specifically, our bootstrapping algorithm to estimate the riskless discount function $D(T)$ uses the following approach. We collect month-end values of the constant maturity Treasury (CMT) rates for 1-month, 3-month, 12-month, 2-year, 3-year, 5-year, 7-year, 10-year, and 20-year maturities from the Federal Reserve H.15 release for each month during the sample period. We then use a standard exponential spline algorithm to bootstrap the discount function and interpolate it to a vector of monthly horizons ranging from one month out to 20 years (for more details on this algorithm, see Longstaff, Mithal, and Neis (2005)). We also take into account that the tranches mature in the middle of the month by discounting the riskfree price by two weeks using the one-month discount factor. In the example, the riskfree price is 102.9887 and the riskfree yield is 6.096. Thus, the credit spread is $7.145 - 6.096 = 1.049$ percent (104.9 basis points).

The next step in calculating time series of month yields and credit spreads is to take the average yields and credit spreads across all triplets or bonds in the sample each month. In computing these averages we include only those triplets with time to maturity greater than or equal to eighteen months. For the period prior to February 2009, the time to maturity is simply the time until the expected maturity date of the A tranche in the triplet. For the months of February, March, and April 2009, we take the following approach (see Section A.4 for details on this period). Specifically, for this three-month period, we assume that the Advanta bonds will be paid off halfway between expected maturity and the final maturity date to account for the uncertainty about whether Advanta would enter early amortization. In May 2009, Advanta announced that early amortization would commence in June 2009. Thus, starting in May 2009 we then assume that all bonds have a maturity of 37 months (which matches the average number of months to maturity in May 2009 in our sample), and we decrease the number of months to maturity by one each month through the end of the sample in December 2010. This assumption is roughly in line with market expectations at the time that senior bondholders would be paid in full as a result of declining charge-offs in the collateral pool (again, see Section A.4 for details on this period).

We next calculate excess returns on Advanta bonds which will then be used in calibrating the Merton (1974) model. Recall that the Merton (1974) assumes that the debt in the issuer's capital structure consists of a zero-coupon bond. To be consistent with the model, we compute the excess return for a hypothetical zero-coupon bond in the following way. First, we assume a representative three-year maturity for the zero-coupon bond. This closely matches the aver-

age maturity of the Advanta bonds during the sample period. Second, for each month, we compute the price of the zero-coupon bond as of the end of the previous month and at the end of the current month. The return on the bond for that month is the simple arithmetic return implied by these two prices. Finally, we compute the excess return for the bond by subtracting the return on a corresponding three-year riskless zero-coupon bond from the return for the Advanta bond. Note that we subtract the return on the riskless bond (rather than the yield on the riskless bond) in order for the difference to be more cleanly interpretable as an excess return in the traditional sense.

To illustrate the procedure, suppose the date is September 30, 2000. The yield of the Advanta bond at the end of the previous month is 7.14%. The corresponding price of a zero-coupon Advanta bond, using 36 months to maturity, is 80.7076. On September 30, 2009 the yield on Advanta bonds is 6.76% and the corresponding price using 35 months to maturity is 82.0960. Thus, the return on the zero-coupon Advanta bond is equal to $(82.0960 - 80.7076)/80.7076 = 1.72$ percent. Next the prices of riskless zero-coupon bonds at the end of the previous month and at the end of the current month are 83.553 and 84.388, respectively. Thus, the return of a risk-free three-year zero coupon bond is $(84.388 - 83.553)/83.553 = 1.00$ percent which implies an excess return on the zero-coupon Advanta bond for the month of September 2000 of 0.72 percent. The result from this procedure is a time series of monthly returns and excess returns on Advanta bonds. We use the average excess return of 1.121 percent over the sample period in calibrating the Merton (1974) model.

A.7 Survey of Private Equity Returns Estimates

This section gives an overview of estimates for the risk and return to private equity investments in the literature. In presenting these estimates, we follow Korteweg (2019) who provides an extensive list of private equity risk-adjusted excess returns (alphas) and (Fama-French three-factor) beta estimates reported in the literature

First, we distinguish between studies using fund-level data and those using deal-level data (e.g., start-up companies in the case of venture capital (VC)). The reason for this is that fund-level returns and deal-level returns involve distinct econometric and data issues. Second, we report different estimates for buyout (BO) deals and venture capital (VC) deals. The main difference between venture capital and buyout deals is that the investments in VC are minority stakes in start-up companies, while BO typically purchases all the equity of established firms (Korteweg (2019)). The reason for this is that research to date has focused mainly on VC and BO. Third, we report estimates of returns to entrepreneurship from studies using survey data separately because these data again come with

distinct issues and challenges. Table A3 presents summary statistics of average returns and standard deviations of returns on private equity

We focus first on studies using fund-level data. From a high level point of view, challenges with fund level data arise because fund data typically consist of fund cash flows that occur at irregular times, with reported net asset values (NAVs) that are often stale and in some cases biased. These issues could be reasons for the wide range of estimates for risk and returns of BO and VC deals in fund level data. As shown in Table A3, estimates of average returns on buyout deals using internal rates of return (IRR) range from about 8.60 percent for the 1990–2010 period (Gupta and van Nieuwerburgh (2018)) to as high as 21.83 percent over the 1981–1994 period (Ljungqvist and Richardson (2003)). Estimates of annualized return volatilties for buyout deals using fund-level data range from around 10.10 percent for the 1990–2010 period (Gupta and van Nieuwerburgh (2018)) to as high as 44.45 percent for the 1980–2007 period (Ewens, Jones and Rhodes-Kropf (2013)). We note that since fund-level returns are estimated net of fees, they are lower bounds for expected returns to portfolio companies.

We turn next to studies using deal-level data. We note first that most papers using deal-level data have considered VC only, as start-up company returns have traditionally been more readily available (Korteweg (2019)). From a high level point of view, challenges with deal-level data arise because deal data are valuations that are observed at the time of investment or when a portfolio company is successfully sold or goes public, but failures are often missing. These issues can lead to success bias, especially in VC, which could be among the reasons for the wide range of estimates for risk and returns in deal-level data. As shown in Table A3, estimates of average returns (IRR) range from about 7.00 percent for the 1987–2005 period (Korteweg and Nagel (2016)) to as high as 56.10 percent over the 1991–2008 period (Cochrane (2005)). Estimates of the annualized volatility of venture capital returns in deal-level data range from about 9.27 percent (Stafford (2017)) to to as high as 107 percent (Cochrane (2005)).

Finally, we focus on returns to entrepreneurship estimated from survey data. One challenge with survey data is that it is self-reported. For instance, a number of recent papers, raise concerns about the reliability of self-reported entrepreneurial income data (see, Tedds (2010), Hurst, Li, and Pugsley (2014), and Astebro and Chen (2014)) which could be among the reasons for the different esimates for risk and return to entrepreneurship from survey data. As shown in Table A3, Astebro and Chen (2014) report average returns to entrepreneurship as high as 42.00 percent for the 1980–1996 period using the Panel Study of Income Dynamics (although their estimate is based on a number of assumptions). By contrast, Moskowitz and Vissing-Jorgensen (2002) report average returns to entrepreneurship of about 15.80 percent for the 1989–1998 period and Kartashova

(2014) finds average returns of around 16.59 percent over the 1960–2010 period.

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Table A1

Data Definitions and Sources. This table summarizes the datasets used in this study. Frequency shows at what intervals the data are available. Description and Source show the data source and its definition. The data are for the period from August 2000 through December 2010.

	Data	Frequency	Description and Source
1	Treasury CMT Data	Daily	Constant maturity Treasury rates from the Federal Reserve H.15 Selected Interest Rates Release for tenors of 1, 3, 6 months and 1, 2, 3, 5, 7, 10, 20 years.
2	Discount Function	Daily	Discount function out to 20 years bootstrapped from Treasury CMT Data as described in Liu, Longstaff, Mandell (2006).
3	Libor Interest Rate Swap Spreads	Daily	Three-month Libor into fixed interest rate swap rates. Cash flows on the fixed leg are semiannual, and the floating leg pays three-month Libor each quarter. Data from the Bloomberg system for tenors of 3, 6, 9, 12, 18 months, and 2, 3, 4, 5, 7, 10, 12, 15, 20 years.
4	Libor Basis Swap Spreads	Daily	Three-month Libor into One-month Libor interest rate basis swap rates. Cash flows on the fixed leg are at quarterly frequency, and the floating leg pays one-month Libor reset monthly each quarter. Data from the Bloomberg system for tenors of 3, 6, 9, 12, 18 months, and 2, 3, 4, 5, 7, 10, 12, 15, 20 years.
5	Libor	Daily	Three-month USD London Interbank Offered Rate (LIBOR) from the Bloomberg system.
6	Treasury Bill Rate	Daily	The three-month Treasury bill rate from the Bloomberg system.
7	Fama-French 5 Factors	Daily	Fama-French 5 Factors and riskfree rate from Kenneth French's website.
8	U.S. Treasury Bond Returns	Daily	The Bloomberg Barclays U.S. Treasury Total Return Index. Data from the Bloomberg system.
9	AAA Spread	Monthly	The spread between yields on AAA corporate bonds and the 10-year Treasury rate. Data from the Bloomberg system.
10	BBB Spread	Monthly	The spread between yields on BBB corporate bonds and the 10-year Treasury rate. Data from the Bloomberg system.
11	High-Yield Spread	Monthly	The spread between yields on high-yield (non-investment-grade) corporate bonds and the 10-year Treasury rate. Data from the Bloomberg system.
12	Monthly Payment Rate	Monthly	Monthly payment rates (MPR) of the credit card master trusts from the Bloomberg system. The monthly payment rate is the ratio of total cash flows collected each month divided by the portfolio balance, expressed as a percentage.
13	Credit Card ABS Tranche Prices	Monthly	Prices of individual A, B, and C tranches of the credit card securitizations in the sample. Data from the Bloomberg system.

Table A1 — Continued

	Data	Frequency	Description and Source
14	Portfolio Yield	Monthly	Monthly portfolio yields of the credit card master trusts from the Bloomberg system and 10-D filings with the SEC. The portfolio yield is the annualized percentage gross return on the portfolio.
15	Excess Spread	Monthly	Monthly excess spreads of the credit card master trusts from the Bloomberg system and 10-D filings with the SEC. The excess spread is the annualized percentage net return on the portfolio.
16	Charge-off Rate	Monthly	Monthly charge-off rates of the credit card master trusts from the Bloomberg system and 10-D filings with the SEC. The charge-off rate is the one-month annualized percentage rate of charge-offs on the portfolio.
17	Risk Retention Ratio	Monthly	Required minimum issuer risk retention ratio for the credit card securitization portfolio. Data collected from 424-B5 filings.
18	Attachment and Detachment Points	Monthly	The average attachment and detachment points for individual tranches, expressed as percentages of the total notional amount of the securitization. Data collected from 424-B5 filings.
19	Consumer Credit Card Spread	Monthly	The credit spread measure on consumer credit card debt from Fleckenstein and Longstaff (2020b).
20	Non-corporate leverage	Monthly	The average annual ratio of total liabilities to total assets for non-financial non-corporate businesses. Data from the Federal Reserve Z.1 Release for the Financial Accounts of the United States.
21	Corporate leverage	Monthly	The average annual ratio of total liabilities to total assets for non-financial corporate businesses. Data from the Federal Reserve Z.1 Release for the Financial Accounts of the United States.
22	Household leverage	Monthly	The average annual ratio of total liabilities to total non-financial assets for households and non-profit organizations. Data from the Federal Reserve Z.1 Release for the Financial Accounts of the United States.
23	Charge-off Rate on Consumer Loans	Monthly	The annualized charge-off rate on consumer loans (net of recoveries) from the Board of Governors of the Federal Reserve System (US), retrieved from FRED, Federal Reserve Bank of St. Louis.
24	Charge-off Rate on Single-Family Mortgages	Monthly	The annualized charge-off rate on single-family mortgages (net of recoveries) from the Board of Governors of the Federal Reserve System (US), retrieved from FRED.
25	Charge-off Rate on Commercial Real Estate	Monthly	The annualized charge-off rate on commercial real estate loans (net of recoveries) from the Board of Governors of the Federal Reserve System (US), retrieved from FRED.
26	Charge-off Rate on Business Loans	Monthly	The annualized charge-off rate on business loans (net of recoveries) from the Board of Governors of the U.S. Federal Reserve System.

Table A1 — Continued

	Data	Frequency	Description and Source
27	Industrial Production	Monthly	The U.S. industrial production index from the Board of Governors of the Federal Reserve System, obtained from the Bloomberg system.
28	Inflation	Monthly	The U.S. non-seasonally-adjusted Consumer Price Index of All Urban Consumers (CPI-U) published by the U.S. Bureau of Labor Statistics.
29	Jobless Claims	Monthly	The monthly average of the Conference Boards weekly initial jobless claims index, obtained from the Bloomberg system.
30	Consumer Sentiment	Monthly	The University of Michigan Consumer Sentiment index, obtained from the Bloomberg system.
31	Consumer Confidence	Monthly	The Conference Boards Consumer Confidence index, obtained from the Bloomberg system.
32	Business Confidence	Monthly	The Conference Boards Business Confidence index, obtained from the Bloomberg system.
33	Economic Confidence	Monthly	The Conference Boards Business Confidence index of leading indicators, obtained from the Bloomberg system.
34	VIX	Monthly	The CBOE Volatility Index of option-implied volatilties from S&P 500 index options, obtained from the Bloomberg system.
35	MOVE	Monthly	The ICE MOVE index of U.S. interest rate volatility implied by one-month over-the-counter options on 2-year, 5-year, 10-year and 30-year Treasuries.
36	Stock Market Return	Monthly	Monthly returns on the CRSP Value Weighted Index, obtained from WRDS.
37	Treasury Market Return	Monthly	Monthly returns on the Bloomberg Barclays U.S. Treasury total return index, obtained from the Bloomberg system.
38	Housing Market Return	Monthly	Monthly percentage changes of the S&P Case-Shiller Housing Price Index, obtained from the Bloomberg system.

Table A2

Summary Statistics for Business Credit and Consumer Credit Cardholders. This table reports summary statistics of Advanta business credit cardholders and consumer credit cardholders. Data for Advanta are from the Advanta Business Card Master Trust. Consumer credit cardholder data are from American Express Credit Account Master Trust, Chase Issuance Trust and Chase Master Trust, and First National Master Note Trust. Values reported in percent. Percentages are based on the total dollar amount of credit card receivables in the credit card master trusts.

	Advanta Business Credit Cards	Consumer Credit Cards
Credit Limits		
0.01 to 5,000	7.31	4.16
5,000 to 10,000	16.36	14.34
10,000 to 15,000	21.32	20.20
15,000 to 20,000	22.11	18.70
20,000 to 25,000	11.65	18.70
Over 25,000	21.26	23.90
Total	100.00	100.00
Account Balances		
Less than 0.00	−0.10	−0.14
0.00	0.00	0.00
0.01 to 5,000	26.61	15.35
5,000 to 10,000	26.88	25.82
10,000 to 20,000	25.69	38.91
20,000 to 25,000	10.52	10.00
Over 25,000 to 50,000	10.40	10.06
Total	100.00	100.00
FICO Scores		
No FICO Score	0.31	0.00
Less than 600	9.34	5.73
600 to 660	7.37	12.36
661 to 719	35.17	37.26
720 and Higher	47.80	44.65
Total	100.00	100.00

Table A3

Estimates of Returns to Entrepreneurial Investment. This table presents estimates of returns to entrepreneurial investment from the literature. The columns titled $E[dS/S]$ and $\mathrm{SD}[dS/S]$ show average returns and standard deviations of returns, respectively, estimated over the periods in the column titled Sample Period. The categories Fund-level Data and Deal-level Data are based on Korteweg (2019). Fund-level Data are portfolios of individual deals, typically organized as 10-year limited partnerships, and Deal-level data are from individual deals. The sources include those from Table 1 in Korteweg (2019). Fund-level estimates are net of fees. The superscript * denotes that expected returns are implied from market betas, using a market risk premium of 8% and a riskfree rate of 4%. The superscript ** denotes that expected return estimates are internal rates of return.

Source	Sample Period	$E\,[dS/S]$	$\mathrm{SD}\,[dS/S]$	Notes
Fund-level Data				
Ang et al. (2018)	1994–2008	17.00	26.00	Buyout Index
	1994–2008	15.00	11.00	Buyout Index (Cambridge Associates)
	1994–2008	14.00	29.00	Buyout Index (LPX)
	1994–2008	17.00	31.00	Venture capital index
	1994–2008	18.00	26.00	Venture capital index (Cambridge Associates)
	1994–2008	11.00	37.00	Venture capital index (LPX)
Anson (2007)	1985–2005	15.40	–	Venture capital*
Barber & Yasuda (2017)	1993–2009	11.10	–	Buyout**
	1993–2009	7.00	–	Venture capital**
Boyer et al. (2018)	2006–2017	20.00	40.00	Secondary market transactions-based index
	2006–2017	13.00	9.00	NAV-based index
Brown, Gredil & Kaplan (2019)	1969–2016	13.80	18.70	Buyout**
	1969–2016	15.00	48.90	Venture capital**
Buchner & Stucke (2014)	1980–2001	24.80	–	Buyout
	1980–2001	26.20	–	Venture capital
Chen, Baierl & Kaplan (2002)	1960–1999	45.00	115.60	Venture capital
Driessen, Lin & Phalippou (2012)	1980–1993	18.00	–	Venture capital**
	1980–1993	14.00	–	Buyout**
Ewens, Jones & Rhodes-Kropf (2013)	1980–2007	13.61	30.69	Venture capital**
	1980–2007	13.84	44.45	Buyout**
Fang, Ivashina & Lerner (2015)	1991–1999	20.67	–	Venture capital**
	1991–2010	5.67	–	Venture capital**
	2000–2010	3.89	–	Venture capital**
Gompers & Lerner(1997)	1974–1989	18.88	5.10	Venture capital*
	1983–1989	34.56	5.68	Venture capital*
	1983–1989	27.36	9.19	Early-stage venture capital*
	1983–1989	18.08	5.80	Balanced venture capital*
	1983–1989	12.56	3.73	Later-stage venture capital*

Table A3 — Continued

Estimates of Returns to Entrepreneurial Investment. This table presents estimates of expected returns and return volatilities to entrepreneurial investment from the literature. This table presents estimates of returns to entrepreneurial investment from the literature. The columns titled $E[dS/S]$ and SD$[dS/S]$ show average returns and standard deviations of returns, respectively, estimated over the periods in the column titled Sample Period. The categories Fund-level Data and Deal-level Data are based on Korteweg (2019). Fund-level Data are portfolios of individual deals, typically organized as 10-year limited partnerships, and Deal-level data are from individual deals. The sources include those from Table 1 in Korteweg (2019). Fund-level estimates are net of fees. The superscript * denotes that expected returns are implied from market betas, using a market risk premium of 8% and a riskfree rate of 4%. The superscript ** denotes that expected return estimates are internal rates of return.

Source	Sample Period	$E\,[dS/S]$	SD $[dS/S]$	Notes
Fund-level Data				
Gredil, Sorensen & Waller (2018)	1979–2008	45.00	–	Venture capital
	1979–2008	15.00	–	Buyout
Gupta & van Nieuwerburgh (2018)	1990–2010	8.60	10.10	Buyout**
	1990–2010	3.30	20.40	Venture capital**
Harris et al. (2018)	1987–2007	7.60	–	Venture capital (Burgiss database)**
	1987–2007	9.10	–	Venture capital (Preqin database)**
Harris, Jenkinson & Kaplan(2014)	1984–2008	14.20	–	Buyout**
	1984–2008	16.80	–	Venture capital**
Higson & Stucke (2012)	1990–1999	10.40	–	Venture capital**
	2000–2008	6.90	–	Venture capital**
Jegadeesh, Kräussl & Pollet (2015)	1997–2008	13.60	–	Venture capital (funds-of-funds)*
	1997–2008	11.70	–	Venture capital (partnerships)*
Kaplan & Schoar (2005)	1980–1994	17.00	34.00	Venture capital**
	1980–1994	19.00	27.00	Buyout**
Korteweg & Nagel (2016)	1979–2008	7.00	35.00	Venture capital**
Lerner et al. (2018)	1987–2017	9.00	–	Venture capital**
Ljungqvist & Richardson (2003)	1981–1993	14.08	26.88	Venture capital**
	1981–1994	21.83	20.33	Buyout**
McCourt (2018)	1995–2015	15.30	–	Venture capital*
McKenzie & Janeway (2011)	1980–2007	47.00	72.00	Venture capital**
Peters (2018)	1990–2011	15.80	–	Venture capital*
Phalippou & Gottschalg (2009)	1980–1993	15.20	–	Buyout and venture capital**
Phalippou (2014)	1993–2010	20.70	–	Buyout
Robinson & Sensoy (2016)	1984–2009	9.00	26.00	Venture capital**
	1984–2009	8.00	43.00	Venture capital**
Woodward (2009)	1996–2008	22.10	–	Venture capital*

Table A3 — Continued

Estimates of Returns to Entrepreneurial Investment. This table presents estimates of returns to entrepreneurial investment from the literature. The columns titled $E[dS/S]$ and $\mathrm{SD}[dS/S]$ show average returns and standard deviations of returns, respectively, estimated over the periods in the column titled Sample Period. The categories Fund-level Data and Deal-level Data are based on Korteweg (2019). Fund-level Data are portfolios of individual deals, typically organized as 10-year limited partnerships, and Deal-level data are from individual deals. The sources include those from Table 1 in Korteweg (2019). Fund-level estimates are net of fees. The superscript * denotes that expected returns are implied from market betas, using a market risk premium of 8% and a riskfree rate of 4%. The superscript ** denotes that expected return estimates are internal rates of return.

Source	Sample Period	$E\,[dS/S]$	$\mathrm{SD}\,[dS/S]$	Notes
Deal-level Data				
Acharya et al. (2013)	1991–2008	56.10	46.60	Venture capital**
Axelson, Sorensen & Stromberg (2014)	1994–2007	12.10	–	Venture capital**
Cochrane (2005)	1987–2000	59.00	107.00	Venture capital
Franzoni, Nowak & Phalippou (2012)	1975–2006	18.00	–	Venture capital**
Groh & Gottschalg (2011)	1984–2004	9.07	25.25	Venture capital
Hwang, Quigley & Woodward (2005)	1987–2003	8.48	27.40	Venture capital
Kaplan (1989)	1980–1985	45.90	23.75	Venture capital
Korteweg & Nagel (2016)	1987–2005	7.00	35.00	Venture capital**
Korteweg & Sorensen (2010)	1987–2005	29.70	–	Venture capital*
Peng (2001)	1987–1998	41.36	27.80	Venture capital
Stafford (2017)	1986–2016	13.20	9.27	Venture capital
Survey-Based				
Astebro & Chen (2014)	1980–1996	42.00	–	Panel Study of Income Dynamics
Kartashova (2014)	1960–2010	16.59	9.28	Survey of Consumer Finances
Moskowitz & Vissing-Jorgensen (2002)	1989–1998	15.80	–	Survey of Consumer Finances
Mueller (2010)	1989–2001	49.80	–	Survey of Consumer Finances