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The Collateral Link between Volatility and Risk Sharing
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ABSTRACT

We show that aggregate volatility affects the extent to which agents can share idiosyncratic risks through the valuation of collateral. Both private and public assets are used in insurance markets as collateral, but their exposure to volatility differs. While aggregate volatility decreases the value of private assets—they are exposed to more variation—it increases the value of public assets—they become more valuable to smooth consumption intertemporally. Hence, a more volatile economy tends to damage risk sharing when the composition of collateral is biased toward private assets. As we show that a stable economy is more propitious to the creation of private collateral, stability makes risk sharing increasingly fragile to volatility shocks. We find empirical evidence that the higher use of private assets in the U.S. has affected the sensitivity of risk sharing to aggregate volatility as predicted by our model.

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1 Introduction

Public assets (government bonds, T-bills, etc.) and private assets (houses, asset backed securities, etc.) have two relevant uses. On the one hand, they back promises to move resources over time, this is to smooth *aggregate shocks intertemporally*. On the other hand, they back promises to move resources across agents, this is to smooth *idiosyncratic shocks intratemporally*. The first use has been widely analyzed by the asset pricing literature (for example, Holmstrom and Tirole 2001 and Bansal et al. 2014). The second has been instead mostly discussed by macroeconomics (for example, Heaton and Lucas 1996, Lustig and Van Nieuwerburgh 2005, Blundell et al. 2008, Schulhofer-Wohl 2011 and Blundell et al. 2016).

These uses, however, are intimately related. The use of assets as collateral affects the prevalence of idiosyncratic risk in the economy and, as such, affects the intertemporal value of assets. In parallel, the intertemporal value of assets determines their value as collateral and thus the prevalence of idiosyncratic risk. In this paper, we explore how the intertemporal properties of an economy (its time series volatility of aggregate consumption) affects its intratemporal properties (its cross-sectional idiosyncratic variance). In other words, we explore the effect of aggregate volatility on the extent of risk sharing across agents in the economy through the valuation of collateral.

Understanding the relationship between aggregate volatility and risk sharing has become particularly relevant during turbulent times, such as the wake of the COVID-19 pandemic. The pandemic constitutes a sudden, unexpected shock to the economy, affecting all countries and vastly increasing the aggregate uncertainty individuals face in the short- and medium-run. How does this shock affect the capacity of individuals, companies, and financial institutions to cross-insure against future idiosyncratic shocks? How does an increase in economic uncertainty affect inequality across agents, the functioning of financial markets, and the allocation of resources?

We show that this relationship is non-monotonic, as an increase in uncertainty—captured by an increase in the *volatility of time-series aggregates* that agents face—can either reduce or increase the possibilities for those agents to share risk. To be more precise, the way in which volatility affects risk sharing depends on the ratio of private to public assets used as collateral in the economy, as their valuations react to volatility in opposite directions. On the one hand, an increase in aggregate volatility increases the intertemporal price of public assets. The reason is that public assets, through the taxation power of governments, have the property of providing noncontingent future payment promises, and a promise of a dollar in the future depends on how uncertain the future is. If agents face high volatility (that is, when the variance of future realizations is high), such a dollar can pay off in very bad periods, with high marginal utility, making the promise more valuable. On the other hand, an increase in aggregate volatility reduces the intertemporal price of private assets. The reason is that their payoffs are tied to the evolution of the aggregate economy, and riskier assets become less valuable when the variance of future realizations is high.

When the ratio of private to public assets used as collateral to back insurance promises is relatively low, the value of public assets is more relevant to determine the value of available collateral. In this case, higher volatility implies more valuable collateral, on average, and better idiosyncratic insurance—a sort of "positive externality" of economic volatility on economic risk sharing. The opposite is true when the ratio of private to public assets is relatively high. In this case, the total value collateral declines, and there is less idiosyncratic insurance—a "negative externality" of economic volatility on economic risk sharing.

The effect of volatility on risk sharing feeds back on the price of government bonds, which also has implications for the cost of public finances. When government bonds are heavily used as collateral to back idiosyncratic insurance, an increase in aggregate volatility increases insurance by reducing idiosyncratic variance, thus moderating the increase in the price of public assets as their risk-sharing premium decreases. In contrast, when private assets are used as collateral to back idiosyncratic insurance, an increase in volatility reduces insurance, consequently adding idiosyncratic risk to consumption and thus providing an extra kick in increasing the price of public assets.

These results naturally apply to insurance markets among financial intermediaries and uncover a novel source of financial fragility. In an economy in which banks insure against idiosyncratic shocks (such as liquidity needs, shocks to specific asset positions, etc.) by using contracts that rely heavily on the use of private collateral, their risk sharing could be negatively affected by an increase in economic volatility. In this case, a large use of private assets as collateral in inter-bank transactions will positively correlate volatile times with financial fragility, as banks will find it harder to hedge their idiosyncratic shocks in interbank markets, putting their activities under stress.

Our results also have long-term implications if there is a trend in financial innovation that induces private assets to be used more intensively as collateral. Indeed, a recent literature has argued that the importance of private assets as collateral has increased dramatically in the U.S. over the past few decades, mostly through financial engineering and deregulation (see Gorton et al. 2012) and that this trend was curbed somewhat by the regulatory reforms that followed the financial crisis at the onset of the Great Recession of 2008 (such as the Dodd-Frank Act and new Basel III restrictions). Combining these results, we predict that risk sharing improved with private asset creation until the Great Recession but at the cost of making risk sharing within financial markets more fragile to sudden increases in aggregate volatility.

As risk sharing is one of the fundamental roles of interbank markets, the fact that an extensive use of private assets makes the system more fragile leads us to study what determines the creation of private assets for collateral. We introduce the possibility of creating private collateral (for example, securitization) and show that creating private collateral is more likely in a stable economic environment, as public debt is less valuable for intertemporal consumption smoothing and thus less valuable as collateral for risk sharing. In this sense, a stable economy is propitious to the creation and use of private assets as collateral, turning the financial system increasingly fragile to a sudden increase in future economic volatility. In short, economic stability plants the seeds of its own instability.

Our setting highlights the intricate relationship between the creation and valuation of private and public assets. On the one hand, it is not always the case that more government bond issuance induces less creation of private assets (the celebrated crowding out effect of government debt). On the other hand, financial innovation that increases the use of private collateral reduces the economy's exposure to idiosyncratic shock and the price of government bonds, increasing the government's cost of raising funds.

Our results have clear testable implications of the relation between the composition of private and public assets and risk sharing, and clear regulatory implications for policymakers that pay attention to financial stability. Unfortunately, directly measuring the composition of assets used as collateral in the financial sector is challenging, both for academics and policymakers, as it requires taking stands on what is considered a private asset given government guarantees, how assets are subject to double counting when they are used in several financial transactions, etc. Similar difficulties exist when directly measuring the extent of risk sharing in financial markets.

We bypass some of these challenges by using prices to infer the extent of risk sharing by measuring the *convenience yield* of safe assets—that is, the additional value assigned to assets (net of their payoff risks), which in our setting is because of their use as collateral in risk sharing contracts. The convenience yield indirectly captures the extent of risk sharing, as more risk sharing makes safe assets less valuable as collateral, and thus a reduction in the convenience yield is a signal that risk sharing is less relevant or is better hedged. We then empirically test whether the sensitivity of the convenience yield to aggregate volatility varies over time and, if so, whether it varies in the expected direction. Specifically, we test whether this sensitivity increases in periods when the literature suggests there was an extensive use of private assets as collateral in financial markets.

We perform two tests. First, using low frequency data, we show that this sensitivity has increased over time. While the convenience yield (and thus risk sharing) was barely reactive to aggregate volatility before the nineties, such sensitivity increased over the past few decades, suggesting a heightened role of private collateral in the economy. Second, we use high frequency data to zoom in on the active period surrounding the Great Recession, characterized by a large increase in private asset creation followed by a large drop after the 2008 financial crisis. We show that the sensitivity of the convenience yield (and thus risk sharing) to aggregate volatility increased dramatically leading up to the crisis and then declined while the crises unraveled. Furthermore, as the sensitivity of risk sharing to volatility has not increased in the years after the crisis, our model suggests that private assets never recovered their role as collateral at the extent seen before the financial crisis.

Related Literature: In this paper, we are particularly interested in the relationship between collateral and risk in the economy (both intertemporal and intratemporal risk). In contrast to most of the literature about the role of collateral, we abstract from traditional models with natural borrowers with investment projects (such as Kiyotaki and Moore 1997) or liquidity needs (such as Holmstrom and Tirole 1998). In our setting, agents are identical in an endowment economy, and so are the risks they face.

Our result is complementary to Gorton and Ordonez (2020), who also study the dual use of public and private assets as collateral but, in that case, to back productive loans. Their focus is on the role of informational fragility that mounts in the economy as private assets (heterogeneous and plagued by asymmetric information issues) become larger vis-a-vis public assets (more homogeneous and less subject to informational frictions). While that work highlights the *informational fragility* of collateral composition for productive reasons, here we study the *valuation fragility* of collateral composition for insurance reasons.

Krishnamurthy (2003), in the spirit of Kiyotaki and Moore (1997), also incorporates insurance markets in which assets serve as collateral to back insurance promises. In his work, the focus is on valuation effects stemming from collateral constrained insurance against aggregate shocks. Our focus is, instead, on insurance against idiosyncratic shocks and how aggregate volatility affects such insurance by affecting collateral valuation. We also introduce private safe asset creation and the relation between exogenous aggregate volatility and endogenous financial fragility.

Our framework is consistent with the work of Greenwood et al. (2015), Krishnamurthy and Vissing-Jorgensen (2015), and Sunderam (2014), who document the crowding out effect of public assets on private asset creation, and with Infante (2020), who points out that this sensitivity depends on whether the underlying collateral is publicly or privately produced. Our framework is also consistent with Mankiw (1986) and Constantinides and Duffie (1996) who show that risk premia increases if idiosyncratic shocks become more volatile during economic contractions. Our framework, however, explores the feedback between volatility and risk sharing through the endogenous value of collateral in the economy and its composition. Indeed, the relevance of housing valuations on the extent of risk sharing has been empirically documented in Hurst and Stafford (2004), Lustig and Van Nieuwerburgh (2010) and Hryshko et al. (2010) but not formally related to its private/public composition.

There is a rich macroeconomics literature that explores how risk sharing affects the relevance of volatility in the economy and, as such, asset prices. Storesletten et al. (2007) add life cycle and capital accumulation to these settings and show that these effects mitigate the role of idiosyncratic shocks to explain asset prices. Our main insight goes in the opposite direction: the effect of volatility on risk sharing.

The valuation effect described in this paper is related to the so called "negative beta" property of U.S. Treasury securities—that is, Treasuries appreciate in times of market stress. Connolly et al. (2005) and Baele et al. (2010) empirically document over recent U.S. history that, when volatility and illiquidity increase, Treasuries tend to appreciate while stocks tend to depreciate. In an international context, Caballero et al. (2017) and He et al. (2019) describe how government bonds issued from economies with reserve currency or safe asset status appreciate as global demand for safety increases. These papers allude to the "flight to safety" feature of government bonds. Our paper introduces a related but distinct channel—namely, government bonds' increased ability to hedge idiosyncratic risks can attenuate or exacerbate the negative beta effect, depending on whether the ratio of private to public assets used as collateral is low or high.

Finally, Brumm et al. (2018) quantitatively study how re-using private collateral increases leverage and volatility in the economy. Rampini and Viswanathan (2010) argue that higher collateralizability increases borrowing capacity, leverage, and risk, which incentivizes more productive borrowers to exhaust their debt capacity. In our setting, we explore the opposite direction, in which volatility affects the value of both private and public collateral to provide insurance against idiosyncratic shocks. Our finding that the sensitivity of risk sharing to volatility depends on the composition of collateral highlights why the relationship between leverage and volatility is not obvious nor monotonic.

The paper proceeds as follows. The next section presents a model with aggregate volatility in which public and private assets can be used as collateral to share idiosyncratic risks. Section 3 presents a tractable CARA-Normal case that allows for very clean comparative statics on the valuation of public and private assets and their use for risk sharing. Section 4 gives agents the ability to create private assets at a cost. In Section 5, we provide empirical evidence on the sensitivity of risk sharing to aggregate volatility and how it has changed over time with private asset creation and financial engineering. Section 6 concludes.

2 Model

In this section, we present the model in which we relate aggregate macro volatility with the extent of idiosyncratic risk sharing in the economy, assuming exogenous supply of private and public assets.

2.1 Set Up

Consider a three period ($t \in \{0, 1, 2\}$) endowment economy with two agents, called Raymond (R) and Shirley (S). Both agents have additive separable utility, with each period's consumption utility $u(\cdot)$ and discount factor β. Agents split equally an aggregate endowment, and additionally each agent receives an idiosyncratic endowment shock which is completely offset by the other agent's shock.[1] Specifically, Raymond (Shirley) receives a positive (negative) shock if it "rains" and a negative (positive) shock if it "shines." To maintain the aggregate endowment separated from idiosyncratic realizations, we assume that the probability of rain and shine are both $\frac{1}{2}$. Formally, agent i has the following endowment process:

$$e_{0i} = \frac{Y_0}{2}; \qquad \tilde{e}_{1i} = \frac{\tilde{Y}_1}{2} + \tilde{y}_i; \qquad \tilde{e}_{2i} = \frac{\tilde{Y}_2}{2},$$

where Y_t represents aggregate endowments (the tilde signifies that endowment shocks are t-measurable random variables). For Raymond, $\tilde{y}_i$ is either $\overline{y}$ if it rains or $-\overline{y}$ if it shines. Shirley has the opposing idiosyncratic endowment shock.

Supply and Demand of Assets: There are three assets in the economy: short-term government bonds, long-term government bonds, and a private asset. While in this section the total *supply* of public and private assets is fixed, the *demand* is determined by agents' maximization of expected payoff.

The government pays short- and long-term government bonds (the public assets) raising lump sum taxes on agents in the period the bonds mature. Because of the government's ability to tax agents, these assets will be considered safe—that is, they will always pay at par when they mature. We denote the face value of the total amount of short-term bonds by Θ_0^{Sh} and of long-term bonds by Θ_0. We assume the private asset's payoff is proportional to the aggregate endowment process, paying a dividend $\tilde{a}_t = \rho\tilde{Y}_t$, with $\rho \in (0, 1)$ in each period. Initially, each agent is endowed with half of a total private asset supply $\hat{\Theta}_0$.[2]

As the supply of public and private assets is exogenous, assets can be "scarce"—that is, there are not enough of them to back all private promises that fully hedge idiosyncratic risks. In period $t \in \{0, 1\}$, each agent will demand θ_{ti}^{Sh} of short-term government bonds, θ_{ti}

[1] We model an endowment process and not a Lucas tree to avoid the possibility of using the tree as collateral.

[2] There is no loss in generality in assuming that the private asset's payoff is correlated with the aggregate endowment process. In equilibrium, this assumption merely implies that agents exposure to aggregate risk is scaled by $(1 + \rho\hat{\Theta}_0)$: a part attributed to the endowment process 1 and a part attributed to agents' optimal holdings of the private asset $\rho\hat{\Theta}_0$. If the private asset payoff were independent of the endowment process, because of market clearing, agents' optimal portfolio holdings would still create a correlation between their consumption path and asset payoffs. Therefore, absent wealth effects, the introduction of additional risks orthogonal to the risk already embedded in the model does not have different qualitative effects.

of long-term government bonds, and $\hat{\theta}_{ti}$ of private assets at the market clearing price p_t^{Sh}, p_t, and p_t^a, respectively. We assume there are no short-sale constraints, thus $\theta_{ti}, \theta_{ti}^{Sh}$, and $\hat{\theta}_{ti}$ must be non-negative.

Risk Sharing and Collateral: At $t = 0$, agents are able to write state-contingent contracts among themselves depending on whether it rains or shines. We model these as "Arrow-Debreu" securities that pay one unit of the consumption good depending on whether it rains or shines. Importantly, we will assume that, if an agent sells an Arrow-Debreu security (effectively selling insurance for that state of the world), they must *fully* collateralize their promise with public or private assets.[3] If we denote by w_i^r and w_i^s—the amount of promises agent i makes in case it rains and shines, respectively (super scripts r and s denote the state "rain" and "shine", respectively)—the need of fully collateralizing the promise implies that

$$w_i^r \leq \theta_{0i}^{Sh} + \underline{p}_1 \theta_{0i} + \alpha \underline{p}_1^a \hat{\theta}_{0i}; \qquad w_i^s \leq \theta_{0i}^{Sh} + \underline{p}_1 \theta_{0i} + \alpha \underline{p}_1^a \hat{\theta}_{0i}. \tag{1}$$

A single contract against rain or shine can be thought of as a collateralized insurance contract.[4] As standard in the literature, the ability of an asset to collateralize a claim depends on the worst value it can take (the safe portion of the payoff that can be pledged in all states of the world). In terms of constraints (1), $\underline{p}_1$ and $\underline{p}_1^a$ are the lowest price the long-term government bond and the private asset can have in $t = 1$, respectively.[5] The parameter α captures the pledgeability of the private asset, which if inferior than public assets to serve as collateral (more limited pledgeability, informational frictions, etc.), implies $\alpha < 1$.

In what follows, we will denote the market trading price of contingent contracts for when it rains and shines by q^r and q^s, respectively.

Consumption: With all these elements, we can write agent Raymond's consumption

[3] Formally, because of collateralizability, these contracts are not real Arrow-Debreu securities. This market incompleteness implies that the first welfare theorem does not hold.

[4] In principle, agents can sell both idiosyncratic contracts, hedging against all states of nature to have a given consumption. This portfolio of contracts can be interpreted as, for example, a repo contract.

[5] Note that, i) collateral constraints are ex post, and ii) restricting by the lowest price of the asset requires that prices are bounded below by zero.

process in each period as,

$$c_{0R} = e_{0R} + a_0 \frac{\hat{\Theta}_0}{2} - p_0 \left(\theta_{0R} - \frac{\Theta_0}{2} \right) - p_0^{Sh} \left(\theta_{0R}^{Sh} - \frac{\Theta_0^{Sh}}{2} \right) - p_0^a \left(\hat{\theta}_{0R} - \frac{\hat{\Theta}_0}{2} \right) + q^r w_R^r + q^s w_R^s \quad (2)$$

$$\tilde{c}_{1R} = \tilde{e}_{1R} + \tilde{a}_1 \hat{\theta}_{0R} - \tilde{p}_1(\theta_{1R} - \theta_{0R}) + \left(\theta_{0R}^{Sh} - \frac{\Theta_0^{Sh}}{2} \right) - \tilde{p}_1^a(\hat{\theta}_{1R} - \hat{\theta}_{0R}) - w_R^r 1^r - w_R^s 1^s \quad (3)$$

$$\tilde{c}_{2R} = \tilde{e}_{2R} + \tilde{a}_2 \hat{\theta}_{1R} + \left(\theta_{1R} - \frac{\Theta_0}{2} \right). \quad (4)$$

The consumption process for Shirley takes the same form.

Timing: In $t = 0$, agents choose the amount of government bonds to purchase and contingent contracts to sign, taking into account the need to collateralize these contracts—that is, satisfy the inequalities of (1). In $t = 1$, agents rebalance their portfolio upon the realization of both the aggregate and idiosyncratic shocks. In $t = 2$, agents consume endowments and proceeds from their portfolio.

Given the symmetry of agents in period 0, each will end up with half the government supply of short- and long-term bonds, which will determine prices p_0^{Sh} and p_0. Each agent, however, has the possibility to rebalance his/her portfolio, demanding, for instance, more long-term bonds in period 1 or more private assets upon the idiosyncratic shock, once agents' endowments become asymmetric. Taxes are then collected at period 2 to redeem those bonds. Since there are no choices in period $t = 2$, the next subsections characterize backwards the optimal choices in periods $t = 1$ and $t = 0$.

2.2 Equilibrium in $t = 1$

In $t = 1$, after the endowment shock is realized and it rains or shines, $\tilde{p}_1 = p_1, \tilde{p}_1^a = p_1^a$, and $\tilde{c}_{1i} = c_{1i}$. Upon this realization, each agent rebalances their portfolio by choosing the optimal amount of long-term bonds (which are now one-period bonds) and private asset holdings. An agent $i \in \{R, S\}$'s first order conditions are

$$\begin{aligned} -p_1 u'(c_{1i}) + \beta \mathbb{E}_1(u'(\tilde{c}_{2i})) &\leq 0 \\ -p_1^a u'(c_{1i}) + \beta \mathbb{E}_1(u'(\tilde{c}_{2i}) \tilde{a}_2) &\leq 0. \end{aligned}$$

Raymond, for instance, holds both long-term bonds and private assets in equilibrium if

$$p_1 = \frac{\beta\mathbb{E}_1(u'(\tilde{c}_{2R}))}{u'(c_{1R})} \tag{5}$$

$$p_1^a = \frac{\beta\mathbb{E}_1(u'(\tilde{c}_{2R})\tilde{a}_2)}{u'(c_{1R})}, \tag{6}$$

which are the standard intertemporal pricing equations. The condition is identical for Shirley. Note that the price will depend on the aggregate shock in $t = 1$ and possibly the idiosyncratic shock. Specifically, if agents cannot fully insure, then their consumption in $t = 1$ will depend on whether it rains or shines, and possibly so will p_1 and p_1^a. Given the full symmetry of the problem, the price will be the same for both realizations of the idiosyncratic shock, as one of the two agents will always have the "good shock" and the other the "bad shock".

Denote by θ_{1i}^r and θ_{1i}^s agent i's portfolio in $t = 1$ when it rains or shines, respectively. Using agents' consumption path expressed in equations (3) and (4), we can write equilibrium conditions so that both agents hold the long-term government bond and private asset whenever it rains by,

$$\begin{aligned} &\frac{\mathbb{E}_1(u'(\frac{\tilde{Y}_2}{2} + \tilde{a}_2\hat{\theta}_{1R}^r + \theta_{1R}^r - \frac{\Theta_0}{2}))}{u'(\frac{Y_1}{2} + a_1\frac{\hat{\Theta}_0}{2} - p_1\left(\theta_{1R}^r - \frac{\Theta_0}{2}\right) - p_1^a\left(\hat{\theta}_{1R}^r - \frac{\hat{\Theta}_0}{2}\right) + (\overline{y} - w_R^r))} = \\ &\frac{\mathbb{E}_1(u'(\frac{\tilde{Y}_2}{2} + \tilde{a}_2\hat{\theta}_{1S}^r + \theta_{1S}^r - \frac{\Theta_0}{2}))}{u'(\frac{Y_1}{2} + a_1\frac{\hat{\Theta}_0}{2} - p_1\left(\theta_{1S}^r - \frac{\Theta_0}{2}\right) - p_1^a\left(\hat{\theta}_{1S}^r - \frac{\hat{\Theta}_0}{2}\right) - (\overline{y} - w_S^r))} \end{aligned} \tag{7}$$

$$\begin{aligned} &\frac{\mathbb{E}_1(u'(\frac{\tilde{Y}_2}{2} + \tilde{a}_2\hat{\theta}_{1R}^r + \theta_{1R}^r - \frac{\Theta_0}{2})\tilde{a}_2)}{u'(\frac{Y_1}{2} + a_1\frac{\hat{\Theta}_0}{2} - p_1\left(\theta_{1R}^r - \frac{\Theta_0}{2}\right) - p_1^a\left(\hat{\theta}_{1R}^r - \frac{\hat{\Theta}_0}{2}\right) + (\overline{y} - w_R^r))} = \\ &\frac{\mathbb{E}_1(u'(\frac{\tilde{Y}_2}{2} + \tilde{a}_2\hat{\theta}_{1S}^r + \theta_{1S}^r - \frac{\Theta_0}{2})\tilde{a}_2)}{u'(\frac{Y_1}{2} + a_1\frac{\hat{\Theta}_0}{2} - p_1\left(\theta_{1S}^r - \frac{\Theta_0}{2}\right) - p_1^a\left(\hat{\theta}_{1S}^r - \frac{\hat{\Theta}_0}{2}\right) - (\overline{y} - w_S^r))}. \end{aligned} \tag{8}$$

If these equilibrium conditions are satisfied, then both agents hold the long-term government bond and private asset whenever it rains. Note that, in order to have interesting equilibria, the idiosyncratic shock has to be large enough so that the maximum amount of insurance is not enough to hedge all idiosyncratic risks. Specifically, whenever (1) binds, we have

$\overline{y} - w_i^j > 0$ for state j. Market clearing is given by

$$\theta_{1R}^r + \theta_{1S}^r = \Theta_0; \qquad\qquad \hat{\theta}_{1R}^r + \hat{\theta}_{1S}^r = \hat{\Theta}_0.$$

Because we are interested in characterizing symmetric equilibria, the analysis is identical for when it shines.

We conjecture that conditions exist in which both agents' optimal private asset holdings are unaffected by the liquidity shock. Specifically, if trading of the long-term government bond is able to equalize agents' marginal rates of substitution, then the original private asset holdings of agents will be optimal. Given the possible reoptimization strategy in period 1, denote the optimal continuation value of Raymond's $t = 1$ utility by

$$U_R(\theta_{0R}^{Sh}, \theta_{0R}, \hat{\theta}_{0R}, w_R^r, w_R^s; \tilde{Y}_1) = Max_{\{\theta_{1R}, \hat{\theta}_{1R}\}} u(c_{1R}) + \beta \mathbb{E}_1(u(\tilde{c}_{2R})).$$

2.3 Equilibrium in $t = 0$

In $t = 0$, Raymond solves the following maximization problem:

$$Max_{\{\theta_{0R}^{Sh}, \theta_{0R}, \hat{\theta}_{0R}, w_R^r, w_R^s\}} u(c_{0R}) + \beta \mathbb{E}_0(U_R(\theta_{0R}^{Sh}, \theta_{0R}, \hat{\theta}_{0R}, w_R^r, w_R^s; \tilde{Y}_1)),$$

subject to the constraint (1) and that $\theta_{0R}^{Sh}, \theta_{0R}, \hat{\theta}_{0R} > 0$. Using the envelope condition, this problem leads to the following first order conditions,

$$\begin{aligned}
\theta_{0R}^{Sh} &: \quad -p_0^{Sh} u'(c_{0R}) + \beta \mathbb{E}_0(u'(\tilde{c}_{1R})) + (\xi_R^r + \xi_R^s) \leq 0 \\
\theta_{0R} &: \quad -p_0 u'(c_{0R}) + \beta \mathbb{E}_0(\tilde{p_1} u'(\tilde{c}_{1R})) + (\xi_R^r + \xi_R^s) \underline{p}_1 \leq 0 \\
\hat{\theta}_{0R} &: \quad -p_0^a u'(c_{0R}) + \beta \mathbb{E}_0((\tilde{a}_1 + \tilde{p_1^a}) u'(\tilde{c}_{1R})) + (\xi_R^r + \xi_R^s) \alpha \underline{p}_1^a \leq 0 \\
w_R^r &: \quad q^r u'(c_{0R}) - \frac{\beta}{2} \mathbb{E}_0(u'(\tilde{c}_{1R}^r)) - \xi_R^r = 0 \\
w_R^s &: \quad q^s u'(c_{0R}) - \frac{\beta}{2} \mathbb{E}_0(u'(\tilde{c}_{1R}^s)) - \xi_R^s = 0,
\end{aligned}$$

where ξ_R^r and ξ_R^s are the Lagrange multipliers associated with the collateral constraint in (1) for w_R^r and w_R^s, respectively; and $\tilde{c}_{tR}^r$ and $\tilde{c}_{tR}^s$ are Raymond's consumption when it rains and shines, respectively. It is natural to assume that in equilibrium, Raymond will buy insurance for when it shines and sell insurance for when it rains. That is, Raymond's collateral constraint will only bind when selling rain insurance. Similarly, Shirley's collateral constraint will only bind when selling shine insurance, thus $\xi_R^s = \xi_S^r = 0$. These state-specific constraints lead to the following pricing:

$$
\begin{aligned}
q^s &= \frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s)}{u'(c_{0R})}\right) \\
q^r &= \frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1S}^r)}{u'(c_{0S})}\right)
\end{aligned}
$$

and Lagrange multipliers

$$
\begin{aligned}
\xi_R^r &= q^r u'(c_{0R}) - \frac{\beta}{2}\mathbb{E}_0(u'(\tilde{c}_{1R}^r)) \\
\xi_S^s &= q^s u'(c_{0S}) - \frac{\beta}{2}\mathbb{E}_0(u'(\tilde{c}_{1S}^s))
\end{aligned}
$$

These conditions show that insurance for a "bad state of the world" (in terms of idiosyncratic shocks) is priced by the agent who needs it most. That is q^j is priced by the agent who suffers a negative idiosyncratic shock in state j. In the symmetric equilibrium, $q^r = q^s$ and $\xi_R^r = \xi_S^s$. In addition, Raymond sells insurance to Shirley and Shirley sells insurance to Raymond—that is, $w_R^r = -w_S^r$ and $w_S^s = -w_R^s$.

To close the model, market clearing is given by

$$
\theta_{0R}^{Sh} + \theta_{0S}^{Sh} = \Theta_0^{Sh}; \qquad \theta_{0R} + \theta_{0S} = \Theta_0; \qquad \hat{\theta}_{0R} + \hat{\theta}_{0S} = \hat{\Theta}_0.
$$

Pricing and Convenience Yield: Period 0 prices in the symmetric equilibrium are

$$
\begin{aligned}
p_0^{Sh} &= \beta\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R})}{u'(c_0)}\right) + \left[\frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s) - u'(\tilde{c}_{1R}^r)}{u'(c_0)}\right)\right] && (9) \\
p_0 &= \beta\mathbb{E}_0\left(\tilde{p}_1\frac{u'(\tilde{c}_{1R})}{u'(c_0)}\right) + \underline{p}_1\left[\frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s) - u'(\tilde{c}_{1R}^r)}{u'(c_0)}\right)\right] && (10) \\
p_0^a &= \beta\mathbb{E}_0\left((\tilde{a}_1 + \tilde{p}_1^a)\frac{u'(\tilde{c}_{1R})}{u'(c_0)}\right) + \alpha\underline{p}_1^a\left[\frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s) - u'(\tilde{c}_{1R}^r)}{u'(c_0)}\right)\right] && (11)
\end{aligned}
$$

The equations (9)–(11) directly show that, if agents cannot fully insure, all assets will have a *convenience yield* related to agents' limited abilities to hedge their idiosyncratic risk. The pricing effect of the convenience yield depends on how useful the asset is to collateralize the sale of insurance. That is, the manner in which the convenience yield affects prices depends on the worst possible price in $t = 1$ and the degree of pledgeability α. In this model, the convenience yield takes the simple expression of

$$CY := \frac{\beta}{2}\mathbb{E}_0 \left(\frac{u'(\tilde{c}^s_{1R}) - u'(\tilde{c}^r_{1R})}{u'(c_0)} \right) > 0 \tag{12}$$

and has an easy interpretation. The convenience yield is equal to the difference in marginal utility when agents suffer a bad idiosyncratic shock relative to a good one. That is, the additional value Raymond places on assets that can help write insurance contracts against "shiny days" is equal to the marginal decrease in consumption Raymond experiences when it shines relative to when it rains.

In equilibrium, the convenience yield will depend on the amount of safe assets in the economy, their degree of pledgeability, and the size of the idiosyncratic shock. From the expression in equation (12), it is clear that the convenience yield is non-negative (being zero only in the case of perfect insurance, thus no difference between the marginal utility of consuming in shiny or rainy days).

It is also useful to define the $t = 0$ price of a theoretical *risk-free security* that pays par in $t = 1$, in absence of idiosyncratic risks. That is, the frictionless value of funds in $t = 1$.

$$p^{rf}_0 := \beta\mathbb{E}_0 \left(\frac{u'(\tilde{c}_{1R})}{u'(c_0)} \right) = \frac{\beta}{2}\mathbb{E}_0 \left(\frac{u'(\tilde{c}^s_{1R}) + u'(\tilde{c}^r_{1R})}{u'(c_0)} \right) \tag{13}$$

3 Special Case with Closed-Form Solutions

In this section, we present comparative statics for a special case with simplifying assumptions that allow us to obtain closed-form solutions:

Assumption A1. *Consider a case with the following simplifying assumptions:*

1. *Preferences are characterized by CARA, with risk aversion γ.*
2. $\tilde{Y}_1 = Y_1 = 0$.
3. $\tilde{Y}_2 \sim N(\mu, \sigma^2)$.

In this simpler setting σ^2, which is the variance of aggregate endowment realizations in period 2, captures the uncertainty agents face in periods 0 and 1 about consumption in period 2 and fully captures aggregate volatility.

3.1 Characterization

This simplified case is useful for the following reasons. First, CARA utility eliminates wealth effects, so agents' optimal risky asset holdings in $t = 1$ do not depend on the idiosyncratic shock, nor does $t = 1$ prices. Second, having a deterministic endowment shock in $t = 1$ means that the $t = 1$ price of the long-term government bond and the private asset are known in $t = 0$. Therefore, the worst-case outcome is merely the price in $t = 1$. Third, normality in the final aggregate shock allows to recover a simple formulation of prices in $t = 1$, facilitating comparative statics.

Allocations: The benefit of this special set of assumptions is to dramatically simplify equations (7) and (8), as the marginal rates of substitution between $t = 1$ and $t = 2$ must be the same for both agents. We conjecture then that the optimal portfolio choice is each agent holding half of the private asset ($\hat{\theta}^r_{1R} = \hat{\theta}^r_{1S} = \hat{\Theta}_0/2$) and that the long-term government bond is used to smooth the idiosyncratic shock. This implies that

$$(1 + p_1)\left(\theta^r_{1R} - \frac{\Theta}{2}\right) = \overline{y} - w,$$

giving the following optimal portfolio holdings for when it rains in $t = 1$:

$$\hat{\theta}^r_{1R} = \hat{\theta}^s_{1R} = \frac{\hat{\Theta}_0}{2}; \qquad \theta^r_{1R} = \frac{(\overline{y} - w)}{1 + p_1} + \frac{\Theta_0}{2}; \qquad \theta^r_{1S} = -\frac{(\overline{y} - w)}{1 + p_1} + \frac{\Theta_0}{2},$$

with $w = \frac{\Theta^{Sh}}{2} + p_1\frac{\Theta_0}{2} + \alpha p^a_1\frac{\hat{\Theta}_0}{2}$. When it rains, Raymond buys some extra long-term government bonds from Shirley. This is intuitive. As Shirley suffers a bad idiosyncratic shock when it rains, she would optimally sell bonds to Raymond to consume more in $t = 1$. Thus, the optimal consumption paths in the symmetric equilibrium are

$$c_{0R} = \frac{Y_0}{2} + a_0 \frac{\hat{\Theta}_0}{2} \tag{14}$$

$$c^r_{1R} = \frac{(\overline{y} - w)}{(1+p_1)}; \qquad c^s_{1R} = -\frac{(\overline{y} - w)}{(1+p_1)} \tag{15}$$

$$\tilde{c}^r_{2R} = \frac{\tilde{Y}_2}{2} + \tilde{a}_2 \frac{\hat{\Theta}_0}{2} + \frac{(\overline{y} - w)}{(1+p_1)}; \qquad \tilde{c}^s_{2R} = \frac{\tilde{Y}_2}{2} + \tilde{a}_2 \frac{\hat{\Theta}_0}{2} - \frac{(\overline{y} - w)}{(1+p_1)}. \tag{16}$$

By mere inspection of the optimal consumption paths, with CARA utility, the difference in consumption between rain and shine cancels out and, thus, does not affect $t = 1$ pricing. As we advanced, without wealth effects, agents' private asset holdings are their original ones, and only long-term government bonds are used to smooth the idiosyncratic shock.

Exploiting that collateral prices are deterministic (that is, the lowest values for the long-term government bond and the private asset are their trading price because $\tilde{Y}_1 = Y_1 = 0$) both agents hold the long-term government bond (that is, $\theta^r_{1R}, \theta^r_{1S} > 0$) if

$$\frac{\Theta_0}{2}(1+p_1) - \left(\overline{y} - \underbrace{\left(\frac{\Theta_0^{Sh}}{2} + p_1 \frac{\Theta_0}{2} + \alpha p_1^a \frac{\hat{\Theta}_0}{2} \right)}_{=w} \right) > 0. \tag{17}$$

To have an interesting result, we need to ensure that the idiosyncratic shock cannot be fully diversified away. That is,

$$\overline{y} - \underbrace{\left(\frac{\Theta_0^{Sh}}{2} + p_1 \frac{\Theta_0}{2} + \alpha p_1^a \frac{\hat{\Theta}_0}{2} \right)}_{=w} > 0. \tag{18}$$

Prices: Using equations (5) and (6), with CARA utility we have,

$$\begin{aligned} p_1 &= \beta \mathbb{E}_1 \left(\exp \left\{ -\frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0\right) \tilde{Y}_2 \right\} \right) \\ &= \beta \exp \left\{ -\frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0\right) \mu + \frac{1}{8} \gamma^2 \left(1 + \rho \hat{\Theta}_0\right)^2 \sigma^2 \right\} \qquad (19) \\ p_1^a &= \beta \mathbb{E}_1 \left(\exp \left\{ -\frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0\right) \tilde{Y}_2 \right\} \rho \tilde{Y}_2 \right) \\ &= \rho \left(\mu - \frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0\right) \sigma^2 \right) p_1, \qquad (20) \end{aligned}$$

where we have used the fact that $\tilde{a}_t = \rho \tilde{Y}_t$ and $\tilde{Y}_2$ is normally distributed. Note that, because there are no wealth effects, the price in $t = 1$ only depends on model parameters.

In what follows, we focus on the case in which the expected return is high enough to have positive pricing but not so high as to make it more valuable than the risk-free rate. Put differently, the private asset's certainty equivalent is less than one, making it less attractive as a store of value. In addition, to put some discipline to the model, we assume that agents' preferences and the private asset's distribution satisfy the Hansen-Jagannathan bounds:

Assumption A2. *Assume the private asset price is positive but lower than the long-term bond price. That is*

$$0 \leq \left(\mu - \frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0\right) \sigma^2 \right) \leq 1 \qquad (21)$$

and the Hansen-Jagannathan bounds for pricing in $t = 1$ hold.[6] *That is*

$$\left| \left(\mu - \frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0\right) \sigma^2 \right) \frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0\right) \right| \leq \exp \left\{ \frac{1}{4} \gamma^2 \left(1 + \rho \hat{\Theta}_0\right)^2 \sigma^2 \right\} - 1.$$

Given these closed-form expressions and related assumptions, the next theorem gives the conditions for the existence of a symmetric equilibrium.

Theorem 1 (Existence of Symmetric Equilibrium)**.** *If Assumption A1 and A2 hold,* $\overline{y} \in [\frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha \frac{\hat{\Theta}_0}{2}, \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha \rho (\mu - \frac{\gamma}{2}(1 + \rho \hat{\Theta}_0) \sigma^2) \frac{\hat{\Theta}_0}{4}]$, $\beta > \frac{1}{2}$, *and* $\frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0\right) \sigma$ *is sufficiently small, there exists a symmetric equilibrium characterized by the consumption paths in equations* (14)–(16) *and prices in equations* (10)–(11) *and* (19)–(20)*.*

[6] See Lemma 3 in appendix B for the derivation of the Hansen-Jagannathan bound in this context.

Proof. We only have to ensure that Raymond and Shirley hold both long-term government bonds and private assets in $t = 1$ and that the idiosyncratic shock is large enough so that agents cannot fully hedge their idiosyncratic risk. That is, the inequalities in (17) and (18) hold. From (19), because of condition (21) and $\rho \in (0, 1)$, $p_1, p_1^a \in (0, 1)$. In addition, if $\frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\sigma$ is sufficiently small enough $p_1 > \frac{1}{2}$. In effect, using the Hansen-Jagannathan bound of Assumption A2 we know that

$$\begin{aligned}
\ln(2p_1) &= \ln(2\beta) - \frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\left(\mu - \frac{1}{4}\gamma\left(1 + \rho\hat{\Theta}_0\right)\sigma^2\right) \\
&= \ln(2\beta) - \frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\left(\mu - \frac{1}{2}\gamma\left(1 + \rho\hat{\Theta}_0\right)\sigma^2\right) - \frac{1}{8}\gamma^2\left(1 + \rho\hat{\Theta}_0\right)^2\sigma^2 \\
&\geq \ln(2\beta) - \left(\exp\left\{\frac{1}{4}\gamma^2\left(1 + \rho\hat{\Theta}_0\right)^2\sigma^2\right\} + \frac{1}{8}\gamma^2\left(1 + \rho\hat{\Theta}_0\right)^2\sigma^2 - 1\right) \geq 0,
\end{aligned}$$

where we have use the Hansen-Jagannathan bound, $\log(2\beta) > 0$, and that $g(x) = \exp(x) + x/2 - 1$ is equal to zero when $x = 0$ and strictly increasing. This ensures that if $\frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\sigma$ is sufficiently small enough $p_1 > \frac{1}{2}$. Therefore,

$$\overline{y} \geq \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}_0}{2} > \frac{\Theta_0^{Sh}}{2} + p_1\frac{\Theta_0}{2} + \alpha p_1^a\frac{\hat{\Theta}_0}{2},$$

guaranteeing condition (18), and

$$\overline{y} \leq \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1 + \rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}_0}{4} < \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2}(1 + 2p_1) + \alpha p_1^a\frac{\hat{\Theta}_0}{2},$$

guaranteeing condition (17).

The parameter space for Theorem 1 is feasible if $\Theta_0 > \alpha\left(1 - \frac{\rho}{2}(\mu - \frac{\gamma}{2}(1 + \rho\hat{\Theta}_0)\sigma^2)\right)\hat{\Theta}_0$. □

It is useful to express the initial prices of all securities as a function CY (see equation (12)), p_0^{rf} (see equation (13)), and $t = 1$ prices. In this case, equations (9)–(11) can be expressed as

$$
\begin{aligned}
p_0^{Sh} &= p_0^{rf} + CY && (22)\\
p_0 &= p_1\left(p_0^{rf} + CY\right) && (23)\\
p_0^a &= p_1^a\left(p_0^{rf} + \alpha CY\right). && (24)
\end{aligned}
$$

That is, prices in $t = 0$ can be decomposed into the traditional convenience yield, the frictionless value of funds in $t = 1$, and the price of assets in $t = 1$.

3.2 Comparative Statics

Having characterized the symmetric equilibrium and ensured its existence, we now study how prices change with changes in the supply of both short- and long-term public assets (Θ_0^{Sh} and Θ_0), the severity of idiosyncratic shocks ($\bar{y}$), the pledgeability of private assets (α), and most importantly, aggregate volatility (σ^2). We defer the comparative statics with respect to the supply of private assets for the next section, in which we endogenize it.

Given that asset prices in (22) – (24) are expressed in terms of the traditional risk-free rate and the convenience yield, the following lemma is useful to generalize the model's comparative statics with respect to any of our parameters of interest,

Lemma 1. *(Sensitivity of risk-free rates and convenience yields). Given the equilibrium characterized in Theorem 1, for any model parameter z, we have the following comparative statics,*

$$
\begin{aligned}
\frac{\partial p_0^{rf}}{\partial z} &= -\gamma CY \frac{\partial c_{1R}^s}{\partial z} + \gamma p_0^{rf} \frac{\partial c_{0R}}{\partial z}\\
\frac{\partial CY}{\partial z} &= -\gamma p_0^{rf} \frac{\partial c_{1R}^s}{\partial z} + \gamma CY \frac{\partial c_{0R}}{\partial z}.
\end{aligned}
$$

Proof. First, note that $\frac{\partial c_{1R}^s}{\partial z} = -\frac{\partial c_{1R}^s}{\partial z}$ for all z (see equations (15)). The result comes from taking the derivative of equations (12) and (13), and noting that for CARA utility $u''(z) = -\gamma u'(z)$. □

The interpretation of these sensitivities with respect to any parameter z is informative of the comparative statics with respect to the needs of risk sharing—what we call *risk sharing* *effects*. Assume, for example, that a change in z increases Raymond's consumption when it shines, then reduces the need for risk sharing. In this case, the price of the frictionless risk-free bond is lower as agents' abilities to transfer wealth from $t = 0$ to $t = 1$ is less relevant, because their exposure to idiosyncratic shocks is lower. The decrease is proportional to the wedge created by agents' limited hedging abilities: the convenience yield, CY. This direct effect of better risk sharing on the price of the risk-free bond is captured by the first term in the first equation of the lemma.

Similarly, if an increase in z reduces the needs of risk sharing, the convenience yield is lower and proportional to the value of funds needed to smooth consumption across states—that is, the price of the risk-free bond. Put differently, as agents need less, or are better at, risk sharing across states, they need less funds across those states, valued at p_0^{rf}. This direct effect of better risk sharing on the convenience yield is captured by the first term in the second equation of the Lemma.

3.2.1 Changes in the Supply of Public Assets

Assume the government increases the total amount of both short- and long-term government bonds. Because the government raises lump sum taxes in periods the bonds mature, and because, in $t = 1$, the difference in consumption between rain and shine cancels out, changes in issuance would not change consumption paths, not affecting prices in $t = 1$ (see equations (19)–(20)). The increase in the supply of public assets, however, affects the possibilities of risk sharing. From equations (14)–(16), we have

$$\frac{\partial c_{1R}^s}{\partial \Theta_0} = \frac{p_1}{2(1+p_1)} > 0 \qquad \texttt{and} \qquad \frac{\partial c_{0R}}{\partial \Theta_0} = 0.$$

The case for Θ_0^{Sh} is similar, except that the partial derivatives with respect to consumption are proportional to $\frac{1}{2(1+p_1)}$.

This partial derivative shows that an increase in short- and long-term government bonds increases consumption in the bad state (of Raymond when it shines and of Shirley when it rains) by improving risk sharing. This is a pure collateral effect: the economy has more collateral to sustain insurance promises, which improves hedging against idiosyncratic shocks. This effect shows up in the price of public and private assets in period 0. The next proposition characterizes these changes.

Proposition 1. *(Asset Pricing Effects of the Supply of Public Assets). Given the equilibrium characterized in Theorem 1, the initial prices of the short-term government bond, long-term government bond, and private asset have the following comparative statics with respect to the supply of government bonds, Θ_0^{Sh} and Θ_0,*

$$\begin{aligned}
\frac{\partial p_0^{Sh}}{\partial \Theta_0^{Sh}} &= -\frac{\gamma\left(p_0^{rf}+CY\right)}{2(1+p_1)}, \quad \frac{\partial p_0}{\partial \Theta_0^{Sh}} = -\frac{\gamma p_1\left(p_0^{rf}+CY\right)}{2(1+p_1)}, \quad \frac{\partial p_0^a}{\partial \Theta_0^{Sh}} = -\frac{\gamma p_1^a\left(\alpha p_0^{rf}+CY\right)}{2(1+p_1)} \\
\frac{\partial p_0^{Sh}}{\partial \Theta_0} &= -\frac{\gamma p_1\left(p_0^{rf}+CY\right)}{2(1+p_1)}, \quad \frac{\partial p_0}{\partial \Theta_0} = -\frac{\gamma p_1^2\left(p_0^{rf}+CY\right)}{2(1+p_1)}, \quad \frac{\partial p_0^a}{\partial \Theta_0} = -\frac{\gamma p_1 p_1^a\left(\alpha p_0^{rf}+CY\right)}{2(1+p_1)}
\end{aligned}$$

Proof. The result comes from directly applying Lemma 1 to (22)– (24). □

The Proposition shows that having more public assets that improve risk sharing makes all assets, both public and private, less valuable as collateral, reducing their price. Also, because $p_1 < 1$, the effect of short-term government bond supply is larger than the impact of long-term government bond supply. Thus, changes in short-term government bond supply are more effective at increasing risk sharing, making the impact on prices larger.[7]

The effect on the price of the private asset is also proportional to the convenience yield and depends on its collateralizability. Interestingly, even if the private asset were not pledgeable at all (that is, $\alpha = 0$), the increase in idiosyncratic risk would still lower its price, since private assets are also useful to transfer wealth to $t = 1$. Still, a higher α makes the private asset more sensitive to changes in the supply of public assets, because of the extra change in their value as collateral.

3.2.2 Changes in Private Asset Pledgeability

Now, assume there is an increase in pledgeability α, perhaps by financial innovation or deregulation. This increase makes risk sharing easier by indirectly increasing the supply of collateral in the economy. As before, prices in $t = 1$ do not change, but allocations, from equations (14)–(16) do. Specifically, we have

$$\frac{\partial c_{1R}^s}{\partial \alpha} = \frac{p_1^a}{(1+p_1)}\frac{\hat{\Theta}_0}{2} > 0 \qquad \texttt{and} \qquad \frac{\partial c_{0R}}{\partial \alpha} = 0.$$

[7]This result is consistent with Krishnamurthy and Vissing-Jorgensen (2012), Greenwood et al. (2015), and Infante (2020), who show that the convenience yield, and thus the price of safe assets, is more sensitive to changes in T-bill outstanding than changes in longer term U.S. Treasury bonds outstanding.

The following proposition summarizes the effect on prices:

Proposition 2. *(Asset Pricing Effects of the Pledgeability of Private Assets). Given the equilibrium characterized in Theorem 1, the initial prices of the short- and long-term government bond and the private asset have the following comparative statics with respect to the pledgeability of private assets α,*

$$
\begin{aligned}
\frac{\partial p_0^{Sh}}{\partial \alpha} &= -\gamma(p_0^{rf} + CY)\frac{p_1^a}{(1+p_1)}\frac{\hat{\Theta}_0}{2}, \\
\frac{\partial p_0}{\partial \alpha} &= -\gamma p_1(p_0^{rf} + CY)\frac{p_1^a}{(1+p_1)}\frac{\hat{\Theta}_0}{2}, \\
\frac{\partial p_0^a}{\partial \alpha} &= -\gamma p_1^a(\alpha p_0^{rf} + CY)\frac{p_1^a}{(1+p_1)}\frac{\hat{\Theta}_0}{2} + p_1^a CY
\end{aligned}
$$

Proof. The result comes from directly applying Lemma 1 to (22)— (24). □

The intuition is parallel to the increase in public assets, but the effect on the private asset price is now ambiguous. On the one hand, private assets become more useful as collateral, becoming more valuable, which operates through $p_1^a CY$. On the other hand, the implied improvement in risk sharing tends to reduce the value of all assets.

3.2.3 Changes in Idiosyncratic Volatility

Assume now that the variance of idiosyncratic shocks increase, which given the binomial structure of idiosyncratic shocks is captured by its size, $\overline{y}$. This increases the needs for risk sharing. As with government bonds, because of CARA utility, the realization of the shock does not affect prices in $t = 1$ (see equations (19)–(20)). In terms of allocations, from equations (14)–(16) we have

$$
\frac{\partial c_{1R}^s}{\partial \overline{y}} = -\frac{1}{(1+p_1)} > 0 \qquad \texttt{and} \qquad \frac{\partial c_{0R}}{\partial \overline{y}} = 0.
$$

Proposition 3 characterizes the effect of more risk sharing needs on prices.

Proposition 3. *(Asset Pricing Effects of Idiosyncratic Volatility). Given the equilibrium characterized in Theorem 1, the initial prices of the short-term government bond, long-term* *government bond, and private asset have the following comparative statics with respect to the size of idiosyncratic shocks $\overline{y}$,*

$$\frac{\partial p_0^{Sh}}{\partial \overline{y}} = \frac{\gamma\left(p_0^{rf} + CY\right)}{(1+p_1)}, \quad \frac{\partial p_0}{\partial \overline{y}} = \frac{\gamma p_1\left(p_0^{rf} + CY\right)}{(1+p_1)}, \quad \frac{\partial p_0^a}{\partial \overline{y}} = \frac{\gamma p_1^a\left(\alpha p_0^{rf} + CY\right)}{(1+p_1)}$$

Proof. The result comes from directly applying Lemma 1 to (22)– (24). $\square$

As idiosyncratic shocks become larger, the extent of idiosyncratic insurance decreases, making all assets more valuable as collateral for risk sharing purposes.

3.3 Changes in Aggregate Volatility

In this subsection, we present our main result. We assume an increase in aggregate volatility, which is captured by an increase in the variance of period 2 aggregate realizations, σ^2. In contrast to the previous cases, this generates a direct impact on prices at $t = 1$. From equations (19)–(20),

$$\begin{aligned}
\frac{\partial p_1}{\partial \sigma^2} &= \frac{\gamma^2}{8}\left(1+\rho\hat{\Theta}_0\right)^2 p_1 \\
\frac{\partial p_1^a}{\partial \sigma^2} &= \rho\left(\mu - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\frac{\partial p_1}{\partial \sigma^2} - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\rho p_1 \\
&= \frac{p_1^a}{p_1}\frac{\partial p_1}{\partial \sigma^2} - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\rho p_1
\end{aligned}$$

The $t = 1$ price of the long-term bond always increases when volatility increases. This can be interpreted as a standard "negative beta" effect of government bonds: as aggregate volatility increases, the need to smooth consumption intertemporally from $t = 1$ to $t = 2$ increases, making long-term government bonds more valuable.

The $t = 1$ price effect on the private asset is, however, more intricate. On the one hand, similar to the long-term government bond, there is a "negative beta" effect proportional to the private asset's certainty equivalent (the first term). On the other hand, more volatility implies that the private asset is less desirable per se, as it encompass part of the aggregate risk, putting downward pressure on its price (the second term). We focus on the economically interesting case in which the second effect dominates and the private asset's price declines with aggregate volatility, $\frac{\partial p_1^a}{\partial \sigma^2} < 0$. The next assumption characterizes the parametric condition for this to happen

Assumption A3. *Assume aggregate volatility depresses the price of private assets,* $\frac{\partial p_1^a}{\partial \sigma^2} < 0$*, which is guaranteed if*

$$\frac{\gamma}{4}\left(1+\rho\hat{\Theta}_0\right)\left(\mu-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right) < 1. \tag{25}$$

This assumption is ensured for $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma$ sufficiently small.[8] We highlight later the role of this natural assumption for the effect of aggregate volatility on risk sharing.

Remark of the plausibility of Assumption A3: Even though this assumption seems natural, usually it is difficult to test given the lack of a purely exogenous shock on aggregate volatility. The recent crisis caused by the outbreak of the COVID-19 virus constitutes, however, a unique shock to aggregate volatility and higher future uncertainty—characterized by being exogenous, unexpected, significant, without an end in sight, and truly aggregate as it affects all countries at once. Given this unique event, we are in a position to test these pricing assumptions. In Figure A.1 of Appendix A, we use VIX as a measure of aggregate volatility, which was relatively stable during 2018 and 2019 and exhibited a large and sudden increase starting in February 2020 with the COVID-19 outbreak. As the VIX was stable, the spread between public and private yields was roughly constant. In February of 2020, as the news about the COVID-19 virus spread, the behavior of public and private yields started moving in opposite directions, consistent with Assumption A3.

How do these changes in $t = 1$ prices affect allocations, in particular the extent of risk

[8] Note that the parameter space assumed in Proposition 4 is non-empty. In effect, the equilibrium in Theorem 1 exists if $\Theta_0 > \alpha\left(1-\frac{1}{2}\frac{p_1^a}{p_1}\right)\hat{\Theta}_0$. This condition can simultaneously hold with $\alpha\frac{p_1^a}{p_1}\hat{\Theta}_0 > \Theta_0$ if $\frac{p_1^a}{p_1} > \frac{2}{3}$, which depends on the total amount of collateralizability of the private asset's certainty equivalent relative to the amount of long-term government bonds. Using the Hansen-Jagannathan bounds, condition (25) holds if $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma < \sqrt{\ln(3)}$.

sharing in the economy? From equations (14)–(16), we have

$$\frac{\partial c_{1R}^{s}}{\partial \sigma^2} = \underbrace{\frac{1}{(1+p_1)}\left[\frac{\partial p_1}{\partial \sigma^2}\frac{\Theta_0}{2} + \alpha\frac{\partial p_1^a}{\partial \sigma^2}\frac{\hat{\Theta}_0}{2} + \frac{(\overline{y}-w)}{(1+p_1)}\frac{\partial p_1}{\partial \sigma^2}\right]}_{\text{Valuation Effect}} \tag{26}$$

$$= \frac{1}{(1+p_1)^2}\left[\overline{y} - \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2}\right]\frac{\partial p_1}{\partial \sigma^2} - \frac{\alpha}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\frac{\hat{\Theta}_0}{2}$$

and $\frac{\partial c_{0R}}{\partial \sigma^2} = 0$. The effect of higher aggregate volatility on Raymond's consumption when it shines comes through changes in the price of assets in $t = 1$, since those assets are used as collateral in $t = 0$ to back promises that mitigate the effects of idiosyncratic shocks. That is, *risk sharing is affected by aggregate volatility purely by a valuation effect.*

The overall impact is mixed. On the one hand, the price of long-term bonds in $t = 1$ increases, improving risk sharing (this is the first term in equation 26) and the amount of funds received when selling the long-term bond in $t = 1$ (this is the third term in equation 26). On the other hand, the price of private assets (under Assumption A3) in $t = 1$ decreases, weakening risk sharing (this is the second term in equation 26). The net impact depends on the relative amount of public and private assets used as collateral, which itself depends on the supply of assets and the private assets' certainty equivalent and pledgablility. If the economy depends heavily on private assets, overall consumption in the bad state decreases, reducing risk sharing. Finally, because aggregate volatility in $t = 2$ does not affect the price of short-term government bonds in $t = 1$, it does not affect risk sharing through the use of short term bonds as collateral. These observations are summarized in the following proposition

Proposition 4. *(Risk Sharing Effects of Aggregate Volatility). Given the equilibrium characterized in Theorem 1 and Assumption A3, if more private assets are used as collateral than long-term government bonds—that is, $\alpha p_1^a\hat{\Theta}_0 > p_1\Theta_0$ (in terms of parameters this condition is $\alpha\rho(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2)\hat{\Theta}_0 > \Theta_0$)— then an increase in aggregate volatility σ^2 reduces risk sharing—that is, $\frac{\partial c_{1R}^s}{\partial \sigma^2} < 0$. Moreover, the decrease in risk sharing is larger if the private asset is more pledgeable—that is, $\frac{\partial^2 c_{1R}^s}{\partial \alpha \partial \sigma^2} < 0$.*

Proof. See Appendix B. □

When agents support risk sharing more with private assets than with public long-term assets, the increase in aggregate volatility decreases insurance through a decrease in aggregate collateralizability, as the most relevant asset to hedge idiosyncratic risk becomes less valuable. In addition, the proposition also shows that, when the private asset becomes more useful as collateral, captured by α, then the negative sensitivity of risk sharing to aggregate volatility becomes even stronger. Intuitively, when private assets become more important as collateral, a reduction in their price triggered by an increase in aggregate volatility becomes more pervasive for risk sharing. Finally, notice that relaxing Assumption A3 implies that the price of both public and private assets would, perhaps counterfactually, increase with aggregate volatility, unconditionally improving risk sharing in the economy.

While Proposition 4 studies how aggregate volatility affects allocations, Proposition 5 characterizes how it affects asset prices at $t = 0$ through its role of facilitating (or not) risk sharing.

Proposition 5. *(Asset Pricing Effects of Aggregate Volatility). Given the equilibrium characterized in Theorem 1, the initial prices of the short-term government bond, long-term government bond, and private asset have the following comparative statics with respect to* σ^2*,*

$$\begin{aligned} \frac{\partial p_0^{Sh}}{\partial \sigma^2} &= -\gamma(p_0^{rf} + CY)\frac{\partial c_{1R}^s}{\partial \sigma^2} \\ \frac{\partial p_0}{\partial \sigma^2} &= -\gamma p_1(p_0^{rf} + CY)\frac{\partial c_{1R}^s}{\partial \sigma^2} + (p_0^{rf} + CY)\frac{\partial p_1}{\partial \sigma^2} \\ \frac{\partial p_0^a}{\partial \sigma^2} &= -\gamma p_1^a(\alpha p_0^{rf} + CY)\frac{\partial c_{1R}^s}{\partial \sigma^2} + (p_0^{rf} + \alpha CY)\frac{\partial p_1^a}{\partial \sigma^2}. \end{aligned}$$

Proof. The result comes from directly applying Lemma 1 to (22)– (24). □

Proposition 5 shows that the effect of aggregate volatility on $t = 0$ prices depends on two forces: a direct effect on the asset itself and an indirect effect on facilitating risk sharing.

The direct, asset-specific effect of aggregate volatility on long-term government bonds and private assets comes from their value changing in $t = 1$. While the value of long-term bonds increases with volatility, the value of private assets decreases (under Assumption A3). The indirect effect of aggregate volatility on risk sharing depends on the composition of collateral. If there are fewer private assets used as collateral, then there is more idiosyncratic insurance and the $t = 0$ value of all securities are lower as their convenience yield declines. Consequently, the impact on prices operates through *risk sharing effects*: if aggregate volatility improves risk sharing, all assets become less valuable as collateral.

These results underscore that the composition of collateral is important to understand the overall impact of aggregate volatility on risk sharing and asset prices. The reason is that public assets also serve to smooth consumption intertemporally, while private assets inherit part of that volatility. When the economy, for some reason, relies heavily on private assets for hedging against idiosyncratic shocks, an increase in volatility may generate a sudden dry up of risk sharing. A relevant question, then, is what determines such composition? In the next section, we endogeneize the private creation of assets and collateral.

4 Private Asset Creation

In this section, we entertain the idea that Raymond and Shirley have the ability to create private assets at a cost. We then give conditions under which supplying public assets can crowd out or crowd in private assets. To build intuition toward tackling this question, we first study the effect of an exogenous supply of private assets on risk sharing and prices, as these considerations will enter into agents' decisions to create private assets.

4.1 Model with private asset creation

Assume the cost of producing x units of private assets is $C(x)$ in terms of consumption goods, with $C', C'' > 0$. Agents incur this cost before choosing their portfolio in $t = 0$ and sell these assets (perhaps to themselves) at the equilibrium price p_0^a. Focusing on Raymond's consumption path, we have

$$
\begin{aligned}
c_{0R} &= e_{0R} + a_0 \frac{\hat{\Theta}_0}{2} - p_0 \left(\theta_{0R} - \frac{\Theta_0}{2} \right) - p_0^{Sh} \left(\theta_{0R}^{Sh} - \frac{\Theta_0^{Sh}}{2} \right) - p_0^a \left(\hat{\theta}_{0R} - \left(\frac{\hat{\Theta}_0}{2} + x_R \right) \right) \\
&\quad + q^r w_R^r + q^s w_R^s - C(x_R) \\
\tilde{c}_{1R} &= \tilde{e}_{1R} + \tilde{a}_1 \hat{\theta}_{0R} - \tilde{p}_1 (\theta_{1R} - \theta_{0R}) + \left(\theta_{0R}^{Sh} - \frac{\Theta_0^{Sh}}{2} \right) - \tilde{p}_1^a (\hat{\theta}_{1R} - \hat{\theta}_{0R}) - w_R^r 1^r - w_R^s 1^s \\
\tilde{c}_{2R} &= \tilde{e}_{2R} + \tilde{a}_2 \hat{\theta}_{1R} + \left(\theta_{1R} - \frac{\Theta_0}{2} \right)
\end{aligned}
$$

where x_R is the amount of private assets Raymond creates. If we assume that each agent does not internalize their effects on prices, through the envelope condition we know that Raymond's optimal production of assets is given by

$$C'(x_R^*) = p_0^a. \tag{27}$$

Thus, given the problem's symmetry (Shirley faces the same problem at $t = 0$), the total stock of private assets is given by $\hat{\Theta} = \hat{\Theta}_0 + 2x_R^*$, and all the previous pricing equations hold simply replacing $\hat{\Theta}_0$ with $\hat{\Theta}$.

Exploiting again the specific case under Assumption A1, optimal consumption paths are

$$c_{0R} = \frac{Y_0}{2} + a_0\frac{\hat{\Theta}}{2} - C(x_R^*) \tag{28}$$

$$c_{1R}^r = \frac{(\overline{y} - w)}{(1 + p_1)}; \qquad c_{1R}^s = -\frac{(\overline{y} - w)}{(1 + p_1)} \tag{29}$$

$$\tilde{c}_{2R}^r = \frac{\tilde{Y}_2}{2} + \tilde{a}_2\frac{\hat{\Theta}}{2} + \frac{(\overline{y} - w)}{(1 + p_1)}; \qquad \tilde{c}_{2R}^s = \frac{\tilde{Y}_2}{2} + \tilde{a}_2\frac{\hat{\Theta}}{2} - \frac{(\overline{y} - w)}{(1 + p_1)}. \tag{30}$$

If we also adopt the parameter restrictions on agents' preferences and the private asset's distribution, described in Assumption A2, we have the following equilibrium characterization:

Theorem 2 (Existence of Symmetric Equilibrium with Private Asset Creation)**.** *If Assumption A1 and A2 hold,* $\overline{y} \in (\frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}_0}{2}, \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1 + \rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}_0}{4})$, $\beta > \frac{1}{2}$, $\frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\sigma$ *is sufficiently small enough, and* $C'(\cdot)$ *is sufficiently large enough, there exists a symmetric equilibrium characterized by the consumption paths in equations* (28)–(30)*, prices in equations* (10)–(11) *and* (19)–(20)*, and the total amount of safe asset creation is given by* $\hat{\Theta} = \hat{\Theta}_0 + 2x_R^*$*, where* x_R^* *solves* (27)*.*

Proof. The proof is exactly as before, except that we have to ensure that the total amount of private assets $\hat{\Theta} = \hat{\Theta}_0 + 2x_R^*$ is such that

$$\overline{y} \in \left[\frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}}{2}, \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1 + \rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}}{4}\right]$$

The equilibrium is characterized by the following system of equations:

$$\begin{aligned}
T_1 &:= C'(x_R^*) - p_0^a = 0 &(31)\\
T_2 &:= p_0^a - \left[\beta \mathbb{E}_0 \left(p_1^a \frac{u'(\tilde{c}_{1R})}{u'(c_0)}\right) + \alpha p_1^a \left[\frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s) - u'(\tilde{c}_{1R}^r)}{u'(c_0)}\right)\right]\right]\\
&= p_0^a - p_1^a \left(p_0^{rf} + \alpha CY\right) = 0 &(32)
\end{aligned}$$

which is given by the relevant bounds of the theorem and C' sufficiently large enough. $\square$

4.2 Changes in the Exogenous Supply of Private Assets

Before studying what determines private asset creation, we discuss how private asset creation affects prices and allocations. Expositionally, the reason we did not tackle these comparative statics in the previous section is because an increase in private assets not only changes the total amount of collateral in the economy, but also increases the amount of risk that agents face in $t = 2$, which directly affects $t = 1$ prices, as was the case with aggregate volatility.

Let's go back to the situation in which the supply of private assets is exogenous, at $\hat{\Theta}_0$, so there is no creation (which can be captured by $C = \infty$ and then $x_R^* = 0$). From equations (19) and (20), we have

$$\begin{aligned}
\frac{\partial p_1}{\partial \hat{\Theta}_0} &= -\frac{\gamma}{2}\rho\left(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2\right)p_1\\
\frac{\partial p_1^a}{\partial \hat{\Theta}_0} &= -\frac{\gamma}{2}\rho\left(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2\right)p_1^a - \frac{\gamma}{2}\rho^2\sigma^2 p_1.
\end{aligned}$$

An exogenous increase in the supply of private assets decreases both the $t = 1$ price of long-term government bonds and of private assets. More private assets in the economy increases agents' asset holdings, making them wealthier in $t = 2$. This puts downward pressure on all assets proportional to private assets' certainty equivalence. In addition, the value of private assets is also depressed because there are more of them in the economy, making them less attractive for agents to hold.

Similar to the analysis of Section 3.3, the partial derivative of c_{1R}^s is the hardest to characterize, because it involves a valuation effect on risk sharing. In this case, there is an additional quantity effect on risk sharing because of the increased asset supply (absent, for instance, in the comparative statics with aggregate volatility). Specifically,

$$\begin{aligned}
\frac{\partial c_{1R}^s}{\partial \hat{\Theta}_0} &= \underbrace{\frac{1}{(1+p_1)}\left[\frac{\partial p_1}{\partial \hat{\Theta}_0}\frac{\Theta_0}{2} + \alpha \frac{\partial p_1^a}{\partial \hat{\Theta}_0}\frac{\hat{\Theta}}{2} + \frac{(\overline{y}-w)}{(1+p_1)}\frac{\partial p_1}{\partial \hat{\Theta}_0}\right]}_{\text{Valuation Effect}} + \underbrace{\frac{\alpha p_1^a}{2(1+p_1)}}_{\text{Quantity Effect}} \\
&= -\frac{\gamma p_1^a}{2(1+p_1)^2}\left[\overline{y} - \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha \frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2}\right] - \frac{\gamma \alpha p_1}{2(1+p_1)}\rho^2\sigma^2\frac{\hat{\Theta}_0}{2} + \frac{\alpha p_1^a}{2(1+p_1)}
\end{aligned}$$

and $\frac{\partial c_{0R}}{\partial \hat{\Theta}} = \frac{\rho Y_0}{2}$. The overall change on risk sharing has two opposing forces. On the one hand, as discussed earlier, an increase in private asset supply increases agents' wealth by increasing their asset holdings, reducing $t = 1$ prices of all assets. This valuation effect reduces the extent of risk sharing and hence, c_{1R}^s is smaller. On the other hand, an increase in private assets improves risk sharing directly as there are more assets that can be used as collateral. This quantity effect depends on the asset's collateralizability. In this case, the trade-off does not come from opposing forces *between private and public collateral valuations* (as in the case of aggregate volatility) but instead by the opposing forces *between the valuation and quantity of available collateral.*

The net effect on risk sharing then depends on how much the economy depends on long-term assets for risk sharing and their pledgeability. For example, if there were few pledgeable long-term assets in the economy (low $\Theta_0, \hat{\Theta}_0$), and private assets were very pledgable (high α), then the valuation effect would be small and the quantity effect would drive an improvement in risk sharing. In contrast, if there were numerous pledgable long-term assets in the economy (high $\Theta_0, \hat{\Theta}_0$) and private assets were not very pledgeable (low α), then the valuation effect would trump the quantity effect, resulting in a decrease in risk sharing. These observations are summarized in the following proposition:

Proposition 6. *(Risk Sharing Effects of Private Assets). Given the equilibrium characterized in Theorem 2, if there are numerous long-term assets used as collateral relative to the pledgablility of private assets—that is,* $\gamma\left(\frac{\Theta_0}{2} + \alpha \frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2}\right) > \alpha$*—then an increase in private asset supply* $\hat{\Theta}_0$ *weakens risk sharing—that is,* $\frac{\partial c_{1R}^s}{\partial \hat{\Theta}_0} < 0$.

Proof. See Appendix B. □

We are now in a position to study the effects of a larger supply of private assets on prices at $t = 0$, which depends on the change in idiosyncratic insurance, the change in intertemporal smoothing, and also the change in consumption in $t = 0$,

Proposition 7. *(Asset Pricing Effects of Private Assets). Given the equilibrium characterized in Theorem 1, the initial prices of the short-term government bond, long-term government bond, and private asset have the following comparative statics with respect to* $\hat{\Theta}_0$*,*

$$
\begin{aligned}
\frac{\partial p_0^{Sh}}{\partial \hat{\Theta}_0} &= -\gamma(p_0^{rf} + CY)\frac{\partial c_{1R}^s}{\partial \hat{\Theta}_0} + \gamma(p_0^{rf} + CY)\frac{\partial c_{0R}}{\partial \hat{\Theta}_0} \\
\frac{\partial p_0}{\partial \hat{\Theta}_0} &= -\gamma p_1(p_0^{rf} + CY)\frac{\partial c_{1R}^s}{\partial \hat{\Theta}_0} + (p_0^{rf} + CY)\frac{\partial p_1}{\partial \hat{\Theta}_0} + \gamma p_1(p_0^{rf} + CY)\frac{\partial c_{0R}}{\partial \hat{\Theta}_0} \\
\frac{\partial p_0^a}{\partial \hat{\Theta}_0} &= -\gamma p_1^a(\alpha p_0^{rf} + CY)\frac{\partial c_{1R}^s}{\partial \hat{\Theta}_0} + (p_0^{rf} + \alpha CY)\frac{\partial p_1^a}{\partial \hat{\Theta}_0} + \gamma p_1^a(p_0^{rf} + \alpha CY)\frac{\partial c_{0R}}{\partial \hat{\Theta}_0}
\end{aligned}
$$

Proof. The result comes from directly applying Lemma 1 to (22)– (24). □

The results in Proposition 7 are similar to those in Proposition 5. Aside from the uninteresting effect of changes in $t = 0$ consumption, the overall effect on prices depends on the indirect impact on risk sharing and the direct impact on the asset price itself.

As discussed, an increase in private asset supply unequivocally reduces all $t = 1$ prices and the value of long-term securities in $t = 0$. At the same time, it increases the quantity of available collateral and the extent of risk sharing. The overall change in $t = 0$ prices then depends on this trade-off. If α is large, for instance, more private assets allows for more risk sharing, reducing asset prices in $t = 0$ further. But the valuation effect can dominate by reducing risk sharing. If this reduction is large enough, the increase in convenience yields can be significant, which may lead to the counterintuitive result that an increase in supply *increases* prices through a deterioration of risk sharing.

4.3 Crowding Out

Here we explore how the production of private assets changes with the provision of public assets. This analysis will shed light on the government's ability to crowd out private asset creation and potentially steer the buildup of financial fragility (in terms of reduction of risk sharing among financial intermediaries) to a sudden increase in uncertainty (in terms of an increase in aggregate uncertainty). In fewer words, the government may want to reduce the relevance of private assets as collateral by creating public assets. Here we study the consequences of such a policy. The following lemma gives the functional form of the model's comparative statics for any model parameter.

Lemma 2. *(Crowding Out). Given the equilibrium characterized in Theorem 2, for any model parameter z, we have the following comparative statics*

$$\begin{pmatrix} \frac{\partial x_R}{\partial z} \\ \frac{\partial p_0^a}{\partial z} \end{pmatrix} = \frac{1}{|D|} \begin{pmatrix} 1 \\ C''(x_R) \end{pmatrix} \frac{\partial \left(p_1^a (p_0^{rf} + \alpha CY) \right)}{\partial z}$$

where $\frac{\partial \left(p_1^a (p_0^{rf} + \alpha CY) \right)}{\partial z}$ are the partial equilibrium sensitivities characterized by the model without endogenous safe asset creation and $|D| = C''(x_R) - 2\frac{\partial p_0^a}{\partial \hat{\Theta}}$ with

$$\frac{\partial p_0^a}{\partial \hat{\Theta}} = -\gamma p_1^a (\alpha p_0^{rf} + CY) \frac{\partial c_{1R}^s}{\partial \hat{\Theta}} + (p_0^{rf} + \alpha CY) \frac{\partial p_1^a}{\partial \hat{\Theta}} + \gamma p_1^a (p_0^{rf} + \alpha CY) \frac{\partial c_{0R}}{\partial \hat{\Theta}}$$

which is the partial derivative of p_0^a characterized by Proposition 7.

Proof. See Appendix B. □

Lemma 2 gives us a generalized view of the model's sensitivities. Let's start with the intuitive case, where in partial equilibrium, $\frac{\partial p_0^a}{\partial \hat{\Theta}} < 0$ and thus $|D|$ is positive. Following the insights from Proposition 1, an increase in long-term government bonds would put downward pressure on all asset prices because of better risk sharing. With private asset creation, this effect reduces private asset creation, marginally decreasing the quantity of collateral, with an overall effect of more risk sharing due to the increase of high quality collateral—that is, government bonds. These effects are the celebrated and well documented crowding-out effect of government debt.

Interestingly, these standard results can be reversed once we factor in general equilibrium forces. Specifically, if the marginal cost to produce private assets does not increase too fast with private asset creation (that is, $C'' \approx 0$), and the increase in public assets reduces risk sharing (this is, the conditions in Proposition 6 such that $\frac{\partial p_0^a}{\partial \hat{\Theta}} > 0$), then $|D| < 0$ and all of the sensitivities would flip. Intuitively, an increase in government bonds would directly increase the amount of collateral used for risk sharing, but the change in $t = 1$ prices could reduce agents' abilities to share risk through the valuation effect. This pushes all $t = 0$ prices upward, consistent with agents' increased incentives to create more private assets. That is, the general equilibrium valuation effect can induce agents to "exaggerate" a response by increasing supply. These effects point at the lesser acknowledged possibility of crowding-in effects of government debt.

5 Empirical Analysis

Our main theoretical result is that the effect of increased aggregate volatility on risk sharing depends on whether agents rely on private or public assets to collateralize their idiosyncratic insurance. While measuring the relative share of private to public assets, and their usefulness as collateral, is challenging (see for instance Gorton et al. 2012), our model predicts that this share determines the sensitivity of risk sharing to aggregate volatility. Unfortunately, measuring risk sharing is also challenging, but we can use the convenience yield as a proxy that captures the changes of risk sharing: when risk sharing is either not an important consideration or can be provided easily, the value of assets as collateral is low, and thus better risk sharing implies a lower convenience yield.

Proposition 5 states that if the amount private collateral is larger than the amount of public collateral ($\alpha p_1^a \hat{\Theta}_0 > p_1 \Theta_0$) then an increase in aggregate volatility decreases risk sharing ($\frac{\partial c_{1R}^s}{\partial \sigma^2} < 0$). Moreover, this sensitivity decreases as the private asset becomes more useful as collateral ($\frac{\partial^2 c_{1R}^s}{\partial \alpha \partial \sigma^2} < 0$). We can test these sensitivities by studying the correlation between the convenience yield and measures of aggregate volatility. More formally, in terms of the convenience yield, which is inversely related to risk sharing, the testable implication of our model is the following:

Proposition 8. *(Testable Implications Based on the Convenience Yield). Given the equilibrium characterized in Theorem 1, if more private assets are used as collateral than long-term government bonds—that is, $\alpha p_1^a \hat{\Theta}_0 > p_1 \Theta_0$—then an increase in aggregate volatility σ^2 increases the convenience yield—that is,*

$$\frac{\partial CY}{\partial \sigma^2} = -\gamma p_0^{Sh} \frac{\partial c_{1R}^s}{\partial \sigma^2} > 0.$$

Moreover, if $\alpha p_1^a \hat{\Theta}_0 < 2$, then the increase in convenience yield is larger if the private asset *is more collateralizable, that is,*

$$\frac{\partial^2 CY}{\partial\alpha\partial\sigma^2} = \gamma p_0^{Sh} \frac{p_1^a}{(1+p_1)} \frac{\hat{\Theta}_0}{2} \frac{\partial c_{1R}^s}{\partial\sigma^2} - \gamma p_0^{Sh} \frac{\partial^2 c_{1R}^s}{\partial\alpha\partial\sigma^2} > 0.$$

Proof. See Appendix B. □

Proposition 8 states that an increase in aggregate volatility reduces risk sharing when private assets are heavily used as collateral, increasing the convenience yield of assets. Furthermore, when the private asset can be used more efficiently to hedge idiosyncratic risks, this effect is magnified and the impact of aggregate volatility on the convenience yield is even larger. The condition $\alpha p_1^a \hat{\Theta}_0 > p_1 \Theta_0$ in Proposition 8 is necessary but not sufficient, implying a positive sensitivity between the convenience yield and aggregate volatility whenever more private assets are used as collateral. The model does not have a clear prediction when the share of public assets is larger.

The insights from Proposition 8 motivates the empirical analysis. Taken literally, the proposition shows that an unexpected increase in the aggregate volatility that individuals expect to face in the future will affect the convenience yield today.[9] These observations motivate us to estimate the following empirical model:

$$\Delta CY_t = \beta_0 + \sum \gamma_j \Delta CY_{t-j} + \beta_V \Delta VIX_t + \beta_F \Delta FedFunds_t + \theta \Delta Gov_t + \epsilon_t, \tag{33}$$

where ΔCY_t is first differences of well-known empirical measures of the safe asset convenience yield and ΔVIX_t is first difference of the Chicago Board Options Exchange VIX index, a measure of implied volatilities of S&P500 index options, to capture changes in aggregate volatility. Lagged changes of the convenience yield are used to control for serial autocorrelation.

This empirical specification is inspired by Nagel (2016), who shows that the level of the convenience yield depends on the level of rates, as it depends on the opportunity cost of holding money. In particular, Nagel shows that, once you control for the level of rates, government asset supply loses its statistical significance in explaining the safe asset convenience

[9] Our stylized three-period model is not designed to capture price changes in response to a fully dynamic, infinite horizon, volatility process. The model is intended to capture how changes in the volatility of future payoffs affect agents' exposure to idiosyncratic risk in the near term—that is, how future aggregate volatility affects risk sharing today.

yield. Therefore, we also control for changes in the level of rates and changes in government bond supply. Importantly, our specification differs as we focus on changes in the convenience yield, rather than its level.

We estimate the empirical model in equation (33) over different time periods in which the literature has highlighted changes in the production and use of private assets as collateral. One of those changes evolved in the long term, spanning several decades, and was given by a process of slow financial innovation and deregulation.[10] The other happened more drastically over a short period and goes in the opposite direction, driven by the Global Financial Crisis (GFC) that put the use of private assets as collateral under distress and was promptly followed by tight regulations.[11] Our model conjecture is that the sensitivity of the convenience yield to changes in aggregate volatility is higher in the 90s and 2000s when compared with the 70s and 80s. Moreover, this sensitivity increased rapidly in the 2000s leading toward the financial crisis, after which it declined as new regulations were implemented.

In what follows, we first perform a long-term analysis that captures the long-run evolution of private assets as collateral since World War II, then we conduct a short-term analysis that captures the more rapid increase in the use of private assets as collateral leading to the Great Recession and the large collapse of such use afterwards.

5.1 Longer-Term Analysis

For the long-term analysis, we use the same data as Nagel (2016).[12] The convenience yield is measured as the spread between the banker's acceptance and the three-month T-bills spread (BA/T-bill spread). The VIX index is only available from 1990 onward, but earlier time periods are estimated using the projection of the VIX on realized S&P Index volatility. We winsorize the changes in convenience yield and VIX at the 1^{st} and 99^{th} percentile to control for outliers. The interest rate is the federal funds rate, and the government's supply of bonds is captured by the total amount of T-bill outstanding and total U.S. debt relative to GDP.

[10]For example, in the 1980s repos were excluded from automatic stay, contributing to the prevalence of these types of contracts.

[11]For example, the Liquidity Coverage Ratio places a larger regulatory burden on private assets that are used to back financial firms' liabilities.

[12]This dataset is available on Nagel's website.

This post-war data *frequency is monthly, from January 1950 to December 2011.*[13] Details of the data can be found in Nagel (2016).

In this estimation, we study how the slow and persistent process of financial innovation, financial engineering (such as securitization), and financial deregulation, which generated an increase in the relative share of private to public assets used as collateral according to Gorton et al. (2012), has changed the sensitivity of the convenience yield to aggregate volatility. Inspired by Proposition (8), we would expected β_V to be larger in the more recent decades. To capture this long-term change in sensitivity, we estimate model (33), splitting the sample in 1990, and use two lags of the dependent variable as controls.

Table 1 shows the estimates for the entire sample, using data before 1990 and using data after 1990. The results show that the statistical significance of ΔVIX_t is much larger in the latter part of the sample. That is, in the more recent time period, when the economy faces an increase in aggregate volatility the convenience yield increases. From the eyes of our model, this happens because of the economy's higher reliance on private assets, which reduces risk sharing in response to an increase in aggregate volatility.

[Insert Table 1]

5.2 Shorter-term Analysis

For the shorter-term analysis that covers the most recent period, we use the same data as Infante (2020). The convenience yield is measured by the spread between the one-month overnight index swap (OIS) rate downloaded from Bloomberg, and the four-week T-bill rate, downloaded from the Federal Reserve H.15 Statistical Release. We again winsorize the changes in the convenience yield and VIX at the 1^{st} and 99^{th} percentile to control for outliers.[14] Government supply is captured by the total amount of T-bill outstanding and total amount of Treasury notes and bonds outstanding, published by TreasuryDirect. The data *frequency is daily and runs from August 2004 and April 2020.* Here we estimate the

[13] Nagel (2016) provides convenience yield data from January 1920, however Gorton et al. (2012) show that the increase in private safe assets began at the start of the 1950s.

[14] We also drop observations on quarter-end dates, and two days surrounding quarter-end, to exclude any changes in short-term rates driven by financial firms' window dressing behavior. See Infante (2020) for more details.

sensitivity of weekly changes using overlapping data to reduce the impact of high frequency variations.[15]

In this estimation, we study how the more stringent regulatory landscape implemented after the Great Recession, which in principle reduced private asset creation and made the use of private assets as collateral more difficult, has affected how the convenience yield reacts to changes in aggregate volatility. Again, inspired by Proposition 8 we would expect the coefficient on $\Delta^5 VIX_t$ to be smaller after the Great Recession.[16] To capture the change in sensitivity, we estimate model (33), splitting the sample in 2009, and use two lags of the dependent variable as controls.

Table 2 shows the estimates for the entire sample, using data before 2009 and using data after 2009. The results show that the statistical significance of $\Delta^5 VIX_t$ is much larger before the Great Recession than after, consistent with the idea that regulatory efforts after the crisis reduced the economy's reliance on private assets. Again, intuitively, the lower reliance on private assets makes risk sharing less responsive to changes in aggregate volatility.

[Insert Table 2]

We can further exploit the high frequency data to estimate the model in shorter time intervals and see the evolution of $\Delta^5 CY_t$'s sensitivity to $\Delta^5 VIX_t$. Specifically, in each quarter, we estimate the empirical model (33) using plus and minus two years of data.[17] With this strategy, we can keep track of the changes in sensitivities over time.

Figure 1 shows the results. We can observe that the point estimate on $\Delta^5 VIX_t$ is positive and statistically significant at the end of 2006. Arguably, this period is the pinnacle of the securitization boom that began in the previous decade. We would expect that this period also coincides with an increase in financial engineering, which allowed agents to use more private assets as collateral. During the period running up to the GFC, the point estimate begins to decline. It turns insignificant at the end of 2011, around the time when new regulatory initiatives took hold and financial firms' ability to use private collateral was less attractive. From the lens of our model, the results in Figure 1 suggest that before the onset

[15] Appendix D shows the results for daily changes, which are qualitatively similar to the analysis with weekly changes.

[16] Where $\Delta^5 x_t = x_t - x_{t-5}$, the first difference operator with five lags.

[17] To increase the length of the time series, we exclude Treasury issuance controls, which, in our data set, only begins at a daily frequency in August 2004. The results are qualitatively similar if we consider Treasury issuance controls over a shorter time span.

of the GFC, the economy relied heavily on private assets as collateral, a trend which reversed and persisted thereafter.

[Insert Figure 1]

6 Concluding Remarks

We have characterized the relationship between aggregate volatility, which determines the cyclical properties of the economy, and risk sharing, which determines its distributional properties. This relation is qualified by the composition of private and public assets that are used as collateral to sustain insurance promises. As both assets are used for intra- as well as intertemporal reasons, aggregate volatility can either improve or weaken risk sharing depending on the importance of (or lack thereof) public collateral. The main linkage is then given by the valuation of collateral, as aggregate volatility affects valuation of private and public assets in different directions. An economy that relies relatively more on private assets to collateralize risk sharing sees insurance decline when aggregate volatility increases.

Financial intermediaries are among the largest players in trading public and private assets to back contracts that insure against idiosyncratic shocks. This paper is particularly relevant in that context, as it shows that changes in aggregate volatility will tend to cause stability problems in derivative markets that rely largely on private collateral.

Our model generates testable implications that relate aggregate volatility and risk sharing depending on the intensity of using private assets as collateral. We overcome the difficulty to measure risk sharing by using the convenience yield of safe assets as a proxy and testing its sensitivity to changes in aggregate volatility. We provide empirical evidence that this sensitivity has increased over the second half of the 21st century, and dramatically so during early 2000s. This trend, however, has sharply reversed after the Great Recession. From the prism of our model, this suggests that the U.S. economy's reliance on private collateral, and thus the added fragility that comes with it, has increased during the second half of the 21st century (consistent with financial innovation and financial deregulation) but declined after the Great Recession, a period indeed characterized by stricter regulations.

We also show that, because of the valuation implications of aggregate volatility, as the economy becomes more stable, the value of public bonds decline relative to those of private assets, prompting more production of private assets and endogenously making them more relevant to back insurance contracts. As such, economic stability endogenously induces a higher dependence on private collateral, making risk sharing more fragile to shocks to aggregate volatility. In short, stability creates a more fertile ground for fragility, planting the seeds of its own instability. This insight provides a novel element—the relevance of private assets relative to public assets to collateralize contracts—that policymakers should follow when assessing the fragility of the economy and when imposing macroprudential safeguards.

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Tables and Figures

Table 1: Volatility versus Convenience Yield Pre- and Post- 1990
This table shows the empirical results of equation (33) using monthly average data. The convenience yield measure is spread between the monthly average of the 3-month bankers acceptance and the monthly average of the 3-month T-bills. ΔVIX_t is the first difference of the monthly average of the VIX Index, and $\Delta FedFunds_t$ is the first difference of the monthly average of the federal funds rate. $\Delta log(TbillOut_t/GDP_t)$ is the log difference of total outstanding of T-bills to GDP, and $\Delta log(Debt_t/GDP_t)$ is the log difference of total U.S. debt to GDP. Two lags of the dependent variable are included as controls (not shown), with reported p-values of lags equal to zero. The sample runs from January 1950 to December 2011. The dependent variable and the ΔVIX are winsorized at the 1% and 99%. Newey-West standard errors with 12 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

	Full Sample	Pre-1990	Post-1990	Full Sample	Pre-1990	Post-1990
$\Delta FedFunds_t$	0.191***	0.195***	0.106***	0.189***	0.194***	0.082**
	(0.029)	(0.030)	(0.036)	(0.028)	(0.030)	(0.034)
ΔVIX_t	0.006**	0.005	0.007***	0.006**	0.005	0.008***
	(0.002)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)
$\Delta log(TBillsOut_t/GDP_t)$				-0.264**	-0.258*	-0.409**
				(0.122)	(0.146)	(0.166)
$\Delta log(USTNotesOut_t/GDP_t)$				-0.648*	-1.107	-0.326
				(0.378)	(0.766)	(0.272)
P-value	0.725	0.864	0.101	0.715	0.840	0.139
Adj RSq	0.190	0.199	0.113	0.196	0.206	0.135
N obs	740	476	264	740	476	264

Table 2: Volatility versus Convenience Yield Pre- and Post- 2009

This table shows the empirical results of equation (33) using overlapping daily data. The convenience yield measure is the spread between the 1-month overnight index swap rate and the 4-week Treasury bills rate. $\Delta^5 VIX_t$ is the 5-day first difference of the VIX Index, and $\Delta^5 FedFunds_t$ is the 5-day first difference of the federal funds rate. $\Delta^5 log(TbillOut_t)$ is the 5-day log difference of total T-bills outstanding, and $\Delta^5 log(USTNotesOut_t)$ is the 5-day log difference total U.S. Treasury notes and bonds outstanding. Two lags of the dependent variable are included as controls (not shown), with reported p-values of lags equal to zero. The sample without U.S. Treasury issuance controls runs from December 2001 to April 2020, and the sample with U.S. Treasury issuance controls runs from August 2004 to April 2020. Estimates exclude quarter-end dates (and $\pm$ 2 days surrounding quarter-end). The dependent variable and the $\Delta^5 VIX$ are winsorized at the 1% and 99%. Newey-West standard errors with 21 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

	Full Sample	Pre- 2009	Post- 2009	Full Sample	Pre- 2009	Post- 2009
$\Delta^5 FedFunds_t$	-0.101**	-0.098	-0.117***	-0.120**	-0.146**	-0.116***
	(0.051)	(0.068)	(0.030)	(0.051)	(0.067)	(0.029)
$\Delta^5 VIX_t$	0.001	0.005*	-0.001	0.002*	0.010***	-0.001
	(0.001)	(0.002)	(0.001)	(0.001)	(0.003)	(0.001)
$\Delta^5 log(TBillsOut_t)$				-0.893***	-1.565***	-0.567***
				(0.261)	(0.510)	(0.114)
$\Delta^5 log(USTNotesOut_t)$				-0.156	-1.196	0.196
				(0.938)	(3.014)	(0.473)
P-value	0.499	0.228	0.000	0.613	0.336	0.000
Adj RSq	0.024	0.034	0.091	0.054	0.088	0.125
N obs	2805	1075	1730	2405	682	1723

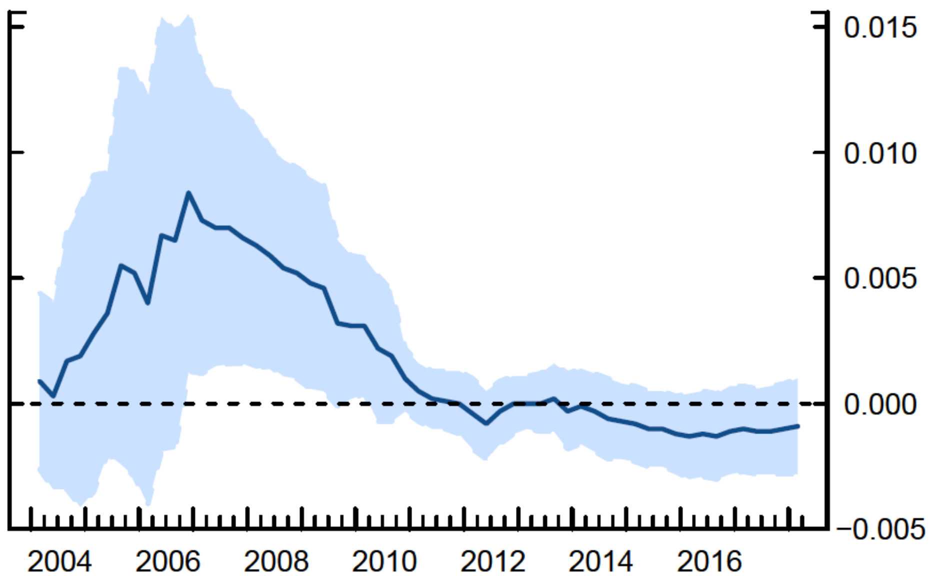

Figure 1: Five-Day Sensitivity of $\Delta^5 CY_t$ to $\Delta^5 VIX_t$
The solid line shows the point estimate of the 5-day estimation of model (33) using daily data and $\pm$ 2 years of data each quarter. The shaded region shows the 95% confidence interval of each estimate.

Appendix

A Figures to back Assumption A3

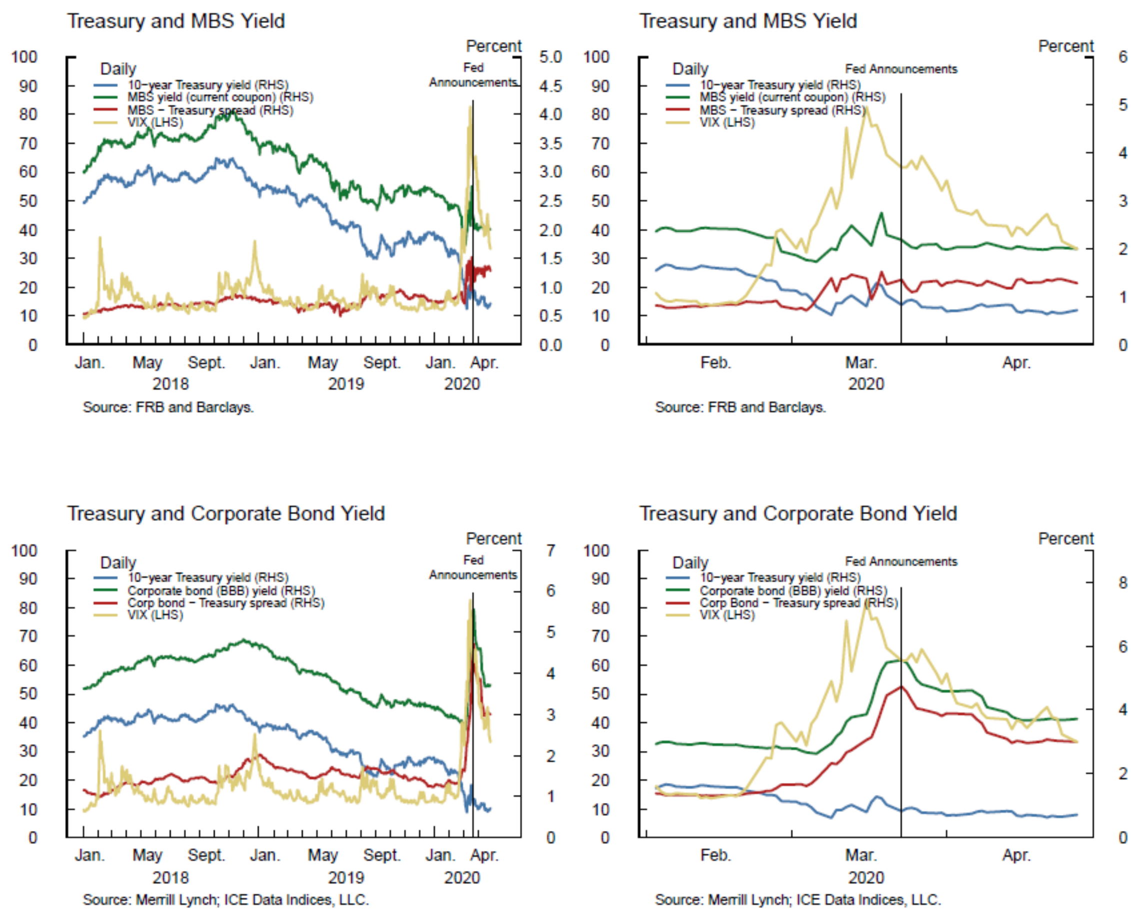

Figure A.1: Ten-Year Treasury, Agency MBS, and Investment-Grade Corporate Bond Yields; Spreads Relative to the Ten-year Treasury Yield and VIX Index

The top two panels show the Treasury and Agency MBS yields, their spread, and the VIX index during January 2018–April 2020 and February 2020–April 2020. The bottom two panels show the Treasury and investment-grade corporate bond yields, their spread, and the VIX index during January 2018–April 2020 and February 2020–April 2020. The tripwire indicates the date the Federal Reserve announced expanded asset purchases and new funding facilities on March 23, 2020.

B Proofs

Proof of Proposition 4

From Theorem 1's hypothesis, we know that equation (17) holds, thus $\overline{y} - \frac{\Theta_0^{Sh}}{2} \leq (1+p_1)\frac{\Theta_0}{2} + p_1\frac{\Theta_0}{2} + \alpha p_1^a \frac{\hat{\Theta}_0}{2}$, and form the Proposition's hypothesis, $\Theta_0 \leq \alpha \frac{p_1^a}{p_1}\hat{\Theta}_0$. Thus, from equation (26), we have

$$\begin{aligned}
\frac{\partial c_{1R}^s}{\partial \sigma^2} &\leq \frac{1}{(1+p_1)}\alpha\frac{p_1^a}{p_1}\frac{3}{2}\hat{\Theta}_0\frac{\partial p_1}{\partial \sigma^2} - \frac{\alpha}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\frac{\hat{\Theta}_0}{2} \\
&= \frac{\alpha}{(1+p_1)}\frac{\gamma}{4}\left(1+\rho\hat{\Theta}_0\right)p_1\hat{\Theta}_0\left[\frac{3}{2}\frac{p_1^a}{p_1}\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right) - \rho,\right]
\end{aligned}$$

where the second equality replaces the expression for $\frac{\partial p_1}{\partial \sigma^2}$. Replacing the expression for p_1^a/p_1, the term in the square bracket is bounded by

$$\frac{3}{2}\rho\left(\mu - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right) - \rho \quad \leq \quad \frac{3}{2}\rho\left[\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\} - \frac{5}{3},\right]$$

where we use the Hansen-Jagannathan bound. Thus, for $\gamma\left(1+\rho\hat{\Theta}_0\right)\sigma$ sufficiently small, we have, $\frac{\partial c_{1R}^s}{\partial \sigma^2} \leq 0$.

Finally, we have to show that $\frac{\partial^2 c_{1R}^s}{\partial\alpha\partial\sigma^2} \leq 0$. From the expression in equation (26), we have

$$\begin{aligned}
\frac{\partial^2 c_{1R}^s}{\partial\alpha\partial\sigma^2} &= \frac{1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left[\frac{1}{(1+p_1)}\frac{p_1^a}{p_1}\frac{\partial p_1}{\partial\sigma^2} - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right] \\
&= \rho\frac{p_1}{(1+p_1)^2}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\frac{\hat{\Theta}_0}{2}\left[\left(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2\right)\frac{\gamma}{4}(1+\rho\hat{\Theta}_0) - (1+p_1)\right] \\
&\leq \rho\frac{p_1}{(1+p_1)^2}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\frac{\hat{\Theta}_0}{2}\left[\frac{1}{2}\left(\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\} - 1\right) - (1+p_1),\right]
\end{aligned}$$

also using the Hansen-Jagannathan bound. Thus, for $\gamma\left(1+\rho\hat{\Theta}_0\right)\sigma$ sufficiently small we have, $\frac{\partial^2 c_{1R}^s}{\partial\alpha\partial\sigma^2} \leq 0$.

■

Proof of Proposition 6

From Theorem 2, equation (18) holds and $\overline{y} - \frac{\Theta_0^{Sh}}{2} > p_1\frac{\Theta_0}{2} + \alpha p_1^a\frac{\hat{\Theta}_0}{2}$. From equation (33),

$$\begin{aligned}
\frac{\partial c_{1R}^s}{\partial \Theta_0} &< -\frac{\gamma p_1^a}{2(1+p_1)}\left[\frac{\Theta_0}{2} + \alpha\frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2}\right] - \frac{\gamma\alpha p_1}{2(1+p_1)}\rho^2\sigma^2\frac{\hat{\Theta}_0}{2} + \frac{\alpha p_1^a}{2(1+p_1)} \\
&= -\frac{p_1^a}{2(1+p_1)}\left[\gamma\left(\frac{\Theta_0}{2} + \alpha\frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2} + \alpha\frac{p_1}{p_1^a}\rho^2\sigma^2\frac{\hat{\Theta}_0}{2}\right) - \alpha.\right]
\end{aligned}$$

Thus, under the Proposition's hypothesis we have $\frac{\partial c_{1R}^s}{\partial\hat{\Theta}_0} < 0$.

■

Proof of Lemma 2

Invoking the implicit function theorem, we have

$$\begin{pmatrix} \frac{\partial x_R}{\partial \Theta_0} \\ \frac{\partial p_0^a}{\partial \Theta_0} \end{pmatrix} = -\underbrace{\begin{bmatrix} \frac{\partial T_1}{\partial x_R} & \frac{\partial T_1}{\partial p_0^a} \\ \frac{\partial T_2}{\partial x_R} & \frac{\partial T_2}{\partial p_0^a} \end{bmatrix}^{-1}}_{:=D^{-1}} \begin{pmatrix} \frac{\partial T_1}{\partial \Theta_0} \\ \frac{\partial T_2}{\partial \Theta_0} \end{pmatrix}$$

We first have to characterize the partial derivatives with respect to the endogenous variables. These are

$$\frac{\partial T_1}{\partial x_R} = C''(x_R); \qquad \frac{\partial T_1}{\partial p_0^a} = -1; \qquad \frac{\partial T_2}{\partial x_R} = -\frac{\partial p_0^a}{\partial \hat{\Theta}}\frac{\partial \hat{\Theta}}{\partial x_R}; \qquad \frac{\partial T_2}{\partial p_0^a} = 1$$

where with a slight abuse of notation, $\frac{\partial p_0^a}{\partial \hat{\Theta}}$ is the partial derivative of the $t = 0$ price of the private asset in the original model—that is, the comparative statics characterized in Proposition 7. Specifically,

$$\begin{aligned} \frac{\partial p_1^a}{\partial \hat{\Theta}} &= -\gamma p_1^a(\alpha p_0^{rf} + CY)\frac{\partial c_{1R}^s}{\partial \hat{\Theta}} + (p_0^{rf} + \alpha CY)\frac{\partial p_1^a}{\partial \hat{\Theta}_0} + \gamma p_1^a(p_0^{rf} + \alpha CY)\frac{\partial c_{0R}}{\partial \hat{\Theta}} \\ &= -\gamma p_1^a(\alpha p_0^{rf} + CY)\left[-\frac{\gamma p_1^a}{2(1+p_1)^2}\left[\overline{y} - \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2}\right] - \frac{\gamma \alpha p_1}{2(1+p_1)}\rho^2\sigma^2\frac{\hat{\Theta}}{2} + \frac{\alpha p_1^a}{2(1+p_1)}\right] \\ &\quad +(p_0^{rf} + \alpha CY)\left[-\frac{\gamma}{2}\rho\left(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2\right)p_1^a - \frac{\gamma}{2}\rho^2\sigma^2 + \gamma p_1^a\left(\frac{\rho Y_0}{2} - C'(x_R^*)\right)\right] \end{aligned}$$

Therefore, we have that

$$|D| = C''(x_R) - 2\frac{\partial p_0^a}{\partial \hat{\Theta}}$$

Inspecting the derivatives of exogenous variables, note that $\frac{\partial T_1}{\partial z} = 0$ and $\frac{\partial T_2}{\partial z} = -\frac{\partial p_1^a\left(p_0^{rf} + \alpha CY\right)}{\partial z}$ is merely the partial equilibrium sensitivities characterized by the model without endogenous safe asset creation giving the Lemma's result.

■

Proof of Proposition 8

The proof of $\frac{\partial^2 CY}{\partial \alpha \partial \sigma^2} > 0$ is a direct consequence of Lemma 1 and Proposition 8.

The expression for $\frac{\partial^2 CY}{\partial \alpha \partial \sigma^2}$ comes from taking the derivative of $\frac{\partial^2 CY}{\partial \alpha \partial \sigma^2}$ then using Proposition 2. Thus, using the expression in equation (26) and in the proof of Proposition 4, we have

$$\begin{aligned}
\frac{\partial^2 CY}{\partial\alpha\partial\sigma^2} &= \gamma p_0^{Sh}\frac{1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left\{\frac{p_1^a}{(1+p_1)^2}\left[\overline{y}-\frac{\Theta_0^{Sh}}{2}+\frac{\Theta_0}{2}+\alpha\frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2}\right]\frac{\partial p_1}{\partial\sigma^2}-\frac{\alpha}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1 p_1^a\frac{\hat{\Theta}_0}{2}\right.\\
&\left.-\left[\frac{1}{(1+p_1)}\frac{p_1^a}{p_1}\frac{\partial p_1}{\partial\sigma^2}-\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right]\right\}\\
&\geq \gamma p_0^{Sh}\frac{1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left\{\frac{p_1^a}{(1+p_1)}\left[\frac{\Theta_0}{2}+\alpha\frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2}\right]\frac{\partial p_1}{\partial\sigma^2}-\frac{\alpha}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1 p_1^a\frac{\hat{\Theta}_0}{2}\right.\\
&\left.-\left[\frac{1}{(1+p_1)}\frac{p_1^a}{p_1}\frac{\partial p_1}{\partial\sigma^2}-\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right]\right\}\\
&\geq \gamma p_0^{Sh}\frac{1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left\{\frac{\alpha p_1^a}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left[2\frac{p_1^a}{p_1}\frac{\partial p_1}{\partial\sigma^2}-\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right]-\left[\frac{1}{(1+p_1)}\frac{p_1^a}{p_1}\frac{\partial p_1}{\partial\sigma^2}-\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right]\right\}\geq 0
\end{aligned}$$

where we used the condition in (8) and the fact that $\alpha p_1^a\hat{\Theta}_0 > p_1\Theta$. Because of the Hansen-Jagannathan bound, the term accompanying $\frac{p_1^a}{p_1}\frac{\partial p_1}{\partial\sigma^2}$ can be made arbitrarily small. Because $\alpha p_1^a\hat{\Theta}_0 < 2$, we have the result.

■

In order to put some discipline on the model, it is important to choose parameters that satisfy then Hansen-Jagannathan bound. The following Lemma characterizes the Hansen-Jagannathan bounds in period $t = 1$ of the model.

Lemma 3. *The Hansen-Jagannathan bounds for the pricing in $t = 1$ is given by*

$$\left|\left(\mu-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\right|\leq\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\}-1.$$

Proof. Given the optimal consumption paths in (15) and (16), the stochastic discount factor is

$$\tilde{S}=\beta\exp\left\{-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\tilde{Y}_2\right\}$$

and the $t = 1$ prices for the for the risk free and risky asset ($\tilde{a}_2 = \rho\tilde{Y}_2$) is,

$$\begin{aligned}
p_1 &= \mathbb{E}(\tilde{S})\\
&= \beta\exp\left\{-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\mu+\frac{1}{8}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\}\\
p_1^a &= \mathbb{E}(\tilde{S}\rho\tilde{Y}_2)\\
&= \rho\left(\mu-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)p_1.
\end{aligned}$$

which written in terms of excess returns implies that $\mathbb{E}\left(\tilde{S}\left(\frac{\rho\tilde{Y}_2}{p_1^a}-\frac{1}{p_1}\right)\right) = 0$. Therefore, the Hansen-

Jagannathan bound requires that

$$\left|\mathbb{E}(\tilde{S})\mathbb{E}\left(\frac{\rho\tilde{Y}_2}{p_1^a} - \frac{1}{p_1}\right)\right| \leq \mathbb{V}(\tilde{S})\mathbb{V}\left(\frac{\rho\tilde{Y}_2}{p_1^a}\right)$$

where

$$\begin{aligned}
\mathbb{V}(\tilde{S}) &= \mathbb{E}(\tilde{S}^2) - \mathbb{E}(\tilde{S})^2 \\
&= \beta^2\mathbb{E}\left(\exp\{-\gamma(1+\rho\hat{\Theta}_0)\}\tilde{Y}_2\right) - p_1^2 \\
&= \beta^2\mathbb{E}\left(\exp\left\{-\gamma\left(1+\rho\hat{\Theta}_0\right)\mu + \frac{1}{2}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\}\right) - p_1^2 \\
&= p_1^2\left(\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\} - 1\right)
\end{aligned}$$

Thus, the bound can be rewritten as

$$\left|\left(\mu - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\right| \leq \exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\} - 1. \tag{B.1}$$

□

C Alternative Government Tax Schemes

In this appendix, we explore the impact of implementing different government policies to tax agents. The motivation is to capture the interaction between altering agents' intertemporal smoothing through taxation and their risk sharing. We show that, if taxes can change the path of consumption, it may have valuation effects on collateral, which affects risk sharing.

Specifically, because the government is the only agent in the economy that can store wealth by raising and holding funds through bond issuance and repayment, it can directly alter agents' consumption paths. In this sense, the role of the government is to store agents' wealth for future periods, as their bonds are the only (safe) way agents can carry wealth from one period to the next, and choose how much agents can transfer. This assumption can be interpreted as a shortcut to the assumption that the government has access to markets that the agents cannot, such as foreign investors.

Therefore, in this case, we can write Raymond's consumption processes—which is identical to Shirley's—in each period as

$$\check{c}_{0R} = e_{0R} + a_0\frac{\hat{\Theta}_0}{2} - \check{p}_0\theta_{0R} - \check{p}_0^{Sh}\theta_{0R}^{Sh} - \check{p}_0^a\left(\hat{\theta}_{0R} - \frac{\hat{\Theta}_0}{2}\right) + q^r w_R^r + q^s w_R^s + \frac{T_0}{2} \tag{C.2}$$

$$\tilde{\check{c}}_{1R} = \tilde{e}_{1R} + \tilde{a}_1\hat{\theta}_{0R} - \tilde{\check{p}}_1(\theta_{1R} - \theta_{0R}) + \theta_{0R}^{Sh} - \tilde{\check{p}}_1^a(\hat{\theta}_{1R} - \hat{\theta}_{0R}) - w_R^r 1^r - w_R^s 1^s + \frac{T_1}{2} \tag{C.3}$$

$$\tilde{\check{c}}_{2R} = \tilde{e}_{2R} + \tilde{a}_2\hat{\theta}_{1R} + \theta_{1R} + \frac{T_2}{2}, \tag{C.4}$$

where we have used $\check{p}_t^{Sh}, \check{p}_t$ and $\check{p}_t^a$ for the equilibrium prices for the short-, long-term bond, and private asset, respectively.[18] In this case, T_0, T_1, and T_2 are aggregate lump sum transfers to agents (negative values are taxes). Note that these consumption equations are identical to the original model (equations (2)–(4)), except that the government returns what it raises (plus interest) when short- and long-term government bonds mature and manages its transfers to agents to balance its budget.

We assume that the government must have enough funds to make payments intertemporally. That is, in each period, the government must have enough funds to make final bond payments and transfers. Specifically,

$$\begin{array}{lrl} t=0: & T_0 & \leq \check{p}_0^{Sh}\Theta_0^{Sh} + \check{p}_0\Theta_0 \\ t=1: & \Theta_0^{Sh} + T_1 + T_0 & \leq \check{p}_0^{Sh}\Theta_0^{Sh} + \check{p}_0\Theta_0 \\ t=2: & \Theta_0^{Sh} + \Theta_0 + T_0 + T_1 + T_2 & = \check{p}_0^{Sh}\Theta_0^{Sh} + \check{p}_0\Theta_0 \end{array}$$

where the last equality ensures that the government has to balance its aggregate budget in $t=2$. These inequalities imply that the government uses its storage technology to transfer aggregate consumption from one period to the next, but it must be able to fulfill its promises in each period. To simplify the analysis, we will restrict the governments choice set by assuming that the government fully balances its budget in $t=1$. This implies that $T_2 = -\Theta_0$, $T_0 = \check{p}_0^{Sh}\Theta_0^{Sh} + \check{p}_0\Theta_0 - T_1 - \Theta_0^{Sh}$, and thus the initial financing constraint implies that $T_0 \leq \check{p}_0^{Sh}\Theta_0^{Sh} + \check{p}_0\Theta_0$.[19]

Thus, in absence of idiosyncratic shocks (i.e., $\overline{y}=0$), the direct impact of the government's issuance and tax policy is on how it affects the cost to transfer wealth from one period to the next.

It is easy to show that in this context, under assumptions A1 and A2, Raymond and Shirley's optimal portfolios are just as in the original model. Specifically, Raymond (Shirley) sells rain (shine) insurance to Shirley (Raymond); and agents hold half of the private asset supply in all periods, half of the government's issuance in $t=0$, and rebalance their long-term government bond holdings in $t=1$ to smooth their idiosyncratic risk exposure. Incorporating the government's tax plan, as a function of T_1, agents optimal consumption is

$$\begin{array}{rcl} \check{c}_{0R} & = & \dfrac{Y_0}{2} + a_0\dfrac{\hat{\Theta}_0}{2} - \dfrac{\Theta_0^{Sh}}{2} - \dfrac{T_1}{2} \\ \check{c}_{1R}^r & = & \dfrac{(\overline{y}-w)}{(1+\check{p}_1)} + \dfrac{\Theta_0^{Sh}}{2} + \dfrac{T_1}{2}; \qquad \check{c}_{1R}^s = -\dfrac{(\overline{y}-w)}{(1+\check{p}_1)} + \dfrac{\Theta_0^{Sh}}{2} + \dfrac{T_1}{2} \\ \tilde{\check{c}}_{2R}^r & = & \dfrac{\tilde{Y}_2}{2} + \tilde{a}_2\dfrac{\hat{\Theta}_0}{2} + \dfrac{(\overline{y}-w)}{(1+\check{p}_1)}; \qquad \tilde{\check{c}}_{2R}^s = \dfrac{\tilde{Y}_2}{2} + \tilde{a}_2\dfrac{\hat{\Theta}_0}{2} - \dfrac{(\overline{y}-w)}{(1+\check{p}_1)}. \end{array}$$

In this version of the model, optimal consumption in $t=1$ has a component attributed to idiosyncratic risk and a component attributed to the government's tax scheme. Thus, in contrast to the original model,

[18] To alleviate excessive notation, all other variables in this model extension take the same form.

[19] This set up nests the original model, where $T_0 = p_0^{Sh}\Theta_0^{Sh} + p_0\Theta_0$ and $T_1 = -\Theta_0^{Sh}$.

changes in c_{1R}^s do not merely reflect the degree of risk sharing.

The equilibrium $\check{p}_t^{Sh}, \check{p}_t$ and $\check{p}_t^a$ take the same functional form as (5)–(6) for $t=1$ and (9)–(11) for $t=0$, however the final expressions will differ because of agents' new optimal consumption paths.

$$\begin{aligned}
\check{p}_1 &= \beta \mathbb{E}_1 \left(\exp\left\{ -\frac{\gamma}{2} \left[\left(1+\rho\hat{\Theta}_0\right) \tilde{Y}_2 - T_1 - \Theta_0^{Sh} \right] \right\} \right) \\
&= \beta \exp\left\{ -\frac{\gamma}{2} \left(1+\rho\hat{\Theta}_0\right) \mu + \frac{1}{8}\gamma^2 \left(1+\rho\hat{\Theta}_0\right)^2 \sigma^2 + \frac{\gamma}{2} \left(T_1 + \Theta_0^{Sh}\right) \right\} \\
&= p_1 \exp\left\{ \frac{\gamma}{2} \left(T_1 + \Theta_0^{Sh}\right) \right\} \qquad \text{(C.5)} \\
\check{p}_1^a &= \beta \mathbb{E}_1 \left(\exp\left\{ -\frac{\gamma}{2} \left[\left(1+\rho\hat{\Theta}_0\right) \tilde{Y}_2 - T_1 - \Theta_0^{Sh} \right] \right\} \rho \tilde{Y}_2 \right) \\
&= \rho \left(\mu - \frac{\gamma}{2} \left(1+\rho\hat{\Theta}_0\right) \sigma^2 \right) \check{p}_1. \qquad \text{(C.6)}
\end{aligned}$$

That is, prices in this model are proportional to the prices in the original ones but scaled by the relative distortion from the government's tax policy. A larger lump sum transfer in $t=1$ increases consumption in $t=1$, and thus increase the need for intertemporal smoothing between $t=1$ to $t=2$, putting upward pressure on $t=1$ prices.

It is easy to check that the same arguments in the proof of Theorem 1 still hold, thus, if $\overline{y} \in [\frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}_0}{2}, \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}_0}{4}]$, $\beta > \frac{1}{2}$, and $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma$ sufficiently small, then there exist a symmetric equilibrium.

Moreover, the convenience yield takes the same functional form as before. Thus, the effect of the governments alternate tax policy on the $t=0$ prices can be expressed as

$$\begin{aligned}
\check{p}_0^{Sh} &= (\check{p}_0^{rf} + \check{CY}) = p_0^{Sh} \exp\left\{ -\gamma \left(T_1 + \Theta_0^{Sh}\right) \right\} \\
\check{p}_0 &= \check{p}_1(\check{p}_0^{rf} + \check{CY}) = p_0 \exp\left\{ -\frac{\gamma}{2} \left(T_1 + \Theta_0^{Sh}\right) \right\} \\
\check{p}_0^a &= \check{p}_1^a(\check{p}_0^{rf} + \alpha\check{CY}) = p_0^a \exp\left\{ -\frac{\gamma}{2} \left(T_1 + \Theta_0^{Sh}\right) \right\},
\end{aligned}$$

where $\check{CY} = CY \exp\left\{-\gamma\left(T_1 + \Theta_0^{Sh}\right)\right\}$ and $\check{p}_0^{rf} = p_0^{rf} \exp\left\{-\gamma\left(T_1 + \Theta_0^{Sh}\right)\right\}$ are the convenience yield and the price of the risk-free security in absence of idiosyncratic risk.

The effect of different tax policies in $t=0$ is the opposite to what happens in $t=1$. As lump sum transfers in $t=1$ increases, there is more consumption in $t=1$ and less in $t=0$. In this case, the government is effectively forcing agents to save more, making it less attractive to do so, putting downward pressure on prices.

Thus, the equilibrium in the case of alternative tax plans are the same as in the original model, scaled by the direct effect of said tax plan. This implies that the comparative statics of all non-governmental variables are as before, scaled by the tax distortion. The only important difference are the comparative statics with respect to the government's $t=1$ lump sum tax decision. These decisions not only have an effect on agents' consumption smoothing across time, but also on the amount of risk sharing. Specifically, we have

$$\frac{\partial \check{c}_{1R}^s}{\partial T_1} = \underbrace{\frac{\gamma}{2(1+\check{p}_1)}\left[\check{p}_1\frac{\Theta_0}{2} + \alpha\check{p}_1\frac{\hat{\Theta}_0}{2} + \frac{(\overline{y}-w)}{(1+\check{p}_1)}\check{p}_1\right]}_{\text{Valuation Effect}} + \frac{1}{2}$$

and $\frac{\partial \check{c}_{0R}}{\partial T_1} = -\frac{1}{2}$. The direct effect due to changes in agents' consumption smoothing is capture by the last term: 1/2. The effect on agents' risk sharing comes through a pure valuation effect: an increase in T_1 increases the price of the long-term bond and the private asset, augmenting agents' ability to hedge idiosyncratic risks. This leads to the following result,

Proposition 9. *Given the equilibrium characterized in Theorem 1 with an alternative tax plan, the initial prices of the short-term government bond, long-term government bond, and private asset have the following comparative statics with respect to T_1,*

$$\begin{aligned}
\frac{\partial \check{p}_0^{Sh}}{\partial T_1} &= -\gamma(\check{p}_0^{rf} + \check{CY})\left(\frac{\gamma}{2(1+\check{p}_1)}\left[\check{p}_1\frac{\Theta_0}{2} + \alpha\check{p}_1\frac{\hat{\Theta}_0}{2} + \frac{(\overline{y}-w)}{(1+\check{p}_1)}\check{p}_1\right] + 1\right) \\
\frac{\partial \check{p}_0}{\partial T_1} &= -\gamma\check{p}_1(\check{p}_0^{rf} + \check{CY})\left(\frac{\gamma}{2(1+\check{p}_1)}\left[\check{p}_1\frac{\Theta_0}{2} + \alpha\check{p}_1\frac{\hat{\Theta}_0}{2} + \frac{(\overline{y}-w)}{(1+\check{p}_1)}\check{p}_1\right] + \frac{1}{2}\right) \\
\frac{\partial \check{p}_0^a}{\partial T_1} &= -\gamma\check{p}_1^a(\alpha\check{p}_0^{rf} + \check{CY})\left(\frac{\gamma}{2(1+\check{p}_1)}\left[\check{p}_1\frac{\Theta_0}{2} + \alpha\check{p}_1\frac{\hat{\Theta}_0}{2} + \frac{(\overline{y}-w)}{(1+\check{p}_1)}\check{p}_1\right]\right) - \frac{\gamma}{2}\check{p}_1^a(\check{p}_0^{rf} + \alpha\check{CY})
\end{aligned}$$

Proof of Proposition

The result comes from noting that $\check{CY} = CY\exp\left\{-\gamma\left(T_1 + \Theta_0^{Sh}\right)\right\}$ and $\check{p}_0^{rf} = p_0^{rf}\exp\left\{-\gamma\left(T_1 + \Theta_0^{Sh}\right)\right\}$ and observing that $\frac{\partial \check{c}_{1R}^s}{\partial T_1} = \frac{\partial c_{1R}^s}{\partial T_1} + \frac{1}{2}$ (where CY, p_0^{rf}, and c_{1R}^s are as in the original model), and applying Lemma 1.

■

The increase in lump sum transfers in T_1 unequivocally makes all assets less valuable in $t = 0$. The direct effect is an increase (decrease) in aggregate consumption in $t = 1$ ($t = 0$), which reduces the need to transfer wealth from $t = 0$ to $t = 1$ and thus reduces $t = 0$ prices. This effect is somewhat muted by the increase in $t = 1$ prices, which affects both long-term bonds and private assets. These effects are somewhat mechanical and well understood. The novel change is the valuation effect on risk sharing. By making prices higher in $t = 1$, assets are more pledgeable, allowing for more risk sharing, making the assets less valuable in $t = 0$.

This indicates that the government can improve the amount of risk sharing by either altering the amount of government bonds or how they pay for them. An important element in these results is that the agents cannot transfer resources intertemporally to undo the effects of government taxes. The only way agents can react is through their demand for government bonds, thus affecting their price and the valuation effect. While the power of the government to change the path of consumption may seem extreme, this assumption should be taken as capturing incomplete markets, or other frictions in which taxation affects paths of consumption in equilibrium.

D Additional Empirical Analysis — Daily Frequency

In this appendix, we show the results for the same empirical strategy described in section 5.2, but use one-day differences rather than five-day differences. For this specification, we consider four lags of changes in the convenience yield as controls.

The results in Table D.1 are qualitatively similar to those in Table 2. There is a positive and statistically significant relationship between changes in the convenience yield and changes in the VIX in the early part of the sample, before the GFC. After the GFC, the relationship loses its statistical power.

The results in Figure D.2 are qualitatively similar to those in Figure 1.[20] We can observe that the sensitivity of ΔVIX_t on CY_t is positive and statistically significant toward the end of 2006 and subsequently loses significance toward the end of 2010.

[20]The scales on both figures are the same to simplify the comparison.

Table D.1: Volatility versus Convenience Yield Pre- and Post- 2009
This table shows the empirical results of equation (33) using daily data. The convenience yield measure is the spread between the 1-month overnight index swap rate and the 4-week Treasury bills rate. ΔVIX_t is the first difference of the VIX Index, and $\Delta FedFunds_t$ is the first difference of federal funds rate. $\Delta log(TbillOut_t)$ is the log difference of total T-bills outstanding, and $\Delta log(USTNotesOut_t)$ is the log difference total U.S. Treasury notes and bonds outstanding. Four lags of the dependent variable are included as controls (not shown), with reported p-values of lags equal to zero. The sample without U.S. Treasury issuance controls runs from December 2001 to April 2020, and the sample with U.S. Treasury issuance controls runs from August 2004 to April 2020. Estimates exclude quarter-end dates. The dependent variable and the ΔVIX are winsorized at the 1% and 99%. Newey-West standard errors with 21 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

	Full Sample	Pre- 2009	Post- 2009	Full Sample	Pre- 2009	Post- 2009
$\Delta FedFunds_t$	0.034*	0.042*	-0.027	0.032	0.038	-0.026
	(0.020)	(0.023)	(0.021)	(0.021)	(0.025)	(0.021)
ΔVIX_t	0.001	0.003**	-0.000	0.001	0.005***	-0.000
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)
$\Delta log(TBillsOut_t)$				-0.332**	-0.549**	-0.211**
				(0.131)	(0.231)	(0.098)
$\Delta log(USTNotesOut_t)$				-0.372	-1.521	-0.015
				(0.448)	(1.559)	(0.247)
P-value	0.000	0.000	0.000	0.000	0.000	0.000
Adj RSq	0.057	0.068	0.110	0.061	0.076	0.112
N obs	3347	1282	2065	2860	802	2058

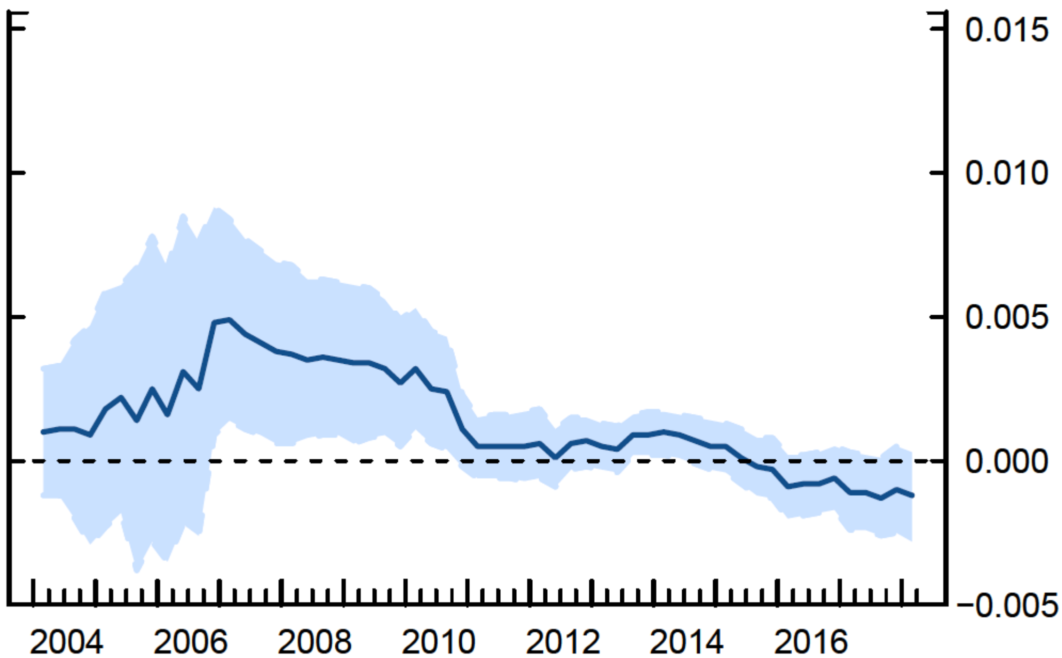

Figure D.2: One-Day Sensitivity of ΔCY_t to ΔVIX_t
The solid line shows the point estimate of the 1-day estimation of model (33) using daily data and $\pm$ 2 years of data each quarter. The shaded region shows the 95% confidence interval of each estimate.