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FORWARD GUIDANCE AND DURABLE GOODS DEMAND

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ABSTRACT

Durable goods attenuate the power of forward guidance. The extensive and intensive margins of durable goods demand are both more sensitive to the contemporaneous user cost than to future user costs. Changes in the contemporaneous real interest rate directly affect the contemporaneous user cost and durable demand, whereas promises of low future real interest rates have weaker effects through equilibrium price changes. Quantitatively, reducing the real interest rate one year from now increases output by only forty percent as much as reducing the real interest rate today. Our results are little affected by the maturity of financial assets that finance durable purchases.

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1 Introduction

Forward guidance plays an increasingly important role in the conduct of monetary policy and is one of the main tools of unconventional monetary policy (Bernanke, 2020). Despite the prominence of forward guidance in modern monetary policy, the theoretical underpinnings of how future interest rates affect aggregate demand are still a matter of debate within monetary economics. Workhorse New Keynesian models are viewed by many as being too forward looking and thereby attributing too much power to forward guidance policies (Carlstrom et al., 2015; Del Negro et al., 2015). Indeed, the predictions of the Euler equation at the heart of the three-equation New Keynesian model illustrates the issue starkly: changes in expected real interest rates at *any horizon* have an equally large effect on the current level of aggregate demand. This implausible prediction has come to be known as the "forward guidance puzzle."

A number of authors have offered modifications to the New Keynesian framework that can reduce the power of forward guidance. These include market incompleteness (McKay et al., 2016; Werning, 2015; Hagedorn et al., 2019; Acharya and Dogra, 2020), behavioral or informational frictions (Farhi and Werning, 2019; Gabaix, 2020; Angeletos and Lian, 2018), and including wealth in the utility function (Campbell et al., 2017; Michaillat and Saez, 2019). In these approaches aggregate demand is solely determined by *non-durable* consumption. However, monetary policy is generally viewed as having a particularly strong influence on durable demand and investment spending (Mishkin, 1995; Barsky et al., 2007; Sterk and Tenreyro, 2018).

In this paper, we characterize the power of forward guidance in an incomplete markets model of durable goods demand subject to fixed adjustment costs. We show that forward guidance is much less powerful relative to contemporaneous interest rate changes.

Real interest rates affect demand for durable goods by changing the user cost of durables. Our main analytical result is that households discount future user costs at both the extensive and intensive margins of durable adjustment. At the extensive margin, this discounting is very stark because the contemporaneous user cost plays a special role in the timing decision. The smooth pasting condition requires that households at an adjustment threshold are indifferent between adjusting now versus waiting a little bit. Consider a household that is contemplating increasing its durable position. Upgrading the durable position now brings a higher utility flow but at the cost of paying the contemporaneous user cost on the addition to its durable stock. On the other hand, waiting a short time to adjust brings a smaller utility flow over that time but saves the contemporaneous user cost. As the contemporaneous user cost enters this trade off, its effect on the extensive margin of durable adjustment is qualitatively different from the effects of future user costs.

At the intensive margin of durable adjustment, the marginal cost of acquiring a larger durable stock is the expected discounted user cost over the time until the next durable adjustment takes place. User costs at longer horizons receive less weight in the intensive margin decision both because of time discounting and because the cumulative probability of a durable adjustment increases with time.

These results imply a steep discounting of future user costs in durable demand decisions. With an upward sloping supply curve for durable goods, however, user costs are themselves forward looking because they depend on the relative price of durables and any expected capital gains on durables. The equilibrium response of durables prices is therefore a channel through which forward guidance can operate to stimulate durable demand. We characterize this channel analytically in a special case of the model. While these price movements make forward guidance more powerful, we show that durable demand continues to discount future real interest rates.

We quantitatively evaluate the power of forward guidance and find it is substantially less powerful than contemporaneous interest rate changes. For example, an interest rate cut at a horizon of one year has an effect on current output that is only about forty percent of the effect of a contemporaneous interest rate cut. Durable goods demand drives our results; forward guidance is essentially as powerful as contemporaneous interest rate changes in a version of the model without durables.

It is often argued that forward guidance is powerful because it affects the (long-term) interest rates on financing for durable goods purchases such as mortgage rates. While our main model abstracts from long-term financing, we show the first order conditions are *the same* in an extension with a long-duration financial asset. Thus, durable decisions continue to discount future user costs. Furthermore, it is the short-term real interest rate that enters the user cost not the long-term real interest rate. For intuition, let us focus on the extensive margin. The benefit of waiting a short time to make a purchase depends on how the long-term rate is expected to change over that time, as opposed to the level of the long-term rate. In every day language, if households expect long-term interest rates to rise they want to lock in the rate now. The expected change in the long-term rate over that short period is closely related to the expected return on the long-term bond, which is equal to the short-term interest rate by no-arbitrage.

Our focus on the fixed-cost model is motivated by the microeconomic evidence of inaction and lumpy adjustments in household durable demand. These patterns of behavior naturally point to (S, s) policies of the type generated by fixed-cost models. Our model is closely related to the fixed-cost model we developed in McKay and Wieland (2020), in which we study how monetary policy shifts durable demand intertemporally. In that paper, we show that the model matches micro-data on durable adjustment hazards and that it accurately describes the transmission of real interest rate changes to durable demand both on the intensive and extensive margins.

2 Model

2.1 Households

Households consume non-durable goods, c, and a service flow from durable goods, s. Household $i \in [0, 1]$ has preferences given by

$$E_0 \int_{t=0}^{\infty} e^{-\rho t} u\left(c_{it}, s_{it}\right) \mathrm{d}t.$$

The service flow from durables is generated from the household's stock of durable goods d_{it} as we describe below. The felicity function is CES,

$$u(c, s) = \frac{\left[(1-\psi)^{\frac{1}{\xi}} c^{\frac{\xi-1}{\xi}} + \psi^{\frac{1}{\xi}} s^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi(1-\sigma)}{\xi-1}} - 1}{1-\sigma}$$

, where ξ is the elasticity of substitution between nondurables and durables and σ^{-1} is the intertemporal elasticity of substitution.

Households hold a portfolio of durables and liquid assets denoted a_{it}. When a household with pre-existing portfolio (a_{it}, d_{it}) adjusts its durable stock, it reshuffles its portfolio to (a'_{it}, d'_{it}) subject to the payment of a fixed cost such that

$$a'_{it} + p_t d'_{it} = a_{it} + (1 - f)p_t d_{it}, \tag{1}$$

where p_t is the relative price of durable goods in terms of nondurable goods, and $fp_t d_{it}$ is a fixed cost proportional to the value of the durable stock.

The stock of durables depreciates at rate δ. A fraction χ of depreciation must be paid immediately in the form of maintenance expenditures so we have

$$\dot{d}_{it} = -(1 - \chi)\delta d_{it}, \tag{2}$$

where a dot over a variable indicates a time derivative. The household must also pay a flow cost of operating the durable stock equal to $\nu p_t d_{it}$. These operating costs reflect expenditures such as fuel, utilities, and taxes.

Liquid savings pay a safe real interest rate r_t. Borrowers pay real interest rate $r_t + r^s$, where r^s is an exogenous borrowing spread. The household is able to borrow against the value of the durable stock up to a loan-to-value (LTV) limit λ

$$a_{it} \geq -\lambda(1 - f)p_t d_{it}. \tag{3}$$

When a household does not adjust its durable stock, its liquid assets evolve according to

$$\dot{a}_{it} = r_t a_{it} + r^s a_{it} I_{\{a_{it}<0\}} - c_{it} + y_{it} - (\chi\delta + \nu)p_t d_{it}. \tag{4}$$

Household after-tax income, y_{it}, is given by $y_{it} = (1 - \tau_t)z_{it}Y_t$, where Y_t is aggregate income, z_{it} is the household's idiosyncratic income share, and τ_t is a time-varying income tax rate. The log income share $\ln z_{it}$ follows the Ohrnstein-Uhlenbeck process

$$\mathrm{d}\ln z_{it} = \rho_z \ln z_{it}\,\mathrm{d}t + \sigma_z\,\mathrm{d}\mathcal{W}_{it} + (1 - \rho_z)\ln \bar{z}\,\mathrm{d}t, \tag{5}$$

where $\mathrm{d}\mathcal{W}_{it}$ is a Brownian motion, $\rho_z < 0$ controls the persistence of the income process, σ_z determines the variance of the income process, and $\bar{z}$ is a constant such that $\int z_{it}\,\mathrm{d}i = 1$.

The service flow of the durable $s_{it} = q_{it}d_{it}$ is modulated by match quality q_{it}. q_{it} equals one when an adjustment takes place but subsequently drops to zero with Poisson intensity

θ. These match-quality shocks stand in for unmodeled life events that cause households to adjust their durable positions such as a new job in a distant city. Match-quality shocks are a source of inframarginal adjustments of the household durable stock, which help the model match the responsiveness of durable demand to monetary policy shocks (see McKay and Wieland, 2020).

2.2 Firms

Nondurable goods are produced with a technology that is linear in labor, $Y_t = L_t$. Nondurable goods are converted to durable goods using a technology with a congestion externality,

$$X_{kt} = M_{kt} \left(\frac{X_t}{\bar{X}} \right)^{-\zeta},$$

where X_{kt} is durable output by firm k, M_{kt} is nondurbale input, $X_t = \int_0^1 X_{kt} dk$ is aggregate durable production, and ζ determines the strength of the externality. Firms are competitive and price equals marginal cost,

$$p_t = \left(\frac{X_t}{\bar{X}} \right)^{\zeta}. \tag{6}$$

The congestion externality yields an upward sloping supply curve for durables with elasticity ζ^{-1}. For our purposes the source of the slope is not important. It could, for instance, arise from aggregate adjustment costs or a fixed factor such as land. An advantage of the congestion externality is that durable firms make zero profit, so we do not have to specify the distribution of dividends across heterogeneous agents.

2.3 Government

We assume that the central bank directly chooses a path for the real interest rate, $\{r_s\}_{s \geq 0}$. Implicitly we assume nominal rigidities allow the central bank to implement this real rate path through an appropriate choice of the nominal interest rates.[1] This is a common way of analyzing forward guidance (e.g. McKay et al., 2016; Werning, 2015).

[1] We select the equilibrium in which the economy returns to steady state. This can be implemented by assuming that the central bank reverts to a standard interest rate rule at some arbitrarily far away date.

Financial assets are in positive net supply due to a fixed supply of real government bonds $A_t = \bar{A}$. The tax rate τ_t adjusts to finance debt payments and maintain a balanced budget,

$$r_t \bar{A} = \int_0^1 \tau_t z_{it} Y_t \, \mathrm{d}i = \tau_t Y_t.$$

2.4 Market Clearing

By integrating over all households we obtain aggregate quantities,

$$C_t = \int_0^1 c_{it} \, \mathrm{d}i,$$
$$D_t = \int_0^1 d_{it} \, \mathrm{d}i.$$

Total durable expenditure (including maintenance), X_t, can be obtained either by summing individual policy functions or from the durable accumulation equation,

$$\dot{D}_t = -\delta D_t + X_t. \tag{7}$$

In equilibrium, aggregate income must equal aggregate expenditure

$$Y_t = C_t + \nu p_t D_t + p_t X_t. \tag{8}$$

It is unnecessary to fully specify the supply side of the model for our analysis of the demand response to a given real interest rate path. In this regard, our approach follows Werning (2015). In equilibrium, Y_t is determined by (8) and then divided among households according to $y_{it} = (1 - \tau_t) z_{it} Y_t$. In McKay and Wieland (2020) we provide a complete supply side that yields these equilibrium relationships. In that formulation, z_{it} is idiosyncratic labor productivity and wages are sticky and set by unions that ration hours equally across households.

3 Durable Adjustment with Fixed Costs

We now show that households place less weight on user costs (and thus interest rates) in the future than in the present. Denote the value function by $V_t(a, d, z)$. When no adjustment takes place, the value function follows the Hamilton-Jacobi-Bellman equation,

$$\rho V_t(a,d,z) = \max_{c_t} \left\{ u(c_t,d) + \mathbb{E}_t \frac{\mathrm{d}}{\mathrm{d}t} V_t(a,d,z) \right\}, \tag{9}$$

subject to the laws of motion for the individual states and the LTV constraint (2)-(5).

When an adjustment takes place, the household picks the optimal durable stock given its cash-on-hand $x_t \equiv a + (1-f)p_t d$ to maximize the post-adjustment value subject to the LTV constraint

$$V_t^{adj}(x,z) = \max_{p_t(1-\lambda(1-f))d' \leq x} V_t(x - p_t d', d', z).$$

Let $d_t^*(x,z)$ be the solution to this problem.

3.1 Extensive Margin

A durable adjustment takes place when the household hits an adjustment threshold. The optimal adjustment thresholds are characterized by the value matching and smooth pasting conditions. The value matching condition simply states that the value pre- and post-adjustment is the same,

$$V_t\left(a - p_t(d_t^* - (1-f)d), d_t^*, z\right) = V_t(a,d,z).$$

The smooth pasting condition requires that the household is indifferent between adjusting and waiting another instant,

$$\mathbb{E}_t \frac{\mathrm{d}}{\mathrm{d}t} V_t(a - p_t(d_t^* - (1-f)d), d_t^*, z) = \mathbb{E}_t \frac{\mathrm{d}}{\mathrm{d}t} V_t(a,d,z).$$

The instantaneous user cost of durables is an important determinant of this indifference condition. It is given by,

$$r_t^d \equiv p_t(r_t + \nu + \delta) - \dot{p}_t, \tag{10}$$

and captures the marginal cost of holding durables for an instant. A unit of durables acquired at p_t costs forgone interest $p_t r_t$, operating costs $p_t \nu$, depreciation $p_t \delta$, and potential capital losses $-\dot{p}_t$.

In Supplementary Appendix A, we derive a characterization of the adjustment thresholds assuming $r^s = 0$,

$$\begin{aligned}
&\frac{1}{V_{a,t}(a_t^*, d_t^*, z)}\left[u(c_t^*, d_t^*) - u(c_t, d)\right] = \qquad (11)\\
&\quad r_t^d\,(d_t^* - d) + \left[r_t^d - (\nu + \delta\chi)p_t\right] fd + (c_t^* - c_t)\\
&\quad + \frac{1}{1-\lambda(1-f)}\left[\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1\right]\left\{\frac{a_t}{p_t}\left[r_t^d - (\nu + \delta\chi)p_t\right] + z(1-\tau_t)Y_t - c_t - (\nu + \delta\chi)p_t d\right\},
\end{aligned}$$

where c_t^* and a_t^* are post-adjustment consumption and assets. The crucial thing to note here is that r_t^d enters (11) in a way that future user costs do not. In this sense, the contemporaneous user cost plays a special role in the extensive margin decision.

To understand this result, begin with the first line of (11), which captures the benefit of a durable adjustment this instant. For concreteness, consider an upward adjustment, $d_t^* > d$. The term $u(c_t^*, d_t^*) - u(c_t, d)$ captures the increased flow utility from upgrading durables, which is converted into nondurable goods units via $V_{a,t}(a_t^*, d_t^*, z)^{-1}$.

The other side of the smooth pasting condition captures the marginal benefit of delaying the adjustment. The second line represents the benefit for a household that is not LTV-constrained. For this household, delaying the purchase $d_t^* - d$ incurs a flow benefit given by the instantaneous user cost: the household earns additional interest, pays lower operating and maintenance costs, and does not incur any capital losses on the purchase. In addition, the household delays the payment of the fixed cost, which is valued at the instantaneous user cost less the operating and maintenance costs.[2] Finally, complementarities in the choices of nondurable consumption and durable expenditure through, for example, the utility function or borrowing constraints, yield an additional benefit of delaying equal to $c_t^* - c_t$.

For an unconstrained household the third line is zero since $\frac{V_{d,t}}{p_t V_{a,t}} = 1$ upon adjustment. However, for a household constrained by LTV, the third line also enters the equation as $\frac{V_{d,t}}{p_t V_{a,t}} > 1$ upon adjustment. This household incurs an additional benefit from waiting since by accumulating more assets it can relax the LTV constraint, with $\frac{1}{1-\lambda(1-f)}$ leveraging up these savings. Savings are valued at the user cost less the operating and maintenance costs: not only does interest accumulate, but waiting delays the depreciation of the purchase and

[2] Subtracting the operating and maintenance costs leaves the interest expense $r_t p_t$ and reduction in resale value $p_t\delta(1-\chi) - \dot{p}_t$.

potentially allows the household to buy the durable for a lower price $(-\dot{p}_t)$ thereby converting liquid assets into a higher durable stock. The household's purchasing power also evolves with its flow income less its expenditures on nondurables and operating and maintenance costs.

The smooth pasting condition implies that the contemporaneous user cost plays a central role in determining durable demand. If the user cost is low, for example because r_t is low, then the benefit of waiting shrinks both for constrained and unconstrained households. We would then expect households to accelerate their durable purchases and a corresponding increase in aggregate durable demand. The contemporaneous interest rate is more powerful in stimulating durable demand than are future interest rates because it directly affects the contemporaneous user cost.

3.2 Intensive Margin

We now turn to the intensive margin, which is more forward-looking, but still discounts far future user costs relative to those in the near future.

To begin, we define the cumulative user cost from t to $t+\tau$ as

$$r^d_{t,t+\tau} = p_t e^{\int_0^\tau r_{t+u}\,\mathrm{d}u} - p_{t+\tau} e^{-\delta(1-\chi)\tau} + (\nu + \delta\chi) \int_0^\tau e^{\int_k^\tau r_{t+u}\,\mathrm{d}u - \delta(1-\chi)k} p_{t+k}\,\mathrm{d}k.$$

This is the cost of buying a unit of durables at t and holding it to $t+\tau$ (exclusive of adjustment costs). The first two terms accumulates lost interest, depreciation, and capital losses over the holding period. The third term accumulates (with interest) the flow payments for operating and maintenance costs over the holding period. Note that over a short interval, $r^d_{t,t+\mathrm{d}t}$ is the instantaneous user cost as $\lim_{\mathrm{d}t\to 0} \frac{r^d_{t,t+\mathrm{d}t}}{\mathrm{d}t} = p_t(r_t + \nu + \delta) - \dot{p}_t = r^d_t$.

In Supplementary Appendix B we show that with $r^s = 0$ the intensive margin first order condition can be expressed as

$$\mathbb{E}_t \int_0^\tau e^{-(\rho+\delta(1-\chi))s} u_d(c_{t+s}, e^{-\delta(1-\chi)s} d)\,\mathrm{d}s = \mathbb{E}_t e^{-\rho\tau} V^{adj}_{x,t+\tau} \left[r^d_{t,t+\tau} + e^{-\delta(1-\chi)\tau} p_{t+\tau} f \right] \qquad (12)$$
$$+ \mathbb{E}_t \int_0^\tau e^{-\rho s} \Psi_{t+s} \left[r^d_{t,t+s} + (1 - \lambda(1-f)) e^{-\delta(1-\chi)s} p_{t+s} \right] \mathrm{d}s$$

where $t+\tau$ is the optimal (stochastic) stopping time when the next durable adjustment takes place and Ψ_t is the Lagrange multiplier on the borrowing constraint at date t. An unconstrained household $(\Psi = 0)$ equates the expected discounted marginal utility of durables over the holding period of the durable to the expected discounted cumulative user cost over the holding period plus the losses from the fixed cost. When borrowing constraints bind for some $t+s$, the household also considers how liquid assets at date $t+s$ are affected by increasing the durable position, which is given by $r^d_{t,t+s}$, and how durables act as collateral to relax the borrowing constraint.

The crucial thing to note about (12) is that the planning horizon stops at the next adjustment date $t+\tau$. Suppose τ were deterministic, then only the cumulative user cost between t and $t+\tau$ enters the equation, whereas user costs after $t+\tau$ get zero weight. Since τ is in fact stochastic, it is integrated out by the expectation operator which weighs the user cost at $t+s$ by the probability the durable position has not been adjusted before that date. At longer horizons, it is quite likely that the household has already adjusted its durable position so user costs at these horizons receive little weight relative to those in the more immediate future.

4 The User Cost in Equilibrium

We have shown that durable demand is particularly sensitive to the contemporaneous user cost and user costs in the near future. However, in equilibrium, user costs are themselves forward looking through the relative price of durables and its expected evolution. We now characterize this channel, which smooths out the relationship between durable demand and interest rates. To do so we use a special case of the model in which there are no fixed costs ($f=0$), durables are fully collateralizable ($\lambda=1$) and there is no borrowing spread ($r^s=0$). In this case, the LTV constraint never binds and households continuously adjust their durable positions. The first order condition is the limit of the intensive margin first order condition (12) as the time between adjustment shrinks to zero, $\tau\to 0$. It implies that all households set their marginal rate of substitution between durables and nondurables equal to the contemporaneous user cost,

$$\left(\frac{\psi}{1-\psi}\frac{c_{it}}{d_{it}}\right)^{\frac{1}{\xi}} = p_t(r_t+\nu+\delta) - \dot{p}_t = r^d_t. \tag{13}$$

Since the contemporaneous user cost is common across households, so is the $c_{it}/d_{it} = C_t/D_t$ ratio. While this first order condition is considerably simpler than those in Section 3, it shares the property with the smooth-pasting condition that the contemporaneous user cost plays a special role in determining durable demand. In this sense, the insights of this special case are relevant to the fixed-cost model.

For given paths $\{r_t, C_t\}_{t=0}^{\infty}$ we can solve for $\{X_t, D_t, p_t, r_t^d\}_{t=0}^{\infty}$ using equations (6), (7), (10), and (13). After linearizing this system, we can use standard differential equation methods to solve for durable expenditure (see Supplementary Appendix C). Let $\hat{k}_t = \frac{K_t - \bar{K}}{\bar{K}}$ denote percent deviation from steady state of variable K_t. Starting from steady state, durable expenditure is given by

$$\hat{x}_0 = -\frac{1}{\zeta} \int_0^{\infty} e^{-\kappa_2 t} \left[(r_t - \bar{r}) + \frac{\bar{r} + \delta + \nu}{\xi} \hat{c}_t \right] \mathrm{d}t \tag{14}$$

where $\kappa_2 = \frac{r+\nu}{2} + \sqrt{\frac{(\bar{r}+\nu)^2}{4} + \frac{\delta[\bar{r}+\delta+\nu]}{\zeta\xi} + \delta(\bar{r}+\delta+\nu)} > \bar{r} + \nu + \delta$ is the positive eigenvalue of the system.

While the first order condition (13) shows that durable demand depends only on the contemporaneous user cost and consumption level, equation (14) shows that durable expenditure is a smooth function of the expected interest rate and the consumption paths. The forward-looking nature of equation (14) reflects equilibrium movements in the relative price of durables that smooth changes in the user cost. Suppose the real interest rate is reduced at some future date t leading to an increase in durable demand and durable prices at that date. Before t, there is an incentive to accumulate durables to exploit the anticipated appreciation. In equilibrium, the contemporaneous price of durables will react to the increase in future demand. The higher price reflects stronger demand for durables, which itself reflects a lower user cost due to the anticipation of capital gains. Therefore the relative price of durables rises upon the news and then rises further as date t approaches. Given the isolestic supply curve, (6), equilibrium movements in durables prices are proportional to movements in durable expenditure so the path for p_t directly maps into the path for X_t.

While equilibrium price movements make durable expenditure forward-looking, future real interest rates are discounted at rate $\kappa_2 > \bar{r} + \nu + \delta$. This discounting makes forward guidance less effective than contemporaneous real rate cuts at stimulating durable demand.

To understand the discounting, again consider the scenario in which the real rate is low at date t, causing an increase in durable demand and prices at t. How strongly prices before t respond to the anticipated appreciation depends on the costs of holding the asset. With higher (steady state) interest rates, operating costs, or depreciation there is a higher flow cost of holding the asset and the price path will need to compensate households for those costs meaning it will need to be steeper and the initial price response will be smaller. Moreover, part of the payoff of acquiring durables is the marginal utility flow they bring. With diminishing marginal utility, taking on a larger durable position to exploit anticipated appreciation brings about a smaller dividend and the price path will need to compensate the household for this, too. Marginal utility drops by more if the the change in the durable stock is large (the supply elasticity $1/\zeta$ is large) or the elasticity of substitution ξ is small. Again, as movements in p_t are proportional to those of x_t, these considerations for prices immediately translate to the path for x_t.

We find that the patterns described here carry over to the numerical solution for the full fixed-cost model, which we turn to next. In particular, the user cost of durables is forward looking in equilibrium, which makes the extensive margin forward looking. Moreover, future real rates are less powerful than contemporaneous real rates in line with the discounting discussed above.

5 Quantitative Results

We now quantitatively assess the power of forward guidance.

5.1 Calibration

The calibration largely follows McKay and Wieland (2020) where we show the model accurately captures the transmission of monetary shocks to the intensive and extensive margins of durable demand as well as nondurable consumption.

We choose an elasticity of substitution between durables and nondurables of $\xi = 0.5$, which is at the lower end of the range of values estimated empirically (Ogaki and Reinhart, 1998; Davis and Ortalo-Magné, 2011; Pakoš, 2011; Davidoff and Yoshida, 2013; Albouy

Table 1: Calibration of the Model

Parameter	Name	Value	Source
ξ	Elas of substitution	0.5	See text
σ	Inverse EIS	4	Estimated C IRF
ψ	Durable exponent	0.581	d/c ratio = 2.64
δ	Depreciation rate	0.068	BEA Fixed Asset Table
χ	Required maintenance share	0.35	See text
ν	Operating cost	0.048	See text
ρ	Discount rate	0.096	Net Assets/Private GDP = 1.12
$\bar{r}$	Real interest rate	0.015	Annual real FFR
$\bar{r}^s$	Borrowing spread	0.017	Mortgage T-Bill spread
f	Fixed cost	0.194	Ann. adjustment prob = 0.19
θ	Intensity of match-quality shocks	0.158	McKay and Wieland (2020)
λ	Borrowing limit	0.8	20% Down payment
ρ_z	Income persistence	-0.090	Floden and Lindé (2001)
σ_z	Income st. dev.	0.216	Floden and Lindé (2001)
ζ	Inverse durable supply elasticity	0.049	See text

et al., 2016). Higher values imply that durable demand is overly sensitive to monetary policy (McKay and Wieland, 2020). We also verified numerically that choosing a higher value for the elasticity of substitution ξ reduces the power of forward guidance relative to contemporaneous interest rates and in this sense our choice is conservative. We set the elasticity of intertemporal substitution to 1/4, which allows the model to match the small response of nondurable consumption to monetary policy shocks (McKay and Wieland, 2020). It is at the lower end of the range typical in calibrations, but on the higher end of traditional time-series estimates (Hall, 1988; Campbell and Mankiw, 1989; Yogo, 2004) as well as recent cross-sectional estimates (Best et al., 2020).

We calibrate the taste for durables to match the value of the stock of durables relative to nondurable consumption from 1970-2019. Durables include housing and consumer durables. The depreciation rate is set to match durable stock depreciation in the BEA fixed asset table. We measure maintenance costs as the sum of intermediate goods and services consumed in the housing output table, the PCE on household maintenance, and the PCE on motor vehicle maintenance and repair. Operating costs include taxes on the housing sector and PCE on household utilities and motor vehicle fuels and fluids.

We calibrate the discount rate, ρ, to match aggregate holdings of financial assets net of mortgage and auto loans. We set the steady state real interest rate to 1.5%, which is the average real federal funds rate between 1991 and 2007. We set the borrowing spread to 1.7%, which is the average spread between the 30-year mortgage and 10-year Treasury rates.

The fixed adjustment cost is set to match the frequency of durable adjustments. Our calibration target is a weighted average of the frequency of moving residence or making a housing addition or substantial repair and the frequency of buying a car. These frequencies are weighted in proportion to the values of the respective durable stocks. In McKay and Wieland (2020), we estimate the arrival intensity of match-quality shocks, θ, from PSID data on durable adjustments using the method of Berger and Vavra (2015). θ is identified by the frequency with which households adjust their durable position despite having a small gap between their existing durable position and their target position. The LTV limit, λ is set to 80% and we take the parameters of the idiosyncratic risk process from Floden and Lindé (2001).

To calibrate the supply elasticity of durable goods we interpret the production externality as reflecting congestion caused by a fixed factor specifically land. This leads us to an inverse durable supply elasticity of $\zeta = 0.049$. This value reflects the share of residential investment in durable expenditure (36%), the share of new permanent-site structures in residential investment (58%), and the cost of land in new permanent-site structures (approx. 24%).[3] An elastic supply is consistent with the muted response of the relative price of durables estimated by McKay and Wieland (2020) and with House and Shapiro's (2008) finding that capital goods production responds significantly to investment stimulus but prices do not.[4] The supply elasticity determines the variation in durable prices and thereby the importance of capital gains in the user cost discussed in Section 4. In Supplementary Appendix E we show forward guidance is only slightly more powerful when we calibrate ζ to the inelastic end of the range estimated by House and Shapiro (2008).

[3]The first two values are from NIPA Table 1.1.5 and NIPA Table 5.4.5, 1969-2007. We calibrate the cost of land in housing prices using the midpoint of new and existing houses in Davis and Heathcote (2007). See McKay and Wieland (2020) for further details.

[4]Goolsbee (1998) finds stronger price responses, but for the categories of goods that also serve as consumer durables (autos, computers, and furniture) he, too, finds little price response.

We solve the model using continuous-time methods from Achdou et al. (2017) and the sequence-space methods from Auclert et al. (2019).

5.2 Results

Figure 1 shows the change in contemporaneous output in response to interest rate cuts at different horizons. The solid line corresponds to our main model. A contemporaneous interest rate cut of 1% for one quarter increases output by 0.75%. Promises of interest rate cuts in the future are less powerful and substantially so at more distant horizons. If the same interest rate change occurs one year from now, then today's output increases by 0.30%, only forty percent as much compared to contemporaneous stimulus. For promises more than four years out, the power of forward guidance settles at a fifth of the effectiveness of a contemporaneous cut. In short, forward guidance is considerably less powerful than contemporaneous interest rate cuts in our model. In Supplementary Appendix D we show that 73 percent of the weaker effects of forward guidance at a one-year horizon reflect a weaker extensive margin response and the intensive margin accounts for 21 percent.

We plot results from two other models for comparison. First, as is well-known, the standard new Keynesian model predicts real rate changes at any horizon have the same effect on output today. This prediction of the model is widely regarded as implausible and at the heart of the forward guidance puzzle (see Carlstrom et al., 2015; Del Negro et al., 2015; McKay et al., 2016). Our model makes several changes from the new Keynesian model, but the addition of durables is particularly important. We plot the effect of forward guidance in a version of our model with only nondurables.[5] In that model, forward guidance effects are only slightly attenuated relative to the three-equation model. For example, an interest rate cut three years from now is 90% as effective as a contemporaneous interest rate cut.

[5] In this model, the durable share in utility is set to zero, $\psi = 0$, rendering $\delta, \xi, f, \theta, \nu$ irrelevant. The inverse intertemporal elasticity of substitution, σ, is calibrated to match the impact response of output to a contemporaneous 1% real rate reduction in our full model. The borrowing limit is set to $-\lambda$ times the 25^{th} percentile of durable holdings in our full model. The parameter ρ is set to match the same net asset to GDP ratio as in the full model. Other parameters are unchanged.

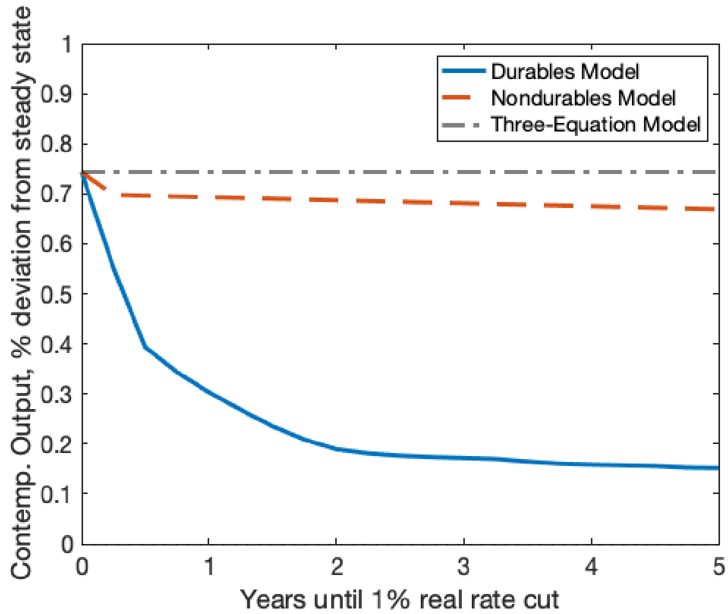

Figure 1: Contemporaneous output response to promises of interest rate cuts at different horizons. At each horizon the real interest rate drops by 1 percentage point for one quarter. The solid blue line represents our main model from Section 2. The dashed red line is a version of the same model without durables and the dashed-dotted yellow line is the standard Three-Equation model. The latter two models are calibrated to yield the same output effects for a contemporaneous real interest rate cut as our main model.

6 Long-Term Financing

Forward guidance is often thought to affect household purchasing decisions by moving long-term financing rates such as mortgage rates. Our model abstracts from this mechanism as households use short-term assets for financing. We now extend the model to include a long-term asset and show that the smooth-pasting condition governing durable adjustment timing (11) and the first order condition for the intensive margin (12) are *unchanged*. Furthermore the interest rate that appears in the user cost is the short-term return on the long-term asset not the yield to maturity.

The long-term asset is a bond that can be bought and sold at price q_t. Each unit of this bond pays a flow coupon $v\,dt$ each instant with the quantity of bonds amortizing at rate $\Gamma\,dt$. The instantaneous return on the bond is

$$r^b_t \equiv \frac{\dot{q}_t + v}{q_t} - \Gamma. \tag{15}$$

Borrowing through the long-term bond incurs an intermediation fee r^s proportional to the value of the debt so the instantaneous cost of borrowing through the long-term asset is $r_t^b + r^s$.

Define total holdings of short-term assets by $\tilde{a}_{it}$ and total holdings of the long-term bond by b_t. Total liquid wealth of the household is $a_{it} = q_t b_{it} + \tilde{a}_{it}$. Absent a durable adjustment, the household's budget constraint is

$$\dot{a}_{it} = r_{it}^a a_{it} + r^s a_{it} I_{\{a_{it}<0\}} - (\nu + \chi\delta) p_t d_{it} - c_{it} + z y_{it} \tag{16}$$

where r_{it}^a is the return on wealth.[6] Let $\omega_{it} = \frac{q_t b_{it}}{a_{it}}$ be the portfolio share of the long-term bond. Then the return on wealth is a portfolio-weighted average of the two asset returns,

$$r_{it}^a = \omega_{it} r_t^b + (1 - \omega_{it}) r_t$$

The short-term return on wealth replaces the short-term real rate in the user cost equation (10), since it captures the return that could be earned by delaying a durable adjustment.

The no-arbitrage condition implies that all assets must pay the same return on a perfect foresight path, $r_{it}^a = r_t^b = r_t$. Combining this condition with (16) yields the budget constraint (4). Therefore, the model with a long-term asset has the same budget constraint conditional on not adjusting as the model with a short-term asset only. The budget constraint conditional on adjusting (1) and the borrowing constraint (3) from the short-term asset model are also unaffected. Therefore, long-term debt does not change the household problem conditional on the household's initial states (a_{i0}, d_{i0}, z_{i0}) and the paths for aggregate variables. We obtain *exactly* the same smooth-pasting condition (11) and intensive margin first order condition (12). Thus, durable demand decisions discount future user costs exactly as in our baseline model. Moreover, as the decision problem is unchanged, so is the definition of the user cost (10). In particular, the user cost depends on the short-term real interest rate, not the yield to maturity.

To understand why the first order conditions are unaffected, let us focus on the extensive margin. Recall that households make a short-term decision near the adjustment threshold—to adjust now or a little later. By adjusting later, the household avoids paying the expected return on the long-term debt over that period. By no-arbitrage, the return on the long-term bond is equal to the short-term rate. Using the same logic, the instantaneous user cost

[6] In writing (16), we incorporate that households will never hold a positive position in one asset and a negative position in the other in equilibrium.

depends on the instantaneous return of the long-term bond, which is equal to the short-term real rate.

Intuitively, financing a purchase is more expensive if q_t falls as the household then has to issue more bonds and commit to more coupon payments. A low expected return on the bond then implies the cost of financing the purchase is expected to rise. Moreover, as the bond price q_t is inversely related to the long-term rate, a low expected return also implies the long-term rate is expected to rise. In this manner, the model captures the desire to lock-in a low long-term financing cost when long-term rates are expected to rise.

Intuitively, financing a purchase becomes more expensive if q_t falls as the household then has to issue more bonds and commit to more coupon payments. If households expect such an increase in the cost of financing, then the expected return on the long-term bond is low, all else equal, and households will find it beneficial to make adjustments sooner. Moreover, as the bond price q_t is inversely related to the long-term rate, an expected decrease in q_t implies that the long-term rate is expected to rise. In this manner, the model captures the desire to lock-in a low long-term financing cost when long-term rates are expected to rise.

While the partial equilibrium decision problem is unaffected by long-term debt, the equilibrium of the economy will reflect a valuation effect on a_{i0} as the asset price q_0 jumps upon news of the real interest rate path. Moreover, the government budget constraint is similarly affected by valuation effects yielding a different path for taxes. In Supplementary Appendix F we quantify the importance of these valuation effects and show that they slightly reduce the power of contemporaneous interest rates but overall our results are little changed.

7 Conclusion

In recent years, forward guidance policies have received considerable attention not only because of their relevance to unconventional monetary policy strategies but also because they raise questions about the plausibility of the strongly forward-looking behavior in workhorse macroeconomic models. We show that incorporating durables goods demand subject to fixed adjustment costs substantially reduces the power of forward guidance. We view this as an attractive approach for modeling forward guidance because durable goods are particularly sensitive to monetary policy and because fixed adjustment costs are supported by the microeconomic lumpiness of durable adjustments.

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A Derivation of Equation (11)

Starting with the smooth-pasting condition

$$\mathbb{E}_t \frac{\mathrm{d}}{\mathrm{d}t} V_t(a - p_t(d_t^* - (1-f)d), d_t^*, z) = \mathbb{E}_t \frac{\mathrm{d}}{\mathrm{d}t} V_t(a, d, z)$$

and substituting the evolution of the value function conditional on not adjusting yields,

$$\mathbb{E}_t \frac{\mathrm{d}}{\mathrm{d}t} V_t(a - p_t(d_t^* - (1-f)d), d_t^*, z) = \rho V_t(a, d) - u(c_t, d)$$

Using Ito's Lemma, we determine the evolution of the left-hand-side,

$$\begin{aligned}
&V_{a,t}(a_t^*, d_t^*, z)[\dot{a} + (1-f)p_t\dot{d} - \dot{p}_t(d_t^* - (1-f)d) - p_t(\dot{d}_t^* + d_{x,t}^*\dot{x} + d_{z,t}^*\mathbb{E}_t\dot{z} + d_{zz,t}^*\frac{\sigma_z^2}{2})] \\
&+ V_{d,t}(a_t^*, d_t^*, z)(\dot{d}_t^* + d_{x,t}^*\dot{x} + d_{z,t}^*\mathbb{E}_t\dot{z} + d_{zz,t}^*\frac{\sigma_z^2}{2}) + V_{z,t}(a_t^*, d_t^*, z)\mathbb{E}_t\dot{z} + V_{zz,t}(a_t^*, d_t^*, z)\frac{\sigma_z^2}{2} + \dot{V}_t(a_t^*, d_t^*, z) \\
&= \rho V_t(a, d, z) - u(c_t, d).
\end{aligned}$$

Conditional on adjusting, the optimal choice of d^* is given by

$$p_t V_{a,t}(a_t^*, d_t^*(x, z), z) \leq V_{d,t}(a_t^*, d_t^*(x, z), z),$$

with equality if the LTV constraint, $d_t^*(x, z) \leq \frac{1}{1-\lambda(1-f)}\frac{x}{p_t}$, is not binding.

A.1 LTV constraint not binding

If the household is not borrowing constrained in making a durable adjustment, then the terms involving the optimal choice of d^* drop out (envelope condition),

$$\begin{aligned}
&V_{a,t}(a_t^*, d_t^*, z)[\dot{a} + (1-f)p_t\dot{d} - \dot{p}_t(d_t^* - (1-f)d)] + V_{z,t}(a_t^*, d_t^*, z)\mathbb{E}_t\dot{z} + V_{zz,t}(a_t^*, d_t^*, z)\frac{\sigma_z^2}{2} + \dot{V}_t(a_t^*, d_t^*, z) \\
&= \rho V_t(a, d, z) - u(c_t, d).
\end{aligned}$$

Next, we substitute the HJB equation post-adjusting,

$$\begin{aligned}
&V_{a,t}(a_t^*, d_t^*, z)[\dot{a} + (1-f)p_t\dot{d} - \dot{p}_t(d_t^* - (1-f)d) - \dot{a}_t^*] - V_{d,t}(a_t^*, d_t^*, z)\dot{d}_t^* + \rho V_t(a_t^*, d_t^*, z) - u(c_t^*, d_t^*) \\
&= \rho V_t(a, d, z) - u(c_t, d).
\end{aligned}$$

Using the value-matching condition, first-order condition for adjustment, and dividing by $V_{a,t}$ yields,

$$\dot{a} + (1-f)p_t\dot{d} - \dot{p}_t(d_t^* - (1-f)d) - \dot{a}_t^* - p_t\dot{d}_t^* = \frac{1}{V_{a,t}(a_t^*, d_t^*, z)}\left[u(c_t^*, d_t^*) - u(c_t, d)\right]$$

Substituting the evolution of liquid assets and the durable stock yields,

$$\begin{aligned}&(r_tp_t + \nu p_t + \delta p_t - \dot{p}_t)(d_t^* - d) + f(r_tp_t + \delta(1-\chi)p_t - \dot{p}_t)d + (c_t^* - c_t)\\ &= \frac{1}{V_{a,t}(a_t^*, d_t^*, z)}\left[u(c_t^*, d_t^*) - u(c_t, d)\right]\end{aligned} \tag{17}$$

Finally, we plug in the definition of the instantaneous user cost, $r_t^d = r_tp_t + \nu p_t + \delta p_t - \dot{p}_t$. This yields equation (11) for the unconstrained case, $p_tV_{a,t} = V_{d,t}$.

A.2 LTV constraint binding

If the household is LTV-constrained, then $d_t^* = \frac{1}{1-\lambda(1-f)}\frac{x_t}{p_t}$, and the smooth pasting condition is

$$\begin{aligned}\mathbb{E}_t\frac{\mathrm{d}}{\mathrm{d}t}V_t(a - p_t(d_t^* - (1-f)d), d_t^*, z) &= \rho V_t(a,d) - u(c_t, d)\\ \mathbb{E}_t\frac{\mathrm{d}}{\mathrm{d}t}V_t(x_t - p_td_t^*, z) &= \rho V_t(a,d) - u(c_t, d)\\ \mathbb{E}_t\frac{\mathrm{d}}{\mathrm{d}t}V_t(-\frac{\lambda(1-f)}{1-\lambda(1-f)}x_t, \frac{1}{1-\lambda(1-f)}\frac{x_t}{p_t}, z) &= \rho V_t(a,d,z) - u(c_t, d).\end{aligned}$$

Using Ito's Lemma,

$$\begin{aligned}&- V_{a,t}(a_t^*, d_t^*, z)\frac{\lambda(1-f)}{1-\lambda(1-f)}\dot{x}_t + \frac{1}{1-\lambda(1-f)}V_{d,t}(a_t^*, d_t^*, z)\frac{1}{p_t}(\dot{x}_t - x_t\frac{\dot{p}_t}{p_t})\\ &+ V_{z,t}(a_t^*, d_t^*, z)\mathbb{E}_t\dot{z} + V_{zz,t}(a_t^*, d_t^*, z)\frac{\sigma_z^2}{2} + \dot{V}_t(a_t^*, d_t^*, z) = \rho V_t(a,d,z) - u(c_t, d)\end{aligned}$$

In the instant after an adjustment takes place, the value function satisfies $u(c_t^*, d_t^*) + V_{a,t}(a_t^*, d_t^*, z)\dot{a}_t^* + V_{d,t}(a_t^*, d_t^*, z)\dot{d}_t^* + V_{z,t}(a_t^*, d_t^*, z)\mathbb{E}_t\dot{z} + V_{zz,t}(a_t^*, d_t^*, z)\frac{\sigma_z^2}{2} + \dot{V}_t(a_t^*, d_t^*, z) = \rho V_t(a_t^*, d_t^*, z)$. Substituting this into our previous equation yields,

$$\begin{aligned}&V_{a,t}(a_t^*, d_t^*, z)\left[-\frac{\lambda(1-f)}{1-\lambda(1-f)}\dot{x}_t - \dot{a}_t^*\right] + \frac{1}{1-\lambda(1-f)}V_{d,t}(a_t^*, d_t^*, z)\frac{1}{p_t}(\dot{x}_t - x_t\frac{\dot{p}_t}{p_t})\\ &-V_{d,t}(a_t^*, d_t^*, z)\dot{d}_t^* - u(c_t^*, d_t^*) + \rho V_t(a_t^*, d_t^*, z) = \rho V_t(a,d,z) - u(c_t, d)\end{aligned}$$

Next we substitute the value-matching condition and $\dot{d}_t^* = -\delta(1-\chi)d_t^* = -\frac{\delta(1-\chi)}{1-\lambda(1-f)}\frac{x_t}{p_t}$ to further simplify,

$$V_{a,t}(a_t^*, d_t^*, z)\left[-\frac{\lambda(1-f)}{1-\lambda(1-f)}\dot{x}_t - \dot{a}_t^*\right] + \frac{1}{1-\lambda(1-f)}V_{d,t}(a_t^*, d_t^*, z)\frac{1}{p_t}(\dot{x}_t + \delta(1-\chi)x_t - x_t\frac{\dot{p}_t}{p_t})$$
$$= u(c_t^*, d_t^*) - u(c_t, d)$$

The evolution of cash on hand conditional on not adjusting is given by,

$$\begin{aligned}
\dot{x}_t &= \dot{a} + (1-f)p_t\dot{d} + (1-f)\dot{p}_t d_t \\
&= r_t a - (\nu + \chi\delta)p_t d - c_t + zy_t + (1-f)p_t\dot{d} + (1-f)\dot{p}_t d \\
&= r_t x_t - c_t + zy_t - [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)]d,
\end{aligned}$$

where we use y_t as compact notation for $(1-\tau_t)Y_t$. Since $a_t^* = -\frac{\lambda(1-f)}{1-\lambda(1-f)}x_t$ and $d_t^* = \frac{1}{1-\lambda(1-f)}\frac{x_t}{p_t}$, we then get

$$V_{a,t}(a_t^*, d_t^*, z)\left[-\frac{\lambda(1-f)}{1-\lambda(1-f)}\{-c_t + zy_t - [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)]d\}\right.$$
$$\left. - \{-c_t^* + zy_t - (\nu + \chi\delta)p_t d_t^*\}\right]$$
$$+ \frac{1}{1-\lambda(1-f)}V_{d,t}(a_t^*, d_t^*, z)\frac{1}{p_t}(\dot{x}_t + \delta(1-\chi)x_t - x_t\frac{\dot{p}_t}{p_t}) = u(c_t^*, d_t^*) - u(c_t, d)$$

Next we distribute terms into distinct benefits and costs of adjusting,

$$V_{a,t}(a_t^*, d_t^*, z)\left\{-[r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)]d + (\nu + \chi\delta)p_t d_t^*\right\}$$
$$+ \frac{1}{1-\lambda(1-f)}V_{a,t}(a_t^*, d_t^*, z)\left\{-zy_t + c_t + [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)]d\right\}$$
$$+ V_{a,t}(a_t^*, d_t^*, z)(c_t^* - c_t)$$
$$+ \frac{1}{1-\lambda(1-f)}V_{d,t}(a_t^*, d_t^*, z)\frac{1}{p_t}(\dot{x}_t + \delta(1-\chi)x_t - x_t\frac{\dot{p}_t}{p_t}) = u(c_t^*, d_t^*) - u(c_t, d)$$

Substituting the evolution of cash-on-hand,

$$V_{a,t}(a_t^*, d_t^*, z)\left\{-[r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)]d + (\nu + \chi\delta)p_t d_t^*\right\}$$
$$+ \frac{1}{1-\lambda(1-f)}V_{a,t}(a_t^*, d_t^*, z)\left\{-zy_t + c_t + [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)]d\right\}$$
$$+ V_{a,t}(a_t^*, d_t^*, z)(c_t^* - c_t)$$
$$+ \frac{1}{1-\lambda(1-f)}V_{d,t}(a_t^*, d_t^*, z)\frac{1}{p_t}([r_t + \delta(1-\chi) - \frac{\dot{p}_t}{p_t}]x_t - c_t + zy_t - [r_t p_t + \nu + \delta - \dot{p}_t - f(r_t p_t + \delta(1-\chi) -$$
$$= u(c_t^*, d_t^*) - u(c_t, d)$$

Collecting terms again,

$$\left(\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t} d_t^* - V_{a,t}(a_t^*, d_t^*, z) d\right)(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)$$
$$+ V_{a,t}(a_t^*, d_t^*, z)\left\{f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)d + (\nu + \chi\delta)p_t(d_t^* - d)\right\}$$
$$+ \frac{1}{1-\lambda(1-f)}\left[V_{a,t}(a_t^*, d_t^*, z) - \frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t}\right]\left\{-zy_t + c_t + [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi)p_t -\right.$$
$$+ V_{a,t}(a_t^*, d_t^*, z)(c_t^* - c_t)$$
$$= u(c_t^*, d_t^*) - u(c_t, d)$$

Divide by the post-adjustment marginal utility of wealth $V_{a,t}(a_t^*, d_t^*, z)$

$$\left(\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} d_t^* - d\right)(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)$$
$$+ f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)d + (\nu + \chi\delta)p_t(d_t^* - d)$$
$$+ \frac{1}{1-\lambda(1-f)}\left[\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1\right]\left\{zy_t - c_t - [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi) - \dot{p}_t)]d\right\}$$
$$+ (c_t^* - c_t)$$
$$= \frac{1}{V_{a,t}(a_t^*, d_t^*, z)}\left[u(c_t^*, d_t^*) - u(c_t, d)\right]$$

Next we separate the first term into a component that is present for all household and one that is only present for constrained households,

$$(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)(d_t^* - d) + \left(\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1\right)(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)d_t^*$$
$$+ f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)d + (\nu + \chi\delta)p_t(d_t^* - d)$$
$$+ \frac{1}{1-\lambda(1-f)}\left[\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1\right]\left\{zy_t - c_t - [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)]d\right\}$$
$$+ (c_t^* - c_t)$$
$$= \frac{1}{V_{a,t}(a_t^*, d_t^*, z)}\left[u(c_t^*, d_t^*) - u(c_t, d)\right],$$

which we can then combine with the other term affecting constrained households only,

$$
\begin{aligned}
&(r_t p_t + \nu p_t + \delta p_t - \dot{p}_t)(d_t^* - d) + f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)d + (c_t^* - c_t) \\
&+ \frac{1}{1-\lambda(1-f)}\left[\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1\right]\Big\{\frac{x_t - (1-f)p_t d_t}{p_t}(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t) \\
&+ z y_t - c_t - (\nu + \delta\chi)p_t d\Big\} = \frac{1}{V_{a,t}(a_t^*, d_t^*, z)}\left[u(c_t^*, d_t^*) - u(c_t, d)\right]
\end{aligned}
$$

Last, we substitute out cash on hand for liquid assets,

$$
\begin{aligned}
&(r_t p_t + \nu p_t + \delta p_t - \dot{p}_t)(d_t^* - d) + f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)d + (c_t^* - c_t) \\
&+ \frac{1}{1-\lambda(1-f)}\left[\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1\right]\left\{\dot{a}_t + a_t\left(\delta(1-\chi) - \frac{\dot{p}_t}{p_t}\right)\right\} \\
&= \frac{1}{V_{a,t}(a_t^*, d_t^*, z)}\left[u(c_t^*, d_t^*) - u(c_t, d)\right]
\end{aligned}
$$

When the household is not borrowing constrained, then $\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} = 1$, and this first order condition coincides with our earlier derivation (17). Thus our derivation given the borrowing constrained nests the unconstrained optimality condition as a special case.

To obtain equation (11), we plug in the definition of the instantaneous user cost (10) $r_t^d = r_t p_t + \nu p_t + \delta p_t - \dot{p}_t$ and the evolution of liquid assets (4).

B Derivation of Equation (12)

Assume that an adjustment is optimal today. Then the integrated HJB equation (9) is

$$
V_t^{adj}(x, z) = \max_{\{c_{t+s}\}, \tau, d} E\left\{\int_0^\tau e^{-\rho s}[u(c_{t+s}, e^{-\delta(1-\chi)s}d)]\,\mathrm{d}s + e^{-\rho\tau} V_{t+\tau}^{adj}(a_{t+\tau} + p_{t+\tau}(1-f)e^{-\delta(1-\chi)\tau}d, z_{t+\tau})\right\}
$$

where τ is the optimal stopping time. If between t and $t+s$ no further adjustment takes place, then liquid assets accumulate as

$$
a_{t+s} = (x - p_t d)e^{\int_0^s r_{t+u}\,\mathrm{d}u} + \int_0^s e^{\int_k^s r_{t+u}\,\mathrm{d}u}[y_{t+k} - c_{t+k} - (\nu + \delta\chi)p_{t+k}e^{-\delta(1-\chi)k}d]\,\mathrm{d}k.
$$

which we substitute into the integrated HJB above equation and the borrowing constraint below,

$$
a_{t+s} \geq -\lambda(1-f)e^{-\delta(1-\chi)s}p_{t+s}d
$$

Letting the Lagrange multiplier on the borrowing constraint be Ψ_{t+s}, then we can rewrite value function as

$$
\begin{aligned}
&V_t^{adj}(x,z) = \\
&\max_{\{c_{t+s}\},\tau,d} \mathbb{E}_t \left\{ \int_0^\tau e^{-\rho s}[u(c_{t+s}, e^{-\delta(1-\chi)s}d)]\,\mathrm{d}s + e^{-\rho\tau} V_{t+\tau}^{adj}\left((x-p_t d) e^{\int_0^\tau r_{t+u}\,\mathrm{d}u} + \right.\right. \\
&+ \int_0^\tau e^{\int_s^\tau r_{t+u}\,\mathrm{d}u}[y_{t+s} - c_{t+s} - (\nu+\delta\chi)p_{t+s}e^{-\delta(1-\chi)s}d]\,\mathrm{d}s + p_{t+\tau}(1-f)e^{-\delta(1-\chi)\tau}d, z_{t+\tau} \Big) + \\
&+ \mathbb{E}_t \int_0^\tau e^{-\rho s}\Psi_{t+s}\left[(x-p_t d)e^{\int_0^s r_{t+u}\,\mathrm{d}u} + \int_0^s e^{\int_k^s r_{t+u}\,\mathrm{d}u}[y_{t+k} - c_{t+k} - (\nu+\delta\chi)p_{t+k}e^{-\delta(1-\chi)k}d]dk \right. \\
&\left. +\lambda(1-f)e^{-\delta(1-\chi)k}p_{t+s}d \right] ds
\end{aligned}
$$

The first order condition for the durable stock is,

$$
\begin{aligned}
&\mathbb{E}_t \int_0^\tau e^{-(\rho+\delta(1-\chi))s} u_d(c_{t+s}, e^{-\delta(1-\chi)s}d)\,\mathrm{d}s = \\
&+ \mathbb{E}_t e^{-\rho\tau} V_{x,t+\tau}^{adj}\left[p_t e^{\int_0^\tau r_{t+u}\,\mathrm{d}u} + (\nu+\delta\chi)\int_0^\tau e^{\int_k^\tau r_{t+u}\,\mathrm{d}u - \delta(1-\chi)k} p_{t+k}\,\mathrm{d}k - (1-f)e^{-\delta(1-\chi)\tau}p_{t+\tau} \right] \\
&+ \mathbb{E}_t \int_0^\tau e^{-\rho s}\Psi_{t+s}\left[p_t e^{\int_0^s r_{t+u}\,\mathrm{d}u} + (\nu+\delta\chi)\int_0^s e^{\int_k^s r_{t+u}\,\mathrm{d}u - \delta(1-\chi)k} p_{t+k}\,\mathrm{d}k - \lambda(1-f)e^{-\delta(1-\chi)s}p_{t+s} \right] \mathrm{d}s
\end{aligned}
$$

Substituting the definition of the cumulative user cost $r_{t,t+s}^d$ yields the equation (12) in the text.

C Derivation of Equation (14)

Linearizing equations (6) through (13) yields the following system of equations,

$$
\begin{aligned}
\frac{\dot{D}_t - \bar{D}}{\bar{D}} &= -\delta\frac{D_t - \bar{D}}{\bar{D}} + \delta\frac{X_t - \bar{X}}{\bar{X}} \\
\zeta\frac{\dot{X}_t}{\bar{X}} &= \zeta\frac{X_t - \bar{X}}{\bar{X}}(\bar{r}+\nu+\delta) + (r_t - \bar{r}) + \frac{\bar{r}+\delta+\nu}{\xi}\frac{D_t - \bar{D}}{\bar{D}} - \frac{\bar{r}+\delta+\nu}{\xi}\frac{C_t - \bar{C}}{\bar{C}}.
\end{aligned}
$$

In matrix form using the definitions of percent deviations

$$
\begin{pmatrix} \dot{\hat{d}}_t \\ \dot{\hat{x}}_t \end{pmatrix} = \begin{pmatrix} -\delta & \delta \\ \frac{\bar{r}+\delta+\nu}{\zeta\xi} & \bar{r}+\delta+\nu \end{pmatrix} \begin{pmatrix} \hat{d}_t \\ \hat{x}_t \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{\zeta}(r_t - \bar{r}) - \frac{\bar{r}+\delta+\nu}{\xi\zeta}\hat{c}_t \end{pmatrix}.
$$

The eigenvalues this system are,

$$
\kappa_{1,2} = \frac{\bar{r}+\nu}{2} \pm \sqrt{\frac{(\bar{r}+\nu)^2}{4} + \frac{\delta[\bar{r}+\delta+\nu]}{\zeta\xi} + \delta(\bar{r}+\delta+\nu)},
$$

and the corresponding eigenvectors,

$$\Omega = \begin{pmatrix} \delta & \delta \\ \delta+\kappa_1 & \delta+\kappa_2 \end{pmatrix}$$

The diagonalized system is,

$$\begin{pmatrix} \dot{z}_{1t} \\ \dot{z}_{2t} \end{pmatrix} = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix} \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} + \frac{1}{\zeta(\kappa_2-\kappa_1)} \begin{pmatrix} -1 \\ 1 \end{pmatrix} (r_t - \bar{r}) - \frac{\bar{r}+\delta+\nu}{\xi\zeta(\kappa_2-\kappa_1)} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \hat{c}_t.$$

We solving z_1 backward:

$$z_{1t} = z_{10}e^{\kappa_1 t} - \frac{1}{\zeta(\kappa_2-\kappa_1)} \int_0^t e^{\kappa_1(t-s)}(r_s-\bar{r})\,\mathrm{d}s + \frac{\bar{r}+\delta+\nu}{\xi\zeta(\kappa_2-\kappa_1)} \int_0^t e^{\kappa_1(t-s)}\hat{c}_s\,\mathrm{d}s,$$

and z_2 forward,

$$z_{2t} = -\frac{1}{\zeta(\kappa_2-\kappa_1)} \int_t^\infty e^{-\kappa_2(s-t)}(r_s-\bar{r})\,\mathrm{d}s + \frac{\bar{r}+\delta+\nu}{\xi\zeta(\kappa_2-\kappa_1)} \int_t^\infty e^{-\kappa_2(s-t)}\hat{c}_s\,\mathrm{d}s$$

Rotating the system back into is original plane yields,

$$\hat{d}_t = \delta(z_{1t}+z_{2t})$$
$$\hat{x}_t = (\delta+\kappa_1)z_{1t} + (\delta+\kappa_2)z_{2t}$$

The initial condition z_{10} is determined by $\hat{d}_0 = \delta(z_{10}+z_{20})$,

$$\delta z_{10} = \hat{d}_0 + \frac{\delta}{\zeta(\kappa_2-\kappa_1)} \int_0^\infty e^{-\kappa_2 s}(r_s-\bar{r})\,\mathrm{d}s - \frac{\delta(\bar{r}+\delta+\nu)}{\xi\zeta(\kappa_2-\kappa_1)} \int_0^\infty e^{-\kappa_2 s}\hat{c}_s\,\mathrm{d}s$$

which we use to derive the solution for the durable stock,

$$\begin{aligned} \hat{d}_t = \hat{d}_0 e^{\kappa_1 t} &+ e^{\kappa_1 t}\left[\frac{\delta}{\zeta(\kappa_2-\kappa_1)} \int_0^\infty e^{-\kappa_2 s}(r_s-\bar{r})\,\mathrm{d}s - \frac{\delta(\bar{r}+\delta+\nu)}{\xi\zeta(\kappa_2-\kappa_1)} \int_0^\infty e^{-\kappa_2 s}\hat{c}_s\,\mathrm{d}s\right] \\ &- \frac{\delta}{\zeta(\kappa_2-\kappa_1)} \int_0^t e^{\kappa_1(t-s)}(r_s-\bar{r})\,\mathrm{d}s + \frac{\delta(\bar{r}+\delta+\nu)}{\xi\zeta(\kappa_2-\kappa_1)} \int_0^t e^{\kappa_1(t-s)}\hat{c}_s\,\mathrm{d}s \\ &- \frac{\delta}{\zeta(\kappa_2-\kappa_1)} \int_t^\infty e^{-\kappa_2(s-t)}(r_s-\bar{r})\,\mathrm{d}s + \frac{\delta(\bar{r}+\delta+\nu)}{\xi\zeta(\kappa_2-\kappa_1)} \int_t^\infty e^{-\kappa_2(s-t)}\hat{c}_s\,\mathrm{d}s. \end{aligned}$$

Plugging the solution for the durable stock into the equation for durable expenditure yields,

$$\begin{aligned} \hat{x}_t &= \frac{\delta+\kappa_1}{\delta}\hat{d}_t + (\kappa_2-\kappa_1)z_{2t} \\ &= \frac{\delta+\kappa_1}{\delta}\hat{d}_t - \frac{1}{\zeta}\int_t^\infty e^{-\kappa_2(s-t)}(r_s-\bar{r})\,\mathrm{d}s + \frac{\bar{r}+\delta+\nu}{\xi\zeta}\int_t^\infty e^{-\kappa_2(s-t)}\hat{c}_s\,\mathrm{d}s, \end{aligned}$$

and setting $t=0$ gives equation (14) in the text.

D Forward Guidance Decomposition: Durable Expenditure and Extensive Margin

In Section 5 we noted that 73 percent of the weaker effects of forward guidance at a one-year horizon reflect a weaker extensive margin response and the intensive margin accounts for 21 percent.

Figure A.1 breaks down the total output response from Figure 1 into the durable contribution (dashed red line), the contribution coming only from the extensive margin (dash-dotted black line), and the contribution coming only from the intensive margin contributions (dash-dotted purple line). A key take-away from this figure is that the weaker output response of forward guidance is almost entirely accounted for by a weaker response of durable spending, which parallels the total output response.

Figure A.1 also shows that both the extensive margin and the intensive margin are less responsive to forward guidance than to contemporaneous interest rate changes. The sensitivity of the extensive margin declines from a 0.479% contribution to output for a contemporaenous interest rate change to a 0.156% contribution to output for a real rate cut one year from now. For the intensive margin the contribution drops from 0.159% to 0.067%. Since the overall responsiveness of output falls from 0.744% for a contemporaenous real rate cut to 0.303% for a real rate cut one year from now, the extensive margin accounts for $\frac{0.156-0.479}{0.303-0.744} = 73.2\%$ and the intensive margin for $\frac{0.067-0.159}{0.303-0.744} = 20.8\%$.

E Forward Guidance and the Durable Supply Elasticity ζ

In Section 4 we argued that an upward sloping supply curve for durables makes durables demand more forward looking through movements in the relative price of durables. In this section we show that reasonable variation in the durable supply elasticity does not significantly change the discounting of future real interest rates. In our calibration the supply elasticity is $\zeta^{-1} = 20$. Supplementary Appendix Figure A.2 plots the response of

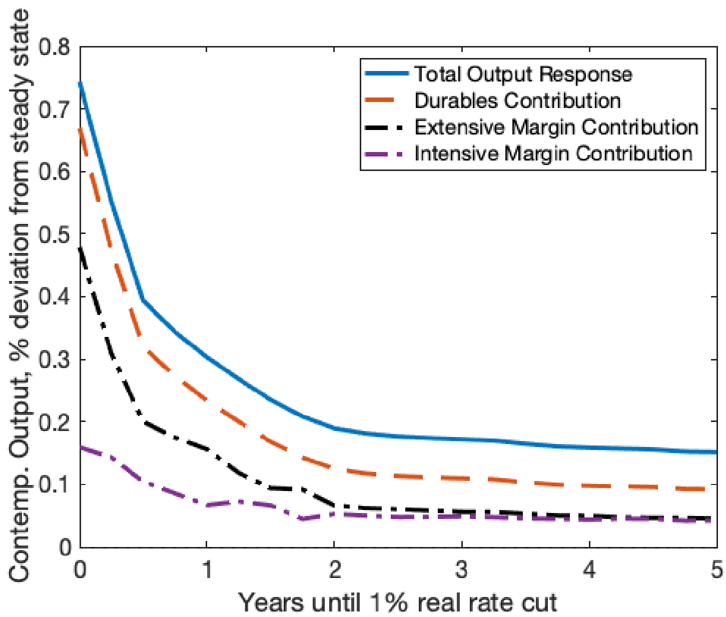

Figure A.1: Contemporaneous output response to promises of interest rate cuts at different horizons decomposed by contributions to durable expenditure. At each horizon the real interest rate drops by 1 percentage point for one quarter. The solid blue line represents the output response in the durables model shown in Figure 1. The dashed red line shows the contribution from total durable expenditure. The dash-dot black line shows the contribution from the change in the extensive margin of durable adjustment. The dash-dot purple line shows the contribution from the intensive margin of durable adjustment.

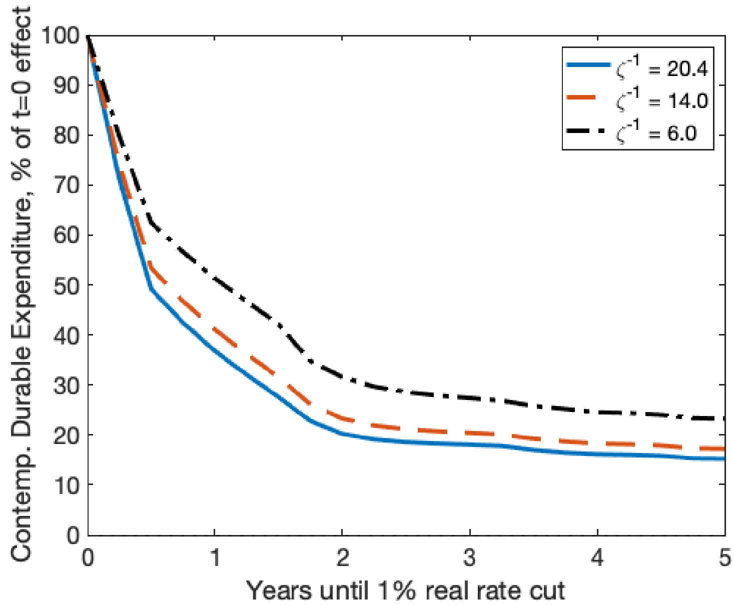

Figure A.2: Contemporaneous durable expenditure response to promises of interest rate cuts at different horizons for various choices of the supply elasticity. In each case, the response is normalized to 100% for a contemporaneous interest rate cut. At each horizon the real interest rate drops by 1 percentage point for one quarter. The solid blue line represents our main model from Section 2, calibrated according to Section 5.1 with $\zeta^{-1} = 20$. The dashed red line is the same model with the supply elasticity equal to $\zeta^{-1} = 14$. The dash-dotted yellow line is the same model with the supply elasticity equal to $\zeta^{-1} = 6$.

durable expenditure to forward guidance expressed as a fraction of the response of durable expenditure to contemporaneous interest rates. For example, a value of 0.5 on the vertical axis indicates that forward guidance is half as effective as contemporaneous interest rates. We consider two alternative values of ζ; $\zeta^{-1} = 14$ and $\zeta^{-1} = 6$, which straddle the range of estimates in House and Shapiro (2008). We again normalize durable expenditure by its response to a contemporaneous interest rate change.[7] As Figure A.2 shows, the relative strength of forward guidance on durable demand is similar to our baseline calibration for these choices.

[7]With less elastic supply, equilibrium durable expenditure responds less strongly to monetary policy at all horizons. This is why we normalize by the response to contemporaneous interest rates in the figure.

F Forward Guidance and Long-term Debt

In Section 6 we argued that the partial equilibrium decision problem is unaffected by long-term financing of durables conditional on initial wealth, but long-term assets create valuation effects so the equilibrium with long-term debt is not identical to the one with short-term debt only. In this appendix we investigate how our results are affected by these valuation effects.

We assume that household portfolios consist entirely of long-term debt, i.e. $\omega_{it} = 1$. The total value of assets for each household is then $a_{it} = q_{it} b_{it}$. Like Farhi and Werning (2019) we then introduce short-term debt at the margin and make sure that households are not better off by including it in their portfolio. This implies that the return on both assets must be equalized, $r_t = r_t^b$. The budget constraint conditional on not adjusting then evolves as in the baseline model (equation (4)).

We normalize dividend payments $\nu = r + \Gamma$ such that the steady state price of debt is $q = \frac{\nu}{r+\Gamma} = 1$. The valuation effect on assets at time 0 is then

$$a_{i0} = \frac{q_{i0}}{q} b_{i0} = q_{i0} b_{i0}$$

with b_{i0} given and the path for q_{it} determined by the no-arbitrage equation

$$r_t = \frac{\dot{q}_t + \nu}{q_t} - \Gamma \equiv r_t^b.$$

To ensure that the valuation effects do not immediately cause households to violate the borrowing constraint, we specify it in terms of the number of long-term bonds held,

$$\frac{q}{q_{it}} a_{it} = b_{it} \geq -\lambda p_{it} d_{it}.$$

Thus, a household that is initially at the borrowing constraint with $q b_{i0} = b_{i0} = -\lambda p_{it} d_{it}$ will continue to satisfy it after the valuation effects take place.

The government maintains a constant quantity of debt $\bar{B}$. This implies that there are no discontinuous changes in tax policy from valuation effects. As in our baseline model, the government balances its budget. This requires raising taxes to finance dividend payments $\nu \bar{B}$ net of debt issuance $\Gamma q_t \bar{B}$ each instant. Thus, the aggregate tax rate is

$$\tau_t = \frac{(\nu - \Gamma q_t)\bar{B}}{Y_t}.$$

Relative to our baseline model, there is one additional parameter Γ governing the duration of the long-term asset (or debt). Setting the duration to $\Gamma^{-1} \to 0$ yields the baseline model as a special case. Next, we calibrate the duration to $\Gamma^{-1} = 4.5$ years based on Doepke and Schneider (2006), Figure 3.

Figure A.3 compares the effectiveness of forward guidance in the model with long-term debt with our baseline model. The output responses are very similar and contemporaneous interest rate reductions remain substantially more powerful at stimulating contemporeanous output than are expected future interest rate reductions.

There are, however, some small difference in the results. First, contemporaneous interest rates are slightly less powerful in the long-term debt model. A lower real rate increases the asset price q_0, which redistributes from debtors to creditors and partially offsets the redistribution from creditros to debtors from lower interest rate payments (Auclert, 2019). The asset price q_0 responds more strongly for more immediate interest rate reductions. Thus, contemporaneous interest rate changes lead to a larger redistribution from debtors to creditors than do future changes. This depresses the expansionary effects of contemporaneous interest rate changes relatively more than forward guidance.

Second, forward guidance is slightly more powerful with long-term debt. With long-term debt, τ_0 falls in response to future interest rate cuts because the revenue the government raises from issuing a unit of bond rises. In contrast, taxes react only to contemporaneous interest rate changes with short-term debt. The reduction in τ_0 in response to future interest rate changes makes forward guidance more powerful.

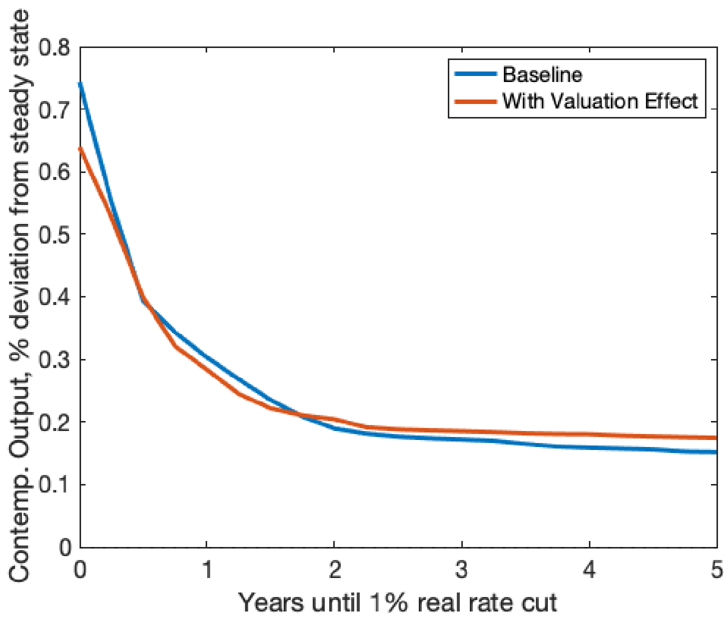

Figure A.3: Contemporaneous output response to promises of interest rate cuts at different horizons in the baseline model with short-term debt (blue line) and the model with long-term debt (red line). At each horizon the real interest rate drops by 1 percentage point for one quarter.