NBER WORKING PAPER SERIES

ESTIMATING THE AGE-PRODUCTIVITY PROFILE USING LIFETIME EARNINGS

Laurence J. Kotlikoff

Working Paper No. 2788

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 1988

I am grateful to the Hoover Institution and the National Institute of Aging grant # 1P01AG05842-01 for research support. Jagadeesh Gokhate and Jinyong Cai provided excellent research assistance. I thank Edward Lazear, Jagadeesh Gokhale, Kevin Lang, Chris Ruhm, and Larry Katz for helpful comments. This research is part of NBER's research programs in Labor Studies and in Aging. Any opinions expressed are those of the author not those of the National Bureau of Economic Research.

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ABSTRACT

Understanding how productivity varies with age is important for a variety of reasons. A decline in productivity with age implies that aging societies must increasingly depend on the labor supply of the young and middle age. It also means that policies designed to keep the elderly in the work force, while potentially good for the elderly, may decrease overall productivity. A third implication is that, absent government intervention, employers may not be willing to hire the elderly for the same compensation as younger workers. Labor economists are particularly interested in the relationship of productivity and age because it can help test alternative theories of the labor market.

This paper assumes risk neutral employers and estimates the age-productivity relationship using the first order condition that the present expected value of total compensation equals the present expected value of productivity; workers hired at different ages have different present expected values of total compensation, and, correspondingly, different present expected values of productivity. Hence, if one parameterizes the age-productivity relationship, the parameters of this relationship can be identified from information on how total present expected compensation varies with age.

The data in the study are earnings histories for over three hundred thousand employees of a Fortune 1000 corporation covering the period 1969-1983. While the results may be subject to several biases and should be viewed cautiously, they are fairly striking. For each of the five sex-occupation groups, productivity falls with age. For young workers, compensation (earnings plus pension accrual) is below productivity and for older workers compensation exceeds productivity. For several worker groups the discrepancy between compensation and productivity is very substantial.

In addition to confirming some features of contract theory, the results lend support to the bonding models of Becker and Stigler and Lazear which suggest that firms use the age-earnings profile as an incentive device.

Laurence J. Kotlikoff
National Bureau of Economic Research
1050 Massachusetts Avenue
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Understanding how productivity varies with age is important for a variety of reasons. A decline in productivity with age implies that aging societies must increasingly depend on the labor supply of the young and middle age. It also means that policies designed to keep the elderly in the work force, while potentially good for the elderly, may decrease overall productivity. A third implication is that, absent government intervention, employers may not be willing to hire the elderly for the same compensation as younger workers.

Labor economists are particularly interested in the relationship of productivity and age because it can help test alternative theories of the labor market. The simplest such theory is the spot market theory in which workers are paid, at least annually, their marginal product. However, there appear to be few, if any, economists who view the spot market theory as reasonable. My research with David Wise (Kotlikoff and Wise; 1985, 1987a, 1987b) presents fairly strong evidence against the spot market theory; we demonstrated that most defined benefit pension plans induce very sharp discontinuities with age in vested pension accrual. Under the spot market theory there should be offsetting sharp discontinuities at these ages in wage compensation. Such offsetting wage discontinuities are not, however, evident in the data.

In contrast to the spot market theory, contract theories of labor markets imply only a present value relationship between compensation and productivity. Consider, for example, the contracts that would be written by risk neutral employers. In these contracts, although earnings in any single year can exceed or be less than that year's productivity, the present expected value of the worker's output will equal the present expected value of her compensation.

Different contract theories have different implications concerning the relationship of productivity and wages as the worker ages. One such theory is the specific human capital model of Mincer (1964) and Becker (1971). It suggests that if firms are free to fire older workers, the age-wage profile will be structured such that earnings exceed productivity when young and vice versa when old. On the other hand, in Lazear's (1979, 1981) agency model of worker shirking, the worker receives less than her marginal product when young, with the difference paid out in the form of wages, accrued pension benefits, or severance pay in excess of the marginal product when old.

The efficiency wage models of Harris and Todaro (1970), Stofft (1984), Yellen (1984), Stiglitz and Shapiro (1984), and Bulow and Summers (1985) represent a third view of the labor market. These models stress the payment of above market clearing wages, rather than the shape of the age-compensation profile as an incentive device. While there is equilibrium unemployment in these models, they, like the spot market theory, predict that workers are paid their marginal products at each age over the work span.

The evidence to date on the age-productivity relationship is limited and mixed. The findings for older workers of Abraham and Medoff (1981) that pay increases although indices of productivity decline, suggest wages in excess of marginal products toward the end of the work span. Lazear and Moore (1983) report that the earnings profiles of the self-employed are flatter than those of employees, also suggesting earnings in excess of productivity among older employees. Kahn and Lang (1986), in contrast, examine responses to questions concerning desired hours of work; they point out that older workers, with earnings in excess of their marginal products, are likely to be hours-constrained by their employers and, therefore, desire to work more. The opposite would be true if earnings of older workers is below their marginal products. Kahn and Lang's empirical findings support the view that marginal productivity exceeds earnings for older workers.

This paper assumes risk neutral employers and estimates the age-productivity relationship using the first order condition that the present expected value of total compensation equals the present expected value of productivity; workers hired at different ages have different present expected values of total compensation and, correspondingly, different present expected values of productivity. Hence, if one parameterizes the age-productivity relationship, the parameters of this relationship can be identified from information on how total present expected compensation varies with age.

The data in the study are earnings histories for over three hundred thousand employees of a Fortune 1000 corporation covering the period 1969 - 1983. While the firm's name can not be disclosed, the firm is involved primarily in sales. These data are advantageous not only because one can control for the firm, but also because one can determine precisely the accrued pension compensation arising under the firm's defined benefit pension plan. At particular ages and amounts of service, pension compensation in this firm is an important component of total compensation.

The results indicate that productivity declines with age and that workers are paid more than they produce when old to offset being paid less than they produce when young. For some sex-occupation groups the difference between productivity and compensation at young and old ages is very sizeable. The results support the bonding models of Becker and Stigler (1974) and Lazear (1979,1981) as opposed to the efficiency wage models cited above and the Becker - Mincer human capital models. The results are, however, compatible with more general efficiency wage models (Akerlof and Katz, 1986).

There are, however, a number of reasons for viewing these results cautiously. First the analysis assumes that the form of contracts remained constant over the sample period. Second, the probability of remaining employed is treated as exogenous and time invariant, rather than an endogenous choice of the employer. Third, the analysis assumes the age-productivity relationship has remained constant over a 16 year period. Fourth, the results may be subject to selectivity bias if (1) different workers within an occupation group have contracts that differ in ways other than their initial wage and (2) if the composition of workers who join or leave the firm at particular ages is correlated with the characteristics of the contract.

The paper continues as follows. The next section, II, presents the basic methodology. Section III presents the data. Section IV examines the results, and Section V states conclusions and suggests additional research.

Section II. Methodology

Consider a firm whose concave production function depends on capital and labor. Labor input is assumed to differ across workers only in terms of effective units; i.e., the labor input of one worker is a perfect substitute for that of any other, but the amount of effective labor units is different for each worker. The firm is assumed to have full knowledge of the worker's productivity at the time he or she is hired. Let Y_t, L_t, and K_t stand for output, labor, and capital in year t, respectively. The concave production function is:

(1) $$Y_t = F(L_t, K_t)$$

where

(2) $$L_s = \sum_{j=s-57}^{s} \sum_{a=18}^{75} N_{j,a}\, q(a+s-j,a,s)\, h(a+s-j,a,s)$$

In (2), total labor input at time s, L_s, equals the sum of the labor input of workers hired this year and in past years. If 18 and 75 are the minimum and maximum ages of workers, respectively, then the firm at time s has no workers hired before s-57. The term $N_{j,a}$ stands for the number of workers hired in year j at initial hiring age a. Of course, not all of the workers hired in the past stay with the firm. The term q(a+s-j,a,s) denotes the fraction of those workers who are currently age a+s-j, who joined the firm at age a, and who have remained with the firm through year s. Finally, h(a+s-j,a,s) denotes the productivity in year s of workers age a+s-j who joined the firm at age a.

The expected present value of real profits of the firm at time t, π_t, is given by:

(3) $$\pi_t = E_t \sum_{s=t}^{\infty} P_s Y_s(L_s, K_s) R^{s-t} - \sum_{s=t}^{\infty} \sum_{a=18}^{75} N_{s,a} e_{s,a} R^{s-t} - \sum_{s=t-57}^{t} \sum_{a=18}^{75} N_{s,a} D_{s,a}.$$

where E_t is the expectation operator at time t, P_s is the real price of output in year s, R is one over one plus the real interest rate, $e_{s,a}$ is the present (discounted to year s) expected value of compensation payments to workers hired in year s at age a, and $D_{s,a}$ is the present expected value of remaining compensation payments to workers hired at age a in year s<t. Equation (3) states that the present expected value of profits equals the present expected value of output, less the present expected value of compensation paid to current and future hires, and less the present expected value of remaining compensation paid to past hires. At time t the future values of P_s are uncertain; as a consequence the future values of Y_s are also uncertain.

In maximizing the present expected value of profits firms are constrained to structure compensation payments to provide workers with competitive levels of expected utility. In addition, they may face anti-shirking constraints, requiring that they structure the time path of compensation to reduce or eliminate worker malfeasance. Regardless of these side constraints, the first order condition for hiring workers age a at time t is that the present expected value of marginal output equals the present expected value of compensation; i.e.,

$$(4) \qquad E_t \sum_{s=t}^{t+(75-a)} P_s(\delta Y_s/\delta L_s)q(a+s-t,a,t)h(a+s-t,a,s)R^{s-t} = e_{t,a}$$

The present expected value of compensation of a worker hired in year t at age a can be expressed in terms of the time path of future annual compensation. Let w(i,a,s) stand for the compensation paid to workers who are age i in year s and who joined the firm at age a. Then:

$$(5) \qquad e_{t,a} = \sum_{s=t}^{t+75-a} w(a+s-t,a,t)q(a+s-t,a,t)R^{s-t}$$

While the length of employment is uncertain, the assumption of risk neutral employers and risk averse workers, whose productive characteristics are fully known by the firm, implies that the actual compensation payments are specified with certainty at the time the worker joins the firm.

Assuming the structure of the compensation contract is constant through time, the ratio of compensation at age i+1 to compensation at age i is independent of time; i.e.,

$$(6) \qquad w(i+1,a,t)/w(i,a,t-1) = \mu(i+1,a)$$

If the age-productivity relationship and the probabilities of departure are also assumed to be time invariant, the third arguments in the functions h(, ,) and q(, ,) can be dropped.

Letting $\theta_s = P_s(\delta Y_s/\delta L_s)$, equations (4), (5), and (6) imply:

$$(7) \quad w(a,a,t) \sum_{s=t}^{t+75-a} \mu(a+s-t,a)q(a+s-t,a)R^{s-t} = \sum_{s=t}^{t+75-a} E_t\, \theta_s q(a+s-t,a)h(a+s-t,a)R^{s-t}$$

The assumption of myopic expectations permits writing $E_t\theta_s = \theta_t$, and (7) can be expressed as:

$$(8) \qquad C(a,t) = \theta_t \sum_{s=t}^{t+75-a} q(a+s-t,a)h(a+s-t,a)R^{s-t} = \theta_t H(a),$$

where C(a,t) stands for the left hand side of (7) and equals the present expected compensation of a worker hired at age a in year t.

To gain some intuition about the relationship between the C(,)s and the h(,)s, consider the simple case in which there is a constant probability p of staying each year with the firm, i.e., $q(i,a) = p^{i-a}$, that h(,) depends only on age, i.e., h(i,a) = v(i), and that θ_t equals unity. In this case $C(a,t) = C^*(a)$, and a little manipulation of (8) leads to:

$$(9) \qquad v(a) = C^*(a) - pRC^*(a+1)$$

and

$$(10) \qquad v(a+1) - v(a) = [C^*(a+1) - C^*(a)] - pR[C^*(a+2) - C^*(a+1)]$$

From (9), if pR equaled unity, v(a) would just equal the difference in the present expected value of compensation of workers hired at age a and at age a+1. In this case the present expected value of compensation of younger hires would always exceed that of older hires (assuming positive values of v(a) at all ages). If, on the other hand, the annual probability of

Table 4 Levels of Total Compensation and Productivity
Assuming a 6 Percent Interest Rate

	Males						Females			
	Office Workers		Salesmen		Managers		Office Workers		Saleswomen	
Age	Comp	Prod	Comp	Prod	Comp	Prod	Comp	Prod	Comp	Prod
35	23290	33020	33218	33721	18027	39033	22616	33993	32836	33951
36	24705	33872	33777	34559	20268	39767	24492	34826	32179	34785
37	26023	34659	34258	35327	22480	40394	26255	35587	31794	35547
38	27249	35375	34659	36020	24632	40905	27890	36269	31613	36231
39	28392	36016	34983	36631	26699	41294	29385	36869	31584	36833
40	29459	36577	35230	37156	28661	41553	30736	37381	31663	37347
41	30460	37052	35403	37591	30507	41676	31940	37799	31816	37768
42	31402	37437	35504	37928	32228	41655	32999	38118	32014	38090
43	32291	37728	35538	38165	33820	41483	33916	38334	32234	38309
44	38027	37918	41751	38294	39931	41154	39843	38440	37954	38419
45	34683	38003	36253	38312	37418	40659	36157	38431	33411	38414
46	35925	37979	36472	38213	39263	39993	37225	38303	33926	38290
47	36825	37840	36405	37992	40567	39148	37830	38050	34253	38042
48	37686	37581	36276	37643	41746	38117	38330	37666	34549	37664
49	38522	37198	36104	37162	42813	36892	38751	37147	34833	37151
50	39282	36686	35834	36543	43719	35468	39057	36487	35057	36497
51	40176	36039	35584	35781	44753	33836	39451	35681	35350	35697
52	40867	35253	35142	34872	45409	31990	39626	34724	35535	34747
53	41472	34323	34599	33809	45884	29922	39737	33610	35696	33641
54	41983	33244	33933	32589	46138	27626	39784	32334	35833	32374
55	71120	32010	58669	31204	77399	25095	67896	30892	61738	30940
56	46071	30618	34550	29651	49446	22320	42811	29277	38965	29334
57	46052	29062	33026	27925	48668	19297	42525	27484	38859	27551
58	45142	27338	30691	26019	46652	16016	41518	25509	38152	25586
59	43832	25440	27962	23930	43994	12472	40315	23345	37246	23433
60	39923	23363	23095	21651	38351	8657	36845	20988	34203	21088
61	35391	21104	18005	19178	31940	4563	33039	18433	30773	18544
62	33111	18655	14645	16505	27796	185	31530	15674	29244	15797
63	30599	16014	11692	13628	23317	-4484	29976	12705	27494	12842
64	31674	13174	10923	10540	22178	-9453	31924	9523	28628	9673
65	21294	10132	3381	7238	10702	-14729	23324	6120	20016	6286

Figure 2

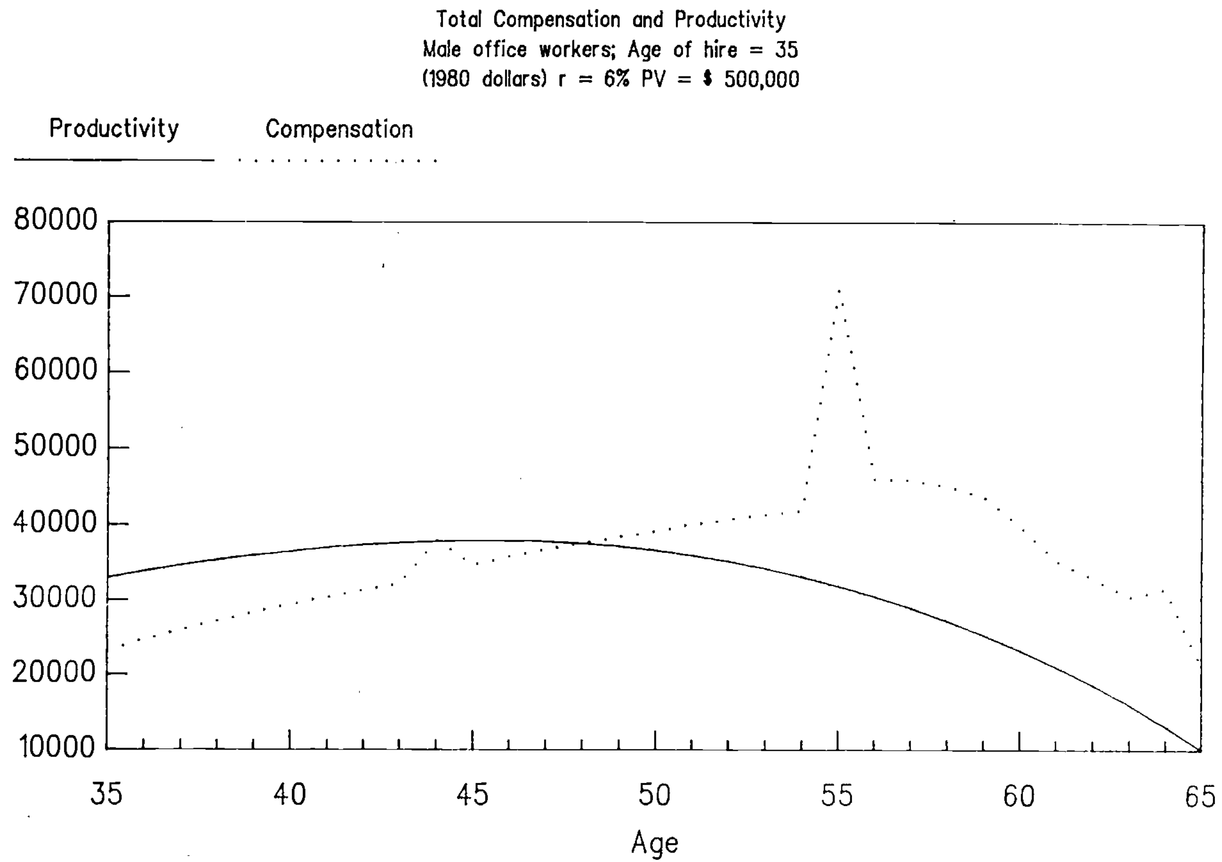

Total Compensation and Productivity
Male office workers; Age of hire = 35
(1980 dollars) r = 6% PV = $ 500,000
Productivity
Compensation
80000
70000
60000
50000
40000
30000
20000
10000
35
40
45
50
55
60
65
Age

Table 3 Age-Productivity Regressions Assuming a 6 Percent Interest Rate

	Males			Females	
Variables	Office Workers	Salesmen	Managers	Office Workers	Saleswomen
α2	50.53035	38.34286	97.88788	27.72098	45.18319
	(1.105)	(0.618)	(3.336)	(0.426)	(9.696)
α3	-0.74407	-0.57257	-1.57805	-0.41612	-0.67761
	(0.166E-1)	(0.942E-2)	(0.561E-1)	(0.645E-2)	(0.145)
D71	-.0095057	0.02396	.00345915	-0.05147	-0.67456
	(0.304E-1)	(0.219E-1)	(0.452E-1)	(0.210E-1)	(0.244)
D72	0.04380	0.01887	-0.02484	0.03571	-0.52057
	(0.291E-1)	(0.222E-1)	(0.454E-1)	(0.208E-1)	(0.238)
D73	0.03226	-0.01416	-0.02531	0.05581	-0.64611
	(0.285E-1)	(0.220E-1)	(0.451E-1)	(0.190E-1)	(0.229)
D74	-0.02681	-0.04849	-0.12467	.00717999	-0.56458
	(0.288E-1)	(0.204E-1)	(0.416E-1)	(0.188E-1)	(0.221)
D75	0.02725	-0.04625	-0.07275	0.09147	-0.48456
	(0.281E-1)	(0.196E-1)	(0.411E-1)	(0.185E-1)	(0.219)
D76	-0.03789	-0.06847	-0.05845	0.03307	-0.42320
	(0.261E-1)	(0.191E-1)	(0.411E-1)	(0.173E-1	(0.217)
D77	-0.03253	-0.08741	-0.05446	0.08427	-0.43532
	(0.256E-1)	(0.184E-1)	(0.406E-1)	(0.171E-1)	(0.215)
D78	-0.07344	-0.17232	-0.05886	0.03560	-0.53027
	(0.257E-1)	(0.179E-1)	(0.396E-1)	(0.168E-1)	(0.215)
D79	-0.13934	-0.26782	-0.22318	-0.01083	-0.60448
	(0.258E-1)	(0.181E-1)	(0.409E-1)	(0.168E-1)	(0.215)
D80	-0.15293	-0.25847	-0.16289	-.0099359	-0.57778
	(0.265E-1)	(0.181E-1)	(0.389E-1)	(0.167E-1)	(0.215)
D81	-0.16023	-0.30280	-0.26470	0.01889	-0.61783
	(0.252E-1)	(0.179E-1)	(0.389E-1)	(0.165E-1)	(0.215)

D82	-0.04405 (0.266E-1)	-0.36594 (0.176E-1)	-0.21860 (0.391E-1)	0.06838 (0.174E-1)	-0.63699 (0.215)
D83	-0.17587 (0.271E-1)	-0.41658 (0.174E-1)	-0.14946 (0.446E-1)	-0.01804 (0.173E-1)	-0.69837 (0.215)
Number of Obser.	7083	19696	2116	20753	3217
R^2	.276	.075	.204	-.134	-.086

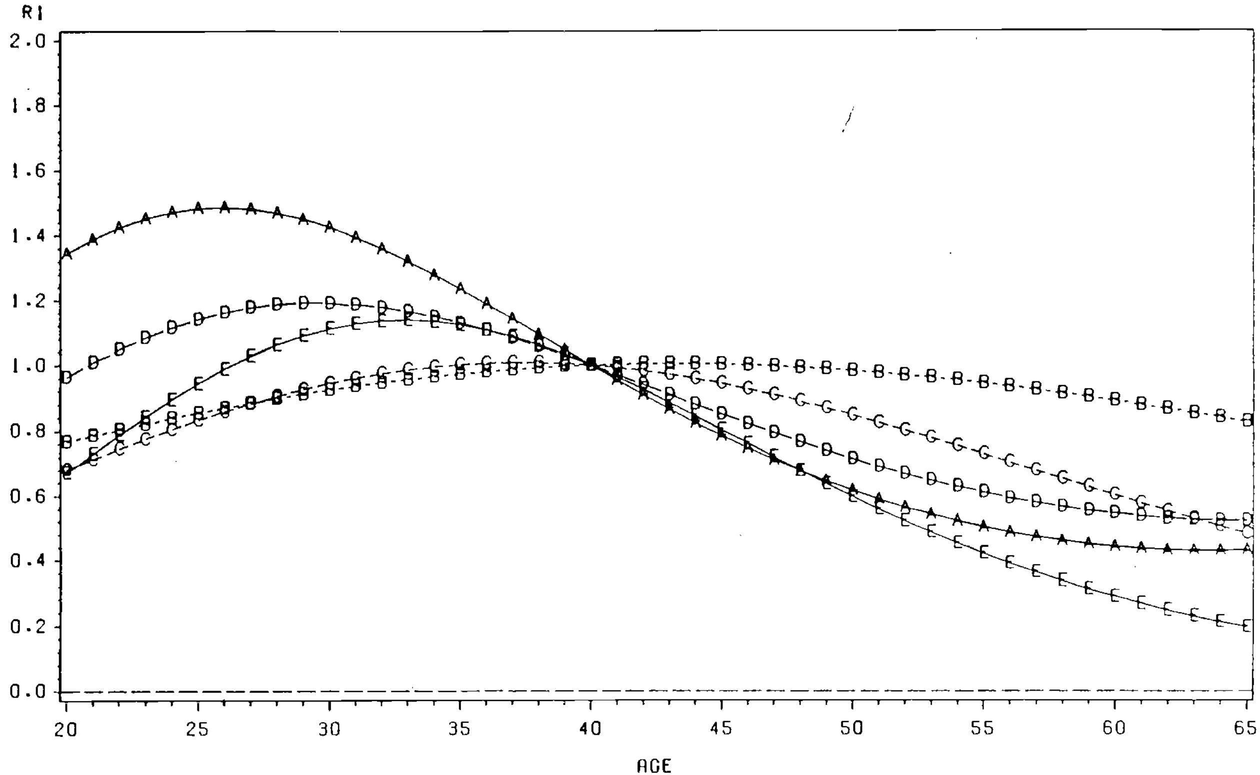
Section IV. Estimates of the Age-Productivity Profile

Table 3 presents the regression results from estimating (11') assuming a 6 percent interest rate. In the regression observations on workers hired only during the years 1970 through 1983 are included, since pension accrual for workers hired prior to 1969 could not be determined. All of the age-squared and age-cubed coefficients reported in the Table are highly significant. Many of the year dummies are also significant, suggesting that the modeling of expectations of future θs may be important. The regression coefficients are little affected by the choice of interest rate; the regressions were repeated assuming interest rates of both 3 percent and 9 percent, and the coefficients are very similar to those reported in Table 3.

Figures 2 through 6 are based on the six percent interest rate regressions of Table 3. They present the age-productivity profiles (dashed lines) predicted by the regressions for the five sex-occupation groups for workers hired initially at age 35. They also present the age-total compensation profile (solid lines) implied by the smoothed $\mu(\; , \;)$s and the pattern of pension accrual. The age 35 initial level of productivity and compensation are chosen to insure that both the present expected value of compensation and the present expected value of marginal product equal $500,000. Table 4 presents the values of compensation and productivity profiles presented in Figures 2 through 6.

While productivity initially rises with age in each diagram, it eventually starts declining with age. For male office workers productivity peaks at age 45 and declines thereafter. For this group age 65 productivity is less than one-third of peak productivity. The female office workers

FIGURE 1: RELATIVE PROFILE OF PRESENT EXPECTED COMPENSATION



A = MALE MANAGERS
B = SALESWOMEN
C = SALESMEN
D = FEMALE OFFICE WORKERS
E = MALE OFFICE WORKERS

Table 1 Smoothed q(,) Functions by Occupation-Sex Group

Age of Hire	Age 25	35	45	55	65
Male Office					
20	.461	.303	.335	.218	.024
30		.699	.485	.215	.017
40			.791	.334	.029
50				.681	.092
60					.435
Female Office					
20	.472	.300	.289	.144	.010
30		.688	.420	.142	.007
40			.792	.298	.018
50				.735	.101
60					.543
Salesmen					
20	.286	.084	.049	.020	.002
30		.420	.149	.054	.007
40			.496	.145	.021
50				.480	.077
60					.379
Saleswomen					
20	.301	.053	.015	.004	.001
30		.373	.083	.023	.005
40			.431	.105	.026
50				.467	.111
60					.474
Male Managers					
20	.622	.505	.488	.215	.013
30		.885	.768	.343	.024
40			.900	.431	.038
50				.657	.079
60					.321

Table 2 Smoothed $\mu(\, , \,)$ Functions by Occupation-Sex Group

	Age				
Age of Hire	25	35	45	55	65
Male Office					
20	1.071	1.028	1.028	1.017	1.005
30		1.047	1.021	.998	.968
40			1.030	1.003	.957
50				1.019	.973
60					1.014
Female Office					
20	1.047	1.027	1.030	1.007	.994
30		1.048	1.008	.990	.985
40			1.043	.999	.987
50				1.034	.998
60					1.019
Salesmen					
20	1.016	.976	.987	.985	.827
30		1.010	.992	.970	.840
40			1.004	.975	.872
50				1.000	.924
60					.996
Saleswomen					
20	1.042	1.072	1.076	1.124	1.128
30		1.012	1.023	1.023	1.005
40			.996	.980	.943
50				.992	.942
60					.962
Male Managers					
20	1.090	1.054	1.062	.991	.770
30		1.079	1.026	.983	.852
40			1.068	1.005	.925
50				1.057	.990
60					1.047

accrual prior to age 55. After age 55 the accrual is much smaller and, indeed, can become negative.

The q(,)s used in constructing $c_{a,t,j}$ and the variables in equations (10′) and (13) were calculated separately for each of the five age-occupation groups in the following manner. First, the fraction of workers at a given age and initial age of hire who remain in the firm from one year to the next was calculated. Next, these annual survival hazards were smoothed using a third order polynomial in age, age of hire, and interaction terms. Finally, the cumulative survival probabilities, the q(,)s, were computed based on the smoothed annual survival probabilities.

The μ(,)s in the above discussion have stood for the growth in total compensation, including pension compensation; but in order to determine the course of pension compensation, one needs first to know the course of nonpension compensation. Hence, the function μ^*(,), which gives the growth in nonpension compensation, was first estimated by regressing observed growth rates in earnings, excluding pension compensation, against a third order polynomial in age, age of hire, and interaction terms. The initial wage together with the smoothed μ^*(,)s provide a path of nonpension compensation that can be used to calculate the path of pension accrual. The path of nonpension plus pension compensation is then used to form the present expected value of total compensation, the $c_{a,t,j}$s.

Tables 1 and 2 present, respectively, the smoothed q(,) and μ^*(,) functions for the different occupation-sex groups at selected ages and ages of hire. Table 1 indicates very substantial differences in job survival rates across the five groups; 34.3 percent of male managers who hire on at age 30 are predicted to remain with the firm 25 years later. For male and female office workers the comparable percentages are 21.5 and 14.2, respectively. For salesmen and saleswomen the respective percentages are 5.4 and 2.3. The Table also demonstrates that workers hired at older ages, at least through age 50, have larger probabilities of remaining in the firm for a given period of time than workers hired at younger ages.

Table 2 indicates that the age of hire also is an important factor in real wage growth. According to the regression, workers hired at later ages often experience greater real wage growth than workers hired at younger ages. In addition, wage growth for female office workers and saleswomen at particular combinations of age and age of hire often exceeds that of their male occupational counterparts.

A reduced form regression can help illustrate the shape of the age-profile of the present expected value of compensation. This regression relates the log of the present expected value of compensation (calculated using the inital wage, the q(,) function, and the μ(,) function), to a set of year dummies and a polynomial in age. The exponent of the coefficients of this polynomial in age multiplied by their respective variables indicates the shape of the age-present expected value of compensation profile. Figure 1 presents this profile for each of the five sex-occupation group normalized by the age 40 level of this profile. Notice that each of the normalized profiles of present expected compensation rises at early ages at a decreasing rate, suggesting, as indicated above, that productivity rises with age at these ages. In addition, each of the profiles, except that of saleswomen, declines a decreasing rate in old age, suggesting that productivity declines with age at these ages for at least the other sex-occupation groups.

problem for estimating the age productivity relationship, the parameter α_1 is normalized to unity. With this normalization and using (12), equation (11) can now be expressed as:

$$(11') \qquad c_{a,t,j} = \log\theta_t + \log[X_1(a) + \alpha_2 X_2(a) + \alpha_3 X_3(a)] + \epsilon_{a,t,j}$$

where $X_1(a)$, $X_2(a)$, and $X_3(a)$ are the respective sums on the right hand side of (12). Equation (11') can be estimated nonlinearly. Since time enters only through the intercept term $\log\theta_t$, data for workers hired in different years can be pooled by simply entering year dummies.

The assumption of myopic expectations may not be justified. An alternative assumption is that firms expect that the value of the marginal revenue product of labor will revert to a value θ^* next period and stay at that level in the future. Under this assumption we have:

$$(13) \qquad c_{a,t,j} = \log[\theta_t h(a,a) + \theta^* \sum_{s=t+1}^{t+75-a} q(a+s-t,a)h(a+s-t,a)R^{s-t}] + \epsilon_{a,t,j}$$

In principal, equation (13) can be estimated nonlinearly to recover values of θ^*, the θ_t, as well as α_2 and α_3. In practice, parameter estimates of this model did not converge because of colinearity of the right hand side variables in (13). The nonconvergence occurs even if one models h(,) as a gradratic, rather than a cubic function of age.

Section III. The Data and Empirical Implementation

The large firm's data used in this study are earnings histories covering the period 1969 through 1983 of workers employed in the firm at some time during the period 1980 through 1983. The workers are classified into three rather broad occupation and sex groups: male office workers, female office workers, salesmen, saleswomen, and male managers. There are too few female managers to warrant their analysis. Unfortunately, there are no additional demographic variables that could be included in the analysis.

The firm has a defined benefit plan with a fairly complex set of age- and service-related benefits. The benefit formula is a percent of earnings formula in which the basic retirement annuity equals a percentage rate times the number of years of service for workers with fewer than 26 years of service. For those with more service, the formula equals 25 times the former percentage rate, plus the additional service beyond 25 times a lower percentage rate. The basic benefit is offset by the amount of Social Security benefits the firm predicts the worker will receive. The predicted Social Security benefit is derived from another age- and service-related formula unique to the firm.

The pension plan's normal retirement age is 65, and its early retirement age is 55. For workers who retire after the early retirement age, but before the normal retirement age, there is a special early retirement benefit reduction table that is based on the workers' age and service. Workers who terminate employment before age 55 are not eligible for the quite generous early retirement benefit reduction rates and face instead actuarially reduced benefits. Another very important penalty for terminating before the early retirement age is that for workers retiring after the early retirement age have their Social Security offset deferred until they reach age 65. These provisions of the firm's pension can produce quite substantial vested pension accrual at age 55, but rather modest departing the firm is high, pR will be much less than unity, and a value of $C^*(a+1)$ in excess of $C^*(a)$ is consistent with positive values of v(a).

The formula for changes in productivity with age is given in (10). In some cases one can read the age-productivity relationship from the slope of the age-present expected compensation profile, $C^*(a)$, and knowledge that $pR<1$. For example, productivity is constant with age in the range of ages over which the $C^*(a)$ profile is flat. One can also tell that productivity rises with age over ranges in which $C^*(a)$ is rising, but at a decreasing rate; the intuition here is that a positive, but flattening slope of $C^*(a)$ means that the immediate positive slope of $C^*(a)$ (the difference in $C^*(a+1)$ and $C^*(a)$) is due to v(a+1) exceeding v(a), rather than to later marginal products exceeding v(a). If $C^*(a)$ is increasing, but at an increasing rate, one can not say whether productivity at age a+1 exceeds or falls short of productivity at age a. Similarly, one can tell that productivity delines with age over ranges of ages in which $C^*(a)$ declines with age at a decreasing rate; however, if $C^*(a)$ declines with age at an increasing rate, one can not tell whether productivity is decreasing or increasing with age.

Returning to the general case, equation (8) can be transformed into an econometric relation by appending a multiplicative error term, $e^{\epsilon_{a,t,j}}$, where the subscript j references the individual worker. The error term can be viewed as a worker-specific productivity factor. Its inclusion in the model means that workers hired at the same age in the same sex-occupation category may have different initial salaries. Hence, the model permits worker heterogeneity as well as selectivity based on the $\epsilon_{a,t,j}$s. While workers hired at particular ages, or in certain years, may be more or less productive than workers hired at other ages or in other years without biasing the results, the model does require the same wage-growth contract and the same departure rates for all workers within a sex-occupation group.

Taking logarithms of the resulting expression yields:

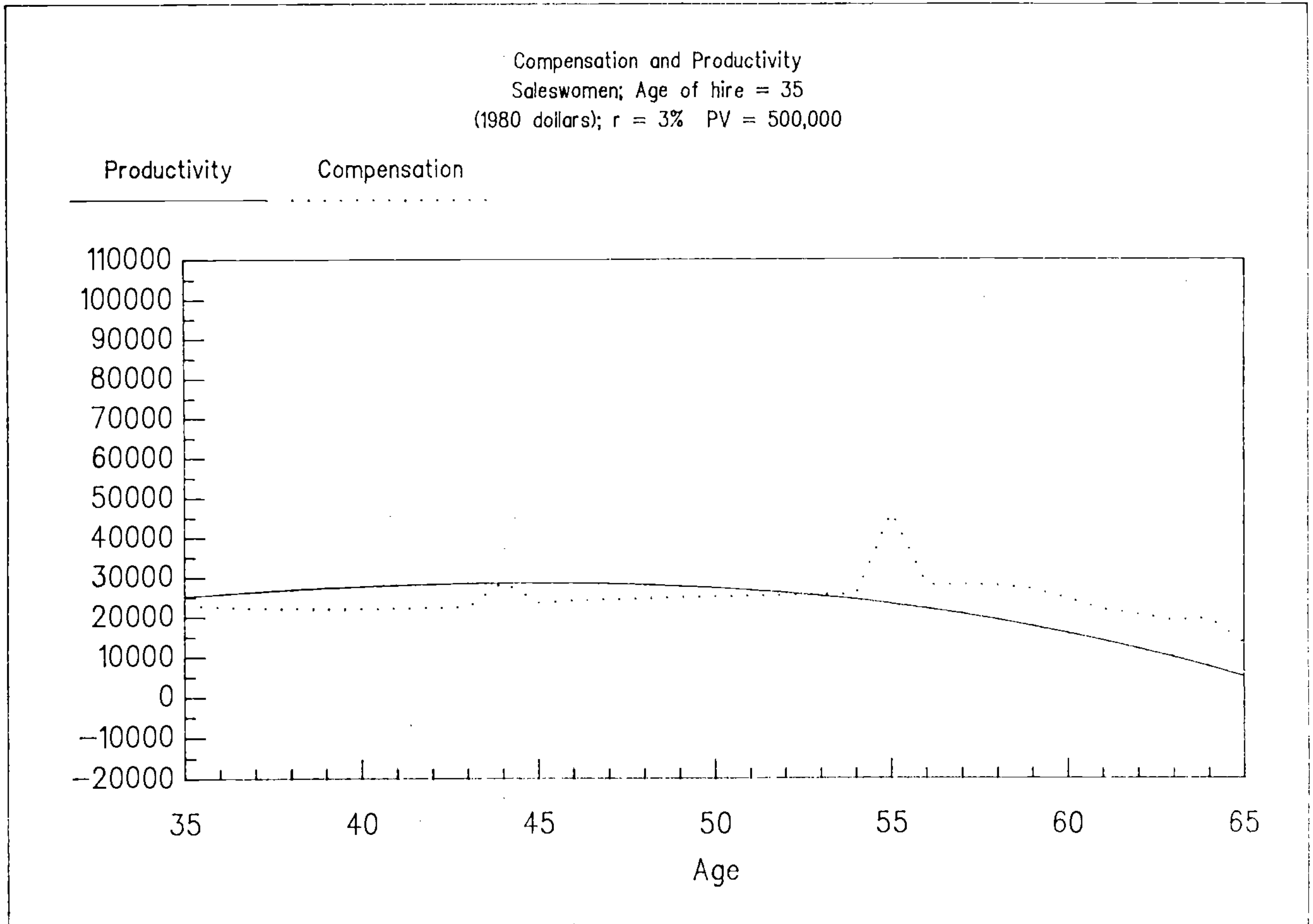
$$c_{a,t,j} = \log\theta_t + \log H(a) + \epsilon_{a,t,j} \tag{11}$$

In (11), $c_{a,t,j}$ is the logarithm of C(a,t) for worker j who is age a in year t. While h(,) can, in principle, be parameterized as a function of service as well as age, in practice, the resulting cummulative age and cummulative service variables are too colinear to estimate separate age and service coefficients. Hence, I parameterize the productivity function h(,) as simply a cubic function of age, and acknowledge that the age-productivity results reported below confound service-productivity effects. Letting $h(k,a) = \alpha_1 k + \alpha_2 k^2 + \alpha_3 k^3$, H(a) can be written as:

$$H(a) = \alpha_1 \sum_{s=t}^{t+75-a} q(a+s-t,a)(a+s-t)R^{s-t} + \alpha_2 \sum_{s=t}^{t+75-a} q(a+s-t,a)(a+s-t)^2 R^{s-t} + \alpha_3 \sum_{s=t}^{t+75-a} q(a+s-t)(a+s-t)^3 R^{s-t} \tag{12}$$

One can not separately identify all four of the parameters in (11) and (12), θ_t, α_1, α_2, and α_3. To see this substitute from (12) into (11) and divide both sides of the resulting expression by α_1; observe that the resulting constant term will equal $\log\theta_t + \log\alpha_1$. Since this poses no

Figure 13


Compensation and Productivity
Saleswomen; Age of hire = 35
(1980 dollars); r = 3%   PV = 500,000
Productivity
Compensation
110000
100000
90000
80000
70000
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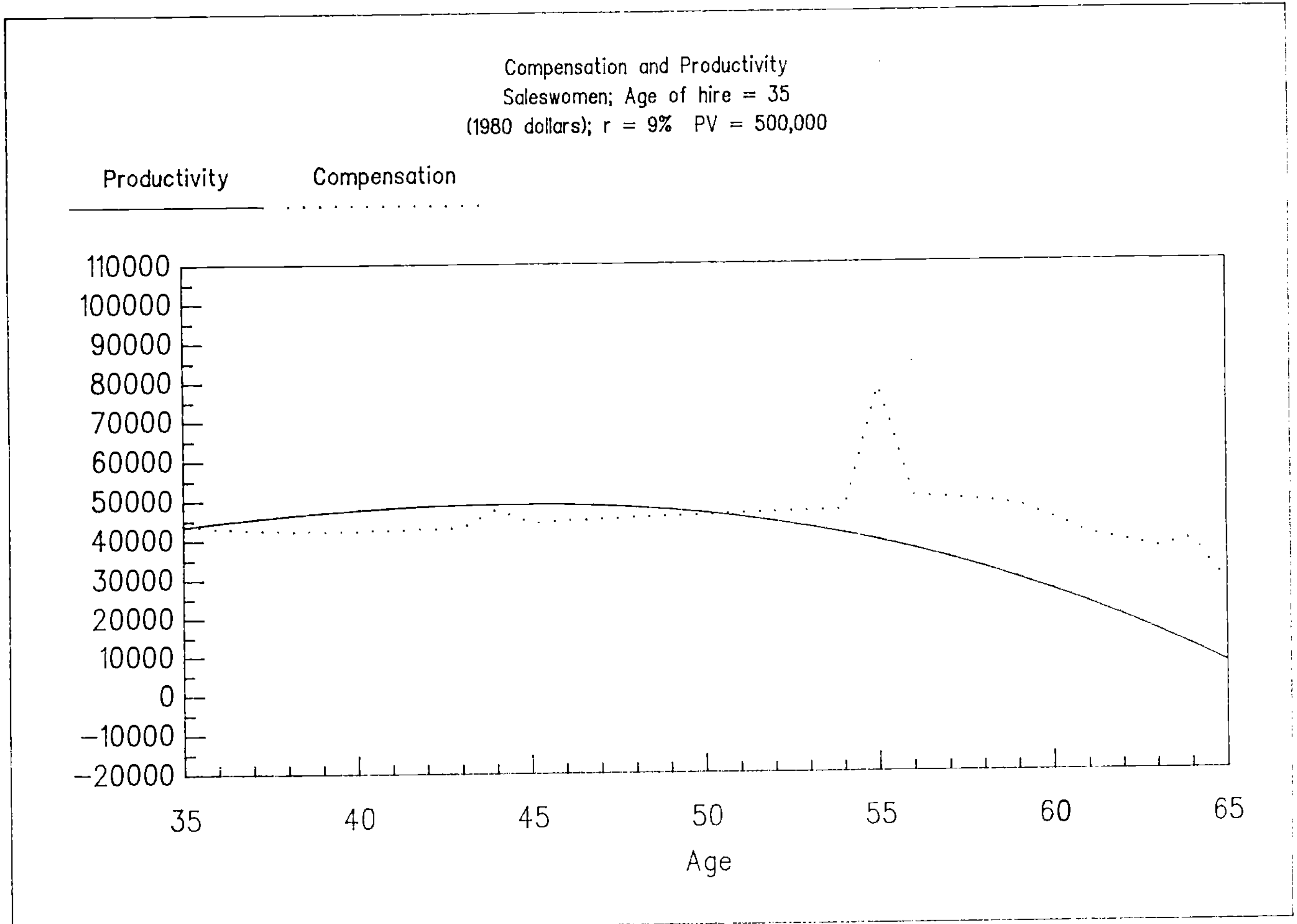
Age

Figure 14


Compensation and Productivity
Saleswomen; Age of hire = 35
(1980 dollars); r = 9% PV = 500,000
Productivity
Compensation
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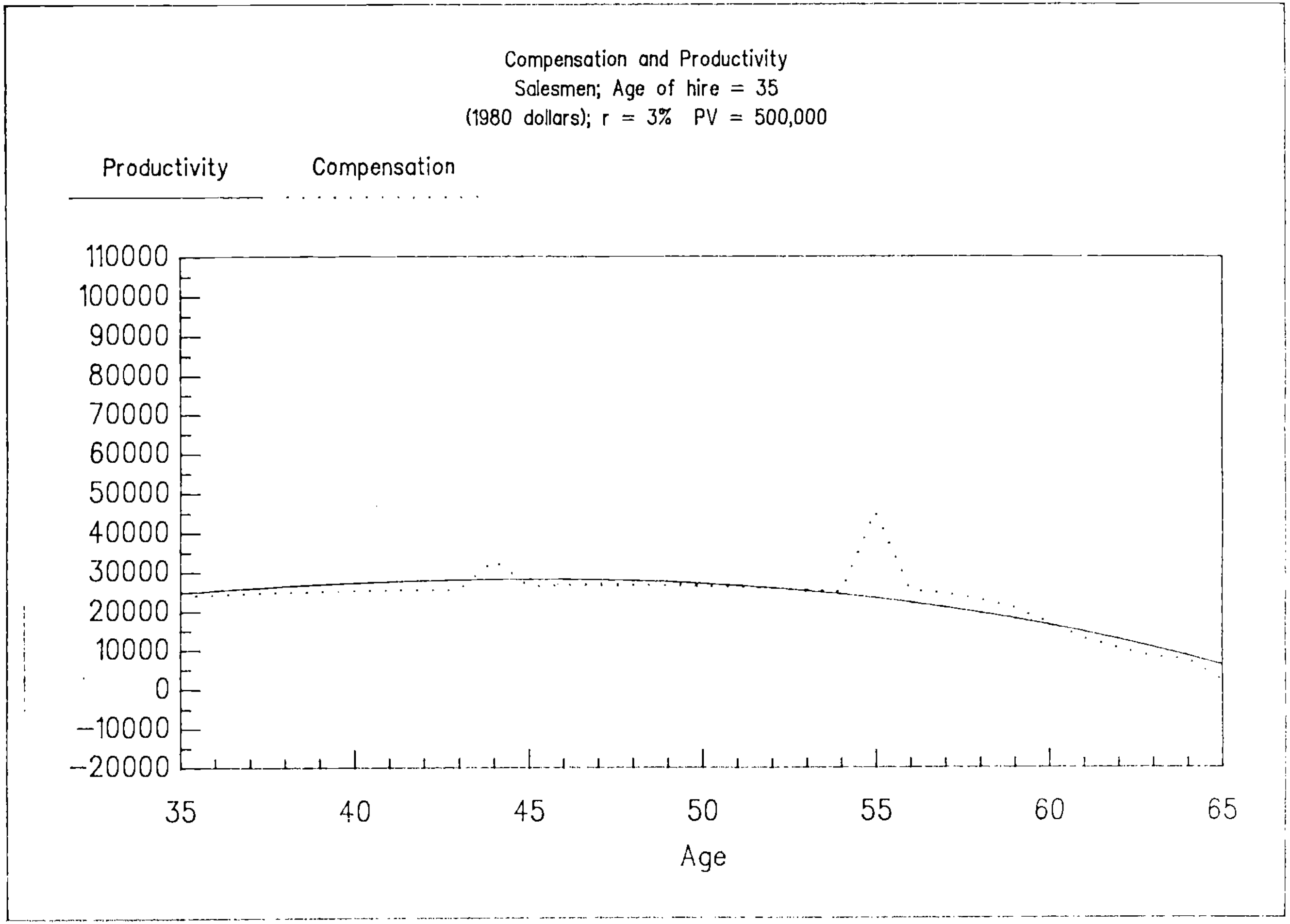
Age

Figure 11


Compensation and Productivity
Salesmen; Age of hire = 35
(1980 dollars); r = 3% PV = 500,000
Productivity
Compensation
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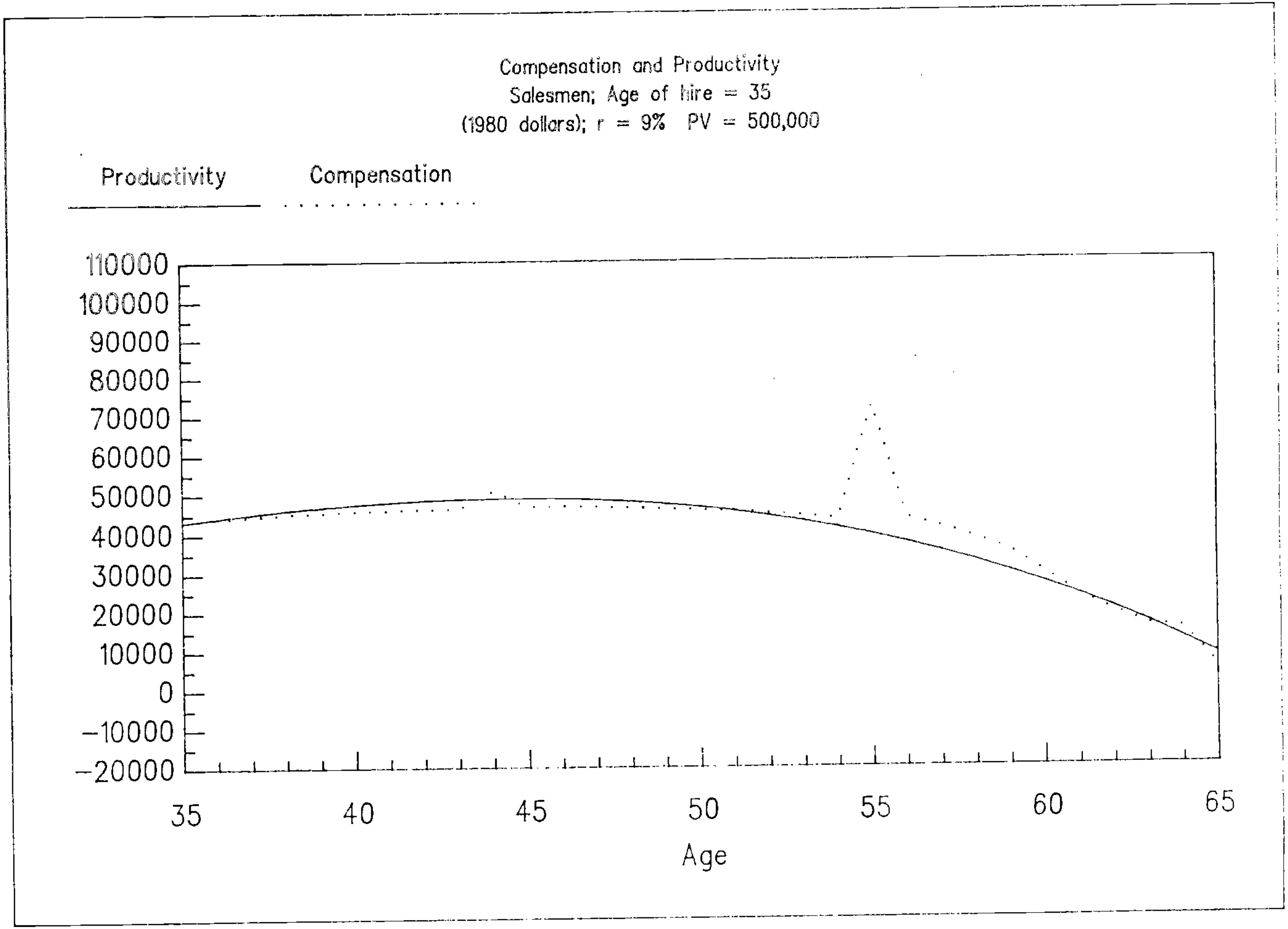
Age

Figure 12



Compensation and Productivity
Salesmen; Age of hire = 35
(1980 dollars); r = 9% PV = 500,000
Productivity
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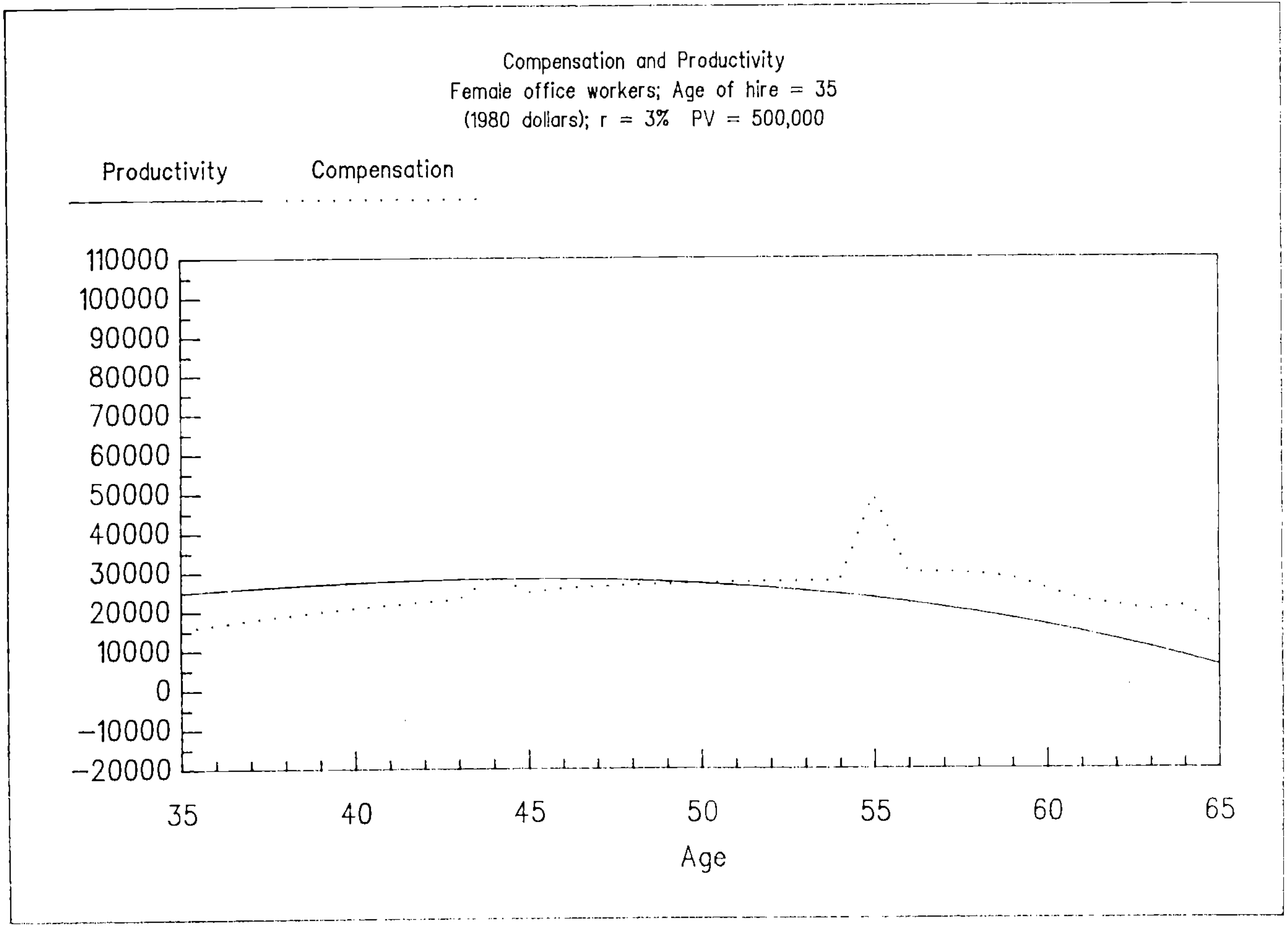
Age

Figure 9



Compensation and Productivity
Female office workers; Age of hire = 35
(1980 dollars); r = 3% PV = 500,000
Productivity
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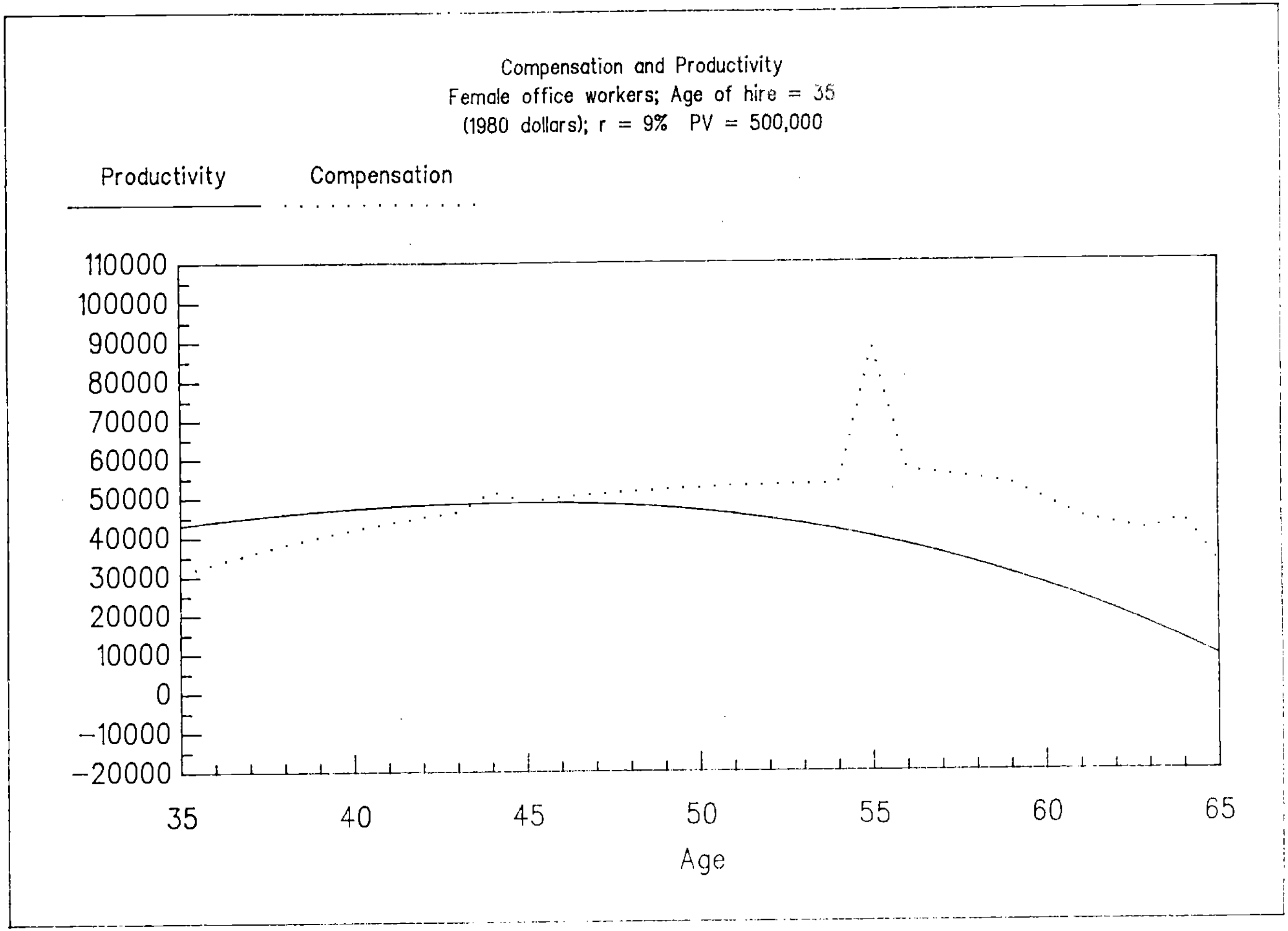
Age

Figure 10



Compensation and Productivity
Female office workers; Age of hire = 35
(1980 dollars); r = 9% PV = 500,000
Productivity
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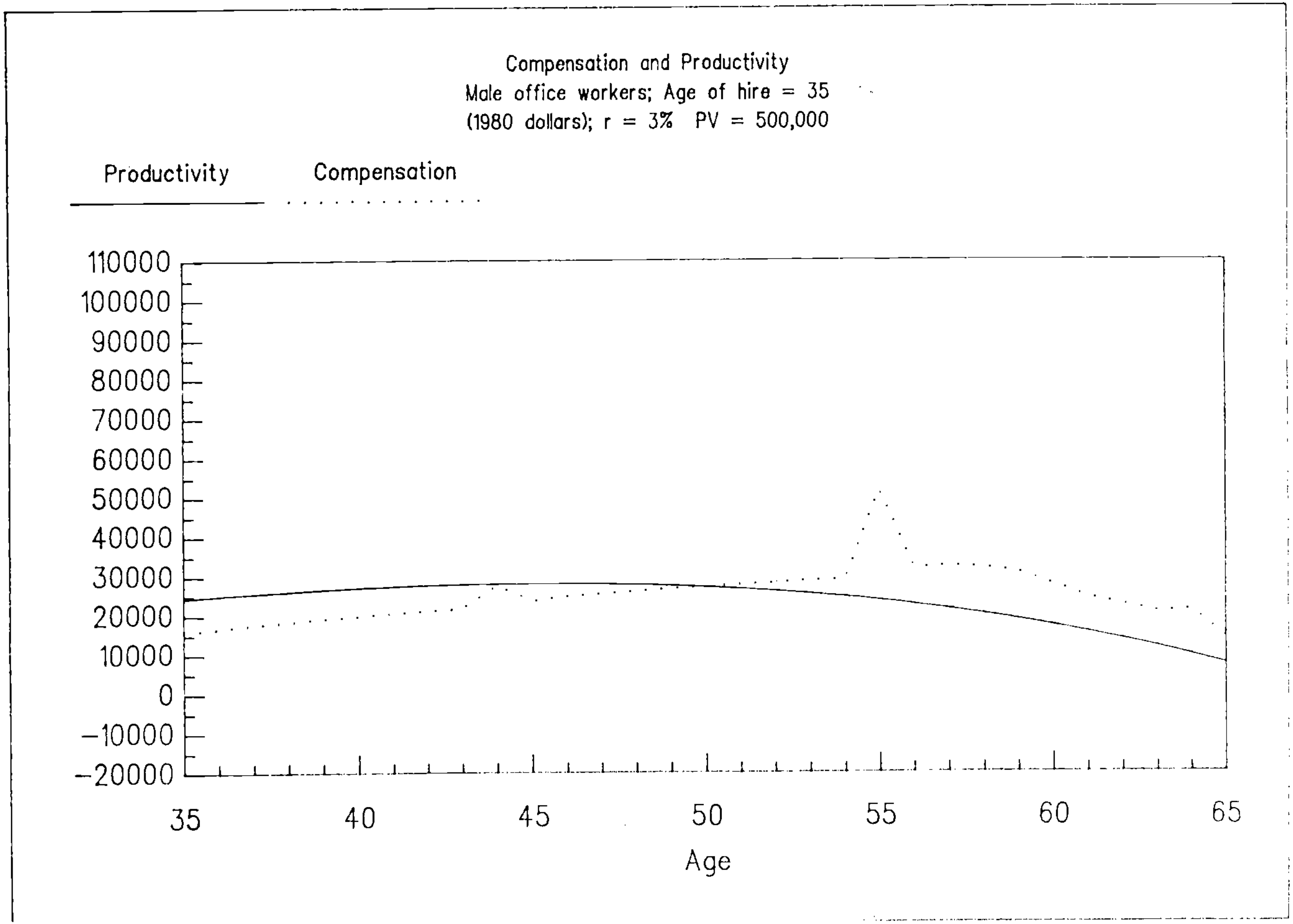
Age

Figure 7


Compensation and Productivity
Male office workers; Age of hire = 35
(1980 dollars); r = 3% PV = 500,000
Productivity
Compensation
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90000
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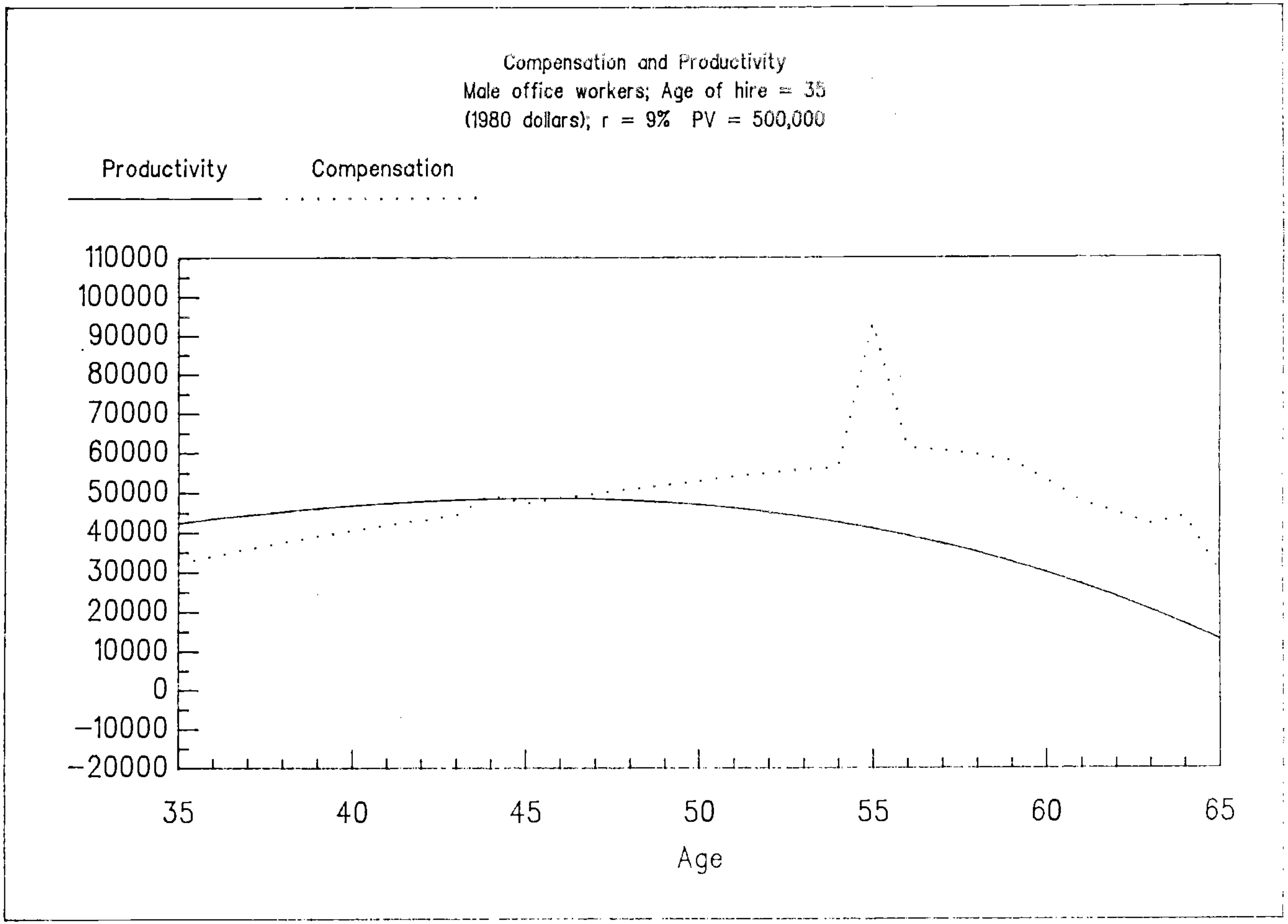
Figure 8



Compensation and Productivity
Male office workers; Age of hire = 35
(1980 dollars); r = 9% PV = 500,000
Productivity
Compensation
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100000
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60000
50000
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Age

productivity profile is quite similar to that of the male office workers. Both the salesmen and saleswomen productivity profiles peak a few years later than those of office workers, but their rate of decline with age is quite similar. Productivity for male managers peaks at age 43; by age 60 productivity is less than one third of peak productivity, and productivity actually becomes negative after age 62.

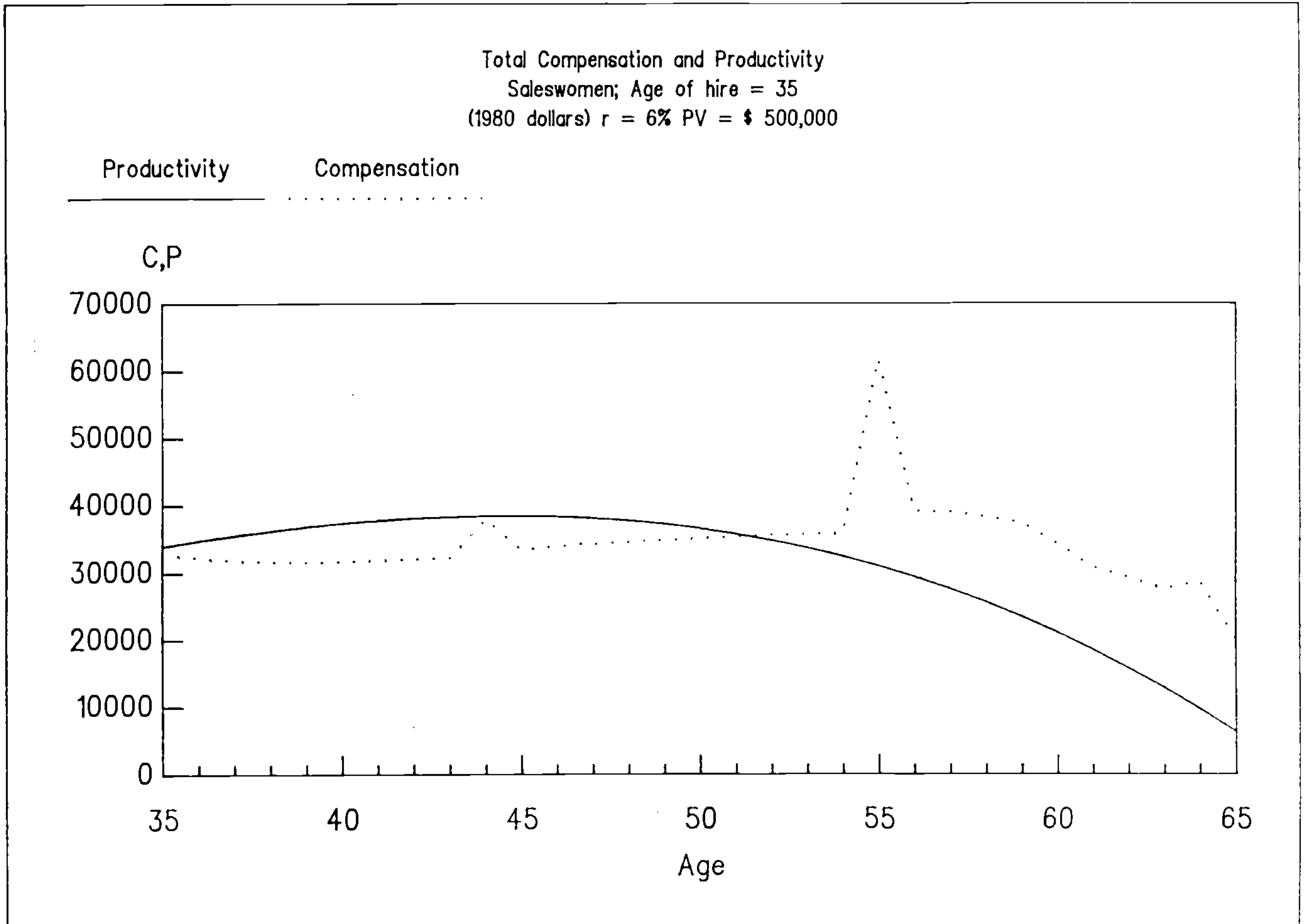
In four of the diagrams productivity exceeds total compensation while the worker is young and then falls below total compensation; in the remaining case, that of salesmen, the relationship of compensation and productivity is quite similar to the other four groups, except after age 61 when productivity again exceeds compensation. Except for the kinks in the age-compensation profiles associated with pension accrual, the age-compensation profiles and age-productivity profiles for salesmen and saleswomen are very close to one another at each age. This is what one would predict since salesworkers in this firm are paid, in large part, on a commission basis.

In contrast, to the results for salesworkers, one might expect the weakest connection between annual earnings and annual productivity among male managers. Figure 4 indicates this is indeed the case. At age 35 productivity for male managers exceeds total compensation by more than a factor of two, while compensation is over twice as high as productivity by age 57. The discrepancies between total compensation and productivity at these ages are somewhat smaller for office workers, but still quite important. For example, age 35 total compensation for female office workers is $22,616, while age 35 productivity is $33,604. In contrast, age 57 total compensation is $42,526, although productivity is only $28,117.

The results depicted in Figures 2 through 6 are not sensitive to the inclusion of pension accrual in total compensation; if one ignores pension accrual in the estimation, the age-earnings and age-productivity profiles have the same relative shapes as those presented. Of course, the age-earnings profile does not exhibit the kinks of the age-total compensation profile since these kinks arise from pension accrual. In the absence of considering pension accrual, one can also use the data on workers hired prior to 1970. While the initial wage of those hired prior to 1969 is not reported, this wage can be inferred based on the wage observed in 1969 and the $\mu(,)$s; i.e., one can inpute backwards the wage at the initial age of hire. The results based on this larger data set are again extremely similar to those presented in Figures 2 through 6.

As indicated in Figures 7 through 16 which present compensation and productivity profiles at 3 percent and 6 percent interest rates, the general shapes of the age-total compensation profiles and age-productivity profiles are also insensitive to the choice of interest rate. For example, compare the 3 percent and 9 percent compensation and productivity profiles of female office workers in Figures 13 and 14. Since the present value of these profiles always equals $500,000, the profiles assuming a 9 percent interest rate are initially higher than those assuming a 3 percent interest rate. The percentage differences between the two profiles is slightly larger at early and late ages, assuming a 9 percent, rather than a 3 percent interest rate; but since the initial levels of the profiles in Figure 14 are higher, the absolute differences between compensation and productivity are considerably larger. The age at which productivity of female office workers falls below compensation also depends on the interest rate. The cross-over

Figure 5


Total Compensation and Productivity
Saleswomen; Age of hire = 35
(1980 dollars) r = 6% PV = $ 500,000
Productivity
Compensation
C,P
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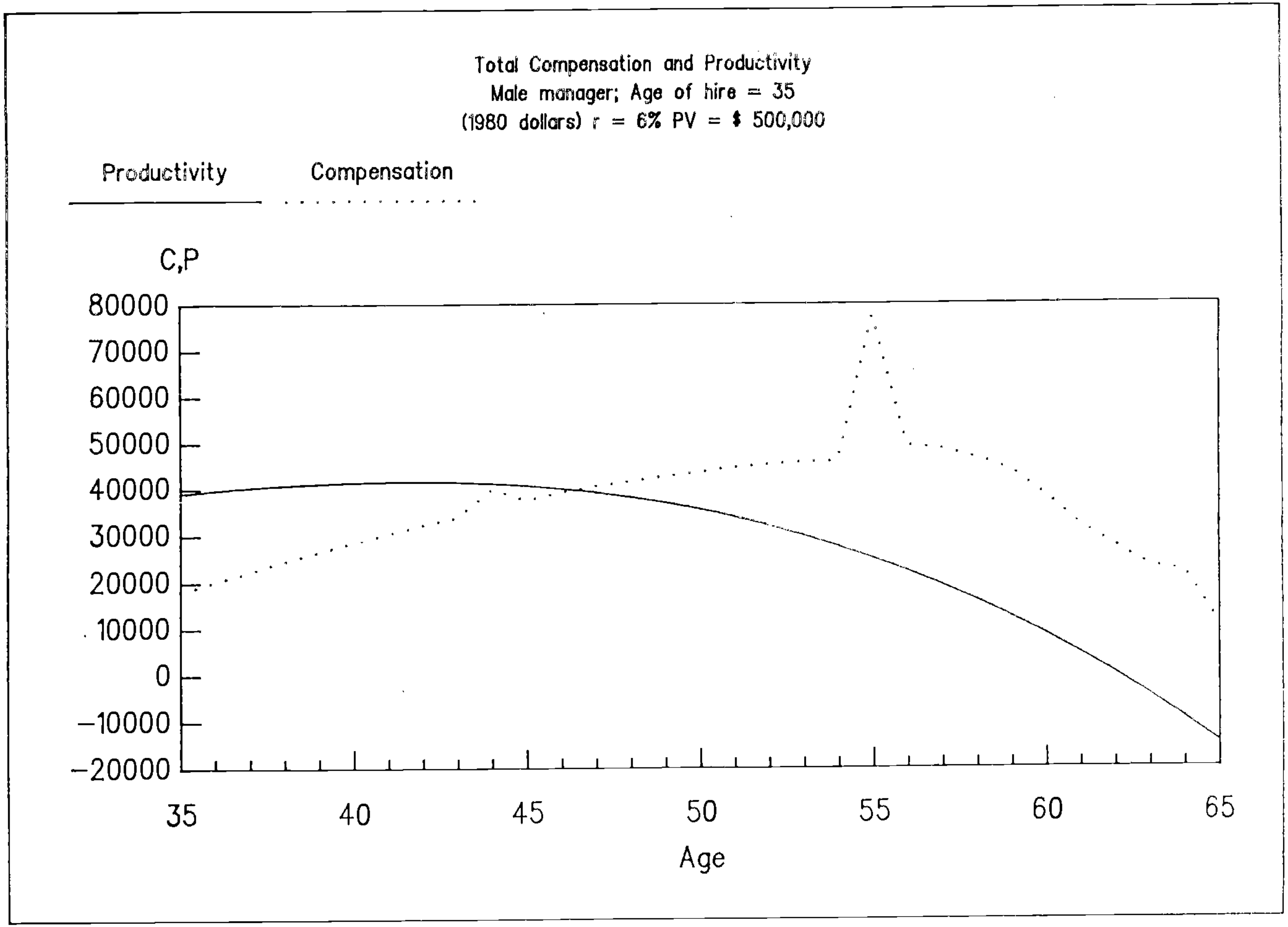
Age

Figure 6


Total Compensation and Productivity
Male manager; Age of hire = 35
(1980 dollars) r = 6% PV = $ 500,000
Productivity
Compensation
C,P
80000
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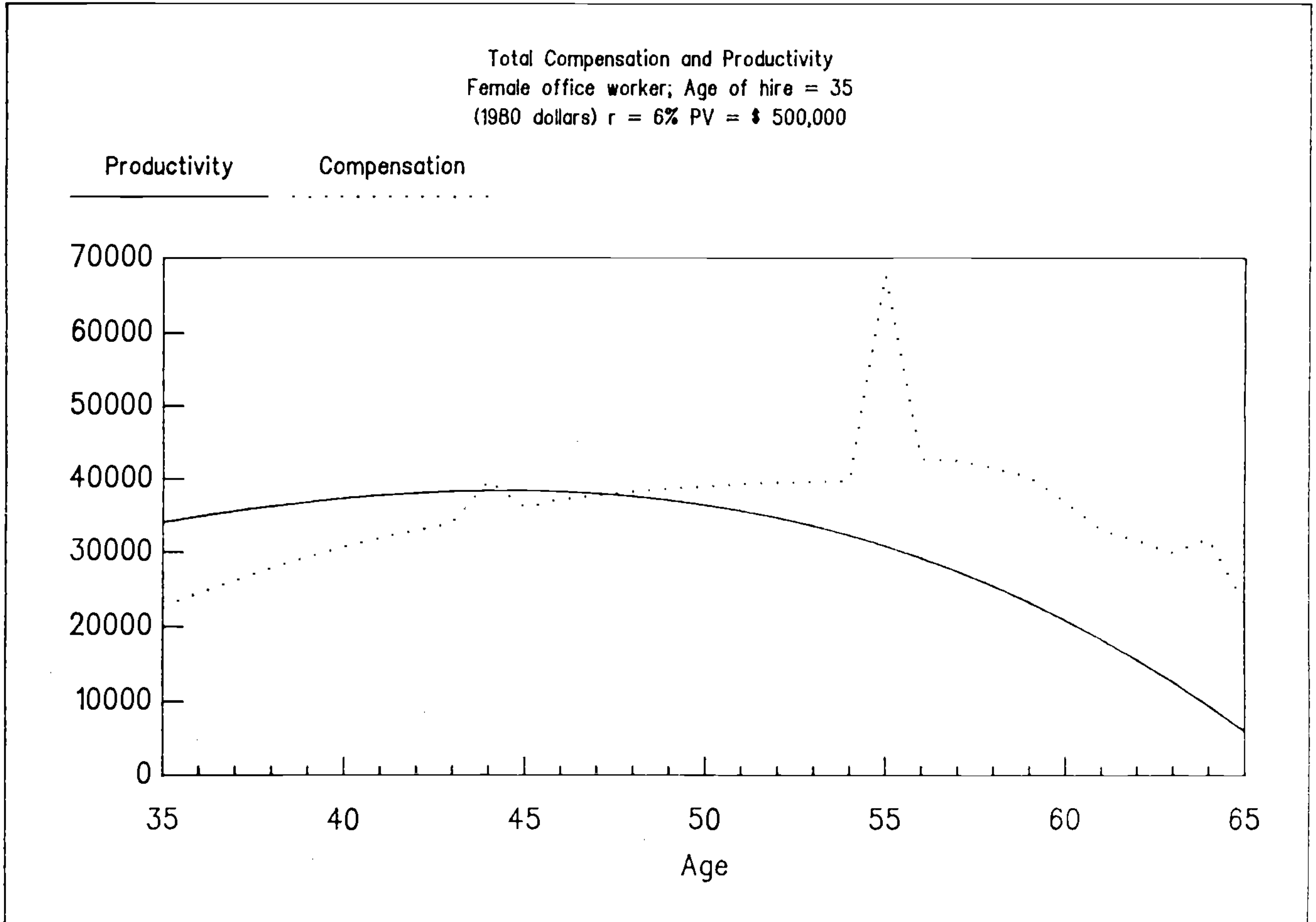
Age

Figure 3


Total Compensation and Productivity
Female office worker; Age of hire = 35
(1980 dollars) r = 6% PV = $ 500,000
Productivity
Compensation
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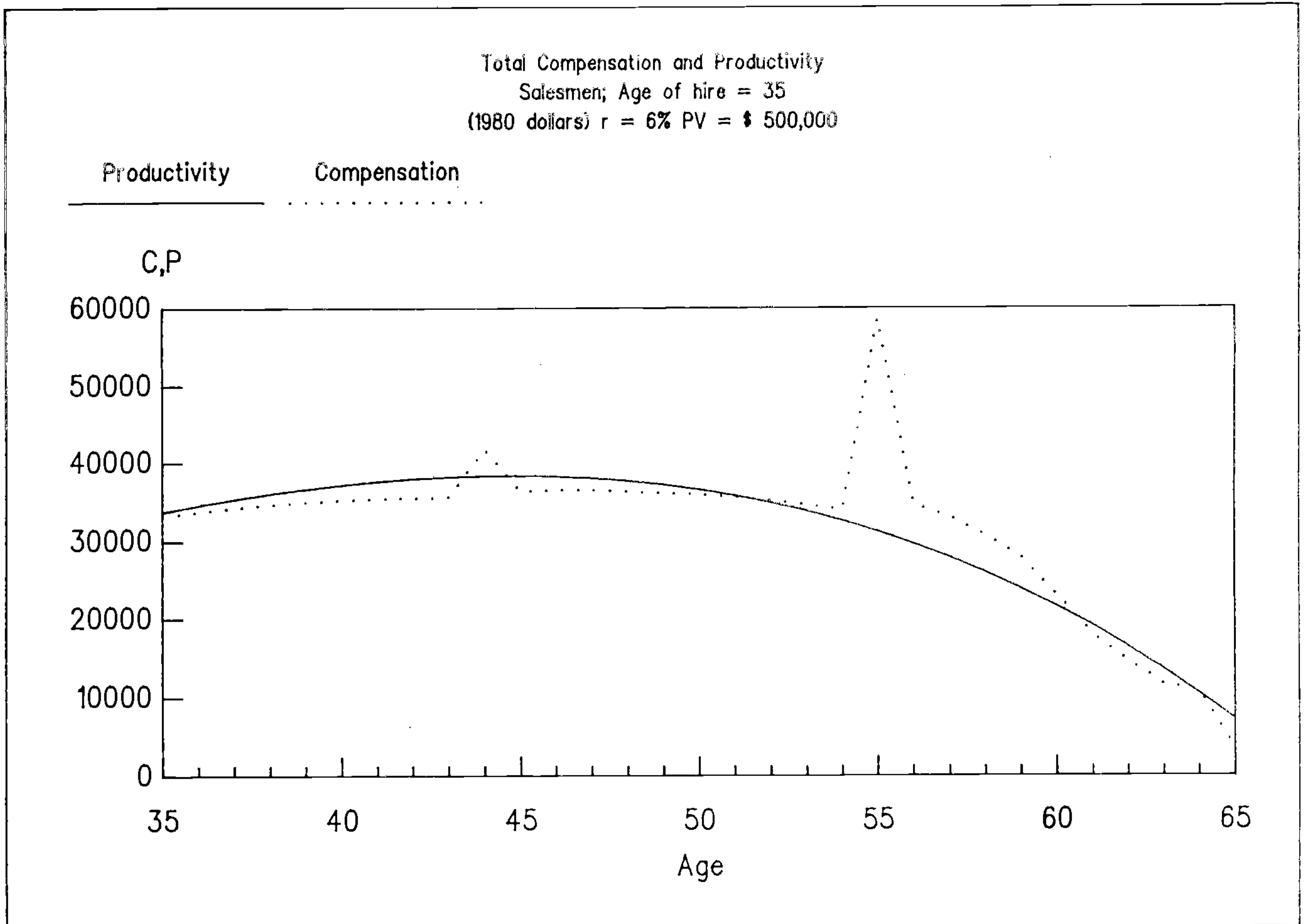
Age

Figure 4



Total Compensation and Productivity
Salesmen; Age of hire = 35
(1980 dollars) r = 6% PV = $ 500,000
Productivity
Compensation
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Age

Figure 15

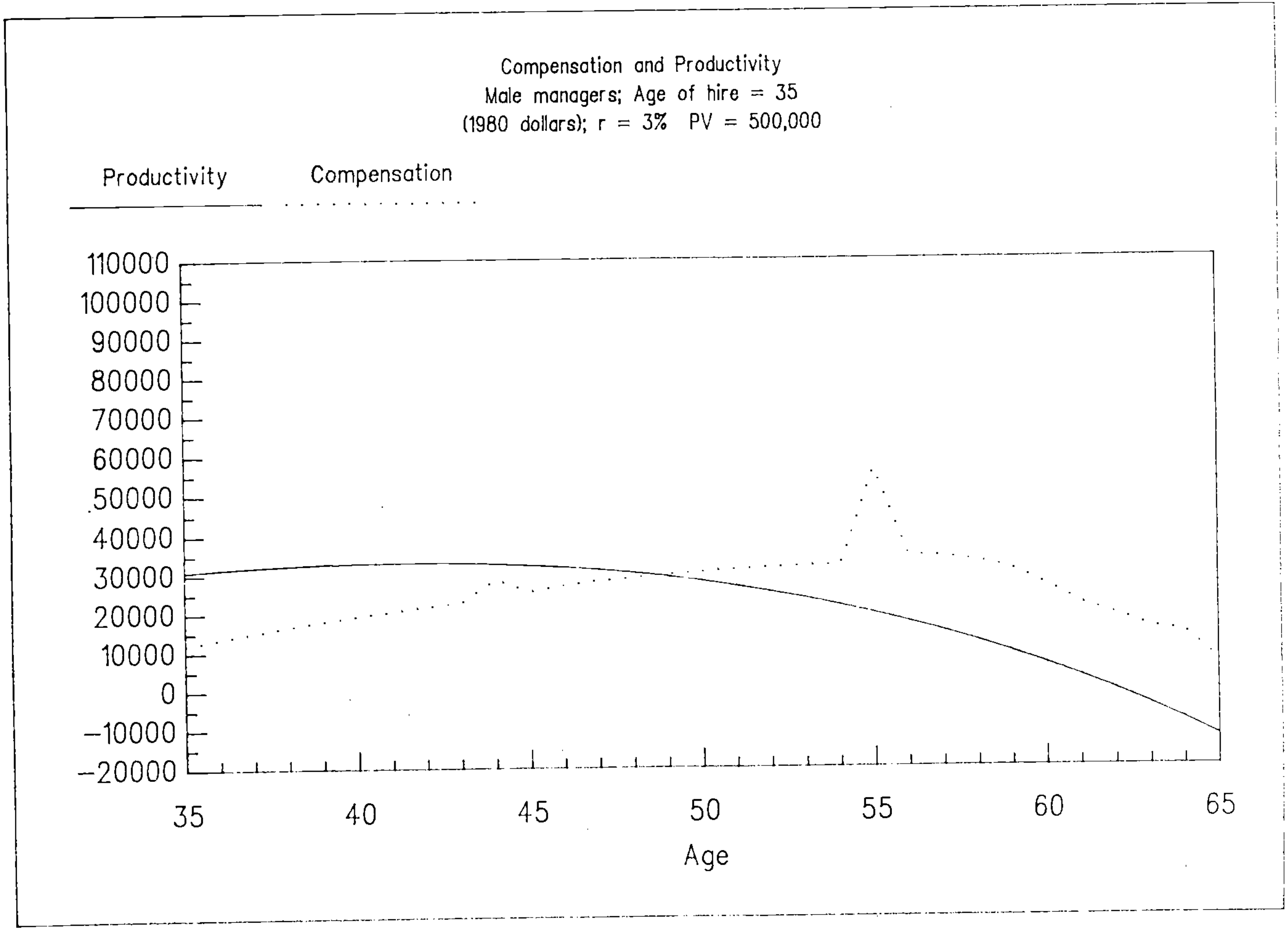
Compensation and Productivity
Male managers; Age of hire = 35
(1980 dollars); r = 3% PV = 500,000
Productivity
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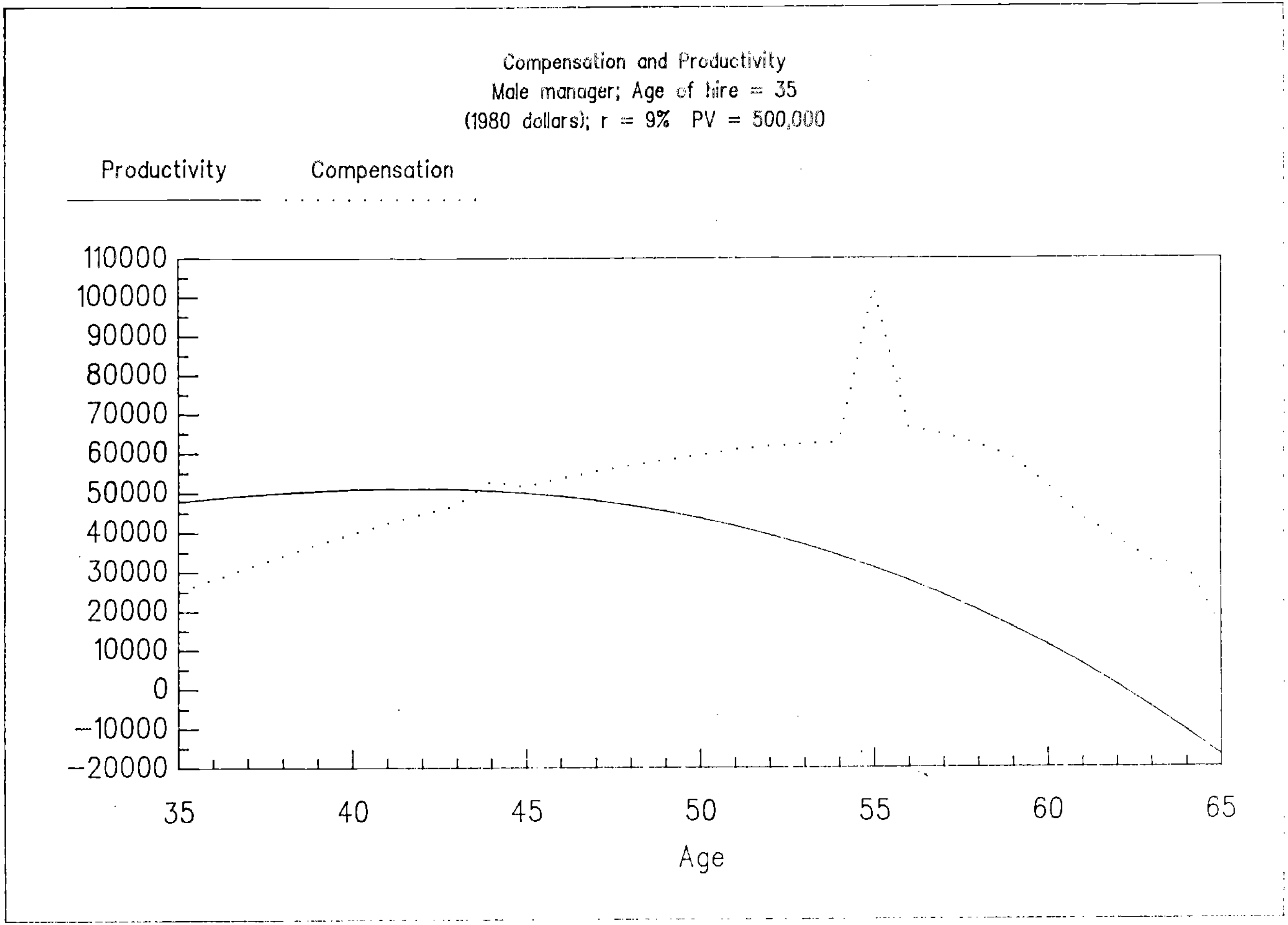
Figure 16


Compensation and Productivity
Male manager; Age of hire = 35
(1980 dollars); r = 9% PV = 500,000
Productivity
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Age

age is roughly 50 assuming a 3 percent interest rate; it is roughly 48 assuming a 6 percent interest rate; and it is roughly 45 (ignoring the vesting kink in the compensation profile) assuming an interest rate of 9 percent.

Another concern is the extent to which the profiles described here as age-productivity profiles confound service-productivity effects. Unfortunately, the colinearity between cumulated service and age variables precludes modeling the h(,) function as a continuous function both of age and age of hire. An alternative way to explore this issue is to model h(,) as depending only on age, but to estimate the model separately for workers hired at different ages. If one estimates the model separately for those hired prior to age 35 and those after age 35, the resulting general shapes of the productivity profiles are quite similar to those based on the entire sample. The post-age 35 profiles are indeed very similar, while the pre-age 35 profiles exhibit a steeper decline in productivity with age, with negative predicted productivity after roughly age 55. This prediction of negative productivity late in the work span may simply represent a poor fit in the tail of the estimated polynomial.

Section V. Conclusions and Suggestions for Additional Research

The findings that productivity decreases with age must be viewed cautiously. Contrary to what has been assumed, it may be that some workers within a sex-occupation category receive different contracts than others. Suppose that within a sex-occupation category there are A and B type workers and that A type workers receive contracts with steeper compensation profiles than B type workers. Also assume that A type workers have smaller values of q(,) than B type workers. If the composition of workers remaining with the firm changes, the estimated $\mu(,)$ and q(,) functions would differ from those for either A or B separately or from those that would arise if the separate $\mu(,)$s and q(,)s for A and B were averaged using constant weights. As a consequence, the age-productivity profile derived using the method presented here could differ from either the profile for group A workers or the profile for group B workers. Similar biases may arise if the composition of type A and type B workers among new hires changes as the age of hire increases. These potential biases need to be explored more formally as does the possible bias arising from assuming static expectations of overall worker productivity.

These concerns notwithstanding, the results are fairly striking. Productivity falls with age, compensation at first lies below and then exceeds productivity, and the discrepancy between compensation and productivity can be very substantial. Interestingly, there is much closer correspondence of productivity to compensation for salesworkers, who are compensated more on a spot market basis, than for other types of workers. Also, the relationship of productivity to compensation is weakest for male managers, who, one would expect, are most likely to be hired on a contract rather than a spot market basis.

In addition to confirming contract theory, the results lend support to the bonding wage models of Becker and Stigler (1974) and Lazear (1979,1981). In contrast, the results contradict the predictions of the standard Mincer-Becker human capital model in which workers receive more than they are worth when young and less than they are worth when old. They also are at odds with the assertion of some efficiency wage models that workers receive their marginal product on an annual basis.

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